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ABSTRACT

Fourteen research reports related to mathematics education are abstracted and analyzed. Two of the reports concern studies funded by the National Science Foundation to provide an assessment of the status of science education; three deal with teaching methods, three with conditional reasoning, two with problem solving, two with intellectual development, and one each with attitudes and sex-related differences. Research related to mathematics education which was reported in RIE and CIJE between April and June 1978 is listed. (MP)

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A note from the editor . . .

This issue of IME contains abstracts of two lengthy documents which may have particular importance to mathematics educators. They concern two of the three studies funded by the National Science Foundation to provide an assessment of the status of science education. The Case Studies (directed by Stake and Easley) and the Survey (directed by Weiss) were supplemented by a third report, a literature review*, which has not been included in IME because this journal does not usually contain abstracts of reviews. The reviewers of the two documents -- Peggy House, Ross Taylor, and Bob Kane -- have provided summaries which are longer than the usual IME abstracts because of the length and type of documents they were reviewing. Our appreciation is extended to them for tackling this time-consuming task. Readers should also be aware that several professional organizations -- e.g., NCTM and ASCD -- have been perusing the set of documents for implications and interpretations pertinent to their specific memberships.

Our appreciation is also extended to the reviewers of the other articles in this issue. IME exists because of the willingness of such reviewers to contribute their knowledge and time.

Marilyn N. Suydam
Editor

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* Suydam, Marilyn N. and Osborne, Alan. The Status of Pre-College Science, Mathematics, and Social Science Education: 1955-1975. Volume II: Mathematics Education. NSF Contract C7620627. 1977.

INVESTIGATIONS IN MATHEMATICS EDUCATION

Summer 1978

CASE STUDIES IN SCIENCE EDUCATION. Stake, Robert; Easley, Jack (Project Directors), A Project for the National Science Foundation conducted by the Center for Instructional Research and Curriculum Evaluation and the Committee on Culture and Cognition, University of Illinois at Urbana-Champaign, January 1978. ERIC: ED 156 498 - ED 156 513.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Peggy House, University of Minnesota, and Ross Taylor, Minneapolis Public Schools.

1. Purpose

The major purpose of this study was to describe the status of pre-college science education in the United States in the 1976-1977 school year. ("Science" is defined by NSF to include the natural sciences, mathematics, and the social sciences.)

2. Rationale

The past 15 years have seen unparalleled federal investment in science education. The National Science Foundation (NSF) has played a special role in the improvement of science education. Recently the Foundation has undergone a reorganization which has included the establishment of seven major directorates, including the Directorate for Science Education; changes in the peer review process; and changes in the proposal processing procedures. The Case Studies in Science Education (CSSE) is one of four studies for which the Directorate for Science Education has contracted to provide baseline information so that the Foundation can target its future science education programs to meet documented needs.

3. Research Design and Procedure

The Case Studies in Science Education project was conducted to provide an in-depth investigation in schools and in their communities of events and circumstances which have an impact on science education. Specifically, the CSSE focus was on "issues," i.e., on circumstances about which people disagree and on those conditions which cause or are believed to cause certain effects. Since these effects are valued differently by different people, there is disagreement on whether or how conditions should be changed. By focusing on the current issues in science education, the investigators sought to determine how much science is being taught and to identify the obstacles to good science teaching.

The design used to approach this task was a case study methodology. Ten sites were originally selected which were considered to be representative of the spectrum of school size, geographic location, type of community, and curricular orientation (innovative, traditional). Five sites were chosen for their proximity to prospective field observers, and an additional five were selected to achieve the balance described above. Later, an eleventh site (Columbus, Ohio) was added because it provided an opportunity to study science education in a crisis situation, namely, a heating fuel shortage which forced the closing of many schools. At each site, one senior high school was identified and then two or three of its feeder schools (junior high and elementary) were chosen to complete the makeup of the site's "school cluster." In general, it was expected that attention be divided as follows: half to the natural sciences and one quarter each to mathematics and the social sciences.

Data were gathered from three sources. First, on-site field observations of 4 to 15 weeks were conducted by persons selected by the project directors. Each field observer determined his or her own method of observation which in most cases consisted of watching and asking questions. Early plans to use a standard check list or rating scale were abandoned when two field observers refused to complete the instruments. The content and format of each site report were determined by the individual observers.

A second source of information was from four- to six-member site visitation teams which spent three days at the eleven sites toward the end of the respective field observations. These site visitations were intended to confirm the existence of phenomena or attitudes and otherwise to overview the site. Teams were usually composed of a member of the local community, a mathematics educator and a science educator, scientists, educational policy people, or experts in evaluation strategies. Each team member wrote a report of the site visit, but, as with the field observation, the content, style, and methodology were left to the individuals. A few samples of site visitation reports are included, but the complete reports of these teams are not presented.

Finally, a national survey of teachers, science supervisors, administrators, high school counselors, high school seniors, and their parents was conducted to provide additional interpretations and information on the extent of the generalizability of the case study data. The sample was divided according to grade level and/or area of responsibility into 22 different respondent groups and each was asked to respond to a variation of the survey questionnaire. That four-page instrument consisted of three sections: (1) a questionnaire designed for the specific respondent group to collect demographic, biographic, and experience-related information and also containing one or more general issue-oriented questions which may be common to more than one respondent; (2) scenario and related questions; and (3) general items regarding science education. In all, eight scenarios were developed which consisted of a contrived illustration designed to establish a particular issue in science education in its proper context, and each was presented to two, three, or four of the respondent groups. The page of general items was designed in three distinct formats, each of which was administered to one-third of the respondents.

In the field observations and site visitations, CSSE observers attempted to sample diverse populations by seeking out persons with different positions, roles, experiences, attitudes, and goals. Their approaches included deliberate sampling of members of different categories of school district personnel, referrals by respondents to other persons who might have desired information, and a search for persons who might hold divergent or opposing views. However, no consistent systematic sampling procedure is reported. In the survey administration, sampling of teachers, supervisors, and administrators was performed by the Research Triangle Institute (RTI) by creating subgroups of samples drawn by RTI for another NSF project surveying pre-college education. Counselor, student, and parent samples were drawn by CSSE from a random sample of 35 schools whose principals were included in the RTI sample. The final survey sample consisted of the following respondent groups:

Superintendents	(n = 149)
3 Principal samples:	K-6 (n = 94), 7-9 (n = 86), 10-12 (n = 87)
5 Supervisor samples:	7-12 science (n = 200), 7-12 mathematics (n = 211), 7-12 social studies (n = 201), K-6 science (n = 210), K-6 mathematics (n = 198)

7 Teacher samples:	10-12 science (n = 150), 10-12 mathematics (n = 150), 10-12 social studies (n = 75), 7-9 science (n = 150), 7-9 mathematics (n = 150), 7-9 social studies (n = 75), 4-6 general (n = 150)
Counselors	(n = 30)
Senior students	(n ~ 750)
Parents of students	(n ~ 750)

The case studies are reported in eleven descriptive narratives prepared by the respective field observers. These vary in length from 13 to 136 pages and display a wide diversity in their focus, organization specificity, and generalizability. Two volumes of findings were prepared by the CSSE staff to assimilate the content of the field observers' and site visitation teams' reports. Survey responses are reported in a separate volume which gives numerical and percentage information for various response categories to each question. An overview volume and an executive summary complete the 16-volume report.

4. Findings

a. Science Education Findings

Teacher Is Key. A child's education for a given year is most dependent on the beliefs, knowledge, and actions of the teacher. In most of the sites the in-service program provided little aid, partly because it was anemic and not aimed directly at improving instruction, and partly because the teachers paid little heed to it.

The Basic Two--Reading and Arithmetic. In school settings greater emphasis was given to reading and arithmetic and to the results of minimum competency testing aimed at the basics. Less emphasis was being given to science, mathematics, and social science concepts and relationships.

Science, Mathematics, and Social Studies Curricula. There were almost no interdisciplinary efforts in the schools of this study. Mathematics was getting greater attention with the national emphasis on "the basics" and vocational preparation. The result was an apprehension almost exclusively about computation from second grade to senior year. Much of the remedial teaching was being done by non-mathematics teachers reassigned for various reasons. The curriculum was "course" and "skill" centered. It was authoritarian. It was external to the interests and experiences of pupils.

Socialization as a Preemptive Aim. A most common and vigorously defended purpose held by teachers was that of socialization. It was intimately related to observance of the mores of the community, conforming to the role of "good student," and getting ready for the next rung on the educational ladder.

Text-bound Teaching. In mathematics at all levels the teaching method was usually one of going over the problems assigned, either teacher or students working a few at the chalkboard where others may observe, the teacher working out the more difficult problems, the teacher and students starting new assignments together, then students working individually on assigned exercises.

Articulation and Uniformity. There was extremely little articulation in the science, mathematics, and social studies curricula among different schools in a district, either between levels or between school buildings at the same level. A large majority of the people in the sites visited felt that classes should be more uniform across the city and more articulated up and down the grades. Teachers regularly expressed dismay that students did not arrive knowing what they were supposed to have learned previously.

Low Priority for Science Education. During the school visits many people were asked about the importance of science and mathematics programs. About half of those questioned agreed that the general public does not put high priority on the teaching of science. However, less than 20% of school people and less than one-third of the parents agreed that the general public does not put a high priority on the teaching of mathematics. The higher support for mathematics was interpreted to be a result of a perceived need for simple computational skills.

b. Other (than science education) Findings

Due Process vs. Ordinary Pedagogy. Efforts to give equal education opportunity to all children are at times at odds with efforts to protect learning spaces from the distraction and disruptions of students who could not or would not learn. Little relief from this situation is in sight.

Technologizing the Curriculum. A nationwide effort has been undertaken to make teaching more explicit and more rational and to make learning more uniform and more measurable. The process usually involves stating objectives and developing criterion tests to measure accomplishment of the objectives. Many administrators spoke highly of these efforts. However, the evaluators reported:

> We did not run into any situations or even any "folklore stories of far-off places" where the objectives-based system had in fact changed the achievement levels of the youngsters.
>
> (CSSE, p. 19:14)

The Management Burden. The increasing number and complexity of federal and state regulations are placing an extra burden on school administrations. School systems are becoming increasingly dependent on state funding and on federal funding for categorical programs. Elaborate information systems have been established to provide information for the local, state, and national levels. However, in the process it appears that affairs of curriculum and instruction are receiving less administrative attention.

Powerlessness and Remedy. Many people in the schools were proud of what they had, but indicated that there were things that they would like to change. However, many of the changes would require a change in the larger system and teachers as well as others felt that they had little power to make the changes. Many have been disillusioned by reforms and there is a tendency to accept things as they are. However, there are people in this society who continue to work to improve the lot close at hand. Agencies such as NSF could do many things to support the efforts of people to remedy ills that they encounter.

5. **Interpretations**

a. Strengths. The most substantial strengths of science programs were seen to be as follows:

(1) Individual teachers are given large responsibility to decide what will be taught and how it will be taught.

(2) The general public shows substantial respect for science and mathematics faculties and for individual teachers.

(3) Teachers have a sincere regard for the well-being of students.

(4) NSF institutes for in-service training provide one of the best opportunities for upgrading the understandings and skills of teachers.

(5) Modern youngsters have a complex and sophisticated epistemology and an intuitive understanding that "truth" changes and that knowledge is tentative.

(6) A vast array of learning resources is available.

(7) The "mathematics wars" are over, and there has been a mellowing of faculty attitude toward science and technology.

b. Problems. The following were considered to be the most serious problems in science teaching and learning:

(1) The proportion of school funding spent for instruction is diminishing at a distressing rate.

(2) There is a diminished concern for scientific ideas (such as Newton's laws) as central to instruction, and schools are pressured to set aside these ideas in favor of an emphasis on basic skills.

(3) The pedagogical support for teachers is poor in relevance and small in quantity.

(4) Opportunities to learn science out of school are not sufficiently supported by teaching in the schools.

(5) The emphasis throughout the K-12 school program is on preparation, not on utilization.

(6) Schools no longer provide a spokesperson for science;

chief administrators are managers rather than a voice for the importance of learning.

c. Non-problems. The following were identified as problems already receiving widespread attention and not deserving of NSF programming attention:

(1) Among teachers and citizens there are great differences in the perceptions of the objectives of the schools.

(2) The quality of reading and other "basic" performances of students is too low.

(3) There is little articulation regarding instruction across classrooms within a building, across buildings, and across levels of instruction from elementary to high school.

(4) Science and the social sciences are seldom taught in an interdisciplinary manner.

(5) The level of work in schools is highly dependent on competition.

(6) There is a diminishing regard for authority.

d. Science education issues. From the case studies a number of issues were identified. These were reduced and refined to eight fairly specific topics which in turn were used to generate the scenarios for the national survey. The eight issue areas so developed as representatives of major current issues were the following:

(1) Budget cuts and their ramifications

(2) Issues of pluralism and uniformity

(3) The back-to-the-basics movement

(4) Problems which arise in diagnostic teaching

(5) Teaching and socialization

(6) Support systems available to teachers

(7) Personal bias in teaching

(8) Elitism in the sciences

e. Need as a basis for policy setting. The model originally conceived for the CSSE project was to determine need by subtracting the description of the status quo from that which is identified as the ideal or goal. CSSE rejected that model on the grounds that it is impossible to reach agreement on what "should be." They recommend instead that actual status and desired status be considered simultaneously, relating both to particular children, particular communities, particular learning tasks, and particular curricula. One does not know where the ideal lies. Rather, one studies present conditions, estimates what improvements would result if conditions were changed in some way, then heads out in the direction of most likely improvement. If NSF is to continue to improve its awareness of current conditions of science teaching and learning from time to time, additional studies should be established to look broadly at the needs, to identify needs directly, and to move to feasibility studies and perhaps pilot operation of one or more program options appearing most favorable.

f. Possible action for NSF Science Education Directorate

(1) At present there is little call for text materials not already available, but teachers are looking for inexpensive and motivating supplementary materials. There is dissatisfaction with basic or remedial arithmetic materials for high school age students. Curriculum developers probably should give less attention to the analysis of skills and more to the contextual utility of skills.

(2) There was substantial need for pedagogical support for teachers, and both teachers and administrators requested that NSF teacher training and "course content" institutes be extended.

