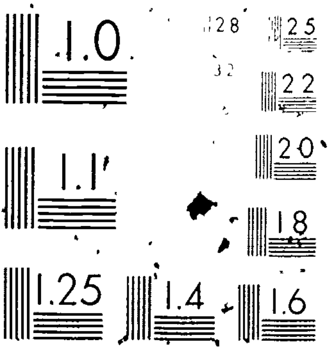
1.0
2.8
2.5
3.2
2.2
2.0
1.1
1.8
1.25
1.4
1.6

DOCUMENT RESUME

ED 146 054 SE 023 273

AUTHOR	Osborne, Alan R., Ed.
TITLE	Investigations in Mathematics Education, Vol. 10, No. 2.
INSTITUTION	Ohio State Univ., Columbus. Center for Science and Mathematics Education.
PUB DATE	77
NOTE	67p.
AVAILABLE FROM	Information Reference Center (ERIC/IRC), The Ohio State University, 1200 Chambers Rd., 3rd Floor, Columbus, Ohio 43212 (Subscription $6.00, $1.75 ea.)
EDRS PRICE	MF-$0.83 HC-$3.50 Plus Postage.
DESCRIPTORS	*Abstracts; College Mathematics; Elementary School Mathematics; Elementary Secondary Education; Higher Education; Instruction; *Learning; *Mathematics Education; *Research Reviews (Publications); Secondary School Mathematics; *Talented Students
IDENTIFIERS	*Krutetskii (V A); *Study of Mathematically Precocious Youth; USSR

ABSTRACT

The majority of this IME issue is devoted to reviews of recent research reports from the project for the Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University. An overview is provided of the book "Intellectual Talent: Research and Development", edited by Keating. Nine studies reported in the book are then abstracted and critiqued. A second feature of this issue of the journal is a review of Krutetskii's "The Psychology of Mathematical Abilities in School Children". (Author/MS)

INVESTIGATIONS IN MATHEMATICS EDUCATION

Published quarterly by

The Center for Science and Mathematics Education
The Ohio State University
1945 North High Street
Columbus, Ohio 43210

With the cooperation of the ERIC Science, Mathematics and Environmental Education Information Analysis Center

Volume 10, Number 2 - Spring 1977

Subscription Price: $6.00 per year. Single Copy Price: $1.75.
Add 25¢ for Canadian mailings and 50¢ for foreign mailings.

A Note from the Editor . . .

The majority of this issue of IME is a set of reviews of recent research articles that have emanated from the project for the Study of Mathematically Precocious Youth (SMPY) directed by Julian C. Stanley. John Harvey provides an overview of the entire book, Intellectual Talent: Research and Development, edited by Keating, that is the source for these studies. Nine studies that are reported in the book are abstracted and reviewed. Since the studies make extensive use of an assortment of tests, a listing of the tests used is provided along with a citation of a source of reviews of the test in Buros' Seventh Mental Measurements Yearbook. I think that you will find the reviews of the SMPY research interesting and stimulating.

A second special feature of this issue of IME is a review of Krutetskii's The Psychology of Mathematical Abilities in School Children edited by Kilpatrick and Wirszup. Bright provides an abstract of the contents of the Krutetskii book and two critical commentaries are offered, one by Goldin and the other by Bright. The Russian approach to the study of mathematical ability provides a sharp contrast to that of the SMPY project.

Spring 1977

INTELLECTUAL TALENT: RESEARCH AND DEVELOPMENT. Edited by Daniel P. Keating. Baltimore, Maryland: The Johns Hopkins University Press, 1976.

An Overview Prepared Especially for I.M.E. by John G. Harvey, University of Wisconsin-Madison.

The goal of the Study of Mathematically Precocious Youth (SMPY) is to "identify, study, and facilitate educationally those youngsters who are especially adept at mathematical reasoning while still in the first two years of junior high school, i.e., grades seven and eight and ages 12 to 14" (Stanley, 1974, p. 197). This study, directed by Julian C. Stanley, is being conducted at Johns Hopkins University with the financial support of the Spencer Foundation of Chicago; it began 1 September 1971. Thus far two comprehensive reports have resulted from this study; they are Mathematical Talent: Discovery, Description and Development (Stanley, Keating and Fox, 1974) and Intellectual Talent: Research and Development.

The origins of SMPY can be traced to two sources. First, it is somewhat akin to and, in a narrow sense, continues the work of Terman et al. It is akin to the work of Terman and his associates in that psychometric instruments are used to identify and study mathematically precocious youths. It continues the work of Terman in that it attempts to study longitudinally those mathematically precocious junior high school students who participate in its program of counselling and educational facilitation. It is narrower in that instead of studying generally precocious individuals, it is studying mathematically or quantitatively precocious ones (see Keating, 1976, p. 24 for a definition of quantitatively precocious). The second impetus for SMPY was the frequent, unsought identification of mathematically precocious youths by Stanley (1976, pp. 6-10).

The SMPY project staff initially thought that informal methods such as parent or teacher referrals would identify for study sufficiently large numbers of junior high school students who were mathematically precocious. However, this proved not to be the case. Thus, in early 1972 the first talent-search test competition was organized. The competition was advertised primarily in the Baltimore area. In March 1972, 167 seventh-grade students (77 girls, 90 boys), 224 eighth-grade students (95 girls, 129 boys), and five accelerated ninth-grade students (1 girl, 4 boys) took two College Entrance Examination Board (CEEB) tests: The Scholastic Aptitude Test - Mathematics (SAT-M) and Mathematics Achievement Level I (M-I). Twenty seventh-graders (7 girls, 13 boys), 33 eighth graders (4 girls, 29 boys) and one ninth-grade girl took the Educational Testing Service Sequential Test of Educational Progress, Series II - Science (STEP II-Science). A list of the tests used by SMPY that are reviewed in Buros' Seventh Mental Measurements Yearbook appears elsewhere in this issue of IME.) On the basis of their scores on SAT-M or STEP II-Science, 35 boys and 10 girls were invited back for further testing; all of the boys and eight of the girls came. These 43 children comprise the first group of mathematically precocious youths identified by SMPY. A complete description of the characteristics of these youths and of the educational facilitation initially offered them is included in Mathematical Talent: Discovery, Description and Development (Stanley, Keating and Fox, 1974); a majority of that data is also included in Intellectual Talent: Research and Development (1976).

In 1973 and 1974 the talent-search test competition (now called the Maryland Mathematics Talent Search) was expanded to include students in all of Maryland; students in the Washington metropolitan area counties were especially sought in 1973. After the 1972 competition two other changes were made. First, in order to participate in 1973 or 1974 a student had to be in the top two percent of his or her grade on national norms in arithmetic reasoning, total arithmetic, quantitative aptitude, or their equivalent. Second, only the SAT was administered to the 1973 and 1974 participants; Table 1 indicates the numbers of seventh- and eighth-grade boys and girls who participated in each year:

TABLE I

NUMBER OF STUDENTS BY GRADE AND SEX WHO PARTICIPATED IN THE 1973 AND 1974 MARYLAND MATHEMATICS TALENT SEARCHES

1973				1974			
7G	7B	8G	8B	7G	7B	8G	8B
88	135	158	286	222	372	369	556

Notes: 7G = seventh-grade girls; 7B = seventh-grade boys; 8G = eighth-grade girls (includes accelerated ninth-graders); 8B = eighth-grade boys (includes accelerated ninth-graders). This table is derived from data given by Keating (1976, p. 27).

In 1974 the 111 students who scored at least 640 on SAT-M were declared the talent-search winners for that year (George and Solano, 1976, p. 63). Similar data are not given for 1973. Using the data given by Keating (1976, p. 27), it can be determined that in 1973 there were 90 students (14 girls, 76 boys) who had a SAT-M score of at least 650. This criterion was not used, however, in choosing students to participate in a special class conducted at Johns Hopkins; for that class, students from Baltimore and Howard counties who had a score of at least 500 on SAT-M and 400 on SAT-V were invited to participate. It cannot be determined what further study and educational facilitation have been given to the other students who participated in 1974 except that 41 of the 1974 participants did receive one-course college scholarships. Intellectual Talent: Research and Development reports the research conducted with the educational facilitation given to youths who had high scores on the tests given by SMPY in the 1972, 1973, or 1974 talent-search test competitions.

Chapter One of Intellectual Talent is a revised version of an address given by Stanley to the American Psychological Association in August 1973. In this chapter Stanley advances the theses that

1. Tests are a prime way--probably the prime way--for the preliminary identification of high-level developed aptitude or achievement.

2. It is even more important than generally realized for tests to have enough "ceiling" (and "floor," too) for each individual tested. This means bold use of tests designed for older persons, . . .

3. The higher an examinee's scores are, the greater his or her potential tends to be. For appropriate criteria, validity does not drop at the upper part of the score range of a test that is difficult enough for the persons tested (Stanley, 1976, p. 5).

The chapter then goes on to recount instances in which its author had encountered precocious or talented individuals and concludes with some of the early outcomes of SMPY.

Chapter Two, by Keating, gives a definition of quantifiative precocity (p. 24) and describes the data collected in the 1972, 1973, and 1974 talent-search test competitions, including a grouped frequency distribution of the SAT-M scores by grade and sex for each of those years (p. 27, Table 2.1). Most of the data previously described in this overview are from or based on data given in this chapter. In this chapter Keating also discusses the need for and value of tests which are adequately difficult in detecting differences between students who would otherwise appear to be the same when tested; for example, in-grade tests on which two students of unequal ability score in the 99th percentile. On the basis of this discussion, Keating concludes that to find out "which of a given group of able twelve-to fourteen-year-olds has attained a level of quantitative reasoning ability comparable with able high school seniors, one need only to give them the same test of mathematical reasoning one would give to a group of high school seniors. The excellent and frequently used test for this purpose is SAT-M" (p. 29). He then goes on to argue that a younger student who has a high score on the SAT-M uses higher-level processes than does the high school senior and that this probably biases the predictive validity positively; that is, that these students are more likely to be successful in learning new material than are high school seniors.

Chapter Three concerns itself with methods and models for the identification and acceleration of gifted junior high school students, especially those who are mathematically talented. Using experience gained through SMPY, Fox proposes that a wide variety of psychometric instruments be used to identify precocious youths, to establish their range and level of abilities, and to determine their interests and motivations. Next she discusses the alternatives which a school could use to accelerate a precocious youth; these include grade skipping, subject-matter acceleration, fast-paced classes, Advanced Placement Courses, and college courses. With these alternatives established, she then proposes four plans; briefly, they are:

Plan I: Seventh Grade to College in Five Years
Plan II: Seventh Grade to College in Four Years
Plan III: Radical Acceleration Alternative
Plan IV: Subject-matter Acceleration Only

It would seem that, except for details, the titles of these plans are self-explanatory except for Plan III. In that plan a student would be placed in tenth grade during the next school year except that all of his or her precalculus mathematics courses would be radically accelerated so that this student would enter an Advanced Placement calculus course during the following school year. Fox concludes this chapter with a short discussion of the need for and ways to monitor the progress of a student who is accelerated in one of these ways.

Chapter Four very carefully details the 1974 Maryland Mathematics Talent Search. Chapters Five through Seven describe research associated with SMPY. This research is abstracted in this issue of IME: thus it will be briefly described here. In Chapter Five, Keating reports an experiment in which psychometric and Piagetian methods for identifying mathematical precocity are compared. Chapter Six reports on the fast-paced classes offered for mathematically precocious students by SMPY. This chapter also reports on the cognitive tests, the interest inventories and the values scales completed by the students in these classes. In Chapter Seven the results of an experiment are reported in which college-level teachers taught special fast-paced mathematics courses to school children. Chapter Eight is a report from the Study of Verbally Gifted Youth. This project, also conducted at Johns Hopkins University and funded by the Spencer Foundation of Chicago, is similar to SMPY in that it is seeking to learn more about giftedness and to develop effective methods of facilitating the education of gifted students.

Chapters Nine through Fifteen are also abstracted in this issue of IME. In Chapter Nine Fox describes an experiment in which a special summer accelerated Algebra I program was used with seventh-grade girls. A study of educators' stereotype of mathematically gifted boys is presented in Chapter Ten. Chapters Eleven, Twelve, Thirteen, and Fourteen report on the nonintellectual correlates of mathematically precocious boys and girls, the career-related interests of those youths, the creative potential of the boys in that group, and the values of these students, respectively. Chapter Fifteen recounts a study which compared the profiles of values reported in Chapter Fourteen to randomly generated values.

In Chapters Sixteen and Seventeen, Page and Bereiter comment upon SMPY and the techniques which that project has used to identify, study, and educationally facilitate mathematically precocious youths. In addition, Page introduces and discusses a measure of intelligence (or mathematical intelligence) analogous to IQ which is calculated using scores from tests designed for older persons. He then goes on to describe some of the uses of this new measure.

These seventeen chapters comprise the papers which were given at the Sixth Annual Hyman Blumberg Symposium held at Johns Hopkins University in October 1974. The concluding chapter of Intellectual Talent is a summary of the general discussion which followed the presentation of the papers.

Critical Commentary

While this overview of Intellectual Talent: Research and Development has not followed the usual format for the abstracts which appear in IME, it still seems appropriate to conclude with the usual "Critical Commentary." Some of the remarks in this part may echo or reinforce those which appear in the abstracts of the research papers.

1. The criteria by which the 1973 group of mathematically precocious youths was chosen from those who participated in the talent-search test competition that year do not seem to be described.

2. The plans for the study and educational facilitation of the 1974 group of mathematically precocious youths do not seem to be included. It is hoped that this does not indicate that the long-range planning for this group of students is not complete or has not been initiated especially because of the negative reactions from schools reported by Fox (1976, p. 203), the less than favorable stereotypes of mathematically gifted boys reported by Haier and Solano (1976), and the belief that

> The need for special efforts . . . to design innovative educational programs for them [talented students] is particularly acute during the junior high school years . . . (Fox, 1976, p. 33).

3. It is often difficult to discern which group(s) of students are being discussed; a more consistent identification of each group and the group from which they were drawn would assist the reader in understanding the SMPY program of identification, study, and facilitation and in evaluating the research which is reported.

