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ABSTRACT

This historical study presents evidence on the status of pre-college mathematics education from 1955 through 1975, based on a review, analysis, and synthesis of the literature. It identifies practices and trends in curriculum, instruction, teacher education, learner performance, and needs assessments during the two-decade period. A systematic search of the literature was conducted using such sources as the ERIC data base, "Dissertation Abstracts International," "Education Index," state educational archives, reports from governmental and institutional studies, journals, monographs, yearbooks, and other available, influential documents. Documents were selected in terms of: (1) evidence of significance, (2) validity and generalizability of conclusions from data, and (3) perception of the quality of the work. Sections of the report correspond to three major themes: (1) existing practices in schools--organization, content and courses, what goes on in classrooms, achievement evaluation, student characteristics, instructional materials, and costs; (2) existing practices in teacher education--teacher characteristics and competence, pre- and in-service education; and (3) assessments efforts--national and state needs and progress assessments. Summaries highlight major conclusions, while a concluding section attempts to integrate major findings and to anticipate trends for the immediate future. (MS)

THE STATUS OF PRE-COLLEGE SCIENCE, MATHEMATICS, AND SOCIAL SCIENCE EDUCATION: 1955-1975
VOLUME II: MATHEMATICS EDUCATION

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Review of Literature on Needs and Practices

TABLE OF CONTENTS

TABLES

MATHEMATICS EDUCATION

1955 - 1975

I. Introduction

Schools and schooling are affected by public educational policy. That policy can be rational, based upon knowledge and wisdom, or it can be based upon popular mythologies and misconceptions. The former state of affairs is preferred. The purpose of this document is to provide evidence of how schools and teachers are performing in their task of the mathematical education of children and youth. It is a study of the state of mathematics education in the schools with the past used as a backdrop of evidence about causes and effects of public educational policy formation. Since the past twenty years have witnessed a dramatic reorientation of the mathematics curriculum, of instructional practice, and of teacher education, the historical evolution of school mathematics is traced through the twenty-year period in the hopes that events of the past can be used to provide guidance for making future decision-making more rational.

The schools are an important social institution in the United States and the expectations of society for the schools have a significant role in determining the perception of effectiveness of the schools and the resulting decisions affecting the nature of school programs. The perception of how well the school mathematics programs have served the interests of learners and society is an important feature in the decision-making process. The programs in mathematics that are sanctioned and supported provide evidence of whether for a given era the expectations of society are in terms of the goals of the utility of mathematics for the learner, helping the scientifically talented looking forward to their contribution to society, or for the school serving as a socializing agent for youth. These goals re-occur with regularity throughout the history of the schools.

The schools are big business. Billions of dollars are invested in salaries, instructional materials, teacher education, and school plants. We recognize that non-federal agencies (not only state and local educational agencies but also industries such as textbook publishers) are concerned with and contribute to this investment. However, this report is particularly directed toward the analysis of the federal role. We believe the recent evidence suggests that the fiscal margin that promotes change and innovation comes from investment of federal monies into the educational system. Prior to 1950, the federal investment and intervention of major significance was the setting aside of school lands in opening the Northwest Territories and the founding of institutions of higher education via the Morrill Grants. These political acts were enabling in character and markedly free of specific guidance for the solution of problems. They established no precedents for the manipulation of content, curriculum, or instruction. The events during the years following World War II have been of a dramatically different character. Money was invested in developing and implementing new curricula, in promoting modifications of teacher education programs, in providing new types of school facilities and instructional materials, and in promoting change generally. The 1965 Elementary and Secondary Education Act addressed the problem of breaking the poverty cycle through an assortment of special programs directed to unique segments of the educational enterprise. Since this recent federal intervention is of such different character than observed previously in the history of American education, and since the current evidence is that this intervention will continue, it behooves us to examine carefully the historical record to gain information to guide public, political decision-making about policies affecting the future of mathematics education in the schools.

This document addresses the following questions:

(1) What were and are current practices in mathematics education for curriculum, instruction, teacher education, performance of learners, and needs assessments during the twenty-year period beginning in 1955?

(2) Was the information about practices used or ignored in decision-making concerning policy in education during the twenty-year period?

Since the period is so recent, the information used in this document is at the same time historical in nature but also descriptive of current practices. Whether to consider this document a purely historical study or a description of current status is compounded by the fact that many practices have not changed appreciably during the two-decade period.

Procedures

The procedures used in this study focused upon searching and analyzing the literature of the period. New information was not generated; rather, existing documents were collected and examined carefully. The evidence from published literature in journals, committee reports, and influential books in the field served as a first source. Pertinent documents were collected from the ERIC data base, education archives of the states, and other institutional archives that present evidence concerning the performance of the schools, teachers, and teacher education institutions. The cooperation of state departments of education provided documents concerning curriculum plans, needs assessments, and teacher certification requirements that are not readily available. Research reported in journals, monographs, dissertations, and other sources was considered.

We did not start from ground zero in surveying existing practices in mathematics education. A recent yearbook of the National Council of Teachers of Mathematics, A History of Mathematics Education in the United States and Canada (Jones, 1970) provides an extensive description of events and existing practices for the first two-thirds of the twenty-year period. The Overview and Analysis of School Mathematics Grades K-12 prepared by the National Advisory Committee on Mathematics Education (1975) provides extensive information about more recent history in mathematics education. In the present report we have attempted to emphasize different sources of information and to complement and up-date the insights of these excellent sources.

Extensive use has been made of other historical and descriptive studies. If a document exists for a particular topic that provides extensive related information of a summary character, we have followed the strategy of trying to capture the highlights of the content, rather than extensively and exhaustively reporting its content. In many instances, the reader may find referring to the original document helpful in completing the perspective for particular findings.

Document selection provided a major problem. Determination of which documents to cite and use, as opposed to ignore and not cite, was a judgment of importance in the writers' opinions. The judgment was exercised in terms of:

(1) Evidence of significance provided by literature in refereed journals, committee reports, and major books for which events and history have indicated a primary influence on the field.

(2) Generalizability of conclusions from documents reporting data. That is, size of populations, sampling procedures, and methods of analysis that provided limitations on the scope and applicability of results led to the rejection of many documents. It should be noted that the majority of the documents cited are status studies or other types of survey research. These provide evidence on the practices or reactions of various samples at a given point in time, and were particularly useful for the purposes of this report. Experimental research is cited when it illustrates a point or provides cumulative evidence.

(3) Perceptions of the quality of the work based upon the writers' experience and knowledge, and using evaluative criteria developed by Suydam (1972). The purpose of this report was not, however, to evaluate research, and thus strengths and weaknesses of the studies are seldom delineated. The intent was to select documents of sufficient quality to warrant citation; it should be recognized that few documents are without limitations,

either in design or in author bias.

Clearly, we may have erred; however, it should be recognized that relatively few writings (and events) withstand the test of historical significance when considered from a long-term historical perspective. Given the additional perspective of another twenty years in the year 2000, the majority of the documents cited may well be deemed inconsequential and irrelevant. But at this point in time, we opine the ideas gleaned from the documents carry major import for decisions bearing on current issues.

Format of the Report

Three major themes are treated in this review of existing practices:

(1) The Schools -- organizational, instructional, and curricular patterns are reported as well as information concerning facilities, equipment, costs, and student characteristics.

(2) The Teachers -- preservice and in-service education are examined as well as information concerning background, competence, and behaviors.

(3) Needs Assessment -- planning documents, systematic needs assessments, and systematic progress assessments useful in policy-making at the national and state levels are described.

Corresponding to each of these themes is a major section in the following pages. Each has summary sections that synthesize highlights reflecting major conclusions derived from the historical record. A final concluding section provides a summary that serves to integrate major findings and to anticipate trends for the immediate future.

Using History in the Study of Education

This is a historical study. It is easy to err in using history to predict the future. No historian limits his or her thinking only to history in making judgments about what-ought-to-be for the future. The careful historian realizes that because the societal ethos is brittle and changing, because the environment is shifted due to the very events of the history being studied and the changes wrought by new technologies and new knowledge, the conditions leading to decisions and actions never repeat themselves precisely.

History does not determine what-ought-to-be. The value questions associated with the determination of goals and objectives in the future, and the present, exceed the prerogative of the historians. At best the social historian can explain how value structures evolved and what they are. The task of determining goals for future activity in mathematics education exceeds the scope of this historical record, although judgments of deficiencies in the present status of mathematics education are reported.

This document provides information about the determination and implementation of educational policy and its rationality, or lack thereof. Determination of educational policy operates at two levels. One operates internal to the education profession and is manifested in the type of philosophical support and the state of knowledge accorded learning or teaching in the schools. The other level is external to the schools and is based upon societal concern and ethos for the schools and their ..als and is realized through the political decision-making process.

As you examine the historical record for mathematics education from 1955, consider the extent to which these two levels of determination of public policy for education interact. To what extent does public policy result from knowledge generated within the profession? Are policy decisions affecting mathematics education made on a basis of sound knowledge concerning the existing practices in the schools? To what extent are practices in the schools and in the profession tempered and affected by the societal ethos or the political climate? If the concerns, issues, and problems for the two different levels exhibit commonality and consistency, is change in practice more likely? Are needs assessments, progress assessments, and descriptions of the status of the schools and teacher education used for rational formulation of policy or merely symptomatic of current societal concerns?

We make few judgments concerning the answers to these questions. For most topics in the following historical record, this task is left to the reader. It is an important task since it involves the rationality of the decision-making process for policies affecting mathematics education.

We suggest that you will observe that educational policy-making does not use knowledge of existing practices to determine policies. We remark that you will also note that the profession's seeking of new knowledge about practice frequently does not necessarily bear on the problems and concerns at issue for the decision-making process until after the decision has been made.

Determination of educational policy must recognize reality. Some aspects of schools and schooling have an inherent stability and resistance to change no matter what the educational policy might be. For example, many student characteristics are unlikely to change as a result of changes in educational policy. Decision-making about policy will not affect the genetic make-up of students nor will it have much impact on student characteristics induced by well-established societal mores. Many traditions concerning how teachers act and the structure of the school derived from many generations of schooling provide an inertia requiring exceptional energy to effect change. But these factors must be described and taken into account in decision-making concerning educational policy; otherwise both energy and resources are likely to be wasted by the formulation of policy addressing the wrong problems. The section describing existing practices in the schools identifies many of these factors that are not subject to significant control through policy formulation.

The Political Setting

Policy decisions for education take place within the political arena. The societal ethos of an era determines the character of the political arena since it incorporates the goals and values displayed by the society. Thus, it is important to recognize some major features of the political and social climate for the period from 1950 to the present before examining the evolution of existing practices for mathematics education.

Our historical perspective is that the decade of the 1950s is best characterized in terms of the interaction between recovery from World War II and the issues related to the Cold War. A relatively stable economy provided freedom for growth in the educational system, a growth necessary because of the influx of children to the schools resultant from the post-war baby boom.

The Cold War factor was of significant import in education since the nuclear arms race made important the extent and the quality of the pool of scientific talent in the United States.[3] At the same time the nurture of scientific talent was at issue, attacks on the remaining vestiges of the Progressive Education Association--and related teacher education programs--was taking place in the setting of higher education (Cohen, 1976). The events of McCarthyism and the concern for scientific talent provided a state of readiness for and acceptance of dramatic changes relative to school mathematics (Osborne and Crosswhite, 1970).

The spirit of the Kennedy presidential years involved a social concern that presaged the educational policy determination of the mid-1960s. The thrust toward helping the less fortunate, the culturally disadvantaged and separated, and the attempt to break the poverty cycle through education exhibited in the good intentions of the Johnson era, all provided a reorientation for policy making in education and the resultant funding patterns. The U.S. Office of Education attempted to become an agent for change rather than an information repository. The imperative for developing scientific talent evaporated.

The political context of the late 1960s and on into the next decade is one of societal discontent reflecting the impact of the conflict in Southeast Asia and the derivative financial hardship. A more hard-nosed, reactionary view toward spending for all social welfare, including education, became apparent. This, coupled with the loss of the imperative for development of scientific talent and the established remaining concerns for the disadvantaged, provided a confused context for policy makers concerning education and for mathematics education in particular. Accountability, divesture of many responsibilities to the states through revenue sharing, and a loss of a clear educational imperative for a particular but limited set of goals created an amorphous, puzzled political consciousness not conducive to establishment of clearly delineated educational policy.

II. Existing Practices in Schools

In this section, evidence from research and other literature describing practices in mathematics education is presented. An attempt was made to trace patterns and to consider the mode of decision-making for aspects of seven areas of concern:

- The organization of schools
- The curriculum
- Classroom concerns
- Evaluation of achievement
- Student characteristics
- Instructional materials
- Costs of instruction

We struggled to trace patterns of practices; only occasionally could patterns within these areas be determined from existing documentation. For most areas of concern, no discernible patterns could be found: in some, practices changed in reaction to some definite stimulus; in others, practices fluctuated without apparent design.

We struggled to determine what the decision-making process was, what created the need for decisions, and on what basis decisions were made; only rarely could these be ascertained. Decisions were and are being made continuously about practices in each of these areas -- but the basis and rationale for these decisions have been documented infrequently.

We conclude that we could conjecture about the change process, and we could cite the conjectures of others, but to document the actual reasons for decisions made regarding practices in teaching mathematics is not feasible for most of these areas. The factors which influence practices are varied and complex; change is not linear.

A. Overview, 1955-1975

To provide a perspective on existing practices, we begin with an overview of concerns in and affecting mathematics education since 1955, noting in particular both the involvement of federal agencies and research efforts that reflected changing concerns.

In 1955, few teachers realized that they were on the brink of a curricular reform movement -- a movement whose origin is frequently cited as 1951, when the University of Illinois Committee on School Mathematics (UICSM) was formed. The process seems to some to be more evolutionary than revolutionary. The scope of the changes in mathematics itself since the turn of the century increasingly demanded changes in the content of school mathematics (e.g., see Price, 1961). Methodological concerns were continuous; the drill orientation of the 1920s had given way to the incidental theory in the 1930s, but by the early 1950s Brownell's (1935) reasoned argument for meaningful instruction had been adopted by consensus -- in thought if not in deed. Even a cursory reading of Brownell indicated that the "discovery" teaching of the 1960s was foreshadowed.

There would seem little need to describe the details of the curriculum movement; it has been extensively documented on other sources. The most recent description is in the Report of the National Advisory Committee on Mathematical Education (NACOME, 1975). The Thirty-second

Yearbook of the National Council of Teachers of Mathematics (NCTM), A History of Mathematics Education (Jones, 1970) provides a thorough account, as does a dissertation by Crespy (1970) and a host of earlier publications {e.g., The Revolution in School Mathematics (NCTM, 1961) and The Continuing Revolution in Mathematics (NCTM, 1968)}. Some of the major events in the process of mathematical curricular reform will be noted, however, to trace patterns for those who might be familiar with them.

In the Thirty-second NCTM Yearbook, Jones and Coxford (1970) note:

> By 1955, partly as a result of the unrest growing out of World War II, the lay public throughout the country had been told in magazine articles and in books that the academic substance of the school curriculum was grossly inadequate. It was said that the content not only of mathematics but of other subjects as well had for too long been determined by professional educators with little or no impact from the scholars of the various disciplines . . . (p. 76)

The stage was being set for change.

In 1955, the College Entrance Examination Board (CEEB), concerned by the need to provide a base for a changed college curriculum, formed the Commission on Mathematics. The Commission was "to review the existing secondary school mathematics curriculum, and to make recommendations for its modernization, modification, and improvement"; its concern was primarily with the "college-capable" student. Although the Commission's report was not published until 1959, a preliminary form was widely circulated, and its recommendations provided the framework for the reform of the curriculum. There was anticipation that the new program could be introduced for the majority of college-bound students within five years, provided adequate attention was given to in-service and preservice teacher education. A report by an NCTM committee on the secondary-school curriculum (NCTM, 1959) echoed the Commission's report, but differed in recognizing the need to consider the below-average student.

The National Science Foundation (NSF) was established in 1950 to develop a national policy for the promotion of basic research and education in the sciences. From the mid-1950s, the major contribution of NSF to elementary and secondary education was in providing support for in-service institutes for teachers of mathematics and science. While such efforts continued into the 1970s {see Krieghbaum and Rawson (1969) for a history of the institutes, 1954-1965}, the flight of Sputnik in 1957 resulted in an acceleration of federal funding that allowed the Foundation to begin the process of curricular reform on a major scale.

The nation's avowed need for scientific manpower and increased scientific literacy was reflected in the curricular efforts of other agencies in addition to NSF. In 1958, Congress recognized the need to improve school mathematics in the provisions of the National Defense Education Act. NDEA Title III authorized payments on a matching basis to state educational agencies for:

(1) The acquisition of laboratory and other special equipment, including audiovisual materials and equipment, and printed materials (other than textbooks) suitable for use in providing education in science, mathematics, and modern foreign languages in public elementary and secondary schools.

(2) Minor remodeling of the laboratory or other space used for such materials or equipment.

(3) The expansion or improvement of State supervisory or related services in the fields of

science, mathematics, and modern foreign languages. (Phillips and Kluttz, 1965, pp.22-23)

By mid-1964, the States had received matching funds for 78,760 projects and 46 states had made supervisory services in mathematics available, an increase from 6 in 1958.

Congress increased appropriations to NSF, and the money for education was immediately put to work in implementing the recommendations of the Commission on Mathematics. As a result of the deliberations of mathematicians at an NSF-sponsored conference concerned with research potential and training, the School Mathematics Study Group (SMSG) was formed and set to work developing materials for secondary-school mathematics.

The establishment of other curriculum development projects followed, most of them with some support from NSF (see Table 1 and Lockard, 1977). Conferences (see Table 2) were used as a primary vehicle for ascertaining needs; the invited experts presumably reflected prevailing opinion tempered with knowledge and thought.

Crespy (1970) commented on 24 projects initiated between 1950 and 1966. Of the 20 projects producing materials, 6 focused on the elementary-school level; 2 on grades 7 and 8; 5 on grades 7-12; 2 on grades 9-12; and 5 focused on K-12. (Lockard (1977) provided a more complete listing of over 60 mathematics projects in operation between 1956 and 1976.) Crespy called attention to three important points about the projects:

- A hallmark of the period was the ability of mathematicians and educators to work as a team. Such cooperation had not taken place since the first part of the twentieth century and was virtually unkown in the 1930s and 1940s.

- UICSM set the pattern that not only were new materials needed but the retraining of teachers was also a necessity.

- The cost of mathematics curriculum development was phenom-

TABLE 1

TIME LINE, 1955-1975

1955	CEEB Advanced Placement Program began
	Commission on Mathematics appointed by CEEB
	Ball State Experimental Program funded
1956	
1957	Sputnik launched
	Madison Project, University of Maryland Mathematics Project, Boston College Mathematics Institute funded
1958	NDEA passed
	School Mathematics Study Group, University of Illinois Arithmetic Project funded
1959	CUP reorganized as CUPM
	Greater Cleveland Mathematics Project, Stanford Mathematics Projects formed
	Commission on Mathematics Report, NCTM Secondary School Curriculum Committee Report issued
1960	Conference Board of the Mathematical Sciences formed
1961	Minnemath started
1962	National Longitudinal Study of Mathematical Abilities began
1963	Cambridge Conference Report issued
	R&D Centers established
	Committee on Mathematics for the Non-College Bound formed
1964	Individually Prescribed Instruction - Mathematics Project began
1965	ESEA passed
	Regional Educational Laboratories established

TABLE 1 (continued)

1966	Secondary School Mathematics Curriculum Improvement Study began
1967	Comprehensive School Mathematics Program established at CEMREL
1968	
1969	
1970	
1971	Unified Science and Mathematics Project for Elementary Schools began
1972	NIE formed
1973	
1974	Conference Board of the Mathematical Science appointed National Advisory Committee on Mathematical Education
	Problem Solving Strategies and Applications of Mathematics in the Elementary School and Project for the Mathematical Development of Children formed
1975	NACOME Report issued

enal compared to the cost prior to 1950. (pp. 319-320)

He might also have noted that concerns about overemphasis on formalism and rigor at the expense of useful techniques and applications were beginning to be expressed by 1962 (DeMott, 1962).

Accompanying the curriculum reform was an explosion in research, generated largely by the need for more doctoral-level manpower and the resulting availability of funds for higher education, and partially by the need for research to support the curriculum development effort. The

TABLE 2

CONFERENCES ON MATHEMATICS EDUCATION (K-12)

Date	Conference	Focus	Reference
1958	Chicago Conference on Research Potential and Training	need for change	
1959	Royaumont Seminar on Secondary School Mathematics	"new mathematics"	OECD, 1961 (ED 055 895)
1962	Cambridge Conference on School Mathematics	pre-college curriculum for the future	Cambridge Conference, 1963 (ED 015 140)
1966	Conference on Secondary School Mathematics	planning for "second round" SMSG development	SMSG, 1966 (ED 059 875)
1967	Conference on Mathematics for Gifted Students	role of SMSG in preparing materials	SMSG, 1967 (ED 083 007)
	National Conference on Needed Research in Mathematics Education	progress of research, guidelines for future	Hooten, 1967 (ED 022 674)
	Cambridge Conference on the Correlation of Science and Mathematics in Schools	mathematics-science curriculum development	Cambridge Conference, 1969 (ED 042 599)
1970	Conference on Mathematics Education in the Inner-City Schools	role of SMSG in inner city	NSF, 1970 (ED 083 008)
1973	Estes Park Conference on Learning Through Investigation and Action on Real Problems in Secondary Schools	problem solving approaches	Estes Park Conference, 1973 (ED 090 065)
	Conference on the K-12 Mathematics Curriculum, Snowmass	K-12 curriculum	Springer, 1973 (ED 081 643)
	Cape Ann Conference on Junior High School Mathematics	junior high content	Cape Ann Conference, 1973 (ED 085 257)

TABLE 2 (continued)

	Conference on the Future of Mathematics Education, Tallahassee	future needs	Tallahassee Conference, 1973
	Orono Conference in the Middle School Mathematics Curriculum	middle-school content	Beard and Cunningham, 1973 (ED 085 258)
1974	Conference on Mathematics Resource Materials, Eugene	instructional materials	Hoffer, 1974 (ED 103 273)
1975	Euclid Conference on Basic Mathematical Skills and Learning	needed mathematical skills	NIE, 1975 (ED 125 908/909)
1976	Conference on Needed Research and Development on Hand-held Calculators in School Mathematics	calculators	NIE/NSF, 1977 (SE 022 565)

amount of research in mathematics education increased startlingly (see Table 3), especially at the dissertation level; publication outlets were limited, so that the increase in the number of articles is not as dramatic. As one indication of the amount of research, data for the year 1975 alone should be compared with that for 1955-59: 368 studies were reported for the one year, contrasted with only 340 for the earlier 5-year period. Appendix A provides additional evidence on some of the areas of concern attacked by researchers since 1955; the extent of attention on particular aspects is evident from the patterns of the data.

In their discussion of the years from 1945 through the 1960s,

TABLE 3

RESEARCH REPORTS ON MATHEMATICS EDUCATION (K-12)

	Summaries	Articles*	Dissertations	Total	
1955	4	20	26	50	
1956	3	33	27	63	
1957	8	29	41	78	340
1958	2	31	40	73	
1959	3	36	37	76	
1960	6	62	51	119	
1961	9	49	41	99	
1962	5	60	64	129	662
1963	5	79	57	141	
1964	7	98	69	174	
1965	7	67	92	166	
1966	9	95	134	238	
1967	6	93	136	235	1224
1968	5	82	151	238	
1969	16	119	212	347	
1970	14	91	223	328	
1971	10	94	322	426	
1972	13	71	335	419	1942
1973	9	76	299	384	
1974	9	112	264	385	
1975	3	99	266	368	
				4536	

* Some articles are reports on previously recorded dissertations.

Jones and Coxford (1970) wrote:

> It may be that this period is harder to describe and seems significantly different from earlier periods because we are so close to it. However, the forces critical of mathematics education and, indeed of all education have never been so varied nor so strong at any other time. Likewise, the range of innovations actually attempted and the energies poured into educational reform in this period -- especially 1952-1962 -- have never before been even approximated. (p. 67)

A new type of attack on the schools evolved during this period of change; scarcely anyone was unaware of the accounts of experience and observations by writers like Holt (1964), Kohl (1967), Kozol (1967), or Silberman (1970). Some of the concerns were humanistic in nature, reactions to how children were being treated and what schools were doing to children. Mathematics and other curricular areas provided illustrations of how instruction was intensifying the problem of children being led or dragged through meaningless content and being "turned off" by schools.

Behavioral objectives and individualized instruction became key words. A strong behavioristic wave started as part of new trends in individualizing instruction by means of educational management systems. Large projects like PLAN and Talent, funded largely by private foundations, addressed the goal of transforming exisiting educational materials into structured sequences by means of task analyses.

By the mid-1960s, it became apparent that the public not only expected academic excellence: that schools help solve societal problems was also demanded. The social-action legislation of the Johnson Years included the Elementary and Secondary Education Act (ESEA) of 1965, sending money for innovation, particularly for the "disadvantaged", into the schools. ESEA had two titles which had specific potential for affecting mathematics instruction: Title I, Programs for the Disadvantaged (poor) and Title III, Supplementary Centers. (Title II was for library resources; Title IV, research, amending the Cooperative Research Act of 1954; and Title V, strengthening state education agencies.)

Administration of ESEA fell almost entirely to the U.S. Office of Education. Largely because of concerns about federal control of education, USOE had assumed an advisory stance over the years, collecting information but rarely initiating action. Its role was now mandated as one of encouraging change through the allocation of funds.

Title I monies, 5/6 of the total amount budgeted under ESEA, were to be spent for improving the education of the disadvantaged, with reading as the primary target and mathematics second. Title III was to be a means of linking research and development with practice. Supplementary Centers were to deliver innovative services not previously available to individual schools. "Actually," reported the NIE Databook (NIE, 1976b), "Title III funds have been used to support development and dissemination of 'exemplary' practice" (p. 17). Of 661 'products' sponsored by NIE in 1975 (NIE, 1976a), mathematics and science were the focus of only 39 -- that is, 6%.

A continuing problem was how to generate impact and effect change in the schools. The establishment of ERIC (Educational Rescources Information Center) in 1966 resulted from one aspect of this need. ERIC provided a repository for information, especially on "innovative practices". Twelve clearinghouses were funded initially, with the number rising to 19 in 1969 and then decreasing as efforts were compacted; in

1975 there were 16 and in 1977, only 11. In addition, 20 Regional Educational Laboratories were established by 1966-67 by USOE (in 1973, 12 remained) to disseminate the results of R&D efforts, especially those of the previously established R&D Centers. Thus the Individually Prescribed Instruction (IPI) program was developed at the R&D Center at the University of Pittsburgh and disseminated by Research for Better Schools, a regional laboratory. Two R&D Centers were funded in 1964; by 1968 twenty had some type of funding, but by 1970 the number had shrunk to 15 and in 1975 there were seven. One widely known among mathematics educators was located at the University of Wisconsin; it produced the Developing Mathematical Processes (DMP) materials, with a measurement orientation to mathematics instruction. Other regional laboratories and R&D centers have produced supplementary materials and materials for minority groups.

A report from NSF (1975) describes four thrusts of the Foundation during this period: curriculum projects; teacher preparation; implementation; and reports, conferences, and research support. The Cooperative College-School Science Program provided a vehicle for collaboration, while the Course and Curriculum Improvement Projects and the Course Content Improvement Program were among the thrusts to promote "grass-roots" implementation.

Concomitant with the needs and demands of the period, second rounds of curriculum development were organized by NSF, to improve on initial efforts and to add new emphases. The mathematically able and talented student was the focus of the first-round curriculum development effort; funds now were also directed into programs for the low achiever. Social forces -- dissent boiling over in riots, spreading from urban centers to university campuses to secondary schools -- created another impetus for change. The curriculum was neither the cause nor the focus of the dissent -- but schools reacted to the stress by changing course structure and content and by developing such scheduling patterns as the module, which allowed learners to put short curricular sequences together in unique patterns.

Changes were also occurring in NSF and USOE, largely as a result of pressures to show the impact of the dollars being directed toward education. The National Institute of Education (NIE) was created by the Education Amendments of 1972, the culmination of several years of efforts to establish a separate organization within HEW devoted to educational research and development exclusively. NIE took over USOE's role in supporting curriculum development. (The early history of NIE is summarized in several publications; e.g., NIE, 1973a, 1973b.) Priorities have included both basic skills and compensatory education. The thrust of the basic skills effort is to discover what reading and mathematics skills are "required for adequate functioning in society", how children "may overcome barriers to learning such skills", and how to improve the teaching of the two areas.

NSF began to consider different patterns of funding to promote in-service education efforts. A systems approach was modelled by the Oregon System in Mathematics Education and by the Delaware Model. The first attempts to work closely with small-scale projects throughout the state; the second is closely allied with higher education agencies.

The Education Amendments of 1974 extensively revised many of the activities authorized by the ESEA of 1965. Several "national priorities" were specified in the Act, including use of the metric system, education of gifted and talented children, career education, consumer education, and women's equity in education. For each of these, a relevance to mathematics was apparent. In essence, Title III of ESEA ceased to exist; it was continued as Title IV of the new legislation, consolidated with six other programs (NACSCS, 1975).

The 1970s brought additional demands for curricular change as headlines projecting "declining scores" and accountability demands increased. "Back-to-the-basics" became the slogan, as Kline (1973) and others led in depicting the "failure of the new math". Needs assessment became a policy as federal agencies demanded better accounting of the funds pouring through their hands. The Conference Board of the Mathematical Sciences appointed the National Advisory Committee on Mathematical Education (NACOME), charging it to provide an overview of mathematics education in the schools, synthesizing reactions and making recommendations for future directions.

It also seemed apparent in the 1970s that technology, which had so great an effect on the quality of life over the 20-year span, took leaps ahead and gave indications that the school, too, could be integrally affected by technological inventions.

> . . . some feel that what is past is only a prelude to a greater revolution yet to come. And others see the events of the past two decades as a natural, although accelerated, evolution from the long sequence of events which has been traced. . . However, few would deny that, as measured against that of any comparable period in the history of mathematics education, both the pace and the extent of change over the past twenty years have been revolutionary. (Osborne and Crosswhite, 1970, p. 235)

Overview, 1955-1975: HIGHLIGHTS

- The past 20 years have witnessed:
  - continuing curriculum reform, with mathematicians and educators working as a team
  - extensive federal funding with federal policy increasingly affecting curricular development
  - changing roles for federal agencies (NSF, OE, NIE) as they assumed varying degrees of responsibility for the cost of curriculum development and teacher retraining
  - an explosion in research as well as development efforts
  - concern for the mathematically able, especially at the secondary level
  - concern for the disadvantaged, especially at the elementary level

- The need for curriculum reform was generated by:

1955 - public dissatisfaction with existing curricular outcomes

- concern from mathematicians and mathematics educators

1965 - concern for the economically and educationally disadvantaged

- reassessment of the need for mathematical rigor

1975 - patterns of declining achievement scores, especially at the college-entrance level

- pressures for accountability

- Needs assessments in mathematics education were conducted through:
  - conferences
  - informed writing, both pro and con
  - opinion polls

- Much analysis of mathematics education has been undertaken, including major efforts by the National Council of Teachers of Mathematics and the Conference Board of the Mathematical Sciences.

B. How Are Schools Organized?

Educators have long-searched for the "perfect" pattern of school and classroom organization to meet the needs of individual students and increase achievement. Much has been written about various organizational patterns. In 1955, there was recurrent discussion of departmentalization and the use of mathematics specialists as the answer to the poor mathematical preparation of many elementary-school teachers. By 1960, various multi-graded and nongraded approaches were tried out, and the core of some of these remain today. Team-teaching was proposed for all levels as an alternative to departmentalization. In the late 1960s, "open-space schools" and "open-classroom environments" were espoused in yet-another attempt to make the school less rigid. Alternative schools (to enable parents and students to select a desired pattern) and various modular scheduling patterns (to enable students to select topics of need and interest) are still available on a small scale. In fact, all of the proposed innovations are evidenced in various locations. But perusal of a wide variety of documents leads to the conclusion that the graded, self-contained classroom at the elementary-school level and the fixed-period-schedule-of-the-secondary school have remained the predominant patterns over the past 20 years.

Data on the number of schools reporting use of various approaches to mathematics instruction have been difficult to locate. One survey of 720 schools in New York (conducted by two New York State bureaus during 1971-72) provided the following information:

Technique	Elementary Schools	Middle Schools	High Schools
Flexible scheduling	21.5%	20.6%	14.6%
Independent study	21.2%	21.2%	34.6%
Team Teaching	31.9%	23.5%	9.9%
Non-graded	24.4%	8.6%	4.9%
Continuous progress	24.4%	15.0%	8.2%

Over the years, a large number of studies has been conducted to ascertain the efficacy or the superiority of one or another organizational pattern; the data in Appendix A reflect 141 such studies for mathematics education alone during the two decades. Such attempts have been hampered by the difficulty of isolating and measuring the effects of the organizational pattern, since such factors as content organization and teacher skill interact with the pattern. Additional confusion results because definitions of the various patterns tend to overlap; thus, what one person labels team teaching another may define as departmentalization.

In reviews of research (e.g., Suydam, 1972; Suydam and Weaver, 1970, 1975), it has been concluded that there appears to be no one organizational pattern which will increase student achievement in mathematics. Proponents of any pattern can find studies which verify their stand, but a large proportion of the studies reported no significant differences in achievement between two or more patterns. Decisions appear to be made on the basis of selective evidence and a hope for improvement. It appears that belief in a particular pattern and a desire to make it succeed may aid in creating an environment conducive to obtaining favorable achievement by students and satisfaction from teachers. The specific components that make any organizational pattern effective and the weaknesses that cause another pattern to seem less effective are rarely documented: rationales rather than evidence abound. Perhaps the most important implication of the various studies is that good teachers can be effective regardless of the nature of the school organizational pattern. Concomitant with this is the frequently noted suspicion that some teachers can be more effective with one pattern than with another; however, research has not explored this suspicion.

Organizational Patterns: HIGHLIGHTS

- There appears to be no one organizational pattern which will increase student achievement in mathematics. Good teachers can be effective regardless of the nature of the school organizational pattern.

- While much has been written about team-teaching, modular-scheduling, and other varied approaches, the self-contained classroom at the elementary-school level and the fixed-period schedule of the secondary school have remained predominant organizational patterns.