(3) Overall, science was seen as having rather limited value to the education of all students. The idea of a better place for science education in general education needs further study.

(4) NSF could assist schools in making arrangements for opportunities for students to learn science out of school through programs integrated into the regular curricular structure.

(5) As schools seem to be moving away from science education (with the exception of computational arithmetic), more programming could be purchased non-school. In particular, special programs might be developed for students whose parents are economically disadvantaged and who do not subscribe to the local school objectives and are not served well by them. These could include special television programming and work with local park districts.

(6) There is need for adult and continuing education in science, and NSF could consider extension programs oriented to this need.

(7) Research on science education is needed, but such research should focus on the context of instruction (i.e., on the pedagogical processes, the administrative processes, and the social-political background) rather than on the learner. Specific topics for research include the support system for teaching and learning, the curriculum supervisor, testing for scientific knowledgeability, and the use of science instruction for socialization.

Critical Commentary

The NSF defines "science" as consisting of natural sciences, mathematics, and the social sciences. However, in the report the term is not used with any consistency. Whether or not "science" is meant to include mathematics and the social sciences is not always readily clear to the reader from the context. Mathematics educators who read the report may tend to find this ambiguity disconcerting.

In reviewing Case Studies in Science Education, one must bear in mind that the purpose of the study was to provide the NSF with information on the status of pre-college science education in the United States. On the basis of the report, we were not able to determine exactly what information was desired by the NSF. For example, page c:4 of the report refers to 17 substantive questions which were raised in the request for proposals (RFP) to guide case study observation and analysis of the project. However, we were unable to find statements of these questions anywhere in the report. We do not understand why these questions are not specifically listed in the report along with answers that can be traced to specific information gained from the site visits and the surveys. In view of our inability to determine from the CSSE report precisely what information was requested, we shall leave it to the NSF to determine if the desired information was provided in the report.

In our opinion, the report contains little information of specific usefulness to mathematics educators. The study tends to focus on general rather than specific education issues. For example, at the start of the study the CSSE staff saw three large foreshadowing problems:

> How is science being taught today?
> What are the current conceptualizations of science in the classroom?
> What are the current encroachments upon the science curriculum?

The list of issues changed over the course of the study. For example, section d under <u>Interpretations</u> (part 5) of this report lists eight science education issues that were identified from the case studies. However, issues of particular interest to mathematics educators such as the impact of hand-held calculators, the metric system, teaching problem solving, or the learning of mathematics by minorities and females did not emerge.

It is important to remember that case study research, in particular multiple case study research, is not a highly refined technique. Furthermore, while most research studies focus on that which is generalizable across instances, case studies tend to focus on the idiosyncrasy and complexity of the particular case. Consequently, the task of documenting generalizations on the basis of a multiple case study is not an easy one.

We also recognize that to identify a set of case study sites which is both manageable and representative is a difficult task, so the project necessarily presents formidable problems. Yet, we were discomforted by what appears to be the random and fragmentary nature of the reports. Much of this seems to result from the lack of uniformity both in the conduct of the case studies and in the reporting of the findings.

While each study is interesting to read, the reader is nonetheless reminded of the blind men and the elephant. It is a bit like being handed a set of snapshots from a single roll of film, some taken with a telephoto and some with a wide angle lens, and trying from this to construct an adequate portrayal of a school system.

While the field observers were well identified, we did not find sufficient identification of the site visitors either with respect to the background and qualifications of individuals or with respect to the composition of the various teams. Thus, we frequently found ourselves questioning the observations, wondering about the perspectives from which the visitors viewed schools. We also were disappointed by the abandonment of plans to utilize a standard instrument for classroom observation in the case studies. Such a systematic approach would have been a strength and it would have provided much more descriptive information about what takes place in classrooms.

We appreciate the investigators' goal of sampling persons on all sides of issues and from the entire spectrum of educational responsibility. However, we could find no documentation that this goal was achieved or even that it was undertaken in any systematic way. We also would have found it useful if the project had included in each case study report certain common and easily collected data such as enrollment trends, instructional costs, curricular offerings, etc. At times, too, it was difficult to differentiate between what derives from other sources such as literature reviews and what is based on actual observations in the case studies.

The case study reports are, for the most part, "chatty" and anecdotal. They reflect a tendency to report vignettes and partial quotes, and the great variation in their length and specificity make it extremely difficult to draw conclusions about the status of education in general or science education in particular. Although the report includes two volumes of "findings" and an executive summary, it is difficult to discern the source of these conclusions. The reports of the site visitation teams, for example, are referred to but not reprinted, A more serious concern is the investigators' admission that the conclusions and recommendations go beyond the reported findings. Despite the general unfamiliarity of educators with case study research of this magnitude, there remains a basic uneasiness with what often appears to be undue reliance on the partially quoted remarks of a few persons within a single school.

The survey also presents concerns. The underlying idea of validating the case study findings through a national survey is a good approach. However, it is often difficult to estimate the validity of the responses. Too often, it seems, respondents react not to the issues or situations presented in the scenarios of the survey, but to their own local needs and experiences. There, likewise, is a concern which arises over the decision to present each scenario to only two or three respondent groups. Can one be sure that the resulting sampling is adequate or appropriate? Further, groups of respondents frequently express very divergent responses to situations, yet no further explanation or interpretation is offered.

In summary, we would recall that the results of case studies are dependent upon subjective observation and are particularly susceptible to the influences of the biases of the observers. Multiple case studies are additionally vulnerable to the biases of the researchers who assimilate the results across case studies. These factors are acknowledged by the CSSE authors and there is sufficient information for readers of the entire report to make judgments about such subjectivity. However, persons who read only the executive summary should be continually aware of the above and should be reminded that if the study had been done by a different set of evaluators with a different orientation, the points emphasized and the general tone of the executive summary might have been significantly different.

In the last analysis, we believe that while one could gain some insights about the conditions that exist in the schools by reading the eleven individual case studies, one's time would probably be better spent visiting schools and personally observing the teaching/learning process.

REPORT OF THE 1977 NATIONAL SURVEY OF SCIENCE, MATHEMATICS, AND SOCIAL STUDIES EDUCATION. Weiss, Iris R. (Project Director). Final Report prepared for National Science Foundation (Contract No. C7619848). Research Triangle Park, North Carolina: Center for Educational Research and Evaluation, Research Triangle Institute, March 1978. ERIC: ED 152 565.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert B. Kane, Purdue University.

The National Science Foundation contracted for a national survey of science, mathematics, and social studies, listing the following interest areas:

(1) What science courses are currently offered in the schools?

(2) What local and state guidelines exist for the specification of minimal science experience for students?

(3) What texts and other instructional materials are being used in science classrooms?

(4) What share of the market is held by specific textbooks?

(5) What regional patterns of curriculum usage are evident and what patterns exist with respect to urban, suburban, rural, and other geographic variables?

(6) What laboratory or activity-centered materials are being used and to what extent and frequency?

(7) What audio-visual materials are used and to what extent and frequency?

(8) How much time, compared with other subjects, is spent on teaching science?

(9) What is the role of the science teacher in working with students and how has this changed in the past 15 years?

(10) What are the roles of science supervisory specialists in school districts and state departments of education?

(11) How have science teachers been influenced in their use of materials by federally-supported in-service training programs?

This review is restricted to those portions of the Report which deal with mathematics education.

Chapter 1. In addition to a statement of purpose, the sample design, instrument development, data collection, analysis, and report structure are described. National probability samples of school districts, schools, and teachers were drawn so that national estimates with respect to each question asked might be made. The sample strata made it possible to derive such estimates for various factors such as community type or geographic region. The sampling design was constructed thoughtfully and carefully, and meets the criteria of a probability sample.

Instrument development included identifying important variables from the literature, reviewing those variables by NSF staff, preparing preliminary questionnaires, and reviewing draft questionnaires by representatives of the Association of State Supervisors of Mathematics. After refinements based on all the above, drafts of the questions were mailed to consultants in both schools and universities. In addition, representatives of AAAS, APA, and EPIE reviewed the draft questionnaires. A second revision of the questionnaires was then developed and reviewed by representatives of the Committee on Evaluation and Information Systems of the Council of Chief State Shcool Officers. The instruments were field-tested and final versions of the questionnaires were approved by the Office of Management and Budget (OMB).

Data collection involved mailing the questionnaires to respondents in the sample and carrying out a variety of mailgram and telephone follow-up steps as needed. Final response rates ranged from 90% for state supervisors down to 72% for district supervisors. Both teachers (76%) and principals (84%) had higher response rates than district supervisors.

<u>Chapter 2</u>. <u>This chapter includes data about state and local supervision of instruction and course requirements.</u>

In summary, 28% of the states reported the existence of guidelines for time spent in mathematics instruction in the elementary school, and between 36% and 40% of the districts in the survey reported guidelines for the minimum amount of time spent per day in mathematics instruction in grades 1-6. Some 23% of the districts responding indicated that there were minimum guidelines for mathematics instruction in the kindergarten. Fifty-seven percent of the states required one year of high school mathematics for graduation. When examining region responses, one notes that the Northcentral region reported 82% of the states requiring a minimum of one year and that this requirement ranged down to 29% in the Northeast region. While most states required at least a year of high school mathematics for graduation, only 7% of the states indicated that they required specific courses in mathematics to satisfy their requirement. On the other hand, 40% of the school districts reporting indicated that they required specific courses. The same districts reported that 93% of their elementary schools were required to use standardized tests to measure mathematical achievement. Sixty-seven percent of the secondary schools of these districts used standardized tests for this purpose. A major use of tests was to report results to individual teachers, expecially in the elementary school. Of the several uses stated for test results, the least important seem to have been to determine topics for in-service education programs for teachers, to report progress for federally funded programs, and to place students in programs for gifted or talented.

Only 58% of the states employed one or more people who spent most of their time on state-wide coordination of mathematics instruction. Across the nation the average amount spent on state-wide coordination was $48,000. The Northeast region, which reported the lowest percentage of required mathematics courses for high school graduation, spends the most on state-wide coordination ($\overline{X}$ = $61,000). Sixty-three percent of the districts reported no district supervisors. Of those districts where supervision is available, it usually is less than full-time. For example, only 26% of the districts reported a person spending 75% or more of his/her time on supervision of elementary school mathematics and only 16% reported a secondary supervisor. District supervisors almost always had prior relevant teaching experience (87%) and certification as a supervisor (80%), as well as a masters degree in a relevant field (65%).

Chapter 3. In this chapter information about mathematics offerings in the elementary and secondary schools is summarized.

Elementary teachers were asked about the number of minutes per day spent teaching mathematics. In K-3 the average was 41 minutes, while in grades 4-6 it was 51 minutes. It is interesting to note that reading is afforded about twice as much time as mathematics which, in turn, is afforded about twice as much time as either social studies or science. The most heavily enrolled secondary school course is elementary algebra (2.8 million), followed by geometry (2.6 million). Ninth-grade general mathematics enrollment is estimated at 1.7 million, while advanced algebra is estimated at 1.4 million. Computer mathematics at 153,000 has substantially larger enrollments than probability or statistics with 40,000. There are an estimated 105,000 enrollments in calculus. Nearly half of the mathematics classes offered in schools having grades 10-12 are homogeneously grouped -- a far higher proportion than in elementary schools. The average class size for the span K-12 ranges from 23.6 in grades 10-12, to a high of 27.7 in grades 4-6. The overall average is 25.5.

Chapter 4. Data related to the dissemination and use of federally funded curricular materials are included in this chapter.

Of the state mathematics supervisors, 77% indicated that they had attended one or more NSF-sponsored institutes, conferences, or workshops. Eighteen percent of the elementary school respondents and 39% of the secondary respondents indicated that they had attended such activities. When asked about major sources of information relative to curricular materials, state supervisors said that meetings of professional organizations, journals, and other professional publications were the most common sources, with publishers and sales representatives also considered useful. Similar response patterns were noted for local-district supervisors. At the district level, teachers were rated as a major source of information, and college courses were also named. Elementary teachers were far more likely to indicate that principals and local in-service programs were major sources of information than were secondary teachers. In general, elementary teachers tended to rely more heavily on local information sources than did secondary teachers. The more frequently disseminated instructional materials by state departments of education were SMSG materials, Stretchers and Shrinkers, Developing Mathematical Processes, Individually Prescribed Instruction, and Unified Science and Mathematics for Elementary Schools. Only SMSG materials had been disseminated in more than half of the states. It is interesting to note that only 14% of the states had disseminated information about more than half of the mathematics materials, compared to 36% of the states in social studies and 64% in science.

Although 58% of the school superintendents indicated that federal support for curriculum development and dissemination has improved the quality of curriculum alternatives, only 27% believe that these efforts have improved the quality of instruction greatly. About two-thirds of the superintendents believe that continued federal support for curriculum development during the next decade is important, with 77% feeling that NSF should continue to help teachers learn to implement NSF-funded curricula and 55% believing that the federal government should direct more attention toward dissemination of new curricula. On the issue of whether or not federal support for curriculum development and dissemination tends to create a nationally uniform curriculum, school superintendents are essentially evenly divided -- 47% agree, 45% disagree.

Prior to 1976-77, 64% of the elementary schools and 71% of the secondary schools asked were using none of the listed mathematics curricular materials. In 1976-77, 92% of the elementary schools and 91% of the secondary schools indicated that they were using none of the listed curricular materials. Schools in the Northeast sector of the nation are significantly more likely to be using one or more of the federally funded curriculum materials than schools in the other three regions of the country. Schools in small cities and suburban areas are significantly more likely to be using one or more of the instructonal materials packages than those in urabn areas. Moreover, large schools are more likely than small schools to be using some federally funded curricular materials, schools in districts with high per-pupil expenditures are more likely to use federally funded materials, and schools with a very small percentage of children who quality for federal free-lunch programs are significantly more likely to be using these materials.

<u>Chapter 5</u>. <u>In Chapter 5 one finds more explicit data about textbooks and programs in mathematics used by the nation's schools</u>.

