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ABSTRACT

Research conducted in grades K-8 on activity-based teaching approaches, including studies on the use of manipulative materials, is reviewed and synthesized in this report. On the basis of the synthesis, it was concluded that lessons using manipulative materials have a higher probability of producing greater mathematical achievement than do non-manipulative lessons. Use of both manipulative materials and pictorial representations is highly effective; symbolic treatments alone are less effective. The use of materials appears to be effective with children at all achievement levels, ability levels, and socioeconomic levels. Activity-oriented programs and the use of mathematic laboratories can be expected to result in achievement at least as high as when actitities are not emphasized. Other conclusions are also presented, plus implications for further research. Implications for classroom practice and five guidelines for those planning projects or selecting programs are presented. (MS)

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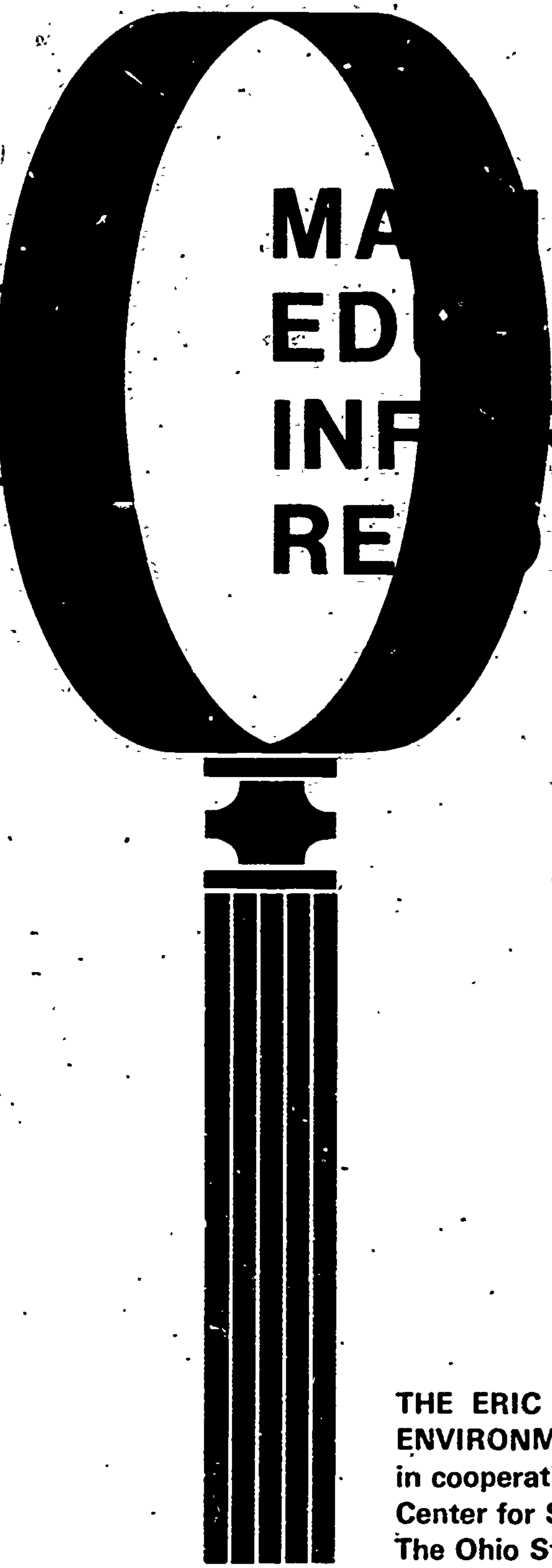

MATHEMATICS EDUCATION INFORMATION REPORT

THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
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MATHEMATICS EDUCATION REPORT

Activity-Based Learning in Elementary School Mathematics: Recommendations from Research

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Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the associate director.

PREFACE

In April, 1975, the National Institute of Education issued a request for proposals to review and synthesize studies of "activity-based approaches to mathematics teaching. The RFP explained,

> In recent years a variety of "activity-based" approaches to the teaching of mathematics have become widespread. Typically, these approaches are characterized by children's active manipulation of physical objects, but beyond that the diversity of programs claiming to be activity-based is enormous.
>
> The research that has been conducted on the effectiveness of such programs has led to results that are often conflicting. There seems to be some evidence, though, that many "activity-based" programs have a beneficial effect on low-achieving children. The purpose of this review of studies is to synthesize and interpret existing data on "activity-based" programs. The primary objective is to identify which of the many dimensions in which the approaches differ are the ones that are particularly important.

This publication is the final report of that review and synthesis (NIE Contract No. 400-75-0063). The synthesis was restricted to studies in which the mathematical content did not go beyond what is commonly found in grades K-8. Furthermore, we were interested on the effect of this teaching approach on elementary school students, especially low-achievers. Therefore, we did not analyze the sizeable number of studies which involve K-8 content taught by an activity-based approach to prospective elementary school teachers in college course.

Within these limits, we believe that this publication represents the most complete and thorough review of research on mathematics laboratories and other activity-based teaching approaches presently available. From this review, we have attempted to draw both practical conclusions for classroom teachers and suggestions for mathematics education researchers.

Jon L. Higgins
Associate Director for
Mathematics Education

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I. Activity-Based Instruction: What Does It Mean?

Educators have varying conceptions of what activity-based instruction is when they hear or use the term. The common element across these conceptions or definitions appears to be student involvement in the process of learning mathematics. This involvement is more than intellectual: the student is actively involved in doing or in seeing something done. On a broad scope, activity-based instruction is concerned with a teaching strategy, may encompass a program, and frequently involves the use of manipulative materials.

The Role of Teaching Strategy

As a teaching strategy, activity-based instruction means that the teacher incorporates activities of some type in planning lessons. But under this general umbrella lie a range of specifics:

- The activities vary widely, from actual real-world experiences to working within groups to accomplish a task.
- The activity may serve to motivate, to introduce, to provide reinforcement or practice, to help children apply a mathematical idea to the real world.
- The activity may be integral to the mathematical content or to the

instructional objectives, or it may be used pro forma, simply because the teacher believes or is told that the use of activities is necessary.

- The activity may or may not involve the use of objects or manipulative materials.

The Role of Program

A program termed activity-based can, in actual practice, refer to almost anything. Students are involved -- but they may be involved with such things as worksheets or workbooks, programmed instruction booklets, teaching machines or computer terminals, audiovisual materials, or calculators. In most activity-based programs, however, such materials form only one component of the system; using manipulative materials and participating in other activities are key components.

To many teachers and educators, the term "activity-based" appears to be synonymous with "mathematics laboratory". Yet mathematics laboratories themselves vary widely. Sometimes they are interest centers or corners containing materials of some type; they may be used every day, or only once or twice a week, or independently during "free" time. Sometimes the term mathematics laboratory refers to a pattern of teaching in which students are free to move around, to experiment, and/or to explore.

In the report of the National Advisory Committee on Mathematical Education (1975), it is noted that "In addition to the availability of varied manipulative materials", activity programs "are generally characterized by the use of varying instructional styles, multimedia approaches, variable-sized groupings, and learning (interest) centers" (p. 61). Most of these attempt to provide "a blend of activity methods, varied concrete materials, [and] student interaction with peers and the environment . . ." (p. 62).

The Role of Manipulative Materials

It should be evident from the above discussion that manipulative materials may or may not be involved in an activity-based program. Manipulative materials are also used in many instances in programs which are not termed activity-based.

The role of teaching strategy as it affects and interacts with the use of manipulative materials cannot be ignored. Through most of this century, and in particular since the mid-1930s, the importance of meaningful instruction[1] in the learning of mathematics has been espoused and accepted. In an analysis of the points of view and of the relevant research on meaningful instruction, Weaver and Suydam (1972) indicated that the use of concrete objects is integrally related with the development of meaning. In many of the studies they cited, the use of materials was an accepted component of the meaningful treatment, while the non-meaningful treatment generally precluded their use. (Few of these studies are cited in this report, since the extent of use of materials was rarely controlled, nor was their effect a clearly discernable factor in the collection and analysis of data.) Van Engen (1949) in characterizing a general theory of meaning, provided an indication of the interrelationship:

[1] In any subject area like mathematics, in which there is a large skill component, teaching can be done in a rote, mechanical way which emphasizes only speed and accuracy. In contrast, teaching can emphasize relationships between different skills and patterns or structures. This latter approach has often been called meaningful instruction. Much writing on this approach was done by William A. Brownell, most notably in the Tenth Yearbook of the National Council of Teachers of Mathematics. Meaningful instruction is a forerunner of cognitive psychology; it underlies the way in which most mathematics educators believe that mathematics should be taught.

> In any meaningful situations, there are always three elements. (1) There is an event, an object, or an action. In general terms, there is a referent. (2) There is a symbol for the referent. (3) There is an individual to interpret the symbol as somehow referring to the referent. . . . It is important to remember that the symbol refers to something outside itself. This something may be anything whatsoever, even another symbol, subject only to the condition that in the end it leads to a meaningful act or a mental image. [p. 323]

Inherent in this statement are two suppositions that have formed the basis for many research studies: (1) that learning proceeds on a concrete-to-abstract continuum and (2) that concrete materials are not essential to every stage in the process of learning: once a concrete referent has been established and has led to the development of understanding with symbols, those symbols may serve as the referent for further learning.

More recently, others (e.g., Bruner, 1960; Ausubel, 1968) have propounded a similar idea; statements such as the following are not uncommon in their writings:

> The essence of the meaningful learning process . . . is that symbolically expressed ideas are related in a non-arbitrary and substantive (nonverbatim) fashion to what the learner already knows, namely, to some existing relevant aspect of his structure of knowledge . . . [Ausubel, 1968, p. 37]

In general, the use of manipulative materials is widely accepted: both mathematics educators and teachers proclaim their belief in the efficacy of such materials. This belief is not always translated into action, however. In surveys of elementary-school teachers, R. Green (1970) and Harvin (1965) found that first-grade teachers reported that they used manipulative materials more than pictorial or symbolic materials. But teachers in grades 2 through 6 indicated little use of manipulative materials. In a survey cited in the NACOME Report (1975), it was noted that 10 per cent of the second- and fifth-grade teachers queried had never used manipulative materials at all.

Of course, the other side of that coin is that 90 per cent of the teachers surveyed have used manipulative materials; data from Thomas (1975) support this conclusion. And many teachers have raised questions about what materials to use, when to use them, how to use them, and with whom to use them. This concern in turn has led to research which attempted to provide answers. We will devote a great deal of attention in this report to these answers, but first it seems plausible to provide an overview of how the review was conducted, as well as a discussion of the limitations which (unfortunately) accrued.

II. Overview of the Report

The primary objective of this review has been to identify the most viable dimensions or facets of activity-based instruction, especially as they might apply to children needing compensatory education. To accomplish this, four procedures were proposed:

(1) study the information data base on activity-based approaches to mathematics instruction in grades K-8,

(2) analyze program components as described by research,[2]

(3) interpret results and conclusions, and

(4) synthesize findings from separate reports and studies into a set of recommendations for activity-based teaching which show the greatest promise for maximizing mathematics achievement and improving attitudes toward mathematics.

The research studies to be analyzed were selected on the basis of:

(1) appropriateness of level and content

(2) appropriateness of the type of material or approach, with emphasis on research reports which present a clear, explicit description of the procedures followed

(3) evaluation of the experimental-type studies in order to reject studies so poorly designed that their findings are meaningless.[3]

Factors to be Considered

To aid in the analysis and synthesis process, an inventory form[4] was

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[2] For this review, research is characterized as an exploration of a question through the collection of data.

[3] To aid in this evaluation, the Instrument for Evaluating Experimental Educational Research Reports was used. Appendix C contains a copy of the instrument, plus information on its development and reliability.

[4] A copy of the form appears in Appendix D.

developed which incorporated nineteen types of distinctions which appeared (in advance) to be relevant:

(1) Mathematical content: the "standard" content of the present elementary-school curriculum, including numeration and place value; computation with whole numbers, fractions, and decimals; measurement; geometry; and verbal problem solving (i.e., word problems).

(2) Grade level: kindergarten through grade 8 (or corresponding ages).

(3) Characteristics of the sample: including socioeconomic factors, cultural factors, geographic location, and type of community.

(4) Length of time: from minutes per day to total number of days, weeks, or months.

(5) Theoretical basis: reason or rationale for using materials or procedures.

(6) Sequence placement of activity: introductory, during development of concept, or for practice.

(7) Use of symbols: initially, throughout instructional period, or for final culmination.

(8) Nature and generality of materials: illustrating a wide variety of concepts, a number of broad concepts, or only one or two narrow concepts.

(9) Manipulative level: level of access, ranging from remote demonstration by teacher to individual manipulation; type of use, ranging from object manipulation to picture study; and purpose of use.

(10) Perceptual variability: the number of different materials used, and the number of different examples or explanations given for each material and/or each concept.

(11) Guidance in the use of materials: ranging from highly structured to free exploration.

(12) Social interaction: large class, groups, individualized, or isolated.

(13) Cost of materials/implementation: ranging from high to low.

(14) Special facilities needed: classroom arrangements, storage, and other factors.

(15) Training of teachers: minimal training, use of training materials, or special in-service sessions (or experimenter-taught).

(16) Extra staffing needed: special teachers, teacher aides, and/or parents.

(17) Special time factors: large segments of time needed, "regular" content not covered, etc.

(18) Correlation with remainder of educational program: ranging from integral to conflicting.

(19) Research and design factors: the nine points included on the instrument in Appendix C represent the salient factors experts on educational research have identified.

As we began to use the form in analyzing the studies, however, a problem became apparent. There was too little stated in many research reports to enable us to complete many of the items. In a few cases, even such aspects as mathematical content and grade level were not clear. While we had never expected that all of these items would be stipulated in each report, we were nevertheless surprised at how few points were presented unequivocably, so that we could be certain that our interpretations were accurate. As a result of this, the synthesis is presented under a more general set of categories than anticipated: type of use of materials, type of learner, level of learner, and mathematical content. While the focus is on achievement, attitude is also considered. Research on programs and modes is considered separately from that explicitly on materials.

Compilation of Studies

Appendix A contains an annotated list of those studies which are discussed or cited on tables in this report. In compiling the list of studies to be reviewed, we attempted to be as comprehensive as possible. However, certain studies originally listed were deleted because of inappropriate level, content, focus, or type. (Appendix B contains a list of those studies which were deleted, and the reasons for their deletion.) Among the types excluded were:

- Piagetian-oriented studies. We at first included Piagetian-oriented research, since there are physical materials involved as cognitive development is probed. But these studies are not concerned with teaching through the use of materials, while our focus is on learning as it relates to effective teaching procedures. The materials in Piagetian research are predetermined by the theory and by protocols, which represent real-world situations involving mathematical ideas, but not materials that the child will use to solve mathematical problems or apply mathematical ideas.

- Studies on activities involving no use of manipulative materials. Thus, research with such materials as computer-assisted instruction programs or calculators was not included.

- Studies so global that the use of materials and/or activities was only one of many factors incorporated and the effect of any one factor was indeterminable.

- Studies in which the materials were used across treatments, so that the effect of use of materials was a controlled variable rather than an independent variable, and their effect was not therefore a factor under investigation.

Research and design factors, item 19 on the list of distinctions, provided a particular difficulty which was not wholly anticipated. We have been aware for many years of the concerns expressed by researchers and others about the quality of research. We realized, therefore, that we would encounter very few studies which could be rated excellent: the difficulty of conducting research in the classroom is well-documented. We knew that the most we could do, if we were to have studies left in the set to be considered, would be to sort out those in which we believe too little confidence can be placed because of design questions. Some studies involved the use of materials, but were not citable for reasons varying from control of variables and confounding of variables to lack of clarity. These studies are listed on Table 1, with the reason for exclusion from the discussion which follows noted. All of these studies are annotated in Appendix A, so that readers can consider their findings, but in terms of the stated reasons for non-inclusion. As one reads this report, one can ascertain the trends across a set of studies with quality of

TABLE 1

STUDIES NOT CITED IN THE REVIEW

Author	Date	Focus/Reason for exclusion
Barragy	1970	type of material: matching photographs with camera position
Barrish	1971	strategy: inductive-discovery vs. deductive-reception; concrete materials used in both as feasible
Cheatham	1970	type of material: compass/straightedge or paperfolding; lack of control of variable
Choate	1975	sequence, conceptualization; paper-folding and diagrams used across groups
Cooke	1971	type of material: attribute blocks; focus on strategy
Coxford	1966	sequence, time; bead frame and other materials used across groups
Crabtree	1966	type of study: case study with number line
Dashiell/Yawkey	1974	type of study: action research with two types of balances
Dawson/Ruddell	1955	focus on algorithm; materials used with both treatments
DeFlandre	1975	type of study: feasibility of developed unit on other number bases
Dilley	1970	focus on algorithm: materials only an adjunct
Eudy	1973	type of material: device not generalizable
Farris	1971, 1973	focus on transfer across modes
Finley	1962	test items with or without materials
Genkins	1971	bilateral symmetry: paper-folding or mirror used
Gipson	1971	type of study: lack of control of variables
Hirschbuhl	1972	focus on transfer across modes
Houtz	1974	type of materials: models, slides, pictures for test items
Jencks	1969	type of study: feasibility of developed text on geoboard
Jones	1975	test items with three types of probability items
Kapperman	1974	type of material: abacus for blind; limited control of variables
Kellerhouse	1975	type of study: poor control of variables
Kratzer	1972, 1973	focus on algorithm; materials used with both
Kuhfittig	1972, 1974	coin conversion; length of time limited (2 days)
McGinty	1973	type of material: attribute blocks; focus on strategy
McLaughlin	1972	type of material: block task
Miller	1964	type of study: poor control of variables
Mott	1959	use of aids not clearly specified
Nicodemus	1970	type of material: attribute blocks; focus on strategy

TABLE 1 (Continued)

Author	Date	Focus/Reason for exclusion
Portis	1973	test items with varying aids
Prindeville	1972	confounded variables
Purser	1973	feasibility study; pictures rather than actual instruments used
Richards	1971	use of ruler
Schott	1957	action research; focus on rationale
Sherer	1968	use of materials not explicitly controlled
Sherzer	1973	number line: focus on algorithmic procedure
Spross	1962	control of materials vague
Swick	1960	type of study: no control group
Tanner	1972	control of variables questionable
Thompson	1974	case study: materials used (only) to elicit responses
Trafton	1971	focus on developmental time
Van Engen/Gibb	1956	use of materials not controlled
Vitello	1972	class inclusion
Weeks	1971	type of material: attribute blocks; focus on strategy
Wilkinson, G.	1971	type of material: films and filmstrips in addition to objects (objects not major focus)

research taken into consideration; one can also check the excluded studies to alleviate the fear of potential bias of the reviewers.

The Task of Synthesis

We began this review with the firm belief that the role of manipulative materials and the effect of activity-based instruction could be made clearer by an intensive analysis of the research -- although we did not believe that all of the answers to teachers' questions could be found in existing research. By the midpoint of the review, we were beginning to realize the appropriateness of an analogy: that of the photographs found in a newspaper. When one looks very closely, the fact that the photograph actually consists of a set of dots with space between them is obvious. It is not until one stands back that the form or outline emerges, and the pattern or picture is apparent. Even so does the meaning of the studies on manipulative materials and activity-based instruction remain unclear until one stands back to see the pattern. Getting "bogged down" in details does not help. Individual studies make a contribution to the whole; unfortunately, however, there are many gaps in the research -- points which have remained unexplored -- so that the entire pattern or picture is not yet obvious.

As in most research studies, the effect of the teacher serves to confound other variables. Because it is so difficult to control, identify, and/or isolate the precise nature of the interaction between teacher and learner, or between the teacher's strategy and the instructional materials, the "teacher effect" has, across research, been held to account for the greatest amount of variance. Thus, Sole (1957) notes that "The effectiveness of the learning of arithmetic depends more on the teacher than on the materials used" (p. 1518). And Reys (1972) emphasizes that "the one common thread among these studies is that learning mathematics depends more on the teacher than on the embodiment [material] used" (p. 490).

The teacher effect in most of the studies we reviewed leads us to the same conclusion. The teacher is, without doubt, a most volatile element in research conducted in a classroom. Research evidence has yet to confirm the factors involved in effective teaching, but many educators nevertheless believe that, if teachers are committed and well-trained in terms of knowing both the child and the curriculum, they will have a very positive effect on a child's learning. For instance, Harshman et al. (1962), in summing up their study, note:

> As one examines the rankings of the twenty-six classes, one is impressed by the fact that some classes ranking low on intelligence have high rankings in arithmetic computation, arithmetic reasoning, and total arithmetic achievement. This might suggest that one of the factors involved was effective presentation of concepts by the teachers. [p. 192]

Because variance in research findings can frequently be attributed to the teacher effect (that is, to differences between teachers), we have searched for categories or sets of studies which cluster. One study may be affected by individual differences of particular teachers; several studies involve a broader base of teachers. When the findings of a set of studies concur, the result is more likely generalizable across teachers. Nevertheless, teachers should test research findings in their own classrooms. Research can only indicate what in general is effective or appears to be "truth".

In the remainder of this report, we will focus on those studies which remained in the set. In Chapter III, studies involving materials are considered, while in Chapter IV research on activity programs and modes of instruction is discussed.

III. Research on Materials

The first four sections of this chapter focus on research specifically related to characteristics of materials and their use, with some form of achievement as the dependent variable generally being measured. Then the focus shifts to the learner, with studies on materials classified by learner characteristics and by age or grade level. Mathematics content provides a seventh focus, with an additional look at the interaction of activity and content in studies on multiplication. The last section considers affective results separately.

Manipulative Materials: Yes or No?

Does the use of manipulative materials help student achievement in mathematics? Both classroom teachers and developmental psychologists generally hold strong beliefs that it does. Classroom teachers often base this belief on experiences with young children, at grade levels where reading and symbolization cause learning difficulties. At these levels, the use of manipulative materials may be the only viable alternative available to the teacher. Developmental psychologists often cite Piaget as providing an example of a theory which places manipulative materials or concrete objects in a special prerequisite function in the development of learning structures. Yet neither teachers nor psychologists are apt to derive their conclusions from research on classroom-based learning and teaching situations.

Nevertheless, a large body of research exists which is related to the question of the effect of manipulative materials on mathematics learning. Many of these studies, however, developed not from theory but out of a folklore of beliefs about activity learning. Perhaps because of this, they present what appears to be an almost random selection of independent variables and treatments. Making sense of the results of this research requires a very careful sifting and categorizing of studies.

Over 20 studies were identified comparing lessons in which manipulative materials were used with lessons in which manipulative materials were not used. These studies covered all levels from grades 1 through 8, and elementary-school mathematics concepts ranging from number recognition to operations with fractions and units of geometry. The results are summarized in Table 2. Of the 23 studies[5], two favored lessons in which manipulative materials were not used; 11 favored lessons in which manipulative materials were used; and in 10 no statistically significant difference in achievement was found between lessons in which materials were used and those in which materials were not used.

Approximately half of the studies comparing use and non-use of materials thus favored the use of materials. A closer look at many of these studies is justified: for a third the data are markedly different between treatments; for another third, the data are clearly different but not markedly so; for the other third, the data indicate a weaker difference. (See Appendix E for pertinent data from each study on Table 2.) But while this variance exists, the fact remains that the differences which were significant on most of the various tests favored the use of materials.

In almost the same number of studies, no significant differences were found. A look at the data indicates that in 6 of the 10 studies, there was a tendency for the group which did not use manipulative materials to score higher than those who used materials -- but this trend is characterized by very slight differences (as one might expect in the case of studies reporting no significant differences). The trend toward favoring

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[5] Tobin (1974) reported data for two samples.

TABLE 2

STUDIES COMPARING USE OF
MANIPULATIVE MATERIALS WITH NON-USE

Author	Date	Content	Grade level	n*	Trend	Strength**
Favoring Use of Manipulative Materials						
Babb	1976	multiplication facts	2	3 c, 76 p		-
Bledsoe et al.	1974	fractions	7	339 p		++
Bolduc	1970	problem solving	1	36 p		+
Bring	1972	geometry	5,6	102 p		+
Brown	1973	fractions	4	12 c		+
Cook et al.	1968	several topics	1	105 p		-
Earhart	1964	several topics	1-3	1088 p		-
Nichols	1972	multiplication, division	3	10 c, 267 p		++
Nickel	1971	problem solving	4	90 p		-
Tobin	1974	remedial	age 9-12	202 p (MRs)		+
Wallace	1974	fractions	4-6	154 p		++
No Significant Differences						
Anderson	1958	geometry	8	408 p	U N	-
Bisio	1971	fractions	5	501 p	N U	-
Coltharp	1969	integers	6	79 p	U N	-
Davidson	1973	several topics	3,4	16 c	N U	-
Dunlap et al.	1971	remedial	4	12 c	N U	-
Macy	1957	several topics	4	2 c	N U	-
McMillian	1973	numeration	7	202 p	N U	-
Tobin	1974	remedial	age 6-9	28 p	unclear	-
Trask	1973	multiplication, division	3	65 p	N U	-
Weber	1970	several topics	1	6 c	U N	-
Favoring Non-use of Manipulative Materials						
Carney	1973	rational numbers	4	8 c		++
Smith, J.	1974	geometry	7	4 c		-

* In tables, c --> classes U --> Use of materials
p --> pupils N --> Non-use of materials
s --> schools

** "Strength" indicates a subjective estimate of the degree of differences in scores, considering the quality of the study and the data, with ++ --> high, + --> average, - --> low. Pertinent data from each study on this table are included in Appendix E, to aid readers in verifying the differences in scores.

the use of materials in 3 other studies is likewise slight. Differences might exist between the treatments in these studies, but their effects were not revealed with the measures used.

Let us propose a way of considering the studies or Table 2. When we consider these studies, we should contrast outcomes with expectations. Suppose that there was, in fact, no difference between the use and the non-use of materials. We might then expect every study to show no significant difference Or we might expect that some studies would favor the use of manipulative materials while others would favor their non-use, according to the effect of uncontrolled variables (including the teacher variable). The effect of these variables should be random, however -- the number of studies favoring use of manipulative materials should be approximately equal to the number favoring non-use (and the number of each should be less than the number of studies finding no significant difference, forming a pattern analogous to that of the normal distribution curve). This is not the pattern found in Table 2. The number of studies favoring the use of manipulative materials greatly exceeds the number favoring the non-use of materials.[6]

Those studies in this latter category are only 2 in number. One (J. Smith, 1974) is categorized by data in which, of 15 geometry tests, significant differences favoring expository instruction over the use of multimodel or unimodel approaches were found in only 4 instances -- but the trend of the remaining data favors the expository group in every instance but 2. We should note, however, that expository instruction

6 It is possible that there could be a single study so well designed and executed that its findings take precedence over all other studies on a given point. This would negate the trend analysis. We searched our list for such a study (or studies). We did not, however, find any that fulfilled this "exceptional" expectation. Therefore, we have had to consider trends.

presented by teachers was compared with manipulative lessons where all instructions were read by students from booklets. It seems possible that a confounding variable, reading ability, may have produced an effect just as strong as the manipulative/non-manipulative variable.

The data from Carney (1973) clearly favors the group taught by an abstract approach (based on, for instance, properties of whole numbers such as the commutative property, 3 + 2 = 2 + 3) over the group taught with materials. In contrast with other studies, the abstract approach was considered to be the experimental treatment; moreover, it is inferred that a large part of the control group's experiences consisted of work with diagrams rather than concrete materials.

Similar types of comments could be made about studies in the other two categories, also. There are reasons which can be hypothesized in almost every instance for why the results of a study were found -- reasons ranging from confounding of variables to experimenter-bias to strong or weak treatments. The overall point remains, however: far more of the studies favor the use of materials than favor non-use of materials.

Considering the evidence summarized in Table 2, we believe that, on a simple manipulative vs. non-manipulative comparison, non-manipulative lessons cannot be expected to produce superior achievement in elementary-school mathematics. The use of manipulative materials has a high probability of producing greater mathematics achievement than do non-manipulative sequences.

The fact that in so many studies no significant differences were found between treatments may indicate that the way manipulative materials are used in lessons may be of critical importance. What are the influencing factors for a strong manipulative activity treatment? Does the sequence of use from concrete to abstract make a difference? Do the number of different physical embodiments or representations of the mathematical concept make a difference? Does it make a difference whether the student handles materials or only watches the teacher demonstrate with materials? Do age or other characteristics of the learner affect the outcome of the treatments? Do some manipulative treatments work better for some mathematics concepts or content than for others? In the following sections, research studies will be classified in an attempt to answer these questions.

Concrete, Pictorial, and Symbolic Sequences: Which? When?

Through the years, the idea that the learning of mathematics progresses through three levels of abstraction has evolved and been propounded (e.g., see Brownell, 1928). It began (in all probability) with Pestalozzi, though its origins can be traced to earlier times.

> Until 1500 A.D., arithmetic throughout Europe was taught altogether by means of objects. After the introduction of the Hindu-Arabic symbols, calculating by means of the abacus and through the use of objects ceased, and the study of numbers became very abstract. Object teaching was discontinued until after the teaching of Pestalozzi began to exert an influence. [Newcomb, 1926, p. 10]

> A great reform in elementary teaching was initiated in Switzerland and Germany at the beginning of the nineteenth century. Pestalozzi emphasized in all instruction the necessity of object-teaching. . . . The child must be taught to count things and to find out the various processes experimentally in the concrete, before he is given any abstract rule, or is put to abstract exercises. [Cajori, 1896, pp. 211-212]

> Reform in arithmetical teaching in the United States did not begin until the publication by Warren Colburn, in 1821, of the Intellectual Arithmetic. This was the first fruit of Pestalozzian ideas on American soil. [Cajori, 1896, p. 218]

By the 1930s, the belief was well-accepted that it was necessary for the young child to progress through three stages -- from concrete objects to pictures to symbols.

More recently, Piaget (1958) and Bruner (1966) have discussed these three stages with new labels attached: enactive, iconic, and symbolic. They suggest that first learners must learn by manipulating physical, concrete objects in an enactive sense. Once this is done, they can learn through pictures or representations of objects in an iconic fashion. Finally they develop the ability to learn by manipulating only abstract symbols. It is not clear whether this sequence is necessary for every new mathematics topic, or whether it can be shortened for some concepts or for some learners at particular ages or stages.

Do concrete, pictorial, and symbolic treatments presented separately have differing effects on achievement? The results of studies directed to this question are shown on Table 3. Only three studies favored a symbolic treatment, and they vary widely in their focus. Carney (1973) worked with fourth graders using the number line with one group and an abstract approach based on the properties of whole numbers for his second treatment. Thus the students in this treatment were expected to -- and did -- exhibit transfer from previously taught ideas.