C. How Are Curriculum and Content Selected?

As it is reflected in textbooks, curriculum guides, and descriptions of courses, the content of school mathematics curricula has changed over the past twenty years. The NACOME Report (1975) noted that

> . . . the common elementary program has undergone substantial change in the past ten years. The label "arithmetic" has appropriately given way to "mathematics" as curricula incorporate varying amounts of geometry, probability and statistics, functions, graphs, equations, inequalities, and algebraic properties of number systems. (p. 11)

At the secondary-school level a comparison of leading commerical texts reveals both change in emphases and inclusion of new content.

Much consistency is noted across the years: computation with whole numbers, fractions, and decimals persisted as the mainstays of the elementary-school curriculum; the secondary-school curriculum for college preparation continued to be based on algebra and geometry. Differences are obvious: the inclusion of geometry at the elementary-school level and computer mathematics at the secondary-school level, for instance, or the change from plane geometry and solid geometry to "geometry" with no modifier. In other instances, changes between 1955 and 1965 have been reversed by 1975. Thus several topics, such as sets and non-decimal numeration systems, are practically non-existent in newer elementary-school curriculum materials.

The elementary-school mathematics curriculum of 1955 was sequenced in great part as a result of the work of Washburne and the Committee of Seven (1931). After thousands of students were tested, the mental age at which each topic could be learned was ascertained; grade placement and sequencing of topics were determined in terms of that data.

Content and courses at the secondary-school level have evolved over a long period of time, largely as content has moved downward from the college level. Algebra, for instance, began to become a mainstay of the secondary-school curriculum when Harvard University required it for admission in 1820. Geometry moved down from the college level just after the Civil War. On the other hand, the general mathematics course was developed to meet the needs of the non-college-bound, as advocated by a National Committee on Mathematical Requirements in 1923.

The curriculum reform movement begun in the 1950s was originally intended to effect changes in the secondary-school curriculum for college-bound students. However, elementary-school curriculum projects were funded in 1958 at the same time as the secondary-school projects were. Most of the early elementary-school projects proposed to develop supplementary materials to enrich and extend the curriculum to incorporate new goals. It became evident to those conducting secondary-school projects, and especially to SMSG, that secondary-school reform would not be successful unless the elementary-school program were changed to provide a better foundation. Accordingly the curricular reform moved downward.

What characterized the new mathematics programs was difficult to define even while the development was occurring, for the variety was great. Some factors seemed common to a majority of these programs:

(1) Increased emphasis on the structure of mathematics.

(2) Increased emphasis on rigorous deductive proof, particularly at the secondary-school level.

(3) Increased emphasis on student exploration, partic-

ilarly at the elementary school level, with discovery and inductive approaches promoted.

(4) Increased emphasis on correct terminology.

(5) Readjustment of grade placement of topics.

(6) Inclusion of topics not usually taught at the level, such as geometry in the elementary school and calculus in the secondary school.

Jones and Coxford (1970) named structure, proof, generalization, and abstraction as "the essence of modern mathematics".

The NACOME Report discusses, in some detail, content innovations, the role of deduction, the role of abstraction, and the role of symbolism and terminology. They conclude:

> The content innovations K-12, the emphasis on student understanding of mathematical methods, the judicious use of powerful unifying concepts and structures, and the increased precision of mathematical expression have made substantial improvement in the school mathematics program. Unfortunately, the innovations have not fulfilled the euphoric promise of 1960, and current debate seems intent on locating blame for failures in real or imagined "new math" programs. (p. 21)

They go on to deplore the dichotomization of curricular issues and note

> a fallacy that seems very difficult to eradicate; that of viewing the "new math" as a monolith, a single phenomenon that one can be for or against. Actually it refers to two decades (1955-1975) of developments that had a general thrust and direction but sprang from many roots, took many different and even opposing forms, evolved and changed with facets disappearing and new ones arising. (p. 21)

Many studies have been conducted to trace the development of the curriculum (see Table 6). Crespy (1970) provides one of the most thorough overviews, including topics taught in various courses from

1950 through 1965 and even the names of members of various committees.

In summary, he concluded that:

- Impetus for reform in the school mathematics curriculum existed in the late 1940s and early 1950s. The availability of Federal funds in the late 1950s and the shock Sputnik brought to national attention a reform movement in school mathematics that was already in existence.

- The earlier reform groups started with limited goals and expanded as they matured.

- The reform groups did not accept the principle of diversity among schools in the nation; rather they worked from the premise that a hard core of mathematical content existed for all and had no basis for varying geographically.

- Much of the mathematics already in the curriculum served as the basis for the mathematical content of the reform. However, it was presented in a new light. Emphasis was on the concepts rather than rules of operation. Content was introduced at earlier levels than under traditional curricula. Some traditional content was dropped or had less time devoted to it; e.g., solid geometry as a separate twelfth-grade course. The major new content was in the area of statistics and probability.

- Much of the energy of the reform movement was directed to a better understanding of the basic concepts of mathematics rather than more computational efficiency.

- Uniformly, all reform groups producing materials made experimental use of them in classrooms prior to revision and final publication. Evaluation was by exposure to actual teaching. Students using the new materials did as well as students using traditional materials on tests measuring traditional content.

- New materials were widely used. By the mid-1960s, SMSG and GCMP each stated that their materials were being used by five million pupils nationally (of 40 million students in K-12).

Content was obviously not the only component for which change was attempted: the methodology was affected, too. And there was an attempt to incorporate a change in goals. Historically, computational skill has been highly valued by society; in the 1950s, mathematical understanding was endorsed as another important goal. The value of this goal is being questioned in the back-to-the-basics movement of the 1970s. Such attacks on the curriculum began at a time when the curriculum was already undergoing adjustment. Two topics which are frequently associated with "new" mathematics are sets and other number bases. By the mid-1960s, it had become apparent from observations by mathematics educators that sets were not being used in a meaningful way in most elementary-school mathematics programs. Non-decimal bases were included in programs because it was presumed that their study would strengthen understanding of base ten. But research clearly indicated that they did not do this: the same amount of time spent on base ten was as effective as the study of non-decimal bases (Glennon and Callahan, 1975; Suydam and Weaver, 1975). Thus both topics were disappearing from elementary-school textbooks.

In the late 1950s and early 1960s, content of the new curricula was the focus of many articles on mathematics in educational journals. Several NCTM yearbooks were devoted to the function of retraining teachers on new content (see Table 4), as the NCTM devoted extensive efforts to support of curricular reform. As might be expected, a large percentage of research in the late 1950s and 1960s focused on the feasibility of teaching various topics (see Appendix A). Thus Suydam and Weaver (1970, 1975) reviewed studies indicating that geometry, graphing, number system properties, integers, probability and statistics, sets, and logic could be taught in the elementary school. At the secondary-school level, functions, vector approaches to geometry, computer techniques, and calculus were among the topics studied (Suydam, 1972).

TABLE 4

NCTM YEARBOOKS, 1955-1977

Date	Title
1957	Insights into Modern Mathematics
1959	The Growth of Mathematical Ideas, Grades K-12
1960	Instruction in Arithmetic
1961	Evaluation in Mathematics
1963	Enrichment Mathematics for Grades
1963	Enrichment Mathematics for High School
1964	Topics in Mathematics for Elementary School Teachers
1969	More Topics in Mathematics for Elementary School Teachers
1969	Historical Topics for the Mathematics Classroom
1970	A History of Mathematics Education in the United States and Canada
1971	The Teaching of Secondary School Mathematics
1972	The Slow Learner in Mathematics
1973	Instructional Aids in Mathematics
1973	Geometry in the Mathematics Curriculum
1975	Mathematics Learning in Early Childhood
1976	Measurement in School Mathematics
1977	Organizing for Mathematics Instruction

The Cambridge Conference on School Mathematics (1963) proposed a curriculum that might be attained by the end of the century. This vision was a shock to many. Outlines of a variety of units were developed to provide evidence that the proposed content could be taught effectively.

The curriculum development projects given national prominence (see Table 2) and those supported at the local level, in large part from Federal funds, have similarly explored a variety of content. Both research and development efforts have provided "existence proofs" on the possibility of teaching many specific topics.

There is little doubt that the number and variety of courses offered at the secondary-school level have increased since 1955. In 1960-61 and again in 1972-73, surveys of secondary-school course enrollments were made by the National Center for Education Statistics. The greater variety of courses offered, the extent to which college-level courses were made available to secondary-school students, and the offering of "traditionally" upper-level high school courses to younger students were noted in comparing data from the two (e.g., see Gertler and Barker, 1972). The NACOME Report (1975) summarized data from the two surveys in terms of size of school. They emphasized the dramatic increase in the variety of courses:

> The impact of Commission recommendations on thinking about proper curricula for schools is evident in the decline of solid geometry offerings (coupled with rise of unified plane and solid geometry courses), growth of the advanced algebra/trigonometry option, and appearance of many different twelfth year options in advanced mathematics. These offering and enrollment data are paralleled by patterns of change in state and local curriculum guides and mathematics objectives... (p. 6)

The data from the national survey are confirmed by more intensive surveys in individual states. For instance, in South Dakota, the number of courses offered increased from 7 to 13 between 1953 and 1963 (Bedwell, 1966), and offerings similarly increased in Iowa between 1954 and 1964 (Hawthorne, 1966). Moreover, in recent guides it is apparent that the variety continues and is, in fact, expanding to some extent as courses designed for the non-college-bound student and consumer-oriented courses are added.

Williams (1970) prepared a "progress report" on the implementation of the recommendations of the Commission on Mathematics, evaluating specific points made by the Commission in terms of the responses obtained for 1,910 seniors in 1965-66. She concluded that

> In view of topics that were taken and grade levels at which certain topics were studied, the mathematics programs . . . probably were not as traditional as might be implied by the pattern of courses taken in grades 9 through 12. A number of the topics that are considered to exemplify contemporary mathematics were studied by more than half of the students in the sample. . . . The data from the survey indicated not only that some of the recommendations of the new experimental programs had begun to permeate the mathematics programs . . . but also that some of the recommended topics were being integrated into the program rather than being attacked in a superficial way. (p. 468)

However, the inclusion of different mathematical content may be illusory. The NACOME Report (1975) raised the question of the extent to which the so-called "new" mathematics was actually implemented, referring particularly to the elementary-school level. The Report noted that relatively small efforts were made to educate elementary-school teachers about the new content and thrusts. This, combined with their lower level of mathematics background, led them to continue to emphasize what they knew best and felt they could teach best: computational skills with whole numbers, fractions, and decimals.

Unfortunately, when efforts were made to update elementary-school teachers' background, the emphasis was placed almost solely on content, and in particular on terminology, on precision, and on non-typical topics. Upgrading background meant acquiring more mathematics -- with comparatively little attention to the rationale for teaching that content, the connection between that content and the elementary-school curriculum, or methods of teaching that content to children. At the secondary-school level, as has been typical in the preparation of teachers at that level, methodology was also considered only coincidentally. Consequently, the underlying goal of helping students to understand mathematics took an adjunct role, and far too many teachers were led to believe that it was not of central importance. Discovery or guided-discovery teaching was discussed but not necessarily implemented.

Some new areas of content have been added in recent years. The decision to make the metric system the primary system of measurement was reflected in the literature of the early 1970s as responses were made to expressed concerns of teachers. Elementary-school teachers, in particular, feared another upheaval in the curriculum. A flurry of activities (e.g., see Szabo et al., 1975) and materials resulted, and continues as the topic is labeled a priority by NIE.

Career education, another new term of the 1970s, has resulted in numerous curriculum guides, units of study, resource materials, information on specific careers, so-called "systems of instruction", bibliographies, lists of objectives, teachers' manuals, interest inventories, guidelines, and activities for kindergarten through the remaining school years and beyond. Questions have been raised about the quality of much of this material. As another priority item on a federal agency agenda, career education presumably will not disappear, although its implications and impact have been questioned.

Curriculum Guides: Scope and Sequence

Curriculum guides from 38 states, or communities within those states, were examined. The guides tended to be of two types. One type included only statements of goals and objectives, possibly sequenced. The second type included specific activities for the teacher to use, similar to a manual for a textbook (but usually with less attention to appearance). Major differences in content are not reflected across curriculum guides and other forms of scope and sequence from states and school districts. Format distinctions are evident but seem minor in importance. Content emphases vary across the years but with limited variance across guides: the same topics appear in virtually all, although the amount of attention given to each varies from state to state or community to community.

One of the most evident changes in curriculum guides is the statement of objectives in behavioral form in many published during the past ten years. The format of the objective makes explicit what is to be taught and how it is to be measured, but at the expense of some higher-level processes which are difficult to state in behavioral form.

In California, a state committee developed a strands approach (California, 1963). Nine strands were proposed: numbers and operations, geometry, measurement, applications of mathematics, statistics and

and probability, sets, functions and graphs, logical thinking, and problem solving. The strands approach has served as a model for numerous other state guides, for testing programs, and for other curriculum development work. {Revision in 1972 led to the Second Strands Report (California, 1972a).}

Courses, Programs, and Projects

Many new courses, programs, and projects were created in response to the goals established by local, state, and national groups. These innovations were frequently encouraged by federal funding, and were often responses to certain technological developments; in some cases a computer led to changes. There was much duplication of effort, with courses developed in one location differing little from those developed around similar ideas at another site. Some educators have voiced the opinion that this duplication of effort may be a needed component accompanying change. It signifies involvement by those actually engaged in the process of teaching. This involvement serves as one form of in-service teacher education, considered to be vital if changes are to be effected in instructional practices.

Until the 1960s, course descriptions existed almost solely within curriculum guides and as textbooks. Yearbooks and journals listed a few projects or programs, usually undertaken at the local level. With the expansion of offerings and with federal funding involved in the development of courses, programs, and projects, reporting and compilation of "what's going on" became more complicated. Numerous collections of "innovative and effective" programs and projects have been prepared; e.g., by Sloan and Loomer (1973), Carasso and Lachat (1974), and Henrie (1974). ERIC also contains an array of reports on specific projects. In addition, NSF and NIE have issued reports on various activities and projects, as have the R&D Centers and Regional Laboratories.

In 1962, the International Clearinghouse on Science and Mathematics Curricular Developments was established at the University of Maryland. Ten reports summarizing curriculum development projects have been produced (in 1963, 1964, and 1965 on only American projects and in 1966, 1967, 1968, 1970, 1972, 1975, and 1977 including international projects). The tenth report (Lockard, 1977) summarized each of the projects active since 1956.

Dissemination of information about projects, as well as about research findings, has been of increasing importance since the mid-1960s. Both NSF and NIE have expended much effort to have the products of funded efforts implemented.

Enrollment Patterns

The statistics on enrollment in mathematics courses at the secondary-school level are buried amid the hordes of data gathered annually in state and federal education agencies. Not infrequently, differing data are cited in different summaries -- though at times documented to the same source! Surprisingly little definitive analysis has been reported on the data: generally only small portions have been summarized (e.g., Brown and Abell, 1966) and used as the basis for making some point related to enrollments.

In the late 1950s and early 1960s, data clearly indicate that enrollment in mathematics courses increased. In 1949, only 65% of all secondary-school students (7-12) were enrolled in a mathematics course; by 1960, 73% were enrolled (NCES, 1960). Truenfels (1961) summarized USOE data from 1958 on 4,254 randomly selected secondary schools (8-12). An increase in mathematics course enrollment was reported by 27.4%, while 1.6% had decreases and 71% reported no change. The emphasized need for mathematics, especially as a prerequisite for college science courses, and the prestige or curiosity involved in participating in experimental courses, probably caused the increase.

Nationwide samplings were supported by data from individual states. For instance, Bedwell (1966) sampled 130 of the secondary schools in South Dakota, representing 68% of the student population and 54% of the mathematics teachers in the state. He reported that the total secondary-school enrollment increased 47.8% from 1953 to 1963, but mathematics enrollment increased 154.5%. In Iowa, enrollment also increased in mathematics between 1954 and 1964, with percentages for trigonometry and algebra 2 increasing "markedly" (Hawthorne, 1966).

The enrollment pattern seems relatively stable in recent years, with a slight decline in some instances. For instance, the New York State list contains 62 courses offered from 1971-76. The data indicate that enrollment declined at least slightly year by year over the period for: Math 7, 8, 10, 11, and 12; Algebra I and II; Trade and Shop Math; Advanced and Analytic Geometry; Problem Solving; and History of Mathematics. The numbers of students in other courses showed an increasing trend; all except Basic Math 9 involved a small proportion of the total number of students, however, and most were at an advanced (12th grade) level.

As noted previously, there is little doubt that the number and variety of courses offered in secondary schools has increased since 1955. In the summary, the NACOME Report (1975) stated:

> Individual increases were prominent in advanced general mathematics, plane geometry, advanced algebra, and trigonometry -- indicating that students were already beginning to seek more extensive preparation for college level science study. Furthermore, the 1960 survey revealed that 2.3% of all twelfth graders were enrolled in advanced mathematics courses such as calculus, . . . probability and statistics, . . . college mathematics, . . . and analytic geometry. (pp. 5-6)
>
> The 1972-73 survey data reveal some very interesting patterns. The number of students taking a second course in algebra or the new integrated algebra/trigonometry course had risen to nearly equal the number of students taking elementary algebra. ... The algebra/trigonometry format captured 40% of the advanced algebra registrations. ... Over 260,000 students were in calculus or other advanced level mathematics courses (four times the 1960 figure). Some 5,000,000 students were described as studying one of the various experimental curricula (SMSG, SSMCIS, UICSM, etc.) (p. 6)

The extent of the increase in enrollments varied from state to state, but studies from different states provide a reflection of the trend in increased offerings which reflected increased demand. For instance, Truenfels (1961) reported that the percentage of schools offering each course during 1958 was:

General Mathematics	34.4%
Algebra I	71.6%
Plane Geometry	46.7%
Intermediate Algebra	37.0%
Solid Geometry	3.9%
Trigonometry	11.5%

Rudnick (1962) obtained data from 354 schools in 109 cities in 38 states and the District of Columbia. He reported that the percentage of schools offering each course was:

Course	1957-58	1960-61
Algebra I, grade 8	13%	13%
grade 9	100%	87%
Plane Geometry	96%	82%
Plane and Solid Geometry	2%	18%
Algebra II	93%	95%
Solid Geometry	87%	65%
Algebra III (College)	43%	37%
Trigonometry	93%	88%
Analytic Geometry	7%	21%
Advanced Placement	2%	6%
Other	8%	55%

Alspaugh and Delon (1967) surveyed a sample of 50 schools in Missouri and conducted a follow-up study three years later (Reys, Kerr, and Alspaugh, 1969). They noted "substantial changes for a three-year period", such as the starred items on the table below.

Course	1964-65	1967-68
Functional Mathematics I	94%	95%
Functional Mathematics II	12%	25% *
Terminal Mathematics	12%	19%
Algebra I	96%	98%
Algebra II	94%	92%
Plane Geometry	57%	45% *
Plane and Solid Geometry	25%	48% *
Solid Geometry	2%	2%
Trigonometry	60%	73% *
Mathematical Analysis	36%	65% *
Elementary Functions	4%	8%
Matrix Algebra	2%	3%
Analytic Geometry	-	10%
Calculus	-	7%
College Algebra	-	2%
Probability	-	3%

Course-offering and enrollment data are, of course, affected by factors other than student demand for courses. In many studies, however, it is not possible to ascertain the reason for the findings. Thus, only conjecture can be made about the results from two studies (Crawford, 1967; Dunson, 1970) in which black secondary schools in the South were surveyed. All offered General Mathematics, Algebra I, and Geometry. Over 50% of the students were enrolled in General Mathematics. Only large schools offered courses beyond Geometry, and enrollments in courses such as Analytic Geometry involved less than 1% of the students. A school must be of sufficient size to warrant the offering of a course, but even in large schools, the number of courses offered was low.

Curriculum and Content: HIGHLIGHTS

- "New math" was not a single phenomenon, but a two-decade series of developments that evolved and changed continuously.
- Initially, curriculum reform focused on the college-bound student at the secondary-school level, while most early elementary-school projects developed supplementary materials. Changes in intent accompanied changing needs (noted in the overview).
- Emphasis was placed on structure, rigorous deductive proof, exploration, and correct terminology, with changes in sequence and inclusion of topics. Methodological emphasis was placed on developing understanding.
- As reflected in print, the content of school mathematics curricula changed. The number and variety of courses offered at the secondary-school level had increased by 1965, but inclusion of "new math" content in the elementary school may be illusory.
- Curriculum guides vary in format and emphases; they have little variance in content, with the impact of the California "strands" approach evident in many. Behaviorally stated objectives distinguish many 1965-75 guides from earlier guides.
- The need to disseminate information to increase implementation of new curricular ideas became apparent.
- Since 1955, data clearly indicate that enrollment in secondary-school mathematics courses has increased, especially in advanced mathematics courses. Thus more students are studying more mathematics. A large percentage of students have studied materials developed by one or

another of the curriculum development projects.

- Enrollment patterns seem relatively stable in the 1970s, with continued small increases in advanced courses and in basic or remedial mathematics.

D. What Goes On in the Classroom?

Class size, time allotment and use, teaching approaches, and the differentiation of instruction are each explored as facets of what goes on in the classroom.

Class Size

Class size has been of continuing concern, but there is little evidence that mathematics achievement is affected in a simple or direct way by total class size; rather, the size of the group with whom the teacher works on a particular topic may be of importance. A ratio of one teacher to one pupil (e.g., Moody et al., 1973), while seeming optimal by some criteria, obviously does not seem optimal by other criteria, not the least of which are fiscal limitations. To a greater extent today than in 1955, class size is negotiable by teachers with school boards. But as school budgets tighten, the number of pupils per teacher, which had decreased by the end of the 1960s, is beginning to climb upward again. In some sets of data, however, this is obscured by including special class, supplementary services, administrative, and other personnel in the equation.

Shetler (1959) reported that 46% of the 574 mathematics classes in his survey of secondary schools in 20 states had an average class size ranging from 16 to 25. For 35%, the range was 26 to 39, while 18% averaged 1 to 15 students. Only 4 schools (less than 1%) indicated the use of large classes averaging 40 or more students. The average size of mathematics classes varied directly with school enrollment.

Furno and Collins (1967) analyzed data from 16,449 pupils enrolled in third-grade classes in Baltimore schools in 1959, and traced their patterns for 1959-1964. Students in smaller classes in the regular curriculum made significantly greater gains in arithmetic achievement over the five-year period in 96 comparisons to 29 for students in larger classes. The advantage of small class size (up to 25 students) was considerably greater for non-white students and for those in the special education curriculum.

In discussing data from the National Longitudinal Study of Mathematical Abilities (NLSMA), Begle (1973) reported that class size (less than 30, or greater than or equal to 30) had an effect on achievement in 8 of 16 instances. He commented:

> Curiously enough, the smaller class size was more advantageous for elementary school students, but the larger class size was more advantageous at the junior high school level. (p. 212)

Salopek (1974) reported that class size was one of three consistent predictors of variance on arithmetic tests in grade 6 in one county in Pennsylvania, and similar correlational data have been reported from a few state assessments of achievement.

Jamison, Suppes, and Wells (1974) concluded, however, that

> where significant differences were found they were about as likely to favor large classes as small and that even when differences were significant they were usually small. (p. 21)

In five studies specific to mathematics classes covered in their review, large classes (usually more than 25 students) were favored in three and class size was not significant in two; in six studies on various subject areas (including mathematics), smaller classes were favored in five, with no significant differences in the sixth.

Time Allotment and Use

The amount of time allocated to mathematics instruction varies across states and across grade levels. From somewhat limited evidence, it appears that the time mandated in various states and communities for mathematics instruction may not be the actual amount of time spent on mathematics instruction, however.

Researchers have considered several questions related to the use of time:

(1) How much time has been allocated to mathematics instruction?

Table 5 indicates evidence from several studies in which respondees were asked to indicate the amount of time on mathematics instruction (Miller, 1958; Jarvis, 1966; Price et al., 1975, 1977). They confirm data on time allotments suggested by various states, and indicate that the lower the grade, the less time spent on mathematics. One of the studies cited observation data which contrast sharply with self-report data; Conant (1973) indicated that far less time may be spent on actual instruction than is reported. Reports from another project (Filby et al., 1976; Fisher et al., 1976a ; Marliave et al., 1976) also indicated discrepancy between allocated time and "engaged" time. From other studies (e.g., Olson, 1971), it appears that approximately 20% of the elementary day has been allocated to mathematics instruction; at the secondary-school level, 200-300 minutes per week.

(2) What is the best use of the time devoted to mathematics instruction?

To determine how the use of class time affects achievement, Shipp (1958) compared four groups, in which 75%, 60%, 40%, or 25% of class time was spent on developmental work while the remainder was

TABLE 5

TIME ALLOCATED TO MATHEMATICS INSTRUCTION

Researcher	Method of collecting data	Source of data	Grade level	Minutes per day Mean	Minutes per day Range
Jarvis (1966)	self-report	165 schools	4-6	45	30-72
Miller (1958)	self-report	34 large-city schools	1	23	0-59
			2	32	0-59
			3	40	20-69
			4-6	45	30-69
		44 small-city schools	1	30	0-69
			2	35	20-69
			3	42	20-69
			4-6	47	30-69
Price et. al (1975, 1977)	self-report	1,220 teachers	2	43	55%, 40 min. or less; 14%, less than 30 min.
			5		80%, 40 min. or more; 5%, less than 30 min.
Conant (1973)	observation	47 teachers	1-4	18	
Filby et al., 1976 Fisher et al., 1976 a,b Marliave et al., 1976	self-report and observation	6 to 33 classes			considerable variance in amount of time on different topics, and also in amount of engaged time

59

spent on individual practice. Higher achievement in computation, problem-solving, and concepts was obtained when more than half the time was spent on developmental activities. In replications of the experiment, Pigge (1964) and Zahn (1966) used other time allocations at varying grade levels. They confirmed the finding that when the greater proportion of time is spent on developmental activities, achievement is higher.

(3) How is time used?

It comes as a surprise to many people that there are actually relatively few studies which describe the actual classroom situation. Goodlad (1977) noted:

> There is only one honest answer to the question, "What goes on in our schools?" It is that our knowledge is exceedingly limited. ... There is not now either a body of data on what transpires in schools from which to begin an enlightened discussion of schooling or a tested methodology for securing these data. (p. 3)

In most studies in the classroom, the setting is described only generally. Comparisons are made with the "traditional" or "usual" classroom, as if everyone knew precisely what that was. There are also some surveys in which teachers were asked to list or to check activities which they use. But only rarely have observers gone into classrooms to see and define what is occurring. Some studies provide information on verbal behaviors (e.g., Fey, 1969a, 1970; Halperin, 1976; Kester, 1969; Mahan, 1971; Meckes, 1972; Stilwell, 1968). Thus it is known that

- the teacher talks about 2/3 of the time
- teachers tend to use a direct, rather structured approach

- over 50% of the questions teachers ask are at the knowledge level, requiring relatively low-level cognitive processes from students

- the teacher initiates most exchanges, with students doing little more than answering questions -- in addition to sitting and listening

- teachers communicate with brighter pupils in a more friendly and encouraging manner than with other students

Evidence from a variety of sources documents a picture of the classroom -- at both elementary-school and secondary-school levels -- that has changed little, despite the innovations advocated in the past 20 years (e.g., see Alspaugh, 1966; Brown, 1974; Conant, 1973; Gates, 1969; Goodlad et al., 1970; Hughes, 1959; Price et al., 1975, 1977; Shetler, 1959):

- teachers spend a large proportion of their time on managerial duties; an "astonishing amount of time" (perhaps up to 70%) is taken up in control, classroom routines, and what appeared to be scarcely more than busywork

- telling and questioning, usually in total-class groups, is the prevailing teaching method

- tell-and-show and seatwork at the elementary-school level, and homework-lecture-new homework at the secondary-school level, are the prevailing patterns

- textbooks predominate as the medium of instruction, with a single text followed closely; some teachers use virtually no other activities or materials

- the pace of lessons is slow, yet teachers allow little time for individual pupils to answer questions

- in the elementary school, the major portion of time is spent on reading and language arts, with mathematics second

- seatwork consumes up to 50% of the time in class; questions and answers or discussion and explaining involve about 25% of the time

- teachers are teaching essentially the way they were taught in school

In an interesting variation on the usual assessment of time allocation, Barley (1975) described the amount of time in formal instruction in terms of the percentage of their waking lives that students spend on each subject. For the majoirty of students K-12, school days consume 16% to 20% of their waking lives in any given school year. The majority of elementary-school students spent about 3% of their waking lives on mathematics (compared with 7.6% on reading and language arts). Secondary-school students spent about 1.8% of their time on mathematics (compared with 2.1% for science).

Harnischfeger and Wiley (1975) concluded that students who spend 190 days in school achieve more than do those who spend only 170 days in school each year. In commenting on this, Goodlad (1977) stated:

> If time spent on learning affects quality and quantity of outcomes . . . then how the days and weeks of the school years are being used and how they might be used differently become first and second items on the agenda of school improvement. (p.4)

Earlier, he had noted:

> To carry on a serious dialogue about, let alone to seek change, American schooling or simply the local elementary school, without a rather substantial body of the information implied seems somewhat bizarre. And yet, to do so is virtually a national pastime. In our pseudowisdom, we know what schools need without knowing what they already have and we know what to put into them without knowing what needs to be replaced. (p. 3)

Several major classroom observation studies are currently being conducted. One, directed by Stake and Easley at the

University of Illinois, is a companion study with the present one, sponsored by NSF. The intent is to ascertain the factors that are involved as students are taught mathematics. In-depth case studies are being made in ten school districts; observations and interveiws with students and teachers are components of the task. Another study is being conducted by Goodlad (1977) for I/D/E/A, and is an extension of his previous studies. A third study has been underway for several years at the Far West Regional Laboratory (Filby et al., 1976; Fisher et al., 1976a,b; Marliave, 1976). Their data, based on a relatively small number of classrooms in grades 2 and 5 actually observed, indicate that there is considerable variance in the amount of time spent on different mathematical topics. There is also considerable variance in the amount of time actually "engaged", or directed to the task, by the students.

(4) <u>Can some students profit from spending more or less time on certain courses?</u>

Possibly how time is used is of more importance than how much time is available. Achievement differences favored students in grades 4-6 spending 60 minutes per day rather than 40 minutes per day on mathematics instruction (Jarvis, 1963; Lawson, 1966). In the Oregon (1976) progress assessment, however, amount of time per day in formal mathematics instruction (16-30 minutes, 31-45 minutes, 46-60 minutes) revealed little or no significant differences in performance. The percentage allocating each amount were 12%, 50%, and 32%. Other studies (e.g., see the literature review in Fisher et al., 1976a,b) have also reported varying results on the relationship of time and achievement. Fisher et al. (1976a,b; Filby et al., 1976; Marliave, 1976) have found that the amount of time is related to achievement when substantial amounts of time difference are observed.

Doubling the length of the class period from 55 minutes to 110 minutes by meeting for only half the number of periods was not found to affect the achievement of secondary-school students (Albers, 1973; Hansen, 1963). Whether lengthening the number of semesters spent on a course has an effect on achievement has been studied with algebra for low achievers; results differed in studies by Buchman (1972), Herriot (1968), and Posamentier (1973).

Acceleration will be considered in a later section; in general it has been reported to be effective for some students.

Teaching Approaches

Many varying instructional approaches have been and are being tried in classrooms. The literature reflects current concern that far too many of them have been promoted as panaceas, rather than as components in a teacher's repertoire, to be used as children, content, and circumstances warrant. The emphasis of research has been on such comparisons as expository _versus_ discovery approaches, incidental _versus_ systematic procedures, or team learning _versus_ independent study. Only limited attention has been focused on the circumstances under which each could be used with optimal outcomes by an individual teacher. Educators generally believe that children learn best in various ways; thus it may follow that individual teachers may teach best in various ways and that specific content may be best taught in certain ways.

The literature on comparisons of various types of approaches is plentiful (see Appendix A). Some reviews of research have provided some syntheses (see Table 6). The topic of most concern for the past two decades, discovery learning, has been the object of several reviews. Tanner (1969) found studies on discovery versus expository instruction provided "insufficient rationale for sweeping changes in curriculum and instruction". In his more extensive study, Weimer (1975) also reported "no clear evidence of a single superior method"; rather, "many effective teaching strategies are available".

Suydam and Weaver (1970, 1975) concurred with this; a summary by Robertson (1971) expresses their point:

> It would appear that no one treatment or mode of instruction can be considered the best approach. The teacher who learns as many instructional modes as possible, identifies and diagnoses pupil needs and abilities, and uses this knowledge to individualize instruction may very well get the best results. (p. 5279)

Research has indicated rather clearly, however, that meaningful instruction (that is, instruction in which the learner is taught understanding of an idea) will lead to higher achievement than will rote instruction (Weaver and Suydam, 1972). This does not preclude all learning by rote, however, for certain skills are particularly amenable to such procedures.

Learning through activity approaches such as use of a mathematics laboratory or other approaches in which materials are used was stressed increasingly in the 1960s. There is evidence that teachers believe that such activities <u>should</u> be used -- but they are actually used by few. Research indicates that the use of manipulative materials appears to be important at all levels at least through grade 8, indeed, even adults.

TABLE 6

SELECTED REVIEWS AND SUMMARIES OF RESEARCH ON TEACHING APPROACHES IN MATHEMATICS EDUCATION

Author	Date	Topic of Research
Anderson	1969	fractions
Austin et al.	1972	summer compensatory education
Bittinger	1968	discovery
Brousseau	1973	mathematics laboratories
Buswell	1960	elementary-school mathematics
Connolly	1973	mentally retarded
Cruiskshank/Arnold	1969	non-decimal numeration
Dessart/Frandsen	1973	secondary-school mathematics
Earp	1971	reading in mathematics
Fennema	1972	models
Fennema	1974	sex differences
Fey	1969b,c	classroom teaching
Gibb et al.	1960	secondary-school mathematics
Glennon et al.	1958,1968 1975	applications for elementary classrooms
Goldstein	1960	homework
Gorman	1968	problem solving
Hatfield	1969	computers
Henderson	1963	overview
Kellogg/Johnson	1961	secondary-school mathematics
Kieren	1969,1971	activity learning
Kilpatrick	1969a,b	problem solving
Noffsinger/Dobbs	1970	mentally retarded
Payne	1965	modern programs
Reys	1972	multiple embodiments
Riedesel/Burns	1973	elementary-school mathematics
Romberg	1969	overview
Spitzer	1962,1970	applications to classrooms
Spitzer/Burns	1961	overview
Suydam	1972	applications for secondary classrooms
Suydam/Dessart	1976	computational skills
Suydam/Higgins	1976,1977	materials and activity approaches
Suydam/Riedesel	1969	applications for elementary classrooms
Suydam/Weaver	1970,1975	applications for elementary classrooms
Tanner	1969	discovery
Vance/Kieren	1971	mathematics laboratories
Vinsonhaler/Boss	1972	CAI drill and practice
Weaver/Gibb	1964	elementary-school mathematics
Weimer	1975	discovery teaching
Willoughby	1969	overview
Zoll	1969	programmed instruction

learn many ideas through the use of materials (Suydam and Higgins, 1976, 1977). Students using activity-oriented programs or units can be expected to achieve as well or better than students using programs not emphasizing activities (Kieren, 1969, 1971). That the mathematics laboratory is one strategy among many, to be used as appropriate, has been noted more frequently in the 1970s than it was in the 1960s.

