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ABSTRACT

Information is provided on the methods, rationale, and tests used by PMDC investigators to obtain data on the status of first and second graders during the first five weeks of the school year. Instruments to assess general intelligence, cognitive mathematical concepts and skills, socioeconomic status, and school environment of students from seven schools in four states are described. Data from up to 279 first graders and 137 second graders are summarized and discussed. Reports of four investigative follow-up studies are also included. (MS)

PMDC Technical Report
No. 1

1974 Fall Testing Program and Analysis of the Data

Edited by Tom Denmark

pmdc

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PREFACE

Ed Begle recently remarked that curricular efforts during the 1960's taught us a great deal about how to teach better mathematics, but very little about how to teach mathematics better. The mathematician will, quite likely, agree with both parts of this statement. The layman, the parent, and the elementary school teacher, however, question the thesis that the "new math" was really better than the "old math." At best, the fruits of the mathematics curriculum "revolution" were not sweet. Many judge them to be bitter.

While some viewed the curricular changes of the 1960's to be "revolutionary," others disagreed. Thomas C. O'Brien of Southern Illinois University at Edwardsville recently wrote, "We have not made any fundamental change in school mathematics."[1] He cites Allendoerfer who suggested that a curriculum which heeds the ways in which young children learn mathematics is needed. Such a curriculum would be based on the understanding of children's thinking and learning. It is one thing, however, to recognize that a conceptual model for mathematics curriculum is sound and necessary and to ask that the child's thinking and learning processes be heeded; it is quite another to translate these ideas into a curriculum which can be used effectively by the ordinary elementary school teacher working in the ordinary elementary school classroom.

Moreover, to propose that children's thinking processes should serve as a basis for curriculum development is to presuppose that curriculum makers agree on what these processes are. Such is not the case, but even if it were, curriculum makers do not agree on the implications which the understanding of these thinking processes would have for curriculum development.

In the real world of today's elementary school classroom, where not much hope for drastic changes for the better can be foreseen, it appears that in order to build a realistic, yet sound basis for the mathematics curriculum, children's mathematical thinking must be studied intensively in their usual school habitat. Given an opportunity to think freely, children clearly display certain patterns of thought as they deal with ordinary mathematical situations encountered daily in their classroom. A videotaped record of the outward manifestations of a child's thinking, uninfluenced by any teaching on the part of the interviewer, provides a rich source for conjectures as to what this thinking is, what mental structures the child has developed, and how the child uses these structures when dealing with the ordinary concepts of arithmetic. In addition, an intensive analysis of this videotape generates some conjectures as to the possible sources of what adults view as children's "misconceptions" and about how the school environment (the teacher and the materials) "fights" the child's natural thought processes.

The Project for the Mathematical Development of Children (PMDC)[2] set out to create a more extensive and reliable basis on which to build mathematics curriculum. Accordingly, the emphasis in the first phase is to try to understand the children's intellectual pursuits, specifically their attempts to acquire some basic mathematical skills and concepts.

The PMDC, in its initial phase, works with children in grades 1 and 2. These grades seem to comprise the crucial years for the development of bases for the future learning of mathematics, since key mathematical concepts begin to form at these grade levels. The children's mathematical development is studied by means of:

1. One-to-one videotaped interviews subsequently analyzed by various individuals.

2. Teaching experiments in which specific variables are observed in a group teaching setting with five to fourteen children.

3. Intensive observations of children in their regular classroom setting.

4. Studies designed to investigate intensively the effect of a particular variable or medium on communicating mathematics to young children.

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[1] "Why Teach Mathematics?" *The Elementary School Journal* 73 (Feb. 1973), 258-68.

[2] PMDC is supported by the National Science Foundation, Grant No. PES 74-18106-A03.

5. Formal testing, both group and one-to-one, designed to provide further insights into young children's mathematical knowledge.

The PMDC staff and the Advisory Board wish to report the Project's activities and findings to all who are interested in mathematical education. One means for accomplishing this is the PMDC publication program.

This publication is intended to share with the reader the information obtained from the Fall 1974 Testing Program, including a summary of the data collected during the program, analyses and/or interpretations of selected facts, and the results of selected investigative studies conducted as follow-ups to the Fall Testing Program. We hope the reader will find this publication to be a rich source of ideas about the mathematical status of first and second grade children.

Those wishing to consult the non-commercial testing and resource materials used by PMDC in gathering the data presented herein are directed to the SMSG Elementary Mathematics Project Technical Reports Nos. 2 and 3, Stanford University, 1971. For a description of the Hollingshead Socioeconomic Index, refer to NLSMA Reports, No. 9, Non-Test Data, 1968. Resource materials, data collections forms, and reports developed by PMDC and referred to in this publication include directions for administering the SMSG Scales, Grades 1 and 2, the "Report On Preliminary Testing Program," Master Record forms for demographic data, the School and Class Profile Questionnaires, and Summaries of First and Second Grade Data By Individual Schools. Any of all materials are obtainable by writing PMDC; please use bibliography.

Many individuals contributed to the activities of PMDC. Its Advisory Board members are: Edward Begle, Edgar Edwards, Walter Dick, Renee Henry, John LeBlanc, Gerald Rising, Charles Shock, Stephen Willoughby and Lauren Woodby. The principal investigators are: Merlyn Behr, Tom Denmark, Stanley Erlwanger, Janice Flake, Larry Hatfield, William McKillip, Eugene D. Nichols, Leonard Pikaart, Leslie Steffe, and the Evaluator, Ray Carry. A special recognition for this publication is given to the PMDC Publications Committee, consisting of Merlyn Behr (Chairman), Thomas Cooney and Tom Denmark. Thanks are due to graduate students who participated in the administration of the tests: Bill Anderson, Pat Campbell, Cynthia Clarke, Marty Cohen, Marsha Fleming, Max Gerling, Fran Logan, Myrtle Manning, Curtis Spikes and Hal Willis. Thanks are also due to the Project administrative assistant, Janelle Hardy, for coordinating the technical aspects of the preparation of this report, to Lucy Kalogera for editing the manuscript, and to Joe Schmeder for the typing.

Eugene D. Nichols
Director of PMDC

I. METHODS, RATIONALE, AND TESTS

During the summer of 1974, the PMDC Advisory Board and the PMDC Planning Committee were establishing (a) specific objectives for PMDC, (b) operational procedures, and (c) proposals for research studies. One outgrowth of these activities was a decision to obtain a core of descriptive information on all pupils who might participate in PMDC research studies. Both the Advisory Board and the Planning Committee felt that such data would contribute to the general PMDC objectives and would support the work of the individual principal investigators. Specifically, the rationale for collecting this base-line data was three-fold:

1. To provide each principal investigator with pertinent information for selecting subjects to participate in an investigative study.

2. To provide each principal investigator with data to construct individual and/or class profiles, both of which might be necessary components of his research report.

3. To provide the principal investigator and other users of the research reports with a basis for making meaningful interpretations of the research findings.

The data to meet the above needs were obtained from a battery of tests administered during the first six weeks of the 1974-75 school year and from information available in school files. After processing, the appropriate raw data were transmitted to the principal investigators for their immediate use.

DESCRIPTION OF BASE-LINE DATA

The specific facts which comprise the core of descriptive data may be classified in one of four general categories: general intelligence, cognitive concepts and skills, socioeconomic status, and school environment. The composite information from these four categories provides an adequate, although perhaps minimal, background for assessing the validity and/or usefulness of the observations and conclusions reported in the various research studies. Specifically, the components of the base-line data core were selected to furnish the following types of information:

1. General intelligence: A measure of each child's mental adaptability provides an indication of the pupil's academic potential. This information serves as a basis for comparing the results of an investigation against predicted outcomes.

2. Cognitive concepts and skills: Measures of each child's acquisition of facts and attainment of concepts as well as problem-solving behaviors indicate the child's prior success in learning school-related concepts and skills. Such information offers a basis for making comparative assessments of the pupil's achievement (past and future) in academic areas and identifies, for diagnostic purposes, areas of deficiency. In order to obtain a more complete picture of the pupil's cognitive development, reading and/or mathematical concepts and skills were assessed. For beginning first graders, data related to cognitive development reflect the child's readiness for first grade instruction. For beginning second graders, such data reflect to some degree the pupil's academic achievement during the preceding school year.

3. Socioeconomic status: An index of the pupil's socioeconomic environment provides a measure of non-academic factors and variables which may influence a child's academic success.

4. School environment: Factual information related to the school organizational structures, to the curriculum, to the instructional strategies employed by the regular classroom teachers, and to the community served by the school provide a description of the educational setting in which the research studies were conducted.

SELECTION OF ASSESSMENT INSTRUMENTS

Once a decision had been reached on the general classifications of data to be collected, members of the PMDC staff considered several alternative procedures for gathering the data. The Planning Committee decided to use, wherever possible, existing evaluation instruments. This decision was primarily based on two considerations. First, an established base was needed to compare the data collected for the PMDC 1974 Fall Testing Program. Second, the development of evaluation instruments and procedures had not been identified as a major theme for PMDC during the first year of operation. The various instruments utilized in the 1974 Fall Testing Program are identified in the following sections. Included in the discussions are a brief description of each instrument and the rationale for its selection:

1. *The Stanford-Binet Intelligence Scale, Form L-M (short version), 1960 revision* was used to obtain a measure of general intelligence at both the first and second grade levels. The short version contains four subtests. For example, the Year VI Scale included the following subtests: vocabulary, differences, number concepts, and composite analogies. For year VII, the subtests were similarities, copying, comprehension, and repeating digits. The Stanford-Binet scale is individually administered. It was selected because it provides a reliable and valid measure of a child's mental adaptability and the derived IQs are comparable at all age levels.

2. *The Metropolitan Readiness Tests, Form A, 1969* was administered to the first grade pupils to obtain a measure of the development of certain skills and abilities: word meaning, listening, matching, alphabet, numbers, and copying. Each subtest is individually timed, with tests 1, 2, 4, and 5 timed item-by-item. The Metropolitan Readiness Tests are group administered and require the pupils to follow directions and handle a paper and pencil test.

This test was selected in preference to other comparable tests because (a) this test was administered as part of the School Mathematics Study Group (SMSG) Elementary Mathematics Project (ELMA), thus providing the potential for making comparison of the PMDC data and the ELMA data, and, (b) over 50% of the schools participating in the PMDC program administer this test as part of their regular testing program.

3. *The Metropolitan Achievement Test, Primary I, Form F, 1970*, was administered to the second grade pupils to obtain a measure of how much the pupils had learned in important content and skill areas of the first grade school curriculum. Consisting of four subtests (word knowledge, word analysis, reading, and math concepts), this test is group administered and timed on each subtest, and was included in the PMDC test battery because (a) it is one of a series of tests covering grades K-9, and (b) it is part of the regular testing program in over 50% of the participating PMDC schools.

4. *The SMSG Elementary Mathematics Project* test battery was administered at the first and second grade levels. The composite test given at the first grade level consisted of four scales: Scale 204, counting members of a given set—picture cards; Scale 205, equivalent sets—dots; Scale 206, ordering geometric shapes; and Scale 211, classifying. These scales are administered one-to-one, but are not timed. The directions for administering these scales and the test items are cited in the bibliography (Item A).

At the second grade level, the composite test included five subtests: Scale 401, number comparison and order; Scale 402, place value; Scale 403, comprehension; Scale 404, applications; and Scale 405 computation--addition. These scales are administered to groups of 6-8 pupils and are not timed. Oral directions are given for each item on the first four scales. The pupils work independently on the fifth subtest. The procedures for administering these scales and the test items are cited in the bibliography (Item B).

These SMSG scales were included in the PMDC test battery because (a) the concepts and skills evaluated on these tests were important to the work of the various PMDC principal investigators, and (b) the existence of the SMSG data on these scales provided the potential for making comparisons with the data collected from the PMDC Testing Program.

5. *The Hollingshead Socioeconomic Index* (SEI) was selected to obtain a measure of each pupil's social class. The Hollingshead scale is a two-factor index utilizing information about parental occupation and education, and was selected because the pertinent information was readily available in existing school records. Also, formulae were available for estimating the SEI, if information about only one of the factors was available in the school records (see bibliography, Item C).

6. *Questionnaires* prepared by the PMDC staff obtained data pertaining to the school and class environments. Information comprising the school profile included total school size, grade levels, the organization of classes at each grade level, a description of the community served by the school, special services available to teachers in the school, and the source of monetary support. For each class from which pupils were chosen to

participate in a PMDC investigative study, descriptive facts obtained were class size, method of assigning pupils to the class, textbook(s), the mode of instruction typically utilized by the regular classroom teachers, the use of materials to supplement the instructions provided in the textbook(s), and the availability of additional instructional assistance. The school profile and class profile questionnaires are cited in bibliography (Item D).

PROCEDURES FOR COLLECTING AND REPORTING DATA

Since data for the 1974 PMDC Testing Program were obtained from seven schools at four geographical sites by seven principal investigators, there was general concern from the outset that guidelines be established to ensure, to the greatest extent possible, uniform methods of data collection and reporting. Thus, detailed instructions were provided for the administration and the scoring of each test, each person involved in the testing program participated in an appropriate training program, and special forms were provided for recording data.

Other major factors considered in the design of the testing program were the amount of time needed to administer the entire battery of tests and the feasibility of obtaining certain data from school records or from parents. In order to minimize the additional demands placed on pupils and teachers by the testing program, data obtained previously for other purposes were utilized. For example, if a test included in the test battery had been previously administered as part of the regular school testing program, the data from this earlier test were collected for PMDC purposes.

The rights and privacy of the pupils were ensured by assigning to each pupil an identification number for the purposes of recording and reporting data. Also, parental permission for a child to take certain PMDC tests and/or for gathering data from the pupil's permanent record was secured. If a parent requested that the child not participate in the Testing Program, his request was honored. In addition, each principal investigator was responsible for gathering data related to school and class environments and for recording same on forms supplied (see bibliography, Item D). A file by schools on all data collected as part of the PMDC 1974 Fall Testing Program is maintained at the Tallahassee site. The data in each school file is recorded on a master record sheet providing comprehensive coverage of data pertaining to each pupil (see bibliography, Item E).

Details on the administration of the 1974 PMDC Testing Program follow:

Stanford-Binet Intelligence Scale. At three of the four PMDC sites, this test was administered by advanced graduate students majoring in psychology, each of whom was certified as being qualified to administer the Stanford-Binet Intelligence Scale. At the fourth site, the PMDC Staff received appropriate training to become qualified to give this test. In two schools, an IQ measure was obtained on each pupil in a participating class. In the other schools, to obtain such a comprehensive coverage of IQ data was not possible because the size of the sample population made the cost of administering the IQ instrument prohibitive, some parents refused to grant permission for the test to be given to their child, and some principals were reluctant to agree to a large-scale IQ testing program in their schools. However, in situations where IQ measures could not be secured on entire classes, the Stanford-Binet was given to most pupils in either an experimental or control group. Due to delays in securing permission from parents and school officials to administer the Stanford-Binet, this phase of the testing program was not completed until February 1975.

Metropolitan Tests. The Metropolitan tests, Readiness for grade one and Primary I for grade two, were administered by the regular classroom teacher or a graduate student working for PMDC. In each case, the tester was instructed to follow the directions provided in the appropriate instructional manual. No special training for administering these tests was given. In four schools, these tests were part of the regular testing program and were scored by PMDC Staff members at the various sites. In one school, the Metropolitan Achievement Tests, Primary I, Form H, had been previously given as part of the regular first grade end-of-year (1973-74) testing battery. Therefore, the Primary I, Form F, test was not given to these second grade students in September 1974. Data obtained from the Metropolitan Primary I tests administered in this school were not included in the statistical analyses given in this report. The administration of this component of the test battery was completed by mid-October 1974.

SMSG Elementary Mathematics Project - First Grade Scales. Materials for these tests were reproduced with permission of the SMSG Director. These scales were administered by PMDC principal investigators and graduate students according to the instructions provided in the SMSG materials; each tester also received special training on administering these scales. At the Tallahassee site, the project staff had an opportunity to practice giving the SMSG scales in a non-participating school during the last part of August 1974. Videotapes were made of this trial testing experience and were used, along with other observational notes, to assist each staff member in analyzing his/her testing behavior. A report on this pilot testing program was distributed to principal investigators at each of the other sites.

The SMSG first grade tests were administered in two schools in Tallahassee during the first two weeks of September 1974. Approximately one-half of the sessions were videotaped for later analyses. Excerpts from these tapes, highlighting significant pupil and/or tester behaviors, were used to make a composite tape of the administration of the SMSG First Grade Scales. This tape was distributed to principal investigators at each of the other sites.

The composite videotape, the report on the preliminary testing program, and the SMSG instructions were used by principal investigators at the other three sites to design a training program for PMDC staff members who would assist in the administration of the SMSG first grade tests. The administration of the SMSG First Grade Scales was completed in all but one school by the end of the first week of October 1974. A delay in reaching an agreement with school officials for PMDC staff members to work in that school postponed administration of this test until mid-October 1974.

