DOCUMENT RESUME

ED 171 525 SE 027 562

AUTHOR Pullman, Howard W.
TITLE The Relation of the Structure of Language to Performance in Mathematics.
PUB DATE [79]
NOTE 23p.; Contains light and broken type

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Geometry; *Language Patterns; *Logical Thinking; *Mathematics Education; Performance; Prediction; *Research; Secondary Education; *Syntax; *Thought Processes

ABSTRACT

The relationship of performance in mathematics to syntactical language factors obtained from speech samples was investigated with 95 senior high school geometry students. Statistical procedures included correlation analysis and stepwise regression. The findings indicated that certain syntactical measures indicating logical thought processes correlated significantly with measures of mathematical performance. In addition, these syntactical measures added a statistically significant amount to intelligence in predicting performance in mathematics. (Author/MP)

The Relation of the Structure of Language to Performance in Mathematics

Howard W. Pullman

Youngstown State University

The Relation of the Structure of Language to Performance in Mathematics

Abstract

The relationship of performance in mathematics to syntactical language factors obtained from speech samples was investigated with 95 senior high school geometry students. Statistical procedures included correlation analysis and stepwise regression. The findings indicated that certain syntactical measures indicating logical thought processes correlated significantly with measures of mathematical performance. In addition, these syntactical measures added a statistically significant amount to intelligence in predicting performance in mathematics.

The Relation of the Structure of Language to Performance in Mathematics

It has long been recognized that verbal and other language factors are related to ability and achievement in mathematics. Most studies investigating these relationships have, however, concentrated on the more accessible language factors of ability and achievement (Aiken, 1971) and have neglected the structure of language itself as a factor. The purpose of this study was to investigate the relationship between the structure or syntax of students' spoken language and their ability and achievement in mathematics. Specifically, certain types of constructions in students' speech were identified as being potentially useful in predicting performance in mathematics. These elements of structure, generally known as indicators of verbal conditionality (Hays, 1966), are characterized by the fact that they imply an "evaluation of contingencies during the symbolic consideration of alternative courses of action [p. 1]." Thus, an individual may verbally consider a situation, indicate that there is more than one method of resolving the situation, and indicate his reason for choosing a particular solution. This process can be identified in whole or part solely by syntactical considerations. Thus, it was believed that syntactical indicators of thought which is characteristic of mathematical reasoning could be used to predict performance in mathematics.

The relationship between language and thought is, for many, self-evident (Brown, 1958; Carroll, 1964; Chomsky, 1968; Church, 1961; Vigotsky, 1962). The nature of this relationship is, however, more elusive. Benjamin Whorf (1956) is responsible for much of the early work in this area and for motivating later theory and investigation concerning language and thought. The Whorfian hypothesis, or the principle of linguistic relativity, states that the "formulation of ideas is not an independent process, . . . but is part of a particular grammar [p. 212]." That is, structural differences in languages are indicative of cognitive differences in the users of these languages. While the Whorfian hypothesis deals specifically with interlanguage differences, more recent concern has been with intralanguage differences (Carroll, 1964; Lantz & Stefflre, 1964; Osgood, Suci, & Tannenbaum, 1957; Vernon, 1967). The conclusion reached is that there has been no generally accepted resolution to the issue of the nature of the relationship between thought and language.

Chomsky (1968) has argued that the syntax of one's speech is related to his thought. Starting with the notion that language and cognition are related, the structure of the former being at least a rough approximation of the structure of the latter, it follows that a relatively simple, and heretofore rarely exploited, method of studying the cognitive structure of the individual is to study the syntactic structure of the individual's speech. This method was used by Videbeck (1965) and Hays (1966) in studies relating verbal conditionality to cognitive structure. These researchers found that verbal conditionality was a useful indicator of cognitive functioning and related specifically to performance on concept acquisition and conceptual differentiation tasks. This study was a direct outgrowth of the studies of Hays (1966) and Videbeck (1965).

It is not surprising that one would hypothesize a relationship between language and mathematics. Formal thought in general and formal mathematical thought in particular are characterized as hypothetico-deductive in nature, abstract reasoning and hypothesis testing depending on combinatorial analysis (Inhelder & Piaget, 1958). In addition, mathematics itself may be considered a language with its own syntax and semantics. It is therefore natural to ask whether constructions which indicate logical reasoning in grammatical structure do, in fact, also indicate logical reasoning in mathematical structure.

Wilson (1967) and Reiss (1963) contend that the logical aspect of reasoning is indicated by syntactical structure and emphasize the necessity of logical thought processes in the field of mathematics. "Words like 'because,' 'therefore,' 'if,' 'since,' and 'although' are essential if we are to carry out any communication above a primitive level, because all reasoning and sustained thinking depends upon them. . . . Logical words are good guides to logical thinking and speaking [Wilson, 1967, p. 29]." It must be emphasized that these words are considered as structural indicators of a process and not as indicators of meaning; that is, they are considered syntactically rather than semantically.