4. Curriculum and grade acceleration of precocious youths do not seem to be good ways to facilitate the development of these students. However, while advancing, these students should learn at a different level of abstraction than do less able students and better problem-solving performances should be expected from them [see Lucas (1972) for definitions of mathematical problems and problem-solving performance]. It cannot be discovered from either Mathematical Talent (Stanley, Keating and Fox, 1974) or this account if these goals are being sought or accomplished.

5. A description of the Advanced Placement examinations in calculus states, "Both Calculus AB and Calculus BC are primarily concerned with an intuitive [emphasis added] understanding of the concepts of calculus and experience with its methods and applications" (College Entrance Examination Board, 1976, p. 3). Since a criterion for success in the calculus courses taught the students in this study has been a score of four or five on one

of the Advanced Placement examinations, it cannot be concluded from this evidence alone that these students are acquiring the kind of knowledge in calculus expected of college honors students. The evidence that these students make good grades in college-taught honors calculus or advanced calculus courses is much more persuasive.

6. Recognizing the difference between prococity and creativity, the SMPY project has tried to determine the creative potential of the youths they have identified. This attempt, while not futile, did not show that the subjects have creative potential. One ability commonly thought to be a necessary prerequisite for creativity is the ability to solve problems. Thus a study of the mathematical problem solving abilities of mathematically precocious youths seems to be suggested (see Kilpatrick, 1967; Lucas, 1972; Zalewski, 1974, and Wearne, 1976).

REFERENCES

College Entrance Examination Board. The 1976-77 Advanced Placement Mathematics. Princeton, New Jersey: Author, 1976.

Fox, L. H. Identification and Program Planning: Models and Methods. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976.

Fox, L. H. Sex Differences in Mathematical Precocity: Bridging the Gap. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976.

George, W. C.; Solano, C. H. Identifying Mathematical Talent on a Statewide Basis. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976.

Haier, R. J.; Solano, C. H. Educators' Stereotypes of Mathematically Gifted Boys. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976.

Keating, D. P. (Ed.) Intellectual Talent: Research and Development. Baltimore, Maryland: Johns Hopkins University Press, 1976.

Keating, D. P. Discovering Quantitative Precocity. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976.

Kilpatrick, J. Analyzing the Solution of Word Problems in Mathematics: An Exploratory Study. Unpublished doctoral dissertation, Stanford University, 1967.

Lucas, J. F. An Exploratory Study on the Diagnostic Teaching of Heuristic Problem Solving Strategies in Calculus. Unpublished doctoral dissertation, University of Wisconsin-Madison, 1972.

Stanley, J. D.; Keating, D. P.; and Fox, L. H. Mathematical Talent: Discovery, Description and Development. Baltimore, Maryland: Johns Hopkins University Press, 1974.

Stanley, J. D. Use of Tests to Discover Talent. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976.

Wearne, D. C. Development of a Test of Mathematical Problem Solving Which Yields a Comprehension, Application and Problem Solving Scores. Technical Report No. 407. Madison Wisconsin: The Wisconsin Research and Development Center for Cognitive Learning, The University of Wisconsin, 1976.

Zalewski, D. L. An Exploratory Study to Compare Two Performance Measures: An Interview-Coding Scheme of Mathematical Problem Solving and a Written Test. Technical Report No. 306. Madison, Wisconsin: The Wisconsin Research and Development Center for Cognitive Learning, The University of Wisconsin, 1974.

A PIAGETIAN APPROACH TO INTELLECTUAL PRECOCITY. Keating, D. P. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating, Baltimore, Maryland: Johns Hopkins University Press, 1976, pp90-99.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Leslie P. Steffe, University of Georgia.

1. Purpose

Keating's purpose was to study the relationship between psychometrically defined brightness and cognitive developmental precocity within Piaget's stage theory in early adolescents. Three questions were investigated regarding the purpose. The first question (and [illegible]portant to the authors) had to do with the relationship between [illegible]ess and precocity. Does brightness as measured by psychometric testing imply developmental precocity? The second question was sparked by the use of psychometric tests to identify precocious students. Is it the case that high scorers on psychometric tests are just "good test-takers?" The third question was whether similar aspects of "intelligent behavior" are being tapped by the psychometric and the Piagetian traditions.

2. Rationale

Two traditions that exert major influence on theories of intelligence are the psychometric and the Piagetian. The basis of the psychometric tradition is the measurement of individual differences through evaluation of representative samples of behavioral products in standardized situations. Variability in the mental abilities is assumed. In contrast, the developmental theory of Piaget is a unified theory that has the goal of identification of universal structures of human thought. The methodology of study is not standardized, but clinical, with the purpose of displaying behavioral symptoms of underlying cognitive processes.

3. Research Design and Procedure

One hundred nine students in grades five and seven from the Baltimore County school system were used as subjects. Of the 109 subjects, 31 were bright seventh graders, 19 were average seventh graders, 37 were bright fifth graders, and 22 were average fifth graders. To be classified as bright (B) a student had to score at the 98th or 99th percentile on the arithmetic section of the Iowa Test of Basic Skills. To be classified as average (A) a student had to score between the 45th and 55th percentile on the same test. All subjects completed five tests: (a) Raven's Standard Progressive Matrices; (b) conservation of volume, (c) displacement of volume, (d) equilibrium in the balance, and (e) period of pendulum. Test (a) was considered as a psychometric test and was used to determine a "psychometric order" among the groups. The prediction was that the groups would be arranged using the Piagetian tests in the same way as the psychometric order. Test (b) was considered an advanced concrete operational task whereas the remaining three were considered to be formal operational tasks. The Piagetian tasks were scored 1 (clearly concrete operational), 2 (a breakdown of concrete operational but no indication of formal operational), 3 (transitional), and 4 (formal operational). A repeated-measure ANOVA was run using Psychometric Level and Grade as classificational variables and the three Piagetian formal operational tasks as repeated measures.

4. Findings

(1) On the Raven's Standard Progressive Matrices, the psychometric order of the groups was 7B = 5B > 7A > 5A using Scheffe's multiple comparison method. Scores were 48, 47, 44, and 38, rounded to the nearest whole number.

(2) Inter-rater reliability on the Piagetian tasks over the four groups was .94. All subjects "passed" the conservation of volume tests; therefore, it was disregarded in subsequent analyses.

(3) The percentages demonstrating formal operations on all three Piagetian formal operational tasks were 62, 47, 23, and 0 for groups 7B, 5B, 7A, and 5A respectively. The order of the groups was reported to be 7B > 5B > 7A > 5A.

(4) The percentages of students demonstrating formal operations on at least one Piagetian task were 85, 93, 63, and 31 for the 7B, 5B, 7A, and 5A groups, respectively. The order was reported to be 7B = 5B > 7A > 5A.

(5) The percentages of students demonstrating formal operations on the displacement task were 77, 70, 38, and 0; on the equilibrium in the balance task, 77, 85, 47, and 8; on the period of pendulum task, 62, 70, 38, and 23. Percentages in all three categories are for 7B, 5B, 7A, and 5A, respectively.

(6) In the ANOVA psychometric level was highly significant ($p < .001$) and age was marginally significant ($.05 < p < .10$). No other factor or interaction was significant.

5. Interpretations

Keating, in his discussion of the results, states:

(a) "The major hypothesis...that brightness...implies developmental precocity in reasoning...was confirmed...".

(b) "...when students are selected for high scores on psychometric tests, those successful are indeed precocious in cognitive development, and not just 'good test-takers'."

(c) "...this research...confirms the empirical relationship of brightness and precocity and does so across differing traditions....It seems that brightness leads to precocity... the brighter individual would be at an advantage in moving through the successive stages more quickly...".

(d) "the absence of a main effect for tasks (in the ANOVA) suggests that development within the formal operational period is not entirely analogous with that in the concrete operational period...instead of a series of structural changes, there may be instead a global structural change...".

Critical Commentary

Keating expressed a goal of cognitive-development research quite aptly in his rationale for the study--the identification of universal structures of human thought. It is well known that the rate of development of such structures varies across individuals within cultures. It seems that Keating's results confirm this fact. The issue is not, then, that individuals differ in quite important ways. The issue is in the interpretation of those differences.

Piaget does not believe that the universal structures of human thought are a priori in the sense of existing prior to experience. Experience plays a major role in the development of such structures. Hence, it is not at all surprising that children in the 5B and 7B groups essentially displayed formal reasoning whereas those children in the 5A and 7A groups displayed formal reasoning only erratically with the results better for the 7A group than the 5A group. So, are the 5B children precocious because they are bright or vice versa? Keating seems to think that brightness implies precocity. As brightness means scoring at the 98th or 99th percentile on the mathematics subtest of the Iowa Test of Basic Skills, there is little basis for attempting to establish brightness as a sufficient condition for developmental precocity. Apparently, the relation could just as well be taken the other way.

Focusing on the psychometric tradition (standardized achievement tests or tests of intelligence) and/or on the universal structures of human thought will not alone lead to an understanding of acquisition of mathematical knowledge. Much more is needed. Keating alludes to the interaction of organism and environment as a prime factor in such acquisition. He seems to believe, however, that brightness is a gift to only a small number of lucky individuals. The psychometric tradition would seem supportive of this alleged belief. But, is it possible for an "average" student to become a "bright" student, and vice versa? Surely we should not ignore this very important question, as the influence of environment on an individual's social, emotional, and intellectual existence is barely understood.

CURRICULUM EXPERIMENTATION FOR THE MATHEMATICALLY TALENTED. George, William C.; Denham, Susanne A. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976, pp103-131.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Richard Crouse, University of Delaware.

1. Purpose

To describe the design of a fast-paced mathematics curriculum which was established to meet the needs of highly gifted junior high school students.

2. Rationale

Julian Stanley has suggested that "the highly able are the most 'disadvantaged' group in schools because they are almost always grossly retarded in subject matter placement". The subject matter retardation can have serious effects on students' mathematical performance not only because of failure to develop their talent but also through the influence on students' attitudes and aspirations toward mathematics. This program was based on the assumption that if students with ability and interest in mathematics were given the opportunity to learn as fast as they could, their achievements and satisfaction would probably be apparent.

3. Research Design and Procedure

The sample for the investigation was 33 students (29 ninth graders, 2 eighth graders, 1 seventh grader and 1 sixth grader) from Howard and Baltimore Counties in Maryland. These students were selected from among 953 Maryland seventh, eighth and under-age ninth graders who scored in the upper 2 percent on standardized mathematical or verbal reasoning aptitude tests. These students were then administered both the mathematics and verbal sections of the Scholastic Aptitude Test. It was decided that those students who obtained a score of at least 500 on the SAT-M and 400 on the SAT-V would be eligible for a class at Johns Hopkins University. A sample of 31 students (22 boys and 9 girls) was thus identified for the class; 2 boys, one ninth grader and one sixth grader, were added later.

From June to August 1973 these 31 students participated in an Algebra II class for one two-hour period per week. Four girls and one boy chose to drop out of the special class at the end of the summer. At the end of the Algebra II segment of the class, before Plane Geometry was started, it was decided to split the class into two sections. Five of the students needed more detail than was given in the regular class. These classes met for the entire 1973-1974 year. During each class the teacher introduced challenging material at a rapid pace. The material covered included all of Algebra II, Plane Geometry and a large portion of Algebra III. In addition several students continued further with the class and completed the four and one-half years of pre-calculus mathematics.

Levels of achievement were measured using Cooperative Achievement Mathematics Tests. In addition, the students were given a battery of Cognitive and Vocational Interests Tests. These included the Raven's Progressive Matrices, Standard and Advanced; Sequential Tests of Educational Progress, Science; Revised Minnesota Paper Formboard Test, forms AA and CC; Revised Scales from Holland's Vocational Preference Inventory; the Strong-Campbell Interest Inventory and the Allport-Vernon-Lindzey Study of Values. Means, standard deviations, and percentile ranks were reported.

4. Findings

(a) In 108 hours of instruction, 28 students learned Algebra II and Plane Geometry at a high level of achievement. Algebra III was completed by 23 students and 13 boys successfully completed the four and one-half years of pre-calculus mathematics.

(b) All 28 students scored at the 85th percentile or higher on the national high school norms as measured by the 80-item ETS Cooperative Mathematics Test in Geometry. Thus in 38 hours of instruction they exceeded the total score earned by 85 percent or more of the students who had studied Plane Geometry for an entire school year.

(c) Trigonometry was completed by 17 students in 16 hours. The mean score for the group on the 40-item ETS Cooperative Mathematics Test in Trigonometry was 28. This was the 96th percentile of national high school norms. No student scored below the 76th percentile.

(d) Analytic Geometry was completed by 16 boys in 14 hours of instruction. The mean score of this group on the Cooperative Mathematics Test in Analytic Geometry was 29, which was the 95th percentile of national high school norms. No one scored below the 75th percentile.

(e) The majority of the students found the new class more productive, more fun, and more competitive. In regard to what the students liked best about the class, the students rated the teacher's teaching style highest. The challenge of the mathematics taught and the students' feelings of accomplishment rated next highest.

(f) Girls scored significantly lower than boys on the SAT-M and on Bennett's Mechanical Comprehension Test, Form AA.

(g) Boys were not significantly higher than girls on the investigative (inquisitive and scientifically oriented) Holland Scales. However, girls were significantly lower ($p < .01$) than boys on science and mathematics interest scales as measured by the Strong-Campbell Interest Inventory scale, but girls were significantly higher ($p < .001$) than the boys on the social service interest scale.

(h) On the Allport-Vernon-Lindzey Study of Values, boys were significantly higher ($p < .001$) than the girls on Theoretical and Economic Values while the reverse was true on the Aesthetic, Social, and Religious Values.

5. Interpretations

(a) In order to conduct a fast-paced mathematics class, careful attention must be paid to: (1) identification of qualified students through appropriately difficult tests of mathematical and non-verbal reasoning (a certain minimum degree of verbal mastery seemed necessary to learn mathematics at a rapid-fire pace); (2) the selection of a dynamic, bright, assertive teacher who can create an atmosphere of fun and productivity while introducing challenging materials; and (3) voluntary participation by the students. It appears that once these considerations are met, the academic and social aspects of such a class will proceed "naturally".