The tests were scored according to SMSG instructions. In addition to SMSG scoring procedures, the problem-solving strategies or techniques used by pupils in responding to each question were noted by each tester. Instructions for coding these pupil behaviors are given on the Pupil Score Sheet cited in bibliography (Item F).

SMSG Elementary Mathematics Project - Second Grade Scales. Materials for these tests were reproduced with permission of the Director of SMSG, and were administered by PMDC staff members at each of the four sites, following instructions provided in the SMSG materials. Procedures for training personnel and for scoring this test were similar to those employed for the SMSG first grade tests. However, a videotape for training purposes was not prepared nor were pupils' behaviors recorded, since the second grade tests was group administered. The second grade SMSG test had been given to all pupils by mid-October 1974.

Socioeconomic Index. The collection of data necessary for the computation of SEIs was the responsibility of each principal investigator. Procedures for gathering this information were available in materials reprinted, with permission, for *NLSMA Reports, No. 9, Non-Test Data* (bibliography, Item C). The required information was generally available in school records. However, in one school permission to use this information was granted only for the pupils involved in the PMDC study, not for every child who participated in the PMDC Testing Program. Another school did not grant permission for the collection of the necessary information. The data collected for computing SEIs were recorded on forms provided to each principal investigator. These forms were forwarded to the Tallahassee site where the SEIs were computed, and each principal investigator was apprised of the SEIs computed for the pupils in his sample.

II. DESCRIPTION OF SCHOOLS

The battery of PMDC tests was administered in schools at four sites: Tallahassee, Florida; Athens, Georgia; Austin, Texas; and Athens, Ohio. A total of seven schools participated in the testing program (three in Athens, Georgia; two in Tallahassee, and one at each of the other two sites), providing a variety of educational and community environments. Pertinent descriptive data on these schools follow, with each school assigned a number from 1 through 7.

Six of the seven schools were primarily elementary schools. Four of these schools served pupils in grades K through 5; two other schools, 1 through 5. The seventh was a comprehensive school encompassing grades K through 12. The enrollments ranged from 274 to 887 pupils. The school with 887 pupils included grades K through 12, with approximately 300 pupils in grades K through 5. Six of the seven schools were part of a local public school system. The seventh is best classified as a university developmental research school. A summary of the data related to school size, grade levels, and support is provided in Table 1.

Table 1

Enrollment, Grade Levels and Support by Schools

School	Enrollment	Grade Levels	Support
1	400	K-5	Public
2	887	K-12	Univ
3	560	K-5	Public
4	274	K-5	Public
5	340	K-5	Public
6	381	1-5	Public
7	418	1-5	Public

Six of the seven schools draw pupils from either urban or suburban communities. Three of these schools serve an inner-city or an ethnic minority of the community. The seventh school serves a small city and its surrounding rural area. In all seven schools, the pupil population was diverse with respect to family income; that is, the pupils attending each school were drawn from neighborhoods with a range of socioeconomic classifications. For reasons previously stated, it was impossible to obtain data necessary for the derivation of a socioeconomic index (SEI) for each pupil participating in the Fall Testing Program. Table 2 contains a summary of the available data.

Table 2

Summary of Available Socioeconomic (SEI) Data by Schools

School	Number Of Observations	Lowest SEI	Highest SEI	Median SEI
1	48	750	145	400
2	48	580	145	252
3	0	-	-	
4	65	724	152	296
5	33	724	200	469
6	74	629	200	400
7	17	724	200	560
Total	285	750	145	360

The data in Table 2 were obtained for pupils in grades 1 and 2. (NOTE: The socioeconomic status ranking and the measure of the socioeconomic standing are inversely proportional. That is, the lower SEI measures denote the higher socioeconomic classifications; the larger numbers reflect a lower socioeconomic status.)

Four schools organized their first and second grades into self-contained classrooms while three other schools structured classes along pod or open-concept lines. In five of the schools, the services of a reading resource teacher were available. Three schools had the service of a mathematics resource teacher, provided by Emergency School Assistance Act (ESAA) funds. Title I assistance was available to pupils in three schools. Most of the schools also had a variety of special resource services in such areas as Educable Mentally Retarded (EMR), speech, art, music, physical education, learning disabilities, and gifted studies. Only one school reported that no special resources were available.

Detailed information on the first grade population and test batteries is given in the third chapter of this report. Similar information on the second grade follows in the fourth chapter. Data reports by individual schools are cited in bibliography (Item G).

III. FIRST GRADE TESTING PROGRAM

ENROLLMENT BREAKDOWNS

Pupils participating in the 1974 Fall First Grade Testing Program were selected from a total of 13 mathematics sections in the various schools. Table 3 indicates the number of sections in each school, as well as the number of pupils per section.

Table 3

Math Sections and Enrollments by Schools (Grade One)

School	Number of Sections	Section			
		a	b	c	d
1	1	29	-	-	-
2	1	23	-	-	-
3	1	30	-	-	-
4	2	20	18	-	-
5	1	41	-	-	-
6	3	23	24	12	-
7	4	12	17	11	19

The combined first grade population (all seven schools) for the 1974 Fall Testing Program consisted of 279 pupils. The sex distribution by schools is given in Table 4.

Table 4

Sex Distribution by Schools (Grade One)

School	Boys	Girls
1	18	11
2	12	11
3	16	14
4	24	14
5	23	18
6	31	28
7	30	29
Total	154	125
Percentages	55.2%	44.8%

Age Distributions. In September 1974, when the battery of tests was administered, the mean age of the pupils was 6 years 4 months (76 months) with a standard deviation of 4.6 months. The ages ranged from 5 years 9 months (69 months) to 7 years 7 months (91 months). The median age was also 6 years 4 months (76 months). The distribution of ages is shown in Figure 1. The date of birth was not available for four children. The data suggest that the children in the composite population were of average age for first grade children, since the median age of the pupils in the sample used to establish norms for the Metropolitan Readiness Test was also 6 years 4 months.

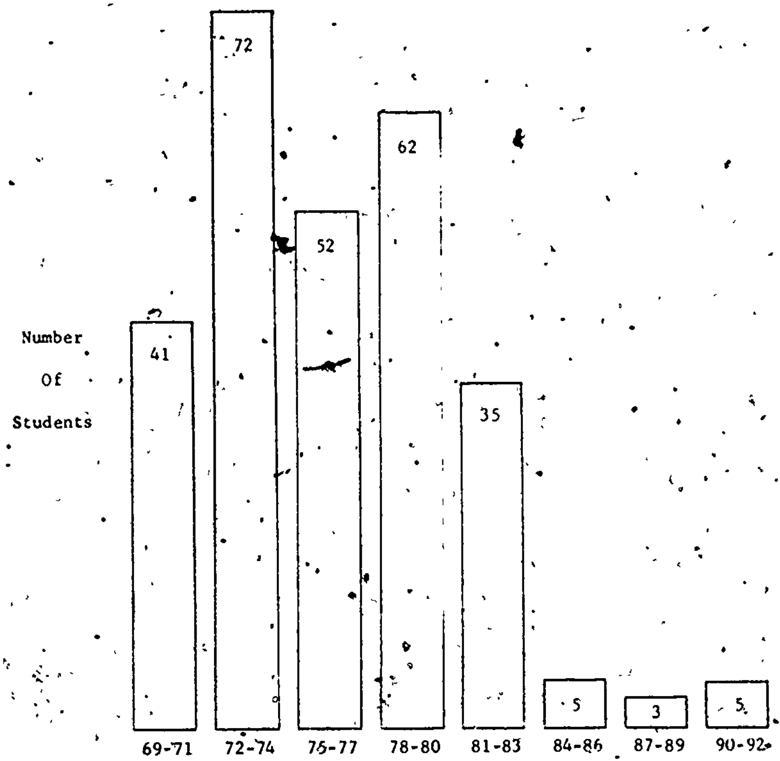


Figure 1

Distribution of Students by Ages in Months (Grade One)

Class Descriptions. All but one of the schools assigned pupils heterogeneously to math sections. The one school which grouped pupils homogeneously did so on the basis of achievement and regrouped the pupils every 2 or 3 weeks. This school followed an open concept structure. The modes of instruction were fairly consistent, with 12 of the 13 sections teaching mathematics primarily in small groups. In only one section did the teacher generally organize the mathematics instruction along the lines of a total class presentation. A total of six different textbooks was used in the various sections although in one section there was no principal textbook; rather, the students worked in one of several textbooks, according to the assignment made by the teacher. Various supplementary materials were also available in each classroom. Table 5 summarizes the type of supplementary materials available in each classroom. The data collected on the availability of supplementary materials suggest that manipulative aids, workbooks, and various games were generally available. Commercial diagnostic tests and cassette tapes, however, were available in only a few schools. The data collected on the use of supplementary materials indicated that such materials were frequently used in 8 of the 13 sections. In only two sections was the use of supplementary materials described as infrequent.

In 10 of the 13 sections, the regular classroom teacher received assistance in implementing the instructional program. Eight sections had the services of university students, five of the sections were assigned teacher aides or para-professionals, and three sections had assistance of either older pupils or parent volunteers.

Table 5

Supplementary Materials Available in Classrooms (Grade One)

School	Section	Workbooks	Manipulative Aids	Diagnostic Tests	Games	Films/ Cassette Tapes
1	a	X	X		X	
2	a	X	X			
3	a	X	X		X	X
4	a	X	X			
4	b	X	X			
5	a		X		X	
6	a		X		X	X
6	b	X	X		X	X
6	c	X	X	X	X	
7	a	X	X	X	X	
7	b		X		X	X
7	c		X		X	X
7	d	X	X	X	X	X

RESULTS FROM THE FIRST GRADE TEST BATTERY

The Stanford-Binet Intelligence Scale was administered to 135 pupils in the combined population. For reasons discussed previously, IQ measures could not be obtained on all pupils in the sample population. The mean IQ for the selected sample was 112.8, with a standard deviation of 17.3. The IQ measures ranged from 76 to 158; the median IQ was 113.6. The distribution of the IQ data is shown in Figure 2.

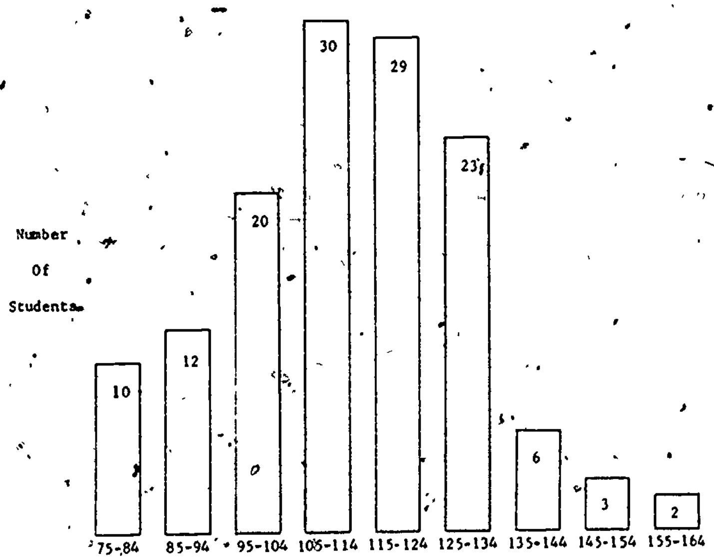


Figure 2

Distribution of IQ Measures (Grade One)

This distribution of IQ measures suggests that the average mental ability of the selected subpopulation is slightly higher than the sample used to establish the Stanford-Binet Norms (1960).

One indication of each child's readiness for first grade instruction was obtained from the pupil's performance on the Metropolitan Readiness Test, which was administered to 267 pupils. Ten pupils did not take the test either because they enrolled in the class after the test had been given or were absent for an extended period of time. The distribution of percentile rankings for those pupils who took the Metropolitan Test is shown in Figure 3.

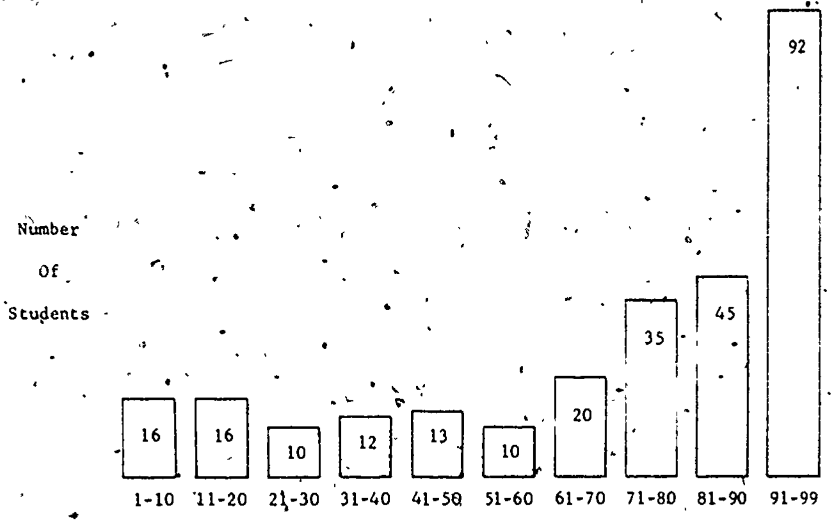


Figure 3

Distribution of Metropolitan Readiness Percentile ~~Rankings~~ (Grade One)

The mean percentile ranking on the Metropolitan Readiness Test was 69.5. The measures ranged from a low of 1 to a high of 99; the median measure was 80.6. This high median measure suggests that, on the average, the pupils in the composite population were better prepared for first grade work than the pupil population used to establish norms for the Metropolitan Readiness Test.

Counting Picture Sets. Pupils in the composite population were also administered four mathematics achievement tests developed for the SMSG Elementary Mathematics Project (see bibliography, Item A). On the first test, counting picture sets, 67.3% of the pupils correctly answered at least seven of the ten items. The mean score was 6.8, with a standard deviation of 2.9. The median measure was 7.8. The distribution of correct scores is shown in Figure 4.

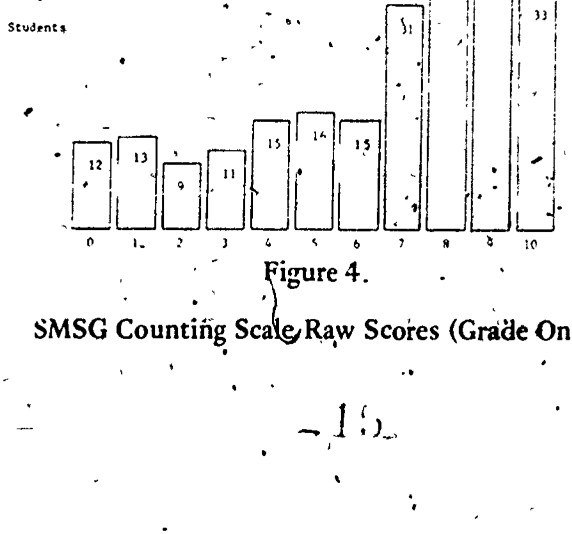


Figure 4.

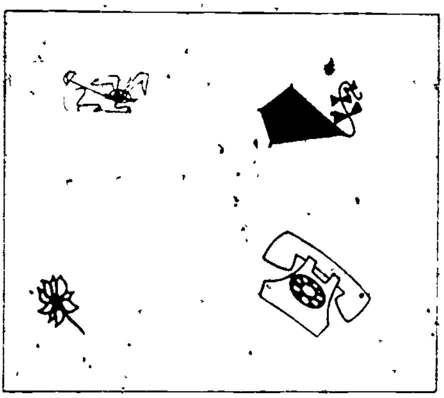
SMSG Counting Scale, Raw Scores (Grade One)

The picture cards used in the administration of this scale are reproduced below[3]. A summary of the data which reflect pupils' performances on a given item is recorded to the left of the appropriate card. Included in the data summary are (a) the number of correct/incorrect responses, and (b) the strategies used by the pupils in determining the number of each set. A discussion of the problem-solving strategies used by the pupils in responding to the counting scale is found on pages 16-18.