On the basis of the considerations above it was hypothesized that verbal conditionality would be positively correlated with mathematical performance. Since the relation of intelligence or general ability to performance in mathematics is well established, it was also hypothesized that verbal conditionality would add significantly to intelligence in predicting performance in mathematics.

Method

The subjects were 95 students, 58 males and 37 females, in geometry classes in a suburban high school. No subject was dropped from the data analysis although, because of incomplete data, not all subjects served in all parts of the final analysis. The age of the subjects ranged from 14 to 18 years with 16.09 years as the mean age; the mean IQ was 117.5. Data on performance in mathematics were obtained from school records; data on verbal conditionality were obtained from individual interviews with the subjects. Interviews lasted between 20 and 30 minutes and took place during the student's free period.

The Interview

Each subject was presented with a relatively unstructured situation and asked to speak freely and at length on the indicated topics. The topics concerned filling a free Saturday, planning a schedule for a foreign exchange student, and spending an unlimited sum of money. The students had no reason to associate the interview with mathematics, and no subject mentioned mathematics in the interview except in the context of the foreign student's program. The situations were assumed to be neutral with respect to intelligence and information. The goal of the interview was to obtain from each subject a verbal sample of approximately 1000 words in which the subject had an opportunity to express ideas using conditionals. Probes were used to increase the size of the verbal sample but not to elicit conditionals. Therefore, it was assumed that each subject could, but was not encouraged to, speak conditionally. Each interview was taped and later transcribed for analysis.

Coding

The protocols were coded by two coders using a schedule developed by Hays (1966). The schedule consists of three major categories each with three subcategories. These are listed below.

1. Hypothetical Mode.

   a. Would and could.

b. May and might.

c. Maybe and perhaps.

2. Alternation (usually "or" as an indication of alternatives).

a. Introductory branch-words.

b. Internal branch-words.

c. Indefinite branch-words.

3. Testing of Contingencies.

a. If clauses.

b. Other conditional adverbial clauses ("whenever," "unless").

c. Descriptive conditionals ("take into account," "depends on").

After coding was done individually, differences were discussed and resolved. Usually discrepancies resulted from missed instances of a conditional rather than different coding of a given conditional. The reliability of the coding procedure was established by Hays (1966), who found inter-judge reliability to be .91 among 29 student judges and .94 between student judges and the experimenter [p. 35]. Coding was performed in terms of all nine categories, but only the three major subdivisions were used in the analysis.

Each individual's score for the three measures of verbal conditionality was the number of occurrences of the respective conditionals. The number of conditionals was preferred to the relative number of conditionals because it was thought to be a more pure measure of the extent to which each subject was willing to elaborate and test his ideas.

Performance in Mathematics

Performance in mathematics was measured by six different variables--algebra final grade, geometry final grade, School and College Ability Tests (SCAT) quantitative score, Preliminary Scholastic Aptitude Test (PSAT) quantitative score, Scholastic Aptitude Test (SAT) quantitative score (two administrations). Course grades were reported as letter grades and were assigned numerical equivalents of 4, 3, 2, 1, and 0 for A, B, C, D, and F, respectively. Since the students were homogeneous with respect to course, geometry, rather than year in school, the standardized scores were not the result of a single administration of the tests.

In addition to the measures of mathematical performance, intelligence measures were obtained from the school records. These scores were a product of the Otis Quick-Scoring Mental Ability Tests administered by the school as a part of its normal testing program.

The hypotheses of the study were tested using correlation analysis and stepwise regression.

Results

Verbal conditionality data were available for only 63 of the 95 subjects in the study. There were several reasons for this. Some students had no study halls and could not come for an interview. Some had study halls only on days when the interviewers could not be at the school, and other students were absent or found that they had to do homework or study for a test during the period which had been scheduled for the interview. Descriptive statistics for the conditionality variables appear in Table 1. The mean number of words indicating the hypothetical mode, branching, and testing of contingencies were 26.40, 7.00, and 10.16, respectively. The mean total output was 725.77 with a standard deviation of 407.35. The range for total output was from 82 to 2069. Several of the subjects, although they volunteered for the interview, were reluctant to say very much.

Insert Table 1 about here

The descriptive statistics for the measures of performance in mathematics appear in Table 2. The mean scores on all standardized measures of mathematical ability are above the national norms, thus indicating that the sample was not typical of the national population in measured ability. The Mean Otis IQ was 117.48 with a standard deviation of 10.28. The range of IQ's was from 96 to 143.