Research has also been concerned with a large number of specific comparisons of techniques to be used in teaching; for instance, subtraction with regrouping, the division algorithm, or algebraic equations (see Table 4). Such studies have been summarized in a variety of documents with interpretations for classroom applications (see Table 6).

Jamison, Suppes, and Wells (1974) provided an overview of research on the effectiveness of various modes of instruction. From their analysis of studies on traditional instruction, categorized by an array of variables, they concluded that "few variables consistently make a difference in school performance" (p. 26). They emphasize that this "does not, however, imply that schools make no difference in the cognitive development of their students" (p. 27).

Differentiation of Instruction

One of the major emphases during the 20-year period has been the concern for individualized instruction. As Schoen (1977) reported:

> In the 1960s and 1970s there has been renewed emphasis on the responsibility of schools to meet the needs of individual students. Not since the peak of the progressive education movement have educators focused so directly on the individual. This phenomenon is reflected in the professional education literature

of recent years. For example, in 1971 the Education Index listed 124 articles on individualized instruction; the average number was about 35 a year during the decade of the sixties. In contrast, only about 4 or 5 (and often fewer) articles on individualized instruction appeared each year in the forties and the fifties (Kozak 1974). (p. 198)

Belief in the need to account for individual needs, combined with ideas from learning theory and from technology, led to the development of systems for individualizing instruction. There is little evidence that the resulting self-paced systems are any more effective than other programs. Schoen (1976a, 1976b, 1977) has documented this point rather extensively; Kozak (1975) and Miller (1976) have also prepared recent reviews. Schoen (1977, pp. 212-213) summarized his findings:

- Results favored traditional instruction (TI) more often than self-paced instruction (SPI), although many analyses resulted in no significant differences.

- Locally developed programs were about as effective as those sold commercially.

- SPI was particularly ineffective in developing computational skills at the intermediate and junior high-school levels.

- High-ability students achieved equally well in SPI as in TI, but most low-ability students were unable to function in SPI.

- On affective criteria, 40 of 55 analyses resulted in significant differences between the TI and SPI groups.

- Teachers in SPI have tended to spend more time on procedural matters and to restrict educational discussion more specifically to the topic at hand than teachers in a typical TI classroom.

- The diagnosis-prescription aspect of SPI has not been shown to be effective.

- Projected over a five-year period, costs for materials alone for SPI were about four or five times that of TI.

68

Two independent studies, one funded by NIE and conducted by the Educational Testing Service (ETS) and one funded by the Office of Education and conducted by American Institutes for Research, also suggested that materials for individualized instruction

> made little or no difference in improving achievement unless the materials are used in a setting where there is one-to-one interaction between teacher and learner . . . {In mathematics in particular,} students who were identified as overachievers, on the average, were members of programs with a more moderate emphasis on innovation. (EPIE, 1977, p. 2)

Many procedures for differentiating instruction have, however, been found to be effective; for instance, grouping for specific needs. While research evidence tends to be equivocal (Suydam and Weaver, 1975), there is evidence from individual users that it is a useful way to provide for individual needs. However, it has been noted in journal articles and other literature that many teachers find it difficult to group for mathematics instruction; in the elementary school, grouping for reading has long been the pattern, but additional grouping for another subject which consumes fewer minutes per day has not been widely accepted. In the secondary school, there is the long-held belief that one or another way of tracking students -- that is, assigning them to classes by ability or achievement levels -- will take care of the need for individualization. Thus, while many variations have been proposed, most elementary schools throughout the past two decades have tended to use heterogeneous grouping procedures, while secondary schools have tended to use one or another form of homogeneous grouping.

The teacher must identify various factors related to pupils' achievement and interest in mathematics, and then decide on appropriate variations in content, materials, method, and time. Related to this is research evidence that, despite the fact that a teacher might be sensitive to and state differences among pupils, teachers frequently do not differentiate instruction, and may frequently select topics and ideas which students already know. Skager (1969) found that teachers selected instructional objectives for low-achieving seventh graders that reflected skills already available to their students, and geared instruction to skills already achieved by students at the time of their entry into the program. Strickmeier (1971) studied patterns of teacher verbal behaviors in seventh-grade mathematics classes grouped by ability; comparisons were made of teachers' perceptions of their verbal behaviors and expectations for classes of different ability levels. Although teachers had different perceptions and expectations for classes of different ability levels, such differences were not reflected by observable differences in the teachers' verbal behaviors.

Stiglmeier (1973) similarly found little relationship between eighth-grade teachers' judgment of student needs and instructional mode. The Educational Products Information Exchange (EPIE, 1976b) reported a pilot study:

> Students tested on the first day of school have achieved a mean score of 64 percent on tests made up of test items taken directly from the major materials from which they were to be instructed for the rest of the school year! (p. 2)

Nelson (1960) interviewed 183 Nebraska secondary school teachers, visited 85 classes, and obtained written responses from 2,185 students.

She concluded that the teachers used a wide variety of methods, but, except for the most capable teachers who used techniques for adapting instruction to students' abilities and needs, differences in teachers' skill rather than the method used were most evident. She observed "few accommodations to individual differences" in the organization of the classes.

The Educationally Disadvantaged Student

The educationally disadvantaged have been a source of concern since long before 1955, or 1965, when federal attention was focused on them. These students, who are labelled slow learners and low achievers, as well as those who are handicapped physically, mentally, or otherwise, began to receive relatively more attention in the early 1960s. For example, the NCTM formed a Committee on Mathematics for the Non-College Bound in 1963 that became primarily concerned with the low achiever. SMSG experimented in the early 1960s with a slow-paced beginning algebra course designed for two-year time span rather then the typical one-year course (Herriot, 1968). The primary concern was for students at the junior and senior high school levels.

With the passage of ESEA Title I in 1965, the attention of education shifted to the elementary school. The apparent assumption that starting the child in school "correctly" yielded achievement benefits in later years interacted with the compensatory education thrust of providing for early success in school by enriching the child's environment. Evans (1971) documents the character of many such programs, including Head Start and Follow Through, that were designed to facilitate success in the early elementary school. Osborne and Nibbelink (1975) identify some of the many evaluative studies of such compensatory programs, noting that those cognitively oriented programs that carefully control the structure of the learning environment and activities appear most successful. One such program is DISTAR (Engleman and Carnine, 1969); it appears so extreme in controlling the environment and activities for learning that many teachers and mathematics educators find it conflicts with their beliefs about teaching and the nature of mathematics.

The concern of the slow learner broadened in the early 1970s to encompass both elementary- and secondary-school students. The attention to the academically disadvantaged child at the early school levels has continued. The evidence on the increased variety and number of courses at the secondary-school level described earlier reflects the design and implementation of special-purpose courses for the low achiever, as well as adapting general mathematics courses to their needs.

The concern for the educationally disadvantaged student encompasses more than simply low achievers. The handicapped have been a major concern throughout the twenty-year period. In 1955, the emphasis was on special education for mentally retarded and other handicapped students, since their needs were not being adequately served in the regular classroom. In the 1970s, however, just as most schools had made provision for such classes, a move in the opposite direction occurred: "mainstreaming" was advocated, since it had seemed apparent that both "special" and other students derived social and psychological benefit from interaction. The need to provide training to help these students became a priority item in several states.

From a review of the research on the academically and environmentally "disadvantaged", Suydam and Weaver (1971; Suydam, 1971) concluded:

a. The disadvantaged, as well as all other pupils, profit from special attention from the teacher, the content of the program, the instructional materials, and the organization for instruction.

b. The mathematical characteristics which distinguish disadvantaged from advantaged pupils appear to exist in degree rather than kind. That is to say, disadvantaged and advantaged pupils have similar abilities and skills, but differ in depth or level of attainment.

c. Rate of learning is but one variable to be considered in providing effective instruction for slower learners. Methods of instruction also must be adapted to these pupils.

d. Social relevance appears to be more crucial to consider in the case of disadvantaged students; however, little research has attended to this topic.

e. The degree of meaning (in the mathematical sense) which is optimal for disadvantaged students is an unknown factor. While there is some evidence that "discovery" approaches are not as effective as rule approaches with low achievers, it may merely be that more-closely-guided discovery and lower levels of meaning are appropriate for these groups.

f. Active physical involvement with manipulative materials, which is believed to be important for all children, may be even more so for the disadvantaged.

g. Pupils who are disadvantaged mathematically may also be disadvantaged in other factors which are related to their mathematical learning (e.g., reading ability). Such things must be taken into account in planning the curriculum for the disadvantaged child.

h. It does little good to report that special programs for disadvantaged students are effective without also reporting in detail the specific nature of those programs. More evidence on "ideas which work", as well as research, is needed.

i. Groups of disadvantaged pupils are not all disadvantaged in the same way. There is as much need to individualize instruction for disadvantaged students as for other groups of students.

The Talented or Gifted Student

The early stages of the revolution in school mathematics focused on college-aspiring youth and the development of curricula appropriate for them. The motivation for and the development of UICSM, SMSG, the Ball State Project, and other curriculum development efforts were in terms of serving those students destined for college work in mathematics and science who were likely to become a part of the scientific talent pool during their mature, contributing years. Some projects and efforts were directed toward the students of exceptional scientific potential within the set of college-aspiring youth. Two specific efforts deserve particular comment: the Advanced Placement Program of the College Entrance Examination Board and the NSF Summer Science Training Program for Secondary School Students.

The Advanced Placement (AP) Program was created to allow the exceptionally talented student in mathematics, who had worked through an accelerated curriculum in secondary school up to work with the calculus, to take examinations set by the CEEB, in order to receive advanced placement or college credit (or both) for mathematics. The CEEB created (and keeps current) a course syllabus for secondary-school mathematics departments desiring to participate in the AP program. This syllabus provides the base from which the AP tests are constructed. The program was established in 1955 with the first test given to 386 students in June 1956. Since this beginning, the program has matured, been modified, and become an accepted means of serving the needs of the talented in mathematics who

74

are in schools that have appropriate curricula for acceleration in mathematics. Heikkinen (1964) and Lefkowitz (1971) presented compelling evidence that advanced placement in mathematics provides a significant advantage to students in college, allowing them to progress through their intended major more expeditiously. Heikkinen's study suggests that the advantage may be greatest for those students who do not major in mathematics at the college level, but are in fields using mathematics. Lefkowitz's survey of 271 students, who had been in the AP program in one high school over a nine-year span, indicated that many students desiring college majors in mathematics felt that the program was not sufficiently theoretical to serve their interests. This was particularly true early in the history of the AP program, but was reduced somewhat with the implementation of two syllabi by CEEB for the school year 1968-69. (The Calculus AB syllabus is directed primarily toward an intuitive understanding of the concepts of the calculus and the skills with methods and applications of the calculus. The BC syllabus addresses the theoretical underpinnings of the calculus to a much greater extent.)

The reaction of colleges and universities to the AP program has made it a part of higher education. For instance, in 1963 the Ohio State Department of Education reported that 90 percent of the colleges and universities in Ohio had accepted the AP Program and would give either advanced placement or credit or both. In Utah, the percentage of those qualifying for advanced standing has risen steadily from 49% to 60% (Utah, 1974).

The Advanced Placement Program in mathematics requires that a school carefully design a curriculum that will accelerate students. The most sucessful schools begin the acceleration process early in the junior high school experience. The AP program does not work well in schools which designed a program affording students AP opportunities only in the last year or two of secondary school.

More than 15,000 students per year are currently involved in AP programs. No direct evidence is available indicating the extent of the effect of the AP program because many students take AP courses but decide not to risk taking the test. But by any reasonable criterion, the AP program must be judged a success in serving the needs of many bright students in school mathematics.

The NSF Summer Science Training Program for Secondary School Students was founded on a different philosophy for serving the interests of the exceptionally able in mathematics (and science). The primary feature of the program is enrichment rather than acceleration. Never serving many students, the program did establish a model for some institutions that continues even today. In 1959, 113 institutions provided summer science and mathematics experiences for approximately 6100 students. The experiences were of four primary types depending upon the institution:

(1) Orientation programs of relatively short duration (two to three weeks) providing general background material in science and mathematics.

(2) Classwork-laboratory programs centered on one or two fields of mathematics and science. These programs were of four to eight weeks duration and provided significant study within a single field.

(3) Classwork-project programs centered on one or two fields of mathematics or science. Similar to the classwork-laboratory programs, these differed primarily in that the students worked on individually conceived research projects.

(4) Full-time research-participation programs in which

students worked as assistants on ongoing research projects at host institutions.

An evaluation of the 1959 summer programs by Science Research Associates (SRA, 1960) indicates that there was a significant impact on participants. In two surveys (one immediately upon completion of the summer program and one after the students had considerable time away from the experience) and extensive interviews with participants at 17 of the sites, evidence was collected indicating that substantial personal re-orientation of career goals had taken place. Significantly, more than half of the students came from homes not representing parents with professional, administrative, or managerial occupations and slightly more than half were from homes with parents of educational levels including no collegiate experience. Thus, the program served a broad spectrum of the population.

espite similar evaluations of summer programs, political and societal concern for the talent in mathematics and science decreased during the 1960s. Very little governmental support was given to this type of program. Currently, two regional programs serve the needs of some talented students. One is the Governor's Honors Program, conducted each summer by the Georgia State Education Department. Designed to serve the needs of the exceptionally able student in the many small, rural schools typical of Georgia, the program involves careful selection of students and provision of an enriched experience in a particular field. The program appears appropriate for other regions with small schools where comprehensive curricular programs are not possible because of budgetary constraints.

The other notable program in mathematics is one that was begun (by

Ross) at the University of Notre Dame with NSF support, moved to The Ohio State University, and is presently sponsored by the University of Chicago Department of Mathematics. Designed to encourage students to realize their potential in mathematics, the program has a remarkable history of encouraging a significant portion of the participants to pursue a career of research work in mathematics. In designing the program, Ross was particularly careful to provide students with a curriculum that allowed significant exploration of deep questions in mathematics that were relatively free of prior, formal experience in mathematics. Topics such as number theory, that would accommodate to personal, exploratory work and the development of mathematical intuitions, were exploited to develop a power with problem solving. Participants who begin the program early in their secondary-school careers can participate for more than one year. Extensive use is made of prior participants as counselors in the program. Eberle (1971) provided detailed information concerning the effect of this summer program on the participants during the years 1964 through 1969. Her follow-up of the participants captured the significant impact that this kind of program can have in nuturing scientific talent and he potential of this type of program in contributing to the pool of research professionals in mathematics.

Acceleration, ability ~~grouping,~~ special courses, and enrichment have always been the obvious means for coping with the talented. However, acceleration and enrichment are the primary alternatives for serving the interests of the exceptionally gifted in mathematics. Research provides little significant evidence that one of these methods is to be preferred, other factors being equal. The constraints imposed by the local school situation and the talent found in the mathematics teaching staff may well determine what is possible at the local school level. Data on the percentage of schools using each varies from survey to survey; in general, however, it appears that special courses are the most frequently used procedures and acceleration the least used. For the junior high school, Begle (1976) reviewed 42 research reports dealing with acceleration for talented and concluded that acceleration was preferable to enrichment at that level. Studies at all levels indicate that care should be taken to select the option that is most appropriate for the individual.

Special curricula for the talented in mathematics have been created to serve the upper tenth of the student population. The Secondary School Mathematics Curriculum Improvement Study (SSMCIS) at Columbia University and the Comprehensive School Mathematics Project (CSMP) at CEMREL both built curricula for the secondary-school student of exceptional talent and represented a move toward realizing the curriculum advocated in the Cambridge Conference Report (1963). Both of these curricula can be successful in a school having a sufficiently large population of talented students and a staff with the mathematical capability for teaching the curricula.

During the 1960s, the orientation to the socially and educationally disadvantaged in society and the resultant lack of political support for programs for the talented contributed to a decline of effort on behalf of the talented student in mathematics. Although the normal distribution of talent would lead one to conclude that there are as many talented individuals at the upper end of the continuum as low-ability individuals at the other end~~, significantly more~~ money is being allocated for the low-ability student than for the talented. A developing concern was evident in the 1970s that the talented are being ignored, although the concern is not nearly so pervasive as that exhibited in the 1950s. The recent work of Stanley and his associates is one example of current and re-awakened interest in this segment of the student population. The Study of Mathematically Precocious Youth (SMPY), begun in 1971 at Johns Hopkins University (Stanley, Keating, and Fox, 1974; Keating, 1976), has undertaken the task of identifying exceptionally talented students in the vicinity of the University and devising educational experiences that best meet the student needs. They have found that extensive acceleration of such students is effective in a number of instances. Grade skipping, part-time enrollment in college courses, supplementary classes, and early entrance to college are some of the procedures used. Pacing, rather than designing special curriculum offerings, is their concern. This work, based upon extensive testing to identify the talented, does not focus on the need of schools for programs for the talented. It works well only for those schools that have convenient local access to institutions of higher education willing to provide opportunities and/or the staff for programs for these students. In addition, critical abstracts and analyses of the SMPY research by a variety of mathematics educators indicated problems with the research and testing design (IME, 1977).

A steady increase in the number of students taking the CEEB Level II Mathematics Achievement Tests can be noted in 1965 through 1976 for students with more than three years of experience in the four-year secondary-school college-bound curriculum (with a corresponding decrease for the Level I test for students with three years or less in the college-bound curriculum) (Jones et al., 1977).

This indicates that some of the needs of the upper third of mathematics students are being served, but the perception of lack of attention to the needs of the exceptionally able is growing. Articles (e.g., House et al., 1977) focus attention on the need to cultivate and nurture the talented student as an important national resource.

What Goes On in Classrooms: HIGHLIGHTS

- Knowledge of what goes on in schools is limited: few studies have described the actual class situation. However, it appears that:
  - Approximately 20% of the elementary-school day is allocated to mathematics, with the number of minutes increasing as grade level increases. At the secondary-school level, approximately 200-300 minutes per week are allocated to mathematics.
  - A large proportion of time is taken up by non-instructional activities.
  - How time is used may be of more importance than how much time is available. Higher achievement is likely to result when more than half of the time is spent on developmental activities.
- Classrooms have changed little over the past 20 years, despite the innovations advocated. Predominant patterns continue to be:
  - instruction with total-class groups
  - tell-and-show followed by seatwork at the elementary-school level, and homework-lecture-new homework at the secondary-school level.
  - use of a single textbook but few other materials
- It appears that no one mode of instruction can be considered best.
  - Meaningful instruction promotes achievement, retention and transfer, all accepted goals of instruction.
  - Teachers believe that activity-oriented instruction should

be used, but few actually use it.

- Few variables consistently make a difference in school performance.
- Teachers frequently do not differentiate instruction. They tend to gear instruction to skills already achieved by their students.
  - Various means can be used to differentiate instruction, including grouping for specific needs. However, many teachers find it difficult to group for mathematics instruction.
  - There is little evidence that self-paced programs for individualized instruction are any more effective than "traditional" instruction; most low-ability pupils find it difficult to function using self-paced programs. Such programs cost much more than traditional instruction costs.
- The disadvantaged student can profit from special attention, but such students differ individually more than as a group.
- The needs of the talented are not being well-served in the 1970s. Enrichment programs are especially needed for those in small schools.
- Advanced Placement serves the needs of those who are going to <u>use</u> mathematics better than the needs of those who are going to <u>major</u> in mathematics.

E. How is Achievement Evaluated?

Evaluation has played an important role in the determination of educational policy throughout the two-decade period beginning in 1955. However, the role of evaluation has shifted. Standardized tests have historically provided a normative effect on curricular content; now evaluation processes have become increasingly influential in determining curricular policy at the local school level.

The period began with the Educational Testing Service, following recommendations of the CEEB and the Commission on Mathematics, exerting a major formative influence on the content of the curriculum for the college-bound. CEEB conducted a status study of the mathematics curriculum and issued the Report of the Commission on Mathematics (CEEB, 1959) with full realization of the dilemmas associated with having a major testing service attempt to influence the curriculum through standardized testing of prospective college students (Jones, 1970, p. 73; Osborne and Crosswhite, 1970, pp. 259-266). Mathematics educators such as Begle (1963, p. 137) identified the impact of the CEEB actions as "the most important step" in the curricular reform in mathematics of the late 1950s.

Evaluation within mathematics education in the 1950s served to provide norms on curricular content; standardized tests were also used to categorize students. Partly in reaction to the mandated evaluation required for ESEA projects and partly due to increased knowledge and sophistication of school personnel about evaluation techniques, evaluation has come to have a more significant role in decision processes for mathematics education. Superintendents and school boards at the local level and educational personnel at the regional, state, and national level have become enamored with the ideas of accountability and verifying the worth of both new and old curricular programs. The NACOME Report (1975) documents the growing pains associated with the increased use of evaluation at all levels. In particular, many of the misuses and consequent issues associated with testing programs in the schools are detailed. The power that tests wield, both in terms of the placement of students in the schools and what they can do after public schooling is completed, is also recognized.

In the 1970s, evaluation encompasses:

(1) Techniques: standardized testing, norm-referenced testing, objective-referenced testing, and criterion-referenced testing.

(2) Processes for particular purposes: formative evaluation, directed toward the redesign of curricular and instructional programs, and summative evaluation, the intent of which is to provide information concerning the performance of established programs.

Most issues and problems associated with the evaluation of mathematics programs arise from misuse of particular techniques or processes in conjunction with misuse of the information derived from them.

Increasingly, there is recognition that scores from standardized tests are misleading -- or are being used in a misinformed fashion. Tests provide a means of sorting students, presumably to aid in the process of instruction. In addition, teachers and public alike appear to believe that the important outcomes of schooling can be adequately appraised by achievement tests.

In a position paper, the National Council of Supervisors of Mathematics (NCSM, 1977) attempts to influence this opinion:

> Standardized tests have several limitations including the following:

a. Items are not necessarily generated to measure a specific objective or instructional aim.

b. The tests measure only a sample of the content that makes up a program; certain outcomes are not measured at all.

> Because they do not supply sufficient information about how much mathematics a student knows, standardized tests are not the best instruments available for reporting individual growth. Other alternatives such as criterion tests or competency tests must be considered . . . There is also need for open-ended assessments such as observations, interviews, and manipulative tasks to assess skills which paper and pencil tests do not measure adequately.

The greatest change in testing over the past 20 years has been the much-publicized concern for objective-referenced or criterion-referenced tests rather than norm-referenced tests. It has been frequently noted, however, that:

- Teacher-made tests are objective-referenced in that they assess achievement on content and procedures teachers consider important.

- Norm-referenced tests are also based on objectives of some type.

- Both norm-referenced and criterion-referenced tests have a purpose -- the first to provide status information and the second to provide learning and instruction information.

Bloom's Taxonomy of Educational Objectives (Bloom, 1956; Krathwohl et al., 1964) was influential in directing thinking about needed evaluation measures (as well as being useful in curriculum construction). Continuing interest is reflected in the 1969 Yearbook of the National Society for the Study of Education (Begle, 1970), in which two chapters focused on evaluation in mathematics instruction; Bloom, Hastings, and Madaus (1971) also provided illustrations of objectives, testing techniques, and sample test items for mathematics evaluation.

Several studies have compared textbooks and instructional objectives with test objectives (Smith, 1966; Bernabei, 1967; Gridley, 1971; Hoepfner, 1974). In general, they compared favorably on content involving computation with whole numbers, fractions, and decimals at the elementary-school level, but fewer items on tests concerned geometry, measurement, and other less "accepted" topics. Gridley cautioned that the meaningfulness of the total score, as well as the subtest scores, was questionable, since frequently several skills or abilities were being measured by a single item. In other words, the distinction of "computation," "concepts," and "problem solving" made on so many tests is often not based on an accurate categorization of items.

The form of objectives has previously been referred to obliquely. In the 1950s, instructional objectives were frequently very general in nature: "to teach addition with carrying" or "to understand algebraic equations." In the 1960s, proponents of behavioral objectives were adamant: they made it appear that until objectives were stated in a precise form, so that they could be measured, no meaningful instruction could proceed. The debate over behavioral objectives flared repeatedly, with "opponents" pointing out that mathematics, which seemed so amenable to statement in behavioral form, actually needed many more objectives than those which could be precisely measured. There was greater danger of "measuring the trivial with precision," while ignoring the long-range goals related to, for instance, "understanding."

The orientation to accountability and performance contracting at the beginning of the 1970s accentuated the problems and issues surrounding the use of behavioral objectives. Many of the state assessments are based on behaviorally stated objectives at the knowledge level, as are many curriculum guides. But the literature of the 1970s reflected less concern with the form and more concern with the intention of objectives.

At all levels, from the public through and including professionals at the federal level, high expectations are held for evaluation. New curriculum development projects and in-service education efforts in the late 1950s could be undertaken with little concern for evaluation; today evaluation is required and expected. It encompasses broad-scale efforts, from the National Assessment of Educational Progress (NAEP) to the local school trying out new mathematics laboratory centers in elementary-school classrooms. The intent is to use information from the evaluation as a guide to the expenditure of resources. As such, it is an expansion of the role of evaluation in educational decision-making.

Evaluation: HIGHLIGHTS

- The scope and role of evaluation has been greatly expanded during the 1955 through 1975 period. Evaluation information is now expected to provide guidance for programmatic decisions, whereas in 1955 the primary use was in terms of standardized tests and decisions concerning individual students.
- Standardized tests have assumed increasing importance. Recognition that scores from tests are being misused has also increased. Many people believe that the important outcomes of schooling can be adequately appraised by achievement tests. That this is a severe limitation on instructional outcomes is being emphasized by many leaders.
- The greatest change in testing has been the increasing use of objective- or criterion-referenced tests, as behavioral objectives were emphasized. In the 1960s, behavioral objectives were an issue. The 1970s brought less concern for the form of objectives and renewed concern for their intentions.
- Instructional objectives and test items compare favorably on content involving knowledge of computation, but not on content concerning geometry, measurement, and other topics. Insufficient attention has been given to the testing of higher-order objectives (e.g., problem solving or analytic thought).

F. What Student Characteristics Influence Achievement?

Student characteristics must be considered as curricula are designed and as a teacher plans for instruction. While many student characteristics could be considered in this section, the discussion has been limited to five that have been of concern to teachers and to researchers as potential factors influencing achievement:

- aptitude
- attitudes
- self concept
- sex differences
- socioeconomic status

Aptitude

Most of the research indicates that aptitude, as measured by intelligence tests, is highly correlated with mathematics achievement. This is hardly surprising, and one wonders why so much attention has been devoted to confirming the correlation between scores from intelligence tests and scores from mathematics tests. What mathematical ability consists of has been the focus of a smaller body of the research.

Feierabend (1960), in a review of research on psychological factors in mathematics education, compared 19 studies concerned with the relationship of general intelligence and special abilities in mathematics. She concluded that

> studies in this area appear to agree on the importance of a general intellectual factor for ability in mathematics, but the investigation of specific abilities is not conclusive and the approach to this problem is perhaps not as meaningful as it could be. . . . The question remains,

90

> unanswered as to whether all persons of sufficient general intelligence have equal potential for mathematics, or whether there may not exist some special abilities, factors, or conceptual approaches which are specific to the field of mathematics or perhaps to creativity in mathematics thinking. (p. 26)

Aiken (1971), in an analysis of studies reported after Feierabend's review, supported her conjecture concerning the importance of special mathematical abilities, in addition to general intelligence, for achievement in mathematics. He found that "only about half the variance in mathematical achievement can be accounted for by differences in abilities." He also suggested that such factors as language, sex, age, and heredity need further study. Among other conclusions Aiken stated were:

- There is some support for a broad group factor of mathematical ability, but generally it appears that mathematical ability, rather than being a unitary trait, consists of a number of factors.

- Individual differences in mathematical ability increase at successive age or grade levels (the range may be as great as seven years).

- Intra-individual changes in mathematical ability as a function of age have been extensively explored by Piaget.

- Such factors as prior experience and verbal ability have been related to mathematical ability, in addition to reasoning ability and spatial ability.

- Most studies on aptitude-treatment interaction have not indicated that, for an individual having a particular pattern of abilities, certain techniques of instruction are more effective than others.

<u>Attitudes</u>

Many people believe that mathematics is disliked by most students -- or that it is just about the least favorite school subject. But in the elementary school

> It is true that in some surveys a significant proportion of pupils rated mathematics as the least liked of their school subjects. But it is equally true that in these surveys {across time} approximately the same proportion of pupils (at least 20%) cited mathematics as the best liked or the second best liked school subject. (Suydam and Weaver, 1970, p. 4)

In a recent study by Ernest and others (1975, 1976), 1324 students in grades 2 through 12 were asked to rank mathematics, English, science, and social studies. Mathematics was liked best by 30% of the boys and 29% of the girls, and liked least by 27% of the boys and 29% of the girls. A statement from one study (Yamamoto et al., 1969) on the attitudes of 800 students in grades 6 through 9 reflects the reactions of even researchers when such a result is apparent: "Rather to our surprise, mathematics fared quite well in students' ratings" (p. 204). To change such impressions has been identified as one of the needs of mathematics education (e.g., NIE/NSF, 1977).

There is limited evidence (e.g., Dutton, 1968) that attitudes toward mathematics were slightly more favorable in the 1960s than they were in the 1950s. Several studies have attempted to analyze the reasons why students like or dislike mathematics (e.g., Dutton and Blum, 1968; Callahan, 1971). They cite frustration with word problems, possibilities of making mistakes, too many rules, and "not being good" as reasons students give for disliking mathematics; reasons for liking it include the points that working with numbers is fun and presents a challenge, mathematics is logical, and there is need for mathematics in practical living.

Suydam and Weaver (1970) reported that their review of research indicated that "boys seem to prefer mathematics slightly more than do girls, especially toward the upper elementary-school grades" (p. 4). In the recent study by Ernest and others (1975), however, mathematics was the <u>only</u> subject in which no sex difference in preferences was observed. This may be evidence that attitudes are changing, but if there is a difference in attitude toward mathematics by boys and girls, it can probably be attributed in large part to a societally induced expectation.

In two long-term studies involving data from the mid-1960s, Anttonen (1968) found that mean attitude scores declined between grades 5-6 and grades 11-12, while Crosswhite (1972), examining measures of attitude, self-concept, and anxiety as one phase of the National Longitudinal Study of Mathematical Abilities (NLSMA), reported that student attitudes toward mathematics peaked near the beginning of junior high school. Aiken (1970) concluded from his thorough review of research that children's attitudes appear to become increasingly less positive as they progress through school; more recent studies continue to support this conclusion.

Mathematics educators and teachers believe that the affective component of learning is important: if children are interested in and enjoy mathematics, they will learn it better. However, research indicates that positive or negative attitudes toward mathematics appear to have only a slight causal influence on how much mathematics is learned. It has been noted, also, that achieving well in mathematics may have the effect of making attitudes more positive.

Suydam (1975) summarized results of 12 studies reported between 1962 and 1973. When significant correlations were found between attitude and achievement, they generally ranged between .20 and .40; that is, no more than 4% to 16% of the variance in achievement could be accounted for by attitudes. There is, however, a rough balance between studies in which no significant differences are reported and those in which a significant correlation was found. There is not, however, any differing pattern across the years.

Teachers are widely believed to be prime determiners of a student's attitude and performance. Smith (1974), for instance, reported that students' perceptions of teachers were significantly correlated with mathematical growth in grades 4 through 6. Rosenbloom et al. (1966) found that teaching effectiveness contributed significantly to the attitude and perceptions of pupils concerning their teachers and their methods, the school, text materials (SMSG), and the class as a group. However, Kester (1969) found that seventh graders' attitudes were not significantly affected by teacher expectations. Perhaps that is good, considering that Ernest et al. (1975) found that, of a small sample of teachers (24 women and three men), 41% felt that boys did better in mathematics, while no one felt that girls did better.

It is also believed that parents determine the child's initial attitudes and affect their child's achievement. Poffenberger and Norton (1959) stated that attitude toward mathematics is a cumulative phenomenon caused by one experience building on another. Attitudes, they believe, are developed in the home and carried to the school; self-concepts in regard to mathematical ability are well established in the early school years, and it is difficult for even the best teacher to change them. Parents influence the child by their expectancy level, by their degree of encouragement, and by their own attitudes toward mathematics. Many parents expect above average work in general, but are satisfied with only average work in mathematics. Many students report that their parents say, "I'm poor in math", and feel that this gives them sanction for being poor students.

Reviews of research on attitudes (Aiken, 1970, 1972; Knaupp, 1973; Neale, 1969; Suydam 1975; Suydam and Weaver, 1970, 1975) have confirmed two other generalizations:

(1) Relatively definite attitudes about mathematics have been developed by the time children reach the intermediate grades (approximately age 9).

(2) There is no evidence that the content or the curriculum per se has particularly influenced attitudes. Evidence has frequently been cited that students like a particular course or program -- but comparative data (do they like one course more than another similar one) are not feasible to obtain.

Self-Concept

How children feel about themselves and their concepts of themselves while doing mathematics are important components of the affective domain. If certain feelings are experienced over a period of time, they can lead to a particular self-image on the part of children, which can influence what they expect of themselves and can affect their performance. Some studies have explored facets of the child's self-concept as it relates to mathematics instruction and learning.

Self-concept and achievement in mathematics were found to be significantly related in studies by Bachman (1969), Hayes (1968), Koch (1972), Messer (1972), Moore (1972), and Stillwell (1969). Moore noted that, while it may be concluded that self-concepts and attitudes toward mathematics influence achievement in mathematics, it is also reasonable to infer a reciprocal cause-effect relationship between these variables.

Correlations between self-esteem achievement may be more positive for girls than for boys. Primavera et al. (1974) suggested that the school plays a greater role in affecting girls' self-esteem because it is a major source of approval and praise for girls, whereas boys can seek approval through athletics and other activities.

