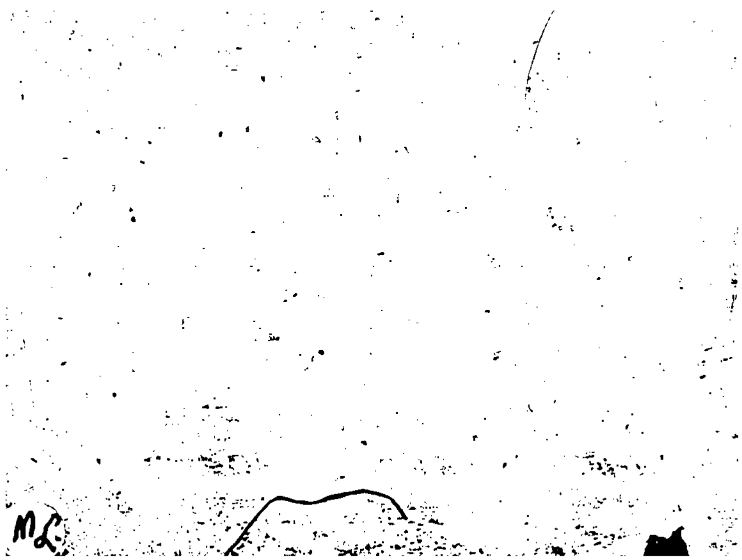

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ABSTRACT

This basic mathematics course is composed of eight lessons. Each lesson contains a pre-test and post-test, problems and examples oriented toward water and wastewater operations, suggested supplemental readings, and estimated time for completion. The topics of the lessons are: (1) numeration; (2) whole numbers; (3) fractions; (4) decimals; (5) percentages; (6) ratio and proportion; (7) squares and square roots; (8) exponents; (9) scientific notation; (10) basic algebra; (11) formulas; (12) roman numerals; (13) the metric system; (14) calculator use; (15) geometry; (16) graphs and charts; and (17) basic statistics. The entire course can be completed in about 40 hours. The prerequisites vary and are specified for each lesson. (BB)

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SELF-PACED BASIC MATHEMATICS FOR WATER AND WASTEWATER OPERATORS

prepared by

Gary Feldman M.S.

Kirkwood Community College

for the

U.S. Environmental Protection Agency
Office of Federal Activities
Water/Wastewater Training Grant
Technical No. IAD-0-74-109-005

1977

CONTENTS

INTRODUCTION

This math course was written primarily for the operator in the field who for one reason or another cannot obtain a basic knowledge of mathematics through regular classroom instruction. Designed with flexibility and adaptability in mind to accommodate the many different situations and circumstances that exist in this field today this book may also serve a variety of other uses as well, such as use as a valuable supplemental tool for regular math instruction, or use as a regular basic mathematics course in itself, suited for classroom use.

This math course is divided into eight sections. Each section covers and represents a general mathematics area important to water/wastewater plant operation. Specialized areas within each section are then discussed at the smaller unit level.

Purposely included in each section are many example problems similar to the type encountered in the operator's daily activities. Each example or set of examples is then followed by numerous practice problems.

It is my sincere belief that after completing this course, you, the operator, will be successful in your daily math applications.

Pre-test

The main purpose of this test is to see if you already possess the knowledge that is presented in the section you are about to study.

This pre-test can act as a time saver. If you already know what is expected, you can skip to the next section, and take the next pre-test. Taking the pre-tests are an important part of this course as many operators are certain to have a thorough knowledge of atleast some of these math sections. There is no sense in going through an entire section when you already know the information presented in it. The needed score to successfully pass a pre-test and thereby "place out" of that section is stated in each pre-test's level of acceptable performance. If you do not pass out of a pre-test you may still wish to compare that score to the post-test score and see how much you improved after completing that section.

Post-test

The post-test serves to see if you have satisfactorily learned the information in the section you have just completed. It determines if you have mastered the material or not. Upon satisfactory completion of this test you may proceed onto the next section. If you do not pass this test the first time, do not become discouraged as everyone is bound to have some problems somewhere along in this course. If you do not pass this post-test, rework the practice problems a second time and then retake the post-test a second time. If additional help is still necessary, read the suggested supplemental readings before retaking the post-test a third time.

Estimated Time

This indicates approximately how long it will take to complete each section. Just because it says 40 hours does not mean it will necessarily take you 40 hours to complete this course. To be sure, it will take some operators longer to complete certain sections than other operators, depending on their backgrounds, strengths and weaknesses, reading speed, and other factors. Do not be alarmed if it takes you longer to complete a section than the stated estimated time for that section. You are not running a race course.

Suggested Supplemental Readings

These readings are available to those operators who have trouble with the section material. These reading are recommended only if the operator still has trouble after reviewing and reworking the practice problems. These resources should be available at your local library.

Available Supplemental Readings

These readings are available to those operators that desire additional support in a specific section or unit area. These readings may cover a similar section or unit area as those in the suggested supplemental readings, or it may cover a section or unit exclusively. Both the suggested and available supplemental readings should be used as tools in conjunction with each other. Select the reading source that is most appropriate for your needs.

SOME HELPFUL TIPS

Please don't jump around!

This mathematics course was put together in such a way that mastery of each section is necessary before progressing onto the next section. Without a knowledge of the basic material, the more advanced material will become increasingly difficult and frustrating as well. Just as we must crawl before we can walk, so we must learn the basics upon which to build. Start at the beginning, taking each section in its proper order as presented, and finist at the end.

Please do the problems!

Doing the problems is an important and necessary part of this course. Reading the course material and examples are not enough, without doing the practive problems as well. Working the problems acts as a check, a self-evaluation that lets you the operator know if you are understanding the material. There is a difference between thinking you understand the material and really understanding the material. So do them!

Please don't cheat!

Looking up the answers to any of the problems in this course before working the problems is prohibited. Doing so will amount to the same problem -- you may think you understand the material without really understanding it. Being curious is fine, but not this curious. Also remember that you are being tested, not you math teacher.

Make a study schedule.

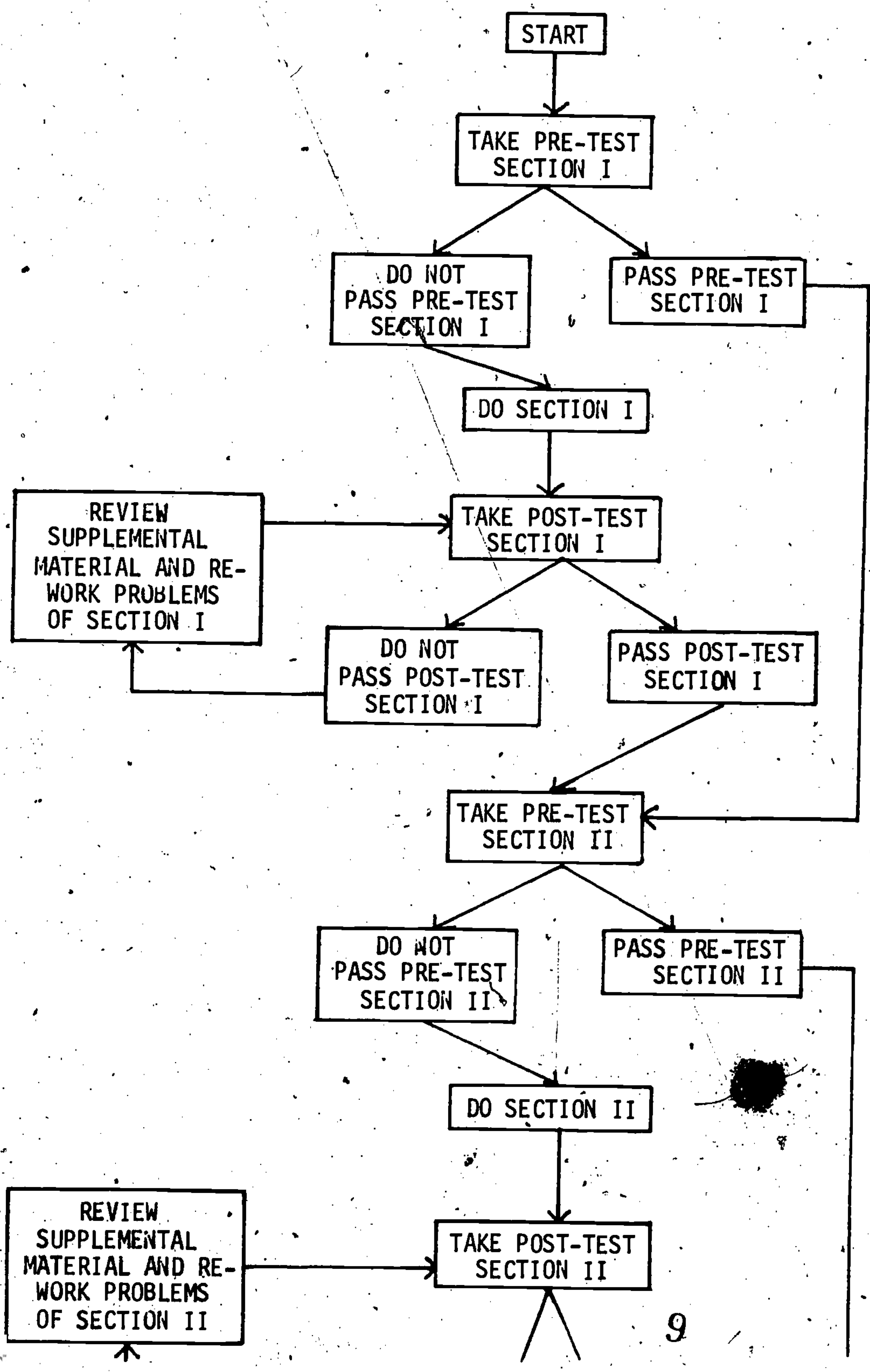
Establishing a regular study schedule is important. Doing so will help you to progress steadily through this course as well as develop good study habits. You may wish to set aside a few hours once or more a week for your homework. Remember that this is a self-study course and you are your own teacher. It is up to you and you alone to do the work.

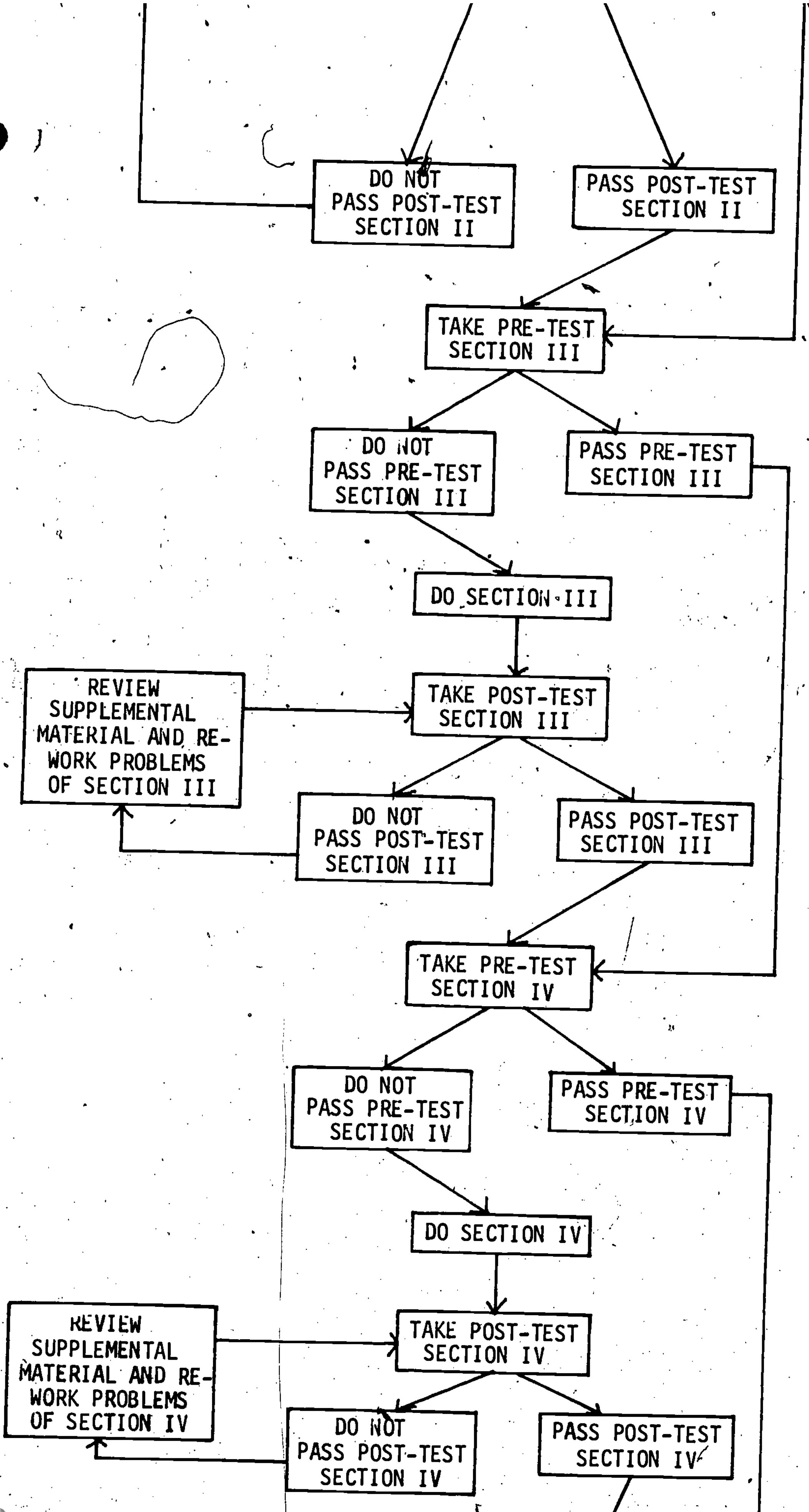
Don't be afraid!

You may feel that by now we have gone over some pretty complicated things, and be having second doubts about even taking this course; but don't be afraid as it is not half as hard as it looks and definitely does not bite! Continue on!!

FLOW CHART OF

STEP BY STEP COURSE SEQUENCE

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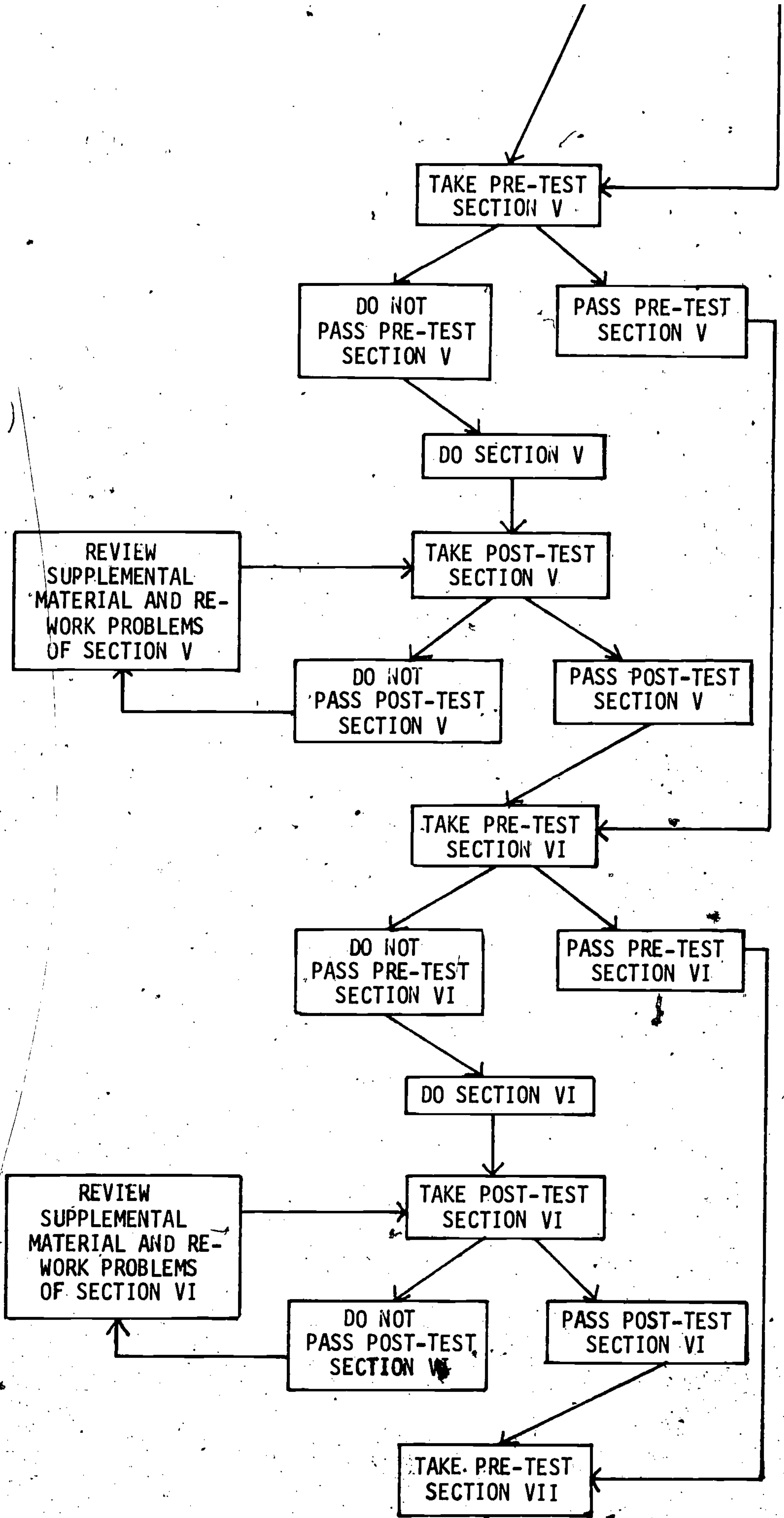

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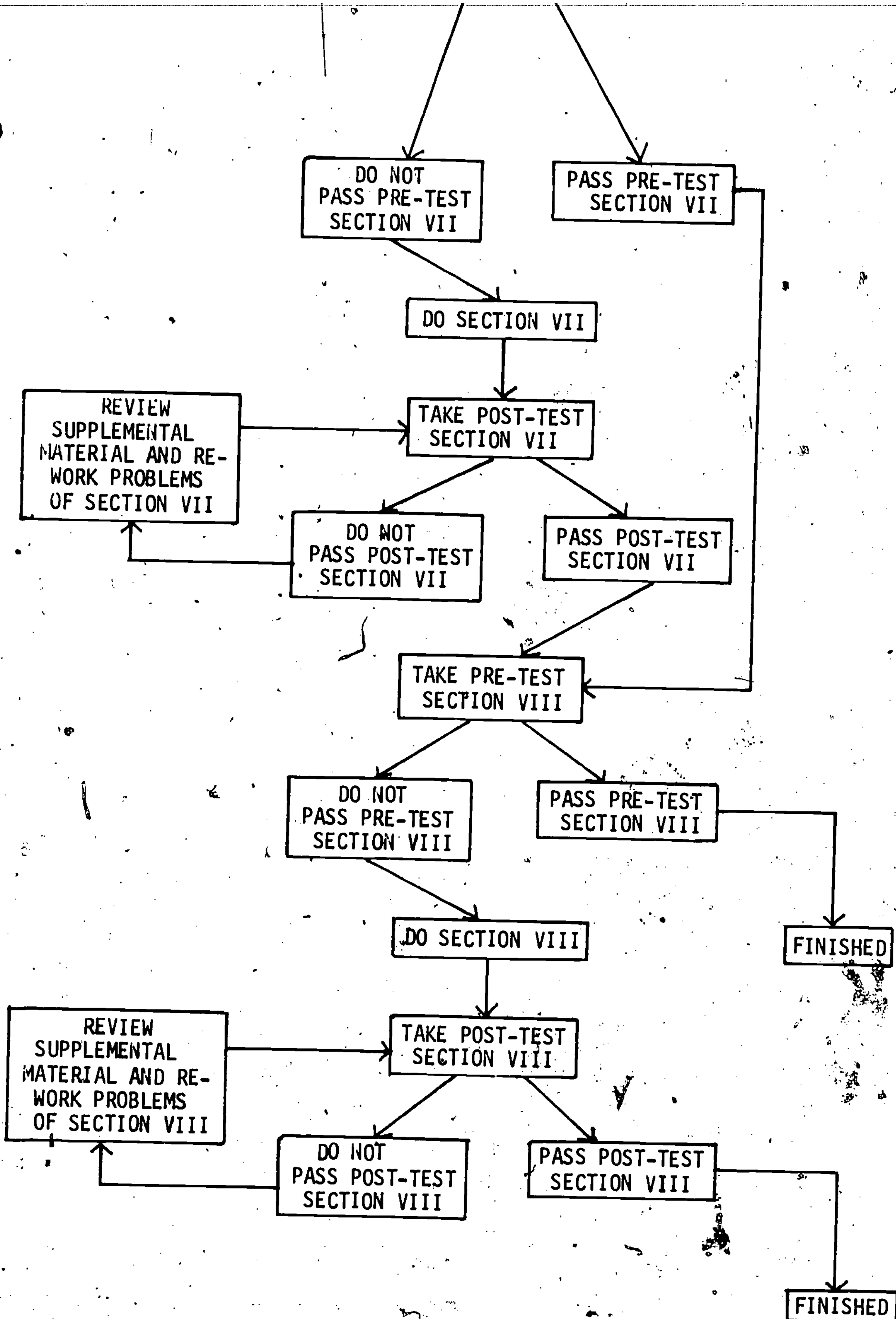
REVIEW SUPPLEMENTAL MATERIAL AND RE-WORK PROBLEMS OF SECTION V
TAKE POST-TEST SECTION V
DO NOT PASS POST-TEST SECTION V
PASS POST-TEST SECTION V
TAKE PRE-TEST SECTION VI
DO NOT PASS PRE-TEST SECTION VI
PASS PRE-TEST SECTION VI
DO SECTION VI
REVIEW SUPPLEMENTAL MATERIAL AND RE-WORK PROBLEMS OF SECTION VI
TAKE POST-TEST SECTION VI
DO NOT PASS POST-TEST SECTION VI
PASS POST-TEST SECTION VI
TAKE PRE-TEST SECTION VII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DO SECTION VII
REVIEW SUPPLEMENTAL MATERIAL AND RE-WORK PROBLEMS OF SECTION VII
TAKE POST-TEST SECTION VII
DO NOT PASS POST-TEST SECTION VII
PASS POST-TEST SECTION VII
TAKE PRE-TEST SECTION VIII
DO NOT PASS PRE-TEST SECTION VIII
PASS PRE-TEST SECTION VIII
DO SECTION VIII
FINISHED
REVIEW SUPPLEMENTAL MATERIAL AND RE-WORK PROBLEMS OF SECTION VIII
TAKE POST-TEST SECTION VIII
DO NOT PASS POST-TEST SECTION VIII
PASS POST-TEST SECTION VIII
FINISHED

COURSE INSTRUCTIONAL PACKAGE GUIDELINE

SUBJECT MATTER: Basic Mathematics for Water and Wastewater Operators

UNIT OF INSTRUCTION: Summary of course material

LESSON NUMBER: Total of 8 Sections

ESTIMATED TIME: 40 hours

JUSTIFICATION FOR THIS INSTRUCTIONAL OBJECTIVE: A knowledge of basic mathematics is important for water and wastewater plant operation.

PREREQUISITES: The prerequisites vary between lessons and are specified in the prerequisites for each lesson.

INSTRUCTIONAL OBJECTIVES:

Terminal Performance Behavior - The learner shall successfully complete this mathematics course. Successful completion of this course shall be demonstrated when the learner through pre-test or post-test written examination has met the specified criterion level based on the behavioral objectives of each section.

Behavioral Objectives - All objectives for each section are stated in that section.

Conditions - The conditions imposed on the learner, (if any), are stated in the conditions for each section.

Criterion - Level of Acceptable Performance - Minimum level of acceptable performance varies between lessons, and is specified in the criterion for each section.

INSTRUCTIONAL APPROACH: Eight individual lessons sequenced in order (1 - 8) utilizing self-paced study of written course material.

Available Supplemental Material - Available supplemental material for each section is stated in that section.

Suggested Supplemental Material - Suggested supplemental material for each section is stated in that section.

SECTION I

UNIT I — **NUMERATION**

- Order
- Place Values
- Measurements
- Rounding

UNIT II — **WHOLE NUMBERS**

- Addition of Whole Numbers
- Subtraction of Whole Numbers
- Multiplication of Whole Numbers
- Division of Whole Numbers

Criterion - Level of Acceptable Performance - Minimum passing score is 90% on either the pre-test or post-test.

INSTRUCTIONAL APPROACH: Individual lesson utilizing self-paced study of written material.

INSTRUCTIONAL RESOURCES:

Available Supplemental Material

Cutler, Ann. The Tractenberg Speed System of Basic Mathematics. Garden City, New York: Doubleday and Company, Inc., 1960. PP. 21-176.
Explains a speed system of the basic mathematical operations.

Thompson, J. E. Arithmetic for the Practical Man. New York: Van Nostrand Reinhold Company, 1962. PP. 7-11.
Briefly explains the four fundamental operations.

Bittinger, Marvin, L. and Keedy, Mervin, L. Arithmetic A Modern Approach. Reading, Massachusetts: Addison-Wesley Publishing Company, 1971. PP. 31-34.
Discusses rounding and estimating.

Assimov, Isaac. Quick and Easy Math. Boston: Houghton Mifflin Company, 1964. PP. 7-109.
Presents short cuts to the four fundamental operations.

Suggested Supplemental Material

Mira, Julio, A. Arithmetic Clear and Simple. New York: Barnes and Nobles, Inc., 1965. PP. 13-78.
Discusses addition, subtraction, multiplication, and division.

Bittinger, Marvin, L., and Keedy, Mervin, L. Arithmetic A Modern Approach. Reading, Massachusetts: Addison-Wesley Publishing Company, 1971. PP. 21-125.
Offers many extra practice problems in the basic operations of whole numbers.

SECTION I

PRE-TEST

1) Add

```
 305
 215
  18
+ 64
```

2) Multiply

```
 520
 x17
```

3) Divide

62/ 1,860

4) Round to the nearest gallon.

185.7

5) Add

```
 57
 25
+18
```

6) Subtract

```
 1605
 -847
```

7 - 8) Place the commas or comma in the appropriate places.

7) 6572

8) 1572111

9) If a small town uses 10 gallons of water per day per person, and has a population of 1000, what would be the total daily need for the town?

10) In 1974 a small town produced four thousand, three hundred metric tons of solid wastes. Write that number noting each place value.

SECTION I

PRE-TEST KEY

1. 602
2. 8,840
3. 30
4. 186 gallons
5. 100
6. 758
7. 6,572
8. 1,572,111
9. 10,000 gallons
10. 4,300

SECTION I

UNIT I

NUMERATION

Order

In the arabic system which we use, there are only ten basic numbers which can be grouped to make a counting and calculation system. In the arabic language ten basic numbers serve as symbols having value.

Figure 1	0	1	2	3	4	5	6	7	8	9
(basic numbers)	zero	one	two	three	four	five	six	seven	eight	nine

These ten basic numbers are put together in an order so that each succeeding number is one value larger than the one behind. Table 1 illustrates the order of numbers and method of counting to 100 which we generally take for granted.

TABLE 1

Numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

In the table these numbers begin with the number 1 at the left of the top row, and increase by the value of 1 for each number to its right, so that 2 is larger than 1 and 3 is larger than 2, etc. This process is repeated in the second row where 12 is larger than 11 and 13 is larger than 12. When we want to represent a number larger than 99 we use the following combination.

101 gives us 1 more than 100
102 gives us 1 more than 101
103 gives us 1 more than 102
etc.

Of course this can be carried on and on in steps of one to any number that is desired. The system has value because it is used as the basis for the manipulation of numbers in addition, subtraction, multiplication, and division. Just as we must crawl before we walk, we must learn to count before we do arithmetic.

Place Values

In order to understand the ordering of numbers let us look at how the basic numbers are put together to make larger numbers. All numbers (including the basic numbers 0 - 9) are expressed by the above symbols or digits (Figure 1). Therefore all numbers consist of basic numbers. The number 10 has the basic numbers 1 and 0. The number 25 has the basic numbers 2 and 5 etc. Zero which has no value serves to fill empty places where there are no other basic numbers. The word place refers to a basic number's location.

When we have a basic number occupying one place we put it in a first column of numbers. This first column of numbers is called the ones column.

0

1

2

3

4

5 All in the ones column

6

7

8

9

When we get a number with 2 places such as 10, we then call the column to the left of the ones column -- the tens column. How many symbols are in the number ten? Ans. two. The symbol 1 and the symbol 0.

When we get a number with three places, we then need a new column because two is not enough. Where do we put the new column? You guessed it-- to the left of the 10's column, making the 100's column.

Hundreds		Tens		Ones	
	1		0		1
	2		3		4
	3		5		7
	4		9		
	5				
	6				
	7				
	8				
	9				

And when we exceed 999 we move left to the thousands column and so on.

This system will make up a picture as below:

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
6,	3	0	2,	4	1	6

So with only ten (10) numerals, we have worked out a system of numbers that could go on indefinitely.

Example 1

748

How many hundreds in this number? 7

How many tens in this number? 4

How many ones in this number? 8

So to read the number it would be seven hundred forty-eight.

Problem 1

Fill in the blank spaces

	656	35	72	0
How many hundreds in these numbers ?	6		0	
How many tens in these numbers ?			7	
How many ones in these numbers ?		5	2	

The basic numbers we use are also called digits.

Arranging any number of digits in a side by side arrangement gives us numbers of different quantities.

7,395 = seven thousand-three hundred ninety-five

The same digits in another arrangement would be a different quantity.

3,975 = three thousand-nine hundred seventy-five

Each group of three digits is separated from other groups of three digits by a comma. These commas are used to make large numbers easier to read.

In placing the commas always go from right to left, after each 3 digits, placing the comma before the next digit to the left.

Example 2

1	No comma	1 digit
25	No comma	2 digits
325	No comma	3 digits, but no digit to the left
6,325	One comma	3 digits and digit to the left
16,325	One comma	5 digits
816,325	One comma	6 digits, but no digit to the left
2,816,325	Two commas	6 digits and digit to the left

Problem 2

Place commas in the following numbers where needed.

2

7654

1000

60743

8972651

When reading each group of three numbers, do not use the word "and". Read each group of three numbers as if it were alone, then join it with the next group of three numbers.

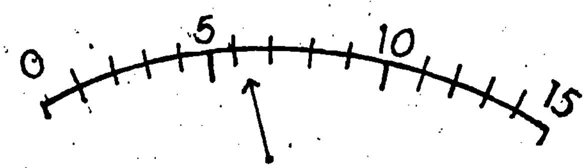
Example 3

	Wrong Way	Right Way
1,054	One thousand and fifty-four	One thousand fifty-four
175	One hundred and seventy-five	One hundred seventy five
10,600,255	Ten million, six hundred thousand and two hundred and fifty five	Ten million, six hundred thousand, Two hundred fifty-five

Measurements

Quite frequently, the operator is required to make measurements. A measurement is the reading off of a number from some sort of gauge or instrument.

These measurements usually involve taking readings off of various scales, balances, various types of meters etc. Every one of the instruments that an operator uses, has a limit below which one cannot distinguish or determine the value he is trying to read. For example if the operator attempts to read the water meter scale below, it is clearly visible that the arrow is past the 5. This is because there are scale lines for every ones unit 0 - 15.



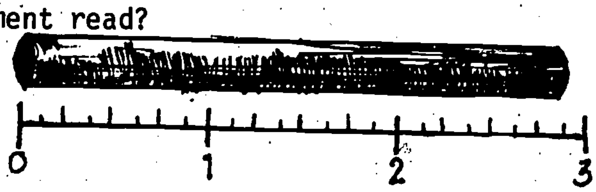
Water meter scale

1 unit = 1 gallon

However, if the next value place was asked for (the tenths place), the operator could only estimate, because it's too hard to read since there are no more lines between 6 and 7. The only figure you can be sure of is the 6. To make note that the .2 in the 6.2 is an estimate, we write the number as $6._2$.

Example 4

You are measuring the length of a pipe in inches. What does the following measurement read?



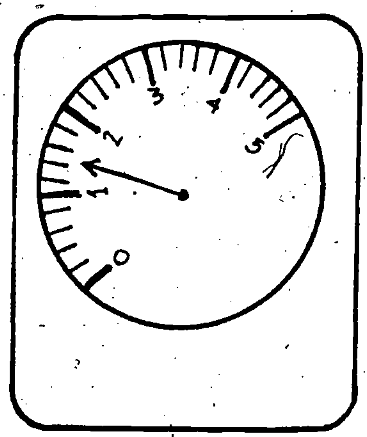
One unit = 1 inch

Here is a case where we know that the value is between 2 and 3. We also know that the value is between 8/10 and 9/10 if we count the smaller lines. But if we take this one step farther we can only estimate how many hundredths there are, because there are no more lines to read. We can only

make an estimate. Since the arrow looks about one half way between 5/10 and 6/10 we assign a value of 50/100 for the hundredths place, and come up with an answer of 1.55. Since the second 5 in the 1.55 is an estimate we write the answer as 1.5_5.

Problem 3

Read the following measurement



Rounding Off

As mentioned earlier, every scale has a limit as to how many places can be read, after which we must estimate. It is at this point of estimation that we often round off. Rounding off is taking the last place or several places furthest to the right, changing them to zero and keeping the place to the left the same or carrying it one value. How often we round off a number depends on the accuracy needed.

In rounding off "the number" 5 is used as the dividing line. If the value is greater than "the number" 5 we raise the place to the left of that value by 1. If it is lower than "the number" 5 we keep the place to the left the same.

Example 5

Round off 7.6

6 is greater than 5 so we get

8.0

Example 6

Round off 9.8

8 is greater than 5 so we get

10.0

Example 7

Round off 17.4

4 is less than .5 so we get

17.0

Occasionally we must round off a number with the number 5 as the last value to the right. The rule here is that if the number to the left of the rounding number is odd keep that number the same while rounding off. If it is even then raise it.

Example 8

Round 6.5 = 7   Round 7.5 = 7

Usually we round off to either eliminate estimated value places or because we need a round number.

Example 9

Round off 1,000,750 to the nearest one thousandth place. The place before the thousands place is 7 in 750. 7 is greater than 5 so we write the number as 1,001,000.

Problems 4 - 8

Round off these numbers to the nearest 100's place

4) 75

5) 149

6) 150

7) 50

Problems 9 - 13

Round off to the nearest tenths place

9) 1.56

10) 15.55

11) 77.09

12) .08

13) 1.50

Problem 14

Round off to the nearest gallon

1,000.9 gallons

26

SECTION I

UNIT II

WHOLE NUMBERS

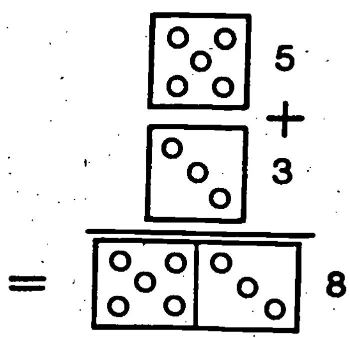
The subject of mathematics is the study of the manipulation of these basic numbers or digits which have been used to represent numbers.

There are only four basic procedures of number manipulation. Each procedure has a symbol which represents that procedure.

Addition	+
Subtraction	-
Multiplication	x
Division	÷

Addition of Whole Numbers

The addition of two numbers can be explained in the following way:



When we add these two boxes (or join them) and count the dots we have eight (8).

Addition thus is the joining or putting together of numbers to get a single new number.

Whole numbers are counting numbers, where each number is one greater than the number behind it, starting from 0 and 1.

It was pointed out earlier that, in counting, each succeeding number was one more than the number behind it. This is the basis for addition. Referring to table 1 if we want to add 5 to 45, we merely have to progress 5 numbers to 50.

The way to visualize this concept is to make a chart of our numerals.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 etc.

The problem 5 + 3 would be solved like this:

ones tens

x

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 etc.

Start with the five (5), then move over to the right three (3) digits and you end up with eight (8).

By using this chart, we can solve simple addition problems.

Do the following problems with this chart:

Problems 15 - 20

15)	16)	17)	18)	19)	20)
1	7	1	8	2	3
+9	+8	+8	+5	+6	+5

When the sum becomes larger than the ones column can handle, we move over to the tens column to the left and start to put numbers in that column.

This also happens in the tens column. When the sum becomes larger than the tens column can handle, we move to the left and use the hundreds column.

Problems 21 - 23 Using the chart add these columns.

21)	22)	23)
1	3	1
7	9	6
5	1	2
+3	+7	+9

This brings up a system of addition that speeds up addition of long columns of figures. It is called adding by tens (10s).

Example 10

1
7
5
+3
16

1 ten
+ 6 left over
16

10
+ 6
16

We pick out the combinations of digits that will add up to ten - put this in the tens column of the answer, then what is left over is added to the total number of tens.

Example 11

3
9
1
+7
20

2 tens
+ 0 left over
20

20
+ 0
20

Example 12

3
9
5
+7
24

1 ten
14 left over
24

10
+14
24

Example 13

4
6
8
9
2
+3
32

2 tens
12 left over
32

20
+12
32

Add these columns of digits in this manner - drawing curved lines to show the operation.

24 (a)
6
4
9
1
+3

24 (b)
9
1
3
7
+4

25) A small sewage plant pumped 100 lbs. of sludge on the 1st shift, 200 lbs. on the second shift and 210 lbs. on the 3rd shift. What is the total amount of sludge pumped for all 3 shifts combined?

This same system of adding by tens can be used for adding columns of more than one digit.

$$\begin{array}{r} 32 \\ 341 \\ 269 \\ 785 \\ 775 \\ +\ 393 \\ \hline 2,563 \end{array}$$

You group the digits in the ones column and you find you have twenty three. Place the 3 in the ones column. Then you take the two tens from there and put it at the top of the tens column. Next group the tens column finding thirty six. Place the 6 in the tens column and place the three hundreds at the top of the hundreds column. Group the hundreds giving you a sum of twenty five. Place the five in the hundreds column and the 2 in the thousands column. The sum of the two numbers is 2 thousand 5 hundred and sixty three.