Insert Table 2 about here

It was predicted that each measure of verbal conditionality would be positively correlated with each measure of performance in mathematics. The correlation coefficients of each of the six measures of performance in mathematics are listed in Table 3. Alternation or branching was positively correlated ($p < .05$) with three of the four standardized measures of mathematical ability, while testing of contingencies was positively correlated with only two of these measures. Hypothetical mode was not correlated with any of the mathematical measures, and course grades were correlated with no measure of verbal conditionality.

Insert Table 3 about here

Further illustration of the relation of the verbal conditionality variables to the SCAT and PSAT variables occurs when subjects are categorized as high, medium, or low on the latter variables. The results presented in Table 4 indicate that there is a direct relationship between verbal conditionality and the SCAT and PSAT in five of the six comparisons.

Insert Table 4 about here

Intelligence or general ability was significantly correlated with each measure of performance in mathematics. Although there correlations were generally moderate to high, they by no means explained all of the variance in performance in mathematics. It was hypothesized that measures of verbal conditionality would add significantly to intelligence in predicting performance in mathematics. Table 5 presents the results of the stepwise regressions performed on intelligence and verbal conditionality to predict each of the six measures of performance in mathematics. In each case intelligence accounted for the greatest part of the variance in performance. In all cases the verbal conditionality variables added a statistically significant amount to the variance common to mathematical performance and intelligence, and thus added to the predictability of performance in mathematics.

Insert Table 5 about here

Discussion

The results presented above lend credence to the relationship of structure of language factors to mathematical reasoning. Two types of reasoning or methods of arriving at conclusions are of particular importance in mathematical problem solving. First is the ability to realize or imagine alternative methods, approaches, or solutions. Second is the ability to test hypotheses, which is closely linked with the perception of alternatives. One is continually met with sequences of testing, finding alternatives, testing, finding alternatives, etc. If indeed speech is a mirror of thought and the words of alternation and testing on contingencies do indicate their respective mental processes, it is understandable that a tendency to use these words is correlated with mathematical performance.

The place of the hypothetical mode in reasoning in elementary mathematics is not as evident as that of alternation and hypothesis testing. Whereas mathematical thought is also concerned with possibilities--indicated by such words as "would," "could," "may," "might," and "perhaps"--it is usually concerned with these possibilities in a quantitative rather than a qualitative manner. That is, mathematical possibility is expressed through numerical probability. It is reasonable to assume, nevertheless, that the hypothetical mode may relate to thinking in mathematics. The data do not, however, bear this out. Perhaps the hypothetical mode would be found to play an important part in more advanced and more creative mathematical thought.

None of the correlations of verbal conditionality with course grades was significant. This is possibly due to the subjectivity and resulting unreliability of the grades.

Although grades do reflect ability, they are possibly confounded with non-ability variables--motivation, neatness, need to succeed, and others--to a larger extent than are standardized measures of performance.

As was noted above, all measures of performance in mathematics were highly correlated with intelligence. Thus the measures of verbal conditionality--although they did add a statistically significant amount to the multiple correlation coefficients, did not add an impressive amount to the predictability of performance in mathematics. However, the results do indicate that additional information about performance in mathematics may be gained from a knowledge of the structure of spoken language and suggest that people who are more conditional in their reasoning, as indicated by the use of conditionals in their speech, are more likely to be successful in mathematics. Thus the structure of language may be a fruitful means for studying reasoning in mathematics.

References

Aiken, L. R., Jr. Verbal factors and mathematics learning: A review of research. Journal for Research in Mathematics Education, 1971, 2, 304-313.

Brown, R. Words and things. Glencoe, Ill.: Free Press, 1958.

Carroll, J. B. Linguistics and the psychology of language. Review of Educational Research, 1964, 34, 119-126.

Chomsky, N. Syntactic structures. The Hague: Mouton, 1968.

Church, J. Language and the discovery of reality: A developmental psychology of cognition. New York: Vintage, 1966.

Hays, D. Syntactic styles and cognitive functioning. Unpublished doctoral dissertation, University of Missouri, 1966.

Inhelder, B. & Piaget, J. The growth of logical thinking from childhood to adolescence. New York: Basic Books, 1958.

Lantz, D. & Steiflre, V. Language and cognition revisited. Journal of Abnormal and Social Psychology, 1964, 69, 472-481.

Osgood, C. E., Suci, G. J., & Tannenbaum, P. H. The measurement of meaning. Urbana, Ill.: University of Illinois Press, 1957.

Pullman, H. W. The relation of the structure of language to mathematical ability. Unpublished doctoral dissertation, Columbia University, 1972.

Reiss, S. On the relation between language and thought. Psychiatric Quarterly, 1963, 37, 264-281.

Vernon, M. Relationship of language to the thinking process. Archives of General Psychiatry, 1967, 16, 325-333.

