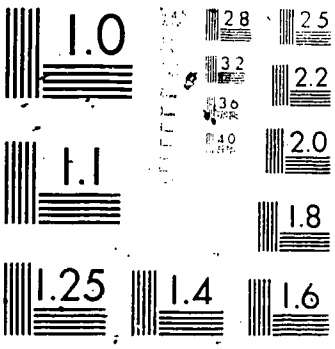

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DOCUMENT RESUME

ED 170 113 SE 026 827

AUTHOR McLeod, Douglas B.; Adams, Verna M.
TITLE Aptitude-treatment Interaction in Mathematics Instruction Using Calculators.
SPONS AGENCY National Science Foundation, Washington, D.C
PUB DATE 79
GRANT NSF-SED-77-18531
NOTE 25p.; Paper presented at the annual meeting [illegible] American Educational Research Association (San Francisco, California, April 8-12, 1979); Not available in hard copy due to marginal legibility of original document

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS *Aptitude; *Calculation; *Educational Research Higher Education; *Instruction; *Mathematics Education
IDENTIFIERS *Calculators

ABSTRACT

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Aptitude-treatment Interaction in Mathematics Instruction using Calculators

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Running head: Aptitude-treatment Interaction

This report is based upon work supported by the National Science Foundation under Grant No. SED 77-18531. Any opinions, findings, and conclusions expressed in this report are those of the author and do not necessarily reflect the views of the National Science Foundation.

Presented at the Annual Meeting of the American Educational Research Association, San Francisco, April 1979.

Abstract

Students in three mathematics classes were assessed on two aptitudes, field independence and general reasoning, and randomly assigned to either an expository or a discovery treatment. The expository treatment used a deductive sequence of instruction and provided maximal guidance for the students. The discovery group used an inductive sequence with minimal guidance, and provided calculators to help students discover concepts and rules independently. The topic of instruction involved errors in measurement and calculations with approximate data. There was a significant interaction with general reasoning on the retention test, as predicted. There were no interactions with field independence.

Aptitude-treatment Interaction in Mathematics Instruction Using Calculators

Aptitude-treatment interaction (ATI) research, generally viewed as an outgrowth of the work of Cronbach (1957), has turned out to be more difficult than originally expected. Simple hypotheses about matching student abilities with appropriate treatments have proven difficult to substantiate. Nevertheless, Cronbach and Snow (1977), in their comprehensive review of the field, confirm that ATI do exist and are important to educational practice.

Cronbach and Snow (see also Snow, 1977) state that the most stable interactions occur with general ability. However, there are a number of interactions in the literature, especially with inductive and deductive instruction (Cronbach & Snow, 1977, p. 320, 371), that do not seem to be related to general ability. General reasoning is one of the aptitude variables that is frequently involved in these more specific interactions.

In mathematics education research, several studies have reported ATI between general reasoning and treatments that differed in the use of an inductive or a deductive sequence of instruction (Eastman & Carry, 1975; McLeod & Briggs, in press). There are also studies that have failed to find the expected interactions (Behr & Eastman, 1975; Eastman & Behr, 1977), but this may have been because the level of difficulty of the treatments was not appropriate for the students.

The theoretical framework for these interactions with general reasoning is not well established. Cronbach and Snow (1977) note that measures of general reasoning are closely related to general ability in mathematics.

In ATI studies, however, general reasoning seems to function quite differently from general ability. For example, tests of general reasoning seem to do a better job of predicting success in a more expository deductive treatment than in an inductive treatment, the reverse of what one usually finds for measures of general ability. To explain these interactions, Cronbach and Snow have suggested that a test of general reasoning might be a measure of crystallized ability, or achievement in traditional school subjects; therefore, it would be expected to produce steeper regression slopes in more traditional deductive instruction (Snow, Note 1). Carroll (1976) has analyzed the aptitude of general reasoning from a different perspective, using the concepts of information processing theory. From this point of view, general reasoning as the ability to perform serial operations, which seems to correspond to the direct sequence (rules followed by examples) of deductive instruction.

