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ABSTRACT
                Abstracts of 28 research reports are provided. The
reports were prepared by investigators for presentation at the 56th
annual meeting of the National Council of Teachers of Mathematics. A
broad range of topics related to mathematics education is covered.
Ten reports deal with problem solving, four are concerned with
instructional methods, three with space and geometry, two with
calculators, two with measurement, and two with conservation. Other
papers deal with games, program evaluation, achievement prediction,
perception of motion in pictures, and learning difficulties related
to numeracy. (MN)

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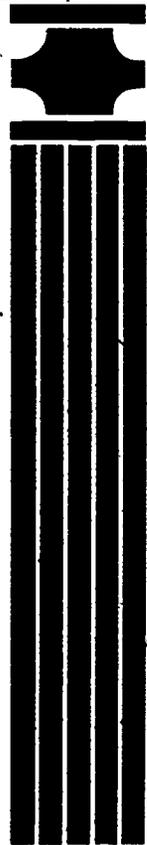
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San Diego, California
12-15 April, 1978

RESEARCH REPORTING SECTIONS

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

56TH ANNUAL MEETING

Jon L. Higgins, Editor

ERIC Center for Science, Mathematics,
and Environmental Education
College of Education
The Ohio State University
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PREFACE

The ERIC Information Center for Science, Mathematics, and Environmental Education has compiled abstracts of the research papers to be presented at the 56th annual meeting of the National Council of Teachers of Mathematics. Selection of the papers was made by Professor Jesse A. Rudnick, Temple University, and members of the Research Advisory Committee of the Council. Minor editing has been done to provide a general format for the papers.

Many of the papers that are abstracted here will later be made available through the ERIC system or published in journals. These will be announced in <u>Research in Education</u> or <u>Current Index to Journals in Education</u>.

March, 1978

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Jon L. Higgins
Associate Director for
Mathematics Education

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An Investigation of A Model of Children's Preferences with Respect to Topological and Geometric Figures

Albert O. Shar
University of New Hampshire

William E. Geeslin
University of New Hampshire

Purpose

,The purposes of this study were to: 1) test one aspect of Piaget's developmental model concerning children's perception of space; and 2) investigate an alternate model of perception. Piaget and Inhelder (1963) contend that young children view space topologically and discuss the child's spatial conceptual development in terms of a transition from the topological to the geometric. Past research in this area has produced contradictory evidence concerning Piaget's contention. Difficulties with previous studies include: inaccuracy and vagueness of mathematical terms; over-reliance on haptic tasks; instrumentation biases; and ignoring within age-level individual differences. Since children's spatial abilities appear to play a significant role in mathematics learning, the question of how children develop spatial concepts and/or perceive figures is an important factor in designing optimal elementary curricula. This research was intended to aid the development of a documented theoretical framework upon which mathematical instruction can be based.

Formal definitions of the terms "topologically equivalent" and "geometrically equivalent" were developed. These definitions appear to cover the intuitive distinctions made by previous authors, including Piaget. An alternate model of perception based on pattern recognition techniques was derived. The significant variable in this model is amount of "distortion." A test package consisting of ten figures and two variations of each figure, one topologically equivalent to the original figure and the other geometrically related to the original figure, was developed. According to the pattern recognition model, a child (or adult) will select the variant which represents the minimum distortion of the given figure. By comparing the relative distortions of the two variants, one can assign selection probabilities to each variant. These probabilities were determined mathematically prior to using the test package with any subjects. The items in the package range in prior probabilities of topological choice preference from approximately 0.1 to 0.9. Items were arranged randomly into four sequences. Similarly, the variants were placed randomly with regards to left/right location.

Procedure

Three hundred forty-five children from the nursery (age 3) level through fourth grade level (from local schools) were selected for testing. Each child was instructed on each item to select the variant most like the given figure. After the ten items had been administered, the child was interviewed concerning his reasons for making selections. To compliment this process and in order for us to be better able to determine key factors in the decision process of the children, a separate group of 50 children from the same levels was videotaped while responding to the items. These children were interviewed after each item response rather than after all ten items had been completed. The interviews were less controlled and ranged over more variety of topics.

Results.

Data analyses included comparison of number of topological selections by age-grade level and sex across the total test and across a subscale of items (the extreme probabilities were eliminated). Total test analyses indicated the pattern recognition model fit the data well (with the exception of the "star" figure). Subscale analyses indicate some support for developmental differences, but the strength of these differences was not in accord with Piaget's findings. Although the interviews are less amenable to statistical analysis, results indicated that children at all levels were well aware of the differences between the given figure and the two variants. Children varied within each level, in spite of observing similar figural elements.

Conclusions

Thus the study provides evidence that children's perception of space is far more complex than Piaget suggests. Individual differences were large at each level. Some children selected almost all the topological variants while others selected mostly geometric variants. Interviews indicated both types of children were consciously and "knowledgeably" making these choices. This indicates children have definite preferences about figures and/or the "correct" answers. Analyses of the test items also indicate that the test item itself does significantly affect children's selections. This no doubt played a role in causing previous studies to arrive at contradictory conclusions. In addition to test item, developmental, and distortion variables, it appears that familiarity, training, and test instruction variables can affect children's selections. Future studies should attempt to locate additional variables affecting children's perceptions. Likewise, it appears that these factors vary in importance. A hierarchy of perception factors is suggested. However, the interviews left us with the feeling that relative importance of the perception factors changes when extreme values are used for a particular factor.

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A Description of How Selected Seven-Year Children Learn
To Reason To Solve Partitive Division Problems

Linnea Weiland
Kean College of New Jersey

Purpose

This teaching experiment examined the procedures used by 21 seven-year old children to solve partitive division problems during discovery-oriented instruction. The goal was to observe and describe the strategies and arguments the children used, and the difficulties they encountered, as they learned to reason out solutions to problems using base ten blocks.

Procedure

The subjects of the experiment attended an independent elementary school in New York City. While the children understood the language of partitive division problems as well as numeration with base ten blocks, they had not received formal instruction in division. During 17 interviews over approximately a year each child solved 66 problems of varying difficulty. Problem difficulty was defined in terms of the size of the dividend and the number of exchanges involved. Divisors ranged from 2 to 9. Each child was asked to reason outloud as he solved the problems. The researcher did not instruct the child in methods of solution or in ways of overcoming difficulties. Running records were kept of the children's behavior during the interviews.

Three coding systems were developed to classify the children's reasoning in terms of (a) the strategies used, (b) the difficulties encountered, and (c) the overall arguments employed in solving the problems. Seventeen distinct strategies of problem solution were identified. These fell in three categories—representational, sequential, and distributional. Representational strategies described how the children represented the dividend; that is, in the standard or a nonstandard way. Sequential strategies referred to the order in which children exchanged and distributed; that is, in the standard order analogous to the standard distributive algorithm or in a nonstandard order. Distributional strategies described how the children distributed the dividend; that is, by concretely passing out objects one by one, or by more abstract methods, for example, guessing, using a measurement approach, halving, or using distributivity. The modes of representation used to express representational and distributional strategies were separately examined. Although the children most often used the concrete mode and expressed their strategies with base ten blocks, they did express some strategies mentally; that is, verbally without accompanying actions, and with pictures and numerals. The

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757 difficulties identified were classified in 20 categories. The frequencies of occurrence of the strategies, their modes of representation, and the difficulties across all the data were examined. To evaluate the effect of instructional time, a comparison was made of the data for five assessment interviews. To evaluate the effect of problem difficulty, a comparison was made of the strategies, modes of representation, and difficulties which occurred for each assessment combination. (The same six combinations representing the total range of problems given were repeated at each assessment.)

Results

The data on the children's reasoning were classified in five argument categories. The standard distributive argument was used most frequently by every subject. The four other arguments were each used at least once by over half the subjects and each accounted for between 3 percent and 5 percent of the arguments. One argument relied on distributivity; another, only appropriate to those combinations with divisors which are powers of 2, involved halving. The measurement argument involved subtracting off bunches the size of the divisor and counting the number of bunches thus made. The last argument relied on recognizing multiplication as the inverse of division.

The behavior of two of the subjects was described in detail. The contrast of these two children in their use of strategies and arguments, and in their reactions to difficulties was illustrative of the variation in sophistication among all the subjects.

The results of this research implied several modifications of the discovery-oriented teaching method. In addition, criteria for the evaluation of problem difficulty and sophisticated thinking about division were proposed, and some directions for further research were outlined.

Games As Aides to Retrieval and Maintenance of
Skill with Basic Multiplication Facts

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Northern Illinois University

Purpose

Gaming activities have been a source of entertainment and instruction for centuries. During the last twenty years, teachers have been enthusiastically encouraged to use an almost bewildering array of mathematics games. The empirical research reporting positive cognitive effects of games on mathematics learning, however, is virtually non-existent (Bright, Harvey & Wheeler, 1977).

Three studies were designed to investigate the effects of games on the retrieval and maintenance of mathematical skills. One of the experimental variations tested the effect of alternative game strategies. All three studies were set in a post-instructional context (Bright, Harvey & Wheeler, 1977).

The games used in these studies were MULTIG and DIVTIG (Romberg, et al., 1974, 1975, 1976) which are similar to CONTIG (Broadbent, 1972) at least with respect to the use of a gameboard, random generation of numbers, and scoring. Each of the studies incorporated the tournament portion of the TGT classroom management model (Edwards and DeVries, 1974). Initial assignment of students to three-person tables was random.

Format for the several instruments was consistent across the studies. For a 20-item test of basic multiplication facts, 10 items were randomly chosen from the 36 facts (4x4, 4x5, ..., 9x9) used in the games and 10 items were randomly selected from the remaining facts. The speed test included all 100 basic multiplication facts, randomly ordered. The placebo test consisted of twenty 2- and 3-digit addition items.

Study I: Retrieval of Basic Multiplication Facts Via Games

The hypotheses of Study I were as follows:

1.   Games are not effective in aiding in retrieval of skills in basic multiplication facts by intermediate grade students.

2. Administration of a basic facts pretest does not alter the effects of games.

3. There is no correlation between posttest performance and final tournament position when a modified TGT structure is imposed on game playing.

Procedure

Subjects were students in 14 classes in grades 4, 5 and 6 in. Rochelle and Rockford, Illinois, and in McFarland, Wisconsin. There were 348 students, but the unit of analysis was the classroom (N = 14).

The treatments were conducted during the first ten days of instruction for the school year. On Day 1 each student took a pretest. Half the students, randomly chosen, took a 20-item test of basic multiplication facts. The other half of the students took the placebo test. Fifteen minutes were allowed. On Day 2 each classroom teacher explained the game. On Days 3 through 9 the game was played for 15 minutes per day within the TGT tournament structure. On Day 10 every student took a 20-item posttest of basic multiplication facts. Both grade 4 classes played MULTIG. Grade 5 and 6 classes were randomly assigned to MULTIG or DIVTIG.

Results

All hypotheses were tested for the total test scores and for the subscores on the ten game-specific items. On the basis of Wilcoxon signed-rank tests, Hypothesis 1 was rejected for total test score ($p < .025$) and for game-specific score ($p < .01$). Hypothesis 2 was not rejected for any of 28 computed F-statistics. The test of Hypothesis 3 is not yet completed, but of the first 16 computed F-statistics, only two are significant at the .05 level.

Conclusions

The two games used in this study were effective in aiding the re-development of skill with basic multiplication facts. Games do provide a legitimate alternative to more standard kinds of drill and practice when used early in a school year to retrieve skills with basic facts. Further, there seems to be no effect caused by pretesting. That is, a pretest neither enhances nor detracts from the effects of the games. Finally, the study provides corroborative data suggesting that the assumptions underlying the TGT model are too simplistic and need to be re-examined.

Study II: The Effect of Alternative Game Strategies

The hypotheses of Study II were as follows:

1. Modifications of the strategy of a game do not have differential effects at the skill utilization level.

2. There is no correlation between posttest performance and final tournament position when a modified TGT structure is imposed on game playing.

Procedure

Students enrolled in grades 3 and 4 from Aurora, Illinois, were subjects. Two hundred fifty students from twelve classrooms used one of two versions of MULTIG to practice basic multiplication facts. In Treatment 1, students used the scoring rule usually associated with MULTIG: namely, count only. In Treatment 2, students used an alternative scoring rule, count-count-multiply.

Treatment 1: Count the number of covered diamonds that touch a side or a corner of the diamond you just covered. This number is your score.

Treatment 2: Count the number of covered diamonds that touch a side or a corner of the diamond you just covered. Count the number of uncovered diamonds that touch a side or a corner of the diamond you covered. Multiply these two numbers. The product is your score.

The treatments were conducted on ten consecutive school days in March 1977. On Day 1, each teacher explained one randomly assigned version of MULTIG. On Days 2 through 8, students played that version of MULTIG within the TGT model. On Day 9 every student took the 20-item power test as the first posttest. On Day 10 the second posttest, the 100-item speed test, was administered to every student. Fifteen minutes were allowed for each posttest.

Analysis

All the data have been collected, but the analysis has not yet begun. Statistical results will be available when the paper is presented.

Study III: Replication of Study I

Study III was conducted in fall 1977 as a replication and extension of Study I. An additional measure of learning was a speed test of all 100 basic multiplication facts. A limit of five minutes was imposed for this test. Results will be reported in the paper presentation.

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A Review of Research in the Learning of Geometric Transformations

Faustine Perham
Central YMCA Community College

and

Zalman Usiskin
University of Chicago

Ten years ago, if the writers' perusal of the research is accurate, there had been few studies in the United States which directly involved the learning of geometric transformations.

Since that time, a rather large number of studies, some indicated below, have been conducted which relate to this subject. They fall into five general types:

A. Natural Evolution of Transformation Geometry Concepts

Piaget and replicators   Moyer (1974)
Shepard and Metzler (1971)   McGlone (1974)
Huttenlocher and Presson (1973)   Martin (1976)

B. Role of Instruction in Learning Transformations-Piagetian-Style Tasks

Turner (1967)   Gardella (1974)
St. Clair (1968)   Kidder (1975)
Shah (1969)   Perham (1976)
Williford (1970)   Russian studies
Morris (1974)

C. Organization of Curriculum for Learning Transformations

Beard (1968)   Shilgalis (1971)
Usiskin (1971, 1975)   Klein (1972)
Olson (1970)   Nuspl (1975)
Hoban (1970)

D. Effects of Learning Transformations

Usiskin (1969, 1972)   Herot (1976)
Kort (1971)   Pitcher (1976)
Solheim (1971)

E. Teacher Attitudes Towards Transformations

Gearhart (1974)   MacDonald (1975)

This session reviews the above research and more recent studies, and due to lack of time and space concentrates on the interpretation of groups of studies. Some attention will be given to those studies (e.g., Kidder, 1975; Solheim, 1971) whose results seem to contradict the results of others.

Some attention is given to foreign work, both of a psychological and of a curricular nature. Of necessity we are brought to this work by Piaget, but others (e.g., the van Hieles, 1959; Freudenthal, 1959) also lead us in this direction.

The goals of this session are (1) to summarize the research in the learning of geometric transformations, (2) to compare and contrast the curricular and psychological studies and discuss the implications which the sociological studies have for researchers in each of the other areas, (3) to suggest avenues for future work, and (4) to generate discussion about all of the above.

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Interest and Its Relationship to Verbal Problem Solving

Martin P. Cohen
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Purpose

The purpose of this study was to investigate the relationship between interests and verbal problem-solving achievement among secondary school mathematics students. More specifically, it was designed to examine tendencies for students to be more successful in solving verbal problems based on situations for which they possessed measured interest than in solving verbal problems based on situations for which they possessed little measured interest.

Impetus for this study developed from the following assumptions:

1. Verbal problem solving is an important function of secondary school mathematics.

2. Verbal problem-solving achievement may be enhanced by the construction of verbal problems whose situational embodiments serve as motivational devices.

The results of studies attempting to secure evidence of a relationship between interests and verbal problem-solving achievement have been inconsistent. Some demonstrate either no relationship or an insignificant relationship. Some others indicate a moderate positive relationship. The lack of a clear, consistent definition of interest is undoubtedly a contributing factor to these contradictory results. However, advances in techniques of measurement and experimental design and the appearance of a carefully researched instrument such as the Kuder General Interest, Form E, designed specifically for secondary school students, should serve to alleviate this problem.