VC = Visual Counting
SP = Systematic Pointing
RP = Random Pointing
A = Automatic
NI = No Information

C: 259
I: 17
O: 3

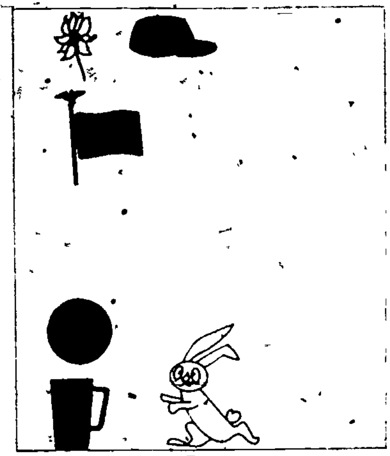
VC: 123/6
SP: 39/0
RP: 2/2
A: 89/7
NI: 6/2



Item 6

C: 196
I: 81
O: 2

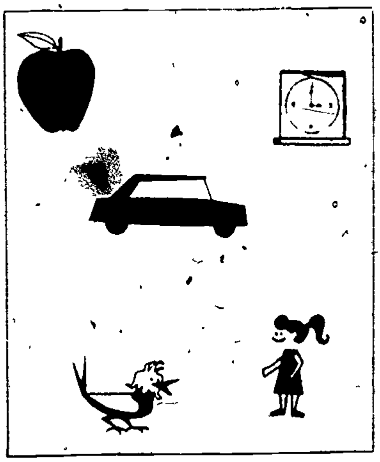
VC: 138/40
SP: 44/5
RP: 3/14
A: 10/14
NI: 1/8



Item 7

C: 247
I: 30
O: 2

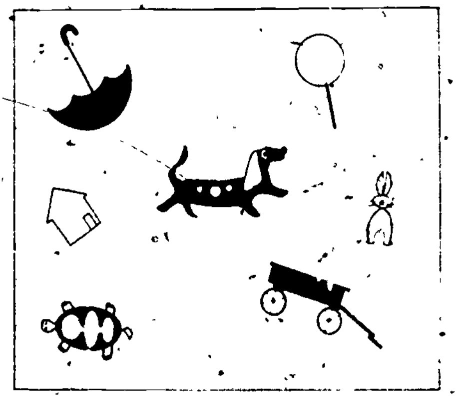
VC: 152/10
SP: 49/1
RP: 2/5
A: 41/10
NI: 3/4



Item 8

C: 207
I: 57
O: 15

VC: 134/31
SP: 61/1
RP: 4/10
A: 5/10
NI: 3/5



Item 9

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[3] Reproduced with permission of the director of SMSG but without the endorsement of the School Mathematics Study Group.

VC = Visual Counting
SP = Systematic Pointing
RP = Random Pointing
A = Automatic
NI = No Information

C: 225
I: 29
O: 25

VC: 140/17
SP: 63/1
RP: 1/6
A: 20/4
NI: 1/1

Item 10

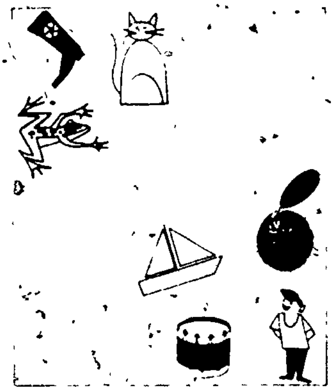
C: 206
I: 37
O: 36

VC: 135/27
SP: 63/2
RP: 4/4
A: 2/2
NI: 2/2

Item 11

C: 185
I: 49
O: 46

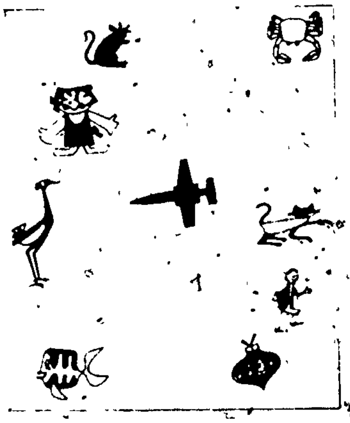
VC: 120/34
SP: 60/0
RP: 0/10
A: 4/3
NI: 1/1



Item 12

C: 155
I: 61
O: 50

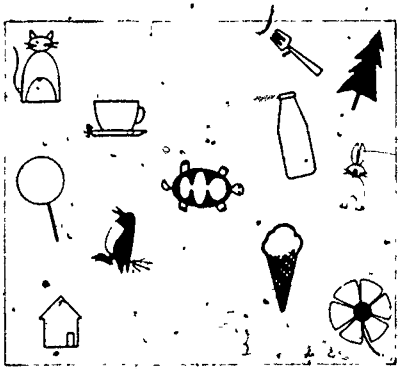
VC: 99/43
SP: 55/2
RP: 2/16
A: 1/3
NI: 1/0



Item 13

C: 134
I: 70
O: 75

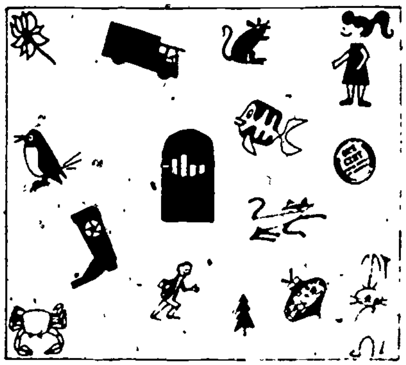
VC: 80/42
SP: 50/3
RP: 3/19
A: 0/1
NI: 1/5



Item 14

C: 85
I: 101
O: 93

VC: 50/55
SP: 30/5
RP: 3/37
A: 1/1
NI: 1/3



Item 15

The first three items (6, 7, and 8) and the fifth item (10) on the test involved sets with six or fewer numbers; only in item 7 were set members not arranged in an orderly pattern. The sets used in items 9, 11, 12, and 13 contained 7, 8 or 9 members, but only in item 12 were the elements of the sets grouped in easily countable subsets. The sets in item 14 and 15 contained 12 and 15 members, respectively, and in these items the members of the sets were not arranged in an orderly pattern. A response was recorded as an "omit" if the pupil did not give an answer. In accordance with the SMSG instructions, the test was terminated after the pupil gave three consecutive incorrect responses. This administrative constraint accounts largely for the marked increase in the number of omits after item 8. The pupils' success in determining the number of members in each set is reflected in Table 6. The item numbers refer to the numbering scheme used on the Pupil Score Sheet (bibliography, Item A).

Table 6

SMSG Counting Scale Responses by Items

Item Number	Number of Members in the set	Number of Responses		
		Correct	Incorrect	Omit
6	4	259	17	3
7	6	196	81	2
8	5	247	30	2
9	7	207	57	15
10	5	225	29	25
11	8	206	37	36
12	7	185	48	46
13	9	158	61	60
14	12	134	70	75
15	15	85	101	93

The data in Table 6 suggest that 80% of the pupils could successfully determine the number of picture sets with five or fewer members and that about 50% of the pupils were proficient in counting picture sets with up to 12 members, regardless of the arrangement of the pictures on the card.

Equivalent Sets. The results on the second test, equivalent sets, show that 75.9% of the students correctly answered at least five of the six items. The mean number correct was 4.8, with a standard deviation of 1.6. The median number correct was 5.3. Figure 5 depicts the distribution of correct scores on this test.

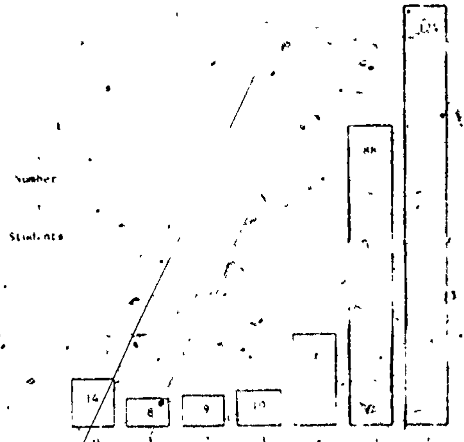


Figure 5

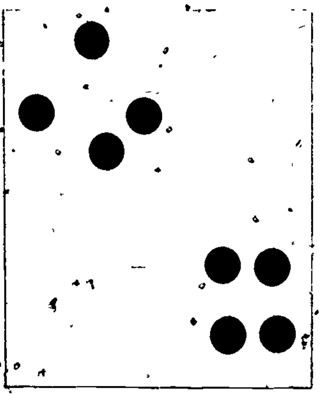
SMSG Equivalent Sets Scale Raw Scores (Grade One)

The dot cards used in this scale follow:[4]

1 - 1 P = 1 - 1 Pattern
1 - 1 NP = 1 - 1 No Pattern
CP = Counting Pattern
CNP = Counting No Pattern
NI = No Information

C: 225
I: 52
O: 2

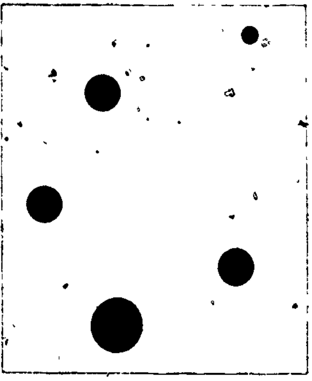
1 - 1 P: 125/18
1 - 1 NP: 1/0
CP: 38/5
CNP: 59/7
NI: 2/22



Item 16

C: 260
I: 17
O: 2

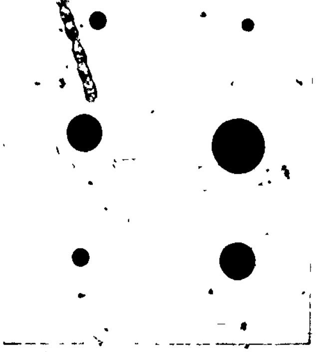
1 - 1 P: 165/4
1 - 1 NP: 0/0
CP: 39/1
CNP: 55/2
NI: 1/10



Item 17

C: 228
I: 48
O: 3

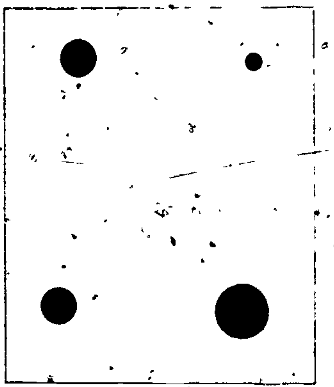
1 - 1 P: 142/16
1 - 1 NP: 0/0
CP: 32/7
CNP: 54/13
NI: 0/12



Item 18

C: 221
I: 43
O: 15

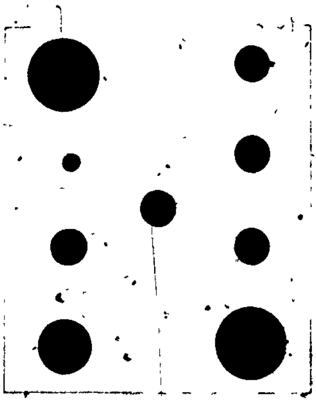
1 - 1 P: 121/27
1 - 1 NP: 1/0
CP: 40/4
CNP: 59/5
NI: 0/7



Item 19

C: 237
I: 20
O: 22

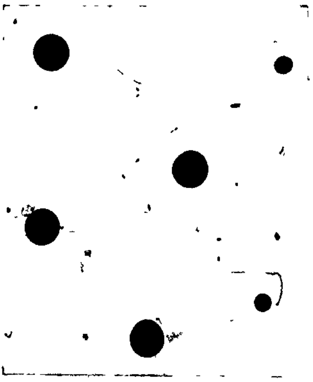
1 - 1 P: 145/11
1 - 1 NP: 1/0
CP: 37/5
CNP: 54/2
NI: 0/2



Item 20

C: 173
I: 73
O: 33

1 - 1 P: 97/47
1 - 1 NP: 0/2
CP: 32/8
CNP: 44/7
NI: 0/9



Item 21

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[4] Reproduced with permission of the director of SMSG but without the endorsement of the School Mathematics Study Group.

A report on the data, item-by-item, is given in Table 7.

Table 7

SMSG Equivalent-Sets Scale Responses by Items

Item Number	Number of Dots	Number of Responses		
		Correct	Incorrect	Omit
16	5	225	52	2
17	4	260	17	2
18	8	228	48	3
19	6	221	43	15
20	6	237	20	22
21	9	173	73	33

The SMSG instructions were to terminate the test after three consecutive incorrect responses. These administrative guidelines largely account for the increase in the number of omissions after item 18.

About 90% of the pupils demonstrated on this test their ability to construct a set equivalent to a given set. The first problem on this scale, item 16, proved to be more difficult than later problems involving a greater number of dots. Several possible explanations for this exist: (1) several pupils did not understand the directions since this was the first item, (2) one dot in the set was considerably smaller in size and placed in one corner away from other dots, making it easy to overlook, and (3) the dots were not arranged in an easily reproducible pattern. In contrast, although item 20 involved six dots, the dots were so arranged that the array was easy to reconstruct. Thus, there were over 60% fewer errors on item 20 than on item 16. The last question, item 21, was answered correctly by only 62% of the pupils, indicating it to be the most difficult item on the scale. This item was the last question on the scale and contained the largest number of dots; furthermore, the dots on the card were so arranged that for a pupil to overlook one of them was possible.

Seriation. The third SMSG scale was a test on seriation; i.e., ordering objects and geometric shapes. On this test, 71.7% of the pupils successfully completed at least five of the six tasks. The mean number correct was 4.7, with a standard deviation of 2.1. The median number correct was 5.7. Figure 6 shows the distribution of scores on this test.

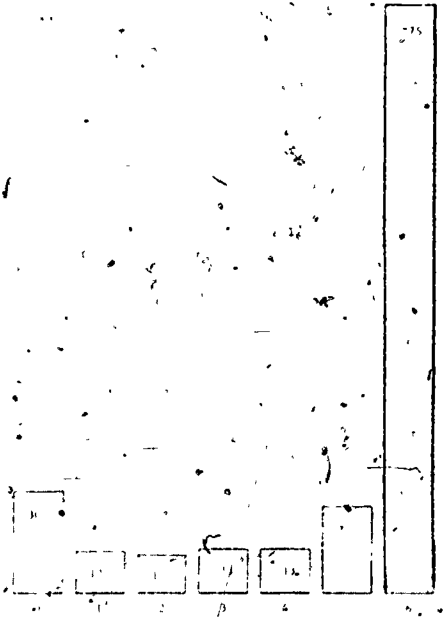


Figure 6

SMSG Seriation Scale Raw Scores (Grade One)

Table 8 gives an item-by-item breakdown on the pupils' responses on the seriation scale. The directions for administering this test are cited in bibliography (Item A).

Table 8

SMSG Seriation Scale Responses by Items

Item Number	Objects	Number of Responses		
		Correct	Incorrect	Omit
22	circles	220	55	4
24	triangles	223	54	2
25	buttons	218	60	1
27	blocks	223	54	2
29	straws	200	76	3
31	rectangles	216	59	4

The pupils' performances on the six items were relatively consistent; approximately 80% of the pupils correctly ordered the objects. The nature of the materials, geometric shapes or objects, did not seem to affect a pupil's ability to complete successfully a seriation task. One possible exception, however, was item 29, in which the pupils were expected to order a set of straws of varying lengths. Many pupils placed the straws end to end, rather than in parallel lines. This technique made it considerably more difficult to discriminate visually between the different lengths. Thus, there were slightly fewer correct answers for item 29.

Most pupils who failed to give a correct ordering did, however, produce a partial ordering. That is, while the end objects were correctly arranged, the middle objects were not. A possible explanation for this response is that the pupil did not fully understand the directions and picked out only the largest and smallest objects. Having thus identified the extremes, the pupil considered the task to be completed.

Classification. The last SMSG scale assessed pupils' abilities to classify objects and geometric shapes by size. The pupils performances on these tasks show that 90% were successful on at least four of the five items. The mean number correct was 4.6, with a standard deviation of 0.7. The median number correct was 4.8. The distribution of number correct is displayed in Figure 7.

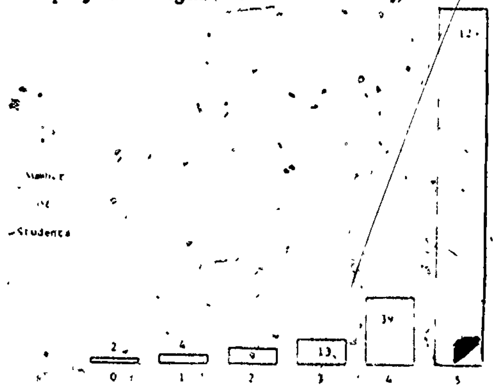


Figure 7

SMSG Classification Scale Raw Scores (Grade One)

Data from pupils' responses to the five items on the classification scale are reported in Table 9. The directions for conducting this section of the SMSG tests are cited in bibliography (Item A).

Table 9

SMSG Classification Scale Responses by Items

Item Number	Objects	Number of Responses Correct	 Incorrect	 Omit
23	circles	262	13	0
26	buttons	265	14	0
28	blocks	229	50	0
30	straws	255	24	0
32	rectangles	266	13	0

The correct response was given by at least 90% of the pupils on all items except item 28. Also, every child attempted to answer each question. Two explanations for the comparatively poor performances on item 28 are possible: (a) in the two previous questions, the pupils had been asked to identify the smallest object; some children might thereby have been conditioned to respond to the smallest object and thus answered without thinking; (b) some children did not consider "largest" to be a synonym for "biggest." Perhaps to them, "large" referred to something smaller than "big" or was a synonym for "smallest."