One reason that Cronbach and Snow (1977) attribute most ATI to general ability is that it is difficult to separate the effects of a specific aptitude from general ability. The difficulties with traditional aptitude constructs led Glaser (1972) to call for research with "new aptitudes", including dimensions that are related to personality variables such as cognitive styles. One cognitive style variable, field independence, has received considerable attention in educational research (Witkin, Moore, Goodenough, & Cox, 1977). In a recent revision of cognitive style theory, Witkin and Goodenough (Note 2) suggest that cognitive restructuring ability and personal autonomy are the two characteristics on which field-dependent and field-independent students differ. Treatments that provide minimal

structure and guidance should be appropriate for field-independent students, since they can provide their own structure and work autonomously. Field-dependent students, however, should excel in a highly structured treatment which provides careful guidance. Some studies in mathematics education have found ATI that support this theoretical position (McLeod, Carpenter, McCornack, & Skvarcius, 1978; McLeod & Adams, in press), but other studies have not produced significant interactions.

In summary, ATI research in mathematics education has found two aptitude variables, general reasoning and field independence, that have produced significant interactions with two dimensions of discovery learning, level of guidance and inductive instruction. The purpose of this study was to search for ATI between these two aptitude variables and treatments that differed in both level of guidance and in use of an inductive or deductive sequence of instruction. The treatment that provided a minimal level of guidance and used an inductive sequence was labeled the discovery treatment; the expository treatment provided maximal guidance with a deductive sequence of instruction. Based on the theoretical background for these two aptitude variables, it was predicted that field-independent students would do best in the discovery treatment, while students who scored well on tests of general reasoning would be better off in the expository group. Rephrasing this hypothesis in terms of regression slopes, it was predicted that the regression of achievement on field independence would be steeper in the discovery group than in the expository group, but the regression on general reasoning would be steeper in the expository group.

Method

Subjects

Students from three sections of a mathematics course for prospective elementary school teachers participated in the study. All three classes met in the afternoons for 75 minutes on two days each week. About 87% of the 60 students in the classes were women. Complete data were obtained for 47 subjects, 24 in the expository group and 23 in the discovery group. Other students were absent for one or more days of instruction and testing. The rate of student absenteeism did not appear to be related to differences in the treatment groups.

Treatments

Two instructional units were prepared on the topic of errors in measurement and their effect on calculations with approximate data. This topic was suggested by the Report of the Conference on Needed Research and Development on Hand-held Calculators in School Mathematics (1976). The treatments included such concepts as precision of measurements, significant digits, and their relationship to adding, subtracting, multiplying, and dividing approximate data. Both treatments covered exactly the same concepts, and students were given about the same amount of practice in solving problems. However, the concepts were presented in different ways in the two treatments.

In the expository treatment, instruction proceeded in a deductive sequence, with definitions and rules followed by examples. Students were given maximal guidance; sample problems were worked out completely before students were asked to do similar problems. The problems were chosen so that they could be worked easily without a calculator. In the discovery treatment,

however, concepts were presented in an inductive sequence. Students first worked out several examples, using a hand-held calculator when it was needed. Students were then encouraged to generalize and produce rules that would follow the examples. Although the students were given an opportunity to discover the rules, the materials did provide the rules to students who did not discover them independently. In both treatment groups, the teacher was available to help answer student questions.

Tests

Field independence was measured using the Group Embedded Figures Test (GEFT) and a version of the Hidden Figures Test (HFT). The GEFT (Witkin, Oltman, Raskin, & Karp, 1971) is the most appropriate group measure of field independence. The version of the HFT that was used (Hidden Figures 2--Form 271) was adapted by the National Longitudinal Study of Mathematical Abilities (NLSMA) from the original of the Educational Testing Service (French, Ekstrom, & Price, 1963). For a complete discussion of this test, see the appropriate NLSMA reports (Romberg & Wilson, 1969; Wilson, Cahen, & Begle, 1968).

The time allowed for the GEFT and HFT was adjusted for this study. Since the GEFT is relatively easy for college students, subjects were given four minutes for each part, rather than five. The version of the HFT that was used was rather difficult, so students were given 15 rather than 10 minutes for that test.

The HFT was used along with the GEFT in order to provide a second measure of field independence, a procedure in line with the multitrait-multimethod approach to measuring aptitude that is recommended by Cronbach

and Snow (1977).

The most common measure of general reasoning in ATI studies is the Necessary Arithmetic Operations (NAO) test (French, Ekstrom, & Price, 1963). In order to distinguish between scores on the NAO test and general ability, students were asked to allow the university to release their SAT scores. Most students agreed to this request, but only 28 of those subjects actually had SAT scores on file.

A 20-item posttest that covered all of the concepts in the unit was used to measure immediate achievement. A subset of 10 items was used to measure retention. The retention test covered only the parts of the unit that had been completed by most participants. Fifteen minutes was allowed for the posttest, and seven minutes for the retention test.