26)
$$\begin{array}{r} 7\ 6\ 2 \\ 3\ 4\ 9 \\ +5\ 2\ 1 \\ \hline \end{array}$$

27)
$$\begin{array}{r} 2\ 3 \\ 1\ 5\ 6 \\ 3\ 2\ 1 \\ +\ \ 7\ 2 \\ \hline \end{array}$$

Three Rules of Addition

Not many people will make a mistake when adding 2 plus 2. However, it is surprising how many cannot correctly add 22 and 19. The reason is they violate one of the main rules of addition or subtraction, and that is;

1. KEEP ALL NUMBERS IN COLUMNS

When the rule is followed correctly, the above addition is easily performed.

$$\begin{array}{r} 22 \\ +19 \\ \hline 41 \end{array}$$

Remember when writing whole numbers to be added, always keep the ones in a column, the tens in the next column to the left and the hundreds in the next colu to the left, etc. These will always form straight columns.

Another common error is made in the following manner:

$$\begin{array}{r} 349 \\ +75 \\ \hline 414 \end{array} \text{ (Wrong)}$$

In this case another rule is violated.

2. WRITE DOWN ALL CARRYOVER NUMBERS

If this rule is followed, the previous problem becomes:

$$\begin{array}{r} 11 \\ 349 \\ +75 \\ \hline 424 \end{array}$$

Carryover numbers should be written lightly over the next column to the left.

Many of us can remember a teacher saying to them, "You can't add apples and oranges". This is our third rule of addition.

3. ALL NUMBERS MUST BE IN THE SAME DIMENSIONAL (ft., lb., sec.) UNITS

If we needed a string 2 feet long and one 6 inches long, we would either say: 2 ft. + 1/2 ft. = 2 1/2 ft. of string, or

$$\begin{array}{l} 2 \quad\;\; \text{ft.} \\ \underline{\;\;1/2 \text{ ft.}} \\ 2\ 1/2 \text{ ft.} \end{array}$$

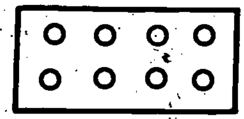
Or we might say:

$$\begin{array}{r} 24 \text{ in.} \\ \underline{6 \text{ in.}} \\ 30 \text{ in.} \end{array}$$

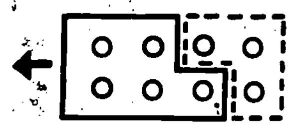
Two and one-half feet and 30 inches are the same length. We must use the <u>same</u> dimensional units when we add any series of numbers.

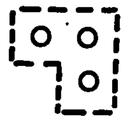
Subtraction of Whole Numbers

Subtraction of two numbers can be explained in this way.

We start with a quantity of 

From this we are going to remove or "take away" a quantity of



Leaving a quantity of 

So 8 - 5 = 3

It was discussed earlier that in counting, each succeeding number was one more than the number behind it. This is the basis for subtraction. Referring to table 1 if we want to subtract 5 from 8 we back off 5 numbers to 3.

Using the same chart of our numerals as we did in addition, we can visualize subtraction (in addition we counted to the right).

In subtraction, since we are reducing or taking away, we count to the left.

8 - 5 = 3

x ones tens

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 etc.

Using the numeral chart, do the following subtractions:

28) 9
 -2

29) 7
 -4

30) 14
 -6

31) $\begin{array}{r} 12 \\ \underline{-7} \end{array}$ 32) $\begin{array}{r} 16 \\ \underline{-9} \end{array}$ 33) $\begin{array}{r} 11 \\ \underline{-5} \end{array}$

The numeral chart we have been using does have some limitation when it comes to larger numbers. The chart would simply be too large to be practical. So we return to the column principal of ones, tens and hundreds as we did in addition.

In the problem 36 - 24 = $\begin{array}{r} 36 \\ \underline{-24} \\ 12 \end{array}$ we take 4 from 6 in the ones column then 2 from 3 in the tens column thus 1 ten and 2 ones = 12

In subtraction there is a problem that comes with the column system. When the number to be reduced in a column is smaller than what we are taking away, we have to borrow a ten or a hundred to use in that column.

$$\begin{array}{rr} 3 & 1 \\ \cancel{4} & 2 \\ \underline{-1} & \underline{9} \\ 2 & 3 \end{array}$$

we borrow 1 ten from 4 tens, leaving 3 tens and use the 1 ten with the 2 ones making 12 and now we can take 9 from 12 leaving 3; then we take 1 ten from 3 tens leaving 2 tens.

The ans. is 2 tens and 3 ones = 23.

This borrowing has to be done any time the amount to be taken away is more than the amount to be reduced in the column: Ones, tens, hundreds, etc.

Working with hundreds, it is done as below:

$$\begin{array}{rrrr} 2 & 11 & 16 & 13 \\ \cancel{3}, & \cancel{2} & \cancel{7} & \cancel{3} \\ - & \underline{3} & \underline{8} & \underline{4} \\ 2, & 8 & 8 & 9 \end{array} \text{ Ans.}$$

When you borrow always cross the number out and write in above a number one less than what you crossed out.

Subtract the following:

34) $\begin{array}{r} 9 \\ \underline{-3} \end{array}$ 35) $\begin{array}{r} 324 \\ \underline{-49} \end{array}$ 36) $\begin{array}{r} 2,376 \\ \underline{1,197} \end{array}$

37) A water storage tank contains 10,000 gallons of water. After discharging 4,500 gallons, how many gallons remain in the tank?

Since subtraction is simply the reverse of addition, the three rules for addition generally apply to subtraction:

1. KEEP ALL NUMBERS IN COLUMNS

Example: Subtract 11 from 25

$$\begin{array}{r} 25 \\ \underline{-11} \\ 14 \end{array}$$

Since subtraction is the reverse of addition, carryovers are not made, but "borrowing" is sometimes necessary.

2. WRITE DOWN ALL BORROWED NUMBERS

Example: Subtract 296 from 485

As before, the numbers should be grouped in columns.

Column Labels

Hundreds	Tens	Units
$\begin{array}{r} 4 \\ \underline{-1} \\ 3 \end{array}$	$\begin{array}{r} 8 \\ \underline{-1} \\ 7 \end{array}$	$\begin{array}{r} 5 \\ \underline{-10} \\ 15 \end{array}$
	$\begin{array}{r} \underline{+10} \\ 17 \end{array}$	
$\begin{array}{r} \underline{-2} \\ 1 \end{array}$	$\begin{array}{r} \underline{9} \\ 8 \end{array}$	$\begin{array}{r} \underline{6} \\ 9 \end{array}$

or

$$\begin{array}{r} 3171 \\ \cancel{4}\cancel{8}5 \\ \underline{-296} \\ 189 \end{array}$$

First Step - Borrow the 1 from the 8 (leaving 7) and add 10 to the 5 to get 15. Subtract 6 from 15 and write down 9.

Second Step - Borrow the 1 from the 4 and add 10 to the 7 to get 17. Subtract 9 from 17 and write down 8.

Third Step - Subtract 2 from 3 and write down 1.

The best way to check a subtraction is by addition. Thus, the preceding problem can be checked by:

$$\begin{array}{r} {\scriptstyle 11} \\ 189 \\ +\underline{296} \\ 485 \end{array} \quad \text{(Check)}$$

The final rule of subtraction is the same as for addition.

3. ALL NUMBERS MUST BE IN THE SAME DIMENSIONAL UNITS

The Three Rules of Subtraction

1. Keep all numbers in columns
2. Write down all borrowed numbers
3. All numbers must be in the same dimensional units

Multiplication of Whole Numbers

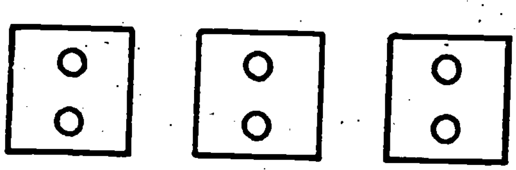
Multiplication is also addition. It is simply a short-cut method of addition. In other words, 3 x 4 is simply:

3 + 3 + 3 + 3 = 12 addition principle

or 4 + 4 + 4 = 12

Thus a multiplication problem can always be checked by addition. In the interest of time, however, every operator should memorize the multiplication table through 10. (See Table 2).

Let us explain multiplication another way. The problem 3 x 2 can be defined as the total number of objects in three (3) sets with two (2) objects in each set.



3 sets with 2 objects per set

3 x 2 = 6

Problems 38 - 40

Multiply the following using the addition principal:

38) 4 x 3 =

39) 5 x 3 =

40) 4 x 6 =

A simpler way to multiply is to use multiplication tables which you can now develop since you know the addition principal.

TABLE 2

0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

As you notice in the table, 3 x 4 = 12 so it doesn't matter which is multiplied by which, it always gives the same answer. (See Diagram Below).

0	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20

0	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16
5	5	10	15	20

Multiplication problems involving larger numbers can be solved by addition also. For example, 24 x 17 can be solved by adding a column of seventeen 24's or a column of twenty-four 17's. This procedure, however, would take considerable time, and therefore the simpler multiplication steps are preferred.

$$\begin{array}{r} 24 \\ \times\ 17 \\ \hline 8 \end{array}$$

1st Step - 7 x 4 = 28. Write down 8 and carry the 2 to the next column.

$$\begin{array}{r} 24 \\ \times\ 17 \\ \hline 168 \end{array}$$

2nd Step - -7 (from 7 in 17) x 2 = 14
14 + 2 (carried 2) = 16
Write down 16.

$$\begin{array}{r} 24 \\ \times\ 17 \\ \hline 168 \\ 4 \end{array}$$

3rd Step - Erase all carryovers. 1 x 4 = 4
Write down four in second row, but <u>one place to left</u>.

$$\begin{array}{r} 24 \\ \times\ 17 \\ \hline 168 \\ 24 \end{array}$$

4th Step - 1 x 2 = 2. Write down 2.

$$\begin{array}{r} 24 \\ \times\ 17 \\ \hline 168 \\ 24 \\ \hline 408 \end{array}$$

5th Step - Add numbers.

Another approach to multiplication is the regrouping concept we illustrated in the subtraction section by placing the number in the appropriate hundreds (H), tens (T), and units (U) columns. The idea behind this approach is that

10 ones or 10 units (U) = 1 ten (T) = 10

and

10 tens or 10 tens (T) = 1 hundred (H) = 100

Example 14

Multiply 24 x 17 or $\begin{array}{r} 24 \\ \underline{\times\ 17} \end{array}$

	H	T	U	
1st Step 7 x 4 =		2	8	= 28 units or 2T + 8U
2nd Step 7 x 2 =	1	4		Unit (7) times Ten (2) makes right digit (4) go in T column or 1H + 4T
3rd Step 1 x 4 =		4		T x U = the Tens column
4th Step 1 x 2 =	2			T x T = the Hundred column
5. Add Columns	3	10	8	
6. Regroup	1			Since 10T = 1H
7. Answer	4	0	8	

To multiply numbers, you may use any method that you understand. These methods are presented to show you different approaches used by many operators which give the same answers.

You will also notice that with any number multiplied by ten, you add a 0. In the same way to multiply by 100 you add two 00s.

Multiply these:

41) 27 x 10 = 42) 9 x 10 = 43) 14 x 100 =

44) 4 x 100 = 45) 100 x 9 = 46) 10 x 4 =

A basic difference between addition and multiplication is that the multiplied numbers do not have to have similar dimensional units.

For this reason it is important to <u>specify the units that go with the numbers</u> and <u>carry them through</u> to the answer.

Example 15

A 20 pound weight on the end of an 8 foot lever would produce

20 lbs. x 8 ft. = 160 ft-lbs.

Example: Three men working five hours each would put in

3 men x 5 hours = 15 man-hours of labor

The multiplication operation is indicated by several different symbols. The most common, of course, is the multiplication sign (x) or times sign. Multiplication also can be indicated by parentheses ( ) or by brackets or simply with a dot .. Thus, the above example can be written five ways.

3 men x 5 hours = 15 man-hours

(3 men) (5 hours) = 15 man-hours

3 men 5 hours = 15 man-hours

3 men • 5 hours = 15 man-hours

(3 men) x 5 hours = 15 man-hours

When solving a problem with parentheses or brackets, always complete the indicated operation within the parentheses or brackets prior to performing the multiplication.

Parentheses ( )

Brackets [ ]

Example 16

(25 - 4) (8 + 2) (3 • 2) =

(21) (10) (6) =

21 x 10 x 6 = 1260

Example 17

$$[15 - (3 + 2)\ (4 - 2)]\ [6 + (7 - 3)] =$$

$$[15 - (5)\ \ (2)]\ [6 + \ 4] =$$

$$[15 \ - \ 10]\ \ 10 = 50$$

The Four Rules of Multiplication

1. Always carry your number.
2. Always move the last number multiplied one place to the left.
3. Multiplied numbers do not have to have the same dimensional units.
4. Multiply all numbers before adding or subtracting them, <u>unless</u> parentheses or brackets are used.

*Any operation in parentheses must be done first.

**Any operations in brackets must be done second.

Division of Whole Numbers

Division offers a means of determining how many times one number is contained in another. It is a series of subtractions. For example, if we say divided 48 by 12, we are also saying, how many times can we take 12 away from 48?

By subtraction:

$48 - 12 = 36$ (one)

$36 - 12 = 24$ (two)

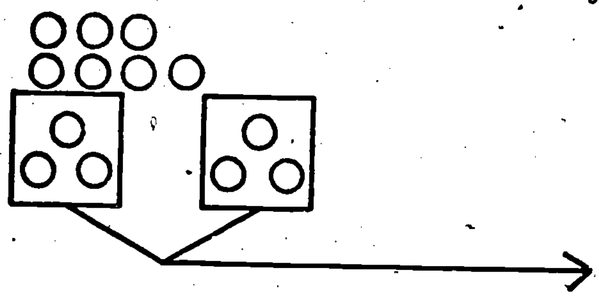
$24 - 12 = 12$ (three)

$12 - 12 = 0$ (four)

Division problems can be written in many ways:

$3\overset{\displaystyle 2}{\overline{)7}}$ with remainder of 1 $\quad$ $7 \div 3 = 2$ with remainder of 1 $\quad$ $\frac{7}{3} = 2$ with remainder of 1 $\quad$ $10/5 = 2$ with remainder of 1

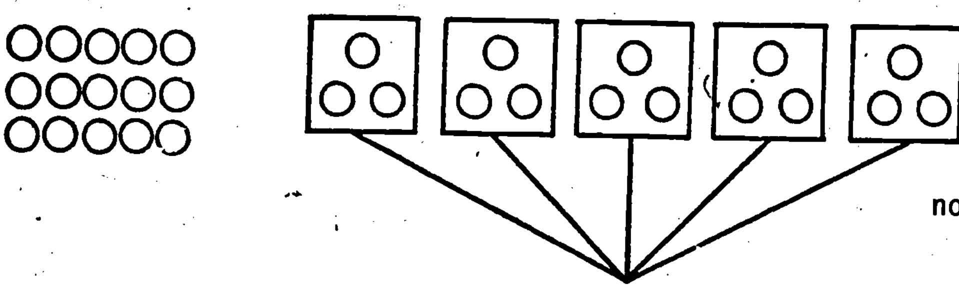
In each case what we are asking is, if we had seven balls and we wanted to put three balls in each box, how many boxes of balls would we have?



+ O left over

Two with a remainder of 1

The same if we have 15 ÷ 3 or 3/‾15



none left over

5 with no remainder

The solving of division problems really can also be a multiplication problem in reverse.

5/‾30 answer 6 $5 \times 6 = 30$

The multiplication table can be used in reverse for division.

Example 18

0	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

By division:

$$12\overline{)48}^{\,4}$$

1st Step - Twelve will not divide into four, but will divide into 48 at least four times.

$$12\overline{)48}^{\,4}$$

2nd Step - Multiply 4 x 12 and write the answer under 48. Remainder is zero. Answer is 4 even.

<u>Example</u>

```
   150 R2
3/ 452
  -3
   15
   15
    02
```

We first divide the 3 into the 4 which will go one time, and place the 1 above the 4. We place the 3 under the 4 and subtract and place a 1 below. Bring down the 5 to make 15. Then we divide the 3 into the 15, getting 5 placing that after the 1 in the answer and subtracting 15 from 15 leaving 0. Bring down the 2. 3 will not go into 2, so we place a 0 after the 5 in the answer and have 2 left over (Remainder of 2). This is the old system of dividing.

Do these division problems:

47) 8/ 125   48) 7/8,530   49) 3/3,005   50) 9/ 7,695

TIP: Remember to place the number above the portion being divided at the time and work from left to right from thousands to ones.

Working with larger numbers is done the same way as smaller numbers. We can also use the new concept of thousands, hundreds, tens and ones, carrying along the zeros to show this.

```
       152  R1
26/ 4,953
    2,600
    1,353
    1,300
       53
       52
        1
```

The only difference to remember is that you are dividing 26 ones into 49 hundreds in the first step.

Second step is 26 ones into 135 tens.

Third step is 26 ones into 53 ones.

Do these Division Problems:

51) $19\overline{)456}$ 52) $32\overline{)7,956}$ 53) $27\overline{)673}$ 54) $21\overline{)5,384}$

Like multiplication when dividing, it is not necessary to use the same dimensional units.

<u>Example 19</u>

How long would it take 4 men to complete a job that required 20 man-hours?

$$\frac{20 \cancel{\text{man}} \text{ hours}}{4 \cancel{\text{men}}} = 5 \text{ hours}$$

When solving a division problem, complete the indicated operations above and below the division line before dividing.

<u>Example</u>

$$\frac{25 - (2)(3) + 18/2}{19 - (3)(4)} + \frac{(4)(9)}{12} - 5 =$$

$$\frac{25 - 6 + 9}{19 - 12} + \frac{36}{12} - 5 =$$

$$\frac{28}{7} + 3 - 5 =$$

$$4 + 3 - 5 = 2$$

The Three Rules of Division

1. Place the answer digits above the portion of the number being divided.
2. Divided numbers do not have to have the same dimensional units.
3. When solving a division problem, complete the indicated operations above and below the division line before dividing.

SECTION I

POST-TEST

1) Add

```
 231
 650
 420
  90
 +71
```

2) Add

```
 73
 22
 97
+38
```

3) Subtract

```
1,749
  958
```

4) In 1972 three billion, six hundred million metric tons of solid wastes were produced in the United States. Write that number being careful to note each place value.

5) Multiply

```
 798
 x53
```

6) Divide

$74\overline{)31,672}$

7 - 8) Place the comma or commas in the appropriate places.

7) 65275

8) 2576154

9) Small towns use an approximate average of 60 gallons of water per day per person. What would be the total daily need for a town of a population of 3954?

10) Round off to the nearest gallon.

115.6

SECTION I

POST-TEST KEY

1. 1462
2. 230
3. 791
4. 3,600,000,000
5. 42294
6. 428
7. 65,275
8. 2,576,154
9. 237,240 gallons
10. 116 gallons

SECTION II

UNIT I FRACTIONS

Fractions and Fractional Numbers

Reduction of Fractions

Addition of Fractions

Subtraction of Fractions

Multiplication of Fractions

Division of Fractions

Available Supplemental Material

Assimov, Isaac. Quick and Easy Math. Boston: Houghton Mifflin Company, 1964. PP. 159-177.
Presents short cuts to fractional operations.

Mira, Julio, A. Arithmetic Clear and Simple. New York: Barnes and Nobles, Inc., 1965. PP. 97-117.
Discusses various aspects of fractions.

Mueller, Francis, J. Arithmetic Its Structure and Concepts. Englewood Cliffs, N. J.: Prentice Hall, Inc., 1964. PP. 205-285.
Gives historical role and explanation of fractions.

Suggested Supplemental Material

Bittinger, Marvin, L. and Keedy, Mervin, L. Arithmetic A Modern Approach. Reading, Massachusetts: Addison-Wesley Publishing Company, 1971. PP. 151-177.
Presents short cuts to fractional operations.

SECTION II

PRE-TEST

1 - 2) Change these improper fractions to mixed numbers.

1) $\frac{13}{8}$

2) $\frac{17}{4}$

3 - 4) Reduce these fractions to lowest terms.

3) $\frac{15}{45}$

4) $\frac{216}{324}$

5) A septic tank has a 1000 gallon capacity. If it is only 1/4 full how many gallons are inside?

6) Three pipes run between four settling basins in a sewage treatment plant. Pipe 1 is 15 1/2 ft. long, Pipe 2, 21 5/8 feet long, and Pipe 3 is 24 5/16 feet long. What is the total length of pipe connecting all 4 settling basins?

7) Subtract

$$\begin{array}{r} \frac{2}{3} \\ - \frac{4}{9} \\ \hline \end{array}$$

8) Divide

$\frac{1}{7} \div \frac{3}{5}$

9) If 1 ounce of chlorine is included in 100 ounces of water, what fraction of the total mixture is chlorine?

10) The detention time of your first primary clarifier is 24/10 more than your second primary clarifier. Change this value to an equivalent mixed number and reduce.

SECTION II

PRE-TEST KEY

1. 1 5/8
2. 4 1/4
3. 1/3
4. 2/3
5. 250 gallons
6. 61 7/16 feet long
7. 2/9
8. 5/21
9. 1/100
10. 2 2/5

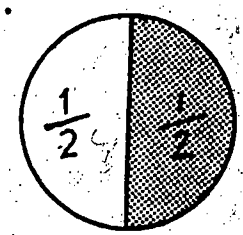
SECTION II

UNIT I

FRACTIONS

Fractions and Fractional Numbers

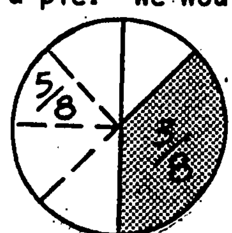
A fraction in its most common form is a part of a whole. For instance, if a pie is divided into two equal parts and one part is eaten, only one half of the pie remains.



1/2 + 1/2 = 1

Thus it can be seen that a fraction is division that has not been completed. As previously explained, in the fraction 1/2, one is called the <u>numerator</u> and two is called the <u>denominator</u>.

If the pie were divided into eight equal pieces and five were eaten, we would have less than one half a pie. We would have 3/8 of a pie remaining.



5/8 + 3/8 = 1

So far, we have used numbers on top of each other with a line drawn between them.

This line is called a fraction bar.

$$\frac{\text{numerator}}{\text{denominator}} \quad \frac{1}{6} \longleftarrow \text{Fraction Bar}$$

In mathematics, this bar - or slash / separating two quantities always means the top quantity is divided by the lower quantity.

$\frac{3}{4}$ or 3/4 really means $3 \div 4$ or $4\overline{)3}$

If the numerator and denominator of a fraction are the same, the result is one (1).

$\frac{4}{4} = 1 \qquad \frac{8}{8} = 1$

With this same fractional concept, we can represent a whole number in fractions.

$8 = \frac{8}{1} \qquad 4 = \frac{4}{1}$

Improper Fraction

An improper fraction has a larger numerator than denominator and is therefore greater than one.

Example 1

$\frac{13}{8}$ (numerator) (denominator)

The following are improper fractions:

$\frac{3}{2} \qquad \frac{4}{3} \qquad \frac{5}{4} \qquad \frac{10}{3} \qquad \frac{21}{6}$

An improper fraction may express a whole number plus a fraction. This is called a mixed number.

Example 2

Improper Fraction $\frac{9}{4}$ $\qquad 9 \div 4 = 2\frac{1}{4}$

$$\begin{array}{rl} 2\frac{1}{4} & \text{mixed number} \\ 4\overline{)9} & \\ 8 & \\ \hline 1 & \text{remainder over} \\ \hline 4 & \text{the denominator} \end{array}$$

The reverse of this operation would be changing a whole or mixed number into an improper fraction. To accomplish this the whole number is multiplied by the denominator, the numerator is added, and this total is written over the denominator.

Example 3

$$2\ 1/4 = \frac{4 \times 2 + 1}{4}$$

$$= \frac{8 + 1}{4}$$

$$= \frac{9}{4}$$

In reducing change any improper fractions to mixed numbers. Some mixed numbers are:

$$2\frac{1}{2} \qquad 6\frac{7}{8} \qquad 1\frac{2}{3} \qquad 16\frac{1}{4} \qquad 23\frac{5}{6}$$

Reduction of Fractions

There are times in this process when we get a mixed number or an improper fraction that can be reduced to simpler terms.

The amount of daily water intake for 1976 to your water treatment plant has increased by 15/6 over the amount in 1975. This number can be changed to a mixed number and reduced.

Example 4

Both numerator and denominator in a fraction may be multiplied by the same number without changing the value of the fraction.

$$\frac{15}{6} \qquad \begin{array}{r} 2 + \frac{3}{6} \text{ or } 2\frac{1}{2} \\ 6\overline{)\,15\,} \\ \underline{12} \\ \frac{3}{6} \end{array} \qquad \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$$

Example 5

$$\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

same

Example 6

$$\frac{1 \times 5}{3 \times 5} = \frac{5}{15}$$

same

Example 7

$$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

same

You may also divide both the numerator and denominator by the same number and not change the value of the fraction.

Example 8

$$\frac{9 \div 9}{18 \div 9} = \frac{1}{2}$$

same

Example 9

$$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

same

Example 10

$$\frac{10 \div 5}{25 \div 5} = \frac{2}{5}$$

same

Dividing the numerator and the denominator by a same number is called "reducing to simplest or lowest terms".

Reducing a Fraction to Lowest Terms

To change a fraction to its lowest terms, divide the numerator and denominator by the largest number that will divide evenly into both:

Example 11

$$\frac{15}{45} = \frac{15 \div 15}{45 \div 15} = \frac{1}{3}$$

NOTE: At this point it should be remembered that the numerator and denominator can be divided or multiplied by the same number without changing the value of the fraction.

Sometimes it will not be possible to reduce the fraction to its lowest terms with the first trial division. In this case, division continues until it can no longer be performed by a number larger than one.

Example 12

$$\frac{216}{324} = \frac{216 \div 3}{324 \div 3} = \frac{72}{108} = \frac{72 \div 9}{108 \div 9} \quad \frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

In solving this problem all of these steps could have been eliminated if we had realized that 108 will divide into the numerator twice and into the denominator three times. This is usually difficult to see, however, and smaller numbers must be used as trial divisors.

Problems 1 - 6

1) What whole number does $\frac{6}{6}$ represent? ________

2) What mixed number does $\frac{26}{5}$ represent? ________

3) What would $\frac{20}{40}$ be when reduced? ________

4) Reduce to simplest terms $\frac{6}{36} =$ ________

5) Change to mixed number and reduce $\frac{21}{6} =$ ________

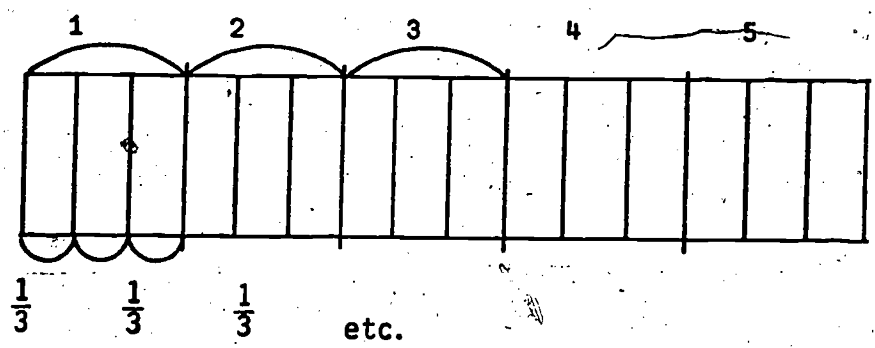
6) What whole number does $\frac{25}{1}$ represent? ________

Addition of Fractions

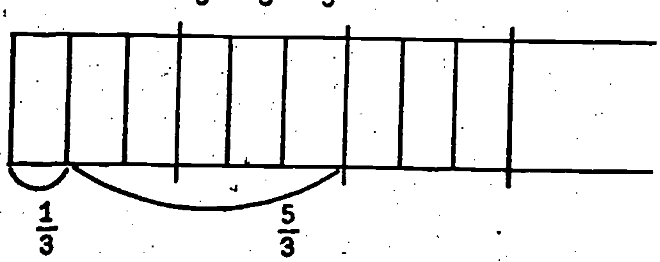
Like fractions all have the same number below the fraction bar.

Such as: $\frac{1}{3}$ $\frac{2}{3}$ $\frac{5}{3}$ $\frac{7}{3}$

When we add like fractions we can use the same concept used with whole numbers except each unit is divided by the number below the fraction bar.



When we add $\frac{1}{3} + \frac{5}{3} = \frac{6}{3}$ or whole number 2



What we do in the case of like fractions is to add the numerator and place the total over the common denominator.

Such as: $\frac{1}{4} + \frac{3}{4} + \frac{6}{4} = \frac{10}{4}$ $\quad 4\overline{)10}$ = 2 R2 = 2 2/4 = 2 1/2
$$\begin{array}{r} 10 \\ \underline{8} \\ 2 \end{array}$$

Don't forget to reduce to mixed number and reduce to lowest terms.

Problems 7 - 9

7) $\frac{1}{8} + \frac{7}{8} + \frac{3}{8} =$

8) $\frac{3}{16} + \frac{2}{16} + \frac{7}{16} =$

9) Three samples of wet sludge were weighed on a pan balance. Sample 1 weighed 7/12 ounces, Sample 2 weighed 3/12 ounces, and Sample 3 weighed 1/12 ounces. What is the total weight of all three samples?

Unlike fractions are fractions with different numbers in the denominators.

Some unlike fractions are: $\frac{1}{7}$ $\quad \frac{3}{5}$ $\quad \frac{1}{2}$ $\quad \frac{3}{4}$

Suppose we want to add $\frac{1}{2} + \frac{1}{3}$

Since the denominators are not alike we cannot use the rule for adding like fractions.

But by using a rule about fractions from previous lessons "You may multiply both the numerator and denominator by the same number and not change the value of the fraction.

Example 13

$\frac{1}{2} \frac{\times 3}{\times 3} = \frac{3}{6}$ (same value as $\frac{1}{2}$)

Example 14

$\frac{1}{3} \frac{\times 2}{\times 2} = \frac{2}{6}$ (same value as $\frac{1}{3}$)

Now both fractions have the same (or common) denominator.

$\frac{3}{6} + \frac{2}{6}$

This we can add with our rule:

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

This process of converting fractions with different denominators to fractions having the same denominator is called finding the <u>common denominator</u>.

The easiest way to find the common denominator is to multiply the two denominators.

$\frac{1}{3} \times \frac{2}{5}$

3 x 5 = 15 the Common Denominator

To get the new numerator do the following 2 things: First take the old denominators 3 and 5 and divide it into the new common denominator 15. Second, take the new numbers 5 and 3 and multiply it by the old numerators 1 and 2.

<u>Example 15</u>

$\frac{1}{3} = \frac{5 \times 1}{15} = \frac{5}{15}$

And likewise we change:

$\frac{2}{5} = \frac{3 \times 2}{15} = \frac{6}{15}$

<u>Example 16</u>

If we want to add $\frac{1}{5} + \frac{3}{4}$

Multiply 5 x 4 = 20 (common denominator)

Divide $5\overline{)20}$ = 4

Divide $4\overline{)20}$ = 5

Thus

$$\frac{1}{5} = \frac{4 \times 1}{20} = \frac{4}{20}$$

$$+\ \frac{3}{4} = \frac{5 \times 3}{20} = \frac{15}{20}$$

$$\frac{19}{20}$$

We usually just write:

$$\frac{4}{20}$$

$$+\ \frac{15}{20}$$

$$\frac{19}{20}$$

So when we look for a common denominator, we are looking for a number that the denominators will divide into evenly.

This last tip should always be kept in mind when looking for a common denominator.

Example 17

Suppose we were determining the total weight of two samples of wet sludge. Sample 1 weighs 1/3 ounce and Sample 2 weighs 5/12 ounce.

In this case the weights of the two samples $\frac{1}{3} + \frac{5}{12}$

Do not multiply 3 x 12 = 36

Common Denominator

12 is the common denominator

Both 3 and 12 divide evenly into 12 -- $3\overline{)12}^{\,4}$ $12\overline{)12}^{\,1}$

$$\frac{1}{3} = \frac{4 \times 1}{12} = \frac{4}{12}$$

$$+\ \frac{5}{12} = \frac{5 \times 1}{12} = \frac{5}{12}$$

$$\frac{9}{12}$$

The problem can be worked either way but do not work with figures larger than are necessary.

So to keep with the smallest common denominators, we take the larger denominator and double it and check if smaller denominators will go into that figure - if not we triple it and try dividing.

Such as: $\frac{1}{6} + \frac{1}{8}$

Will 6 go evenly into 8 - No
Will 6 go evenly into 16 - No
Will 6 go evenly into 24 - Yes - This would be the Common Denominator.

Problems 10 - 11

10) $\frac{1}{3} + \frac{3}{4}$ ______

11) $\frac{5}{6} + \frac{3}{4}$ ______

Problems 12 - 13

Now add these unlike fractions.

12) $\frac{5}{6} =$

$+\frac{1}{3} =$

13) $\frac{4}{5} =$

$+\frac{2}{3} =$

We can add any number of unlike fractions as long as we find a common denominator.

Example 18

Determine the total length of three weirs. Weir 1 = 10 1/3 ft., weir 2 = 20 1/4 ft., weir 3 = 30 1/5 ft.

$10\frac{1}{3} = 10\frac{20}{60}$ ft.

$20\frac{1}{4} = 20\frac{15}{60}$ ft.

$30\frac{1}{5} = 30\frac{12}{60}$ ft.

$60\frac{47}{60}$ ft.

Problems 14 - 15

Add these groups of unlike fractions.

14) $\frac{1}{3}$

$\frac{1}{8}$

$+ \frac{3}{8}$

15) $\frac{1}{6}$

$\frac{3}{5}$

$+ \frac{1}{10}$

We will now consider adding fractional numbers represented by mixed numerals.

Suppose we want to add $4\frac{1}{7} + 3\frac{3}{7}$

First change each number to an improper fraction.

$4\frac{1}{7} = \frac{29}{7}$ and $3\frac{3}{7} = \frac{24}{7}$

Add the improper fractions:

$$\frac{29}{7} + \frac{24}{7} = \frac{53}{7}$$

Change the answer to a mixed number $7\frac{4}{7}$

60

This is a lot of work for a simple addition problem when we know that $4\frac{1}{7}$ really means $4 + \frac{1}{7}$ and $3\frac{3}{7}$ means $3 + \frac{3}{7}$

If we add the whole numbers first 4 + 3 = 7, then add fractions

$\frac{1}{7} + \frac{3}{7} = \frac{4}{7}$

The answer is $7 + \frac{4}{7}$ or $7\frac{4}{7}$

So the rule for adding mixed numbers is to add the fractions separate from the whole number and then add the two answers.

There are times when the fractions will total a mixed number. So we must then add that mixed number in with the whole number.

Example 19

$$\begin{array}{llll} 13\frac{4}{9} = & \frac{8}{18} & & \\ 73\frac{5}{6} & \frac{15}{18} & & \\ \hline 86 & + \frac{23}{18} & \text{or } 1\frac{5}{18} & \end{array}$$

$$= \begin{array}{r} 86 \\ + \ 1\frac{5}{18} \\ \hline 87\frac{5}{18} \end{array}$$

Problems 16 - 17

16) $\begin{array}{r} 79\frac{7}{12} \\ + \ 13\frac{7}{9} \\ \hline \end{array}$

17) $\begin{array}{r} 45\frac{3}{3} \\ + \ 17\frac{15}{16} \\ \hline \end{array}$

Subtraction of Fractions

When subtracting like fractions we really subtract the numerators and keep the denominator which is common.

Let us subtract $\frac{3}{6}$ from $\frac{5}{6}$

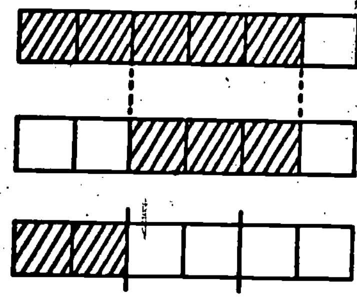
We have a chart

The shaded area represents $\frac{5}{6}$

If we take away $\frac{3}{6}$

We would have $\frac{2}{6}$ or $\frac{1}{3}$

Or



Example 20

$$\frac{11}{18} - \frac{5}{18} = \frac{6}{18} \qquad \text{Lowest terms } \frac{1}{3}$$

Problems 18 - 19

Subtract the following examples:

18) $\frac{13}{16} - \frac{3}{16} =$ ________

19) $\frac{10}{13} - \frac{7}{13} =$ ________

When subtracting with unlike fractions, we follow the same rule as adding unlike fractions. We must first find the common denominator and express the fractions in relation to this common denominator.

Example 21

$$\frac{5}{6} - \frac{4}{9}$$

Common Denominator $\frac{15}{18} - \frac{8}{18}$ Now we subtract the numerators and keep the common denominator giving us $\frac{7}{18}$

The most common way to write subtraction of fractions is as follows:

$$\frac{5}{6} = \frac{15}{18}$$

$$-\ \frac{4}{9} = \frac{8}{18}$$

$$\frac{7}{18}$$

Problems 20 - 21

20) $\frac{3}{4}$

$-\ \frac{1}{16}$

21) $\frac{7}{8}$

$-\ \frac{1}{6}$

When subtracting mixed numerals we subtract using the same rules as we did when adding mixed numerals. First convert the fractions to fractions with a common denominator, subtract the fractions, subtract the whole numbers, and combine the whole number and fractions for the answer.

Example 22

A sample of wet sludge weighs $26\frac{13}{20}$ ounces. If $12\frac{12}{20}$ ounces of this sample is water, what is the weight of the remaining sludge?

$$26\frac{13}{20} \qquad 26 + \frac{13}{20}$$

or

$$-\ 12\frac{3}{5} \qquad -\ 12 + \frac{12}{20}$$

$$26 \quad + \quad \frac{13}{20} \text{ ounces}$$

$$-\ 12 \quad + \quad \frac{12}{20} \text{ ounces}$$

$$14 \quad + \quad \frac{1}{20} = 14\frac{1}{20} \text{ ounces}$$

Problems 22 - 23

22) $5\frac{3}{5}$

$-\ 2\frac{1}{4}$

23) $53\frac{7}{12}$

$-\ 12\frac{3}{8}$

When we subtract mixed numerals we sometimes have a problem much like subtracting whole numbers.

Let's subtract the following: $7\frac{1}{6} - 2\frac{5}{6}$

$7\frac{1}{6}$

$-\ 2\frac{5}{6}$

In the fractional portion of the problem we have a common denominator, but we can't take 5 from 1 in the numerator, so we borrow a whole 1 from the 7 leaving 6.