The statistical data presented above provide an overview of the pupils' abilities to perform certain tasks. They do not provide any information about the problem-solving procedures utilized by the pupils or about the nature of the errors made. However, such information is provided in the following analyses of the data from the SMSG tests.

Problem-Solving. As a pupil's response to each item on the SMSG Counting Picture Sets Scale was recorded, the technique used by the pupil in determining the numbers of members in the set was also indicated. The various techniques were then grouped into four basic categories. Some pupils employed a technique classified as *visual counting.* Students using this method exhibited obvious signs of a counting process. Other pupils actually pointed to (touched) the pictures, either systematically or randomly. Thus, two additional categories of counting techniques were *pointing systematically* and *pointing randomly*. The fourth category includes *automatic responses*. That is, the pupil gave a response without any evidence of counting. The summary of data collected for this analysis is given in Table 10. The item numbers correspond to the numbering scheme used in the SMSG test (see bibliography, Item A). In Table 10, correct and incorrect responses are broken down by the four response technique categories; numbers indicate the number of pupils using a particular strategy to solve a given item. If a pupil attempted to answer the item, but the tester failed to record the strategy employed by the pupil, the pupil's response is included in the "No Information" category. The "omitted" classification includes those cases in which the pupil made no visible attempt to answer the question and those instances in which the item was not given to the pupil. In accordance with the SMSG directions for administering the counting test, the testing was terminated when the pupil did not correctly answer three consecutive items.

An analysis of these data reveals several interesting facts and trends. First, most children use the visual counting strategy. While it is true that children using the visual counting technique gave most of the correct answers, it is equally true that most errors on the more difficult items were made by children using the visual technique. It should also be noted that the incidence of correct visual counting tends to decrease as the problems become more difficult, whereas the number of cases involving correct systematic pointing remains relatively constant. Very few incorrect answers are associated with systematic pointing. Apparently, the visual counting strategy becomes less reliable as the items become more difficult. That is, as the number of members in the item sets increased, the ratio of correct-to-incorrect responses tended to decrease. Students using the visual counting strategy seemingly made an excessive number of errors on items 7 and 9, considering the apparent lack of difficulty of the tasks. Some errors on item 7 might be explained by the fact that one object in the set, the flag, could be construed as two objects. The drawings on the card used in item 9 were not arranged in an easy counting pattern; thus, the number of errors is consistent with the pupils' responses on similarly arranged items.

Table 10

Counting Picture Sets: Problem-Solving Techniques

Item Number	Visual		Pointing Systematic		Pointing Random		Automatic		No Information		Total		
	Right	Wrong	Right	Wrong	Right	Wrong	Right	Wrong	Right	Wrong	Right	Wrong	Omit
6	123	6	39	0	2	2	89	7	6	2	259	17	3
7	138	40	44	5	3	14	10	14	1	8	196	81	2
8	152	10	49	1	2	5	41	10	3	4	247	30	2
9	134	31	61	1	4	10	5	10	3	5	207	57	15
10	140	17	63	1	1	6	20	4	1	1	225	29	25
11	135	27	63	2	4	4	2	2	2	2	206	37	36
12	120	34	60	0	0	10	4	3	1	1	185	48	46
13	99	41	55	2	2	16	1	2	1	0	158	61	60
14	80	42	50	3	3	19	0	1	1	5	134	70	75
15	50	55	30	5	3	37	1	1	1	3	85	101	93

In terms of the correct-to-incorrect response ratio, the systematic pointing strategy was consistently a more reliable technique for determining the number of a set. The fluctuations in the correct/incorrect ratios for systematic pointing strategy are attributable in part to the small number of incorrect responses resulting from the use of this technique. Very few correct responses involved random pointing, considerably more incorrect responses were associated with random pointing, especially on the more difficult tasks. These observations indicate that the random-pointing strategy is not an effective means of determining the number of a set.

In addition, automatic responses were found to be infrequent, except for items 6 and 8. Children understandably could respond without counting on these items, since item 6 had only four members and item 8 had five members arranged in a familiar pattern.

The problem-solving techniques utilized by the pupils in responding to the tasks involving the construction of equivalent sets were classified into four groups. Pupils responded by matching the buttons to the dots one-to-one, and in the process reproduced the dot pattern. In another category, the pupil matched the buttons and dots one-to-one, but did not reproduce the dot pattern. As a third strategy, the pupil counted the dots, then counted the buttons, and then reproduced the dot pattern. The fourth category is similar to the third, except the dot pattern was not reproduced. Recording pupils' responses to a given item included indicating the problem-solving techniques used by each pupil. The data related to the problem-solving procedures utilized in responding to the equivalent set items are given in Table 11.

The data obtained on the various strategies pupils use to solve tasks involving the construction of equivalent sets indicate that considerably more pupils used the matching strategy than the counting strategy. The data also suggest that pupils using the matching technique have a slightly greater change of giving an incorrect response than those using the counting strategy. Among the pupils using the counting strategy, the ratio of those not reproducing the pattern to those reproducing the pattern was about three to two. The data presented in Table 11 also suggest that the pupils were relatively consistent in their procedures, regardless of the difficulty of the item.

No attempt was made to classify the problem-solving strategies used by the pupils in answering the questions on the seriation and classification scales. The seriation scale presented so many variables that a practical

Table 11

Equivalent Sets: Problem-Solving Techniques

Item Number	1-1 Matching and Pattern		1-1 Matching no Pattern		Counting and Pattern		Counting no Pattern		No Information		Total		
	Right	Wrong	Right	Wrong	Right	Wrong	Right	Wrong	Right	Wrong	Right	Wrong	Omit
16	125	18	1	0	38	5	59	7	2	22	225	52	2
17	165	4	0	0	39	1	55	2	1	10	260	17	2
18	142	16	0	0	32	7	54	13	0	12	228	48	3
19	121	27	1	0	40	4	59	5	0	7	221	43	15
20	145	11	1	0	37	5	54	2	0	2	237	20	22
21	97	47	0	2	32	8	44	7	0	9	173	73	33

scheme for codifying and describing each strategy could not be devised. At the other extreme, the classification scale tasks did not call for the utilization of a problem-solving strategy. Thus, only correct and incorrect responses were recorded.

IV. SECOND GRADE TESTING PROGRAM

ENROLLMENT BREAKDOWNS

The second grade pupils participating in the PMDC 1974 Fall Testing Program were selected from five schools with a total of seven sections. At least one school was located at each of the four PMDC sites. See Table 12 for a breakdown of the number of sections per school and the enrollments in each section.

Table 12

Math Sections and Enrollments by Schools (Grade Two)

School	Number Of Sections	Sections	
		a	b
1	1	23	
2	1	25	
3	1	29	
4	2	16	17
6	2	15	12

A total of 137 children participated in the Testing Program. Distribution by sex among the five schools is shown in Table 13 on the page following.

Age Distributions. In September 1974, the mean age of the second grade was 7 years, 4 months (88 months). The standard deviation of the distribution was 3.8 months, with a range in age from 6 years 9 months (81 months) to 8 years 1 month (97 months). The median age was 7 years 5 months (89 months). The date of birth was not available for five children. The data suggest that the children in the sample population were of normal age for second grade children. The distribution of ages is shown in Figure 8 on the page following.

Table 13

Sex Distribution by Schools (Grade Two)

School	Boys	Girls
1	18	5
2	13	12
3	16	13
4	19	14
6	14	13
Total	80	57
Percentages	58.4%	41.6%

Note: The numbering scheme used to identify schools with a participating second grade class is identical to that used with the first grade program. The schools assigned an identifying number of either 5 or 7 in the above Table did not have second grade students participating in the PMDC Fall Testing Program.

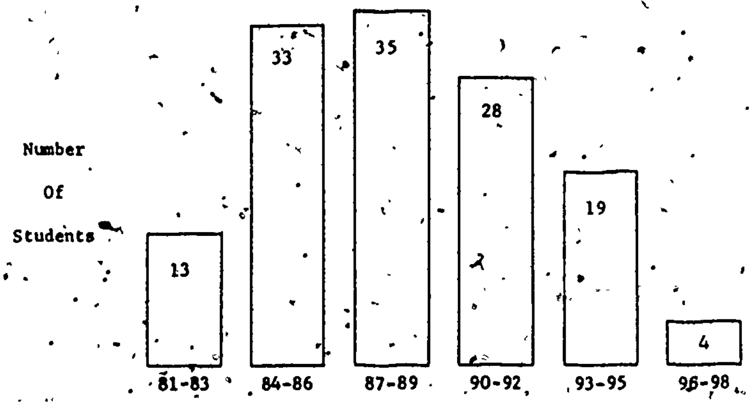


Figure 8

Distribution of Students by Ages in Months (Grade Two)

Class Descriptions. All but one of the schools assigned pupils heterogeneously to math sections. The one school which grouped pupils homogeneously did so on the basis of mathematics achievement tests constructed by the teachers in that school. Since in this school mathematics was the only subject for which homogeneous grouping was employed, the pupils changed classes for the mathematics period.

The mode of instruction was fairly consistent, with five of the seven sections teaching mathematics primarily in small groups. In one section, instruction was organized along the lines of total class presentation, while in the other sections an individualized instructional program was employed. A total of three different textbooks were used in the various sections, but in each section the principal textbook was supplemented with a variety of materials and aids. Table 14 on the page following summarizes the types of supplementary materials available in each classroom.

Table 14

Supplementary Materials Available in Classrooms (Grade Two)

School	Section	Workbooks	Manipulative Aids	Diagnostic Tests	Games	Films/ Cassette Tapes
1	a	X	X	X	X	X
2	a	X	X	X		X
3	a			X		
4	a	X	X			
4	b	X	X			
6	a	X	X		X	
6	b	X	X		X	

The data collected on the availability of supplementary materials suggest that workbooks and manipulative aids were generally available. The other types of materials, such as games and cassette tapes, were maintained in only a few classrooms. Data on the use of supplementary materials indicate that teachers generally made only occasional use of these aids. Frequent use of available supplementary material by pupils or teachers was indicated in only two sections. In four of the seven sections, the regular classroom teacher received assistance from either university students or teacher aides. In one section older pupils were available on a regular basis to assist with the mathematics instruction.

RESULTS FROM SECOND GRADE TEST BATTERY

This section reports data for the total grade sample. The report of data by individual schools is cited in bibliography, Item G.

The Stanford-Binet Intelligence Scale was administered to 97 of the 137 pupils. The mean IQ for this group was 113.3, with a standard deviation of 16.2. The IQ measure ranged from 74 to 162. The median IQ measure was 112. Figure 9 displays the distribution of IQ measures.

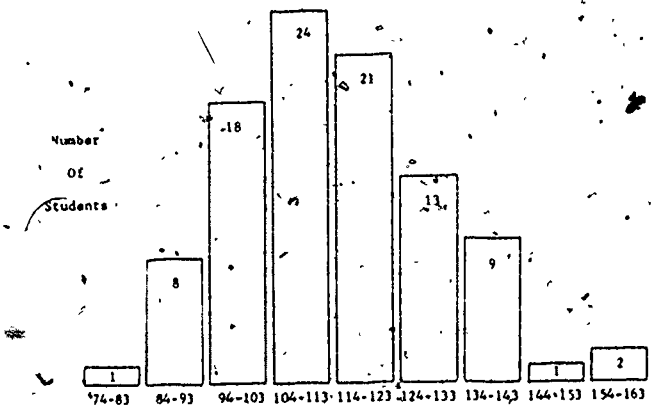


Figure 9

Distribution of IQ Measures (Grade Two)

The data collected on these students suggest that this group was not a representative sample of second grade pupils; the children evidenced ability slightly higher than might normally be expected as indicated by the sample used to establish norms for the Stanford-Binet Scales (1960).

The Metropolitan Achievement Tests, Primary I, was administered to the second grade subjects in order to obtain a measure of their achievement on basic reading and mathematics skills. The test was given at only four schools (three sites) since at the fifth school an equivalent form of this test had been administered at the completion of the first grade. The data reported below were obtained only from those four schools in which the Metropolitan test was given as part of the 1974 Fall Testing Program. The composite Metropolitan Achievement Test, Primary I, provides measures on a number of subtests, among which are (a) total reading measure; (b) word analysis, and (c) mathematics. These scales provide a comprehensive picture of a pupil's abilities in these skill areas.

Metropolitan Reading. The pupils who took the Metropolitan Achievement Test as part of the 1974 Fall Testing Program had a mean percentile ranking of 70.3 on the total reading subtest. The total reading score reflects the pupil's reading vocabulary and comprehension of written material. The percentile scores ranged from 6 to 98. The median was 78.8. Figure 10 depicts the distribution of these scores.

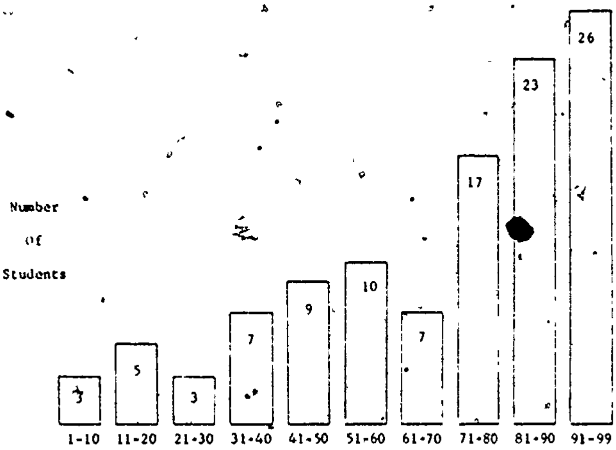


Figure 10

Distribution of Metropolitan Total Reading Percentile Rankings (Grade Two)

The statistical analysis of the reading concepts and skills data suggests that the composite population was definitely above average when compared to the population used to establish norms for the Metropolitan Achievement Tests.

Word Analysis. A related subtest assessed a pupil's knowledge of sound-letter relationships or skill in decoding. The mean performance on this scale was 61.2%, with a standard deviation of 25.6. The median ranking was 62.3%; the range was from the sixth percentile to the 96th percentile. Figure 11 provides a graphic display of the distribution of percentile ranking sample population.

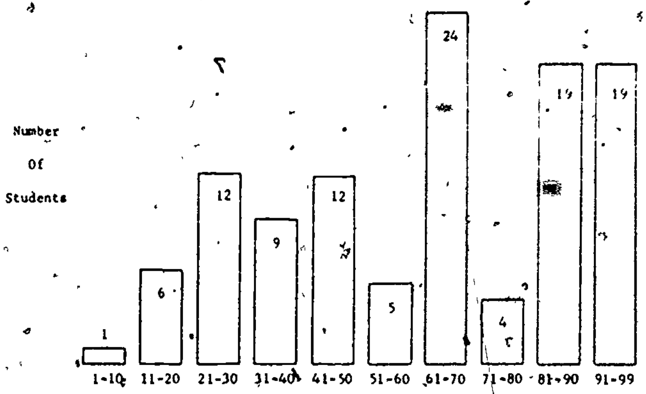


Figure 11

Distribution of Metropolitan Work Analysis Percentile Rankings (Grade Two)

The pupil's performance on the word analysis subtest was slightly above average compared to the results from the Metropolitan sample.

Mathematics. The third scale on the Metropolitan Achievement Test measured pupils' understandings of basic mathematical concepts and computational skills related to addition and subtraction. The pupils' performance on this scale resulted in a mean percentile ranking of 69.7. The percentile scores ranged from 4 to 99. The median percentile was 77. Figure 12 shows the distribution of measures.

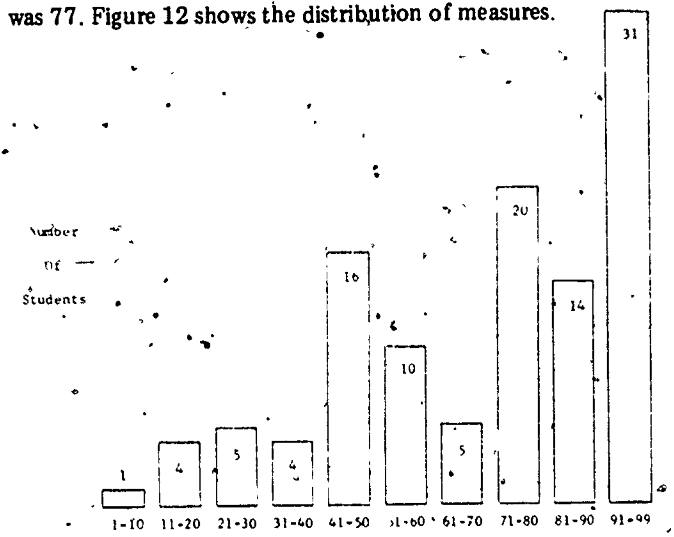


Figure 12

Distribution of Metropolitan Mathematics Percentile Rankings (Grade Two)

The descriptive statistics on the mathematics subtest results suggest that compared to the sample used to establish norms for the Metropolitan test the population was definitely above average in its acquisition of mathematical concepts and skills generally taught in the first grade.