Fennema (1970, 1972) found the result favoring the symbolic treatment only on a transfer test with symbols and not on a test of immediate achievement. She introduced multiplication to second graders as repeated-addition facts (symbolically) or as trains of Cuisenaire rods (concretely). It seems possible that lessons intervening between the immediate achievement test and the symbolic transfer test might have reinforced the symbolic treatment more than they did the concrete treatment (see also pages 48-54). As she states, both treatments were developed meaningfully: this might have been the key variable.

In the third study, Rathmell (1973) was investigating the effect of sequencing differences as first graders began to work with numbers greater

TABLE 3

STUDIES INVOLVING COMPARISONS
OF CONCRETE, PICTORIAL, AND/OR SYMBOLIC TYPES OF MATERIALS

Author	Date	Content	Grade level	n
		Favoring Concrete		
Armstrong	1972	sets	MA 2-4	20 p (TMRs)
Clausen	1972	several topics		8 c, 177 p
Ekman	1967	addition, subtraction	3	27 c, 584 p
Johnson, Robert	1971	geometry	4-6	96 p
Nichols	1972	multiplication, division	3	10 c, 267 p
Shoecraft	1972	problem solving	7 (9)	12 c, 366 p
Wood	1974	multiplication (transfer)	1-3	40 p
		Favoring Pictorial		
Gibb	1956	subtraction	2	36 p
		Favoring Symbolic		
Carney	1973	rational numbers	4	8 c, 240 p
Fennema	1970, 1972	multiplication (transfer)	2	95 p
Rathmell	1973	numeration	1	110 p
		Concrete, Pictorial > Symbolic		
Bohan	1971	fractions (equivalent)	5	6 c, 171 p
Carmody	1971	numeration	6	3 c
Curry	1971	numeration	3	3 c
LeBlanc	1968	problem solving (subtraction)	1	338 p
Nickel	1971	problem solving	4	90 p
Norman	1955	division (transfer)	3	24 c
Scott/Neufeld	1976	multiplication	2	9 c
Steffe	1967, 1970	problem solving (addition)	1	132 p
Steffe/Johnson	1970	problem solving (addition)	1	111 p
		No Significant Difference		
Armstrong	1972	sets	MA 5-12	67 p (EMRs)
Bohan	1971	fractions (operations)	5	6 c, 171 p
Denman	1975	addition, subtraction	5,6	33 c, 455 p

TABLE 3 (Continued)

Author	Date	Content	Grade level	n
		No Significant Difference (continued)		
Fennema	1970, 1972	multiplication (other tests)	2	95 p
Green, G.	1970	fractions	5	120 p
Norman	1955	division (immediate)	3	24 c
Trask	1973	multiplication, division	3	65 p
Wood	1974	multiplication (immediate)	1-3	40 p

than 9. Having pupils read and write numerals before giving them experiences in regrouping with materials appeared to have a positive effect on achievement.

Gibb (1956) reported differences favoring the pictorial treatment over the concrete. She found that a higher level of performance on subtraction problems was associated with problems which were presented with stimuli in the form of squares and/or circles mounted on cards. Abstract contexts were the most difficult. However, popular belief appears to be mirrored in her conclusion that

> Children have less difficulty solving problems if they can manipulate objects or at least think in the presence of objects with which the problems are directly associated than when solving problems wholly on a verbal basis. [p. 77]

Concrete treatments were found to be superior to both pictorial and symbolic treatments by Ekman (1967) at the third-grade level and by Robert Johnson (1971) in grades 4 through 6, as well as by Shoecraft (1972) for some problems in grade 7. For the concrete treatment, Ekman used cardboard disks and holders on a number line while teaching addition and subtraction. Johnson used geometric models to teach perimeter, area, and volume concepts. These are only two studies which illustrate the point that quite different types of concrete objects have resulted in findings of superior achievement in differing areas of mathematics.

Both concrete and pictorial treatments were found to be superior to symbolic treatments in effecting achievement by Carmody (1971) in grade 6, Curry (1971) in grade 3, and Scott and Neufeld (1976) in grade 2. In all three studies there was no statistically significant difference between the concrete and pictorial treatments. Denman (1975), Green (1970), and Wood (1974) found no statistically significant differences between the three types of treatments. In Green's study, involving multiplication with fractions in grade 5, a difference favoring diagrams over materials was indicated. However, Green points out that the difference in achievement was very small, and that the use of diagrams on the achievement test might have favored the pictorial treatment group. Indeed the difficulty of testing for achievement in ways that do not favor one treatment over another is a serious problem in many research studies.

On the whole, the studies on Table 3 show that symbolic treatments are probably at a disadvantage when used alone. This finding is consistent with the trend favoring the use of manipulative materials noted on Table 2. (Nevertheless, many mathematics lessons involve the use of number symbols only, particularly after grade 1.) The studies which found pictorial treatments to be superior to symbolic treatments confirm that pictures and diagrams can also be important in designing mathematics lessons. However, pictures are rarely superior to concrete experiences; thus this importance is a relative one.

Do different sequences of concrete, pictorial, and symbolic treatments produce differential effects on achievement? Such questions of sequence are not as easy to research as the previous question was. On the one hand, one can vary sequence by omitting a type of treatment or by changing the order of the three types of treatments. On the other hand, one can vary sequence by repeating certain treatments in such a way as to place more relative emphasis on one type of treatment than another. When these possibilities are combined, the number of possible sequences that can be investigated is infinite.

Only three studies were clearly directed at the question of sequence. Olley (1974), in a five-lesson study with numeration ideas in grades 3 and 7, found that, for transfer, concrete-to-symbolic sequences were preferable to pictorial-to-symbolic or symbolic (alone) sequences. (No significant differences were found, however, on a retention test.) Punn (1974) studied third graders as they worked with multiplication using concrete-to-symbolic, pictorial-to-symbolic, or concrete-pictorial-symbolic sequences. Higher achievement was attained by those moving in the concrete-to-symbolic sequence, followed by the concrete-pictorial-symbolic. Poorest was the pictorial-symbolic sequence.

St. Martin (1975), in contrast, reported no significant differences between fifth graders using a concrete-pictorial-symbolic sequence and those using a pictorial-symbolic sequence on fractions. He noted, however, that use of concrete materials resulted in some higher subtest scores.

Thus, again, the concrete stage appears to have an important role.

Embodiments: One or Many?

Dienes (1961) argues that one key to mathematics learning and teaching is the presentation of a mathematical concept in several different physical forms or embodiments. The use of different representations causes the child to rocus on what is constant across them -- the mathematical idea. In other words, the child is expected first to transfer a learning from one context to another, and then, more importantly, to generalize across contexts and thus develop a firmer understanding of the mathematics. This is a very complex process for children. Gau (1973) and Beardslee (1973), in studies of concepts of equivalent fractions with fifth- and sixth-grade students, found no difference on tests of either transfer or generalization between treatments involving one, two, or three embodiments.

Their lack of strong findings, however, should not be interpreted as meaning that there is no connection between use of embodiments and mathematics learning. Wheeler (1972), for instance, tested second-grade children and found that those who could correctly represent single-digit addition and subtraction problems with three or four representations of physical objects could also solve more multidigit addition and subtraction problems with paper and pencil than children who could only demonstrate one representation. This correlation was not affected when the data were blocked by age, IQ, or knowledge of basic addition or subtraction combinations.

Wheeler's correlational study cannot, of course, identify which variable is the causal factor. It may be that ability to represent multiple embodiments causes a higher performance on multidigit addition and subtraction problems, or it may be that greater facility with addition and subtraction gives more insight into constructing physical representations. Research attempting to pin down this causal relationship by designing teaching treatments using varying numbers of multiple embodiments is generally inconclusive. In the one study with a directional finding, J. Smith (1974) reported that an expository approach was more effective in teaching area concepts than was use of either one or several models (see page 17 for a comment on his study). But in addition to Beardslee and Gau, Sole (1957) found no effect due to varying the number of embodiments at the third-grade level when teaching addition with carrying. In a study conducted during the past year in connection with the Project for the Mathematical Development of Children at The Florida State University, half a dozen embodiments were used in developing each of several mathematical ideas. Behr (1976) indicated that, although the data analysis was not yet completed, preliminary analysis indicated that the finding of no significant differences between number of embodiments would probably be found.

Although the argument for multiple embodiments seems plausible, the indication of these few studies that it is of little practical consequence may in fact be correct. The process of finding constant ideas across a variety of situations requires both reflective and retroactive thinking. We know from other research which focused on transfer (e.g., Kolb, 1967; Sawada, 1972) or on generalization (e.g., Swenson, 1949) that children must be taught specifically to transfer and to generalize. Children must be made aware of the point that what they learn may be applied in similar -- and not-so-similar -- situations. For most children, neither process is spontaneous. They must be given strong guidance and focus by the teacher. Without the teacher strongly focusing the attention of the learners, relatively little learning might be expected from a variety of embodiments. The Beardslee and Gau studies utilized programmed instruction formats, in which one cannot ascertain whether learners were actually focusing on the vital or relevant components of the instruction; their results reflected weak effects on achievement by their treatments, perhaps attributable to this factor.

It is extremely difficult to design experiments which vary the number of embodiments presented without also varying the amount of practice, time, or intensity of presentation. In fact, many classroom teachers would find it easier to manipulate time, the amount of practice, or the amount of drill. It should also be noted that multiple embodiments have actually been used in many other studies than these few in which they were identified as the factor of concern. Perhaps children who count using both markers and hops on the number line are being given a basic multiple embodiment experience. Considered in this sense, children used several types of materials in many of the studies considered in the previous two sections. But until the interaction of all these variables is better understood, the expectation that use of multiple embodiments, as a single factor, will affect achievement does not seem to be warranted.

There are other criteria by which multiple embodiments may be justified. To help children understand applications of mathematics or appreciate the utility of mathematics in today's world, presenting mathematics in a variety of physical and practical embodiments certainly seems logical. The inclusion of multiple embodiments may be important to consider as course objectives and long-range curriculum goals are determined. Until research can further clarify the relationship of embodiments to immediate learning, it would seem that programs should not be limited to the use of only one type of embodiment.

Who Shall Manipulate?

Does it make a difference who manipulates physical materials in the classroom? Several studies have been concerned with comparisons between individual student use of materials and teacher demonstration with materials. Achievement outcomes of these studies are summarized in Table 4. Only one study favors the demonstration mode (Trueblood, 1968, 1970). It is important to note, however, that in Trueblood's teacher-demonstration treatment pupils were asked to focus on thinking about manipulation of the materials as the teacher worked. He termed this "covert manipulation", and the strong focusing on vicarious participation sets this treatment apart from the usual idea of teacher demonstration.

These results seem quite consistent with those who advocate activity learning approaches. Learning is believed to take place because of mental activity by the learner. Physical manipulations by the learner increase the probability of this mental activity. However, the four studies which

TABLE 4

STUDIES INVESTIGATING PUPIL OR TEACHER USE OF MATERIALS

Author	Date	Content	Grade level	n
		Favoring Pupil Use		
Branch	1974	Integers (addition, subtraction)	6	36 p
Gilbert	1975	Addition, subtraction (two-digit numbers) (one school)	3	124 p
Toney	1968	Several topics	4	2 c
		No Significant Difference		
Bisio	1971	Fractions	5	29 c, 501 p
Gilbert	1975	Addition, subtraction (two-digit numbers) (second school)	3	124 p
Jamison	1964	Numeration	7	3 c, 94 p
Knaupp	1972	Numeration	2	4 c
		Favoring Teacher Demonstration		
Trueblood	1968, 1970	Numeration	4	7 c

found no significant differences between methods remind us that individual manipulation is not the only way to achieve desired mental activity. When other factors such as cost or classroom facilities become important, there may be ways to create demonstrations by the teacher that can be as effective as individual manipulation.

Type of Learner

Various types of learner characteristics have been considered as factors on which to block and analyze data. One of these is achievement: what pattern of achievement have learners exhibited in the past? While many educators believe that low achievers need to use materials more than high achievers do, that point is not borne out by research. Denman (1975) and G. Green (1970) reported no significant differences: that is, the use of materials appeared to be as effective at one achievement level as at another, at least for students in grades 5 and 6. Shoecraft (1972), however, stated that the use of materials did not affect achievement <u>except</u> for the seventh graders at the low achievement level in his study: there the use of materials seemed particularly effective.

Trask (1973) analyzed data by ability level; he found that students in grade 3 who were below average in ability benefited more from the symbolic method, while those with above-average ability were helped more by manipulative materials. He points out that the materials were confusing to the low-ability students. Yet one study is not sufficient to have a clear understanding of the effect of materials in relation to ability. Other studies (see Appendix A) which have considered the IQ variable have generally reported no significant differences. Armstrong (1972) presents the strongest evidence that manipulative materials help mentally retarded children.

Wallace (1974) included approximately 50 students (one-third of her sample) who were "classified as Title I students or welfare recipients" in a study on the use of varied manipulative materials versus expository instruction. (There was no clarification of how many of the students were in Title I programs and how many were welfare recipients.) She reported that "the achievement of the welfare recipients was not significantly different from the achievement of the non-welfare recipients", and concluded that

> It can be inferred that any student, regardless of socioeconomic background, if given an appropriate learning environment, can evidence growth as was indicated by the parallel performance of the welfare and non-welfare recipients. [p. 2899]

In other studies, including some on activity-oriented programs cited in Chapter IV, the findings for students from low socioeconomic levels are mixed. Crowder (1966) and Passy (1964b) found that achievement with the Cuisenaire program increased as socioeconomic level increased. Bisio (1971) reported no significant differences between manipulative use versus non-manipulative use for low socioeconomic groups, while watching a teacher demonstrate with materials was better than non-use for students from higher socioeconomic levels. Schippert (1965) reported no significant differences between students using materials in a laboratory approach or having regular instruction. Weber (1970) indicated a trend favoring manipulative materials for first graders from low socioeconomic levels.

Hankins (1969) developed a special program for fourth graders "from impoverished areas". Like many other programs developed for special purposes, it was found to be better than the "usual" program. The motivational effect of being involved in a project, of being provided with special help and instructional materials, can be attested to by thousands of students and teachers. Students generally profit from the attention directed toward them in special projects: that is, they have positive reactions and sometimes even make achievement gains. Teachers should take advantage of this motivational effect -- and attempt to collect data on how strong and lasting an effect it is.

Age of Learner (Grade Level)

Many mathematics educators feel that the relative importance of activity with manipulative materials decreases as age increases. They reason that materials are important until the child develops symbolic skills, but that once those skills are developed in one major area they may be applied to other areas. For instance, once addition is mastered symbolically, multiplication can be learned symbolically by treating it as repeated addition.

Because of this widespread belief, the need to consider results by age level is apparent. Research studies on manipulative materials were therefore grouped according to the grade level involved, since most used this designation rather than age. Summaries of these groupings are found in the tables which follow. Since not all grade levels are equally represented by studies, it was necessary to group together studies by pairs of grades, in order to get a clearer projection of the results.

Table 5 lists 16 studies conducted in grades 1 or 2. Five studies (Babb, 1976; Bolduc, 1976; Cook et al., 1968; Earhart, 1964; Steffe and Johnson, 1970) favor the use of concrete materials over the use of pictorial and/or abstract modes. In two studies (LeBlanc, 1968; Steffe, 1967, 1970), no significant difference was found between concrete and

TABLE 5

RESULTS OF STUDIES IN GRADES 1 AND 2

Author	Date	Results
Babb	1976	manipulative materials > imagery or textbooks for multiplication facts
Bolduc	1970	visual aids > no aids in problem solving
Cook et al.	1968	manipulative materials > television or textbook
Earhart	1964	abacus > workbook
Fennema	1970, 1972	symbolic > concrete on transfer test for multiplication
Gibb	1956	pictorial > concrete > abstract for subtraction problems
Harshman	1962	teacher-made program with aids > Numberaids (commercial) program
Knaupp	1971, 1972	NSD between individual use and teacher demonstration for numeration
LeBlanc	1968	concrete = pictorial > symbolic on subtraction problems
Muckey	1971	Dienes = non-Dienes materials at high SES level; non-Dienes slightly favored at middle SES level for numeration
Scott/Neufeld	1976	NSD between concrete, pictorial, symbolic for multiplication
Steffe	1967, 1970	concrete = pictorial > symbolic on addition problems
Steffe/Johnson	1970	concrete > no materials for addition problems
Weber	1970	NSD between concrete and paper-and-pencil follow-up activities
Wheeler	1972	significant correlations between number of embodiments and achievement on multi-digit addition and subtraction.
Wood	1974	NSD between concrete and pictorial for multiplication

pictorial treatments, but either was more effective than use of only symbols. Three other studies also indicated no significant difference between concrete and other treatments (Scott and Neufeld, 1976; Weber, 1970; Wood, 1974). Only one study (Fennema, 1970, 1972) indicated that a symbolic treatment was more effective than a concrete one, while one other study (Gibb, 1956) found that a pictorial treatment was more helpful than a concrete one, but either was better than use of only symbols. [The remaining studies focused on the type of use (Knaupp, 1971, 1972), compared two types of materials (Harshman, 1962; Muckey, 1971), or considered multiple embodiments (Wheeler, 1972).] The importance of manipulative materials in grades 1 and 2 seems apparent from these results.

Twenty studies were conducted in grades 3 or 4 (see Table 6). In 5 (Earhart, 1964; Ekman, 1967; Johnson, 1971; Nichols, 1972; Wallace, 1974), the use of concrete materials was favored over the use of pictorial and/or abstract modes. And in 4 others (Brown, 1973; Curry, 1971; Nickel, 1971; Norman, 1955), use of concrete and pictorial materials was better than use of only abstract materials. An abstract treatment was found to be more effective than concrete or pictorial treatments in only one study (Carney, 1973). No significant difference between modes was reported by 3 studies (Davidson, 1973; Trask, 1973; Wood, 1974). [Four studies (Gilbert, 1975; Prigge, 1974; Toney, 1968; Trueblood, 1968, 1970) compared types of use; two (Olley, 1974; Punn, 1974) focused on sequences; while another (Sole, 1957) focused on number of embodiments.] Thus, the picture at grades 3 and 4 again indicates an advantage when manipulative materials are used.

In Table 7, 12 studies with children in grades 5 and 6 are cited. Use of concrete materials was favored in 3 instances (Bring, 1972; Johnson, 1971; Wallace, 1974), and use of either concrete or pictorial treatments

TABLE 6

RESULTS OF STUDIES IN GRADES 3 AND 4

Author	Date	Results
Brown	1973	text + manipulatives + film > other combinations for equivalent fractions
Carney	1973	abstract > objects + number line for addition and subtraction with rational numbers
Curry	1971	concrete or pictures > verbal for clock arithmetic
Davidson	1973	NSD in achievement, but better conservation for materials + textbook rather than textbook alone
Earhart	1964	abacus > workbook
Ekman	1967	concrete > pictorial or abstract for addition and subtraction
Gilbert	1975	individual manipulation > teacher demonstration for addition and subtraction in one school; NSD in another school
Johnson, Robert	1971	physical models and instruments + text > text without models or text without drawings for perimeter, area, and volume concepts
Nichols	1972	materials with discovery > traditional exposition for multiplication and division
Nickel	1971	materials + pictures > verbal for problem solving
Norman	1955	concrete + pictorial > text or conventional on transfer/retention test; NSD on immediate test for division
Olley	1974	concrete-to-abstract sequences > pictorial-abstract or abstract sequences on transfer test on numeration concepts; NSD on retention
Prigge	1974	teacher demonstration and student manipulation of solids > paper and pencil or paper-folding, geoboard activities
Punn	1974	concrete-symbolic > concrete-pictorial-symbolic > pictorial-symbolic sequences for multiplication

TABLE 6 (Continued)

Author	Date	Results
Sole	1957	NSD on number of embodiments used for addition with carrying
Toney	1968	individual manipulation > teacher demonstration for several concepts
Trask	1973	NSD between materials and traditional lessons for multiplication
Trueblood	1968, 1970	teacher demonstration > individual manipulation for exponents and bases
Wallace	1974	materials > traditional for fractions
Wood	1974	NSD between concrete and pictorial for multiplication

TABLE 7

RESULTS OF STUDIES IN GRADES 5 AND 6

Author	Date	Results
Beardslee	1973	NSD on number of multiple embodiments for fractions on generalization tests
Bisio	1971	NSD between pupil use and teacher demonstration, but teacher use > no use of materials for fractions
Bohan	1971	diagrams or paper-folding > abstract for equivalent fractions; NSD for operations with fractions
Branch	1974	individual use > teacher demonstration for integers
Bring	1972	concrete > no materials for geometry
Carmody	1971	concrete > symbolic on transfer test on numeration concepts; pictorial > symbolic on two other transfer tests and immediate test
Denman	1975	NSD between concrete, pictorial, and symbolic for remedial addition and subtraction
Gau	1973	NSD on number of multiple embodiments for fractions on transfer tests
Green, G.	1970	NSD between materials and diagrams for fractions
Johnson, Robert	1971	physical models and instruments + text > text without models or text without drawings for perimeter, area, and volume concepts
St. Martin	1975	NSD between concrete-pictorial-abstract sequence and pictorial-abstract sequence for fractions
Wallace	1974	materials > traditional for fractions

was better than a symbolic treatment in 2 others (Bohan, 1971; Carmody, 1971). No significant difference between modes was reported by 2 studies (Denman, 1975; G. Green, 1970), as well as by one cited above on a second measure (Bohan, 1971). [Other studies focused on type of use (Bisio, 1971; Branch, 1974), sequences (St. Martin, 1975, or embodiments (Beardslee, 1973; Gau, 1973).] That beneficial effects may accrue from the use of concrete materials at these grade levels is suggested by these results.

Two of the 8 studies at the seventh- and eighth-grade levels (see Table 8) favor the use of concrete materials (Bledsoe et al., 1974: Shoecraft, 1972), one reported "no negative effects" from using concrete materials (Rich, 1972), while one favored the use of an abstract approach (J. Smith, 1974). No significant difference between modes was reported by 2 (Anderson, 1958; McMillian, 1973). [One study focused on type of use (Jamison, 1963, 1964) and one on sequence (Olley, 1974).]. While the total number of studies limits the generalization, nevertheless these studies provide some evidence that the use of concrete materials can be effective in upper grade levels.

The variety of approaches, topics, and designs involved when studies are grouped by grade level makes interpretation of age trends very difficult. We believe, however, that the studies do not support the notion that activity lessons with manipulative materials are important at early elementary-school levels, but not at upper elementary-school levels. Studies at every grade level support the importance of manipulative activity lessons -- and more studies support the use of materials than use of only pictorial or abstract procedures.

TABLE 8

RESULTS OF STUDIES IN GRADES 7 AND 8

Author	Date	Results
Anderson	1958	NSD between materials and no materials for geometric concepts
Bledsoe et al.	1974	materials > paper-and-pencil units for fractions and decimals
Jamison	1963, 1964	NSD between individual use and teacher demonstration for numeration
McMillian	1973	NSD between use or non-use of physical model for numeration
Olley	1974	concrete-to-abstract sequences > pictorial-abstract or abstract sequences on transfer test on numeration concepts; NSD on retention
Rich	1972	multi-base blocks + Cuisenaire rods did not negatively affect achievement on fractions
Shoecraft	1972	concrete or abstract > pictorial for number, coin, and age problems; concrete > pictorial for work and mixture problems
Smith, J.	1974	expository > unimodel or multimodel approach for area concepts

Mathematics Content

Is a manipulative materials approach more appropriate for some mathematics concepts than for others? To answer this question we grouped studies according to the mathematics topic involved, and examined the outcomes. A few studies [e.g., Branch (1974) and Toney (1968)] dealt with so broad a range of content that they could not be placed on any table. The remaining studies have been grouped into six broad content areas: numeration, addition-subtraction, multiplication-division, fractions, geometry, and verbal problem solving.

Nine studies on various numeration topics are included on Table 9. Of 5 on number bases, use of concrete or pictorial materials was favored on one (Carmody, 1971), no significant difference was noted on one (McMillian, 1973), while 2 others focused on type of use also reported no significant difference (Jamison, 1963, 1964; Trueblood, 1968, 1970), and one compared two types of materials (Muckey, 1971). Of the remaining studies on numeration, 2 favored use of concrete or pictorial materials (Curry, 1971; Earhart, 1964), while one focused on type of use (Knaupp, 1971, 1972) and one on sequeces (Olley, 1974). The variance in focus is too great to enable any firm conclusion from this set of studies on numeration topics.

Studies which dealt with addition and subtraction are listed on Table 10. These studies can be divided into two categories: those which dealt with the use of materials in problem solving (all in grade 1) and those which dealt with algorithms involving regrouping, carrying, and borrowing (in grades 2 and 3). The 4 studies involving problems (Bolduc, 1970; LeBlanc, 1968; Steffe, 1968, 1970; Steffe and Johnson, 1970) are also included in Table 14; all provide some evidence indicating that the use of materials is helpful in problem solving involving addition and subtraction.

TABLE 9

RESULTS OF STUDIES ON NUMERATION TOPICS

Author	Date	Results
Carmody	1971	concrete > symbolic on transfer test on number bases, properties of odd and even numbers, and divisibility; pictorial symbolic on two other transfer tests and on immediate achievement in grade 6
Curry	1971	concrete or pictures > verbal for clock arithmetic in grade 3
Earhart	1964	abacus > workbook, for varied numeration topics in grades 1-3
Jamison	1963, 1964	NSD between individual use of abacus and teacher demonstration for number bases in grade 7
Knaupp	1971, 1972	NSD between individual use of Dienes blocks or sticks and teacher demonstration for numeration concepts in grade 2
McMillian	1973	NSD between use or non-use of physical model for number bases in grade 7
Muckey	1971	Dienes = non-Dienes materials at high SES level; non-Dienes slightly favored at middle SES level for study of number bases in grade 2
Olley	1974	concrete-to-abstract sequences > pictorial-abstract or abstract sequences on transfer test on numeration concepts related to even/odd, zero, and one or permutations; NSD on retention test in grades 3 and 7
Trueblood	1968, 1970	teacher demonstration > individual manipulation for exponents and number bases in grade 4

TABLE 10

RESULTS OF STUDIES ON ADDITION AND SUBTRACTION TOPICS

Author	Date	Results
Bolduc	1970	visual aids > no aids on addition problems in grade 1
Denman	1975	NSD between concrete, pictorial, and symbolic for remedial addition and subtraction in grades 5 and 6
Ekman	1967	concrete > pictorial or abstract for addition and subtraction in grade 3
Gibb	1956	pictorial > concrete > abstract for subtraction in grade 2
Gilbert	1975	individual manipulation > teacher demonstration for addition and subtraction in one school; NSD in another school (grade 3)
LeBlanc	1968	concrete = pictorial > symbolic on subtraction problems in grade 1
Sole	1957	NSD on number of embodiments in grade 3
Steffe	1968, 1970	concrete = pictorial > symbolic on addition problems in grade 1
Steffe/Johnson	1970	concrete > no materials for addition problems in grade 1
Wheeler	1972	significant correlations between number of embodiments and achievement on multi-digit addition and subtraction in grade 2

The remaining studies vary in focus; the findings of the three comparing modes (Denman, 1975; Ekman, 1967; Gibb, 1956) reveal discrepant findings. Thus the evidence on the effect of the use of manipulative materials in teaching addition and subtraction is somewhat limited, but made stronger by the positive effect on addition and subtraction problems.

Seven studies dealt with the use of manipulative materials to teach multiplication (see Table 11). These all concern introduction of the multiplication concept in the primary grades. Three favored the use of concrete and/or pictorial materials (Babb, 1976; Nichols, 1972; Norman, 1955); one favored the use of the symbolic mode (Fennema, 1970), and 3 reported no significant difference between modes (Scott and Neufeld, 1976; Trask, 1973; Wood, 1973). [One (.ann, 1974) focused on sequence.] Thus the use of concrete materials when introducing multiplication seems rather effective.

The 11 studies listed on Table 12 looked at the use of manipulative materials to teach various fraction concepts. Use of the concrete and/or pictorial mode was favored on 4 (Bledsoe et al., 1974; Bohan, 1971; Brown, 1973; Wallace, 1974); one reported "no negative effects" (Rich, 1972); one favored the abstract mode (Carney, 1973); and no significant differences were reported by 2 (Bohan, 1971, also cited above for another measure; G. Green, 1970). [Two others focused on embodiments (Beardslee, 1973; Gau, 1973), one on type of use (Bisio, 1971), and one on sequence (St. Martin, 1975)]. Again, the use of concrete materials in teaching fractional concepts seems rather effective.

Table 13 lists 5 studies concerned with the teaching of geometric topics. These studies span grades 3 through 8, and are very diverse in the specific geometric concepts which they include. They are also diverse

TABLE 11

RESULTS OF STUDIES ON MULTIPLICATION AND DIVISION TOPICS

Author	Date	Results
Babb	1976	manipulative materials > imagery or textbooks for multiplication facts in grade 2
Fennema	1970	symbolic > concrete on transfer test for for multiplication in grade 2
Nichols	1972	materials with discovery > traditional exposition for multiplication and division in grade 3
Norman	1955	concrete + pictorial > text or conventional on transfer/retention test; NSD on immediate achievement test for division in grade 3
Punn	1974	concrete-symbolic > concrete-pictorial-symbolic pictorial-symbolic sequences for multiplication in grade 3
Scott/Neufeld	1976	NSD between concrete, pictorial, symbolic for multiplication in grade 2
Trask	1973	NSD between materials and traditional lessons for multiplication in grade 3
Wood	1974	NSD between concrete and pictorial for multiplication in grades 1-3

TABLE 12

RESULTS OF STUDIES ON FRACTION TOPICS

Author	Date	Results
Beardslee	1973	NSD on number of embodiments on generalization tests in grades 5 and 6
Bledsoe et al.	1974	materials > paper-and-pencil units for fractions and decimals in grade 7
Bisio	1971	NSD between pupil use and teacher demonstration, but teacher use > no use of materials in grade 5
Bohan	1971	diagrams or paper-folding > abstract for equivalent fractions; NSD for operations with fractions in grade 5
Brown	1973	text + manipulatives + film > other combinations in grade 4
Carney	1973	abstract > objects + number line for addition and subtraction with rational numbers in grade 4
Gau	1973	NSD on number of embodiments on transfer tests in grades 5 and 6
Green, G.	1970	NSD between materials and diagrams in grade 5
Rich	1972	multi-base blocks + Cuisenaire rods did not negatively affect achievement in grade 7
St. Martin	1975	NSD between concrete-pictorial-abstract sequence and pictorial-abstract sequence in grade 5
Wallace	1974	materials > traditional in grades 4-6

TABLE 13

RESULTS OF STUDIES ON GEOMETRIC TOPICS

Author	Date	Results
Anderson	1958	NSD between materials and no materials in grade 8
Bring	1972	concrete > no materials in grades 5 and 6
Johnson, Robert	1971	physical models and instruments + text > text without models or test without drawings in grades 4-6
Prigge	1974	teacher demonstration and student manipulation of solids paper and pencil or paper-folding, geoboard activities
Smith, J.	1974	expository > unimodel or multimodel approaches in grade 7

in findings, and no firm conclusion can be reached.

