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ABSTRACT

The purpose of this study was to determine how students' perceptions of the proportionality schema are related to achievement in introductory chemistry. A total of 309 tenth-grade students, comprised of an ALCHEM materials sample (N=168) and a CHEM study sample (N=141), enrolled in four urban senior high schools, was tested under normal classroom conditions. A set of four collectively administered neo-Piagetian Reasoning Tests served as a measure of students' cognitive functioning level. The General Proportionality Test served to indicate students' general proportional reasoning. Four subtests of a Chemistry Proportionality Test measured achievement in chemistry. In terms of cognitive functioning, the sample was classified as concrete operational (28.8%), transitional (22.3%), early formal (27.2%), and late formal (21.7%). Among additional findings were that significant correlations existed between proportional reasoning in chemistry and (1) achievement in chemistry, (2) cognitive level, and (3) proportional reasoning in a non-chemistry context, and that instruction in proportional reasoning in chemistry did not appear to enhance general proportional reasoning. (MH)

PROPORTIONAL REASONING IN INTRODUCTORY HIGH SCHOOL CHEMISTRY

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Introduction

If curriculum design is to respect the sequence of cognitive development of students in general and of individual students in particular, its nature as related to specific aspects of science curriculum needs examination. Piaget and his co-workers (Inhelder and Piaget, 1958) have identified a sequence of mental actions leading to students' understanding of principles and relationships in several phenomena taught in science courses. Identification of characteristics of the student's thought processes within a stage or substage of cognitive functioning specific to a given concept or principle may supplement these descriptions in ways helpful to science curriculum development. Within a development stage the schema described by Piaget provide a means to identify the types of understandings and thought processes available to individual students. The purpose of this study was to ascertain the extent to which students' perception of the proportional reasoning schema are related to achievement in introductory chemistry at the high school level.

Proportional Reasoning

One of the eight formal operational schema described by Piaget (Inhelder and Piaget, 1958, p. 307-329), the proportionality schema is the internalized structure or mental representation which enables the individual to act on the mathematical equality of two ratios, e.g. $a/b = c/d$, in order to determine their equivalency. The proportionality schema is operational in the sense that it directs the subject towards a specific action or transformation upon a given domain of objects, quantities or symbols. The ratio may arise from the manipulation of concrete objects, as in the case of the paper clip tasks used by Karplus (1974, 1972) or it may arise from symbolic manipulation as in the case of solving a chemical equation for a given mass or volume.

According to Piaget the notion of quantitative relationships in proportional reasoning does not appear until the formal substage IIIA, and only rarely appears before substage IIIB (Inhelder and Piager, 1958, p. 173). This finding also appears to be the case in tasks other than the balance scale experiments (Lawson and Blake, 1976). For the present study, aspects of proportional reasoning were measured by three tasks, namely the Balance Problem (BP), the Ratio Task (RT) and the Metric Puzzle (MP). These tasks were chosen because they all involve direct application of the proportionality schema, have been widely used, and permit scoring techniques based on established criteria. A fourth task, the Islands Puzzle (IP) served as a more general measure of formal operational thought.

Student responses to individual tasks were examined in view of criteria considered indicative of concrete and formal thought. Students meeting the minimum established criteria for each task were considered successful on that task. Category Concrete (C) included only those students who did not meet the minimum requirements on three or all four tasks.

Students failing to satisfy minimum requirements on any two of the four tasks presented were said to be operating at a transitional (T) level for purposes of the study. Students meeting criteria on three of the four tasks or all four tasks were said to be operating at the early formal (F1) and late formal operational (F2) levels, respectively. The overall cognitive level categorization was made on the basis of a composite four-task performance (using 0-4 scale) ranging from 0 for those students who fail to meet minimum requirements for every task to 4 for successful performance on all tasks.

Proportional Reasoning in Chemistry

To study the relationship of the schema of proportionality to selected concepts in introductory high school chemistry, four major areas of investigation were identified.

1. Chemical nomenclature and the writing of formulae. These topics are normally introduced early as basic tools for further application in more advanced areas in chemistry. Forms of proportionality exhibited at this level are related to the basic laws of chemical change (Law of Definite Proportions, Law of Multiple Proportions) and to fundamental aspects of the nature of matter (e.g., atomic masses). To illustrate, atomic masses are based on the isotopic composition of elements and may be said to represent a proportional average.