Then we express the borrowed 1 as $\frac{6}{6}$ and add that to the $\frac{1}{6}$ giving us $\frac{7}{6}$

Or $4\frac{1}{3}$

Now we can subtract $\frac{5}{6}$ from $\frac{7}{6} = \frac{2}{6}$ or $\frac{1}{3}$

Subtract 2 from 6 = 4

Adding 4 and $\frac{1}{3}$ giving a total of $4\frac{1}{3}$

Problems 24 - 25

Now with this principle of borrowing a whole one and expressing it in a fraction, do the following subtractions.

24) $9\frac{2}{7}$

$-\ \frac{3}{7}$

25) $25\frac{3}{5}$

$-\ 14\frac{2}{3}$

Multiplication of Fractions

When one of the factors is a whole number, such as $\frac{2}{3} \times 2$, we

multiply the whole number by the numerator of the fraction and put the product over the denominator.

Example 23

$$\frac{2}{3} \times 2 = \frac{4}{3}$$

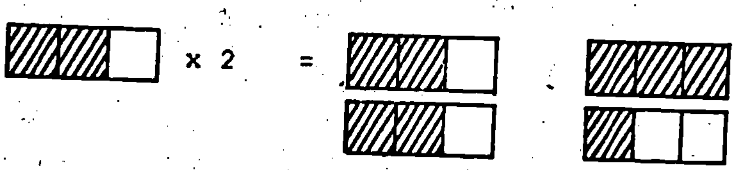
Then we convert this to a mixed number by dividing the denominator into the numerator.

Example 24

$$3\overline{)4}\quad 1\tfrac{1}{3}$$
$$\underline{3}$$
$$\tfrac{1}{3}$$

mixed number

To use a graphic picture of this procedure:



$$\frac{2}{3} \qquad \times 2 \qquad = \qquad \frac{4}{3} \qquad \text{or} \qquad 1\frac{1}{3}$$

Problems 26 - 28

26) $3 \times \frac{3}{11} =$

27) $5 \times \frac{3}{7} =$

28) A 500 gallon water tank is only kept 1/10 full. How many gallons are inside?

Multiplication when Both Numerals are Expressed in Fractions

The rule, in this case, is to multiply the numerators, then multiply the denominators, giving you a new numerator and denominator.

Example 25

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

Sometimes the product can be reduced to simpler terms. When it can be reduced it should be reduced.

Example 26

$$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$$

Problems 29 - 30

Now with this principle known we can do the following multiplications of fractions:

29) [illegible] $\times \frac{3}{5} =$

30) $\frac{8}{9}$

$\times \frac{5}{7}$

Next we will multiply a mixed number by a whole number.

The rule for this is to first multiply the whole number by the whole number part of the mixed number. Then we multiply the whole number by the numerator of the fractional part of the mixed number, then add the two.

Example 27

9 men on a first shift of a wastewater plant each work $4\frac{1}{6}$ hours. How many man hours is this?

$$9 \text{ men} \times 4\frac{1}{6} \text{ hrs} = 9 \times 4 + 9 \times \frac{1}{6} = 36 + \frac{9}{6} = 36 + 1\frac{3}{6}$$

$$= 36 + 1\frac{1}{2} \quad = 37\frac{1}{2} \text{ man hours}$$

Or

$$\begin{array}{lcl} 4\frac{1}{6} \text{ hrs} & = & 4 + \frac{1}{6} \\ \underline{\times 9 \quad \text{men}} & = & \underline{\times 9} \\ & & 36 + \frac{9}{6} = 36 + 1\frac{1}{2} = 37\frac{1}{2} \text{ man hours} \end{array}$$

Multiplying a Mixed Number by a Mixed Number or a Fraction

With the above example you can see we have two multiplications and one addition to do to solve the problem. If we were to multiply two mixed numbers we would have to do the process twice which makes six operations to solve the problem. Large numbers would make the procedure very awkward.

If we were to change all numbers into fractions (proper or improper) and multiply the fractions, the procedure would be much simpler.

Example 28

$1\frac{2}{3} \times 4\frac{1}{5}$

$\frac{5 \times 21}{3 \times 15} = \frac{105 \div 15}{45 \div 15} = \frac{7}{1} \quad 7$

Example 29

$\frac{1}{3} \times 2\frac{2}{5}$

$\frac{1}{3} \times \frac{12}{5} = \frac{12 \div 3}{12 \div 3} = \frac{4}{5}$

There is only one remaining problem with this new system and that is the simplifying at the end.

There is a way to lessen this problem by what we call cross division or canceling out.

The method is that with the multiplication of fractions we can divide the numerator of one fraction and the denominator of the other fraction with a number and not change the value of the fractions as a whole. We can simplify the factors of a fraction in the same way:

We divided both numerator and denominator by 5.	$\frac{1}{\cancel{5}} \times \frac{\cancel{10}}{3} = \frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$
We divide the 3 and the 21 by 3 and the 5 and the 5 by 5	$\frac{\cancel{5}}{\cancel{3}} \times \frac{\cancel{21}}{\cancel{5}} = \frac{1}{1} \times \frac{7}{1} = \frac{7}{1}$ or 7
We divide the 5 and the 10 by 5 and the 2 and the 4 by 2	$\frac{\cancel{5}}{\cancel{10}} \times \frac{\cancel{2}}{\cancel{4}} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

This above process changes any whole numbers into fractions by putting the number over 1. Then change any mixed numbers to improper fractions. Next, use cross division or canceling out and then multiply the numerators and multiply denominators. Last, if necessary, simplify to lowest terms or change to a mixed number

Example 30

$$2\frac{2}{3} \times 6\frac{1}{4} = \frac{\cancel{8}}{3} \times \frac{25}{\cancel{4}} = \frac{50}{3} = 16\frac{2}{3}$$

Following these rules do Problems 31 - 32

31) $3\frac{3}{4} \times 1\frac{3}{5} =$ ________

32) A first sample of suspended solids weighed 2/3 of an ounce. A second sample weighed $3\frac{1}{5}$ times more than the first. How much did the second sample weigh? ________

Division of Fractions

In a division problem with fractions, we have a numerator which will be divided by a denominator to give you an answer.

Example 31

$$\frac{1}{2} \div \frac{4}{5} = \frac{\frac{1}{2}}{\frac{4}{5}}$$

To divide $\frac{1}{2}$ by $\frac{4}{5}$ we first find the reciprocal of the denominator $\frac{4}{5}$.

The reciprocal of any fraction is the fraction turned upside down or inverted.

Example $\frac{4}{5}$ has a reciprocal of $\frac{5}{4}$

To divide $\frac{1}{2}$ by $\frac{4}{5}$ we simply invert $\frac{4}{5}$ (turn it upside down) and multiply as usual.

$$\frac{1}{2} \div \frac{4}{5} \quad \frac{\frac{1}{2}}{\frac{4}{5}} \qquad \frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$$

Remember we follow all the rules of multiplying fractions when we do this.

Example 32

An operator had a small 1/2 gallon steel pail containing 1/2 gallon of untreated water. He emptied 1/2 of the pail's contents; that is he made the amount in the pail twice as small. How much untreated water was left in the pail?

1/2 ÷ 1/2 Invert and multiply 1/2 x 1/2 = 1/2

Example 33

$\frac{9}{16} \div \frac{15}{8}$ Invert and multiply $\frac{\cancel{9}}{\cancel{16}} \times \frac{\cancel{8}}{\cancel{15}} = \frac{3}{10}$

(use cancelling)

Example 34

$1\frac{13}{15} \div 6\frac{2}{5}$

$\frac{28}{15} \div \frac{32}{5}$ Change to improper fractions.

$\frac{\cancel{28}}{\cancel{15}} \times \frac{\cancel{5}}{\cancel{32}}$ Invert the divisor and cancel out.

Now we can do Problems 33 - 35

33) $\frac{3}{11} \div \frac{2}{3}$ ________________

34) $2\frac{1}{10} \div 3\frac{1}{2} =$ ________________

35) A dosage of 4/5 ounces of flourine added to 1000 ounces of water at a water treatment plant was considered to be too strong. How much flourine would be added if the 4/5 ounces of flourine was $3\frac{1}{2}$ times smaller?

70

SECTION II

POST-TEST

1 - 2) Reduce these fractions to lowest terms.

1) $\frac{7}{21}$

2) $\frac{176}{42}$

3) The table at the right shows the number of gallons of water that different sizes of pipe will hold in a 100 foot length. How many total gallons will 100 feet of each of the three types hold?

Pipe Diameter	Gallons
1"	4 7/10
1½"	9 1/5
2"	16 3/10

4) Subtract

$$\begin{array}{r} \frac{3}{5} \\ -\ \frac{7}{12} \\ \hline \end{array}$$

5) About 1½ gallons of water is needed each day for just physical subsistence, and not sanitation or other uses. What would be the minimum requirement for a city of 250,000 just for that purpose?

6) In a 2 compartment septic tank the first compartment contains 2/3 of the total volume. What would be the maximum volume of the first compartment of a 900 gallon tank?

7) Divide

$\frac{1}{7} \div \frac{3}{5}$

8) If 5 ounces of a disinfectant are included in 50 ounces of water, what fraction of the total mixture is disinfectant?

9) Your first anaerobic digester can handle 5/2 the sludge level that your second anaerobic digester does. Change this improper fraction to a mixed number.

10) If your first anaerobic digester handles 25/7 the sludge load as your second anaerobic digester, what would be the equivalent mixed number?

SECTION II

POST-TEST KEY

1. 1/3
2. $\frac{88}{21} = 4\frac{4}{21}$
3. 30 1/5 gallons
4. 1/60
5. 375,000 gallons
6. 600 gallons
7. 5/21
8. 1/10
9. 2 1/2
10. 3 4/7

SECTION III

UNIT I DECIMALS

Decimals

Addition of Decimals

Subtraction of Decimals

Multiplication of Decimals

Division of Decimals

Decimal and Fraction Equivalent

Decimal Fractions

UNIT II PERCENTAGES

Percentages

Percentage and Fractional Equivalent

Percentage and Decimal Equivalent

UNIT III RATIO AND PROPORTION

SECTION INSTRUCTIONAL PACKAGE GUIDELINE

SUBJECT MATTER: Basic Mathematics for Water and Wastewater Operators

UNIT OF INSTRUCTION: Decimals; percentages, ratio and proportion

LESSON NUMBER: Section 3

ESTIMATED TIME: 4½ hours

JUSTIFICATION FOR THIS INSTRUCTIONAL OBJECTIVE: A knowledge of decimals, percentages, and ratio and proportion is important for water and wastewater plant operation.

PREREQUISITES: The learner shall have successfully completed sections 1 - 2.

INSTRUCTIONAL OBJECTIVES:

Terminal Performance Behavior - The learner shall successfully complete this mathematics section. Successful completion of this section shall be demonstrated when the learner through pre-test or post-test written examination has met the specified criterion level based on the behavioral objectives of this section.

Behavioral Objectives - At the completion of this section the learner will be able to:

Demonstrate in writing the proper placing of the decimal point.

Name the six places to the right and left of the decimal point.

Convert in writing decimals to their equivalent value as a fraction and vice versa.

Identify decimal fractions.

Express in writing a number as a percentage and vice versa.

Convert in writing a percentage to its equivalent value as a fraction and vice versa.

Solve percentage problems which involve multiplication and division.

Identify a ratio.

Identify a proportion.

Solve ratio and proportion problems using addition, subtraction, multiplication, and division where necessary.

Perform the following operations on the types of numbers:

Addition with respect to decimals

Subtraction with respect to decimals

Multiplication with respect to decimals

Division with respect to decimals

Conditions - None

Criterion - Level of Acceptable Performance -- Minimum passing score is 90% on either the pre-test or post-test.

INSTRUCTIONAL APPROACH: Individual lesson utilizing self-paced study of written material.

INSTRUCTIONAL RESOURCES

Available Supplemental Material

Assimov, Isaac. Quick and Easy Math. Boston: Houghton Mifflin Company, 1964. PP. 112-158.
Presents short cuts to decimal operations.

Bittinger, Marvin, L. and Keedy, Mervin, L. Arithmetic A Modern Approach. Reading, Massachusetts: Addison-Wesley Publishing Company, 1971. PP 341-357.
Offers extra practice problems in ratio and proportion.

Mueller, Francis, J. Arithmetic Its Structure and Concepts. Englewood Cliffs, N. J. Prentice Hall, Inc. 1964. PP. 205-207.
Gives historical account and description of percentages.

Thompson, J. E. Arithmetic for the Practical Man. New York: Van Nostrand Reinhold Company, 1962.
Briefly explains meaning and rules of ratio and proportion.

Suggested Supplemental Material

Mira, Julio, A. Arithmetic Clear and Simple. New York: Barnes and Nobles, Inc., 1965. PP. 117-158.
Gives a thorough review of the various types of decimals and decimal operations.

SECTION III

PRE-TEST

1) If a water supply contains 20 mg. of sodium per liter, and has an average daily consumption of 75 liters, how much sodium would be consumed through drinking water each day?

2) Add

$$\begin{array}{r} 3.74 \\ 6.251 \\ 80.4 \\ \underline{7.62} \end{array}$$

3) In 1970, 218.3 billion gallons of fresh water were used by industry each day. It is estimated that in 1980, 394.2 billion will be needed. How much of an increase is that in the ten years?

4) Divide

$$\frac{.460}{.80}$$

5) Convert the following decimal to a fraction.

$$\frac{.62}{97}$$

6) Influent BOD to a clarifier is 200 mg/l. Effluent BOD is 100 mg/l. What is the percent removal in the clarifier?

7) What percent is 40 of 50?

8) Percent removal of BOD in a clarifier is 40%. If 80 mg/l are removed, what is the influent BOD?

9 - 10) Solve these proportions.

9) $100 \times A = 1{,}000 \times 1{,}000$

10) $\frac{40}{A} = \frac{100}{10}$

SECTION III

PRE-TEST KEY

1. 1,500 mg
2. 98.011
3. 175.9 billion
4. .575
5. $\frac{62}{9700} = \frac{31}{4850}$
6. 50%
7. 80%
8. 200 mg/l
9. 10,000
10. 4

SECTION III

UNIT I

DECIMALS

Decimals

In our number system the regular whole numbers are grouped in ones, tens, hundred, thousands, and so on, increasing in multiples of 10 as we move left from the decimal point.

Example:

1. = ones

10. = tens

100. = hundreds

1000. = thousands

etc.

The fractions of numbers are grouped to the right of the decimal point and decrease in multiples of tenths as we move to the right.

Example:

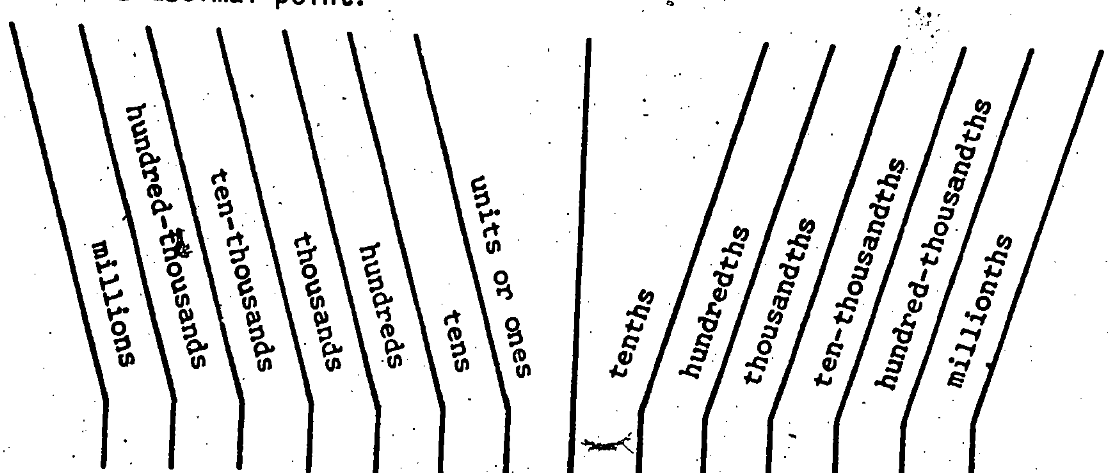
$.1 = \frac{1}{10}$ = tenths

$.01 = \frac{1}{100}$ = hundredths

$.001 = \frac{1}{1000}$ = thousandths

Thus a decimal point serves to separate whole and fractional part of a number.

The following chart names the six places to the left and to the right of the decimal point.



The decimal system is also partially illustrated in Table with regard to placement of the decimal point:

TABLE 3

PLACEMENT OF THE DECIMAL POINT

6 zeros	1,000,000.	one million
5 zeros	100,000.	one hundred thousand
4 zeros	10,000.	ten thousand
3 zeros	1,000.	one thousand
2 zeros	100.	one hundred
1 zero	10.	ten
	1.	one
1 place	0.1	one tenth
2 places	0.01	one hundredth
3 places	0.001	one thousandth
4 places	0.0001	one ten thousandth
5 places	0.00001	one hundred thousandth
6 places	0.000001	one millionth

Some examples of how decimals are related to fractions are as follows:

Example 1

2.5
two and five tenths

$$2 + \frac{5}{10}$$

Example 2

16.34
sixteen and thirty four hundredths

$$16 + \frac{34}{100}$$

Example 3

321.296

three hundred twenty one and two hundred ninety six thousandths

$321 + \frac{296}{1000}$

Addition of Decimals

When we add whole numbers we put all the ones in a column, all the tens in the next to the left, and so on. Then it is easy to add all the ones, tens, etc.

Example 4

Calculate the total length of weir.

```
    5 ft.      Add    5
  216 ft.           216
   23 ft.            23
+ 642 ft.         + 642
                    886
```

In decimals we do the same thing with all the tenths, hundredths, thousandths, etc.

Example 5

Add - 0.3 + .0592 + .167 + .203 =

```
  .3
  .0592
  .167
+ .203
  .7292
```

To make the addition even easier we add zeros before whole numbers and after decimal numbers to square up the problem. These zeros do not change the value of the numbers.

This will give you all the decimal points in 1 column no matter how many numbers we have before or after the decimal points.

Example 6

```
03.5600
10.7320
01.7000
03.0784
```

Then we add in the usual way but remembering to include the decimal point in the answer in the same column as the other decimal points.

```
  03.5600
  10.7320
  01.7000
  03.6784
  -------
  19.6704
```

Now we can do the following problems:

1) Four samples of digested sludge weigh 13.6, 9.7, 3.5 and 10.4 grams respectively. What is their total weight?

   13.6 + 9.7 + 3.5 + 10.4 =

2) 9.5 + .016 + 32.3 + 7.0312 =

3)
```
   0.3
   4.0
   0.7
   5.0
   0.6
 + 8.0
 -----
```

The Rules of Adding Decimals

Keep all decimal points and numbers in columns.

The same rules for addition of whole numbers will still apply in decimal addition.

Subtraction of Decimals

The same rules we used for addition are used in subtraction so long as you keep the columns in line and the decimals in line.

Then we subtract in the usual manner.

Example 7

```
   6.57
 - 2.34
 ------
   4.23
```

The same rules of borrowing for subtraction of whole numbers will still apply in decimal subtraction.

Example 8

$$\begin{array}{r} 4.365 \\ -\ 2.780 \\ \hline 1.585 \end{array}$$

Now we can do these problems:

4) $\begin{array}{r} 4.9 \\ -\ 3.0 \\ \hline \end{array}$ 5) $\begin{array}{r} 39.1 \\ -\ 5.431 \\ \hline \end{array}$ 6) 17.369 - 4.29 =

The Rules of Subtracting Decimals

Keep all decimal points and numbers in columns.

The same rules for subtraction of whole numbers will still apply in decimal subtraction.

Multiplication of Decimals

In multiplication with whole numbers we went over the principle of the powers of 10. Any number multiplied by ten would be that number plus a 0 on the right end.

Example 9

$$\begin{array}{r} 35 \\ \times\ 10 \\ \hline 350 \end{array}$$

Every whole number has an unwritten decimal point to the right. Thus, when we multiply by ten we are really moving the decimal point one place to the right.

Example with decimals 10

$$\begin{array}{r} 35. \\ \times\ 10 \\ \hline 350. \end{array}$$

This same principle still applies no matter where the decimal point is located.

Example 11

32.6 x 10 = 326.

326.597 x 10 = 3265.97

0.0621 x 10 = 0.621

When we multiply by 100 we move the decimal point two places to the right.

3.7632 x 100 = 376.32

.01623 x 100 = 1.623

This goes on through the millions if necessary.

Now do these problems:

7) 43 x 10	8) .35 x 10	9) 356.1 x 100

Now we will multiply decimal numbers by decimal numbers. In this we do not have to keep the decimals in a column. We proceed to multiply just as we do with whole numbers and put in the decimal point (in the answer) later.

Example 12

```
  1.17
x  2.4
   468
  234
  2808
```

Now to find where the decimal point should be. There are two places to the right of the decimal point in 1.17 and one place to the right of the decimal point in 2.4. The sum of these places 2 + 1 is 3 and we must insert the decimal point three places to the left of the last digit.

2808 = 2.808

Example 13

```
    16.3
 x 2.104
     652
   1630
  326
  342952
```

.3 — 1 decimal place
.104 — 3 decimal places

and .3104 — 4 decimal places

342952. 34.2952

Now we can do these problems:

10) .809 x 9.15

11) 7.3 x .9

12) During a peak flow period chlorine demand reached 7.21 mg/l at a water treatment plant. How much is 3.9 times this amount?

The Two Rules of Multiplication of Decimals

When multiplying a decimal by 10 or a multiple of 10 such as 100, move the decimal point in the answer one place to the right for each 0 in that number.

When multiplying a decimal by a decimal, sum up the places to the right of these decimals and insert the new decimal point that many places to the left of the last digit.

Division of Decimals

When we divide any number by 10, 100, 1,000, 10,000 etc., we use the tens principle as in multiplication except we move the decimal to the left.

Example 14

341 ÷ 10 = 34.1

341 ÷ 100 = 3.41

341 ÷ 1000 = .341

3.41 ÷ 10 = .341

.0341 ÷ 100 = .000341

With this principle we can now do these problems:

13) The sewage flow through your plant is 64.3 MGD. If the flow was 100 times less, what would it be?

14) The sewage flow through your plant is .146 MGD. If the flow was 10 times less, what would it be?

15) $4 \div 1{,}000 =$

16) Water flow at a small treatment plant is 465.1 gallons per hour. During a backup period the flow was 100 times less. How many gallons per hour is this?

Now we will do division with decimals when the denominator is a whole number.

The division is done the same way as if using all whole numbers except we have to be sure to put the decimal in the answer.

Example 15

```
                   7.19    answer
denominator    9/ 64.71    numerator
                  63
                  17
                   9
                   81
                   81
                    0
```

Put the decimal in the answer directly above the decimal in the numerator before doing the numerical division.

Example 16

$9\overline{)\,64.71}$

With this we can do these problems:

17) $7\overline{)34.86}$

18) 640.5 pounds of dry sludge was spread over an area of 21 acres. How much sludge per acre is there?

19) $45\overline{)188.1}$

<u>Now for division when both the divisor and dividend are expressed in decimals.</u>

<u>Example 17</u>

$\frac{2.10}{0.06}$ or $.06\overline{)2.10}$

The procedure here is to move the decimal to the right in the denominator to make a whole number. At the same time we move the decimal in the numerator the same number of points to the right.

.06 2.10

Giving us $6\overline{)210}$ then divide. This does not change the problem at all but it does make it easier to solve.

Reasoning: $.06\overline{)2.10}$ can be expressed as a fraction. $\frac{2.10}{.06}$

You can multiply both the numerator and denominator by 100 without changing the fraction.

$\frac{2.10 \times 100}{.06 \times 100}$ Giving us $\frac{210}{6}$ which is $6\overline{)210}$

With this multiplication by tens we can solve a decimal division problem the same way we did when the denominator was a whole number.

Just remember to multiply both the numerator and denominator by the same power of ten.

Move decimal (or multiply)

$3.569.\overline{)534.769.5}$

Giving us: $3569.\overline{)534769.5}$

Now we are ready to do these problems.

20) .12/47.64

21) .28/.8876

22) It takes 3.7 hours to move 1494.8 gallons of sewage through a grit chamber. How many gallons per hour is this?

The Three Rules of Division of Decimals

When dividing a decimal by 10 or a multiple of 10 such as 100, move the decimal point in the answer one place to the left for each 0, in that number.

When dividing a decimal by a whole number place the decimal in the answer directly above the decimal in the numerator.

When dividing a decimal by a decimal multiply both the numerator and denominator by the 10 or the multiple of 10 that will make the denominator a whole number.

Decimal and Fraction Equivalent

Numbers can be expressed in many ways. Two of the ways that numbers can be expressed (which we have already learned) are decimals and fractions. For example, if we have a pie and cut it into four parts, we call each piece a fraction consisting of 1/4 the pie or we may say that the whole pie is 1.00 so that each of the four pieces is .25. Since the numbers "¼" (a fraction) and ".25" (a decimal) represent the same value, we say that they are equivalent, and can convert one form to the other.

Let us first convert or change a fraction to a decimal. To change any fraction to a decimal, divide the numerator by the denominator. If you <u>complete</u> the division of any fraction, it comes out even.

<u>Example 18</u>

$$\frac{3}{4} = 3 \div 4 = 4\overline{)3.000}$$

```
      0.750
  4/3.000
    2 8
     20
     20
```

Example 19

$$\frac{5}{6} = 5 \div 6 = 6\overline{)\,5.000}\quad 0.833\ldots \text{ (not even)}$$

```
        0.833 . . . . (not even)
   6/ 5.000
      4 8
        20
        18
         20
         18
          2
```

Problems 23 - 24

23) Change 9/10 to a decimal

24) 7/8 of a chlorine tank is full. Change this to a decimal.

Now let us change a decimal to a fraction. To change a decimal to a fraction, multiply the decimal by 10/10, 100/100, 1000/1000 etc. It should be noted that multiplying by these factors is multiplying by one (100/100 = 1) and does not change the value of the answer.

Example 20

$$0.25 \times \frac{100}{100} = \frac{25}{100} = \frac{1}{4}$$

Example 21

$$0.375 \times \frac{1000}{1000} = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$$ Reduce to lowest terms

Problems 25 - 26

25) Change .3 to a fraction.

26) Change .17 to a fraction.

Decimal Fractions

Decimal fractions are fractions which have 10, 100, 1000, etc., for denominators. They are usually called decimals.

$\frac{5}{10}$ = 0.5 = five-tenths

$\frac{15}{100}$ = 0.15 = fifteen-hundredths

$375 \frac{25}{1000} = 375.025 =$ three hundred seventy-five and twenty-five thousandths

or = three hundred seventy-five point zero two five

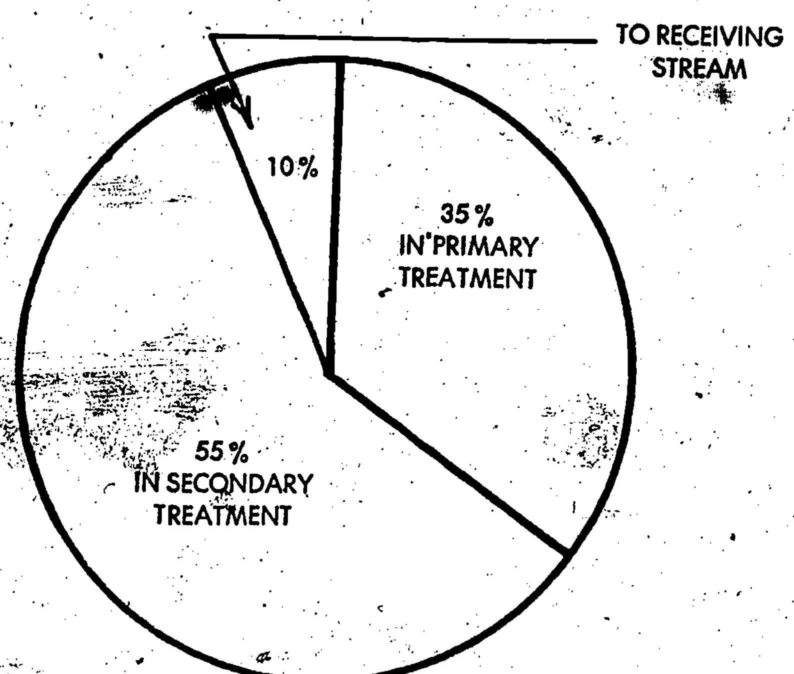
Which of the following are decimal fractions? $\frac{7}{10}$ $\frac{8}{17}$ $\frac{30}{100}$ $\frac{45}{70}$

PERCENTAGES

Percentages

Expressing a number in percentage is just another, and sometimes simpler, way of writing a fraction or a decimal. It can be thought of as parts per 100 parts, since the percentage is the numerator of a fraction whose denominator is always 100. Twenty-five parts per 100 parts is more easily recognized as 25/100 or 0.25. However, it is also 25%. In this case, the symbol % takes the place of the 100 in the fraction and the decimal point in the decimal fraction.

Percentage can best be illustrated by a pie chart showing BOD removals of waste where full circle is 100%.



Thus in percentages we are comparing a number to 100. So 20% compares 20 to 100.

This comparison is called a ratio and we usually represent a ratio in the form of a fraction:

$$20\% = \frac{20}{100}$$

So 10% is the same as $\frac{10}{100}$

What is the ratio for 35%?

Percentage and Fractional Equivalent

To change percent to a fraction, divide by 100%.

Example 22

$$15\% \div 100\% = 15\% \times \frac{1}{100\%} = \frac{15}{100} = \frac{3}{20}$$

Example 23

$$0.4\% \div 100\% = 0.4\% \times \frac{1}{100\%} = \frac{.4}{100} = \frac{4}{1000} = \frac{1}{250} \quad \text{reduced}$$

In these examples note that the two percent signs cancel each other.

To change a fraction to percent, multiply by 100%.

Example 24

$$\frac{2}{5} \times 100\% = \frac{200\%}{5} = 40\%$$

Example 25

$$\frac{5}{4} \times 100\% = \frac{500\%}{4} = 125\%$$

Problem 27

Wet sludge is found to contain 25% solids. Change to a fraction and reduce.

Problem 28

A piece of wastewater equipment malfunctions 1/3 of the time. What percentage does this represent?

Percentage and Decimal Equivalent

If you want to convert a percentile directly to a decimal the procedure is very simple. You take the percentile and move the decimal two places to the left. (This is dividing by 100) from previous lessons.

Example 26

A digester destroys 61% of raw sludge. Express this value as a decimal.
61% = .61

261% = 2.61

If you want to convert a decimal to a percentile you merely do the reverse. We move the decimal two places to the right. (This is multiplying by 100).

Example 27

4.65 = 465%

A wastewater plant has a daily flow capacity of 1,000,000 MGD. Since it operates at 50,000 MGD, only .05 of the plant's capacity is used. Express this as a percentage. .05 = 5%

Now we can change % to decimal and decimal to %.

Convert these values

Problems 29 - 30

29) 21% = 30) 7.9 =

Following is a table comparing fractions, decimal fractions, and percent to indicate their relationship to each other:

TABLE 4

Common Fraction	Decimal Fraction	Percent
$\frac{285}{100}$	2.85	285%
$\frac{100}{100}$	1.0	100%
$\frac{20}{100}$	0.20	20%
$\frac{1}{100}$	0.01	1%
$\frac{1}{1000}$	0.001	0.1%
$\frac{1}{1,000,000}$	0.000001	0.0001%

Sample Problems Involving Percent

Problems involving percent are usually not complicated since their solution consists of only one or two steps. The principal error made is usually a misplaced decimal point. The most common type of percentage problem is finding:

What percent one number is of Another

In this case, the problem is simply one of reading carefully to determine the correct fraction and then converting to a percentage.

Example 28

Influent BOD to a clarifier is 200 mg/l. Effluent BOD is 140 mg/l. What is the percent removal in the clarifier? (NOTE: 200 - 140 = the part removed in the clarifier.)

$$\frac{200 - 140}{200} = \frac{60}{200} = 0.30 \text{ of the original load is removed}$$

$$0.30 \times 100\% = 30\% \text{ removal}$$

$$\text{Therefore \% removal} = \frac{(\text{In} - \text{Out})}{\text{In}} \times 100\%$$

Another type of percentage problem is finding:

Percent of a Given Number

In this case the percent is expressed as a decimal, and the two numbers are multiplied together.

Example 29

Find 7% of 32.

0.07 x 32 = 2.24

Example 30

What is the weight of dry solids in a ton (2000 lbs.) of wastewater sludge containing 5% solids and 95% water?

NOTE: 5% solids means there are 5 lbs. of dry solids for every 100 lbs. of wet sludge.

Therefore

2000 lbs. x 0.05 = 100 lbs. of solids

A variation of the preceding problem is:

Finding a number when a given percent of it is known

Since this problem is similar to the previous problem, the solution is to convert to a decimal and divide by the decimal.

Example 31

If 5% of a number is 52, what is the number?

$$\frac{52}{0.05} = 1040$$

A check calculation may now be performed --

0.05 x 1040 = 52 (Check)

Example 32

Percent removal of BOD in a clarifier is 35%. If 70 mg/l are removed, what is the influent BOD?

$$\text{Influent BOD} = \frac{70}{0.35} = 200 \text{ mg/l}$$

UNIT III

RATIO AND PROPORTION

All math problems can be understood by knowing two principles--ratio and proportion. Understanding these two principles are the key to solving all math problems. Through a fundamental knowledge of ratio and proportion any math problem can be solved. If you learn anything in this math course, learn and understand these two principles of ratio and proportion. With this knowledge you can manipulate numbers such that you can solve any math problem. With the use of these two tools you can master any math problem. Ratio and proportion are the two most important concepts and pieces of knowledge in this course. Master it!

Ratio is the comparison of two numbers with the same denominator. For example, 1 inch compared to 3 inches, or 3 boxes compared to 7 boxes. Ratios are written either as fractions, 1/3, or as 1:3 (which is read "the ratio of one to three").

Proportion is the equating of ratios. For example, 3/6 is equal to 1/2. While the digits (3, 6) in "3/6" are different then the digits (1, 2) in 1/2, the values of the two 3/6 and 1/2 are equal. A proportion is usually written in the form a/b - c/d, or a:b - c:d (which is read as a is to b as c is to d).

In solving a proportion the basic rule to remember is that any operation using any value may be performed as long as the same operation and value is used on both sides of the equal sign.

Once again in solving an equation, any operation using any value may be performed as long as the same operation and value is used on both sides of the equal sign.

What do we mean by operation? The operations that may be used are one of the following manipulations. These have all previously been discussed.

Addition	+
Subtraction	-
Multiplication	x
Division	÷

What do we mean by values? Any number has a value 1/2, 5, 7, 3, 6, etc. These numbers each have a sense of worth. The amount of worth of these numbers are expressed through relations of comparing numbers to each other. What is one? One is one more than zero. This was previously discussed in the section on numeration.

Numbers are not the only values. Actually numbers have little value until applied to something. When we speak of numbers we usually do so in terms of something; such as with money we say we have 10 dollars or 10 cents; in terms of weight 1000 lbs. of sludge; or in terms of volume, 30,000 gallons of water.

<u>Example 33</u>

Division

$1 \times 8 = 4 \times A$

To solve this proportion we use division. Both sides of the equal sign are divided by 4. This leaves or gets the unknown variable A alone by itself, so we can solve for it.

$$1 \times 8 = 4 \times A$$

$$8 = 4 \times A$$

$$\frac{8}{4} = \frac{4}{4} \times A$$

$$2 = 1 \times A$$

$$2 = A$$

Problem 31

Two rectangular clarifiers have equal surface area. The first clarifier is 10 ft. wide by 20 ft. long or 10 ft. x 20 ft. The second clarifier is 25 ft. long. How wide is the second clarifier?

Hint: Set up a proportion.

Example 34

Multiplication

$\frac{A}{3} = \frac{14}{21}$

To solve this equation we use multiplication. Multiplying both sides of the equal sign by 3 will get rid of the 3 in the denominator and get "A" alone so we can solve for it.

$\frac{A}{3} = \frac{14}{21}$

$3 \times \frac{A}{3} = 3 \times \frac{14}{21}$

The $3 \times \frac{A}{3}$ cancels to A

$A = 3 \times \frac{14}{21}$

$A = \frac{28}{21}$

$A = 1\frac{7}{21}$

Reduced, the answer becomes...

$A = 1\frac{1}{3}$

In multiplication you should remember that multiplying any number by 1 does not change that number's value.

Example 34.5

3 x 1 = 3	3 x 3/3 = 3	3 x 100/100 = 3	3 x 15/15 = 3
0 x 1 = 0	0 x 1/1 = 0	0 x 10/10 = 0	0 x 100/100 = 0

Another way to solve this type of problem (Example 34) is to cross multiply and divide.

Example 34.75

To solve the proportion a/b = c/d, we multiply diagonally across

$$\frac{a}{b} = \frac{c}{d}$$

Therefore, a x d = b x c. This procedure is sometimes called cross multiplication.

This can be proved by substituting numbers for these letters a/b = c/d and solving for a. (Example 34.75)

$\frac{a}{6} = \frac{.1}{2}$	First cross multiply diagonally
$a \times 2 = 6 \times .1$	To get this:
$a \times 2 = .6$	Now divide both sides by 2,
$a \times \frac{2}{2} = \frac{.6}{2}$	
$a = \frac{.6}{2}$	
$a = .3$	To get the answer

Problem 32

You are renovating the plumbing of your water treatment plant and need 500 ft. of tubing. If 1 foot of pipe cost 24 cents, how much would 500 feet of pipe cost?

Example 35

Addition

A - 40 = 281 - 1

A - 40 + 40 = 281 - 1 + 40

A = 281 - 1 + 40

A = 280 + 40

A = 320

Here we want to add 40.

By adding 40, A is left by itself on one side of the equation.

Problem 33

Solve this proportion

A - 75 = 340 - 115

Example 36

Subtraction

A + 40 = 280 + 0

A + 40 - 40 = 280 - 40 + 0

A = 280 - 40 + 0

A = 240 + 0

A = 240

Here we wish to subtract 40.

Problem 34

Solve this proportion.

A + 50 = 850 + 200

When one complete ratio is known and one term of the second ratio is known, the proportion relationship indicates what the unknown number should be.

As discussed previously, if one number from the previous example was missing, the number could be found by cross multiplying.

Example 37

$$\frac{a}{6} = \frac{1}{2}$$

$$a \times 2 = 6 \times 1$$

$$\frac{a \times 2}{2} = \frac{6 \times 1}{2}$$ Divide both sides of equation by 2.

$$a = \frac{6 \times 1}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

A few more example problems should better indicate how to deal with ratios and proportions.

Example 38

Certain bolts cost 90 cents a dozen. How much would 3 bolts cost?