Procedure

The Kuder General Interest Survey (GIS), Form E, designed to measure an individual's preferences in ten broad areas of interest, was administered to 223 eighth grade mathematics students (114 males, 109 females) in the Waco (Texas) School District. The three interest areas utilized in this investigation are outdoor, computational, and scientific.

Three parallel forms of a verbal problem-solving test, corresponding to the interest areas of outdoor, computational, and scientific, were constructed by the investigator. Each form consisted of ten verbal problems. It was desired that only one feature of the problems

on the three parallel forms, that of the context in which they were
placed) would vary. In other words, the first problem in each of
the three forms was similar except for context. Likewise, this pro-
cedure was used for problems two through ten. Care was exercised to
control for other aspects of the "equivalent" problems (e.g., reading
level, verbal clues, mathematical operations involved, computational
difficulty).

Separate norms are established for males and females on the GIS.
Hence, students were randomly assigned, by sex, to each of three pro-
blem settings (outdoor, computational, and scientific). The verbal
problem-solving test, with context reflecting outdoor interest, was
administered to those students who had been randomly assigned to the
outdoor problem-setting group. A similar procedure was followed for
those students randomly assigned to the computational or scientific
problem-setting groups. Reliability coefficients (KR-20) for the
outdoor, computational, and scientific verbal problem-solving tests
were .76, .79 and .79, respectively.

In designing this study, the investigator sought answers to the
following questions:

Question 1: For males or females, will there be a difference
in mean scores on the three verbal problem-solving tests?

Question 2: Based on the knowledge of a student's interests
alone, it is possible to predict on what type (context) of
problems that student will be most successful as measured
by a verbal problem-solving test?

Analysis

In order to answer the questions, hypotheses were formulated and
tested by multiple linear regression. The specific hypotheses tested
were:

1. The mean scores on the verbal problem-solving test across
   problem settings will not be significantly different.

2. When scores on the verbal problem-solving test are
   regressed on an outdoor interest variable; the regression
   lines across groups (problem settings) will not be parallel.

3. When scores on the verbal problem-solving test are
   regressed on a computational interest variable, the
   regression lines across groups (problem settings) will
   not be parallel.

4. When scores on the verbal problem-solving test are regressed
   on a scientific interest variable, the regression lines
   across groups (problem settings) will not be parallel.

Each of the above hypotheses was tested twice, once for males and once for females. The investigator chose a significance level of .05 for all stated hypotheses.

Conclusions

The answers to the questions relative to the conditions of the investigation are discussed below.

Answer to Question 1: The mean scores on the three verbal problem-solving tests were not significantly different for males or females.

Answer to Question 2: There was no evidence in the data obtained to support the expectation that it would be possible to predict on what type (context) of problems a student would be most successful as measured by a verbal problem-solving test if prior knowledge of his interests were known.

In view of the related research and the literature on motivation, it was somewhat surprising not to find affirmative evidence for Question 2. It may be the case that if interests serve as motives, they tend to be very weak as predictors of verbal problem-solving achievement.

For future research in the area of this investigation, it is recommended that additional work be conducted in developing the verbal problem-solving tests. More evidence is necessary to claim that the tests actually do reflect a specified area of interest.

It is the opinion of the researcher that interest areas in which students have had more hands-on experience (e.g., sports, auto mechanics, music) will serve as stronger motives than those used in this study. It may be necessary to use interest measures other than the GIS. A suggested test may be Ewen's Activity Experience Inventory.

Inventory

With appropriate changes, future investigations may illustrate the usefulness of designing instructional materials to suit students' interest profiles in order to facilitate the development of verbal problem-solving skills.

A Strategy for Inducing Reclassification of Problem Types According to Heuristic Schemes.

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Massachusetts College of Pharmacy

Purpose

This research study investigates changes in problem solving schemes that occur as a result of a course in heuristic problem solving. Recent studies by Chartoff (1976) and Silver (1977) have isolated a number of problem sorting schemes including: the heuristic sort, the generic sort, the contextual sort and the question-posed sort. Silver's research seems to indicate that those students who sort according to heuristic cues are more efficient problem solvers than those who sort otherwise. If such evidence is correct, then teaching students to solve problems heuristically may well increase performance in problem solving.

An examination of current high school and college texts reveals that for the most part problems are categorized with respect to content (mixture problem, age problem etc.) or algebraic type (quadratic equation problem, system of linear equations problem, etc.). Students are then encouraged to look for cues in problems that will allow them to place them in one of the above categories. A major criticism is that no instruction is given in how to conceptualize or approach problems that do not closely resemble previously encountered problems. The heuristic approach, on the other hand, emphasizes problem solving strategies rather than specific problem types. Strategies include arriving at a contradiction, specializing, generalizing and establishing patterns, to name a few.

Procedure

This study has both an instructional and a research component. Involved in the instructional component is the design of a set of materials used in the teaching of a 10-week course in heuristic problem solving. The course is organized around three heuristic strategies: (1) Using the symbolic language of algebra as a problem solving tool, (2) Establishing and continuing patterns as a way to solve problems and (3) Using contradiction as a method of problem solving. The research component involves the analysis of data obtained from a class of freshman students enrolled at the Massachusetts College of Pharmacy in Boston who took this course. The basic questions investigated by the study are: (1) does the proposed course improve problem solving performance? (2) does the course effect changes in problem sorting schemes? (3) are changes in problem sorting schemes related to increased problem solving performance? (4) do students with

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certain patterns of abilities (i.e. convergent and divergent thinking and deductive reasoning) benefit most from such a course? (5) does the course increase performance on basic mathematical skills?

The research design is the Solomon four group design as described by Campbell and Stanley (1963). The subjects (n = 100) were a group of freshmen students enrolled in a required sequence of college algebra courses at the Massachusetts College of Pharmacy. These students could be described as primarily middle income students of average mathematical ability residing chiefly in the New England area. The control group studied the traditional course in college algebra and trigonometry. Pre-testing was of two types: pre-testing of initial "abilities" and pre-testing for initial problem sorting schemes. The pre-tests for initial ability consisted of five tests from the "Kit of Factor Referenced Cognitive Tests" developed by Ekstrom, French, Harman and Dermin. A specially designed problem solving test and accompanying problem sorting questionnaire was also administered as a pre-test. The post-test consisted of a 30-item test of which 15 items were on specific mathematical content and 15 items were on problem solving. All students also completed the problem sorting questionnaire at the end of the 10-week period. Multidimensional scaling was used to analyze the data from the problem sorting questionnaire and multivariate regression analysis was the statistical means of relating the studied factors (initial problem sorting scores, cognitive test scores, treatment vs. control) to the response variables (post-test problem solving scores and post-test mathematical content scores).

Results

Results will be reported when the paper is presented.

An Investigation of the Hand Calculator As a
Mathematical Problem Solving Tool

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Purpose

The hand-held calculator is purported to have tremendous
potential for the development of problem solving abilities. Among
the studies currently found in the literature, however, little
research related to use of the calculator in problem solving is
present; almost all studies relate to attitudes, achievement and
computational skills. The NACOME report (1975), Shumway (1976),
and Suydam (1976) suggest that the calculator may have the potential
to "free a subject's mind" to focus on the reasoning processes and
problem solving strategies. If this is true, the immediacy and
generality of outcomes could still be seen in attitudes, achievement,
and computational skills. Hilton and Rising (1975) however, already
point out that "singular dogmatic answers" and "irrational reactions"
have and will continue to focus attention away from the use of cal-
culators in school settings. Suydam (1976) indicates an immediate
need for research as well as development, in order to mitigate against
criticisms and in order to determine the extent and nature of the
calculator's potential as a problem solving tool.

To investigate the development of problem solving abilities,
analytic examinations of the development of mathematical processes
as opposed to products is frequently cited as critically important
(Kilpatrick, 1967; Suydam, 1976; Scheffler, 1975; Kantowski, 1974).
This focus on process is not only appropriate for investigating
problem solving but also is necessary in exploratory studies
involving the calculators as well. As Suydam (1976) and the NACOME
Report (1975) suggest, the calculator virtually assures the product
in the treatment of problem solving provided accurate reasoning and
analysis of the problem leading to the development of process precede
the calculations.

Procedure

In view of the need and the state of research on the calculator
as a problem solving mediator, a combined experimental and clinical
study was carried out. This study employed a Pretest-Posttest
Control-Group Design involving an experimental group and two control
groups. Further, selected subjects from each group were clinically
studied during the experiment to determine the extent and nature of
the development of mathematical processes. The three groups, con-
sisting of from 9 to 16 subjects each, are defined as follows:

G_1 is defined as those subjects who were given specific instruction in: 1) The Texas Instruments produced Introductory Algebra Calculator Mathematics, 2) selected heuristic reasoning techniques, 3) inductive reasoning techniques, and 4) analytic-synthetic reasoning, along with 5) continued content instruction.

G_2: Those subjects who were given specific instruction and practice with: 1) specific heuristic reasoning techniques, 2) inductive reasoning techniques, and 3) analytic-synthetic reasoning, along with content instruction, but no work with the calculator.

G_3: Those subjects who were given specific instruction in the use of the calculator for learning Algebra I, including the Texas Instruments produced Introductory Algebra Calculator Mathematics.

The treatment involved topics taught during the last three months of Algebra I. Initial pretest data was collected in mid-March, 1977 and posttest data was collected in mid-June, 1977.

Dependent measures of content knowledge and selected "reasoning" abilities were used to collect data to assess differences among groups. They included:

1. Lankton First-Year Algebra Test,

2. Necessary Arithmetic Operations (ETS),

3. Number Sequence Test, and

4. Nonsense Syllogisms Test (RL-1 by ETS).

More critically, selected subjects from each of the three groups were periodically given "typical Algebra I" word problems and complex novel problem to solve in a clinical setting. Tape recordings of their "thinking aloud" were gathered for protocol analysis. The tentative answers to several questions were sought through the analysis of these protocols. An example of the questions is: Do process sequences exhibit increased frequencies of the use of estimation and successive approximation as problem solving abilities develop under calculator mediation?

Data were analyzed through analysis of covariance, with the pre-test data being used as the covarate. Additionally, protocols were analyzed through the process coding scheme developed by Kilpatrick (1967) and refined by Kantowski (1974). (A slight modification of this scheme was necessary for this particular study.) The protocols arose from the administration of six verbal problems to eight subjects from each of the three groups at the outset of the study and from the administration of six verbal problems to the same subjects at the end of the experimental period. Data arising from the protocol analyses were used to compare the individual's performance on a particular problem with his/her own process-product median score as well

as to compare performances of an individual and a group with the process-product scores of other groups.

Results

Results of the analysis of covariance for Algebra I achievement indicate significant differences among treatment groups. The F-ratio, $F(2,31) = 4.49$ is significant at $P < .05$. Adjusted mean scores were: 32.27 (G_1); 27.20 (G_2) and 30.55 (G_3).

Results of the analysis of covariance for Necessary Arithmetic Operations did not indicate significant differences among treatment groups. The F-ratio was $F(2,31) = 2.11$. Adjusted mean scores were: 14.27 (G_1); 12.55 (G_2) and 15.27 (G_3).

Results of the analysis of covariance for the Number Sequence Test do not indicate significant differences among treatment groups. The F-ratio was $F(2,31) < 1$. Adjusted mean scores were: 12.48 (G_3); 13.20 (G_2) and 12.74 (G_3).

Results of the analysis of covariance for the Nonsense Syllogisms Test indicate significant differences among treatment groups. The F-ratio, $F(2,31) = 11.14$ was significant for $p < .01$. Adjusted mean scores were: 16.42 (G_1); 11.46 (G_2) and 16.84 (G_3).

The results of protocol analysis indicate trends toward greater usage of specific heuristics under a combined problem-solving and calculator treatment. For those subjects in G_1 and G_3 who showed increased frequency in the use of specific heuristics, there was a corresponding increase in median process-product scores and complete solutions on the final battery of tests. The questions that guided the analysis of protocol data along with tentative answers to those questions based on the data is included in the full report of the study.

Conclusions

Within the context of this experiment, a combination of calculator mediated and problem solving oriented Algebra I instruction appears to produce greater improvement in Algebra I achievement than problem solving oriented instruction alone. In contrast, however, on the variable that has been designated as more closely related to deduction, significant differences favored treatment groups in which problem solving oriented instruction was provided. Protocol analyses produced evidence of trends toward improved usage of heuristics and more formal reasoning strategies. However, the data from protocol analysis did not provide evidence of marked improvement in completed solutions nor of different effects on problem solving processes resulting from the different treatments.

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The Evaluation of A First-Year Algebra Program: Fundamental
Issues in Educational Evaluation

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and

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Purpose

During the 1976-77 school year, the National Science Foundation
sponsored a field evaluation of the experimental first-year algebra
materials, ALGEBRA THROUGH APPLICATIONS. These materials were
developed over a two-year period by Zalman Usiskin under a grant
from NSF. In them, the usual skills and concepts are developed
through applications and models rather than from the properties.
The traditional skills associated with first-year algebra are pre-
sented but with the following exceptions: factoring of polynomials,
fractional expressions and simplification, and artificial word pro-
blems. In their place, greater attention is given to operations,
linear expressions, sentence solving, and problems arriving from
real situations. Elementary notions from probability and statistics
are integrated into the course. The course is designed for the
average student as a substitute for the traditional first-year
algebra course.

Procedure

Twenty (20) schools throughout the United States were selected
from volunteer schools on the basis of a geographic and community-
size distribution. The schools ranged geographically from California
to New York and from Urban centers to rural pockets. Each school
selected to participate was contacted in the summer of 1976 and
requested to submit the names of two equally-capable teachers, both
of whom would be willing to teach the experimental materials. By a
random process, one of the two teachers was selected as the experi-
mental teacher. Each participating school was also requested to
provide four first-year algebra classes, two control classes, and two
experimental classes. Schools were further requested to randomly
assign students in these four classes to their particular section.
The two classes assigned to the experimental teacher constituted the
experimental classes. These were taught using the experimental
materials which were provided at no cost to the school by NSF. With
the exception of a teacher's guide, "Notes to the Teacher," no guidance
or in-service was provided to the experimental teacher. The control
teacher taught the two control classes using whatever first-year
algebra materials were normally used in the school. In all, there
were 2,446 students participating in the study.

In the fall of 1976, the following four tests were administered to all classes participating in the study:

1. A 25-item, Likert-type Opinion Survey developed by the project staff incorporating items from Aiken, NAEP, and others;

2. The Mathematics Computation sub-test of the Stanford Achievement Test: Advanced Battery, Form A (1973);

3. The ETS Cooperative Mathematics Test: Algebra I, Form A (1962); and

4. A 28-item Consumer Test developed by Kepner.

In the spring of 1977, the following four tests were also administered to all classes:

1. A 25-item Opinion Survey containing items from the fall Opinion Survey together with items modified to focus specifically on algebra or the algebra text;

2. A Consumer Test, Form A or B, each consisting of 11 and 10 problems, respectively, from the fall Consumer Test;

3. The EST Cooperative Mathematics Test: Algebra I, Form A (1962); and

4. A First-Year Algebra Test developed by Kepner.

The computation test and ETS test were given in the fall in order to determine the relative ability levels of the students and the equivalency of the control and experimental classes. The ETS Algebra I Test was given as a post-test measure of achievement on many of the objectives common to a broad spectrum of first-year algebra courses. The First-Year Algebra Test was developed to measure achievement on those objectives of the control and experimental materials not measured by the ETS Algebra I Test. The Consumer Test was used to compare improvement in the consumer problem-solving skills of regular first-year algebra students with those presented an applications orientation. The consumer problems themselves did not require algebraic skills. The Opinion Survey was administered to monitor changes in attitude relative to the enjoyment and usefulness of mathematics, as well as to obtain feedback from the students on the texts. All tests were administered by the classroom teachers according to a schedule suggested by the evaluators.

In addition to student testing, a site visit was made to each participating school, textbook evaluation forms were completed by both control and experimental teachers, and end-of-chapter reports and chapter tests were submitted by the experimental teachers.