An additional measure of pupil mathematical achievement was obtained by administering selected scales prepared for the SMSG Elementary Mathematics Project. The composite second grade population (all five schools) took five of these scales, including number comparison, place value, comprehension, applications, and computation. Each scale was designed to be administered at the beginning of the second grade.

Number Comparison. The number comparison scale consisted of seven items. The mean score was 5.8 with a standard deviation of 1.6. The measures ranged from none to seven correct. There was a median measure of 6.4. Figure 13 shows the distribution of the data.

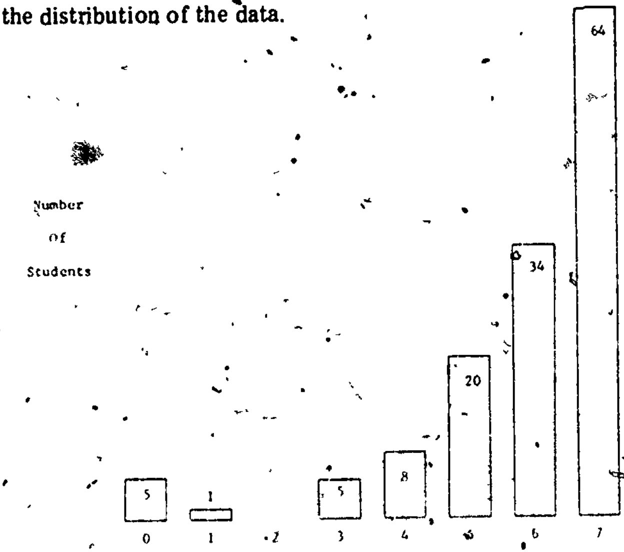


Figure 13

SMSG Number Comparison Scale Raw Scores (Grade Two)

Approximately 72% of the pupils correctly answered at least six of the seven questions. The pupils' performances on each item is reported in Table 15.

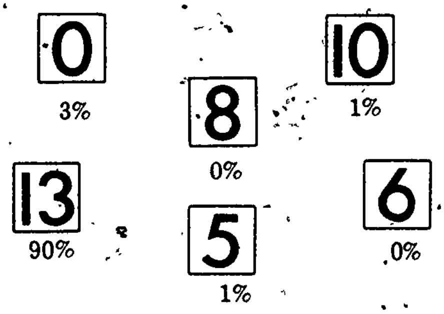
Table 15

SMSG Number Comparison Scale Responses by Items

Item Number	Number of Responses		
	Correct	Incorrect	Omit
1	123	7	7
2	99	32	6
3	102	26	9
4	107	20	10
5	126	6	5
6	123	8	6
7	120	9	8

Each of the items which assessed the meaning of largest (1), more (5), greatest (6), and least (7) was correctly answered by more than 87% of the pupils. However, approximately one-fourth of the sample failed to find a correct answer for the questions involving the concepts of fewer (2) and between (3 and 4). Each item on the number comparison scale was a multiple-choice question. The errors were generally distributed over the various distractors, except for questions 2, 3, and 4. On item 2, 19% of the pupils selected the square with the greatest number of objects. On both items related to betweenness, about 10% of the pupils selected the distractor which was smaller than either of the two given numbers.

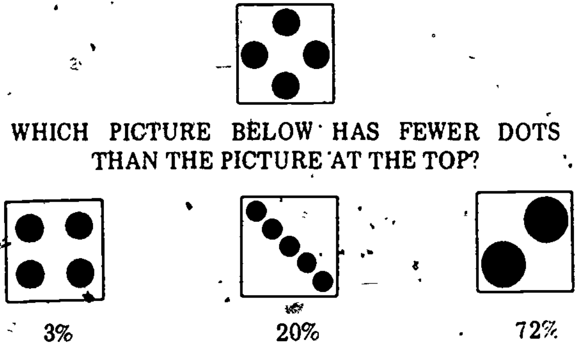
The items from the tester's guide are reproduced below[5]. Next to each item, the numbers of correct, incorrect, and omitted responses are listed. The percentage given below each answer choice indicates the number of pupils who selected that choice as the answer to the question.



WHICH NUMBER IS LARGEST?

No. correct: 123
No. incorrect: 7
No. omit: 7

Item 1



WHICH PICTURE BELOW HAS FEWER DOTS THAN THE PICTURE AT THE TOP?

C: 99
I: 30
O: 6

Item 2

---

[5] Reproduced with the permission of the Director of SMSG but without the endorsement of the School Mathematics Study Group.

WHICH NUMBER IS BETWEEN EIGHT AND FIVE?

3	5	7
12%	5%	74%

C: 102
I: 16
O: 9

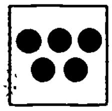
Item 3

WHICH NUMBER IS BETWEEN FOUR AND SEVEN?

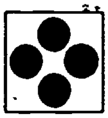
8	6	3
10%	78%	3%

C: 107
I: 28
O: 10

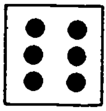
Item 4



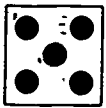
WHICH PICTURE BELOW HAS MORE DOTS THAN THE PICTURE AT THE TOP?



2%



92%



2%

C: 126
I: 6
O: 5

Item 5

WHICH MEANS THE GREATEST NUMBER OF THINGS?

38	29	5	0
90%	2%	1%	3%

C: 123
I: 8
O: 6

Item 6

WHICH MEANS THE LEAST NUMBER OF THINGS?

12	9	2	53
2%	1%	88%	4%

C: 120
I: 9
O: 8

Item 7

Place Value. The scale on place value consisted of eight items. The mean score was 5.2 with a standard deviation of 2.1. The median measure was 5.4 in a distribution of scores which ranged from 0 to 8 correct. A display of the distribution of scores is given in Figure 14.

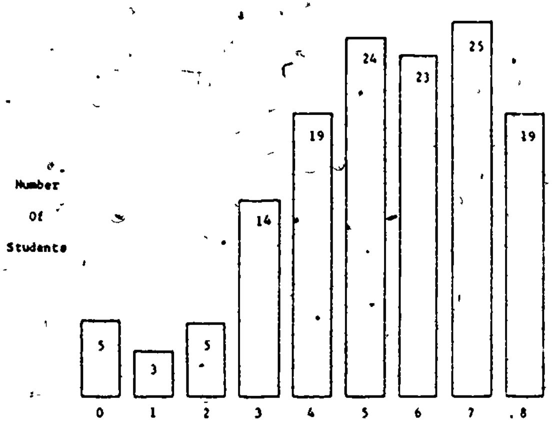


Figure 14

SMSG Place Value Scale Raw Scores (Grade Two)

Approximately one-third (32%) of the pupils correctly answered at least seven of the eight questions. About two-thirds (66.4%) selected the correct response for five or more items.

Table 16 shows the correct/incorrect responses to each item.

Table 16

SMSG Place Value Scale Responses by Items

Item Number	Number of Responses		
	Correct	Incorrect	Omit
8	29	100	8
9	119	10	8
10	77	40	20
11	110	18	9
12	117	12	8
13	93	23	21
14	101	26	10
15	70	46	21

Questions which assessed a pupil's ability to relate a given set with specific numerals (9, 13, and 14) were correctly answered by more than two-thirds of the pupils. An analysis of the incorrect responses suggests that most errors were attributable to mistakes in counting the set or to a misunderstanding of the question. Mastery of the concept of place value was assessed in items 8, 10, and 15. As a group, the pupils did not perform well on these items. The percent of correct responses ranged from 21% (8) to 56% (10). Low scores suggest that at least 50% of the pupils do not have an adequate understanding of the meaning (i.e., significance) of the digits in a two-digit numeral. Items 11 and 12 tested the pupil's ability to translate an expanded numeral into a standard numeral; at least 80% of the pupils gave the correct response on both items.

The items which comprise the place value scale are reproduced below[6]. The pertinent correct/incorrect data

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are listed for each item, with the percentage of pupils selecting each choice on the multiple-choice items given below each response. For the free-response items, the correct answer and the most frequently given incorrect answers are listed, along with the percentage of pupils giving each response.

WHICH NUMBER HAS A FIVE IN THE TENS PLACE?

C: 29
I: 100
O: 8

15

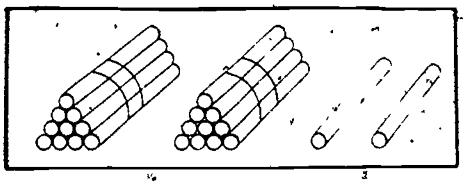
5

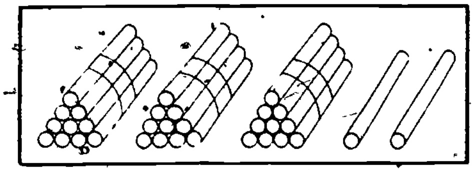
51

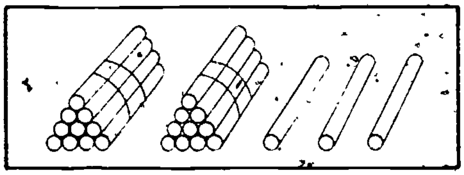
Item 8

C: 119
I: 10
O: 8

WHICH PICTURE SHOWS 32







Item 9

C: 77
I: 40
O: 20

28

WHICH NUMBER TELLS HOW MANY TENS?

RESPONSE

2	56%
8	7%
28	2%
10	4%
9	2%

Item 10

WHICH NUMBER MEANS FIVE TENS AND TWO ONES?

C: 110
I: 18
O: 9

52 25 7

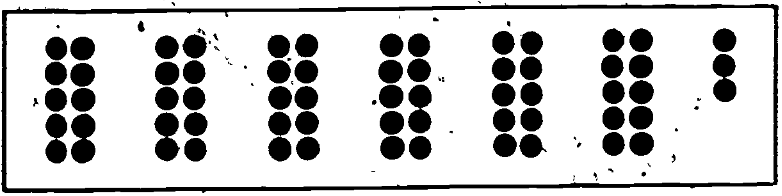
Item 11

WHICH NUMBER MEANS ONE TEN AND THREE ONES?

C: 117
I: 12
O: 8

4 13 31

Item 12



C: 93
I: 23
O: 21

HOW MANY DOTS ARE IN THE PICTURE?

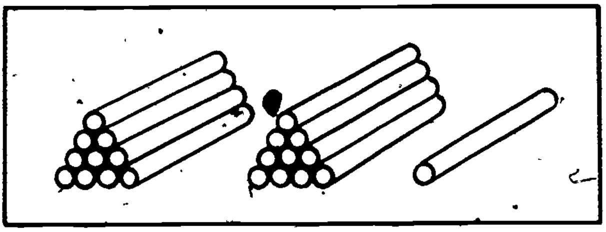
MARK THE NUMBER AT THE BOTTOM THAT TELLS HOW MANY DOTS.

(Pause, but not long enough for the children to count all the dots.)

MARK THE NUMBER THAT TELLS HOW MANY DOTS ARE IN THE PICTURE

7	36	63	70
2%	2%	68%	10%

Item 13



.C: 101
I: 26
O: 10

LOOK AT THE PICTURE. HOW MANY TENS ARE THERE?

RESPONSE

2	74%
20	3%
10	2%
21	7%
30	2%
— or —	
??	2%

_______ TENS

Item 14

C: 70
I: 46
O: 21

37

WHICH NUMBER IS IN THE ONES PLACE?

RESPONSE

7	51%
3	17%
37	7%

Item 15

Comprehension. The scale on comprehension consisted of four items. The mean score was 2.3, with a standard deviation of 1.1. The median score was 2.4. The range was from 0 to 4 correct. Figure 15 gives the distribution of scores.

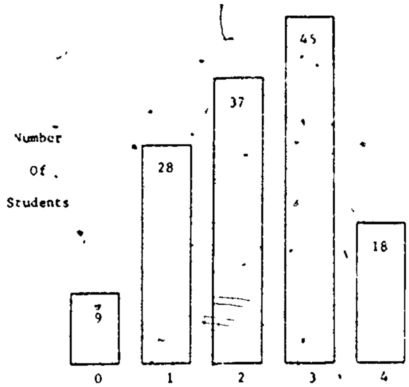


Figure 15

SMSG Comprehension Scale Raw Scores (Grade Two)

Almost one-half (46%) of the pupils correctly answered three or more questions on this scale.

A report on the data obtained on each item is given in Table 17.

Table 17

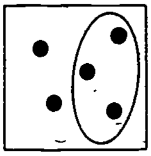
SMSG Comprehension Scale Responses by Items

Item Number	Number of Responses		
	Correct	Incorrect	Omit
16	107	15	15
17	90	33	14
18	81	48	8
26	88	39	10

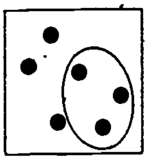
Over three-fourths (78%) of the pupils correctly related a subtraction (minus) statement to a picture (16)[7]. On the other hand, less than one-fourth of the sample (24%) could relate a multiplication (times) statement to a model (17). In response to this item, most pupils (56%) selected the model which depicts "three plus four." The solution to item 18 required the pupil to identify an instance of the commutative property of addition embedded in the context of a verbal problem. Almost 60% of the pupils did this successfully. The last item in the comprehension scale was related to fractions. By correctly answering this question, over 64% of the pupils exhibited some understanding of the concept of fractions.

WHICH PICTURE SHOWS FIVE MINUS THREE EQUALS TWO?

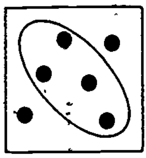
C: 107
I: 15
O: 15



78%



6%

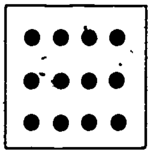


5%

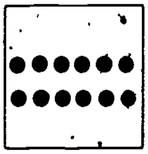
Item 16

WHICH PICTURE SHOWS THREE TIMES FOUR?

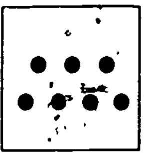
C: 90
I: 33
O: 14



24%



9%



56%

Item 17

C: 81
I: 49
O: 9

TOM AND JIM SHARE A BAG OF MARBLES. ONE DAY TOM TAKES TWENTY-FIVE OF THE MARBLES TO SCHOOL AND JIM TAKES THE OTHER SEVENTEEN. THE NEXT DAY TOM TAKES SEVENTEEN MARBLES. HOW MANY MARBLES ARE THERE FOR JIM TO TAKE?

17	42	25	8
6%	6%	60%	23%

Item 18

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C: 88
I: 39
O: 10

MRS. JONES BOUGHT SIX EGGS. SHE USED ONE HALF THE EGGS TO MAKE A CAKE. HOW MANY EGGS DID SHE USE?

2	3	4
31%	54%	10%

Item 26

Applications. The scale on applications consisted of seven items. The mean score was 5.2, with a standard deviation of 1.8. The median measure was 5.7 in a distribution which ranged from 0 to 7 correct. The distribution of measures is shown in Figure 16.

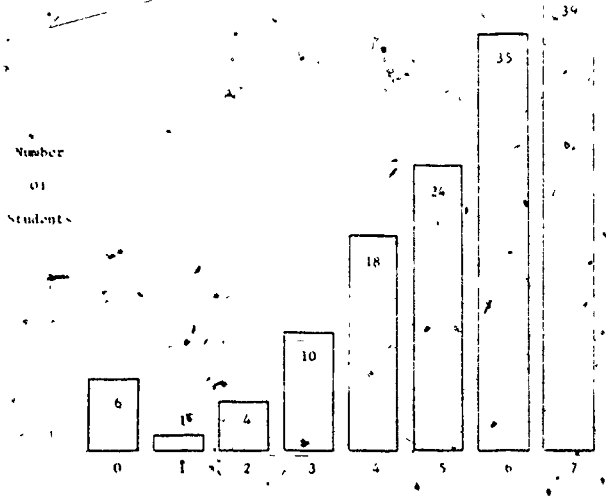

Figure 16

SMSG Applications Scale Raw Scores (Grade Two)

The descriptive statistics indicate that 56% of the pupils successfully solved at least six of the seven verbal problems on this scale.

An item-by-item report on the data collected on the applications scale is given in Table 18 on the page following. Items 19, 20, and 21 involved simple addition or subtraction problems. The percent of pupils selecting the correct response for these questions ranged from a low of 89% to a high of 93%. Sixty-five percent of the pupils successfully solved the verbal missing addend problem (22) and 73% of the pupils solved the missing minuend (sum) problem (23). Item 24 involved a comparative subtraction situation; 63% of the pupils selected the correct response. The last item in this scale (25) involved the concept of a fraction. Fifty-four percent of the pupils were able to answer this question. However, almost one-third of the pupils selected "2" as the correct response, suggesting that for many of these pupils, experience with fractions is limited primarily to the concept of one-half. This observation is supported in part by the pupils' more successful responses to item 26 (see Table 17) a problem involving the concept of one-half.