In setting up this proportion, we would say: 12 bolts cost 90 cents; 3 bolts cost x cents. Therefore, the proportion is written either as 12/3 = 90/x or 12/90 = 3/x.

$$\frac{12}{90} = \frac{3}{x}$$

$$12 \times x = 90 \times 3$$

$$x = \frac{90 \times \cancel{3}}{\cancel{12}}$$

$$= \frac{90}{4}$$

$$= 22\ 1/2 \text{ or } 23¢ \text{ (to the nearest penny)}$$

Example 39

If 3 lbs. of salt are added to 10 gallons of water to make a solution of a given strength, how many pounds would be added to 129 gallons to make a solution of the same concentration?

$$\frac{3 \text{ lbs.}}{10 \text{ gal.}} = \frac{x}{129 \text{ gal.}}$$

$$x\ (10 \text{ gal.}) = 3 \text{ lbs. } (129 \text{ gal.})$$

$$x = \frac{3 \text{ lbs. } (129\ \cancel{\text{gal}}.)}{10\ \cancel{\text{gal}}.}$$

$$= \frac{387 \text{ lbs.}}{10}$$

$$= 38.7 \text{ lbs.}$$

Note: Gallons in the numerator and gallons in the denominator can be canceled without changing the value of the solution.

Although proportions are usually not difficult to solve, some care must be taken when using them. Some varying quantities are inversely proportional to each other. Their products, rather than their ratios, are constant. This can be easily explained by an example.

Example 40

If 3 men can do a certain job in 10 hours, how long would it take 5 men to do the same job?

This problem is inversely proportional. If this fact were not noticed, many would solve it by direct proportion.

$$\frac{3 \text{ men}}{10 \text{ hours}} = \frac{5 \text{ men}}{x \text{ hrs.}}$$

$$x = \frac{5\ \cancel{\text{men}} \times 10 \text{ hrs.}}{3\ \cancel{\text{men}}}$$

$$= \frac{50 \text{ hrs.}}{3}$$

$$= 16\ 2/3 \text{ hrs. (Wrong)}$$

The solution is wrong since <u>increasing the manpower</u> should <u>decrease the time</u> required to do the job. The problem is therefore inversely proportional and the products of the varying quantities should be equated.

$$3 \text{ men} \times 10 \text{ hours} = 5 \text{ men } (x \text{ hrs.})$$

$$x = \frac{3 \cancel{\text{men}} \times 10 \text{ hrs.}}{5 \cancel{\text{men}}}$$

$$= 6 \text{ hrs.}$$

It is important for the operator to remember that gas pressure-volume problems are also inversely proportional. The higher the pressure, the smaller the volume of gas.

<u>Example 41</u>

A vessel contains 100 cubic feet of gas at 5 lbs. per square inch pressure. What is the pressure if the volume is reduced to 40 cubic feet?

$$100 \text{ cu. ft.} \times 5 \text{ psi} = 40 \text{ cu. ft.} (x \text{ psi})$$

$$x = \frac{100 \cancel{\text{cu. ft.}} \times 5 \text{ psi}}{40 \cancel{\text{cu. ft.}}}$$

$$= \frac{500 \text{ psi}}{40}$$

$$= 12.5 \text{ psi}$$

Note: In this problem the temperature was assumed to remain the same.

If you had trouble with the practice problems in this unit, go back and try them again.

SECTION III

POST-TEST

1) The preferred amount of fluoridation in public or private water supplies should be about .8 parts per million. How much more than the preferred amount would a 1.7 ppm be?

2) Multiply

7.25 x 6.743

3) In a city of 36,500 population the raw sewage sludge amounts to 14.6 tons per day. How much per person per day does this represent?

4) Add

```
   3.756
   2.4
  18.74
+201.7560
```

5) Convert the following fraction to a decimal.

$\frac{64}{1000}$

6) Find 42% of 95.

7) What is the weight of dry solids in 1000 lbs. of wastewater sludge containing 10% solids and 90% water?

8 - 10) Solve these proportions

8) 120 x A = 60 x 40

9) A x 15 = 20 x 30

10) In a water laboratory facility .003 ounces of chlorine is added to 1 gallon of water. How many ounces should be added to 1,000,000 gallons?

SECTION III

POST-TEST KEY

1. .9 ppm
2. 48.88675
3. 2,500
4. 226.6520
5. .064
6. 39.9
7. 100 lbs. of solids
8. 20
9. 40
10. 3,000 ounces

SECTION IV

UNIT I	SQUARE AND SQUARE ROOTS
UNIT II	EXPONENTS
UNIT III	SCIENTIFIC NOTATION

SECTION INSTRUCTIONAL PACKAGE GUIDELINE

SUBJECT MATTER: Basic Mathematics for Water and Wastewater Operators

UNIT OF INSTRUCTION: Square and square roots; exponents; scientific notation

LESSON NUMBER: Section 4

ESTIMATED TIME: 3½ hours

JUSTIFICATION FOR THIS INSTRUCTIONAL OBJECTIVE: A knowledge of square and square roots, exponents, and scientific notation is important for water and wastewater plant operation.

PREREQUISITES: The learner shall have successfully completed sections 1 - 3.

INSTRUCTIONAL OBJECTIVES:

Terminal Performance Behavior - The learner shall successfully complete this mathematics section. Successful completion of this section shall be demonstrated when the learner through pre-test or post-test written examination has met the specified criterion level based on the behavioral objectives of this section.

Behavioral Objectives - At the completion of this section the learner will be able to:

Express whole numbers or decimals in scientific notation.

Perform in writing the following mathematical operations on the following types of numbers:

Squaring with respect to whole numbers

Obtaining square roots with respect to whole numbers

Simplification and reduction of exponents with respect to whole numbers

Multiplication of exponents with respect to whole numbers

Division of exponents with respect to whole numbers

Conditions - None

Criterion - Level of Acceptable Performance - Minimum passing score is 90% on either the pre-test or post-test.

INSTRUCTIONAL APPROACH: Individual lesson utilizing self-paced study of written material.

INSTRUCTIONAL RESOURCES

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Available Supplemental Material

Bittinger, Marvin, L. and Keedy, Mervin, L. Arithmetic A Modern Approach. Reading, Massachusetts: Addison-Wesley Publishing Company, 1971. PP. 363-374. Gives extra practice problems for squares and square roots.

Dressler, Isadore. Preliminary Mathematics. New York: Amsco School Publications, Inc., 1965. PP. 343-349. Gives a short account of scientific notation.

Heywood, Arthur, H. A First Program in Mathematics. Encino, California and Belmont, California: Dickenson Publishing Company, Inc., 1972. PP. 87-109. Short lesson on how to calculate exponents.

Sparks, Fred, W. A Survey of Basic Mathematics. Philadelphia: W. B. Saunders Company, 1975. PP 23-29. Discusses various methods of obtaining square roots.

Thompson, J. E. Arithmetic for the Practical Man. New York: Van Nostrand Reinhold Company, 1962. PP. 54-64. Briefly explains squares and square roots.

Suggested Supplemental Material

Mira, Julio, A. Arithmetic Clear and Simple. New York: Barnes and Nobles, Inc., 1965. PP. 185-225. Explains a speed system for squares and square roots.

SECTION IV

PRE-TEST

1) Find the square of 25.

2) Find the square root of 1521.

3 - 4) Simplify the exponent.

3) 6^3

4) 2^6

5 - 6) Multiply these exponents.

5) $10^2 \times 7^3$

6) $4^2 \times 3^4$

7 - 8) Divide these exponents.

7) $\frac{4^3}{4^2}$

8) $\frac{5^3}{2^4}$

9) The daily water intake for your water treatment plant is 6 MGD or 6,000,000 gallons per day. Express this number in scientific notation.

10) A new chemical is determined to be effective in testing wastewater when used in a dosage of $\frac{6}{1 \times 10^{-6}}$.

How many ppm or parts per million is this?

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SECTION IV

PRE-TEST KEY

1) 625
2) 39
3) 216
4) 64
5) 34,300
6) 1,296
7) 4
8) $7\frac{13}{16}$
9) 6×10^6
10) 6 ppm

SECTION IV

UNIT I

SQUARE AND SQUARE ROOTS

Squaring a number simply means multiplying a number by itself. For instance, in squaring two we obtain four ($2 \times 2 = 4$). In squaring three, the answer is nine. A short way of writing 2×2 is by using the exponent 2 in the following manner, 2^2. Thus, if we were trying to indicate the squaring of numbers we would write:

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$, and so on

A reverse of this process is to take a number that has been squared and find the number which was multiplied by itself to form the square. This process is called finding the square root. The sign $\sqrt{}$ indicates square root. The square root of 4 is written, $\sqrt{4}$, and the answer is 2. The reverse of the previous column would then be:

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$, and so on

A difficulty arises when the square root of a number does not result in a whole number. Such is the case for $\sqrt{20}$. Since the $\sqrt{16}$ is 4, and the $\sqrt{25}$ is 5, the answer is between 4 and 5. Two solutions are available to the operator who does not possess a calculating machine, slide rule, table of square

roots, or a logarithm table. One method is an exact method which is similar to a long division problem. For this method, the operator must refer to a mathematical textbook. Quite frankly, this method is cumbersome and difficult to remember if you do not work with it frequently.

The other method is a trial and error method. This method is shown here because it is a method which will enable the solution of square root problems using only the knowledge of multiplication.

Example 1

Find the square root of 20.

As previously discussed, the answer is between 4 and 5. Therefore, simply guess a number and square it.

Assume 4.3:

$4.3 \times 4.3 = 18.49$

$$\begin{array}{r} 4.3 \\ \underline{4.3} \\ 129 \\ \underline{172} \\ 18.49 \end{array}$$

Next assume 4.4:

$4.4 \times 4.4 = 19.36$

Since $(4.4)^2$ is close to 20, next try 4.44 (these numbers are picked because they are quickly multiplied).

$4.44 \times 4.44 = 19.7136$

Next assume 4.46:

$4.46 \times 4.46 = 19.8916$

Next assume 4.47:

$4.47 \times 4.47 = 19.9809$

For most purposes, 4.47 would be sufficiently close to use as the answer.

For most numbers the trial and error solution takes more time than the exact solution. Its advantage is that it requires no memorized steps for solution, except multiplication.

Problems 1 - 2

1) Find the square of 7 (7^2)

2) Find the square of 12.5 $(12.5)^2$

Problems 3 - 4

3) Find the square root of 100 ($\sqrt{100}$)

4) Find the square root of 100,000 ($\sqrt{100,000}$)

Problem 5

5) Find the square root of $\sqrt{5^2 + 6^2}$

Hint: First perform all operations in square root sign. In other words first square the 5 and square the 6, then add those 2 answers together. Now take the square root of all of that.

SECTION IV

UNIT II

EXPONENTS

In the previous unit we learned that squaring a number was multiplying that number by itself. Thus to square the number 10 would be to multiply 10 x 10 or 10^2. When this number (10) is squared several times, it may be represented by exponents (a small subscript after the number multiplied). Thus an exponent simply tells us how many times a number is being multiplied by itself.

Example 2

$10'$ tells us that we are multiplying 10 by itself one time.

$10' = 10$

Example 3

8^2 tells us that we are multiplying 8 by itself two times.

$8^2 = 8 \times 8 = 64$

Example 4

7^3 tells us that we are multiplying 7 by itself three times.

$7^3 = 7 \times 7 \times 7 = 343$

Example 5

3^4 tells us that we are multiplying 3 by itself, four times.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

Example 6

2^5 tells us that we are multiplying 2 by itself five times.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Thus when a number is multiplied by itself several times, the group of multiplied numbers may be represented by an exponential number.

A rule to remember about using exponents is that any number with an exponent of 0 is equal to 1.

Example 7

$10^0 = 1$

Example 8

$5^0 = 1$

Example 9

$100^0 = 1$

Example 10

$1{,}275^0 = 1$

Problems 6 - 10. Simplify these exponents by multiplying them out.

6) 26^0

7) 5^2

8) 12^2

9) 12^3

10) 6^4

Problems 11 - 15

11) 7^4

12) 2^6

13) [illegible]^5

14) 10^4

15) 20^6

Multiplying Similar Numbers Having Exponents

When multiplying two or more exponents of the same number,

1) Add the exponents

2) Write the number with the resulting exponent of Step 1 adding of the exponents

Example 11

1) $4^2 \times 4^3 =$

2) $2 + 3 = 5$

3) 4^5

The exponents 2 and 3 are added to get 5. Number 4 is written using 5 as the exponent.

Example 12

1) $8^4 \times 8^5$

2) $4 + 5 = 9$

3) 8^9

The exponents 4 and 5 are added to get 9. Number 8 is written using 9 as the exponent.

Problems 16 - 20. Simplify these exponents by multiplying them out.

16) $10^2 \times 10^5$

17) $7^3 \times 7^2$

18) $5^2 \times 5^2$

19) $2^3 \times 2^6$

20) $4^2 \times 4^3$

Dividing Similar Numbers Having Exponents

When dividing similar numbers having exponents

1) Multiply the numerator and denominator out.

2) Reduce by canceling out similar numbers.

Example 13

$$\frac{10^3}{10^2}$$

$$\frac{10^3}{10^2} = \frac{10 \times 10 \times 10}{10^2}$$ $(10^3 = 10 \times 10 \times 10)$

$$\frac{10 \times 10 \times 10}{10^2} = \frac{10 \times 10 \times 10}{10 \times 10}$$ $(10^2 = 10 \times 10)$

$$\frac{\cancel{10} \times \cancel{10} \times 10}{\cancel{10} \times \cancel{10}}$$ The similar tens are canceled out

$$\frac{10}{1}$$ Leaving $\frac{10}{1}$ or 10 as the answer

Problems 21 - 22. Simplify and reduce these exponents.

21) $\frac{7^4}{7^2}$

22) $\frac{8^5}{8^2}$

Multiplying or Dividing Different Numbers Having Exponents

When multiplying or dividing different numbers having exponents,

1) Multiply out the exponents.

2) Carry out the multiplication or division of the resulting new numbers.

Example 14

Multiply the exponents

$2^2 \times 3^3$	First multiply out the exponents
$2 \times 2 \times 3^3$	$(2^2 = 2 \times 2)$
$2 \times 2 \times 3 \times 3 \times 3$	$(3^3 = 3 \times 3 \times 3)$
4×27	$(2 \times 2 = 4$ and $3 \times 3 \times 3 = 27)$
108	$(4 \times 27 = 108)$

<u>Example 15</u>

$5^3 \times 2^5$	
$5 \times 5 \times 5 \times 2^5$	$(5^3 = 5 \times 5 \times 5)$
$5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2$	$(2^5 = 2 \times 2 \times 2 \times 2 \times 2)$
$125 \times 2 \times 2 \times 2 \times 2 \times 2$	$(5 \times 5 \times 5 = 125)$
125×32	$(2 \times 2 \times 2 \times 2 \times 2 = 32)$
4000	$(125 \times 32 = 4000)$

<u>Problems 23 - 24.</u> Multiply the exponents

23) $4^3 \times 2^2$

24) $8^2 \times 4^3$

<u>Example 16</u>

Divide the exponents

$\frac{8^2}{2^3}$	First multiply out the exponent
$\frac{64}{2^3}$	$(8^2 = 8 \times 8 = 64)$
$\frac{64}{8}$	$(2^3 = 2 \times 2 \times 2 = 8)$
8	

Example 17

$\frac{10^4}{5^2}$ $(10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000)$

$\frac{10{,}000}{5^2}$ $(10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000)$

$\frac{10{,}000}{25}$ $(5^2 = 5 \times 5 = 25)$

400 $\frac{10{,}000}{25}$

Problems 25 - 26. Divide the exponents

25) $\frac{5^4}{3^3}$

26) $\frac{6^3}{2^2}$

SECTION IV

UNIT III

SCIENTIFIC NOTATION

In the water and wastewater field, very large and very small numbers are both encountered daily. For example some larger water and wastewater treatment plants have daily water intakes of over 10,000,000 gallons. At the same time chemical treatment dosages are considered in parts per million or parts times .000001. Multiplying, dividing, adding, or subtracting such large and small numbers can be awkward, time consuming, and often a cause for error, such as misplacing the decimal point. On the other hand using scientific notation in place of these large and small numbers is both quick and accurate.

In scientific notation all numbers are and can be expressed as an exponent of ten, multiplied by another number between and including 1 and 10. What is an exponent of ten? An exponent of ten is the number ten multiplied by itself a specified amount of times. Thus in the figure 10', the first number 10 is the number multiplied, while the second number (1) (the exponent) tells how many times ten is to be multiplied by itself. Similarly 10^2 tells us that 10 is the number to be multiplied by itself, while (2) (the exponent) tells us how many times ten is to be multiplied by itself.

Why are all the numbers expressed as an exponent of 10 in scientific notation? Why not 9 or 8?

What makes 10 so special? Ten is special because it is the basic group of numbers used in our number system. That is it is a basic group of repeating numbers. The numbers 10, 20, 30, 40, 50, 100 are all repeating numbers or multiples of ten (10 x another number). Why then is ten our basic group of numbers? Probably because we have 10 fingers!

Now familiarize yourself with the (exponents of ten) and what they equal.

TABLE 5

$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$

$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

$10^3 = 10 \times 10 \times 10 = 1{,}000$

$10^2 = 10 \times 10 = 100$

$10^1 = 10 = 10$

$10^0 = 1$ $= 1$

$10^{-1} = \frac{1}{10}$ $= .1$

$10^{-2} = \frac{1}{10^2} = \frac{1}{10 \times 10} = \frac{1}{100}$ $= .01$

$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1000}$ $= .001$

$10^{-4} = \frac{1}{10^4} = \frac{1}{10 \times 10 \times 10 \times 10} = \frac{1}{10{,}000}$ $= .0001$

How to Express Numbers in Scientific Notation

When expressing a number in scientific notation:

1) Place the decimal point so that there is only one digit to the left of the decimal point.
2) Count how many spaces to the left or right you have to go compared to the original number. Going to the left makes the number positive and to the right negative.
3) Take the number of spaces you have counted and use it as an exponent of 10.
4) Multiply the number obtained in the first step by the exponent of 10.

Example 18

Express 846 in scientific notation

First, place the decimal point so that there is only one digit to the left of the decimal point.

8.46

Second, count how many spaces to the left or right you have to go compared to 846. In this problem we go 2 spaces. Since it is also to the left we make it a positive 2 (+ 2).

Thirdly, we take this positive 2 (+ 2) and make it an exponent of 10 (10^2).

Finally, we multiply the number obtained in the first step where we moved the decimal point to the left (8.46), and multiply by the exponent number 10^2. So 8.46×10^2 is the answer.

Example 19

Express 15,500 in scientific notation.

1) 1.55

2) +4

3) 104

4) 1.55×10^4

Example 20

Express 180,000 in scientific notation.

1) 1.8

2) +5

3) 10^5

4) 1.8×10^5

Example 21

Express .00057 in scientific notation.

1) 5.57

2) -4 (Notice here we use a negative number to indicate that we are moving 4 spaces to the right.)

3) 10^{-4}

4) 5.57×10^{-4}

Two rules to remember on scientific notation are:

1) When counting spaces or going to the right, always make the exponent negative (-).

2) Any number with an exponent of (o) is equal to 1 $10^0 = 1$

Problems 27 - 36

Express the following numbers in scientific notation.

27) 1005

28) 47

29) 1,800,000

30) .01

31) .00574

32) 25

33) .0001075

124

34) 68,000

35) 10,000,000

36) 5,500

SECTION IV

POST-TEST

1) Find the square of 13.

2) Find the square root of 196.

3 - 4) Simplify the exponent.

3) 5^4

4) 8^3

5 - 6) Multiply these exponents.

5) $5^2 \times 3^3$

6) $6^2 \times 5^3$

7 - 8) Divide these exponents.

7) $\frac{5^3}{5^2}$

8) $\frac{10^3}{5^2}$

9) A water storage tank has a water storage capacity of 100,000 gallons. Express this number in scientific notation.

10) A flow of 2×10^6 gallons per day flows through a 30 inch pipe. How many gallons per day is this?

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SECTION IV

POST-TEST KEY

1) 169
2) 14
3) 625
4) 512
5) 675
6) 4,500
7) 5
8) 40
9) 1×10^5
10) 2,000,000 GPD

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SECTION V

UNIT I — **BASIC ALGEBRA**

Solving for the Unknown

Problem Solving

UNIT II — **FORMULAS**

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SECTION INSTRUCTIONAL PACKAGE GUIDELINE

SUBJECT MATTER: Basic Mathematics for Water and Wastewater Operators

UNIT OF INSTRUCTION: Basic algebra; formulas

LESSON NUMBER: Section 5

ESTIMATED TIME: 7 hours

JUSTIFICATION FOR THIS INSTRUCTIONAL OBJECTIVE: A knowledge of basic algebra and formulas is important for water and wastewater plant operation.

PREREQUISITES: The learner shall have successfully completed sections 1 - 4.

INSTRUCTIONAL OBJECTIVES:

Terminal Performance Behavior - The learner shall successfully complete this mathematics section. Successful completion of this section shall be demonstrated when the learner through pre-test or post-test written examination has met the specified criterion level based on the behavioral objectives of this section.

Behavioral Objectives - At the completion of this section the learner will be able to:

Solve for one unknown using the basic algebraic operations of addition, subtraction, multiplication and division.

Solve story problems in basic algebra using the six rules for problem solving.

Solve for one unknown in a formula using the four rules of solving an unknown in a formula.

Conditions - None

Criterion - Level of Acceptable Performance - Minimum passing score is 90% on either the pre-test or post-test.

INSTRUCTIONAL APPROACH: Individual lesson utilizing self-paced study of written material.

INSTRUCTIONAL RESOURCES

Available Supplemental Material

Dressler, Isadore. Preliminary Mathematics. New York: Amsco School Publications, Inc., 1965. PP. 183-185.
Explains how symbols are used to represent numbers and operations.

Hill, T. H. Ward, Mathematics for the Layman. New York: Philosophical Library, 1958. PP. 204-219.
Explains how to work a formula and the symbols used in algebra.

Suggested Supplemental Material

Dressler, Isadore. Preliminary Mathematics. New York: Amsco School Publications, Inc., 1965. PP. 183-233.
Gives a thorough explanation of algebra, and solving for equations, along with a large selection of practice problems.

SECTION V

PRE-TEST

<u>Problems 1 - 6 Solve for the unknown.</u>

1) $x + 15 = 27$

2) $x - 101 = 25$

3) $P \times 40 = 200$

4) $16.5 + 4.2 + x = 21$

5) $\frac{50 + (20 \div 4)}{11} = x$

6) $\frac{20 \times 7}{70} = Y$

7) 2,000 lbs. of solids enter a digester daily. 80% are volatile solids and 20% are fixed solids. What is the weight of volatile solids and fixed solids entering the digester per day?

8) The daily average flow to a water treatment plant is 6 MGD. If the population of the district it serves is 100,000 what is the GPD flow per capita?

9) In the formula $A = W \times L$, find the value of A when $W = 10$ and $L = 25$.

10) In the formula $V = L^3$, find the value of V when $L = 5$.

SECTION V

PRE-TEST KEY

1. 12
2. 126
3. 5
4. .3
5. 5
6. 2
7. 1,600 lbs. volatile solids, 400 lbs. fixed solids
8. 60 GPD per capita
9. 250
10. 125

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SECTION V

UNIT I

BASIC ALGEBRA

Solving for the Unknown

Algebra is the combining together of symbols or letters with numbers in solving equations. In algebra we solve for unknown quantities using equations. The solving of algebraic problems is based on the principles of ratio and proportion.

Therefore, in solving for an unknown number, the same methods used in ratio and proportion may be applied.

1) In solving an equation any operation using any value may be performed as long as the same operation and value is used on both sides of the equal sign.
2) Use that operation and value which will get the unknown variable you wish to solve for alone on one side of the equation. Ask the question what operation using what value will get the unknown variable alone by itself?

In solving for an unknown number the operations used are:

1. Addition
2. Subtraction
3. Multiplication
4. Division

As a rule of thumb in solving for an algebraic equation, use that operation which is the opposite of the operation in the problem. The opposite operations are:

1. For addition, use subtraction
2. For subtraction, use addition
3. For multiplication, use division
4. For division use multiplication

Example 1 (Addition)

What number is added to 20 to equal 50? This statement can be written as:

? + 20 = 50

In Albebra rather than use the symbol ? the symbol x is used so:

? + 20 = 50 is changed to x + 20 = 50

Let us look again at the problem

x + 20 = 50

Therefore: x = 30

In this problem we wanted to get x alone on one side of the equation so we subtracted 20 from each side.

x + 20 = 50

x + 20 - 20 = 50 - 20

x = 50 - 20

x = 30

Solve for the unknown in the following algebra problem. (The letters stand for the unknown variable).

Problems 1 - 5

1) x + 40 = 70

2) x + 30 + 105 + 15 = 200

3) P + 4 = 8

4) $A + 41 = 47$

5) $x + 15.5 = 17.1$

Example 2 (Subtraction

What number can 5 be subtracted from to get 25? This problem can be written $x - 5 = 25$

The answer is 30

Let us look again at this problem

$X - 5 = 25$

What do we have to do to get the x alone by itself? Get rid of the -5 on the left hand side of the equation. This can handily be done by adding 5 to both sides of the equation.

$x + 5 - 5 = 25 + 5$

$x = 25 + 5$

$x = 30$

Problems 6 - 10

6) $P - 50 = 20$

7) $x - 101 = 60$

8) $x - 14.5 = 15$

9) $R - 5 = 1.5$

10) $x - 5 = 15$

<u>Example 3</u> Multiplication

What number is multiplied by 5 to get 30? This statement can be written,

$$X \times 5 = 30$$

The answer is 6

Let us look again at this problem.

$$X \times 5 = 30$$

In this problem we wanted to get X alone by itself on one side of the equation so we divided both sides by 5.

$$X \times 5 = 30$$

$$X \times \frac{5}{5} = \frac{30}{5}$$

$$X = \frac{30}{5}$$

$$X = 6$$

<u>Problems 11 - 15</u>

11) $X \times 10 = 100$

12) $P \times 35 = 140$

13) $A \times 1{,}000 = 15{,}000$

14) $X \times 29 = 58$

15) $P \times 7 = 126$

<u>Example 4</u> Division

What number is divided by 7 to get 8?

$x \div 7 = 8$

The answer is 56.

Let us look again at this problem.

$x \div 7 = 8$

What do we have to do to get the x alone by itself? Get rid of the 7 on the left hand side of the equation. This can handily be done by multiplying by 7 on both sides of the equation.

$x \div 7 = 8$

$\frac{x}{7} = 8$

$\frac{x}{7} \times 7 = 8 \times 7$

$x = 8 \times 7$

$x = 56$

<u>Problems 16 - 20</u>

16) $x \div 4 = 25$

17) $P \div 5 = 6$

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18) $M \div 4 = 120$

19) $X \div 3 = 17$

20) $x \div 11 = 20$

Combination algebra problems - solving for an unknown

The following algebra problems are a combination of addition, multiplication, subtraction, and division. In solving these problems you may wish to review Section I and the rules of the 4 basic operations. In solving algebra problems which involve a combination of operations, the order in which you perform the operation is very important. It can mean the difference between the answer being right or wrong.

Example 5

$$X = \frac{10 - 1}{2 + 1}$$

In this problem we have 3 operations to perform; namely, subtraction, addition, and division. First remembering one rule of division we know that in a division problem, the operations above and below the division line must be carried out first.

$X = \dfrac{10 - 1}{2 + 1}$ So starting with,

$X = \dfrac{9}{2 + 1}$ $(10 - 1) = 9$ We perform the subtraction operation and get,

$X = \frac{9}{3}$ $(2 + 1) = 3$ Then we perform the addition operation and get,

$X = 3$ Then we finally divide and get,

Example 6

$5 \times 4 + 4 = x$

In this problem we have multiplication and addition. From our previous rules we know that multiplication is performed before addition. So starting with

$5 \times 4 + 4 = x$

We perform the multiplication first,

$20 \times 4 = y$ $(s \times L) = 20$

Then the addition.

$24 = x$

Example 7

$$\frac{(6 \times 7) + (16 \div 2)}{10} = x$$

In this problem we have all the operations addition, subtraction, multiplication and division. From our previous rules we remember that all operations in parentheses are performed before all other operations.

So starting with our problem

$$\frac{(6 \times 7) + (16 \div 2)}{10} = x$$

We perform the operations in the parentheses first

$$\frac{42 + (16 \div 2)}{10} = x \qquad (6 \times 7) = 42$$

$$\frac{42 + 8}{10} = x \qquad (16 \div 2) = 8$$

Now from previous rules we know that all other operations above and below the division line are performed before dividing.

So $\frac{50}{10} = x$ $(42 + 8) = 50$

Now we finally divide $\frac{50}{10} = 5$.

$5 = x$

Problems 21 - 30

Solve for the unknown in the following algebra problems.

21) $5 \times 7 + 4 = P$

22) $10 \times 20 - 15 = P$

23) $\frac{10}{4} + 5 = x$

24) $\frac{100}{20} + 5 \times 10 = x$

25) $\frac{15 \times 7}{105} = P$

26) $\frac{15 + 5}{2 + 8} = x$

27) $\dfrac{15 \times 2 \div 3}{5 + 6} = x$

28) $\dfrac{\frac{10}{2} + 5 - 1 \times 6}{29}$

29) $\dfrac{60 - 10}{30 + 20}$

30) $3 + (5 \div 2)$

SECTION V

UNIT I

BASIC ALGEBRA

Problem Solving

Many times the operator is confronted with a problem or situation that involves straight forward computation, and which the operator readily understands. For instance, if your plant has 100 feet of weir and then obtained another 100 feet of weir, and you were told to figure out the total amount of weir that your plant now has, you add the two sets of weir together 100 feet of weir + 100 feet of weir to obtain the new total amount of 200 feet of weir. You could easily figure this out because it involves simple addition. You could recognize the problem as addition, because you work with this type of a problem often and are probably experienced at working it.

You are also used to the words stated in the problem and what they mean. However, these conditions are not always true.

That is what if you could not understand what the problem asked. How would you recognize the problem if you had no experience working it? If you couldn't recognize the problem and know what it asked, you probably couldn't solve it. However, let us now look at a sure way to approach and solve the mathematic story problems you will deal with as an operator.

Six Rules for Problem Solving

1. Write down the problem whether its a formula, equation or word description.
2. List all the information given in the problem.
3. Set up the equation. Draw a mathematical relationship between the desired unknown and the information given, placing the desired unknown, variable or letter on one side of the equation.
4. Plug in the values given for all the unknown variables.

5. Perform all needed operations to solve the problem.
6. Check your answer.

By following these six rules you will know both how to express an algebra problem mathematically and how to solve it. The problems we will work with are algebraic in nature; that is we will solve for an unknown quantity.

Example 8 (using percentage)

800 lbs. of total solids enter a digester daily. 60% of these 800 lbs. are volatile solids while 40% are fixed solids. What is the weight of the volatile solids and fixed solids entering the digester per day?

Rule 1 State the Problem

800 lbs. of total solids enter a digester daily. 60% of these 800 lbs. are volatile solids while 40% are fixed solids. What is the weight of the volatile solids and fixed solids entering the digester per day?

Rule 2 List all the information given in the problem

800 lbs. of total solids enter a digester daily. 60% of the 800 lbs. are volatile solids while 40% of the 800 lbs. are fixed solids.

Rule 3 Set up the equation. Draw a mathematical relationship between the desired unknown and the information given, placing the desired unknown on one side of the equation.

Here we ask the question, "What do we want to know?" We wish to know the weight of the volatile solids and fixed solids. From the information given (Rule 2) we know that,

1. The weight of the volatile solids is equal to a percentage of the total solids.
2. The weight of the fixed solids is equal to a percentage of the total solids.

Now we put these sentences in mathematical terms.

1. Wt. of volatile solids = x% x total solids.
2. Wt. of fixed solids = y% x total solids.

Rule 4 Plug in the values given for all the unknown variables into the mathematical sentences above.

We then look at the information in Rule 2 and that tells us that,

1. Wt. of volatile solids = 60% x 800 lbs.
2. Wt. of fixed solids = 40% x 800 lbs.

Rule 5 Perform all needed operations to solve the problem.

In this problem we need only multiply.

1. 60% x 800 lbs. = 480 lbs. = wt. of volatile solids.
2. 40% x 800 lbs. = 320 lbs. = wt. of fixed solids.

Rule 6 Check your answer.

Since we multiplied to obtain this answer we use division, its opposite operation to check it.

1. $\frac{480 \text{ lbs.}}{.60} = 800$ lbs.   2. $\frac{320 \text{ lbs.}}{.40} = 800$ lbs.

Example 9 (using addition and multiplication)

Three drying beds at a plant are used for sludge drying. Bed 1 can hold 1,000 lbs. of sludge; bed 2 can hold twice as much sludge as bed 1; and bed 3 can hold three times as much sludge as bed 1. What is the total amount of sludge that can be dried on all three drying beds?

Rule 1 State the problem

Three drying beds at a plant are used for sludge drying. Bed 1 can hold 1,000 lbs. of sludge; bed 2 can hold twice as much sludge as bed 1, and bed 3 can hold three times as much sludge as bed 1. What is the total amount of sludge that can be dried on all three drying beds?

Rule 2 List all the information given in the problem

Three drying beds are used for sludge drying. Bed 1 holds 1,000 lbs. of sludge; bed 2 holds twice that of bed 1; bed 3 holds three times that of bed 1.

Rule 3 Set up the equation. Draw a mathematical relationship between the desired unknown and the information given, placing the desired unknown on one side of the equation.

Here we ask the question, "What do we want to know?" In this problem we wish to know the total amount of sludge that can be dried on all three drying beds. From the information given (in Rule 2) we know that,

1. Bed 1 holds x lbs. of sludge.
2. Bed 2 holds y times that of bed 1.
3. Bed 3 holds z times that of bed 1.

Now we put these sentences in mathematical terms.

1. Bed 1 = x lbs. of sludge.
2. Bed 2 = y x X lbs. of sludge (in bed 1)
3. Bed 3 = z x X lbs. of sludge (in bed 1)

The total wt. = Bed 1 + Bed 2 + Bed 3 = x lbs. + y x X lbs. + Z x X lbs.

Rule 4 Plug in the values given for all the unknown variables into the mathematical sentences above.

We look at the information in Rule 2 and that tells us that,

1. Bed 1 = 1,000 lbs. of sludge.
2. Bed 2 = 2 x 1,000 lbs. of sludge (in bed 1).
3. Bed 3 = 3 x 1,000 lbs. of sludge (in bed 1)

The total wt. = bed 1 + bed 2 + bed 3 = 1,000 lbs. of sludge + 2 x 1,000 lbs. of sludge + 3 times 1,000 lbs. of sludge.

Rule 5 Perform all needed operations

  1,000 lbs. of sludge
+ 2,000 lbs. of sludge
+ 3,000 lbs. of sludge
= 6,000 lbs. of sludge (total)

Rule 6 Check your answer.

This involves simple subtraction.

6,000 lbs. of sludge - (2,000 + 3,000) lbs. of sludge

6,000 lbs. of sludge - 5,000 lbs. of sludge = 1,000 lbs. of sludge

Problem 31

A rapid sand filter at a water treatment plant receives an average daily flow of 7 MGD. During a backup period this filter becomes clogged up, causing daily flow to become ten times less than normal. What is the flow during this backup period?

Problem 32

Organic loading may be expressed mathematically as equal to

$$\frac{\text{lbs. BOD/day}}{\text{cu. ft. volume}}$$

$\frac{\text{15 lbs. BOD/day}}{\text{1,000 cu. ft. volume}}$ is considered normal organic loading for standard rate filters.

While $\frac{\text{200 lbs. BOD/day}}{\text{1000 cu. ft. volume}}$ is considered normal for high rate filters.

How much greater is the organic loading in a high rate filter than in a standard rate filter?

Problem 33

The average daily flow to a water treatment plant is 4 MGD. If the population of the county it serves is 50,000 what is the GPD flow per capita?

Problem 34

The number of tons of solid waste generated from different sources in 1972 in the United States are listed below. How many total tons were generated that year?

Municipal	297,080,000
Manufacturing	127,320,000
Mineral	1,315,450,000
Agricultural	2,503,960,000

Problem 35

A water treatment plant has an average inflow rate of 10 gallons per second. How many GPD is this? How many gallons per year is this?

This problem involves multiplying units and cancellation.

Hint: Remember that multiplying a number by one does not change its value.

$$\frac{60 \text{ seconds}}{\text{minute}} = 1$$

$$\frac{60 \text{ minutes}}{\text{hour}} = 1$$

$$\frac{24 \text{ hours}}{\text{day}} = 1$$

$$\frac{365 \text{ days}}{\text{year}} = 1$$

Play around with these units and cancel where necessary to get the appropriate answer.

SECTION V

UNIT II

FORMULAS

A formula is an equation expressing a mathematical rule. The purpose of a formula is to help you the operator solve the unknown value of a particular quantity you wish to know. This quantity that you wish to solve can be in terms of weight, flow rate, distance, area, volume, or many other dimensions. A formula has at least one letter or unknown variable that is to be solved. Sound familiar? A formula is an equation. Usually the variable or letter you wish to solve is put on one side of the equation with the other numbers and a known variable on the other side. To find the value of an unknown variable substitute the values of the other letters or unknown variables in the formula and carry out the necessary operations.

Example 10

In the formula $A = \Pi \times D$ find the value of A when $\Pi = 3.14$ and $D = 10$.

First write the formula

$A = \Pi \times D$

Then substitute numbers for the unknown variables or letters

$A = 3.14 \times D$	$(\Pi = 3.14)$ 3.14 replaces Π
$A = 3.14 \times 10$	$(D = 10)$ 10 replaces D
$A = 31.4$	

Sometimes the unknown you wish to solve is not alone on one side of the equation.

Example 11

In the formula $A = LW$ find the value of W when $A = 10$ and $L = 20$.

First write the formula

$A = LW$

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Then substitute numbers for the unknown variables or letters.

$10 = LW \quad (A = 10)$ — 10 replaces A

$10 = 20W \quad (L = 20)$ — 20 replaces L

Now what? What's wrong? W is not alone on one side of the equation by itself. This can sometimes be a problem. With some insight you could divide both sides of the equation by 20 to get W alone by itself.

$$10 = 20W$$

$$\frac{10}{20} = \frac{20W}{20}$$

$$\frac{10}{20} = 1W$$

$$\frac{10}{20} = W$$

$$\frac{1}{2} = W$$

An easier way, however, would have been to put the variable you wish to have solved on one side of the equation.