In almost an equal number of studies, no significant relationship between self-concept and achievement has been found (e.g., Birr, 1969; Hunter, 1974; Phelan, 1974; Zander, 1973).

Sex Differences

Among other student characteristics of increasing concern during the 1970s is that of sex differences. When sex has been incorporated as a factor in the design of a mathematics education study, there is a pattern in the findings across grade levels. As reported in a review by Fennema (1974):

> No significant differences between boys' and girls' mathematics achievement were found {in 36 studies examined} before boys and girls entered elementary school or during early elementary years. In upper elementary and early high school years significant differences were not always apparent. However, when significant differences did appear they were more apt to be in the boys' favor when higher-level cognitive tasks were being measured and in the girls' favor when lower-level cognitive tasks were being measured. No conclusion can be reached concerning high school learners. (from abstract, p. 113)

This confirms findings of other reviewers (e.g., Suydam and Weaver, 1970, 1975). Data from the National Assessment of Educational Progress (NAEP) on mathematics also indicate that "neither sex has a clear advantage in computational ability since results for males and females varied at the different age levels" (NAEP, 1975a, p. 35).

Fennema and Sherman (1976) discussed variables hypothesized to be related to achievement of women in general and to mathematics learning and studying in particular. They considered verbal ability, spatial visualization ability, confidence in learning mathematics, mathematics as a male domain, attitude toward success in mathematics, perceived attitudes of parents and teachers toward one as a learner of mathematics, usefulness of mathematics, and motivation. Four conclusions were drawn: (1) sex-related differences in mathematics achievement are not universal, (2) many fewer females than males study mathematics in eleventh and twelfth grades, (3) the relationship between cognitive factors and differential learning of mathematics by the sexes is unclear, and (4) differential mathematics study and achievement is at least partially caused by socio-cultural factors mediated through sex-role expectations.

Increased sex-typing in mathematics and science has been documented by several researchers, and many have documented the fact that there are fewer and fewer women in mathematics as age level increases from junior high school through college.

Whether different provisions for instruction for males and females should be made is another question entirely. Fox (1975) has collected evidence on this; it appears that even when special classes are provided the attrition rate for females is high. Differences in aptitude and achievement seem to vary more with individuals than by sex. Societal expectations, which have changed dramatically in the past ten years in terms of women's roles, as yet seem to have little influence at the secondary-school level, where peer interrelationships are so important.

Socioeconomic Status

There has been so much written on the effect of socioeconomic differences that it seems pointless to belabor the point here. The conclusion of the Coleman Report (1966) has been widely cited: that, in general, the public schools exert very little influence on the achievement of children independent of their own family background and social context. When socioeconomic status has been incorporated as a factor in designing mathematics education studies, students from high socioeconomic levels tend to achieve better than students from low socioeconomic levels (e.g., see Dunkley, 1965; Johnson, 1970; Montague, 1964; NAEP, 1975a; Passy, 1964; Unkel, 1966). When racial and ethnic minorities have been considered specifically, the members of these groups in general achieve as well as or less well than members of the majority (white/Caucasian) group: they rarely achieve better (e.g., see Asbury, 1970; Casteneda, 1968; Centrone, 1973). Data from the National Assessment of Educational Progress (1975a) indicated that:

> Blacks appeared to have difficulty with computations, their performance being generally below that of the nation as a whole. . . . The difference in performance between Whites and Blacks was smallest at age 9 and increased for 13- and 17-year-olds, with no appreciable change in relative performance between ages 13 and 17. (p. 36)

98

When type of community has been incorporated as a factor, students from urban areas tend to achieve slightly better than do students from rural areas; finer distinctions are evident in the seven types of communities assessed by NAEP (1975a).

Assessment data from various states parallel the Utah (Ellison et al., 1975) finding: socioeconomic status was highly related to mathematics achievement, with students from high-income neighborhoods generally having higher mathematical scores. Freda (1976), in his study of 244 California schools, reported that education of parents and income of fathers were the two "input characteristics" most highly correlated with mathematics achievement.

In a reassessment of Coleman's data to consider comparative contributions of verbal, nonverbal, reading, mathematical, and general informational achievement, Boardman et al. (1973) reported that both the home and the school were important for all achievements, especially verbal and general informational. The explanatory variables considered, however, appeared to be less important for mathematics than for other achievement. Bredemeier (1967) also analyzed data from Coleman and data from Project Talent. The differential achievement of secondary-school students in mathematics had low correlations with any measured characteristics of the schools they attend, and only slightly higher correlations with family background.

In yet another reassessment of the data from Coleman and from Project Talent, Jencks and Brown (1975) reported some implications:

> Some high schools are more effective than others in raising test scores. Nevertheless, the gains are never large relative to the variance of initial scores, and schools that boost performance on one test are not

> especially likely to boost performance on other tests. Moreover, high-school characteristics such as social composition, per-pupil expenditure, teacher training, teacher experience, and class size had no consistent impact on cognitive growth between ninth and twelfth grades. . . . Our data tell us nothing about what methods might be most effective. They tell us only that more money, more graduate courses for teachers, smaller classes, socioeconomic desegregation, and other traditional remedies are unlikely to have much effect. (p. 32I)

They caution also that legislatures and school boards who want to hold high schools accountable for their students' acheivement should be "extremely careful to specify the outcomes that really interest them" (p. 321).

To extend their point further, they state:

> So far as we can discover, SES has no significant effect on cognitive growth between ninth and twelfth grades . . . equalizing high-school quality cannot reduce the correlation between SES and twelfth-grade scores. . . . One would actually have to move high- and low-SES students into the same communities and neighborhoods to eliminate the source of inequality. (p. 322 ff.)

Emphasis is given to another point which deserves consideration:

> . . . high-school quality accounts for only 1.0 to 3.4 percent of the variance in twelfth-grade test scores, 0.2 to 2.4 percent of the variance in educational attainment, and 2.5 to 4.8 percent of the variance in occupational status and career plans. This means that even if we knew how to eliminate all disparities in high-school quality, which we clearly do not, we could reduce the standard deviations of these outcomes by only one or two percent. (p. 32)

In a review of evaluations of compensatory education programs at national, state, local, and program levels, Stickney (1976) found "very little evidence that compensatory education has been able to arrest the accumulative achievement deficit that exists between advantaged and disadvantaged pupils." He suggests that "as long as schools remain marginal institutions they are unlikely to compensate for environmentally determined differences in academic acheivement" (p. 2088).

In short, the evidence seems to indicate that SES and achievement in mathematics are correlated, but that the school has little hope of narrowing the achievement differential between socioeconomic levels.

Student Characteristics: HIGHLIGHTS

- Not surprisingly, intelligence and mathematical achievement are highly correlated.
- There may be a general intellectual factor for ability in mathematics, but it is suggested that mathematical ability consists of a number of factors. Prior experiences, verbal ability, reasoning, and spatial ability are related to mathematical ability. The role of language, sex, age, and heredity need further study.
- The range of mathematics achievement scores increases as age or grade level increases.
- Attitudes toward mathematics are generally positive in the elementary school and appear to peak at approximately age 12.
- While mathematics educators and teachers believe that attitude toward mathematics is related to achievement in mathematics, there appears to be no meaningful or significant relationship between the two.
- Whether self-concept is significantly related to mathematics achievement has not been definitively ascertained.
- Sex-related differences are not universal across the factors related to mathematical ability; differences in aptitude and achievement vary more with individuals than by sex.
    - Girls and boys at the early elementary-school level do not differ significantly in mathematical achievement. In upper elementary and early high-school years, differences were not always apparent; when they did

occur, they were likely to favor boys on high-level tasks and girls on computation.

- No conclusions regarding sex differences can be reached concerning secondary students; fewer girls take mathematics, however. Socio-cultural factors appear to be involved.

- Socioeconomic factors appear to account for much of the variance in mathematical achievement.

G. What Use Is Made of Instructional Materials?

As has been noted, textbooks, supplemented by workbooks and other materials for seatwork or homework, are heavily relied upon in mathematics teaching. But other types of materials are also endorsed for use in mathematics classrooms; for instance, the NCTM has published a yearbook and several supplementary publications which tried to focus attention beyond the textbook.

Textbooks and Other Print Materials

The textbook is the primary determinant of mathematical curricula throughout schools in this country. State curriculum guides present an outline that can be filled in by use of a textbook; local guides resemble textbooks in scope and sequence. Over half the states have mandated textbook adoption lists, with more states having multiple-text adoptions than was observable 20 years ago. But within most classrooms, the evidence indicates that a single textbook is used with all students, rather than referring to multiple textbooks or varying text use by group or individual needs. There appears to be rather firm adherence to "covering the material" in the text, although sections which teachers do not consider important (and which may not be included on standardized tests) may be ignored. Elementary-school geometry has suffered this fate for years. That the textbook influences what is learned was supported by Begle (1973), who reported that different patterns of achievement were associated with the use of different textbooks.

In a report on an unpublished study of most-used instructional materials, EPIE (1976a) stated:

> The ten most used materials in mathematics are clearly traditional programs, all quite similar to each other in terms of instructional design. They are also traditional in terms of the way they were developed. . . . If we look at the first 32 mathematics materials listed, only one program is the result of nontraditional development and this development was federally funded. This material is rank-ordered 24th. Of the remaining 31 materials, at best two could be considered to have even a modicum of an R&D base . . . built upon an empirical data base, as opposed to "conventional wisdom" . . . (p. 1)

Among other highlights of the survey of 12,389 teachers, including 4,455 mathematics teachers (K-12), were (EPIE, 1976b):

- Instructional materials, print and nonprint, are used during 90 to 95 per cent of all K-12 classroom instructional time. Schools spend about 1 per cent of their budgets on these materials . . . (p. 1)

- Teachers tend to be unclear about how good a "fit" there may or may not be among their teaching, the materials they are using, and the needs and abilities of their students . . . (p. 2)

On the list of mathematics materials were 18 elementary and 14 secondary. Twelve companies produced the elementary materials, with none clearly dominating. One company accounted for 9 of the secondary materials; five companies accounted for the remaining five. Sixty-two per cent of the teachers said they would "willingly" use the same materials again.

PRIMES, the Pennsylvania Retrieval of Information on Mathematics Education System, has collected some of the most extensive information on the contents of textbooks (Creswell and Berger, 1968). Groups of teachers and mathematics educators, working with Department of Education personnel, have since the mid-1960s developed a list of 300 content-related items for grades K-6, and analyzed textbook series in terms of that list. These data are stored in a computer; a school staff can determine the content and sequence they desire and compare their plan with the analyses of the textbook series, to aid in selecting a textbook or combination of textbooks.

Many textbook analyses have been reported, spanning the years (e.g., Buchalter, 1969; Burns, 1960; Clason, 1969; Dahle, 1970; Folsom, 1960; Kahn, 1974; Maura, 1957; Neatrour, 1969). Some points seem especially relevant:

- Low-level cognitive processes -- knowledge and comprehension -- are used far more frequently than high-level processes.

- There is considerable agreement on grade placement, sequence, and presentation of basic topics.

- There is wide variance in the total number of concepts and the amount of space devoted to the various topics.

- Relationships are found between textbook emphases and social or psychological trends.

- An emphasis on computational skill is apparent.

- The appearance of textbooks changed since 1955, with marketing considerations and appeal of obviously increasing importance by the late 1960s.

- At the elementary level, teachers' guides vary with textbook series; most continue to provide suggestions for differentiating instruction. Such facets as the form of stating objectives have changed across the 20 years.

- Secondary-school teachers' guides have expanded since 1955, although most are not as extensive as those for elementary-school teachers.

Stevens (1966) found that, for elementary-school textbooks published between 1955 and 1964, the total vocabulary load increased by more than 40%, except for grade 3. Hater and Kane (1975), Shaw (1967), Smith (1969), and others were similarly concerned with readability at various levels. This has led to some textbooks and project materials being revised to prepare versions with more appropriate vocabulary and reading levels.

Dooley (1959) studied 153 series of elementary-school textbooks published between 1900 and 1957, attempting to ascertain the effect of research on the content and methods suggested in them. She found that when recommendations were "clear, concise, and exact" they were incorporated into many textbooks within five years. Since the late 1950s, it has taken some ideas a far shorter amount of time to appear in the majority of textbooks.

Brown (1974) conducted an in-depth study on the use of textbooks made by teachers and students in Geometry and Algebra 2. Very heavy dependence on the textbook was found:

> Teachers followed the textbook very closely with regard to content selection and sequencing. The major objective of observed lessons tended to be completion of the exercises presented at the end of the section of the textbook under discussion. (p. 5795)

Teachers made little use of special features, such as historical and bibliographic information or enrichment exercises. They rarely presented topics not in the textbook. Typically, they progressed through the text, section by section. Brown concluded that, for the teachers and classes in the study, mathematics did not extend beyond that which was presented in the textbook: "the subject was resolved into a sterile sequence of homework/discussion/new homework" (p. 5796).

Programmed Instruction

Through the late 1950s and into the early 1960s, programmed instruction (PI), with or without a teaching machine, was considered a panacea for educational problems. The work of Skinner and Pressey gave impetus to the use of small-step increments and immediate feedback. PI was used in many studies because it allowed the researcher to control the teaching variables, ensuring that every student had the same treatment.

The foremost claim for programmed instruction was that it would allow each pupil to progress at his or her own rate. Some studies ascertained the feasibility of using programmed instruction to teach specific content. When compared with conventional instruction, the results were equivocal (Suydam, 1972). It was evident that programmed materials were most useful when used to supplement, rather than replace, the teacher (Lackner, 1967). In his review, Zoll (1969) concluded:

> It is not clear from these {35} studies that the strongest single claim for the use of programmed instruction, that each individual learns at his own rate, has been supported. (pp. 107-108)

Bobier (1964) noted that low-achieving students of limited ability were not sufficiently motivated to use programmed textbooks independently. For many teachers, it became apparent that programmed instruction was not a panacea. Most could probably agree with the conclusion of Jamison, Suppes, and Wells (1974):

> . . . PI is generally as effective as TI {traditional instruction} and may result in decreasing the amount of time required for a student to achieve specific educational goals. (p. 41)

Nevertheless, teaching machines from the 1950s gather dust, and programmed instruction is rarely discussed. However, it is actually still apparent in computer-assisted instruction programs and in self-paced "individualized instruction" programs.

Manipulative and Other Materials

In 1955, the primary-grade teacher was more likely to use manipulative materials than teachers at other levels. Emphasis on the use of materials at all levels was emphasized in the 1960s. Yet the pattern of 1955 continues in 1977: the primary-grade teacher is still most likely to use materials, and little use is reported at other levels.

It was not uncommon is 1955 for a teacher to make or collect inexpensive instructional materials for use in the classroom. The enactment of NDEA in 1958 began the years of availability of federal funds for a wide variety of materials. Evidence from a range of sources indicates that this money was not always spent with frugality and extensive care in selecting appropriate educational materials. Part of the reason for this stemmed from the fact that money frequently became available at short notice, "to be spent within 30 days"; also, its use was not accounted for specifically. As budgets have tightened over the past several years, teachers have at times resorted to the plea of "unavailability of funds" to explain failure to use materials.

A review of research on the use of materials in elementary school mathematics (K-8) was conducted by Suydam and Higgins (1976, 1977). They reported:

> (1) In almost half of the considered studies, students having instruction in which manipulative materials were used scored significantly higher on achievement tests than students who had instruction in which manipulative materials were not used. In almost the same number of studies, the two groups scored much the same; few instances were found in which the group not using materials scored higher. Thus, lessons using manipulative materials have a

higher probability of producing greater mathematics achievement than do non-manipulative lessons.

(2) Only 3 of 28 findings favored the use of symbols alone; only one study favored pictorial treatments used alone. In 7 instances use of manipulative materials was favored over sequences in which manipulative materials were not used. In 9 instances, use of manipulative materials and pictorial representations resulted in higher achievement than use of symbols alone. The concrete materials thus appeared to play an important role.

(3) Research in which the number of embodiments for a mathematical idea has been the focus resulted in no significant differences in achievement in 3 of 4 studies.

(4) In three of 8 studies, manipulation of materials by students was favored over having students watch the teacher demonstrate with materials. In 4 other studies, no significant differences were found. It appears that individual manipulation by the learner is not the only way children learn: it can be effective to watch the teacher demonstrate.

(5) Across a variety of mathematical topics, studies at every grade level support the importance of the use of manipulative materials. Little evidence was found that manipulative materials are effective <u>only</u> at lower grade levels.

(6) The use of materials appears to be as effective at one achievement level as at another -- that is, high achievers profit from the use of materials as much as low achievers.

(7) The use of materials appears to be as effective at one ability level as at another -- that is, those of high ability profit from the use of materials as much as those of low ability do.

(8) Although the data are sparse, the use of materials appears to be at least as effective at one socioeconomic level as at another.

The <u>extent</u> to which materials are used has been considered in several surveys. Johnston (1962) found that few teachers in grades 1-8 used any material other than the textbook. Green (1970) reported that first-grade teachers used more materials and used materials more frequently than sixth-grade teachers. Haladyna (1975), in a study with 4400 Oregon teachers, also found that with

> the use of manipulatives the tendency was for moderate to frequent use in the primary grades to a minimal use at the intermediate, high school, and junior high school levels. (p. 8)

In another report on Oregon projects which focused on various materials, Thomas (1975b) found that in no instance were either manipulative materials or games the basis of a significant percentage of programs. However, teachers' attitudes toward the use of both was very positive; the use of manipulative materials was not favorably viewed by students, however.

The Developing Mathematical Processes program developed at the R&D center at the University of Wisconsin-Madison integrates a variety of materials in its measurement-oriented approach. Necessary materials are available in kits; nevertheless, many teachers do not make use of them. The same response shows up in connection with materials provided with a variety of other programs.

The NACOME Report (1975) indicated that

> in spite of the recent publicity and emphasis it is not at all clear that manipulative materials are widely used. For instance, 37 percent of the elementary school teachers in the NCTM survey had never used the mathematics laboratory, and ten percent had never used manipulative materials at all (in grades 2 and 5). (pp. 62-63)

The research evidence lends support to the belief that additional means must be found to encourage teachers to use materials. But the literature contains many references indicating that it is also necessary to consider carefully <u>what</u>, <u>when</u>, <u>how</u>, <u>why</u>, and <u>by whom</u> the material will be used.

Schoen (1977), in his review of research on self-paced instruction, confirmed this:

> There is consistent evidence that the use of various media and supplementary teaching materials increased the effectiveness of SPI. There is also consistent evidence that media and materials in a typical SPI program have been restricted to printed audio materials. In addition, the various media and supplementary materials often have not been used, even when available. (p. 213)

There is relatively little evidence on the amount of use of various audiovisual devices. Generally, they are collectively studied as one of a variety of instructional materials. Many reports indicate the <u>availability</u> of equipment for using films, film loops, filmstrips, television, overhead projectiles, and the like, but their actual <u>use</u> is not yet an everyday occurrence.

Computer-Aided Instruction

In 1955, schools and computers were separated entities: availability and cost prohibited their merger. In the early 1960s, however, some schools bought or leased computers or computer time, usually first for administrative purposes, and inevitably, after a time, for mathematics instruction. The Dartmouth model, funded by NSF, has been extensively copied.

Three modes of computer use have evolved:

(1) computer-aided: non-tutorial, problem-solving aid

(2) computer-assisted: tutorial instruction with the computer taking a teacher's role

(3) computer-managed: courses of study are developed, sequenced, and/or monitored for students, with the computer storing information.

Two large-scale national surveys (Darby et al., 1970; Bukoski and Korotkin, 1976) of computing activities have been conducted by the American Institutes for Research. A stratified random sample of 25% of the secondary schools was selected for the second study, plus a sample of the schools participating in the 1970 study; responses were received from 3,643. Since 1970, the fraction of secondary schools reporting <u>some</u> computing activity has steadily increased, from 34.4% in 1970 to 58.2% in 1975. Mathematics classes used the computer most frequently, although the percentage dropped from 46.7% to 43.2%.

The researchers projected:

> Though the continued growth of computer-based education seems assured, the specific future of <u>instructional computing</u> is unclear. Based upon the growth over the last five years (1970-1975), it is projected that within the next decade the majority of secondary schools in the country will have some type of instructional computer-based application. . . it is probable that computer science and problem solving will remain prominent instructional applications through the next decade . . . (Bukoski and Korotkin, 1976, p. 20)

Despite the growth in computing activities, they indicated that the relative costs remained virtually the same.

Among the other studies on the extent of computer use is one by Buchman (1969), who found that in 1967-68 only 5% of New York secondary schools had computer access for mathematics classes; only 13% had desk calculators. Rudolph (1972) found that one-third of the 647 Illinois secondary schools she surveyed used computers, with 54% of these using computers for both instruction and administration, and only 5% solely for instruction. Problem solving in mathematics and science, and data processing, accounted for over 80% of the time. Bishop (1971) reported that 30% of the secondary schools in the Missouri region offered technically oriented computer-related courses in their mathematics curriculum; 20% used computer time for enriching and supporting courses.

Moran (1974) reviewed current practices and trends. He noted that time-sharing has grown in importance; however, the minicomputer and programmable calculators have had and will continue to have an impact on school use of computing power.

Studies on the effectiveness of the use of computers were reviewed by Kieren (1973) and by Hatfield (1973). In general, the results are equivocal: higher general achievement is not a foregone outcome of the use of computers, but it does aid in promoting problem-solving achievement. Batch processing appeared to be at least as effective as having direct computer access: the important factor may be experience in writing programs rather than the time it takes to receive computer solutions.

Jamison, Suppes, and Wells (1974), surveying some studies using mathematics content, stated:

> . . . no uniform conclusions can be drawn about the effectiveness of CAI. At the elementary-school level, CAI is apparently effective as a supplement to regular instruction . . . At the secondary-school level, a conservative conclusion is that CAI is about as effective as TI when it is used as a replacement. It may also result in substantial savings of student time in some cases. (p. 55)

Vinsonhaler and Boss (1972) reviewed seven major studies on drill and practice programs using CAI. They indicated that higher achievement could be anticipated when CAI was used to augment regular instruction. They noted that

> There are indications that the effects obtained with CAI might be obtained through less expensive means. For example, one of the studies reported by Suppes and Morningstar (1969) suggests that an additional 30 minutes of ordinary classroom drill and practice can accomplish the same results as a 15-minute CAI program. (p. 31).

In a study concerned with students' reactions, Hess and Tenezakis (1973) reported that students who had used a remedial drill-and-practice program in basic arithmetic for one or two years regarded the computer in more positive terms than the teacher did. Non-CAI students also regarded the computer significantly more favorably: they had a less favorable image of the teacher than did the CAI group. For both CAI students and non-CAI students, the computer had a more favorable image than did either the teacher or textbooks.

Calculators

The hand-held calculator has been on the market since the early 1970s. In 1975, the cost of calculators dropped sharply, and as a result "for millions of people, everyday arithmetic will never be the same" (McWhorter, 1967). Desk calculators had been used in some secondary-school mathematics classrooms before 1955, but their use was largely restricted to low achievers, and they generated little excitement. Having calculators readily available for each and every child changed the story.

A position statement of the NCTM (1974) reflects the immediate concern of mathematics leaders:

> . . . Mathematics teachers should recognize the potential contribution of this calculator as a valuable instructional aid. In the classroom,

> the mini-calculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes proficient in mathematics.

Other groups throughout the country also recognized the potential of the calculator. The NACOME Report (1975) severely questioned the strong trend to emphasize computation; the case for decreasing emphasis on manipulative skills was seen as stronger than ever before because of the impending universal availability of calculators. They noted that many low-achieving students have been condemned

> to a succession of general mathematics courses that begin with and seldom progress beyond drill in arithmetic skills. Providing these students with calculators has the potential to open a rich new supply of important mathematical ideas for these students . . . at the same time breaking down self-defeating negative attitudes acquired through years of arithmetic failure. (pp. 41-42).

Therefore, they recommended use of calculators "beginning no later than the end of the eighth grade", with the student permitted to use the calculator during all mathematical work including tests. The development of instructional materials and curricular revision or reorganization "in light of the increasing significance of computers and calculators" were also recommended.

The Euclid Conference (NIE, 1975) participants also indicated concern with the effect of calculators on the curriculum, stressing the need for developing new sequences of instruction. The National Science Foundation, concerned about the potential impact of the calculator, funded a critical analysis (Suydam, 1976). All existing literature was studied, and a survey conducted to ascertain the arguments for and against use of calculators and the ways in which calculators should be used. Frequently cited reasons for using calculators included: aid in computation; facilitation of understanding and concept development; lessening of the need for memorization; help in problem solving; motivation; aid in exploring, understanding, and learning algorithmic processes -- and the fact that they exist, and are appearing in the hands of increasing numbers of students.

The most frequently cited reasons for not using calculators were that: they could be used as substitutes for developing computational skills, they are not available to all, and they may give a false impression of what mathematics is. The first concern was expressed most frequently by parents and other members of the public; few educators, however, believed that children should use calculators in place of learning basic mathematical skills.

Analysis of the studies published up to August 1977 in which calculator and non-calculator groups were compared indicates that, of 40 findings, in 19 instances the calculator group achieved significantly higher on paper-and-pencil tests (with which the calculator was not used). No significant differences were found in 18 studies; in only three instances was achievement significantly higher for the non-calculator group.

A conference on the uses of hand-held calculators in education was held in June 1976 by NIE and NSF to produce a planning document "that will provide a well-defined framework for future research and development efforts" (NIE/NSF, 1977). The participants noted:

> These small, portable, and inexpensive machines have the potential for replacing the paper and pencil calculations that have been the major (and

often the sole) component of elementary school arithmetic. (p. 2)

> Educators are faced with a dilemma. Their experience and instincts tell them to research, test, and proceed with caution. Yet calculator technology is progressing rapidly, and marketing pressures are great. The evolutionary pace traditionally associated with curriculum change is too slow to fit the present situation. (p. 3)

The conference report summarizes discussion about many aspects of present-day school mathematics, and the opportunities and dangers presented by calculators. The recommendations that emerged from those discussions called for the establishment of an information collection and dissemination center, surveys of existing materials and practices, both short-term and long-term curriculum development with related research, and teacher-training efforts.

A Calculator Information Center was established by NIE in early 1977, supplementing continuing efforts by the NCTM. Both are involved in the task of collecting and disseminating information to and from schools as more and more teachers incorporate calculators in the teaching of mathematics. Requests for proposals exploring calculator use were issued in 1977 by both NSF and NIE, beginning the task of research and curriculum development.

How extensively the calculator will influence the mathematics curriculum is unclear. Conflict is obvious between those who see computational skills as the most vital task for mathematics teachers and those who see the calculator allowing a change in direction -- a change feasible for the first time in history. In the past three years, opinions have changed, and the calculator is being used with increasing frequency, but the curriculum has not changed noticeably.

Other technological developments are on the near horizon. The dividing line between calculators and computers is already tenuous; existing calculators have the computing power of computers of twenty years ago. Interaction between student and machine will be increasingly feasible.

Instructional Materials: HIGHLIGHTS

- The textbook is the primary determinant of mathematics curricula, and many teachers use no instructional materials except the textbook and the chalkboard.
- About half the states have mandated textbook adoption lists, with more listing multiple texts; however, a single text is used in most classrooms.
- While there is variance across textbooks at the elementary-school level, the basic components of the curriculum have become standardized, so that the variance is largely in terms of amount of space allocated to a topic, approach, and design. At the secondary-school level, wider variance is obvious as the type of course varies.
- Teachers tend to follow the textbook closely with regard to content selection and sequencing, though they may skip or ignore components which they do not consider essential.
- Readability has been of specific concern for at least ten years.
- Use of programmed instruction may save time in achieving specific goals, but it is unclear whether pupils actually progress at individual rates. Use of manipulative materials decreases as grade level increases; however, use of such materials appears to be effective with certain content at all age levels and with all types of children.
- Computers are used more widely in mathematics classes than in any other classes, although the percentage of use for mathematics declined slightly between 1969 and 1974. The problem-solving mode was most widely used, followed by simulation and then tutorial CAI.
- The hand-held calculator has the potential to change the curricular

focus on computation. Both short- and long-term research and curriculum development need to be undertaken, in addition to teacher-training efforts.

121

H. What Is the Cost of Instruction?

It is virtually impossible to ascertain the actual amount of money spent for education -- different bases are used and figures cited in one report differ from those in another. The amounts allocated by the federal government would seem to be easiest to ascertain -- but alas, the figures are reported in such a variety of ways that the services of many accountants could probably be engaged for years to sort things out. The NIE document prepared by Nelson et al. (1977) illustrates the dilemma. They report:

> It is impossible to state the precise total spent on educational R&D in the U.S. Analysis is hampered by a lack of data series needed for such an estimate, conceptual incompatibilities in the definitions of existing series . . ., and differences in the range of functions recognized . . . Such ambiguities are compounded by differing reporting procedures . . . (p. 15)

Thus NSF, OMB, and NIE, for instance, all produce data which are difficult (if not impossible) to correlate. Most of the statements which follow will be made in general terms: therefore, an interpretation of what the data seem to indicate is given.

There is little doubt that both the costs of instruction and the amounts allocated to instruction have increased since 1955, over and above the inflation rate.

> Each annual survey of the Cost of Education Index, based on a sampling of approximately 1,200 school districts of various sizes and locations, reflected record spending, increasing year by year from 1958 to 1972. (Morrison, 1973).

NCES data indicate that total spending by state and local governments for education rose from about $24 billion in 1962-63 to $65 billion in

1971-72, $70 billion the following year, and $72 billion in 1973-74. During the decade 1962-72, education was consistently the largest item in the budgets of state and local governments, accounting for 37 to 39 percent of their budgets (NCES, 1975, 1976).

The Gross National Product Index rose from approximately $285 billion in 1950 to $504 billion in 1960 to $977 billion in 1970; the percentage spent for education also rose, from 3.4% to 5.3% to 7.7%. Yet the amount spent for all research and development in education may be as low as 1% of the total: compared with other enterprises, education spends a relatively limited amount for such efforts.

The average per-pupil cost of instruction has risen; from a number of references in various sources it appears that:

- in 1955, the range was from less than $100 to about $200
- in 1965, the range was from $300 to $850, with an average of $500 ($455 in 1957 dollars)
- in 1973, the average was approximately $1200 ($766 in 1957 dollars)
- in 1976-77 the range was approximately $1000 to $3000, with an average of about $1450 ($793 in 1957 dollars)

Some states spend less than 1% of personal income on education; others spend over 5%. Unfortunately, the states with less total income are likely to be the same states that spend less proportionately.

It is obvious that funds for education come from four sources -- local, state, and federal governments and, to a small extent, private funding. But the amount of these funds devoted to mathematics instruction is obscure. Perusal of document after document yielded largely aggregate figures, or amounts for reading <u>and</u> arithmetic: the few precise amounts are relatively meaningless isles in the sea of data.

A rough estimate appears to be the most feasible figure to use. Dexter Magers, Mathematics Counsultant at the U.S. Office of Education, provided the data typed as Table 7, and indicated:

> I have talked to some of our Title I staff and examined some of the annual reports for several other programs including NDEA. Based on these sources it appears that 20% is a good estimate of the proportion of funds that could be counted as devoted to mathematics instruction from these sources. . . . {However}, since most of the Federal programs are targeted on groups of persons rather than subject matter areas, I suggest you use 18% of the amounts in column 3 of the table. (letter, 25 May 1977)

TABLE 7

REVENUE RECEIPTS
OF PUBLIC ELEMENTARY AND SECONDARY SCHOOLS
FROM FEDERAL, STATE, AND LOCAL SOURCES

(Portion of table with data from
National Center for Education Statistics)

School year	Total	Federal	State	Local (including intermediate)
1	2	3	4	5
AMOUNT IN THOUSANDS OF DOLLARS				
1955-56 . . .	9,686,677	441,442	3,828,886	5,416,350
1957-58 . . .	12,181,513	486,484	4,800,368	6,894,661
1959-60 . . .	14,746,618	651,639	5,768,047	8,326,932
1961-62 . . .	17,527,707	760,975	6,789,190	9,977,542
1963-64 . . .	20,544,182	896,956	8,078,014	11,569,213
1965-66 . . .	25,356,858	1,996,954	9,920,219	13,439,686
1967-68 . . .	31,903,064	2,806,469	12,275,536	16,821,063

TABLE 7 (continued)

1969-70 . . .	40,266,923	3,219,557	16,062,776	20,984,589
1971-72 . . .	50,003,645	4,467,969	19,133,256	26,402,420
1973-74 . . .	58,230,892	4,930,351	24,113,409	29,187,132
1975-76		5,346,000 (est.)		

Using 18% as the estimate, it appears that the amount of federal funding which might have been directed toward mathematics education might be:

	direct percentage	converted to 1957 dollars
1955-56	$ 79,460,000	$ 79,460,000
1957-58	87,567,000	89,405,000
1959-60	117,295,000	115,536,000
1961-62	136,976,000	131,497,000
1963-64	161,452,000	151,281,000
1965-66	359,452,000	327,101,000
1967-68	505,164,000	434,441,000
1969-70	579,520,000	453,764,000
1971-72	804,234,000	570,201,000
1973-74	887,463,000	574,189,000
1975-76	962,280,000	532,140,000

It should be reemphasized that these data are estimates, and possibly only of the amount that <u>should</u> be spent on mathematics education. (Earlier it was noted that 20% was the estimate for the amount of <u>time</u> spent on mathematics instruction, so the estimates <u>could</u> be appropriate.) But there is no way to determine how much money has <u>actually</u> been spent on mathematics instruction, either with or without federal funding.

The federal sources of funds for elementary- and secondary-school mathematics have come largely from the National Defense Education Act, Title III (1958) and the Elementary and Secondary Education Act, Titles

I and III (1965) and Title IV (1974), both administered by the U.S. Office of Education, and from education-specific funds of the National Science Foundation. Other federal legislation, including other titles of NDEA and ESEA, the Office of Economic Opportunity, and School Assistance for Federally Affected Areas (SAFA) have also provided funds which may have been used for mathematics instruction. Ginsburg and Killalea (1975) reported that funds from the major program areas reached their intended audiences; that is, ESEA Title I funds went to districts with lower family income, SAFA funds went to districts with low tax bases because of parents employed by or living on federal installations, and State Discretionary Federal grants went more heavily to urban districts in more urban regions and rural places in more rural regions. No assessment of whether any subject area was affected was made, however.

In a report on the use of Title I funds by the Bureau of Indian Affairs in New Mexico, Ramey and Sileo (1975) reported that 3.5% of the more than $7.9 million allocated in 1973-74 were spent for mathematics. It is also interesting to note that gains in language arts, which accounted for 80% of the funded projects in the state, were 7 months; gain for mathematics was 1.1 years.