(b) From their SAT-M score it appeared that from the outset boys had more mathematical reasoning ability than girls, even though a greater percentage of girls than boys had taken Algebra I already. It seems that boys acquire some of their mathematical skills from sources outside the classroom.

(c) The higher scores of girls on the social service interest scale may be of practical educational significance. Their high interests in social sciences and mathematics in combination with their social-investigative orientation may lead them into the teaching field, medicine, psychology, or similar careers. On the other hand, the boys were far more scientifically oriented, pointing to possible careers as scientists, mathematicians, or computer designers.

(d) An investigative orientation toward pursuing goals and choosing activities is helpful if one is to survive in an investigative environment. Thus placing a socially (but not investigatively) oriented student in a highly investigative environment may not allow for the effective use of the individual's talents. It is worth considering whether social classroom environments should be constructed for the benefit of social-type students and investigative environments should be constructed for those students who can benefit from them most. This would imply considerable segregation by sex.

Critical Commentary

Meeting the needs of highly gifted students in mathematics via an appropriate curriculum is an extremely important problem. This program is certainly an interesting one that should stimulate both researchers and classroom practitioners to emulate and/or refine the principles and practices developed. The predictive potential of this program holds considerable promise, but only limited generalizations and/or interpretations can be made from this investigation. Any conclusions drawn and/or implications made must be tempered by the following facts and questions.

The authors report that the material covered included all of Algebra II, Plane Geometry, Algebra III, et cetera. However, what does this mean? Exactly what was included in Algebra III? It would have been clearer if the authors described the material covered by listing the topics included.

Not all tests were described in detail; in particular, no reliabilities were reported for the Cooperative Achievement Mathematics Tests which were used to measure levels of achievement. It has been this reader's experience that these tests have low ceilings, thus raising the question of just how well or how much these students really did learn. Several improvements in this investigation could have been made, such as including a pre- and post-test component for each subject which would have provided valuable baseline as well as change data. Retention tests might also provide useful information.

The authors report that "the teacher's style and ability are vital to the success of such a program". This conclusion may indeed be true but further data or experimentations are needed to substantiate this claim. It would be interesting to investigate whether these students could have learned the same material with another teacher with a different teaching style or by independent study with appropriate mathematics material.

In general this report is clearly written. However, the authors could have done a better job in organizing the material. It would have been better to describe the levels of achievement of the students immediately after the description of the program instead of being separated by a discussion of cognitive and vocational interests tests. Trying to figure out which students were in which class at what time was also quite confusing. In spite of these minor criticisms, this report is certainly one that should be read. Its potential for meeting the needs of highly gifted mathematics students is considerable.

SPECIAL FAST-MATHEMATICS CLASSES TAUGHT BY COLLEGE PROFESSORS TO FOURTH THROUGH TWELFTH GRADERS. Stanley, Julian C. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976, pp132-159.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Arthur F. Coxford, The University of Michigan.

1. Purpose

(a) To develop and evaluate, within a single school, a program for teaching algebra to mathematically apt students earlier and faster than usual.

(b) To apply the fast-mathematics teaching techniques to supplementing calculus instruction for apt students in order to improve performance on the BC Level Advanced Placement Program examination.

2. Rationale

The author and his colleagues have illustrated the effectiveness of special fast-paced mathematics instruction for extremely apt mathematics students in situations which drew students from large populations. Such populations contained a relatively large number of talented mathematics students. In a local single school building, the number of talented students is significantly less. Thus, the participants in SMPY (Study of Mathematically Precocious Youth) wished to test their procedures under the more difficult conditions existing in a single building. That is, they wished to determine whether the principles and practices developed in semi-laboratory settings could be used under more typical school conditions.

3. Research Design and Procedure

The school used to test the fast-mathematics procedure for teaching algebra enrolled 67 fourth-, 63 fifth-, 68 sixth-, 370 seventh-, and 360 eighth-grade students. An initial screening of fourth- through seventh-grade students was done by examination of scores on the arithmetic reasoning section of the Iowa Tests of Basic Skills achievement battery. A sliding scale was used. Twenty-three girls and 17 boys were identified.

These students were given the Academic Promise Test (APT) and Raven's Standard Progressive Matrices (SPM) on consecutive days. The subtests Numerical (N), Verbal (V), Abstract Reasoning (AR), and Language Usage (LU) of APT were used to select students. This procedure produced 12 girls for an all-girls class and 12 boys for an all-boys class. Seven girls were in grade 7 and 5 in grade 6. Six boys were in grade 7, 3 in grade 6, 2 in grade 5, and 1 in grade 4. All were highly talented mathematics students.

The procedure called for the boys to be taught by a man and the girls by a woman. The teaching in the classes was to be fast--no pacing adjustment was allowed for students lagging behind; rather the students were to fill gaps by completing carefully designed homework. The standards were high and the teacher was bright and alert with mathematical background well beyond the level taught. The two classes each met for a two-hour block each week. In each class, a total of 37 hours of instruction was provided before giving ETS-Cooperative Algebra I test in June 1974.

The same students were to resume fast-algebra study in Fall 1974. However, due to a variety of factors only 5 boys and 9 girls continued, and they were put in a single class. These 14 students participated in 24 additional hours of fast-Algebra I study and were retested. Following this test, the class studied fast-Algebra II and were given the Cooperative Algebra II test in March 1975 and again in June.

The fast-supplementary calculus teaching work began in September 1974. The class was composed of students studying regular calculus in school. The teaching took place on Saturdays for two hours. The purpose was to prepare for the BC Level Advanced Placement Examination in calculus. Fifteen boys participated initially on a volunteer basis. Thirteen continued until February 1975, at which time they completed the Cooperative Calculus test. The same test was administered again in May 1975 and all thirteen completed the BC Level Advanced Placement Calculus Test on May 13, 1975.

4. Findings

For the fast-Algebra I class tested in June 1974, 7 of 21 scored at or below the 49th percentile rank on national eighth-grade norms, 6 scored at or above the 90th percentile, and 8 scored between these extremes. When compared with 66 eighth-grade Algebra I students ($\approx$ 18% of the 360 eighth-grade students), the fast-algebra I students fared as follows: 5 scored higher than any of the 66, and all scored higher than twenty-three (35%) of the 66 eighth-graders.

Of the 14 continuing fast-Algebra I students taking an alternate form of the Cooperative Algebra I test, 50% scored on the 90th percentile or above after 24 additional hours of fast-Algebra I. In March 1975, eleven of the 13 continuing fast-Algebra II students took a form of the Cooperative Algebra II Test; the other form was given in June 1975. In March nearly 50% scored at or above the 79th percentile on national norms.

For the fast-calculus supplementary teaching class, the results on the Cooperative Calculus Test given in February 1975 showed only 2 of 13 scoring below the 90th percentile for national high school norms and showed no one below the 90th percentile in national college norms. Two of these students were in grade 9, 7 in grade 10, one in grade 11, and 3 in grade 12. In the May administration of the alternate form of the Cooperative Calculus test, all 13 students scored at or above the 99.1 percentile for national college norms. In the AP Level BC calculus examination, 9 students earned a 5 rating, 3 earned a 4 rating, and 1 had a 3 rating.

5. Interpretations

In regard to the fast-algebra I and II in a single large school, the author concludes:

> The most important factors that produce results. . . seem to be as follows: a teacher who knows mathematics well, is enthusiastic, has high standards, and moves the group fast; students who have considerable mathematical and verbal aptitude, as determined by standardized tests, and are fairly homogeneous in these respects but not necessarily alike in grade placement or chronological age; interest in learning mathematics quickly and well, which (especially among girls) does not always accompany aptitude; facilitative parents who value the unusual educational opportunity the special class represents and therefore encourage their children to do well, and helpful school personnel who do not try to obstruct progress because they feel threatened by it.

> By all criteria the course was a resounding success. In just 30 two-hour supplemental meetings with Dr. McCoart these able young men learned college calculus I and II splendidly, and a great deal of calculus III also.

The author concludes in general that the results of the fast-mathematics instruction imply that the type of class, homogeneity of student, and equality of instruction are vital considerations for learning. "In far fewer hours the students: . .have learned far more mathematics well than they would have in a regular classroom. . ." one or more years later. Finally, the author suggests that the techniques used in fast-mathematics classes may be applicable for other subjects in other schools, and that until these classes are instituted the intellectually gifted students ". . .will for the most part continue to get little that effectively meets their real intellectual needs".

Critical Commentary.

There is no doubt that some extremely able youngsters attained high levels of mathematical achievement in Algebra I and in Algebra II at young ages. It is also true that a significant part of the students did not complete the program. Whether or not the same youngsters would have learned more or less under a different procedure has not been answered in this report. Also the issue of whether there are more effective ways to attain better results with the "also able" dropouts has similarly gone unanswered.

In a sense the report verifies a tautology: Those who can learn under certain conditions do so. Now that the author and his colleagues have illustrated this tautology, it would be extremely worthwhile to vary their procedures in order to try to reach more of the able youngsters they so obviously wish to educate. For example, the results of the Algebra II co-educational class were quite good. Does this not suggest that teaching single-sex groups may not be necessary? And what about the pacing? It was not made clear how fast was "fast". Can the pace be varied to obtain better results for more of the talented students?

With regard to the supplementary teaching of calculus, let it be noted that 30 two-hour extra sessions is 60 hours of instruction. Sixty hours of instruction is what a college student gets in a 15-week, 4-hour course. Thus it is not surprising that these very able students did well on a test of two semesters of calculus having studied calculus for a time equivalent to three semesters.

In general the author seems to be crusading for his brand of "fast-math". I would suggest that he consider it as one approach and examine the positive effects (as was done in this article) and the negative effects. For example, were there any ill effects for those students who could not keep up? Before exporting this procedure to other areas, the author should experimentally verify that the features he thinks vital actually are, for it may be that the highly able will respond to any stimulating learning environment, not just this particular one.

SEX DIFFERENCES IN MATHEMATICAL PRECOCITY: BRIDGING THE GAP. Fox, Lynn H. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976, pp183-214.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Peggy A. House, University of Minnesota.

1. Purpose

The investigator hoped to accelerate by one year the mathematics program of bright seventh-grade girls by having them study Algebra I for three months in a special summer class designed to focus on the girls' social interests.

2. Rationale

It is generally recognized that there are sex differences in average mathematical aptitude and achievement among adolescents and adults. Previous research by the Study of Mathematically Precocious Youth (SMPY) showed boys to be more successful than girls in accelerating their mathematics learning through special out-of-school mathematics courses. The investigator hypothesized that the girls' more limited success may rest on two factors: first, the SMPY class, taught by a male ex-physicist, was theoretical while the girls were social by nature and did not like the classroom atmosphere or the required independent study; and, second, the program did not attempt to emphasize the relevance of mathematical study to the educational and career goals of the girls. An underlying assumption of the present study was that if one is to succeed in a mathematics-related field, then at an early age one must recognize the possibility of career success in that field and must begin to aspire positively toward developing one's talents. The special class was designed to appeal to the girls' social interests as a means of accelerating their achievement in mathematics.

3. Research Design and Procedure

Seventh-grade girls from Baltimore County, Maryland, who scored 370 or above on the Scholastic Aptitude Test-Mathematics (SAT-M) during SMPY's 1973 Talent Search were invited to participate in the experimental class. Invitations went to 32 girls selected on the above criteria and to two others referred for other reasons. Twenty-six accepted and enrolled in the class; 18 completed the program. For each girl who accepted, two control Ss, one girl and one boy, were selected from among the remaining SMPY contestents. Control subjects were matched on mathematical ability (SAT-M), verbal ability (Scholastic Aptitude Test-Verbal, SAT-V), educational level of mother, and education and occupation of father. An analysis of variance showed the three groups to be significantly different ($p < .01$) on SAT-M, and Tukey comparisons of the differences between means indicated that the boys scored higher than girls in both experimental ($p < .01$) and control ($p < .05$) groups. However, the decision was made to accept the boys as the best control group available. The groups were not significantly different on the other variables. Mothers' education was hypothesized to be related to the expectations which they hold for their daughters. No rationale was given for the inclusion of fathers' education and occupation as variables.

Experimental Ss studied Algebra I for three months during late spring and early summer of 1973, meeting approximately four hours per week. Control Ss took Algebra I in regular school classes during 1973-74. No information is given about the distribution of the control subjects in Baltimore County schools or about the nature of either their Algebra I classes or the teachers of these classes. None of the subjects had studied Algebra I in seventh grade. A pretest of Algebra I using the Cooperative Mathematics (COOP) Test, Form A, showed no significant differences among the three groups on knowledge of Algebra I prior to the experimental class.

The experimental class was taught by three women. No further information is given about the teachers. The class was organized around small-group and individualized instruction and was conducted informally with a stress on cooperative rather than competitive activities. Whenever possible and appropriate, teachers emphasized ways in which mathematics could be used to solve social problems. No information is provided on the number or frequency of these talks, on the background of the speakers, or on the approach used in addressing the girls. Finally, efforts were made to develop the study habits and skills of the experimental Ss by strongly encouraging them to read their mathematics texts, to use the test as a resource, and to set and meet self-imposed deadlines. How these efforts were carried out is not described.

Three questions were addressed in the study: Was an emphasis on social interests effective in recruiting girls to participate in the accelerated programs? To what degree did the girls master Algebra I in the accelerated program? Did the program actually accelerate the progress of the girls in their studies of mathematics in school?