This area also included numerous instances in which students apply basic rules to obtain chemical formulae for specific compounds. Given the chemical symbols and valences for the ions concerned, the student determines the simplest whole number ratio of ions such that the net sum of ionic charges is zero. It is necessary that the student recognize the invariance of the combining ratio as given by the simplest or empirical formula. This is especially important when dealing with ionic solids where the number of ions of the given elements in the aggregate may vary without affecting the fixed ratio of ions. It seems unlikely that the study of chemistry and chemical transformations can begin until the student acquires the notion of invariance of chemical substances (i.e., composition) in spite of certain physical or other superficial changes that may be observed.

2. Gravimetric stoichiometry. The study of quantitative relationships implied by a chemical reaction (stoichiometry) contains instances in which

proportional reasoning is of central importance. In particular, the concept of the 'mole' has wide applicability in this area and serves to illustrate a number of types of proportionality problems.

The concept of a gram-molecular weight of a substance (mole) or molar mass is usually presented early in high school chemistry. Often the mole concept proves to be a troublesome one for students. Part of the difficulty may arise from the fact that while a mole constitutes a certain mass for a given substances, its real significance emerges when it is connected with the number of particles (Avogadro's number of atoms, ions, molecules, etc.) That is, while the actual mass of a mole varies for different substances, the number of particles remains invariant. Furthermore, in the case of gaseous substances the mole is often dealt with in terms of a given volume under standard temperature and pressure conditions. Clearly conservation concepts (i.e., conservation of mass for particular substances, conservation of number and volume for all substances) are prerequisite schema for the 'mole' concept.

Stoichiometric problems dealing with the mole concept may be categorized into three main types:

1) <u>Conversion problems</u>

Here students given an initial mass convert directly to moles of a substance (A) using the relationship,

$$\text{moles of A} = \frac{\text{mass of A}}{\text{molar mass of A}} = \frac{\text{grams}}{\text{grams per mole}}$$

Conversely a number of instances require the conversion of a given mass of substance (A) into moles as given by:

$$\text{mass of A} = (\text{moles of A}) \times (\text{molar mass of A}).$$

The notion of reversibility seems particularly relevant for the ability to apply proportional reasoning when dealing with conversion problems of this nature. For example the operation of decreasing the mass of a given substance has a negating (or inversion) effect on the number of moles of substance present. However a similiar decrease in the number of moles could result via a reciprocal operation for the same mass of substance, if the substance dealt with posessed a greater molar mass. The realization of these two forms of reversibility (negation and reciprocity) may lead to a firmer understanding of the 'mole' concept and its application to a number of areas in chemistry. Other important conversions may require the expression of moles in terms of volumes under certain conditions (e.g., when dealing with gaseous substances) or in terms of the number of particles (ions, atoms, molecules, etc.)

ii) Ratio problems

Application of the mole concept in gravimetric stoichiometry is often discussed in relation to the reacting mole ratios implicit in a balanced chemical reaction. Consider the chemical equation represented by:

$$aA + bB = cC + dD,$$

Where a, b, c, and d represent the numerical coefficients of the balanced chemical reaction. Here the mole is treated in terms of a reacting ratio dependent upon the substances involved. That is, in this case 'a' moles of substance A reacts with 'b' moles of B to form 'c' and 'd' moles, respectively, of the products C and D.

Given, for example, x moles of A, the number of moles of reactant B required for reaction is determined by taking into consideration the mole ratio in which the two reactants combine, namely

$$\frac{\text{moles of B}}{\text{moles of A}} = \frac{b}{a}.$$

Hence the required number of moles of B required for reaction with x moles of A is given by (x moles of A) x (mole ratio, $\frac{b}{a}$).