In other states, the monies expended for compensatory education were also deemed successfully spent. In Michigan, for instance, more than half the students in federally funded projects gained one month in achievement score per month in the program (which presumably was greater than could have been expected); 28% gained 200% and 12% gained 300% -- that is, 3 months gain for each month in the program. In this case, however, gains were greater in reading than in mathematics.

Cost Effectiveness

If it is difficult to determine how much money was actually spent for mathematics instruction, then it follows that it is difficult to determine cost-effectiveness. We shall report selected studies that pertain to the question.

Generally, the few studies conducted before 1960 (e.g., Furno, 1956) involved rather nebulous "quality indicators" and uninterpretable correlations. Nevertheless, the conclusions usually indicated that the amount of money spent influenced achievement. Findings are not specific to mathematics instruction, however.

Stock (1974) reported that

> More recent studies in the Sixties published conflicting findings regarding the impact of expenditure levels upon achievement, "quality", or other education program characteristics. (p. 26)

He cited three studies from the 1960s in which expenditures were related to quality or achievement, and six studies in which no relationship was found. Among the latter was the study involving 645,000 students in grades 3, 6, 9, and 12 directed by Coleman (1966). Achievement measures and statistical procedures have been questioned by many, but the Coleman Report documents the case for those who believe that per-pupil expenditure shows "virtually no relation to achievement if the 'social' environment of the school -- the educational backgrounds of other students and teachers -- is held constant."

Results from studies in the late 1960s and 1970s fail to indicate that expenditure and achievement are highly correlated. For example:

- Data from the Missouri Assessment (1971) indicated that the amount of money spent per student was not related to achievement.

127

- In the Oregon (1976) progress assessment, district per-pupil expenditure revealed little or no significant difference in performance.

- Stock (1974) found that school districts in Ohio in 1971-72 which spent a greater amount of money per pupil did not exhibit significantly higher scores on mathematics achievement tests than did districts which spent less money per pupil.

- Morrison (1973) compared the relationship between instructional cost for 1968-69 and the performance of third graders in 1969 in 702 school districts in New York. Instructional costs were not significantly related to the quality of education in mathematics.

- Tallmadge (1973) analyzed achievement gains and pupil expenditures in 1972 California Title I projects. In schools less than 75% of the pupils eligible for Title I participation, there was no relationship between achievement gains in mathematics and any combination of regular and supplementary expenditures. In saturated schools (above 75%), a significant relationship was found between achievement gains and Title I per-pupil expenditures for reading but not for mathematics.

A few studies indicate some (limited) variance which was statistically attributed to expenditures:

- Vlahos (1975) reported that revenue and total current expenses were related to mathematics achievement in grades 6 and 9 of 172 school districts in Colorado during 1972-73. The financial variables as a group made the most unique contribution to sixth graders' scores, while administrative and total expenses per pupil were the significant unique contributors.

- In Wisconsin, assessment results for 1969 reported by Coulson (1974) indicated that pupils from high-expenditure districts (over $800 per pupil) scored significantly higher than pupils from medium- or low-expenditure districts (under $600); however, pupils from low-expenditure districts outscored pupils from medium-expenditure districts.

- In a study of 1,900 sixth graders in eight suburban and rural school districts in Erie County, Pennsylvania, Salopek (1974) reported that school system characteristics had a significant impact on student achievement for average and low IQ groups. Teacher experience, class size, and costs of textbooks and supplies were the most consistent predictors of variance on arithmetic subtests.

It appears to be a plausible conclusion, given the data available, that the amount of money spent per pupil has not generally been significantly related to mathematics achievement. There are indications that socioeconomic factors outside the control of the school exert a greater influence. For instance, Hawaii, one of the two states in which finances are equalized across schools (California changed to this basis in June 1977) has found that achievement test scores in mathematics "show much the same close relationship to family background as they do elsewhere in the country" (Education Summary, 1975, p. 2).

Federal Funding Impact

Beginning in 1968, increased emphasis was placed on evaluation of federally funded projects. Reports from those receiving federal funds indicate that they felt the projects had an impact. Thus McDaniel (1973) indicated that teachers and supervisors in 57 secondary schools with 4 or more NDEA Title III projects "observed improvement in teachers and students" as a result of use of NDEA Title III-funded materials and equipment. No data are reported.

Several assessments of the impact of funding were reported in which findings were at some variance with official statements. Thus DeShields (1973) reported that students in Title I schools performed at significantly lower levels than those not in Title I schools (but who may have been eligible) and Ordonez (1971) reported that pupils in Title I schools had significantly less positive attitudes toward arithmetic. In both instances, however, effects of pre-existing conditions might have been measured, rather than effects resulting from Title I funds.

A Rand Educational Policy Study (McMaughlin, 1975) is perhaps the most widely quoted analysis of a federal program. It is a confirmation of conclusions reached as reports pertaining to Title I and Title III projects (as well as similar ones) were perused for this report.

McLaughlin traced the evaluation requirements of ESEA Title I, noting that, because of political concerns, "framers of Title I purposely left ambiguous parts of the bill that might generate conflict and weaken support" (p. 17). The LEA receiving funds was required to report annually to the state education agency, who in turn was required to make periodic reports to the Commissioner of Education.

> . . . an implicit decision was made not to set uniform reporting standards, not to require measurement by standardized tests, and not to suggest what the preferred components of "effectiveness" might be. More sophisticated methodological notions, such as the provision of control groups, were rejected as running against the grain of legislative intent. (p. 19)

Consequently, reports for 18,000 LEAs and 50 SEAs for the first two years

> . . . painted the success of Title I in glowing terms, and suggested that the local school administrators were moving quickly to devise effective compensatory strategies. Title I seemed to be working beyond anyone's highest expectations . . . (pp. 22-23)

McLaughlin found that evaluation was not being used to aid in decision-making about curriculum and instruction nor to determine priorities at any level -- local, state, or federal -- nor were they used by SEAs or USOE to determine funding approvals. Because of reactions to reports, however,

> federal interest in the results of the mandated reporting scheme ended with the publication of <u>Title I/Year II</u>. There is no evidence that local reporting practices have improved with time

> (although states are required to turn in reports from time to time). Reviews undertaken by the American Institutes for Research (AIR) and the Center for Educational Policy Research, Harvard University, found that these evaluations were as unsatisfactory in 1972 as they were in 1966. . . . if one were to rely solely on these required reports in judging the impact of Title I, one would have to conclude that it has been an astonishing success -- a conclusion that . . . finds little support in other efforts to evaluate Title I. (p. 23)

McLaughlin believed that LEAs wanted general aid, not categorical aid targeted for disadvantaged children. Both USOE and the SEAs seemed unwilling to destroy good working relationships "over the relatively trivial matter of Title I data collection and evaluation" (p. 25). As noted previously in this report,

> An attempt to trace the flow of Title I dollars to specific programs and outcomes is beset with problems . . . it is difficult if not impossible to trace the course of Title I dollars through the school system. (p. 40)

It is also noted, however, that experience with other social programs (particularly health care) suggests that social programs may have "high impact or high coverage, but not both," implying that "measurable benefits from large-scale social action programs such as Title I can be expected to be marginal" (p. 40). McLaughlin noted that academic achievement is but one of many objectives of Title I: therefore to conclude on the basis of standardized test scores that Title I is not (or is) 'working' is not justified (p. 41). However,

> Ironically, another major impact of the outcome of Title I evaluation has been the spawning of more evaluation. No one has stood back and reassessed the value of the process of evaluation itself or the assumptions underlying the evaluation models, or wondered if the cost of acquisition was in this instance worth paying. If the evaluations being done at present are a yardstick of what has been

learned from 7 years and over $50 million of Title I evaluation, the conclusion must be that we have learned very little.

> But information gathering has become a necessary activity . . . in the policy system, and faith in the science of systems analysis remains undiminished at the higher echelons of the federal government. The Title I evaluations have generally set to rest the uncritical optimism of the mid-sixties concerning the effects of school and the role of education as an antipoverty strategy. But the scientific movement in education . . . continues on unperturbed by the experience of Title I. (p. 118)

At another point, McLaughlin noted that "a federal evaluation policy that conflicts in fundamental ways with local priorities is unlikely to succeed" (p. 119). That federal policy on evaluation of funding efforts can be implemented when public opinion coincides with federal need can be noted as needs assessments are considered in a later section of the report.

Costs of Instruction: HIGHLIGHTS

- For at least 15 years, education has been the largest item in the budgets of most state and local governments; the amount of federal funding for education has increased dramatically.
- The amount of money devoted to mathematics instruction is difficult to determine; 18% to 20% seems plausible but cannot be verified from available data.
- The amount of money spent per pupil has not been found to be significantly related to mathematics achievement in most studies.
- Since 1968, increased emphasis has been placed on evaluation of federally funded projects.
- The reports from those receiving funds almost invariably indicate that they feel the funded activity was successful; in few cases are there hard data or a controlled research design. Evaluation from outside reviewers rarely indicates the degree of success that those involved in a project or activity declare.
- Federal policies which conflict with local priorities are not unlikely to be fully implemented.

III. Existing Practices and Procedures in Teacher Education

A. Overview and Beginnings, 1955-1965

Dramatic changes in the nature and quality of preservice and in-service education for both elementary- and secondary-school mathematics teachers have transpired during the 20 years following 1955. Table 8 highlights, but over-simplifies, some of the trends associated with changes during the period. It also indicates some of the factors prevalent immediately prior to 1955.

The role of the societal/political ethos cannot be underrated--it is the driving force that couples values with willingness to fund teacher education. The political reality of 1955 was McCarthyism, keeping up with the Russians, and concern for the scientific talent pool. Schaffter (1969) and Krieghbaum and Rawson (1969) documented the political realities in establishing the National Science Foundation. They also indirectly document the societal pressures; both these and the political realities produced an optimistic, enthusiastic ethos for teacher education in the mid-1950s. Osborne and Crosswhite (1970) and Cohen (1976) focus more particularly on the conflict between teacher educators and other academics concerning the goals of education being focused on all American youth for science and mathematics. In 1955, the schools were coping with large numbers of children from the post-war baby boom and the resultant teacher shortage. Particularly in the non-urban areas, there were many small, non-comprehensive high schools requiring teachers who could operate in many subject-matter areas.

The need for change in 1955 was urgent. The prevailing mind-set was in terms of a national emergency. The schools were not producing,

TABLE 8

TREND HIGHLIGHTS IN TEACHER EDUCATION, 1950-1975

Factors	Period		
	1950-1955	1955-1965	1965-1975
Societal Educational Climate	1. Recovery from World War II 2. Fulfilling roles	1. Staying ahead of the Russians 2. Building a pool of scientific talent	1. Disenchantment with science 2. Concern for the non-scientifically talentrd
Teacher Supply	1. Shortage 2. Many small schools need multi-talented teachers	1. Shortage	1. Moving toward over-supply 2. Mostly larger comprehensive high schools requiring specialists
Elementary Teacher Characteristics	1. Many without BA 2. Some with only one college mathematics course; many with none	1. Many without BA 2. Little background in mathematics	1. Large majority with BA; one-third with MA 2. Most with one-mathematics course
Secondary Teacher Characteristics	1. Most with BA 2. Many teaching out of field of training 3. Colleges require as many pre- as post-calculus courses in a 27-semester-hour major	1. Some improvement in background	1. More than half with MA 2. Most teaching in field of teaching 3. Many colleges do not count pre-calculus courses for certification requirements in a 32-semester-hour major
Teacher Education Program Thrusts	1. Mathematical literacy for all	1. Up-dating mathematics background 2. Discovery learning theory 3. In-service is the major thrust 4. Federally funded institutes	1. Computer usage grows to be expected for secondary 2. Activity or laboratory learning 3. Field experience prior to student teaching 4. Flirtation with CBTE

according to the popular press, the politicians, and the academics. The orientation was for immediate action to change the schools, rather than for change in preservice education that might yield long-range effects. During the 1955-1965 period, in-service education was the focus of attention and action. Consequently, information about teacher education for this ten-year period is about in-service education. The attention accorded in-service was so consuming that the majority of conclusions t be made about preservice are inferential and based on information collected relative to in-service needs.

B. Teacher Education, 1955-1965

This section begins by examining the nature of teacher competence and characteristics, shifts to considering the in-service programs and the effect of the in-service programs on teachers, and concludes by considering the effect on preservice teacher education.

Teacher Competence and Characteristics, 1955-1965

A teacher's competence was defined in terms of the teacher's course background until recently, when the additional factor of the performance of the teacher's students has become significant. Thus, throughout the 1955 to 1965 period, knowledge of teacher competence is largely inferential, stemming from the characteristics inferred from the course and degree background of teachers. Schumaker (1960) provided a relatively thorough description of the graduation requirements for a mathematics major in the 140 institutions graduating the largest numbers of secondary mathematics (identified from 314 AACTE members). He surveyed college catalogues for these schools and found that in 1957 the median requirement for a major was 27 semester-hours of mathematics. The major included calculus and roughly as many hours of pre-calculus courses as post-calculus courses. One infers from the titles of the post-calculus courses that they were a hodgepodge not reflecting the current mathematics of the period in spirit or content. Thirty-two percent of the schools required college geometry; 28 percent, theory of equations; and 31 percent, differential equations. No other post-calculus courses were required by even 20 percent of the institutions. Eighteen hours were required for the minor. For both, 24 hours in professional education was required, with 5 hours of student teaching the median minimum requirement. Shumaker reported that teachers colleges tended to offer professionalized subject-matter courses more frequently than did four-year colleges or state universities. A striking lack of influence of the recommendations by professional groups is noted.

The evidence collected by Shumaker suggests that in 1955 secondary teachers of mathematics were competent, if judged on the basis of the type of background they were required to acquire in the colleges and universities. Clearly the mathematics was neither "modern" nor extensive.

But were teachers working within the field for which they were trained? Several kinds of information suggest not. The end of the 1955-1965 period finds a severe teacher shortage in mathematics. The NEA Research Division (NEA, 1966) estimated a total need for new teachers of mathematics to be more than 12,000, but the number of newly certified mathematics teachers was just below 10,000, with only about 65 percent of them expected to enter teaching. This suggests that many teachers were operating out of their fields of specialization. Obourn and Brown (1963) found that nearly 15 percent of the mathematics and science teachers in the United States taught only one period per day in these academic areas -- one suspects their undergraduate background to be other than mathematics or science. The National Association of State Directors of Teacher Education and Certification (NASDTEC, AAAS, 1961) published a study indicating for 1961 the percentage of mathematics classes taught by teachers in terms of hours of credit in mathematics; Table 9 summarizes the results.

TABLE 9

PERCENT OF SECONDARY MATHEMATICS CLASSES TAUGHT BY TEACHERS WITH A GIVEN NUMBER OF SEMESTER HOURS IN MATHEMATICS

Hours in Mathematics	Percent of Classes, Grades 7 and 8	Percent of Classes, Grades 9 through 12
Less than 9	34	11
9-17	19	12
18-29	26	32
30 or more	21	45

Brunsvold (1966) made a careful examination of 90% of the secondary teaching staff in the 452 secondary school districts (98.5%) in Iowa, operating from state department records. He found 754 teachers of mathematics, of whom 73 percent were male. The mathematics teachers were of average age 34.3, with males being the youngest for all curricular areas. They averaged 8.9 years teaching experience, with female teachers averaging almost nine more years of experience than males had. Approximately 28 percent of the teachers held MA degrees, with more males than females having the degree. Almost 80 percent of the teachers with mathematics majors were teaching 100% time in their major areas. However, 9 percent were teaching in two areas and 7 percent in three areas. These tended to be in small schools. Larger schools had better utilization of teaching staffs in terms of the teachers' background (or competence) and had teachers with better backgrounds. The data that Brunsvold exhibited are consistent with that reported in his extensive review of the literature.

The characteristics of secondary mathematics teachers have to be inferred from studies like the above and generalized from background data. Brunsvold studied teachers in a decidedly rural setting; studies in urban settings provide additional insights worth noting. Rudnick (1962) identified several general background characteristics of 1,425 teachers of college preparatory mathematics from schools in the 193 cities in 1959 with more than 75,000 population. Contrasting curricula of 1957-58 and 1960-61 (before and during the major impact of SMSG and teacher institutes), he found that all teachers had a bachelor's degree and 58.2 percent had a master's degree. They had an average of 16 years teaching experience and an average course background in mathematics of 39 semester hours. Moreover, 67 percent had taken graduate work in mathematics and 76 percent in education, for an average of 16 hours and 21 hours respectively. A total of 49.7 percent of the teachers had taken programs sponsored and paid for by institutions, rather than paying for it themselves.

Shetler (1959) provided insight into the kinds of issues and problems concerning teachers. He surveyed a sample of teachers representing 10 percent of all of the secondary schools in the 20 states of the North Central Association. The teachers' perceptions of aims in teaching mathematics were in general agreement with authorities in mathematics education (thus reflecting the general orientation prior to the perception of a need to develop a pool of scientific talent). Multi-track programs were noted to be on the increase and rural school practices tended to be traditional. Many teachers indicated a concern that their curricula were inadequate. Shetler indicated the same contrast between rural and urban as can be observed in the studies by Rudnick and Brunsvold.

Elementary teachers' background and characteristics early in the 1955-1965 era are not as well-documented as those of the secondary teacher. Ruddell et al. (1960) provided the most comprehensive information. During the 1950s, state requirements were shifting toward requiring a bachelor's degree to teach in the elementary school; in 1951 only 17 states had this requirement, but by 1957, 35 states did. Ruddell and his associates point out that about 30 percent of the elementary teachers in 1957 held provisional certificates. In only 12 states was there a specific mathematics requirement for certification, seven required a mathematics course, and five required a methods course. Examination of college catalogues for 96 institutions revealed that 39 percent required no mathematics course and 29 percent required no course on methods of teaching mathematics. Evidence from a survey taken in 1966 about 1962 requirements indicated that 23 percent of the colleges graduating elementary teachers required no mathematics (Dubisch, 1970).

There are relatively few studies during the 1955-1965 era that focused on what mathematics elementary teachers knew or what their attitudes were about mathematics, either directly or by inference from course background. However, respected mathematics educators like Grossnickle and Dutton had conducted studies in the late 1940s that indicated this was a major problem. With the publication of CUPM Level I guidelines for elementary teachers of mathematics, a spate of studies was conducted, but results were not published until after 1965.

In-Service Education, 1955-1965

In-service education prior to 1955 was the responsibility of the individual teacher of mathematics or the teacher's school system. Most teachers acquired their in-service education through an institution of higher education, studying for a master's degree to enhance their earnings.

The history of in-service education, especially at the secondary-school level, during the 1955-1965 era is highly related to the history of the National Science Foundation's development of in-service programs. Krieghbaum and Rawson's An Investment in Knowledge (1969) is a history of NSF's development of summer institutes for secondary teachers during the first 12 years of the institute program. In the process of spinning an enthusiastic, entertaining history of the summer programs, considerable background on other forms of NSF in-service activities is described. Thus, their book reports on the establishment of academic-year institutes, in-service institutes for part-time study during the school year, implementation institutes directed toward the major new curricula (UICSM and SMSG), and parallel institutes f· elementary teachers.

The NSF institutes reached an estimated 35 percent of the mathematics and science teachers (Krieghbaum and Ransom, 1969). Mostly disciplinary in orientation (a typical summer institute was about 80 percent mathematics and 20 percent methods), they established a precedent of paying mathematics teachers' university fees, tuition, and/or living expenses. Further, the mathematics and methods were "packaged" for the teacher by the institution. NSF institutes became almost the only in-service activity for secondary teachers of mathematics.

The National Science Foundation became concerned with the question of whether the institutes really were up-grading the competence of all types of teachers. Thus, a study of the 16,000 applicants to the 1957 and 1960 institutes (Blanche et al., 1963) was initiated to examine differences between those accepted and those rejected for the various kinds of in-service activities. Berger (1961) reported differences between the acceptance and rejection groups for each type of institute for secondary teachers. Academic-year institutes and summer institutes accepted individuals with better academic credentials in terms of the number of hours and the grade point average. This apparently contributed to the later establishment of institutes for different levels of student. The institutes could not be successful in upgrading the competence of teachers if only teachers with better backgrounds were included.

The Foundation was also concerned about the types of teachers who were <u>not</u> applying to institutes. The American Institutes for Research (Orr, 1962) conducted a study of the non-applicants, sampling teachers in 491 secondary schools selected on a stratified random basis. Acceptees, non-acceptees, and non-applicants in the schools were compared. Teachers were sent questionnaires and a subsample was interviewed. The acceptees were more likely to have participated because of wanting to know more of the subject matter and teaching methods; rejectees were motivated to apply for reasons of financial gain more often than the acceptees. Rejectees appeared to have as high a "drive" as the acceptees, but a lower ability level. Females were a significantly larger portion of the non-applicant group than of the applicant group and often mentioned family obligations as the interfering factor. However, the primary factor for non-applicants was identified as lack of drive, a characteristic extending to and pervading most aspects of the non-applicants' work in the schools. The non-applicant felt inadequate for teaching in the subject field more frequently than the applicant, but prized a self-perceived ability to get along with students more often than the applicant. The non-applicant was more likely to be a woman teaching in a small school in a rural area or small town that served a low-cost housing area. The non-applicants perceived the environment in which they worked as supportive of neither education nor science.

A conclusion that seems apparent from the non-applicant study is that there was a segment of teachers whom the in-service institute programs could not reach no matter what modifications were made in availability, stipend support, and the like.

Few follow-up studies of institutes independent of the NSF in-house evaluations were conducted prior to 1965. The reports were positive, optimistic, and full of promise (e.g., Krieghbaum and Ranson, 1969). Many teachers were being changed and were excited about their participation. The 24 summer institutes oriented to UICSM and the 40 organized around SMSG curricular materials seemed particularly powerful mechanisms for establishing new curricula in the schools (and received better participant evaluations than the non-curricular-oriented institutes).

Preservice Education, 1955-1965

The content of preservice mathematics education changed significantly, but professional experiences in education generally retained the same structure. The most significant changes for secondary teacher education programs were in terms of shifting the content of the mathematics courses to be more current and to encompass a greater portion of post-calculus mathematics and a lesser amount of pre-calculus mathematics. For elementary teachers, the shift was more dramatic; it was primarily an increase in the number of required hours of mathematics. It seems that mathematics educators' energies were devoted primarily to in-service education, so that preservice programs were adjusted only in terms of content.

Examination of the two leading methods books for secondary education during this era supports this contention. Reeve's <u>Mathematics for the Secondary School</u> (1954) and Butler and Wren's <u>The Teaching of Secondary Mathematics</u> (1960) are both written in terms of the curricula of the 1950s. The elementary-school mathematics methods books also showed little significant change.

If there was a particular methodological emphasis in the early 1960s, it was in terms of the new curricula and the discovery processes implicit in the UICSM materials. However, this was not a major emphasis in available text materials for methods classes. Mathematics educators came to realize there was a problem in preservice teacher education.

The in-service education effort was essentially retraining and updating the teacher's knowledge of mathematics and preparing them for the new curricula. Preservice teacher education needed comparable attention; otherwise the new teachers would require retraining immediately. Several groups formulated guidelines for revision of undergraduate preservice programs; Gibb, Karnes, and Wren (1970) and Dubisch (1970) provided listings of guidelines and content.

The guidelines of the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM, 1961a, b) were the most used. This is probably for two reasons:

(1) CUPM periodically conducted regional conferences for educators concerned with program design, requirements, and certification.

(2) CUPM provided extensive recommended course outlines specifying content and intent. In addition they indicated available published materials fitting the courses they had described.

CUPM recognized that the undergraduate curriculum was at least as out-of-date in many institutions as the school mathematics curriculum. The CUPM recommendations were unique in that they considered three levels of secondary-school teacher preparation. A summary of their 1961 recommendations for school mathematics appears in Table 10. The CUPM course guides and level recommendations provided standards for mathematics educators. Initially CUPM did not consider methodology.

C. Teacher Education, 1965-1975

Mathematics education changed significantly in the 1960s; much of this change profoundly affected teacher education. In other sections of

TABLE 10

1961 CUPM COURSE RECOMMENDATIONS

Level	Description	High School Prequisites	Courses: Numbers	Analysis	Algebra	Geometry	Probability and Statistics	Electives
1	Elementary School	2 years of college-preparatory mathematics	2		1	1		
2	Elements of Algebra and Geometry--Junior High School	Pre-calculus		2	1	2	1	1
3	High School	Pre-calculus		2	2	3	2	2
4	Elements of Calculus Linear Algebra and Probability	Pre-calculus		4	2	3	2	7

of this report, the points are made that:

(1) Curricular changes were accomplished in the secondary school and were initiated in the elementary schools by the mid-1960s.

(2) The aims of mathematical instruction were enlarged in the mid-1960s to fit concerns for the learner who was not college-aspiring or college-talented.

It should also be noted that the number of researchers in mathematics education changed dramatically by the mid-sixties, partly as a result of NSF academic-year institute programs and partly because collegiate-level mathematics education was a growth industry. Many young professionals had new research degrees and positions in higher education. They were doing research concerning teaching and learning mathematics at a never-before-attained rate. Many of the studies related directly to teacher education. These "new" mathematics educators who had grown to professional maturity in the institute programs and in learning about "modern" mathematics became a new generation of teacher educators with a mind-set quite different than that exhibited by their colleagues trained in the pre-1955 era.

Mathematics educators working in teacher education during the 1965-1975 era felt that they could safely extend their programs beyond the paramount, consuming aim of mathematical competence prevailing in the 1955-1965 era. Most elementary and secondary schools had at least one staff member with a contemporary mathematical background and were using curricular materials of a modern character. The undergraduates in preservice programs had more extensive mathematical backgrounds and teacher-training materials reflected the nature of the instructional materials in the schools. By the 1970s, the students in preservice programs had a history of contemporary mathematics in their school experience before entering college. This is not to say that mathematical competence was no longer a concern or issue; rather, teacher educators had evidence that progress on the mathematical competence problem had been made, and there was a conviction that other factors in teacher education were in need of attention.

The societal and political concern and support for science and for building a pool of scientific talent eroded, to be replaced with a concern for the socially disenfranchised and a perception of the schools as a constructive mechanism for social change reaching all levels of society. In particular, the schools were perceived as a means of breaking the poverty cycle. Thus, the efforts of teacher educators came to encompass more than simply mathematical competence.

The economics of in-service education changed dramatically. After a twenty-year period of massive federal support for in-service education, primarily through NSF institutes, federal support for in-service education was, to all intents and purposes, terminated for mathematics and science teachers. During the peak three years of support, 1962-1965, the level of federal investment was approximately $37,000,000 per year (equivalent to approximately 70 million dollars in 1975 dollars). Ten short years later, on 28 November 1975, Walter Gillespie of the National Science Foundation wrote an open letter to the mathematics and science education community declaring that no funds were available for institutes during the coming fiscal year. Teachers' expectations and attitudes about in-service education built over the twenty-year period were upset, as well as the roles and functions that school systems and institutions of higher education had established. For a period of time, this traumatized the mathematics education in-service effort.

In the following sections, the effects of these general trends and how they came about will be examined. Shifts in teacher competence will be considered, followed by an examination of in-service programs and trends in preservice education.

Teacher Competence and Characteristics, 1965-1975

Much more information concerning teacher competence and characteristics is available for 1965-1975 than for the preceding ten-year period. Rather than having to operate from a basis of judgment about teachers inferred from limited information on their course background, a considerable store of research evidence has been amassed. One of the effects of the societal emphasis on science and education was the development of many doctoral programs on mathematics education. The production of research studies concerning teacher education during the entire ten years from 1955 to 1965 is roughly equivalent to the research production per year in the 1965 to 1976 period. Many of these studies described teacher characteristics; few described teacher competence.

One of the major questions raised by the massive federal intervention into science education was whether the investment was worth it when the major goal of upgrading teachers' understanding was considered. A large number of studies have examined whether an increased number of courses and/or grade point in collegiate mathematics contributed to improved performance of students in mathematics. One of the larger studies of this type was reported by Begle and Geeslin (1972) as part of the NLSMA research effort. For the first year of the NLSMA studies, 1405 teachers participated, with 1478 in the second year. The students of these teachers were given pretests and, then, at the end of each year, tests on computation and comprehension. Eleven different measures of teacher characteristics were used in stepwise regression analyses to discover relationships between the teacher characteristics and the performance of their classes. Although substantial variance was found in the performance of their classes, the teachers' characteristics did not account for a significant portion of the variance. The percentage of the variance accounted for was too low to be useful for school decision-making. Further, the measures of teacher effectiveness were not stable across the two different years of data collection.

What accounts for the lack of relationship between teachers' background and students' performance? One attractive interpretation of the NLSMA study described above is that the information gathered from transcripts may be ambiguous; professors grade in markedly different ways, standards vary from institution to institution, a B grade earned in 1955 may not mean at all the same thing as a B grade in the same course in 1972. Begle (1972) investigated the performance of the students of 308 teachers who were participants in NSF institutes. Measures of the teachers' understandings of algebra were taken from their performance on two algebra tests. Their students were given pre- and post-tests of knowledge of algebra. He found no significant correlation between teachers' knowledge and the performance of their students. Eisenberg (1977) replicated the study with a smaller but more typical set of algebra teachers who were not participants in NSF institutes, and therefore had not been selected on some criteria which might produce "ceiling effects." The results of the Eisenberg study are consistent with those of the Begle study. Moreover, these results are consistent with the findings of other studies concerned with the performance of students at different levels in the school curriculum; see Eisenberg (1977) for a listing of eight other studies of this nature.

Willson and Garibaldi (1976) reported a study of 112 senior high and 99 junior high school teachers in school districts in Mississippi, South Dakota, and Wyoming. Teachers' backgrounds, institute participation, and scores on the National Teachers Examination in Mathematics were related to their students' achievement on a mathematics achievement test (40 items selected from the NLSMA item pool). The teachers' abilities in mathematics were not related to their students' achievement, but their participation in in-service institutes was related. According to the authors, the results were strong enough to warrant prescriptive remarks recommending continued participation in in-service activities throughout the professional lives of teachers.

The intuitions of most mathematics teachers, mathematics educators, and mathematicians are not in accord with the findings reported in these studies. Most want to claim that the more a teacher knows about the subject being taught, the better the teaching that can be done. Clearly a minimal level of understanding of the subject matter is necessary. The explanations of the lack of significance for mathematical background typically hinge on the identification of potential interactive effects with other characteristics of the teacher. Several characteristics of teachers have been identified that do affect learning of mathematical topics. These are candidates for having potentially significant interactions with the teacher's knowledge of mathematics in affecting learning. Among the more significant of these are:

(1) The teacher's verbal facility and behavior: Studies by Fey (1969, 1970), Gregory (1972), Hernandez (1973), and those reported in <u>Teaching Strategies: Papers from a Research Workshop</u> (Cooney, 1976) all noted verbal

factors in teachers' performance in the mathematics classroom that contribute to learning of mathematics. None of these studies, however, considered interactive effects with the teachers' knowledge of mathematics.

(2) The teacher's expectations of student performance: Heller (1974) and Lockheed (1976) identified the characteristic of expectation of the teacher for student performance as being a critical factor in the classroom. Other replications of the Rosenthal and Jacobson study reported in Pygmalion in the Classroom (1968) did not produce significant results. No studies have considered the expectation characteristics in conjunction with the teacher's mathematical competence.

(3) The cognitive style of the teacher: Engelhart (1973), Stone (1976), and Story (1973) reported that matching the cognitive style of the teacher and the cognitive style of the student can affect learning in mathematics. Since cognitive style of the teacher is a factor in the teacher's learning and doing of mathematics, this may be a potentially useful characteristic to explore in examining the role of knowledge of mathematics in the performance of children.

None of the studies cited above defined the competent teacher. Rather they indicated some characteristics of teachers that appear to affect learning of mathematics, and thus might have significant interactive effects with the knowledge of mathematics possessed by the teacher. They presented evidence that teachers vary significantly in a variety of characteristics that affect learning.

The attitude of the teacher about mathematics is another characteristic that might be expected to affect the learning of students in mathematics. Suydam's A Categorized Listing of Research on Mathematics Education (K-12): 1964-1973 (1974) listed 39 studies concerning preservice teachers' attitudes and 34 studies concerning preservice teachers' attitudes about mathematics. Unfortunately the number of these studies that examine the relationships between teachers' attitudes and the performance in mathematics of the teachers' students is relatively few. Van de Walle (1973) found at the third-grade level that comprehension of mathematics was related to the positive attitudes of teachers and that teachers' negative attitudes were associated with computational ability. At the sixth-grade level, no significant relationships were found. Two of the NLSMA Reports {Begle and Geeslin (1972) and Travers (1971)} examined the relationship between teachers' attitudes about mathematics and mathematics teaching and student achievement. No significant relationships were reported.

The design of teacher education programs is predicated upon some strong assumptions concerning teachers' attitudes about mathematics and their knowledge of mathematics. Intuitively it seems apparent that these are critical factors in competence. The research evidence does not support these assumptions. We note that neither of these assumptions has been researched carefully in a manner that accounts for possibly significant interactions with other variables. Most of the studies of attitude have had other purposes that have determined the design.

Many of the studies of teacher attitudes reported in Suydam (1974) indicated a relationship between the achievement of teachers in specific in-service or preservice mathematical experiences and students' attitudes about mathematics. Most such studies indicated a weak association between success in mathematics and a positive attitude about mathematics for elementary-school teachers. Elementary teachers who prefer teaching at the upper grade levels appear to enjoy greater success in mathematics and more positive attitudes about mathematics. Although in-service experiences and institutes for elementary teachers attract the teachers who feel more positive about mathematics, their attitudes are enhanced.

Attitudes and mathematical background or understanding are not characteristics that yield simple measures of a teacher's competence or effectiveness in promoting student learning. Some studies of effectiveness that appear to have promise are those that incorporate many variables into the description of teacher behaviors and that account for classroom environmental factors. Some variables that appear to be significant have been identified but have not been studied in conjunction with baseline characteristics of teacher attitude toward mathematics or understanding of the mathematics being taught. Some of the variables appear to be dependent on an understanding of mathematics. Rosenbloom et al. (1966) identified the most effective teachers in a group of 127 who were field-testing SMSG curricular materials. The most effective teachers produced a greater variety of ideas about success and failure in their teaching and offered a greater variety of alternative ways of teaching mathematical concepts. These observations of the teachers were based upon the logs which the teachers kept concerning their teaching. Good and Grouws (1975) examined achievement in fourth-grade mathematics in terms of the teachers' use of various teaching strategies and classroom environmental factors. Clusters of variables that were associated with effectiveness were (1) general clarity of instruction, (2) a non-evaluative and generally relaxed environment, (3) higher achievement expectations, (4) classrooms that were relatively free of major behavior disorders, (5) characteristics of whole-class instruction, and (6) student initiated behavior.