Results

Due to incomplete data, only seventeen schools were retained in the final achievement analysis. In the across-school analysis, t-tests for 17 matched pairs (control matched with experimental in each of 17 schools) showed no significant difference between the two treatments on the Stanford Achievement Test and ETS Algebra I Test administered in the fall and again on the ETS Algebra I Test and First-Year Algebra Test administered in the spring. School-by-school t-test analysis yielded significant differences in favor of the experimental group in 8 of the 17 schools on the First-Year Algebra Test, and in favor of the control group in eight schools on the ETS Algebra I Test. Item analysis of two achievement post-tests across schools showed significant differences favoring the experimental group on 13 items on the First-Year Algebra Test and significant differences favoring the control group on three items on the First-Year Algebra Test and 16 items on the ETS Algebra I Test. Overall, achievement between the control and experimental groups was comparable when the entire spectrum of objectives and school situations is considered.

Attitude data was analyzed by item across 19 schools. In the fall, there was a significant difference between the responses of the experimental and control groups on only one of 25 items in the survey. In the spring, there was a significant difference favoring the experimental on four items and favoring the control on one item. The four items dealt either with the importance of algebra in everyday life, the enjoyment of word problems, or the explanations in the text. From fall to spring, there was a decline in attitude in both groups on 10 out of 19 repeated items, with the experimental group experiencing a significantly greater decline on one item. Overall, the control group found algebra more interesting while the experimental group enjoyed word problems more.

The Consumer Test data was also analyzed by item across 19 schools. Gains from fall to spring showed a significant difference in favor of the experimental group on five items and in favor of the control group on two items. The experimental group showed a significant decline on one item. Overall, the performance of first-year algebra students on consumer-related problems was disappointing.

Fundamental to any curriculum evaluation are a number of basic issues in educational research concerning a fair yet unbiased testing of experimental materials. These include:

1. How do you get schools and teachers which are truly representative to test new materials?

2. How do you insure that teachers teach in the spirit of the new materials without extraordinary assistance and in-service?

3. What degree of control can the evaluator exercise over participating teachers and still maintain an authentic classroom setting?

4. Does the first year of use of a set of new materials constitute a valid test?

5. What is the magnitude and effect of the status quo bias on the success of experimental materials?

Results of the present evaluation project must be interpreted in light of a discussion of these questions and with a consideration of the unique circumstances in the schools involved.

Pocket-Sized Calculators Versus Seventh Grade Math Students

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Purpose

According to a recent report in Changing Times magazine, due to nominal pricing, pocket-sized electronic calculators have put remarkable problem-solving capabilities in the hands of millions of citizens.

This phenomenon was not taken lightly by the Mathematics Education Action Research Center (MEARC) of Temple University.

First of all there is little available evidence to support the contention that these small computing devices make a difference in terms of how young students function in basic arithmetic computation.

Under the direction of Dr. Jesse A. Rudnick of the MEARC staff a cooperative research project commenced during the fall of 1975. This research project was planned to extend a one-year period and to include a mutual participation from industry, the University and a public school system. The major question of concern was - What impairment if any do calculators pose for students learning arithmetic?

Consequently, the Monroe Calculator Company became involved and made approximately $10,000 worth of their hand-held mini-calculators available. The West Chester area public school system involved six classroom teachers and more than 700 students in their seventh grades as a population for the study.

The calculator study was initiated to determine what impact if any the availability and use of a hand-held calculator would have upon the achievement and attitude toward mathematics of seventh grade students.

Specifically the study was designed to answer the following questions:

1. Would the availability and use of a hand-held calculator, over time, affect the students ability to perform the basic computational skills without the calculator?

2. Would the availability and use of a hand-held calculator, over time, affect the students' attitude toward mathematics?

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3. Would the availability and use of a hand-held calculator, over time, affect the students' overall achievement in mathematics?

4. Is there any relation between student dexterity and the students' achievement with the calculator?

In addition the study attempted to assess the attitude of parents toward the inclusion of the calculator into school programs. More specifically:

5. What are the attitudes of parents toward the use of a calculator by their children in the school mathematics class?

6. What change, if any, occurs in parent attitude toward the inclusion of a calculator in the school mathematics classroom after the parent's child has used the calculator for an entire school year?

Procedure

The MEARC staff organized the study, involved the West Chester faculty in the planning, and was responsible for the collection, interpretation and dissemination of the data and findings under the direction of Drs. L. Waldo Rich and David Kapel.

Students participating in the study attended the North Junior High School and the Stetson Junior High School. Approximately one-half of the seventh grade students in these schools were assigned to the experimental group, while a similar number were designated as a control group. The selection and assignment was done randomly by the school administration under the direction of Dr. Bruce Burt, Director of Mathematics. Each of the six teachers participating in the study had four classes: two experimental and two control.

The experimental group had mini-calculators available to them each day for use in their mathematics classes during the fall of 1975 and spring of 1976. During the summer 1975 the MEARC staff, under the direction of Drs. Krulik, Wilderman and Mrs. Nola Blye, developed a training manual for the calculators.

The curriculum for both the experimental and control groups was constant, namely, the topics which the district felt could be covered in the seventh grade Holt School Mathematics text.

After a brief period with the calculators at the beginning of the school year, fall 1976, the students were "on their own" as to how and when they would use the mini-calculators. The students kept a log of when and for what operations he or she used the calculator during the day.

All the students were pre-tested in several important areas. One question under investigation was whether or not manual dexterity enters into the eye-hand coordination. Since this can easily affect the student's use of the calculator, the MEARC staff utilized a form of the Gestalt Bender Test to measure this dexterity, and hoped that the study would reveal some correlation between these factors and the use of the calculator.

Since student attitude toward mathematics is an important factor in performance, students were tested on a pre-post basis to determine any change in their attitude. For this purpose, a form of the attitude instrument found in the International Study of Achievement in Mathematics was employed (Husen, 1967).

The parents of all the students in both the experimental and control groups were surveyed for attitude and all of the mathematics teachers in the West Chester schools were also queried in this area.

Standardized tests in both problem solving and computational skills were administered to participating students by the West Chester faculty, and the same tests were given at midsemester and again at the conclusion of the study in an attempt to determine what effect the availability and use of the calculator had upon the computational and problem solving skills of the students. Specifically, the COOP Test of Arithmetic Skills (ETC) was administered to both the experimental and control groups before, halfway through, and after the school year 1975 to 1976. Three parallel forms were employed. Fifty items comprised this test which are distributed unequally in problem solving and arithmetic computation.

A computation test designed by the West Chester School District comprised of 26 items dealing with basic computation facts was administered at the end of the year permitting those students who received the calculator treatment to use their calculators during the test while the control group could not.

The research design was established to permit pre-interim-post analysis of all data collected on main effect variables related to achievement and attitude. Provisions were also made for the collection and evaluation of summative data. Test analysis included t-tests, Pearson correlations and analysis of variance. All computations were facilitated by Temple University CDC 6400 computer.

Results

Although the work of the MEARC staff continues it is felt reasonable and honest to report the following results as sound and conclusive.

Analyzing the results on an administered computational test to both experimental and control groups and allowing the experimental group (the group which learned how to use the calculators) to take the test with a calculator, it was found that the groups did not appreciably differ in arithmetic achievement and computational skills. Further, the MEARC staff feels on good grounds to conclude that calculators don't harm children. It can further be offered that total performance differences leaned in the direction of the group (experimental group) which used the calculator.

Although some parents felt consternation at the outset, their attitude moved in a positive direction at the end of the study.

Slight positive correlations were uncovered with relation to dexterity and achievement indicators; however, these correlations were too weak to merit conjecture.

The correlations between control and experimental group in all areas of achievement were high, which suggested that the performance of the control group was not greatly dissimilar to that of the experimental group.

Further, it was found that at the end of the study both groups improved across the study; but surprisingly enough, the improvement was at the same rate.

In terms of student attitude, the control group leaned in the direction of positive change from beginning to end while the experimental group seemed to be leaning in a negative direction. This could be explained by the extra work it took to acclimate students to calculators, associated with a self-fulfilling prophecy on the part of students that they "must" use the device. Students were informed that using or not using calculators was their decision. However, this observation was not supported by statistical significance and so should not be taken out of the context of the total data to which it was a part.

Finally, one popular viewpoint suggests that aids to computation stifle the understanding of basic arithmetic processes. Yet another, asserts the divine principle that with calculators our boys and girls will be saved from the wrath of math. However, this study rather conservatively suggests that calculators neither exceptionally enhance nor do they hinder growth in mathematics. To say the least, some learn with and some learn without calculators.

The Nature of Spatial Ability and Its Relationship to
Mathematical Problem Solving

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Purpose

Interest in problem solving has clearly grown in the past five years. Research studies have attempted to: (1) refine the definition of problem solving; (2) evaluate the complex strategies involved in problem-solving behavior; and (3) assess the abilities needed to be successful in problem-solving situations.

This investigation attempted to analyze one specific ability that has arisen as a possible factor in successful problem-solving behavior, namely spatial ability. The investigator attempted to answer the following general questions:

1. In analyzing the intellectual abilities of an individual using existing instruments, can a distinct, indivisible quantity, called spatial ability, be detected?

2. Is there a significant relationship between spatial ability and problem-solving performance?

3. What effect, if any, will instruction in certain perceptual techniques have on spatial ability? On problem-solving performance?

Procedure

The study consisted of two phases. The first phase explored the nature of spatial ability and how it related to problem solving, based on pretest data information. The second phase was an instructional phase, followed by a posttest period used to assess the effects of the instruction.

The study was carried out over an 11-week period during the first semester of the 1976-77 academic year. The sample population consisted of four intact fifth-grade classrooms, totaling 145 students. All four classes were given the pretest and posttest batteries; two of the four classes were randomly selected as experimental classes and received the instruction in perceptual techniques.

The pretest and posttest batteries were the same set of six tests. Five of the tests were spatial ability tests that had been

selected after an extensive review of factor analytic studies and after pilot testing to determine if the tests were appropriate for fifth-grade students. The five spatial tests chosen were: Punched Holes Test, Card Rotations Test, Form Board Test, Figure Rotations Test, and Cube Comparisons Test. The sixth test was a Problem-Solving Inventory designed by the investigator. This inventory consisted of ten problems: four spatial problems (i.e. problems solved most efficiently by pictures, graphs, diagrams, tables, and lists), three analytic problems (i.e. problems solved most efficiently by computational techniques and number sentences) and three equally spatial and analytic problems.

The spatial ability score was computed by summing the z-scores on the five spatial tests. The problem-solving performance score was computed by the number of correct responses to the ten problems on the Problem-Solving Inventory. The degree of visuality score (an indication of how much an individual uses pictures, graphs, tables, etc. in solving problems) was computed by assigning a score of 0, 1, or 2 for each item on the Problem-Solving Inventory and then summing across all ten items.

The pretest data collection occurred during the first week of the study. The instructional period lasted for nine weeks. The posttest data collection occurred during the 11th week.

In addition, several EEG measures were taken in order to corroborate the results: The EEG measures brain hemisphere activity, where left hemisphere activity is associated with analytic thinking and right hemisphere activity is associated with spatial thinking.

Results

1. Four of the five spatial tests hung together as an indivisible construct based on: (a) Pearson product-moment correlations between each other; (b) factor analyses; and (c) regression analyses. The fifth test, namely Cube Comparisons, did not correlate significantly with the other four tests and came out on a separate factor. The EEG data served to confirm the author's hypothesis concerning this finding; the other four tests were pure spatial tests (right hemisphere activity) while Cube Comparisons could be solved analytically (left hemisphere activity).

A second and third administration of seven spatial tests (the five original tests plus two additional tests) indicated that spatial ability was an indecomposable ability, contrary to previous factor analytic results showing that spatial ability could be separated into two components.

2. Spatial ability was found to correlate significantly with problem-solving performance, and, to a lesser extent, with degree of visuality;

however, problem-solving performance and degree of visuality did not
correlate significantly (based on Pearson product-moment correlations).
An individual with high spatial ability will do well in problem-
solving situations but will not always use visual solution processes.
Again, EEG data confirmed the author's hypothesis about this finding.
The EEG data indicated that the high spatial ability individual may
not be writing down visual images, but they are taking place in
his brain, as the right hemisphere activity indicated. In other
words, the high spatial ability individual may not find the need to
write down these visual processes. Since the degree of visuality
score was based on written work, the statistical results are not
surprising.

3.) The classification of problems as spatial, analytic, or equally
spatial and analytic is a meaningful and consistent characterization.
The ten problems in the Problem-Solving Inventory were originally
classified according to individual oral interviews in a pilot testing
situation. The problems were re-classified, judging a problem to
be spatial if; (a) it has a high mean degree of visuality score based
on written work; (b) its mean score correlates significantly with
spatial ability; and (c) EEGs show much right brain hemisphere activity
by students trying to solve the problem. In general, problems
classified as spatial using one definition were found to be spatial
by all four definitions.

4.   The instruction affected significantly spatial ability and
problem-solving performance on spatial problems, but did not affect
degree of visuality (all based on analyses of covariance, using
pretest measures as covariates, in order to factor out initial
differences between control and experimental classes). This finding
indicates that spatial ability is a modifiable quantity (contrary to
research studies claiming that spatial ability is genetically deter-
mined). The finding also indicates that problem-solving performance
can be improved by spatial instruction; it would behoove mathematics
educators to consider including spatial instruction as part of the
regular mathematics curriculum.

No differences were found in instructional effects between
males and females or between high and low spatial ability students.

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An Exploratory Study of Fourth Graders'
Heuristic Problem Solving Behavior

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Purpose

The primary purpose of this investigation was (1) to study how
young children (fourth graders) attempt to solve word problems, and
(2) to study whether young children can use heuristics appropriately
and effectively in solving word problems. Heuristics were considered
as questions or advice a problem solver may conscientiously consider
when solving a problem.

Conceptual Framework

This study considered three aspects of an educational situation:
when to teach, what to teach, and how to teach. Bruner (1963),
Piaget (1964), and Polya (1957) emphasize that the learning situation
should be centered around the learner, particularly, the learner's
cognitive developmental stage. Polya's heuristics (1957) were adapted
considering the developmental stage of the subjects (concrete opera-
tional) and the problems found in the fourth grade textbooks. Polya's
teaching method (1957), imitation and doing, was used. The clinical
method was chosen for this study. This method allowed the investigator
to observe each subject involved in problem solving and to ask probing
questions intended to uncover the inner dynamics of the process of
problem solving.

Procedure

The investigator selected two groups of subjects, experimental
and control. The 16 subjects were selected on the basis of 1) two
Piagetian problems, Equilibrium in the Balance and Oscillation of a
Pendulum, and 2) teacher's recommendations. Eight of the 16 subjects
were recommended as average achievers and satisfied Piaget's criteria
of II-A cognitive level on both problems. The other eight subjects
were recommended as high achievers and satisfied Piaget's criteria
of II-B cognitive level on both problems.

The experimental groups, four II-A and four II-B, received 20
sessions of instructions in the use of heuristics when solving word
problems. Each instructional session lasted about 45 minutes and
was completed in eight weeks. Two pre-experimental interview problems
and six post-experimental interview problems were given to all 16
subjects. Four weeks after the post-experimental interview, two word

problems were given to the experimental subjects as part of the
delayed post-experimental interview. Each interview session was held
in an individual clinical setting where audio recordings, student
worksheets, and investigator remarks were collected. These were the
basis of the quantitative and qualitative analysis of the study.

Results

In the post-experimental interview, the experimental groups solved
35 of 48 problems successfully or 73 percent. The control groups
solved only 3 of 48 problems or 6 percent. The delayed post-experimental
interview indicated that the experimental subjects retained the appro-
priate usage of heuristics in solving problems. Nearly 80 percent of
the problems were solved successfully.

The control groups showed no change in frequency of usage of heur-
istics from pre- to post-experimental interview. The experimental
groups' use of heuristics increased noticeably from the pre- to post-
experimental problem solving. The experimental subjects were able to
select an appropriate heuristic for nearly all the post-experimental
interview problems. There was a clear difference in the usage of heur-
istics between the II-A and the II-B experimental groups. A more
detailed reporting of results, particularly qualitative results, will
be included in the presentation of the paper.

Since this study was exploratory, definitive conclusions cannot
be provided. The goal was to generate hypotheses based on the observed
results and the theoretical rationale of the study. The generated
hypotheses were as follows:

1. Modification of Polya's heuristics can be effectively incor-
   porated into the problem solving experience of fourth grade
   students in the sense that the children (a) are capable of
   using the heuristics when attempting to solve problems not
   encountered before and (b) will be more successful in solving
   the problems attempted.