The most commonly used textbooks/programs by subject and grade are shown in tables, and in the appendix one finds a list of all the textbooks and programs which are being used by 2% or more of the classes in each subject/grade category. Uniformly across the grades from kindergarten through 12, the textbooks in use were more frequently seen to have a copyright between 1974 and 1977 rather than an earlier date. Between one-half and two-thirds of the mathematics classes in each grade are using textbooks/programs which have accompanying supplementary materials. The teacher's manuals which typically accompany elementary and secondary textbooks are the most extensively used of the listed supplementary materials. Two-thirds of the elementary teachers queried made use of teacher's manuals or teacher's editions of their mathematics textbooks. The next most popular supplementary material was the student workbook. In grades K-3 almost 60% of the classes used workbooks. In no other grade category was workbook usage found in more than one-third of the classes. Except for grades K-3, roughly one-third of the mathematics classes utilized published testing materials. Thirty-six percent of the K-2 mathematics classes made use of manipulative materials which accompanied textbooks. No other grade range category showed usage in as many as 20% of the classes.

With respect to the selection of textbooks/programs, principals, superintendents, and teachers were quite similar in their perceptions about who does the selection. All three groups agree that students, parents, and school board members have low involvement in this process. About half the principals indicated that they, themselves, are heavily involved in textbook selection; only 2% indicated lack of involvement. About half the superintendents indicated that principals are heavily involved in textbook selection. Elementary teachers were significantly more likely to indicate that principals are heavily involved in this process than were their secondary school counterparts, even though all groups were asked about the textbook selection process in their school district as a whole, not just in a specific grade range. District supervisors are heavily involved in textbook selection in about one-third of the schools and somewhat involved in about one-fourth of the remaining schools. All groups indicated that teacher committees and individual teachers are the groups most heavily involved in textbook selection. Only 3% of the schools indicated that individual teachers are not involved in textbook selection, while about two-thirds of the schools indicated that individual teachers are heavily involved.

When teachers were asked to indicate the textbook/program they would use in mathematics for their classes, if given free choice, 63% of the mathematics teachers indicated they would choose the one they were currently using. Approximately one-fourth of the teachers indicated they would choose a different textbook/program.

<u>Chapter 6</u>. <u>In this chapter information about instructional techniques and classroom activities in mathematics classes was reported</u>.

Each teacher was asked a series of questions about instruction in a single randomly selected mathematics class; then each teacher was given a list of materials appropriate to mathematics instruction and asked to indicate the availability and use of each material. And, finally, teachers were asked to focus on the most recent mathematics lesson they had taught, and asked about instructional arrangements and activities used in that lesson. About half of all mathematics classes were reported to include lectures "just about daily", while an additional 20% of the teachers indicated that lectures were included at least once a week. On the other hand, 23% of the teachers indicated that they never used lecturing in their mathematics classes. The vast majority of this latter group were elementary school teachers. About 72% of the teachers indicated that lecturing was used in their most recent lesson. The range was from 58%, in grades K-3, up to 89% in grades 10-12. Class discussion was the most popular instructional mode; 71% of the mathematics classes included discussions on a daily basis. Seventy-two percent of the mathematics classes in the sample included chalkboard work at least once a week. Individual assignments were very common in mathematics classes; 59% of the classes included them daily. Thirty-nine percent of the mathematics classes included manipulative materials in their most recent lesson. While the percentage was much higher in K-3 (58%) than at higher grades, it is interesting to note that even at grades 10-12 approximately 25% of the most recent lessons included manipulatives. Televised instruction, programmed learning, and computer-assisted instruction are infrequent or rare occurrences in mathematics classrooms. Except in grades K-3, the vast majority of mathematics classes included tests and/or quizzes; 62% of the classes included tests or quizzes once a week or oftener. Some 38% of the classes in grades 4-6 incorporate some sort of contract approach to teaching assignments; about half of these classes utilized the contract approach at least once a week. At other grade levels only minimal use of contracts was reported. Almost two-thirds of the mathematics classes used teacher demonstrations at least once a week. Surprisingly, this compares with 38% of the science classes.

Teachers reported that the percentage of time spent in various instructional arrangements in mathematics classes was as follows: entire class working as a group, 43%; small groups, 23%; students working individually, 34%. The use of the small-group technique remained relatively constant across grade levels, while the entire class as a group increased in percentage as the grade level increased and individual work decreased in percentage as the grade level increased.

The only audio-visual device (exclusive of the chalkboard) employed at least once a week in more than a very small percentage of the mathematics classes was the overhead projector. Frequent use was made of games and puzzles in mathematics classes up through grade 9. In grades 10-12 there is a marked decrease in the use of such devices. The same pattern of diminishing use with senior high school classes is found with such materials as activity cards, kits, numeration and place value manipulatives, rods or blocks, and the like.

Chapter 7. Information about facilities, equipment, and supplies was collected from superintendents, principals, and teachers and reported here.

There were two principle external sources of funds for mathematics instruction. One of these was funds available under NDEA titles (26% of the districts) and the other under ESEA titles (52% of the districts). Other federal, state, private foundation, and organization grants were few in number. A computer or computer terminal was available to 36% of the senior high schools, 16% of the junior high schools, and about 7% of the elementary schools. Hand-held calculators were a more common item. The percentage of availability ranged from 28% at grades K-3 up to 77% at grades 10-12. Mathematics laboratories were found most frequently at the junior high school level (31% of the schools). At other grade levels the frequency of availability of a mathematics laboratory ranged from 13% to 10%. In general, suburban schools were the best equipped, with urban schools in second place. Schools in small cities and rural areas appear to be the least well equipped. When asked about the availability of hand-held calculators, 77% of the teachers at grades K-3 indicated they were not needed, and 44% of those at grades 4-6, 42% at grades 7-9, and 33% at grades 10-12 agreed.

Chapter 8. Teachers were asked about their experience, training, and qualifications for teaching various subjects and about areas in which they felt the need for additional assistance.

There was surprising uniformity in the average number of years of teaching experience among all categories of teachers. The average was

12.2 years. The greatest deviation was among the senior high school teachers whose average was 11.2 years. About 50% of the teachers working in grades 7-12 had earned a degree beyond the bachelor's, while 34% of those teachers working in grades K-6 had done so. Almost as many individuals indicated that the last course for college credit taken was accomplished in 1976-77 as indicated that their last course was taken prior to that time. Ninety-five percent of the elementary teachers believe themselves to be adequately or very well qualified to teach mathematics; only four percent indicated that they felt not well qualified to do so. (Essentially the same distribution of teachers was noted for the area of reading -- some 63% felt themselves to be very well qualified and an additional 32% felt themselves to be adequately qualified in reading.)

Although most educational placement directors and many people hiring teachers are asking that individuals prepare to teach in more than one subject, the data do not indicate that secondary teachers very often are required to do this. Seventy-six percent of the junior high school teachers and 85% of the senior high school teachers teach all of their courses within a single subject area.

Eleven percent of the junior high school mathematics teachers and 5% of the senior high school mathematics teachers feel inadequately qualified to teach one or more of their courses. The remainder, the overwhelming majority, do not feel inadequate for any of their assignments. On the other hand, many teachers felt the need for assistance in one or more areas in which they teach. For example, only about one out of four teachers indicated that there were no areas in which assistance would be appreciated. Among the kinds of assistance most frequently requested were: learning new teaching methods, obtaining information about instructional materials, implementing the discovery/inquiry approach, and using manipultive or hands-on materials. A consistent result across all the grade ranges was that the large majority of teachers indicated they do not need assistance in lssson planning, teaching lessons, and maintaining discipline. Two out of three teachers felt the need for assistance in obtaining information about instructional materials, yet only one in three felt adequate assistance was provided in this area. Well over half of the teachers indicated a need for assistance in learning new teaching methods; about four out of ten are not currently receiving such assistance. About half of the teachers indicated they would like assistance in the use of manipulative materials and only 14% felt they were receiving an adquate levelof assistance in this area.

Chapter 9. Teachers, principals, district-level supervisors, and state supervisors were given a list of sources of information about new developments and asked to rate the utility of each.

Teachers were perceived as an important source of information by fellow teachers and, to a somewhat lesser degree, by principals and (with diminishing fervor) by district supervisors and state supervisors. Interestingly, principals felt that other principals were more important sources of information than teachers. District and state-level supervisors did not feel that principals were an important source of information. Elementary teachers thought that subject matter coordinators at the district level were more important to them than did secondary teachers. State supervisors were more inclined to rate local or district supervisors as an important source of information, while both principals and other local supervisors were somewhat less enthusiastic. State supervisors found other state department personnel to be very useful sources of information. On the other hand, very small percentages of principals, district supervisors, or teachers considered state department personnel very useful. Teachers rated college courses as very useful sources of information. Principals and local supervisors were somewhat less enthusiastic. Interestingly, those principals and local supervisors who were more concerned with secondary education felt that college courses were more useful sources of information than did those working primarily at the elementary level. Very few state supervisors found college courses a useful source of information about new developments in education. In-service programs were more valued by elementary than secondary personnel. Fewer principals and teachers thought federally-sponsored workshops an important source of information. On the other hand, state supervisors were more likely to rate such workshops useful. Very few educators consider teacher union meetings a useful source of information. Seventy-nine percent of the state supervisors in mathematics found professional meetings very useful as a source of information about new developments in education. Percentages were lower for principals and district supervisors, and very low for teachers. On the other hand, secondary teachers are significantly more likely than elementary teachers to rate professional meetings as very useful. Journals and other professional publications were considered very useful sources of information by many people in all categories. The trend was that state supervisors, district supervisors, principals, and teachers (in that order) rated such publications as very useful. Within the teacher group, elementary teachers were least likely to find journals useful. There is a clear increase in the percentage finding journals useful associated with increase in grade range of mathematics teaching. One out of three state supervisors indicated that publishers and sales representatives were an important source of information. For teachers, principals, and district-level supervisors, the percentages were substantially lower.

Chapter 10. Teachers, principals, and supervisors were given a list of "problems" and asked to rate the seriousness of each one.

The "problems" were generated from a list of factors which have potential for affecting instruction in mathematics. Neither the belief that mathematics is less important than other subjects nor compliance with federal regulations was considered a serious problem by any of the respondent groups. Fewer than 15% of the respondents in each category felt that inadequate facilities was a problem for mathematics instruction. All respondent groups felt that insufficient funds for purchasing equipment and supplies was a serious problem in mathematics instruction. Lack of materials for individualizing instruction in mathematics was considered a serious problem by 18% of the teachers and principals, almost half of the district-level supervisors, and over 60% of the state supervisors. The use of out-of-date teaching materials was not considered a problem in mathematics instruction nor was there an insufficient number of textbooks according to each category of respondents. Lack of student interest in mathematics appears to be more of a problem in secondary schools than in elementary schools. Each respondent group indicated significantly greater concern about this for secondary students than for elementary students. Inadequate student reading ability was a problem in the secondary schools, according to all the groups questioned. For example, about one-fourth of the mathematics teachers rated this as a serious problem. Principals' and supervisors' ratings were quite similar to those of teachers'. Lack of teacher interest in mathematics was not considered a serious problem, either in the elementary schools or the secondary schools. State supervisors of mathematics consider inadequately prepared elementary teachers to be a serious problem. However, elementary teachers do not share this opinion. Fewer than 10% of them rated inadequate teacher preparation a serious problem. Elementary teachers regarded lack of teacher planning time as a serious problem. It was significantly less of a problem as perceived by secondary teachers. Principals tended to agree, as a group, with the teachers. However, district supervisors were not nearly so likely to consider lack of teacher planning time a serious problem. Inadequate time to teach mathematics was not considered a serious problem at any level. Teachers rate large class sizes as a serious problem with the percentages increasing as grade levels increase. Principals and supervisors do not perceive class size to be as serious a problem as do teachers. Fewer than 10% of the teachers, principals, and district supervisors regarded difficulty in maintaining discipline as a serious problem for mathematics instruction. The most serious problem identified by state supervisors was inadequate articulation of instruction across grade levels. Principals and district supervisors also consider articulation to be a problem, but not nearly as serious as state supervisors consider it. Only about 10% of the teachers rate inadequate articulation as a serious problem.

Evaluative Summary

The narrative portions of the Report amount to 166 pages. They are followed by almost twice as many pages of appendices which include complete tabular presentations of the responses to all questions by all groups. What was abstracted above was based almost exclusively on data which found their way into the narrative chapters. In short, there is a wealth of survey information on elementary and secondary mathematics education in the United States in this Report. The Report is extremely valuable. Since the complete Report covers science and social studies as well, comparison data are readily available. These data are at least as useful as the information about mathematics education per se.

As stated earlier, the sampling design meets the criteria of a probability sample, and thus national estimates and estimates for various levels may be inferred for each question. In a survey of this magnitude, it is always possible to take exception to the scope of the items included. Were unimportant questions asked? Were important questions left unasked? In this case it would be meaningless to undertake such an exercise. The Report is one of seven documents summarizing the state of pre-college mathematics, science, and social studies education in the United States. None of the individual documents should be expected to be completely comprehensive. Taken as a whole, they provide as comprehensive a review of mathematics education in the elementary and secondary schools of the United States as has ever been available.

INTELLECTUAL DEVELOPMENT AND REVERSIBILITY OF THOUGHT IN EQUATION SOLVING. Adi, Helen. Journal for Research in Mathematics Education, v9 n3, pp204-213, May 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Leland F. Webb, California State College, Bakersfield.

1. Purpose

The purpose of the study was to investigate how students of different developmental levels would perform when solving equations that required repeated applications of different reversible processes called inversions and compensations.

2. Rationale

Solving equations is necessary in many mathematical problems. The inability to solve equations correctly is common with many high school and early college students. Reversibility of thought is necessary in equation solving, and according to Piaget's theory two forms of reversibility are applicable at the concrete operational level: (1) negations (inversions) and (2) reciprocities (compensations). If equation-solving strategies are identified that require repeated applications of inversions or compensations (but not both mixed), then "success in learning these different methods of equation solving may be very much dependent on the developmental stage of the learner."

3. Research Design and Procedure

The following hypotheses were tested:

a. "On reversal (inversion) equation solving, subjects at the early formal operational stage will perform at least as well as those at the late concrete formal operational stage who will in turn perform as well as those subjects at the early concrete operational stage.

b. "On formal (compensation) equation solving, subjects at the early formal operational stage will perform at least as well as those at the late concrete operational stage who in turn will perform better than those subjects at the early concrete operational stage.

c. "Differences in mean performance scores between reversal and formal equation solving are greater for the early concrete group than for the early formal group."