Six studies investigated the use of materials in mathematical problem-solving situations (see Table 14). Although these studies span grades 1, 4, and 7, they all agree that the use of manipulative materials or visual aids is an advantage in problem solving. Perhaps this finding is not too surprising. Since most problem-solving involves first understanding the situation in which the problem is embedded, representing that situation with objects or pictures, should obviously be helpful. Perhaps what is surprising is that the use of physical materials in problem solving in elementary-school mathematics is not more widespread.

Our initial analysis of studies on manipulative materials indicated that the use of manipulative materials either increased achievement in mathematics, or provided achievement as good as that in lessons with which materials were not used. This analysis of studies according to the type of content involved does not change that conclusion; rather, it adds some detail to the picture. While no firm conclusions can be drawn from the studies on numeration or geometry, the other topics (addition and subtraction, multiplication, fractions, and problem solving) are generally positive toward the use of manipulative materials. We have suggested that this is entirely plausible for problem solving. It also seems quite reasonable for the other topics, particularly at the introductory points for new content. But as concepts become increasingly difficult, more prior learning must be considered as studies are designed. We shall now turn to an exploration on procedures, considering four of the cited studies on multiplication as examples.

TABLE 14

RESULTS OF STUDIES ON PROBLEM SOLVING

Author	Date	Results
Bolduc	1970	visual aids > no aids with addition problems in grade 1
LeBlanc	1968	concrete = pictorial > symbolic on subtraction problems in grade 1
Nickel	1971	materials + pictures > verbal approach in grade 4
Shoecraft	1972	concrete or abstract > pictorial for number, coin, and age problems in grade 7; concrete pictorial for work and mixture problems
Steffe	1968, 1970	concrete = pictorial > symbolic on addition problems in grade 1
Steffe/Johnson	1970	concrete > no materials for addition problems in grade 1

for the addition-subtraction category. These topics are taught at a time when the child is first coping with symbolism, and materials which remove symbolism difficulties should be expected to produce greater achievement. Perhaps, however, the fact that manipulative materials provide less relative advantage for multiplication, fractions, and geometry is a function of the fact that these concepts become increasingly difficult, and more prior learning must be considered as studies are designed. We shall now turn to an exploration on procedures, considering four of the cited studies on multiplication as examples.

Multiplication Revisited: The Interaction of Activities and Content

Sometimes the use of manipulative materials aids in the learning of mathematics content; sometimes it does not. To try to gain insight into reasons that might underlie these differences we will examine the treatments used in five studies involving the teaching of multiplication in grades 2 and 3. As we have seen from Table 11, the results of studies in this area are varied.

One of the most important studies on multiplication and activity learning is that of Fennema (1970, 1972). In a carefully controlled study, second-graders were taught multiplication products less than or equal to 10 by either a meaningful symbolic method or by a concrete manipulative method. In both cases, multiplication was taught as a mapping of an ordered pair of numbers onto a single number. The multiplication fact "two times three equals six" was written as 2,3-->6. The method for justifying this mapping was varied according to treatment. The justification used in the symbolic treatment was that of repeated addition: 2,3 maps onto 6 because 3+3=6. In the concrete treatment, the justification was made using Cuisenaire rods: 2,3 maps onto 6 because 2 rods with a length of 3 when placed end to end are as long as a rod with a length of 6.

Six classes involving 148 second-graders from the elementary school in Oregon, Wisconsin were used in the study. Children from each of six classrooms were randomly divided into two groups for the two treatment lessons. The unit of analysis used in the study was classes (6). Four outcome measures were obtained. The first of these, an immediate recall test of the multiplication facts taught, showed no significant difference between the two treatment groups. A second test (Symbolic Transfer I) assessed students on multiplication facts whose products ranged from 11 to 16 (facts which had not been specifically taught by either treatment). No significant difference was found on this test, although trends favored the symbolic treatment group slightly. A third test (Symbolic Transfer II) was given one week after the second test. This test was identical to Symbolic Transfer Test I, except that all students were provided with discrete counters to use. A significant difference at the .03 level favored the symbolic treatment group on this test. Finally, a concrete transfer test was given, in which children were asked to use multiplication facts to balance a balance beam. No significant differences were found on this fourth test.

The finding of a significant difference on the third test is particularly puzzling since that test can be interpreted as measuring at least two different outcomes. Because it was given seven days after the original symbolic transfer test, it can be interpreted as a retention test, leading one to conclude that the symbolic treatment provided greater retention of learning. However, this test was unique in that all students were provided with a manipulative aid which they had not used before, namely counters. These counters were all identical. Students in the concrete treatment had previously used Cuisenaire rods. But Cuisenaire rods are not all identical: they vary by length, with different lengths representing different numbers. It seems just as plausible to explain the differences on this test as being due to interference between the manipulative aid provided and the manipulative aid used by the concrete group. (Since the symbolic group used no aids they should have experienced no interference, and therefore have been at an advantage, as the results indicate.) Unfortunately, there is no evidence in the study to indicate which of the two interpretations is correct.

The conjecture that performance is affected by the nature of the manipulative aid used may also provide an explanation for the slightly poorer showing of the concrete treatment on the first symbolic transfer test. On this test the concrete group was provided with Cuisenaire rods as well as pencil and paper. This test (as did the second symbolic transfer test) involved multiplication products ranging from 11 to 16. The largest Cuisenaire rod has a length of 10. In the original lessons (which involved products less than or equal to 10), any required answer could be represented by a single rod. But in the transfer test, no answer could be represented by a single rod. To find an answer to 3,4→ _?_, three four-rods should be placed end-to-end. These are longer than a ten-rod. One must then find another rod to add to the ten-rod to construct an equivalent length. Technically this calls for a new procedure not taught in the original lessons. Thus the symbolic transfer test allows children in the symbolic treatment to transfer their procedure to new problems, but requires children in the concrete treatment to invent a new step to extend their procedure to apply to the new problems. Whether or not this extension is trivial is certainly open to question. It seems plausible that it might account for the slight advantage noted for children in the symbolic treatment.

It might also account for part of the disadvantage shown by children from the concrete treatment on Symbolic Transfer Test II. As a consequence of Symbolic Transfer Test I, the successful concrete-treatment children learned to apply one process to products less than or equal to ten, and a slightly different process to products greater than ten. Yet on Symbolic Transfer Test II they were provided with a manipulative aid (discrete counters) which allowed a single process to be applied to all problems.

Perhaps the only conclusion that can be drawn from Fennema's study is that the use of Cuisenaire rods to teach multiplication raises potential problems! Comparisons of Cuisenaire programs with other elementary-school programs show that the Cuisenaire programs lose their advantage by grade 3 (see Table 19). The introduction of multiplication as an important topic in grade 3 makes it possible that earlier advantages could be canceled out by difficulties encountered in teaching multiplication with Cuisenaire rods.

Are there similar difficulties in teaching multiplication with discrete objects such as counters? The study by Nichols (1972) seems to indicate that this is not the case. Her Treatment A used 5/16-inch cadmium-plated metal nuts with felt mats marked into patterns to correspond with number combinations and booklets for the recording of answers. Treatment B used semi-concrete materials such as pictures, drawings, flannelboards, and other materials of a non-manipulative nature. She also characterized Treatment A as one involving pupil discovery, while Treatment B used teacher explanation and exposition. In Treatment A children worked in pairs according to assignment by the teachers. Treatment A utilized specific language for the multiplication and division signs. These pupils were taught to read "x" as "sets of" or "groups of". Thus 2 x 3 is read as "2 sets of 3". The division symbol ÷ was read as "has how many sets (groups) of". Thus 6 ÷ 2 was read as "6 has how many sets of 2". No mention was made of similar vocabulary treatment in Treatment B.

Both treatments involved fifteen 45-minute periods of instruction on the multiplication and division combinations with 6, 7, and 8. A total of 267 third-grade pupils and ten teachers from six schools in the LaMesa-Spring Valley (California) schools were involved. Results strongly favored Treatment A pupils. They raised scores on a multiplication test from 45 per cent before the lessons to 85 per cent after the lessons, while Treatment B pupils were only able to increase scores from 42 per cent to 54 per cent. Gains on a division test were even more spectacular for Treatment A pupils, rising from 28 per cent to 74 per cent as compared with a gain from 33 per cent to 43 per cent for Treatment B pupils.

Unfortunately, it is impossible to credit this success to any single factor. Treatment A differed from Treatment B in its use of manipulative materials. But it also differed in its use of pupil discovery, pupil-pairing, pupil recording of results in booklets, and use of specific and consistent vocabulary for multiplication and division symbols. When mixed together by Nichols, these factors all combined to create a potent treatment.[7] But we know neither the effect of varying the proportion or the emphasis of these factors, or of the mixing process itself. (Nichols mentions that each teacher in Treatment A was supplied with a kit of materials and instructed in their use, but gives no details about this instruction.)

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[7] The reader might be interested in noting that Babb (1976) indicates the use of Nichols' manipulative treatment, but does not attain the strength of her results. However, it is unclear just how closely Babb replicated the procedures, beyond the use of cadmium-plated nuts . . .

We can conclude that a treatment involving counters can be very effective in teaching multiplication and division facts, but we really have no insight into what makes it effective. (Could it be because the metal nuts were cadmium-plated?)

Punn (1974) also found an advantage for using manipulative materials to teach multiplication in grade 3. His first treatment involved a wide variety of manipulative materials such as number lines, arrays, grids, chips, bottle caps, pegboards, geoboards, and matched sets of blocks and cut-out felt objects. Treatment 2 used pictures, charts, and transparencies. A third treatment used both manipulative materials and pictorial representations. Ninety pupils from three schools in the Cherry Creek School District in Colorado were involved. Three randomized samples were drawn from the schools, and each received one treatment for a period of nine weeks (with 30 to 40 minutes of instruction per day). Three different teachers were involved, and each had a teacher aide. The criterion test was an investigator-constructed multiplication usage test including 20 missing-factor problems and ten word problems involving multiplication. The first and third treatments (involving manipulative materials) were superior to the second treatment (involving only pictorial representation) at the .05 level of significance.

In contrast, Trask (1973) was not able to show any advantage for the use of manipulative materials in teaching multiplication at grade 3. He was concerned about keeping treatment times equal over the 12-week period of the study, and because the non-manipulative treatment group was proceeding at a faster pace, he provided each pupil in that group with a set of flashcards. The manipulative treatment group did not receive similar drill and practice. Trask followed the textbook closely with both treatment groups. He used the manipulative materials to illustrate textbook situations, and in so doing used concrete devices as representations of other concrete devices. Thus "pebbles were used to represent keys, pennies, or shoes; pegs on a pegboard often represented fruit trees or basketball players" (pp. 32-33). This concrete representation of a thought-image seems to be a more abstract way to use manipulatives, and clearly sets Trask's study apart from Nichols' or Punn's. Whether Trask's different findings are due to this distinction or to the different emphasis on drill and practice can only be a matter of sheer speculation.

What then can one conclude from these studies? In the course of this discussion, we have probably raised many questions in the reader's mind about the credibility of their findings. The reader is urged to be aware that questions can be raised about (almost) every single study ever done, when one delves into them to a greater or lesser degree. Yet we can also learn something from (almost) every single study ever done. We have in this review by and large attempted to stand back in order to glimpse the meaning of the picture, rather than staring at the dots. The look at the dots of these four studies was done to illustrate the point (in case the reader was unaware!) that research is a <u>very</u> complicated process. Every single detail has potential implications, from the choice of material to the choice of test. These researchers passed a first test of their own, however: they studied important questions. They raised other questions that future research should clarify.

<u>Attitudes: The Forlorn Variable</u>

Up to this point we have focused on studies which considered how well students achieve when manipulative materials are used. Now let us consider (briefly) the relatively few studies in which attitudes have been of concern. Table 15 includes nine studies in which attitudes were assessed. Because of the varied comparisons, a firm conclusion seems to use to be unwarranted. At most we note that the use of manipulative materials and pictorial aids may be better than the use of symbols (or no materials) in promoting more positive attitudes.

Analyses of research which have attempted to discern the relationship between attitude and achievement have indicated that at best the correlation is positive but low (Suydam and Weaver, 1975). Nevertheless, there is a popular belief that a stronger relationship between the two exists; it has been suggested that perhaps the instruments and scales used for measuring attitudes are not as effective as we had once assumed them to be. This comment should be kept in mind as the findings on the table are noted.

Summary of Research on Materials

What can we conclude about the use of manipulative materials? Embedded one by one in the previous sections are these points:

Manipulative Materials: Yes or No? Does the use of manipulative materials help student achievement in mathematics? In almost half (11/23) of the considered studies (see Table 2), use of manipulative materials was favored; that is, students having instruction in which manipulative materials were used scored significantly higher on achievement tests than students who had instruction in which manipulative materials were not used. In 10 additional studies, no significant differences were found; that is, the group using manipulative materials and the group not using manipulative materials scored much the same. Therefore, in a simple manipulative vs. non-manipulative comparison, non-manipulative lessons cannot be expected

TABLE 15

RESULTS OF STUDIES OF ATTITUDE

Author	Date	Findings
Anderson	1958	NSD between multisensory aids or none
Babb	1976	manipulative non-manipulative
Bohan	1971	paper-folding group > abstract
Green, G.	1970	diagrams favored more than materials
Harshman	1962	NSD between three types of material
Knaupp	1971, 1972	NSD between use of Dienes blocks or sticks
Nichols	1972	manipulative > non-manipulative
Punn	1974	concrete-symbolic sequence > concrete-pictorial-symbolic > pictorial-symbolic
Scott/Neufeld	1976	manipulative and pictorial > abstract

to produce superior achievement. Lessons using manipulative materials have a higher probability of producing greater mathematics achievement than do non-manipulative lessons.

Concrete, Pictorial, and Symbolic Sequences: Which? When? Does the sequence of use from concrete to abstract make a difference? Symbolic treatments are at a disadvantage when used alone (see Table 3); only 3 of 28 findings favored the use of symbols alone. So are pictorial treatments at a disadvantage, as the fact that only one study favored them suggests. In 7 instances use of concrete manipulative materials was favored over sequences in which manipulative materials were not used. In 9 instances, use of manipulative materials and pictorial representations resulted in higher achievement than use of symbols alone. Pictures and diagrams can be important in designing mathematics lessons, especially when used in conjunction with manipulative materials. In 8 instances, no significant differences were found; that is, all treatments resulted in equivalent achievement.

Do different sequences of concrete, pictorial, and symbolic treatments produce differential effects on achievement? The evidence of three studies shows:

(1) concrete-symbolic > pictorial-symbolic or symbolic

(2) concrete-symbolic > concrete-pictorial-symbolic > pictorial-symbolic

(3) concrete-pictorial NSD pictorial-symbolic

Thus again the concrete stage appears to have an important role.

Embodiments: One or Many? Do the number of different physical embodiments or representations of the mathematical concept make a difference? Research in which the number of embodiments for a mathematical idea has been the focus has resulted in no significant differences in achievement in 3 of 4 studies (see pages 25-28). It may be that the reflective and retroactive thinking required is too difficult at the age levels tested. It may be that the teaching did not clearly focus children's attention on the idea behind the use of various embodiments. On the other hand, it may be that it really doesn't make any difference how many embodiments are used, providing the teaching, whether on one or many, is effective.

Thus, a program should not be selected only because it uses multiple embodiments for each or many mathematical ideas. Varying embodiments may aid children in making mathematical applications, but no studies have measured this specific achievement.

Who Shall Manipulate? Does it make a difference whether the student handles materials or only watches the teacher demonstrate with materials? As might be expected on the basis of evidence from Tables 1 and 2, student manipulation was favored -- but only in 3 of 8 studies. In 4 others, no significant differences were found. This is somewhat surprising to many. It appears that individual manipulation by the learner is not the only way children learn: it can be as effective to watch the teacher demonstrate. The only study favoring teacher demonstration was one in which the teacher strongly focused learners' attention, asking them to "think along" in vicarious participation. More research is needed in this area before a conclusion can be drawn firmly. For the present, it appears that effective lessons can be designed even if monetary or space factors preclude the use of materials directly by children -- though this should not be taken to mean that all teaching/learning should proceed without the use of materials (see previous sections of this summary!).

Type and Age of Learner. Do age or other characteristics of the learner affect the outcome of treatments involving materials? In grades

1 and 2, 7 studies favored the use of materials over abstract modes, with no significant differences in 3 cases (see Table 5), and only 2 instances in which the concrete mode was less effective than a pictorial or a symbolic treatment. In grades 3 and 4, 5 studies favored the use of materials, and in 4 other reports the use of concrete and pictorial modes was favored. Three studies reported no significant difference, while one study favored an abstract treatment (see Table 6). In grades 5 and 6, three studies favored the use of concrete materials, two favored either concrete or abstract treatments, with no significant differences characterizing three others (see Table 7). In grades 7 and 8, two studies favored the use of materials, two reported no significant differences, and one favored an abstract approach.

Thus, across a variety of mathematical topics, studies at every grade level support the importance of the use of manipulative materials. Additional studies support the use of both materials and pictures. We can find little conclusive evidence that manipulative materials are effective only at lower grade levels. The use of an activity approach involving manipulative materials appears to be of importance for all levels of the elementary school.

The use of materials appears to be as effective at one achievement level as at another -- that is, high achievers profit from the use of materials as much as low achievers do, at least below grade 6. One study indicates that using materials in grade 7 was particularly effective for low achievers. (For evidence on this and the following studies, see pages 30-32.)

The use of materials appears to be as effective at one ability (IQ) level as at another -- that is, those of high ability profit from the use of materials as much as those of low ability do. One caution, however, is that low-ability students must be able to see the reason for using the materials and understand how they are to be used -- but that caution is true for children at all other ability levels, too!

Although data are sparse, the use of materials appears to be at least as effective at one socioeconomic level as at another -- providing the materials are not Cuisenaire rods! (Two studies with these indicated that achievement increased as socioeconomic level increased. Why this might be a factor found with the use of the rods is unclear.) There is some indication -- a trend rather than a significant difference -- that children from low socioeconomic levels find manipulative materials particularly helpful.

Special programs for children from low socioeconomic levels -- or of low ability -- or low in achievement -- are frequently reported to be successful. (So are special programs for those from high- or middle-level pupils!) When someone takes care to consider the needs of a particular group of children, they respond -- generally favorably! The program may be "exportable" -- providing there are those in another community who will shape it to fit the needs of <u>their</u> children. One of the joys in reading project reports is that sometimes the enthusiasm is transmitted. One of the despairs is the lack of specific information in so many, so that one cannot try out the idea elsewhere. (Just as research should be replicated, so should effective projects!)

<u>Mathematics Content</u>. Do some manipulative treatments work better for some mathematics concepts or content than for others? While no firm conclusion can be drawn for studies on numeration (see Table 9) or geometry (see Table 13), studies on the other topics (see Tables 10, 11,

12, and 14 for addition-subtraction, multiplication, fractions, and problem solving) are generally positive toward the use of manipulative materials.

Attitudes: The Forlorn Variable. Because of the paucity of evidence (considering how rarely attitude was assessed in relation to the total number of studies), a firm conclusion on how children like mathematics or like using materials is unwarranted. At most, we can note that the use of manipulative materials and pictorial aids may be better than the use of symbols (or no materials) in promoting more positive attitudes.

Note: We had originally hoped that we could make some conclusions about types of materials which seem particularly effective. A wide variety of materials — egg cartons, bottle caps, markers, geoboards, etc. -- were used in these studies. Except for one study reporting that inexpensive materials were as effective as expensive ones, we were unable to find any more specific trends.

IV. Research on Activity Programs and Modes of Instruction

In the introduction to this review, activity programs and activities embedded in a mode of instruction were discussed. The key point emphasized was that programs termed "activity-based" vary widely. Given that as a problem in reviewing studies, we have nevertheless grouped in this chapter research (and, in some cases, field-study evidence) under several topics. Various activity-oriented programs are discussed first, leading into a discussion of studies on games. Then current developmental projects are discussed, followed by discussion of research on programs based on the use of Cuisenaire rods. Finally, the focus is on the mathematics laboratory. It should be noted that in many specific studies there is no clear distinction between activity-oriented programs and laboratories: what one researcher labels a "mathematics laboratory" another identifies as "activity-oriented instruction". We have separated them in terms of the label the researcher used, but recognize that the findings cannot be considered discretely.

Activity-Oriented Programs

During the 1930s and 1940s, many studies were conducted to evaluate the effect of then-current activity programs (e.g., Harap and Barrett, 1937; Harap and Mapes, 1934, 1936; Harding and Bryant, 1944; Hopkins, 1933; Jersild, 1939; Passehl, 1949; Wrightstone, 1944). In most of these studies, the efficacy of the activity program in question was supported. But the educational scene changed, and the social utility emphasis which had spawned many of these programs waned. It was not until the early 1960s that the term activity program again assumed popularity, as such experimental programs as the Madison Project and the Nuffield Project (the latter an English project) began to have an impact. The National Defense Education Act provided the needed money for schools to spend on materials; research and development dollars were made available for the development of materials; sudden awareness of Piaget and his theory of developmental stages was attained: these and other forces interacted to strengthen once again our belief that children can learn most effectively through being involved in the learning process.

During the past 10 to 12 years many researchers have investigated the effect of activity-oriented programs and units. Table 16 contains a list of 21 such studies. (None of these studies has been included on previous tables; although many involved the use of manipulative materials, the effect of those materials can rarely be determined apart from their interaction with a variety of other instructional materials, devices, and activities.) About half report achievement differences favoring the use of activities; while the other half report no significant differences. The variance in focus is wide; undoubtedly (as in other research) an experimenter-bias effect or a halo effect has crept in; yet the fact remains that, despite questions about quality, the evidence indicates that students using activity-oriented programs or units can be expected to achieve as well as or better than students using programs not emphasizing activities.

The evidence on attitudes is not clear, largely because so few researchers considered it. Whether the belief that children favor the use of activities can be substantiated is a moot question (see also Table 20 citing research on mathematics laboratories for additional findings on the attitude question). In the survey reported by Thomas (1975), students indicated a preference for activities like games, but were less positive about manipulative materials.

TABLE 16

STUDIES ON ACTIVITY-ORIENTED PROGRAMS/UNITS

Author	Date	Grade level	n	Focus	Achievement	Attitude
Activity-Centered Program	1973	elem.		school-developed program	acceptable	
Beal	1973	7-9, 12	30 c	NCTM materials (low achievers)	activities > non-NCTM text on various units	
Becklund	1969	3-5	18 c	activities vs. regular program (geometry)	differences for each on some measures	
Castaneda	1968	1		activities with Mexican-American disadvantaged	activities > textbook (gain scores)	
Dunlap et al.	1971	4	12 s	learning and behavior problems (remediation)	manipulative aids > paper-and-pencil	
Ebeid	1964	7, 8		SMSG texts with or without self-selection	NSD	NSD
Fitzgerald	1965	7,8		self-selection	NSD below 115 IQ, regular better above 115 IQ	
Hall	1967	4,5		summer camp program; concept method	grade 5 > grade 4	positive attitudes in both groups
Hankins	1969	4		disadvantaged	multi-faceted program better	

TABLE 16 (Continued)

Author	Date	Grade level	n	Focus	Achievement	Attitude
Johnson, Randall	1971	7	160 p	activity units	NSD	
Jones	1971, 1972	ages 7-11	6 p	emotionally disturbed	improved with activities	improved with activities
Koch	1973			varied program	NSD	
Lerch	1972	k		activities on number concepts	activities $>$ control	
Macy	1957	4	2 c 56 p	varied program, alternated groups	NSD	
Matthews	1974	7-9	42 c	NCTM materials	some significant increases, but NSD on competencies	
Moody et al.	1971	3		activity vs. rote	NSD	
Polz	1975	ages 7-10	48 p	learning disabled	NSD	
Snyder	1967	7,8		self-selection	NSD (gains greater for self-selection)	
Stanford	1970	7		games, problems, self-selection	significant increase	
Tobin	1974	ages 6-12		mentally retarded	NSD, 6-9 years; materials $>$ non-materials, 9-12 years	
Unkel	1971	elem.	29 p	tutoring under-achievers	significant gains	

A Note on Research on Games

The research explicitly on the use of games has focused on relatively few games, with "Equations" used most frequently (see Table 17). Played at the seventh- and eighth-grade levels, "Equations" appears to result in better attitudes (Allen and Ross, 1974a; DeVries and Edwards, 1972, 1973; Edwards and DeVries, 1972, 1974), better reasoning (Allen and Main, 1973; Edwards et al., 1972), better computation (Allen and Ross (1974b), and better social interaction (DeVries and Edwards, 1974). In only one instance (Henry, 1974) were findings of no significant difference noted between game and non-game users.

Another logic game, WFF'N PROOF, was used in two studies. Bowen (1970) reported greater increases in logic scores in comparison with textbook instruction. Wolff (1970) found that first and second graders using an adapted version did not react well to the cooperative game format unless they were allowed to move at their own pace. With "Equations" at upper levels, team competition and reward were favored (Edwards and DeVries, 1972; Hulten, 1974), though Faris (1971) reported better achievement when the game was played cooperatively rather than competitively.

In two studies involving a number of games, positive results were reported with kindergarteners and first graders (Wynroth, 1970) and with mentally retarded children aged 4 to 10 (Ross, 1970). A card game was effective for aiding in learning factorization (Karlin, 1972), while a division game was not as effective as anticipated (Fishell, 1972).

So what have we learned? Certain games can be used to promote specific learning outcomes. Students like to play games. Older students like to play competitively. The "Equations" game may be used as a vehicle for

TABLE 17

STUDIES ON GAMES

Author	Date	Grade level	n	Game	Finding
Allen/Main	1973	7,8		Equations	game --> better use of logical reasoning
Allen/Ross	1974a	7,8	29 p	Equations	game --> better attitudes
Allen/Ross	1974b	3	10 c 237 p	Equations	game --> better computation
Bowen	1970	4-6	3 c	WFF'N PROOF	game > textbook for logic gain scores
DeVries/Edwards	1972, 1973	7	110 p	Equations	game --> more peer tutoring, positive affective reactions
DeVries/Edwards	1974	7		Equations	game --> reduced race and sex barriers
Edwards/DeVries	1972	7	117 p	Equations	game --> better attitudes; team reward > individual reward
Edwards/DeVries	1974	7	128 p	Equations	game --> positive attitudes
Edwards et al.	1972	7	96 p	Equations	game > conventional instruction for computation, divergent solutions
Fishell	1975	5	8 c	trading game (division)	NSD
Freitag	1974	6	4 p 63 p	function game Bingo	case study: descriptive
Henry	1974	7	9 c 182 p	Equations TacTickle	NSD
Hulten	[illegible]	7	8 c 240 p	Tuf	team competition > individual competition
Karlin	1972	5	8 c	card game (factoring)	game > textbook (factor trees) in learning theorem
Paris	1971	5	302 p	Equations	cooperation (teams) > competition; NSD in attitude

TABLE 17 (Continued)

Author	Date	Grade level	n	Game	Finding
Ross	1970	ages 4-10 (MRs)		"program of games"	games $>$ special class
Wolff	1970	1,2	66 p	WFF'N PROOF On-Sets	preference for games declined; fixed pace, NSD; own pace, cooperation $>$ competition
Wynroth	1970	k,1		25 "competitive games"	game $>$ conventional instruction

research. The research appears to focus on justifying the inclusion of commercial games in the curriculum. Little attention has focused on games for most basic topics. Taken as a whole, this set of studies is disappointing. Not yet explored are such questions as what makes a game effective and for whom and when are games effective.

DMP and Other Programs

Several large-scale developmental projects are currently underway, producing new curriculum materials. One that incorporates a belief that the use of activities and materials is essential to learning is the Developing Mathematical Processes project of the Wisconsin Research and Development Center for Cognitive Learning. DMP is based on a measurement approach to elementary-school mathematics, rather than a counting approach. As the developers noted, "If you're basing a program on measurement, the child must do things." Other strong purposes of DMP are to teach children to solve problems and to teach them to work together, interacting about mathematics.

While the theories of Piaget, Bruner, and Gagné can be cited to support the developmental work of the curriculum project, the actual approach is far more pragmatic. Previous research was done with a different curriculum, so the findings are not all useful, noted one DMP-developer. The manipulative and other materials used in the program have been evaluated in terms of their usefulness in developing several concepts (that is, their variability); on their cost, safety, and durability; on the way children react -- as well as whether they lend themselves to the mathematics to be taught.

DMP has had research associated with various factors (e.g., questions of sequence or of alternative algorithms) related to its development, but few summative evaluation studies have thus far been conducted -- or, perhaps, not only conducted but also published in some form. This is not surprising, since at the time of this writing, the sixth-grade program was still being developed! Various external field tests (e.g., Hubbard and Buchanan, 1972; Schall et al., 1974, 1975) indicate that pupils in the primary grades achieve successfully with DMP. Teachers have reported that they have had some difficulty in adjusting to the program, but their reactions to it are favorable. (Perhaps the strong in-service structure of workshops which are associated with adoption of the program aid in promoting their positive attitudes.)

Other programs involving activities are listed on Table 18. Two (Clausen, 1972; Plummer, 1972) studied a Montessori program. This program involves a highly structured approach to the use of sequential materials, developed by Maria Montessori. The manner and the sequence in which materials are to be used are precisely specified. Clausen reported significant differences favoring the use of the program with children in kindergarten and grade 1. (It should be noted that there are many variations of Montessori programs; not all use the materials, or use them in the specified way.)

While many other investigations of the varied activity programs being used in this country have been conducted, most of these have not been included in our final set of studies. In a great number of instances, the effect of the activities simply could not be determined, because of the confounding variables. In others, no data were provided in the reports.