$A = LW$ — Dividing both sides by L we get

$$\frac{A}{L} = \frac{LW}{L}$$

$$\frac{A}{L} = 1W \qquad \frac{L}{L} = 1$$

$\frac{A}{L} = W$ — This would have made the problem easier.

Problems 36 - 40

36) In the formula $A = \frac{1}{2}bh$ find the value of A when $b = 20$ and $h = 40$

37) In the formula $V = 1/3\,\pi R^2h$ find h when $V = 1000$, $\pi = 3.14$, and $R = 10$

38) In the formula $A = 2\pi Rh$, find A when $\pi = 3.14$, $R = 6$, and $h = 10$.

39) In the formula $M = \frac{a}{b}$, find a when $M = 25$ and $b = 5$.

40) In the formula $A = L^2$, find A when $L = 25$.

4 Rules of solving an unknown in a formula:

1) Make sure the unknown you are solving for is alone on one side of the equation. If not perform the necessary operation to put it there.
2) Write down the formula.
3) Substitute the other letters or unknown variables with the known values.
4) Carry out the necessary operations to solve the desired unknown.

IMPORTANT FORMULAS FOR WATER AND WASTEWATER OPERATORS

1. Pounds of chemical required per day = $\frac{8.34 \times \text{flow in MGD} \times \text{dosage in PPM}}{\%\ \text{availability}}$

2. Pounds of chemical required per day = $\frac{143 \times \text{flow in MGD} \times \text{dosage in GPG}}{\%\ \text{availability}}$

3. Dosage in parts per million (PPM) = $\frac{\text{lbs. of chemical used}}{\text{flow in MGD} \times 8.34}$

4. Dosage in grains per gallon (GPG) = $\frac{\text{lbs. of chemical used}}{\text{flow in MGD} \times 143}$

5. Pump efficiency = $\frac{\text{water horsepower}}{\text{brake horsepower}}$

6. Water horsepower = $\frac{\text{GPM} \times \text{head in feet}}{3960}$

7. Brake horsepower = $\frac{\text{GPM} \times \text{head in feet}}{3960 \times \text{efficiency of pump}}$

8. Motor power input = $\frac{\text{brake horsepower}}{\text{motor efficiency}}$ MPI x .746 = the units in kilowatts

9. Motor efficiency = $\frac{\text{GPM} \times \text{TDH} \times .746}{3960 \times \text{Kw}}$

10. Gallons per minute = MGD x 700

11. MGD = $\frac{\text{GPM}}{700}$

12. Percent BOD removal = $\frac{\text{PPM BOD influent} - \text{PPM BOD effluent}}{\text{PPM BOD influent}}$

13. Population equivalent BOD = $\frac{8.34 \times \text{PPM} \times \text{MGD}}{.17}$

14. Population equivalent suspended solids = $\frac{8.34 \times \text{PPM} \times \text{MGD}}{.20}$

15. Rate of filtration = $\frac{\text{MGD}}{\text{acres}}$ This answer will show MGD per acre.

16. Average rapid sand filter rate = 2 gallons per minute per square foot.

17. Rapid sand filter wash rate: .625 GPM per square foot of area for 1 inch rise and 15 GPM per square foot of area for a 24 inch rise.

18. Capacity required in gallons = $\frac{\text{Daily flow in gallons x detention period in hours}}{24}$

19. Detention time in hours = $\frac{\text{capacity of tank in gallons x 24}}{\text{daily flow in gallons}}$

20. Detention time in hours = $\frac{\text{capacity of tank in cubic feet}}{\text{5570 x sewage flow in MGD}}$

21. Detention time in minutes = $\frac{\text{capacity of tank in gallons}}{\text{flow in GPM}}$

22. Tank volume in cubic feet = 5570 x flow in MGD x detention period in hours.

23. The national Board of Fire Underwriters has adopted the following formula for determining the total quantity of water required for fire service. In the formula, "G" is the required amount of water in gallons per minute and "P" is the population expressed in thousands.

$$G = 1020 \times VP\,(1.0 - 0.01 \times VP)$$

24. Velocity = $\frac{\text{distance}}{\text{time}}$ $\frac{\text{inches, feet or miles}}{\text{hours, minutes or seconds}}$

25. Fluid discharge = $\frac{\text{volume of flow}}{\text{time}}$ $\frac{\text{gallons or cubic feet}}{\text{any unit of time}}$

26. Displacement velocity = $\frac{\text{93 x sewage flow in MGD}}{\text{cross-section in square feet}}$

27. Weir overflow rate = $\frac{\text{gallons per day}}{\text{length of weir in feet}}$

28. Surface loading in gallons per day per square foot = $\frac{\text{180 x tank depth in feet}}{\text{detention time in hours}}$

29. psi = depth of water in feet x .434

30. Depth of water = psi x 2.31

SECTION V

POST TEST

Problems 1 - 6 Solve for the unknown.

1) $\frac{x}{8} = 10$

2) $P + 40 = 62$

3) $15.5 - 7 = x$

4) $\frac{(20 \times 4)}{(40 \div 2)} = x$

5) $(40 \div 2) + 7 - 5 = P$

6) $\frac{(10 \times 4)}{2} = Y$

7) Two drying beds at a wastewater plant are used for sludge drying. Bed 1 can hold 2,000 lbs. of sludge; bed 2 can hold three times as much sludge as bed 1. What is the total amount of sludge that can be dried on both beds?

8) The average daily flow to a water treatment plant is 1 MGD. If the population of the county it serves is 20,000, what is the GPD flow per capita?

9) In the formula $A = \pi r^2$, find the value of A when $r = 10$, assuming $\pi = 3.14$.

10) In the formula $P = W + W + W + W$, find the value of P when $W = 25$.

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SECTION V

POST TEST KEY

1. 80
2. 22
3. 8.5
4. 4
5. 22
6. 20
7. 8,000 lbs. of sludge
8. 50 GPD per capita
9. 314
10. 100

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SECTION VI

UNIT I ROMAN NUMERALS

UNIT II METRIC SYSTEM

UNIT III CALCULATOR USE

SECTION INSTRUCTIONAL PACKAGE GUIDELINE

SUBJECT MATTER: Basic Mathematics for Water and Wastewater Operators

UNIT OF INSTRUCTION: Roman numerals; metric system; calculator use

LESSON NUMBER: Section 6

ESTIMATED TIME: 3½ hours

JUSTIFICATION FOR THIS INSTRUCTIONAL OBJECTIVE: A knowledge of roman numerals, the metric system, and calculator use is important for water and wastewater plant operation

PREREQUISITES: The learner shall have successfully completed sections 1 - 5.

INSTRUCTIONAL OBJECTIVES:

Terminal Performance Behavior - The learner shall successfully complete this mathematics section. Successful completion of this section shall be demonstrated when the learner through pre-test or post-test written examination has met the specified criterion level based on the behavioral objectives of this section.

Behavioral Objectives - At the completion of this section the learner will be able to:

Express in writing the Roman notational value of various whole numbers and vice versa.

Express in writing the metric equivalents of similar English values in various dimensions; length, volume, weight, area, and rates of flow.

Perform the following mathematical operations on the following types of numbers on a calculator:

Addition with respect to whole numbers and decimals

Subtraction with respect to whole numbers and decimals

Multiplication with respect to whole numbers and decimals

Division with respect to whole numbers and decimals

Percentage computations with respect to whole numbers and decimals

Squaring with respect to whole numbers

Obtaining square roots with respect to whole numbers

Conditions - The learner will accomplish these objectives with the aid of a calculator and a metric ruler.

Criterion - Level of Acceptable Performance - Minimum passing score is 90% on either the pre-test or post test.

INSTRUCTIONAL APPROACH: Individual lesson utilizing self-paced study of written material.

INSTRUCTIONAL RESOURCES

Available Supplemental Material

Dressler, Isadore. Preliminary Mathematics. New York: Amsco School Publications, Inc., 1965. PP. 123-126.
Gives a brief explanation of the metric system.

Kibbe, Richard, R., Meyer, Roland, O. Neely, John, E., and White, Warren, T. Machine Tools and Machining Practices. New York: John Wiley and Sons, 1977. PP. 16-19.
Explains how to select and use an electronic calculator. Note: (Calculator direction manuals for the model being used will offer the most complete operating directions for your particular model and are most useful).

Mira, Julio, A. Arithmetic Clear and Simple. New York: Barnes and Nobles, Inc., 1965. P 7.
Briefly explains Roman numerals.

Suggested Supplemental Material

Kirkpatrick, Joane. Mathematics for Water and Wastewater Treatment Plant Operators. Ann Arbor: Ann Arbor Science Publishers Inc., 1973. PP 56-72.
A good source for learning the metric system. Emphasizes principles and methods of conversion. Has many illustrated examples.

SECTION VI

PRE-TEST

Problems 1 - 3

Find the Roman notational values of

1) 15

2) 21

3) 30

4) What is the numerical value of XV?

5) Convert 10 kilograms into pounds.

6) Your water treatment plant receives a shipment of 100 liters of chlorine. How many gallons is this?

Problems 7 - 10

Using a calculator, perform the following operations on the following problems.

7) 10 + 26 + 37

8) 624 ÷ 8

9) 500 - 220 - 47

10) 50 x 4 ÷ 10 + 25 - 17

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SECTION VI

PRE-TEST KEY

1. XV
2. XXI
3. XXX
4. 15
5. 22 pounds
6. 26.42 gallons
7. 73
8. 78
9. 233
10. 28

SECTION VI

UNIT I

ROMAN NUMERALS

There are two common methods for notations and writing numbers. One of these is the Arabic notation, which we have already learned. This system uses the basic numbers zero through nine. The other method for writing numbers is the Roman notation. Unlike the Arabic notation which is used primarily for calculating, the Roman notation is used for numbering chapters in books, or time pieces etc.

The operator may encounter Roman numerals when reading reports using Roman numerals as subheadings for sections. Roman numerals are thus worthwhile to know and can come in handy on several occasions.

In Roman notation, letters are used as symbols instead of numbers. The following is a list of the Roman notation letters and their values.

Numbers 1 - 9

I	II	III	IV	V	VI	VII	VIII	IX
1	2	3	4	5	6	7	8	9

Groups of Ten

X	XX	XXX	XL	L	LX	LXX	LXXX	XC
10	20	30	40	50	60	70	80	90

Group of 100

C
100

The basic Roman notations are:

I	V	X	L	C
1	5	10	50	100

For the operator's purposes, learning Roman notation up to 30 should be more than sufficient.

The following general rule should be remembered about Roman numerals. Smaller numbers placed in front of larger numbers are subtracted from the larger. Smaller numbers placed behind larger numbers are added to the larger.

Example 1

Write the Roman notation for 11.

XI

X is ten

I is 1

XI is 11 because I is placed behind the X, the I is added to the X

Ten + one = eleven

Example 2

Write the Roman notation for 9.

X is 10

I is 1

IX is 9 because I is placed before the X, the I is subtracted from the X

Ten - one = nine

Example 3

Write the Roman notation for 15.

X is ten

V is five

XV is 15 because V is placed after X, the V is added to the X.

Ten + five = fifteen

TABLE 6

The following are a list of common Roman notational values.

I = 1	XI = 11	XXI = 21
II = 2	XII = 12	XXII = 22
III = 3	XIII = 13	XXIII = 23
IV = 4	XIV = 14	XXIV = 24
V = 5	XV = 15	XXV = 25
VI = 6	XVI = 16	XXVI = 26
VII = 7	XVII = 17	XXVII = 27
VIII = 8	XVIII = 18	XXVIII = 28
IX = 9	XIX = 19	XXIX = 29
X = 10	XX = 20	XXX = 30

XL = 40	L = 50	LX = 60
LXX = 70	LXXX = 80	XC = 90
C = 100		

Without looking at the previous list of Roman notational values complete the following problems.

Problems 1 - 4

Find the Roman notational values of:

1) 6
2) 12
3) 25
4) 19

Problems 5 - 8

Express in Roman notational values

5) 30

6) 17

7) 50

8) 8

Problems 9 - 10

9) This section is Section IV. What is the numerical value?

10) While filling out a State report concerning water treatment you came across Section XIII. What is the numerical value?

SECTION VI

UNIT II

METRIC SYSTEM

The metric system is a system of measurement based on the meter. Most of us at one time or another have heard of the word meter, but not all of us know what it means. A meter is a unit of length, slightly greater than a yard. Its major importance is that it is the foundation of a system of measurement using different units then our own English system. The metric system is important because it is the standard system of measurement for the world. It is very possible that in our lifetimes all numbers and measurements will be gaged in metric terms. Thus the need for the operator to be at least aware of this system, even if not presently using it, is apparent.

The metric system is not difficult to learn. In general each unit size is 10 times larger or smaller than the unit size before or after it.

PREFIXES USED IN THE METRIC SYSTEM

Prefixes	Meaning
Milli	1/1000 or 0.001
Centi	1/100 or 0.01
Deci	1/10 or 0.1
Unit	1
Deka	10
Hecto	100
Kilo	1000

Measures of Length

The basic measure of length is the meter.

1 kilometer (km) = 1000 meters (m)

1 meter (m) = 100 centimeters (cm)

1 centimeter (cm) = 10 millimeters (mm)

Kilometers are usually used in place of miles, meters are used in place of feet and yards, centimeters are used in place of inches, and millimeters are used for fractions of an inch.

To get a feel for metric units take a ruler and measure the lengths of various items, your finger, a dime, a pipe, using the centimeter scale. This, with enough practice, will help give you experience and skill in dealing with the metric system. Measure the length of this line.

Length Equivalents

1 kilometer = 0.621 mile	1 mile = 1.64 kilometers
1 meter = 3.28 feet	1 foot = 0.305 meter
1 meter = 39.37 inches	1 inch = 0.0254 meter
1 centimeter = 0.3937 inch	1 inch = 2.54 centimeters
1 millimeter = 0.0394 inch	1 inch = 25.4 millimeters

NOTE: The above equivalents are reciprocals. If one equivalent is given, the reverse can be obtained by division. For instance, if one meter equals 3.28 feet, one foot equals 1/3.28 meter, or 0.305 meter.

Measures of Capacity or Volume

The basic measure of capacity in the metric system is the liter. For measurement of large quantities the cubic meter is sometimes used.

1 kiloliter (kl) = 1000 liters (l) = 1 cu meter (m^3)

1 liter (l) = 1000 milliliters (ml)

Kiloliters, or cubic meters, are used to measure capacity of large storage tanks or reservoirs in place of cubic feet or gallons. Liters are used in place of gallons or quarts. Milliliters are used in place of quarts, pints, or ounces.

Capacity Equivalents

1 kiloliter = 264.2 gallons

1 liter = 1.057 quarts

1 liter = 0.2642 gallon

1 milliliter = 0.0338 ounce

1 gallon = 0.003785 kiloliter

1 quart = 0.946 liter

1 gallon = 3.785 liters

1 ounce = 29.57 milliliters

Measures of Weight

The basic unit of weight in the metric system is the gram. One cubic centimeter of water at maximum density weighs one gram, and thus there is a direct, simple relation between volume of water and weight in the metric system.

1 kilogram (kg) = 1000 grams (gm)

1 gram (gm) = 1000 milligrams (mg)

1 milligram (mg) = 1000 micrograms (mg)

Grams are usually used in place of ounces, and kilograms are used in place of pounds.

Weight Equivalents

1 kilogram = 2.205 pounds

1 gram = 0.0022 pound

1 gram = 0.0353 ounce

1 gram = 15.43 grains

1 pound = 0.4536 kilogram

1 pound = 453.6 grams

1 ounce = 28.35 grams

1 grain = 0.0648 gram

Area

Metric system—not generally used

1 square inch (sq. in.) (in^2) = 6.45 sq. cm.

1 square foot (sq. ft.) (ft^2) = 144 sq. in.

1 square yard (sq. yd.) (yd^2) = 9 sq. ft.

1 acre (ac) = 43,560 sq. ft.

1 square mile = 640 ac = 2.59 sq. kilometers

Rate of Flow

Metric system-not generally used

Gallons per minute (GPM)

Gallons per hour (GPH)

Gallons per day (GPD)

Million gallons per day (MGD)

Cubic feet per second (CFS)

The most common units are:

1 GPM = 60 GPH

1 GPM = 1440 GPD

1 MGD = 694 GPM = 41440 GPH

1 CFS = 60 CFM = 3600 CFH

= 1440 CFD

= 646,300 GPD = 0.6463 MGD

1 MGD = 1.547 CFS

Example 4

Convert 20 kilograms into pounds.

To solve this problem we set up a ratio and proportion. First we must use the unit of conversion for 1 kilogram; that is, what does 1 kilogram weigh using pounds? By referring to our chart we see that 1 kilogram = 2.2 pounds. Consequently,

1 kilogram is to 2.2 pounds as 20 kilograms is to an unknown amount of pounds.

This may be mathematically stated as

$$\frac{1 \text{ kilogram}}{2.2 \text{ pounds}} = \frac{20 \text{ kilograms}}{x \text{ pounds}}$$

We then can multiply

$1 \times X = 2.2 \times 20$

$1 \times X = 44$

Example 5

Convert 20 pounds into kilograms.

By referring to our chart we see that 1 pound = .45 kilograms.

Consequently,

1 pound is to .45 kilograms as 20 pounds is to an unknown amount of pounds.

$$\frac{1 \text{ pound}}{.45 \text{ kilograms}} = \frac{20 \text{ pounds}}{X \text{ kilograms}}$$ We then cross multiply

$1 \times X = .45 \times 20$

$1 \times X = 9$

$X = 9$ kilograms

Problems 11 - 12

11) Your water treatment plant receives a shipment of 1000 liters of chlorine. How many gallons is this?

12) A wastewater intake valve has a diameter of 10 centimeters according to your specifications manual. It must, however, be at least 3.9 inches wide for proper flow. Is the valve wide enough?

CONVERSION FACTORS

FOR OPERATORS

MULTIPLY	BY	TO OBTAIN
Acres	43,560	Square feet
Acre-feet	43,560	Cubic feet
Acre-feet	325,851	Gallons
Centimeters	0.3937	Inches
Cubic feet	1,728	Cubic inches
Cubic feet	7.48052	Gallons
Cubic feet	28.32	Liters
Cubic feet/second	448.831	Gallons/minute
Cubic feet/second	0.646317	Million gallons/day
Cubic yards	27	Cubic feet
Degrees (angle)	60	Minutes
Feet	30.48	Centimeters
Feet	12	Inches
Feet	0.3048	Meters
Feet	1/3	Yards
Feet of water	0.4335	Pounds/square inch
Gallons	0.1337	Cubic feet
Gallons	3.785	Liters
Gallons	8	Pints (liq.)
Gallons	4	Quarts (liq.)
Gallons, Imperial	1.20095	U. S. Gallons
Gallons, U. S.	0.83267	Imperial gallons
Gallons, Water	8.3453	Pounds of water
Gallons/Min.	2.228×10^{-3}	Cubic feet/sec.
Gallons/Min.	8.0208/area (sq. feet)	Overflow rate (ft./hr.)
Grains/U. S. Gal.	17.118	Parts/Million
Grains/U. S. Gal.	142.86	Lbs./Million Gal.
Grams	0.03527	Ounces
Grams	2.205×10^{-3}	Pounds
Grams/Liter	58.417	Grains/Gal.
Grams/Liter	1,000	Parts/Million
Horse-Power	33,000	Foot-Lbs./Min.
Horse-Power	0.7457	Kilowatts
Horse-Power	745.7	Watts
Inches	2.540	Centimeters

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ENGLISH SYSTEM

Unit	Abbreviation
Feet	ft. or '
Inches	in or "
Square feet	sq ft or ft^2
Acre	ac
Cubic feet	cu ft or ft^3
Gallons	gal
Cubic feet per second	cfs or cu ft/sec
Gallons per minute	gpm or gal/min
Gallons per hour	gph or gal/hr
Gallons per day	gpd or gal/day
Thousand gallons per day	tgd
Million gallons per day	mgd
Pounds	lb or #
Parts per million	ppm

METRIC SYSTEM

Unit	Abbreviation
Millimeter	mm
Centimeters	cm
Square centimeters	sq cm or cm^2
Cubic centimeters	cu cm or cm^3
Liters	l
Milliliters	ml
Grams	gm
Milligrams	mg
Milligrams per liter	mg/l
Kilograms	kg
Meter	m

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SECTION VI

UNIT III

CALCULATOR USE

The benefit of using the electronic calculator in the field of water and wastewater technology cannot be overestimated. With an electronic calculator mathematical operations are performed at a much higher rate of speed than the same operation done in ones head, with the result of saving time. Calculators are also highly precise instruments; with greater accuracy there is also less chance of errors in computation. Thus the main advantages of a calculator are speed and accuracy.

If you do not already have a calculator, you may wish to consider obtaining one. There are many different types of calculators available today, some with advanced scientific capabilities, and others with basic capabilities. The basic capabilities (necessary for solving water and wastewater math problems), include the performing of the four basic mathematical operations: Addition, subtraction, multiplication, and division. Some basic models also include square roots, percentage operations and even pi.

An appropriate calculator should also possess certain capabilities that will permit flexibility in the operator's work. The calculator should for instance be able to perform operations to the sixth decimal point. This sixth place is necessary in problems dealing with parts per million, such as in water treatment. The unit should also have a floating decimal point, where the decimal point is automatically placed in the answer when calculated.

Finally, consideration should be given to the power supply. Battery operated or rechargeable units are highly advantageous as they permit mobility. For some operators this is a necessity as their work takes them into the field, where electrical service is unavailable.

As previously mentioned all popular calculators perform the basic four mathematical operations: Addition, subtraction, multiplication, and division, while some others will also perform square root, percentage and pi operations. How the calculator performs these operations, however, may vary from calculator to calculator.

The following calculator practice procedures are based on a unisonic model 1011, a fairly popular brand and may or may not work with your calculator. The best advice that can be given for operating a calculator is to read the instructions supplied by the manufacturer, as the order of operations between calculators may vary.

Operating a calculator

Step 1 - turn power on.

Step 2 - performing operations

Example 6

Addition: 10.52 + 25 = 35.52

Enter	Depress	Display
	C	0
10.52	+	10.52
25	+	35.52 answer

Example 7

Subtraction: 15 - 7 = 8

Enter	Depress	Display
	C	0
15	-	15
7	- or =	8

Example 8

Multiplication: 12 x 5 = 60

Enter	Depress	Display
	C	0
12	x	12
5	x or =	60

Example 9

Division: 40 ÷ 10 = 4

Enter	Depress	Display
	C	0
40	÷	40
10	÷ or =	4

Example 10

Addition by constant 10 + 7 + 7 + 7

Enter	Depress	Display
	C	0
10	+	10
7	+ or =	17
	+ or =	24
	+ or =	31

Example 11

Subtraction by constant 100 - 10 - 10 - 10

Enter	Depress	Display
	C	0
100	-	100

Enter	Depress	Display
10	- or =	90
	- or =	80
	- or =	70

Example 12

Multiplication by constant 20 x 10 x 10 x 10

Enter	Depress	Display
	C	0
20	[illegible]	20
10	x or =	200
	x or =	2,000
	x or =	20,000

Example 13

Division by constant 1,000 ÷ 2 ÷ 2 ÷ 2

Enter	Depress	Display
	C	0
1,000	÷	1,000
2	÷ or =	500
	÷ or =	250
	÷ or =	125

Example 14

Chain operation 100 ÷ 5 x 3 - 10 + 20

Enter	Depress	Display
	C	0
100	÷	100
5	x	20

Enter	Depress	Display
3	-	60
10	+	50
20	=	70

Example 15

Percentage 50% x 75

Enter	Depress	Display
	C	0
75	x	75
50	%	37.5

Example 16

Squaring $7^2 = 49$

Enter	Depress	Display
	C	0
7	x	7
	x	49

Example 17

Square roots $\sqrt{64} = 8$

Enter	Depress	Display
	C	0
64	$\sqrt{\ }$	8

Problems 13 - 22 Perform the following operations on a calculator.

13) 16.75 + 10

14) 25 - 11

15) 100 x 5 + 4

16) 20 + 10 + 10 + 10

17) $\sqrt{196}$

18) 25^2

19) 70 x 5 x 5

20) 104 ÷ 2 ÷ 2

21) 20% x 250

22) 50% x 180

SECTION VI

POST-TEST

Problems 1 - 2

Find the Roman notational values of

1) 16

2) 20

Problems 3 - 4

Find the numerical values of

3) XX

4) V

5) Convert 100 kilograms into pounds.

6) A small wastewater pipe has a measurement of 2.54 centimeters according to your specifications manual. How many inches is this?

Problems 7 - 10

Using a calculator, perform the following operations on the following problems:

7) 25 - 13

8) 30 x 40

9) 25 + 17 - 10

10) 25 x 10 ÷ 2 + 5

SECTION VI

POST-TEST KEY

1. XVI
2. XX
3. 20
4. 5
5. 220 lbs.
6. 1 inch
7. 12
8. 1200
9. 32
10. 130

SECTION VII

UNIT I GEOMETRY

Perimeter

Area

Volume

182

SECTION INSTRUCTIONAL PACKAGE GUIDE

SUBJECT MATTER: Basic Mathematics for Water and Wastewater Operators

UNIT OF INSTRUCTION: Geometry

LESSON NUMBER: Section 7

ESTIMATED TIME: 7 hours

JUSTIFICATION FOR THIS INSTRUCTIONAL OBJECTIVE: A knowledge of geometry is important for water and wastewater plant operation.

PREREQUISITES: The learner shall have successfully completed sections 1 - 6.

INSTRUCTIONAL OBJECTIVES:

Terminal Performance Behavior - The learner shall successfully complete this mathematics section. Successful completion of this section shall be demonstrated when the learner through pre-test or post-test written examination has met the specified criterion level based on the behavioral objectives of this section.

Behavioral Objectives - At the completion of this section the learner will be able to:

Determine the perimeter of the following geometric figures: Triangle, quadrilateral (including squares and rectangles), pentagon, hexagon, septagon, octagon, nonagon, decagon, circle.

Determine the area or surface area of the following geometric figures: Triangle, quadrilateral (including squares and rectangles), circle, cylinder, cone, sphere, various combinations of the above.

Determine the volume of the following geometric figures: Cube, rectangular solid, triangular solid, trapezoidal solid, cylinder, cone, sphere, various combinations of the above figures.

Conditions - The learner will accomplish these objectives with the aid of the geometric formulas provided in this section.

Criterion - Level of Acceptable Performance - Minimum passing score is 90% on either the pre-test or post-test.

INSTRUCTIONAL APPROACH: Individual lesson utilizing self-pacing of written material.

INSTRUCTIONAL RESOURCES

Available Supplemental Material

Hill, T. H. Ward, Mathematics for the Layman. New York: Philosophical Library, 1958.
Discusses magnitude and linear measurements.

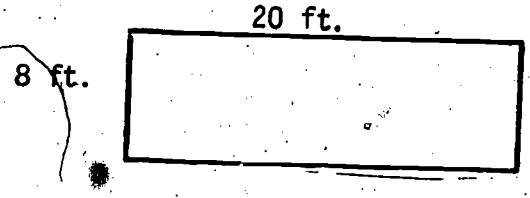
Kirkpatrick, Joanne. Mathematics for Water and Wastewater Treatment Plant Operators. Ann Arbor: Ann Arbor Science Publishers Inc., 1973. PP 81-106. Covers area and volume measurements for various geometric figures.

Suggested Supplemental Material

Bittinger, Marvin, L., and Keedy, Mervin, L. Arithmetic A Modern Approach. Reading, Massachusetts: Addison-Wesley Publishing Company, 1971. PP291-337. An excellent source for measurements in geometry as well as for additional practice problems.

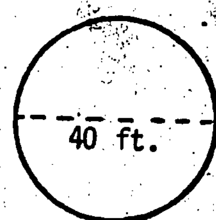
PRE-TEST

1. A rectangular clarifier has the following surface dimensions: A hand rail has to be installed around the perimeter of this tank. How much railing is required?

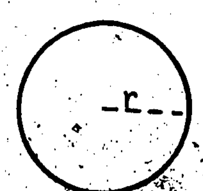


2. What is the surface area of the tank above?

3. What is the circumference of a circular trickling filter with a 40 foot diameter?

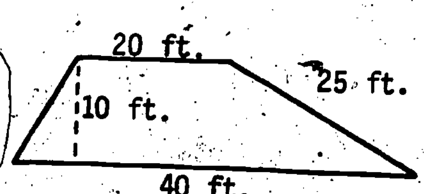


4. What is the area of a circular trickling filter with a 20 foot radius?

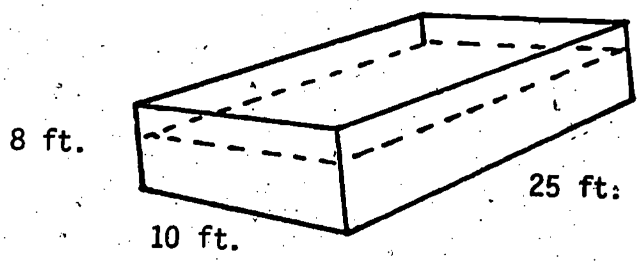


$A = \pi r^2$

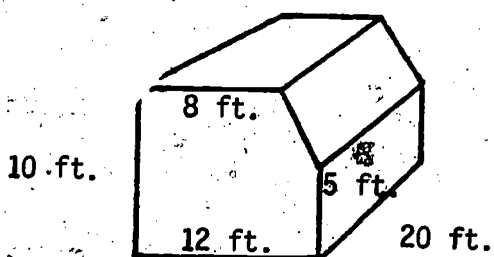
5. What is the area of this trapezoid?



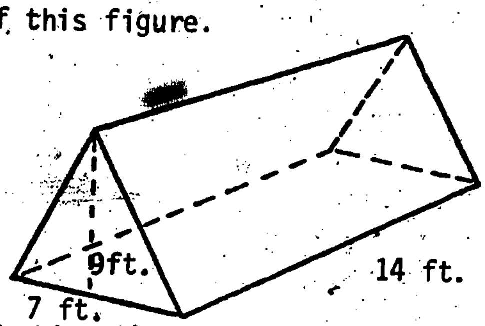
6. The primary clarifier is in the form of a rectangle. It is filled 3/4 full. What is the volume of liquid in the tank?



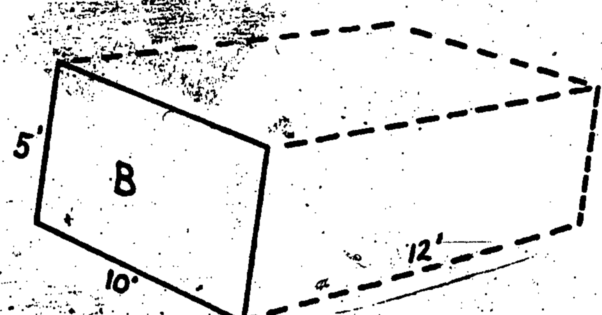
7. Your grit chamber has the following dimensions: When it is flowing full what is the volume of water in the tank?



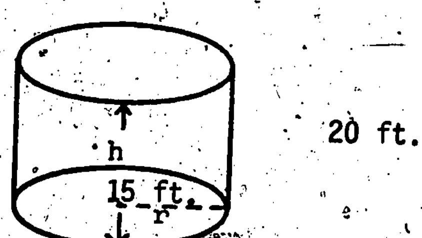
8. Find the volume of this figure.



9. Find the volume of this primary clarifier.



10. What is the volume of this cylindrically shaped water storage tank?



SECTION VII

PRE-TEST KEY

1. 56 ft.
2. 160 $ft.^2$
3. 125.6 ft.
4. 1256 $ft.^2$
5. 300 $ft.^2$
6. 1500 $ft.^3$
7. 2200 $ft.^3$
8. 441 $ft.^3$
9. 600 $ft.^3$
10. 14,130 $ft.^3$

GEOMETRY

Man has always been interested in questions such as "How Much?," "How Far?," "How Long?," and "How Big?". All of these questions are answered by measuring the distances involved.

We are going to measure the length of lines. These lines may be straight or curved. They may be up, down, across, over, under, through, or around. These lines are called by different names; some of which are length, width, depth, perimeter, radius, diameter, and circumference.

Sometimes it is not possible to determine a distance by direct measurement, such as across water, and the distance must be determined mathematically.

In order to be accurate in our measurement and, at the same time, be consistent, we must use a standard system.

There are two standard systems of measure used in waste water treatment. They are the English system and the Metric system. The English system is widely used and is the one with which we are all familiar. The metric system has always been used in the laboratory and is now being gradually adopted as the world standard of measure.

Tables of the English system, the Metric system, and the conversion factors to change from the English to the Metric system are included on Pages 167 - 172.

Two commonly used comparisons of measure are the inch in the English system and the centimeter in the Metric system. Comparing the two standards the inch is approximately 2 1/2 times as long as the centimeter.

Perimeters

Linear distances are important by themselves. We need to know many different distances. For example: How much sewer pipe do we need? How long

is the grit chamber? How much fencing do we need to surround and protect our plant? How much chain do we need to operate our scrapers?

We are not only interested in the length of a treatment unit but also in the distances around it. The distance around a triangular or rectangular unit is called the perimeter. The perimeter or distance around a circular unit has a special name and is called the circumference. We need to be able to measure or calculate the length of a straight, rectangular or circular weir.

Remember distances are measured in linear units i.e. inches (in.), feet (ft.), centimeters (cm.), meters (m.), etc.

We also must be concerned with the measurement of surface areas. For the purpose of processing we will limit this discussion to areas of flat surfaces. We need to know: How much surface do we have for settling? How much surface do we have on our trickling filter? Are our drying beds large enough?

Areas

Areas are measured in two dimensions or square units [illegible] square inches ($in.^2$), square centimeters cm^2), square feet $ft.^2$)*

*Incidentally this little superscript or exponent ^{2} means to multiply by itself.

Example 1

$ft.^2$ = ft. x ft. = feet x feet

r^2 = r x r = radius x radius

D^2 = D x D = Diameter x Diameter

The surface of the structures that we will study are in one plane or in other words have a flat surface.

The distance around this surface is called perimeter or, in the case of a circle, it is called the circumference.

The number of sides of a geometric figure may be used as a general means of classification.

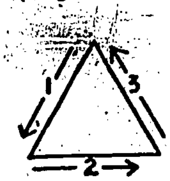
The following is a list of this general classification.

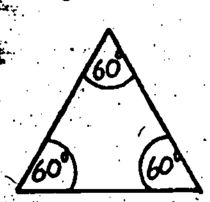
Name of Figure	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Septagon	7
Octagon	8
Nonagon	9
Decagon	10
Circle	No Sides

The triangle, quadralateral and circle are the three general classifications with which we are going to work.

Perimeter of Triangles

All triangles have three sides and three angles. An angle is the space formed between two lines that meet. The perimeter of a triangle is the distance around the figure or the sum of the lengths of the three sides.





Formula: Perimeter = Side 1 + Side 2 + Side 3

$$P = S_1 + S_2 + S_3$$

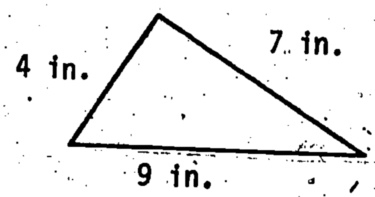
This formula applies to all triangles.

Example 2

$P = S_1 + S_2 + S_3$

$P = 4 \text{ in.} + 7 \text{ in.} + 9 \text{ in.}$

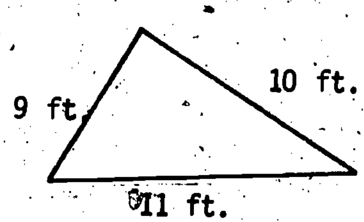
$P = 20 \text{ in.}$



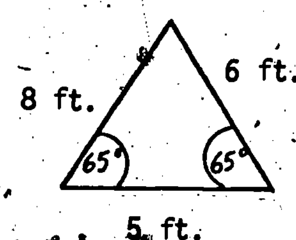
Problem 1 - 3

Find the perimeter of these triangles:

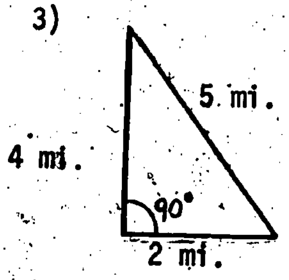
1)



2) How many feet of steel wire is needed to fasten a large triangularly shaped crate containing pipes?



3)



Triangle

The area of a triangle is equal to one half the base multiplied by the height. This is true for any triangle.

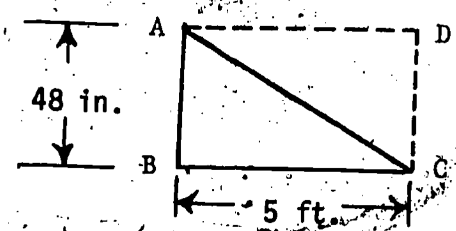
Area of a Triangle

We also need to know the area of a triangle. This area is the number of square units contained within the triangle.

We do not have a simple way to measure the area of a triangle directly. Using dimensions which we can measure and a formula (formulas are directions or instructions) we can calculate the area.

Example 3

Find the area of triangle ABC:



The first step in the solution is to make all the units the same. In this case, it is easier to change inches to feet.

$$48 \text{ in.} = 48 \cancel{\text{in.}} \times \frac{1 \text{ ft.}}{12 \cancel{\text{in.}}} = \frac{48}{12} \text{ ft.} = 4 \text{ ft.}$$

NOTE: All conversions should be calculated in the above manner. Since 1 ft./12 in. is equal to unity, or one, multiplying by this factor changes the form of the answer but not its value.

Area, sq. ft. = 1/2 (Base, ft.) (Height, ft.)

$$= 1/2 \times 5 \text{ ft.} \times 4 \text{ ft.}$$

$$= \frac{20}{2} \text{ ft.}^2$$

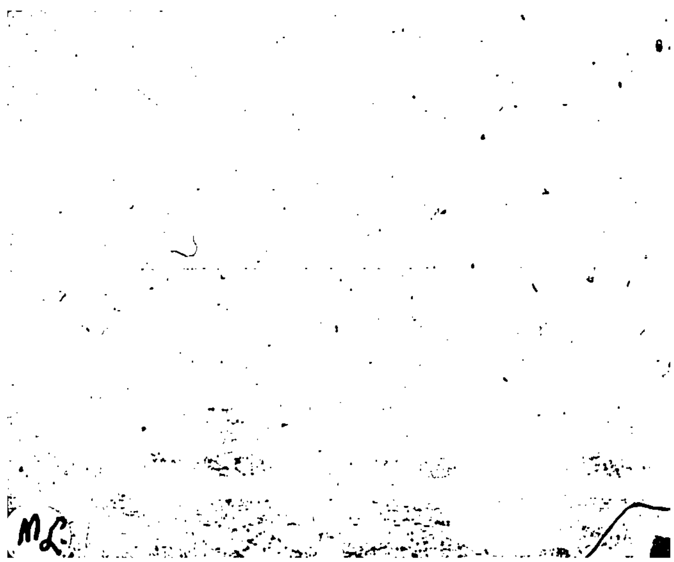
$$= 10 \text{ sq. ft.}$$

NOTE: Triangle ABC is one half the area of rectangle ABCD. The triangle is a special form called a <u>Right Triangle</u> since it contains a 90° angle at point B.