Seventy-five elementary education majors (6 males, 69 females) participated in the study. Prior to the study all students had at least one mathematics course which contained a unit on solving equations. Using a 15-item paper-and-pencil Piagetian task (developed by the experimenter) involving keeping a balance beam in equilibrium, students were grouped into three developmental levels: (1) Early concrete operational thought (n = 37), Group IIA; (2) Late concrete operational thought (n = 26), Group IIB; and (3) Early formal operational thought (n = 12), Group IIIA. The balance beam task was selected because it requires reversibility of thought. The coefficient of reproducibility of the test was established prior to its administration at .96.

To measure proficiency, a five-item pretest on equation solving was administered prior to the instructional treatment, but was later rejected because 68 of 75 students scored zero on the test. The experimenter used five fifty-minute instructional periods to teach three classes two objectives: (1) When an equation with one unknown occurring only once in the equation is presented, the student will find the solution by repeated applications of inversions; and (2) When an equation with one unknown is presented, the student will apply compensations to both members of the equation, determine a solution, and check to see if the solutions satisfy the given equation. Worksheets and exercises were provided during the instruction.

A twelve-item posttest was developed, requiring students to solve six equations using inversions and six equations using compensations. The split-halves reliability of the test was .84. Data were analyzed by one-way analysis of variance.

4. Findings

The average difficulty level on the first six items was .96 and for the second six items, .74. Success on each of the two methods was defined as correctly solving five of six problems. The computed coefficient of contingency between developmental levels and equation-solving success was $C = .23$ ($p < .05$), which indicated a positive relationship between developmental levels and success in the two methods of equation solving.

The ANOVA results on reversal equation solving of the three groups yielded a significant F-ratio ($F[2,72] = 5.8$, $p < .01$). Further analysis by pairwise contrasts indicated that Group IIB scored significantly higher than Group IIA. Similar results were found for the ANOVA results on formal problem solving.

A t-test was used to compare whether differences in mean performance scores between reversal and formal equation solving were equal for Groups IIA and IIIA. The difference was significant ($t[47] = 4.32$, $p < .01$).

5. Interpretations

The study supports the hypotheses tested:

a. There is a significant positive relationship between the developmental levels of learners and their performance on equation solving when different processes are applied.

b. Subjects scored significantly better on reversal equation solving than on formal equation solving.

c. The results confirmed that the three developmental levels performed well on reversal equation solving.

d. Performance of IIA subjects on formal equation solving was significantly lower than IIB and IIIA subjects.

e. A significant t-value confirmed that for Group IIA reversal equation solving was easier than formal equation solving. This did not hold true for Group IIIA.

Critical Commentary

This study seems to be well-constructed and carefully executed. The experimenter seemed to be quite careful about controls placed on the tests devised and the instructional treatment. Good validation procedures for the paper-and-pencil test and the equilibrium-in-a-balance test (EBT) were used; it seemed appropriate to reject the paper-and-pencil pretest because so many subjects received zeroes on it; and there appeared to be careful restrictions applied to the generalizability of the results in that the IIA, IIB, and IIIA categorization was assumed to be specific to the EBT test.

There are, however, several methodological questions that might be raised, as well as a few questions regarding the use of the statistics:

(1) The experimenter indicated that data were analyzed by one-way analysis of variance, yet the first table presenting data (Table 2) uses a contingency coefficient which is a chi-square analysis.

(2) No significance level was mentioned when the hypotheses were stated. It appears that the .05 level was utilized, but all of the levels of significance reported are .01 except one, which is .05. To some this may seem a minor point; however, establishing a significance level in advance eliminates any further need for reporting levels of significance, and it also eliminates the need for making determinations of significance "after the fact."

(3) It appears that a paper-and-pencil test with a diagram was used for the EBT. Why wasn't an actual balance beam utilized? This could have been explained.

(4) It is not clear why the formal equational problems are more difficult. One can merely look at the problems and see that they are more difficult; however, the experimenter does not indicate if this was intentional or merely because the reversal equation-solving method does not lend itself to problems with variables on both sides of the equation.

(5) It is interesting to note that there are three F-ratios that are identical, two of which have the same degress of freedom, F [1,72] = 7.1, and one which does not, F [2,72] = 7.1. One might wonder whether this is coincidental, but it cannot be checked unless the individual scores are supplied by the experimenter.

(6) The experimenter noted that "success on each method of equation solving was defined by correctly solving at least five equations out of six", and yet, in the subsequent paragraph, she reported that mean performances are reported on a total of 60 points, with the following table reporting values such as 51.08, 58.46, and 53.85. The abstractor understands what was done in each of these cases; however, one wonders why there seem to be inconsistent methods applied in various situations. In the first case, percentages were used to calculate the contingency coefficient to determine whether a positive relationship exists between developmental levels and success on the two methods of equation solving. In the other case, the F ratios were calculated and the abstractor assumes that a more accurate value was required.

Two things seem apparent: (a) if a contingency coefficient is calculated to determine a relationship, it should be stated that it will be used and the suggested relationship should be stated as an hypothesis; and (b) it appears more accurate to consider the percentage of the total items correct for each method and each level, rather than considering five out of six as a criterion for each person and then counting the number of individuals reaching this arbitrary criterion.

(7) Any reader should note that the number of individuals at each level varies greatly, that the standard deviations are not reported in Table 1 (and could have been done quite easily), and that for some of the F-tests the standard deviation of the group scores had to be zero (everyone received a perfect score of 60) or close to zero (58.46 and 58.33). One wonders what effect this had on the statistical results?

(8) Rather than conduct a series of tests using analysis of variance, an interesting alternative would be to use the EBT as a predictor in equation solving, allowing the students to select the method of equation solving, and look for interactions between level and method. This might be an alternative for future research.

(9) Hypothesis 1, as stated, was not confirmed, because there were statistically significant differences between the three groups. Hence, all groups did not "perform at least as well." However, the experimenter implies this when she says in her conclusions that "It was theoretically expected to have the three developmental groups perform well on reversal problem solving. The results confirm this expectation." What does the experimenter mean by "perform well?"

(10) Much of the criticism outlined above is a result of the report of the experiment. As with the content of the study, a great deal of care also is required in writing the report to make it as clear as possible. Unfortunately, in the case of this report, the clarity of the presentation detracted from the uniqueness of the study itself.

IMPROVING SKILL IN APPLYING MATHEMATICAL IDEAS: A PRELIMINARY REPORT ON THE INSTRUCTIONAL GAMING PROGRAM AT PELHAM MIDDLE SCHOOL IN DETROIT. Allen, Layman E.; Ross, Joan. Alberta Journal of Educational Research, v23 n4, pp257-267, December 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward J. Davis, University of Georgia.

1. Purpose

The authors had in mind to investigate the question "...whether skills in applying mathematical ideas can be improved by learning procedures which emphasize exposure to situations that are rich in opportunities for such application at levels of complexity appropriate for each learner." (p. 258) The context in which this general question was investigated asks the specific question: Does the experience of playing EQUATIONS and studying an IMP (Instructional Math Play) kit improve students' computation test scores and scores on test items that require students to select appropriate operations and apply mathematical principles taught in the IMP kit?

2. Rationale

It is not enough to be able to perform a given computation. Students must also understand when it is appropriate or necessary to employ a given computation and be able to utilize principles associated with that computation. The principles related to the order of operations, commutativity, associativity, and the distributive property. The authors state their experiences indicated two-thirds of those students who solve a computation problem fail to solve a corresponding application problem.

3. Research Design and Procedure

Five groups of eighth-grade students from an inner-city middle school were selected. Entire classes were placed as follows:

Group Name	Number of Classes	Treatment (2 weeks between pre- and posttests--10 instructional days)
I	1	IMP Kits (programmed instruction) used 5 of the 10 days; EQUATIONS tournament for 2 days
E	1	EQUATIONS played 5 of the 10 days; no instruction in IMP content

Group Name	Number of Classes	Treatment (2 weeks between pre- and posttests--10 instructional days)
TE	2	EQUATIONS played 5 days; teachers explicitly taught content in the IMP kits
O	3	Regular class schedule followed no EQUATIONS-no IMP material
T	3	Regular class 5 days teacher led IMP instruction 5 days

Two pretests were given:

Test C-b (Computation - before)
Test R-b (Recognition and Application - before)

After the two-week treatment two posttests were administered:

Test C-a (Computation - after)
Test R-a (Recognition and Application - after)

The following are given as examples of a C item and an R item respectively (these are the only examples given):

Computation:

1. 6-(1-3)=____?____.

Recognition and Application:

	A	B	C	Yes	No
2.	--136	8			

By writing an X in the Yes or No column, indicate whether or not all of the numbers and operations in Column A can be appropriately ordered and grouped (inserting parentheses wherever necessary) to form an expression equal to the number in Column B. If your answer is Yes, write that expression in Column C.

4. Findings

(Total possible 21)

Group	$\bar{X}$C-b	$\bar{X}$C-a	$\bar{X}$R-b	$\bar{X}$R-a
I	7.70	9.26	3.39	6.78

Group	$\bar{X}$C-b	$\bar{X}$C-a	$\bar{X}$R-b	$\bar{X}$R-a
E	7.39	8.35	2.88	4.12
TE	5.33	7.48	1.65	2.84
O	4.31	4.74	0.83	0.95
T	4.02	5.32	0.59	1.41

The authors also report the following scores from the Stanford Arithmetic Advanced Computation for 112 of the 237 subjects (this test was given approximately two years before the reported experiment was undertaken):

Group							Equations Groups	Non Equations Groups
	All	I	E	TE	O	T	I+E+TE	O+T
N	112	22	14	25	23	28	61	51
$\bar{X}$	46.54	49.95	52.07	44.36	44.96	43.50	47.79	44.16

5. Interpretations

The authors made the following statement based mainly on R-a scores for all five groups and (R-a) - (R-b) figures for all five groups.

> This study provides strong support for the proposition that skills in applying mathematical ideas can be improved by learning procedures that are rich in opportunities for such application at appropriate levels of complexity for each student. Interpreted most favourably, the results show that the combination of playing EQUATIONS over a two-year period and then working intensively with the IMP kits for two weeks enables students to apply mathematical ideas (in the sense studied in this experiment) better than any of the other four sets of conditions do: better than just playing EQUATIONS alone, better than playing EQUATIONS and being taught explicitly by the teacher the 21 ideas presented in the IMP Kits, better than being taught the 21 ideas in an ordinary traditional mathematics class, and better than being in an ordinary traditional class without any special teaching of the ideas - and, furthermore, better in each case by a highly conservative test at an extreme level of significance (.0001). It should be acknowledged immediately that there are some questions with respect to this most favourable interpretation which require further investigation. At the start of the experiment the IMP Kit group was clearly performing at a higher level of achievement than were the other groups. This superior performance seems linked to their two-year experience in playing EQUATIONS.

> The available evidence indicates that the EQUATIONS and nonEQUATIONS groups were not different upon their entry to the seventh grade, but that after two years of different experience with respect to whether or not they played EQUATIONS, the EQUATIONS group was significantly better in both computing with and applying these 21 mathematical ideas. (pp. 266-267)

Critical Commentary

First it should be noted that we are dealing with mean scores from intact classes, and more importantly, that on tests of 21 items we have:

Pretest Computation averages in a range of approximately 4-8
Posttest Computation averages in the range of approximately 5-9
Pretest Application averages in the range of approximately .6-3.4
Posttest Application averages in the range of approximately 1-7

Thus we are asked to base conclusions on achievement scores that are "low". This is not an uncommon happening in educational research but disturbing to this reviewer. I am suspicious of the validity of any pretest and posttest combination wherein the "best" group of students average 3.39 and 6.78 out of a possible 21.0. This is the case for group I on tests R-b and R-a on which rests the bulk of the interpretation reported above.

Second, it should be apparent to anyone familiar with the game EQUATIONS that the sample R-type test item given is going to be very familiar to groups I, E, and TE. If (and it apparently was) this is the case, then groups I, E, and Te had a distinct "home court" advantage on tests R-b and R-a over groups O and T.

Third, a question should be raised as to how familiar the teachers of groups TE and T were with teaching the content of the IMP kit (21 ideas). Was this the first time these teachers had a chance to teach the content? This is a relevant question since Allen and Ross claim students learn better from the kits of programmed lessons than from teachers. This claim is even more questionable when one realizes the kits are commercially available and undoubtedly have undergone development, trial, and revision.

But far more important is the question of how appropriate is it to claim superior performance for group I when it is clear from the pretests that group I (and groups I, E, and Te collectively) was considerably more able at the beginning of the two-week experiment? It is not entirely clear what the treatment is that the authors claim is having the effect. Do we have two-year treatments on each of the five groups or do we have two-week treatments on each of the five groups? Their conclusions indicate the former, but the confinement of instruction in the IMP content to a two-week span with pretests and posttests at the outset and conclusion of this interval indicates that the two-week period is critical to the study. If indeed we are to consider two-year treatments on each of five groups, then we need to know a great deal more about what transpired in classrooms for the entire period and something about the teachers of these treatments. After all, a claim is made that these groups were equal in computational ability two years earlier. Are we to conclude that the substantial (by the authors' admission) differences between the groups on the computation and the application pretests are solely the result of playing EQUATIONS for two years? Apparently this is the case.

If we have a two-year study, then what were the treatments that gave us vastly different groups at the start of the two-week testing period? If we have a two-week study, then how do we ignore the significant differences in the groups at the outset of the study? In either case these problems make the study of questionable scientific value to this reviewer.

DISTRACTORS IN NONVERBAL MATHEMATICAL PROBLEMS. Bana, Jack; Nelson, Doyal. Journal for Research in Mathematics Education, v9 n1, pp55-61, January 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Cecil R. Trueblood, The Pennsylvania State University.

1. Purpose

To investigate the effects of distractors in four different settings involving the same problem.

2. Rationale

Problem solving research, child development theories and research involving concept formation indicates that many children seem to experience difficulties in solving problems in "nonverbal" settings because of their attention to irrelevant aspects (distractors) present in the problem situation. Problem-solving researchers believe that clinical studies of individual subjects is a viable approach to investigate the role of distractors in young children's problem-solving behavior. Furthermore, they maintain that this approach should be used as precursors to larger research efforts because the results will help fill in the gaps in the knowledge base concerning how children think when solving problems.