These two studies offer examples of the variables that appear to affect learning to a significant degree. The variables are characteristics of the teacher in that they indicate behaviors of the teacher, some of which are learned. The problem with most research that examines teacher behaviors in the classroom is that the behaviors are seldom examined in terms of both the performance of learners and the background characteristics of the teachers. As Rosenshine and Furst (1973) point out in reviewing more than 120 instruments or systems for classroom observation, only about one in ten is related to student achievement in any way. Although they were examining observational systems across all fields of teaching, the same conclusions obtain for the teaching of mathematics.

The question of teacher competence or effectiveness is more complex than the accomplished research would lead one to believe. Few studies have accounted for the many factors that have been identified as potentially significant. Clearly it will take an investment in research of at least an order of magnitude greater than has been invested in the problem heretofore. Turner (1976) described the many different factors that should be taken into account in order to extend the research domain for teacher-effectiveness studies. Controlling the many variables he identified is necessary if the knowledge of teacher characteristics that yield effectiveness is to be other than the observation of symptoms of effective behavior.

The discussion of teacher characteristics to this point has focused on effectiveness or competency in promoting growth of students in mathematics. Other characteristics of teachers are significant in that they indicate factors in the professional attitudes and makeup of teachers that should be taken into account in planning teacher education programs and/or in acquiring a sense of the progress that has been made in teacher education.

The mathematics teacher at the elementary and secondary levels is more of a professional in 1975 than in 1965. This can be deduced from evidence of the change in the backgrounds of teachers. Osborne and Bowling (1977a) surveyed a national sample of secondary and elementary teachers in 1975 for the NCTM In-Service Project. The teachers were selected on a stratified random basis to reflect all areas of the country and the various types of public schools. Fifty-six percent of the secondary teachers and 35 percent of the elementary teachers reported that their highest degree was a master's. Only 11 percent of the elementary teachers and 12.4 percent of the secondary teachers were teaching with no methods course in mathematics; indeed, 52 percent of the elementary and 58 percent of the secondary teachers reported more than one methods course for mathematics in their background. The secondary teachers reported the following when queried about the number of post-calculus mathematics courses:

Number of courses	0 to 3	4 to 7	8 to 11	12 to 15	more than 15
Percent of teachers	15.7%	24.3%	21.3%	14.1%	24.9%

Only 10.5 percent of the elementary teachers indicated they had only one or no mathematics courses at the college level. This background is consistent with that reported in other studies, although there appears to be regional variation, with urban areas having a higher concentration of teachers with extensive backgrounds (Bertram, 1971; Biggs, 1969; Bradshaw, 1968; Haigh, 1970; Schubert, 1975; Woods, 1973). Thus, there are many more teachers in the schools who have a background approaching that recommended in the CUPM guidelines for mathematics teacher education than ever before. However, a significant subset of the teachers do not possess the recommended levels of training--estimates of those not having CUPM-recommended backgrounds range from 10 to 37 percent, depending upon the region of the country and the type of community served.

Teachers in the mid-1970s not only have a better background in mathematics and methods; they are earning their second professional degree at a younger age, are less likely to interrupt their professional service, and will stay in the teaching profession longer than teachers at any point earlier in the 20-year period which is the concern of this report. The NCTM in-service surveys indicate that teachers are satisfied with their choice of profession, with approximately 80% stating they would elect teaching as a profession if they had an opportunity to start over again. For elementary teachers, 83 percent indicated experiences in in-service programs during the two years prior to the survey; for secondary teachers, 71 percent had participated in in-service during this period. The following indicates the sources of in-service education for these teachers:

	Elementary	Secondary
My school system	88.3%	82.0%
A state educational agency	24.5%	23.7%
A district or regional educational agency	32.7%	25.7%
A college or university	37.9%	35.9%
A private concern such as a publisher	35.6%	13.0%
A professional group such as the NEA or NCTM	20.6%	29.4%

Although approximately 36 percent of the elementary teachers and 46 percent of the secondary teachers reported their prior experience with in-service had not been positive, approximately 80 percent reported a need for in-service and approximately 60 percent felt it should be required of all mathematics teachers. Close to 50 percent of all respondents felt the requirement should be for maintenance of certification.

The NCTM survey results provide strong evidence that the majority of teachers are professionals desirous of continuing education, concerned with currency of their knowledge, and more desirous of in-service for methodology then for mathematics content. A strong concern for having in-service that related specifically to their curricular and instructional programs was evident in the responses. If teachers at either level participated in identification of topics and in planning the in-service program, then they were much more likely to feel that the in-service experience was satisfying.

The survey data indicated that most teachers were positive and optimistic about in-service education and simply wanted to be treated as professionals. Some of the respondents were not so positive about in-service education--a negativism pervaded their responses to a large number of the items concerned with in-service on the 147-item survey form. One question that tended to show a relationship with a positive view of all aspects of the respondents' professional perceptions was the following: "Are students as excited about learning mathematics as they ever were?" Sixty-eight percent of the elementary teachers responded "yes", but only 44 percent of the secondary teachers responded positively.

To summarize, the characteristics of teachers that stand out most dramatically in the mid-1970s are a reasonably extensive background in mathematics and professional courses. Most teachers are participating in some form of in-service education, prefer more in-service education, and have relatively high hopes for in-service education. A significant factor accounting for teachers holding a positive view of past in-service is whether they have particpated in decisions about the in-service program and whether it fits the school's mathematics program. Relatively little evidence relating teachers' background with their students' performance in mathematics is to be found, although there is some promise in looking for interactive effects of background with variables of teachers' verbal behavior, expectations, and attitude, particularly if school environmental factors are controlled.

Preservice Program Design, 1965-1975

The five major developments in this period for preservice program design are:

(1) Increasing the mathematics requirements for secondary and elementary programs

(2) Competency-based teacher education (CBTE)

(3) Increasing the amount of pre-student-teaching field experience

(4) Incorporating an emphasis on laboratory and/or activity learning into the teacher education program

(5) The supply and demand factors in the prospective teacher population

Other developments in the design and implementation of teacher education programs are so limited as to have little effect nationally.

Requirements: The increase in mathematics course requirements is evident in the content of the preceding teacher characteristics section. Perhaps the most significant comment is to point out that the recommendations of the various professional groups -- CUPM (1961, 1968, 1971), NCTM (1973), and AAAS (1961, 1971) -- have had some effect. They are used by the national and regional accrediting agencies (NCATE and state departments of education). CUPM guidelines have had the most effect primarily because of the regional conferences directed toward their implementation and because they were developed with detailed course outlines. The NCTM Guidelines focus on the professional training in addition to the mathematics background and specify some institutional responsibilities, but are so recent that they are only beginning to have an effect. They are constructed to accommodate to the teacher-training institution that operates with a CBTE program design as well as the more usual program design. The AAAS Guidelines have had little effect on mathematics teacher education program designs.

The evidence from a series of dissertation studies indicated a progression of incremental steps for both secondary and elementary teachers toward the implementation of the CUPM guidelines throughout the 1965-1975 period (Bompart, 1968; Brown, 1972; Cook, 1970; Copley, 1971; Dayoub, 1974; Fisher, 1967, 1968; Foster, 1971; Garnett, 1969; Hunkler, 1971; Johnson, 1976; Lightner, 1968; McCowan, 1976; Ray, 1967; Smith, 1971: Thompson and Poe, 1968; Tilton, 1967; Vinskey, 1971; Withnell, 1968). The studies suggest that this progress toward implementation of the guidelines holds across all types of institutions that train teachers and is being realized in certification laws as well as degree requirements. The evidence also hints that the more recent NCTM Guidelines are beginning to be used also. Both NCTM and CUPM Guidlines are recommended by NCATE for institutional evaluations. There is little evidence of differences in program design observable between NCATE and non-NCATE institutions.

The implementation of recommendation by CUPM for three mathematics courses for elementary teachers led to a discovery of the prospective elementary teacher as an object of research. We have learned that (a) the more mathematics courses taken in high school and/or the better the grade point average in high school mathematics, the better the prospective teacher does in CUPM-style courses; (b) the more CUPM courses the prospective teacher takes in college, the more mathematics the teacher is likely to know; (c) gains in mathematics achievement are a result of taking CUPM-style courses; and (d) attitudes about mathematics are slightly higher after successful experience in a mathematics course. These results are not unexpected!

A number of the studies cited immediately above and some others (Collier, 1972; Gibney et al., 1970 a, b; Reys, 1968a, b; Reys and Delong, 1968) have tested prospective elementary teachers' mathematical understanding and/or other factors relating to their attitudes about mathematics and what led them to teaching. Generally, prospective elementary teachers do not perform significantly different than junior high school students on standardized tests and make relatively the same kinds of errors. They do not find mathematics their favorite activity. Their performance on geometric and algebraic topics needs to be improved. Students completing their collegiate mathematics teacher education courses are more likely to perceive mathematics as informal and non-rigid than before entering the courses.

For secondary preservice programs, the effect reported in the studies cited above has been for an increase in post-calculus mathematics and a decrease in pre-calculus university-level courses. Theory of equations is no longer a required course; in some institutions, it is not a topic given ch treatment. Modern algebra, group theory, and linear algebra are the favored algebraic experiences for prospective junior and senior high school teachers.

Johnson (1976), whose analysis of course and topic requirements for prospective secondary teachers is based on a thorough survey of 60 percent of the AACTE institutions, indicated a trend toward acceptance of the Level II (junior high) and Level III (senior high) recommendations in that a difference in mathematics requirements is occurring. The typical junior high course requirement is for 31.42 semester hours on the average, with 33.28 hours for senior high. Nine percent of the institutions have a special course for prospective junior high school teachers.

The role of the computer in mathematics teaching is a matter of concern for all of the groups making recommendations for teacher education. We have little evidence that requirements in teacher education institutions or that state certification laws honor the recommendations of professional groups for prospective mathematics teachers to have computer literacy, let alone having it as a specific aspect of their teacher education experience. This is perhaps the most significant failure relative to implementation of teacher education guidelines that we have found.

Probability and statistics courses are seldom required, although most institutions think they should be, according to several surveys. Wong (1970) reported that transformational geometry is an intergral part of most preservice secondary teachers' course work and that the majority of institutions require at least one geometry course.

The CUPM recommendations for secondary preservice teacher education have not led to the same sort of testing of preservice teachers as they did at the elementary level. Thus, no characteristics of preservice secondary teachers are reported other than those to be inferred from the course requirements of institutions of higher education.

Competency Based Teacher Education: CBTE as a feature in preservice teacher education programs came on the scene in the early 1970s with the thrust toward accountability and performance contracting. It was an extension of the behavioral objectives philosophy of many generalists in state departments and institutions of higher education. Maurer (1973) reported that 19 states could award teaching certificates through competency assessment; that is, competency assessment teaching certificates were possible but not required. His survey data, representing 49 states, indicated that nine states decided not to use CBTE, five were undecided, and the remaining 35 intended to implement CBTE certification programs. Thirty-seven states indicated that the responsibility for implementing CBTE would be thrust by the state agencies onto institutions of higher education.

Evidence for the present status of the trend to CBTE is limited and somewhat "soft," but our opinion is that the orientation to and interest in CBTE peaked about 1973. We find less evidence of interest in the literature; indeed, there is practically a cessation of CBTE articles. Only one of the 448 institutions responding to Johnson's (1976) survey noted a CBTE program in 1974. Although certification laws based on CBTE are on the books in some states, our perception is that they are being ignored or not being implemented. CBTE is expensive and a significant number of scholars in mathematics education and mathematics are philosophically opposed to CBTE. Given the present state of finances in higher education, it appears that in the immediate future CBTE will not be widely applied.

Field Experiences: The third major trend in preservice education in mathematics is of a more significant character than CBTE, in our opinion. This is the trend toward increasing significantly the amount of field experience prior to student teaching. Promising Practices in Mathematics Teacher Education (Higgins, 1972) reports 64 innovative preservice teacher education programs. Thirteen of the 21 secondary programs have required pre-student-teaching field experience; the experiences extend beyond passive observation to working with learners toward specific objectives of the teacher education programs. Of the 43 elementary programs, 16 incorporate significant amounts of closely supervised field experiences with children prior to student teaching. The popularity of field experiences prior to student teaching during the early seventies is indicated by the proportion of claims of innovativeness based upon field experience.

Pre-student teaching field experience in teacher education programs is used for a variety of purposes. Early experience with children and within schools provides the undergraduates with a realistic base to decide if teaching is to be their life's work before committing a large portion of their undergraduate registration to education courses. The early experience establishes a touchstone of reality for professional course work and establishes a relevance for the mathematical topics being learned. Many of the component skills of teaching, such as tutoring or diagnosis, can be established and practiced under supervision and hence are learned more efficiently. Finally, it provides a clinical and/or laboratory setting for learning about learning and teaching.

There is considerable amount of mysticism and folklore about early field experience. Prospective teachers and teacher educators generally say they feel the early experience has a positive effect and is good for teacher education. For mathematics teacher education there is a paucity of evidence that provides evaluative information or that identifies specific effects of the early experience. Graening (1972) described effects of early field experiences for a secondary mathematics preservice program, noting that there were appreciable gains and changes for the preservice teachers prior to student teaching on a number of measures of effectiveness and attitude. His measures incorporated evidence of both the preservice teachers and the students with whom they worked. The student teaching experience tended to decrease these gains and to dampen the enthusiasm acquired by the prospective teacher in the prior experience. Erb (1972) analyzed the effect on junior-level prospective teachers tutoring at the junior high school level. Significant changes were noted in the behavior of the preservice teachers and in the improved attitudes of those being tutored. Although field experience has been a component of some experiments in teacher education (e.g., Thornton, 1977), almost no direct evidence of the effects of early experience and how or what it contributes to a total program in teacher education is available beyond these two studies. It is not known what constitutes sound pre-student teaching field experience or what does not. This is a major arena for needed research in mathematics teacher education, since cost figures for such programs are appreciably higher than for traditional teacher education programs.

Laboratory Learning: The fourth major trend for teacher education during 1965-1975 is the incorporation of laboratory or activity learning into the preservice experience, either in a mathematics setting or in the methods setting. This move in the design of teacher education programs is interpretable as an attempt to adjust teacher education programs to the orientation toward activity learning in many school mathematics curricula. Fuson (1975) pointed out that few instructional materials of this type were available for teacher education until the 1970s; she also remarked on the extremely limited amount of research. Her exploratory evaluation indicated that prospective teachers (1) used manipulative materials to a considerable extent in student teaching after the course experience, (2) increased the extent to which they behaved in learner-focused ways, and (3) thought they had gained appreciabl" in their understanding of mathematics and enjoyment of mathematics.

The identification of the increased use of manipulative materials for laboratory or activity learning as a component of teacher education during 1965-1975 is based upon limited, "soft" evidence. No survey data exist supporting this contention. This perception of increased use is based upon the significant increase in available books and other teacher education materials incorporating this approach and the evidence of the increasing popularity of this topic for teacher education sessions at professional meetings.

The activity or laboratory emphasis in teacher education is related to another development, the integration of the mathematics and methods course content. Stemming in part from a belief that teachers teach as they were taught, several institutions have implemented such combination courses, often with joint staffing by content and methods personnel. A noticable developmental program of this sort has been the Mathematics Methods Project at Indiana University (Thornton, 1977). The MMP design has a significant field-experience component, with a significant emphasis on activity learning. It has been adopted for implementation at many institutions. Such programs will not be widely adopted, in our opinion, until better relationships are commonplace between content teachers in mathematics departments and methods teachers in education departments. This factor keeps such integration from being labelled as a trend.

Teacher Supply and Demand: The fifth major factor affecting preservice teacher education during the 1965-1975 era is that of supply and demand. During the 1955-1965 era, shortage prevailed as the orienting factor for school people and for teacher educators. During the 1965-1975 era, the supply factor reversed dramatically. A state of oversupply of teachers existed in the early 1970s according to all analyses {see Carroll and Ryder (1974) for a listing and comparative analysis of several supply and demand studies}. The most interesting factor identified in the supply and demand studies, apart from the oversupply factor in the 1970s, is the development of some new trends in the occupational choices of undergraduates. Carroll and Ryder (1974) reported a significant decrease in the number of freshmen (both men and women) indicating teaching as a career choice, based on surveys from 1967 through 1974 conducted by the American Council on Education (ACE). There was a decline by 1972 to between on third and one half that observed in 1968. In 1972, for example, only 12.1 percent of the entering freshmen were considering teaching as a career choice. This trend holds for both elementary and secondary levels, but for the latter does not reflect data specific to mathematics teaching. Carroll and Ryder do indicate some problems with the ACE survey techniques but p -ject that this factor may contribute significantly to the supply of teachers in the 1980s. Most projections of teacher supply and demand figures, however, have assumed that the pattern of approximately 35 percent of the bachelor's degree holders being trained teachers would continue.

Another assumption implicit in projections of supply and demand made in the early 1970s for the 1980s is that unemployed, trained teachers in the "reserve pool" would be willing to enter the teaching profession; this assumption may be specious. Little evidence exists indicating the portion of people in the reserve pool who are willing to enter the teaching profession five or more years after their training was completed. Even if they are willing to enter teaching at this point in their lives, the effect on the schools of their out-of-date training is not projected.

There are no projections of teacher supply and demand factors that are specific to secondary-school mathematics. One can infer from the present characteristics of secondary teachers that 14 percent of the teachers in the secondary schools will be mathematics teachers (Magers, 1977). However, this does not provide much evidence concerning supply and demand. We do know that the cohort of undergraduate majors graduating in the mathematical sciences--the source of beginning secondary teachers--peaked in 1969-70 at 27,400, decreased until 1973, but apparently has maintained a constant level of approximately 25,500 through 1975 (NSF, 1976; Simon and Frankel, 1975; Simon and Fullam, 1970). The increased percentage taking training in computer science fields in the early 1970s suggests an increasing number of mathematically trained personnel (a) are entering industry and (b) are not to be counted as potential secondary-school teachers of mathematics. We suggest that the evidence weakly indicates that the oversupply of mathematics teachers is not nearly so dramatic for secondary mathematics teachers as for other secondary teaching fields or for elementary teachers. Carroll and Ryder (1974) warned that "if and when the surplus ends, the inertia in the system of supply and demand will lead to the almost immediate onset of a teacher shortage." They projected a continuation of the trend of decreasing production of new teachers that is observable from 1967 to 1974. The limited evidence of declining undergraduate enrollments in mathematical sciences and of an apparently growing portion of those majors entering industry suggests that if a teacher shortage develops, then secondary mathematics teaching will be among the earliest fields specifically affected. Unfortunately, the supply and demand data for secondary teachers provide no information specific to the various disciplinary fields; projections are based upon "guesstimates" at best.

The writers find the lacunae in the teacher supply and demand figures upsetting and startling. We have found no firm data concerning the number of mathematics teachers serving in the schools at the secondary level and have no idea of how many of the undergraduate majors in mathematical sciences are certified for teaching. Supply and demand data for secondary mathematics teachers are non-existent. Projections must be inferred from non-field-specific secondary teacher preparation data and from manpower supply data that generally treats the mathematical sciences. These two sources of data seem of doubtful validity when one realizes the variation from year to year concerning the same facts reported in annual reports by the same agencies. We conjecture that no trustworthy set of data exists, even reflecting the historical facts that could be verified, that is within a ten-percent level of accuracy. During the early 1960s there was a U.S. Registry of Junior and Senior High School Science and Mathematics Teaching Personnel that provided a glimmer of what was happening in the schools. No comparable data pool presently exists.

By way of summary of the trends in the development of preservice education programs, five areas of import are apparent in the literature reflecting the 1965 through 1975 era. The following conclusions appear warranted:

(1) There has been a significant increase in the mathematical requirements for both prospective elementary and secondary school preservice teachers matched to a limited extent by increases in the professional components required for graduation and certification. Little evidence of the new

secondary-school mathematics teacher acquiring computer literacy as a requirement for certification can be found.

(2) Competency-based teacher education (CBTE) enjoyed a brief but significant moment of influence in the design of teacher education programs. Present, limited evidence indicates that interest in and commitment to CBTE and its implementation is on the wane.

(3) The trend toward requiring more field experience prior to student teaching that began in the late sixties is becoming a norm in the design and redesign of teacher education programs for both prospective elementary and secondary school teachers. This is the case even though no significant research base supporting an increase in the field experience or information concerning its effect on prospective teachers exists.

(4) Incorporating an emphasis on laboratory and/or activity learning in both the mathematics and the professional education portions of teacher education programs at the elementary school levels has increasingly become a feature of teacher education programs.

(5) The trends in supply and demand indicate that during this period we have moved from a state of undersupply to a state of oversupply of elementary-school teachers and that the supply of secondary-school mathematics teachers is about five years out of phase. The trend of fewer freshman-level students in higher education indicating a desire to enter teaching as a career, coupled with fewer students majoring in the mathematical sciences, suggests that the state of oversupply of secondary-school mathematics teachers may change rapidly to a state of undersupply.

In-Service Education, 1965-1975

In 1965, the National Science Foundation invested $37,000,000 in the in-service education of science and mathematics teachers; in 1975, funding of in-service education efforts through the Foundation was terminated. This dramatic turnabout in the ten-year period is the single most significant factor in setting the trends and patterns in the in-service education of mathematics teachers during this period.

The publication of the Foundation entitled Science Education--

The Task Ahead for the National Science Foundation (NSF, 1970) delineated the points at issue. Evidence was presented (p.14) that the new curricula were being implemented massively across the nation; that is to say, no more effort need be expended for curricula implementation since it was happening. The task of teacher education is specified as primarily a matter of subject-matter "upgrading", and the Advisory committee recommended continued institute work as long as new participants can be found and the subject matter was "genuinely upgrading." Otherwise, it was recommended that teacher education effort of the Foundation be limited to the innovative (p.13). The Advisory Committee further recommended that the important place to modify teacher education was at the preservice level, since without attention to this factor the nation must automatically be locked into a "retread job" of teacher education at the in-service level (p.28).

Interestingly, the Advisory Committee failed to recognize the effect of the academic-year institute programs while condemning professional educators and schools of education for "enormous resistance" (p.28) to dramatically improving preservice science education. An NSF staff paper (NSF, 1972) showed that 58.4 percent (approximately 9,300) of the academic-year institute graduates for the period 1956-69 were significantly involved in teacher education, with only 20 percent being limited to in-service work within their own school system. That is to say, the academic-year institutes had dramatically changed the staffing patterns (and the values) of teacher educators in the institutes of higher education. Thus, we concluded that the judgment of the Advisory Committee for Science Education in 1970 was specious to say the least.

Retrospect provides additional insight, however, into the effects of the cessation of federally funded activity in in-service education after almost twenty years of heavy involvement. The NCTM publication An In-Service Handbook for Mathematics Education (Osborne, 1977) identified several factors stemming from the federal involvement that were of import in the mid-1970s. Primary among these is that mathematics teachers came to expect an institution of higher education to prepackage in-service work and thereby lost the skills of identifying needs and planning in-service to fit those instructional and curricular needs. Second, teachers came to expect that not only would in-service work be designed for them, it would be provided and paid for by someone other than themselves or their school system. Third, the national surveys reported in the Handbook indicated that teachers expect in-service education and want it. Thus, by 1975, the twenty years of summer, academic-year, and in-service institutes had established several precedents and firmed teachers' (and school systems) expectations such that in-service education became an issue.

If the precedents and expectations are coupled with the fact (according to the Handbook) that the learnings required for effective teaching -- in terms of the knowledge of mathematics, research-based theories of learning and teaching mathematics, and the skills of teaching -- are far in excess of what is possible in a short four- or five-year preservice program, then in-service education becomes significantly important. The compelling evidence of the NCTM In-Service Project surveys is that many teachers attain their second professional degree before ten years of their professional life have passed and that they have a strong perception of need for further in-service experience throughout their remaining 25 to 35 years in the profession.

As noted previously, the NCTM In-Service Project surveys indicated that the critical factor in determining teachers' perceptions of the effectiveness of in-service education is the extent to which planning is participatory. If teachers' judgments of need are incorporated into planning a program fitting their curriculum and their instruction, then they are significantly more likely (the chi-square statistical tests were at the .00005 level of significance) to feel their in-service experiences were satisfying and to feel positive about them. The respondents were highly critical of in-service programs that were so general that little help in teaching mathematics was provided. There was a pronounced discontent with programs that were either too mathematical or too methodological.

The evidence of this survey, and a prior pilot survey, indicated that teachers are interested in in-service education that helps them deal with motivation and helps students with attitudinal problems. For the majority of elementary- and secondary-school mathematics teachers, topics of a purely mathematical bent were not as popular as those incorporating aspects of the teaching and learning of mathematics.

The survey evidence indicated that if teachers were employed in a school system having an individual responsibility for in-service education in mathematics and/or a developed in-service program, then they were more likely to have participated in in-service, to have found it useful, and to have fewer gripes. They were also more likely to recommend that in-service be required of all teachers of mathematics at either the elementary or secondary levels.

The evidence is that teachers who work in a school system that encourages in-service education by one means tend to be in schools that encourage it by several means. The major factor that teachers would like to see encouraged in in-service education is released time, but only 44 percent of the elementary respondents and 39 percent of the secondary respondents reported that their schools can or do provide this. Follow-up activities in their school and in their classrooms for in-service activities was a key factor in assuring the teachers' perceptions of success of in-service programs.

We can conclude that the surveys present a picture of a typical mathematics teacher, at both the elementary- and secondary-school levels, as one who wants to behave as a professional sharing in professional decisions. This attitudinal factor of in-service education is important and one that should be capitalized upon according to the evidence of the surveys. One senses a positive expectation for in-service that must not be compromised and that helping teachers realize their professional expectations through in-service has an attitudinal impact extending beyond the specifics of what is learned in in-service education.

In fact, little evidence exists that in-service education makes a difference in children's learning. The studies addressing this problem are few and far between. We do know that the NSF institute effort changed teachers' mathematical competencies and was a significant factor leading to the rapid implementation of the new curricula such as SMSG and UICSM. This does not say that the teachers became more skillful and/or more effective in teaching mathematics. A large number of studies have evaluated the institute programs of the various institutions of higher education (for example, see Bradberry, 1967; Connellan, 1962; Corbet, 1976, Davis, 1973; Fields, 1970; Gray, 1971; Hand, 1967; Heideman, 1962; Irby, 1967; Jolley, 1972, Martinen, 1968; Moore, 1972; Roye, 1968; Schlessinger, 1958; Schlessinger and Helgeson, 1969; Schuler, 1963; Stokes, 1971; Swadener, 1970; Whitaker, 1962; Wiersma, 1962; Wilson, 1967; Yon, 1960). The typical study either (a) was a follow-up of institute participants asking them to evaluate their experience in the institute, or (b) inquired about their professional life following the institute. Overwhelmingly, the evidence is that participants were positive about the institute experience. There is considerable evidence of significant professional life in mathematics education following institutes and some evidence that participation led to curricular changes in participants' schools. The professional stature of participants was improved in their schools. However, the majority of studies offer little generalizability; they are simply one-shot case studies of little import. Generally, the writers concluded that the overall evaluation of the institutes is a positive evaluation of in-service; it should be noted, however, that the design of the studies seldom allowed for other than this outcome.

The two institute follow-up studies (Zeddies, 1972; Joyner, 1974) that examined the attitudes and achievement of students of teachers who participated in institutes provided weak supportive evidence that in-service participation helps students' achievement. Neither indicated related changes in student attitude. The Willson and Garibaldi (1976) study described earlier also provided evidence that participation in in-service promoted student achievement in mathematics.

The effect of in-service on student growth in mathematics is shown most convincingly by an evaluation of an in-service program conducted for elementary teachers by the State Board of Education in California (California, 1972b). This large study matched elementary teachers participating in an in-service education program for one, two, and three years with peers who did not. The results show/improvement in the performance of the institute teachers' students in mathematics. In a study of in-service programs in Maine, Greene et al. (1976) report that summer in-service had an effect if there was a carefully designed follow-up in the schools during the academic year, but not if that feature was missing.

Thus, the evidence is supportive of in-service education making a difference to teachers and their backgrounds. However, it is only weakly supportive of changing the performance of the teachers' students and does not reflect change in teachers' instructional practice. In fact, the research to collect evidence of the effect of in-service on student performance or change in classroom practice of teachers simply has not been done.

The pattern for in-service program design in 1965 was that established by the National Science Foundation. During the late sixties, the Foundation and the USOE, on a limited basis, both experimented with involvement of local school people in the planning of in-service programs. For the Foundation, this was the Cooperative College-School Science Program, requiring cooperation and support of local schools with the institutions of higher education. These programs were basically oriented toward the academic advancement of teachers coupled with salary incentives derived from improved degree status. By 1975, with the cessation of federal funding for in-service in mathematics, schools were becoming involved in designing and conducting in-service on their own. Some interesting and potentially significant precedents and trends are being established. Some of these are:

(1) Minnesota and Pennsylvania have both passed laws formalizing a mechanism for locally designed and implemented in-service programs providing master's level equivalency credit for teachers toward enhanced salary status without participation in an advanced-degree program at an institution of higher education.

(2) With the commitment to metrication, some states participated in a consortium-designed effort for in-service education that utilized a multiplier effect. That is, at the state level a cadre of professionals was trained to train other professionals to conduct in-service on teaching the metric system in the classroom. The design ultimately trickled down to representatives of each school building in the state.

(3) Several states began experimenting with systematic efforts to provide in-service education. The National Science Foundation established comprehensive systems utilizing and encouraging cooperation between institutions of higher education, the schools, and the state department in Oregon and Delaware (see Stufflebeam (1974) for a relatively complete description and evaluation of the systems approach). A comparable system design with variations is in evidence in Arkansas and West Virginia. Utilizing the services and cooperation of many professionals in mathematics education in many institutional and agency roles, the systems approach appears successful in serving the needs of many teachers.

(4) Several states have used ESEA 1965 funds to establish intermediate school districts that offer services in in-service education for mathematics teachers across local school district boundaries. Georgia, Florida, Iowa, and Pennsylvania are among the states following this organizational administrative pattern in serving non-city school system teachers. No comprehensive evaluation of this regional design has been conducted.

(5) Institutions of higher education are experimenting with different registration and course arrangements to attract

teachers--some spread courses out to span the entire school year, others tailor in-service experiences to the needs of a local school system, and others are experimenting with marketing services to schools without the requirement of academic credit. The latter arrangement is not possible for many institutions because of the fee structure that provides the livelihood for the university.

We opine that the experimentation with different structures and mechanisms for providing in-service education is a healthy state of affairs. Clearly, the traditional academic master's degree route to in-service is not serving the needs of many teachers--particularly those who earn their second professional degree at an early age. The evidence (Osborne, 1977) is that the in-service aspirations and needs perceived by elementary and secondary teachers with significant professional backgrounds is as profound as for the less experienced and less adequately trained.

Factors Affecting Locally Sponsored School In-Service Programs, 1965-1975

Toward the end of the 1965-1975 era, it became apparent that school systems would have to assume increasing responsibility for in-service education. Several factors have been noted that have profound implications for locally designed and implemented in-service programs:

(1) Frye and Dalton (1977) noted problems in the leadership capability of individuals in local school settings. Indeed, for secondary schools they identify the ineffectiveness of mathematics department chairpersons as a major weakness in assuring in-service education for secondary teachers. By reasons of administrators retaining power and not delegating time and responsibility to department chair-

persons, and because of inadequacy of training for leadership, department chairpersons are ineffective as in-service educators.

(2) The NCTM In-Service Handbook (Osborne, 1977) identified the design and implementation of training in-service program managers as a matter of high national priority. The advisory committee for the NCTM In-Service Project based this conclusion on the thrusting of in-service education into the schools, the evidence that teachers respond better to locally designed programs based on identified programmatic needs, and the evidence that many supervisors for mathematics education have little training specific to their responsibilities.

(3) There is compelling evidence that many supervisors of mathematics, the primary implementors of in-service in the schools, are finding their positions in jeopardy. The NCTM In-Service Project surveyed supervisors of mathematics as well as [illegible]chers (Osborne and Bowling, 1977b). Thirty-seven percent reported that recently their school system had seriously considered doing away with their position because of budgetary problems. There is also an alarming tendency to replace subject matter specialists with generalists--a trend encouraged by many state laws concerning the certification of supervisors.

(4) Finally, there is the matter of budget for in-service education. The NCTM In-Service Project survey of

supervisors had a 74 percent response rate--the 549 supervisors responding served approximately 150,000 teachers who work with almost 9 million students. Following are the percentages of their responses to the query: "How much money does your school invest in in-service education (exclusive of supervisor's salary) per individual teacher?"

0¢ per teacher	10.4%
25¢ per teacher	14.6%
50¢ per teacher	10.2%
$1 to $5 per teacher	41.3%
more than $5 per teacher	23.6%

One must question both the quantity and quality of in-service education in mathematics that can be provided by the supervisors (35.2%) who invest less than a dollar per year per teacher in in-service education. And only 28 percent of the supervisors indicated any control of discretionary funds for in-service education.

The budgetary factors associated with in-service at the local school system level is reflected in the data concerned with the time supervisors can devote to in-service education. The percentages of supervisors responding to the questions, "What percent of your time is given to in-service?", and "What percent of your time is given to administrative tasks?" are given below:

Percent Time of Supervisors	Given to In-Service	Given to Administration
10%	53.7%	10.7%
20%	26.3%	28.3%

40%	10.3 %	32.0%
55%	7.0%	16.5%
70%	2.7%	12.4%

Another factor impinging on the capability of the supervisors to provide in-service and the budget for in-service is the increased teacher militancy concerning salary and welfare issues. In-service education was reported by 35.2 percent of the supervisors as a point of negotiation when teacher groups bargain for new contracts. Sometimes the bargaining concerned the kind of in-service program and its content; more often the teacher groups (and the school's administration) appeared to be willing to trade the money for in-service and supervisors' salaries for salary and benefits.

In conclusion the evidence indicates that the supervisor is a critical factor in providing quality in-service education at the local level. The surveys of teachers indicated that local leadership is a key variable affecting their perception of in-service education. With the decline of federal funding for in-service education, the supervisor becomes a critical factor in providing in-service for mathematics teachers. The evidence is that few programs exist that are devoted to the training and education of supervisors or that provide them with significant help in dealing with their admittedly political responsibilities.