4. Findings

Experimental Ss were compared with girls in two previous mixed-sex SMPY classes (SMPY-I, SMPY-II) for recruitment and dropout rates. The acceptance rate of girls invited to the all-girl class was higher than for girls invited to either SMPY-I or SMPY-II. No indication is given as to the statistical significance of this difference. Further, the criteria for selection differed among the three classes. The dropout rate was about the same as for SMPY-I and lower than for SMPY-II. Again, the statistical significance of the differences is not reported. The 18 girls who completed the program were reported to be more interested in investigative careers and to have stronger liking for mathematics than the eight who dropped out, but the report does not specify what instruments were used to obtain these ratings of attitude and career interest. Neither does it report the reliability or validity characteristics of the instruments. Other differences (girls who dropped out tended to come from homes where one or both parents were college graduates) are reported but not interpreted.

The 18 who completed the program were tested in July 1973 using Form A of an Algebra I test. The reference to the Algebra I test is not explicit, but it appears to refer to the COOP test used earlier as a pretest. The mean score of 30.6 was at the 89th percentile of national ninth-grade norms. Experimental and control subjects were retested in January 1974 using the COOP Algebra I, Form B. Scores from 23 matched triads were analyzed using an analysis of covariance with premeasures of SAT-V, SAT-M, and algebra achievement (COOP Algebra I, Form A) as covariates. The difference in performance among the three groups on the tests of algebra knowledge was significant ($p < .001$); Tukey comparisons of the differences between means showed the experimental Ss to be significantly higher ($p < .005$) than either control group. The control groups were not significantly different. These comparisons treated the entire experimental group ($N = 23$) without distinction between those who completed and those who did not complete the program. At the time of testing in January, control subjects were enrolled in regular Algebra I classes in their respective schools; experimental Ss who had completed the program were enrolled in Algebra II or were repeating Algebra I. The mathematics program of experimental Ss who dropped the course is not reported.

Eleven girls completed Algebra II during the year following the experimental course, nine of them receiving grades of A or B. Reasons why others did not complete Algebra II are complex and primarily related to difficulties with administrators, teachers, and counselors in the home schools. A second factor may have been that the criterion for success in Algebra I (65th percentile on ninth-grade national norms) was not high enough.

5. Interpretation

It is possible to motivate mathematically talented girls to attend a special accelerated program when social aspects of the program are emphasized. It is also possible to teach them Algebra I in less time than the typical school year. It is still difficult, however, to accelerate their progress in school. Further, the impact of accelerated programs appears less successful for bright girls than for bright boys.

Research is needed to investigate the impact of learner style and interests on achievement when aptitude is relatively constant. Research also is needed to investigate the nature and causes of sex differences in mathematical ability, particularly at higher levels of achievement. Comparisons should be made between accelerated programs and programs which supplement traditional classes with career education components and between sex-segregated classes and interest- (but not sex) segregated classes.

Critical Commentary

The investigation calls attention to two areas of significance in mathematics education: the needs to develop the mathematical abilities of the gifted and to encourage girls to pursue mathematics. Efforts to find viable alternatives for the education of gifted girls need to be encouraged and supported.

While it raises some important questions in these areas, the study as reported here cannot be considered an experiment. Questions must be raised about the investigator's attempts to compare nonequivalent groups. Most questionable is the attempt to compare experimental and control groups on Algebra I achievement at a time (January) when control subjects would have been in the middle of their Algebra I courses while most experimental subjects had completed Algebra I and an additional half-year of study at or beyond that level.

There are further difficulties in interpreting the results and conclusions because needed information is not reported. Other questions arise from design considerations: two girls were included in the sample for reasons other than the stated selection criteria; it appears that the same instrument was used as both pretest and posttest to measure Algebra I achievement; retest scores of Algebra I were analyzed for experimental Ss without differentiating between those who completed the program and those who dropped; some subjects were allowed to take the January test at different times and under different test conditions.

Variables are suggested, but their relationship to the study is not clear. Experiences of girls in the SMPY classes were cited to suggest the need to focus on the social interests of girls, but other factors (teacher male, teacher an ex-physicist; course theoretical, independent study required; et cetera) are not systematically controlled. Variables introduced into the experimental program (female teachers, more than one teacher, outside speakers, informal class organization, attention to study skills, et cetera) are not measured or evaluated for their effect on achievement or acceleration. It also appears that no attempt was made to control these variables for subjects in the control groups.

The study does provide evidence that, under certain conditions, talented girls can learn algebra in a brief period of time. It is hoped that future studies will be designed to identify and investigate those conditions which contribute significantly to that success. However, this reviewer would find it impossible to replicate the study as reported here because of the many unknowns indicated above.

Two additional questions must be raised: first, the study seems to assume that acceleration in mathematics is the most desirable outcome for mathematically gifted girls. This assumption is open to challenge. Second, the study rests on the assumption that the key in motivating the girls to succeed and accelerate in mathematics is through emphasis on their career interests. However, it is not at all clear that seventh-grade students are highly motivated by career goals. To ask the subject which of several careers she or he would prefer can suggest career preferences; it does not necessarily follow that the subject is conscious of or motivated by those preferences. This assumption needs investigation.

A significant contribution which the study makes is to call attention to the problems encountered when the investigator sought to make provisions for the advanced placement of the experimental Ss. The negative attitudes of teachers, counselors, and principals are extremely disconcerting and they serve to emphasize the urgent need not only for more research but also for major changes in the attitudes of educators toward the gifted and in their priorities for meeting the educational needs of this portion of the school population.

EDUCATORS' STEREOTYPES OF MATHEMATICALLY GIFTED BOYS. Haier, Richard J.; Solano, Cecilia H. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976, pp215-222.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas R. Post, University of Minnesota.

1. Purpose

Previous experiences of the SMPY (Study of Mathematically Precocious Youth, originated in 1971 at Johns Hopkins University under the direction of Julian C. Stanley) indicate that a negative stereotype of the gifted child exists in academic circles. This survey explores the prevalence and content of stereotypes of gifted male students in two groups of educators, one unfamiliar with a specific group of gifted students, the other personally familiar with these mathematically precocious youth. Identification of the nature and extent of this stereotype is the primary focus of this paper.

2. Rationale

The success and continuation of programs designed for exceptionally gifted students is dependent to a large extent on the approval and cooperation of principals, teachers, guidance counselors, and other school officials. Resistance to implementation of specialized programs for the gifted is sometimes substantiated on grounds of lack of money, difficulties with bureaucratic channels, scheduling problems, and existence of adequate programs for enrichment. Although these factors undoubtedly contribute to such reluctance, it also appears that an undertone of negativism pervades some thinking about gifted students. The attitudes or stereotypes that educators hold toward gifted students are critically important. Unfounded negative stereotypes can needlessly impede efforts made on behalf of such students. Although it has been shown repeatedly that negative stereotypes of the gifted have little empirical basis, there exists some evidence that such negative stereotypes continue to exist.

3. Research Design and Procedure

Two hundred principals, teachers, and guidance counselors from 50 public junior high schools in Pennsylvania were selected to represent a population of educators having no prior contact with the Study of Mathematically Precocious Youth (SMPY). An attempt was made to select schools from those counties in Pennsylvania whose demographic characteristics were similar to the counties in Maryland in which SMPY high scorers attend school. Principals of selected schools were asked to disseminate survey materials to one male and one female mathematics teacher, one guidance counselor, and him- or herself.

Each of the four educators in each school received a form containing case descriptions which briefly described four "real" boys identified by the SMPY as mathematically gifted. Each form contained descriptions of three gifted and one exceptionally gifted student. These descriptions were neutrally worded and included test scores, college courses taken, and grades received. Nine such descriptions of gifted students were prepared. Three additional boys who were described as exceptionally gifted were also "written up." The twelve descriptions were randomly assigned to one of three forms. Forms were subsequently sent to a randomly selected third of the schools contacted along with an attitude measure consisting of 150 personality-relevant adjectives, both positive (i.e., active, adaptable,... wise) and negative (i.e., affected, aloof,...whiny) in nature. Survey participants were instructed to read the case descriptions and then check those adjectives which he or she thought applied to the group of gifted boys. "The Pennsylvania sample therefore represented a population of educators who described mathematically highly talented boys on the basis of little or no direct experience with such students."

These data were compared to data from a sample of teachers, guidance counselors, and principals of 46 SMPY high scorers. This group is referred to as the Maryland educators. The same checklist was completed but with reference to "the specific student of his/her acquaintance."

Scores from both groups were standardized and individuals were subsequently classified as holding a positive or negative stereotype toward gifted students; positive if the standard score on the favorable scale exceeded the score on the unfavorable scale and negative if the reverse were true.

4. Findings

The frequency of negative stereotypes was found to be higher among Pennsylvania educators than Maryland educators. Negative stereotypes are more common with educators unfamiliar with such students than with those having had personal experience with gifted boys. Fifty-two percent of the Pennsylvania educators surveyed held negative stereotypes of mathematically gifted boys. The same was true for 32% of the Maryland educators.

The percentages of negative stereotype educators checking each of the 150 adjectives were calculated. Higher percentages of endorsement on favorable as compared to unfavorable adjectives was observed. The authors concluded that there exists more agreement on the favorable attributes of gifted students than on the unfavorable attributes. Even educators holding negative attitudes endorse many favorable adjectives. This was true in all groups.

The Pennsylvania educators (those not personally familiar with specific SMPY students) have higher percentages of endorsement on both favorable and unfavorable adjectives when contrasted to their Maryland counterparts.

A large degree of consistency with respect to the endorsement of specific adjectives was observed. The most popular favorable adjectives were: alert (96%), intelligent (95%), capable (94%), and ambitious (89%); the most frequently checked unfavorable descriptors were: opinionated (44%), argumentative (43%), impatient (38%), and egotistical (36%).

5. Interpretations

Although many educators held negative stereotypes, these are neither extremely hostile nor derogatory and may have some basis in fact. Since familiarity appears to mitigate negative opinions, the negative stereotypes are likely to be troublesome but temporary obstacles in facilitating the education of gifted boys.

No suggestions for further research were made.

Critical Commentary

It is difficult for this reviewer to react initially to the findings of this survey because of a number of procedural and design questions which cast some doubt as to the validity of the findings. The question of validity of the direct comparison of Pennsylvania and Maryland teachers needs further consideration.

Why did the authors choose to identify populations of educators in two different states? Such geographically diverse populations might very well have large-scale, "a priori" attitudinal differences ingrained perhaps due to region, educational climate, or a number of other uncontrollable or unidentified variables. Such differences might invalidate comparisons, even though an attempt was made by the authors to match counties on the basis of demographic variables. Would it not have been better for comparison purposes to select subjects from the same geographic location, ideally at the school level? Given such discrepancies as might exist in the Pennsylvania and Maryland populations, it would have been desirable to establish that Ss in both groups were in fact comparable in their professional perspectives except for the fact that one group had prior extensive experience with mathematically gifted boys. This was not done.

Pennsylvania and Maryland educators were given different directions prior to responding to the adjective checklist (ACL). The Pennsylvania group was asked to "check those adjectives on the ACL which he thought applied to the group of gifted boys" described by the four case summaries. These four summaries varied in length from two to six sentences, obviously an extremely small data-base from which to formulate a globalized opinion of mathematically gifted boys. The Maryland group, on the other hand, was asked to check "those adjectives that were descriptive of the specific student of his or her acquaintance." The validity of observed differences is suspect given the differential stimulus conditions at the time of "testing".

The authors further assume that "the Pennsylvania sample represented a population of educators who described mathematically highly telanted boys on the basis of little or no direct experience with such students." The authors provided no evidence that this assumption had been further substantiated. Is it reasonable to assume that mathematically gifted boys exist only in Maryland?

The ACL scores were standardized, by converting obtained raw scores using "adult normative data in the manual." It is not clear to this reviewer how such normative data could have been developed and/or could be useful unless the specific prompt used, i.e., mathematically gifted boys, is identified with the specific norms. If such was the case, it was not explicitly stated.

The manner used to identify positive and negative stereotypes seemed to lack precision. In reality the difference between being classified as having a positive or negative stereotype could have been the difference between the S checking or not checking a single adjective. Such classification is therefore potentially spurious. An alternative would have been to establish some percentage differential between the number of favorable and unfavorable responses as a prerequisite for classification. Such a procedure would undoubtedly result in a number of midrange individuals who would not be classifiable in either the positive or negative category, but the remaining persons would have expressed clear-cut attitudes toward the population in question. Such a procedure would serve to reduce the number of classification errors.

The actual results obtained in this survey are quite disconcerting to this reviewer if it is assumed for the moment that the issues addressed above have not in any way had an influence on the validity of the data. For example, it would be appalling if in reality 52% of educators who have not had direct contact, and 32% of educators who have had direct contact with gifted youth, have predominantly negative attitudes toward them. Such a finding, if valid, would indeed have tremendous implications for the education and re-education of school personnel. Clearly, as the authors indicate, the attitudes of educators toward a specific student population will play a crucial role in the effectiveness of programs designed for that population. If educators do in fact hold large-scale negative attitudes toward the gifted, as this study implies, a more precise understanding of those attitudes would provide a knowledge or conceptual base from which subsequent investigations might be launched. More research is needed.

CAREER-RELATED INTERESTS OF ADOLESCENT BOYS AND GIRLS. Fox, Lynn H.; Pasternak, Sara R.; and Peiser, Nancy L. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976, pp242-261.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by John C. Peterson, The Ohio State University.

1. Purpose

To obtain more information concerning the interests and career choices of seventh-grade boys and girls of high ability who were matched on mathematical aptitudes, verbal aptitudes, and sociometric level.

2. Rationale

Cognitive ability and personal interests and values are important psychological factors. Previous research by one of the authors indicated a relationship between extremely high cognitive ability, values, scientific interests, and career choices of young adolescents. Results of other studies indicated a relationship between masculine interests, scientific career choices, and achievement in mathematics. Another study has shown that highly precocious boys (scores of 640 or higher on the SAT-M) showed a greater interest (as measured by the Allport-Vernon-Lindzey Study of Values) in investigative careers and theoretical values than less precocious boys or girls.

3. Research Design and Procedure

Three matched groups of equal size (n = 26) and cognitive ability were formed: One experimental group of girls, one control group of girls, and one control group of boys. All students were gifted seventh graders in the upper two percent of their grade level on the Iowa Test of Basic Skills.