In addition to the more obvious numerical relationships required to solve quantitative problems of this nature, the student must be able to freely move from a microscopic or molecular perspective of the reaction (as implied by the balanced equation) to the macroscopic or molar interpretation normally associated with laboratory work. That is, the impracticality in most cases of discussing individual atoms or molecules in a reaction entails an implicit understanding of the 1:1 proportionality existing between the molecular and molar connotation of a chemical reaction. Unless the student understands that the number of particles represented by a mole of any substance (Avogadro's number or 6.02×10^{23}) remains invariant, this relationship will undoubtedly be a major source of difficulty. Certain items of the Chemical Proportionality Test (CPT) used in the present study were designed to measure students' understanding of the relationship between the micro and macro interpretations associated with chemical formulae and equations.

iii) Proportionality problems

Proportionality implies the understanding of the equality of two ratios, $\frac{x}{y} = \frac{x^1}{y^1}$. According to Piaget the discovery of the equivalence of two ratios is intimately related to notions of inversion and reciprocity found in the INRC group model and does not occur before the formal level (Inhelder and Piaget, 1958, p. 314). In introductory chemistry, a number of examples of the proportionality schema are present in the form p, p*, q, q* described by Inhelder and Piaget. This schema of metrical proportions implies an equivalence between ratios such that:

$$\text{if } \frac{p}{q} = \frac{p^*}{q^*}, \text{ then } \frac{p}{p^*} = \frac{q}{q^*}.$$

Using the previous illustration from chemistry,

$$aA + bB = cC + dD,$$

a number of proportions consisting of equivalent mole ratios can be obtained, e.g.,

$$\frac{a}{b} = \frac{a^1}{b^1}, \frac{b}{c} = \frac{b^1}{c^1}, \frac{a}{d} = \frac{a^1}{d^1}, \text{ etc.}$$

The realization of the equivalency between respective ratios as determined by a balanced chemical reaction appears prerequisite to a sound chemical understanding of stoichiometry. This would apply in instances where stoichiometric problems are presented algebraically from data derived directly from the balanced reaction or in terms of reacting moles ratios. The relative effectiveness of both approaches to the treatment of proportionality in stoichiometric calculations was examined in the present study. The Chemistry Proportionality Test (CPT) covered aspects of the proportionality schema as it relates to introductory chemistry concepts. The inclusion of a General Proportionality Test (GPT) consisting of items measuring aspects of the proportionality schema similar to those in the CPT served as a means of determining the extent to which findings with respect to proportional reasoning in chemistry are specific to the subject matter studies.

Purpose

The main purpose of this investigation was to ascertain the extent to which students' perceptions of the proportionality schema are related to achievement in introductory chemistry. The central question of the study is: Is the ability to apply proportional reasoning a significant factor in achievement in selected areas in introductory high school chemistry?

Sample

The sample consisted of 168 Grade 10 Chemistry students in two large urban high schools in Edmonton, Alberta, Canada. The seven cooperating teachers were selected for their involvement in the development of the Alberta Chemistry Materials Program (ALCHEM) which formed the course of studies for the sample.

Procedure

The following test instruments provided the variables examined:

1) Chemistry Achievement Test (CAT) - a 60 item multiple choice, four distractor open book achievement test based on the Chemistry 10 program. The CAT included items equivalent to those present on the subtests of the Chemistry Proportionality Test (CPT), e.g., chemical nomenclature, chemical reactions, mole concept, and gravimetric stoichiometry.
2) General Proportionality Test (GPT) - The two logical and statistically equivalent forms of the GPT (Form A and Form B) dealt with ratio and proportion as they applied to common situations.
3) Chemistry Proportionality Test (CPT) - a test devised for the study consisting of short answer and open-ended questions in introductory chemistry in which students are required to show all their work in arriving at their solutions. The CPT is composed of four subtests each one dealing with one of the following topics:
   a) Chemical nomenclature and the writing of formulae (CPT[1]).
      e.g., Given two hypothetical elements X and Y with valence +6 and -2 respectively, suggest a formula for the compound formed between X and Y.

b) Chemical reactions (CPT[2]).

e.g., Write a balanced chemical equation from the statement: Aluminum sulphide reacts with water to form aluminum hydroxide and hydrogen sulphide.

c) The 'mole' concept (CPT[3]).

e.g., Determine the number of moles in an 80 g. sample of NaOH (1 mol = 40 g.)

d) Gravimetric stoichiometry (CPT[4]).

e.g., Given a balanced chemical equation represented by: $aA + bB = cC + dD$ where a, b, c, and d represent the numerical coefficients of the substances A, B, C, and D, students are asked to calculate, for example, the number of moles of B required for complete reaction with X moles of A.