2. There is a relationship between cognitive levels II-A and
   II-B and mathematical achievement at the fourth grade.

3. In a problem solving situation where multiplication is
   appropriate, II-A children use additive procedures and
   II-B children use multiplicative as well as additive pro-
   cedures.

4. Some proportionality problems can be solved by concrete oper-
   ational children if they learn to use proper heuristics.

5. Some combinational problems can be solved by concrete opera-
   tional children if they learn to use appropriate heuristics.

6. There are differences in the interpretation of certain heur-
   istics by fourth graders.

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The Effects of Three Key Organizers on Mathematical Problem
Solving Success with Sixth, Seventh and Eighth
Grade Learners

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Purpose

Problem solving ability as demonstrated by a learner has been fre-
quently cited as an essential goal of secondary and elementary school
mathematics education. Nevertheless the major emphasis of the role of
problems in a mathematics curriculum has been relegated to what are
commonly referred to as word or verbal problems and translating these
into number sentences. Also, activities exhibited by the learner in
solving mathematical problems frequently focused on the narrow range
of possible answers and not on the possible means or methods of solv-
ing these problems. Recent research seems to support the hypothesis
that specific instruction in certain problem solving behaviors or
activities can enhance problem solving success. These behaviors can
be described as organizers of information that facilitate perceptive
conjectures and provide an orientation and focus for the learner.
Organizers can be considered prerequisite abilities for certain
heuristic techniques that have been shown to be effective in solving
problems.

A partial list of organizers that embody some potential for
increasing success in problem solving is given below. This list will
be identified as key organizers since they possess the greatest poten-
tial for opening the blocked goal to the solution of the problem.

The key organizers are the following:

1. Choosing appropriate notation to represent information.
2. Identifying the available, explicitly stated information.
3. Recall of related, implicitly stated information.
4. Constructing a chart or table.
5. Drawing a diagram or picture.
6. Constructing and/or using a physical model.
7. Approximating and verifying.
8. Making a simpler problem.
9. Using resources.
10. Making an equation, inequality.

This study will focus on only three of the key organizers. These
will be drawing a diagram, approximating and verifying, and construct-
ing a chart or table.

The three key organizers studied may be described as follows:

Drawing a diagram.--Given a problem, a sketch is drawn of the situation that includes many of the conditions of the problem.

Approximating and verifying.--Given a problem, a reasonable approximation of a possible solution is made. This is followed by a verification of this approximation with the conditions stated in the problem.

Constructing a chart.--Given a problem, a table or chart is constructed involving most of the possible conditions of the problem. This chart could show a pattern developing from the conditions of the problem.

Related to the description of the key organizers is the question concerning instruction in these organizers. In other words, is it possible to instruct students in these key organizers? Also, if students learn these organizers will they employ them in problem solving situations? Are there certain learning styles which reflect successful problem solvers? What key organizers do students possess before instruction? Is there any relationship between effective application of these key organizers and success in problem solving? Is there any relationship between success in problem solving and practical judgment?

The purpose of the study is to determine if the three key organizers—drawing a diagram, approximating and verifying, constructing a chart—can be taught to students and exhibited by the students as an effective tactic of solving problem tasks.

Procedure

Situation:	within a school organization
Grade levels:	6th, 7th, 8th
Selection of Ss:	random within grade level
Number of Ss:	7 for each grade level, total 21
Description of Ss:	instructional history, academic achievement
Instructional topics:	drawing a diagram, approximating and verifying, constructing a chart.
Mode of instruction:	teaching experiment, student guidesheets for each presentation.
Number of presentations:	6 presentations for each topic, total 18.
Instructional sequence:	presentation of individual topics is in a lateral movement in a cyclic manner; that is, within each topic the presentation is sequenced for difficulty and the presentation for a particular topic occurs every third situation.
Length of Instruction:	14 weeks.

Preassessment

Assessment by teacher(s) of S's math ability.

Learning Style Inventory

Problem Solving Test I: set of three problems that could be solved
                          by involving at least one of the three key
                          organizers.

    Mode: individually administered;
                          written response;
                          talk aloud-audio record

    Scoring: process code;
                          product code.

Postassessment

Practical Judgment Test

Problem Solving Decision Test

    Format: forced choice; six, problems with the three key
                          organizers as choices; same problems as Problem
                          Solving Test II

    Mode: group administered;
                          written responses

    Scoring: weighted response score.

Problem Solving Test II: set of six problems that could be solved
                            by involving at least one of the three
                            key organizers.

    Mode: individually administered;
                          written responses;
                          talk aloud-audio record

    Scoring: process code;
                          product code.

Results

Although data analyses are incomplete at this time, preliminary
findings indicate that a teaching experiment with problem solving key
organizers can increase the effectiveness of solving problems by sixth,
seventh and eighth grade learners. A more detailed reporting of results
will be possible at the time of the presentation.

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Conservation of Equation, Function, and Structure and Its
Relationship to Formal Operational Thinking

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Purpose

The primary purpose of the research reported in this paper was to
investigate the relationship between the ability to conserve equation,
function, and structure under a transformation of literal variable and
the ability to use formal operations in solving typical Piagetian tasks.

Conceptual Framework

A pattern is a relationship among two or more elements which is
invariant under certain transformations of at least one element. A
geometric pattern, for instance, is invariant under certain transfor-
mations of the objects, materials, or colors which comprise the pattern.
A musical pattern, or melody, is invariant under transposition to
another key. A behavior pattern may be repeated either by the same
person under different circumstances or by different people under the
same circumstances. Abstract mathematical structures, such as groups
or fields, are patterns which are invariant under certain transforma-
tions of elements, operations, or variables.

To exemplify a pattern, it is only necessary to select particular
elements related according to the pattern. To define a pattern, on the
other hand, it is necessary to represent individually, yet simultaneously,
all elements which might comprise the pattern. The symbols used to
represent these various elements individually and simultaneously are
variables, as used in mathematics. Because a pattern depends only upon
the relationship common to all of its exemplars, and not upon the symbols
used to define it, any pattern is invariant under a transformation of
the variables used in defining it.

The algebraic patterns of equation and function are among the first
patterns which students typically see defined mathematically using varia-
bles. Although equations and functions, like all other patterns, are
invariant under transformations of variable, this fact may not be imme-
diately clear to students encountering the use of variables for the
first time. Because the symbols most often used as variables in
mathematics are letters of the alphabet, students may at first tend to
associate the alphabetical order of letters with the numerical order of
the numbers represented by the letters used as variables. It is only
when the student realizes that letters in the context of algebra are
completely arbitrary and interchangeable symbols that the student may
fully comprehend the concepts of equation and function.

Similarly, students first encountering mathematical definitions of abstract structures cannot appreciate the essence of a structure until they realize that the structure pattern is invariant under transformations, not just of variables, but also of elements and operations.

Just as Piaget has used conservation tasks to measure children's understanding of certain concepts, one way of measuring a student's understanding of equation, function, or structure is to test the student's ability to conserve equation, function, or structure under changes of variable and other appropriate transformations. The methods used to test for conservation of pattern in this research are compatible with the techniques used by Piaget and others and may, therefore, eventually lead to an extension of Piaget's model of cognitive development into the realm of highly abstract mathematical thinking.

Procedure

The research reported herein includes the results of two separate studies, one already completed and one to be completed in the near future. Tasks were devised to test for conservation of equation, function, and structure. Other tasks involving concepts such as proportion, syllogism, and Cartesian product were designed to be similar to those used by Piaget and others to identify formal operational thinking.

In the first study, six tasks were administered in individual interviews to 72 New York City public-school students, 12 boys and 12 girls at each of three age levels: 12, 14 and 17 years of age. The order of the tasks was varied to control for practice effects among the Piagetian tasks and learning effects among the conservation tasks. All interviews were conducted by the investigator and were audio-tape-recorded. In addition, written notes were made on each student's responses to the tasks. The responses were then classified and scored from the written protocol according to criteria consistent with Piaget's theory of development.

The second study is an extension of the first to include college students with a variety of mathematical backgrounds and tasks which may require a higher level of mathematical sophistication than some of those used in the first study.

Results

Using a chi-square goodness-of-fit test, it was found in the first study that (a) significantly ($p < .01$) more students conserved equation but not function than conversely; (b) significantly ($p < .001$) more students conserved equation but were not formal operational on the proportion task than conversely; (c) significantly ($p < .001$) more students conserved equation but were not formal operational on the syllogism task than conversely; (d) significantly ($p < .01$) more students conserved function but were not formal operational on the proportion task than conversely; (e) significantly ($p < .001$) more students were

formal operational on the combinations task but did not conserve function than conversely.

In addition, it was noted that nearly 50 percent of the 12-year-olds and about 83 percent of the 14-year-olds in this study conserved equation, whereas about 20 percent of the 12-year-olds and 50 percent of the 14-year-olds conserved function. The performance of the 17-year-olds was similar to that of the 14-year-olds in this study.

Data analysis for the second study has not been completed.

Conclusions

The results of this research suggest that the ability to conserve equation may be acquired, in general, prior to the ability to conserve function and prior to the onset of formal operational thought as presently defined. Further studies may clarify the relationship between formal thought and the ability to conserve function and structure under various types of transformations.

The results of this research clearly show that it is not necessary for a student to have formal training in algebra in order to conserve equation and function. Moreover, the responses to the conservation tasks indicate that students who do not conserve equation and/or function may indeed associate alphabetical order with numerical order when they first encounter letters used as variables. Teachers who are aware of this tendency may be able to dispel some confusion by emphasizing that letters used as variables are completely arbitrary and interchangeable symbols, that alphabetic order is irrelevant in the context of algebra.

The Ability of First, Third, and Fifth Grade Children to Perform Rigid Transformation Tasks

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Purpose

The purpose of this study was to investigate the ability of first, third, and fifth graders to perform rigid transformation tasks. The tasks required the child to reassemble parts of a meaningful configuration showing what the configuration would look like if it were displaced by translations, reflections, and rotations.

Conceptual Framework

The conceptual framework for this study has been developed in large part through involvement in the Space and Geometry Working Group of the Georgia Center for the Study of Learning and Teaching Mathematics. In particular, it was influenced by Lesh (1977) and the research of the other Working Group members scheduled to appear Spring 1978 as a monograph published through the ERIC Information Center.

An earlier investigation (Schultz, 1977) found that an intuitive understanding of transformations was already present in the elementary school child's thinking. This investigation also obtained criteria for predicting relative difficulty of transformation tasks. The present study attempted to further analyze the child's thinking of translations, reflections and rotations. In particular, it represents a continuing study of the figurative and operative aspects of concrete manipulative materials and activities for transformation geometry in the elementary school classroom. It is important to have a theoretical base for the use of materials and accompanying activities in any area of mathematics learning, especially when the development of spatial skills is involved. It is through these skills that complex mental structures are formed so as to facilitate the process of abstracting.

The present study asked the following questions: (1) are there differences in children's understandings among the three transformations? (2) does the direction of the displacement effect the child's performance? (3) what strategies are used to solve the transformation tasks? (4) does the child's ability to perform the tasks relate to mathematics achievement as determined by standardized tests? (5) is there a difference in performance between boys and girls on the tasks?

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Procedure

The experimenter individually interviewed 120 students in a suburban Atlanta, Georgia, elementary school. There were 40 first graders, 40 third graders, and 40 fifth graders with 20 boys and 20 girls in each grade. The subjects were each given five translations, five reflections, and five rotations displacing a configuration in each of five different directions. The distance of each displacement was four cm.

The subject was first shown a sailboat made of two flat wooden pieces differing from each other in shape and color. This sailboat configuration was glued on an eight cm square sheet of clear plexiglass and then covered by another similar sheet. The experimenter moved the top sheet as a translation, reflection, or rotation. The subject was then asked to place two wooden pieces like the others on the displaced sheet of plexiglass to show what the configuration would look like if it were moved in like manner.

Analysis

The subject's placement of the wooden pieces for each of the 15 tasks was recorded by an observer. Each task response was then evaluated using a scale of zero to four according to the location and position of the shapes. A score of four indicated that both pieces of the sailboat were in the correct location and orientation on the plexiglass sheet.

The data were analyzed using an analysis of variance with repeated measures. Variables considered were grade, sex, isometry (slide, flip, and turn), and direction of isometry. Significantly more first grade boys were able to perform the tasks than first grade girls. No other sex differences were found. A significant isometry by direction interaction indicated that children had considerable difficulty with diagonal flips. Slide subscores did not differ across grades, but flip and turn subscores did differ.

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An Examination of Student Perceptions of Relatedness Among Mathematical Word Problems

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Purpose

The purposes of this study were (1) to examine the problem similarity dimensions perceived by a student when classifying mathematical word problems as related, and (2) to examine the relationship between a student's perceptions of problem similarity and his performance on a variety of individual difference measures.

Conceptual Framework

In his classic treatise on mathematical problem solving, How to Solve It, Polya (1957) has suggested a series of heuristic precepts that may be useful in the solution of a mathematical problem. For example, Polya has suggested that when one is devising a plan for solving a problem, it is often useful to think of a "related" problem.

It is reasonable to assume that many mathematics teachers suggest this advice to their students. But how do students interpret the word "related"? If they consider "related" problems when it is suggested, are the problems related in a mathematically useful way?

The answers to these questions may be fundamental to an understanding of the psychological implications of Polya's logical suggestion. In fact, the answers may help to provide increased understanding of the modes of information processing that are helpful in mathematical problem solving.

Research evidence related to students' perceptions of problem relatedness is scarce. Krutetskii (1976) discerned both qualitative and quantitative differences between the memories of good problem solvers and the memories of poor problem solvers. Specifically, he found that good problem solvers tend to recall the structure of a problem for a long time, whereas poor problem solvers tend to remember, if anything at all, the details of the problem statement. His research has pointed to differences in the way good and poor problem solvers perceive the important aspects of a problem.

First Phase

Procedure

The principal technique used to examine students' perceptions of problem similarity was a card-sorting task (CST). The CST problems were written so that they varied systematically along two problem similarity dimensions—mathematical structure and contextual details. Students formed groups of problems that they thought were "mathematically related" and explained their basis for categorizing them.

The CST was administered to 98 eighth grade students on two occasions: Once before and once after they attempted to solve the problems. In the time between the two sorts, each student attempted to solve each problem. Students were also assessed using measures of verbal IQ, nonverbal IQ, mathematics concepts knowledge, computational ability, and problem-solving ability.

Results

Analyses of students' criteria for categories formed in the CST indicated that they were associating problems along four problem similarity dimensions. The structure and context dimensions were evident in their sorts, and two new dimensions were identified—question form and problem pseudostructure. Many students appeared to associate problems on the basis of the quantity measured in the problem, such as age, time, or weight. This latter characteristic of problems was called pseudostructure since it often takes on the appearance of the mathematical structure dimension.

The tendency to sort problems according to their mathematical structure was significantly positively correlated with all ability variables. The tendency to sort problems according to their contextual details was negatively correlated with all ability variables.

The significant relationship ($r = .45$; $p < .001$) between a student's problem-solving ability and his tendency to associate problems on the basis of structure was analyzed using partial correlations. When the effects of verbal IQ, nonverbal IQ, mathematics concepts knowledge, and mathematics computation ability were simultaneously controlled, the relationship remained significant ($r = .28$; $p < .01$.

Second Phase

Procedure

Interpretation of certain results of the first phase was difficult since the question and pseudostructure dimensions overlapped with the structure and context dimensions within the CST. In the second phase,

some of the CST problems were rewritten so that the set of revised problems varied systematically along all four dimensions and contained little dimensional overlap. The revised set of problems was used with 58 eighth grade students who had not participated in the first phase of the study. The revised CST task was administered using the same procedures as in the first phase. Data related to students' IQ, concepts knowledge, and computational ability were also obtained.

Results

The significant positive relationship, noted in the first phase, between a student's mathematical ability and his tendency to associate problems on the basis of mathematical structure was supported by the results of the second phase. Also supported was the negative relationship between the mathematical ability variables and tendency to associate problems according to context.