The two matched control groups were used for the first part of this study. The career interests of these 26 boys and 26 girls were compared with each other and with those of 75 ninth-grade boys and 75 ninth-grade girls from two junior high schools and for whom summary data were already available. Interests were measured using the Strong-Campbell Vocational Interest Inventory (SCII). The ninth graders are referred to as the average adolescents and the seventh graders the gifted adolescents.

Scores from the SCII are reported in three ways. Part Two, the Basic Interest Scales, which show the consistency of one's interests in 23 specific areas, were the main criteria used in this study. These 23 scales are grouped into six General Occupational Themes: Realistic, Investigative, Artistic, Social, Enterprising, and Conventional. A score above 50 on any of the 23 scales indicates above-average interest for that particular theme. An individual's interests are well differentiated if he or she earns high scores in numerous unrelated areas. Such differential profiles are not uncommon among adolescents.

Students in the three seventh-grade groups were asked to rate eight occupations on a seven-point scale of 16 adjective pairs in the form of a semantic differential, called a "See Myself Scale." The eight occupations selected consisted of four typically female occupations (homemaker, nurse, professor of English, and elementary-school teacher) and four primarily male occupations (physician, professor of science, mathematician, and computer programmer).

4. Findings

The girls in the gifted groups scored above a mean of 50 for 17 of the 23 interest scales; the boys in the gifted group scored above 50 for nine of the interest scales. The girls scored above 50 on every scale except mechanical activities (Realistic) and the five Enterprising scales (public speaking, sales, law/politics, merchandising, and business management). The boys scored above 50 on all the Realistic scales except nature and all the Investigative scales except medical service. The boys scored below 50 on all the Artistic scales, the Conventional scale, and all the Social and Enterprising scales except athletics (Social) and public speaking (Enterprising).

An analysis of variance of the two groups on the 23 scales showed the sex difference was significant ($p < .05$), as were the differences of rating on the interest scales ($p < .001$) and the interaction of sex and interest scales ($p < .001$).

Tukey tests of mean comparisons showed that the girls scored significantly higher than the boys on the following interest scales: domestic arts, art, social service, music/dramatics, teaching, writing, nature, office practice ($p < .005$), religious activity ($p < .01$), and medical service ($p < .05$). The boys scored significantly higher than the girls on mechanical activities and science ($p < .005$).

The students in the sample of average adolescents were almost two grades older than the gifted sample. The basic interest scale scores of the average groups were lower than those of the gifted group on most scales, especially the scientific and artistic ones. The average ninth-grade girls scored above 50 on only nine of the 23 interest scales as compared with 17 for the gifted seventh-grade girls. The ninth-grade boys scored above 50 on only five of the scales, as compared with nine for the gifted seventh-grade boys. The ninth-grade girls scored higher than the ninth-grade boys on the following scales: domestic arts, office practice, social service, art, medical service, music/dramatics, and nature. The boys scored higher on the following scales: mechanical activities, adventure, military activities, and science.

All three groups of seventh graders were used in the second part of the study. For the semantic differential, three groups were used--two groups of girls and one group of boys. A mean score of 64 indicates a neutral position with respect to that career; a score above 64 is considered positive. Group II girls rated all eight of the occupations above 64. Group I girls rated all occupations except nurse above 64. The boys rated all occupations except nurse, homemaker, and professor of English above 64. In an ANOVA of ratings of the eight careers by the three groups, the careers were rated significantly different ($p \leq .001$), there were significant differences between the groups ($p < .05$), and the interaction of groups and careers was also significant ($p < .001$).

Tukey tests of multiple mean comparisons were used to determine which careers were rated significantly different by the three groups. For the male careers, boys were significantly higher than girls on three of the eight comparisons. On the female careers, boys were significantly lower than the girls on seven of the eight comparisons.

Tukey tests of means with groups across careers were computed in order to compare the ratings of each of the four male careers with each of the four female careers. Boys rated every male career significantly higher than every female career, except elementary-school teacher. Group I girls did not rate any male career significantly lower than any female career and did rate mathematician significantly higher than nurse. Group II girls rated professor of science significantly lower than elementary-school teacher and homemaker, and rated computer programmer significantly lower than every occupation (male or female) except professor of science.

5. Interpretations

Intellectual ability and scientific career interests appear to be highly related. Gifted girls and boys have stronger interests in mathematics, science, medical science, writing, and public speaking than do somewhat older students of more average ability. This result would seem consistent with the fact that gifted students can more realistically aspire to academic careers.

The gifted girls are somewhat more like gifted boys than average girls with respect to interests that are fairly predictive of adult career choices. Although gifted girls do differ from average girls with respect to investigative interests, the gifted girls had somewhat less interest in these areas than gifted boys. What appears to be true is that gifted girls make fewer clear distinctions between preferences for male and female career interest areas than gifted boys and appear more drawn to male interest areas than girls of average ability. These data suggest that high cognitive ability leads to more conflict for gifted girls than gifted boys or average girls with respect to future career choices.

Critical Commentary

This study has the potential to serve as an impetus for several other studies. How do the career interests of gifted students differ from the interests of non-gifted students who are the same age or in the same grade? How do students' (gifted and non-gifted) interests change as they progress through school (e.g., what changes occur between seventh grade and ninth grade?)? Are these changes the same for males and females? How indicative are the career interests of students at various grades of their occupation at age 25? age 30?

This study compared the career interests of gifted seventh graders with the career interests of average ninth graders. Thus, students differed not only in cognitive ability but in age. What results were due to the differences in cognitive abilities? in ages? These questions should have been addressed. Why did the researchers use average ninth graders rather than average seventh graders?

Perhaps the most important concern of the abstractor is in the selection of the Strong-Campbell Vocational Interest Inventory (SCII). This instrument was, apparently, still in the developmental stage at the time of this study. A current catalog indicates that the instrument is for grade 11 and up. Did the researchers have any data on the reliability and validity of the instrument? Why did they select an instrument that is not recommended for students below grade 11?

The SCII was developed by combining the Strong Vocational Interest Blank (SVIB) for Men and the SVIB for Women. Development of SCII was prompted by a great deal of concern about the possible sex-bias of the SVIB. No data were available to the abstractor to indicate if the SCII is a sex-fair instrument. This would seem to have been a prime consideration of this study. Since the instrument was still under development, the experimenters should have addressed this issue.

More studies need to be conducted into the vocational interests of adolescents. Hopefully, any future studies will take into consideration these questions and comments.

CREATIVE POTENTIAL OF MATHEMATICALLY PRECOCIOUS BOYS. Keating, Daniel P. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976, pp262-272.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Otto C. Bassler, George Peabody College.

1. Purpose

To investigate potential creativity, values, and vocational preferences of youths with great mathematical ability.

2. Rationale

This investigation is a part of the Study of Mathematically Precocious Youth (SMPY) which has been primarily concerned with identifying students who possess a high level of mathematical reasoning ability and then helping these students to further this ability. Identification of such students presented the opportunity to study variables such as creativity, values, and vocational preferences within this group. These variables may become meaningful and potential indicators of the future productivity and creativity of academically talented mathematics students.

3. Research Design and Procedure

The subjects were 72 junior high school boys who were the top scorers in two mathematics competitions held one year apart. These boys all had demonstrated high ability and achievement on tests designed for high school seniors.

The students were administered a battery of paper-and-pencil measures at several testing sessions. Fifty-seven subjects took all measures and this group was used as the base group. When boys not in the base group were compared to those in the base group, there were no significant differences on the scores that the two groups attained.

The instruments administered were:

a. Study of Values - It assesses values denoted as theoretical, political, economic, aesthetic, and religious. The "classic" value structure of the creative scientist is high theoretical, high aesthetic, and low religious.

b. Biographical Inventory-Creativity - It is a self-report of past behavior and self-ratings that yields scores on "art and writing" and "mathematics and science."

c. Barron-Welsh Art Scale - It assesses preference for certain figures and may discriminate between creative and less creative mathematicians.

d. The California Psychological Inventory - It is used to predict creativity.

e. Strong-Campbell Interest Inventory - It assesses vocational interests.

f. Vocational Preference Inventory - It assesses vocational preference.

g. Raven's Advanced Progressive Matrices - It is an IQ instrument that measures non-verbal reasoning ability.

4. Findings

a. Study of Values: The theoretical value was rated highest or second highest by 77% of the subjects; only 8% rated aesthetic as highest or second highest; and 43% rated religious last.

b. Biographical Inventory-Creativity: In comparison with the college norm group, the mathematically precocious boys had a mean score equivalent to the 58th percentile on the arts and writing scale and to the 68th percentile on the mathematics and sciences scale.

c. Barron-Welsh Art Scale: The mean score of subjects in this study was 17.9, which when compared to a non-artist group (mean of 15.06) was non-significant.

d. The California Psychological Inventory: Using a previously developed regression equation for the scales of this test, the subjects as a group appear to be less creative than a group of randomly-selected eighth graders as well as a high school norm group.

e. Strong-Campbell Interest Inventory and the Vocational Preference Inventory: The subjects most frequently selected occupations in the investigatory category as their first (61%) or second (24%) choice on one scale. On the other scale, 93% chose investigative occupations as their first or second choice. Most occupations in this category are science-oriented and require advanced educational degrees.

f. The Advanced Progressive Matrices: The mean score of subjects in the study (29.51) is above the 95th percentile of adult norms.

To determine the potential creativity of an individual student, his score on each measure was compared to a criterion specified to be the mean score of the group plus one standard deviation. This criterion was established within the group of precocious boys as well as for the norm group of each test. Within-group comparisons indicated that 56% of the subjects were above criterion on one or more measures; 26% on two or more measures; 7% on three or more measures; and 2% on four measures. Norm group comparisons were 96% above criterion one or more measures; 77% on two or more measures; 32% on three or more measures; 10% on four or more measures; and 2% on five measures.

5. Interpretations

From the group averages for the various instruments, it is not clear if the group of mathematically talented boys can be characterized as creative or not. In general, the group as a whole does not stand out from the norm groups on any measure except the Raven Advanced Progressive Matrices test where it markedly exceeds the norm group. The group possesses a strong theoretical-investigative orientation but a low aesthetic orientation; these are mixed results with respect to creativity.

Since almost one-third of the students tested were substantially above the mean of the norm group on three or more measures, it was concluded that these individual students had high creative potential. Further it was hypothesized that some students who do not appear to be particularly creative at this time may in the future come up to the criteria used in this investigation. This seems plausible since most of the instruments were normed on much older subjects.

Explanations advanced for the lack of agreement of creativity-related measures in this group were:

a. One or more of the measures may not bear any deep relationship to creativity.

b. There is a restriction in range within this group and this may provide too little variation on measures that are even slightly correlated with creativity.

c. It may be that each of the measures does bear some relationship to creativity and that each of them is measuring a different aspect of creative potential. For an individual to be creative, it may be necessary to possess all or nearly all of these traits.

Critical Commentary

This study was exploratory in nature and was designed to describe the attributes of a very select group of boys on factors thought to be related to creativity. As such it added to our knowledge of the characteristics of academically talented junior high school boys. Comparisons between the studied population and the groups for which the tests were normed were casual rather than statistical. This is as it should be since the instruments were generally designed for different age groups and different types of individuals.

The questions of what creativity is and how can it be measured were left open in this study. In fact, the author points out that the measures used here are indirect measures of creativity and are not presumed to be in themselves measures of the construct. Thus the validity of this approach can only be determined by a longitudinal study which the author says is planned.

Investigations of this type are needed to provide better information about the capabilities of precocious mathematics students, to further our understanding of constructs such as creativity, and to seek relations between constructs such as ability and creativity. Only after exploratory studies like this will we be able to construct valid instruments and to facilitate the learning of precocious youth.

THE VALUES OF GIFTED YOUTH. Fox, Lynn H. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: Johns Hopkins University Press, 1976, pp273-284.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lewis R. Aiken, University of California at Los Angeles.

1. Purpose

The study was undertaken to learn something about the values and interests of individuals counseled in the Study of Mathematically Precocious Youth. Such information would hopefully assist in deciding what methods of facilitation--college courses, advanced work in high school mathematics, special fast-paced mathematics courses, or rapid-paced independent study in mathematics--would be best for a given student.

2. Rationale

The investigation was a part of the Study of Mathematically Precocious Youth conducted by Julian Stanley, Daniel Keating, and their colleagues at the Johns Hopkins University. It was one of several sub-studies focusing on the relationships of affective variables to mathematical ability. One prediction from previous research was that scores on the theoretical scale of the Study of Values (SV) would be positively related to mathematical ability.

3. Research Design and Procedure

The Allport-Vernon-Lindzey Study of Values was administered to 655 boys and girls who participated in the 1973 SMPY mathematics talent search and to boys who were winners or near-winners in the 1972 and 1974 contests. The results were compared with those of the normative sample of male and female high school students (grades 10-12) given in the SV manual.

4. Findings

Girls in the 1973 Talent Search scored higher than high school girls in the normative sample on the social, theoretical, political, and aesthetic scales, but lower on the religious and economic scales. Boys in the 1973 Talent Search scored higher than high school boys in the normative sample on the theoretical, social, and political scales, but lower on the religious, aesthetic, and economic scales. The rank ordering of the values was also different for girls in the gifted sample than for those in the normative sample, although the ordering of the values for the gifted boys was quite similar to that of the boys in the normative group. The seventh- and eighth-grade students of the same sex in the gifted group had quite similar value profiles. Among the gifted students, the mean scores for boys were significantly higher than those for girls on the theoretical, economic, and political scales, but significantly lower on the social, aesthetic, and religious scales. Girls tended to score highest on the social and religious scales, and boys on the theoretical and political scales. In general, boys who had the highest scores on the theoretical and religious scales of the SV scored highest on the SAT-M, whereas girls who had the highest scores on the aesthetic scale scored highest on the SAT-M.

5. Interpretations

More mathematically precocious students appear to value theoretical pursuits more than less mathematically able students. However, this was less true of girls than of boys, and certainly not true of all boys. Mathematically talented students who are highly motivated are likely to succeed in accelerated mathematics courses even when their theoretical scores on the SV are not particularly high.