The KR-20 reliability coefficients were judged to be satisfactory on all tests. They ranged from .61 on the posttest to .82 on the NAO.

Procedures

The HFT and NAO tests were administered during the first week of class as a part of the regular course procedures. During the middle of the term, 90 minutes of class time was devoted to the study.

Students were randomly assigned to treatment groups within each class. Students assigned to the discovery treatments were asked to go to a room equipped with calculators. Students in the expository group stayed in the regular classroom. They were told that they would get their chance to work with the calculators later, since there were not enough calculators for the entire class to use them at the same time. Since no calculators were needed for the expository treatment, the lack of a calculator caused no problems

for that group.

At the beginning of the treatments, students were given a brief introduction to the materials and were encouraged to work independently, directing their questions to the teacher. At the end of the first day of the study, the materials were collected and graded. Most students were not able to complete the treatments in the 75 minutes allowed. The posttest was administered two days later at the next class meeting. Four weeks later students were assessed again to measure retention. On the same day, students took the GEFT.

Results

Descriptive statistics are presented in Tables 1, 2, and 3. Table 1 includes the means and standard deviations for all tests; scores ranged widely among students, but there were no large differences between groups. Table 2 presents the correlation matrix for the aptitude and achievement tests. Correlations between the NAO test and the two measures of field independence were somewhat higher than one usually expects. Also, there was a strong correlation between the posttest and retention test. Table 3 presents the regression equations for each group, using HFT and NAO as predictors. Substitution of the other measure of field independence (GEFT) for the HFT scores produced similar results.

Insert Tables 1-3 about here

Tests for Interaction

The data were analyzed using multiple regression techniques. The two dependent variables were treated separately. For the main analyses, the full model included vectors for field independence (HFT, GEFT, or their sum), NAO, treatment, and the interaction of treatment with each of the aptitude vectors. As these vectors entered the equation (in the specified order), the change in R^2 due to each interaction vector was calculated. On the retention test, the interaction of NAO and treatment was significant (see Table 4) and in the predicted direction.

Insert Table [illegible] here

Figure 1 presents the interaction of NAO and treatment for the retention test. In the figure, the regression equations are calculated for each group using the NAO scores as the only predictor. The slope for the expository group was .42; in the discovery group it was .09. This difference in slopes is significant, $F(1, 43) = 6.96$, $p = .011$.

Insert Figure 1 about here

The data were analyzed further in several different ways. Scatterplots of each aptitude variable with the two achievement measures were constructed; in each case the use of linear models seemed appropriate.

Other measures of field independence (GEFT, the sum of HFT and GEFT) were included in the main analysis along with NAO. The results were essentially the same as those reported in Tables 3 and 4. There was still an interaction with NAO on the retention test, but not on the posttest. There were no interactions at all with field independence.

Since there was no interaction on the posttest, it was appropriate to test for a difference between treatment group means, when using HFT and NAO as covariates. No difference was found, $F(1, 43) = .67$, $p = .418$.

The importance of class effects has been emphasized by Cronbach (Note 3), so the data were reanalyzed taking into account the student's class and possible interactions of class with treatment, NAO, and the treatment-by-NAO interaction. On the retention test, the interaction with NAO occurred consistently across classes. On the posttest, only one class produced this type of interaction effect; in the other two classes the NAO slopes were about the same in both treatment groups.

Source of the Interaction

The data were analyzed further to determine whether the interaction with NAO could be attributed to general reasoning alone, or whether it should be thought of as an interaction with general ability or crystallized ability. The analysis began by considering the 28 subjects on which SAT data were available. The sum of the verbal and quantitative parts of the SAT were used as a measure of general ability. There was no evidence of any interaction with SAT, either by itself or in conjunction with the other aptitude variables. When SAT and NAO were put in the same regression equation with the retention test as the dependent variable, the NAO-by-treatment vector

accounted for about 3% of the variance, substantially more than the 1% due to the SAT-by-treatment vector. Of course, neither of these interactions was significant, since there were only 28 subjects in this analysis. However, these data provide some support for attributing the interaction with NAO to the aptitude of general reasoning rather than to general ability.