Table 18

SMSG Applications Scale Responses by Items

Item Number	Number of Responses		
	Correct	Incorrect	Omit
19	123	9	5
20	115	16	6
21	122	5	5
22	89	39	9
23	100	29	8
24	86	41	10
25	74	56	7

The items for the applications scale are reproduced below[8] with the available data on the pupils' responses.

C: 103
I: 9
O: 5

SUE HAD ONE CRAYON. MARY GAVE HER TWO MORE CRAYONS.
HOW MANY CRAYONS DOES SUE HAVE NOW?

1	4%	2	1%	3	90%	4	2%

Item 19

C: 115
I: 16
O: 6

MARY HAD SOME MONEY. SHE SPENT THREE CENTS FOR CANDY AND ONE CENT FOR A GUM BALL. THEN HER MONEY WAS ALL GONE. HOW MUCH MONEY DID MARY HAVE BEFORE SHE SPENT ANY?

1	2%	2	2%	3	8%	4	84%

Item 20

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C: 127
I: 5
O: 5

PATTI HAS THREE COOKIES. IF SHE EATS ONE OF THEM, HOW MANY COOKIES WILL SHE HAVE LEFT?

1 1% 2 93% 3 2% 4 2%

Item 21

C: 89
I: 33
O: 9

TONY HAD SOME BLOCKS. DAVID GAVE HIM FOUR MORE BLOCKS. NOW TONY HAS SEVEN BLOCKS. HOW MANY BLOCKS DID TONY HAVE BEFORE DAVID GAVE HIM MORE?

3 65% 4 15% 5 6% 6 8%

Item 22

C: 108
I: 29
O: 8

JOHN HAD SOME PENNIES. HE LOST THREE OF THEM. NOW HE HAS FOUR PENNIES. HOW MANY PENNIES DID JOHN HAVE BEFORE HE LOST ANY?

3 10% 5 7% 7 73% 4 3%

Item 23

C: 86
I: 41
O: 10

BILL HAS FIVE PENCILS. JOHN HAS THREE PENCILS. HOW MANY MORE PENCILS DOES BILL HAVE THAN JOHN?

1 6% 3 2% 5 21% 2 63%

Item 24

C: 74
I: 56
O: 7

WHEN SOMETHING IS CUT IN FOURTHS, HOW MANY PIECES ARE THERE?

2	3	4	5
4%	4%	64%	20%

Item 25

Computation. The fifth SMSG scale of ten items assessed the pupils' proficiencies in performing basic addition computational tasks. The mean score was 8.5 with a standard deviation of 2.0. The scores ranged from 0 to 10 correct. The median measure was 9.0. The distribution of data is shown in Figure 17:

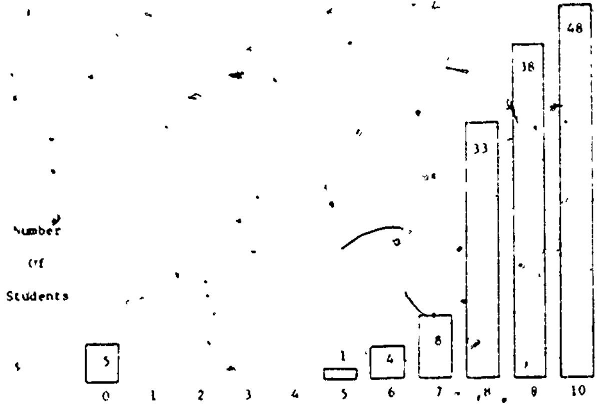


Figure 17

SMSG Computation Scale Raw Scores (Grade Two)

The data indicate that almost 62% of the pupils correctly computed the sum for nine of the ten problems.

An item-by-item report of the data is provided in Table 19.

Table 19

SMSG Computation Scale Responses by Items

Item Number	Number of Responses		
	Correct	Incorrect	Omits
27	131	1	5
28	132	0	5
29	130	1	6
30	131	1	5
31	129	2	6
32	120	8	9
33	127	4	6
34	112	10	15
35	80	36	21
36	79	29	29

As the test results show, approximately 95% of the pupils were successful in computing sums less than 10. The pupils, as a group, were somewhat less proficient in computing sums for basic facts greater than ten. However, almost 82% of the pupils solved "7 + 7 = ." This problem was more difficult than the other basic fact problems, since both addends were greater than five; the greater difficulty accounts in part for the increase in the number of omissions. Close to 60% of the sample population were successful in finding the sum of two-digit numbers with no regrouping. Five pupils in the sample did not attempt to answer any of the questions.

The problems on the computation scale are reproduced below[9]. (Each item is a free-response question.) Listed with each problem are the responses, correct and incorrect, most frequently given by the pupils, and the percentage of pupils giving each answer. Random errors are not listed:

C: 131
I: 1
O: 5

3+3=☐

RESPONSE:
6 96%

C: 132
I: 0
O: 5

4+1=☐

RESPONSE:
5 96%

C: 130
I: 1
O: 6

0+9=☐

RESPONSE:
9 95%

C: 131
I: 1
O: 5

2+5=☐

RESPONSE:
7 96%

Items 27-30

C: 129
I: 2
O: 6

1+7=☐

RESPONSE:
8 94%

C: 120
I: 8
O: 9

6+4=☐

RESPONSE:
10 88%

C: 127
I: 4
O: 6

9+2=☐

RESPONSE:
11 93%

C: 112
I: 10
O: 15

7+7=☐

RESPONSE:
14 82%

Items 31-34

---

[9] Reproduced with the permission of the Director of SMSG but without the endorsement of the School Mathematics Study Group.

		RESPONSE:	
C:	79	90	58%
I:	29	9	4%
O:	29	09	3%
		89	1%
		91	1%

$$50 + 40 = \square$$

		RESPONSE:	
C:	80	65	58%
I:	36	55 or 64	2%
O:	21	75 or 66	3%
		56	2%
		11	4%

$$\begin{array}{r}42\\+23\\\hline\end{array}$$

Items 35-36

V. ADDITIONAL OBSERVATIONS

In the two previous sections, summaries of descriptive data on nine first-grade variables and eleven second-grade variables were presented. The report of the data by individual schools is cited in bibliography (Item G) In the following section, relationships among some of these variables are discussed.

Readiness for First Grade Work. Measures or indicators of entering first graders' knowledge of certain basic mathematical concepts and skills were obtained through the administration of two instruments, the Metropolitan Readiness Test (MRT) and the SMSG scales. The items on the MRT number subtest cover a wide variety of topics including time, money, fractions, verbal problems, numeral recognition and betweenness. Only two items on this test are explicitly related to counting and only one classification item appears. However, none of these three items tests precisely the same concepts and skills that are measured by the SMSG Counting and Classification Scales. For example, none of the items on the MRT number subtest assesses a child's proficiency in the areas of equivalent sets and seriation. Since the common content coverage of the two instruments is minimal, inferences about a child's performance on one test could not necessarily be drawn from his performance on the other test. However, if the pupil's achievement on basic mathematical topics as measured by the two instruments are highly related, then the data from either instrument could be a sufficient predication of a child's readiness for first grade work. Thus, to investigate the relationships among pupils' performances on the MRT number subtest and their performances on each of the four SMSG scales, appropriate scattergrams were constructed. Summaries of the data from these scattergrams are reported in Table 20 on the page following. The percentage given in each cell denotes that part of the total PMDC first grade sample which had a particular pair of corresponding scores on the two tests.

Analyses of the data from the scattergrams (Table 20) suggest that there is, to some degree, a moderately high association between the pupils' performances on the MRT number subtest and each of the four SMSG scales. However, it must be noted that those pupils who had a raw score of at least six on the MRT number subtest generally did quite well on the SMSG scales. That is, a relatively high score on a SMSG scale does not imply a similarly high score on the MRT number subtest. The implication is that many pupils who score relatively high on the MRT number subtest have in fact acquired a reasonable level of competency with concepts and skills related to counting, equivalent sets, seriation, and classification. The distribution of points on the MRT-SMSG counting scattergram suggests a lower association between these variables than among MRT and the other SMSG variables. The scattered distribution of measures could reflect a greater degree of real variability in the counting skills possessed by the pupils in the PMDC sample. Or, this distribution could reflect the affects on the pupils' performances attributable to the testing situations. Regarding the 1974 PMDC test data, the latter appears to be the most reasonable explanation, because the counting scale was the first scale

Table 20

Scattergrams: MRT Number Raw Scores vs. SMSG First Grade Scales

MRT number raw score	0-3	4-7	8-10
21-26	0%	3%	10%
16-20	1%	8%	29%
11-15	3%	9%	13%
6-10	5%	5%	5%
0-5	5%	3%	1%

SMSG Counting

MRT number raw score	0 2	3-4	5-6
21-26	0%	1%	12%
16-20	2%	2%	34%
11-15	1%	3%	22%
6-10	3%	5%	[illegible]
0-5	5%	3%	1%

SMSG Equivalent Sets

MRT number raw score	0-2	3-4	5-6
21-26	0%	1%	12%
16-20	2%	2%	34%
11-15	4%	3%	18%
6-10	6%	2%	7%
0-5	7%	1%	1%

SMSG Ordering

MRT number raw score	0-1	2-3	4 5
21-26	0%	0%	13%
16-20	0%	1%	[illegible]
11-15	0%	1%	24%
6-10	0%	4%	11%
0-5	2%	2%	5%

SMSG Classification

administered in the SMSG component and many children at first seemed uneasy working in a new situation.

Regarding content, the four SMSG scales measured distinct variables. To determine the extent to which children having acquired one concept or skill have also attained a satisfactory level of proficiency in another area, scattergrams for the six possible combinations among the four variables (counting, equivalent sets, ordering, classification) were constructed. Data which summarize the results from these scattergrams are reported in Table 21 on the page following. An analysis of the data presented in Table 21 suggests that about 50% of the pupils who did quite well on the counting scale (correctly answered at least eight of the ten items), also performed exceptionally well (80% or above) on each of the other scales. Pupils scoring at the 80% level or above on the counting test demonstrated a mastery of counting picture sets with more than 10 members. The scattergrams suggest that entering first graders who have acquired this level of proficiency with counting skills stand a 9 to 1 chance of also doing well (80% level or above) on the equivalent sets scale, an over 5 to 1 change that they will do well on the ordering scale, and a 23 to 1 chance that their performances on the classification scale will be at least at the 80% level. Since the testing situation had some adverse effects on the pupils' performances on the counting scale, these odds are probably understated and therefore do not reflect accurate relationships among these variables. The data reported in Table 21 indicate that from one-fourth to one-third of the pupils had scores on the counting scale below the 80% level and were at the 80% performance level on each of the other tests. Thus, it is possible that under different conditions many of these pupils could have scored at the 80% level or above on the counting scale making the odds stated above somewhat higher. Generally, pupils who scored low on the counting scale (scores in the 0-3 range) were as likely as not to perform very well on the other tests.

The odds that a pupil performing very well (80% level) on the equivalent sets scale would perform at the same level on the counting test were less than 2 to 1, less than 5 to 1 on the ordering test, and 37 to 1 on the classification test. For those pupils having low scores on the equivalent sets test (0-2 range), the likelihood of having a similarly low score on the counting test was about 1 to 1, 1 to 2 on the ordering test, and 1 to 4 on

Table 21

Scattergrams: First Grade SMSG Scales

SMSG Counting	0-1	2-3	4-5
8-10	0%	2%	56%
4-7	0%	4%	24%
0-3	2%	2%	10%

SMSG Classification

SMSG Equivalent Sets	0-2	3-4	5-6
5-6	9%	5%	62%
3-4	1%	1%	8%
0-2	6%	3%	2%

SMSG Ordering

SMSG Counting	0-2	3-4	5-6
8-10	2%	4%	52%
4-7	3%	5%	20%
0-3	6%	4%	4%

SMSG Equivalent Sets

SMSG Counting	0-2	3-4	5-6
8-10	6%	3%	4.
4-7	6%	4%	18
0-3	7%	2%	5

SMSG Ordering

SMSG Equivalent Sets	0-1	2-3	4-5
5-6	0%	2%	74%
3-4	0%	2%	11%
0-2	2%	4%	5%

SMSG Classification

SMSG Ordering	0-1	2-3	4-5
5-6	0%	2%	70%
3-4	0%	2%	7%
0-2	2%	4%	13%

SMSG Classification

the classification test.

An interpretation of the data with respect to the ordering scale shows that 2 out of 3 pupils who performed at the 80% level on this test performed similarly well on the counting scale. Likewise, about 5 out of 6 pupils performed at the 80% level on both the ordering and equivalent sets tests, while 7 out of 9 pupils had equally high scores on both the ordering and classification scales. At the other extreme, the odds that a pupil performed very poorly on both the ordering scale and the counting scales were about 1 to 2. About 2 out of every 3 pupils with low scores on the ordering scale did better on the equivalent sets scale. Similarly, 8 out of 9 pupils with low scores on the ordering scale performed better on the classification scale.

The data reported in Table 21 indicate that most pupils (80%) performed at the 80% level on the classification scale. The ratios of success (80% level) on the classification scale to success on the counting, equivalent sets, and ordering scales were less than 2 to 1, 5 to 1, and 4 to 1, respectively. However, those pupils scoring low on the classification scale (0-1 range) also scored low on the other scales.

Overall, the data from the scattergram analyses of the results from the SMSG tests indicate that the vast majority of the pupils who did quite well (80%) on the counting, equivalent sets, and ordering scales also had high scores on the classification scale. While the converse relationships among these variables are not as evident, it was observed that those pupils with very low scores on the classification scale also scored very low on the other tests. The data also suggest that a high score (80% level) on the counting, equivalent sets, or ordering scales generally implies a similar score on each of the other scales; however, in each case a sizeable part of the test population did well on one test and less well on one or both of the other tests. Pupils who had a very low score on either the counting, equivalent sets, or ordering tests did not necessarily have a low score on the other tests. In fact, the opposite was usually the case. That is, pupils not performing well on one test were more likely to have performed better on one or both of the other tests. It should also be noted (see center cell of scattergram) that very few pupils performed moderately well on two or more tests. These observations suggest considerable variation among the pupils' acquisitions of readiness skills related to counting, equivalent sets, and ordering as measured by the SMSG tests.

Since girls generally develop school-related skills sooner than boys, the data obtained from the 1974 PMDC Testing Program were analyzed by sex to determine if there existed differences in boys' and girls' readiness to do first grade work. A summary of the data obtained from these analyses is reported in Table 22.

Table 22

Means of Major First Grade Variables by Sex

Variable	Boys	Girls
Age In Months	76.3	76.4
IQ	113.4	112.1
SEI	390	418
MRT Percentile	68.6	70.5
Counting Picture Sets Raw Score	6.8	6.8
Equivalent Sets Raw Scores	4.7	5.0
Ordering Raw Scores	4.6	4.7
Classification Raw Scores	4.6	4.6

The data reported in Table 22 indicate only slight variances in the boys' and the girls' acquisitions of readiness concepts and skills measured by the instruments used in the PMDC Testing Program. The girls had slightly higher scores on the MRT, the SMSG Equivalent Sets Scale, and the SMSG Ordering Scale. These differences, however, do not indicate that the girls have a marked advantage over the boys in any one readiness area.

In section II of the report, it was noted that certain children used a physical pointing strategy to determine the number of a picture set, but that most of the other pupils employed a visual counting technique. It was also observed that the pointing strategy was considerably more reliable than a visual counting technique. To determine whether or not the children in one of these two groups were better prepared for first grade work as determined by the pupils' performances on the PMDC test battery instruments, analyses of the data pertaining to each of the major variables were done for both groups. A summary of the results of these analyses is reported in Table 23 on the page following.

The data reported in Table 23 indicate that except for their ability to determine the number of picture sets, the pupils who employed a pointing strategy were slightly less ready to pursue first grade work. Since the non-pointers are generally the more able group, it is quite likely that the results on the SMSG Counting Scale would have been somewhat higher had the test directions explicitly stated that the pupil could touch the pictures.

Table 23

Means of Major First Grade Variables by Counting Techniques

Variable	Pointers	Non-Pointers
Age In Months	75.7	76.7
IQ	110.8	114.1
SEI	444.0	379.5
MRT Percentile	66.9	70.8
MRT-Math Subtest Raw Scores	13.7	15.0
Counting Picture Sets Raw Scores	8.1	6.1
Equivalent Sets Raw Scores	4.6	4.9
Ordering Raw Scores	4.4	4.8
Classification Raw Scores	4.5	4.6

Readiness for Second Grade Work. The instruments used in the PMDC test battery measured, for the most part, what the pupils had learned during the first grade. Two tests were used to assess a child's acquisition of mathematical concepts and skills. One instrument was the Metropolitan Achievement Test, Primary I (MAT), and the other was prepared by SMSG. These two tests cover somewhat similar content-areas. Topics unique to the MAT include time, money, ordinal number names, measurement, and a variety of skills which are usually taught during the first part of grade one. Also, the MAT includes a more comprehensive coverage of computation problems, including subtraction, problems with three addends and problems with missing addends. The SMSG test places more emphasis on applications, verbal problems, number properties, betweenness, and the relationships between sets and numbers, especially in connection with more than, less than, and place value. Although in some respects the topical coverage on both tests is somewhat similar, considerable variance in the items used within each overlapping strand exists. To determine the extent to which the pupils' performances on one test were related to their performances on the other test, scattergrams were constructed to show the relative distribution of scores on the two tests. Since the SMSG test is sub-divided into five parts, five separate scattergrams were constructed. Each scattergram relates the pupil's percentile ranking on the MAT mathematics subtest and the pupil's raw score on one of the five SMSG scales: number comparison, place value, concepts, applications, and computations. Summaries of these distributions are reported in Table 24 on the page following. The percentage in each cell denotes that part of the second grade sample which took the MAT, with a particular combination of scores on each test pair.