Critical Commentary

This is basically a correlational study which shows that scores on the Study of Values differ significantly for mathematically talented and mathematically non-talented groups and for girls and boys within those groups. Certainly the finding that the theoretical scale scores are higher in the mathematically talented boys in no way gives assurance that the high theoreticals will be most successful if certain approaches to instruction in mathematics are utilized. Hence, the purpose of the investigation is not actually realized. The results do not provide a sound rationale for counseling and placing students in particular intervention or facilitative procedures.

This report, as with the book as a whole, is primarily descriptive rather than explanatory. It provides no prescriptions for the treatment of mathematically talented youth. Furthermore, the reviewer found this chapter a bit long-winded and indirect in its message. Thus, the reader has to search in order to find out what statistical tests were used in determining the significance of the findings. And in a situation that literally cries for correlation coefficients and multiple regression procedures, none are found. Also, the norms on the SV are probably inappropriate for comparison purposes for a number of reasons, cohort and age differences being the primary ones. The observed sex differences in this investigation were among the most interesting findings of this investigation, but they also require a much clearer interpretation than that given.

RANDOM VS. NONRANDOM STUDY OF VALUES PROFILES. Linsenmeier, Joan A. W. In Intellectual Talent: Research and Development. Edited by Daniel P. Keating. Baltimore, Maryland: The Johns Hopkins University Press, 1976, pp285-292.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lewis R. Aiken, University of California at Los Angeles.

1. Purpose

This study was designed to determine whether the Study of Values (SV) profiles of mathematically talented junior high school students were likely to result from random responding to the test items.

2. Rationale

The investigator briefly describes the statistical characteristics of ipsative measures such as the SV and the use of ipsative tests for intraindividual comparisons. Some of the previous statistical work on ipsative measures is summarized, and the theoretical rationale of the SV is described.

3. Research Design and Procedure

The SV was administered to the 35 top-scoring students in the 1972 Mathematics Talent Search, all of whom were boys aged 12-14 selected on the basis of their SAT-M and STEP-Mathematics (Level I) Scores. Three sets of 100 random SV profiles were also generated by Monte Carlo methods. Frequency distributions of scores on the six SV scales and profile standard deviations were computed for the profiles of the 35 students and the three sets of random profiles.

4. Findings

The distributions of the random profiles were nearly normal, and none of the scale means were significantly different from expectation. The differences among scale variances within each of the three sets of random profiles were also non-significant. In contrast, the variances of scores on the six values for the actual profiles were statistically significant, the variance of actual scores being greater than the variance of random scores on all scales except political. Furthermore, for the group of actual scores the means on the theoretical, economic, aesthetic, and religious scales were significantly different from the expected means on the respective scales. Finally, the mean profile standard deviations were significantly greater for the actual profiles than for the random profiles.

5. Interpretations

The fact that the variance of the individual SV profiles of the mathematically talented group was greater than that of the randomly generated profiles is interpreted as indicating that the profiles will remain stable over time and that it is appropriate to use the profiles in describing the characteristics of the students. The greater variances of the actual profiles were produced primarly by high scores on the theoretical and economic scales and low scores on the aesthetic and religious scales, a finding which the investigator interprets as reflecting true characteristics of the students.

Critical Commentary

This paper and the previous one (Fox, 1976, pp. 273-284) have gone to considerable effort to demonstrate that the Study of Values, an instrument originally developed many years ago and renormed on a nationwide sample of 6,000 high school students in 1968, is an appropriate measure of the values of mathematically talented junior-high students. It may well be so, but no concrete evidence of the reliability and validity of this instrument for the target group has been presented in either chapter. As intimated in the previous abstract, this reviewer is not impressed by comparisons of SV profiles of junior high students in the 1970s with those of senior high students in the 1960s, the former group consisting of mathematically talented and the latter group of presumably average students. Neither am I comfortable with computer-generated random profiles as baseline data aginst which to compare actual profiles. It is a fairly safe bet that if the investigators had asked the same or another group of junior high students to respond randomly to the SV, the results would have been substantially different from the computer-generated profiles. Furthermore, even if the SV is unreliable and invalid for this particular group, the occurrence of random data would certainly be unexpected. The main point, however, is that lack of randomness in the data in no way guarantees that the test is appropriate and valid for the target group.

Although the results of administering the SV to the mathematically talented students are interesting and heuristic, any comparisons that are made from the data must--like the SV itself--be ipsative. The "normative" data from average high school students in the 1960s and computer-generated data in the 1970s hardly qualify as satisfactory frames of reference for interpreting the SV profiles of mathematically gifted junior high students or for validating the test for these students.

Tests Used in the SMPY Studies

The SMPY studies reviewed in this issue of IME are heavily dependent upon test instruments. Not all of the instruments are commonly used by mathematics educators. Many of the abstractors did not want to use limited space to consider the appropriateness or the characteristics of the tests. Consequently we have listed for your convenience reviews of the tests that appear in Buros' Seventh Mental Measurements Yearbook (1972). Each is cited by an ordered pair (x,y) where x is the number of the test and y is the page number in the Buros volumes.

Academic Promise Test (672,1046)

Adjective Checklist (38,74)

Advanced Placement Examinations (662,1009)

Advanced Progressive Matrices (376C2,695)

Alpha Biographical Inventory (975,1370)

Art Scales (41,81)

Bennett's Mechanical Comprehension Test (1049,1483)

California Psychological Inventory (49,87)

Cooperative Mathematics Tests - Algebra I & II (500,894)

Cooperative Mathematics Tests - Analytic Geometry (532,926)

Cooperative Mathematics Tests - Calculus (531,924)

Cooperative Mathematics Tests - Trigonometry (543,934)

Iowa Test of Basic Skills, Modern Mathematics Supplement to the, (481,870)

Modern Language Aptitude Test - Elementary (255,542)

Personality Inventory, Eysenck (76,159)

Remote Associates Test (445,825)

Revised Minnesota Paper Form Board Tests (1056,1487)

Scholastic Aptitude Test - Verbal and Mathematics (344,640)

Social Insight Test, Chapin (51,98)

Study of Values: A Scale for Measuring the Dominant Interests in Personality, Third Edition (146,350)

Vocational Preference Inventory, Sixth Revision (157,384)

THE PSYCHOLOGY OF MATHEMATICAL ABILITIES IN SCHOOL CHILDREN. Krutetskii, V. A. Edited by J. Kilpatrick and I. Wirszup. Translated by J. Teller. Chicago: University of Chicago Press, 1976.

Expanded Abstract Prepared Especially for I.M.E. by George W. Bright, Northern Illinois University.

1. Purpose

The basic goal was "to create psychological foundations for an active pedagogy of abilities." The specific goals were (a) "to characterize the mental activity of mathematically gifted pupils as they solve various mathematical problems," (b) "to create experimental methods of investigating mathematical giftedness that might have an independent value", (c) to reveal "typological differences in the structure of abilities," and (d) to determine whether the development of mathematical abilities is related to age.

Several specific hypotheses regarding mathematical abilities were also made:

(1) Pupils with different mathematical abilities are characterized by differences in degree of development of both the ability to generalize mathematical materials and the ability to remember generalizations.

(2) Able and less-able pupils differ in their rate of "curtailment" of reasoning.

(3) Pupils with different mathematical abilities are characterized by different degrees of the ability to switch from a direct to a reverse train of thought.

During the course of the study additional hypotheses were generated.

(4) Able pupils are characterized by an ability to switch rapidly from one mental operation to another.

(5) An ability for spatial concepts is expressed in different ways and may be related to the presence of different types of mathematical abilities.

2. Rationale

The foundation for the study is built very carefully through an extensive review of non-Soviet as well as Soviet research literature. Ability is not viewed as an inborn trait. Rather, certain typological properties are considered to be inborn. The manifestations of these mental properties, however, are determined by the circumstances in which an individual is reared.

Seven basic assumptions are stated.

(1) Abilities are abilities for a definite kind of activity, exist only within a specific activity, and must be studied within that activity.

(2) Ability is dynamic and develops only in a specific activity.

(3) There are optimal age periods for the development of abilities.

(4) Progress in an activity depends on a complex of abilities.

(5) High achievement in an activity can be conditioned by different combinations of abilities.

(6) Relative weakness in one ability can be compensated by other abilities.

(7) General and specific giftedness are related, though the nature of the relationship is not well understood.

3. Research Design and Procedure

The research was conducted over the twelve-year period 1955-66. In all, 201 subjects were studied, some briefly and some for several years. In addition, groups of 62 and 56 mathematics teachers and 21 mathematicians were surveyed, biographies of 84 prominent mathematicians and physicists were studied, and data which permitted correlation of progress in various school subjects were examined for more than 1000 students in grades 7 to 10 in Moscow schools. Further data were gathered from several local mathematics contests and from examination of notebooks of a "large number" of students in grades 6 to 8.

Seven basic principles underlay the methods of the study. First, activities were chiefly mathematical in order to highlight mathematical abilities. Second, experimental problems were designed to reflect various degrees of difficulty. Each problem type was represented by a series of problems of increasing complexity and difficulty. The simplest problems were designed to be accessible even to pupils of indifferent abilities. Third, solving the problems should help clarify the structure of mathematical abilities. That is, features of mental activity specific to mathematical activity should be manifested. Fourth, processes for solving problems were more important than the fact of a final solution. Fifth, to measure ability rather than past habits, experience, and skills, the problems that were selected were non-standard and required little particular previously learned information. Sixth, experimental methods were used that were instructive as well as diagnostic. The pupil's rate of progress was observed in two situations: (1) independently and (2) with slight help from the experimenter. Seventh, quantitative as well as qualitative methods were used. Counting data (e.g., number of problems solved with and without help, number of different solutions) were maintained, and factor analysis was applied to aid in the interpretation of the data.

Twenty-six series of problems were developed. Briefly described they are as follows: unstated question, incomplete information, surplus information, isolation of parts of a figure, inductive generalization, common mathematical structure, increasing abstraction, generalization from a single instance, proof, composition of equations, unrealistic situations, artificial concepts, multiple solutions, changing content, reconstructing a process, unconscious restrictions, direct and reverse processes, heuristics, logical reasoning, series, sophisms, complex data, visual, verbal and visual, spatial concepts, visual-pictorial versus verbal-logical.

Preliminary experimentation was conducted with selected students. Labeling of subjects as very capable (V C), capable (C), average (A), or incapable (I) was accomplished in accordance with broadly stated guidelines. Experiments of the longest duration were conducted with the capable and very capable students. Six major studies involving individual experimentation were conducted.

Years	Grades (Age)	Ability Level VC	C	A	I.
1956-58	6 - 8		9	6	4
1958-65	(6 - 14)	16			
1960-61	6 - 7			9	10
1960-64	5 - 9		36	22	8
1962-64	9 - 10	11	8	11	
1963-65	2 - 4	7	14	9	12
1956-65	2 - 10	34	67	57	34
				TOTAL	192

Subjects were told that the purpose of the experiment was to collect material for new problem books. Usually experiments were conducted individually, during out-of-class time, and after a good rest. Indisposition, fatigue, low spirits, or lack of interest in solving the problems were sufficient cause for postponement of a session.

4. Findings and Interpretations

Several components of mathematical thought were identified by at least 50% of each of the two groups of mathematics teachers surveyed: logical thinking, resourcefulness in studying mathematics, stable mathematical memory, and ability to generalize. Ability to generalize and abstracting essential features of a problem were cited most frequently by the mathematicians surveyed.

On the basis of selected cases of mathematical giftedness, several conclusions were made. First, mathematical abilities can take shape early, usually in the form of computational skills. Second, characteristics that develop include ability to generalize, flexibility in processes, striving for economical solutions, memory for generalizations, curtailment of reasoning, and mathematical perception of the environment. Third, giftedness at an early age is relatively independent of support of such development. ["Flexibility in processes" means an ability to switch rapidly from one operation to another or from one train of thought to another. "Economical solutions" are the easiest, clearest, or most direct. "Curtailment of reasoning" refers to the shortening of processes of solution when the processes are used more than once. The logical jumps between explicit steps in these processes become larger as the intermediate steps are accepted as 'obvious'.]

From the data of all subjects (but with primary emphasis given to the data of capable and very capable students) the following conclusions were reached:

(1) Capable pupils perceive the mathematical material of a problem analytically (different elements are assessed differently) and synthetically (relationships are sought for among the elements). Average and incapable students perceive only disconnected facts and have difficulty synthesizing concrete data. There seems to be an ability to extract from the given terms of a problem the information maximally useful for its solution.

(2) Capable pupils generalize quickly and broadly. They generalize not only the content of the problem but also the method of solution.

(3) Capable pupils can with very limited exposure to similar problems curtail their processes for solving such problems. Average pupils do this only after repeated exposure. Incapable pupils experience great difficulty in producing such curtailment.

(4) Capable pupils easily switch from one mental process to another qualitatively different one, approach problem-solving from different aspects, are free from conventional solution techniques, and easily reconstruct established thought patterns. Incapable pupils are marked by inertness, sluggishness, and constraint in their thinking, and they are impeded by previous solution techniques.

(5) Capable pupils strive for the clearest, simplest, shortest, and most elegant solution to a problem.

(6) Capable pupils can reverse their reasoning processes easily.

(7) When able pupils solve a hard problem their trials seem to be a means of thoroughly investigating it rather than direct attempts at solving it.

(8) Capable pupils remember generalized and curtailed structures. These structures are created from the data and the relationships of particular problems.

(9) There appears to be an identifiable "mathematical cast of mind" which is formed "as a particular synthetic expression of mathematical giftedness and includes cognitive, emotional and volitional aspects." Further there appear to be three types of "mathematical minds:" analytic, geometric, and harmonic (combination).

(10) Sudden inspiration among capable pupils is frequently explainable by the ability to generalize and the ability to think in curtailed structures.

(11) Capable pupils tire much less during mathematics lessons than during other kinds of lessons.

(12) The ability to generalize appears to develop first. Curtailing reasoning, generalizing memory, and striving for elegance in solutions appear to be formed later.