The items on the CPT subtests were constructed so as to illustrate aspects of the proportionality schema analogous to those found in the General Proportionality Test (GPT). For example:

i) Conversion type problems.

e.g., Find the number of atoms in 16 g. of sulphur. (at. wt. of S = 32)

ii) Ratio type problems.

e.g., Determine the reacting ratio of two substances in a given balanced Chemical reaction.

iii) Direct proportionality problems.

e.g., Given that one mole of any element contains 6.02×10^{23} atoms, calculate the weight of one atom of oxygen (at. wt. of oxygen = 16).

4) Neo-Piagetian Reasoning Tasks (NRT) - The four group-administered paper and pencil reasoning tasks which served as a measure of students' cognitive

functioning level were:

a) Balance Problem (BP): (Inhelder and Piaget, 1958; Lovell, 1961; Lunzer and Pumfrey, 1966).

b) Ratio Task (RT): a modification of the Form B paper clip ratio task used by Karplus et al. (1974).

c) Metric Puzzle (MP): a direct proportionality task requiring students to predict a distance in kilometres given basic data necessary for the conversion. The Metric Puzzle is a modification of that used by Collea, 1975.

d) Islands Puzzle (IP): a paper and pencil formal operational task requiring students to answer questions concerning possible plane routes among a series of four islands, given certain constraints.

The tasks were administered prior to the commencement of instruction in chemistry, followed by the GPT(A) during regular classroom periods. Subtests of the Chemistry Proportionality Test (CPT) followed the conclusion of each appropriate unit throughout the semester period. At the conclusion of semester, all students (N = 168) were administered the alternate form of the General Proportionality Test, GPT(B), followed by the CAT.

Results

The classification of the study sample in terms of cognitive functioning level was as follows:

	Students	(%)
Concrete operational (C)	55	32.7
Transitional (T)	40	23.8
Early Formal (F_1)	36	21.4
Late Formal (F_2)	37	22.1
N =	168	100.0 %

Seventy-three students (43.5%) were found to be capable of using formal operations in their thinking with respect to these tasks and 55 students (32.7%) appeared to be limited to concrete operations. The remaining 40 students (23.8%) were deemed to be at an intermediate or transitional level of thought. This categorization relies heavily on the assumption that the group administered tasks as used in the study give a valid indication of the level of cognitive functioning (Rowell and Hoffman, 1975). These broad proportions are in basic agreement with other investigations conducted in the area (Hobbs, 1975; Renner and Lawson, 1973; Field and Cropley, 1968) indicating that a considerable proportion of high school students may not be functioning at the formal operational level.

Table I presents the phi correlation coefficients among the four tasks. as expected, all four formal operational tasks were found to be significantly intercorrelated, ($p < .01$)

Table I

Phi correlation coefficients for
BP, RT, MP and IP *N = 309

	BP	RT	MP
BP			
RT	.42		
MP	.30	.32	
IP	.34	.22	.23

* In addition to the ALCHEM study sample (N=168) the four neo-Piagetian Reasoning tasks were also administered to 141 CHEM Study students. Total N = 309.

This is understandable in that three of the tasks (BP, RT and MP) dealt directly with the proportionality schema (or at least the understanding of the equivalency between stated ratios) while the fourth, the Islands Puzzle (IP) required the generation of all possible combinations of a hypothetical situation given certain parameters. As such, it possibly taps another subset of formal abilities. For purposes of the present investigation, performance of all four tasks, rather than any particular task, served as a general measure of cognitive level.

A further categorization was attempted using the Ratio Task (RT) as the criterion variable based on the system of categorization described by Wollman and Karplus (1974). Table II presents the categorization results on the Ratio Task by per cent.

Table II

Sample Categorization
on Ratio Task
N = 168

Category	Description	Percent
N	No explanation	1.9
I	Intuition	0.7
IC	Intuitive Computation	4.2
A	Addition	23.3
S	Scaling	2.9
P	Proportionality	67.0
		100.0%

As expected, the majority of students (67.0%) were able to apply proportional reasoning on the Ratio Task (RT). However, the percentage using an additive approach was unexpectedly high (23.3%). Wollman and Karplus (1974) in their investigation of the same task (Ratio Task, Form B) placed approximately one-third of students tested (450, Grades 7 and 8 students) in Category A (Addition). While results of the present study reflect a smaller proportion of students responding in category A, attributable to the higher grade level, the number is considerable and appears to represent a relatively persistent mode of reasoning which students apply in a consistent fashion. Students using additive reasoning tend to treat given data with a simple, well-defined strategy directing their attention toward difference in information given rather than toward their numerical ratios. For example, in the Ratio Task (RT) the focus is on the height difference instead of the height ratio. e.g.,

> "In the small paper clips it took 6 to get 4 large paper clips so you add 2 small paper clips to the height of the large flask." (answer 8).