Cuisenaire Program

Much research has been focused on the use of Cuisenaire materials and the Cuisenaire program, in attempts to answer the question, "How effective is it". The Cuisenaire rods have been used as a material in some

TABLE 18

STUDIES ON OTHER ACTIVITY PROGRAMS

Author	Date	Program	Grade level	n	Result
Abernatha/Wiles	1975	DMP	ages 7-12 (EMRs)	10 p	appropriate and effective for mentally retarded children
Adamson	1976	DMP, Distar	1		Distar > DMP
Clausen	1972	Montessori	k,1	8 c 177 p	Montessori > worksheet/text approach
Coltharp	1969	GCMP, "abstract"	6	4 c 79 p	NSD between GCMP number line approach and abstract approach
Frary	1967	IMS	elem.		"meeting most of its goals"
Hubbard/ Buchanan	1971	DMP	2	8 s	NSD between DMP or conventional programs (but mastery level of 81% found with DMP)
Krairajananan	1974	USMES	elem.		"effective"
Lucas	1967	GCMP, attribute blocks	1	8 c	some differences favoring each
Plummer	1972	Montessori	1	109 p	NSD between Montessori and non-Montessori
Schall et al.	1974	DMP	1		"encouraging results"
Schall et al.	1975	DMP	k-3		no significant gains found, but program rated favorably by teachers, parents, and pupils

*DMP = Developing Mathematical Processes,
GCMP = Greater Cleveland Mathematics Program
IMS = Individualized Mathematics System
USMES = Unified Science and Mathematics in the Elementary School

studies (e.g., Fennema, 1970, 1972). Here we focus n their use with the Cuisenaire program, which is based on use of the rods in specifically defined ways. The rods are

> all equal in cross dimension [1 cm] but differing in length and color. The rods are calibrated in length to represent the [numbers] 1 to 10 and are identified at first only by color. Thus, the rod for five is called, not the <u>five rod</u>, but the <u>yellow rod</u>. Stress is put on the discovery of relationships rather than on early mastery of the number combinations and early proficiency in computation. [Brownell, 1968, p. 153]

Table 19 indicates the general variance across these studies. Crowder (1966) reported that a group of first graders using the Cuisenaire program (1) learned more conventional subject matter and more mathematical concepts and skills than pupils taught by a conventional program; (2) average and above average pupils profited most from the Cuisenaire program; and (3) socioeconomic status was a significant factor in relation to achievement with the rods.

The first finding is reflected in several other studies (Hollis, 1965a, 1965b; Nasca, 1966; Use of Coloured Rods . . ., 1964). Children learned traditional subject matter with the Cuisenaire program as well as they did with the conventional method, while they acquired additional concepts and skills through the Cuisenaire program beyond the ones taught in the conventional program.

Brownell (1966a, 1966b, 1968) used tests and extensive interviews in an analysis of the effect on underlying though processes of three mathematics programs, with British children who had studied those programs for three years. He concluded that (1) in Scotland, the Cuisenaire program was in general much more effective than the conventional program in developing meaningful mathematical abstractions; and (2) in England, the conventional program had the highest over-all ranking for effectiveness in promoting

TABLE 19

STUDIES ON THE CUISENAIRE PROGRAM

Author	Date	Grade level	n	Result
Brownell	1963, 1967, 1968	age 7	1406 p	Cuisenaire > conventional (Scottish schools); conventional > Cuisenaire, Cuisenaire = Dienes (English schools)
Callahan/ Jacobson	1967	ages 7-10	1 c	Cuisenaire helpful for mentally retarded
Crowder	1966	1	425 p	Cuisenaire > conventional
Dairy	1969	k-2	19 p	Cuisenaire in kindergarten > conventional
Davies/Williams	1972	age 11	36 c	NSD, conventional/Cuisenaire/ multimodel
Fedon	1967	1	2 c 26 p	NSD, Cuisenaire/eclectic
Haynes	1964	3	5 c 106 p	NSD, Cuisenaire/conventional
Hollis	1965a	1	12 c	Cuisenaire > conventional on "modern" test;
Hollis	1965b	1,2	9 c	NSD on traditional test
Lucow	1963, 1964	3	12 c	Cuisenaire > conventional
Nasca	1966	2	2 c	Cuisenaire > conventional on Cuisenaire content; NSD on traditional content
Passy	1963a, 1963b, 1964	3	1800 p	conventional > Cuisenaire
Williams	1972	elem.		conventional > Cuisenaire (first year); also Cuisenaire > Dienes Cuisenaire > conventional (second year); Cuisenaire = Dienes.
Use of Coloured Rods . . .	1964	1,2	5-8 s	Cuisenaire > conventional on Cuisenaire content NSD on traditional content

conceptual maturity, with the Dienes and Cuisenaire programs ranked about equal to each other. Brownell inferred that the quality of teaching was decisive in determining the relative effectiveness of the programs.

Other studies have been concerned with the effect of use of the Cuisenaire program on a particular topic, for shorter periods of time. Lucow (1964) and Haynes (1964) studied use of the program to teach multiplication and division concepts for six weeks in third grade. Lucow attempted to control the effect of prior work in grades 1 and 2. He concluded that the Cuisenaire method was as effective as regular instruction in general, and seemed to operate better in a rural setting, especially with those of high and middle IQ, than in an urban setting. Haynes used pupils who were unfamiliar with the materials; no significant differences in achievement were found between pupils who used the Cuisenaire program and those who did not.

One may note from Table 19 that, of 7 studies at the first- and second-grade levels, all but one favor the Cuisenaire program (although more often on special content tests than on traditional tests). However, the studies at grade 3 do not continue this trend: one favors the Cuisenaire program, one favors a traditional program, and one reports no significant difference. Perhaps the Cuisenaire approach is more effective in grades 1 and 2, with its effectiveness dissipating during grade 3. The earlier discussion of the difficulties of using Cuisenaire rods for multiplication (see pages 49-52) would certainly be consistent with such a conjecture.

Mathematics Laboratories

The mathematics laboratory is one type of activity-based approach, and is the mode of instruction used more frequently to promote the goals of activity-based learning. It must be clearly recalled that the term "laboratory" is used to denote a wide variety of procedures: some of the studies cited on Table 16 (activity-oriented studies) might well have been tagged as mathematics laboratories. In both cases, concrete materials plus a variety of other activities are used. Tables 20 and 21 list studies or approaches which the researchers themselves identified as mathematics laboratories.

On Table 20 are 13 studies in which a laboratory approach was compared with a non-laboratory approach. Although the content varies, it seems evident that at least equivalent achievement can be expected when mathematics laboratories are used, while the attitudes of students using laboratories were comparable to the attitudes of students not using laboratories. The fact that achievement is equivalent occurs even though time was taken from regular instruction; that is, students who were exposed to different content in the mathematics laboratory achieved as well on regular content as students who continued to study regular content; Ropes (1973) and Vance and Kieren (1972) note this specifically. The finding of no significant difference in attitudes is somewhat surprising to many, for it is generally believed that more positive attitudes are promoted by laboratory experiences. When one considers that sometimes only one lesson a week (or only part of a lesson) is spent in a laboratory situation, the finding may not be as obscure. Little note is taken of individual reactions -- whether a child works well in a laboratory setting or whether some might feel more "comfortable" in a setting where limits are controlled and where expectations are fully known has not been detailed by researchers.

Table 21 lists studies in which laboratory approaches were used, but there was no comparison with a non-laboratory approach. In some, the

TABLE 20

STUDIES COMPARING MATHEMATICS LABORATORIES WITH NON-LABORATORY INSTRUCTION

Author	Date	Grade level	n	Focus	Achievement	Attitude
Cohen	1971	7,8	28 p	lab vs. conventional in inner-city special school	conventional had significant increase, but NSD on common items	significant difference on some items
Dunlap	1971	4	147 p 24 c 24 s	lab vs. textbook	NSD on standardized test; on non-standardized test of concepts, lab better; on computation, textbook better	NSD, but lab better in more schools
Hollis	1972	4-6	230 p 2 s	lab vs. non-lab	NSD between pupils; differences significant between schools	NSD in one school; lab favored in other
McLeod	1971	2,4	550 p	lab vs. teacher demonstration vs. no instruction on probability	NSD	
Nowak	1972	4-6		lab vs. non-lab	lab better in grades 5, 6; conventional better in grade 4	
Ropes	1973	2,6	88 p	lab vs. non-lab	NSD	NSD

TABLE 20 (Continued)

Author	Date	Grade level	n	Focus	Achievement	Attitude
Schippert	1965	7	4 c	SMSG texts with student manipulation of models or abstract approach in inner-city schools	significant skill growth favoring lab; still favored 2 years later	NSD
Silbaugh	1972	7	36 c	lab vs. non-lab vs. non-lab in school (but no manipulative materials in any)	lab had favorable effect	
Simpson	1974	7	87 p	lab approach with NCTM book vs. traditional text approach (slow learners)	significant differences favoring lab on 1 of 6 units, favoring non-lab on 2 units	NSD
Smith, E.	1974	6-8	235 p	lab vs. non-lab	NSD (significant correlation between achievement and attitudes for lab group)	NSD
Vance/Kieren	1972	7,8		pairs in lab vs. large-class discovery vs. regular program	NSD, except lab group better on higher level thinking and cumulative achievement	strong preference for lab
Whipple	1972	8	93 p	lab vs. individualized instruction units	lab better than non-lab	

TABLE 20 (Continued)

Author	Date	Grade level	n	Focus	Achievement	Attitude
Wilkinson, J.	1971	6	232 p	lab units with worksheets and manipulative materials vs. lab units with cassette tapes vs. conventional approach	NSD	NSD

TABLE 21

OTHER STUDIES ON MATHEMATICS LABORATORIES

Author	Date	Grade level	n	Focus	Achievement	Attitude
Bronder	1973	elem.		individualized diagnostic units on fractions	increased, but students did not meet criterion	
Gray	1973	elem.	3 s	field study of lab approach	contributed to improved achievement and attitudes	
Hicks	1975	7	120 p	identifying factors related to success with manipulative lab approach	no single factor identified	
Higgins	1969, 1970	8		study of attitudes after instruction in lab		significant difference on 6 scales; no significant relationship between attitude and achievement
Howard	1970	5	12 p	field study of lab (in rural culturally deprived area)	achievement and attitude gains	
Kaiser	1974	jr. hi.	5	teacher-pupil interaction (in inner-city schools)	types of behaviors identified	
McClure	197[illegible]	8	6 c 146 p	pre- and post-lab experiences	effect differed among classes	favorable attitudes

focus is on making finer distinctions -- e.g., Higgins (1969, 1970) on attitudes; Kaiser (1974) on teacher-pupil interactions; and Hicks (1975) on predictive factors.

Several factors have been proposed in attempts to analyze the failure of mathematics laboratories to fare better in research studies. We believe that the wide variations in type of content, type of use, time, and such variables play a role; in many laboratories, non-standard content has been emphasized, rather than interesting and challenging activities more related to the regular program. The laboratory is at times treated as a mere "fun-time", rather than an integral part of the program. There seems little reason to us why laboratories cannot be aimed at promoting both curricular and motivational goals.

Summary of Research on Activity Programs/Modes

What can we conclude about the use of activity programs and strategies? In the previous sections, these points have been made:

Activity-Oriented Programs. About half of the studies reported achievement differences favoring the use of activities, while the other half reported no significant differences. Despite questions about quality, the evidence (see Table 16) indicates that students using activity-oriented programs or units can be expected to achieve as well or better than students using programs not emphasizing activities. The evidence on attitudes about mathematics is again unclear.

Games. Certain games can be used to promote specific learning outcomes. Children like to play games. Older students like to play competitively. The "Equations" game may be used as a vehicle for research (see Table 17). But research has not yet clearly focused on the effectiveness of games for teaching basic elementary-school mathematics topics.

DMP and Other Programs. Field-test data indicate that pupils in the primary grades achieve successfully with the Developing Mathematical Processes program (DMP). Because the use of materials and activities is so integrated in the program, it is not possible to separate the effect of them from the effect of the measurement approach, sequence differences, and a host of other factors.

The same problem exists with other programs: there are simply too many confounding variables.

Cuisenaire Program. Use of the Cuisenai: program has been found to be more effective than use of a conventional program (that is, one in which the Cuisenaire rods were not used) in about half the studies cited (see Table 19). Research has focused on its effect in the primary grades, with no discovered attempt to assess its effect beyond third grade. (Note: the Cuisenaire rods have been used in other studies as a material.)

Mathematics Laboratories. Denoting a wide variety of procedures, the mathematics laboratory usually (but not always) involves use of manipulative materials plus a variety of other activities. At least equivalent achievement can be expected when mathematics laboratories are used (see Table 20). The attitudes of students using laboratories were comparable to the attitudes of students not using laboratories; almost all studies reported no significant differences in attitudes. Laboratories may be a strategy not amenable for use by all students or by all teachers.

Thus, the evidence provided by Shipp and Deer (1960), Shuster and Pigge (1965), and Zahn (1966) that a greater amount (at least 50 per cent) of concentrated developmental conceptual work in lessons has a strong positive effect on achievement and understanding, is strengthened by the findings on activity-oriented programs.

V. Conclusions, Implications, and Suggestions

Conclusions

In the concluding sections for the previous two chapters, the evidence from research focused on the use of manipulative materials and other activities included in activity-oriented programs was summarized. But redundancy seems to be a function of reviews, so the conclusions are succinctly restated here. In each case, the previous summaries should be referred to as the source of documenting evidence.

Manipulative Materials: Yes or No? Lessons using manipulative materials have a higher probability of producing greater mathematical achievement than do non-manipulative lessons.

Concrete, Pictorial, and Symbolic Sequences: Which? When? Symbolic treatments are not as effective as those in which manipulative materials are used. Use of both manipulative materials and pictorial representations is also highly effective. The inclusion of the concrete stage in sequences of instruction -- that is, leading from concrete to pictorial to symbolic -- is highly plausible.

Embodiments: One or Many? Research has generally indicated that there is no difference in achievement whether one or several embodiments of mathematical ideas are presented.

Who Shall Manipulate? It appears to be as effective for children to watch the teacher demonstrate with materials as it is for them to use materials themselves, particularly if their attention is focused and they "think along" with the demonstration. (However, not all lessons should proceed without the use of manipulative materials by students -- see above conclusions!).

Type and Age of Learner. The use of materials appears to be effective with children at all achievement levels, ability levels, and socio-economic levels, although data are sparse. Special programs may, however, produce special effects!

The use of manipulative materials is of some impostance for all grade/age levels in the elementary school.

Mathematics content. The use of materials appears to be effective for addition-subtraction, multiplication, fractions, and problem solving, with no clear conclusion for numeration and geometry.

Attitude. Little research included in this review assessed the attitudes of students in studies on manipulative materials. The use of many materials and pictorial aids may be better than the use of no materials in promoting more positive attitudes.

Activity-Oriented Programs. Students using activity-oriented programs or units can be expected to achieve as well or better than students using programs not emphasizing activities. Attitudinal evidence is unclear.

Games. Certain games (like "Equations") can be used to promote specific learning outcomes. Little research has focused on basic elementary-school mathematics content.

DMP and Other Programs. Developing Mathematical Processes (DMP) appears to be effective in promoting desired achievement. The effect of incorporation of manipulative materials and activities in this and other programs cannot be precisely determined.

Cuisenaire Program. The Cuisenaire program has been found to be more effective than conventional programs in about half the studies cited. (It should be noted that the Cuisenaire rods have been used in other studies as a material.)

Mathematics Laboratories. At least equivalent achievement and attitudes can be expected when mathematics laboratories are used.

The answer to the question, "Should and how should manipulative materials and activities be used with children in compensatory education programs?" is no different from the answer for other children. Care in deciding on how to use them is of importance: the teacher should attempt to focus the attention of the children on the mathematical purpose for using the material or lesson. This may come through most clearly when the lesson proceeds from a problem-solving basis. That is, a situation (or verbal problem) is presented; the children mirror the problem with materials, as they (a) develop a mathematical idea and (b) realize the applicability of mathematics to the real world.

Other Reviews

Thus far in this review, we have ignored other reviews of research. Table 22 lists these, along with other descriptive studies on such topics as historical background and extent of use of materials. Of particular interest are reviews of research on manipulative materials and/or on mathematics laboratories developed by Fennema (1972), Fitzgerald (1972), Kieren (1969, 1971), Vance and Kieren (1971), and Wilkinson (1974). In almost all cases there is similarity between their conclusions and certain of ours.[8] For instance, Vance and Kieren (1971) summarize the results of the seven

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[8] We did not reread their reviews until after the preceding sections of this report were written!

TABLE 22

LIST OF REVIEWS AND OTHER DESCRIPTIVE STUDIES

Reference	Focus
Adkins, 1957	historical study on use of counting aids
Bernard, 1973	historical study on development of laboratory approach
Brousseau, 1973	review of research on mathematics laboratories
Dexter, 1975	guidelines for developing a manipulative material
Dittmer, 1972	guidelines for developing a mathematics laboratory
Eidson, 1956	checklist for role of instructional aids
Fennema, 1972	review of research on models
Fennema, 1973	discussion of findings on manipulative materials
Fitzgerald, 1972	review of research on mathematics laboratories and manipulative materials
Green, R. W., 1970	survey of extent of use of materials
Harvin, 1965	survey of extent of use of materials
Holz, 1972	critique of a study
Kerr, 1974	evaluation of devices used by the blind
Kieren, 1969	review of research on manipulative activity learning
Kieren, 1971	review of research on manipulative activity learning
Lesh, 1974	discussion of research-psychological theory on mathematics laboratories
Lewis, 1970	survey of computational aids for visually handicapped
Reys, 1972	rationale for use of a variety of materials
Snyder, 1976	model for developing a mathematics laboratory
Thomas, 1975	survey on use of games and manipulative materials
Trimmer, 1974	review of research on materials and activities
Vance and Kieren, 1971	review of research on mathematics laboratories
Wilkinson, 1974	review of research on mathematics laboratories

studies on mathematics laboratories which they reviewed (including one with ninth graders) by stating:

> 1. The research indicates that students can learn mathematical ideas from laboratory settings. However, in maximizing achievement on cognitive variables, other meaningful instruction appears to work as well if not better. . . .
>
> 2. One generally held feeling about mathematics laboratories is that they promote better attitudes toward mathematics. There is only limited evidence of this in the careful evaluations of activity-oriented mathematics, although most students seem to prefer laboratory approaches to more class-oriented approaches. . . .
>
> 3. The "gains" made through a laboratory approach appear to be practical. . . .
>
> In summary, the research and evaluation literature suggests that laboratory approaches can be used practically and effectively. However, any effective utilization takes organization. . . . [pp. 588-589]

Fennema (1972), in her review of research on manipulative materials (termed a model in her review) indicates:

> Although the evidence, both theoretical and empirical, appears to indicate that the ratio of concrete to symbolic models used to convey mathematical ideas should reflect the developmental level of the learner, it should not be inferred that either model [concrete or symbolic] can suffice at any level in the elementary school. [p. 638]

Since the set of studies reviewed in this report differs, few exact comparisons can be expected. That there is a need for more research is recognized by all.

Implications for Further Research

We believe that research clearly favors the use of manipulative materials over the non-use of materials when compared on general measures of elementary-school mathematics achievement over a wide range of mathematics topics, grade levels, and classroom settings. When summed together, the research reviewed shows the use of materials to be at least equally effective, and often superior, to the non-use of materials. Cases where non-use of materials is superior are very few, and usually include the presence of confounding variables.

Unfortunately, the implications of research for classroom adoption and use of manipulative materials are not at all clear, despite the optimistic statements above. The difficulty is that, while the existing research can be summed to provide a broad and general picture of effectiveness, it provides practically no specific details about the components or aspects of materials-use that lead to that effectiveness. If every research study concluded that every use of every manipulative material increased student achievement, there would be little need to worry about the details of that use. We could conclude that the mere presence of manipulative materials in a mathematics program would be beneficial. In fact, research results show us to be as far away from this comfortable conclusion as possible. No matter how we classify studies -- by mathematics topic, by type of learner, by age of learner -- we find the number of studies showing no significant advantage for the use of materials to be about equal to the number significantly favoring the use of materials. Our optimism about the effect of materials stems from the absence of studies showing the non-use of materials to be more effective than the use of materials. The crucial question is obvious: what factors made some treatments involving materials so much more effective than others?

As indicated in the section "Multiplication Revisited", we can find no definitive answers to this question in the present body of research. Most studies have been designed to show only whether or not the use of materials can be effective. We believe that this question has been answered, affirmatively. Future research must be designed to isolate and identify those factors which are present in and characterize the effective use of materials. Such factors obviously include the nature of the materials themselves, but also involve their use within the curriculum on both short- and long-term bases.

This effort calls for more sophisticated research design than has been typical of past studies. Most of the studies we reviewed embedded the use of manipulative materials in a broader mathematics program without isolating the wide range of potentially confounding variables. These other variables are seldom clearly identified, let alone measured. On pages 7 and 8 of this report we identify eighteen categories of variables that would seem to have some a priori effect on the success of the use of materials. We anticipate the use of regression analysis techniques ultimately to elaborate the interactions and relative importance of these categories. However, we believe that the lack of specificity and understanding of variables within these categories makes discussion of this research approach premature. We believe that the following seven areas of investigation are simple enough to have more immediate consequence. (Our seven areas, and specific suggestions within each, are meant to be illustrative rather than exhaustive.)

1. Degree of Guidance. Many studies use manipulative materials in an inductive or discovery mode. However, the degree to which this induction is guided by prescribing specific steps and procedures for handling the materials is rarely made explicit. It should be relatively easy to construct one treatment where the desired final goal is generally described but the intermediate steps are not specified and free exploration is encouraged. A contrasting parallel treatment with specific procedures for manipulation of materials given to students at each step could then be compared.

2. Use of Symbols. Symbols of mathematics are an effective way to record the results of various manipulations and configurations of materials. This is obvious to an experienced adult, but it may not be at all obvious to children. Do children need a high degree of guidance and instruction in transferring concrete manipulations to symbolic records? The contrast of (a) a treatment where the child is given detailed and explicit instruction in the use of appropriate symbols with (b) a parallel treatment where children are encouraged to use symbols only as they need and invent them should suggest additional investigations in this area.

3. Role of Social Interaction. A child may manipulate materials in relative isolation, or the materials may pose problems to be discussed among children. Is social interaction beneficial in conjunction with the use of manipulative materials? Is there an optimal size for the interaction group -- for example, do small groups of children learn more effectively from materials than large groups or individuals? The construction of treatments differing only on group size should be relatively easy. Incredibly, we found no studies focusing on manipulative materials in which group size was the primary independent variable.

   Classroom social interaction is also determined by the teaching strategy employed by the teacher. Do different types of teacher questions, or different forms of teacher interaction with student groups, affect the success of manipulative material use? Perhaps initial exploration with materials is better carried out in groups, while the drawing of conclusions and transfer of knowledge to symbols would be done more efficiently by individuals interacting with the teacher. The design of experiments where grouping patterns of children are alternated should also provide interesting data.

4. Sequence Placement of Activities. Does the point at which manipulative materials are introduced in the development of a concept make a difference? For example, is it better to use materials to introduce and define multiplication combinations, or is it better to define multiplication more generally (and abstractly), and later introduce materials to show the applications of multiplication? This question lies at the heart of spirited philosophical arguments between mathematicians and educators, yet the very general approach taken in past research on manipulative materials gives no data relevant to the question.

   Hand-held calculators now provide an abstract, but accessible, way to introduce multiplication. Comparisons on the use of manipulative materials before the calculator is introduced with the use of manipulative materials only after work on the calculator should shed light on this question.

5. Length of Use. Are manipulative materials more effective if used for longer periods of time? Is there a benefit to be gained

from using similar equipment to illustrate different concepts at different points of time in the curriculum? Or should specific materials be tied to specific applications so that a wide variety of differing materials is used for different applications? These and other questions relate to the effect of time on the use of manipulative materials.

6. Training of Teachers. We suspect that some teachers may be more effective in their use of materials than others. Does a thorough knowledge by teachers of the possibilities and application of a set of manipulative materials affect this effectiveness? Most studies mention that teachers were trained to some degree in the use of manipulative materials in the research treatments. But no studies varied the amount of this training as a first step toward measuring a specific teacher effect.

7. Type of Student. The initial purpose of this report was to determine the effectiveness of manipulative materials for students in compensatory education programs. We could find little evidence that materials were either more or less effective for students with learning difficulties or socioeconomic disadvantages. In fact, we found very few studies designed to investigate directly this question, and fewer instances of sufficient data or analysis upon which to draw confident conclusions. The measurement and description of student abilities and characteristics is crucial to this type of study. Because of the complexity of providing enough data to understand the interaction of student characteristics with the use of manipulative materials, the use of a large population with appropriate sampling techniques may be required. Such studies might best be designed by teams of researchers funded for relatively large-scale investigations.

Many mathematics educators believe intuitively in the importance of using manipulative materials in the elementary-school classroom. Some believe that there is no longer any need for further research on the use of manipulative materials. This report tends to support the first position, but strongly repudiates the second. There are too many studies where the use of manipulative materials is "only as good as" regular instruction to believe that we know all that is needed about the use of materials. Our understanding of the details of effective use if shockingly scant. There is an obvious need for new research efforts on the use of materials in activity learning in elementary-school mathematics.

Implications for Classroom Practice

As we developed this report, we generated certain hypotheses about effective classroom practice. Some were developed on the basis of the findings of one or more studies. Unlike the items in the section of this chapter on conclusions, however, they are more conjectural in nature; they may serve as general guidelines for teachers as well as for others.

We believe that lessons involving manipulative materials will produce greater mathematical achievement than will lessons in which manipulative materials are not used if the manipulative materials are used well. What does it mean to use materials well? Although much further research is needed to answer this question, our analysis of effective and ineffective treatments suggests the following points:

1. Manipulative materials should be used frequently in a total mathematics program in a way consistent with the goals of that program.

   For example, the study by Ropes (1973) shows that casual (in this case, once a week) use of manipulative materials in mathematics laboratories is not effective. Simpson (1974) is able to trace weak treatments to units which devoted small percentages of time to manipulative activities. He concluded that for his units (at the seventh-grade level) at least 50 per cent of the lessons should have involved manipulative materials for the units to be effective. It may not be reasonable to expect a fixed percentage of time for all mathematics topics, but the amount of time devoted to manipulative activities should be substantial.

2. Manipulative materials should be used in conjunction with other aids, including pictures, diagrams, textbooks, films, and similar materials.

   An example of a study which supports this is Brown (1973), who found that a program involving manipulative devices together with textbooks and films produced superior achievement when compared to programs using textbooks and films, or manipulative materials and textbooks, or textbooks alone. Several other studies have have found that treatments combining different approaches appears to be more effective than single-focus approaches.

   Manipulative devices are effective in promoting learning, but we must not be lulled into believing that they are the long-sought panacea for solving the problems of mathematics education. Visionaries and prophets periodically arise to espouse the cause of manipulative materials. We should beware of tunnel vision that focuses solely on manipulative materials to the exclusion of everything else.

3. Manipulative materials should be used in ways appropriate to mathematics content, and mathematics content should be adjusted to capitalize on manipulative approaches.

Some algorithms may be effective without manipulative materials, while other alternative algorithms are preferable when used in conjunction with manipulative activities. As hand-held calculators affect the classroom, this matching of algorithm to approach will become even more significant.

4. Manipulative materials should be used in conjunction with exploratory and inductive approaches.

Materials used in arbitrary and rote procedures do not seem to be particularly effective. Nichols (1972) constructed one of the strongest treatments using manipulative materials of any study we analyzed. She consciously built a discovery approach in with the use of materials. Strong treatments in other studies also contain inductive approaches, although they may not be overtly identified as such. If the ultimate purpose of physical manipulative activities is to stimulate mental activities, some degree of freedom seems desirable. It is possible to manipulate objects according to preset, fixed patterns -- without thinking.

5. The simplest possible materials should be used in direct relationship to the mathematical attribute being studied.

The effectiveness of using Cuisenaire rods to teach multiplication seems to be questionable (e.g., see Fennema, 1970). Although the Cuisenaire rods reduce the number of pieces that must be handled, the child must count a number of non-unit rods, convert these rods to trains, then establish a train of tens or unit rods of equivalent length. Although elegant from an adult point of view, this procedure is apparently much more complicated to the child than simply counting objects. Thus, for introducing multiplication, materials like egg cartons and counters seem to be more effective. Simple, direct materials are more effective than elaborate special constructions. Materials should make mathematical procedures easier, not more complex.

6. Manipulative materials should aid in organizing content.

When materials provide organizational schemes children seem to be helped, even with symbolic recall and achievement. For instance, Trask (1973) points out that the number of objects used in introducing multiplication can be overwhelming to children. Successful studies have not only provided counters, but also other aids to help in organizing them and the ideas. Punn (1974) provided array-boards and pegboards in addition to the usual chips and bottle caps commonly used with multiplication. His treatment was effective, whereas Trask's was not. Nichols (1972) provided felt mats and record booklets, and organized her treatments so that children explored multiplication combinations for only one "family" at a time. The strength of her treatment seems to have been characterized by freedom within organization.

7. Manipulative materials should be used with programs which encourage results to be recorded symbolically.

   Treatments which stress connections between materials and symbols seem to be more effective than those which do not. The key to using manipulative materials is ultimately the transfer of understanding generated by those materials to the successful use of and facility with a symbolic system.

The use of manipulative materials and other objects is helpful in promoting understanding of a variety of mathematical ideas (it seems to us). In some way, whether by informal, every-day observations or by more systematic instruction, the child must develop a base or foundation on which to develop such understandings. The ideas, or knowledge and skills derived from the ideas, are applied in the real, physical, concrete world. Concomitant elements or factors which strengthen the effect of the use of materials, and facilitate the understanding of mathematical ideas developed from a concrete base, include meaningful instruction and teaching for transfer and generalization.

Some Suggested Guidelines

As we analyzed the studies and developed conclusions, we also developed a list of suggestions for those who are planning mathematics projects or selecting elementary-school mathematics programs:

1. Mathematics projects and programs at the elementary-school level should be scanned to ascertain whether manipulative materials are incorporated. The use of manipulative materials appears to increase mathematics achievement. Projects not using manipulative materials should justify

non-use of them: there are exceptions which can be justified as, for instance, when no meaningful material exists.

2. Mathematics projects and programs should be scanned to assure that there is a high probability that manipulative materials will be well-used.

3. Projects and teachers should have a budget item or "petty-cash fund" to purchase ordinary, low-cost materials rather than relying exclusively on commercially prepared materials. One cannot judge the quality or effectiveness of materials on the basis of cost alone.

4. Programs should not be chosen on the basis of single factors such as multiple embodiments or manipulative materials without supporting materials.

5. When mathematics laboratories, or other activity-oriented programs, are proposed, the coordination of the laboratory and activities with the total mathematics program should be planned. Objectives, scope, sequence, and evaluation must be coordinated.