Final Reflections on In-Service Education, 1955-1975

In-service education has in some sense served as the impetus for teacher education throughout the 1955-1975 era. The in-service programs of the 1950s and 1960s served to specify the nature of the preservice program redesign. As teachers became more concerned with the non-college-bound segments of the school population, and as teachers also acquired better backgrounds in mathematics, a disenchantment with the traditions of upgrading and retreading their mathematical training became apparent in the teachers' attitudes and perceptions. Indeed, there is some evidence that teachers are beginning to distrust in-service through institutions of higher education. Post, Ward, and Willson (1976) found that teachers' (and principals') perceptions of an idealized mathematics teacher were not congruent with mathematics educators' and mathematicians' perceptions of an idealized mathematics teachers. Teachers have a profound distaste for the administrative hassles of in-service red tape in institutions of higher education. They want in-service specific to their instructional and curricular needs. A significant majority already have a second professional degree. In-service education in the 1970s appears to be more effective if adjusted to accommodate to the local school setting and if the participation of higher education is controlled accordingly.

We are at variance with the NACOME Report's emphasis on preservice education and comparatively light-weight treatment of the problems of in-service education. The evidence suggests that NACOME reverses the priorities if teachers' performance and attitudes are to be improved. In like manner, we argue that the decisions of the Science Education Advisory Committee for NSF in 1970 ignored the evidence of needs in the schools and the characteristics of the teachers doing the majority of teaching of mathematics to school-age children.

Teacher Education: HIGHLIGHTS

- The mathematical background of students completing preservice programs for elementary- and secondary-school teaching has increased significantly during the twenty-year period, with the character of that mathematical experience reflecting the current curricula in the schools.
- Teachers are acquiring a second professional degree in greater percentages and at an earlier age than ever before.
- Teachers want in-service education and prefer that it be related to programmatic and instructional needs in their schools.
- Teachers prefer in-service education that is neither purely mathematical nor purely methodological.
- The massive sponsorship and support of in-service education provided by the federal government during the 1950s and 1960s has changed the expectations of teachers relative to in-service education.
- Leadership for in-service education at the local school level can appreciably change the character of in-service education and the teachers' perception of the worth of in-service education.
- Research provides little evidence that participation in in-service education improves the effectiveness of teachers.
- Competence of teachers, when assessed in terms of promoting mathematical growth in students, is apparently related to a complex interaction of an assortment of factors rather than being simply related to a limited number of factors in linear combination. Mathematical background and attitude toward mathematics as characteristics of teachers do not account

for a substantial amount of the variance in the performance of the teachers' students.

- CBTE does not appear to be a significant factor of sustained impact on teacher education programs, at least for the immediate future.
- Computer literacy and the background to use the computer in the teaching of mathematics is not a component of certification requirements in most states or in the institutions that train teachers.
- The most significant trend in teacher education at the preservice level is the move toward incorporating pre-student-teaching field experience in mathematics education as a major modification in program design. This trend is being accomplished because it seems "sensible" rather than because its effects on the prospective teacher are known or verified.
- There is a significant trend toward including laboratory or activity learning emphases in both the mathematical and the methodological phases of prospective elementary teachers' academic preparation for teaching.
- The teacher shortage characteristic of the 1950s and 1960s has given way to oversupply in the 1970s, but the evidence suggests that the oversupply of secondary teachers in particular may rapidly give way to undersupply in the near future. Significantly fewer freshman-level students are indicating teaching as a career choice.

IV. Needs Assessment Efforts

In 1955, "needs assessment" was not a term common in every educator's vocabulary. That did not mean that needs assessments were not conducted; however, efforts were largely informal and unheralded by the term. Needs were assessed in terms of a particular purpose, used for that purpose, and not necessarily preserved once the purpose had been achieved.

Reflections of needs are evident in a variety of sources, including journal articles, conference reports, legislation, committee recommendations, guidelines, trend analyses, and achievement test data. All but the last tend to involve goals, and this is the type of assessment to which the term "needs assessment" will be applied in this section. The term "progress assessment" will be used in referring to achievement and other status test data.

Thus there is correspondence with two definitions of educational need in current use:

(1) What is thought should be minus what is thought to be ---› needs assessment

(2) Desired learner status minus current learner status ---› progress assessment

A. Needs Assessments: National Concerns

Planning documents and other evidence of concern for needs assessment have been cited throughout this report. The year 1955 saw the appointment of the CEEB Commission on Mathematics as a response to the needs being expressed by two groups. Jones and Coxford (1974) noted that the public was being told in magazine articles and books that the curriculum was not sufficiently academic. Mathematicians and mathematics educators were increasingly aware of the need to restructure the curriculum to meet both mathematical and methodological needs.

The Report of the Commission identified specific needs and proposed a set of recommendations to upgrade the secondary-school curriculum, emphasizing

> a balanced preparation in concepts and skills, deductive reasoning throughout the high school, the display and use of mathematical structure, correlation of equalities and inequalities, stressing of unifying ideas in mathematics such as set and function, and special suggestions for reorganizing geometry, trigonometry, and twelfth-year mathematics. (Jones and Coxford, 1970, p. 73)

Evidence (e.g., Williams, 1970; NACOME, 1975) has been presented that the recommendations were largely implemented.

Action on the recommendations, and analysis of other needs at both the secondary-school level and the elementary-school level, was pursued in great part through conferences and committees, backing curriculum development efforts. Table 2 presented a list of some of the major conferences, most of which identified specific needs relevant to a particular focus. Thus the Snowmass Conference on the K-12 Mathematics Curriculum (Springer, 1973) identified the need to:

- improve cooperation between the mathematics education community in the university and that in the schools
- examine societal needs and delineate the goals of mathematics education to provide a basis for curriculum development
- support promising innovative preservice and in-service teacher training
- improve implementation of basic research findings into the curricula for teacher education and for school students

- prepare topics with significant applications of mathematics suitable for K-12

- provide instruction in statistics at all levels

- establish computer literacy as one of the objectives of mathematics education

- develop new techniques for assessing programs and student performance

The Tallahassee Conference (1973) cited the need to strengthen problem-solving abilities, meaningful applications, interdisciplinary or integrated curricula, probability and statistics, the place and role of computers and calculators, research on cognitive development and learning processes, linkage of research and curriculum development, identification of goals and objectives of mathematics instruction for general education, evaluation, and teachers' professional competency.

Each conference could be considered in turn, and the needs identified by each listed. But it became apparent that each has delineated needs pertaining to one or more components of a basic model:

needs of society	->	real-life applications impact of technology
needs of the subject	->	content methods
needs of the child	->	psychological environmental

Many of the points which were cited by the Snowmass and Tallahassee Conferences had been cited in previous conference reports. The value judgment of the relative importance of the needs and how to cope with the needs were the real issues.

Conferences sponsored by mathematics organizations and by federal agencies have had varying impact. The Georgia Conference on Needed

Research (Hooten, 1967), for instance, gave an impetus to mathematics education research which, it is widely felt, has been felt continuously since then. Many of the conferences led directly to curriculum development programs. The report on the Cambridge Conference on School Mathematics (1963) shocked many into discussion -- yet appears to have had little direct impact on any but a few experimental projects. The Euclid Conference on Basic Skills (NIE, 1975) attempted to explore the wide variability in defining such skills -- but what type of impact the conference report might have is as yet unclear.

Various surveys have also provided an assessment of needs. Not the least of these is the Gallup Poll. Those interested in mathematics education are prone to believe that the public is highly concerned about the teaching of mathematics. They are concerned -- but, comparatively, mathematics and other academic subject concerns rank below many other factors. As was noted in the 1975 NCER Report,

> Educators, the Congress, and the American public voice many concerns from different perspectives. One listing of problems is provided by the annual Gallup poll of public views on education. The 1975 poll lists the following in the order reported: lack of discipline; integration/segregation/busing; lack of proper financial support; difficulty of getting good teachers; size of school classrooms; use of drugs; poor curriculum; crime/vandalism/stealing; lack of proper facilities; and pupils' lack of interest. Another list might include such problems as the failure of education to relate to employment needs. (NCER, 1976, p. II)

Curricular concerns are noted in general; instructional concerns are far lower on the list. Mathematics per se is not cited. The same pattern prevailed across the years in such polls, which, admittedly, are not intended to assess concerns about any specific curricular matters. Similar concerns about federal control have also typically been noted through the years. (For an analysis of eight years of cumulative results from the polls, see Smith and Gallup, 1977.)

Recommendations about mathematics have frequently come from mathematics educators. Thus Mayor (1966) solicited recommendations from 22 mathematics education leaders in all parts of the country. The needs mentioned most frequently were:

(1) Improved programs of pre- and in-service education in mathematics for elementary teachers

(2) Increased use of teachers with some specialization in mathematics

(3) Research in the learning of mathematics

Among needs cited by fewer respondees were:

(4) Articulation of mathematics with other subjects, and across grades

(5) Goals stated in behavioral terms

(6) Supervision of mathematics programs in all grades

(7) Grade placement of topics

(8) Assistance with methods of teaching

(9) Special curricula for slower pupils

(10) Evaluation

(11) Use of new technology

It is interesting to note that almost all of these items have had some attention directed toward their resolution -- yet most would probably turn up in a similar polling in 1977.

Harding (1969) identified groups of mathematics educators; secondary-school mathematics teachers; school administrators; scientists, engineers, and mathematicians; professors of education; students; and parents (for a total of 625 persons). Seventy-six objectives were identified, and each person rated the importance of each objective. The upper third of the objectives as perceived by mathematics educators included 5 objectives which were in the lower half as perceived by one or more other groups; 5 objectives ranked in the upper quarter by two or more groups not in the upper third for mathematics educators. Objectives thus identified involved:

reading and interpreting charts and graphs	ranked
common formulas	higher
proofs	by
deductive reasoning	mathematics
attaining attitude of respect for knowledge	educators.
capability for logical, rational action	ranked
organizing statements in logical sequence	lower
algebraic terms and symbols	by
basic structure and principles of real numbers	mathematics
applying arithmetic to business and personal finance problems	educators

Such discrepancy in the selection or ranking of goals is not uncommon. It can be of particular concern when the rankings of educators and taxpayers are widely divergent. Thus in the 1970s there is a discrepancy between public concern for "the basics" and educators' concern for "mathematical understanding". Position statements are one way in which an attempt is made to bring two positions closer by influencing the thinking of the "opposing" group.

Recently, the National Council of Supervisors of Mathematics issued such a Position Paper on Basic Mathematical Skills (NCSM, 1977). They noted that.

> Mathematics educators find themselves under considerable pressure from boards of education, legislatures, and citizens' groups who are demanding instructional programs which will guarantee acquisition of computational skills. Leaders in mathematics education have expressed a need for clarifying what are the basic skills needed by students who hope to participate successfully in adult society. (p. 1)

As a rationale for their expanded definition, they state:

> There are many reasons why basic skills must include more than computation. The present technological society requiresdaily use of such skills as estimating, problem solving, interpreting data, organizing data, measuring, predicting and applying mathematics to everyday situations. The changing needs of society, the explosion of the amount of quantitative data, and the availability of computers and calculators demand a redefining of the priorities for basic mathematics skills. In recognition of the inadequacy of computation alone, NCSM is going on record as providing both a general list of basic mathematical skills and a clarification of the need for such an expanded definition of basic skills. (p. 1)

Comments on minimal essentials, methods for developing skills, and evaluating student progress are also included.

Many position statements were prepared for a conference, or issued by a mathematics organization. Most are reactions to identified needs, rather than statements of need. For instance, the NCTM has adopted position statements on a broad spectrum of topics, ranging from the nature of basic skills to the role of computers and calculators. There are few controversial recommendations in these statements; they focus on what <u>might</u> be done rather than what <u>should or should not</u> be done, presumably identifying as official policy that which a majority of the membership in the Council already believed. They take the form of reasoned arguments, stressing the need for thoughtful appraisal and study. It is perhaps a way of reflecting a collective opinion and therefore of influencing non-members which is the highest expectation of such guidelines: the identified need is that of the non-member.

Several organizations have been particularly active in the development of guidelines for mathematics education. In 1947, the NCTM appointed a Commission on Post-War Plans which published a checklist for assessing basic competence in mathematics. In the 1970s, to meet the need for increased competency demanded by present-day society, an ad hoc committee developed a list of "basic mathematical competencies and skills" (Edwards et al., 1972). They included points related to content, the nature of mathematics, and the role of mathematics in society.

Other guidelines have been issued for metric education and on the use of calculators; of particular impact, however, were guidelines for teacher education which have been cited in another section of this report. One set of guidelines on the use of computers, issued in 1972 by the Conference Board of the Mathematical Sciences (CBMS, 1972) has rece·ed much attention (e.g., NACOME, 1975). Recommendations have come from a variety of other sources, including state education associations and state mathematics councils. Their concerns are reflected in the national statements, although the ranking of priorities may differ at times. Thus the Ohio Education Association recently called for more planning time for mathematics teachers and placed the use of applications, activity-oriented modes, computers, calculators, and metric system lower on their list. The Montana Council of Teachers of Mathematics

(Montana, 1972) listed 50 recommendations, covering the range from conditions of instruction, curriculum construction, teacher training, research, and rights and responsibilities of teachers and students.

To mention the Report of the National Advisory Committee on Mathematical Education (NACOME, 1975) is redundant: it is evident that it assessed needs and provided documentation on a range of problems facing mathematics education. It is both a response to a need (for evaluation and a status report) and a delineator of needs (reflected in the recommendations).

Throughout the years, general statements on educational policy have had a "windfall effect" on mathematics education. One such report on educational policy in the next decade, now being prepared by Keppel and others, is to be published shortly (Warren, 1977). One conclusion it reaches is that the responsibility for education should continue to rest with the schools themselves, with the federal government content to identify needs and stimulate action. Promoting equity through compensatory aid should continue, as well as specific programs for including continued research and development designed to improve the educational process and to provide a steady flow of capability in, for instance, the sciences. The states should set policy and oversee programs, including consumer education, accountability, and basic level of education for all adults. In addition, the report calls for the establishment of minimum performance standards in "the basic subjects", especially at the junior and senior high school levels. There is little that is radical; rather, there is support for continuing in directions over which some questions have been raised. Thus, it may be concluded that needs assessments most typically are reporting symptoms of what has already transpired.

Trends in mathematics education have been analyzed from many perspectives. The Thirty-second NCTM Yearbook (Jones, 1970) and the NACOME Report (1975) provide two excellent recent analyses; reports prepared for international congresses provide others (e.g., see UNESCO, 1972). Overviews in the Encyclopedia of Educational Research (e.g., Willoughby, 1969) or the Handbook of Research on Teaching (e.g., Dessart and Frandsen, 1973) are noteworthy. The October 1969 issue of the Review of Educational Research contained summaries by Romberg, Kilpatrick, Fey, Kieren, and Heimer. Many dissertations which trace the changes in the mathematics curriculum should also be noted (see Table 11).

Rather than assess the trends prophesized in the past, however, we choose to cite evidence from a recent survey (Fairbairn, 1976). Mathematics educators, department heads, and supervisors were asked to comment on future events that could have implications for mathematics education, and to generate consensus on what should receive priority, in light of this envisioned future. The event areas considered to be most important were:

(1) Back-to-the basics movement
(2) Continued acceleration in computer technology
(3) Increasing complexity of our society
(4) Continued demand for relevancy in mathematics
(5) An increase in community involvement in schools
(6) Increasing demand for school accountability, both in programs and expenditures.

The curriculum priorities which were deemed most important or desirable

TABLE 11

STUDIES OF PRACTICES AND TRENDS

Author	Focus	Period Studied
Bell, 1971	Traced influence of psychology on secondary school mathematics curriculum, noting impact on SMSG and SSMCIS	1893-1970
Byham, 1970	Surveyed secondary school geometry texts *Noted more indirect proof used, less direct proof	1955-1969
Fishman, 1966	Traced secondary school mathematics curriculum in relation to educational theories and social changes *Since 1950s noted: subject matter reorganized instruction accelerated, academically talented emphasized	1893-1964
Hancock, 1961	Traced recommendations for secondary school mathematics, analyzed current projects *Methods received little attention from either national committees or current projects *Elementary algebra for grade 9 recommended for 70 years, demonstrative plane geometry for grade 10, greater variety for grades 11, 12	1893-1960
Hoffman, 1973	Surveyed recommendations for content of secondary school geometry *Geometry should be developed as part of an integrated mathematics course	1969-1972
Huber, 1963	Traced proposals for mathematics at junior high school level *Extending algebra and geometry to grades 7 and 8 repeatedly recommended	1890-1961

196

*Trends

TABLE 11 (Continued)

Author	Focus	Period Studied
Hunte, 1966	Traced role of demonstrative geometry *Minor changes in Euclidean geometry noted *New curricula included variety of geometries	1900-1965
Kelley, 1960	Summarized addresses, recommendations re secondary school mathematics Prepared tables showing relative emphasis on given topics (e.g., sets, ordered pairs, geometry, trigonometry) by grades *Noted stress on mathematics for mathematicians rather than consumer mathematics for laymen	1955-1960
Krause, 1969	Surveyed literature to trace reform movement Compared implications with 23 states guides *Guides evidenced effects of reform movement	1936-1968
Quast, 1968	Traced recommendations of committees, leaders *Noted need to change teaching of geometry	1890-1966
Stubblefield, 1964	Traced development of secondary school mathematics curriculum in Chicago *From 1938-1961 courses in essential mathematics appeared *Since 1958 courses for gifted appeared	1856-1962
Yasin, 1962	Traced secondary school reform movements, defined stages *Geometry must be changed, scientifically relevant mathematics needed	1900-1960

were:

(1) Mathematics should involve more activity learning.

(2) Mathematics should involve more use of computers and calculators.

(3) Real applications (some involving metric dimensions) should illustrate the utility of mathematics.

(4) More emphasis should be placed on developing creative thinking in and via mathematics.

(5) Probability and statistics should receive more emphasis in school mathematics programs.

(6) The mathematics curriculum should be continually revised and updated to conform with the present and future need of the students. (Fairbairn, 1976, p. 5111)

It should also be noted that local control of schools was closer to being a reality in 1955 than it is in 1977. Increasingly, legislation by states and funding policies by the federal government have been determining what the schools may do and should do. Schools are being used to achieve national social goals (e.g., desegregation and equal opportunity). Schools are focusing attention on nationally determined needs and goals; perhaps, having "tasted" federal funds, they are loathe to turn away. A serious attempt needs to be made to look at the possible negative aspects of various policies and trends: perhaps future analyses that say, "That was a mistake," can be avoided.

198

National Concerns: HIGHLIGHTS

- Needs which have been repeatedly discussed and cited include the need to:

  - examine mathematical goals in relation to societal needs
  - examine implications of technology, including computers and calculators
  - establish minimal competencies (as a basis for accountability)
  - restructure the curriculum (to resequence, extend, enrich, or one or another specific purpose)
  - increase attention to applications, statistics and probability, problem solving, the metric system, and basic mathematical skills
  - provide for individual needs, particularly of less-able pupils and the talented
  - improve articulation of mathematics with other subjects and across grades
  - conduct research on the learning of mathematics, link research and curriculum development, and improve the implementation of research
  - improve pre- and in-service teacher education, to strengthen teacher competency, both in knowledge of content and methods of teaching
  - develop better evaluation techniques
  - improve cooperation between mathematics educators in universities and schools

- Discrepancy in the selection or ranking of goals -- between educators and public, college personnel and classroom teachers, students and teachers -- is common.

- Increasingly, federal and state legislation has been encroaching on local control of schools.

B. Needs Assessments in the States

The availability of planning documents and statements across the states is by no means complete, especially for the earlier part of the 20-year period. Rarely do the various state agencies have these available except in a state library or archive, and in most cases it was not possible to trace the patterns. Information on legislation, even for recent years, must be culled from various documents. Few summaries exist -- especially summaries related to mathematics education. (This probably reflects the scarcity of mathematics specialists in state agencies, and the extent of the tasks assigned to those who do exist.)

In most of the documents perused from the individual states, mathematics concerns were either not cited, or were only one of several or scores of concerns cited. In relatively few states were specific documents available on planning for mathematics education. As far as can be determined from the documents surveyed, the main identified concerns did not differ from those at the national level. Slight differences in priorities were found, as was noted previously.

Many have assumed that recent needs assessments in the various states came about solely because of pressure from parents and the public (i.e., taxpayers) to make schools accountable for meeting desired goals. Assessments are seen as groundswells. But a recurrent response to a stimulus may involve more than a "bandwagon" effect. Assessments were a logical step in the progression from behavioral objectives to performance contracting; they were logical responses to concerns over the degree to which basic skills were being learned -- or not learned. But they are also a required response to a charge from the USOE made to the 50 state education agencies in July 1968:

> The state Plan shall identify the critical educational needs of the state as a whole and the critical educational needs of the various geographic areas and population groups within the state, and shall describe the process by which such needs were identified. The process shall be based upon the use of objective criteria and measurements and shall include procedures for collecting, analyzing and validating relevant data and translating such data into determinations of critical educational needs.
>
> Section 118.8, U. S. Office of Education regulations for administering ESEA Title III programs.

The state agencies approached the task in various ways. Some created commissions to conduct a goals assessment; some created committees to respond; some collated the results of previous surveys. In many states legislative action was spurred, although this was more frequent with regard to progress assessment than to needs assessments.

In most states, the needs assessment was not specific to mathematics. Thus "the ten most critical needs of education" were identified by surveys in Kansas (1970) as:

(1) Development of positive student self-image

(2) A renewed effort to develop learning patterns based upon student needs

(3) Place new and increased emphasis on the importance of the elementary school

(4) Strengthen programs for noncollege-bound students

(5) Teacher-training in student motivation

(6) Programs for the potential dropout, unmotivated students, or the school-alienated student

(7) Analyze total reading program and success of students in reading

(8) Provide a more positive, wholesome attitude toward quality education

(9) More effective student evaluation and assessment of achievement

(10) More meaningful student involvement in learning situations (pp. 4-6)

In such surveys, mathematics is merely a component of one or more goals.

In other states, mathematics was specifically cited in a goal, as in Oregon: students need to acquire early mastery of the fundamental skills such as reading and mathematics. The public ranked it 6th; educators, 18th; students, 14th; dropouts, 12th (Clemmer, 1970).

In relatively few instances, statewide needs assessments specific to mathematics education were conducted. In at least one instance, what appeared from a state report (Maryland, 1975b) to be a statewide survey instead involved a small group of mathematics educators who confirmed NAEP-related goals:

(1) Recall and/or recognize mathematical definitions, facts, and symbols

(2) Perform mathematical manipulations

(3) Understand mathematical concepts and processes

(4) Solve specific mathematical problems

(5) Use mathematical reasoning and processes to meet personal and societal needs

(6) Appreciate and use mathematics

An actual statewide needs assessment was conducted in Maryland, however. Hershkowitz, Shami, and Rowan (1975) reported that two goals ("knowledge of concepts" and "mastery of computational skills") were ranked low in a needs assessment of 23,990 persons. "Ability to apply knowledge and skills to real-life problems" was, however, ranked very high.

In another mathematics-specific assessment, this one conducted by the Oregon System of Mathematics Education (Thomas, 1975a), discrepancies across samples were noted:

> (1) There is some difference of opinion between respondent groups in what is considered to be important.
>
> (2) Items which are agreed upon as important reflect what is "typically" thought of as a mathematics curriculum (+,-,x,÷,%, and fractions).
>
> (3) If a curriculum modification has been made in public education the response would tend to indicate that the respondent groups haven't adopted the same things as important.
>
> (4) While many educators in the state seem to feel that hand calculators will substantially change mathematics, the respondents didn't find these innovations to be especially important.
>
> (5) The general public doesn't appear to have sufficient information to make other than neutral response possible.
>
> (6) The extremely low responses provided by university professors also suggests some questions as to the basis for their response. (p. 7)

Data in many other research studies support the findings of Smith (1972), who found that the four primary needs of students involved: basic operations, topics for individual needs, consumer mathematics, and applications to the real world.

Many other states as well as local communities have conducted needs assessments, although these have not always been documented. Frequently they involve the informal collecting of opinion rather than a systematic procedure. Schools cannot be operated in a vacuum: needs assessments provide a means of ascertaining what is perceived to be desirable in designing instructional programs.

Needs Assessments in the States: HIGHLIGHTS

- Relatively little attention has been given in most states to documenting the history, status, or needs of mathematics education.
- Mathematics education per se is seldom cited in state goals; it is most frequently one aspect of a "competency in basic skills" goal.
- Where needs assessments specific to mathematics have been conducted, both "knowledge of basic skills" and "applications of skills to real-life problems" have been high on the list of needs.
- Discrepancy among concerned groups was apparent in the priority assigned to mathematical goals.

C. Progress Assessments at the National Level

Within the 20-year period, comparisons of "new" and "traditional" mathematics programs focused attention on the need to develop more appropriate means of assessment. In these studies, students using traditional programs tended to score slightly better on traditional tests, while students using new programs scored higher on tests of the newer content.

As a direct result of such findings with SMSG materials, SMSG planned and conducted the National Longitudinal Study of Mathematical Abilities (Wilson et al., 1968-72). It was the first large-scale testing program in mathematics; although not primarily concerned with assessment, many of the procedures parallel those used in later assessments.

NLSMA was conceived as a study of the effects of various kinds of mathematics textbooks on the learning of mathematics. Schools were recruited to participate at the 4th, 7th, and 10th grade levels, and students in these initial samples were followed for five years, in order to detect long-term as well as short-term effects of curricula (Begle, 1975). SMSG exerted no influence on the choice of textbooks, nor were any consultant services or materials provided. Data on various characteristics of students and teachers were gathered, in addition to cognitive and affective scores. The mathematics tests were constructed in terms of computation, comprehension, application, and analysis objectives: an item bank was developed which has been used, in actuality or as a model, for myriad other studies.

The major findings of NLSMA can be summarized briefly:

(1) Different patterns of mathematical achievement were associated with the use of different textbooks.

(2) Mathematics achievement is a multivariate phenomenon.

(3) "Students are more likely to learn what they have been taught than something else." Each group performed best in those areas stressed in their particular textbooks.

(4) Great variability in pupil achievement was found when teacher 'effectiveness' was considered.

(5) The attitudes of both sexes deteriorated during the secondary-school grades, but the decline was greater for girls.

(6) Teacher characteristics did not account for a significant percentage of the variance; it was too low to be of value in practical school decisions.

(The Summer 1975 issue of Investigations in Mathematics Education (IME) contained abstracts and critques of the NLSMA reports.)

At the time NLSMA was being planned, the goal of a national assessment across educational levels and subjects was coming to reality. The National Assessment of Educational Progress, conducted by the Education Commission of the States, began assessment of various subject areas in the late 1960s. The first mathematics assessment by NAEP was conducted during 1972-73; the second is scheduled for 1977-78. The assessment included six major content areas: numbers and numeration, measurement, geometry, variables and relationships, probability and statistics, and consumer mathematics. About half the exercises will be repeated from one assessment to the next, so that the first assessment provided baseline data for later comparisons.

Four reports on the first testing have been published (NAEP, 1975a, b,c, 1976), in addition to a series of interpretive articles (Carpenter et al., 1975-76), a general statement of objectives (Norris and Bowes, 1970), yearbooks, and newsletters. About 90,000 students at ages 9, 13, and 17, plus 4000 young adults aged 26-35, were tested. In addition to age levels, data were also analyzed in terms of sex, race, region, level of parental education, and community size and type.

Carpenter et al. (1975-76), writing for the NCTM Project for Interpretive Reports on National Assessment, indicated that the data showed "a mixed picture of strengths and weaknesses". Students' performance was "strong or at the level of reasonable expectation in terms of the mathematics curriculum" for:

whole-number computation

knowledge of numeration concepts

analysis of simple (one-step word problems)

intuitive or practical measurement concepts

recognition of basic geometric figures and relationships

Weaknesses were indicated in the areas of:

percent

development of fraction concepts

complex word problems

measurement tasks

understanding of geometry topics

Reacting to current concerns, they noted:

> The modern mathematics movement of the 1960s has been accused by its critics of destroying pupils' computational skills. These NAEP mathematics data argue that whole-number computation is not a lost art and, in fact, 13-year-olds perform at about the same level as adults (and 17-year-olds perform better). The current retrenchment of mathematics programs into emphasis on arithmetic skills should be examined for finding a proper balance between skill and understanding, or between arithmetic skills and skills in measurement and geometry. (1975a, p. 449-450)

In another summary, they indicated that 13- and 17-year-olds need to develop more problem-solving skills, estimation skills, understanding of percents, and skills with fractions. In regard to consumer mathematics, they noted:

> Although performance varied among the consumer exercises, it seemed generally low. One can take little satisfaction from findings that suggest only about one half of the 17-year-olds and young adults can usually solve typical consumer problems. Continuous gains in performance were made from the 13-year-olds to the young adults. The most dramatic gains were made from the 13- to the 17-year-old groups; this was expected because of the direct influence of the mathematics curriculum. Young adults performed consistently higher than 17-year-olds on all types of consumer exercises ... These gains may simply be the result of maturation and experience in solving consumer-related problems. On the other hand, these consistent differences cannot help but raise questions regarding current mathematics programs. (1975b, p. 469)

Bright (1978, in press) has compared data from a number of assessments for which computational examples have been published, including NLSMA, NAEP, and several state assessments. He reported the level at which stabilization is reached -- that is, where 80 percent to 90 percent of the students have reached mastery. He concluded:

> Overall, several patterns in the data seem to support clear conclusions. First, there is general improvement in performance across grades. This result is not unexpected, and it is consistent with the results of the grade-equivalent studies discussed earlier. Second, the levels of performance decrease as the items become more complex. Third, performance tends to stabilize. For the areas discussed in this article, stabilization seems to occur during the junior high school years . . . Fourth, stabilization of performance for whole number computations occurs earlier and at a higher level than for fractional number computation. Fifth, for all computation skills considered, there is no decline -- or at least no important decline -- in the performance of adults in comparison to that of high school students. In the context of improvement of skill performance across grades, this suggests that once skills are mastered, they are not forgotten.

> ... (it is observed) that computation skills are not acquired on the basis of initial instruction. Instruction over several years is needed to reach stability, and in every area examined there is still room for improvement...
>
> It is important to note that the data presented refute the notion that students generally do not acquire basic computation skills. In fact, some skills (e.g., addition and subtraction without regrouping) are almost universally acquired, whereas others (e.g., division of decimal fractions) are not. Any meaningful discussion of the performance of students in basic computational skills must be a discussion of specific skills rather than skills in general. (p. 163)

Results from national assessments of achievement seem to reach the headlines (especially) if they are low or declining: similarly do the results of international studies. The International Study of Acheivement in Mathematics (Husén et al., 1967), conducted in the early 1960s, is the prime example. The IEA mathematics survey involved 133,000 students in 5450 schools in 12 countries; 13-year-olds and pre-university students (grade 12) were sampled. <u>The New York Times</u> headlined "United States Gets Low Marks in Math". The most-quoted findings in the news media were: the U.S. 13-year-olds ranked 11th in mathematics achievement among students from the 12 countries, while high school seniors ranked last. Both liked school and school learning less than students in other countries. Little attention from the media, but much on the part of mathematics educators, was paid to further considerations of the data {e.g., the <u>Journal for Research in Mathematics Education</u> focused an issue on IEA (JRME, 1971)}. Husén (1973) indicated that the arithmetic means had to be considered in terms of the "recruitment bases" or "retentivity" of schools in the various countries; when that was done and equal proportions of students considered, the variations turned out to be considerably less.

In the special issue of JRME, Postlethwaite (1971) reported on procedures used in the IEA and cited data on tests and scales. Among the many findings he stressed were: (1) age of entry into school was not an important variable in mathematics achievement, (2) reducing class size was not likely to increase mathematical attainment significantly, (3) type of school affected the achievement of 13-year-olds, and (4) correlations between achievement and attitude were small but positive. Other articles in the issue provided a critique of the study and the presentation of many specific interpretations related to the data. (A second international survey is being planned.)

In another type of national survey, Okada et al. (1969) reported on the Educational Opportunity Survey, citing data on the achievement of black and of white students. Black students did not attain the sixth-grade achievement level for mathematics until grade 8. From grades 6 through 12, there is a gradually increasing gap between black and white students, with similar lags in achievement observed for other disadvantaged groups. Evidence from NAEP (Carson, no date) also showed that

> Blacks performed 14 to 21 percentage points below the national average . . . Whites performed from 3-4 points above the national average. . . The difference in performance between Blacks and Whites was smallest at age 9 and increased for 13- and 17-year-olds with no appreciable change in relative performance between ages 13 and 17. (p. 39) {On consumer-math problems Blacks were 20 percentage points below at age 13, 24 points below at age 17, and 29 points below as adults; Whites were 4-5 points above the national level.}

Standardized achievement tests have been given for years, but only occasionally were data compared across time. (Table 12 includes some of the little-published evidence of such studies for certain states.) One highly publicized instance at the national level is that of the Scholastic

Aptitude Test scores. Decreases in scores were observed; the average score of mathematical ability was 502 in 1963 and only 472 in 1975. While the test scores were for college-aspiring students, much of the mathematics tested was of a basic nature. Therefore declines in scores are presumed to be symptomatic of a failure to establish competency in mathematics, though it was pointed out that

> The ultimate blame may rest with the influence of television, permissive parents or dozens of factors beyond the control of schools. (U.S. News and World Report, Nov. 24, 1975, p. 34)

It should be noted that scores on the language (verbal ability) portion of the SAT were even more depressed.

Harnischfeger and Wiley (1975) analyzed scores from nine widely-used testing programs, including both elementary-school and secondary-school tests: the SAT, the American College Testing Program, the Iowa Tests of Basic Skills, and the Comprehensive Tests of Basic Skills, plus five others. Nearly all reported data showed declines for grades 5-12 over the past decade. Both the verbal and mathematics scores on the SAT peaked in 1963 and then declined steadily. On the ACT a similar pattern was found, and on the ITBS, the pattern was one of general increase from 1955 to 1963, then consistent decline to 1970.

They hypothesized likely causes for the drop in achievement levels to be both the school and the home, but they believe the school-related causes can be more closely studied and more easily influenced. School-related factors whose developments closely parallel the decline in the achievement scores seemed to be:

- high school students are taking fewer "basic" courses like English and mathematics, and fewer college preparatory courses like algebra, first-year foreign languages, chemistry and physics {note that this conflicts with data cited in this report}

- increasing numbers of students are absent from school, and

- fewer students are dropping out, resulting in a larger percent of drop-out-prone students taking the tests.