> "The diagram was 2 clips more than they said it was in the upper part (referring to the written section) so it should be 2 clips higher also." (answer 8).

Additive reasoning was also evident on both the Balance Problem (BP) and the Metric Puzzle (MP) and appeared to be associated with concrete thought.

For example, on the MP, 16 students or 8.6% were classified as additive in their approach to the problem. Eleven of these 16 students were classified as concrete thinkers (C) on the basis of their performance on all four tasks, while the remaining 5 were classified as transitional (T). Typical explanations for responses from these students included:

"If you subtract 288-180 you will get 108 difference so I just added 108 to 350."

"I minused the miles from the kilometers and got 108. Then I added 108 to 350 to get 458."

"Since the number is 108 higher from miles to kilometers in Calgary it should be the same for Saskatoon."

Preliminary analysis of the nature and extent of student responses suggests that the additive mode does represent a widespread, stable strategy which is clearly not randomly applied. It appears to be more prevalent among concrete thinkers than formal thinkers. However, the present scoring system does not lend itself to any further analysis.

The means, standard deviations, intercorrelations, and internal consistency reliabilities of the study tests are presented in Table III.

Significant correlations were found between proportional reasoning in chemistry (CPT) and (i) achievement in chemistry (CAT), (ii) cognitive level (PT) and (iii) proportional reasoning in a non-chemistry context (GPT).

Overall mean differences for the two GPT forms were noted to be non-significant. Despite the somewhat low intercorrelation, the GPT forms were considered to be both logically and statistically equivalent on the basis of preliminary investigations. Hence the consistency and non-significance of the pretest (GPT[A]) and postest (GPT[B]) findings as reported in Table III are particularly noteworthy. Students' exposure to prolonged instruction in chemistry, which draws heavily on all levels of the proportionality schema, appears to have little or no effect on students' ability to apply the schema in a more general or common sense.

The study sample (N = 168) was divided into three nearly equal achievement groups (high, middle and low) on the basis of performance on the CAT.*

* One-way analysis of variance revealed that the three achievement groups were significantly different with respect to each of the following study variables: GPT(A), GPT(B), PT, CPT(1), CPT(2), CPT(3) and CPT(4).

Table III

Pearson Product-Moment Intercorrelations, Means and Standard Deviations for Test Scores on the Neo-Piagetian Reasoning Tasks (NRT), General Proportionality Tests - GPT(A) and GPT(B), Chemistry Proportionality Tests - CPT(1) Nomenclature and Formulae Writing; CPT(2) Chemical Reactions; CPT(3) The Mole; CPT(4) Gravimetric Stoichiometry, and the Chemistry Achievement Test (CAT)

N = 168

	KR-20	NRT	GPT(A)	GPT(B)	CPT(1)	CPT(2)	CPT(3)	CPT(4)	CPT(T)	CAT
NRT										
GPT(A)	.69	.48								
GPT(B)	.74	.36	.52							
CPT(1)	.70	.45	.38	.33						
CPT(2)	.61	.55	.46	.44	.67					
CPT(3)	.77	.54	.50	.40	.55	.65				
CPT(4)	.70	.42	.37	.29	.57	.64	.58			
CPT(T)	-	.58	.50	.44	.83	.86	.83	.82		
CAT	.87	.48	.46	.41	.65	.70	.66	.67	.79	
Mean		2.3	23.9	22.1	16.3	12.8	13.3	9.2	51.6	36.7
Standard Deviation		1.1	4.0	4.2	4.1	3.0	3.8	3.1	12.1	9.9

Table IV shows the distribution of male and female students in the study sample within the three achievement groups. The sample was 53.6 per cent male and 46.4 per cent female. A chi-square test of independence performed on the CAT data indicated that there was no significant difference in performance for the three groups according to sex.