Like any plane figure, we should be able to find out two things about these figures. They are the perimeter or distance around, and the area which is the square units enclosed by the sides.

The Perimeter of a Quadrilateral

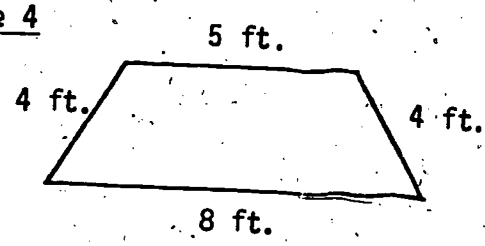
All quadrilaterals have four sides and four angles. The perimeter of a quadrilateral is the total length of all four sides.



The formula for this is $P = S_1 + S_2 + S_3 + S_4$

This formula works on all types of quadrilaterals.

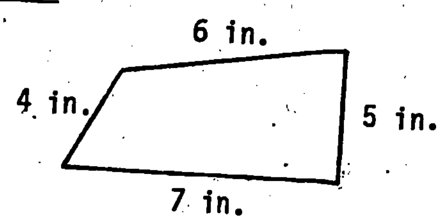
Example 4



P = 5 ft. + 8 ft. + 4 ft. + 4 ft.

P = 21 ft.

Example 5



P = 4 in. + 6 in. + 7 in. + 5 in.

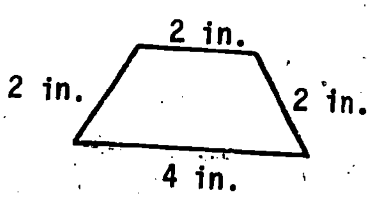
P = 22 in.

When all four sides of a quadrilateral (a four sided figure) are equal it is a square.

Problems 4 - 6

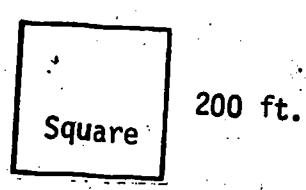
Find the perimeter of these quadrilaterals.

4)



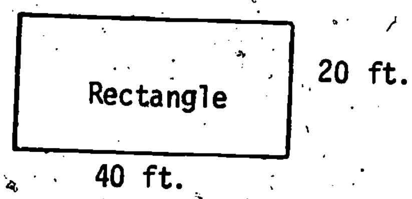
5) We need a new fence to protect our new water treatment plant. It is calculated that 200 ft. for one side of a square shaped fence pattern is ideal. How much total fence is needed to enclose the plant?

Hint: In a square, all sides are equally long.



6) A wastewater plant has one drying bed 20 feet wide and 40 feet long. What is the perimeter of this rectangular drying bed?

Hint: In ~~a rectangle~~, opposite sides are equally long.

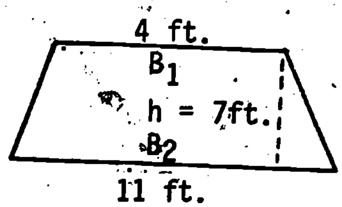


The Area of a Quadralateral

Area of a Trapezoid

Since the only constant in a trapezoid is that the two bases are always parallel (or even), we must have a more complicated formula to figure the area of a trapezoid.

Example 6



$$A = \frac{B_1 + B_2}{2} \times h$$

$$A = \frac{9 \text{ ft.} + 11 \text{ ft.}}{2} \times 7 \text{ ft.}$$

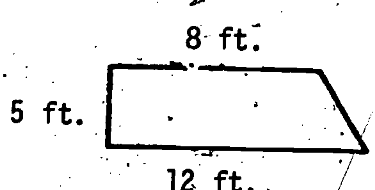
$$A = 10 \text{ ft.} \times 7 \text{ ft.}$$

$$A = 70 \text{ ft.}^2$$

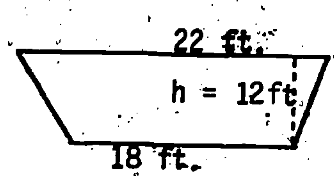
These 2 lines are parallel or even

Problems 7 - 8

7) Find the area of this small grit chamber



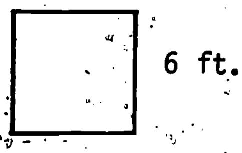
8) Find the area of this large grit chamber.



Area of a Square

Since we know a square has four equal sides, we simply square the length of the side and get the Area. (Area is always expressed square units)

Example 7



$A = S^2$

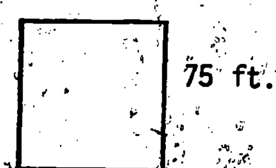
$A = 6 \text{ ft.} \times 6 \text{ ft.}$

$A = 36 \text{ ft.}^2$

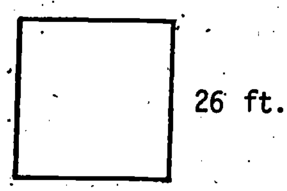
Problems 9 - 10

Find the area of these squares.

9)



10) Find the area of this lift station wet well.



We must also be able to find the areas of quadralaterals. The formulas change with the shape of the quadrilaterals.

Area of a Rectangle

The rectangle has two different lengths of sides. One side is called the length and the other is called the width. If we multiply these two we come up with square units of the figure.

Example 8

Find the area of a rectangle if the length is 5 feet and the width is 3.5 feet

Area, sq. ft. = Length, ft. x Width, ft.

$= 5 \text{ ft.} \times 3.5 \text{ ft.}$

$= 17.5 \text{ ft.}^2$

$= 17.5 \text{ sq. ft.}$

Example 9

The surface area of a settling basin is 330 square feet. One side measures 15 feet. How long is the other side?

$A = L \times W$

$330 \text{ sq. ft.} = L \text{ ft.} \times 15 \text{ ft.}$

$\frac{L \text{ ft.} \times 15 \text{ ft.}}{15 \text{ ft.}} = \frac{330 \text{ ft.}^2}{15 \text{ ft.}}$ Divide both sides of equation by 15 ft.

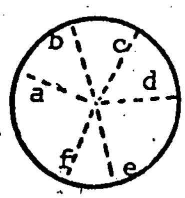
$L \text{ ft.} = \frac{330 \text{ ft.}^2}{15 \text{ ft.}}$

$= 22 \text{ ft.}$

Circles

A circle is a figure with all the points around the outside the same distant from a center point.

Example 10

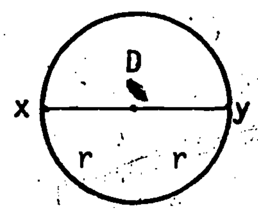


a, b, c, d, e & f are equal in length.

This distance from the center point to a point on the outside is called the radius.

The straight distance from one side of a circle to the other passing through the center is called the diameter. The diameter is twice the radius.

Example 11



$D = 2\ r$

The diameter of any circle is twice as long as its radius.

We have a mathematical constant (a never changing value) that we use with circles. This is called pi (a Greek letter) (the symbol is π). Pi never changes in value and is the circumference (distance around the outside or perimeter) of a circle divided by the diameter of the circle.

$$pi = \frac{circumference}{diameter}$$

This may be expressed as a fraction or as a decimal.

$$\pi = \frac{22}{7} \text{ or } 3.1416$$

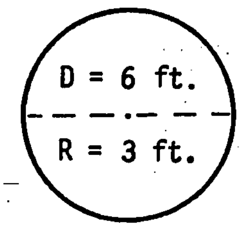
Circumference of a Circle

Formula: $C = 2\ \pi\ r$ or $\pi\ D$ Remember $D = 2\ r$ so $\pi\ D = \pi\ 2\ r$ or $2\ \pi\ r$

Circumference = $2\ \pi$ x radius or π x Diameter same

Example 12

Determine the circumference of a well with a radius of 3 ft., and diameter of 6 ft.



$C = 2\,\pi\,r$ or $C = \pi D$

$C = 2 \times 3.14 \times 3$ ft.

$C = 6.28 \times 3$ ft.

$C = 18.84$ ft.

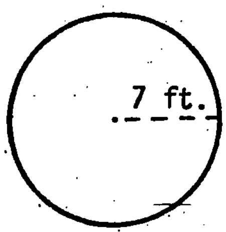
$C = \frac{22}{7} \times 6$ ft.

$C = 18.85$ ft.

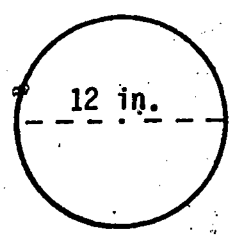
Problems 11 - 12

Find the circumference.

11) How many feet is it around a circular clarifier if you measure its radius to be 7 feet?



12)



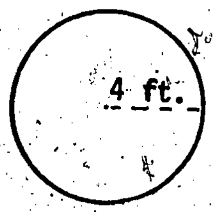
Area of a Circle

The area of a circle is pi times the radius squared, or pi times the diameter squared divided by 4.

Formula: $A = \pi r^2$

or $A = \pi \frac{D^2}{4} = \frac{3.14\,D^2}{4} = .785\ D^2$

Example 13



$A = \pi \frac{D^2}{4}$

$A = 3.14 \times \frac{8\text{ ft.} \times 8\text{ ft.}}{4}$

$A = 3.14 \times 16\text{ ft.}$

$A = 50.24\text{ ft.}^2$

Example 14

What is the area of a circle with a diameter of 20 centimeters? In this case, the formula using a radius is more convenient since it takes advantage of multiplying by 10.

$\text{Area, sq. cm.} = \pi\ (\text{R, cm})^2$

$= 3.14 \times 10\text{ cm.} \times 10\text{ cm.}$

$= 314\text{ sq. cm.}$

Example 15

What is the area of a trickling filter with a 50-foot radius? In this case, the formula using diameter is more convenient.

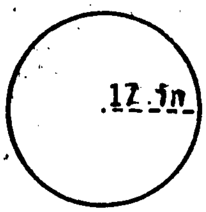
$\text{Area, sq. ft.} = 0.785\ (\text{Diameter, ft.})^2$

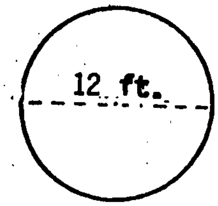
$= 0.785 \times 100\text{ ft.} \times 100\text{ ft.}$

$= 7850\text{ sq. ft.}$

Problems 13 - 14

Find the area

13) 

14) What is the area of the filter? 

Occasionally the operator may be confronted with a problem giving the area and requesting the radius or diameter. This presents the special problem of finding the square root of the number.

Example 16

The surface area of a circular clarifier is approximately 5000 square feet. What is the diameter?

$A = 0.785\ D^2$, or

Area, sq. ft. = 0.785 (Diameter, ft.)^2

5000 sq. ft. = 0.785 D^2 -- To solve, substitute given values in equation.

$\frac{0.785\ D^2}{0.785}$ $\frac{5000 \text{ sq. ft.}}{0.785}$ -- Divide both sides by 0.785 to find D^2.

$D^2 = \frac{5000 \text{ sq. ft.}}{0.785}$

= 6369 sq. ft.

Therefore:

D = square root of 6369 sq. ft. or

Diameter, ft. = $\sqrt{6369 \text{ sq. ft.}}$

As previously mentioned, it is sometimes easier to use a trial and error method of finding square roots. Since 80 x 80 = 6400, we know the answer is close to 80 feet.

Try 79 x 79 = 6241.

Try 79.5 x 79.5 = 6320.25.

Try 79.8 x 79.8 = 6368.04.

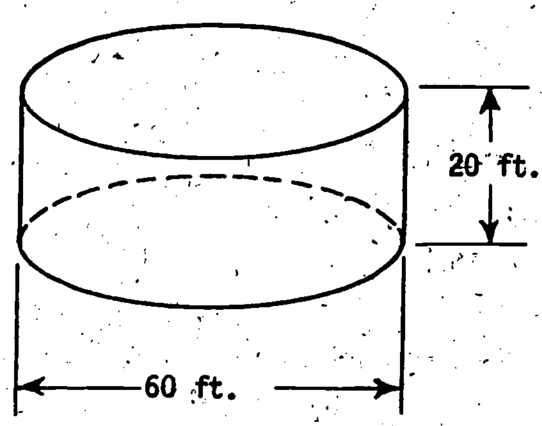
The diameter is 79.8 ft. or approximately 80 feet.

If it is too difficult to figure in your head, use a calculator or a table to compute your result.

Cylinder

With the formulas presented thus far, it would be a simple matter to find the number of square feet in a room that was to be painted. The length of each wall would be added together and then multiplied by the height of the wall. This would give the surface area of the walls (minus any area for doors and windows). The ceiling area would be found by multiplying length times width and the result added to the wall area gives the total area.

The surface area of a circular cylinder, however, has not been discussed. If we wanted to know how many square feet of surface area are in a tank with a diameter of 60 feet and a height of 20 feet, we could start with the top and bottom.



The area of the top and bottom ends are both $\Pi \times R^2$

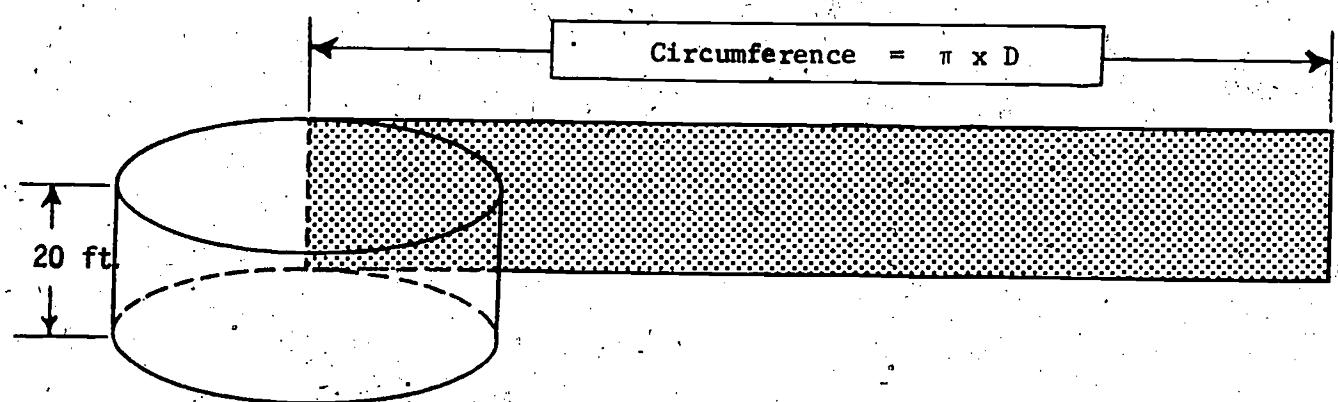
Area, sq. ft. = 2 ends (Π) (Radius, ft.)^2

= 2 x (Π) (30 ft.)^2

= 5652 sq. ft.

Area of top and bottom ends = 5652 sq. ft.

The surface area of the wall must now be calculated. If we made a vertical cut in the wall and unrolled it, the straightened wall would be the same length as the circumference of the floor and ceiling.



This length has been found to always be Π x D. In the case of the tank, the length of the wall would be:

Length, ft. = (Π) (Diameter, ft.)

= 3.14 x 60 ft.

= 188. 4 ft.

Area would be:

A, sq. ft. = Length, ft. x Height, ft.

= 188.4 ft. x 20 ft.

= 3768 sq. ft.

Length of wall = 3768 sq. ft.

Outside Surface Area to Paint, sq. ft. = Area of top and bottom, sq. ft. + Area of wall, sq. ft.

= 5652 sq. ft. + 3768 sq. ft.

= 9420 sq. ft.

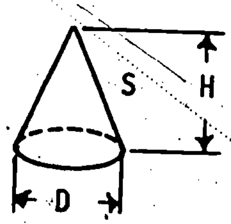
A container has inside and outside surfaces and you may need to paint both of them. In that case just double the area.

Total outside surface area to paint = 9420 sq. ft.

Inside and outside surface area = 2 x 9420

= 18,840 sq. ft.

Cone



The outside area of a cone is equal to 1/2 of the slant height (S) multiplied by the circumference of the base.

$$A, \text{outside} = 1/2\ S \times \pi \times D = \pi \times S \times R$$

In the case the slant height is not given, it may be calculated by:

$$S = \sqrt{R^2 + H^2}$$

Example 17

Find the entire outside area of a settling tank with a diameter of 30 feet and a height of 20 feet.

$$\text{Slant Height, in.} = \sqrt{(\text{Radius, ft.})^2 + (\text{Height, ft.})^2}$$

$$= \sqrt{(15\text{ ft.})^2 + (20\text{ ft.})^2}$$

$$= \sqrt{225\text{ ft.} + 400\text{ ft.}^2}$$

$$= \sqrt{625\text{ ft.}^2}$$

$$= 25\text{ ft.}$$

Area of Cone, sq. ft.

$$= \Pi\,(\text{Slant Height, ft.})\,(\text{Radius, ft.})$$

$$= 3.14 \times 25\text{ ft.} \times 15\text{ ft.}$$

$$= 1177.5\text{ sq. ft.}$$

Since the entire area was asked for, the area of the base must be added.

$$\text{Area, sq. in.} = 0.785\,(\text{Diameter, ft.})^2$$

$$= 0.785 \times 30\text{ ft.} \times 30\text{ ft.}$$

$$= 706.5\text{ sq. ft.}$$

$$= \text{Area of Cone, sq. ft.} + \text{Area of Bottom, sq. ft.}$$

Total Area, sq. in.

$$= 1177.5\text{ sq. ft.} + 706.5\text{ sq. ft.}$$

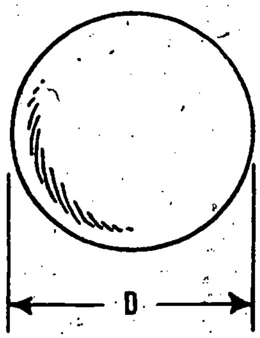
$$= 1884\text{ sq. ft.}$$

Problem 15

Find the outside area of a settling basin (do not include the base), that has a slant height of 10 ft. and a diameter of 15 ft.

2

Sphere



The surface area of a sphere or ball is equal to Pi multiplied by the diameter squared.

$A_S = \pi D^2$

If the radius is used, the formula becomes:

$A_S = \pi D^2 = \pi \times 2R \times 2R = 4\pi R^2$

Example 18

What is the surface area of a sphere shaped methane gas container 20 feet in diameter?

Area, sq. ft. = π (Diameter, ft.)^2

= 3.14 x 20 ft. x 20 ft.

= 1256 sq. ft.

Problem 16

What is the surface area of a sphere shaped chlorine container 10 ft. in diameter?

Volumes

We must know how much sewage each treatment unit will hold. This is called the volume. Volume is expressed in cubic units i.e. cubic inches (in.^{3}), cubic feet (ft.^{3}), cubic yards (yds.^{3}), and cubic meters (m.^{3}), gallons, etc.

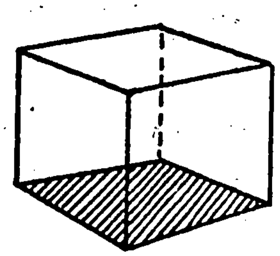
The volume of a solid figure is the number of cubic units of space the figure occupies or in our case the volume of liquid it will hold.

In this section we will concern ourselves with six of the most common solids.

1. Cube
2. Rectangular solid
3. Triangular solid
4. Trapizoidal solid
5. Cylinder
6. Cone
7. Sphere
8. Combination figures

The basic principle of finding the cubic units in a solid is:

Find the square units of area in the figure that carries the base of the solid figure and multiply that by the third dimension.



Cube

Shaded area is the base.

Since the Base is a square we find the area by multiplying length by width or 3 in. x 3 in. This results in 9 in.^{2}.

To find the volume we multiply the 9 in.^{2} by 3 in., the height, giving us, 27 in.^{3}.

This exponent ^{3} means multiplying units x units x units and is called cubing.

For example: $m^3 = m \times m \times m$ = meter x meter x meter

$in^3 = in \times in \times in$ = inch x inch x inch

$ft^3 = ft \times ft \times ft$ = feet x feet x feet

The simplest Formula for volume of a cube is: $V = S^3$ (or $V = S \times S \times S$) where S = the length of a side.

Example 19

The volume of a 4 inch cube is

$V = S \times S \times S$

V = 4 in. x 4 in. x 4 in.

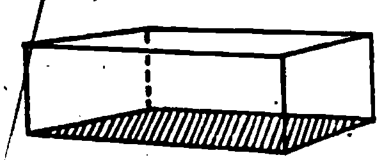
Problems 17 - 18

Find the volume of the following:

17) A wastewater storage room (10 ft. cube) =

18) Volume of a 2 yd. cube =

Rectangular Solid



Shaded area is the base

Volume of a rectangular solid is area of base times the height.

The area of the base is length times width which we multiply by the height to get the volume.

The formula then is $V = L \times W \times H$

From the figure above $V = 7 \text{ in.} \times 3 \text{ in.} \times 4 \text{ in.}$

$$V = 84 \text{ in.}^3$$

Problems 19 - 20

Find the volume of the following rectangular solids:

19) A rectangular tank has the following dimensions. What is its volume?

L = 5 yds.

W = 2 yds.

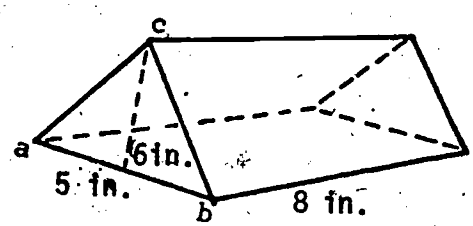
H = 3 yds.

20) L = 6 ft.

W = 4 ft.

H = 3 ft.

Triangular Solid



The base of this solid would be the triangle formed by points a, b, & c.

From a previous lesson we know the area of a triangle is ½ the base times the altitude. In the figure, 5 in. in the base and y in. is the altitude.

The area then is $\frac{5 \text{ in.}}{2} \times 6$ in.

Area is 15 in.^{2}

Formula: Volume = ½ base x height x length

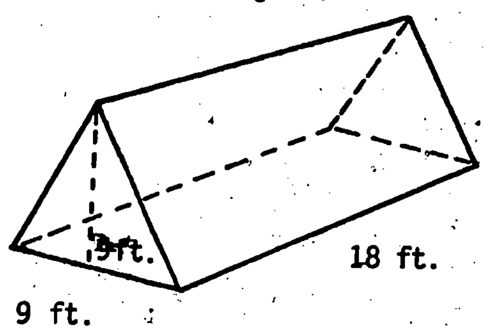
$V = \frac{5 \text{ in.}}{2} \times 6 \text{ in.} \times 8 \text{ in.}$

$V = 15 \text{ in.}^2 \times 8 \text{ in.}$

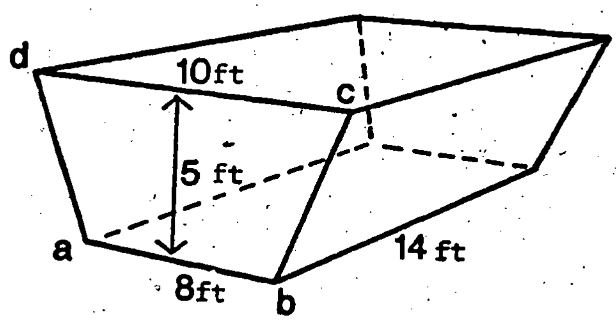
$V = 120 \text{ in.}^3$

Problem 21

Find the volume of this figure.



Trapezoidal Solid



The base of this solid would be a trapezoid formed by points a, b, c, & d. From a previous lesson we know the area of a trapezoid =

$\frac{\text{base}_1 + \text{base}_2}{2} \times \text{height}$

209

$A = \frac{10 \text{ ft.} + 8 \text{ ft.}}{2} \times 5 \text{ ft.}$

$A = 45 \text{ ft.}^2$

Formula for volume $= \frac{\text{base}_1 + \text{base}_2}{2} \times \text{height} \times \text{length}$

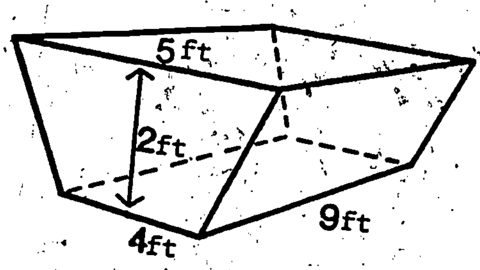
$V = \frac{10 \text{ ft.} + 8 \text{ ft.}}{2} \times 5 \text{ ft.} \times 14 \text{ ft.}$

$V = 9 \text{ ft.} \times 5 \text{ ft.} \times 14 \text{ ft.}$

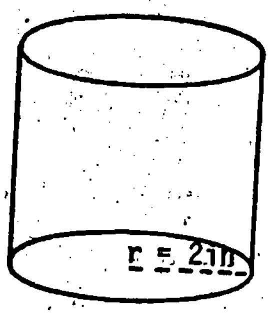
$V = 45 \text{ ft.}^2 \times 14 \text{ ft.}$

Problem 22

Find the volume of this small grit chamber.



Cylinder



h = 6 in.

The base of this solid would be a circle.

The r representing the radius and h representing the height.

We know that the formula for the area of a circle is πr^2

$A = \pi r^2$

$A = 3.14 \times (2 \text{ in.})^2$

$A = 12.56 \text{ in.}^2$

Formula for Volume of a cylinder $= \pi r^2 \times h$

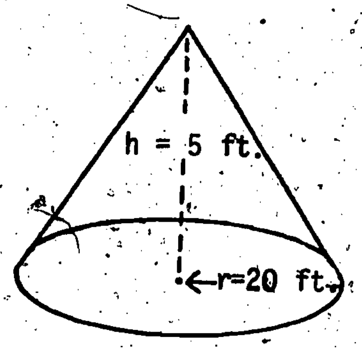
$V = 3.14 \times (2 \text{ in.})^2 \times 6 \text{ in.}$

$V = 75.36 \text{ in.}^3$

Problem 23

Find the volume of a cylindrically shaped wastewater storage tank when the measurements are:

$r = 7$ ft. $H = 20$ ft.



Cone

The base of this solid is a circle. The r represents the radius of the circle and the h the height of the cone.

We know that the formula for the area of a circle is Πr^2

$A = \Pi r^2$

$A = 3.14 \times (20 \text{ ft.})^2$

$A = 3.14 \times 400 \text{ ft.}^2$

$A = 1256 \text{ ft.}^2$

The formula for volume of a cone is $1/3 \; \Pi \; r^2h$

$V = 1/3 \; \Pi \; r^2h$

$V = 1/3 \times 3.14 \times (20 \text{ ft.})^2 \times 5 \text{ ft.}$

$V = 1/3 \times 1256 \text{ ft.}^2 \times 5 \text{ ft.}$

$V = 1/3 \times 6280 \text{ ft.}^3$

$V = 2093.3 \text{ ft.}^3$

Problem 24

Find the volume of a cone when the measurements are:

$r = 30$ ft. $h = 54$ ft.

Sphere

The volume of a sphere is equal to $\pi/6$ times the diameter cubed.

$$V = \frac{\pi}{6} \times D^3$$

Example 20

How much gas can be stored in a sphere with a diameter of 12 feet? (Assume atmospheric pressure.)

$$\text{Volume, cu. ft.} = \frac{\pi}{6} \times (\text{Diameter, ft.})^3$$

$$= \frac{\pi}{6} \times 12 \text{ ft.} \times 12 \text{ ft.} \times 12 \text{ ft.}$$

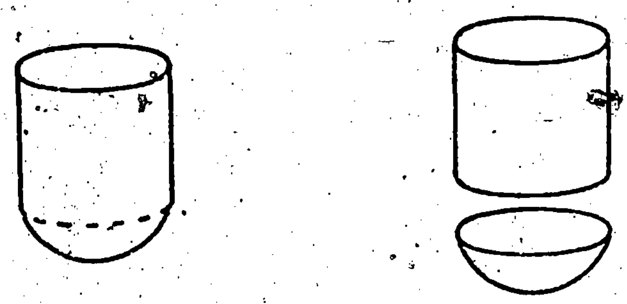
$$= 904.32 \text{ cubic feet}$$

Problem 25

How much chlorine gas can be stored in a sphere with a diameter of 10 feet?

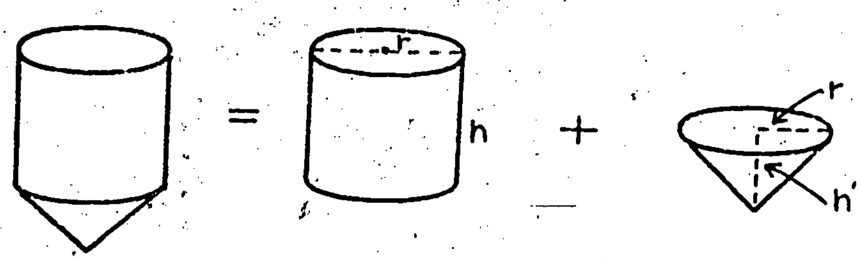
Combination Figures

There are some geometric figures that are formed by conbinations of other geometric figures. An example of this is an anaerobic digester filled with sludge and occupied by bacteria. This geometric figure looks like a cylinder with a round bottom but may be divided into a cylinder and dish as shown in the picture below. Thus the volume of this digester can be expressed as the sum of the volume of the cylinder plus the volume of the dish.



For all practical purposes, however, sufficient accuracy can be obtained by considering the dish as a cone rather than a part of a sphere.

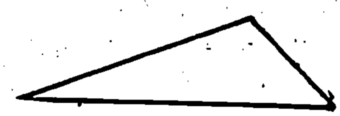
Therefore, for practical considerations the volume of the digester becomes the volume of the cylinder plus the volume of the cone.



Volume of digester = $\Pi r^2 h$ (vol. of cylinder) + $\frac{1}{3} \Pi r^2 h'$ (vol. of cone)

PERIMETER FORMULAS (One-dimensional - perimeter always in single units)

Triangle



$P = S_1 + S_2 + S_3$

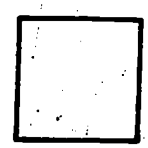
Quadrilaterals

Any 4-sided figure

$P = S_1 + S_2 + S_3 + S_4$

Square

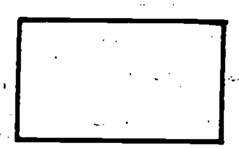
(Same as quadrilateral)



$P = S_1 + S_2 + S_3 + S_4$

Rectangle

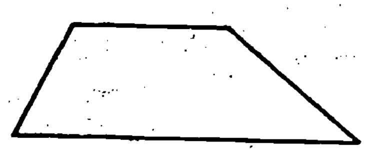
(Same as quadrilateral)



$P = S_1 + S_2 + S_3 + S_4$

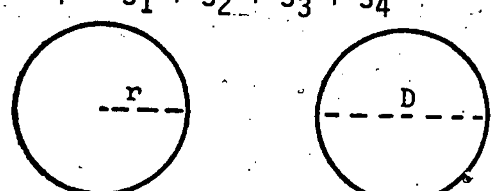
Trapezoid

(Same as quadrilateral)



$P = S_1 + S_2 + S_3 + S_4$

Circle



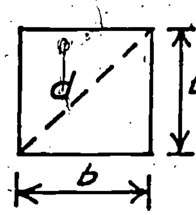
Perimeter = The circumference

$C = 2\pi R$ $\quad$ $C = \pi D$

AREA FORMULAS (Two-dimensional - area always in square units.)

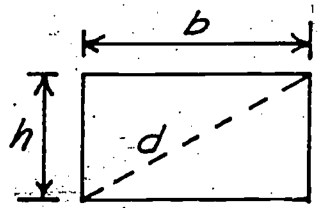
Square

$$\text{Area} = (b)(h) = b^2$$
$$d = (b)\sqrt{2} = 1.414\,(b)$$



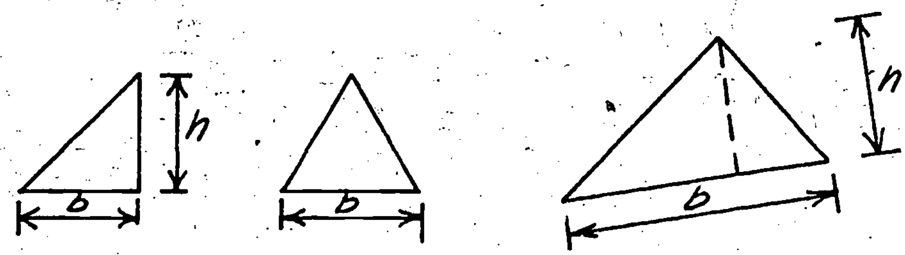
Rectangle

$$\text{Area} = (b)(h)$$
$$d = \sqrt{b^2 + h^2}$$



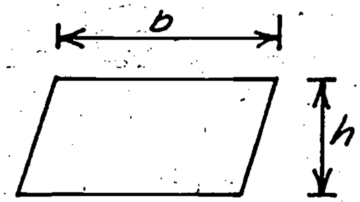
Triangle

$$\text{Area} = \frac{(b)(h)}{2}$$



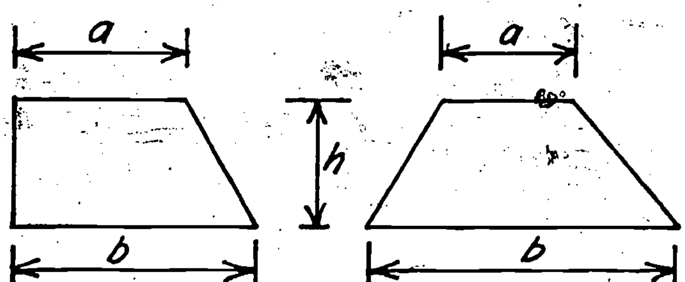
Parallelogram

$$\text{Area} = (b)(h)$$



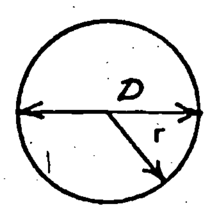
Trapezoid

$$\text{Area} = \frac{h\,(a + b)}{2}$$



Circle

$$\text{Area} = \pi R^2 = \frac{\pi D^2}{4}$$
$$= 3.1416\, R^2$$



R = RADIUS
D = DIAMETER
C = CIRCUMFERENCE
C/D = π = 3.1416

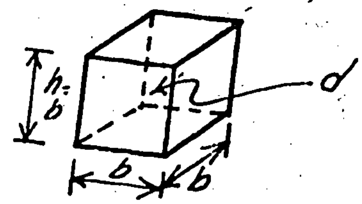
VOLUME FORMULAS (Three dimensional - volume alwyas in cubic units.)

Cube

Volume = (b) (b) (h) $= b^3$

$d = b\sqrt{3} = 1.732\ (b)$

Surface = $6\ (b^2)$

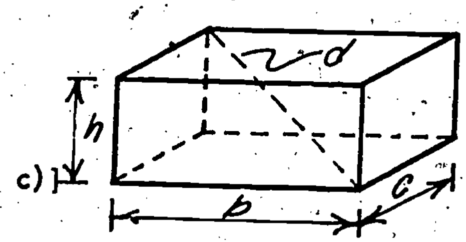


Rectangular Solid Parallelopiped

Volume = (b) (c) (h)

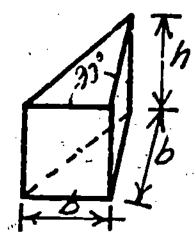
$d = \sqrt{b^2 + c^2 + h^2}$

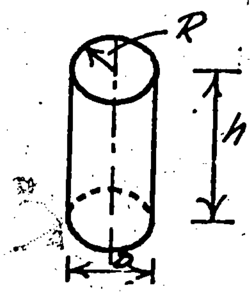
Surface = $2\ [(b)\ (h) + (c)\ (h) + (b)\ (c)]$



Triangular Solid or Cylinder

Volume = (base area) (height)



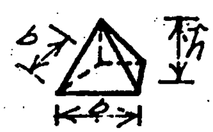


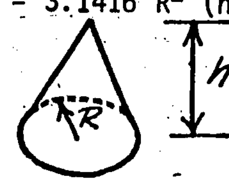
$V = \frac{(b)\ (b)}{2}\ (h)$

$V = \pi R^2\ (h)$
$= 3.1416\ R^2\ (h)$

Pyramid or Cone

Volume = 1/3 (base area) (height)





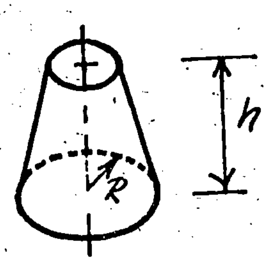
$V = 1/3\ (b)\ (b)\ (h)$

$V = 1/3\ \pi R^2\ (h)$

Frustrum of Pyramid or Cone

Volume = $(A_1 + A_2 + \sqrt{A_1 \times A_2})\ h/3$

A_1 and A_2 = Area base and top



VOLUME CONTINUED

Trapezoidal Solid

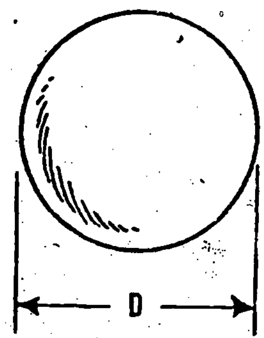
$$\text{Volume} = \frac{\text{base}_1 + \text{base}_2}{2} \times \text{height} \times \text{length}$$

$$V = \frac{(b_1 + b_2)}{2} (h) (l)$$

Sphere

$$\text{Volume} = \frac{\pi}{6} \times D^3$$

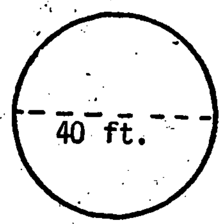
$$\text{Surface} = \pi D^2 \text{ or } 4 \pi R^2$$



SECTION VII

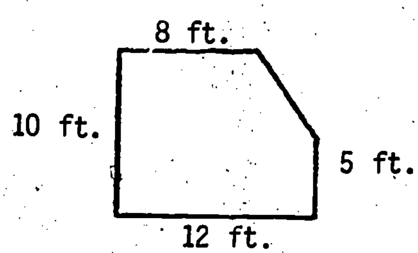
POST-TEST

1. A circular clarifier has a diameter of 40 ft. to the outside weir. What is the weir length?

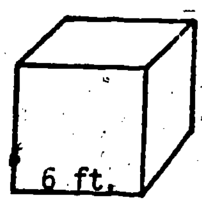


2. What is the total area of the tank above?

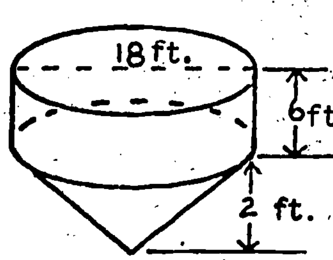
3. The cross section of a grit chamber is in the form of a trapezoid on top of a rectangle as pictured. What is the total area of this cross section?