(13) There is no difference in qualitative characteristics of mathematical thinking of boys and girls.

Summary conclusions were as follows:

(1) There appears to be a basis for speaking of specific abilities (including mathematical abilities) rather than general abilities that are only "refracted in a unique way in mathematical activities."

(2) In some people, the brain is uniquely attuned toward isolating from the environment stimuli of the type of spatial and numerical relationships and symbols and toward optimal work with precisely this kind of stimuli. That is, some people have inborn characteristics in the structure of their brains which are extremely favorable to the development of mathematical abilities.

Critical Commentary I

Prepared Especially for I.M.E. by Gerald A. Goldin, Philadelphia, Pennsylvania.

Krutetskii's investigations are not quantitative, rigorous analyses of data tending to confirm or deny specific experimental hypotheses. Consequently many U.S. readers may experience a certain initial resistance to his approach. For me, however, the study of mathematical abilities based on the observation of problem solving processes, rather than the statistical interpretation of test scores, is sensible and enormously refreshing.

Too often researchers in mathematics education propose hypotheses haphazardly, without developing conceptual foundations, or else in scattershot fashion seek to correlate lists of supposedly dependent variables with lists of independent variables. While such studies may enjoy a claim to statistical and methodological rigor, their eventual outcomes lend credence to no particular theory or model for mathematical learning, because the initial hypotheses were themselves unmotivated by such a theory or model. Krutetskii's studies, on the other hand, may be seen as principally clinical investigations aiming toward the creation of a model for the structure of mathematical abilities. Such a model, he points out, should consist of more than a list of independent or partially independent "factors;" in fact Krutetskii has harsh words for the preoccupation of Western psychologists with factor-analytic methods. He devotes a major section of the book to his critical review. Rather he maintains with justification that the components in a model for mathematical abilities ought to be interrelated and comprise a coherent system, an organizational whole.

A meaningful model, emerging as it may from qualitative or semi-quanitatative clinical data, can then be subjected to a more rigorous quantitative verification. This is the scientific context with respect to which I am evaluating Krutetskii's contribution. Therefore, the following comments focus primarily on the problem-solving instruments (test series) designed by Krutetskii, and the model which he develops as a consequence of his investigations.

The rich and varied collections of mathematical problems in this book alone make it worthwhile for purchase, even by the reader who is uninterested in Krutetskii's theories. The problems are organized into 26 groups or series, based on various shared problem characteristics within each series (see Table 1). Many of the problems are ingenious, and are suggestive in themselves of teaching objectives and motivational strategies.

These problem sets constitute the experimental instrumentation for the development and substantiation of Krutetskii's model for the structure of mathematical abilities. As such they are subject to certain criticisms which, in view of the scope and significance of the problem sets, ought not to be construed as detracting from their value.

1. Krutetskii presents only the most cursory discussion of the content validity and reliability of his test instruments. For example,

TABLE I

THE SYSTEM OF EXPERIMENTAL PROBLEMS FOR INVESTIGATING SCHOOLCHILDREN'S MATHEMATICAL ABILITIES*

Category	Group	Series
Information gathering	Perception (interpretation of a problem)	I. Problems with an unstated question
		II. Problems with incomplete information
		III. Problems with surplus information
		IV. Problems with interpenetrating elements
Information processing	Generalization	V. Systems of problems of a single type
		VI. Systems of problems of different types
		VII. Systems of problems with gradual transformation from concrete to abstract
		VIII. Composition of problems of a given type
		IX. Problems on proof
		X. Composition of equations using the terms of a problem
		XI. Unrealistic problems
		XII. Formation of artificial concepts
	Flexibility of thinking	XIII. Problems with several solutions
		XIV. Problems with changing content
		XV. Problems on reconstructing an operation
		XVI. Problems suggesting "self-restriction"
	Reversibility of mental processes	XVII. Direct and reverse problems
	Understanding; reasoning; logic	XVIII. Heuristic tasks
		XIX. Problems on comprehension and logical reasoning
		XX. Series problems
		XXI. Mathematical sophisms
Information retention	Mathematical memory	XXII. Problems with terms that are hard to remember
Typology	Types of mathematical ability	XXIII. Problems with varying degrees of visuality in their solution
		XXIV. Problems with verbal and visual formulations
		XXV. Problems related to spatial concepts
		XXVI. Problems that expose correlations between visual-pictorial and verbal-logical components of nonmathematical intellectual activity

*Krutetskii, pp. 100-104, abridged

The validity (suitability, legitimacy) of the experimental problems was established before the beginning of the experimental study. As is well known, the validity of test problems is determined by demonstrating their results in practice. The trial experiments showed rather persuasively that the more mathematically able the examinees, the more successfully they solved the experimental problems . . . (p. 91).

There is no discussion of such important related questions as the validity of the classification of problems into the various series. Would all experts agree that each problem designated as an example of "Composition of Equations Using the Terms of a Problem" indeed belongs in that group? Similarly, we have:

The reliability of the problems was confirmed selectively (using the series that yielded numerical scores) from the standpoint of the stability of the scores (p. 91).

However, Krutetskii is concerned not mainly with test scores, but with the use of certain problem-solving processes. It would have been useful to measure the reliability of the problems from the standpoint of the stability of the processes which the problems are designed to elicit. We are given no detailed data on either validity or reliability.

2. The selection of problems, and more particularly the decisions about the characteristics of the various problem series, are of necessity influenced by the researcher's preconceptions as to the nature of the information being sought. Yet the influence on Krutetskii's findings of the structure of the problem sets themselves is enormous. One might go so far as to say that many of the conclusions of the study are built into the problem series, and it is not clear to what extent the author appreciates this fact. It is useful at this point to refer to Krutetskii's general outline of the structure of mathematical abilities:

1. Obtaining mathematical information
   A. The ability for formalized perception of mathematical material, for grasping the structure of a problem.

2. Processing mathematical information
   A. The ability for logical thought in the sphere of quantitative and spatial relationships, number and letter symbols; the ability to think in mathematical symbols.
   B. The ability for rapid and broad generalization of mathematical objects, relations, and operations.
   C. The ability to curtail the process of mathematical reasoning and the system of corresponding operations; the ability to think in curtailed structures.
   D. Flexibility of mental processes in mathematical activity.
   E. Striving for clarity, simplicity, economy, and rationality of solutions.
   F. The ability for rapid and free reconstruction of the direction of a mental process, switching from a direct to a reverse train of thought (reversibility of the mental process in mathematical reasoning).

3. Retaining mathematical information
   A. Mathematical memory (generalized memory for mathematical relationships, type characteristics, schemes of arguments and proofs, methods of problem-solving, and principles of approach).

4. General synthetic component
   A. Mathematical cast of mind. (pp. 350-351)

The quantitative substantiation for component 1.A. above, for example, is based on problem series I-III (pp. 225-226). It is certainly plausible to assert that these three problem series (unstated question, incomplete information, and surplus information) measure the same component of mathematical ability. Krutetskii's data support the existence of a common factor that accounts for success in these three groups of problems. However, this result is not obtained from an analysis of scores on all of the test problems grouped together, as Kilpatrick and Wirszup note in their introduction (p. xv). It is based rather on the correlation matrix for series I-III exclusively. Thus Krutetskii has not really isolated a "factor;" he has simply developed an instrument for the evaluation of a trait presupposed to be a component of mathematical ability, and has demonstrated the internal consistency of this instrument. This limitation applies in turn to the quantitative verification of each component of the structure, and highlights the sense in which the organization of the problem into series has largely, though not entirely, determined the resulting structure of mathematical abilities.

Krutetskii also presents important qualitative data in support of his model: data from research mathematicians, analyses of individual cases of mathematically gifted children, and excerpts from children's problem-solving protocols. I found the latter to be rather unsatisfying due to the shortness and selectivity of the excerpts. It is almost always impossible to reach an independent conclusion as to the correctness of the author's interpretations, and much has to be accepted on faith.

We may now ask whether or not Krutetskii has succeeded in his objective of elucidating the structure of mathematical abilities. He has certainly identified several distinct components. He has demonstrated that each component "hangs together;" that is, it can be measured consistently by means of a variety of problems developed for the purpose of doing so. He has neither asserted nor demonstrated his components to be independent of each other; on the contrary, he views them to be interrelated, and to correspond in a broad way to sequential problem-solving stages. In these respects I believe that a meaningful structure has been asserted.

Krutetskii uses some of the language of information processing theory, but he does not build on the research in this field. He is ultimately not trying to characterize the individual problem-solving process, but to identify and study the organization of the shared characteristics (traits) of successful vs. unsuccessful children as problem solvers.

Krutetskii's work will inevitably be compared with that of Piaget. Just as many educators and psychologists have sought to accelerate children's progression through Piaget's developmental stages, one response to Krutetskii will be to try to teach the various components of "mathematical ability." In the case of Piaget, these efforts often ran counter to the spirit of the theory; but for Krutetskii, I believe this to be the very object of his work. Granting, for example, that successful problem solvers perceive the problem in relation to its elements, grasping initially the problem's structure, can we not teach this ability or at least facilitate its development? Krutetskii's problem series are suggestive of ways to do exactly that.

This commentary has omitted mention of much that is interesting and valuable in Krutetskii's book: the discussion of various "types" of mathematical ability, its relationship to personality, age and sex differences, and the discussion of certain skills or components of mathematical ability which (perhaps surprisingly) turned out to be inessential to the structure. Suffice it to say that this work is an important contribution to the literature which ought to inspire research in mathematics education and mathematical problem solving for many years to come.

Critical Commentary II

Prepared Especially for I.M.E. by George W. Bright, University of Northern Illinois.

It seems virtually certain that Krutetskii's volume will become one of the most quoted mathematics education reports published in 1976. The research is apparently important historically in the Soviet study of abilities, and its distinction within the USSR is sufficient cause for serious examination of the report by all mathematics educators. However, a completely open-minded reading is difficult, at least for this reviewer, because of the conflict between the value system implicit in the Western view of acceptable empirical research and the value system associated with the Soviet view.

The prominence of statistically based, empirical research in the United States tends to cause some automatic suspicion of studies that are not quantitative. Soviet researchers, in contrast, doubt the value of studies for which interpretations are based primarily on statistical manipulation; e.g., factor analysis. Open-minded acknowledgement and evaluation of the Soviet criticisms of the basic orientations of Western empirical research are difficult, because the process of analyzing such criticism causes considerable dissonance. This dissonance, in turn, causes a distraction so severe that some important conclusions are overlooked, simply because they are not permissible within the value system of Western research.

The comments that follow, therefore, should be interpreted with the knowledge that this reviewer has experienced a significant conflict of values. Negative criticisms are potentially overreactions to Krutetskii's different values. Positive criticisms, on the other hand, are potentially over-compensations for the recognized conflict. The main purpose of all the comments is to provide a set of guideposts for the reader's personal study of the book.

1. The research is a massive undertaking, and Krutetskii is to be lauded for the clarity of exposition and for the level of synthesis, not only of his own work but also of the published literature. Krutetskii's view of Western literature is substantively different from that of any literature review published in this country. His unusual view is, by itself, an important contribution to the study of mathematical abilities and evolves from the value system implicit in Soviet research. The position of Soviet political and social philosophy is that individual differences are not significant. Consequently Soviet psychology has denied the importance of differential performance, at least as measured by tests, and has focused instead on the processes by which problems are solved by different people. The implementation of this focus has been the development of the "teaching experiment," which is basically a one-on-one, open-ended, and loosely structured interview.

2. The most obvious deficiency in Krutetskii's report is the lack of detail in explaining what happened in the experiments. Since the problem-solving sessions were designed as potential if not actual "teaching experiments," complete details of specific procedures, of course, could not be reported in only a single volume. Each trial was unique and was determined

not only by the problems presented but also by the responses of the subject. It is important for adequately interpreting the results, however, to know how much direction was provided by the experimenters. The reader is at a loss to determine this. Too, since so much importance was given to the data of the capable and very capable pupils, there is at least a possibility that the quality of aid given to those pupils was higher than that given to average and incapable pupils. More information is needed before a judgment can be made of whether such a bias existed.

3. The most critical problem in interpreting Krutetskii's research arises from the lack of descriptions of criteria by which supportive data were retained and non-supportive data were discarded. In comparison to dialogues reported by Piaget, for example, Krutetskii's selections are quite short. As less and less verbatim dialogue is reported, of course, the more important become the criteria for selection of the quoted passages. In this reviewer's opinion, Krutetskii becomes suspect of over-zealous selection of supportive data when he quotes a nine-year-old as saying, "Oh, what an example! But it only seems hard. It is apparent at once that there is a common factor in all 3 parts" (p. 250). This child is admittedly bright in mathematics, though at age 8 "she writes badly and does not read very readily" (p. 193). It seems unreasonable that by age nine she would have acquired use of such sophisticated language. It is of course true that her response has been translated twice; first, into the original report and second, from Russian to English. Nonetheless, the sophisticated use of language by subjects who excel only, or at least mainly, in mathematics creates some suspicion. One wonders whether editing has been done to the transcripts in order to support the hypotheses. More information about the details of selection of data should have been provided.

4. It seems that throughout the manuscript the data from capable and very capable pupils are viewed as positive and those from average and incapable pupils are viewed as negative. In part this is explainable in terms of the philosophy of Soviet research. Since mathematical ability is only expressable in mathematical activity, then those pupils who cannot perform mathematical activities cannot be viewed as possessing mathematical abilities. From the Western view, however, it is unreasonable always to interpret the data of average and incapable pupils in terms of deficiencies relative to the data of capable and very capable pupils. Western researchers are more inclined toward making positive interpretations of data. Too, a theory of mathematical abilities ought not to be determined in its positive aspects wholly by the behavior of capable pupils. Average pupils may have different kinds of abilities rather than only a lack of abilities.

5. The factor analyses that were performed seem to have employed data only from capable pupils. Consequently there is a possibility that the factors that were identified were related as much to characteristics unique to the subjects in that sub-population as to mathematical ability factors. After all, if subjects are selected for statistical study because of similar behavior it is no surprise that statistics verify that similarity.