A Final Word

Present research provides no more than tantalizing ideas and suggestions about how to use manipulative materials successfully. It assures us that "it can be done", but not how to do it. If research can provide definitive answers, it must be the research of the future, not that of the past. Research is a very complicated process. Every single detail of a study has potential implications. Every single detail of a teacher's planning also has implications . . .

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National Advisory Committee on Mathematical Education. Overview and Analysis of School Mathematics Grades K-12. Washington: Conference Board of the Mathematical Sciences, 1975.

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Piaget, Jean and Inhelder, Barbel. The Growth of Logical Thinking from Childhood to Adolescence. New York: Basic Books, Inc., 1958.

Sawada, Daiyo. Toward a Theory of Sequencing: Study 3-1: Curriculum Hierarchies and the Structure of Intelligence: A Strategy of Organizing Instructional Objectives into Mathematical Systems Employing Basic Piagetian Constructs. (The Pennsylvania State University, 1971.) Dissertation Abstracts International 32A: 6221-6222; May 1972.

Suydam, Marilyn N. and Weaver, J. Fred. Using Research: A Key to Elementary School Mathematics. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1975.

Swenson, Esther J. Organization and Generalization as Factors in Learning, Transfer, and Retroactive Inhibition. Learning Theory in School Situations. University of Minnesota Studies in Education, No. 2. Minneapolis: University of Minnesota Press, 1949.

Van Engen, Henry. An Analysis of Meaning in Arithmetic. Elementary School Journal 49: 321-329, 395-400; February/March 1949.

Weaver, J. Fred and Suydam, Marilyn N. Meaningful Instruction in Mathematics Education. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1972.

APPENDIX A

ANNOTATED LIST OF REFERENCES

Abernatha, Evelyn and Wiles, Clyde A. A Three Month Trial of Developing Mathematical Processes (DMP) with Ten Educable Mentally Retarded Children. Technical Report No. 336. Madison: Wisconsin Research and Development Center for Cognitive Learning, 1975. ERIC: ED 113 204.

Ten children aged 7 to 12, from an intact class, used the DMP program for three months. Data attest to the appropriateness and effectiveness of the materials for mentally retarded children.

Adamson, Geraldine Y. Mathematics Achievement Between First-Grade Students Using Developing Mathematical Processes and Distar Arithmetic Mathematics Instruction. (Brigham Young University, 1975.) Dissertation Abstracts International 36A: 4211; January 1976.

For small groups of first graders, mean differences in scores significantly favored the Distar program, using deductive logic and direct instruction, when compared with the discovery-oriented DMP program using manipulative materials.

Adkins, Julia Elizabeth. An Historical and Analytical Study of the Tally, the Knotted Cord, the Fingers, and the Abacus. (The Ohio State University, 1956.) Dissertation Abstracts 16: 2083; November 1956.

Man's use of physical devices through the ages was studied, with trends and practices of schools from 600 B.C. to 1955 A.D. briefly surveyed.

Allen, Layman E. and Main, Dana B. The Effect of Instructional Gaming upon Absenteeism: The First Step. ERIC: ED 113 159. (1973)

Seventh- and eighth-grade students used the "Equations" game, with twice-a-week tournaments using teams. The mean absentee rate in nongame classes was significantly higher than in game classes.

Allen, Layman E. and Ross, Joan. Instructional Gaming as a Means to Achieve Skill in Selecting Ideas Relevant for Solving a Problem. ERIC: ED 113 158. (1974)

Twenty-nine junior high students in a high-ability mathematics class played the "Equations" game for five sessions. Significant increases in ability to detect the relevance of a particular idea for solving a problem and in evaluating a mathematical expression involving that idea were found.

Allen, Layman E. and Ross, Joan. Improving Skill in Applying Mathematical Ideas: A Preliminary Report on the Instructional Gaming Program at Pelham Middle School in Detroit. ERIC: ED 113 163. (1974)

Ten eighth-grade mathematics classes (n = 237 students) were tested on computation problems after using the "Equations" game; scores increased.

Anderson, George R. Visual-Tactual Devices: Their Efficacy in Teaching Area, Volume and the Pythagorean Relationship to Eighth Grade Children. (The Pennsylvania State University, 1957.) Dissertation Abstracts 18: 160-161; January 1958.

In eighth-grade classes (n = 408 students), a kit of 16 visual-tactual devices (multi-sensory aids) were used by the teachers of one group in making presentations for a unit on areas, volumes, and the Pythagorean relationship; these devices were also available at all times to students. In a comparison with matched groups, no significant differences were found in achievement or attitude, although all stated that they found the devices to be helpful.

Armstrong, Jenny R. Representational Modes as They Interact with Cognitive Development and Mathematical Concept Acquisition of the Retarded to Promote New Mathematical Learning. Journal for Research in Mathematics Education 3: 43-50; January 1972.

Trainable mentally retarded children (MA 2 to 4) (n = 20) exhibited greater mathematical learning when using manipulative materials than when using drawings. For educable mentally retarded children (MA 5 to 12) (n = 67), no significant differences were found between the two types of materials except for learning which required representative thought, where those using manipulative materials scored higher.

Babb, James Herman. The Effects of Textbook Instruction, Manipulatives, and Imagery on Recall of the Basic Multiplication Facts. (University of Southern Florida, 1975.) Dissertation Abstracts International 36A: 4378; January 1976.

Instruction with textbooks, manipulative materials, or imagery (a mneumonic method) were compared with three second-grade classes learning multiplication facts. The use of materials was more effective than the use of imagery or textbooks.

Barragy, Sister Micheleen. The Effect of Varying Object Arrangement and Number on Children's Ability to Coordinate Perspectives. (George Peabody College for Teachers, 1970.) Dissertation Abstracts International 31A: 2730; December 1970.

Randomly selected children from kindergarten, third, and sixth grades (n = 60) were asked to match photographs of object groups with camera position. The number of objects and the type of arrangement had no measureable effect on the difficulty level of the coordination of the perspective task, although there were significant age differences in performance.

Barrish, Bernard. Inductive versus Deductive Teaching Strategies with High and Low Divergent Thinkers. (Stanford University, 1970.) Dissertation Abstracts International 31A: 4029; February 1971.

For 20 days, 125 children from grades 4, 5, and 6 were taught by either a deductive-reception strategy or an inductive-discovery strategy, in each of which concrete manipulation was used "where feasible". For the learning of low-cognitive mathematical material, a deductive-reception strategy was found to be more effective than an inductive-discovery strategy; no differences were found for high-cognitive material.

Beal, Jackie Lee. An Evaluation of Activity Oriented Materials Developed to Help the Low Achiever Attain Basic Mathematical Competencies. (University of Nebraska, 1972.) Dissertation Abstracts International 33A: 3249-3250; January 1973.

Thirty classes in grades 7, 8, 9, and 12 used NCTM materials for low achievers with emphasis on active involvement of the learner. Some differences in achievement and attitude favoring classes using various units were found.

Beardslee, Edward Clarke. Toward a Theory of Sequencing: Study 1-7: An Exploration of the Effect of Instructional Sequences Involving Enactive and Iconic Embodiments on the Ability to Generalize. (The Pennsylvania State University, 1972.) Dissertation Abstracts International 33A: 6721; June 1973.

Forty-nine pupils from grades 5 and 6 were randomly assigned to programmed units on equivalent fractions using either (1) disks, (2) disks and circles, or (3) disks, circles, and rectangles. An additional 29 pupils who had already achieved criterion on two of three sets of objectives received instruction on only the final set of objectives. No significant differences ($p = .15$) were found between the use of one, two, or three concrete embodiments on tests of generalization. [See Gau for a companion study.]

Beardslee, Edward C.; Gau, Gerald E.; and Heimer, Ralph T. Teaching for Generalization: An Array Approach to Equivalent Fractions. Arithmetic Teacher 20: 591-599; November 1973.

The companion studies by Beardslee and Gau provided the basis for this article, which explicates the procedures used in the studies so that teachers can apply them in their classrooms. Attention is focused on the use of arrays (with disks and circles).

Becklund, Lester Albert. Independent Study: An Investigation of the Effectiveness of Independent Study of Novel Mathematics Materials in the Elementary School. (University of Minnesota, 1968.) Dissertation Abstracts 29A: 3452; April 1969.

For content on vectors, groups, and transformation, pupils in 18 classes in grades 3, 4, and 5 used standard materials or activity-oriented materials either with or without teacher direction. On various measures, each of the groups scored significantly higher in some instances.

Bernard, Richard Paul. The Historical Development of the Laboratory Approach to Elementary School Mathematics. (Indiana University, 1972.) Dissertation Abstracts International 33A: 5028; March 1973.

In this analysis of the development of the laboratory approach, it was noted that between 1966 and 1971 the approach was used, discussed, and advocated more than at any previous time.

Bisio, Robert Mario. Effect of Manipulative Materials on Understanding Operations with Fractions in Grade V. (University of California, Berkeley, 1970.) Dissertation Abstracts International 32A: 833; August 1971.

Twenty-nine fifth-grade classes (n = 501 pupils) from two socio-economic levels were compared on an experimenter-developed test before and after 33 lessons on addition and subtraction with like fractions. Pupils using manipulative materials (flannelboard fraction kits) or watching the teacher demonstrate with materials achieved at least as well as pupils taught with no manipulative materials. No significant differences were found among lower SES groups; passive use was significantly better ($p < .01$) than non-use for pupils in the higher SES groups. Interviews with 100 pupils indicated that as difficulty level increased, more errors were made with addition and subtraction and with reducing to lowest terms.

Bledsoe, Joseph C.; Purser, Jerry D.; and Frantz, Nevin R., Jr. Effects of Manipulative Activities on Arithmetic Achievement and Retention. Psychological Reports 35: 247-252; August 1974.

With seventh graders, use of learning packages on fractions and decimals, using manipulative materials, produced greater gain on post and retention tests than packages using only paper-and-pencil exercises.

Bohan, Harry Joseph. A Study of the Effectiveness of Three Learning Sequences for Equivalent Fractions. (The University of Michigan, 1970.) Dissertation Abstracts International 31A: 6270; June 1971.

In one approach, equivalent fractions were introduced with diagrams and sets of objects; in another approach, paper-folding activities were used; a third approach used multiplication to develop an applicable generalization. Two fifth-grade classes were assigned to each six-week treatment (n = 171 students). No significant differences between groups were found on tests of addition, subtraction, or multiplication with fractions. On posttests on equivalent fractions, the groups using diagrams or paper-folding scored significantly higher than those using the "property of one" procedure, while the paper-folding group scored significantly higher on this retention test and an attitude measure.

Bolduc, Elroy Joseph, Jr. A Factorial Study of the Effects of Three Variables on the Ability of First-Grade Children to Solve Arithmetic Addition Problems. (The University of Tennessee, 1969.) Dissertation Abstracts International 30A: 3358; February 1970.

Thirty-six randomly selected first-grade pupils were tested on problems in which (1) the question preceded or followed the data; (2) the elements had like or different names; and (3) direct, indirect, or no visual aids were used. No significant differences were found for (1) or (2), but problems presented without a visual aid were more difficult than those with either type of visual aid.

Bowen, James Joseph. The Use of Games as an Instructional Media. (University of California, Los Angeles, 1969.) Dissertation Abstracts International 30A: 3358-3359; February 1970.

Three classes of intermediate grade honor students were involved in this study with the game WFF'N PROOF. Those who used the game had significantly higher gain scores than those who used a textbook to study logic.

Branch, Robert Charles. The Interaction of Cognitive Style with the Instructional Variables of Sequencing and Manipulation to Effect Achievement of Elementary Mathematics. (University of Washington, 1973.) Dissertation Abstracts International 34A: 4857; February 1974.

Nine sixth-grade pupils classified as high analytic and nine classified as low analytic were randomly assigned to each of four treatment groups pairing inductive and deductive sequencing with or without manipulative use of number lines, for four lessons on addition and subtraction with integers. On a retention test one day later, pupils using number lines scored significantly higher than pupils watching the teacher use a number line on the chalkboard ($p < .05$). Inductive sequencing with use of the number line was better ($p < .005$) than deductive sequencing without materials.

Bring, Curtis Ray. Effects of Varying Concrete Activities on the Achievement of Objectives in Metric and Non-Metric Geometry by Students of Grades Five and Six. (University of Northern Colorado, 1971.) Dissertation Abstracts International 32A: 3775; January 1972.

For one week, 102 pupils in grades 5 and 6 used semi-programmed units on metric and non-metric geometry. Classes using concrete materials (sugar cubes and models) achieved higher mean scores than classes without materials, but the difference was significant on only posttest II. Caucasians achieved significantly higher means than students of other ethnic backgrounds (in Colorado), but the difference in gain scores was not significant.

Bronder, Cecilia Colette. The Application of Diagnostic Teaching and a Mathematics Laboratory to a Middle School Individualized Unit on Fractions. (University of Pittsburgh, 1973.) Dissertation Abstracts International 34A: 1579; October 1973.

An individualized unit on fractions which incorporated diagnostic teaching and a mathematics laboratory increased achievement, although the elementary-school students did not meet criterion on the test.

Brousseau, Andre R. Mathematics Laboratories: Should We or Should We Not? School Science and Mathematics 73: 99-105; February 1973.

Research on the use of mathematics laboratories is reviewed; it is suggested that laboratories be considered one approach to be used to meet individual needs.

Brown, Claude Kenneth. A Study of Four Approaches to Teaching Equivalent Fractions to Fourth-Grade Pupils. (University of California, Los Angeles, 1972.) Dissertation Abstracts International 33A: 5465; April 1973.

Twelve classes from grade 4 were taught equivalent fractions for 18 days. Use of manipulative materials and/or a film with the textbook resulted in higher achievement than use of the textbook alone.

Brownell, William A. Arithmetical Abstractions: Progress Toward Maturity of Concepts Under Differing Programs of Instruction. Arithmetic Teacher 10: 322-329; October 1963.

Brownell, William A. Arithmetical Abstractions: The Movement toward Conceptual Maturity under Differing Systems of Instruction. University of California Publications in Education, Volume 17. Berkeley: University of California Press, 1967.

Brownell, William A. Conceptual Maturity in Arithmetic Under Differing Systems of Instruction. Elementary School Journal 69: 151-163; December 1968.

To ascertain the progress toward abstractness and the maturity of arithmetical concepts of children who had been exposed for three years to different instructional programs, 1406 seven-year-olds in English and Scottish schools were interviewed. They were asked to provide answers to combinations, their mathematical rationale for each operation, and how they solved word problems. In the Scottish schools (n = 478 pupils), the Cuisenaire program was more effective than the conventional program. Children using the Cuisenaire rods had quicker responses and more mathematically mature solutions. In the English schools (n = 928 pupils), the order of the three programs in use there was the conventional program, the Dienes program, and the Cuisenaire program, but "without much difference between the last two."

Callahan, John J. and Jaocbson, Ruth S. An Experiment with Retarded Children and Cuisenaire Rods. Arithmetic Teacher 14: 10-13; January 1967.

Use of Cuisenaire rods with mentally retarded pupils in one class (ages 7 to 10) increased knowledge and understanding of number facts and properties.

Carmody, Lenora Marie. A Theoretical and Experimental Investigation into the Role of Concrete and Semi-Concrete Materials in the Teaching of Elementary School Mathematics. (Ohio State University, 1970.) Dissertation Abstracts International 31A: 3407; January 1971.

Three sixth-grade classes studied units on number bases, properties of odd and even numbers, and divisibility for 11 days. Significant differences on the numeration test and on two transfer tests were found favoring the group using semi-concrete materials over the group using only symbols. On one transfer test, differences favored the group using concrete materials over the group using symbols.

Carney, Harold Francis. The Relative Effectiveness of Two Methods of Teaching the Addition and Subtraction of Rational Numbers. (New York University, 1973.) Dissertation Abstracts International 34A: 659-660; August 1973.

Eight fourth-grade classes (n = 240 pupils) were taught 28 lessons on addition and subtraction with rational numbers. On experimenter-developed tests, the procedure using the field postulates and other properties of whole numbers (such as commutativity: 2 + 3 = 3 + 2) was more effective than a procedure using objects and a number line.

Castaneda, Alberta Maxine Mondor. The Differential Effectiveness of Two First Grade Mathematics Programs for Disadvantaged Mexican-American Children. (The University of Texas, 1967.) Dissertation Abstracts 28A: 3878-3879; April 1968.

Disadvantaged Mexican-American first graders taught by a special program involving activities showed greater gains in mathematics achievement than those taught by the textbook-oriented mathematics program.

Cheatham, Ben H., Jr. A Comparison of Two Methods of Introducing Selected Geometric Concepts to Seventh Grade Students. (University of Florida, 1969.) Dissertation Abstracts International 31A: 1132; September 1970.

For six classes of seventh graders, gains in geometric concepts were not significantly different for those who constructed models with compass and straightedge or with paperfolding techniques.

Choate, Stuart Alan. The Effect of Algorithmic and Conceptual Development for the Comparison of Fractions. (The University of Michigan, 1975.) Dissertation Abstracts International 36A: 1410; September 1975.

In a study with eight classes of mixed fourth- and fifth-grade students (n = 200) on sequences for developing the algorithm for comparing fractions, all groups used paper-folding and/or diagrams. A sequence in which conceptual work was followed by late presentation of the algorithm appeared to be better than three other sequences.

Clausen, Thomas Greenwood. A Developmental Study of Children's Responses to Multi-Sensory Approach in Mathematics. (University of Southern Mississippi, 1971.) Dissertation Abstracts International 32A: 4830; March 1972.

Eight classes (n = 177 pupils) in kindergarten and grade 1 used a multi-sensory (Montessori) approach or a worksheet-textbook approach for six months. The multi-sensory groups achieved higher than the others. When comparisons were made at three mental-age levels, no differences between the two approaches were found.

Cohen, Martin Seymour. A Comparison of Effects of Laboratory and Conventional Mathematics Teaching upon Underachieving Middle School Boys. (Temple University, 1970.) Dissertation Abstracts International 31A: 5026-5027; April 1971

Fourteen seventh- and eighth-grade boys in a special inner-city school were taught fractional concepts and computation for 34 days through a laboratory approach which used a variety of manipulative and multi-sensory materials along with a student-centered teaching approach. A control group of 14 boys was taught the same content through a conventional textbook/chalkboard/discussion approach. The conventionally taught group had a significant increase in achievement, but no significant differences were found on a subtest of commonly taught content, nor were attitude scores significantly different.

Coltharp, Forrest Lee. A Comparison of the Effectiveness of an Abstract and a Concrete Approach in Teaching of Integers to Sixth Grade Students. (Oklahoma State University, 1968.) Dissertation Abstracts International 30A: 923-924; September 1969.

For 79 students in four sixth-grade classes, integers were taught either by a concrete approach using the Greater Cleveland Mathematics Program material, which relied on the number line and other visual procedures, or an abstract approach using materials developed by the researcher. No significant difference was found in achievement.

Cook, Doris M. and others. Research and Development Activities in R & I Units of Two Elementary Schools of Janesville, Wisconsin, 1966-67. Report from Project Models. ERIC: ED 023 175. (1968)

In a study with 105 first graders, a group using manipulative materials performed better than groups using television and/or textbook approaches.

Cooke, Gary Edward. Conceptual Learning in Young Children: A Comparison of the Effects of Rote, Principle, and Guided Discovery Strategies on Conceptualization in First Grade Children. (University of Oregon, 1971.) Dissertation Abstracts International 32A: 2904; December 1971.

Twenty-four pairs of first graders were presented with a series of five pairs of block designs; one group was given attribute cues, a second group was told the organizing principle and directed to model the designs, and a third group was questioned about the attributes and the ordering of the design. Students only questioned about the attributes and design scored significantly better than those told the organizing principle or given attribute cues.

Coxford, Arthur Frank, Jr. The Effects of Two Instructional Approaches on the Learning of Addition and Subtraction Concepts in Grade One. (University of Michigan, 1965.) Dissertation Abstracts 26: 6543-6544; May 1966.

The effects of immediate and delayed symbolization of addition and subtraction concepts were studied in six first-grade classes for 30 weeks. Delayed symbolization of subtraction led to greater transfer and applicability than did immediate symbolization.

Crabtree, Joseph Farris, II. An Investigation of the Ability of Specially Selected Children in Grades K-2 to Learn Certain Concepts, Operations and Applications of Directed Numbers. (University of Virginia, 1965.) Dissertation Abstracts 26: 5907-5908; April 1966.

Two pupils each from kindergarten, grade 1, and grade 2 were tutored for 18 thirty-minute sessions. All could construct a number line, arrange whole numbers in order, and cite some properties for addition using a number line. All had difficulty learning to subtract using a number line or a slide rule.

Crowder, Alex Belcher, Jr. A Comparative Study of Two Methods of Teaching Arithmetic in the First Grade. (North Texas State University, 1965.) Dissertation Abstracts 26: 3778; January 1966.

A group of 242 first-grade pupils using the Cuisenaire program was compared with a group of 183 pupils using a conventional program. The Cuisenaire group's arithmetic achievement was significantly greater ($p > .01, .001$). Upper and middle socioeconomic groups scored higher than lower SES groups.

Curry, Richard Dean. Arithmetic Achievement as a Function of Concrete, Semi-Concrete, and Abstract Teaching Methods. (George Peabody College for Teachers, 1970.) Dissertation Abstracts International 31A: 4032-4033; February 1971.

Clock arithmetic was taught to students aged 8 to 10. Methods providing concrete materials or pictures resulted in greater computational achievement and understanding of properties than did a verbal method.

Dairy, Lorna. Does the Use of Cuisenaire Rods in Kindergarten, First, and Second Grade Upgrade Arithmetic Achievement? ERIC: ED 032 128. (1969)

Children who used Cuisenaire rods in kindergarten scored significantly higher ($p > .01$) in grade 2 than did a group taught without using rods.

Dashiell, William H. and Yawkey, Thomas D. Using Pan and Mathematics Balances with Young Children. Arithmetic Teacher 21: 61-65; January 1974.

Forty first-grade pupils who used a mathematical balance as a physical model for adding single-digit whole numbers solved a greater number of problems correctly than did pupils who had used a pan balance.

Davidson, James Edward. The Impact of Selected Concrete Materials on the Understanding of Certain Mathematical Concepts by Grade 3 and Grade 4 Students. (Columbia University, 1972.) Dissertation Abstracts International 33A: 6323; May 1973.

Pupils in grade 3 who were below the grade median in IQ had significantly better conservation responses after use of materials with the textbook than did those who used only the textbook. In grade 4, the high IQ group using materials had better conservation of length. No achievement test differences were found.

Davies, Rhys and Williams, Phillip. A Comparison of Three Methods of Teaching Fractions to Older Slow-Learners. Educational Research 14: 236-242; June 1972.

No significant differences were found between groups of English slow learners (mean age 11) who used a formal traditional program, the Cuisenaire program, or multi-model materials, except in attitude toward fractions, where the traditional program scored lowest.

Dawson, Dan T. and Ruddell, Arden K. An Experimental Approach to the Division Idea. Arithmetic Teacher 2: 6-9; February 1955.

For fourth graders, use of manipulative materials and visualization procedures for division seemed to aid in achievement. A greater understanding of division and its interrelationships with other operations resulted from the study of division using the subtractive concept and manipulative materials.

DeFlandre, Charles, Jr. The Development of a Unit of Study on Place-Value Numeration Systems, Grades Two, Three, and Four. (Temple University, 1974.) Dissertation Abstracts International 35A: 6434; April 1975.

A place-value numeration unit consisting of 27 activity cards was developed; children were led from the manipulation of concrete objects to the process of symbolization. Used for nine weeks with pupils in grades 2, 3, and 4, the units were found to help children to apply order and equivalence relations.

Denman, Theresa Irene. The Effects of Special Remedial Classes and Various Multisensory Learning Packages on the Mathematics Achievement of Pupils. (The University of Michigan, 1974.) Dissertation Abstracts International 35A: 7025-7026; May 1975.

No significant differences in achievement on addition and subtraction computational skills were found between 455 pupils taught 22 lessons with concrete, pictorial, or symbolic materials and having or not having remedial after-school sessions in grades 5 and 6. Use of a visual aid appeared helpful, however.

DeVries, David L. and Edwards, Keith J. Learning Games and Student Teams: Their Effects on Classroom Processes. ERIC: ED 070 019. (1972)

DeVries, David L. and Edwards, Keith J. Learning Games and Student Teams: Their Effects on Classroom Process. American Educational Research Journal 10: 307-318; Fall 1973.

Using the game "Equations" with teams of seventh graders (n = 110 students) resulted in more peer-tutoring and other affective benefits.

DeVries, David L. and Edwards. Keith J. Student Teams and Learning Games: Their Effects on Cross-Race and Cross-Sex Interaction. Journal of Educational Psychology 66: 741-749; October 1974.

Administering team rewards to heterogeneous groups of seventh graders playing the "Equations" game helped to reduce race and sex barriers inhibiting interaction.

Dexter, John Harry. The Development of a Product for the Concrete Manipulation of Negative Numbers. (Columbia University, 1975.) Dissertation Abstracts International 36A: 1267-1268; September 1975.

Implications of theories were examined to determine a set of guidelines for constructing a manipulative material.

Dilley, Clyde Alan. A Comparison of Two Methods of Teaching Long Division. (University of Illinois at Urbana-Champaign, 1970.) Dissertation Abstracts International 31A: 2248; November 1970.

Ten fourth-grade classes were randomly selected and assigned to be taught division using the distributive algorithm "as a method of keeping records of a natural manipulation of bundles of sticks", or using the successive subtractions algorithm as it was presented in a textbook. On an applications test, significant differences favored the successive subtractions algorithm; differences on a retention test favored the distributive algorithm. No differences were found on measures of speed and accuracy, nor did either algorithm appear better for any SES level.

Dittmer, Karen Ann. Guidelines for Developing a Mathematics Laboratory. (University of Alabama, 1971.) Dissertation Abstracts International 32A: 5083-5084; March 1972.

Responses to specific questions from state supervisors and from teachers in grades 7 through 12 who used a mathematics laboratory approach were presented.

Dunlap, William Phillip. A Comparison of the Effects of Diagnostics and Remedial Arithmetic Programs upon the Achievement and Attitude Development of Fourth Grade Children. (University of Oregon, 1971.) Dissertation Abstracts International 32A: 2905-2906; December 1971.

For 29 days, 147 fourth-grade pupils used either a textbook approach ($n = 74$) involving paper-and-pencil activities or a laboratory approach ($n = 73$) involving "extensive use of games, puzzles, patterns, and the manipulation of physical objects by children". On a standardized test, no significant differences were found for concepts ($p > .05$) or computational skills ($p > .01$), although differences favored the laboratory group on the concepts test. Data analyzed by schools indicated that the laboratory approach resulted in better attitudes toward arithmetic in more schools.

Dunlap, William and others. Differential Effects of Activity-Oriented vs. Textbook-Oriented Mathematics Instruction for Elementary School Children with Learning and Behavior Problems. Monograph No. 4. ERIC: ED 057 546. (1971)

Remediation was done with manipulative aids or with paper-and-pencil procedures. In fourth grades in 12 schools, significant differences were found between schools on almost all achievement data.

Earhart, Eileen. Evaluating Certain Aspects of a New Approach to Mathematics in the Primary Grades. School Science and Mathematics 64: 715-720; November 1964.

The group taught with an abacus in grades 1 through 3, and tested in grade 3, performed significantly better on the fundamentals test than did the workbook-aided group.

Ebeid, William Tawadros. An Experimental Study of the Scheduled Classroom Use of Student Self-Selected Materials in Teaching Junior High School Mathematics. (University of Michigan, 1964.) Dissertation Abstracts 25: 3427-3428; December 1964.

For pupils in grades 7 and 8, no significant differences in achievement or attitude were found between groups using School Mathematics Study Group (SMSG) texts with or without self-selected activities using a variety of mathematical materials.

Edwards, Keith J. and DeVries, David L. Learning Games and Student Teams: Their Effects on Student Attitudes and Achievement. ERIC: ED 072 391. (1972)

Seventh graders ($n = 117$) using the "Equations" game had more positive attitudes toward mathematics classes than did pupils having quizzes. For low- and average-ability pupils, team rewards were viewed more positively than individual rewards.

Edwards, Keith J. and DeVries, David L. The Effects of Teams-Games-Tournament and Two Instructional Variations on Classroom Process, Student Attitudes, and Student Achievement. Report Number 172. ERIC: ED 093 883. (1974)

For seventh graders (n = 128), some significant positive effects were noted for various scoring conditions when playing the "Equations" game.

Edwards, Keith J.; DeVries, David L.; and Snyder, John E. Games and Teams: A Winning Combination. ERIC: ED 067 248. (1972)

Seventh graders (n = 96) using the "Equations" game in four-member teams twice a week for nine weeks achieved significantly greater gains on computation and divergent solutions tests than did groups having only regular instruction. Low achievers did especially well on the divergent solutions test.

Eidson, William P. The Role of Instructional Aids in Arithmetic Instruction. (The Ohio State University, 1956.) Dissertation Abstracts 16: 2095-2096; November 1956.

A checklist was developed to determine the role of instructional aids in the elementary-school mathematics program. It was noted that instructional aids themselves seldom teach arithmetic; the role of the teacher in their use is paramount.

Ekman, Lincoln George. A Comparison of the Effectivenss of Different Approaches to the Teaching of Addition and Subtraction Algorithms in the Third Grade. (Volumes I and II.) (University of Minnesota, 1966.) Dissertation Abstracts 27A: 2275-2276; February 1967.

For 18 days, 27 third-grade classes (n = 584 pupils) were taught addition and subtraction by (1) presentation of the algorithm form immediately, (2) developing the ideas using pictures before presenting the algorithm, or (3) developing the ideas using child-manipulated cardboard disks before presenting the algorithm. The classes using manipulative materials scored higher on experimenter-designed tests of understanding, transfer, and computational skill.

Eudy, Elaine Holland. The Effectiveness of a Mathematical Device Called a Tryab on the Arithmetic Achievement of Primary Students. (The University of Mississippi, 1973.) Dissertation Abstracts International 34A: 1479-1480; October 1973.

Use ~~of~~ the Tryab, a pegboard-flannelboard-chalkboard device, did not result in higher arithmetic achievement for 32 first-grade pupils who were compared on a standardized test with another group of 32 pupils not using the Tryab.

Farris, Dan Curry. Toward a Theory of Sequencing: Study 1-2: An Exploration of Selected Relationships Among the Enactive, Iconic and Symbolic Modes of Representation. (The Pennsylvania State University, 1970.) Dissertation Abstracts International 31A: 4618; March 1971.