Table IV

Distribution of Main Study Sample by Sex in High, Middle and Low Achievement Groups N = 168

	Achievement Group				
Sex	High	Middle	Low	Total	
Male	35	27	28	90	(53.6%)
Female	22	27	29	78	(46.4%)
Total	57	54	57	168	(100.0%)

(χ^2 = 2.0, df = 2, .30 < p < .50)

Table V presents the sample distribution by sex and cognitive level.

Table V

Distribution of Sample by Sex and Cognitive Level N = 168

Sex	Concrete (C)	Transitional (T)	Early Formal (F1)	Late Formal (F2)	Total
Male	19	21	21	29	90
Female	36	18	16	8	78
Total	55	39	37	37	168

(χ^2 = 17.3, df = 3, p < .001)

Only 19 male students (11.4%) were considered concrete in their thinking compared to 36 (21.4%) for the female students. Further, 17.2 per cent of the male sample were categorized as late formal (F2) compared to only 4.7 per cent for the female group.

A stepwise regression analysis for the prediction of achievement in chemistry revealed that the chemistry proportionality subtests were the only significantly contributing predictor variables accounting for 63.4 per cent of the total variance of the chemistry achievement test (CAT) scores. Table VI presents the results of the regression analysis.

Table VI

Prediction of Chemistry Achievement (CAT) Scores from Sections of the General Proportionality Test, GPT(A), the Balance Problem (BP), the Ratio Task (RT), the Metric Puzzle (MP), the Islands Puzzle (IP), the Total Score on the Neo-Piagetian Reasoning Tasks (NRT) and the Chemistry Proportionality Subtests, CPT(1), CPT(2), CPT(3), CPT(4).

Prediction Variable Entering	F Value for Variable Entering	Total F Value	Probability Level	R^2 (per cent)
CPT(2)-Chemical Reactions	159.2	159.2	.001	48.9
CPT(4)-Gravimetric Stoichiometry	31.3	109.8	.001	57.1
CPT(3)-'The Mole'	16.6	85.7	.001	61.0
CPT(1)-'Nomenclature and the writing of formulae'	10.6	70.7	.0014	63.4

Regression equation in raw-score form is given by:

$$\hat{Y}_{CAT} = 0.80\ X_{CPT(2)} + 0.63\ X_{CPT(4)} + 0.75\ X_{CPT(3)} + 0.53\ X_{CPT(1)} + 2.7$$

The first varialbe to enter the regression equation was the CPT(2) subtest score, 'Chemical Reactions', followed by CPT(4), 'Gravimetric Stoichiometry', which increased R^2 to 57.1 percent. CPT(3), 'The Mole' and CPT(1), 'Nomenclature and the Writing of Formulae', entered next increasing the total variance accounted for by the four CPT subtests to 63.4 per cent. Results of the stepwise regression analysis are in agreement with the correlational data. That is, CPT(2), 'Chemical Reactions', the most significant predictor of chemistry achievement had the highest correlation coefficient with CAT (r = 0.70) while CPT(1) 'Nomenclature and the Writing of Formulae', the last significant predictor to enter the regression equation, had the lowest (r = 0.65).

Results of a principal-factor solution performed on the intercorrelation matrix of the eighteen study variables are presented in Table VII.

Three major factors were identified. The chemistry proportionality subtests CPT(1), CPT(2), CPT(3), CPT(4) and the achievement criterion, CAT, loaded heavily on the first factor. The existence of a secondary factor associated with the four Reasoning tasks, BP, RT, MP and IP, was noted to be similar to a finding reported by Bart (1971) on the bifactor structure of formal thought. That is, the second factor seemed to distinguish the Reasoning Tasks (BP, RT, MP and IP) from the proportionality subtests (CPT[1], CPT[2], CPT[3], and CPT[4]). The third factor, containing heavy loadings for the verbal reasoning and verbal analogy sections of the two forms of the General Proportionality Test, CPT(A) and CPT(B) was interpreted as a further indication of the statistical equivalency of these instruments.