The tendency to associate problems on the basis of their pseudostructure appeared to be unrelated to mathematical ability; in fact, students who were classified as high in nonverbal IQ, concepts, knowledge, or problem-solving ability actually had more pseudostructure associations before solution of the problems than students who were classified as low for the variables. On the other hand, the tendency to associate according to question form was negatively correlated with all ability variables.

As in the first phase, the significant positive correlation ($r = .69$; $p < .001$) between problem-solving ability and the tendency to associate problems on the basis of structure remained significant ($r = .30$; $p < .01$) even when the effects of the four general ability variables were simultaneously controlled.

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The Effect of Problem Structure and Cognitive Level on the
Processes Students Use to Solve Verbal Algebra Problems

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Purpose

The purpose of this study was twofold: first, the study compared
the processes used by concrete- and formal-operational students to solve
problems defined as having either simple or complex structures. Second-
ly, the study attempted to determine if the problems defined as having
complex structures were more difficult than those defined as having
simple structures.

A widely held goal of mathematics teaching is the development of
students' ability to solve verbal problems. Knowledge of the processes
and strategies used by students to solve various types of verbal problems
may provide teachers with information for improving instruction in mathe-
matical problem solving. Additionally, knowledge of some of the
relationships which exist between task variables, subject variables, and
problem-solving performance could aid in determining how problems should
be sequenced in instruction and in our textbooks.

Conceptual Framework

Several researchers (Dalton, 1974; Flaherty, 1975; Grady, 1975;
Hollander, 1974; Kantowski, 1975; Kilpatrick, 1967, Webb, 1975) used
the Think Aloud Technique to identify and study the problem-solving
processes used by mathematical problem solvers. These researchers iden-
tified a fairly long list of processes as being frequently used by
mathematical problem solvers. Research also suggests that certain
subject variables may affect the amount of success subjects will have
when using a given process. For example, junior high students tend to
be more successful at problem solving when using trial-and-error and
less successful when attempting to use equations (Kilpatrick, 1967),
and high school algebra students tend to be more successful at problem
solving when they use equations and less successful when using trial-
and-error (Webb, 1975).

Tyszkowa (1973) obtained results which suggest that the processes
students use to solve problems at one level of difficulty may be
entirely different from those used to solve problems at another level
of difficulty. Research (Barnett, 1974; Ingle, 1975; Jerman, 1970,
1972; Suppes, Loftus & Jerman, 1967) has shown that certain structural
variables are good predictors of problem difficulty. Thus, one could

conclude that structural variables are probably good predictors of process-use also. If the use of certain processes depends on characteristics of the task, then the processes which are most useful on a given type of problem should be identified.

Piaget and Inhelder (1969) described four stages of cognitive development in children. The last two of these stages are <u>concrete operations</u> and <u>formal operations</u>. According to Piaget's Theory, subjects in the formal operational stage can perform certain reasoning processes which cannot be performed by concrete subjects. Thus, formal-operational individuals should have a larger repertoire of problem-solving processes than concrete subjects.

Grady (1975) found that students classified as formal-operational used significantly more means-end heuristics (pictures, diagrams, equations) while solving verbal algebra problems than those classified as concrete operational. Thus, learner characteristics such as cognitive level, age, or mathematical experience may affect the processes subjects use to solve verbal problems. Additionally, there may be an interaction between subject variables, task variables, and process use.

Procedure

Two eighth grade general mathematics classes were randomly selected from each of three junior high schools. A modified version of Longeot's test was administered to each class to classify potential subjects as concrete operational or formal operational. After students in a given school had been classified, ten subjects were randomly selected from the concrete group and ten from the formal group. Each of these 60 subjects were scheduled for a two-hour interview.

In the interview, students were asked to solve ten world problems while "thinking aloud." Two of these problems served as practice problems and eight as experimental problems. Four of the experimental problems were defined as having simple structures and four were defined as having complex structures. A mathematical model of a given problem was used to define it as having a simple or complex structure. The interviews were audio-tape recorded and a written record was kept of what was being said or done. The protocols obtained in the interviews were used to determine the processes or strategies the students used to solve the given problems.

Each subject was given two scores for each variable. One score was based on his performance on the simple structured problems and the other on his performance on the complex structured problems. Some of the dependent variables were: range, process, strategy, understanding, representational, and evaluation scores. The range score was defined as the number of different processes used to solve each group of problems. The process and strategy scores were defined as the number of problems on which the process or strategy was used. The understanding, representational, and evaluation scores were obtained by summing the process scores which fell under the given category.

Analysis

A 2(cognitive level) x 2(problem structure) factorial design with
repeated measures on problem structure was the experimental design
employed in this study. Analysis of variance and Wilson's two-way
analysis of variance based on medians were used to analyze the data.

Results

Analysis of the range score revealed that formal subjects used a
larger variety of problem-solving processes than concrete subjects,
and both groups used a larger variety of processes on the complex
structured problems than on the simple structured problems $(p < .001)$.

In general, understanding, representational, deduction, and eval-
uation processes were used more frequently on the complex structured
problems than on the simple structured problems.

The understanding, representational, and recall scores of concrete
and formal subjects did not differ significantly, but formal subjects
had significantly higher deduction (complex) and evaluation (complex)
scores than concrete subjects $(p < .01)$. The use of certain evaluation
and deduction processes on the complex structured problems may have
required formal operations whereas their use on the simple structured
problems did not. This would explain why the deduction and evaluation
scores of the two groups differed on the complex structured problems
but not on the simple structured problems.

As expected, formal subjects used systematic trial-and-error on
more problems than concrete subjects.

Finally, problem structure did not have as much effect on problem
difficulty in the concrete group as it did in the formal group. Addi-
tional results, conclusions, and implications will be presented.

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The Interaction of Field-Dependence-Independence and
the Level of Guidance of Mathematics Instruction

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Purpose

Attempts to individualize instruction have traditionally involved
varying the rate of instruction and relatively little attention has
been given to adapting the method of instruction to student character-
istics. Cronbach (1957) recommended that researchers try to find
aptitudes which interact with variations in instructional treatments
and to design instructional treatments to fit particular aptitudes of
groups of students.

The search for ways of adapting instructional treatments to indi-
vidual differences, known as aptitude-treatment-interaction (ATI)
studies, has in general been unsuccessful. For reviews, see Berliner
and Cahen (1973), Cronbach (1975), and Cronbach and Snow (1977). How-
ever, there have been recent successes in finding ATI's in mathematics
education (Carpenter, McLeon & Skvarcius, 1976; McLeon & Adams, 1977;
McLeod & Briggs, 1977) that have used cognitive style as an aptitude
variable. The present study attempted to design two treatments tailored
to fit the cognitive style of field-dependence-independence and to search
for other interactions.

Field-dependence-independence is a cognitive style thought to have
implications for educational issues (Witkin, Moore, Goodenough & Cox,
1977). Individual differences in field-dependence-independence are
identified on a continuum determined by the extent a person perceives
analytically. Students who are relatively field-dependent find it
difficult to solve problems which depend on the ability to take a
critical element out of context and restructure the problem in order
to use that element in a different context. Field-independent students
are likely to restructure a situation in order to solve a problem or to
impose structure on material when structure is lacking. Another aspect
of field-dependence-independence which may be important in developing
instructional materials is cue sampling behavior. The effect of cue
salience is greater for field-dependent than field-independent students.
Field-dependent students also favor more of a spectator approach to
learning than field-independent students.

A significant disordinal interaction between field-dependence-independence and the level of guidance in mathematics instruction was found by Carpenter, McLeod and Skvarcius (1976) and later by McLeod and Adams (1977). In view of these findings, the purpose of this study was to investigate further the interactions between levels of guidance and field-dependence-independence.

Procedure

Two inductive treatments were developed, one being a compensatory treatment (Salomon, 1971) for field-dependent students and the other, a preferential treatment for field-independent students. In the compensatory treatment, called the high-guidance group, organization was provided for the student through the use of partially completed tables and rules, and underlining of key words. This treatment was typed using double spacing. In the second treatment, called the low-guidance group, students were expected to make their own tables and discover rules on their own. This treatment was typed using single spacing. Both of the treatments presented the same content on networks, used the same problems, and provided about the same amount of practice. The low-guidance materials actively involved the student by including short questions in the materials. These questions were excluded from the high-guidance materials, where students were given the same information in an expository fashion.

Subjects were 97 students from four sections of a class for prospective elementary school teachers. Within each section, students were randomly assigned to the two groups. Of the 97 students, 51 were in the low-guidance group and 46 were in the high-guidance group. The two instructors for the four sections participated in the study and were randomly assigned to groups so that each had two low-guidance groups and two high-guidance groups. In each case, the low-guidance group remained in the regular classroom and the high-guidance group moved to another room.

On the second day of regular class, a pretest was administered to determine whether or not students had mastered materials covered in the first semester of this two-semester course. This was used as an aptitude variable along with the measurement for field-dependence-independence. Two regular 75-minute class periods were used by the students to complete the treatments and to take an achievement posttest immediately following completion of the treatments. The posttest consisted of three subsections designed to measure achievement at three levels: comprehension, applications and analysis. The posttest was administered again five weeks later to measure retention. The Group Embedded Figures Test, a measure of field-dependence-independence (Witkin, Oltman, Raskin & Karp, 1971), was also administered at that time.

Results

The data were analyzed using multiple regression procedures (Karlinger & Pedhazur, 1973). There was a significant interaction between the pretest scores and the level of guidance of the treatments for the retention test, but no other significant interactions were found. On the immediate posttest, the high-guidance group scored significantly better than the low-guidance group.

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Degrees of Teacher Vagueness and Pupil Participation as They
Relate to Student Learning of Mathematical Concepts
and Generalizations

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Purpose

Numerous studies have investigated the relationships between various
teacher behaviors and student achievement. Many of the studies, however,
have been descriptive rather than experimental in nature. For example,
Hiller, Fisher and Kaess (1969) and Smith (1977) found negative correla-
tions between the frequency of teacher vagueness terms and student
achievement. Shuster and Pigge (1965), Scandura and Behr (1966), Zahn
(1966), and Robitaille (1975) reported that mathematics teachers were
more effective when they allowed their students to participate actively
in developmental activities rather than to engage solely in passive
intake of information.

The present research was designed to extend the results of these
prior studies by experimentally investigating the joint effects of
different degrees of teacher vagueness and student participation on
the learning of mathematical concepts.

Procedure

Subjects for this study were 204 students enrolled in introductory
psychology and sociology courses at Augusta College. Each student was
randomly assigned to one of six groups (n = 34 each) defined by the possi-
ble combinations of three vagueness conditions (no vagueness, moderate
vagueness, high vagueness) and two student activity conditions (passive
learning, active learning).

Each of the six groups was shown a 20-minute videotaped mathematics
lesson on sums of consecutive possible integers (SCPI's). The same con-
cepts, generalizations, and processes discussed by Frielipp and Kuenzi
(1975) were presented in each lesson. None of the students had prior
knowledge of SCPI's. The lessons were videotaped so that the teacher's
presentation was read from a script while the camera focused on corres-
ponding definitions, examples, and applications that were shown on
transparencies with an overhead projector. For each lesson, the same
person read the script and the same sequence of concepts, generaliza-
tions, and examples was presented. Each of the six lessons represented
a different combination of teacher vagueness and degree of student
participation. Each lesson was viewed on a television monitor. Student
comprehension of the lessons was determined by administering a test
focusing on the content of the lessons.

Two of the lessons contained a high degree of teacher vagueness terms (144 per lesson), two of the lessons contained a moderate degree of teacher vagueness terms (72 per lesson), and two lessons contained no teacher vagueness terms. The vagueness terms were obtained from vagueness categories defined by Hiller, et al. (1969).

In three of the lessons the students were first shown examples of how to identify SCPI's and write numbers as SCPI's and then the students were required to work similar examples individually. After the students were given time to work the problems individually, they were informed on the videotaped lesson concerning the correct way to work the problems. These three lessons were defined as "pupil active participation" lessons.

The remaining three lessons were defined as "pupil passive intake" lessons. The same procedure was used in these lessons as in the pupil active participation lessons, except that students were not given the opportunity to work problems individually. Instead, a detailed analysis of each problem and its solution was presented.

Results

A 2 (pupil active participation vs. pupil passive intake) x 3 (high vagueness vs. moderate vagueness vs. no vagueness) analysis of variance was performed on the student achievement scores. The mean achievement scores for the six experimental conditions are presented in Table 1. Table 2 indicates that neither the main effect due to degree of pupil participation nor the interaction between vagueness and pupil participation was significant. The vagueness main effect was significant, $F(2,198) = 3.66$, $p < .05$. The Tukey (b) tests revealed a significant difference ($p < .05$) between the high vagueness groups and the no vagueness groups.

Table 1. Group Mean Scores

	High Vagueness	Moderate Vagueness	No Vagueness
Active Participation	N = 34 $\bar{x} = 8.765$	N = 34 $\bar{x} = 9.412$	N = 34 $\bar{x} = 11.029$
Passive Intake	N = 34 $\bar{x} = 8.647$	N = 34 $\bar{x} = 9.559$	N = 34 $\bar{x} = 9.647$

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Table 2. Results of the Analysis

Factor	df	SS	MS	F
Vagueness (A)	2	90.66	45.33	3.466*
Degree of Pupil Participation (B)	1	10.37	10.37	1
A x B	2	22.71	11.36	1
Error	198	2589.24	13.08	

*$p < .05$

Conclusions

The findings of this study fail to support the conclusions made in prior studies that the degree of student participation significantly influenced achievement. One possible reason for this result is that many studies concerning pupil participation have involved elementary school or high school students rather than college students. It may be that most college students have become accustomed to playing a passive role during lesson presentations and thus do not improve significantly, at least at first, if they are allowed to take a more active part in learning. A second possible reason for this finding is that there is no guarantee that students are really "passive" learners when they are not encouraged to work problems during a lesson. Likewise, students are not necessarily "active" learners when they do try to work problems during a lesson. Each student may process information in his own way and the format for reinforcement of this process may be of secondary importance, particularly for college students.

The findings of this study support the conclusions of Hiller et al. (1969) and Smith (1977) that the frequency of vagueness terms negatively influences achievement. The present study reveals that a high degree of vagueness per se affects achievement, whereas the studies of Hiller, et al. and Smith were not able to determine this relationship because they did not control variables correlated with vagueness. For example, Smith (1977) reported a positive correlation between the frequency of teacher vagueness terms per lesson and the number of irrelevant examples the teacher presented per lesson. Also, Smith reported a negative correlation between the frequency of teacher vagueness terms per lesson and the degree to which the teacher met lesson objectives in his presentation. To summarize, when student achievement is based on short-term learning of specific subject matter, there appears to be a negative relationship between the frequency of teacher vagueness terms and the degree to which the teacher has prepared and organized his lesson.

As previously mentioned, in the present study such factors as organization of the lesson, the substantive content of the lesson, and the frequency and sequencing of examples were held constant. Thus, it may be that a high frequency of teacher vagueness terms causes pupils to perceive the teacher as being disorganized regardless of the content and the examples the teacher presents.

Future research on teacher vagueness should focus on the relationship between vagueness and other variables that seem to be related to teacher competence. Further, studies of teacher vagueness terms have focused on students' short-term learning of specific subject matter and have involved the teacher's use of direct or deductive methods of presentation rather than the use of indirect or inductive teaching styles. Future research should investigate the role of teacher vagueness terms in settings where long-term learning, pupil inquiry and discovery or "learning how to learn" are goals.

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The Effects of Inductive-Deductive Teaching Methods and Field-
Dependence-Independence Cognitive Style Upon Student
Achievement in Mathematics

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Purpose

A recurring theme throughout a vast majority of the literature
written on education in the last ten years has been the need to indi-
vidualize education to meet the needs of the students at all levels of
education (Cronbach and Snow, 1969; Bloom, Hastings and Madaus, 1971;
Glaser, 1972; Coop and Sigel, 1971). In fact, "individualization" has
become an educational slogan for many schools and numerous curricular
materials. However, few of these programs have examined carefully
"the inter-individual variability of the learners who will be exposed
to their educational stimuli" (Coop and Sigel, 1971, p. 152). Thus,
research needs to be done on the many consistent individual differences
of students and the interactions between these differences and instruc-
tional procedures.