Farris, Dan C. Study 1-2: An Exploration of Selected Relationships Among the Enactive, Iconic, and Symbolic Modes of Representation. Journal for Research in Mathematics Education 4: 104-105; March 1973.

In a study with 24 fifth graders, mastery of antecedent objectives varying in terms of which modes (enactive, iconic, symbolic) were used apparently did not induce mastery of consequent objectives for which no explicit instruction was provided. Order of acquisition of antecedent objectives did not significantly affect the implicit mastery of the consequent objectives.

Fedon, John Peter. A Study of the Cuisenaire-Gattegno Method as Opposed to an Eclectic Approach for Promoting Growth in Operational Technique and Concept Maturity with First Grade Children. (Temple University, 1966.) Dissertation Abstracts 27A: 3771-3772; May 1967.

In a study with two classes of first graders (n = 26 pupils), the Cuisenaire program was compared with an eclectic approach using such materials as Cuisenaire-like but non-colored rods, centimeter rulers, and strings of beads. It was concluded that: (1) the use of color inhibited the initial study of rod relationships; (2) teaching of mathematical concepts was more effectively developed when the approach utilized a multiplicity of experiences with a maximum emphasis on manipulation; and (3) while no significant difference in mean performance was found, a trend favored the eclectic group.

Fennema, Elizabeth Hammer. A Study of the Relative Effectiveness of a Meaningful Concrete and a Meaningful Symbolic Model in Learning a Selected Mathematical Principle. (University of Wisconsin, 1969.) Dissertation Abstracts International 30A: 5338-5339; June 1970.

Fennema, Elizabeth H. The Relative Effectiveness of a Symbolic and a Concrete Model in Learning a Selected Mathematical Principle. Journal for Research in Mathematics Education 3: 233-238; November 1972.

For 14 days, 95 second-grade pupils studied multiplication ideas using either symbols only or Cuisenaire rod "trains". No significant differences were found on a recall test, on a concrete transfer test, or on symbolic transfer test I, but the symbolic treatment resulted in significantly higher achievement on symbolic transfer test II.

Fennema, Elizabeth H. Models and Mathematics. Arithmetic Teacher 19: 635-640; December 1972.

In this research review, it was concluded that research appears to indicate that the ratio of concrete to symbolic models used to convey mathematical ideas should reflect the developmental level of the learner. It is suggested that alternative models be available so that the learner can select the most meaningful one for him.

Fennema, Elizabeth. Manipulatives in the Classroom. Arithmetic Teacher 20: 350-352; May 1973.

The use of manipulative materials in the classroom is discussed, with reference to research findings and implications.

Finley, Carmen Joyce. Arithmetic Achievement in Mentally Retarded Children: The Effects of Presenting the Problem in Different Contexts. (Columbia University, 1962.) Dissertation Abstracts 23: 922; September 1962.

In a study with 108 pupils in third grade, test items presented with concrete materials tended to be more difficult for retarded pupils than those either pictorially or symbolically presented, but differences were not significant. For normal children (n = 54), the pictorial item was significantly easier than either the concrete or the symbolic item.

Fishell, Frank E. The Effect of a Math Trading Game on Achievement and Attitude in Fifth Grade Division. (Michigan State University, 1975.) Dissertation Abstracts International 36A: 3382; December 1975.

Use of a trading game on division for 15 days did not significantly improve achievement or attitude for fifth graders (n = 8 classes).

Fitzgerald, William. Self-Selected Mathematics Learning Activities. ERIC: ED 003 348. (1965)

Bright students in grades 7 and 8 did not learn as much in the self-selection classes as did those in conventional classes. Slower students (below 115 IQ) learned equally well in both classes.

Fitzgerald, William M. About Mathematics Laboratories. ERIC: ED 056 895. (1972)

Research on mathematics laboratories is reviewed, including research on manipulative materials.

Frary, Robert B. Formative Evaluation of the Individualized Mathematics System (IMS). ERIC: ED 059 096. (1971)

IMS, in which reusable workpages guide students individually, "often using manipulative materials", was evaluated early in its development. From the reports of two outside evaluators, teacher reactions, coordinator comments, and extrapolated achievement test results, it was reported that IMS is meeting most of its goals.

Freitag, Richard Alan. Case Studies of a Teaching Model: Teaching Through Games. (State University of New York at Buffalo, 1974.) Dissertation Abstracts International 35A: 98; July 1974.

Six case studies using games were conducted. A function game was used with six fourth graders and a form of Bingo was used with two sixth-grade classes (n = 63 pupils). Reactions and scores are discussed in relation to intervening variables and hypotheses.

Gau, Gerald Elmer. Toward a Theory of Sequencing: Study 1-6: An Exploration of the Effect of Instructional Sequences Involving Enactive and Iconic Embodiments on the Attainment of Concepts Embodied Symbolically. (The Pennsylvania State University, 1972.) Dissertation Abstracts International 33A: 6728; June 1973.

This study was conducted in conjunction with the study by Beardslee, using the same basic design; 81 students not given treatments by Beardslee, but from the same fifth and sixth grades, were involved. After instruction over each set of objectives, students were given a transfer test of the symbolically embodied objectives for that set. No significant differences ($p = .15$) were found between the use of one, two, or three concrete embodiments on tests of transfer.

Genkins, Elaine Frances. A Comparison of Two Methods of Teaching the Concept of Bilateral Symmetry to Young Children. (Columbia University, 1971.) Dissertation Abstracts International 31A: 1355-1356; September 1971.

In a study on bilateral symmetry, 185 children from kindergarten and grade 2 were individually trained and tested. The paper-folding method was more effective than the mirror method at the kindergarten level; the mirror method was effective in teaching second graders to discriminate more types of figures.

Gibb, E. Glenadine. Children's Thinking in the Process of Subtraction. Journal of Experimental Education 25: 71-80; September 1956.

The ways in which 36 second-grade pupils thought while performing subtraction was explored. It was found that performance was better on problems in a semi-concrete context than in a concrete context, while the children achieved least well on problems in an abstract context.

Gilbert, Robert Kennedy. A Comparison of Three Instructional Approaches Using Manipulative Devices in Third Grade Mathematics. (University of Minnesota, 1974.) Dissertation Abstracts International 35A: 5189-5190; February 1975.

In one school, third graders (n = 124) manipulating materials individually for addition and subtraction with two-digit numbers scored significantly higher than students watching the teacher manipulate materials or handling materials in groups of four or five children. In a second school, no significant differences were found.

Gipson, Joella. Use of the Environment and Discovery in Teaching Decimals to Second Grade Children. School Science and Mathematics 71: 737-741; November 1971.

Use of familiar situations and manipulative materials resulted in a mean score of 14.75 on a 20-item test on decimals for the second-grade class studied.

Gray, Theresa Marie. A Field Study of Mathematics Laboratory Development in Youngstown, Ohio. (University of Pittsburgh, 1973.) Dissertation Abstracts International 34B: 1184-1185; September 1973. ERIC: ED 085 267.

Evaluation of an elementary-school laboratory program in three schools indicated that it "was contributing to the improvement of attitudes and achievement of quite a few mathematically deficient students".

Green, Geraldine Ann. A Comparison of Two Approaches, Area and Finding a Part of, and Two Instructional Materials, Diagrams and Manipulative Aids, on Multiplication of Fractional Numbers in Grade Five. (University of Michigan, 1969.) Dissertation Abstracts International 31A: 676-677; August 1970.

In a 12-day study, 120 fifth graders were taught multiplication with fractions by one of two approaches, one based on the area of a rectangle and the other dependent on finding the fractional part of a region or set, using either diagrams or cardboard strips. In general, the area/diagram combination was most successful, the fractional-part/cardboard-strips combination was second, while the fractional-part/diagram combination was poorest.

Green, Robert Wesley. A Survey of the Mathematical Instructional Materials Used in Teaching Culturally Disadvantaged Children Grades 1 through 6 Throughout the United States. (Indiana University, 1969.) Dissertation Abstracts International 31A: 1101; September 1970.

Questionnaires and interviews with 232 elementary-school teachers provided data on the availability, purpose, and extent of use of 59 instructional materials. An average of 38 per cent were furnished by the schools, with most (43%) furnished to grade 1 and least (35%) to grades 2 and 6. An average of 49% of the materials were used by teachers, with 28% used frequently. Sixth-grade teachers used materials more for demonstration, while in other grades they were used most frequently for student manipulation. Teachers in grades 1, 2, and 3 listed more materials and activities used successfully with culturally disadvantaged children than did teachers of the upper grades.

Hall, E. Leona. Methods and Materials of a Mathematics Program for the Disadvantaged and Underachieving Child. (Michigan State University, 1966.) Dissertation Abstracts 28A: 154-155; July 1967.

Teaching by a "concept" method using models and aids in a five-week summer camp was more effective for fifth graders than for fourth graders when achievement scores were considered, while attitude changed positively for both groups.

Hankins, Donald David, Jr. A Fourth Grade Mathematics Program for Children from Impoverished Areas and Its Effect upon Learning. (United States International University, 1969.) Dissertation Abstracts International 30A: 2249; December 1969.

A program designed for fourth-grade disadvantaged pupils, stressing success, concrete-to-abstract development, simple language, reduced reading, and activity, resulted in significant differences from a control group in learning concepts and in overall achievement.

x Harap, Henry and Barrett, Ursala. Experimenting with Real Situations in Third Grade Arithmetic. Journal of Educational Methods 16: 188-192; January 1937.

A third-grade class (n = 43) taught through activity units attained a mastery of 93% of the steps set up as a goal for the grade.

x Harap, Henry and Mapes, Charlotte E. The Learning of Fundamentals in an Arithmetic Activity Program. Elementary School Journal 34: 515-525; March 1934.

In an activity program based on real situations, fifth-grade pupils (n = 37) mastered 84% of the processes on a test of denominate numbers and multiplication and division with fractions.

x Harap, Henry and Mapes, Charlotte E. The Learning of Decimals in a Arithmetic Activity Program. Journal of Educational Research 29: 686-693; May 1936.

In a program of 13 units based on real situations incorporating fundamentals of decimals, sixth-grade pupils (n = 39) attained mastery of 96% of the 27 basic processes. A "control group" of 12 pupils matched with 12 pupils in the activity group achieved only 67% mastery.

Harding, Lowry W. and Bryant, Inez P. An Experimental Comparison of Drill and Direct Experience in Arithmetic Learning in a Fourth Grade. Journal of Educational Research 37: 321-337; January 1944.

The fourth-grade class taught by functional experiences achieved slightly higher gain scores than the class taught by drill.

Harshman, Hardwick Wilton. The Effects of Manipulative Materials on Arithmetic Achievement of First-Grade Pupils. (University of Michigan, 1962.) Dissertation Abstracts 23: 150; July 1962.

Harshman, Hardwick W.; Wells, David W.; and Payne, Joseph N. Manipulative Materials and Arithmetic Achievement in Grade 1. Arithmetic Teacher 9: 188-192; April 1962.

Twenty-nine first grade classes (n = 654 pupils) were taught by (1) a commercial set of materials of high comparative cost, (2) a set of inexpensive manipulative materials, or (3) "teacher-selected", homemade manipulative materials. No significant differences between the three programs were found in mean scores of classes on achievement or attitude measures. Using individual scores, significant differences favored the use of the teacher-selected, homemade materials on some subtests, even though more content was covered in the other two programs.

Harvin, Virginia Raines. Analysis of the Uses of Instructional Materials by a Selected Group of Teachers of Elementary School Mathematics. (Indiana University, 1964.) Dissertation Abstracts 25: 4561; February 1965.

Questionnaires were used to secure information from 51 elementary schools. Teachers in grade 1 indicated more use of manipulative materials than pictorial or symbolic materials, which were used more by teachers in grades 2 through 6. Nevertheless, teachers at all levels checked "student manipulation" as the most frequent use of materials.

Haynes, Jerry Oscar. Cuisenaire Rods and the Teaching of Multiplication to Third-Grade Children. (Florida State University, 1963.) Dissertation Abstracts 24: 4545; May 1964.

Five third-grade classes (n = 106) were included in this six-week comparison of the Cuisenaire program with the conventional program for instruction on multiplication. In general, the Cuisenaire program was found to be no more effective than conventional instruction.

Henry, Kermit Maxson. The Effect of Games on Cognitive Abilities and on Attitudes Toward Mathematics. (Oregon State University, 1974.) Dissertation Abstracts International 34A: 4025; January 1974.

No significant differences in achievement or attitude were found between groups of seventh graders (n = 182) who used or did not use the games "Equations" or "Tac-Tickle".

Hicks, Enfield Thomas. The Relationships of Student Characteristics to Achievement in a Junior High School Mathematics Laboratory. (West Virginia University, 1974.) Dissertation Abstracts International 36A: 2077-2078; October 1975.

No single factor could be used to identify those students in junior high school who achieved well in a manipulative approach to mathematics; the best model provided for optimal placement of only 72% of the students.

Higgins, Jon L. The Mathematics Through Science Study: Attitude Changes in a Mathematics Laboratory. School Mathematics Study Group Report Number 8. ERIC: ED 064 174. (1969).

Higgins, Jon L. Attitude Changes in a Mathematics Laboratory Utilizing a Mathematics-Through-Science Approach. Journal for Research in Mathematics Education 1: 43-56; January 1970.

Significant differences were found on six attitude scales after eighth-grade instruction in a laboratory setting. When data were analyzed in terms of naturally occuring attitude groups, however, no significant relationship to achievement was found.

Hirschbuhl, John Joseph. Toward a Theory of Sequencing: Study 1-5: An Exploration of Selected Transitivity and Conjunctive Relationships Among the Enactive, Iconic and Symbolic Modes of Representation. (The Pennsylvania State University, 1971.) Dissertation Abstracts International 32A: 6202; May 1972.

With 42 fourth graders, transfer to related but untaught objectives was not found to occur for six of eight clusters of objectives, varying in terms of which modes (enactive, iconic, symbolic) were taught.

Hollis, Loye Yvorne. A Study to Compare the Effects of Teaching First Grade Mathematics by the Cuisenaire-Gattegno Method with the Traditional Method. (Texas Technical College, 1964.) Dissertation Abstracts 26: 905-906; August 1965.

In a six-month study with 12 classes in grade 1, the Cuisenaire program was compared with the traditional program. Pupils in the

Cuisenaire program learned as much traditional subject matter as pupils having the traditional program; they also acquired additional concepts and skills.

Hollis, Loye Y. A Study to Compare the Effect of Teaching First and Second Grade Mathematics by the Cuisenaire-Gattegno Method with a Traditional Method. School Science and Mathematics 65: 683-687; November 1965.

With first and second grades (n = 9 classes), the Cuisenaire method taught traditional subject matter as well as did the traditional method, while those taught by the Cuisenaire method also acquired additional concepts and skills.

Hollis, Loye Y. A Study of the Effect of Mathematics Laboratories on the Mathematical Achievement and Attitude of Elementary School Students. Final Report. ERIC: ED 066 315. (1972)

No significant difference was found between scores of pupils in grades 4, 5, and 6 using or not using laboratories for 36 weeks, but differences between schools were noted.

Holz, Alan W. Comments on the Effect of Activity-Oriented Instruction. Journal for Research in Mathematics Education 3: 183-185; May 1972.

This critique of Moody, Abell, and Bausell (1971) raises questions about the validity of that study for testing either any established theory or activity-oriented instruction.

Hopkins, L. Thomas. Learning Essentials in an Activity Curriculum. Journal of Experimental Education 1: 298-303; June 1933.

Children in grades 2 through 8 (n = 2,434) taught in an experience curriculum achieved scores comparable to the norms established for those taught in a traditional curriculum.

Houtz, John Charles. Problem-Solving Ability of Advantaged and Disadvantaged Elementary School Children with Concrete and Abstract Item Representations. (Purdue University, 1973.) Dissertation Abstracts International 34A: 5717; March 1974.

Models, slides, and picture-book forms of problem items resulted in higher performance than the abstract form for 1,203 pupils in grades 2 and 4.

Howard, Vivian Gordon. Teaching Mathematics to the Culturally Deprived and Academically Retarded Rural Child. (University of Virginia, 1969.) Dissertation Abstracts International 31A: 294-295; July 1970.

Mathematics laboratory experiences, planned to facilitate learning a hierarchy of needed concepts, were successful, resulting in both achievement and attitude gains for the 12 elementary pupils involved.

Hubbard, W. Donald and Buchanan, Anne E. Developing Mathematical Processes: 1972-73 Field Test Report. Technical Report No. 324. Madison: Wisconsin Research and Development Center for Cognitive Learning, 1975. ERIC: ED 113 203.

This field test of DMP was conducted with second graders in eight schools. A mastery level of 81 percent was found, but data on standardized achievement tests did not conclusively favor either DMP or conventional programs.

Hulten, B[illegible]rma H. Games and Teams: An Effective Combination in the Classroom. ERIC: ED 090 927. (1974)

Eight classes (n = 240 pupils) in grade 7 played a modified game of "Tuf" using individual or team competition, with individual or team practice. Those having team competition showed significantly greater improvement on a standardized achievement test.

Jamison, King Wells, Jr. The Effectiveness of a Variable Base Abacus for Teaching Counting in Numeration Systems Other Than Base Ten. (George Peabody College for Teachers, 1962.) Dissertation Abstracts 23: 3816; April 1963.

Jamison, King W. An Experiment with a Variable Base Abacus. Arithmetic Teacher 11: 81-84; February 1964.

A variable-base abacus was used for five days in one seventh-grade class by the pupils, and for demonstration by the teacher in another class, while a third class used only the chalkboard. On experimenter-developed tests, no significant differences were found between groups.

Jencks, Stanley Morris. The Construction and Validation of Geoboard Investigations, a Programmed Approach to Laboratory Materials in Elementary Mathematics. (University of Utah, 1968.) Dissertation Abstracts 29B: 2975; February 1969.

A text on geoboard investigations was written by the researcher and validated with fifth-grade students.

Jersild, A.T. and others. An Evaluation of Aspects of the Activity Program in New York City Elementary Schools. Journal of Experimental Education 8: 166-207; December 1939.

Students in grades 4 through 6 in a non-activity program maintained a substantial advantage over those in the activity program.

Johnson, Randall Erland. The Effect of Activity Oriented Lessons on the Achievement and Attitudes of Seventh Grade Students in Mathematics. (University of Minnesota, 1970.) Dissertation Abstracts International 32A: 305; July 1971.

Activity-oriented instruction did not appear to be more effective for seventh graders (n = 160 students) than instruction with little or no emphasis on activities for units in number theory, geometry, measurement, and rational numbers.

Johnson, Robert Leo. Effects of Varying Concrete Activities on Achievement of Objectives in Perimeter, Area, and Volume by Students of Grades Four, Five, and Six. (University of Colorado, 1970.) Dissertation Abstracts International 31A: 4624; March 1971.

For four weeks, 96 pupils in grades 4, 5, and 6 were taught concepts on perimeter, area, and volume using (1) a semi-programmed text plus two sets of physical models and instruments, (2) the same text, but no models, or (3) only the text with drawings and illustrations deleted. Significant differences ($p < .01$) favored the group using the physical materials on both immediate learning and retention tests.

Jones, Graham Alfred. The Performances of First, Second and Third Grade Children on Five Concepts of Probability and the Effects of Grade, I.Q. and Embodiments on their Performances. (Indiana University, 1974.) Dissertation Abstracts International 35A: 4272-4273; January 1975.

Most pupils (n = 162) in the primary grades had begun to acquire some concepts of probability, tested with three types of manipulative materials. Use of varied settings appeared necessary.

Jones, Rowen Cox. A Diagnostic-Manipulative Instructional Program for Teaching Addition and Subtraction to Six Emotionally Disturbed Children: A Case Study Approach. (University of Oregon, 1971.) Dissertation Abstracts International 32A: 5071; March 1972.

Jones, Rowen C. and others. A Diagnostic and Activity Based Arithmetic Program for Emotionally Disturbed Children. ERIC: ED 057 547. (1971)

In a 38-day study with six emotionally disturbed children aged 7 to 11, physical models, manipulative aids, games, and other materials were used by teachers and pupils. Achievement and attitudes both improved.

Kaiser, Virginia Ruth Stone. An Exploratory Study of Selected Spoken and Unspoken Behaviors in an Inner-City Mathematics Laboratory for Underachieving Students. (The Ohio State University, 1974.) Dissertation Abstracts International 35A: 2652-2653; November 1974.

Using an observation system, 125 mathematics laboratory lessons were observed. More than 65% of time in the junior high school laboratory was spent in individual student substantive behaviors and teacher interactive substantive behaviors. As students reached higher levels of cognitive performance, teacher behaviors seemed to be more indirect.

Kapperman, Gaylen Gerd. A Comparison of Three Methods of Arithmetic Computation by the Blind. (University of Northern Colorado, 1974.) Dissertation Abstracts International 35A: 2810-2811; November 1974.

After five weeks of instruction with 16 blind students from grades 5 through 12, use of the Cranmer Abacus did not appear as effective as use of the Braillewriter and mental computation, which had been used for years.

Karlin, Marvin William. The Development and Utilization of a Card Game for Teaching Prime Factorization in the Fifth Grade. (University of Colorado, 1971.) Dissertation Abstracts International 33A: 80; July 1972.

Use of a card game with fifth graders in two schools was as effective as a textbook-oriented approach in fostering recognition of the Fundamental Theorem of Arithmetic.

Kellerhouse, Kenneth Douglas, Jr. The Effects of Two Variables on the Problem Solving Abilities of First Grade and Second Grade Children. (Indiana University, 1974.) Dissertation Abstracts International 35A: 5781; March 1975.

For first graders, problems with sets with three different names were more difficult than problems with sets having the same name. For second graders, use of a visual aid affected difficulty level.

Kerr, Joseph Jackson, Jr. British and American Arithmetic Devices for the Blind -- An Analytical Description. (Temple University, 1974.) Dissertation Abstracts International 35A: 3553; December 1974.

Devices used by the blind from 1700 to the present were evaluated.

Kieren, Thomas E. Activity Learning. Review of Educational Research 39: 509-522; October 1969.

Studies on discovery learning and teaching in mathematics and on manipulative learning in mathematics are discussed, with a critique and recommendations.

Kieren, Thomas E. Manipulative Activity in Mathematics Learning. Journal for Research in Mathematics Education 2: 228-234; May 1971.

In this research review, the place of manipulative activity in the instructional sequence and its value in promoting learning are discussed.

Knaupp, Jonathan Elmer. A Study of Achievement and Attitude of Second Grade Students Using Two Modes of Instruction and Two Manipulative Models for the Numeration System. (University of Illinois at Urbana-Champaign, 1970.) Dissertation Abstracts International 31A: 6471; June 1971.

Knaupp, Jonathan. A Study of Achievement and Attitude of Second Grade Students Using Two Modes of Instruction and Two Manipulative Models for the Numeration System. Illinois School Research 8: 27-33; Winter 1972.

For second graders in four classes, both teacher-demonstration and student-activity modes with either Dienes blocks or sticks resulted in significant gains in achievement, but there were no significant differences between the two in either achievement or attitude.

Koch, Richard R. MICA, 1972-73. Outcome Evaluation Report. ERIC: ED 092 381. (1973)

Manipulative materials were used in conjunction with small group instruction, listening stations, and other procedures. No significant differences in achievement or attitude were found.

Krairojananan, Sompop. The Mathematical Behaviors Derivable from the Program of Unified Science and Mathematics for Elementary Schools. (Michigan State University, 1973.) Dissertation Abstracts International 34A: 5746; March 1974.

It was felt that the four USMES units studied for nine weeks by the elementary-school pupils were effective.

Kratzer, Richard Oren. A Comparison of Initially Teaching Division Employing the Distributive and Greenwood Algorithm with the Aid of a Manipulative Material. (New York University, 1971.) Dissertation Abstracts International 32A: 5672; April 1972.

Kratzer, Richard O. and Willoughby, Stephen S. A Comparison of Initially Teaching Division Employing the Distributive and Greenwood Algorithms with the Aid of a Manipulative Material. Journal for Research in Mathematics Education 4: 197-204; November 1973.

Six classes of fourth graders were taught the distributive algorithm for division while six classes were taught the successive subtractions (Greenwood) algorithm, both as a method of keeping records of manipulating bundles of sticks. No significant difference was found between the two algorithms on achievement of familiar problems on immediate or retention tests. For unfamiliar problems, the distributive group had better understanding of the process.

Kuhfittig, Peter Kurt Friedrich. The Effectiveness of Discovery Learning in Relation to Concrete and Abstract Teaching Methods in Mathematics. (George Peabody College for Teachers, 1972.) Dissertation Abstracts International 33A: 1323; October 1972.

Kuhfittig, Peter K. F. The Relative Effectiveness of Concrete Aids in Discovery Learning. School Science and Mathematics 74: 104-108; February 1974.

In a two-day study with 40 seventh graders on currency conversion, low-ability students benefited more from using concrete aids and from intermediate guidance than did high-ability students. Use of concrete aids with intermediate guidance appeared preferable to use of aids with maximal guidance.

LeBlanc, John Francis. The Performances of First Grade Children in Four Levels of Conservation of Numerousness and Three I.Q. Groups When Solving Arithmetic Subtraction Problems. (University of Wisconsin, 1968.) Dissertation Abstracts 29A: 67; July 1968.

The performances of 338 first-grade children in solving problems involving subtraction situations were analyzed. Problems with no aids and no transformation were significantly more difficult than all other problem types; problems with aids (either physical or pictorial) and a transformation were significantly easier. Children with low levels of conservation and those with low IQs were more dependent on aids and transformations.

Lerch, Harold H. An Activities and Materials Based, Non-Text Mathematics Program for Kindergarten. ERIC: ED 063 973. (1972)

Activities and use of manipulative materials were stressed in this number program for kindergarten pupils. The experimental group had significantly better ($p < .05$) matching skills and greater number knowledge and skills.

Lesh, Richard (editor). Cognitive Psychology and the Mathematics Laboratory. Columbus: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1974. ERIC: ED 108 893.

In this collection of papers from a symposium, research on mathematics laboratories is discussed in relation to psychological theories.

Lewis, Marian. Teaching Arithmetic Computation Skills. Education of the Visually Handicapped 2: 66-72; October 1970.

A survey of computational aids used in classes for the visually handicapped is reported.

Lucas, James Stanley. The Effect of Attribute-Block Training on Children's Development of Arithmetic Concepts. (University of California at Berkeley, 1966.) Dissertation Abstracts 27A: 2400-2401; February 1967.

Attribute-block training was compared with the Greater Cleveland Mathematics Program in eight first-grade classes. Attribute-block users were better able to conceptualize addition and subtraction, were not as good in computation, were no better on verbal problems, and were slightly better in multiplication.

Lucow, William H. Testing the Cuisenaire Method. Arithmetic Teacher 10: 435-438; November 1963.

Lucow, William H. An Experiment with the Cuisenaire Method in Grade Three. American Educational Research Journal 1: 159-167; May 1964.

In a six-week study with 12 classes of third graders, the Cuisenaire method resulted in significantly higher achievement than the traditional method in teaching multiplication and division; however, these differences could be attributed to the previous experience given the Cuisenaire group. Both methods produced significant gains.

Macy, Murray. The Effectiveness of Representative Materials and Additional Experience Situations in the Learning and Teaching of Fourth Grade Mathematics. (New York University, 1956.) Dissertation Abstracts 17: 533-534; March 1957.

Pupils from one fourth-grade class (n = 28) were matched with those from another fourth-grade class, with each group alternating use and non-use of manipulative materials and an "enriched experience program" for two-week periods. No significant difference ($p < .05$) in achievement was found between the groups either during or following the treatment period.

Matthews, Larry Allen. An Evaluation of the Effect of Using Supplementary Activity-Oriented Materials on Student Achievement of Mathematical Competencies for Enlightened Citizens. (The University of Nebraska-Lincoln, 1974.) Dissertation Abstracts International 35A: 2543-2544; November 1974.

Forty-two classes in grades 7 through 9 used NCTM activity-oriented materials for low achievers. After pretesting, teachers of experimental groups were sent information on the competency areas in which each student was deficient, plus lists of activity-oriented

materials to help students achieve these competencies. While significant increases in achievement were found for some subgroups, in none of the 48 competency areas was there a significant difference in achievement between experimental and control groups.

McClure, Clair Wylie. Effectiveness of Mathematics Laboratories for Eighth Graders. (Ohio State University, 1971.) Dissertation Abstracts International 32B: 4078; January 1972.

The effectiveness of pre- and post-laboratory experiences on achievement differed for 146 eighth graders in six classes in two schools. Generally, student attitude was favorable toward the lessons on fractions, geometry, and ratio and proportion.

McGinty, Robert LeRoy. The Effects of Four Methods of Instruction upon the Ability of Second and Third Grade Students to Derive Valid Logical Conclusions from Verbally Expressed Hypotheses. (Michigan State University, 1972.) Dissertation Abstracts International 33A: 4686; March 1973.

Pupils in grades 2 and 3 given instruction on attribute blocks, pictorial logic, or set theory scored higher on logic and classification tests than pupils not having logic instruction.

McLaughlin, Lynn Mary James. Age and Observational Learning of a Multiple Classification Task. (St. Louis University, 1972.) Dissertation Abstracts International 33B: 1271; September 1972.

Pupils in grades 2 and 4 were able to reproduce behavior on a block task which they had seen demonstrated; only sixth graders could transfer the information to a different but structurally similar task. Watching a model demonstrate was more effective than practicing by trial-and-error.

McLeod, Gordon Keith. An Experiment in the Teaching of Selected Concepts of Probability to Elementary School Children. (Stanford University, 1971.) Dissertation Abstracts International 32A: 1539; September 1971.

Most pupils in grades 2 and 4 ($n = 550$) were able to apply the concepts on the likely-to-unlikely probability continuum before instruction for seven to ten days. No clear treatment effect was found for groups having laboratory participation, teacher demonstration, or no instruction.

McMillian, Joe Adair. Learning a Mathematical Concept With and Without a Physical Model as Predicted by Selected Mental Factors. (University of Houston, 1972.) Dissertation Abstracts International 33A: 4182; February 1973.

No significant differences in learning, retention, or transfer were found for seventh-grade students who used individual materials for a unit on non-decimal numeration with or without a physical model.

Miller, Jack W. An Experimental Comparison of Two Approaches to Teaching Multiplication of Fractions. Journal of Educational Research 57: 468-471; May-June 1964.

For four classes of sixth graders (n = 114 students), use of written lesson plans plus automated practice machines was superior to use of the textbook with concrete materials.

Moody, William B.; Abell, Roberta; and Bausell, R. Barker. The Effect of Activity-Oriented Instruction upon Original Learning, Transfer, and Retention. Journal for Research in Mathematics Education 2: 207-212; May 1971.