4. A grease pit is in the form of a cube. It is required that you record the cu. ft. of grease removed daily. It is filled to the top before it is emptied. What is the volume of grease removed?



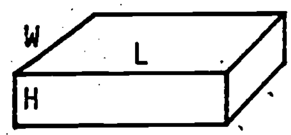
5. The primary clarifier is in the form of a cylinder with a cone shaped bottom. The cylindrical section contains raw sewage. What is the volume of the sewage?



6. The conical section in the above diagram contains sludge. What is the volume of the sludge?

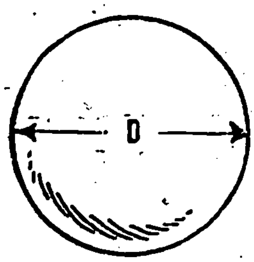
7. What is the volume of a rectangular clarifier 10 ft. high, 20 ft. wide, and 40 ft. long?

Retangular Tank



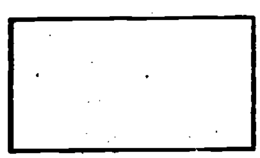
V = L x W x H

8. A spherical chlorine storage tank needs to be painted. What is the surface area of the tank if you know that it is 15 feet wide? (Diameter = 15 ft.)



9. What is the volume of the chlorine storage tank above?

10. What is the perimeter of this rectangle?



80 yds.

200 yds.

SECTION VII

POST-TEST KEY

1. 125.6 ft.
2. 1256 $ft.^2$
3. 110 $ft.^2$
4. 216 $ft.^3$
5. 1526.04 $ft.^3$
6. 169.55 $ft.^3$
7. 8000 $ft.^3$
8. 706.5 $ft.^2$
9. 1766.25 $ft.^3$
10. 560 yards

SECTION VIII

UNIT I GRAPHS AND CHARTS

UNIT II BASIC STATISTICS

Mean

Median

Geometric Mean

SECTION INSTRUCTIONAL PACKAGE GUIDELINE

SUBJECT MATTER: Basic Mathematics for Water and Wastewater Operators

UNIT OF INSTRUCTION: Graphs and charts; basic statistics

LESSON NUMBER: Section 8

ESTIMATED TIME: 3½ hours

JUSTIFICATION FOR THIS INSTRUCTIONAL OBJECTIVE: A knowledge of graphs and charts, and basic statistics is important for water and wastewater plant operation.

PREREQUISITES: The learner shall have successfully completed sections 1 - 7.

INSTRUCTIONAL OBJECTIVES:

Terminal Performance Behavior - The learner shall successfully complete this mathematics section. Successful completion of this section shall be demonstrated when the learner through pre-test or post-test written examination has met the specified criterion level based on the behavioral objectives of this section.

Behavioral Objectives - At the completion of this section the learner will be able to:

State in writing the five steps that are used in preparing a graph, as illustrated in Example 1 of Section VIII.

Transform numeric data onto graphs or charts in a diagram, picture or form that is representative of that numeric (number) form.

Define a mean.

Calculate and determine a mean.

Define a median.

Calculate and determine a median.

Define a geometric mean.

Calculate and determine a geometric mean.

Conditions - None

Criterion - Level of Acceptable Performance - Minimum passing score is 80% on either the pre-test or post-test.

INSTRUCTIONAL APPROACH: Individual lesson utilizing self-pacing of written material.

INSTRUCTIONAL RESOURCES

Available Supplemental Material

Bittinger, Marvin, L., and Keedy, Mervin, L. Arithmetic A Modern Approach. Reading, Massachusetts: Addison-Wesley Publishing Company, 1971. PP 359-368. Gives a brief explanation of mean and median, as well as practice problems on these two topics.

Hill, T. H. Ward, Mathematics for the Layman. New York: Philosophical Library, 1958. PP 116-130.
Explains graphs and how to read them.

Suggested Supplemental Material

Cameron, A. J. A Guide to Graphs. Oxford: Pergamon Press, 1970. PP 1-147. An excellent source for a review of graphs, charts, their types, and uses. Gives a thorough explanation of how to read and make a graph, along with many practice problems.

Kirkpatrick, Joane. Mathematics for Water and Wastewater Treatment Plant Operators. Ann Arbor: Ann Arbor Science Publishers Inc., 1973. PP 32-29. Gives brief description and explanation of mean, median, and mode.

SECTION VIII

PRE-TEST

1. Graph the following sample volume data.

TIME	SAMPLE VOLUME
8:00 A.M.	0 ml
9:00 A.M.	300 ml
10:00 A.M.	500 ml
11:00 A.M.	650 ml
NOON	700 ml
1:00 P.M.	650 ml
2:00 P.M.	450 ml
3:00 P.M.	250 ml
4:00 P.M.	200 ml
5:00 P.M.	150 ml
6:00 P.M.	120 ml
7:00 P.M.	100 ml
8:00 P.M.	100 ml

2. Find the average amount of sludge your digester pumps in one day.

2,000 lbs/day

6,000 lbs/day

8,000 lbs/day

5,000 lbs/day

3. What is the mean of the following numbers?

42, 37, 6, 105, 62

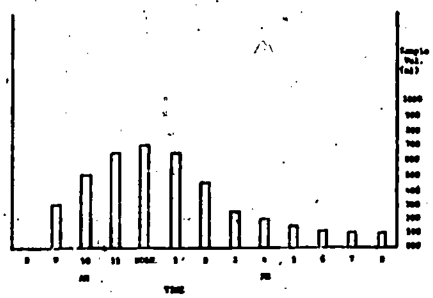
4. Two days of sampling resulted in a most probably number (MPN) of coliform group bacteria per 100 ml of 40, 110, 650, 1100, and 1800. Find the mean coliform content.

5. Find the median coliform content for the above problem.

SECTION VIII

PRE-TEST KEY

1.



2. 5250 lbs/day

3. 50.4

4. 740 coliform

5. 650 coliform

SECTION VIII

UNIT I

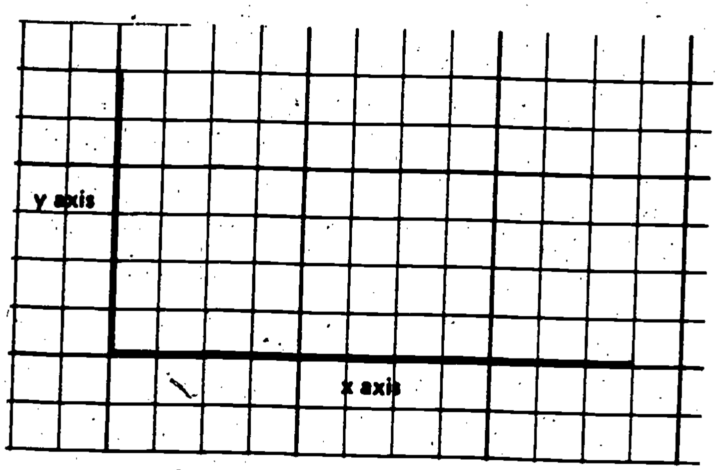
GRAPHS AND CHARTS

Graphs

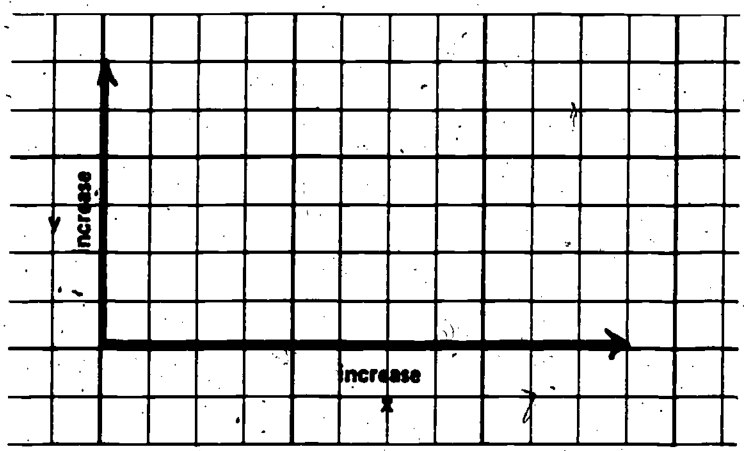
A graph is a diagram or a chart which visually shows a series of changes. These changes may be indicated by curves, lines, dots, dashes, bars, etc. This method of representation indicates how things are changing in proportion to each other.

In this case, we are going to limit ourselves to preparing and understanding line graphs. They are easy to prepare and understand. The flow charts in our treatment plants are line graphs. Charts or graphs which are drawn automatically have the advantage of making and recording continuous measurements. The graphs which we will prepare must be done manually by recording intermittent values, plotting them, and then drawing lines between these points.

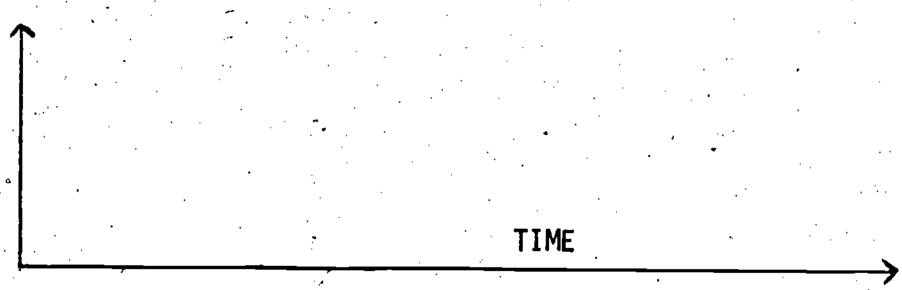
These graphs will meet two conditions. The points to be recorded will be measured in two directions, one measurement horizontally across the paper (the x axis), and the other vertically up and down the paper (the y axis). The lines of the graph paper will be the same distance apart both across and up and down the paper.



The size of the numbers will increase as we move up from the point where the x and y axes meet or to the right of where the x and y axes meet.



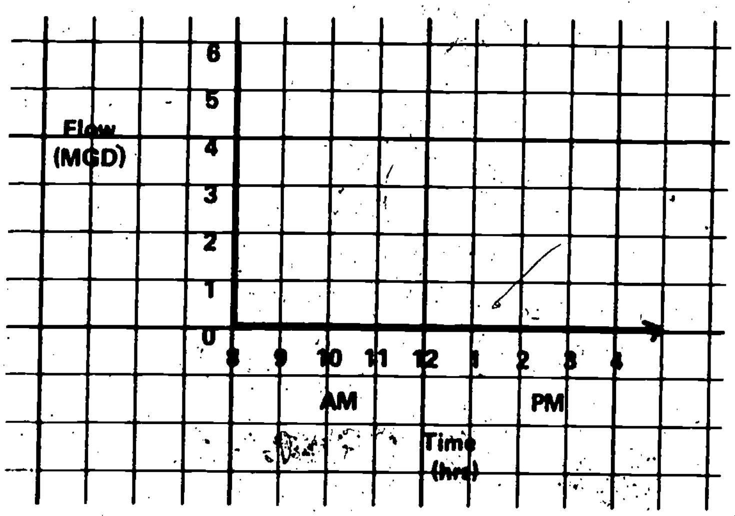
When one of the measurements to be plotted is time, time will be plotted on the horizontal or x axis. Time will start at the point where the y axis meets the x axis and progress toward the right.



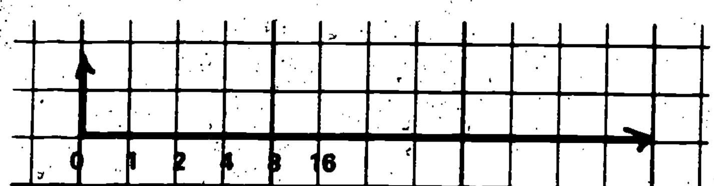
Start					End
Mon	Tues	Wed	Thur	Fri	Etc.
Jan.	Feb.	March	April	May	Etc.
0 min.	10 min.	20 min.		30 min.	Etc.

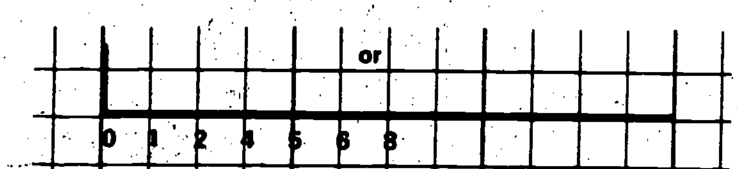
The scales of the graph should be meaningful, accurate, and easily understood.

To plot flow against time, for example, we might use values such as these:



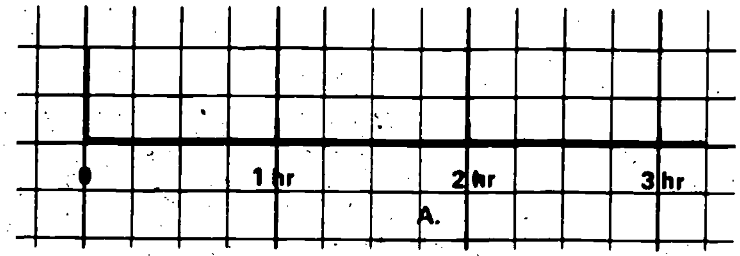
DO NOT vary the units of the scales like this:



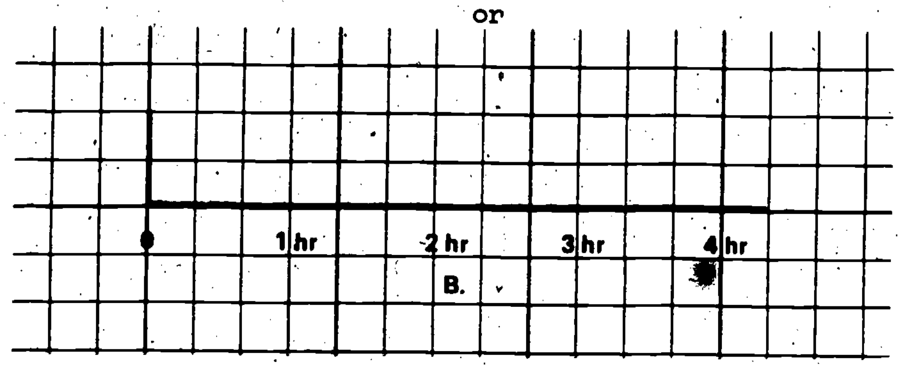


Make your numbers large enough for easy interpretation. Use scales which permit you to easily determine values between the recorded numbers. Use scale in keeping with the numbers and values actually encountered.

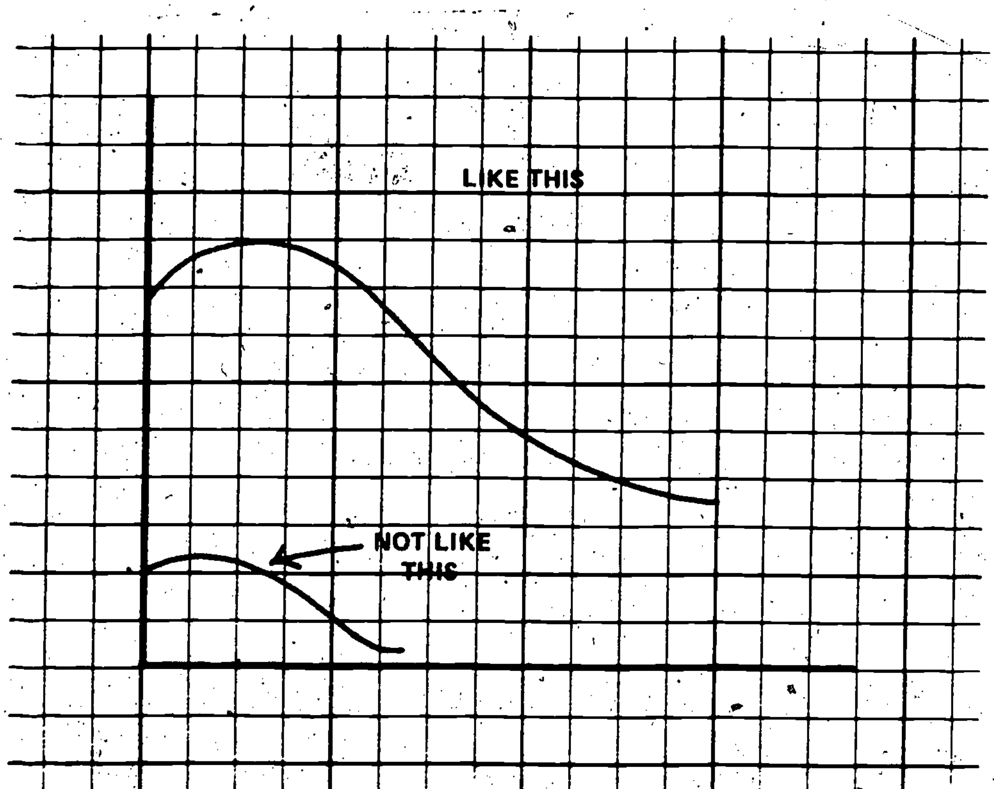
For example:



or



Prepare your scales so that the range of recorded values will occupy the major portion of your graph.



Example 1

The flow in your plant is as follows:

Time (hr)	Flow (MGD)
8 A.M.	1.2
9 A.M.	1.4
10 A.M.	1.8
11 A.M.	2.9
12 Noon	3.1
1 P.M.	2.8
2 P.M.	2.0
3 P.M.	1.7
4 P.M.	1.5

Prepare your graph by using the following steps:

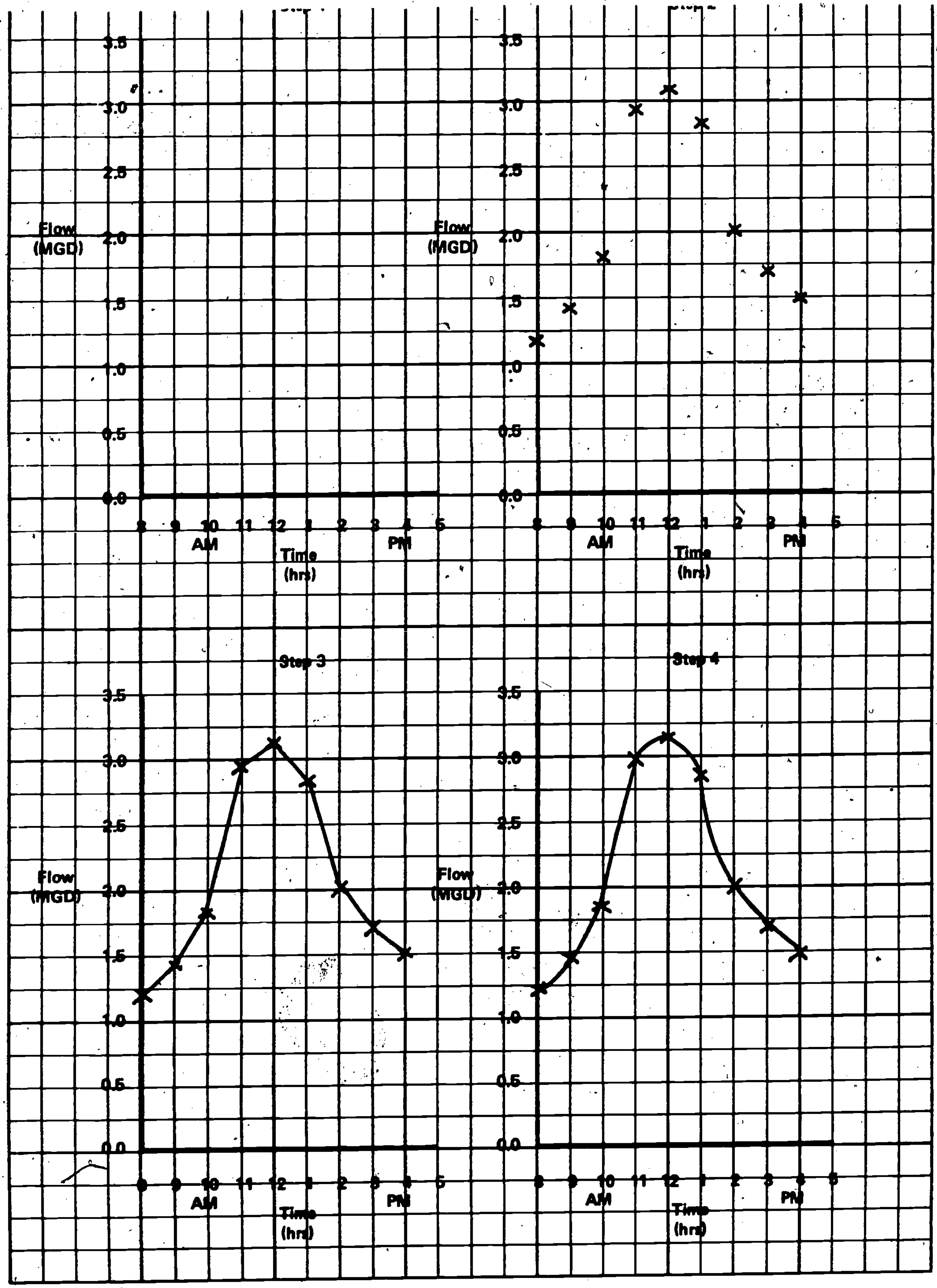
Step #1. Since time is to be plotted on the x axis, determine the scale of values for both time and flow.

Step #2. Plot the points from the above data.

Step #3. Connect the points with straight lines.

Step #4. More meaningful results may be obtained by drawing a smooth curve between the points.

Step #5. Interpret the graph from Step #4.


Step 3
Step 4
Flow
(MGD)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
8
9
10
11
12
1
2
3
4
5
AM
PM
Time
(hrs)

Step #5. Interpretations

(a) When does peak flow occur? Ans. 12 Noon

(b) What is the flow at 9:30? Ans. 1.6 MGD

(c) How long does the peak flow last? Ans. 2 Hrs.

(d) When does the rate of flow increase the fastest?

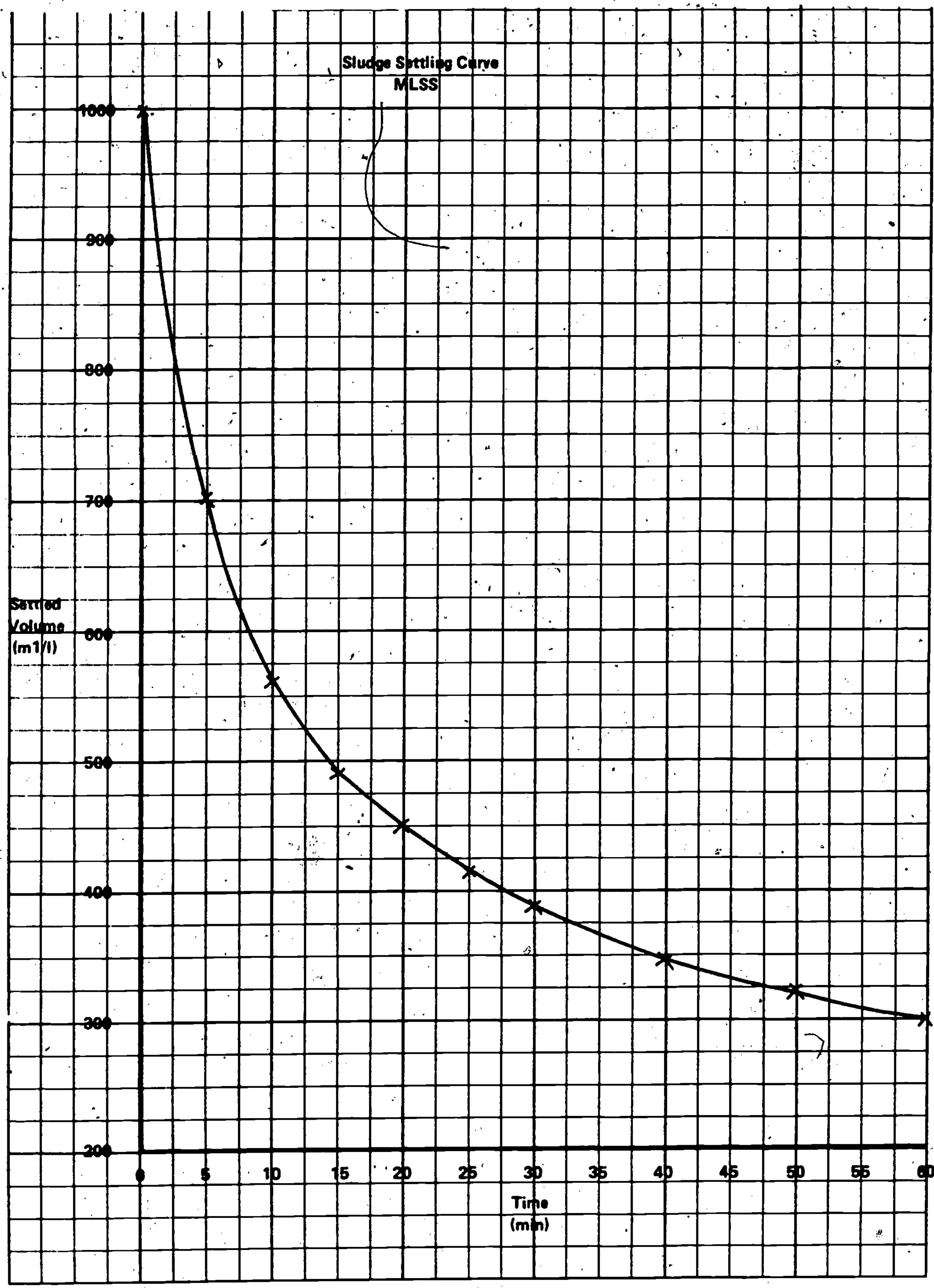
Ans. Between 10 and 11 A.M.

Example 1.5

A 1 (one) liter sample of mixed liquor suspended solids was taken to the laboratory and allowed to settle in a 1 (one) liter graduated beaker. The volume of settled sludge was measured at 5 (five) minute intervals for the first 30 minutes and every 10 minutes from 30 minutes to 1 (one) hour. The volumes recorded below are plotted on the following page.

Settling Time (min.)	Sludge Volume (ml/l)
0	1000
5	700
10	560
15	490
20	450
25	410
30	390
40	350
50	330
60	305

This is a typical curve of good settling activated sludge.

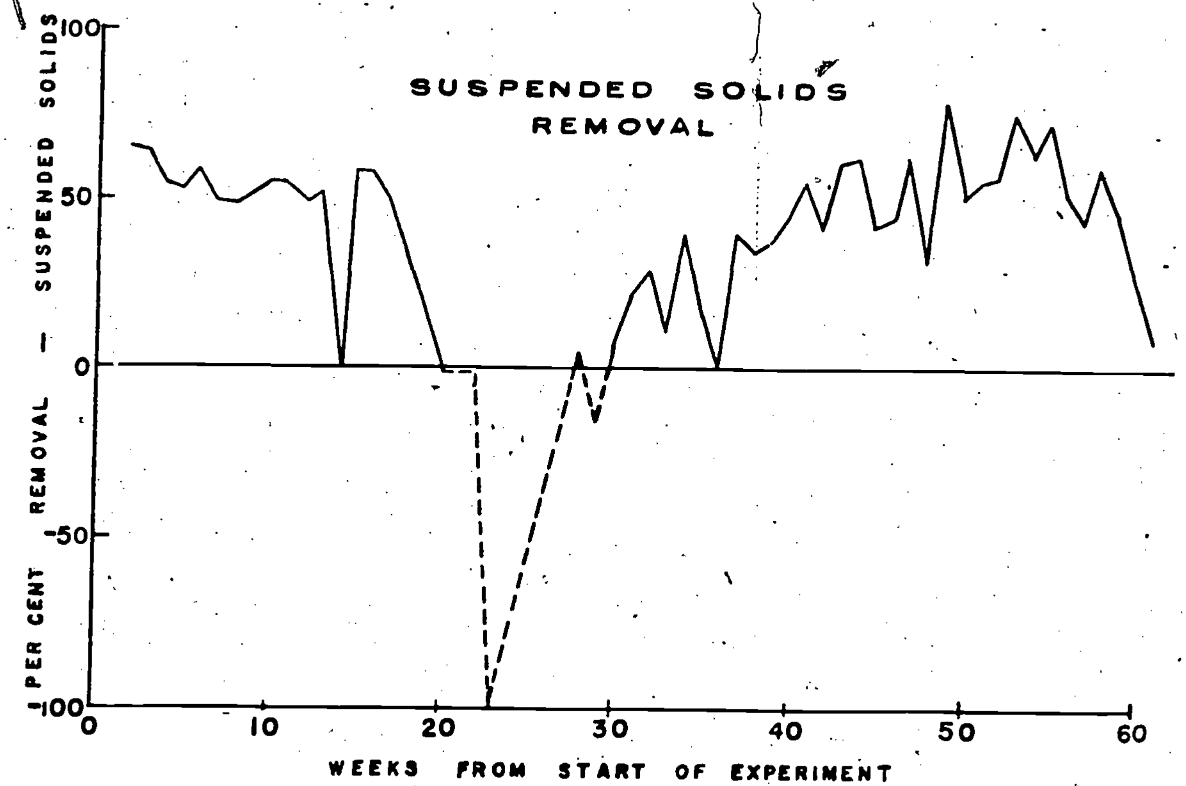


Sludge Settling Curve
MLSS
1000
900
800
700
600
500
400
300
200
Settled Volume (ml/l)
0
5
10
15
20
25
30
35
40
45
50
55
60
Time (min)

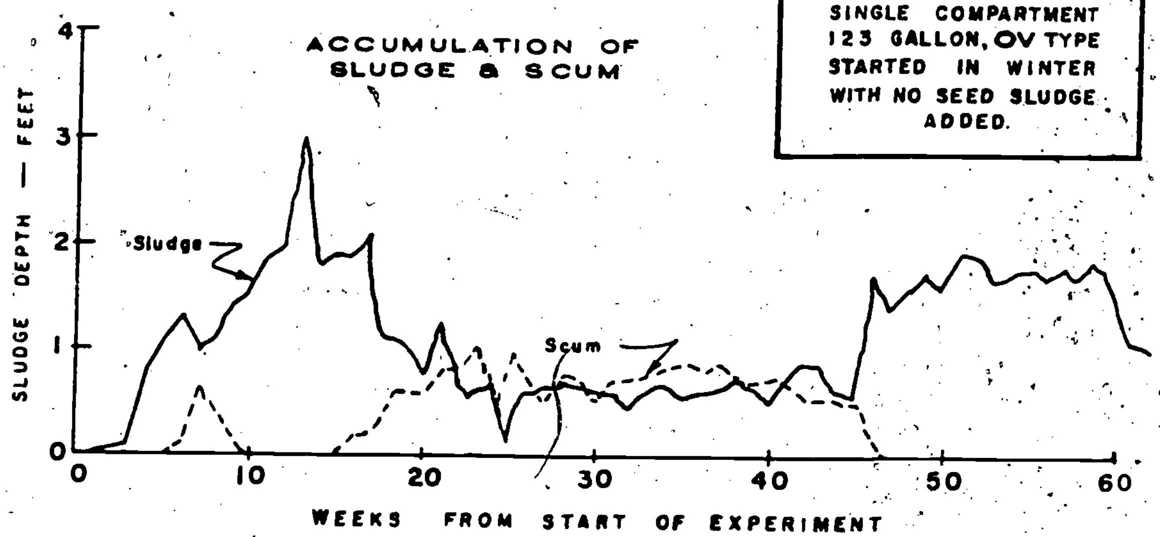
Problem 1 Graph the following flow data:

TIME	FLOW MGD
8:00 A.M.	3.1
9:00 A.M.	3.2
10:00 A.M.	3.3
11:00 A.M.	3.7
12:00 NOON	3.8
1:00 P.M.	3.7
2:00 P.M.	3.4
3:00 P.M.	3.6
4:00 P.M.	3.7
5:00 P.M.	3.9
6:00 P.M.	4.0
7:00 P.M.	4.1
8:00 P.M.	4.4
9:00 P.M.	4.3
10:00 P.M.	3.8
11:00 P.M.	3.5
12:00 P.M. Midnight	3.2

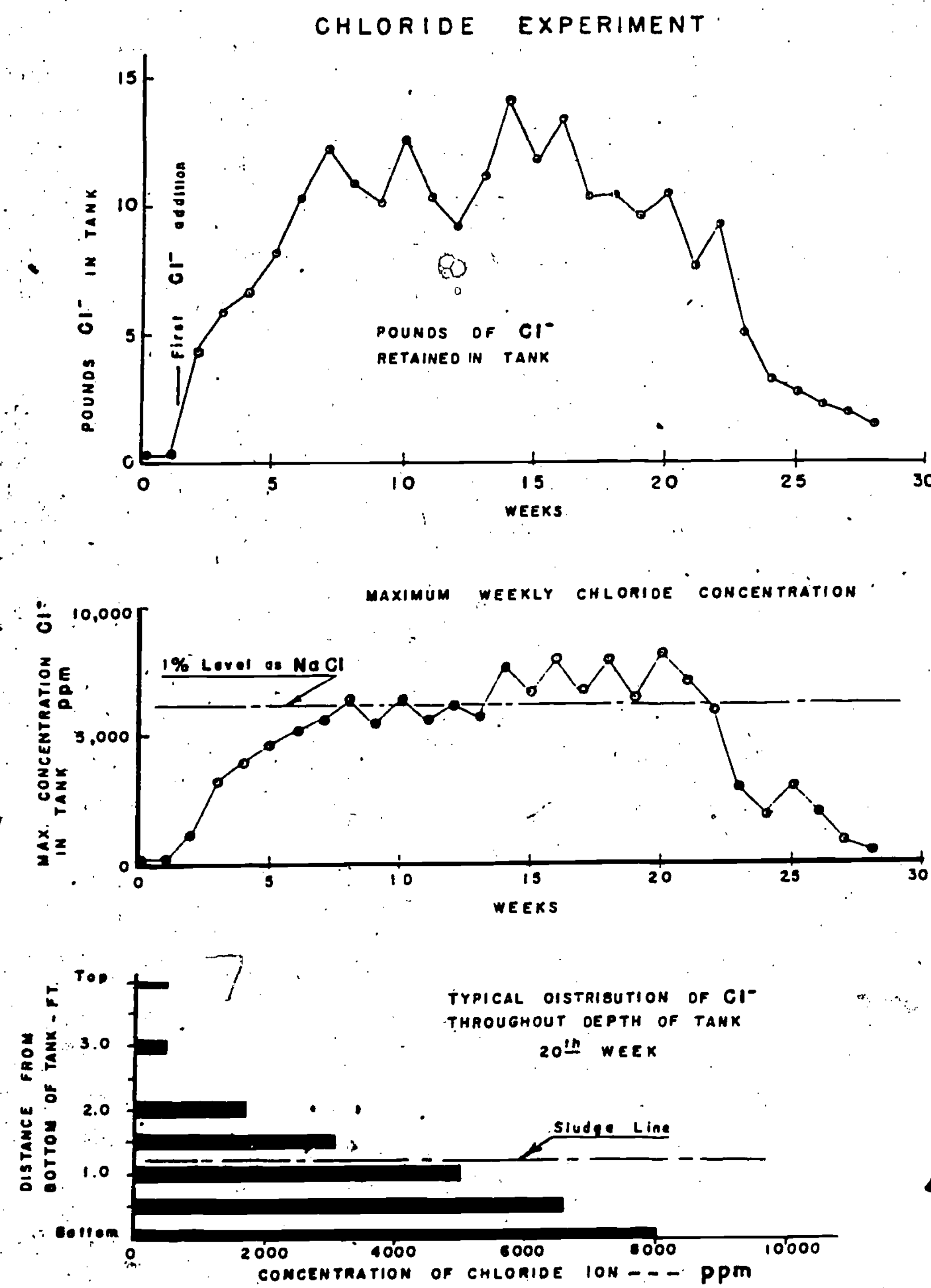
The following series of graphs are representative of the variety typically encountered in every day plant operation. You should notice that there are many ways to draw lines on a graph and that the typical solid lines are not always used.