Further information on the nature of the NAO interaction was obtained by considering the difference of the standardized scores for HFT and NAO. Cronbach and Snow (1977, p. 84) state that two predictors behave differently if their standard-score difference interacts with the treatment dimension. The interaction between treatments and difference scores was not significant, $F(1, 45) = 1.97$, $p = .168$. The sum of the standardized scores for NAO and HFT, however, did interact with treatment, $F(1, 43) = 4.804$, $p = .034$. Since the combination of NAO and HFT should act more like general ability than general reasoning, the analysis of sum and difference scores provides some support for attributing the interaction to general ability rather than to the more specific aptitude of general reasoning.

Regions of Significance

Regions of significance for the interaction represented in Figure 1 were calculated in two ways. Following Cronbach and Snow (1977), confidence intervals were computed about each of the regression lines, using a confidence level of 68%. The confidence intervals overlapped for NAO scores of 13 to 17; therefore, the regions of significance for this interaction were for NAO scores of less than 13 and more than 17. These two regions included 55% of the students. Students with NAO scores of 17 or more did better in the expository group, as predicted, while students who scored less than 13

achieved more in the discovery group.

The Johnson-Neyman technique (Porich, Godbout, & Wunderlich, 1976) is another method of calculating regions of significance. For a level of significance of .10, this technique found the regions of significance for the interaction in Figure 1 to be almost the same as in the analysis using confidence intervals. For the Johnson-Neyman analysis, the upper region of significant differences included scores of more than 18. The lower region was found to be the same as in the analysis using confidence intervals. The regions of significance in the Johnson-Neyman analysis included 49% of the students.

Discussion

This study tested the hypothesis that ATI would occur between two aptitudes, field independence and general reasoning, and treatments that differed in dimensions of discovery learning in mathematics. Field independence was expected to interact with the treatments since they differed in the level of guidance provided to the students. General reasoning was expected to interact with the treatments since they differed in the use of deductive or inductive sequences of instruction. The ATI with general reasoning occurred as predicted on one of the two dependent variables. Therefore, this study helps to confirm the existence of an ATI that has appeared in several other studies (Cronbach & Snow, 1977; Eastman & Carry, 1975; McLeod & Briggs, in press).

Although a number of studies have found ATI with general reasoning, as measured by the NAO test, it is still not clear whether this interaction can be attributed to this specific aptitude, or whether it is the result of general or crystallized ability (Cattell, 1971). Data from the present

study were not conclusive on this point. Further investigation using an information processing approach may help to explain the effects of this aptitude variable. It seems likely that sequence differences in treatments may be related to fixed, as opposed to flexible, sequences of information processing. In this study, it appeared that students with high NAO scores were less flexible in terms of adapting to instruction using an inductive sequence where students were supposed to make generalizations with the assistance of hand-held calculators. In this interpretation, the ATI of this study fits nicely into Snow's recent work (Snow, Note 1) on the relationship of crystallized ability to ATI. Since the interaction occurred only on the retention test, it may be that these differences in information processing are only important when they involve retrieval from long-term memory.

The expected ATI with field independence did not occur. The major reason for this appeared to be that the treatments provided more guidance than was originally intended. This extra guidance was provided partly because the students requested, even demanded, considerable help from the instructor in the classroom. Also, treatments frequently need to be "tuned" in order to produce ATI, and appropriate revisions of the treatments used in this study could result in instruction that provides sufficient, but minimal, support. Such a revision might produce the expected interaction with field independence.

In summary, this study identified the expected ATI with general reasoning (as measured by the NAO test) but not with field independence. Further research on the topic seems appropriate. It used to be sufficient in ATI

research just to find an interaction; no one worried a great deal about whether the ATI could be attributed to a specific aptitude as opposed to general ability. But now more detailed information is necessary as researchers try to build a theory of aptitudes and interactions. These higher expectations seem to be a sign that ATI research is making substantial progress.

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Table 1

Means and Standard Deviations of

All Tests for Each Treatment Group

Test	Maximum possible score	Range	Discovery Mean	Discovery SD	Expository Mean	Expository SD
HFT	16	0-16	5.6	3.6	4.9	3.5
GEFT	18	0-18	9.8	4.5	8.8	5.7
NAO	30	3-24	13.8	3.9	13.8	4.5
Posttest	20	0-15	5.9	2.6	6.3	3.1
Retention	10	0-9	4.6	1.5	4.4	2.7

Table 2

Correlation Matrix for All Tests

Test	Correlation 1	2	3	4	5
1. HFT	1.00	.54	.53	.39	.43
2. GEFT		1.00	.43	.50	.41
3. NAO			1.00	.61	.54
4. Posttest				1.00	.72
5. Retention					1.00