For third graders, no advantages for activity-oriented instruction were observed on learning, transfer, or retention when compared to rote procedures. [This study was critiqued by Holz, 1972.]

Mott, Edward Raymond. An Experimental Study Testing the Value of Using Multi-Sensory Experiences in the Teaching of Measurement Units on the Fifth and Sixth Grade Level. (The Pennsylvania State University, 1959.) Dissertation Abstracts 20: 1678-1679; November 1959.

Multi-sensory aids were used "at every opportunity" to teach measurement to 70 pupils in grades 5 and 6, while the control group (n = 157) "proceeded as usual". No significant differences in attitude or achievement were found.

Muckey, Roy William. Using Decimal and Non-Decimal Numeration Systems to Effect Change in the Ability of Beginning Second Grade Students to Add and Subtract in Different Bases. (University of Minnesota, 1971.) Dissertation Abstracts International 32B: 3510; December 1971.

No significant differences were found between groups of second graders (n = 251) studying base ten only, non-ten bases, or many bases including base ten, in ability to add base ten numbers. The use of Dienes or non-Dienes materials made no difference at the high SES level; only slightly improved scores favored the use of non-Dienes materials at the middle SES level. [The low SES level was not studied.]

Nasca, Donald. Comparative Merits of a Manipulative Approach to Second-Grade Arithmetic. Arithmetic Teacher 13: 221-226; March 1966.

In two second-grade classes, Cuisenaire and traditionally taught groups were compared. The Cuisenaire group achieved significantly better on a test based on the more extensive content of the Cuisenaire program, while there were no significant differences between the two groups on a test of traditional content.

Nichols, Edith June. A Comparison of Two Methods of Instruction in Multiplication and Division for Third-Grade Pupils. (University of California, Los Angeles, 1971.) Dissertation Abstracts International 32A: 6011; May 1972.

Ten classes of third graders (n = 267) were randomly assigned to be taught multiplication and division combinations for 15 days by (1) manipulative materials with a guided discovery approach or (2) abstract and semi-concrete materials with teacher explanation and exposition. Significant differences on achievement and attitude measures favored groups taught by pupil discovery with manipulative materials.

Nickel, Anton Peter. A Multi-Experience Approach to Conceptualization for the Purpose of Improvement of Verbal Problem Solving in Arithmetic. (University of Oregon, 1971.) Dissertation Abstracts International 32A: 2917-2918; December 1971.

Ninety students were drawn from fourth-grade classes for six weeks of instruction on verbal problem solving using (1) concrete materials and pictures; (2) only words, written and oral; or (3) an unspecified control treatment. The approach using materials and pictures was more effective than the verbal approach. Differences between these groups and the control group were not significant (p = .10); it appeared from analysis of logs that the control group teachers spent a disproportionate amount of time on problem solving, expecially using materials and pictures. No significant differences between any groups were found on the retention test.

Nicodemus, Robert B. Order of Complexity in Attribute Blocks. School Science and Mathematics 70: 649-654; October 1970.

In an in-depth exploration with 46 pupils in grade 5, it was concluded that performance of complex behaviors was facilitated by experience with simple subordinate behaviors with attribute blocks.

Norman, Martha. Three Methods of Teaching Basic Division Facts. (State University of Iowa, 1955.) Dissertation Abstracts 15: 2134; November 1955.

In a study with 24 third-grade classes, a developmental method using such aids as the number line, counters, drawings, and number charts plus generalization procedures was compared with a textbook method and a conventional method in which story settings and problems were used to introduce division facts. No significant differences were found on an experimenter-developed test given immediately after instruction; the developmental method was superior on a delayed recall test of untaught facts.

Nowak, Betty Adams. A Study to Compare the Effects of Mathematics Laboratory Experiences of Intermediate-Grade Students on Achievement and Attitudes. (Brigham Young University, 1972.) Dissertation Abstracts International 33A: 2697; December 1972.

Pupils in grades 5 and 6 did better in a laboratory program than in a non-laboratory program, while fourth graders did better in a conventional program. An individualized laboratory program was more effective than an individualized non-laboratory program.

Olley, Peter George. The Relative Efficacy of Four Experimental Protocols in the Use of Model Devices to Teach Selected Mathematical Constructs. (Washington State University, 1973.) Dissertation Abstracts International 34A: 4993; February 1974.

Four teachers in grade 3 each taught four groups of randomly assigned students five lessons on operations on a finite field (even/odd, zero, one); four teachers in grade 7 similarly used five lessons on a permutation group. For transfer, use of concrete-to-abstract sequences were preferable to pictorial-abstract or abstract sequences. No significant differences were found on a retention test.

Paris, John August. The Relation of a Personality Trait and Game Conditions to Participant Learning. (Syracuse University, 1970.) Dissertation Abstracts International 32A: 102; July 1971.

Fifth-grade students (n = 302) scored higher on an achievement test after playing "Equations" in a cooperative rather than a competitive situation.

X Passehl, George. Teaching Arithmetic Through Activity Units. Peabody Journal of Education 27: 148-152; November 1949.

For 30 sixth graders in one class, use of activity units resulted in mastery of 79% of the arithmetical steps in learning to perform operations with common and decimal fractions.

Passy, Robert Albert. How Do Cuisenaire Materials in a Modified Elementary Mathematics Program Affect the Mathematical Reasoning and Computational Skill of Third-Grade Children? (New York University, 1963.) Dissertation Abstracts 24: 1506-1507; October 1963.

Passy, Robert A. The Effect of Cuisenaire Materials on Reasoning and Computation. Arithmetic Teacher 10: 439-440; November 1963.

Third-grade classes using Cuisenaire materials achieved significantly less on tests of reasoning and computation than classes not using Cuisenaire materials (n = 1800).

Passy, Robert A. Socio-Economic Status and Mathematics Achievement. Arithmetic Teacher 11: 469-470; November 1964.

In a study with third graders using either textbooks or the Cuisenaire program, significant differences were found among the various levels of socioeconomic status regardless of which program was used. Mean scores increased with increasing level of education and skill of parent.

Plummer, Sister Mary Jean. The Effect of Preschool Experience on Spatial Perception. (University of Cincinnati, 1971.) Dissertation Abstracts International 32A: 4493-4494; February 1972.

No difference on a block-counting task was found between Montessori and non-Montessori pupils in grade 1 (n = 109).

Polz, Sister Albina. Evaluation of the Effectiveness of Activity Mathematics for Primary Mathematics Learning Disabled Students. Unpublished master's thesis, University of Saskatchewan, 1975.

In a study with 48 mathematics learning disabled students, the relative merits of activity and traditional mathematics programs for a 20-hour unit on number and numeration were examined. No significant differences between groups were found.

Portis, Theodore Roosevelt. An Analysis of the Performances of Fourth, Fifth and Sixth Grade Students on Problems Involving Proportions, Three Levels of Aids and Three I.Q. Levels. (Indiana University, 1972.) Dissertation Abstracts International 33A: 5981-5982; May 1973.

From grades 4, 5, and 6, 138 pupils were given tests accompanied by physical, pictorial, or symbolic aids. Mean performance when physical and pictorial aids were used was significantly higher than when only symbolic aids were available.

Prigge, Glenn Russell. The Effects of Three Instructional Settings on the Learning of Geometric Concepts by Elementary School Children. (Volumes I and II.) (University of Minnesota, 1974.) Dissertation Abstracts International 35A: 3307; December 1974.

In a ten-day study conducted with 169 third-grade pupils, selected basic geometric concepts were taught using a programmed format. Use of both demonstrations by the teacher and manipulation of geometric solids by pupils was more effective than use of paper-and-pencil activities or use of manipulative materials (such as paper-folding, geoboard, and georuler).

Prindeville, Ann Catherine. A Program for Teaching Selected Mathematics Concepts to First-Grade Children Using Manipulanda, Language Training and the Tutor-Tutee Relationship. (University of California, Los Angeles, 1971.) Dissertation Abstracts International 32A: 6111; May 1972.

Pupils in three first-grade classes were given 24 supplementary lessons on concepts of place value, order of numbers to 400, and two-place addition and subtraction. One group received the enrichment instruction in the usual large-group setting; sixth-grade tutors were used to present the enrichment instruction in a second class; the third class used the textbook program in the usual large-group setting. Pupils using manipulative materials with language training scored significantly higher ($p < .05$) than those using the textbook program with workbook and drill sheets. Use of sixth-grade tutors did not result in better achievement.

Punn, Avtar Kaur. The Effects of Using Three Modes of Representation in Teaching Multiplication Facts on the Achievement and Attitudes of Third Grade Pupils. (University of Denver, 1973.) Dissertation Abstracts International 34A: 6954-6955; May 1974.

Ninety third-grade pupils were taught for 27 days to solve missing-factor multiplication facts and word problems using (1) enactive-symbolic, (2) iconic-symbolic, or (3) enactive-iconic-symbolic modes. Pupils using manipulative materials and symbols (1) had significantly higher ($p < .05$) achievement and attitudes than those using materials, symbols and pictures (3), which in turn was better than using only pictures and symbols (2).

Purser, Jerry Donaldson. The Relation of Manipulative Activities, Achievement and Retention, in a Seventh-Grade Mathematics Class: An Exploratory Study. (University of Georgia, 1973.) Dissertation Abstracts International 34A: 3255-3256; December 1973.

In a study on fractions and decimals with 339 students in grade 7, the use of pictures of rulers and micrometers was found to be feasible; significant differences favored groups using such materials. (No instruments were actually used.)

Rathmell, Edward Cary. The Effects of Multibase Grouping and Early or Late Introduction of Base Representations on the Mastery Learning of Base and Place Value Numeration in Grade One. (University of Michigan, 1972.) Dissertation Abstracts International 33A: 6071-6072; May 1973.

No significant differences were found between using various bases or only base ten in grouping objects with pupils in first-grade classes ($n = 110$ pupils). The group having reading and writing experiences before grouping experiences (with objects) achieved better than the group given grouping experiences first.

Reys, Robert E. Mathematics, Multiple Embodiment, and Elementary Teachers. Arithmetic Teacher 19: 489-493; October 1972.

The rationale for using a variety of concrete materials to develop a mathematical idea is discussed and activities for the classroom are presented.

Rich, Littleton Waldo. The Effects of a Manipulative Instructional Mode in Teaching Mathematics to Selected 7th Grade Inner-City Students. (Temple University, 1972.) Dissertation Abstracts International 33B: 330; July 1972.

Use of multi-base blocks and Cuisenaire rods in teaching fraction concepts did not negatively affect achievement for the nine seventh-grade classes studied.

Richards, Kenvyn Barrett. A Comparison of the Effects of Verbal-Manipulative Forms of Programed Instruction in Teaching Measurement Skills to Sixth Grade Pupils. (University of Maryland, 1970.) Dissertation Abstracts International 31A: 5818; May 1971.

No significant differences in achievement or retention were found between groups of sixth graders (n = 72) using verbal or verbal-manipulative programs on reading a ruler.

Ropes, George Hardcastle. The Effects of a Mathematics Laboratory on Elementary School Students. (Columbia University, 1972.) Dissertation Abstracts International 33A: 4250; February 1973.

Twenty-two pupils from grade 2 and 22 from grade 6 were randomly selected to participate in a mathematics laboratory; 22 other pupils from each grade formed control groups. Experimental group pupils spent one 45-minute period per week for 14 weeks in the laboratory, which "contained a variety of manipulative materials and activity sheets related to each". No significant differences ($p < .05$) were found in attitude or achievement (although laboratory group pupils spent 20% less time on the regular content tested).

Ross, Dorothea. Incidental Learning of Number Concepts in Small Group Games. American Journal of Mental Deficiency 74: 718-725; May 1970.

Retarded pupils aged 4 to 10 using a game program for nine months improved significantly more than a group using a special-class program.

Schall, William E. and others. Developing Mathematical Processes (DMP). Field Test Evaluation, 1972-1973. ERIC: ED 097 290. (1974)

The Developing Mathematical Processes (DMP) program was evaluated in its initial period of use; encouraging results were reported.

Schall, William and others. Developing Mathematical Processes (DMP), Field Test Report, 1973-74. ERIC: SE 019 475. (1975).

The DMP program was field-tested in the kindergarten and first three grades of six schools. The previous years' standardized test scores were used to predict achievement; no significant achievement gains were found. Teachers, parents, and pupils all gave the program favorable ratings.

Schippert, Frederick Arthur. A Comparative Study of Two Methods of Arithmetic Instruction in an Inner-City Junior High School. (Wayne State University, 1964.) Dissertation Abstracts 25: 5162-5163; March 1965.

Four seventh-grade classes from an inner-city (Detroit) school were taught with SMSG texts (reviewing operations with non-negative rational numbers) for five months, using either an abstract approach involving verbal and written descriptions of models of mathematical principles or student manipulation of actual models or representations in a laboratory approach. A significant difference in growth of arithmetic skills favored the groups taught by the laboratory approach. After two-and-one-half years, differences still significantly favored the laboratory group. Differences in attitudes were not significant.

Schott, Andrew F. New Tools, Methods for Their Use, and a New Curriculum in Arithmetic. Arithmetic Teacher 4: 204-209; November 1957.

The development and trial testing of a n ber aid was discussed and partial data for grades 1, 2, and 3 were presented, showing increased achievement when these aids were used. [Note that this material is the one considered "expensive" in the Harshman (1962) study.]

Scott, Lloyd F. and Neufeld, Herman. Concrete Instruction in Elementary School Mathematics: Pictorial vs. Manipulative. School Science and Mathematics 76: 68-72; January 1976.

No significant difference in concept knowledge was found between either manipulative or pictorial groups and the abstract group in nine second-grade classes involved in the study. Affective responses favored the first two groups over the third.

Sherer, Margaret Turner. An Investigation of Remedial Procedures in Teaching Elementary School Mathematics to Low Achievers. (University of Tennessee, 1967.) Dissertation Abstracts 28A: 4031-4032; April 1968.

Pupils in grades 3 through 7 (n = 47) tutored with 20 author-developed lessons, using instructional aids such as drawings, counters, and number lines and charts, showed significantly greater gain in achievement than those taught by a traditional procedure.

Sherzer, Laurence. Effects of Different Methods of Integer Addition Instruction on Elementary School Students of Different Grade and Aptitude Levels. (University of Miami, 1973.) Dissertation Abstracts International 34A: 2465; November 1973.

The "correspondence" method for teaching integers was more effective in grades 3 through 6 than was a number line method.

Shipp, Donald E. and Deer, George H. The Use of Class Time in Arithmetic. Arithmetic Teacher 7: 117-121; March 1960.

For three classes of pupils in grades 4, 5, and 6, there was a trend toward higher achievement when the percentage of class time on developmental activities was increased from 25% to 75%. It was concluded that more than 50% of class time should be spent on developmental activities, which included use of materials and discussion.

Shoecraft, Paul Joseph. The Effects of Provisions for Imagery Through Materials and Drawings on Translating Algebra Word Problems, Grades Seven and Nine. (University of Michigan, 1971.) Dissertation Abstracts International 32A: 3874-3875; January 1972.

Twelve seventh-grade classes (n = 366) (and ten ninth-grade classes) were taught number, coin, and age problems for eight days and work and mixture problems for four days. Two approaches used high imagery with (1) concrete materials or (2) pictures, while a control group stressed direct translation of the problems. The materials approach and the direct approach were significantly better than the picture approach in grade 7 for number, coin, and age problems. For work and mixture problems, the materials approach was better than the picture approach. For retention and transfer, differences also favored the materials approach. Low achievers in particular achieved better when they could use materials.

Shuster, Albert and Pigge, Fred. Retention Efficiency of Meaningful Teaching. Arithmetic Teacher 12: 24-31; January 1965.

For six classes of fifth graders, spending betwen 50% and 75% of the time on developmental activities resulted in better retention.

Silbaugh, Charlotte Vance. A Study of the Effectiveness of a Multiple-Activities Laboratory in the Teaching of Seventh Grade Mathematics to Inner-City Students. (George Washington University, 1972.) Dissertation Abstracts International 33A: 205; July 1972.

Mathematics laboratories were attended twice a week by twelve seventh-grade classes; 12 classes in the same school did not use the laboratory, and 12 classes were in another school with no laboratory. The laboratory appeared to have a favorable effect on achievement scores.

Simpson, Clifford James. The Effect of Laboratory Instruction on the Achievement and Attitudes of Slow Learners in Mathematics. (Lehigh University, 1973.) <u>Dissertation Abstracts International</u> 34A: 6959-6960; May 1974.

In a 101-day study, 87 seventh-graders classified as slow learners were taught by a laboratory approach (using activities with an NCTM book) in one school or by a traditional approach (using a commercial text) in another school. For two of six units, significant differences favored the teacher-taught group; for one unit, the laboratory group achieved better.

Smith, Emma Drucilla Breedlove. The Effects of Laboratory Instruction upon Achievement in and Attitude Toward Mathematics of Middle School Students. (Indiana University, 1973.) <u>Dissertation Abstracts International</u> 34A: 3715-3716; January 1974.

Eighty-two students from grades 6, 7, and 8 were taught by a laboratory approach two days a week for one-and-one-half semesters, while 153 students were identified as the control group. No significant differences in achievement or attitudes were found between groups.

Smith, Jimmy Eugene. The Effect on Achievement and Attitude of Three Approaches for Developing Area Concepts. (The University of Texas at Austin, 1973.) <u>Dissertation Abstracts International</u> 34A: 5497-5498; March 1974.

The expository approach was found to be superior to the unimodel and multimodel approaches on most of the area topics studied in four classes at grade 7.

Snyder, Henry D. A Comparative Study of Two Self-Selection-Pacing Approaches to Individualizing Instruction in Junior High School Mathematics. (University of Michigan, 1966.) <u>Dissertation Abstracts</u> 28A: 159-160; July 1967.

No significant differences were found in achievement or in the characteristics of pupils in grades 7 and 8 who selected either of two independent work approaches; however, gains were greater for the independent groups than for the control classes.

Snyder, Patricia Kay. Development of a Model of a Mathematics Laboratory for Secondary Schools. (University of Denver, 1975.) Dissertation Abstracts International 36A: 4236-4237; January 1976.

A model for laboratories was developed, including physical facility and equipment, teacher involvement, student activities and roles, laboratory techniques and procedures, conceptual framework, and mathematical concepts.

Sole, David. The Use of Materials in the Teaching of Arithmetic. (Columbia University, 1957.) Dissertation Abstracts 17: 1517-1518; July 1957.

Twelve third-grade classes (n = 240 pupils) were taught using either a variety of materials or one material. No differences were found between the two groups. If more time was spent in using either one or several materials, then higher achievement resulted. The effect appeared to depend more on the teacher than on the materials used.

Spross, Patricia McNitt. A Study of the Effect of a Tangible and Conceptualized Presentation of Arithmetic on Achievement in the Fifth and Sixth Grade. (Michigan State University, 1962.) Dissertation Abstracts 23: 1293; October 1962.

"Tangible manipulative items that had cultural significance" were used in this study with fifth and sixth graders (n = 166) "whenever possible". Significant differences favored the experimental group over a group having "routine" presentations on both standardized tests used except for one subtest on fundamentals.

Stanford, Thomas Enos. Effects of and Teacher Evaluation of Supplementary Activities on Seventh Grade Boys' and Girls' Achievement in and Preference for Mathematics. (University of Mississippi, 1970.) Dissertation Abstracts International 31A: 2798-2799; December 1970.

Seventh-grade groups using games, non-verbal problems, or self-selection of activities had significant increases in achievement. A local control group also had a significant increase in achievement, while a remote control group did not.

Steffe, Leslie Philip. The Performance of First Grade Children in Four Levels of Conservation of Numerousness and Three I.Q. Groups When Solving Arithmetic Addition Problems. (University of Wisconsin, 1966.) Dissertation Abstracts 28A: 885-886; August 1967.

Steffe, Leslie P. Differential Performance of First-Grade Children When Solving Arithmetic Addition Problems. Journal for Research in Mathematics Education 1: 144-161; May 1970.

Randomly selected first graders (n = 132) were categorized into four levels of conservation of numerousness, and given addition problems with physical, pictorial, or no aids. Problems with no accompanying aids were significantly more difficult than problems with either physical or pictorial aids, which did not differ. Significant correlations were obtained between scores on an addition facts test and problems with aids (r = .46) or without aids (r = .41).

Steffe, Leslie P. and Johnson, David C. Problem Solving Performances of First Grade Children. ERIC: ED 041 623. (1970)

One hundred eleven children were given a 48-item problem-solving test, with six problems from each of eight types presented in a randomized sequence. Half of the children in each ability group were randomly assigned to use of no manipulative objects, while the other half were provided with manipulative objects referred to in the problems and were allowed to use them any way they wanted to help solve the problems. No IQ differences were found, but those using materials scored significantly higher than those not using materials.

St. Martin, Allen H. An Analysis of the Relationship Between Two Alternate Procedures for the Utilization of Teaching Aids in Piaget's Developmental Theory During the Initial Introduction of Selected Fifth Grade Mathematical Topics. (University of Houston, 1974.) Dissertation Abstracts International 35A: 7037-7038; May 1975.

No significant difference in achievement was found between fifth-grade groups using a concrete-semiconcrete-abstract sequence and those using a semiconcrete-abstract sequence. Use of concrete materials resulted in some higher subtest scores.

Swick, Dana F. The Value of Multi-Sensory Learning Aids in the Teaching of Arithmetical Skills and Problem Solving -- An Experimental Study. (Northwestern University, 1959.) Dissertation Abstracts 20: 3669; March 1960.

Students in grades 2 through 5 (n = 404) made significantly greater gains during the nine-week period when a variety of materials was used than were expected on the basis of their scores for the previous nine weeks.

Tanner, Verdelia Jane Lindsey. A Mathematics Program for Primary-Age Children: Concrete-Operational Approaches to Number Concepts. (Brigham Young University, 1972.) Dissertation Abstracts International 33A: 2674; December 1972.

A workbook developed for the beginning concrete-operation stage of number development was found to be superior to commercial programs for developing concepts; no differences were found for computation skills. The manipulative approaches developed positive attitudes toward arithmetic, with greater retention of learning.

Thomas, Gregory P. Field Impact Evaluation. Monmouth, Oregon: Teaching Research, Oregon State System of Higher Education, December 1975.

Fifteen Oregon projects in which teachers had been exposed to mathematics manipulative materials, games, and the laboratory approach were evaluated through a series of field-site interviews. A random sample of 25 percent of the teachers in each project was selected; each teacher was matched with a teacher who had not participated in any of these projects, but was located in the same building and taught at the same level. In addition to the 120 matched pairs of teachers, four students were selected at random from each teacher's class. Both participant and control groups were very similar in their use of various materials. In no instance were either games or manipulative materials found to be the basis of a significant percentage of programs. However, both were highly preferred by teachers, and were used quite frequently, particularly by the participant group (games, 93% and manipulative materials, 87% for participants; for controls, corresponding data were 85% and 69%). Both students and teachers had positive attitudes toward games, while manipulatives were not as highly rated by students.

Thompson, Charles Stanley. The Learning of Multiplication and Other Mathematical Concepts and Skills by Four Children in a Fourth Grade Open Classroom: A Case Study. (The Ohio State University, 1973.) Dissertation Abstracts International 34A: 4584; February 1974.

Four fourth-grade children were observed to determine the mathematical concepts and skills with which they dealt, as well as the processes they used. Tasks involving materials were administered to assess understanding of three interpretations of multiplication.

Tobin, Alexander. An Experimental Study of Teaching Mathematics to Retarded Educable Children in Elementary School Through the Use of Concrete Materials in an Activity-Centered Environment. (Temple University, 1974.) Dissertation Abstracts International 35A: 3412; December 1974.

No significant difference in achievement was found for mentally retarded children aged 6 to 9, but those in the group aged 9 to 12 who had the concrete materials program achieved significantly higher than a group not using the program.

Toney, Jo Anne Staley. The Effectiveness of Individual Manipulation of Instructional Materials as Compared to a Teacher Demonstration in Developing Understanding in Mathematics. (Indiana University, 1968.) Dissertation Abstracts 29A: 1831-1832; December 1968.

Fourth graders were randomly assigned to be taught for 69 days by (1) individual manipulation of materials or (2) teacher demonstration with materials. No significant differences in class means were found, but the group using individually manipulated materials made greater gains in proficiency.

Trafton, Paul Ross. The Effects of Two Initial Instructional Sequences on the Learning of the Subtraction Algorithm in Grade Three. (University of Michigan, 1970.) Dissertation Abstracts International 31A: 4049-4050; February 1971.

In a study with eight third-grade classes, more extensive development of the decomposition algorithm was found to be more effective than a procedure which included work with concepts and use of the number line before the algorithm was taught.

Trask, Marvin Wellington. A Study on Interaction Between Aptitudes and Concrete vs. Symbolic Teaching Methods as Presented to Third-Grade Students in Multiplication and Division. (University of Oklahoma, 1972.) Dissertation Abstracts International 33A: 4253-4254; February 1973.

One group of randomly selected third graders was taught for 49 days by a symbolic method, using the textbook, chalkboard, and multiplication flashcards, while a second group also manipulated concrete objects (egg cartons, pebbles, counting boards) (total n = 65). No significant differences in achievement were found. Regression analysis indicated that pupils of above-average ability were aided more by the materials approach, while those of below-average ability benefited more from the symbolic method.

Trimmer, Ronald G. A Review of the Research Relating Problem Solving and Mathematics Achievement to Psychological Variables and Relating These Variables to Methods Involving or Compatible with Self-Correcting Manipulative Mathematics Materials. ERIC: ED 092 402. (1974)

This research review focuses on determining the psychological variables related to problem solving and presents arguments for self-correcting manipulative materials to teach problem solving. Studies on Cuisenaire rods and other materials and studies involving use of activities are discussed.

Trueblood, Cecil Ross. A Comparison of Two Techniques for Using Visual-Tactual Devices to Teach Exponents and Non-Decimal Bases in Elementary School Mathematics. (The Pennsylvania State University, 1967.) Dissertation Abstracts 29A: 190-191; July 1968.

Trueblood, Cecil R. A Comparison of Two Techniques for Using Visual-Tactual Devices to Teach Exponents and Non-Decimal Bases in Elementary School Mathematics. Arithmetic Teacher 17: 338-340; April 1970.

Pupils in seven fourth-grade classes were randomly assigned to be taught a unit on exponents and nondecimal bases by (1) manipulating visual-tactual aids or (2) observing and telling the teacher how to manipulate such aids. Pupils observing the teacher manipulating materials scored higher ($p = .10$) than pupils manipulating materials themselves. No significant difference in retention was found.

Unkel, Esther R. A Study of the Laboratory Approach and Guided Discovery in the Teaching-Learning of Mathematics by Children and Prospective Teachers. ERIC: ED 056 986. (1971)

Twenty-nine elementary-school pupils were tutored with some use of materials. Some significant gain scores were reported at most grade levels.

Vance, James H. and Kieren, Thomas E. Laboratory Settings in Mathematics: What Does Research Say to the Teacher? Arithmetic Teacher 18: 585-589; December 1971.

In this research review it was concluded that children can learn from and like mathematics laboratory approaches.

Vance, James H. and Kieren, Thomas E. Mathematics Laboratories -- More Than Fun? School Science and Mathematics 72: 617-623; October 1972.

For ten weeks, laboratories were used once a week with some seventh and eighth graders. No significant differences in achievement of work covered in the regular program were found, although one-fourth of mathematics class time was spent in informal exploration. Students strongly preferred the laboratory method. Both laboratory and class-discovery groups scored higher than students in the regular program on cumulative achievement, transfer, and divergent-thinking tests.

Van Engen, Henry and Gibb, E. Glenadine. General Mental Functions Associated with Division. Educational Service Studies, No. 2. Cedar Falls: Iowa State Teachers College, 1956.

In this study on the efficacy of the successive subtractions and distributive algorithms for division, materials were suggested but use was not carefully controlled. Some advantages were found for each algorithm.

Vitello, Stanley John. The effect of three Variables on the Solution of Verbal Problems Requiring Class Inclusion Among Educable Mentally Retarded Children. (The University of Connecticut, 1972.) Dissertation Abstracts International 33A: 2795; December 1972.

The group with a mental age of 10 performed statistically better when given two pictures and iconic presentations; differences at other MA levels (7 through 9) were not significant.

Wallace, Pearlena. An Investigation of the Relative Effects of Teaching a Mathematical Concept Via Multisensory Models in Elementary School Mathematics. (Michigan State University, 1974.) Dissertation Abstracts International 35B: 2989-2999; December 1974.

Pupils from grades 4, 5, and 6 (n = 154) were taught fraction concepts for three weeks using Cuisenaire rods and magnetic fraction parts or with a traditional approach. The materials group scored significantly higher than the traditional group on both achievement and manipulative tests. No significant differences were found between welfare and non-welfare recipients.

Weber, Audra Wheatly. Introducing Mathematics to First Grade Children: Manipulative vs. Paper and Pencil. (University of Calfironia, Berkeley, 1969.) Dissertation Abstracts International 30A; 3373-3374; February 1970.

With six classes of first graders, three from low and three from high SES schools, mathematical concepts were reinforced for one month through use of (1) paper-and-pencil follow-up activities or (2) manipulative and concrete materials for follow-up activities. No significant difference in achievement was found between groups, although a trend favored the use of materials, especially for low SES pupils.

Weeks, Gerald Malcolm. The Effect of Attribute Block Training on Second and Third Graders Logical and Perceptual Reasoning Abilities. (University of Georgia, 1970.) Dissertation Abstracts International 31A: 5681-5682; May 1971.

In a study with second and third graders, attribute block training was found to have a strong positive effect on logical and perceptual reasoning ability.

Wheeler, Larry E. The Relationship of Multiple Embodiments of the Regrouping Concept to Children's Performance in Solving Multi-digit Addition and Subtraction Examples. Dissertation Abstracts International 32A: 4260; February 1972.

A random sample of 144 second-graders was categorized according to three levels of abstraction by testing their performances in regrouping two-digit addition and subtraction examples on the abacus, sticks, place-value chart, and multi-base blocks. There was no significant difference between their performance in solving two-digit examples in the symbolic mode. However, children proficient in regrouping on three or four embodiments scored significantly higher on the multi-digit examples than children not proficient using concrete materials. Significant correlations were found between the number of embodiments children were able to regroup for two-digit addition and subtraction and their performance on multi-digit addition and subtraction.

Whipple, Robert M. A Statistical Comparison of the Effectiveness of Teaching Metric Geometry by the Laboratory and Individualized Instruction Approaches. (Northwestern University, 1972.) Dissertation Abstracts International 33A: 2699-2700; December 1972.