This study was an investigation of two instructional treatments,
inductive and deductive teaching methods. In addition to testing the
main effects of these treatments, this study also examined the inter-
action between these treatments and the cognitive style dimension of
field-dependence-independence. The criterion measures were an overall
achievement test and its subtests made up of the knowledge, applica-
tion and analysis items.

Few areas of educational research produce as many contradictory
results as do the studies on inductive versus deductive teaching. This
lack of consistent results may be due to the existence of aptitude-
treatment interactions which were not hypothesized or systematically
researched. This study was an investigation of the possible interac-
tion of these treatments with one dimension of cognitive style.

The inductive and deductive treatments utilized in this study
were differentiated according to Good's (1959) definitions of the two
methods. Introductory material and basic definitions necessary to the
unit were the same in both treatments. The deductive treatment
followed a rule-example paradigm, while the inductive treatment
followed basically an example-rule paradigm. In the inductive treat-
ment, following the examples, the students were encouraged, via
questions, to formulate a rule or principle of their own.

The aptitude of cognitive style has gained the attention of many researchers in recent years. One dimension of cognitive style is that of field-dependence-independence, first studied by Witkin and his colleagues in 1954. People may be placed along a continuum with field-dependence and field-independence at the extremes on the basis of the ease with which they perceive items as discrete from their backgrounds. Field-dependent persons are characterized by a more global, undifferentiated approach in processing information, while field-independent persons process information and their surroundings in a more analytic, differentiated manner (Witkin, et al., 1962). The Group Embedded Figures Test (GEFT) was the instrument utilized to place the students along the field-dependent-independent continuum. This dimension of cognitive style would appear to relate very highly to the inductive and deductive methods considered earlier.

Procedure

The sample for the study was made up of the students in two sections of a mathematics content course designed for preservice elementary education teachers. There were 71 students in one section of the course and 47 in the other. All four class standings from freshmen to senior were represented in both groups.

Studies designed to investigate the outcomes of two or more instructional procedures can be designed in several ways. Often the researcher chooses to use individual packets due to the control these materials afford the experiment. This researcher, however, chose to teach both classes, in an effort to make the experimental setting resemble the classroom situation as closely as possible. Although "individualized" materials are being used in schools today, most instruction takes place in a classroom setting in which one teacher controls and manages the learning activities of a group of students. Thus, due to the more realistic setting a teaching situation furnishes, this researcher taught a transformational geometry unit to the two intact university classes.

The criterion measure for this teaching unit was an overall achievement test. The researcher developed the instrument by consulting the Taxonomy of Educational Objectives (Bloom, et al., 1956) to write items representing the levels of Bloom's taxonomy. This compilation of possible questions was then given to people knowledgeable in mathematics and education who were asked to furnish suggestions. The final instrument consisted of 22 objective items. For purposes of analyzing the data, these items were classified by this researcher, in agreement with those consulted, into knowledge, application and analysis items based upon Bloom's (1956, 1971) criteria. The application and analysis items were grouped together for the data analysis and referred to as the transfer subtest.

Results

An analysis of variance was employed to test for the main effects of the inductive and deductive teaching methods. A one-way ANOVA was done on each of the five criterion measures (overall achievement, knowledge subtest, application subtest, analysis subtest and transfer subtest). Differences, significant beyond the .05 level, were found between the group means on the transfer and analysis subtests. Both differences were in the direction favoring the inductive treatment. It should be noted, however, that these results are not independent since the transfer subtest was made up of the application and analysis items of the overall achievement instrument. These findings indicate that the inductive teaching method was more effective than the deductive method in producing transfer of the student's knowledge to new situations where the student is required to discover a new relationship.

Linear regression analysis was utilized in testing for the existence of possible aptitude-treatment interactions between field-dependence-independence and the instructional procedures. For the criterion measures of overall achievement and transfer, the data indicated strong support $(.05 < p < .1)$ for such ATI's. For both criterion measures, the lines intersected within the range of scores for the GEFT. In each case, the point along the X-scale was computed below which the treatments were significantly different. When overall achievement was the criterion measure, it was found that all individuals having GEFT scores of five or below out of a possible 18 performed better if they were taught by the inductive method. For other individuals, the choice of method (inductive or deductive) did not result in a difference in achievement. When the transfer subtest was the criterion measure, further analysis of the data indicated that all individuals having a GEFT score of nine or below performed better if they were taught by the inductive method. For other individuals, the choice of method did not result in a difference in achievement.

Conclusions

A number of conclusions can be drawn from the results of this study. The inductive teaching method appeared to be more effective than the deductive method in producing transfer. This result is especially important, due to the lack of experimental evidence supporting either instructional method (Hermann, 1969). In the inductive treatment used in this study, the students spent most of the class time working problems and discussing the results. The statement of rules and principles was absent and knowledge level understanding was not stressed. The activity in class (i.e., working unusual problems) was very similar to solving transfer-type problems. Students in the inductive class had experience solving problems in an unfamiliar setting, while the students in the deductive class did not have this experience. The students in the deductive treatment were accustomed to being given all the necessary steps and relationships

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prior to working the problems. Having to solve unusual problems in unfamiliar settings was a new experience for them and may have affected their achievement on the transfer subtest.

In this study, when a student's GEFT score was utilized it was possible to provide the optimum learning situation for him. The field-dependent students achieved optimally on the overall achievement test when taught by the inductive method. Also, the data analysis indicated that the field-dependent students achieved optimally on items dealing with transfer of learning to new situations if taught inductively.

These findings are important due to the lack of experimental results indicating the existence of ATI's in education (Bracht, 1970). These interactions support the conjecture that how a person perceives his surroundings does affect how he learns under inductive and deductive teaching methods. It must be remembered that this is true only when the measures of success were overall achievement and the transfer of learning to new situations.

An Assessment of the Processes Used By Community College
Students in Mathematical Problem Solving

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Purpose

Classroom efforts to develop the problem-solving ability of Community College students have focused primarily on increasing the number of mathematical concepts students know. There is mounting evidence, however, that another factor—the ability to use problem-solving processes —is related to success in mathematical problem solving. Having taught mathematics at the Community College level for four years, I was personally distressed when a review of the literature in 1975 yielded no information on the problem-solving processes of Community College students. With no such information available, it was impossible for me to design an effective instructional unit on problem-solving processes for these students.

In 1976 I conducted an exploratory study of the processes used by Community College students in mathematical problem solving. The primary purpose was to reveal the processes used by these students in mathematical problem solving and to discover ways in which the processes might be improved and extended in the classroom. After comparing the student characteristics (age, race, sex, IQ, etc.) of my sample to the characteristics of Community College students nationwide, I am convinced that the findings of this research study should generalize well to other urban Community Colleges.

Procedure

Sixty students from the three campuses of the Community College of Denver and from Arapahoe Community College were randomly selected from the mathematics rosters to participate in one and one-half hour interviews with the researcher. During each interview measures for IQ, mathematics achievement, conceptual tempo and mathematical problem-solving ability were administered. Information regarding age and sex was also collected. Students were asked to think aloud while solving the eight items of a mathematical problem-solving inventory developed by the researcher. These sessions were taped and coded for processes.

Results

The most popular processes with Community College students were found to be deduction, trial-and-error, and equations. Significant correlations ($p = .05$) were found between total score on the mathematical

problem-solving inventory and use of the processes of exploratory manipulations (r = -.34), successive approximation (r = .37) and deduction (r = .30). Conceptual tempo, age, sex and IQ were not significantly related to mathematical problem-solving ability, but a significant correlation (p = .05) of .35 was found between mathematics achievement and mathematical problem-solving ability.

Conclusions

The following recommendations were made for the improvement of process use: (1) more time and greater emphasis should be placed on teaching Community College students to translate word problems into appropriate sets of equations, and (2) in lieu of trial-and-error, Community College students should be taught to use successive approximation. It was found that the number of processes used by Community College students could be extended by instruction in the use of successive approximation, checking, analogy, specialization, generalization, algorithm, reduction-combination, and working backwards. Future research studies investigating the relationship between the mathematical problem-solving ability of Community College students and IQ or mathematics achievement should include a measure of reading comprehension of mathematical English.

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A Description of the Difficulties Encountered by Selected Six Year Olds in the Process of Learning Numeration with Base Ten Blocks

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Purpose

It was the purpose of this study to begin to describe how selected six year old children develop numeration concepts with base ten blocks. In particular, the concern focused on the difficulties encountered by these children during discovery-oriented instruction.

The research was based on an approach to learning arithmetic which allows children maximum opportunity to count to solve story problems and then to discover short cuts to counting in the form of arguments expressed with base ten blocks. Later, children are encouraged to use pictorial symbols for the blocks to support their arguments. Standard algorithms are taught only after extensive experience with original arguments.

Especially in such a program, numeration concepts are prerequisites for learning arithmetic algorithms. Thus, a knowledge of difficulties children have in the process of learning these concepts is necessary for teachers of young children. Clinical interviews with individual children were used to probe children's thinking during instruction in numeration. It is believed that examination of children's processes during instruction will in the long run yield more information than an examination of the products of that instruction.

Procedure

In this study 14 six year old children at an independent elementary school in Greenwich Village, New York, were interviewed six times over a period of five months. In the interviews the children were asked to give number names to various collections of blocks and conversely to represent with the blocks numbers that were named by the interviewer. Also, they were asked to solve orally-presented story problems, most of which involved addition. Running records of the actions and speech of the children were kept. The researchers assisted children in prescribed ways. On the basis of these records, descriptions of the difficulties encountered were made.

Results

Thirteen categories were generated to describe the numeration difficulties encountered by the children in the instructional interviews. Each difficulty category describes errors made by more than one subject. One hundred sixty-nine difficulties with numeration were identified across all the data, and 162 of these could be placed in the 13 categories. There were other difficulties encountered by the children which related to their solution of the story problems, but did not directly relate to numeration concepts. Some of the numeration difficulties have been arranged in four groups for discussion here.

One group of difficulties, accounting for 26 percent of the total, includes three categories which all seem to reflect the problem young children have in keeping track of various aspects of a given situation at once. There were 27 times when children switched the digits in a number. For example, one child who had successfully used the blocks to sum 35 cookies and 22 cookies reported orally 75 rather than 57 as her answer. An interesting error which occurred 11 times involved summing digits to create a new number. For example, one child who had 2 hundreds and 1 unit (201 out in front of him; he reported his answer as 203. In this case the child seemed to be summing the 2 (hundreds) and the 1 (unit) to get 3 in the ones' place. Six errors were made by children when they neglected to report one place in their answers, for example, reporting 107 instead of 117 for 1 hundred, 1 ten, and 7 ones.

Another group of difficulties, one which accounted for almost 21 percent of the total number of errors, reflects a counting problem very common to children of this age. They often count hundreds as tens or ones, tens as ones or hundreds, and ones as tens or hundreds. For example, a child who had 5 tens and 1 one in front of her counted, "10, 20, 30, 40, 50, 60." Another child counting 2 hundreds and 1 ten said "100, 200, 300." Here the children seemed just carried away by the pattern of the counting. In one case a child verbalized a rather strange reason for making a somewhat different error. She was counting 1 flat, 9 longs, 10 units. She got to 199, and then with the last unit, said, "299." When asked how she got that, she replied, "'cause this block (the last unit) makes another 100. It's the 100th one." Here the last unit was counted as another 100.

There were still other language-based difficulties. This grouping accounted for about 17 percent of the total. There were 18 times when children made errors such as these:

Two flats, 6 longs and 3 units were named "one hundred sixty three one hundred."

One flat and ten longs were counted "one hundred, one hundred ten... one hundred ninety, one hundred ninety ten."

Six errors occurred when children represented a spoken "one hundred" as 1 unit and 1 flat, that is as 1 and 100. Similarly, in four instances a child named 2 flats as "one hundred and 2" or 4 flats as "one hundred and four." 。

One category accounting for 18 percent of difficulties involved the child making non-standard representations of numbers or counting smaller units than necessary. Often these difficulties occurred when the numbers were in the teens; the vocabulary for naming numbers in the teens is inconsistent with the usual procedure. For example, 17 was represented in blocks as 17 units instead of 1 ten and 7 units, and 1 ten and 8 units were counted by ones on the long. This difficulty is interesting since it can be interpreted as evidence of immature thinking about numeration, and yet, in other contexts, as an indication of flexibility. Thus, a child who is to share 24 cars among 8 children needs to think of the 24 as 24 units rather than as 2 tens and 4 ones to solve the problem. In this case, a non-standard representation of the 24 as 24 units anticipates a necessary exchange.

Conclusion .

When children encounter difficulties in learning, they reveal their knowledge and misconceptions. By analyzing this information, teachers can scientifically modify their instructional methods to make for more effective intervention.

The Relationship Among Piagetian Area Task Scores, Teacher
Grades, and Standardized Test Scores for Grade Two

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Purpose

There were three major objectives for this research study. The
first.was to investigate relationships of Piagetian area scores to
teacher-determined grades, IQ test scores, and standardized achieve-
ment test scores to determine whether second graders' performance on
Piagetian area tasks (Piaget et al., 1960) correlated with or pre-
dicted their performance in mathematics. The second objective was
to replicate the doctoral study of the investigator (Taloumis, 1973)
to determine whether the conclusions reported could be substantiated
again for a different sample group selected from a wider range of
population in grade two. The two main questions considered in the
replication were (a) did sequencing of the presentation of three area
conservation tasks and two area measurement tasks significantly affect
the scores of either set of area tasks, conservation or measurement
(the sequences.being conservation-measurement and measurement-
conservation)? and (b) did second graders assessed in 1976 perform
similarly to the second graders assessed in 1971? The third objec-
tive of the study was to compare scores of subgroups to see if
environmental factors (socio-economic or geographical) influenced
scores significantly.

Conceptual Framework

Since the study was a replication and extension of the investi-
gator's reported doctoral study (Taloumis, 1975), the same five
Piagetian area tasks, administered to 168 children classified as
lower middle class (as defined by Havighurst and Neugarten, 1966) in
grades one, two and three in the spring of 1971 (56 children per grade
level), were administered to the 169 children in grade two from a
wider range of schools in the spring of 1976. The seven schools
involved in the 1976 study included the original three schools used
in 1971, the three remaining elementary schools in the same community,
and a New York City public elementary school.

Research on the prediction of achievement in mathematics from
performance on Piagetian tasks has been conducted in primary grades
within a span of one year of that grade (Steffe, 1966; Stommel, 1966;
LeBlanc, 1968; Nelson, 1969; Kaminsky, 1971; Kaufman and Kaufman,
1972; Siliphant and Cox, 1972; Rohr, 1973; Smith, 1973; and Backus,

1974). The present study investigated whether the performance of children on Piagetian area tasks predicted their teacher grades in mathematics and Stanford Achievement Test (SAT) scores in mathematics over a two-year period. Moreover, the study examined whether Piagetian area task scores contributed significantly to the predictability of mathematics achievement after the prediction of mathematics achievement was known from the Otis-Lennon Mental Ability Test.

The children in the present study had at least three socioeconomic backgrounds that could be identified. The study examined whether these differences in background paralleled differences in performance on Piagetian area tasks. There are several Piagetian studies that compared performance levels of young children from middle-class and lower-class and found that children from middle-class performed higher (Berlin, 1964; Almy, 1966; Rothenberg, 1969; Biot, 1970).

Procedure

The sample consisted of 114 white children and 55 black children, ranging in age from seven years five months to eight years four months, selected from the seven schools. A random number table and an alphabetical listing of children were used to select children from each of the 21 classrooms involved in the study.

The suburban community had two slightly different socio-economic populations. Three of the six schools enrolled more children of parents on welfare and two were geographically located nearer the main shopping area located in the older part of the community. Two of those three schools also had a higher enrollment of the black population. These three schools will, henceforth, be designated as Set B Schools. The other three suburban schools were the same ones as used in the 1971 study and will be denoted as Set A Schools. The inner city school population was designated as being 85 percent black, 1 percent white, and 14 percent other.

The following three substudies were constructed:

1. The scores of 60 children in Set A Schools assessed in 1976 were compared to the scores of 56 second graders assessed in the same Set A Schools in 1971.

2. The scores of 60 children in Set A Schools were compared to the scores of 63 children in Set B Schools.

3. The scores of 27 children in Set B Schools (in two of the three schools) were compared to the scores of 26 children in the inner-city school.