Table VII

Principal - Factor Solution

Varimax Rotation

Tests	Factors 1.	2.	3.	Communalities Original	Estimated
1. General Proportions - PT(A)	.41	.36	.38	.43	.45
2. Verbal Reasoning - GPT(A)	.21	.08	.44	.25	.31
3. Verbal Analogy - GPT(A)	.01	.17	.50	.27	.27
4. Numerical Series - GPT(A)	.17	.16	.18	.09	.19
5. Balance Problem - BP	.12	.57	.02	.34	.51
6. Ratio Task - RT	.29	.54	.15	.38	.54
7. Metric Puzzle - MP	.36	.58	.08	.47	.67
8. Islands Puzzle - IP	.20	.49	.30	.37	.63
9. Neo-Piagetian Reasoning Tasks-NRT	.28	.92	.19	.96	.88
10. Chemistry Proportionality - CPT(1)	.69	.25	.17	.56	.55
11. Chemistry Proportionality - CPT(2)	.74	.34	.23	.72	.68
12. Chemistry Proportionality - CPT(3)	.63	.36	.30	.61	.60
13. Chemistry Proportionality - CPT(4)	.72	.19	.18	.58	.55
14. General Proportions - GPT(B)	.25	.24	.48	.35	.37
15. Verbal Reasoning - GPT(B)	.12	.03	.53	.30	.29
16. Verbal Analogy - GPT(B)	.12	.02	.46	.23	.24
17. Numerical Series - GPT(B)	.14	.20	.21	.10	.19
18. Chemistry Achievment - CAT	.78	.24	.24	.72	.67
VARIANCE	3.21	2.73	1.80		
% TOTAL VARIANCE	17.83	15.14	9.96		
% COMMON VARIANCE	41.53	35.38	23.20		
SUM OF COMMUNALITIES			7.73		
TOTAL VARIANCE ACCOUNTED FOR =			43.92		

Table VIII presents the categorization of the study sample according to cognitive level based on the CAT achievement scores.

Table VIII

Distribution of Sample by Cognitive Level and Achievement Scores (CAT)

N = 168

Cognitive Level	Achievement Group High	Middle	Low	TOTAL
Concrete (C)	6	19	30	55
Transitional (T)	11	12	16	39
Early Formal (F1)	13	18	6	37
Late Formal (F2)	27	5	5	37
TOTAL	57	54	57	168

$\chi^2 = 47.7$, df = 6, $p < .001$

A highly significant chi-square ($\chi^2 = 47.7$, df = 6, $p < .001$) was noted for the relationship between cognitive level and Chemistry Achievement scores (CAT) with only 11 students (19.3%) in the low group classified as formal thinkers compared to a total of 40 students (70.1%) in the high group.

Student interpretations of chemical equations and the use of dimensional or unit analysis in gravimetric stoichiometry were examined by analyzing responses to the open-ended section's of the CPT subtests. Considerable misunderstanding surrounding the chemical interpretation of an equation was apparent. For example, in assessing students' perception of a balanced chemical reaction the following question was asked:

CPT(2) Question 2.

> One student maintains that the correct chemical interpretation of the reaction below is that one atom of A reacts with 4 atoms of B to form one molecule of AB_4
>
> His friend claims that a more accurate statement would be, one mole of A reacts with 4 moles of B to form one mole of AB_4. Are either (or both) of these interpretations correct? Discuss your answer.

(equation)	A	+	4B	→	AB_4
Student 1	one atom	+	4 atoms	→	one molecule
Student 2	one mole	+	4 moles	→	one mole

Only 35 percent of the sample (N = 168) felt that both interpretations presented were valid. Thirty-nine students (23.2%) felt that the 'micro' or atomic chemical interpretation presented by Student 1 was the most accurate. Typical explanations included:

> "Student 1 is right because there is only 1 molecule in that part of the formula, there isn't one mole."
>
> "Student one is correct because you are dealing with molecule and atoms, not with moles."

Forty-five students (26.8%) chose the 'macro' or molar interpretation presented by student 2:

> "Yes, student 2 is the most correct one because the moles are more accurate."
>
> "The Student 2's argument would be correct because of the coefficients represent moles, or rather 4 x 6.02×10^{23} atoms of B & 1 x 6.02×10^{23} atoms of A."

Preliminary findings of this nature suggest that a considerable portion of students fail to realize the 1:1 correspondence between atoms and molecules and moles of atoms and molecules.

Dimensional-analysis refers to the commonly used strategy whereby unit factors or dimensions are utilized in converting a measure in one unit to an equivalent measure expressed in another unit.