Videbeck, R. A syntactic approach to the study of cognitive functioning. Studies in cognitive processing and natural language, Report Number 1. Syracuse N.Y.: Syracuse University, 1965.

Vigotsky, L. S. Thought and language. Cambridge, Mass.: MIT Press, 1962.

Whorf, B. Language, thought, and reality. Cambridge, Mass.: MIT Press, 1956.

Wilson, J. Language and the pursuit of truth. Cambridge: The University Press, 1967.

Table 1

DESCRIPTIVE STATISTICS FOR THE VERBAL CONDITIONALITY VARIABLES
(N=63)

Verbal Conditionality	Statistic			
	Mean	SD	Maximum	Minimum
	Hypothetical Mode			
Would and could	23.33	12.97	71	0
May and might	1.62	1.64	6	0
Maybe and perhaps	1.44	2.49	13	0
Total	26.40	15.12	74	0
	Alternation			
Introductory branch-words	.98	1.41	6	0
Internal branch-words	3.35	2.69	11	0
Indefinite branch-words	2.67	2.79	12	0
Total	7.00	4.57	24	0
	Testing of Contingencies			
If clauses	8.14	5.76	31	1
Other conditional adverbial clauses	.38	.72	3	0
Descriptive conditionals	1.21	1.25	5	0
Total	10.16	6.88	37	1

Table 2

DESCRIPTIVE STATISTICS FOR THE MEASURES OF MATHEMATICAL ABILITY

Mathematical Ability	Statistic				
	Mean	SD	N	Maximum	Minimum
Algebra grade	2.30	1.07	93	4	1
Geometry grade	2.64	.93	91	4	1
SCAT	64.31	25.74	88	99	1
PSAT	52.95	10.89	74	75	28
SAT-1	546.51	113.16	81	780	302
SAT-2	558.25	127.54	73	800	309

Table 3

CORRELATION OF VERBAL CONDITIONALITY
WITH MATHEMATICAL ABILITY

Verbal Conditionality	Mathematical Ability					
	Algebra grade (N=62)	Geometry grade (N=62)	SCAT (N=60)	PSAT (N=51)	SAT-1 (N=55)	SAT-2 (N=55)
Hypothetical mode	.16	.12	.15	.14	.06	.09
Alternation	.21	.17	.28*	.36*	.37*	.16
Testing of contingencies	.10	.17	.25*	.24*	.19	.21

*$p < .05$.

Table 4

RELATION OF VERBAL CONDITIONALITY VARIABLES TO SCAT AND PSAT

Verbal Conditionality	SCAT			PSAT		
	High (N=23)	Medium (N=19)	Low (N=18)	High (N=18)	Medium (N=15)	Low (N=18)
			Hypothetical Mode			
Mean	29.30	28.00	22.06	26.67	29.27	26.00
SD	16.89	14.03	13.85	16.06	5.85	16.42
			Alternation			
Mean	8.91	6.32	5.44	9.50	5.93	5.67
SD	5.54	2.49	3.72	5.70	5.12	2.51
			Testing of Contingencies			
Mean	12.43	9.95	8.00	13.78	9.00	8.50
SD	9.26	3.71	4.68	9.76	5.18	4.65

Table 5

STEPWISE REGRESSION OF INTELLIGENCE AND VERBAL CONDITIONALITY ON MATHEMATICAL ABILITY

Independent Variable	Mathematical Ability		
	Multiple R	R squared	Increase in R squared
	Algebra Grade (N=60)		
Intelligence	.4082	.1667	.1667
Testing of contingencies	.4196	.1769	.0094
Alternation	.4261	.1816	.0055
Hypothetical mode	.4283	.1834	.0018
	Geometry Grade (N=60)		
Intelligence	.5215	.2720	.2720
Alternation	.5293	.2802	.0082
Testing of contingencies	.5389	.2904	.0102
	SCAT (N=60)		
Intelligence	.5705	.3255	.3255
Testing of contingencies	.5727	.3280	.0025
Alternation	.5734	.3288	.0007

Note. - Level for inclusion is $p < .05$.

Table 5, continued

Independent Variable	Mathematical Ability		
	Multiple R	R squared	Increase in R squared
	PSAT (N=50)		
Intelligence	.7522	.5658	.5658
Testing of contingencies	.7542	.5688	.0029
Hypothetical mode	.7552	.5703	.0016
Alternation	.7559	.5713	.0010
	SAT-1 (N=54)		
Intelligence	.7393	.5465	.5465
Hypothetical mode	.7427	.5516	.0051
Alternation	.7460	.5565	.0048
Testing of contingencies	.7474	.5586	.0022
	SAT-2 (N=50)		
Intelligence	.7182	.5159	.5159
Alternation	.7349	.5400	.0242
Hypothetical mode	.7362	.5420	.0019

Note. - Level for inclusion is $p < .05$.