3. Research Design and Procedure

Four matched groups of four boys and four girls each were selected from each of grades one through three in a large, suburban elementary school in Edmonton, Alberta. The total number of children was 96. Each of the four groups at each grade level was randomly assigned to the problem in one of the following settings:

(a) Three identical yellow toy trucks and 12 similar yellow toy cars (minimal distraction).

(b) Trucks and cars as in the instance above; a factory building with parking area and road; a loading ramp; and three toy men—one near the ramp and one on the back of each of two trucks (situational distractions).

(c) One yellow truck, 1 blue truck, and 1 red truck, all of the same shape; and 3 blue cars, 4 yellow cars, and 5 red cars, all of similar size and shape (color-attribute distraction).

(d) Three yellow trucks with six bays marked on the back of each of two of them and five bays marked on the other; 12 similar yellow cars (spatial-numerical distraction).

These settings were based on the different types of distractions identified by Bourgeois (1976).

Each child was presented with the problem individually. The subject was put at ease and familiarized with the setting. Then the experimenter said, "We want to load all of these cars onto the three trucks. Every truck must take the same number of cars. How many cars will be on each truck?" This was repeated once, then as often as necessary to ensure that the subject understood, or in response to any questions asked by the subject. When the subject gave a solution, the experimenter asked, "Are you sure that's right?" This question was repeated until the subject was committed to a solution. Then the experimenter asked, "How did you know there would be ____ on each truck?" The subject was then questioned further to determine the process used to arrive at his or her solution and to ascertain whether or not the subject was distracted. The interview was recorded on a cassette tape recorder, and the behaviors were recorded by the experimenter on a data sheet. Each subject was allocated a score consisting of one point for each of the following: the use of a viable process, a correct solution, and confirmation of a correct solution. Thus each subject could score 0, 1, 2, or 3 points.

4. Findings

Some children attempted the problems by manipulating the cars and trucks. Others did not. They gave only verbal responses. The manipulative solvers were more likely ($p < .005$) to be distracted than the verbal solvers. This was particularly true among the third graders of the students who attained maximum scores; only one-third attended to distractors. A two-way analysis of variance showed that the effects of grade level and problem setting were significant main effects. The "studentized" range statistic (Winer, 1971, pp. 185-187) was used to test for differences between means within categories. In Grade 1, all four groups found the problem to be relatively difficult, and there were no significant differences between means. In Grade 2, both the minimal and the situational distraction groups scored significantly higher than the spatial-numerical distraction group. Although the four means in Grade 3 showed a similar trend to those of the second graders, the differences were not significant. For all three grades considered together, the group in the first problem setting scored significantly higher than the group in the fourth setting.

An interview that focused upon the processes used by students to solve the problems was also analyzed. The analysis revealed that the majority of the third graders could explain the process used as being one of division or partition into equivalent sets, but few children in the other two grades could do this. In the case of spatial-numerical distractors, many subjects simply stated that each truck should take five (or six) cars because there was room for five (or six) or because there should be one car in each marked bay. Subjects who were distracted by color wanted to match cars with trucks of the same color, but were generally unable to give a reason for this.

5. Interpretations

The results of this exploratory study seem to indicate that distractors do affect the problem-solving behaviors of young children. Spatial and numerical distractors appear to be the strongest in the type problems used in this study. The difficulty level of the problems was a significant factor. Interviews that focused upon the students' problem-solving process suggest the distractors could be a function of problem type and/or the mode of representation.

Critical Commentary

This study demonstrates the type of information that can and should be collected using clinical-type studies of students' problem-solving behavior. The investigative procedures used are sound and the interpretations are accurate. In follow-up studies the authors should give consideration to the students' levels of cognitive development and how they interact with the modes of representation. Do concrete operational students respond differently than pre-operational students? In addition to collecting descriptions of the problem-solving strategies children use, the interviews should be used to ask students to justify why they use the strategies they select.

INTEREST AND ITS RELATIONSHIP TO VERBAL PROBLEM-SOLVING. Cohen, Martin P.; Carry, L. Ray. International Journal of Mathematical Education in Science and Technology, v9 n2, pp207-212, May 1978.

Note: Although no reference is cited, this report is apparently based on Cohen's doctoral dissertation, "Interest and Its Relationship to Problem-Solving Ability Among Secondary School Mathematics Students," at the University of Texas at Austin in 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jeremy Kilpatrick, University of Georgia.

1. Purpose

The purpose of the study was to examine the tendency for secondary school students "to be more successful in solving verbal problems based on situations in which they showed some interest, than they were in solving verbal problems based on situations in which they showed little interest" (p. 207).

The authors sought answers to the following two questions:

"(1) Will there be a difference in the mean scores of the three verbal problem-solving tests?

(2) Based on the knowledge of a student's interests alone, is it possible to predict at which type (context) of problem that a student will be most successful, as measured by a verbal problem-solving test?" (p. 209)

2. Rationale

Mathematics educators assume that motivation plays an important part in mathematics learning, and appeal to interests is regarded as one such motive. Although one might logically expect a substantial correlation between an interest and the learning of mathematical content in the area of that interest, previous research attempting to link interest and verbal problem-solving ability has yielded inconsistent results. The lack of clear definitions of constructs may have contributed to the inconsistency; advances in measurement techniques, experimental design, and instruments may alleviate the problem.

3. Research Design and Procedure

The Kuder General Interest Survey, Form E, was administered to 223 eighth-grade mathematics students in the Waco (Texas) School District, and each student's score was calculated in each of three interest areas: outdoor, computational, and scientific.

Three parallel forms of a 10-item problem-solving test were constructed. In one form the context of the problem was outdoor activities, in a second it was computational activities, and in the third it was scientific activities. The only difference in parallel items across forms was to be the context; they were to be as similar as possible in length, style of phrasing, verbal clues, vocabulary level, mathematical operations, and computational difficulty. Problem difficulty was judged appropriate by two eighth-grade mathematics teachers; context validity was judged by an educational psychologist; and parallelism was judged by two mathematics educators. Reliability coefficients (KR-20) for the three forms were 0.76, 0.79, and 0.79, respectively.

The students were sorted by sex and randomly assigned to three groups; each group was given one form of the problem-solving test.

4. Findings

Analyses of variance for the two sexes separately yielded no significant differences ($p = .05$) in the mean scores on the three test forms. Multiple linear regression for the two sexes separately was used to test the parallelism of regression lines across test-form groups when scores on the test were regressed on each of the three interest scores, in turn. There was no significant difference ($p = .05$) in parallelism, suggesting no interaction between problem setting and interest area in the prediction of problem-solving performance. Secondary analyses of regression weights failed to yield predicted patterns.

5. Interpretations

The investigators were somewhat surprised not to find evidence that one could predict the type of context on which a student would be most successful from a knowledge of the student's interests. They suggested that interests may be weak predictors of problem solving, that the problem-solving forms may not have reflected the interest areas very well, and that the moderate reliability of the forms may have reduced their correlations with interests.

The investigators noted that "many students were eliminated from the study for obtaining low scores on the verification scale on the GIS" (p. 212), and that for the remaining students, who tended to have good scores on the California Achievement Test, interest may have played a negligible role in their performance. The investigators recommended additional work in developing problem-solving tests and better interest measures.

Critical Commentary

In constructing the parallel forms of the problem-solving test, Cohen and Carey made a commendable effort to equate the problems on various dimensions other than problem context, and they were sensitive to problems of validity. Unfortunately, they provide no sample problems in their report so the reader can see what sort of problems were used and can judge the equating process. Also, details of sample selection and test administration are lacking, and the loss of an unknown part of the sample owing to low verification scores on the Kuder instrument ought to have been detailed and its effects evaluated.

The Kuder was a poor choice for the study, not just because the scales relate to vocational interests of questionable relevance to motives for solving mathematics problems, but also because the Kuder is notorious for presenting problems of score interpretation. For example, my percentile scores of 90 on the Outdoor Scale and 80 on the Computational Scale do not mean I necessarily have more of an outdoor interest than a computational interest, nor do they mean I have more of an outdoor interest than my classmate who has an 80 on the Outdoor Scale. These problems vitiate the elegant multiple linear regression analysis of the data.

It is difficult to understand Cohen and Carry's call for further research of this type using more refined instruments. Both the empirical findings and common sense suggest that although students' interests may relate to their choices of material for study and ultimately to what they learn, interest in a verbal mathematics problem's context is not likely to account for much of one's ability to solve it.

SEX-RELATED DIFFERENCES IN MATHEMATICS ACHIEVEMENT AND RELATED FACTORS: A FURTHER STUDY. Fennema, Elizabeth H.; Sherman, Julia A. Journal for Research in Mathematics Education, v9 n3, pp189-203, May 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Mary Grace Kantowski, University of Florida.

1. Purpose

To continue the study of cognitive and affective variables that influence males and females to learn mathematics at different levels. Cognitive variables include: computational skill, knowledge of concepts, problem solving ability, verbal ability, and spatial visualization. Affective variables include: attitude toward success in mathematics, mathematics as a male domain, perceived attitude of parents and teachers, effectance motivation, confidence, and usefulness.

2. Rationale

Several recent studies have suggested that the widely held belief of male superiority in mathematics is not as prevalent as had been believed and is, moreover, age-related. In earlier reported studies including only grades 9 through 12, the authors found that sex-related differences were found in only half of the school population sampled when the number of years of studying mathematics was controlled. This research is a follow-up study designed to look at the same variables in the feeder schools for those used in the studies reported earlier.

3. Research Design and Procedure

A battery of tests was administered to 1320 sixth-, seventh-, and eighth-grade students in middle schools in Madison, Wisconsin that were the feeder schools for the population of the previous study. The sample included only students in the top 85 percent in mathematics achievement. The tests included the Romberg-Wearne Problem Solving Test, vocabulary tests from the Verbal Battery of the Cognitive Abilities Test, the Space Relations Test, Mathematics Concepts Test, Mathematics Computation Test, and the Fennema-Sherman Mathematics Attitude Scales.

Means of the 15 measures were computed for males and females in each of the three grade levels and for each of four areas of the city. An ANOVA was performed on each variable, with sex, grade, and area used as sources of variance. Correlation coefficients between measures were computed for each sex and for the students combined over area and grade. A principal component factor analysis was also performed on all variables combined over area and grade.

4. Findings

Means and standard deviations for all measures were reported by area, sex, and grade, in addition to the usual F ratios of ANOVAs (Sex x Grade x Area, and Sex x Grade for each Area). Significant sex-related differences were found in only two affective variables, in each case "favoring" the male: "Confidence in Learning Mathematics" and "Mathematics as a Male Domain." As expected, significant area effects were found for all variables, and significant grade effects were found for the cognitive variables.

The results of the Sex x Grade for each Area data analysis showed the following: (1) significant differences in all areas for "Mathematics as a Male Domain"; (2) significant difference in Computation (favoring females) in Area 4; (3) significant differences in favor of males in Romberg-Wearne Application and Romberg-Wearne Problem Solving and for six of the eight affective variables in Area 3. Only the "Teacher" and "Effectance in Motivation" variables showed no significant sex differences in Area 3.

5. Interpretations

The findings "strongly suggest that there are no universal sex-related differences in mathematics learning." The authors note that the results of this study agree with the NAEP results of lack of differences in mathematics achievement before age 17, but are in conflict with the NLSMA conclusions that males are superior on tasks of high cognitive complexity. They suggest that the heightened interest in women in mathematics in the interim years could be at least partially responsible for some of the differences in results found in NLSMA studies and in this one.

One surprising result was the lack of significant difference in spatial visualization in males and females, a finding that would, if further substantiated, dispel the long-held belief that males are superior to females in spatial ability.

A very interesting aspect of the discussion of results is the comparison of the findings of this study with those of the above-mentioned study in grades 9 through 12. This is particularly true where sharp differences were observed in the affective measures. Especially noteworthy are the "Confidence in Learning Mathematics" and the "Teacher" variables and the relationship between these variables.

Critical Commentary

The Fennema-Sherman studies are a valuable contribution to the search for reasons for the dearth of women in mathematics-related fields. More such well-designed and carefully conducted research is needed to provide hard data to substantiate hypotheses or to dispel popular myths. In addition to presenting status information, these studies provide a baseline with which to compare the results of future studies.

A careful study of the tables and discussion suggests the following questions and comments:

(1) How much a function of Madison, Wisconsin are the results? Although a socioeconomic mix does exist, any university town is an atypical sample. Comparative studies are needed.

(2) What are the socioeconomic characteristics of Areas 1, 2, and 4? It would help the reader to have some demographic information on each of the areas since one purpose of such studies is to provide information to support hypotheses for probable causes of lack of participation and for achievement differences.

(3) The lack of significant sex differences in the spatial test was an especially interesting finding in need of further investigation. Since it is generally accepted that more than one space factor exists, further substantiation of the findings with other measures of spatial ability are indicated.

(4) Some of the graphs on page 199 of the article are misleading. A perusal of the table of means (pages 192-193) suggests some interesting discrepencies. The "Confidence in Mathematics" graph would lead the reader to believe that female confidence was consistently lower. In fact, this was not the case in four of the nine classes studied. Likewise, the "Usefulness of Mathematics" graph shows males consistently higher. Yet the means for females in five of the nine classes are higher. As the authors noted (page 198), great differences in favor of males expecially on the affective variables occurred in Area 3. These large discrepancies in one area could account for an inaccurate picture.

(5) The high correlations between the students' confidence in mathematics and their perceived attitudes of parents and teachers toward them as learners of mathematics should provide hypotheses for further study.

COMPARISON OF LECTURE AND LABORATORY STRATEGIES IN A MATHEMATICS COURSE FOR PROSPECTIVE ELEMENTARY TEACHERS. Flexer, Roberta J. Journal for Research in Mathematics Education, v9 n2, pp103-117, March 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Donald M. Blais and Douglas A. Grouws, University of Missouri-Columbia.

1. Purpose

The purpose of this study was to evaluate the effectiveness of a laboratory approach to instruction by comparing it with a lecture approach in a mathematics course for prospective elementary teachers. Effectiveness was operationally defined using measures of achievement, attitude, and student teaching performance.

2. Rationale

Often cited advantages of the laboratory method include its impact on achievement and its potential for improving attitudes toward mathematics. For the most part such claims have not been substantiated by empirical evidence, although there is some research support for using the laboratory method with slow learners. The review of the literature included five recent studies of the effectiveness of the laboratory strategy in mathematics courses for prospective elementary teachers. The results from these studies were inconclusive.