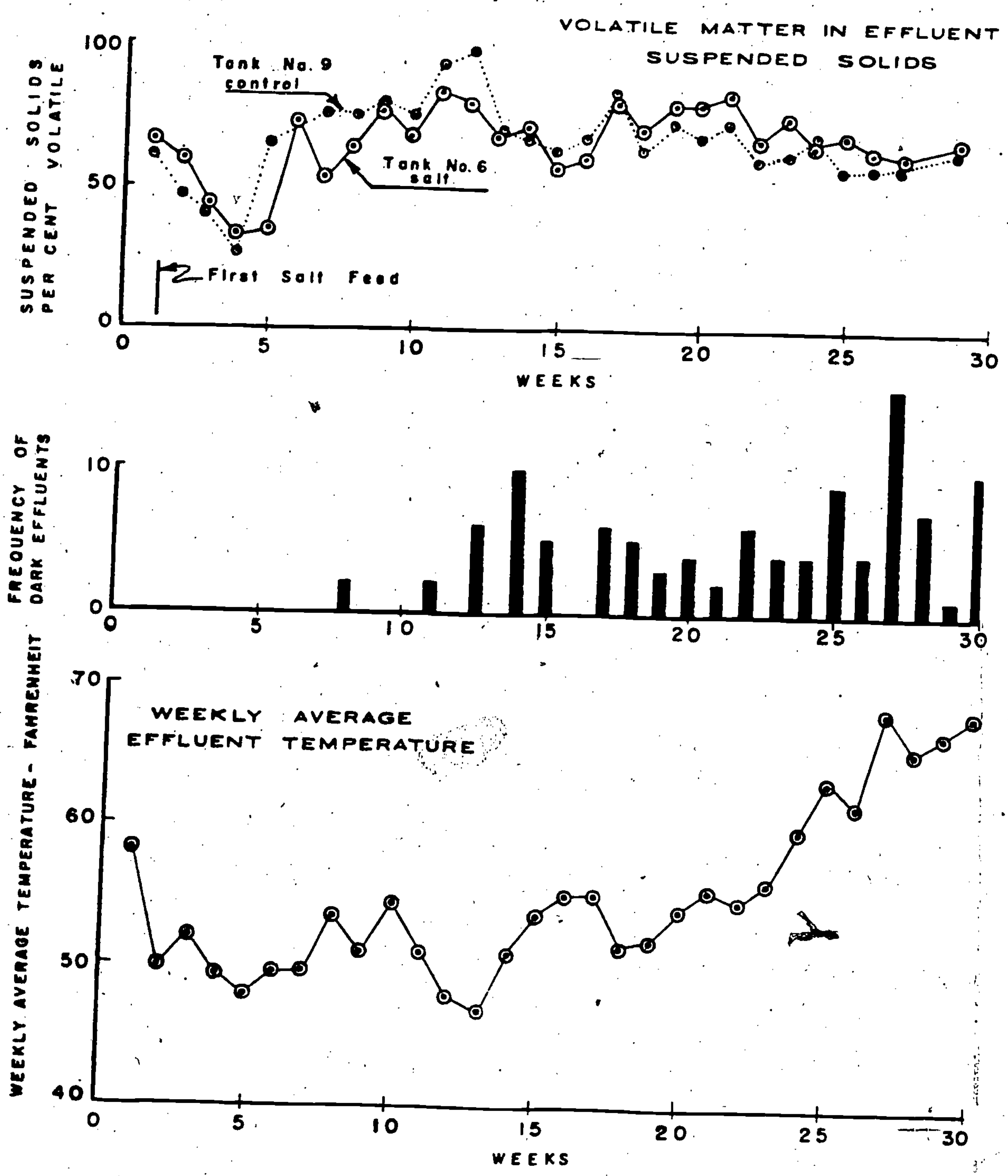
PERFORMANCE CURVES
SINGLE COMPARTMENT
123 GALLON, OV TYPE
STARTED IN WINTER
WITH NO SEED SLUDGE ADDED.



In the case of block graphs the area of the bar shows the size of the number. Since most bars are equally wide, it is the heighth of the bar that indicates the number. Block graphs unlike line graphs, show the totals by means. Thus each bar stands for the average of a group of numbers, and includes a range of values.

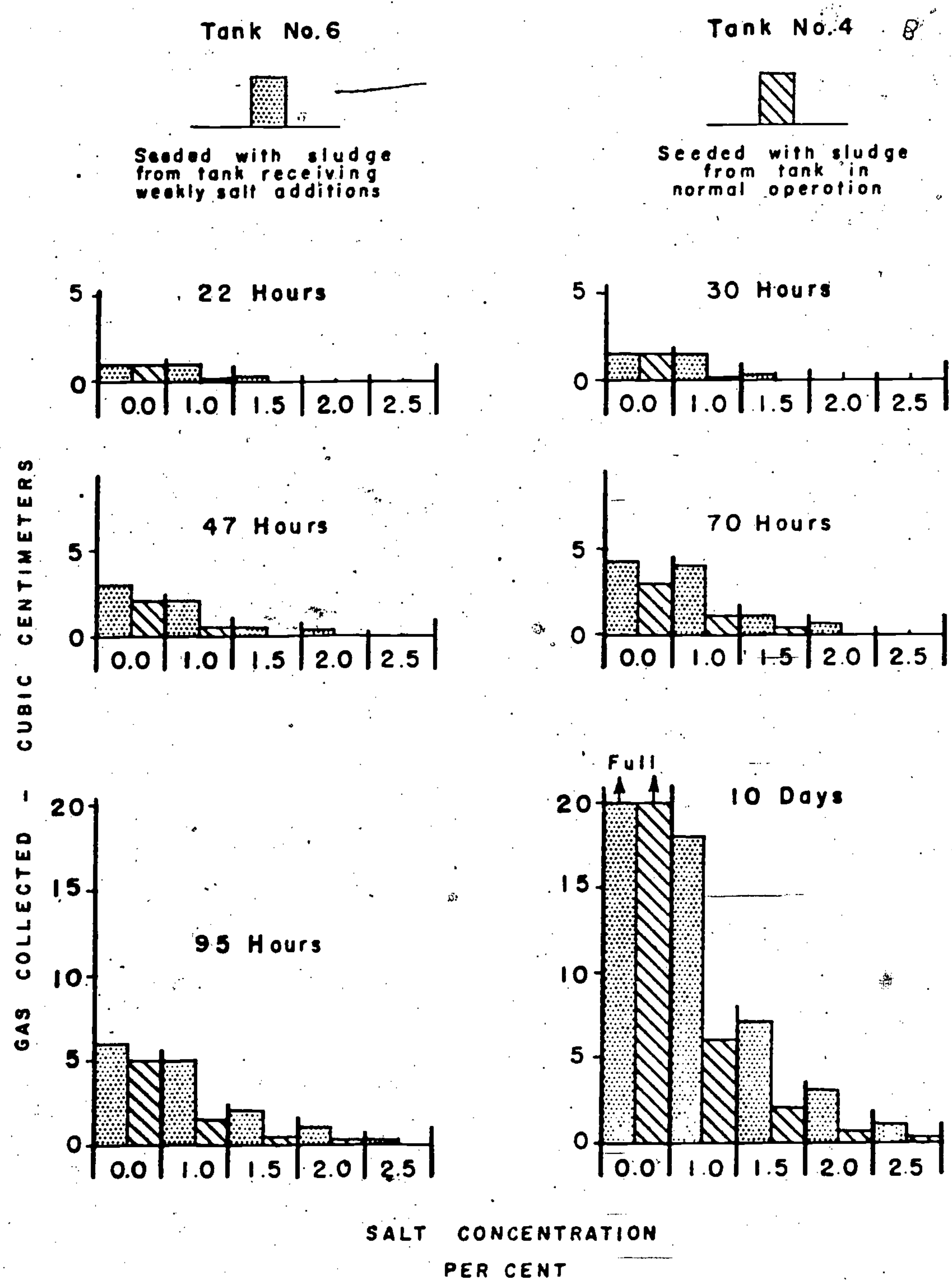


CHLORIDE EXPERIMENT



CHLORIDE EXPERIMENT

Gas Production v.s. Salt Concentration



BASIC STATISTICS

Statistics

What is statistics? Statistics are numbers or facts about numbers put together that produce important information. Why is statistics important? Statistics can help operators figure out much information about plant operation that could be of value to them. Let's look at some of this valuable information.

The Mean

The mean in statistics simply is the average, the average of a list of values. Let's put this in mathematical terms.

$$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$

Example 2

Find the mean of the values 1, 2, 9.

First sum up the values $1 + 2 + 9 = 12$

Now divide by the number of values $\frac{12}{3} = 4$

Here we had 3 values or numbers so we divided by 3. Let's try another.

Example 3

Find the mean of 40 60 100 600

First sum up the values $40 + 60 + 100 + 600 = 800$

How many values do we have

40	60	100	600
1st value	2nd value	3rd value	4th value

4 of course!

So $\frac{\text{sum of the values or 800}}{\text{number of values or 4}}$ = Mean or 200

So the mean or average is 200

It is important to know that every group of numbers has a mean or average.

Another thing to remember is that extreme values have a big effect on the mean.

Example 4

The mean of 1, 2, 3 is 2

The mean of 1, 2, 102 is 35

Example 5

Calculate the amount of sludge your average digester pumps in one day.

Digester A pumps 4,000 lbs/day

Digester B pumps 5,000 lbs/day

Digester C pumps 6,000 lbs/day

4,000 lbs/day + 5,000 lbs/day + 6,000 lbs/day = 15,000 lbs/day

$\frac{\text{15,000 lbs/day}}{\text{3 digesters}}$ = 5,000 lbs/day/digester

Problem 2

Find the amount of sludge your average digester pumps in one day.

Digester A pumps 1,000 lbs/day

Digester B pumps 2,000 lbs/day

Digester C pumps 3,000 lbs/day

Digester D pumps 11,000 lbs/day

Problem 3

What is the mean of the following numbers?

6, 7, 25, 30, 75, 11, 105, 695

Averages and Median

Computing an average from a set of data offers a way of simplifying the data or comparing one set of data with another.

Example 6

Influent BOD's at a treatment plant are determined every day. The following composite values were obtained during one week: 210, 180, 175, 215, 195, 155, and 200. What is the arithmetic mean for the week?

$$\text{Average} = \frac{\text{Sum of items or values}}{\text{Number of items or values}}$$

$$= \frac{210 + 180 + 175 + 215 + 195 + 155 + 200}{7}$$

$$= 190 \text{ mg/l}$$

$$\begin{array}{r} 210 \\ 180 \\ 175 \\ 215 \\ 195 \\ 155 \\ 200 \\ \hline 1330 \end{array}$$

$$\begin{array}{r} 190 \\ 7\overline{)1330} \\ 7 \\ \hline 63 \\ 63 \\ \hline 0 \end{array}$$

Weekly average or mean BOD = 190 mg/l

Another arithmetic tool to analyze a set of data is the median. The median in a set of data is the middle value. There are just as many values above a median as there are below.

To determine the median, the data should be written in ascending or descending order and the middle value identified.

Example 7

What is the median BOD in the following problem?

215
210
200
195 - Median   Weekly median BOD = 195 mg/l
180
175
155

Problem 4

What is the median BOD   165
175
255
345
360

Median coliform numbers are sometimes used as a standard by regulatory agencies to avoid allowing too much weight to large coliform values.

Example 8

Five days of sampling resulted in most probable number (MPN) of coliform group bacteria per 100 ml of 23, 5, 2, 2300, and 16. Find the mean and median coliform content.

$$\text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}}$$

$$\text{Mean MPN/100 ml} = \frac{23 + 5 + 2 + 2300 + 16}{5}$$

$$= \frac{2346}{5}$$

Mean MPN/100 ml = 469 coliform

$$\begin{array}{r} 2300 \\ 23 \\ 16 \\ 5 \\ \underline{2} \\ 2346 \end{array}$$

$$\begin{array}{r} 469 \\ 5\overline{)2346} \\ \underline{20} \\ 34 \\ \underline{30} \\ 46 \\ 45 \end{array}$$

Median MPN/100 ml = 16 coliform

The above example indicates that the median value completely eliminates the effect of the one large sample, while the mean value is affected a great deal. Most agencies feel that the minimum and maximum values of a group of data should always be stated along with a mean or median. The difference between the maximum and minimum values is called the range.

Geometric Mean

The operator on occasion will have need of another type of mean called the geometric mean. This mean is important because it is used in averaging rates of change and also in constructing index numbers.

The geometric mean is the Nth part of the product of any group of numbers. What does this mean?

$GM = \sqrt[N]{(X_1)\ (X_2)\ (X_N)}$

Let's look at an example.

Example 9

Find the geometric mean of 100, 400.

To find the geometric mean of 100, 400, let us first write out the equation.

Geometric mean = $\sqrt[2]{100 \times 400}$	The two numbers here, 100 and 400 are multiplied together
Geometric mean = $\sqrt[2]{40{,}000}$	Since there were two numbers 100 and 400, we take the square root of 100 x 400.
Geometric mean = 200	

Example 10

Find the geometric mean of 1, 3, 9

To find the geometric mean of 1, 3, 9 let us first write out the equation.

Geometric mean = $\sqrt[3]{1 \times 3 \times 9}$ — The three numbers here, 1, 3, and 9, are multiplied together

Geometric mean = $\sqrt[3]{27}$ — Since there are three numbers 1, 3, and 9, we take the cube root of 27.

Geometric mean = 3 — (The cube root is that number that when multiplied by itself 3 times will give you the cube.

3 (cube root)	3 x 3 x 3 multiplied by itself 3 times	= 27 the cube

Logarithms can also be used to find the geometric mean

Example 11

A wastewater plant has an effluent discharge rate of 2, 4 and 8 gallons per minute. What is the geometric mean for these values?

First let us put the values 2, 4 and 8 in the equation.

Geometric mean = $\sqrt[3]{2 \times 4 \times 8}$ — Then take the log of both sides

$$\text{Log Geometric} = \log \sqrt[3]{2 \times 4 \times 8} = \log (2 \times 4 \times 8)^{1/3}$$

$$= 1/3 \quad \log 2 + \log 4 + \log 8$$

Log 2 = .3010

Log 4 = .6021

Log 8 = .9031

1.8062 — Then add up the logs

So log geometric mean = 1/3 (1.8062) = .60206

So geometric mean = 4

Problem 5

A small water treatment plant has an intake rate of 40, 80, and 160 gallons per minute. Calculate the geometric mean.

For one number Geometric Mean = that number

For two numbers Geometric Mean = $\sqrt[2]{\text{1st number x 2nd number}}$

For three numbers Geometric Mean = $\sqrt[3]{\text{1st number x 2nd number x 3rd number}}$

For four numbers Geometric Mean = $\sqrt[4]{\text{1st No. x 2nd No. x 3rd No. x 4th No.}}$

For N numbers Geometric Mean = $\sqrt[N]{(X_1) \times (X_2) (X_N)}$

SECTION VIII

POST-TEST

1. Graph the following flow data.

TIME	FLOW
8:00 A.M.	1 MGD
9:00 A.M.	3 MGD
10:00 A.M.	5.1 MGD
11:00 A.M.	6.5 MGD
NOON	7 MGD
1:00 P.M.	6.5 MGD
2:00 P.M.	4.75 MGD
3:00 P.M.	3.1 MGD
4:00 P.M.	2.1 MGD
5:00 P.M.	1.9 MGD
6:00 P.M.	1.5 MGD
7:00 P.M.	1.2 MGD
8:00 P.M.	1.2 MGD

2. A large water treatment plant has a discharge rate of 400, 300, and 250 gallons per minute. Calculate the geometric mean.

---

3. 15 days of sampling resulted in a most probably number (MPN) of coliform group bacteria per 100 ml of 10, 30, 125, 800 and 1400. Find the mean coliform content.

4. Calculate the BOD median.

   300 g/ml

   250 g/ml

   248 g/ml

   164 g/ml

   110 g/ml

   80 g/ml

   70 g/ml

5. Find the average amount of sludge your digester pumps in one day.

   6,000 lbs/day

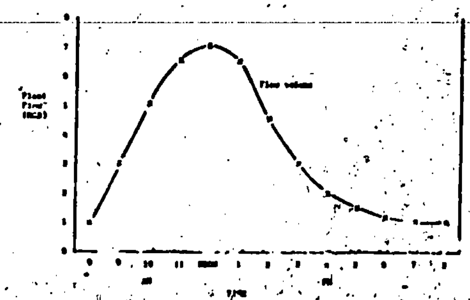
   3,500 lbs/day

   2,600 lbs/day

   1,800 lbs/day

SECTION VIII

POST-TEST KEY

1. 

2. 310.7232 gallons per minute

3. 473 coliform

4. 164 g/ml

5. 3475 lbs/day

PROBLEM ANSWER KEY

SECTION I

UNIT I

1.

656	35	72	0
6	0	0	0
5	3	7	0
6	5	2	0

2.

2
7,654
1,000
60,743
8,972,651

3. 1.5
4. 100
5. 100
6. 200
7. 0
8. 100
9. 1.6
10. 15.6
11. 77.1
12. .1
13. 1.5
14. 1,001 gallons

UNIT II

15. 10
16. 15
17. 9
18. 13
19. 8
20. 8
21. 16
22. 20
23. 18
24. a)

6	2 tens	20
4	3 left over	+ 3
9	23	23
1		
+ 3		

b)

9	2 tens	20
1	4 left over	+ 4
3	24	24
7		
+ 4		

25. 510 lbs. of sludge
26. 1,632
27. 572
28. 7
29. 3
30. 8
31. 5
32. 7
33. 6
34. 6
35. 275

36. 1,179

37. 5,500 gallons

38. 12

39. 15

40. 24

41. 270

42. 90

43. 1,400

44. 400

45. 900

46. 40

47. 15 R 5

48. 1,218 R 4

49. 1,001 R 2

50. 855

51. 24

52. 248 R 20

53. 24 R 25

54. 256 R 8

PROBLEM ANSWER KEY

SECTION II

UNIT I

1. 1
2. $5\frac{1}{5}$
3. $\frac{1}{2}$
4. $\frac{1}{6}$
5. $3\frac{3}{6}$ or $3\frac{1}{2}$
6. 25
7. $1\frac{3}{8}$
8. $\frac{3}{4}$
9. $\frac{11}{12}$
10. $\frac{13}{12}$ or $1\frac{1}{12}$
11. $\frac{19}{12}$ or $1\frac{7}{12}$
12. $\frac{7}{6}$ or $1\frac{1}{6}$
13. $1\frac{7}{15}$
14. $\frac{5}{6}$
15. $\frac{13}{15}$
16. $93\frac{13}{36}$
17. $63\frac{5}{16}$
18. $\frac{10}{16}$ or $\frac{5}{8}$
19. $\frac{3}{13}$
20. $\frac{11}{16}$
21. $\frac{17}{24}$
22. $3\frac{7}{20}$
23. $41\frac{5}{24}$
24. $8\frac{6}{7}$
25. $10\frac{14}{15}$
26. $\frac{9}{11}$
27. $2\frac{1}{7}$
28. 50 gallons
29. $\frac{18}{25}$
30. $\frac{40}{63}$
31. 6
32. $\frac{32}{15}$ or $2\frac{2}{15}$ ounces
33. $\frac{9}{22}$
34. $\frac{42}{70}$ or $\frac{3}{5}$
35. $\frac{8}{35}$ ounces

PROBLEM ANSWER KEY

SECTION III

UNIT I

1. 37.2
2. 48.8472
3. 18.6
4. 1.9
5. 33.669
6. 13.079
7. 430
8. 3.5
9. 35,610
10. 7.40235
11. 6.57
12. 28.119
13. .643 MGD
14. .0146 MGD
15. .004
16. 4.651 gallons per hour
17. 4.98
18. 30.5 lbs. sludge per acre
19. 4.18
20. 397
21. 3.17
22. 404 gallons per hour
23. .9
24. .875
25. $\frac{3}{10}$
26. $\frac{17}{100}$

UNIT II

27. $\frac{1}{4}$
28. 33.33%
29. .21
30. 790%

UNIT III

31. 8 ft.
32. 120 dollars
33. 300
34. 1,000

PROBLEM ANSWER KEY

SECTION IV

UNIT I

1. 49
2. 156.25
3. 10
4. 316.22
5. 7.81

UNIT II

6. 1
7. 25
8. 144
9. 1,728
10. 1,296
11. 2,401
12. 64
13. 243
14. 10,000
15. 64,000,000
16. 10,000,000
17. 16,807
18. 78,125
19. 512
20. 1,024
21. 49
22. 512
23. 256
24. 4,096
25. 23.14
26. 54

UNIT III

27. 1.005×10^3
28. 4.7×10^1
29. 1.8×10^6
30. 1×10^{-2}
31. 5.74×10^{-3}
32. 2.5×10^1
33. 1.075×10^{-4}
34. 6.8×10^4
35. 1×10^7
36. 5.5×10^3

PROBLEM ANSWER KEY

SECTION V

UNIT I

1. 30
2. 50
3. 4
4. 6
5. 1.6
6. 70
7. 161
8. 29.5
9. 6.5
10. 20
11. 10
12. 4
13. 15
14. 2
15. 18
16. 100
17. 30
18. 480
19. 51
20. 220
21. 39
22. 185
23. 7.5
24. 55
25. 1
26. 2
27. $\frac{10}{11}$
28. $\frac{4}{29}$
29. 1
30. $5\frac{1}{2}$
31. .7 MGD
32. $\frac{\text{185 lbs. BOD/day}}{\text{1,000 cu. ft. volume}}$
33. 80 GPD per capita
34. 4,243,810,000
35. 864,000 gallons per day
315,360,000 gallons per year

UNIT II

36. 400
37. 9.55
38. 376.8
39. 125
40. 625

PROBLEM ANSWER KEY

SECTION VI

UNIT I

1. VI
2. XII
3. XXV
4. XIX
5. XXX
6. XVII
7. L
8. VIII
9. 4
10. 13

UNIT II

11. 264.2 gallons
12. Yes

UNIT III

13. 26.75
14. 14
15. 504
16. 50
17. 14
18. 625
19. 1,750
20. 26
21. 50
22. 90

PROBLEM ANSWER KEY

SECTION VII

UNIT I

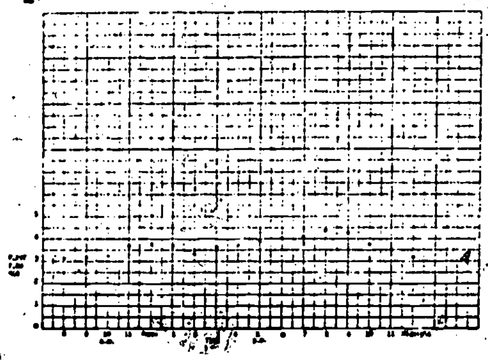
1. 30 ft.
2. 19 ft.
3. 11 mi.
4. 10 in.
5. 800 ft.
6. 120 ft.
7. 50 ft.^{2}
8. 240 ft.^{2}
9. 5,625 ft.^{2}
10. 676 ft.^{2}
11. 43.96 ft.
12. 37.68 in.
13. 907.46 in.^{2}
14. 113.04 ft.^{2}
15. 235.5 ft.^{2}
16. 314 ft.^{2}
17. 1,000 ft.^{3}
18. 8 yd.^{3}
19. 30 yds.^{3}
20. 72 ft.^{3}
21. 729 ft.^{3}
22. 81 ft.^{3}
23. 3,077 ft.^{3}
24. 5,086.8 ft.^{3}
25. 523.33 ft.^{3}

PROBLEM ANSWER KEY

SECTION VIII

UNIT I

1.



UNIT II

2. 4,250 lbs/day

3. 119.25

4. 255

5. 80

261

PROBLEM ANSWER KEY

SECTION VII

UNIT I

1. 30 ft.
2. 19 ft.
3. 11 mi.
4. 10 in.
5. 800 ft.
6. 120 ft.
7. 50 ft.^{2}
8. 240 ft.^{2}
9. 5,625 ft.^{2}
10. 676 ft.^{2}
11. 43.96 ft.
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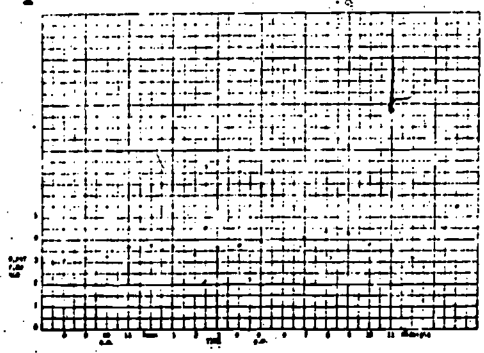
262

PROBLEM ANSWER KEY

SECTION VIII

UNIT I

1.



UNIT II

2. 4,250 lbs/day

3. 119.25

4. 255

5. 80

GLOSSARY

ACCURACY - Correctness, usually referring to numerical compulations.

ADDITION - The act or process of adding or uniting.

ALGEBRA - The mathematical act of reasoning about relations whereby letters representing numbers are combined according to the rules of arithmetic.

ALGEBRAIC EQUATION - An expression containing or using only algebraic symbols and operations.

ALTITUDE - A line indicating the height of a figure or the length of such a line.

AMOUNT - Quantity; value.

ANGLE - The space within two lines coming apart from a common point.

ARABIC NOTATION - The arabic system of writing numbers using the numbers 0-9.

ARABIC SYSTEM - The system of numbers in use today in which there are ten basic numbers, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

AREA - Amount of surface.

ARITHMETIC - The act of computation with figures.

ASCENDING ORDER - Increasing direction; becoming greater.

AVERAGE - Same as mean.

BAR GRAPH - A graph with bar or column representation.

BASE - A base of a geometric figure is usually a side upon which an altitude is constructed.

BASIC - Fundamental.

BASIC ALGEBRA - Fundamental algebra.

BASIC NUMBER - A fundamental number, 0-9, which forms the arabic language counting system.

BASIC STATISTICS - Fundamental statistics.

BLOCK GRAPH - A graph using bar representation.

BORROWING - In subtraction, to take from one number to add to the next lower number.

BRACKET - Means that the terms enclosed are to be treated as a single entity.

CALCULATE - to carry out some mathematical process.

CALCULATOR - A machine that performs mathematical operations.

CANCELLATION - The act of dividing like factors out of the numerator and denominator of a fraction.

CENTER POINT - A point in the exact middle of a circle or sphere.

CENTIMETER - A unit of measurement in the metric system equal to .01 meter.

CHAIN OPERATION - An operation performed in sequence.

CHART - A map or diagram exhibiting information usually in tabular form.

CHECK CALCULATION - Reworking of a problem for the purpose of proving to be right.

CIRCLE - A plane curve consisting of all points at a given distance (the radius) from a fixed point (the center).

CIRCULAR - Of or pertaining to a circle; having the form of a circle; round.

CIRCUMFERENCE - The outer boundary of a circle or circular area.

COLUMN - A vertical row of terms used in addition and subtraction.

COMBINATION - The joining or bringing together. In algebra, the occurrence of several operations in one problem.

COMMON DENOMINATOR - A common multiple of the denominators for two or more fractions.

COMMON FRACTION - A fraction which is part of a whole; 2/3, 1/2, etc.

COMPOSITE VALUE - The total worth or value.

CONE - A geometric solid whose surface is formed by straight lines joining a fixed point to a plane curve.

CONSTANT - A digit or symbol that maintains its same value throughout a sequence of mathematical operations.

CONVERSION - Act of converting; changing.

CONVERT - To change, usually from one mathematical form to another.

COUNT - To name a set of whole numbers in their natural order usually beginning with 1.

COUNTING NUMBERS - All whole numbers including 0 and 1.

CROSS DIVISION - Division of fractions.

CROSS MULTIPLICATION - Multiplication of fractions.

CROSS SECTION - The area of a plane.

CUBE - A six sided geometric solid consisting of 6 perfect squares, which when put together form perpendicular lines. The 3rd power.

CUBE ROOT - A cube root of a number is that digit, when cubed, is equal to that number.

CUBIC UNITS - A specific quantity or magnitude that consists of measurements in 3 directions usually length, width, and height.

CUBING - Multiplying a number by itself 3 times.

CYLINDER - A geometric solid having 2 parallel planes and a curved surface of straight lines connecting the 2.

CYLINDRICAL - Pertaining to a cylinder.

DATA - Facts, figures, information, etc.

DECAGON - A ten sided geometric figure.

DECIMAL - Pertaining to tenths or to the number ten.

DECIMAL EQUIVALENT - A decimal fraction. A decimal equal to a percentile.

DECIMAL FRACTION - A fraction whose denominator is some power of ten, "1/10", which can also be expressed as a decimal, .1.

DECIMAL POINT - A single period that is used to seperate the one's place from the tenth's place.

DEGREE - 1/360 part of a complete angle or circle.

DENOMINATOR - The term usually written below the line of a fraction which shows the number of equal parts into which the unit is divided.

DESCENDING ORDER - Decreasing direction; becoming lesser.

DIAGRAM - A pictorial represent-ation.

DIAMETER - A line joining 2 points of a circle, and traveling thru the center point of that circle.

DIGIT - Any one of the numbers, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

DIMENSION - A magnitude measured in a particular direction.

DIRECT PROPORTION - Where two ratios are exactly equal to each other.

DISPLAY - Show.

DISTANCE - The extent of space there is between things or points.

DIVIDEND - A quantity which is to be divided by another quantity.

DIVIDING LINE - That line usually separating a numerator and denom-inator of a fraction.

DIVISION - The operation opposite or inverse to multiplication. The result of dividing one number (the divident) by another (the divisor) is there (quotient).

ENGLISH SYSTEM - Refers to the English system of measurement.

ENTER - To put in or insert.

EQUAL - Like or alike in quantity, degree, value, etc; expressed mathematically by the symbol, "=."

EQUAL SIGN - "="; a statement of equality.

EQUATE - To form or write the algebraic statement of equality which state that the two expressions are equal; to make equal.

EQUATION - A statement of equality between two expressions.

EQUIVALENT - Equal in value; the same.

EVEN - Any whole number that is divisible by 2.

EXPONENT - A number placed at the right of and above a symbol or number; a power.

EXPONENTIAL NUMBER - An exponent with a number value.

EXPONENT OF TEN - A number placed at the right of and above the number 10.

FACTOR - One of two or more numbers which when multiplied together produce a product or new number.

FLOATING DECIMAL POINT - A term applied in calculator computation when the decimal is placed by the machine as each operation is performed.

FURMULA - A general rule of principle stated in mathematical language.

FRACTION - The ratio between any two numbers, the top part being referred to as the numerator, and the bottom part, the denominator.

FRACTIONAL EQUIVALENT - A common fraction equal to a decimal fraction.

FRACTION BAR - A line dividing the numerator value by the denominator value.

FRUSTUM - The part of a geometric solid between the two parallel planes.

FUNDAMENTAL - Basic.

FUNDAMENTAL OPERATIONS - Addition, subtraction, multiplication, and division.

GEOMETRIC FIGURE - Any combination of points, lines, planes, circles, etc.

GEOMETRIC MEAN - The geometric average. The GM of N numbers equals that Nth root of their product.

GEOMETRY - The science that treats the size and shape of things.

GRAM - The standard unit of weight for the metric system.

GRAPH - The geometric representation of a relationship between numbers.

GRAPHIC PICTURE - Picture of a graph.

HEIGHT - The extent upward; altitude.

HEXAGON - A six sided geometric figure.

HORIZONTAL - A position parallel to the plane; going across.

HUNDRED - 100.

HUNDREDTH - 1/100 or .01.

HUNDRED-THOUSAND - 100,000.

HUNDRED-THOUSANDTH - 1/100,000 or .00001.

IMPROPER FRACTION - A fraction which is greater than a whole, 7/6, 8/5, etc.

INCH - A unit of length; 1/12 foot.

INSIDE SURFACE AREA - Amount of area on the inside of a plane or region.

INVERSE PROPORTION - Where one ratio is equal to the reciprocal of another ratio.

INVERT - To change or convert a number to its reciprocal.

LENGTH - The linear magnitude or distance of anything.

LETTER - A symbol or variable.

LIKE FRACTIONS - Fractions with similar denominators.

LINE - A straight entity represented by a mark extended in 2 directions indefinitely.

LINEAR DISTANCE - Distance between objects covering a straight path, and in 1 direction.

LINE GRAPH - A graph with line or linear representation.

LITER - Metric system. A unit of capacity equal to 1.0567 liquid quarts.

LOG - The exponent of any particular base, such as 10 in the base 10.

LOGARITHM - A log.

LOWEST TERMS - Reduced as much as possible.

MANIPULATION - To change and rearrange numbers to suit one's own purpose.

MATHEMATICAL CONCEPT - An idea referring to mathematics.

MATHEMATICAL RELATIONSHIP - A linking of mathematical ideas or entities.

MATHEMATICS - The science that treats the measurement, properties, and relations of quantities including arithmetic, geometry, algebra, etc.

MEAN - The average of a value or set of values.

MEASURE - The extent or quantity of something by comparison with a standard.

MEASUREMENT - The act of measuring.

MEDIAN - The middle measurement when items are arranged in order of size.

METER - Standard unit of measurement on the metric system.

METRIC SYSTEM - International system of measurement based upon the meter and gram as units of length and weight.

MILLION - 1,000,000.

MILLIONTH - 1/1,000,000 or .000,001.

MIXED NUMBER - A number consisting of both a whole number and a fraction, combined.

MIXED NUMERAL - A mixed number.

MOST PROBABLE NUMBER (MPN) - A number of most likely occurrence.

MULTIPLE OF TEN - Any number divisible by ten.

MULTIPLICATION - The operations opposite or inverse to division. The process of obtaining a product, resulting from addition of a number a given amount of times.

NEGATIVE - Less than zero; opposite of positive.

NEGATIVE NUMBER - Any number less than zero.

NONAGON - A nine sided geometric figure.

NOTATION - A system for writing numbers, consisting of symbols.

NUMBER - Any whole number; numeral.

NUMERAL - Number.

NUMERATION - Act of numbering or counting.

NUMERATOR - The term usually written above the line of a fraction which shows how many parts of a unit there are.

NUMERICAL VALUE - A number.

OCTAGON - An eight sided geometric figure.

ODD - Any whole number that is not divisible by two.

ONE - 1.

OPERATION - Any process that combines two or more symbols into a single symbol.

OPPOSITE OPERATIONS - The reverse. (addition and subtraction), (multiplication and division), are opposite operations.

ORDER - A condition in which everything is in proper place with reference to other things and their purpose.

OUTSIDE SURFACE AREA - Amount of area on the outside of a plane or region.

PARALLEL - Straight lines lying in the same plane, but never meeting, no matter how far they are extended.

PARALLELOGRAM - A 4 sided geometric figure having atleast 2 of its sides parallel to each other.

PARENTHESIS - Means that the terms enclosed are to be treated as a single entity.

PEAK - The maximum value of a quantity.

PENTAGON - A five sided geometric figure.

PERCENT - Hundreths; %.

PERCENTAGE - The result found by taking a certain percent of a number.

PERCENTAGE EQUIVALENT - Conversion of a decimal directly to a percentile. Move decimal 2 places to the right.

PERCENTILE - Is a set of division points that divide a set of data intc 100 equal parts.

PERIMETER - The length of a curve within a circle.

PI - 3.14; Π; the ratio of the curcumference of a circle to its diameter.

PLACE - The location of a digit in a particular column, ones, tens, hundreds, etc.

PLACE VALUE - The place value of any digit depends upon its place in a string of digits.

PLANE FIGURE - A geometric figure which lies entirely in one plane, considered two dimensional, consisting of square units.

PLOT - To locate points, geometrically, graphically, etc.

POINT - A specific position or location.

POSITIVE - Greater than zero. Opposite of negative.

POWER OF TEN - An exponent of ten.

PROBLEM SOLVING - Finding the solution to a particular problem.

PRODUCT - Is the result of the operation of multiplication.

PROPER FRACTION - A common fraction.

PROPORTION - The equating of ratios.

PYRAMID - A geometric solid having a triangular base, and triangular sides all meeting in a point.

QUADRILATERAL - A four sided geometric figure.

QUANTITY - An amount.

RADIUS - A straight line going from the center point of a circle to a point on the circle. The radius is equal to ½ the diameter.

RANGE - A span; the extent to which variation is possible.

RANGE OF VALUES - A span of numbers.

RATIO - A comparison of two numbers through division, having the same denominator units.

RECIPROCAL - The reciprocal of any number is 1 divided by that number. The reciprocal of 5 is 1/5.

RECTANGLE - A quadrilateral having all right angles, whose opposite sides are parallel and equal in length.

RECTANGULAR - Pertaining to rectangles.

RECTANGULAR SOLID - A 3-dimensional rectangle having volume.

REDUCE - To make lesser in size, ~~to~~ make simpler.

REDUCTION - The act of reducing.

REPEATING NUMBERS - A number with an unending amount of digits following the decimal point.

RIGHT TRIANGLE - A triangle having a right angle (90 degrees).

ROMAN NOTATION - The writing of Roman numerals whereby letters are used in place of numbers.

ROMAN NUMERAL - Numbers used in the ancient Roman system of notation, that are still used today.

ROUNDING - Modification or the changing of a decimal number by dropping the digits after a certain place.

SCALE - A displayed arrangement of numbers in proper order.

SCIENTIFIC NOTATION - The expression of a decimal number as a decimal number between 1 and 10, multiplied by a power of 10.

SEPTAGON - A seven sided geometric figure.

SERIES - A succession of numbers; a sequence of numbers.

SIMPLER TERMS - A numerical expression made more simple; reduced.

SIMPLIFY - To reduce; make lesser; to use numbers of smaller value.

SLANT HEIGHT - The distance on the surface of a cone from its base to its apex or top.

SOLUTION - Answer.

SOLVE - To figure out; find the answer to.

SPHERE - A solid geometric figure whose surface is at all points at an equal distance to the center.

SQUARE - A 4 sided geometric figure consisting of 4 right angles with all sides equally long.

SQUARE FOOT - A unit of measurement for area consisting of 1 foot multiplied by itself ($ft.^2$).

SQUARE UNIT - A specific quantity or magnitude that consists of measurement in two directions; length and width, etc.

SQUARING - The act of taking a value and multiplying it by itself.

STANDARD SYSTEM - An ordered body of information that is used as a basis for comparison.

STATISTICS - The science that deals with the collection, classification, and use of numerical facts or data.

STORY PROBLEM - A mathematical problem primarily stated in words more so than mathematically.

SUBSCRIPT - An exponent.

SUBSTITUTE - To take the place of; replace.

SUBTRACTION - The act or process of taking away.

SUM - The result of addition.

SUPERSCRIPT - An exponent.

SURFACE AREA - Amount of area on a plane region.

SURFACE DIMENSION - Any particular magnitude related to a surface.

SYMBOL - A letter, figure, or other sign used to represent something else, including numbers.

TEN - 10.

TENTH - 1/10 or .1.

TEN-THOUSAND - 10,000

TEN-THOUSANDTH - 1/10,000 or .0001.

THOUSAND - 1,000.

THOUSANDTH - 1/1000 or .001.

TRAPEZOID - A quadrilateral (4 sided figure) with exactly 2 parellel sides.

TRAPEZOIDAL SOLID - 3 dimensional trapezoid having volume.

TRIANGLE - A 3 sided geometric figure.

TRIANGULAR SOLID - A 3 dimensional triangle, having volume.

UNIT - An entity or magnitude usually representing forms of measurement such as distance, volume, capacity, weight, etc.

UNKNOWN - In algebra, a variable representing a value that we do not know, and are often trying to solve.

UNKNOWN NUMBER - In algebra, the number which we do not know and are trying to determine.

UNKNOWN VARIABLE - A variable whose value we do not know.

UNLIKE FRACTION - Fractions with different denominators.

VALUE - In mathematics, the worth or amount of some number, or a variable representing a number.

VARIABLE - A symbol representing some value.

VERTICAL - A position perpendicular to the plane; straight up.

VOLUME - Capacity; the amount of anything in three dimensions.

WHOLE NUMBER - Counting numbers starting with 1, with each number 1 greater than the number behind it.

WIDTH - The extent or distance from side to side.

X AXIS - Is the horizontal line running across a graph intersecting the y axis at 0.

Y AXIS - Is the vertical line running up a graph intersecting the X axis at 0.

ZERO - 0; a number having no value.

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