6. The report creates the impression that, for Krutetskii, ability and giftedness are synonymous. Less dissonance would have been created for this reviewer if the title had been The Psychology of Mathematical Giftedness in Schoolchildren, although such a book might receive considerably less attention. Western researchers would probably not accept the

equivalence of these two terms. The use of Krutetskii's work in support of future research, therefore, will have to be handled very carefully. Notions about giftedness may not apply to studies of non-gifted pupils.

7. Krutetskii's research should be viewed as developmental, rather than experimental. His conclusions are somewhat conservative in light of the data presented, but such a stance is desirable in the context of current knowledge. The conclusions seem more useful as suggestions of hypotheses for future research than as well-substantiated results of careful experimentation.

8. The task of defining a theory of mathematical abilities is both very important and very complex. Krutetskii has greatly illuminated some aspects of that task, and he has suggested ways of illuminating other aspects. As one step in the development of a full theory, Krutetskii's work deserves a very good reputation. However, this reviewer sincerely hopes that enthusiasm over the work will not cause it to assume unjustified importance.

THOUGHT PROCESSES INVOLVED IN SETTING UP EQUATIONS. Doblaev, L. P. Soviet Studies in the Psychology of Learning and Teaching Mathematics, v3, pp103-183, 1969.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward G. Begle, Stanford University, Stanford, California.

1. Purpose

This study was an attempt to ascertain the mental processes used by students in deriving algebraic equations from word problems.

2. Rationale

The design of the study was influenced by the experimenter's belief that the only logical way to solve a problem is to first recall the general rule for that kind of problem and then to insert the specifics of the problem into the rule.

3. Research Design and Procedure

Two experiments were performed.

The first used ten seventh graders. Five had been taught by one teacher (B) and the other five by another teacher (N). Each set of five consisted of one "star" student, two average students, and two weak students.

A 21-item instrument was used. Each item asked for the formulation of an algebraic expression or equation. The instrument was administered to the subjects individually. They were asked to tell, after solving the problem, what went on in their minds during the solution, whether they recognized the general proposition, how they decided what to designate by x, etc.

If a student was unable to provide an answer for an item, he was given variants of it until he did produce an answer.

4. Findings

Out of the 210 answers, 180 were correct. In only 28 of these cases did the student report complete, or even partial, recognition of the general proposition. In 21 cases, the general propostion had been taught to them. In seven cases the students generated the proposition.

5. Interpretations

The experimenter uses the term "association" for the mental operation used by subjects to obtain correct answers without recalling the general rule. He provided lengthy discussions of these associations but did not come to any pedagogically useful conclusions.

3_2. Research Design and Procedure

The second experiment used the original ten students, another set of five of B's students, and ten ninth graders. They were given a set of ten more complex word problems to solve and to report on as in the first experiment.

(It should be noted that each of these problems was easy to solve by means of two linear equations in two variables. However, in each of the solutions quoted in this report, only one equation was used.)

Two methods were observed of choosing which unknown to denote by x. In the first method, the unknown required by the statement of the problem was chosen. In the other method, some other unknown was denoted by x.

Similarly, two methods of setting up the equation were observed. One method proceeded by forming new expressions involving the known quantities of the problem and the basic unknown. In the other method, it was decided, perhaps even before choosing x, what was to be on each side of the equation.

Of the 250 problem attempts there were 202 correct responses. Of these, it was possible to determine the method of choosing the variable to be denoted by x and also the method of setting up the equation in 160 cases.

4_2. Findings

(a) The 160 cases were distributed as follows:

		Method of Choosing Variable	
		First	Second
Method of setting up equation	First	31	37
	Second	45	47

In the remaining findings, the method of choosing the variable is not considered any further, and only the two methods of setting up the equation are compared.

(b) The percentage of solutions using the two methods varied from problem to problem. The first method was not used at all on one problem but was used 69 percent of the time on another.

(c) The percentage of solutions by the two methods was not the same at the two grade levels.

		Grade	
		Seventh	Ninth
Method	First	51	33
	Second	49	67

(d) The percentage of solutions using the two methods was not the same for different ability levels.

		Ability Level		
		High	Average	Weak
Method	First	28	47	68
	Second	72	53	32

(3) The percentage of solutions using the two methods was not the ~~same for~~ students of different teachers.

		Teacher	
		B	N
Method	First	32	65
	Second	68	35

(Note: Teacher B concentrated on the second method of problem solving in class, while Teacher N concentrated on the first.)

(f) The percentage of solutions using the two methods was not significantly different when comparing those students included in the first experiment and those not included.

5_2. Interpretations

The experimenter concludes from findings (c) and (d) that the second method of formulating the equation is preferable, but, because of finding (b), it should not be taught exclusively.

Critical Commentary

The quantitative results listed above are not surprising. But even if they had been, the small number of subjects, as is not uncommon in Soviet mathematics education research, would not allow much confidence in them.

The extensive discussion of "association" is based on a view of the problem-solving process that will seem too narrow and too rigid to most Western educators and too much based on the author's opinions rather than on the data he collected, and consequently will not be of much interest to them.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN RESOURCES IN EDUCATION
January - March 1977

ED 128 166 Everest, M. Inez. Community College Students' Academic Achievement in Mathematics and Attitudinal Change as a Function of Instructional Methodology. 106p. MF and HC available from EDRS.

ED 128 182 Austin, Howard. Teaching Teachers LOGO, The Lesley Experiments. Artificial Intelligence Memo Number 336. 27p. MF and HC available from EDRS.

ED 128 189 Burt, Gordon J. The Detailed Evaluation of Mathematics Courses at the Open University. Report No. 1: The Unit on "Functions" in the Mathematics Foundation Course. 28p. MF and HC available from EFRS.

ED 128 195 Fennema, Elizabeth (ed.) Mathematics Learning: What Research Says About Sex Differences. Mathematics Education Reports. 51p. MF and HC available from EDRS.

ED 128 196 Johnson, Carl S. An Analysis of the Required Mathematical Preparation for Secondary School Mathematics Teachers in the United States, A Summary. 19p. MF and HC available from EDRS.

ED 128 197 Brown, Stephen W.; Wunderlich, Kenneth W. The Effect of Open Concept Education and Ability Grouping on Achievement Level Concerning the Teaching of Fifth Grade Mathematics. 17p. MF and HC available from EDRS.

ED 128 198 Flake, Janice L. Interactive Computer Simulations for Sensitizing Mathematics Methods Students in Questioning Behaviors. 18p. MF and HC available from EDRS.

ED 128 201 Eastman, Phillip; Behr, Merlyn. Interaction Between Structure of Intellect Factors and Two Methods of Presenting Concepts of Logic. 23p. MF available from EDRS. HC not available from EDRS.

ED 128 202 Hungerman, Ann D. 1965-1975: Achievement and Analysis of Computation Skills, Ten Years Later. 11p. MF available from EDRS. HC not available from EDRS.

ED 128 226 Osborne, Alan R.; Bradbard, David A. (eds.) Models for Learning Mathematics, Papers from a Research Workshop. 210p. MF and HC available from EDRS.

ED 128 228 MacKay, Irene Douglas. A Comparison of Students' Achievement in Arithmetic with Their Algorithmic Confidence. Mathematics Education Diagnostic and Instructional Centre (MEDIC) Report no. 2-75. 42p. MF available from EDRS. HC not available from EDRS.

ED 128 229 Robitaille, David F. A Comparison of Boys' and Girls' Feelings of Self-Confidence in Arithmetic Computation. Mathematics Education Diagnostic and Instructional Centre (MEDIC) Report No. 3-76. 22p. MF available from EDRS. HC not available from EDRS.

ED 128 230 Feghali, Issa. Interviews with Students of High Confidence and Low Achievement. Mathematics Education Diagnostic and Instructional Centre (MEDIC) Report No. 5-76. 19p. MF and HC available from EDRS.

ED 128 323 Dodd, Carol Ann. An Evaluation Model Applied to a Mathematics-Methods Program Involving Three Characteristics of Teaching Style and Their Relationship to Pupil Achievement. Teacher Education Forum; Volume 3, Number 4. 15p. MF and HC available from EDRS.

ED 128 400 Forbes, Dean W. The Use of Rasch Logistic Scaling Procedures in the Development of Short Multi-Level Arithmetic Achievement Tests for Public School Measurement. 19p. MF and HC available from EDRS.

ED 128 465 Virgin, A. E.; Darby, L. M. 1974 Replication and Follow-Up of a Survey of Mathematics and Reading Skills. 29p. MF and HC available from EDRS.

ED 128 466 Virgin, A. E.; Rowan, M. 1975 Replication of a Survey of Mathematics and Reading Skills. 25p. MF and HC available from EDRS.

ED 129 613 Bukowski, Joseph E. A Survey of Attitudes on the Use of Calculators in the College Classroom. 20p. MF available from EDRS. HC not available from EDRS.

ED 129 623 Graening, John Jay. An Evaluation of a Secondary Mathematics Teacher Education Program Emphasizing School Experiences in Contrasting Cultural Settings. 266p. MF available from EDRS. HC not available from EDRS.

ED 129 628 Poulsen, Sten C. The Study-Methods in Mathematics Project. I. Background Assumptions and Perspectives. Report No. 18. 36p. MF available from EDRS. HC not available from EDRS.

ED 129 631 Damarin, Suzanne K. Problem Solving: Polya's Heuristic Applied to Psychological Research. 40p. MF and HC available from EDRS.

ED 129 633 Fennema, Elizabeth; Sherman, Julia. Sex-Related Differences in Mathematics Learning: Myths, Realities and Related Factors. 24p. MF available from EDRS. HC not available from EDRS.

ED 129 636 Mathematics Technical Report: Summary Volume. National Assessment of Educational Progress. Education Commission of the States, Denver, Colorado. 181p. MF available from EDRS. HC not available from EDRS.

ED 129 868 Crist-Whitzel, Janet L.; Hawley-Winne, Barbara J. Individual Differences and Mathematics Achievement: An Investigation of Aptitude-Treatment Interactions in an Evaluation of Three Instructional Approaches. 42p. MF and HC available from EDRS.

ED 129 886 Dwyer, Carol Anne. Test Content in Mathematics and Science: The Consideration of Sex. 9p. MF and HC available from EDRS.

ED 129 949 Fitz-Gibbon, Carol Taylor. The Role Change Intervention: An Experiment in Cross-Age Tutoring. 340p. Available from University Microfilms. Not available from EDRS.

ED 130 618 Brown, O. Robert, Jr. A Comparison of Test Scores of Students Using Programed Instructional Materials with Those of Students Not Using Programed Instructional Materials. Technical Report No. 3. 25p. MF and HC available from EDRS.

ED 131 115 Calfee, Robert C.; Calfee, Kathryn Hoover. Reading and Mathematics Observation System: Description and Measurement of Time Usage in the Classroom. 19p. MF and HC available from EDRS.

ED 131 116 Elias, Patricia; And Others. The Reports of Teachers About Their Mathematics and Reading Instructional Activities. 14p. MF and HC available from EDRS.

ED 131 117 McDonald, Frederick J. A Report on the Results of Phase II of the Beginning Teacher Evaluation Study: The Effects of Teaching Performances on Student Learning. 12p. MF and HC available from EDRS.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
January - March 1977

EJ 142 959 Vitello, Stanley L. Quantitative Abilities of Mentally Retarded Children. Education and Training of the Mentally Retarded, v11 n2, pp125-129, April 1976.

EJ 143 182 Lim, James K.; Tseng, M. S. The Electronic Pocket Calculator--A Significant Factor in Students' Performance of Pharmaceutical Calculations? American Journal of Pharmaceutical Education, v40 n1, pp14-16, February 1976.

EJ 143 298 Scandura, Alice M.; And Others. Using Electronic Calculators with Elementary School Children. Educational Technology, v16 n8, pp14-18, August 1976.

EJ 143 675 Suydam, Marilyn N.; Weaver, J. F. Research on Mathematics Education Reported in 1975. Journal for Research in Mathematics Education, v7 n4, pp193-257, July 1976.

EJ 143 797 Wood, Robert. Sex Differences in Mathematics Attainment at GCE Ordinary Level. Educational Studies, v2 n2, pp141-160, June 1976.

EJ 144 007 Dunn, J. A. Discovery, Creativity and School Mathematics: A Review of Research. Educational Review, v28 n2, pp102-117, February 1976.

EJ 144 133 Olson, A. T.; Freeman, E. The Objectives for Teaching Mathematics in the Junior High School as Perceived by Parents, Students, Teachers, and Professional Educators. Alberta Journal of Educational Research, v22 n1, pp59-70, March 1976.

EJ 145 403 Abkemeier, M. K., Bell, F. H. Relationships Between Variables in Learning and Modes of Presenting Mathematics Concepts. International Journal of Mathematical Education in Science and Technology, v7 n3, pp257-270, August 1976.

EJ 146 106 Johnson, Virginia G. Comparing Attitudes toward Mathematics of Female Elementary Education Students with Other Female Students. College Student Journal, v10 n3, pp213-216, Fall 1976.

EJ 146 242 Kratochwill, Thomas R.; Demuth, Dennis M. An Examination of the Predictive Validity of the Keymath Diagnostic Arithmetic Test and the Wide Range Achievement Test in Exceptional Children. Psychology in the Schools, v13 n4, pp404-406, October 1976.

EJ 146 365 Osguthorpe, Russell T. The Effects of Pre-Remedial Instruction on Low Achievers' Math Skills and Classroom Participation. Reading Improvement, v13 n3, pp147-150, Fall 1976.

EJ 146 613 Morgali, Ronald R.; Lamon, William E. Using the Papy-Lamon Minicomputer to Teach Basic Addition Facts and Related Concepts to Visually Handicapped Children: A Pilot Study Report. Education of the Visually Handicapped, v8 n2, pp33-43, Summer 1976.

EJ 147 074 Carlson, Gaylen R. Location of a Point in Euclidean Space by Children in Grades One Through Six. Journal of Research in Science Teaching, v13 n4, pp337-343, July 1976.