3. Research Design and Procedure

The sample was composed of four classes (n = 90) of preservice teachers in a "Structure of the Number System" mathematics course. The classes met as usual except during the three two-week treatment periods. Each class was randomly divided into an experimental and control group which met separately during the treatment periods. Each instructor used the lecture method with the control group from his or her own section and taught the experimental group from another section using the laboratory method. Thus, there was a total of four experimental and four control classes during each treatment period. The topics covered during the treatment periods included sets, nondecimal bases, rational numbers, and functions and relations.

In the lecture strategy, content was presented in an expository manner although questions were discussed and dialogue encouraged. In the laboratory strategy, students worked in small groups using written exercises and worksheets to guide their use of manipulative materials (attribute blocks, abaci, pattern blocks, cubes, and geoboards) and to lead them to synthesize the concepts under study. The role of the instructor was to help with problems, answer queries, and pose leading questions.

Dependent measures used in the research included an hourly examination given after each of the three treatment periods, a final examination given at the end of the course and reused six months later to assess retention, an attitude scale (Aiken-Dreger Revised Mathematics Attitude Scale) administered at the beginning and end of the course, three subtests formed from the final examination by pooling the items related to the content covered during each treatment period, and the sum of these subtests.

Two statistical designs were used. The first design involved a three-way ANCOVA on each dependent measure using teaching team, instructor, and method (lecture or laboratory) as independent variables. The covariates in each case were rank in high school class, grade point average, number of mathematics courses, SAT (Mathematics) scores, and attitude pretest scores. The second design was analyzed using a two-way ANOVA on achievement scores that examined the interaction between three levels of mathematical ability (determined by SAT scores) and the two instructional methods (laboratory and lecture).

A follow-up study was carried out to determine if there was an interaction between the treatments and student teaching performance as measured by a Likert-type questionnaire.

4. Findings

Analyses of the achievement data showed no significant differences between the laboratory and lecture strategies and no interaction effects with ability. Positive changes in attitudes were reported for both methods but favored neither group. The instructional strategy to which students were exposed, under the conditions of this study, did not influence student teaching performance.

A course evaluation questionnaire given to the experimental group ($n = 38$) showed that 36 percent preferred to have the material presented by the laboratory method, whereas 48 percent preferred lectures. Also, many students (49 percent) felt that working with concrete materials made learning mathematics easier. A majority of the students (68 percent) felt the laboratory exercises helped their understanding of mathematical topics, while most (90 percent) thought the laboratory exercises gave them ideas for presenting mathematical concepts in elementary school.

5. Interpretations

Experimental and control groups did equally well; each appeared to have learned the same body of information as measured by the instruments used. On this basis, the researcher concluded that the laboratory approach should be regarded as a viable and reasonable alternative to the traditional lecture approach.

Two interpretations were suggested to account for positive changes in attitude towards mathematics. Both methods are effective in causing a positive change or the instructors were capable of changing attitudes irrespective of method used. The author suggests that given reasonably good teachers either a lecture strategy or a laboratory strategy can produce positive gains in attitude toward mathematics.

The instructional strategy to which a student was exposed in a preservice mathematics course did not influence performance in teaching mathematics as a student teacher. The author points out that this conclusion should be considered only tentative considering the limitations involved. These include the fact that one to three years elapsed between treatment and student teaching and the lack of standardized instruments available to measure teaching ability in mathematics.

Critical Commentary

The topic chosen for research is an important one. Past research, however, has seldom been able to furnish consistent empirical evidence supportive of a particular teaching methodology. This does not imply that superior methodologies do not exist, but it does suggest that if one teaching strategy has advantages over another, then these advantages can be expected to be difficult to demonstrate and measure. Hence, powerful experimental treatments which tend to maximize hypothesized differences should be employed. The relatively short treatment period in this study could hardly be expected to produce the differences hypothesized.

Other limitations and information not included in the article raise other questions and issues. Was there a theoretical basis for the development of the laboratory activities or was a random collection of "hands-on" activities used? Were the ideas of perceptual and mathematical variability employed? Were activities presented in series and parallel modes (as suggested by Z. P. Dienes)? No indication was given about the quality of the treatment or the extent to which the treatment was implemented. How much difference did it make that the instructors had no previous exposure to or experience with the laboratory method? Does "three two-week periods" mean 18 one-hour sessions? How much of this time was used for testing?

The long interim period (from one to three years) between the treatment and student teaching must be considered a serious limitation of the follow-up study. Student experiences during this period should be expected to reduce substantially differences between the control and experimental groups. Notable in this respect was the fact that during the interim period each student was enrolled in a mathematics course and a methods course which emphasized the use of manipulatives. In the future it may also be helpful if follow-up studies of this type make a distinction between ability to use and willingness to use a particular method.

CHILDREN'S CONDITIONAL REASONING: PART I: AN INTUITIVE APPROACH TO THE LOGIC OF IMPLICATION. Hadar, N.; Henkin, L. Educational Studies in Mathematics, v8 n4, pp413-438, December 1977.

CHILDREN'S CONDITIONAL REASONING: PART II: TOWARDS A RELIABLE TEST OF CONDITIONAL REASONING ABILITY. Hadar, N.; Henkin, L. Educational Studies in Mathematics, v9 n1, pp97-114, February 1978.

CHILDREN'S CONDITIONAL REASONING: PART III: A DESIGN FOR RESEARCH ON CHILDREN'S LEARNING OF CONDITIONAL REASONING AND RESEARCH FINDINGS. Hadar, N.; Henkin, L. Educational Studies in Mathematics, v9 n1, pp115-140, February 1978.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by John W. Gregory, University of Florida.

1. Introduction

The investigation under review has been extensively reported in three articles (referenced above). The first article details the development and implementation of an experimental, instructional treatment, the intent of which was to improve student conditional reasoning ability. The second article discusses the development of the research instrument used to measure children's conditional reasoning ability. Particular emphasis of the article is placed on difficulties inherent in establishing a reliable instrument for measuring conditional reasoning ability. The third article presents analyses techniques employed to determine the effectiveness of the instructional treatment and discusses the findings and implications relative to elementary mathematics education.

This review will be presented in the typical research format and sequence although this does not reflect the same sequence employed in the original articles. It is hopeful that examination of elements in the original papers identified by this review may be made on the basis of the article descriptions given above.

2. Purpose

Several investigators have provided information relative to the logical reasoning abilities of children, their growth and development as well as identification of particular logical forms which provide most difficulty. Some of these same investigators have sought to enhance logical reasoning abilities of students through instruction in the application of symbolic, syntactical algorithms of logic. The investigation under review sought to enhance conditional reasoning in particular through an instructional unit which was void of the usual algorithms. Instead of teaching formal logic, the unit sought to teach informal, intuitive application of conditional reasoning rules. Specifically the investigation sought to "increase student awareness of the existence of non-valid as well as valid conclusions, and to change student initial inability to separate out those cases in which given information was insufficient to necessitate a certain conclusion" (p. 416).

Hypotheses

The following hypotheses have been inferred from the three articles:

(a) Subjects in the experimental group will show significant growth in conditional reasoning ability between pretest and posttest measures.

(b) There are significant differences between the experimental and control groups in growth of conditional reasoning ability from pre- to posttest.

(c) There are no significant differences between pre- and posttest performance of the experimental group on items differing in logical form.

(d) There is no correlation between growth in conditional reasoning ability and performance on the Stanford Achievement Test or sex.

3. Population and Sample

Four elementary school teachers who had received a 12-hour pretraining workshop given by the investigator, and four additional elementary school teachers from a different school led to the identification of the sample. The four fifth-grade classes of the pretrained teachers constituted the experimental group; the other four fifth-grade classes comprised the control group. Selection of subjects and assignment of treatments to classes was not random.

A final sample of eight classes totaling 210 students (104 experimental, 106 control) who had taken both pre- and posttests was achieved in this manner.

4. Procedures

The four experimental classes received a unit of instruction in the process of making valid deductions. This experimental unit is described below. The instruction was presented four to five times per week totaling 23-25, 30- to 40-minute sessions. The instruction was provided by the regular teachers who had received workshop training concerning the logic of conditional reasoning using the experimental unit materials. The preparation of teachers of the control group and the content of instruction over the same five-week period for the control classes is not described.

Prior to, and following the period of time during which the experimental unit was presented to the experimental group, the same investigator-prepared instrument (described below) was administered to both experimental and control groups. Two mixed order versions of the same test were used at each administration, which lasted from 20 to 25 minutes although no time limit was established.

5. The Experimental Unit

A description of the experimental unit can be found in the first research article. This description is quite complete relative to excerpts from the materials themselves as well as with regard to the basis of the material development.

In brief, the unit consisted of seven chapters. Each chapter was comprised of teacher/large group activity during which material was introduced and discussed followed by small group activity (6-8 students). These materials reflect the pre-established guidelines which included:

(a) Algorithms for applying principles of conditional logic are not presented. Generalizations equivalent to these principles were to have been discovered, but it was not expected that subjects would be able to express such discoveries verbally.

(b) The unit should lead students to recognize the syntactical and semantical differences between a conditional sentence and (1) its converse, and (2) its inverse.

(c) The unit should lead to recognition of the logical equivalence of a conditional sentence and 1) its contrapositive, and 2) the compound sentence 'p-and-q or p-and-not q or not-p-and-not-q' (more simply p—q negates p-and-not-q).

(d) The four logical forms of affirming the antecedent (AA), denying the consequent (DC), affirming the consequent (AC), and denying the antecedent (DA) are presented together rather than one case at a time. Note that this means presented in the sense of being inherent in the materials, not necessarily being presented formally as different forms, even in name.

(e) The development of conditional reasoning ability should be at the intuitive level, dealing only with content that does not contradict everyday experience and makes sense to the population receiving the instruction.

6. The Measuring Instrument

A 32-item test was developed which consisted of four 8-item sets reflecting the four logical forms (AA, DC, AC, DA). Each set consisted of four pairs of items differing in use of negation in the conditional premise: 1) no negation in either the antecedent or the consequent (PP);

2) negation in the consequent only (PN); 3) negation in the antecedent only (NP); and 4) negation in both antecedent and consequent (NAC).

Each item started with a conditional sentence followed by a statement of its antecedent, consequent, or one of these denied. These two premises were labeled as "clues a and b." The item was completed by the word "question" followed by a question related to the two premises for which a conclusion of "yes" or "no" could be determined (AA and DC) or for which only "not-enough-clues" should be selected (AC and DA). Responses were to be made as a choice from among "yes," "no," and "not enough clues" (specific manner of choice identification is not described).

Particular concerns and problems encountered in the development of a conditional reasoning test are raised and, for the most part, rationale is presented for decisions made relative to logical form, use of negation, content inclusion, statement of questions, and other item format qualities. One major basis for decisions made in constructing the instrument in this study was concern for keeping item content meaningful to the fifth-grade subjects. For example, taking a conditional sentence and forming all sixteen structures (4 logical forms crossed with the 4 uses of negation) leads to difficulty in terms of maintaining equally meaningful content, in the opinion of the investigators. Thus it was decided to begin each of the 32 items with a new conditional sentence. Meaningful content as defined by the investigators means the content is easy to picture, is familiar and is not contradictory to the experiences of the subjects.

Even the use of the answer choice "not-enough-clues" is explained on the basis of psychological differences between indicating "can't tell," "maybe" and "not-enough-clues." Investigators desiring to construct similar tests should become familiar with the bases of decisions which led to the construction of the instrument used in this investigation.

A split-half reliability formula was applied to pretest total scores of the 210 students, yielding a 0.79 reliability coefficient.

7. Results

Very briefly stated, the results were:

(a) Experimental and control groups were found to be equivalent with regard to profiles of yes/no/not-enough-clues answers to identical items on the pretest (using Chi-square test), and with regard to Stanford Achievement Test performance.

(b) Subjects in the experimental group showed significant growth from pre- to posttest performance only on items for which "not-enough-clues" was the correct response (logical forms AC and DA). Analysis was achieved through application of Bowker's test on each combination of pretest and posttest performance levels (low-average-high).

(c) Experimental subjects showed significantly more gain than the control group from pre- to posttest performance on AC and DA items. Analysis was achieved through application of Chi-square on distributions of performance level combinations (above) and ANOVA of mean-gain scores of the two groups.

(d) No significant correlation was found between experimental group conditional reasoning test performance or gain and sex or Stanford Achievement Test scores.

Correlation coefficients computed between 1) sex, S.A.T. subtests, and 2) logical form performance on pretest and posttest and gain scores.

Analysis of responses for the four logical forms is also presented in the last article. Attitudes of students and teachers toward the instructional unit are also discussed.

8. Interpretations

Results of the study call for further investigation of the value and usefulness of teaching logic as part of the elementary mathematics curriculum. Particular concern should be given to development of teacher-implementable units in intuitive logic for grades lower than grade 5 which focus on a variety of logical patterns which are more adequate for mathematical argument. Changes in the college preparation of prospective teachers, transferability of training in mathematical logic on the part of students and comparison of concentrated training versus training spread over time should also be investigated.

Critical Commentary

This well-conducted investigation provides particularly fruitful information to the literature domain of conditional reasoning ability due to the completeness of the report. Descriptions of materials and instrument construction will be especially helpful to other investigators of logical reasoning abilities.

Reading of the complete three-part report does lead to some questions which may be beneficial to further investigations. These questions include:

1) In determining the efficacy of a particular instructional unit, might evaluation of unit content be better achieved in relation to other units rather than the application of a treatment/no treatment design? For the study under review one thing shown is that students who are taught correct responses to conditional reasoning items perform better than students who are not taught. The content of the unit was, in some cases, identical in format to the test items. The study

does show that fifth-grade students can learn certain forms of mathematical logic by intuitive means, something which others have failed to show by teaching formal logic. This is a fine contribution to the state of knowledge.

Comparing different instructional units with the same intentional outcome might also reduce possible Hawthorne effect.

2) Might investigations employing a case study approach provide additional helpful information relative to the learning process itself with regard to logical reasoning development? Decisions with regard to statistical treatment of data such as those made in the study under review are always difficult to make. Under a case study approach, interpreting continuous data presented as discrete data, computing reliabilities for subtests as well as the total test, determining when and when not to use raw gain scores or when percentages of students in each group scoring particular percentages on the tests is a valid treatment of data, all can be eliminated thus enabling an investigator to focus more on the individual's interaction with the material.