Students in grade 8 (n = 93) who used a laboratory approach with manipulative materials scored higher than students using individualized instruction units on geometric ideas.

Wilkinson, Gerald Glendel. The Effect of Supplementary Materials upon Academic Achievement in and Attitude Toward Mathematics Among Eighth Grade Students. (North Texas State University, 1971.) Dissertation Abstracts International 32A: 1994; October 1971.

Students in grade 8 (n = 136) using supplementary materials (objects, filmstrips, and films) did not show a significant gain in attitude over those using a traditional method, but achievement increased in heterogeneously grouped classes using supplementary materials.

Wilkinson, Jack Dale. A Laboratory Method to Teach Geometry in Selected Sixth Grade Mathematics Classes. (Iowa State University, 1970.) Dissertation Abstracts International 31A: 4637; March 1971.

Sixth graders (n = 232) were taught geometry for 20 days using (1) laboratory units, with worksheets and manipulative materials requiring experiments and data collection, (2) laboratory units which included cassette tapes, or (3) a "more conventional approach". No significant differences in achievement or attitude were found between the three approaches.

Wilkinson, Jack. A Review of Research Regarding Mathematics Laboratories. In Mathematics Laboratories: Implementation, Research and Evaluation (William M. Fitzgerald and Jon L. Higgins, editors). Columbus: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1974. ERIC: ED 102 021.

Research related to the use of mathematics laboratories is reviewed.

Williams, John D. The Evaluation of Three Math Courses. Journal of Structural Learning 3: 41-79; 1972.

Data from a study (1961-1963) are reported for groups using the Dienes or the Cuisenaire program for one and two years, each compared with control groups using a conventional British program. Attrition and various other sampling problems occurred which reduced the experimental groups markedly. After one year's use of the Dienes program, scores were slightly depressed by comparison with those of their control groups. The first-year Cuisenaire groups obtained slightly higher achievement scores than their control groups. After a second year, both groups improved significantly above the level of their respective control groups.

Wolff, Donald J. An Instructional Game Program: Its Effect on Task Motivation. (Rutgers University The State University of New Jersey, 1974.) Dissertation Abstracts International 35A: 3535-3536; December 1974.

For 66 pupils in grades 1 and 2, preference for the cooperative game format (using adaptations of "WFF'N PROOF" and "On-Sets") declined significantly. No evidence was found for a format effect on interest in the subject matter, set theory. When pupils were free to move at their own pace, cooperative procedures appeared better than competitive ones.

Wood, Carolyn M. A Comparison of the Effects of Sequence and Mode upon the Initial Acquisition, Retention, and Transfer of Elementary Multiplication Concepts. (University of Pittsburgh, 1974.) Dissertation Abstracts International 35A: 2068; October 1974.

No significant differences in achievement were found between small groups of pupils in grades 1, 2, and 3 who were introduced to multiplication rules and concepts through inductive or deductive modes and with concrete or pictorial representations. On the retention test, the first graders scored better using the inductive program and concrete materials.

Wrightstone, J. Wayne. Evaluation of the Experiment with the Activity Program in the New York City Elementary Schools. Journal of Educational Research 38: 252-257; December 1944.

As part of an evaluation of a six-year experimentation period, it was found that arithmetic scores of those in the activity group were significantly lower than for those in the non-activity group on one test, and not significantly different on another test.

Wynroth, Lloyd Z. Learning Arithmetic by Playing Games. (Cornell University, 1970.) Dissertation Abstracts International 31A: 942-943; September 1970.

Kindergarten and first-grade groups taught new concepts verbally through a series of competitive games, followed by self-paced written work later, had significantly higher scores on achievement tests than those who had a "normal" program.

Zahn, Karl G. Use of Class Time in Eighth-Grade Arithmetic. Arithmetic Teacher 13: 113-120; February 1966.

Students in grade 8 (n = 120) who spent 56% or 67% of their time on developmental activities scored higher than those who spent the greater proportion of their time on practice.

Activity Centered Math Program. ERIC: ED 093 676. (1973)

Descriptive statistics were used to substantiate the claim that the program, modeled after the Nuffield Project with an emphasis on activity learning, produced acceptable results.

The Use of Coloured Rods in Teaching Primary Number Work. ERIC: ED 028 823. (1964)

In grades 1 and 2 in Canadian schools, significant differences were found on the Cuisenaire test but not on a standardized survey test.

APPENDIX B

DELETIONS FROM LIST OF REFERENCES

Reference	Reason for deletion
Adams, 1971	content: linear ordering
Adkins, 1959	type of study: list of "instructional aids"
Aldrich, 1970	content: Piagetian classification
Allison, 1965	focus: form and color, not specific to materials
Atkinson, 1973	content: identification of geometric shapes
Austin and Jesson, 1974	not research
Bailey, 1974	content: polygonal paths
Baker, 1971	level: grade 9
Bass, 1971	content: topological understandings
Bennett and Walker, 1971	content: categorization
Biancoviso, 1971	content: Piagetian conservation
Biot, 1970	content: Piagetian conservation
Boersig, 1973	level: grade 9
Bowers, 1972, 1973	level: secondary
Branca, 1971, 1974; Branca and Kilpatrick, 1972	content: mathematical structure of Klein Four group
Brumbaugh, 1970	content: geometric representations
Brush, 1973	focus: materials only incidentally involved in survey
Burgess, 1970	level: secondary
Camp, 1971	content: Piagetian conservation
Carey and Steffe, 1971	content: measurement, not focused on materials
Carlson, 1972	content: Piagetian conservation
Chilewski, 1974	content: mathematical structure of Klein Four group
Collins, 1971	focus: mastery learning

Reference	Reason for deletion
Collis, 1971	focus: designing research using card-sorting task
Colvin, 1973	not research
Cowan, 1964	content: Piagetian transformations
Crist, 1969	content: telling time
Davis, 1967	level: grade 9
Deans, 1973	not research
Devor and Stern, 1970	level: preschool
Downs, 1969	focus: building birdhouses
Engel, 1967	focus: use of automated devices
English, 1961	focus: use of color in printed materials
Esty, 1971	content: topology
Finch, 1972	type of study: survey of materials cited in PLAN
Fink, 1974	focus: role of imaginative play and effect on Piagetian tasks
Finnell, 1973	level: grade 9
Folsom, 1959	focus: teachers' manuals
Fortson, 1970	focus: stimulus activities, including rhythms, creative activities
Galtier, 1973	not research
Gatz, 1973	content: copying geometric figures
Gavzy, 1974	focus: not manipulative materials
Goforth, 1938	type of study: drill procedure, 1938
Gorman, 1943	type of study: list of laboratory equipment
Grafft, 1970	focus: not manipulative materials
Gray, 1965	focus: meaningful instruction
Greenberg, 1970	content: geometric forms
Greenes, 1970	content: geometric forms

Reference	Reason for deletion
Greer, 1972	focus: effect of body movement
Gubrud, 1971	content: vectors
Gururaja, 1971	content: Piagetian
Haring and Berman, 1972	focus: not manipulative materials
Heard, 1954	unable to obtain copy
Hilliard, 1972	content: Piagetian
Howard, 1957	type of study: British teachers' reactions
Hutcheson, 1973	level: grade 9
Johanson, 1972	level: grade 9
Johnson, 1972	content: Piagetian classification and seriation
Johnson, 1970	content: categorizing behavior
Jones, 1971	level: preschool
Jones, 1968	level: grade 9
Kamps, 1971	content: Piagetian conservation
Kerr, 1973	focus: only one reference to activities
Khan, 1973	level: preschool
Kidder, 1973	content: transformation, not focused on materials
King, 1973	focus: two forms of Skemp's test, not directly related to classroom activities
Krulik, 1974	not research
LaCrosse, 1967	content: Piagetian
Lamon, 1969	content: mathematical structure of Klein Four group
Lane, 1964	focus: programmed instruction
LaRoche, 1970	not research
Lerch and Mangrum, 1965	type of study: list of "instructional aids"
Lewis, 1969	content: measurement, not focused on materials

Reference	Reason for deletion
Light, 1972	focus: program not stressing manipulative materials
Lindvall and Light, 1974	not research
Loomis, 1965	not research
McCune, 1971	level: preschool
Miller, 1972	not research
Miller et al., 1969	content: Piagetian topology
Murray, 1970	content: Piagetian conservation
Owens and Cooney, 1972	content: Piagetian transitivity
Page, 1971	level: secondary
Papert and Solomon, 1970	not research
Pereira, 1973, 1974	content: mathematical structure of Klein Four group
Phillips, 1968	content: Piagetian
Porteus, 1972	type of study: no data reported
Prater, 1968	focus: study of color in textbooks
Price, 1951	unable to obtain copy
Rea and French, 1972	focus: varied mental computation activities
Reavis, 1973	focus: parent program, with games included but use not controlled
Reeves, 1972	level: secondary
Regula, 1973	focus: technique for mentally retarded, with no emphasis on manipulative materials
Schell, 1965	focus: not manipulative materials
Schnur, 1970	focus: not manipulative materials
Shah, 1971	content: topological concepts
Shively and Asher, 1971	critique of grade 9 study
Smith, D. D., 1974	focus: paper-and-pencil example as a "model"

Reference	Reason for deletion
Smith, 1975	content: Piagetian conservation
Steffe, 1972	content: Piagetian classification
Steinway, 1918	research design: 1918
Stratford, 1971	focus: description of program using workcards
Syer and Ingeneri, 1949	not research
Tate, 1965	focus: audiovisual materials
Waters, 1972	level: secondary
Weiner, 1972	content: Piagetian more/less
Wilderman, 1974	not research
Williams, 1970	level: preschool
Winkelmann, 1974	content: Piagetian conservation
Young, 1974	content: seriation
ED 049 062	focus: no data on effect of use of materials
ED 061 721	focus: simulation/gaming
ED 067 430	focus: not manipulative materials
ED 069 458	focus: no results cited for a range of instructional services
ED 075 561	focus: outcomes specific to activities unclear
ED 077 768	focus: effect of materials not studied
ED 078 611	focus: not on use of materials or room
ED 087 793	focus: on general program, not effect of manipulative materials
ED 087 835	focus: not on manipulative materials
ED 097 204	focus: use of supplementary materials developed to accompany textbook not controlled

APPENDIX C

INSTRUMENT FOR EVALUATING EXPERIMENTAL EDUCATIONAL RESEARCH REPORTS

Since research efforts vary widely in quality, the question of the degree of confidence which can be placed in the findings of a study is one of considerable importance. Many lists of suggestions have been developed to aid in making this judgment; such lists, however, lend themselves best to casual evaluation, and the results are often inconsistent and unreliable. Perusal of the literature (at the time the instrument was being developed) disclosed six instruments for evaluating research. For three of these, no reliability data were available. For one, the reliability was so low that the usefulness of the instrument is questionable. Two were found to be helpful: Johnson (1957), with inter-rater reliability of .75 to .79, and Gephart (1964), with inter-rater reliability of .74. But neither seemed entirely suitable for evaluating experimental studies in elementary school mathematics. More information is needed to support the items on Johnson's list. Gephart's instrument sacrifices the time-consuming rating of each subitem to a purely subjective final rating.

Therefore, 24 lists of suggestions for evaluating educational research proposed by writers in the field were compiled. Nine points were found to be consistently repeated, and these form the basis for the Instrument for Evaluating Experimental Educational Research Reports. The nine questions focus attention on the vital points, and the value of the research in terms of these is specified. Possible flaws which lead to incorrect conclusions and may negate the value of the research are analyzed. The sum of the numerical values assigned to each question provides a basis for comparison [providing the user can overlook the question of summing ordinal numbers].

Two investigations of the degree of reliability or inter-rater agreement which may be expected in the use of the instrument with studies on elementary school mathematics have been reported by Suydam (1968a, 1968b). In one study, inter-rater reliability was found to be .91 (Analysis of Variance reliability formula). (The coefficient estimates the correlation between the combined ratings of the judges in the study and the combined ratings of another hypothetical random sample of judges taken from the same population and rating the same articles.) The coefficient of reliability which provides a measure of the consistency probable with a single rater using the instrument was .77 (Snedecor's formula). In a second study, with a more diverse population of judges, the inter-rater reliability was found to be .94, while the consistency level was .57. In another analysis of the reliability of the instrument when used with reports of research on oral reading, Spire (1974) obtained an inter-rater reliability of .72 (Z-test).

It must be recongized that the instrument has limitations. It is partially subjective, and use of it demands some background in methodology and statistics, as well as in the subject matter field. [For a fuller critique, see Romberg (1970).] However, use of the instrument is more reliable for more people than is a list of suggestions--and certainly better than merely reading a research report without any criteria. A few possibilities for plausible use of the instrument are:

(1) In many reviews of the literature, every study seems to be considered as good as any other--and this is not true. Such reviews could and should reflect careful evaluation. The instrument will aid in directing attention to those studies done in the past whose findings may be most applicable or most questionable.

(2) Use of the instrument can help researchers to identify studies

done so poorly that replication with increased precision is needed.

(3) The instrument seems to be plausible as a guide for evaluative planning of research as well as evaluative reading of research.

(4) The researcher should find the instrument valuable in writing reports of research, using it as a guide to completeness of vital information to include.

Johnston and Burns (1970) concur with such points in their discussion of the nine points of the instrument.

References Cited

Gephart, William J. Development of an Instrument for Evaluating Educational Research Reports. Cooperative Research Project No. S-014, U. S. Office of Education, 1964.

Johnson, Granville B., Jr. A Method for Evaluating Research Articles in Education. Journal of Educational Research 51: 149-151; October 1957.

Johnston, A. Montgomery and Burns, Paul C. Research in Elementary School Curriculum. Boston: Allyn and Bacon, 1970.

Romberg, Thomas A. Review of "An Instrument for Evaluating Experimental Educational Research Reports". Investigations in Mathematics Education 3: 79-82; January 1970.

Spire, Ronald Duane. An Evaluation of Journal Published Research Reports on Oral Reading in the Elementary School, 1900-1970. (The University of Tennessee, 1973.) Dissertation Abstracts International 34A: 6960; May 1974.

Suydam, Marilyn N. An Instrument for Evaluating Experimental Educational Research Reports. Journal of Educational Research 61: 200-203; January 1968.

Suydam, Marilyn N. An Evaluation of Journal-Published Research Reports on Elementary School Mathematics, 1900-1965. (The Pennsylvania State University, 1967.) Dissertation Abstracts 28A: 3387-3388; March 1968.

Instrument for Evaluating Experimental Research Reports

Marilyn N. Suydam
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1. How practically or theoretically significant is the problem? (1-2-3-4-5)

   a. Purpose (important---non-important)
   b. Problem origin
      1) Rationale (logical---illogical)
      2) Previous research (appropriate---inappropriate)

2. How clearly defined is the problem? (1-2-3-4-5)

   a. Question (operational---vague)
   b. Hypothesis(es) (relevant---irrelevant) (logical---illogical)
   c. Independent variable(s) (relevant---irrelevant)
   d. Dependent variable(s) (operational---vague) (relevant---irrelevant)

3. How well does the design answer the research question? (1-2-3-4-5)

   a. Paradigm (appropriate---inappropriate)
   b. Hypothesis(es) (testable---untestable)
   c. Procedures (clear---unclear)
   d. Treatments (replicable---unreplicable) (appropriate---inappropriate)
   e. Duration (appropriate---inappropriate)

4. How adequately does the design control variables? (1-2-3-4-5)

   a. Independent variable(s) (uncontaminated---contaminated)
   b. Administration of treatment (rigorous---unrigorous)
   c. Teacher or group factors (controlled---uncontrolled)
   d. Subject or experimenter bias (controlled---uncontrolled)
   e. Halo effect (controlled---uncontrolled)
   f. Extraneous factors (controlled---uncontrolled)
   g. Individual factors (controlled---uncontrolled)

5. How properly is the sample selected for the design and purpose of the research? (1-2-3-4-5)

   a. Population (appropriate---inappropriate)
   b. Drawing of sample (random---unspecified)
   c. Assignment of treatment (random---unspecified)

d. Size (appropriate---inappropriate)
e. Characteristics (appropriate---inappropriate)

6. <u>How valid and reliable are the measuring instruments or observational techniques?</u> (1-2-3-4-5)

a. Instrument or technique
   1) Description (excellent---poor)
   2) Validity (appropriate---inappropriate)
   3) Reliability for population (excellent---poor)
b. Procedure of data collection (careful---careless)

7. <u>How valid are the techniques of analysis of data?</u> (1-2-3-4-5)

a. Statistical tests
   1) Basic assumptions (satisfied---unclear)
   2) Relation to design (appropriate---inappropriate)
b. Data
   1) Treatment (appropriate---inappropriate)
   2) Presentation (clear---unclear)
   3) Level of significance (appropriate---inappropriate) (specified---unspecified)
   4) Discussion (accurate---inaccurate)

8. <u>How appropriate are the interpretations and generalizations from the data?</u> (1-2-3-4-5)

a. Consistency with results (excellent---poor)
b. Generalizations (reasonable---exaggerated)
c. Implications (reasonable---exaggerated)
d. Limitations (noted---not noted)

9. <u>How adequately is the research reported?</u> (1-2-3-4-5)

a. Organization (excellent---poor)
b. Style (clear---vague)
c. Grammar (good---poor)
d. Completeness (excellent---poor) (replicable---unreplicable)

APPENDIX D

TREATMENT DESCRIPTION INVENTORY

Reference: ____

____

____

____

☐ Research ☐ Research review

Data detail: ☐ little ☐ much

Re-analysis: ☐ possibly ☐ no

☐ Non-research

☐ Theoretical analysis

☐ Position paper

☐ Teaching idea

Notes:

1. Math. content/concept: ______________

____

____

____

2. Grade level: __________ Age: ______

3. Theoretical basis: ☐ no ☐ yes

Reference: ____

4. Characteristics of sample: Title I - yes ☐ no ☐ SES level ______

cultural factors ____

achievement expectation ______________________________________

geographic region ______________ urban ☐ rural ☐

5. Time: for entire study - weeks _____ days _____ minutes/day _____

for treatment - weeks _____ days _____ minutes/day _____
(material-use phase)

for testing - pre __________ post __________ retention __________

6. Special time factors: ______________________________________

7. Sequence placement of activity: ☐ initial ☐ development ☐ practice

____

8. Use of symbols: ☐ initial ☐ throughout ☐ final culmination

____

9. Social interaction: ☐ large class ☐ groups ☐ individuals ☐ isolated

____

10. Materials used: ______________________

____

Generality of use: high . . . . low

11. Variability: number of different examples or explanations ______

number of different materials ______

12. Manipulative level:

a. Level of access: [ ] remote demonstration

[ ] cooperative demonstration

[ ] large group action

[ ] small group action

[ ] individual

b. Type of use: [ ] object manipulation

[ ] object study

[ ] picture study

[ ] situation-object study

[ ] object-symbol progression

[ ] paper-pencil progression

c. Why manipulated: [ ] game [ ] puzzle [ ] experiment [ ] procedure

13. Guidance:

	highly structured	varying	moderately structured	minimally structured	free exploration
a. Teacher:	[ ]	[ ]	[ ]	[ ]	[ ]
b. Material:	[ ]	[ ]	[ ]	[ ]	[ ]

c. Rules: [ ] arbitrary [ ] material-determined [ ] non-relevant

14. Cost of materials: commercial - [ ] high [ ] moderate [ ] low

non-commercial - [ ] high [ ] moderate [ ] low

15. Special classroom facilities: ______________________

____

16. Teacher training: [ ] experimenter taught [ ] special sessions [ ] training materials [ ] minimal training

17. Extra staffing: [ ] no [ ] yes, ______________________

18. Total cost of implementation: [ ] high [ ] moderate [ ] low

19. Correlation with educational program: ______________________

____

Part II. Study Description Inventory

Number: ____

1. Purpose of study: ______________________________

2. Type of study: [ ] experiment [ ] ex post facto [ ] action [ ] case study
[ ] survey Comments: ______________________________

3. Sample size: total - classes __________ students __________
per group ______________________________

4. Sampling procedures: [ ] selected [ ] random [ ] unspecified

5. Variables: independent: ______________________________
dependent: ______________________________
tests/measures used: ______________________________
controlled: ______________________________

6. Statistical procedures: ______________________________

7. Procedures: ______________________________

8. Findings: ______________________________

9. Prediction: discuss - [ ] definitely [ ] probably
cite - [ ] definitely [ ] probably

APPENDIX E

DATA FROM STUDIES ON TABLE 2

Study by Anderson, 1958

Test	Data: Means Materials	Control	t-test	p
Progress Test (experimenter-constructed) 40 items r = .87 (KR)	26.43	26.67		
Surfaces Test 30 items r = .87; Solids and Right Angles Test 26 items r = .89 (both experimenter-constructed)	28.47	27.70	1.15	n.s.
Retention Test (experimenter-constructed) 28 items r = .89	11.25	10.68	1.42	n.s.

Study by Babb, 1976

Test	Data: Gain score*				
	Materials	Imagery	Text	F	p
Recall of basic facts (6 versions of Stanford Diagnostic Arithmetic Test) 40 items $r = .90$	89.17	70.26	78.56	4.86	.01**

* Sum of scores on 5 tests during treatment minus initial test score, after adjusting with covariate of score on Metropolitan Achievement Test.

** M-I is significant, T-M is not (p. 71).

Study by Bisio, 1971

Test	Data: Comparisons of means	Use	Passive use	Non-use	t-test	p
Posttest in Fractions (experimenter-constructed)	East school	19.21	17.60		1.85	n.s.
27 items		19.34		19.46	.378	n.s.
			17.41	19.32	1.79	n.s.
	West school	19.87	20.13		.766	n.s.
		19.64		20.84	2.19	n.s.
			19.41	22.22	3.24	.01

E-3

176-A

Study by Bledsoe et al., 1974

Test	Data: Adjusted means (Covariance: pretest)		F ratio	p
Basic Skills in Arithmetic Test	Materials	Paper-pencil		
Posttest	30.70	29.19	27.39	.001
Retention test	30.88	26.28	63.50	.001

E-4

Study by Boldúc, 1970

Test	Data: Means			F ratio	p
	Direct	Indirect	Verbal		
Experimenter-constructed test 12 items r = .65 (KR)	2.97	3.19	2.03	10.79	.01

Study by Bring, 1972

Test	Data: Means		t-test	p
	Materials	Control		
Posttest I (Experimenter-constructed) 16 items $r = .85$ (KR)	21.94	19.92	1.42	$.01 < p < .05$
Posttest II (Experimenter-constructed)	25.02	20.79	2.82	.005

Study by Brown, 1973

Test	Data: Means Text-only	Text+film	Text+mat.	Text+film +mat.	p*
California Test of Basic Skills	30.79	35.17	39.81	36.67	
Test of Basic Arithmetic Skills (needed to understand concepts in film) 24 items	18.53	20.13	19.92	20.11	
Experimenter-constructed test 30 items					
Pretest	7.80	8.04	9.38	8.18	
Posttest	9.01	11.92	12.82	13.91	
Mean gain	1.21	3.88	3.44	5.73	
Effectiveness index	.0546	.1873	.2156	.2597	

* No significance levels were reported by the researcher.

Study by Carney, 1973

Test	Data: Mean gains	Materials	Field axioms	F ratio	p
Fractions test by Fincher (1963)	IQ 108+	16.880	26.875		
	IQ 96-108	10.230	19.980	33.78	.05
	IQ 96-	10.940	13.555		

E-8

Study by Coltharp, 1969

Test	Data: Means Concrete	Abstract	t-test	p
Achievement test (experimenter-constructed)	28.30	27.54	0.57	n.s.

Study by Cook et al., 1968

Test	Data: Means				F ratio	p
	Text	TV	TV+text	Materials		
Teacher-constructed test	87.85	82.78	88.17	88.47	2.536	.10
Metropolitan Readiness Test (used as covariate)	16.21	16.52	15.85	15.51		

Study by Davidson, 1973

Test	Data: Mean gain in months			p
	Grade	Materials	Control	
Iowa Test of Educational Achievement (Arithmetic Concepts section)	3 av/lo IQ	5.05	7.58	n.s.
	3 hi IQ	3.86	7.74	n.s.
	4 av/lo IQ	7.62	7.88	n.s.
	4 hi IQ	8.04	7.74	n.s.

Note: Significant differences were found on conservation tests.

Study by Dunlap et al, 1971

Test	Data: Means	Materials	Text	p
Comprehensive Test of Basic Skills: Arithmetic, Form 01				
Computational Skills Subtest	pre	35.41	36.71	
	post	42.42	43.71	n.s.
Knowledge of Concepts Subtest	pre	15.53	16.81	
	post	17.51	18.50	n.s.
Applications Subtest	pre	10.33	10.55	
	post	10.90	11.47	n.s.
Total	pre	61.27	64.07	
	post	70.83	73.69	n.s.
Arithmetic Achievement Test (experimenter-constructed)	posttest			
Manipulation		11.01	10.26	.05
Computation		18.31	21.41	n.s.
Applications		1.72	1.57	n.s.
Total		31.04	33.24	n.s.

Study by Earhart, 1964

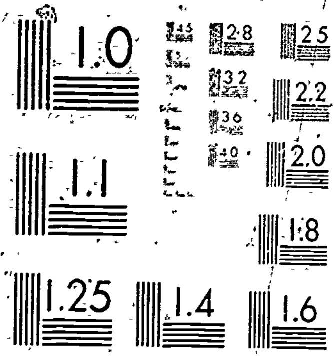
Test	Data: Grade level equivalents		t-test	p
California Achievement Test, Upper Primary	Materials	Control		
Reasoning	4.15	4.27	1.95	n.s.
Fundamentals	4.16	3.84	-5.82	.01

Study by Macy, 1957

Test	Data: Means Materials	Control	t-test	p
Inventory of Mathematical Concepts (NYC)				
for Grade 3 (pre)	29.2	29.0		
50 items				
r = .944 (KR)				
for Grade 4 (post)	30.2	34.6	1.45	n.s.
56 items				
r = .929 (KR)				
Teacher-made tests:				
Calendar (19 items)	13.6	14.7		
Addition facts (81)	71.4	69.4		
Subtraction facts (81)	61.8	58.7		
Measures (13)	10.9	11.4		
Decimal system, fractions (17)	12.6	13.0		
Place value (13)	9.4	10.3		
Fractions (8)	5.3	6.1		
Problems (10)	4.8	6.5		
Total	23.7	23.8		n.s.

Study by McMillian, 1973

Test	Data: Means Materials	Control	t-test	p
Learning test (experimenter-constructed) 59 items	33.12	36.19	1.47	n.s.
Transfer test (experimenter-constructed)	3.40	5.28	1.35	n.s.
Retention test (experimenter-constructed)	29.85	30.15	0.14	n.s.



MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS 1963-A

Study by Nichols, 1972

Test	Data:	Means Materials	Control	F ratio	p
Multiplication	pre	21.56	19.96	6.12	.05
	post	40.80	26.08	604.74	.01
	retention	42.68	24.14	541.25	.01
covariance:	pre-post difference	19.40	6.04	551.85	.01
	post-retention diff.	1.76	-1.86	26.74	.01
Division	pre	12.58	14.75	7.36	.01
	post	33.31	19.36	429.56	.01
	retention	36.52	16.26	544.00	.01
covariance:	pre-post difference	22.97	4.64	711.88	.01
	post-retention diff.	0.74	-2.39	26.80	.01

NOTE: Separate covariance is also shown for pupils whose IQ is less than 105. Trends are the same as above.

Study by Nickel, 1971

Test	Data: Means				p
Stanford Achievement Test: Arithmetic Applications	Materials A_3	Verbal A_2	Control A_1	Tukey test:	
Form X, posttest	14.43	11.41	13.16	$A_2 - A_1 = -1.94$	n.s.
				$A_3 - A_1 = 1.41$	n.s.
				$A_3 - A_2 = 3.35$	.10
Form W, retention	14.59	14.86	13.96	F = 0.092	n.s.

E-17

Study by J. Smith, 1974

Test	Data: Mean gains			p
Experimenter-developed tests r = .825 (KR)	Multimodel	Unimodel	Expository	
Area formulas:				
square	.5833	.4614	.8888	n.s.
right triangle	.0000	.1999	.7692	m/e, .0012 u/e, .029
parallelogram	.2499	.7999	.8799	m/e, .035
triangle	-.0002	.4002	.5714	n.s.
trapezoid	.0000	.0000	.9231	n.s.
Applying with integers:				
square	.0000	.0000	.2221	n.s.
right triangle	.0000	.2000	.5384	m/e, .0077
parallelogram	1.2500	1.7999	1.5999	n.s.
triangle	.0000	.0000	.4286	n.s.
trapezoid	.0000	.0000	.5384	n.s.
Applying with rational numbers:				
square	.4583	.1538	.3704	n.s.
right triangle	.0500	.1499	.3077	n.s.
parallelogram	.6249	.2000	.8000	u/e, .0259
triangle	.0000	.0000	.2857	n.s.
trapezoid	.0000	.0000	.3846	n.s.

E-18

Study by Tobin, 1974

Test	Data: Between-groups F ratios	Pre	Post	p
Individual Arithmetic Achievement Test for Retarded Children (given to ages 9-12) 99 items	Total test:	<1	21.47	.01
	Applications:	<1	14.7	.01
	Operations:	<1	26.74	.01
	Contents:	2.8	56.79	.01
Individual Arithmetic Test for EMR Children Ages 6-9 95 items	Total test:	<1	3.55	n.s.
	Applications:	<1	2.23	n.s.
	Operations:	1.68	<1	n.s.
	Contents:			n.s.

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194-A

Study by Trask, 1973

Test	Data: Means		t-test	p
	Materials	Symbolic		
Computation test (Experimenter-constructed) 24 items r = .8048 (KR)	14.90	16.14	.765	n.s.
Application test (Experimenter-constructed) 16 items r = .9061 (KR)	6.73	7.14	.405	n.s.
Total test 40 items r = .8923 (KR)	21.63	23.14	.580	n.s.

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195-A

Study by Weber, 1970

Test	Data:	Materials	Paper-pencil	F ratio	p
Metropolitan Readiness Test-Arithmetic	Means (pre)				
	Mid SES	44.02	47.44		
	Low SES	48.46	44.86		
	Mean differences (pre-post)			Methods, .492	
	Mid SES	2.78	2.45	SES, .015	n.s.
	Low SES	2.82	2.53	Interest, .003	
Oral Test of Understanding (interview)	number correct	901	656		
	number incorrect	155	301		
	percent correct	.853	.685	Chi square = 80.6233	.001
	percent incorrect	.147	.315		

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