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The five area tasks were given in two different sequences: the sequence (conservation-measurement) was given to 84 children, and the sequence (measurement-conservation) was given to 85 children. There were 84 boys and 85 girls randomly selected. The area tasks were administered individually to each child in a room apart from the classroom. Each of the area tasks contained from 5 to 15 parts. A scoresheet was used to record the responses of each child. A score of one or two was given for each subtask within each of the area tasks.

Statistical analysis included use of a Pearson product-moment coefficient of correlation and a multiple-regression for IQ scores, teacher grades, Stanford Achievement Test subscores (for computation, concepts, and applications), area conservation scores, area measurement scores, and sex differences. The level of significance accepted was $p < .05$ (unless indicated otherwise).

Results

1. Piagetian area task scores were positively related to teacher grades, SAT scores, and Otis-Lennon IQ scores and the correlations were statistically significant, but not for sex differences ($p < .01$).

2. Conservation and measurement scores were affected by sequence of presentation; the second group of area tasks presented in either sequence resulted in higher scores.

3. In comparing 60 second graders' area task scores (1976) to 56 second graders' area task scores (1971), where all children were selected from Set A Schools, there was no significant difference.

4. In comparing 60 second graders' area task scores in Set A Schools to 63 second graders' area task scores in the remaining Set B Schools, there was no significant difference.

5. In comparing 27 black second graders in two Set B Schools to 26 black second graders in the inner-city school, the suburban children performed significantly higher on two conservation tasks, but not on the third conservation task nor on the two measurement tasks.

Conclusions

Conclusions will be more appropriately and accurately drawn when additional data analyses have been completed and related to classroom practices. These will be presented with the paper.

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The Influence of Selected Artistic Variables on
Children's Perception of Mathematical Pictures

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Purpose

Current first grade mathematics textbooks use dynamic pictures,
in conjunction with written numeral statements, as one method of commun-
icating mathematical concepts to children. Portraying motion, these
pictures may serve as a reference to aid the children when solving a
related problem, or they may provide a setting in which a concept may
be interpreted. However, it is initially necessary for the children
to relate the characters depicted in the picture and to perceive the
action portrayed before they can associate the pictures with the addi-
tion or subtraction of whole numbers.

An examination of dynamic illustrations in current primary mathe-
matics textbooks revealed that both single pictures and sequences of
three pictures are presented in textbooks with motion portrayed by use
of either postural cues (i.e., picturing legs of the characters in a
state of non-equilibrium) or conventional cues (i.e., using lines,
vibration marks, or clouds of dust about the characters). The purpose
of this study was to investigate the effects of the artistic cue
(postural or conventional) used to portray motion and the number of
pictures (single or sequence of three pictures) used to depict an
event on first grade children's perception of the mathematical rela-
tionships (either addition or subtraction) portrayed by a dynamic
textbook-type illustration. Further, the study investigated the
possible correlation between first grade children's level of conserva-
tion, their interpretation of the illustrations, and their ability to
characterize the depicted mathematical relationships by number sen-
tences. The procedure used assessed the children's interpretation of
the illustrations within two levels of description, either verbal or
spatial.

Conceptual Framework

Research concerning children's perception of pictures indicates
that children initially perceive only discrete items in a picture
and gradually develop the ability to describe a picture in terms of
overt action and to perceive relationships between the illustrated
characters (Amen, 1941; Travers, 1969). Motion is one of the more
difficult representations for children to perceive in static, two-
dimensional pictures (Friedman & Stevenson, 1975; Schnall, 1968;
Travers, 1969). However, for a child to derive from a textbook

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illustration all the information which the artist has attempted to transmit, the child must convert the still picture into a dynamic scene. This requires interpretation of features within the illustration by an artistic code which deviates from the real optical code.

Friedman and Stevenson (1975) stated that preschool and first grade children do not readily classify pictures with conventional motion cues as "moving." However, one may interpret their study to indicate that, if faced with an apparent contradiction between conventional and postural cues, young children rely on the postural cues. Campbell (1976) noted that first grade children were more likely to observe motion in textbook-type pictures which were characterized by use of both postural and conventional cues than in pictures which utilized only postural cues.

Given a sequence, first grade subjects seem to be able to interpret each group of characters within the pictures as being a single group undergoing a change, with the sequence serving to display the group's characters over time. Results of a study by Campbell (1976) revealed that the number of pictures presented (single, sequence, or combination of single and sequence pictures) had no significant effect on first grade children's perception of the mathematical relationships depicted by the pictures. However, initially viewing and describing sequences did provide a learning experience which significantly affected the interpretation of single pictures.

Procedure

Ten textbook illustrations were chosen as models: five of the illustrations depicted addition events and five of the illustrations depicted subtraction events. For each model, two similar sequences each consisting of three pictures were drafted. One sequence depicted characters with legs (e.g. frogs, pelicans, personified apples) using postural cues to portray motion. The other sequence depicted characters without legs (e.g., fish, snails, personified buses) using conventional cues to portray motion. No sequence portrayed more than seven characters. A set of single pictures was obtained by using the middle picture of each sequence.

Ninety-six children were randomly selected from six first grade classes within three elementary schools in suburban-rural Indiana. Initially each child was administered the Purdue Concept Formation Test. This test consists of 23 Piagetian conservation-type items.

As determined by random assignment, the subjects were shown either single pictures or sequences of pictures; these pictures depicted motion by either postural cues or conventional cues. Upon presentation of a single picture or a sequence of three pictures, the students in the verbal description condition were told that the picture(s) told a story,

and they were then asked to tell a story that went with the picture(s). Students in the spatial description condition were also told that the picture(s) told a story. The toys corresponding to the characters in the picture(s) were placed in a pile before these students. After telling or showing the story, each child was asked to choose a number sentence card, from a set of five cards, to go with the picture(s). This procedure was repeated for each of the ten items presented to each child. Each subject was tested individually.

Analysis

A procedure was defined to assign a score for each story description and number sentence response (0,1,2,3 or 4). Each subject's description was also coded as to whether it revealed perception of motion (0 or 1). Summing these scores over the 10 items, each student had three total scores: a characterization of his descriptions (0-40), his overall perception of motion (0-40), and his number sentence responses (0-40).

A 2(number of pictures) x 2(description) x 2(motion cues) design was used to examine the main effects and interactions using analysis of variance techniques on each of the three sets of data (description scores, motion perception scores and number sentence scores). A Chi-square analysis was used to assess whether there was a relationship between the type of number sentence response which the students consistently chose and the number of pictures or motion cue which they viewed. Pearson correlation coefficients were also computed to assess the possible relationship between the cognitive level of the subjects as measured by the Concept Formation Test and their responses to the pictures, as well as the possible relationship between description and number sentence responses.

Results

Analysis of variance procedures revealed significant main effects due to the number of pictures (single or sequence) ($p = .002$) and the type of artistic motion cue ($p = .004$) on the description responses. The form of the description (verbal or spatial) had no significant effect on the description responses ($a = .05$); no interactions were significant. On the motion perception scores, the type of artistic motion cue had a significant effect; however, neither the form of the description nor the number of pictures had a significant effect on the motion perception scores. No significant interaction effects were noted with respect to the motion perception scores. Analysis of the number sentence responses revealed a significant effect due to the number of pictures ($p = .001$), but no significant effect due to the type of artistic motion cues.

Chi-square analysis revealed a significant relationship ($p = .0037$) between the number of pictures (single or sequence) used to depict a mathematical event and the type of number sentence response chosen to abstractly describe the event. A significant association ($p = .0326$) was also noted between the type of number sentence response and the artistic motion cue.

No evidence of a strong linear correlation between cognitive level and responses to the pictures or between descriptions and number sentence responses was noted.

Conclusions

Based upon the results of this study, some conclusions are suggested.

Significant effects due to single or sequence pictures were noted in both the subjects' descriptions of the pictures and their character-ization of the pictures by a number sentence. It is recommended that illustrated sequences be presented within the primary mathematics curriculum, and that they should be presented prior to single pictures.

The form of the artistic motion cue (postural or conventional) had a significant effect on both motion perception and description scores. Examination of the mean scores and the Chi-square analysis suggest that first graders are able to interpret both types of artistic cues but are more likely to perceive motion and correctly identify number sentences in pictures utilizing postural cues, rather than conventional cues.

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Linear Measurement Processes in Young Children

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Purpose

The present study is concerned with exploring the development of children's concept of linear measurement. The exploration of such a concept implies that both the mathematical aspects of the concept and the psychological processes of concept development must be considered.

Interest in the psychological processes for the development of the measurement concept has, for the most part, been inspired by the research of Piaget (Piaget, Inhelder & Szeminska, 1964). This research indicates that the psychological processes for measurement develop in several stages over a period of 3-4 years. As children pass through these stages, there is a gradual development of a cognitive structure ending in the synthesis of certain mental operations that eventually enables children to measure.

The Piagetian research seems to indicate that the developing cognitive structures are paralleled by increasing knowledge of the mathematical nature of measurement. A critical problem in mathematics education is to determine the state of a child's knowledge with respect to the mathematical aspects of the measurement concept at a given developmental stage.

The purpose of the present study was twofold. The first purpose was to analyze theoretically the possible integration of the psychological processes concerning the development of the linear measurement concept and certain features of the mathematical aspects of the concept. The consequence of this analysis was a set of research questions (stated in the next section). The second purpose was to explore empirically the research questions through the administration of measurement tasks to young children. The data generated from the responses of the children were used to suggest answers to the research questions.

Conceptual Framework

The mathematical framework for the linear measurement process was drawn from the Mathematical Concepts of Elementary Measurement by Blakers (1967). The measurement of length can be conceived as a function from a domain space D of objects (say "sticks", all having the attribute of length) into the positive reals R. The domain space has the following structure: There is an equivalence relation "the same

length as" and an order relation "is longer (shorter) than" denoted by $\sim$ and $>(<)$, respectively. These relations allow one to partition and order the domain space.

There is also a binary operation * for "joining" objects in D. This operation has the following properties for any a, b, c, and d Σ D: (1) if a$\sim$b and c$\sim$d, then a*c $\sim$ b*d; (2) if a$\sim$b and c<d, then a*c < b*d; and (3) if a<b and c<d, then a*c < b*d.

A function ℓ can be defined on D by selecting an arbitrary element $d_o \Sigma$ D and designating $\ell(d_o) = 1$; d_o is called the unit for ℓ. The function ℓ assigns measures to other members of the domain space by (1) direct comparison with the unit or a multiple of the unit or (2) by iteration of the unit or a multiple of the unit. Of particular importance is the fact that ℓ is additive (i.e., for any d_1, $d_2 \Sigma$ D $\ell(d_1*d_2) = \ell(d_1) + \ell(d_2)$).

The psychological framework was drawn from the cognitive development theory of Piaget as presented in The Child's Conception of Geometry by Piaget, et al. Piaget's studies indicate that children's ability to measure emerges during the period of concrete operations. Children's thinking in this period is characterized by eight groupings (i.e., types of mental operations). Piaget has characterized the mental operations required for measurement as follows:

> To measure is to take out of a whole one element, taken as a unit, and to transpose this unit on the remainder of the whole: measurement is therefore a synthesis of subdivision and change of position. (Piaget, et al., 1964, p. 3.)

The mental operation of "subdivision of a whole" is an operation from Piaget's Grouping I (i.e., the additive composition of classes), whereas the operation of "change of position" is an operation from Grouping V (i.e., the addition of asymmetrical relations). Subdivision requires that a child be able to imagine a line segment as a series of contiguous parts composing a whole. Change of position requires that a child be able to mentally establish length relations between segments when the positions of the segment have been changed. When these two operations have been synthesized, the child is capable of iterating a unit (i.e., the child can measure).

Given this conceptual framework, the following four exploratory research questions were posed concerning the possible integration of the psychological processes and the mathematical nature of linear measurement:

1. What does acquisition of selected aspects of the mathematical aspects of measurement imply about the child's state of development with respect to the mental operations of change of position and/or subdivision (e.g., if the child believes the length function is additive, does this mean he has acquired subdivision)?

2. If a child possesses change of position and/or subdivision, what does he know about the mathematical aspects of linear measurement (e.g., has the child mastered the domain structure of the measurement function)?

3. Is there specific order in which children acquire an understanding of the various mathematical aspects of measurement (e.g., must the child master the structure of the domain space before he can use the length function)?

4. Is it required that children possess both of the mental operations of change of position and subdivision before they can iterate a unit (e.g., must a child conserve length before he can iterate)?

Procedures

Subjects were 60 children, 20 each from grades 1, 2 and 3 of the Monroe Primary School in Monroe, Georgia. Ages of the children ranged from 6 years and 5 months to 10 years and 4 months.

Each child was interviewed individually by the author in two separate sessions. During the course of the interviews, each child was given various collections of sticks and asked to perform tasks that required him to do such things as compare sticks directly and indirectly, compare combinations of sticks indirectly, measure sticks via unit iteration, measure sticks with a ruler. As the child attempted each task, his verbal responses and physical behaviors were recorded.

The tasks were designed to measure the development of the child's concept of the mathematical aspects of measurement as well as the development of the requisite mental operations. Tasks related to the former measured the child's ability (1) to infer a length relation between sticks via transitive reasoning, (2) to infer a length relation between two polygonal paths by employing properties of the join operation, (3) to measure a stick by unit iteration, (4) to measure a stick with a multiple (a ruler) of a designated unit, (5) to employ the additive property of the length function.

Tasks to measure the development of the mental operations were adapted from The Child's Concept of Geometry by Piaget, et al. (1964). Change of position was measured by conservation of length tasks similar to ones described in Chapter 3, while subdivision was measured by the tram tasks described in Chapter 6.

Analysis

The analysis proceeded in two steps. First, the responses and common behaviors for each task were classified across grades. The intent of this classification was to enable the author to discuss the integration of the mathematical concepts and the mental operations. Second, response patterns between selected tasks were analyzed. The intent of this analysis was to search for (1) possible hierarchical orderings among the six mathematical abilities mentioned, (2) possible ordering of acquisition of the two mental operations, and (3) relations between the mathematical abilities and the mental operations.

Results

Following are some tentative findings that are related to the four exploratory questions posed earlier:

1. The mathematical abilities show consistent developmental trends (i.e., for most of these tasks more than 75% of the third graders succeeded while only 25% of the first graders did).

2. The mathematical abilities mentioned in (1) do not seem to be totally ordered. Rather, several of them appear to emerge simultaneously.

3. Children that can measure in the Piagetian sense appear to have mastered all of the mathematical abilities measured.

4. Change of position (conservation of length) does not seem to be as important for measurement as claimed by Piaget.

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The Development of Length Conservation
Through Measurement Training

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Purpose

The purpose of this study was to investigate how young children
learn to conserve length. The study is a replication of a training
study reported by Inhelder, Sinclair and Bovet (1974) which attempts
to analyze the specific effects of training on children at different
levels of cognitive development. This training procedure facilitates
the identification of specific mechanisms that lead to the develop-
ment of conservation of length.

Conceptual Framework

For Piaget, conservation is the central concept which underlies
all measurement. Recent evidence, however, suggests that the appli-
cation of measurement operations may contribute to, rather than
depend upon, the development of conservation (Carpenter, 1976). A
key study in this area, that has been widely cited in the literature,
is the study reported by Inhelder, Sinclair and Bovet (1974). In
addition to the significance of this study for understanding the
development of measurement concepts, it is also cited by the Genevans
to support their theory that cognitive development proceeds through
the resolution of cognitive conflict. This critical study involved
only 16 subjects from a very select population and suffers from the
general lack of standardization inherent in Genevan research. Clearly
this study warrants replication.

Procedure

Subjects were 29 kindergarten children attending public school in
a rural Wisconsin community. All children were pretested on number
conservation, length conservation, and length transitivity. Several
days later they received the instructional treatment which was followed
by the posttest, consisting of the same items as the pretest. The
tasks in the instructional treatment required subjects to construct
roads using cuisenaire rods that were the same length as roads construc-
ted by the experimenter. The difficulty of the tasks resulted from
several constraints. In some instances the subjects were asked to
build their roads in a different direction than the experimenter's
roads. In others their roads were to be parallel to but beginning

at a different point than the experimenter's roads. In still other cases the subjects were required to build straight roads corresponding to the experimenter's crooked roads, but were given rods which were shorter than the experimenter's. Each subject was presented with several situations, all of which remained in front of the subject throughout the training. After completing the problems, the subjects were asked to explain their solutions, and to reconsider earlier solutions in order to resolve the conflict potentially existent in their initial solutions.