3) In the evaluation of any instructional unit, enough research evidence is available to at least consider differences in the qualities of verbal interaction patterns employed in the delivery of that unit. Might there have been particular qualities in the verbal behaviors of the teachers who received training in logical reasoning which were not only different from the teachers not receiving training but which also led to the richness of instruction in all areas of the fifth-grade curriculum beyond the unit on logic?

4) Might analysis of performance on items involving negation yield information relative to difficulties incurred by the students? Negation has been identified by other investigations as being a source of difficulty on logical reasoning test performance. If the experimental unit was found to reduce the level of difficulty associated with items involving negation, this alone would be a significant finding.

CONSTRUCTION AND VALIDATION OF AN INSTRUMENT MEASURING CERTAIN ATTITUDES TOWARD MATHEMATICS. Michaels, Linda A.; Forsyth, R. A. Educational and Psychological Measurement, v37, pp1043-1049, Winter 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lewis R. Aiken, University of the Pacific.

1. Purpose

This study was designed to develop paper-and-pencil instruments to measure four facets of attitude toward mathematics: Enjoyment of Word Problems (EW), Enjoyment of Pictorial Problems (EP), Appreciation of the Utility of Mathematics (U), and Security with Mathematics (S). Evidence pertaining to the reliability and validity of these scales was also collected.

2. Rationale

Paper-and-pencil instruments designed to assess attitudes toward mathematics have been mainly of the single-score (hodgepodge) type, with a few efforts being made to develop multidimensional inventories. It is suggested that diagnosis of students' specific problems with mathematics would be more effective if reliable and valid measures of several facets (dimensions) of attitudes toward mathematics were available.

3. Research Design and Procedure

Three attitude inventories were constructed to measure four facets of attitude toward mathematics in seventh-grade students. The U and S scales, each consisting of ten Likert-type (Very Important, Fairly Important, Barely Important) items, were designed to measure Appreciation of the Utility of Mathematics (U) and Security with Mathematics (S). Each of the twelve items on the third inventory consisted of two parts. On the first part of an item, the examinee was directed to choose between a word problem and a figural (pictorial) problem. On the second part of the item, the examinee indicated how difficult the choice was and whether the choice was easy or difficult to make. Scoring of each item on this third inventory was quasi-ipsative, in that the examinee was assigned scores on two variables--Enjoyment of Word Problems (EW) and Enjoyment of Pictorial Problems (EP). Depending on his or her answer to the item, the examinee received a score of one point on either scale and zero on the other, one point on each scale, or zero points on both scales. After being administered initially to a small group of seventh graders, the scales were modified slightly and then administered to 299 seventh graders. Split-half reliability coefficients for the four scales and correlations of the four measures of attitude with students' self-ratings on the four constructs skills were computed. Correlations of the four scales with three-part scores on the Iowa Tests of Basic Skills were also obtained.

4. Findings

The reported split-half reliabilities of the four scales ranged from .51 to .78, and correlations with self-ratings on the same concepts ranged from .11 to .25. Four of the correlations between the four attitude measures and the three ITBS part scores were small but statistically significant (.16 to .28), three of the four being correlations between ITBS scores and the Security with Mathematics (S) scale.

5. Interpretations

The findings are interpreted as supporting the multidimensional nature of attitude toward mathematics and as providing some evidence for the reliability and construct validity of the four attitude measures. The writers view the correlations between the scale scores and self-ratings as promising evidence of the validity of the scales. The fact that S-scale scores also had statistically significant correlations with ITBS part-scores was considered to be additional evidence for the validity of the S scale.

Critical Commentary

Several good points are made in this paper, among which is the need for reliable multidimensional measures of attitude. It is also worth noting that greater objectivity of measurement could be obtained by making these multidimensional attitude scales behaviorally oriented --rather than feeling-oriented, as in the EW-EP scales of the present study. Finally, the reviewer agrees that the criteria used in studying the validities of such attitude inventories should not be limited to measures of achievement in mathematics.

There are several difficulties that make this study fall somewhat short of well-designed investigations of attitudes toward mathematics. The assertions that single-score instruments are "unidimensional" (p. 1044) and the incomplete descriptions of procedure are minor faults, but a major problem of the study is the instruments themselves. Although the subjects were seventh graders, ten-item scales and three-point Likert-scale items are still too abbreviated to expect substantial reliability and validity coefficients. Also, serious statistical problems exist in using correlational techniques with ipsative (forced-choice) measures such as the EW and EP scales. These scales would also be expected to have a narrower band width but greater fidelity than the U and S scales, so U and S should correlate with a wider range of criteria. The investigators, however, entirely neglected this point. Other shortcomings pertain to the restricted nature of the sample (predominantly urban middle-class white seventh graders) and the low split-half (?) reliability coefficients. The question mark refers to the reviewer's concern as to why the investigators used split-half rather than alpha coefficients with multipoint items.

Regarding the construct validation of the scales, even the largest coefficients in Table 1 are unimpressive and certainly not practically significant. Incidentally, a coefficient of .11 based on 247 cases is not statistically significant. The investigators are to be encouraged for approaching the matter of validity from a multitrait-multimethod matrix standpoint, but much more empirical work remains to be done with these scales before they can be declared superior to single-score instruments of greater length but also greater reliability and validity than the EU, EP, U, and S scales.

STRATEGIES USED BY FIRST-GRADE CHILDREN IN ORDERING VARYING NUMBERS OF OBJECTS BY LENGTH AND WEIGHT. Smith, Edward L.; Padilla, Michael J. Journal of Research in Science Teaching v14, pp461-466, 1977.

Expanded Abstract and Anlysis Prepared Especially for I.M.E. by Thomas P. Carpenter, The University of Wisconsin-Madison.

1. Purpose

The primary purpose of the study was to identify the strategies used by first-grade children to perform seriation tasks for length and weight. The study also examined the relative difficulty of length and weight seriation tasks and the effect of the number of objects to be ordered on children's seriation performance.

2. Rationale

Smith has proposed a descriptive framework for conducting research on transfer within which specific effects of the concepts involved, the tasks to be performed, and the strategies by which the tasks might be carried out are assessed. A central assumption of this paradigm is that learners can acquire systematic strategies for performing a given task and that such strategies will transfer to related concepts and more complex tasks. This study fits within this framework by attempting to demonstrate that children do in fact possess systematic strategies that they apply to a variety of problem situations.

The specific content of the study is derived from Piaget's research on children's seriation concepts. But whereas Piaget is concerned with children's misconceptions and what their errors imply about the development of logical reasoning processes, Smith and Padilla focus exclusively on the characterization of correct seriation strategies.

3. Research Design and Procedure

The sample consisted of 96 first-grade children randomly selected from four elementary schools. Each subject seriated one set of objects. Between subjects, problems differed with respect to the seriation variable (length and weight) and the number of objects to be seriated (four, six, eight, and ten). Thus, there was a total of eight seriation problems, each of which was administered to 12 subjects. The length seriation tasks involved comparisons of 3/8-inch dowels that varied in length from 9 to 16 cm; in the weight seriation tasks, styrofoam cups filled with different quantities of paraffin and lead shot to produce weights ranging from 10 to 2100 g were compared by hefting.

Based on the results of pilot studies and an earlier study by Baylor and Gascon (1974), two primary strategies were identified. The Extreme Value Selection (EVS) strategy involved placing the unordered object with the greatest value next in the sequence. The Insertion (INS) strategy involved placing randomly selected objects in their appropriate place in the sequence. A third strategy, which involved successive rearrangements of partially ordered sequences, has been observed mainly with adults and did not figure prominently in this study.

For each subject, a Task Score was derived by correlating (Kendall Tau) the rank of each object in the subject's series with its true rank. For subjects whose task score was greater than .70, solution strategies were inferred from the order in which objects were selected to place in the series and the position in which they were placed.

A 4x2x4 factorial design was employed in which the factors were number of objects, problem type, and school. A three-way analysis of variance was used to test for differences due to number of objects and problem type.

4. Findings

Fifty subjects had Task Scores greater than .70. Of these, 39 used identifiable strategies. The EVS and INS strategies were used with approximately equal frequency for length (10 and 12 subjects respectively), but only three subjects used the INS strategy for weight while 11 used the EVS strategy. These differences were not found to be statistically significant at the .05 level using a chi-square test. Length seriation tasks were significantly easier than weight seriation tasks ($p < .0001$), but differences due to the number of objects to be seriated did not approach significance.

5. Interpretations

The authors conclude that basic seriation strategies are relatively stable across both problem type and number of objects. They propose that the most important implication of these results is the viability of strategy as a construct in educational theory. The fact that even young children use systematic strategies suggests that strategy instruction may be practical.

Critical Commentary

The general goals of this study in science education are consistent with a promising trend in research in mathematics education of focusing on children's problem-solving strategies and the processes they use in performing mathematical operations. This study might have benefited by employing procedures that are gaining acceptance within mathematics education research. The attempt to establish standard criteria for identifying given strategies resulted in seemingly arbitrary and artificial criteria. Using think-aloud protocols, asking subjects to explain their solutions, or carefully analyzing videotapes would provide a richer description of children's solution strategies than simply recording the order in which objects are seriated. There is undoubtedly variation within each basic strategy in terms of trial and error, use of transitivity, etc.: a great deal is also lost by only considering correct solutions. There is substantial evidence that children frequently use systematic strategies even when they generate incorrect solutions. A careful analysis of these strategies can provide valuable insights and a better understanding of how correct strategies develop and are applied.

The authors' conclusion that basic seriation strategies are stable across problem type is unfounded, based on their evidence. Four times as many subjects used the INS strategy with length as used it with weight. The fact that these differences were not found to be significant very likely results from lack of power of the statistical test. It is certainly inappropriate to accept the null hypothesis in this case. Furthermore, a repeated measures design in which individual subjects reviewed several seriation problems would be more appropriate to test stability of strategies over problem type. The between-subject design does not test whether a given child will use the same strategy in different problem situations.

The failure to find significant differences due to number of objects should also be viewed with some caution. One would not expect such differences to be large, and the authors never discuss what magnitude of difference they have the power to detect. In addition, there appears to be a possible ceiling effect, at least for length, that would mitigate against finding such differences.

INVESTIGATIVE TEACHING OF MATHEMATICS AND ITS EFFECT ON THE CLASSROOM LEARNING ENVIRONMENT. Talmadge, Harriet; Hart, Alice. Journal for Research in Mathematics Education, v8, pp345-358, November 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by David F. Robitaille and Thomas O'Shea, University of British Columbia.

1. Purpose

The study was conducted to seek evidence in support of the authors' hypothesis that "changes in the classroom learning environment would result if teachers were involved in defining an instructional approach, identifying its characteristics, and studying the implementation procedures and effects on their own classes" (p. 357).

2. Rationale

The authors contend that there is a trend in mathematics education toward research on aspects of teaching and learning rather than on curricular innovations, citing several examples to support this view. They state that among the several reasons for the fact that the "research literature has yet to document the greater effectiveness of a psychologically based curriculum and instruction over and above those predicted on the inherent logic of mathematics," are that attempts to measure affective phenomena are very recent and that there is a considerable lack of agreement among researchers and practitioners about the precise definitions of the terminology involved in such studies. In this latter regard, they cite discovery teaching as a prime example.

3. Research Design and Procedure

Twenty-three teachers in grades 1-8 who participated in an NSF Elementary Mathematics Implementation program taught the experimental classes. Each of these "identified teachers in their respective schools with classes similar to theirs and obtained consent to use them as control classes"(p. 348). During the year, the teachers of the experimental classes met for four hours every Saturday. Part of their activities consisted in developing a working definition of "investigative teaching," and in identifying characteristics of such teaching. They were then expected to practice "investigative teaching" in their mathematics classes and to study its effects.

Pretest and posttest data on learning environment and cognitive process were obtained using My Class Inventory and the Cognitive Activities Rating Scale. The learning environment variables were measured by having the children rate five environmental attributes: Satisfaction, Friction, Competitiveness, Difficulty, and Cohesiveness. The children's ratings of their perceptions of the dominant learning process in the class were set out in two scales: Lower Thought Processes and Higher Thought Processes.

Student t-tests were used to compare the 14 pairs of experimental and control group means obtained from the test administrations. Analysis of covariance, with pretest results as the covariate, was used to discover the relative importance of group membership (experimental-control), system (public-parochial), grade (primary-intermediate), and sex on the learning environment and cognitive variables. Multiple regression analysis was utilized to compute the amount of variance contributed by the independent variables to the posttest score.

4. Findings

None of the 14 t-tests resulted in significant differences. The analysis of covariance produced significant F statistics for five of the dependent variables: Friction, Competitiveness, Difficulty, Cohesiveness, and Lower Thought Processes. Group membership was significant ($p < 0.01$) only on the Cohesiveness variable. Grade level was significant in three of the seven cases. The regression analysis showed the pretest score to be the most important predictor of posttest performance on all seven variables.

5. Interpretations

The authors conclude that their study shows that experienced teachers can learn new approaches to mathematics teaching, that they can be involved in developing such new approaches, and that they can be provided with a mechanism for transferring these approaches to their own classrooms. Moreover, they conclude that their results show a significantly higher degree of Cohesiveness in the experimental classes than in the control classes. The other four environmental variables appeared to be more affected by grade level than by group membership.

Critical Commentary

The premise that factors such as the learning environment in the mathematics classroom may be important in determining the success or failure of the mathematics curriculum is a persuasive one. Research designed to investigate aspects of such learning environments seems clearly to be needed. It is unfortunate that the authors were not able to carry out a study which might have indicated some directions for others to follow. Their study suffers from several weaknesses, both in design and execution, which invalidate the conclusions which the authors have drawn.

(a) The authors list 14 characteristics of "investigative teaching." Of these, several are described in vague and non-specific terms which would make it very difficult for observers either to decide when the behavior in question had occurred, or to obtain a high degree of interrater reliability. Among the vaguely defined characteristics are the following [emphasis added].