The data presented in Table 24 indicate a somewhat positive relationship between the scores on the MAT and the SMSG scales. The only deviation from this general trend is that relatively few of the students with high scores on the MAT had a top score on the SMSG Concepts Scale. A possible explanation for this exception is that one item on the concepts tests was related to multiplication. Thus, it is to be expected that most beginning second graders, regardless of overall achievement, would miss this item. Pupils who did quite well on the various SMSG Scales generally had MAT Percentile ranking at the 50% level or above. The most notable exception to this trend was on the SMSG Computation Test. On this test, pupils at all achievement levels on the MAT exhibited some mastery of basic addition skills. This trend is probably due to the fact that first graders are likely to spend more class time on this skill than on any other topic. The data in Table 24 also indicate that pupils with high scores on the MAT also did well on the place value and applications scales. This suggests that knowledge of place value and applications (verbal problems) accounts for much of the differentiation among scores on the MAT.

By definition, the five scales which comprise the SMSG second grade test assessed different concepts and skills. Since all of the topics covered by those scales are included in most first grade curricula, analyses of the data obtained from these scales were made to determine to what extent the pupils' acquisitions of concepts and skills in one area were related to their achievements in each of the other areas. These analyses were made

Table 24

Scattergrams: MAT Math vs. SMSG Second Grade Scales

MAT Math Percentile	0-2	3-5	6-7
80-99	0%	7%	31%
60-79	2%	2%	23%
40-59	0%	5%	20%
20-39	0%	6%	4%
0-19	0%	1%	1%

SMSG Number Comparison

MAT Math Percentile	0-3	4-6	7-8
80-99	1%	14%	23%
60-79	1%	15%	19%
40-59	8%	11%	6%
20-39	4%	6%	0%
0-19	2%	0%	0%

SMSG Place Value

MAT Math PERCENTILE	0-1	2-3	4
80-99	4%	27%	7%
60-79	6%	16%	3%
40-59	5%	18%	2%
20-39	5%	5%	0%
0-19	0%	2%	0%

SMSG Concepts

MAT Math Percentile	0-2	3-5	6-7
80-99	0%	4%	34%
60-79	0%	11%	14%
40-59	2%	12%	11%
20-39	2%	5%	3%
0-19	1%	0%	1%

SMSG Applications

MAT Math Percentile	6-3	4-8	9-10
80-99	0%	2%	36%
60-79	0%	8%	17%
40-59	0%	15%	10%
20-39	0%	4%	6%
0-19	0%	1%	1%

SMSG Computation

by constructing scattergrams for each test pair. Summaries of the data from these scattergrams are reported in Table 25 on the page following. The percentage in each cell denotes the part of the second grade sample with that particular pair of scores on the two tests.

Interpretations of the data presented suggest a somewhat strong positive relationship between the pupils' achievements in each of the five areas. Pupils with high scores (80% level or above) on the number comparison scales also tended to have high scores on applications and computation scales, but this group also exhibited more diversity in scores on the place value and concepts scales. However, the pupils who exhibited evidence of only moderate achievement on the number comparison scale generally had low scores on the place value and concepts scales, but relatively high scores on the applications and computation scales, especially the latter.

Overall, the pupils' achievements on the place value scale were not as high as they were on the other tests, excepting the concepts scale. Pupils whose achievements were at the 80% level on the place value scale tended to show similarly high achievements on the other scales, excepting concepts. Pupils with low scores on the place value scale, however, generally had much better scores on each of the other tests, excepting concepts.

The concepts scale contained only four items, one of which was related to multiplication. Since the vast majority of the second graders missed this item, the mean percentage score on this test was somewhat lower than the mean percentage scores on the other scales. However, almost all of the pupils who correctly answered the multiplication question had very high scores on each of the other scales. Pupils whose achievements on the concepts scale were low did not, in most cases, have low scores on the other scales.

The pupils' achievement patterns on the applications and computation scales were somewhat parallel. In both cases, pupils with high scores on one test also exhibited high achievements on the other test and on the number comparison scale, with somewhat lower means on the place value and concepts scales. Furthermore, the pupils who had low scores on either the applications or the computation scale had correspondingly low scores on the place value and concepts scales, but generally their achievements on the other scales were much higher.

Table 25

Scattergrams: SMSG Second Grade Scales

SMSG Number Comparison			
6-7	10%	32%	32%
3-5	6%	15%	5%
0-2	0%	0%	0%
	0-3	4-6	7-8

SMSG Place Value

SMSG Number Comparison			
6-7	17%	47%	10%
3-5	10%	15%	1%
0-2	0%	0%	0%
	0-1	2-3	4

SMSG Concepts

SMSG Number Comparison			
6-7	2%	19%	53%
3-5	2%	18%	6%
0-2	0%	0%	0%
	0-2	3-5	6-7

SMSG Applications

SMSG Number Comparison			
6-7	0%	26%	48%
3-5	0%	9%	17%
0-2	0%	0%	0%
	0-3	4-8	9-10

SMSG Computation

SMSG Place Value			
7-8	6%	24%	7%
4-6	13%	31%	3%
0-3	8%	7%	1%
	0-1	2-3	4

SMSG Concepts

SMSG Place Value			
7-8	0%	6%	31%
4-6	2%	23%	22%
0-3	2%	8%	6%
	0-2	3-5	6-7

SMSG Applications

SMSG Place Value			
7-8	0%	7%	30%
4-6	0%	19%	28%
0-3	0%	9%	7%
	0-3	4-8	9-10

SMSG Computation

SMSG Concepts			
4	0%	2%	9%
2-3	4%	19%	39%
0-1	0%	16%	11%
	0-2	3-5	6-7

SMSG Applications

SMSG Concepts			
4	0%	1%	10%
2-3	0%	22%	40%
0-1	0%	12%	15%
	0-3	4-8	9-10

SMSG Computation

SMSG Applications			
6-7	0%	13%	46%
3-5	0%	19%	18%
0-2	0%	3%	1%
	0-3	4-8	9-10

SMSG Computation

The data reported in Table 25 also indicate that the pupils with an achievement score in the middle range on one test generally had a similar score on the number comparison, place value, concepts and applications scales. However, most pupils whose achievements were in the middle range on the other tests scored in the high range on the computation scale.

From the data obtained in ~~the scattergram~~ analyses, it appears that most pupils in the 1974 PMDC second grade testing sample were able to acquire proficiency in the areas of number comparison, applications and computation without attaining a similar level of proficiency, as measured by the SMSG scales, in the areas of place value and concepts. This gap is especially evident in the area of computation, suggesting that many pupils can and do learn computation skills by rote methods.

Most of the instruments used in the PMDC test battery tended to measure the pupils' achievements in the first grade. Since the degree of success in learning the concepts and skills usually taught in the first grade might not be uniform for both boys and girls, the data obtained from the 1974 PMDC Testing Program were analyzed separately for each sex. A summary of the data from these analyses is reported in Table 26.

Table 26

Means of Major Second Grade Variables

Variable	Boys	Girls
Age In Months	88.6	88.2
IQ	112.2	114.5
SEI	328.4	388.6
MAT-reading percentile	65.1	78.1
MAT-math percentile	71.8	66.6
SMSG Number Computation Raw Scores	5.8	5.9
SMSG Place Value Raw Scores	5.3	5.2
SMSG Concepts Raw Scores	2.2	2.3
SMSG Applications Raw Scores	5.2	5.2
SMSG Computations Raw Scores	8.6	8.5

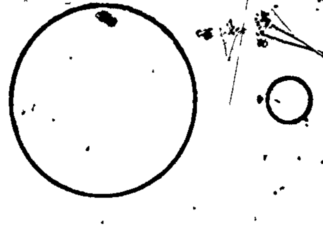
The data presented in Table 26 indicate that both groups were approximately equivalent with respect to age, IQ, SEI, and performances on the five SMSG scales. The girls as a group exhibited somewhat higher achievement in the area of reading on the MAT, whereas the boys had slightly higher scores on the MAT math subtest.

VI. CASE STUDIES AND FOLLOW-UP STUDIES

The data obtained from the SMSG First Grade Scales left unanswered certain questions pertaining to the pupils' understandings of and proficiencies with the concepts and skills being assessed. In an effort to explore further the depth of the children's knowledge in these areas, several PMDC principal investigators conducted follow-up studies to the regular testing program. Reports of four such studies are presented in this section. Two of the reports are related to seriation tasks and two to equivalent sets.

CASE STUDY 1: P'S ABILITY TO SERIATE[10]

Along with other first graders participating in PMDC studies, P was tested with the seriation items under the standard conditions. (See bibliography A.) Responding to directions, P consistently chose only two—the smallest and the largest—of the four or five objects handed to him and placed them from left to right before him on the table in the order mentioned by the experimenter (E). He simply ignored the remaining two or three objects. For example, after being given the four circles and being asked, "Can you put these on a line so they go from largest to the smallest?" he did this:



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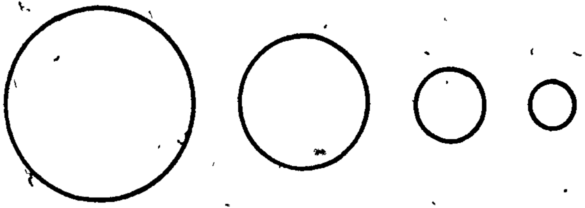
He was successful each time when asked by E to hand him either the largest or the smallest object.

During the analysis of the videotape made of P's behavior under the standard testing conditions, the main question, which cannot be completely answered, was whether P was in fact unable to seriate (arrange objects from largest to smallest, or *vice versa*) or whether P responded to the oral directions as he understood them. That is, while P might have the concept of seriation, he did not know that he was being asked to seriate.

To explore this, E re-interviewed P two weeks later. There is no evidence that P could have received any instruction about seriation during the interim. The second interview was open-ended; the standard directions were not followed.

During the second interview, P was handed four circles and told to do with them whatever he liked. Quickly and without hesitation, P arranged them on the table before him like this:



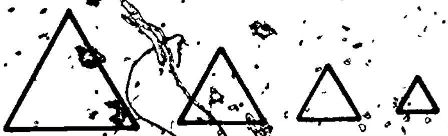
When asked to describe what he had done, P pointed to each circle, starting with the largest, and described them as "Big, middle size, little, teeniest."

Next P was given four triangles.

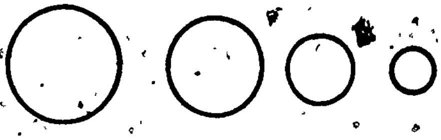
E. Can you do the same thing with the triangles as you did with the circles?

P. Yeah [quite confidently].

After P had arranged the triangles as follows,



he was asked to describe what he had done. He replied, "Biggest, middle size, little, littlest." When asked to do the same thing with four buttons, P arranged them thus,



and described them as "Biggest, middle size, little, teeny."

On the basis of this open-ended interview, one is justified in concluding that P does indeed have the concept of seriation, seemingly in contradiction of the conclusion of the testing two weeks earlier when standard test conditions and directions were used.

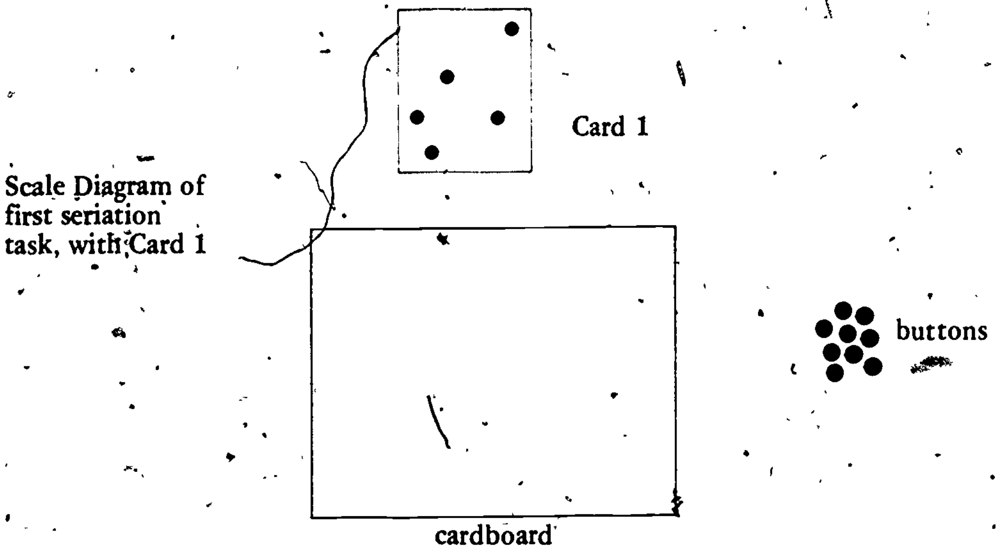
This episode poses several important questions about the effectiveness of communication between adults and children. Do the oral directions to which children respond communicate what we wish the children to do? Are we drawing erroneous conclusions about children and their particular concepts because children respond to the directions as they—not we—understand them? Are there more effective non-verbal ways to communicate with children? If so, how can they be employed in classroom situations?

CASE STUDY 2: J'S CONCEPT OF EQUIVALENT SETS[11]

As part of the total PMDC Testing Program, all first graders who were involved in PMDC studies and those in the corresponding control groups were administered the SMSG Equivalent Sets Test. In order to check out the concept of equivalent sets acquired by second graders, some second grade students were administered the same test, but in an open-ended interview. In his responses, J is typical of the several second graders interviewed. J is judged by his teachers to be a good student. In the first grade, he was taught the usual concept of equivalent sets: i.e., two sets are equivalent when they have the same number of elements. Of course, only finite sets were considered.

The experimenter (E) interviewed J in the Fall of 1974 to discover what the term "equivalent sets" meant to J. E began the interview using materials from Grade 1 test batteries developed by School Mathematics Study Group (see pp. 12-14). Since the specific intention was to administer the test not in accordance with the standard conditions, the prescribed directions were not followed.

A rectangular-shaped piece of cardboard (10"x13") and about 20 buttons were placed on the table before J. Then Card 1 was placed before him.



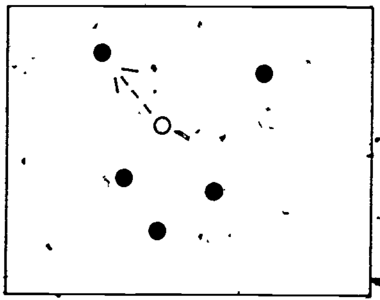
E gave the following directions:

I want you to make on this paper here with these buttons a set which is equivalent to this set [pointing to Card 1].

Rather thoughtfully, J duplicated with the buttons the configuration found on Card 1. Then E said:

Now I am going to move this button here.

E moved one of the buttons to obtain the following configuration:



E's arrangement of buttons from the Card 1 pattern

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The interview continued:

E. Is this set here [points to the cardboard] equivalent to this set [points to Card 1]?

J. No.

E. Would you fix it so it is?

J. [Slides the button back to its original position.]

E. Now this set [points to the cardboard] is equivalent to this one [points to Card 1]?

J. Yes.

E. O.K. Do you know what "equivalent" means?

J. Equivalent? [Shrugs his shoulders.]

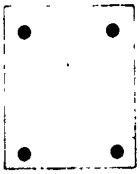
E. Did you ever use this in any class?

J. In first grade.

E. Did you use it in math?

J. Yeah.

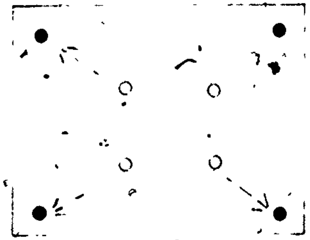
J was then presented with Card 2.



Card 2

J again reconstructed meticulously the pattern of the test card. When E moved one of the buttons out of the pattern, J said that the sets were no longer equivalent. Then E spread the buttons by placing them in the four corners of the cardboard.

E. Now I will change it like this. [Spreads the buttons apart; he no longer needs to repeat the question.]



E's rearrangement of buttons from the Card 2 pattern

J. Sort of.

E. Sort of but not quite.

J. No, no.

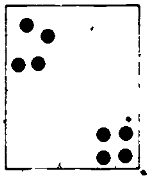
E. Would you fix it so it is not sort of?