The correlation between changes in performance and increased federal spending raises some questions. As has been noted, federal funding has had an impact on mathematics education throughout the past 20 years. Much of that impact has been positive: the effect on curriculum development and teacher training, with the involvement of the National Science Foundation in particular, has been documented. The establishment of priorities across agencies, however, has not consistently resulted in mathematics education being given due attention. For instance, immediately following the publication of the IEA results, in which the performance of the American students was poor, the USOE began decreasing the number of mathematics education specialists who could provide services to schools and who could monitor government-sponsored projects concerned with mathematics. Given the large amounts of money which might have been expended on mathematics education through such programs as ESEA Titles I and III, it is unfortunate that the investment was not guarded and maintained.

For several years, NIE had also elected to give little attention to mathematics education, assigning greater priority to other segments of the curriculum and in particular to reading. Although one cannot quarrel with the identification of reading as a matter of very high priority, it seems appropriate to attend to other areas of the critical basic skills needed for the well-being of the country. The Euclid Conference on

Basic Skills (NIE, 1975) indicated changing awareness within NIE and of attention to mathematics, as did the conference on needed research and development with calculators (NIE/NSF, 1977).

Progress Assessment at the National Level: HIGHLIGHTS

- NLSMA was not a progress assessment, but it focused attention on the need for longitudinal assessment and improved evaluation techniques.
- NAEP data have indicated specific strengths and weaknesses, although the real function of NAEP is to provide longitudinal information on the status of mathematical achievement.
- IEA provided data on the achievement of American students compared with students in 12 other countries, but results are difficult to interpret in view of the many varied cultural and school factors involved.
- A comparison of computational skills data from NAEP, NLSMA, and several other assessments indicated that these skills are not acquired on the basis of initial instruction, but performance tends to stabilize during the junior high school years. Stabilization occurred earlier for whole-number examples than those with fractions; level of performance decreased as items became more complex.
- College-entrance and some other standardized tests scores have indicated declines in achievement across the years, with more extensive decreases for verbal portions than for mathematical portions of the tests.

D. Progress Assessments in the States

Keeping track of what is going on in the states is not an easy task. Numbers vary and documents are difficult to secure. This section is more a picture of "what could be tracked down" than a complete overview.

The movement toward accountability has resulted in both minimal competency requirements and assessments of achievement in many states. Clark and Thomson (1976) provided an overview on minimal competencies which cited the following reasons (drawn from other sources), for "the public's determination to define the high school diploma":

- Scores on the Scholastic Aptitude Test have fallen . . . .

- The National Assessment of Education Progress in 1975 reported a decline . . . {in some subjects}

- NAEP also has reported in a nationwide survey of 17-year-old students and young adults that "many consumers are not prepared to shop wisely because of their inability to use fundamental mathematical principles such as figuring with fractions or working with percents."

- The American College Testing (ACT) program also has reported a decline in the average scores of students applying for college admission.

They also noted:

> Secondary education has, of course, been moving toward competency-based, criterion-referenced education for a decade. Beginning with programmed instruction in the early 1960s, then moving to a focus on behavioral objectives, and followed by the current interest in "outcomes" . . . (p. 5)

Pipho (1977), maintaining the Education Commission of the States' tally of the states which have minimal competency testing for high-school promotion or grade-to-grade promotion, reported that by mid-April 1977 the status was:

Legislation Enacted (1975-76): 8 states

(California, Colorado, Florida, Louisiana, Maryland, New Jersey, Virginia, and Washington)

State Board of Education Rulings (1975-77): 10 states

(Arizona, Georgia, Delaware, Idaho, Michigan, Missouri, Nebraska, New York, Oregon (1972), and Vermont)

Legislation Pending (1977): 15 states

(Alabama, Arizona, Arkansas, California, Florida, Iowa, Illinois, Kansas, Maine, Maryland, Massachusetts, Minnesota, Nevada, North Carolina, and South Carolina)

In some instances, only reading is considered; in most, mathematics and reading are both included; in a few cases, other goals are also considered.

In Virginia, for instance, the General Assembly listed basic skills in reading, communications, and mathematics first in a set of ten "standards of quality" (Virginia, 1976). The pattern in Virginia is one reportedly occurring in other instances: the legislature enacted legislation mandating the development of minimum competency objectives and tests with which to assess them with little interaction with educational agencies in the state. State departments of education and local school districts were given a relatively short period of time to implement the legislative mandate. Educators had no direct role in the decision-making process, nor was the rationale for the decision-making process clear.

Some local school districts across the country are also adopting minimal competency standards; the total is difficult to determine, but known instances total less than 50. Denver led the way, with competency tests administered there since 1962. No reports on the decision-making process associated with these adoptions were located, so the pattern cannot be determined.

The changing status of accountability legislation is also being monitored. As of Fall 1972, 23 states had accountability legislation (Hawthorne, 1973); as of June 1974, this number had risen to 30 (Hawthorne, 1974). She reported that these took the following form:

state assessment/evaluation: state testing programs	18
modern management techniques	16
professional personnel evaluation	13
performance-based school accreditation	3
performance contracting	2

The 30 states cited by Hawthorne are indicated in Appendix B, which also contains a synthesis of available information on needs and progress assessments. Unfortunately, information and documents were not obtained from all states, nor were materials available in the ERIC system. (The NACOME Report (1975) also provided information related to assessments.)

In regard to the assessments, it should be noted that:

(1) There is great variability in the objectives being assessed. For instance, one state included these two objectives for grade 2:

- Pupils will indicate ability to analyze by constructing a market value continuum on a given set of objects or pictures of objects.
- Pupils will indicate application in using the addition and multiplication algorithms by applying those rules to solve addition problems through two 7-digit numbers and multiplication problems of 2-digit numbers.

Other states have restricted the objectives to minimal competencies.

(2) Both standardized and non-standardized norm-referenced and criterion-referenced tests were used.

(3) Reporting procedures vary widely: some states provide a summary, some present data alone, some provide data plus

interpretation. Criterion levels, percentages, grade level norms, and a variety of other statistics are provided.

A summary of the contents of the information in Appendix B is somewhat meaningless, since so many gaps exist and since the data are from a variety of tests, grade levels, and years. Nevertheless, a few general comments seem appropriate about data which were available:

(1) The topics with which difficulty (or weakness) were reported can be ranked in this order of frequency:

fractions
division
subtraction with regrouping
decimals
geometry
measurement
proof
estimation
statistics and probability
problem solving

This corresponds with information from previous error-analysis studies and studies on difficulties and the need for remediation.

(2) Status was reported as "at norm" and "below norm" in approximately an equivalent number of instances; fewer instances of "above norm" were noted.

(3) Trends are unclear: in the few instances where data from the same test administered for 2 or 3 years could be checked, improvement was noted on four of 5; in the fifth case, scores remained at about the same level.

218

In other state, local, and regional assessments, some comparing data across a period of years, no clear trend could be observed (see Table 12). Roderick (1974) provided an example of one difficulty in assessment across decades. Not all of the items administered to the 1973 students involved content still being taught: thus, many items were transfer items for the 1973 students. It is also apparent from a scan of the items that mastery levels were by no means achieved on many of the items by the 1936, 1951-55, and 1965 pupils, any more than they were achieved by the 1973 students. Where an item was passed by 80 percent or more of the earlier students, it tended to be an item on which 1973 students also scored high.

TABLE 12

SMALLER-SCALE ASSESSMENTS

Study	State	Grade	n	Results
Barnes, 1973	Atlanta, GA		city	78% of schools in defined effectiveness range in 1971-72; 74% were at national norm.
Beckman, 1969, 1970	NB	9	1345p	Students at beginning grade 9 in 1965 scored as well as those at end of grade 9 in 1951. Mean score on 109-item test was 45.7 in 1951; 54.9 in 1965.
Brown, 1957	LA	12	28 sch.	Indices of achievement were low.
Cramer, 1975	NB	12.	1430p	Means score on 95-item test was 74.2; students had attained 30.6 of 48 competencies.
Dambacher, 1972	Berkeley, CA	4-12	city	Upward trend in mathematics achievement noted for 1967 through 1972.
Hammons, 1972	LA		6903p	Scores generally declined for computational skills between 1960 and 1969.
Hieronymus, 1965	IA	6,8	state ITBS scores	Students scored higher on the Iowa Test of Basic Skills in 1965 than in 1940 on concepts and problem-solving but not on computation. However, the data are revealing: note the degree of difference.

computation	grade 6	1940	6.65
		1965	6.63
	grade 8	1940	8.60
		1965	8.66
concepts	grade 6	1940	6.50
		1965	7.67
	grade 8	1940	8.35
		1965	9.09

TABLE 12 (continued)

Study	State	Grade	n	Results
				problem solving grade 6 1940 6.45 1965 7.40 grade 8 1940 8.48 8.92
Hieronymous, 1973	IA	6,8		1965 scores were higher than 1972 scores. Median on concepts dropped to 43.5, on problem solving to 42.9 in grade 6; correspondingly, 44.0 and 41.5 in grade 8. (Computation not noted.)
Horn, 1969	IA	6	1200p	1963-64 scores not significantly different from 1967-68.
Hungerman, 1975	MI	6		No significant difference in computational skills between 1965 and 1975 in total score. However, the 1975 group scored higher on whole number computation but lower than the 1965 group on fractions and decimals.
Leonard, 1967		9	2430p	1966 algebra students significantly better in solving equations also attempted by a group in approximately 1926.
Niemann, 1974	NB	7-9	1239p	Mean scores were 48.8 in grade 7, 57.8 for grade 8, and 58.4 for grade 9 on a 96-item test.
Roderick, 1974	IA	6,8	7665p	Achivement levels were lower in 1973 for 4 of 8 areas tested than in 1936, lower for 2 areas at each grade level than in 1951-55, and lower for one area than in 1965. 1973 pupils were inferior to 1951-55 pupils on number computation and fractions in grade 6; decimals, percentage, and problem solving in grade 8. 1973 pupils were

TABLE 12 (continued)

Study	State	Grade	n	Results
				inferior to 1965 pupils on problem solving in grades 6 and 8. Not all content was still being taught.
Rudd, 1975	Bloomington MN	ages 9,13,17	city	Good computation skills, concepts, and problem solving facility were found. 9-year-olds had acceptable or strong performance (compared with state) on 96% of objectives; 13-year-olds, 86%; 17-year-olds, 93%.
Schrader, 1968	IA			Students at 50th percentile on old forms would only be at 41st percentile on new forms (indicating "modest gain in pupil achievement over time").
Thurlow, 1965		7,8		Average scores were 31.05% for grade 7 and 39.76% for grade 8, compared with 1948 scores of 12.5% in grade 7 and 14.01% in grade 8.

Progress Assessments in the States: HIGHLIGHTS

- As of April 1977, eight states had minimal competency legislation, 10 had state board of education rulings, and legislation was pending in 10 states.
- As of June 1974, thirty states had accountability legislation.
- State progress assessments vary greatly in scope of objectives, type of test, and reporting procedures.
- The content for which weaknesses were identified are ones which have been known to be difficult. Fractions, division, and subtraction with regrouping head the list.
- Trends across years are unclear as yet.

V. Synthesis and Conclusions

The avowed purpose of this document was to describe the evidence bearing on the rationality of decision-making for educational policy that influences mathematics education in the schools. This section identifies some major deficiencies that we have found for the process of policy formation as we examined the record of the past twenty years in mathematics education, and offers commentary on those deficiencies.

The evidence of the report shows that progress and change have been the result of federal intervention into the domain of mathematics education. Indeed, some would claim that the federal investment in mathematics education has often been the vital margin determining whether a change would be realized or not. We see little evidence that the future will be otherwise. Thus, the capability for thoughtful and careful policy formation at the federal level is critical since it guides the investment of dollars for mathematics education.*

It is not sufficient simply to recommend increasing the magnitude of the investment in mathematics education if change is desirable. Many segments of society and many non-educational problems have legitimate claims on federal resources. More money is not the universal solvent for educational problems; problems are not solved simply with a greater investment of resources. To argue simply for more money as the solution to educational problems ignores present realities. At issue is investing money wisely in order to accomplish change expeditiously and efficiently

*The impact of the private sector (e.g., textbook publishers) is not denigrated; rather, that federal policy affects the full range of educational activities is the point at issue.

in the areas of greatest need in mathematics education. The recognition of the deficiencies in the policy formation processes is an important first step toward improving the payoff of the investment and toward improving the learning and teaching of mathematics in the schools.

Three primary sources of difficulty or failure in the processes of policy formation for mathematics education are apparent to the writers from the recent history of mathematics education. These failures are:

(1) Educational policy is frequently determined without collecting enough information to allow the process to be rational.

(2) Educational policy is frequently constructed without using information that is readily available.

(3) The point at which values enter into policy formation, and the effects of the differences in the values held by various groups concerned with the schools, is frequently not recognized in determining the priorities within educational policy.

There are numerous notable examples of the first type of failure in the segments of this report that concern existing practices in the schools and in teacher education. Some that stand out in the authors' opinions are:

Practices in the schools

(1) We do not know enough about what happens in the typical classroom. The classroom practices of teachers, ranging from such simple things as how much time the typical elementary-school teacher gives to mathematics instruction

to the more complex and subtle questions pertaining to what guides teachers' choices of instructional strategies, are largely undocumented.

(2) We know little about the extent to which teachers differentiate instruction for children with different characteristics and needs.

(3) We do not know enough about the extent and nature of teachers' use of instructional materials and tools. Although activity learning has been advocated strongly in teacher education and in professional activities and materials, the extent to which teachers involve students with non-text materials is largely unknown. We also do not know what guides teachers in the use of non-text learning materials and how teachers select mathematical topics for this style of teaching.

(4) The extent of teachers' dependence on drill-and-practice teaching strategies is not known. The factors that teachers use to guide their selection of teaching strategies other than drill and practice are not known.

Practices in teacher education

(1) The data concerning supply and demand of secondary mathematics teachers are only conjectural.

(2) There is little evidence available concerning the characteristics of the small but significant portion of teachers who refuse to participate in in-service

activities and/or about program characteristics that may keep them from participating.

(3) Early field experience prior to student teaching as a component in preservice teacher education programs appears to be a sensible new feature in program design. However, there is little evidence concerning how much, what kind, or when such field experience is best or how it actually contributes to helping the prospective teacher become competent.

(4) The characteristics of teachers that contribute to the effective learning of mathematics by students are not well-described nor verified.

The sections on existing practices describe many other blank spots in the knowledge base for effective policy formation. A major difficulty is that these missing segments in the knowledge base are not used to define priorities for information collection or for deciding what research to support and fund.

There are some sources of information concerning existing practices that are difficult to use. Considerable information was found about existing practices in the schools that was either hard to access or in a form that was difficult to interpret. There is a lack of commonality from state to state in what information is collected and how and who stores the information. Many states do not consider the potential uses of information in designing their collection and storage processes and thereby have no convenient means of retrieving the information.

One major characteristic of the information base is that research activities have not been coordinated. There are many examples of highly similar studies on a given topic within a given area of research interest, but for other topics within the same area little or no research has been accomplished.

Failures of the second type -- formation of policy without using available knowledge -- are also readily apparent in the preceding sections. For all areas of practice relevant to this study, the amount of information at the end of the twenty-year period is greater than at the beginning. But often the collection of information confirms what has been known previously. Some characteristics of performance and practice appear to have significant stability over the years. (For example, recent progress assessments reveal that fractions are difficult for children; they were also difficult in 1920. Another example is that of teacher verbal behavior: research conducted in every decade of this century reveals that the typical teacher makes two-thirds of the utterances in the classroom.) The formulation of policy frequently has not recognized the apparent and verified stability of practices. This may be evidence of a lack of information dissemination, failure to do sufficient summative literature analyses, or simply testimony of the youth of the field of mathematics education and its resulting lack of academic traditions.

The third type of failure, not recognizing the point at which the values of various groups enter into policy formation, is also quite evident. McLaughlin (1976), in studying the process of change, concluded that change has little permanence in the schools if the need for a project or program is based on an entrepreneurial motivation rather than a perception of a problem in need of solution by the primary personnel of a project. The discrepancy between the practitioners' (teachers and principals) and mathematics educators' perceptions of the "ideal" mathematics teacher described in the teacher education section of this report is symptomatic of potential difficulties in promoting change and the varying perceptions of the importance of the area of development and research. Thus, a development or research effort will fail at the point of implementation or application of the results if discrepancies are not resolved.

The shifts in interest (and in the funding levels) in a variety of areas, such as mathematics for the talented or for low achievers, activity learning, discovery learning, or basic skills, provides evidence of shifting priorities. However, it often appears that the shifts in priorities for development, research, and implementation have little to do with the evidence of existing practices. We feel that needs assessments often have simply served to confirm already existing problems and issues in mathematics education. That is, they are not anticipatory of developing problems but simply confirm that activity and interest in the area has already begun. Needs assessments are seldom informed judgments based upon the evidence of existing practices and are seldom generated in such a way that allows professionals to indicate which of two or more problems or issues is of greater importance. At issue is whether activities in development, research, and teacher education must be fad-like in character as opposed to a reasoned attack on problems and issues of mathematics education in the schools.

In the introduction, policy making was described in terms of operating at two levels, one of which incorporates professional judgments and is based upon information and the other that is political and reactive to the prevailing societal attitudes and values. We have purposefully delimited the reporting of historical events to descriptions of existing practices, leaving to the reader the judgment of the contrast of the contribution of the two levels to the policy formation for mathematics education.

The evidence of change results only when there is significant agreement across the two levels that is apparent in the policy formulation process, the political/societal ethos, and the professional level internal to education. Since teachers are elements of both sets of individuals, the public and the professionals, they are major barometers of change. That is, if teachers sense agreement between the two levels of decision-making, change takes place. If teachers sense incongruence and disagreement between the levels, then they are dissatisfied and this dissatisfaction is the evidence that significant change will not take place. This dissatisfaction or satisfaction provides a measure of what the teacher is willing to do to accomplish change. This is the critical attitudinal variable relative to teachers' performance in the schools.

We would argue that current evidence indicates that teachers are exhibiting this order of dissatisfaction, and the resulting lack of purpose that compromises significant rapidity of change, and that this is reflected in current disquietude about basic skills. The nature of innovation and change in the schools as studied by McLauglin (1976) suggests that the teacher is the key and that implementation of change must reflect curricular and programmatic needs perceived by the teacher and supported by commensurate teacher education activities. This tells but half of the story, since activity directed toward promoting change must respect the two levels involved in policy formulation. Thus, needs assessment endeavours must systematically garner information not only relative to the schools and their performance, but also on the prevailing societal ethos that is a necessary condition for teachers' acceptance (and support) of the endeavour.

Policy formation at the federal level typically has ignored existing practices in the schools except as mirrored in the disquietude of society. Often, if additional information was needed for the formulation of educational policy, it was collected after-the-fact of policy decision for the purpose of confirming the actions taken. The amazing, significant conclusion indicated by this study is that progress has been made without systematic information collection relative to existing practices. Apparently, the societal/political ethos is sensitive enough to the goals, aims, and objectives of education -- and their attainment -- to provide substantial direction to Amerlean education. Thus we conclude that the problem for professionals is a matter of efficiency in promoting change. The implication is that not only must professionals collect appropriate kinds of information concerning practices in the schools, they must also make sound application of this information.

234

APPENDIX A

CATEGORIZED LISTING OF SELECTED RESEARCH

IN MATHEMATICS EDUCATION

The categories included in this appendix appeared relevant to existing practices. Data on journal-published articles and dissertations were compiled for this table using Suydam's files. Two limitations should be noted:

(1) Some studies are counted in more than one category, reflecting primary and secondary scopes of concern. The categorization system (Suydam, 1974) includes categories in addition to those included on this table. Thus, all research in the field of mathematics education is not listed in this table.

(2) In some instances, a dissertation and one or two aritcles reflect essentially the same research, but have been counted separately in this table. Thus, there is a (small) "inflationary" factor; nevertheless, the table indicates the approximate level of interest in research topics related to this literature review.

235

APPENDIX A

SELECTED RESEARCH IN MATHEMATICS EDUCATION

Topic	1955-59	1960-64	1965-69	1970-74	1975	Total
Historical developments	2 *	2	8	4	1	59
	10	12	16	4	0	
Organizational patterns	3	20	30	40	9	141
	0	3	12	21	3	
Teaching approaches	3	18	44	91	14	365
	17	28	56	81	13	
Instructional procedures:						
Drill and practice	3	4	8	19	4	52
	1	0	3	9	1	
Problem solving	16	35	35	49	9	204
	9	7	11	18	15	
Estimation	0	1	0	3	0	6
	1	0	0	1	0	
Mental computation	8	4	4	5	0	23
	0	0	0	1	1	
Specification of objectives	0	0	2	36	5	70
	0	1	5	21	0	
Attitude and self-concept	10	18	24	62	10	234
	14	14	28	49	5	
Content organization and inclusion	1	4	8	50	8	181
	13	19	36	37	5	

* Top numeral, elementary; bottom numeral, secondary.

Appendix A (Continued)

Topic	1955-59	1960-64	1965-69	1970-74	1975	Total
Grade placement	5 1	22 1	4 3	5 2	0 0	43
Time allotment	7 0	5 3	8 3	6 6	2 1	41
Content and methods:						
Number properties and relations	2 4	7 4	10 4	35 4	2 1	73
Addition with whole numbers	5 0	2 1	14 0	27 1	10 2	62
Subtraction with whole numbers	8 1	4 0	10 0	22 0	11 1	57
Multiplication with whole numbers	3 0	5 2	17 1	20 1	7 1	57
Division with whole numbers	5 1	6 0	3 0	10 1	5 1	32
Fractions	13 1	22 0	27 0	40 8	8 1	120
Decimals	2 1	3 0	4 0	8 2	0 1	21
Negative numbers (integers)	0 1	1 3	5 2	8 2	4 2	28
Geometry in elementary school	3 0	11 0	25 0	83 0	11 0	133

Appendix A (Continued)

Topic	1955-59	1960-64	1965-69	1970-74	1975	Total
Sets	0 0	4 4	4 2	8 1	0 1	24
Logic and proof	1 3	4 3	19 13	36 20	8 6	113
Decimal system	2 0	5 2	6 1	8 0	0 3	27
Other numeration systems	1 1	4 3	8 3	19 6	1 1	47
Probability and statistics	2 2	2 3	6 5	7 8	4 6	45
General mathematics course	0 3	0 6	0 20	0 18	0 6	53
Algebra course	0 22	0 39	0 59	0 56	0 9	185
Geometry course	0 18	0 21	0 46	0 81	0 5	171
Trigonometry course	0 1	0 0	0 2	0 8	0 1	12
Calculus course	0 0	0 1	0 4	0 9	0 0	14
Other courses in secondary school	0 0	0 1	0 10	0 0	0 6	17

Appendix A (Continued)

Topic	1955-59	1960-64	1965-69	1970-74	1975	Total
Materials:						
Textbooks	15	13	21	19	3	131
	9	7	15	28	1	
Manipulative devices	11	17	22	69	10	177
	6	5	5	30	2	
Audiovisual devices (including calculators)	3	5	9	30	4	81
	2	8	8	11	1	
Programmed instruction	1	21	40	38	10	238
	0	33	58	31	6	
Computer-aided instruction	0	0	5	41	6	122
	0	2	7	54	7	
Readability and vocabulary	4	11	9	26	4	79
	3	4	3	13	2	
Developmental projects	0	3	20	35	6	156
	0	25	41	25	1	
Individual differences:						
Error analysis	3	0	2	5	2	22
	6	0	1	3	0	
Diagnostic procedures	1	4	4	8	2	29
	3	1	1	5	0	
Remediation	1	4	4	12	4	46
	4	0	8	8	1	
Slow learner, low achiever	4	3	5	13	5	106
	1	9	22	39	5	

Appendix A (Continued)

Topic	1955-59	1960-64	1965-69	1970-74	1975	Total
Mentally retarded	1	8	16	52	9	110
	1	1	6	11	5	
Tutoring	1	0	1	25	7	47
	0	1	2	10	0	
Enrichment	3	6	4	4	0	51
	3	11	8	12	0	
Acceleration	1	8	7	3	0	39
	1	11	5	2	1	
Grouping procedures	3	30	17	49	19	185
	4	12	16	32	3	
Motivation	0	1	6	17	5	61
	3	7	6	10	6	
Physical, psychological, social characteristics	8	13	26	91	18	234
	7	7	23	36	5	
Sex differences	1	6	6	18	5	65
	2	5	10	9	3	
Socioeconomic differences	1	3	29	70	4	137
	1	4	9	16	0	
Evaluating progress:						
Analysis of tests	2	5	18	49	7	176
	10	15	20	42	8	
Status testing	2	1	4	10	2	44
	9	3	6	6	1	
Achievement evaluation	35	49	58	78	13	321
	11	19	18	37	3	

Appendix A (Continued)

Topic	1955-59	1960-64	1965-69	1970-74	1975	Total
Teacher education:						
Preservice competency	10	17	23	19	1	
levels	1	3	5	4	1	84
Preservice preparation	7	11	29	95	25	
procedures	8	9	15	31	4	234
In-service competency	3	6	12	13	2	
levels	3	1	10	7	1	58
In-service procedures	2	5	28	29	4	
	4	11	15	26	5	129

APPENDIX B

PROGRESS ASSESSMENTS IN THE STATES

The information in this appendix was compiled from <u>obtained</u> documents, and does not purport to be totally comprehensive. That is, there (in all likelihood) exist other documents on state progress assessments which we were unable to obtain.

Some bibliographies are appearing (and more will undoubtedly be published) which compile information on the state programs (e.g., Porter and Wildemuth, 1976). In the reference column in this appendix, ERIC douments are noted, since they are readily available. Other documents on progress assessments for states on which information is noted may be requested from those states.

APPENDIX B

PROGRESS ASSESSMENTS IN THE STATES

State	*	Progress Assessment				
		Mandated	Conducted	Levels	n	ERIC reference numbers
Alabama						
Alaska	A					
Arizona	A		1971-72 1974-75	8 5	35,866	P: ED 077 935 (1972)
Arkansas	A		1971-72 1972-73	3,4,8,9 3,8	28 districts	
California	A		1974-75	2-12		O: ED 022 767 (Kelley et al., 1968) ED 059 910 (1972a) P: ED 129 594 (Hoffman and Tardif, 1976) ED 124 592 (1975)
Colorado	A		1970	k,3,6,9. 12		P: ED 050 135 (Helper, 1970)
Connecticut	A					
Delaware			1970 1971 1972 1973 1974 1975	1,4,8 1,4,8 1,4,8 1,4,8 1,4,8 1,4,8		O: ED 100 057 (1975) P: ED 104 945 (Wise et al., 1975) ED 118 608 (Handrick, 1975)

* Accountability legislation enacted as of June 1974.

** Code: N = Needs assessment; O = Objectives; P = Progress assessment
Where no author is given, document is in References under name of state.

Progress Assessment

State	*	Mandated	Conducted	Levels	n	ERIC reference number
District of Columbia						N: ED 104 902 (1972)
Florida	A		1971 1972-73 1973-74 1974-75	 3,6,9 3,6,9	 2,000 20,000 15,000 120,000 over 120,000	N: ED 100 045 (1972)
Georgia	A			4,8,11	240,000	N: ED 107 695 (1974) P: ED 078 121 (Barnes, 1973)
Hawaii	A		1965-1970 1970-71 1971-72	4,8 4,6,8, 10,12	 65,000	P: ED 074 441 (1971) ED 081 839 (1972)
Idaho				9,11		
Illinois	A					
Indiana	A	(not mathematics)				
Iowa	A		1975-76	5,8		P: ED 125 894 (Morrison, 1976)

Progress Assessment

State	*	Mandated	Conducted	Levels	n	ERIC reference number
Kansas	A		1970 ?	3,5,8,10,12 6,12		
Kentucky			1973 1974 1975	4 8 11 8 2 4 8 11	3741 3389 2702 7226 3981 7067 7076 6019	O: ED 081 793 (1971)
Louisiana						
Maine						
Maryland	A	1973-73 1974-75		3,5,7,9		N,P: ED 118 635 (1975b) 637 (1975a) 638 (1976)
Massachusetts	A	1971	1974-75	ages 9,17	17,600	N: ED 109 769 (1971)
Michigan	A	1969	1970-71 1971-72 1972-73 1973-74 1974-75 1975-76	 4,7 4,7 1,4,7 1,4,7,10	 320,000 +70,000 in gr. 1	O: ED 053 217 (1971a) ED 059 255 (1971b) ED 059 257 (1971c) ED 073 139 (Donovan et al., 1972) ED 104 897 (1973a) ED 104 898 (1972a) ED 104 899 (1973b) ED 104 966 (1972b)

Progress Assessment

State	*	Mandated	Conducted	Levels	n	ERIC reference number
Michigan continued						ED 104 967 (1974a) ED 111 832 (Roeber and Huyser, 1975) ED 120 216 (Donovan, 1973) ED 120 217 (Donovan et al., 1973) ED 120 218 (Mehrens, 1975) ED 120 219 (Fisher et al., 1973) ED 120 220 (1974b) ED 120 221 (1975b) ED 120 225 (Fisher et al., 1974) ED 120 226 (Roeber et al., 1974) P: ED 117 173 (1975a) ED 120 242 (1974c)
Minnesota			1974-75	age 9 13 17	12,000 17,000 16,000	ED 084 657 (Pyecha, 1973)
Mississippi			1971-7[illegible]	5,8		
Missouri			1970-71 1974	4 6	8,034 8,266	P: ED 070 056 (1971a) ED 077 990 (1971b)
Montana						
Nebraska	A			5		

Progress Assessment

State	*	Mandated	Conducted	Levels	n	ERIC reference number
Nevada	ref.		1971-72 1972-73 1973-74	3 3 4 3 5 7	 2392 2315 2420 2376 2750	N: ED 079 822 (Howard and Ogg, 1971)
New Hampshire			1967			N: ED 097 352 (Schweiker, 1974) P: ED 039 147 (Austin, 1969)
New Jersey	A	1972	1972-73 1976 1978	4,12 4 7 10 3,6,9 11	 96,000 109,000 109,000	P: ED 074 129 (1973) ED 097 396 (Gurwitz, 1974)
New Mexico	A	yes	1969 1973-74	5 5,8,12	 70,000	N: ED 077 938 (Klein, 1972) ED 079 422 (1973) ED 095 631 (1974)
New York	A	yes: Regents PEP	PEP-1966 on PEP-1973	3,6,9		P: ED 071 162 (1972) ED 080 591 (1973)
North Carolina		1973	1973-74 1974-75 1975-76	6 3 9	 5,000	N: ED 106 294 (1975) ED 108 974 (1974)
North Dakota						

Progress Assessment

State	*	Mandated	Conducted	Levels	n	ERIC reference number
Ohio	A					N: ED 096 745 (1973)
Oklahoma	ref.					
Oregon	A		1976	4	8,000	N: ED 109 207 (Thomas, 1975a) SE 022 559 (1976)
Pennsylvania	A		1970			O: ED 090 252 (Kendig, 1974) ED 093 943 (Coldiron, 1974) P: ED 068 471 (1971)
Rhode Island	A		1972 1973	4,8 4,8		
South Carolina				4,7,9,11		
South Dakota						
Tennessee						
Texas	A		1971	6	22,055	P: ED 079 879 (1972)
Utah	A	1974	1975	5 11	4,000 3,000	N: ED 079 825 (Campbell and Forsgren, 1970)

239

Progress Assessment

State	*	Mandated	Conducted	Levels	n	ERIC reference number
Vermont						
Virginia	A					
Washington	A		1971	4 6	6,763 6,881	P: ED 086 725 (Brouillet, 1973)
West Virginia				3,6,9,11		
Wisconsin	A	1971	1973 1969	3,7		O: ED 051 186 (Henderson, 1971) ED 069 475 (Henderson, 1973) P: ED 096 320/325/328 (1974a,b,c)
Wyoming						

GLOSSARY OF ACRONYMS

AAAS	American Association for the Advancement of Science
AACTE	American Association of Colleges of Teacher Education
ACE	American Council on Education
ACT	American College Testing Program
AIR	American Institutes for Research
AP Program	Advanced Placement Program, CEEB
CBMS	Conference Board on the Mathematical Sciences
CEEB	College Entrance Examination Board
CEMREL	Central Midwestern Regional Educational Laboratory
CSMP	Comprehensive School Mathematics Program
CUPM	Committee on the Undergraduate Program in Mathematics
EPIE	Educational Products Information Exchange
ERIC	Educational Resources Information Center
ESEA	Elementary and Secondary Education Act
ETS	Educational Testing Service
GCMP	Greater Cleveland Mathematics Project
HEW	Department of Health, Education and Welfare
IEA	International Study of Educational Achievement
IME	Investigations in Mathematics Education
IPI	Individually Prescribed Instruction
JRME	Journal for Research in Mathematics Education
CBTE	Competency Based Teacher Education
LEA	Local Education Agency
MAA	Mathematical Association of America
MMP	Mathematics Methods Project, Indiana University
NACOME	National Advisory Committee on Mathematical Education

NAEP	National Assessment of Educational Progress
NASDTEC	National Association of State Directors of Teacher Education and Certification
NCATE	National Council for Accreditation in Teacher Education
NCES	National Center on Education Statistics
NCER	National Council on Educational Research
NCTM	National Council of Teachers of Mathematics
NDEA	National Defense Education Act
NEA	National Education Association
NIE	National Institute of Education
NLSMA	National Longitudinal Study of Mathematical Abilities, SMSG
NSF	National Science Foundation
OMB	Office of Management and Budget
PRIMES	Pennsylvania Retrieval of Information on Mathematics Education System
SAFA	Security Areas Federal Assistance
SAT:M	Scholastic Aptitude Test: Mathematics
SEA	State Education Agency
SES	Socioeconomic Status
SMPY	Study of Mathematically Precocious Youth
SMSG	School Mathematics Study Group
SSMCIS	Secondary School Mathematics Curriculum Improvement Study
UICSM	University of Illinois Committee on School Mathematics
USMES	Unified Science and Mathematics in the Elementary School
USOE	United States Office of Education

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Notes:

1. ED numbers are given for ERIC documents where possible. If the acquisition of the document was too recent to allow complete processing, then the SE number of the document is given.

2. Although many dissertations were read in their entirety, citations are given in terms of Dissertation Abstracts (DA) or Dissertation Abstracts International (DAI), since that source is more readily available to most readers.

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