The instructional treatments were explicitly designed to create conflict. Children fail to conserve length because they center on a single dominant dimension and fail to recognize the inconsistency of their responses. Certain tasks were designed so that children would naturally center on endpoints, while in others they would focus on the number of cuisenaire rods. It was hypothesized that by attempting to resolve the conflict between these two types of responses, children would learn to conserve.

Results

The results of the study are summarized in Table 1. Ten of the children who failed to conserve length on the pretest improved their performance. Four went from no evidence of conservation to the intermediate stage, four went from the intermediate stage to full conservation, and two went from no conservation to complete conservation. Six of the ten children who showed improvement did not conserve number, one was intermediate, and three conserved number.

TABLE 1

Comparison of Pretest and Posttest Performance

		Number Conserved	Length Conserved	Length Transitivity
	Unsuccessful	19	16	17
Pretest	Intermediate	1	9	
	Successful	9	4	12
	Unsuccessful	19	10	17
Posttest	Intermediate	0	9	
	Successful	10	10	12

Conclusions

Although 40 percent of the subjects showed gains in conservation of length as a result of the training session, the results of this study are not entirely consistent with the Genevans' theory. They maintain that learning is only possible within the limits imposed by development. Specifically they propose that conservation of number is a prerequisite for the learning of length conservation or other more advanced operational structures. In fact they only included in their study subjects who conserved number but failed to conserve length. The current study included both number conservers and non-conservers. Subjects who did not conserve number were as successful in the training session as the number conservers. Approximately the same proportion of each group showed improvement on the posttest.

The current study also failed to find the same levels of performance during the training sessions identified by the Genevans. Although most subjects recognized the conflict in their responses, few of them were able to resolve this conflict. It is interesting that although few children showed much success on the instructional tasks, such a significant number showed gains on the posttest. Since there were not similar gains in number conservation or transitivity, it is unlikely that these gains can be attributed solely to retesting factors.

In general these results tend to support Carpenter's (1976) hypothesis that children seem to benefit from instruction that would appear to be beyond their level of cognitive development. In other words, although the children did not really recognize the significance of using different units of measure (different size cuisenaire rods), they did benefit from instruction using different units. This is consistent with the results of several other studies reviewed by Carpenter (1976). Perhaps the best explanation for children's success is Gelman's (1969) hypothesis that training directs children's attention to the relevant attribute.

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The Relationship of Field-Independent-Dependent Cognitive
Style and Two Methods of Instruction in Mathematical Learning

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Purpose

The purpose of the study was to investigate the relationship between
field-independence-dependence and two instructional methods in terms of
mathematical concept attainment. It was hypothesized that field-
independent students would benefit more from a "guided discovery" treat-
ment and field-dependent students more from a "meaningful didactic"
treatment.

Conceptual Framework

Recent reviews of aptitude-treatment interaction (ATI) studies
(Berliner and Cahen, 1973; Cronbach, 1975) indicate that ATI research
results have generally been inconsistent and inconclusive. Glaser
(1972) has suggested that the lack of solidly demonstrated ATI effects
is in part due to the aptitude constructs employed, and that process
variables such as cognitive styles might prove to be more promising
aptitudes to consider in interaction studies. Yet the value of cogni-
tive style as an aptitude variable for assigning students to
differential instructional treatments has received only minor consi-
deration (Kogan, 1971; Satterly, 1976).

Witkin's field-independence-dependence dimension is perhaps the
most widely known and thoroughly investigated of cognitive styles.
(For a review of investigations, see Witkin, Morre, Goodenough and
Cox, 1977.) This dimension was originally identified as a difference
in perceptual styles. However, subsequent research revealed a more
generalized analytic orientation associated with a learner's percep-
tual ability to overcome or restructure the organization of a field.
Within this context, field-independent individuals can be character-
ized by their ability to perceive, differentiate, and analyze relevant
components of a stimulus. On the other hand, field-dependent indi-
viduals are more passive and less analytic in their processing of a
stimulus, and tend to preserve the holistic nature of an encountered
field.

The stability and consistency of field-independence-dependence
would seem to make this variable ideally suited for ATI research.
Yet the question of appropriate treatments for students at either
end of this continuum is currently a matter of speculation. Kogan
(1971) has suggested that a discovery method of instruction might

prove beneficial to field-independent children, while field-dependent children might learn more from a didactic mode of teaching. The present investigation has attempted to determine whether such an interaction might exist.

Procedure

Sixty seventh grade students participated in this study. These participants were originally selected from a pool of 198 students on the basis of scores obtained on the NLSMA simplified version of the Hidden Figures Test (Wilson, Cahen and Begle, 1968). Half of the participants scored higher than one standard deviation above the mean and were categorized as field-independent. The remaining participants scored lower than one standard deviation below the mean were designated field-dependent. Students from each half were randomly assigned to one of the two treatments, resulting in a 2 x 2 factorial design with 15 subjects in each cell. IQ scores of the participants were obtained from school records and used as the covariate measure.

Two treatment presentations were developed for this study. The meaningful didactic presentation was characterized by deductive methods with stated rules followed by examples, the exclusion of irrelevant information, and immediate feedback on the correctness of responses. The guided discovery presentation was basically inductive, requiring students to derive rules from examples and to sort relevant from irrelevant information with no substantive feedback provided. A script was prepared for each treatment to insure consistency of presentations. The students were taught in groups of ten, with each of the six groups including field-independent and field-dependent participants. These six groups were independently taught the treatment material during two class periods lasting 45 and 30 minutes, respectively. Immediately subsequent to the second instructional period, each group was independently administered the posttest.

The topic chosen for instruction was the traversability of graphs, presented as a highway network problem. This choice was guided by the desire to minimize the effects of prior learning while providing a topic accessible to seventh graders. The 20-item posttest required students to identify new complex traversable networks and apply traversability rules to new situations.

Analysis

Two-way analysis of covariance was applied to the data, using IQ scores as the covariate and posttest scores as the criterion measure. Homogeneity of variance and homogeneity of regression were both tested prior to the analysis and found to be tenable.

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Results

Field-independent students achieved significantly higher (p < .05) posttest scores than did field-dependent students. This finding is consistent with previous research demonstrating the superiority of field-independent individuals in terms of concept learning tasks (e.g., Ohnmacht, 1966) and mathematical ability measures (e.g., Satterly, 1976). The concept learning tasks in this study required students to perceptually organize and conceptually categorize networks and traversability rules. Field-dependent behavior apparently inhibited recognition of those components critical to the identification of traversable networks. On the other hand, field-independent students demonstrated an ability to recognize and evaluate relevant attributes of network stimuli. These behaviors were consistent within each cognitive style group, irrespective of instructional treatment.

The main effect of instructional method was not statistically significant. The high interest level demonstrated by the students in the content of the units, with its possible effect on motivation, may have influenced learning to the extent that it outweighed the effects of differences in instructional formats. This evaluation, although speculative, suggests one possible explanation for the lack of significant difference.

Instructional treatments were not found to interact with field-independence-dependence in this study. However, the possibility of such a relationship continues to exist, with the concommitant possibility for adaptation of instruction to individuals in classroom settings. Level of guidance has been found to interact with this cognitive style in a college setting (Carpenter, McLeod and Skvarcius, 1976), and it is possible that this variable might interact with field-independence-dependence with younger children. It is recommended that additional research be conducted in this area, with consideration of alternative instructional variables and criterion measures. Furthermore, it is also recommended that research commence in the area of compensatory treatments for field-dependent students.

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A Study of the Child's Use of Perceptual and Numerical Cues with Respect to Area Measurement

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Purpose

Little research evidence of a developmental nature exists concerning the child's conception of area measure. Piaget and colleagues (1960) did initial work in this domain. Follow-up work was done by Beilin and Franklin (1961). Wagman (1975) studied the child's conception of area measure with a primary emphasis on the conception of unit. With the exception of Wagman (1975), there is no reported evidence of how children incorporate number into their concept of area measure. The present study was designed to look at children's area measurement behavior with emphasis on which cues, unit covering or multiplicative, children attend to as they consider the area of certain regions.

The task design was similar to that used by Piaget (1960) in his subdividing a straight line task, but with appropriate changes for area instead of length. Children of elementary school age were interviewed using six sets of three area comparison tasks. Analysis of the children's behavior provides valuable information concerning measure behavior in general and area in particular. This information should give direction for instructional research concerning area concepts and related studies of the physical models used for teaching the operations for rational numbers.

Procedure

A sample of 106 students from grades three, four, five and six were interviewed using sets of area items (initial practice items were employed). The students interviewed came from schools in Austin, Texas and Columbus, Ohio. (Thanks are extended to Alan R. Osborne of The Ohio State University for his assistance in the collection and analysis of data.) A sample item is presented below:

Display Region

Answer Region

Figure 1, Sample Item
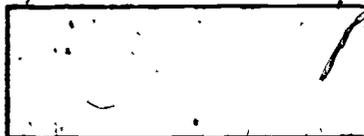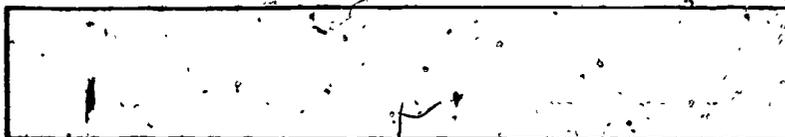
After being shown the Display Region, the child was asked to draw a line on the Answer Region to construct a rectangular region of equal area (see Figure 1). Analogous items were based upon a grid network and an indicated grid in order to provide differing perceptual bases for cuing children about area judgments (see Figure 2).
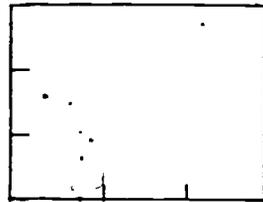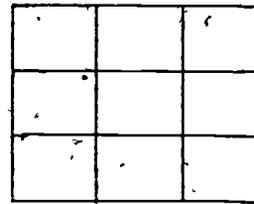
Indicated Grid   Grid

Figure 2.   Display Regions.

Any unusual problem-solving behavior or comments by the child were recorded. Particular attention was given to reporting the nature of children's errors.

Analysis

Analysis was done in a clinical manner because of the nature of the interview items. Therefore, the results and conclusions are anecdotal rather than statistical. This method was chosen so as to provide more direct suggestions to teachers of young children. In the presentation it will be possible to report informative statistics such as percentages of errors and frequencies of types of errors.

Conclusions

Results and conclusions can best be summarized by looking at general item performance and listing some common misconceptions about area.

General Performance. As expected, all children had some competence in area, with older children outperforming the less experienced ones. The three item types were, from easiest to hardest, grid, no-grid, and identical grid. Children who appeared to understand area could operate interchangeably with unit covering and multiplicative approaches to area. Younger children appeared to rely more heavily on the unit covering approach.

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Common Misconceptions. Five types of area misconception seemed common in the sample:

1. Judgment of area based upon length of one dimension;

2. Compensation using primitive perceptual methods;

3. Area determined by point counting;

4. Counting around the corner; and

5. Point counting using only linear units (Hirstein, Lamb and Osborne, 1978).

These results are to be used to design remedial activities in the classroom as well as to generate further instructional research. During the presentation, it will be possible to suggest teacher activities generated by children's errors.

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A Clinical Study of the Heuristic Strategies Utilized by High School Students

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Purpose

The study of heuristic strategies has been a continuing concern among mathematics educators (e.g. Kilpatrick (1967), Lucas (1972), Vos (1976), Blake (1976). While the studies tend to employ distinct, but not disjoint sets of heuristic strategies, it seems reasonable to conclude that most researchers utilized Polya's heuristic process (1957) as the initial framework for the heuristic strategies. One overall implication of such research seems to be clear, namely that students do utilize, and can be taught to utilize various heuristic strategies (e.g. drawing diagrams, making a table and looking for a pattern) in solving novel mathematical problems. With this support for the significance of heuristics in the problem solving process this researcher wanted to investigate the following questions:

1. Without any specific training in heuristics, what are the heuristic strategies utilized by students in solving novel mathematical problems?

2. What are the most commonly utilized heuristic strategies?

The works of Polya (1957, 1954 (a), 1954 (b)) and MacPherson (1973) provided the major frameworks for identifying those heuristics that might be utilized by students.

Procedure

Twenty high school students (grades 9 - 11) were selected from local schools. These students were all in the honors mathematics program and were students who the teachers identified as the 'better' problem solvers.

Each student had four problems to solve. The problems were presented one at a time and the student was allowed 15 minutes to solve each problem. If at the end of that period he had failed to find the solution he continued with another problem. If he completed the problem within the 15 minutes he was allowed to continue with the next problem. Two of the problems used were:

1. The Chinese Tangram puzzle. The student had to arrange
the seven pieces to form a square (see figure 1). The
seven pieces were placed in an envelope in front of him.

Solution:
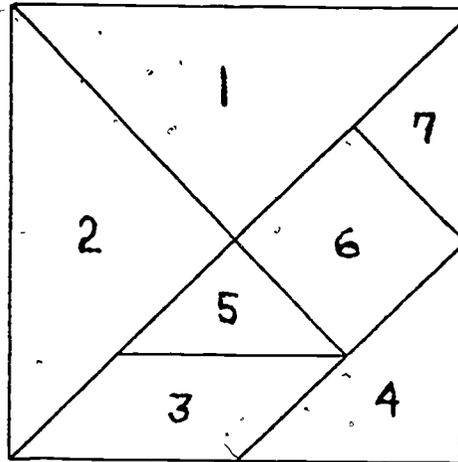
Figure 1

2. Given 1000 squares (1 cm x 1 cm) how can they be arranged
in a plane so that the total perimeter of the figure formed
is a minimum? Find the perimeter. In front of the student
there was a box containing a large number of 1 cm x 1 cm
squares.

Solution: Perimeter is 128 cm.
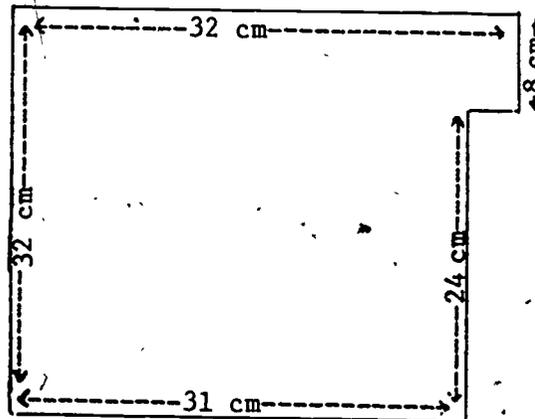
Figure 2

There were three key components to the experimental procedure.

1. The students were videotaped while solving the problems.

2. The students were thinking aloud while solving the problems.

3. The students were given physical materials to work with while solving the problems.

The use of physical materials and videotaping added a dimension to the data not present in most studies. It enabled the judges (see analysis) to determine whether the students were physically modeling heuristics such as symmetry.

Analysis

Using videotapes retained from the pilot stage of the study, the judges were trained in the use of the coding procedure. Primarily the coding procedure consisted of recording the heuristics used and the order in which they were used. At the end of the training sessions the analysis of the actual data tapes was undertaken. Each judge coded each tape separately, and then they met to discuss the results. The discussion centered around segments of the various tapes on which there were disagreements. Only results on which all the judges could agree were included in the interpretative stage of the analysis.

Results

The most commonly employed heuristic was examination of cases (enumeration of cases). Students tended to use both random cases and systematic cases. For example, the perimeter problem some students would try to build up from a small number of squares while others would just try random numbers of squares.

Symmetry was also used by many students. This was particularly obvious in the tangram puzzle. For example, the students would try to arrange the pieces in a symmetric manner (see figure 3).

Figure 3

Other heuristics, such as preservation of rules and analogy only
appeared very rarely.

Note. The use of videotapes proved to be extremely valuable.
The video portion of the tape provided evidence of heuristics that
was not present in the audio portion. Furthermore, the researcher
feels that it would have been impossible to record all the nuances
involved in the problem solving process without the tape.

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