1. The problem is presented in a way that encourages learner input.

3. There is a potential for alternative solutions.

9. The learning environment is conducive to exploration.

(b) The composition of the experimental and control groups appears calculated to maximize the possibility of finding systematic differences. The members of the experimental group were sufficiently concerned about the teaching of elementary mathematics to enroll in a year-long NSF-sponsored program; the members of the control group, who were selected by the experimental group teachers, taught "similar" classes.

(c) Insufficient information is provided about the testing instruments used in the study. This is unfortunate because several important questions concerning them require answers. In the first place, no reliability information is supplied. Secondly, one wonders how such instruments were administered to children in the primary grades. How, for example, would a child at that level respond to an item which says, "Most of the children use what they learn in new situations." A search of the 1974 edition of Tests in Print as well as the Sixth and Seventh Mental Measurement Yearbooks failed to uncover any reference to either of the testing instruments.

(d) Although the study proposes to chart differences in classroom learning environments, no mention is made of any attempt to observe classrooms in operation. In other words, even if significant differences between experimental and control groups were found, it would not be possible to state that such differences were due to the effects of "investigative teaching," since there was no way of verifying what actually transpired in the classrooms.

(e) In general, although there are some positive features such as the use of class as the experimental unit, the statistical analyses employed are questionable. In particular, it is unsatisfactory to suggest that further study be given to the one variable out of 14 which had the greatest t-value ($p = 0.063$). Furthermore, it would have been more appropriate to use a multi-variate analysis of variance in a repeated measures design than to carry out seven different analyses of covariance with a changing covariate, and multiple regression analyses. Actually, the data reported in Table 2 (Analysis of Covariance) and Table 3 (Multiple Regression Analysis) are redundant. Finally, since the authors had made the decision to employ analysis of covariance, it was not appropriate for them to then report, as they do in Figures 3 and 4, interactions using the pre- and posttests as repeated measures.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN RESOURCES IN EDUCATION
April - June 1978

ED 146 885 The Use of the Computer in Teaching Secondary School Subjects. Organization for economic Cooperation and Development, Paris (France), Centre for Educational Research and Innovation. 74p. MF and HC available from EDRS.

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MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
April - June 1978

EJ 169 940 Blasingame, Marsha; McManis, Donald. Classification, Relative Thinking, and Transitivity Performance by Retarded Individuals. American Journal of Mental Deficiency, v82 n1, pp91-4, July 1977.

EJ 170 334 Leahey, Thomas H. The Development of Conservation Abilities: An Intertask Analysis of Continuous Quantity. Merrill-Palmer Quarterly, v23 n4, pp215-26, October 1977.

EJ 170 368 Hatano, Giyoo; Suga, Yasuko. Understanding and Use of Disjunction in Children. Journal of Experimental Child Psychology, v24 n3, pp395-405, December 1977.

EJ 170 533 Shann, Mary H. Evaluation of an Interdisciplinary, Problem Solving Curriculum in Elementary Science and Mathematics. Science Education, v61 n4, pp491-502, October-December 1977.

EJ 170 535 Lashier, William S.; Wren, Eugene L. Effect of Pretest Feedback and Mathematics Skills Overview on IPS Achievement. Science Education, v61 n4, pp513-8, October-December 1977.

EJ 170 538 Za'Rour, George I. Identity Versus Equivalence in Weight Conservation. Science Education, v61 n4, pp541-7, October-December 1977.

EJ 171 164 Sethi, Santosh S.; Dickerscheid, Jean D. Effect of Planned Seriation Experiences on Acquisition of Seriation Abilities in Four-Year Old Children. Home Economics Research Journal, v6 n2, pp107-14, December 1977.

EJ 172 429 Lester, Frank K. Ideas About Problem Solving: A Look at Some Psychological Research. Arithmetic Teacher, v25 n2, pp12-4, November 1977.

EJ 172 433 Wheatley, Grayson H. The Right Hemisphere's Role in Problem Solving. Arithmetic Teacher, v25 n2, pp36-9, November 1977.

EJ 172 434 Suydam, Marilyn N.; Weaver, J. Fred. Research on Problem Solving: Implications for Elementary School Classrooms. Arithmetic Teacher, v25 n2, pp40-2, November 1977.

EJ 172 436 Underhill, Robert G. Teaching Word Problems to First Graders. Arithmetic Teacher, v25 n2, pp54-6, November 1977.

EJ 172 446 Coward, E. M. LAMP Project Case Study--Materials. School Science Review, v59 n206, pp5-12, September 1977.

EJ 172 520 Survey Finds Scientists Staying With Careers. Chemical and Engineering News, v55 n49, pp5, December 1977.

EJ 172 854 Mitchelmore, Michael C. Experiments in Individualized Instruction at a Jamaican Teachers' College. Caribbean Journal of Education, v4 n1, pp1-20, January/April 1977.

EJ 173 654 McGlannan, Frances, Ed. Learning Disabilities and Arithmetic Achievement. Journal of Learning Disabilities, v10 n8, pp508-10, October 1977.

EJ 173 870 Stanley, Julian C. The Predictive Value of the SAT for Brilliant Seventh- and Eighth-Graders. College Board Review, n106, pp30-7, Winter 1977/1978.

EJ 174 035 Gillman, Irene S.; Formanek, Ruth. Memory in Relation to Conceptual Ability in a Seriation Task with Children. Child Study Journal, v7 n4, pp211-30, 1977.

EJ 174 036 Chiseri, Michael J. The Effect of the Perceptual Cue in Paradigms of Identity and Equivalence Conservation. Child Study Journal, v7 n3, pp107-16, 1977.

EJ 174 082 Worner, Martha; and Others. Conservation of Weight with the Deaf. Journal of Genetic Psychology, v131 n2, pp323-4, December 1977.

EJ 174 083 Brekke, Beverly; and Others. Conservation of Weight with Social Deviants. Journal of Genetic Psychology, v131 n2, pp325-6, December 1977.

EJ 174 115 Protinsky, Howard; Hughston, George. Conservation in Elderly Males: An Empirical Investigation. Developmental Psychology, v14 n1, pp114, January 1978.

EJ 174 143 Allen, L. E.; Ross, J. Improving Skill in Applying Mathematical Ideas: A Preliminary Report on the Instructional Gaming Program at Pelham Middle School in Detroit. Alberta Journal of Educational Research, v23 n4, pp257-67, December 1977.

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EJ 174 195 Vollrath, Hans-Joachim. The Understanding of Similarity and Shape in Classifying Tasks. Educational Studies in Mathematics, v8 n2, pp211-24, August 1977.

EJ 174 196 Jurdak, Murad E. Structural and Linguistic Variables in Selected Inference Patterns for Bilinguals in Grades Six to Ten. Educational Studies in Mathematics, v8 n2, pp225-38, August 1977.

EJ 174 234 Starr, Robert J. Modern Math Plus Computational Drills: Affective and Cognitive Results. School Science and Mathematics, v77 n7, pp601-4, November 1977.

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EJ 174 239 Kulm, Gerald. The Effects of Two Summative Evaluation Methods on Achievement and Attitudes in Individualized Seventh-Grade Mathematics. School Science and Mathematics, v77 n8, pp639-47, December 1977.

EJ 174 242 Hollander, Sheila K. The Effect of Questioning on the Solution of Verbal Arithmetic Problems. School Science and Mathematics, v77 n8, pp659-61, December 1977.

EJ 174 246 Jamski, William D. A Comparison of One Aspect of the Certification of Secondary Mathematics Teachers from 1957 to 1974. School Science and Mathematics, v77 n8, pp681-2, December 1977.

EJ 174 272 Sorge, D. H.; Wheatley, G. H. Calculus in High School--At What Cost? American Mathematical Monthly, v84 n8, pp644-7, October 1977.

EJ 174 302 Kren, Sandra R.; Huntsberger, John P. Should Science Be Used to Teach Mathematical Skills? Journal of Research in Science Teaching, v14 n6, pp557-61, November 1977.

EJ 174 344 Boyce, Max W. Some Comments on the Use of the Cloze for Classroom Mathematics Materials. School Science and Mathematics, v78 n1, pp9-12, January 1978.

EJ 174 350 Trent, John H. Need for In-Service and Pre-Service Metric Education. School Science and Mathematics, v78 n1, pp45-52, January 1978.

EJ 174 352 Hollander, Sheila K. A Review of Research Related to the Solution of Verbal Arithmetic Problems. School Science and Mathematics, v78 n1, pp59-70, January 1978.

EJ 174 358 Gussett, James C. What Does Industry Really Want in a Mathematics Major? MATYC Journal, v11 n3, pp157-60, 1977.

EJ 174 362 Davis, Edward J.; Cooney, Thomas J. Identifying Errors in Solving Certain Linear Equations. MATYC Journal, v11 n3, pp170-8, 1977.

EJ 174 372 Price, Jack; and Others. "New Math" Implementation: A Look Inside the Classroom. Journal for Research in Mathematics Education, v8 n5, pp323-31, November 1977.

EJ 174 373 Post, Thomas R.; and Others. Teachers', Principals', and University Faculties' Views of Mathematics Learning and Instruction as Measured by a Mathematics Inventory. Journal for Research in Mathematics Education, v8 n5, pp332-4, November 1977.

EJ 174 374 Talmage, Harriet; Hart, Alice. Investigative Teaching of Mathematics and Its Effect on the Classroom Learning Environment. Journal for Research in Mathematics Education, v8 n5, pp345-58, November 1977.

EJ 174 375 Hirsch, Christian R. The Effects of Guided Discovery and Individualized Instructional Packages on Initial Learning, Transfer, and Retention in Second-Year Algebra. Journal for Research in Mathematics Education, v8 n5, pp359-68, November 1977.

EJ 174 376 Campbell, N. Jo; Schoen, Harold L. Relationships Between Selected Teacher Behaviors of Prealgebra Teachers and Selected Characteristics of Their Students. Journal for Research in Mathematics Education, v8 n5, pp369-75, November 1977.

EJ 174 377 Wheatley, Grayson H.; McHugh, Daniel O. A Comparison of Two Methods of Column Addition for Pupils at Three Grade Levels. Journal for Research in Mathematics Education, v8 n5, pp376-8, November 1977.

EJ 174 378 Eastman, Phillip M.; Behr, Merlyn J. Interaction Between Structure of Intellect Factors and Two Methods of Presenting Concepts of Logic. Journal for Research in Mathematics Education, v8 n5, pp379-81, November 1977.

EJ 174 379 Kilpatrick, Jeremy; Weaver, J. Fred. The Place of William A. Brownell in Mathematics Education. Journal for Research in Mathematics Education, v8 n5, pp382-4, November 1977.

EJ 174 380 Clark, Robert C. A Note on the Power of Statistical Tests. Journal for Research in Mathematics Education, v8 n5, pp385-9, November 1977.

EJ 174 386 Becker, Jerry P.; Young, Courtney D., Jr. Designing Instructional Methods in Mathematics to Accommodate Different Patterns of Aptitude. Journal for Research in Mathematics Education, v9 n1, pp4-19, January 1978.

EJ 174 387 Wheatley, Grayson H.; and Others. Hemispheric Specialization and Cognitive Development: Implications for Mathematics Education. Journal for Research in Mathematics Education, v9 n1, pp20-32, January 1978.

EJ 174 388 Barr, David C. A Comparison of Three Methods of Introducing Two-Digit Numeration. Journal for Research in Mathematics Education, v9 n1, pp33-43, January 1978.

EJ 174 389 Brush, Lorelei R. Preschool Children's Knowledge of Addition and Subtraction. Journal for Research in Mathematics Education, v9 n1, pp44-54, January 1978.

EJ 174 390 Bana, Jack; Nelson, Doyal. Distractors in Nonverbal Mathematical Problems. Journal for Research in Mathematics Education, v9 n1, pp55-61, January 1978.

EJ 174 391 Juraschek, W. A. The Interpretation of the Connective OR in Disjunctive Arguments. Journal for Research in Mathematics Education, v9 n1, pp62-6, January 1978.

EJ 174 392 Easterday, Kenneth E.; Henry, Loren L. The Effect of Maturation or Education on Sentential Logic. Journal for Research in Mathematics Education, v9 n1, pp67-9, January 1978.

EJ 174 393 Becher, Rhoda McShane. The Effects of Perceptual Transformation Experiences and Numerical Operational Experiences on Numerical Correspondence and Equivalence. Journal for Research in Mathematics Education, v9 n1, pp69-74, January 1978.

EJ 174 394 Coburn, Terrence G. Criteria for Judging Research Reports and Proposals. Journal for Research in Mathematics Education, v9 n1, pp75-8, January 1978.

EJ 174 395 Friedman, Morton. The Manipulative Materials Strategy: The Latest Pied Piper? Journal for Research in Mathematics Education, v9 n1, pp78-80, January 1978.

EJ 174 446 Nesher, Pearla; Katriel, Tamar. A Semantic Analysis of Addition and Subtraction Word Problems in Arithmetic. Educational Studies in Mathematics, v8 n3, pp251-69, October 1977.

EJ 174 447 Lunkenbein, Dieter. Rationalizing Teacher Interventions--A Working Model of a Process of Research in Mathematics Teaching. Educational Studies in Mathematics, v8 n3, pp271-93, October 1977.

EJ 174 448 Shaughnessy, J. Michael. Misconceptions of Probability: An Experiment with a Small-Group, Activity-Based, Model Building Approach to Introductory Probability at the College Level. Educational Studies in Mathematics, v8 n3, pp295-316, October 1977.

EJ 174 449 Af Ekenstam, Adolf. On Children's Quantitative Understanding of Numbers. Educational Studies in Mathematics, v8 n3, pp317-32, October 1977.

EJ 174 452 Scott, C. C.; Fensham, P. J. Student and Teacher Perceptions of the Degree of Difficulty in a Mathematics Course. International Journal of Mathematical Education, v8 n4, pp375-84, November 1977.

EJ 174 459 Stephens, Larry J. The Effect of the Class Evaluation Method on Learning in Certain Mathematics Courses. International Journal of Mathematical Education, v8 n4, pp477-9, November 1977.

EJ 174 470 Bell, Alan. The APU and the 1978 Mathematics Survey. Mathematics Teaching, n80, pp24-7, September 1977.

EJ 174 676 Adejumo, Dayo. The Reliability and Validity of Modified Hall's Matrices (MHM) for Predicting Mathematics Achievement Among Nigerian Children. Educational and Psychological Measurement, v37 n2, pp501-3, Summer 1977.