J. Um...maybe it's a little smaller. [Puts the four buttons into their original position, so they accurately duplicate distances between the dots on the card.]

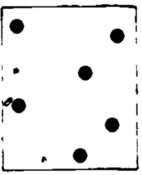
By this time, it was rather clear that J's concept of equivalent sets called for the objects of two sets to be arranged according to the same pattern. Of course, he demonstrated all along that they must have the same number of elements. One wonders whether these two conditions constitute all of the conditions for the equivalence of sets for him.

The interview continued. J was presented with Cards 3 and 4; each time one button was moved out of the configuration, J put it back to restore the "equivalence" of the sets.

Card 3

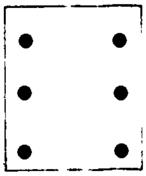


Card 4

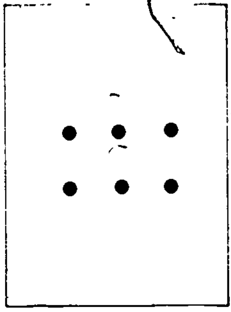


Next J was presented with Card 5.

Card 5



After he constructed the set, reproducing the pattern on Card 5, E turned the cardboard clockwise approximately 90 degrees, so that the button set appeared to J as follows:



New position of buttons from J's viewpoint, after card was turned by E

E. Is this set [points to the cardboard] equivalent to this [points to Card 5]?

J. Doesn't look like it.

E. Doesn't look like it?

J. No.

E. Would you want to fix it so that it is equivalent to this?

J turned the cardboard 90 degrees counterclockwise to restore it to the original position and nodded his head "yes" when asked, "Now it is?"

At this point, E asked J to tell him what was meant by "equivalent sets." J, very thoughtfully, stated that they have the same number and the same shape and they are in the right order and are circles. After enumerating these four conditions, J said, "That's all."

This interview revealed that J had internalized his own notion of equivalent sets and he was able to act upon this notion quite consistently. Later in the interview, when J was asked to pretend that two sets are equivalent "if they have the same number, and that's all," J declared two sets with the same number but different configurations to be equivalent, but the videotapes reveal he was reluctant to do so. Apparently, it is not easy for him to act upon a hypothesis which does not agree with the concept of equivalent sets he has developed on his own. This case study raises an important question for teachers to consider: How different are the concepts which children form from the concepts that the teacher intends them to have?

FOLLOW-UP STUDY OF PERFORMANCE ON SMSG FIRST GRADE EQUIVALENT SETS SCALE

What would the term "equivalent sets" mean to a child who has not been explicitly taught, as in the SMSG kindergarten curriculum, that in order for two sets to be equivalent they must have the same number of members? Prior to the administration of the PMDC test battery, in particular the SMSG test on equivalent sets (pp. 12-14), several PMDC principal investigators hypothesized that the answer to the above question would be "nothing." Nevertheless, while most of the pupils who participated in the PMDC Testing Program had not been exposed to an explicit treatment of equivalence, their performance on the SMSG Equivalent Sets Scale did not bear out the principal investigators' conjectures. Eighty-five percent of the PMDC sample population correctly answered at least one-half of the items, with 75% of the total sample giving correct responses to at least five of the six items in the scale. Furthermore, it was necessary to give the alternate direction, "Make a set with the same number," to only a small fraction of the pupils. Even fewer pupils asked, "What is 'equivalent'?" Thus, without having studied a formal definition of equivalent sets, the pupils in the PMDC testing population were able to do remarkably well on tasks involving the construction of a set equivalent to a given set. A secondary question was thus formulated; i.e., What did the term "equivalent sets" mean to these pupils?

In accordance with the scoring guidelines used by PMDC testers, the method used by a pupil to solve an equivalent sets task was recorded on the pupil answer sheet (bibliography Item A). Basically, the problem-solving techniques were grouped into two major categories: a matching strategy and a counting strategy. In solving the six items on the scale, the counting strategy was used approximately 35% of the time and the matching strategy in about 55% of the cases. The counting strategy was slightly more effective, having a correct/incorrect ratio of 8 to 1 compared to a 6 to 1 success ratio for the matching strategy.

By utilizing a counting strategy to construct a set equivalent to a given set, the pupils exhibited an understanding that equivalent sets had to be equal in number. Approximately one-third of the pupils using a counting strategy attempted to reproduce the configuration of dots on the card. This last suggests that for these pupils, equivalent sets must have the same design (members of the sets arranged in identical patterns) as well as be equal in number. The other pupils who used a counting strategy, about 25% of the total sample population, made no effort to reproduce the dot configuration. For these pupils, then, equality in number was the sole criterion for establishing the equivalency of two sets.

The meaning that "set equivalence" had for the pupils who used a matching strategy was not entirely clear from the data obtained during the administration of the equivalent sets scale. Therefore, a further study was undertaken in an effort to uncover possible meanings of "equivalent sets" among beginning first grade children. The study was conducted four weeks after the completion of the PMDC testing program. During the interim, the children had completed exercises in their textbook drawing lines to pair the members of two sets and then deciding whether or not the sets were equal in number. However, throughout the unit on matching sets the term "equivalence" had not been used.

The follow-up study was conducted in a school which serves a predominately low socioeconomic community. However, the eleven pupils (six boys, five girls) involved in this study were from varied socioeconomic backgrounds. The median SEI for the group was 400. This measure was close to the median SEI of 393 for the entire PMDC sample. The pupils also varied in their readiness for first grade work as measured by their performances on the Metropolitan Readiness Test and for the SMSG scales. The data from these instruments are reported in Table 27, on the page following, along with the corresponding data for the total PMDC testing sample.

Although the pupils in the follow-up sample did not exhibit readiness concepts and skills on a par with those of the total PMDC testing population, their average achievement was slightly above that of the pupils in their class who did not participate in the follow-up study. (See bibliography, Item C). Of the eleven pupils, six used a counting strategy in solving the tasks on the SMSG Equivalent Sets Scale. Four of these six pupils attempted to reproduce the dot configuration after they counted the buttons. The remaining five pupils in the follow-up sample employed a matching strategy to solve the problems.

The regular classroom teacher randomly selected the pupils to participate in the follow-up interviews. Each interview was individually administered, and averaged about ten minutes in length. The interviews were structured as follows:

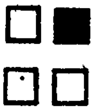
Table 27

Data on Follow-up Study and PMDC Sample

Instrument	Mean Measures	
	Follow-up Sample	Total PMDC Sample
Metropolitan Readiness	51%	69%
SMSG Counting	7.1	6.8
SMSG Equivalent Sets	3.9	4.8
SMSG Ordering	3.5	4.7
SMSG Classification	4.3	4.6

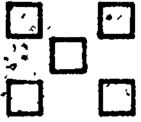
(a) The tester gave the pupil about 20 one-inch cardboard squares, some white and some black.

(b) The tester used similar squares to construct the following pattern;



and then asked the pupil to use his/her squares to make an equivalent set. When the pupil had constructed a set, the interviewer asked the pupil why the set was equivalent.

(c) The pupil was asked to make a set and was told that the interviewer would make an equivalent set. In constructing the set, the interviewer used the same number of squares, but with a different combination of black and white squares and in a significantly different design. For example, one pupil made the following set,



and the interviewer responded as follows:

The pupil was asked, "Did I make a set equivalent to your set?" If the pupil answered, "No," the interviewer asked the pupil to explain how the set could be changed to make it equivalent.

(d) The third task was similar to the first (b), except that the interviewer made the following pattern:

(e) The fourth and final task was a replication of the second (c), except that the interviewer used a different number and color combination to make a design similar to the pupil's. For example:

Thus, the pupils were presented with four situations in which they had to explain why two sets were or were not equivalent.

The explanations given by the pupils were quite clear. In each case, the pupil cited specific reasons as to why the sets were or were not equivalent. One of the eleven pupils based his justifications solely on the design (arrangement of objects) of the set, giving no indication that equality of number was a condition for equivalence. For example, in response to the first task (b) this pupil made a square array with nine members and said that the sets were equivalent. All of the other pupils cited a number condition as being necessary for the equivalence of two sets.

These ten pupils, however, differed in the way they used tbe number property. Two of the pupils focused on the number of the whole set. That is, in responding to the fourth task (e), one pupil said, "They are not equivalent because this set has 6 and yours 5." He directed the interviewer to place one more square in the row with only two squares. He then was satisfied that the sets were equivalent. The remaining eight pupils focused their attentions on the number properties of subsets of the given set. For example, in justifying his response to the first question (b), one pupil said, "You have one black and I have one black. You have 3 whites and I have 3 whites." Another pupil gave this explanation: "I have 2 here [pointing to top row] and you have 2; [pointing to bottom row] and you have 2." When presented with a situation similar to the example in (e), one pupil answered, "No. I have 3 [pointed to a subset with 3 squares], but you have 2 [pointed to a subset with 2 squares]." Thus, this group of 8 pupils established the equivalence or non-equivalence of two sets by comparing the number property of their subsets. In each case, the pupils partitioned the sets into subsets with four or fewer members. While four of these pupils usually insisted that the squares in both sets be arranged in the same pattern, only one pupil identified equivalent subsets, regardless of the arrangement of the item. Although none of the eleven pupils indicated that sameness in color combinations was a necessary condition for equivalence, the eight pupils who compared subsets frequently used color in identifying subsets.

Not all of the pupils who made comparisons with subsets were always successful in identifying equivalent sets. Three of these pupils consistently focused on only one pair of subsets. If that particular pair of subsets were equivalent, then they responded that the sets were equivalent. For example, in responding to the second task (c), one pupil placed only 3 squares in a line and said, "Three here, three here; they are equivalent." The interviewer pointed to the entire set he had made and asked, "Is my set equivalent to your set?" The pupil reaffirmed that the sets were equivalent. In another case (the fourth task), the pupil made a set with seven squares. The interviewer made a set similar in design, but with only six squares. The pupil identified a subset of three squares in each arrangement and responded that the sets were equivalent. In a sense, the pupil had a correct answer because his attention was focused on only one pair of subsets, each with three members.

The data from this investigation into what the term "equivalent sets" means to first grade pupils suggest that similarity in the arrangement of objects within a set is likely to be a necessary condition for equivalence. This requirement, which is self-imposed by the pupil, could be an adverse factor in the learning of other mathematical concepts and skills such as addition and subtraction. The data also suggest that many pupils have developed—on their own—a technique for comparing the number property of sets by partitioning a larger set into subsets. This capability on the pupil's part could be capitalized upon in teaching addition and subtraction. However, the teacher must exercise caution by insuring that the pupil follows this technique through to completion, and does not terminate the process after one or two comparisons, as some pupils did in the examples cited above.

FOLLOW-UP STUDY OF PERFORMANCE ON SMSG FIRST GRADE ORDERING SCALES

Approximately 72% of the first grade pupils in the PMDC testing sample successfully completed at least five of the six seriation tasks in the SMSG Ordering Scales. At the other extreme however, 11% of the pupils did not succeed with any of the items. Further, almost 24% of the pupils in the testing population failed correctly to answer at least one-half of the seriation items. There are several possible explanations as to why certain pupils did not exhibit, in their performances on the SMSG scale, an understanding of seriation concepts. Included among the most probable are: (a) the child had not developed a concept of seriation, (b) the child did not understand the directions and/or the vocabulary used in presenting the tasks, or (c) the conditions of the testing situation could have adversely affected the child's willingness to respond. The study described below was conducted for the purpose of obtaining additional insights as to why some children did not exhibit a greater knowledge of seriation concepts and skills than their performances on the SMSG scale indicated.

All twelve students selected to participate in this follow-up study were from one first grade class and represented 41% of the class. With the exception of two pupils, individual rankings on a socioeconomic scale could be described as low. For this group, the median socioeconomic index on the Hollingshead Scale was 650, with an index of 750 representing the lowest socioeconomic status. The group's average performance on the Metropolitan test was also low. On this test, only four pupils had test scores above the twentieth percentile. On the SMSG Counting Scale, eight of the twelve pupils gave correct responses to at least seven of the ten items. The other four pupils correctly answered, at most, two questions. Overall, this group of children did quite poorly on the SMSG Equivalent Sets Scale. Only two students successfully completed a total of five or six tasks. The remaining ten pupils gave correct responses, at most, to two of the six items on the scale. Eight of the pupils correctly identified at least half of the five objects in the SMSG Classification Scale; the other pupils answered, at most, one question. Based on the data provided by these various evaluation instruments, this group of twelve pupils appeared to be less ready or capable of doing first grade math than most of the pupils in the PMDC sample population.

The follow-up study was conducted through one-to-one interviews with the twelve pupils. The typical interview took about five minutes, with only two interviews extending to twelve minutes. The same basic format was followed in conducting all interviews. However, modifications were made to accomodate different pupil responses. A description of the interview procedures and a summary of the pupils' performances on each task follow.

Each interview began with administration of the first item on the SMSG Ordering Scale which the child had missed during the regular testing program. For nine pupils it was the first item (circles)[12], for two pupils it was the second item (triangles), and for one pupil it was the third item (buttons). Only three pupils successfully performed their first tasks in the follow-up study.

The next step was to place the set of five blocks in a pile on the table and to instruct the pupil to put them in order. Ten of the twelve pupils did not perform this task correctly. Of the three pupils who were successful on the first test, two were successful on the second task. At this point, the interview structure was modified for these two pupils. They were given the remaining sets of objects (objects not used in task one or task two), and told "Put these in order." Both pupils successfully completed all remaining seriation tasks. One of these pupils had correctly answered three of the six items during the original SMSG test administration and the other pupil had correctly ordered two sets of objects. Apparently, the performance of these two pupils on the seriation tasks administered during the original testing program can be attributed largely to conditions of the testing situation, such as the presence of video equipment, the one-to-one interview, the unfamiliar interviewer, and/or the unfamiliar room.

The follow-up procedure for the ten pupils who did not respond to the instructions "Put these in order" was for the interviewer to order the straws longest to shortest on the table and to say to the child, "Order the blocks like I have ordered the straws." Six of the remaining ten pupils were successful in this task. To these children, the interviewer gave each remaining set of objects (buttons, circles, straws, rectangles and triangles), and said, "Put these in order." Each of the six pupils was able to order correctly the objects in at least four of the five sets. The correct responses suggest that these children were unsuccessful in previous seriation tasks because they had not understood the directions. One pupil in this group had correctly answered three items on the SMSG test. For this child, the conditions of the testing situation may have been the major factor influencing his previous responses.

For the pupils who had not successfully completed the above tasks, the interviewer ordered the buttons from largest to smallest, gave the child the set of circles, and said, "Order the circles like I have ordered the buttons." Two pupils correctly performed the task in response to this model. Further, given the directions "Put these in order," they were then able to order correctly the objects in at least four of the six sets. One child did not correctly order the rectangles and straws; the extremes were correct, but two of the middle objects were interchanged. A possible reason for these errors is that the child placed the objects end-to-end, thus making it more difficult to detect differences in length. Although these two pupils were eventually successful in performing seriation tasks, the explanation for their inability to do the prior seriation tasks is not clear. It could be that these children needed the reinforcement of several models before they gained sufficient confidence to respond, or that the seriation test itself could in fact have been a learning situation for them.

For the two pupils who were not successful on any of the above tasks, the follow-up interview continued with the interviewer ordering the straws and asking the pupil to place the rectangles on the table in a like

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[12] see pp. 14-15.

manner. Neither of the pupils performed this task successfully. In the basic plan for the follow-up study, the interview was to be terminated at this point. However, during the follow-up session, one child commented that the longest rectangle was the "daddy" and the shortest rectangle was the "baby." Following up this lead, the interviewer asked the child to think of the rectangles as members of a family and to place them in order. The child did so and explained why the arrangement was correct. Further, this pupil was able to order correctly the objects in each of the other sets when instructed to think of the objects as a family. This technique was also used in testing the other child with the same results. Thus, these two children apparently did in fact possess the essential seriation concepts and skills in the context of a concrete situation.

From the results of this follow-up study, one can conclude that the data from the original SMSG test did not give a totally valid assessment of these twelve children's understandings of seriation concepts. Rather, it appears that the testing situation and/or the test directions were the root causes for the pupils' poor performances. Teachers should be reluctant to accept a child's poor performance on a typical seriation test as a valid indication that the child does not possess the basic seriation concepts and skills. Rather, in these situations, the teacher should vary the mode of presenting the tasks to determine whether some external factor, such as vocabulary, is obscuring the child's real abilities.

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D.	School Profile Questionnaire and Class Profile Questionnaire	$ .25
E.	Master Record Forms	$ .25
F.	Pupil Score Sheet—Report on Preliminary Testing Program	$ .25
G.	Summaries of First and Second Grade Data by Individual Schools	$ .50
A-G		$3.00

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Tallahassee, Florida 32306

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