Table 3

Regression Equation Data for Each Dependent Variable

Dependent variable	Group	Intercept	Regression coefficients HFT	NAO
Posttest	Discovery	.66	.07	.35
	Expository	.08	.12	.41
Retention	Discovery	3.40	.12	.04
	Expository	-1.34	.08	.38

Table 4

Tests for Interaction

Dependent variable	R^2 for full model	Source	Change in R^2	F	p
Posttest	.391	HFT X Treatment	.003	.20	.657
		NAO X Treatment	.001	.10	.753
Retention	.419	HFT X Treatment	.020	1.25	.270
		NAO X Treatment	.076	5.33	.026

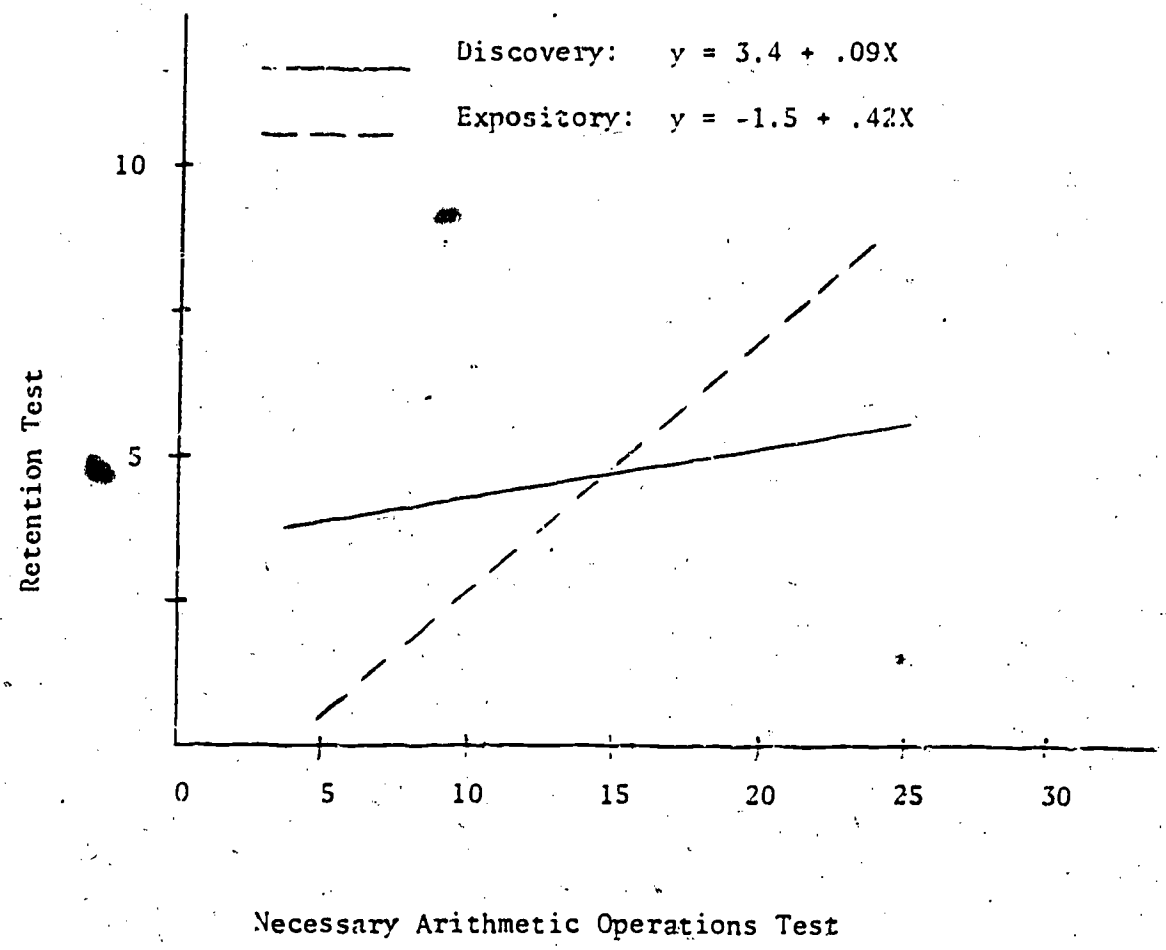

Figure 1. Interaction of NAO test with discovery and expository treatments on the retention test.