The use and effectiveness of the unit or dimensional-analysis technique was examined for both the ALCHEM (N = 168) and CHEM Study (N = 141) sample. Dimensional analysis represents a central problem solving approach in ALCHEM but is not emphasized in CHEM Study. The majority of ALCHEM students (over 90 per cent) tended to apply the dimensional analysis approach to problems on CPT(3) and CPT(4) while approximately 60 per cent of the successful CHEM Study students either set up a proportionality expression based on the mole ratio or directly applied the mole definition.

Discussion

Data on the cognitive categorization of the sample are in general agreement with other students (Lawson and Renner, 1974; Lawson, 1973; Higgens - Trenk and Gaite, 1971; Field and Cropley, 1968) which indicate that as many as 50 per cent of senior high school students may be non-formal in their thinking. Piaget's contention that acquisition of the proportionality schema must await formal operational thought appears to be supported. The present study related this schema to science curriculum by investigating its manifestation in selected areas of introductory chemistry. The relationships observed suggest that chemical proportionality, like metric proportionality, may be an intrinsically higher level of ordering experience involving the matching of relations. While chemistry instruction does not appear to enchance proportional reasoning in a non-chemistry context, there appears to be a significant relationship between the student's ability to apply general proportional reasoning and achievement in chemistry.

Performance in the neo-Piagetian Tasks does not appear to be strongly related to general proportional reasoning as evidenced by loadings on in different factors.

Findings revealed that a sizable proportion (over 20 per cent) of students used an additive reasoning mode in solving selected proportionality tasks. The relatively high degree of preference placed on the additive mode of reasoning raises a number of questions concerning its application to other situations. Is it, for example, a function of the given ratios themselves? That is, are problems involving simple ratios like 1:1, 2:1, 2:3, etc. more susceptible to additive approaches than problems with more complex ratios?

A considerable proportion of students failed to realize the 1:1 correspondence between atoms and molecules and moles of atoms and molecules. For these students, it appears as thought the mole is a conveniently contrived concept which bears little or no relationship to the reality of the chemical reaction itself. This inability to perceive the basic 1:1 proportionality underlying much of chemistry may account for some of the difficulty encountered by students in their efforts to apply the mole concept in problem situations. While students had little difficulty in arriving at the correct coefficients for a balanced equation, only about one-third seemed to have an adequate understanding of its significance. Many students appeared baffled by equations in which a given number of moles of reactants yields fewer (or greater in some cases) moles of products. This apparent discrepancy in the minds of some students may be symptomatic of deeper misunderstandings associated with conservation concepts in elementary chemistry (Hall, 1973).

The relationship between cognitive level and use of the additive mode of reasoning warrants further study. Does the additive approach perhaps represent a means for the concrete thinker to achieve some measure of practical "success" on tasks requiring formal thought? It may be that given the typical numerical ratios used in chemistry problems, additive reasoning may enable the student to arrive at the numerically correct response in a sufficient number of instances to warrant its use. It is likely, however, that use of such reasoning patterns will remain unnoticed unless the teacher uses open-ended questions which reveal student problem solving strategies rather than the more common multiple choice variety. In any event, preliminary analysis of the nature and extent of student responses suggests that the additive mode may represent a fairly widespread, stable concrete strategy which is clearly not randomly applied.

Results also suggest that various aspects of proportional reasoning are not available equally to the respondent in all situations. As Lunzer (1965) has pointed out, both the content and nature of the problem, as well as its structure, are important in instances where formal thought is required. Although not investigated in the present study, tasks involving the application of inverse proportionality (e.g., in volumetric stoichiometry) may well place more serious demands on students than do simple conversions or ratio problems (Lunzer and Pumfrey, 1966). Further, the identification of an essentially separate factor associated with the proportionality schema in a neo-Piagetian task sense, as opposed to more conventional paper and pencil measures involving proportions such as the subtests of the CPT, appears to reinforce the importance of the relationship between the schema and the context of the problem presented.

As suggested by Herron (1975) there appears to be a relationship between the effectiveness of dimensional analysis and cognitive level.

Well over half the students classified as concrete thinkers in the ALCHEM sample, where dimensional analysis is emphasized, responded correctly to selected questions. It may be that dimensional analysis represents a strategy which reduces some of the difficulties in proportional reasoning encountered by students who are not yet at the level of formal thought. Although the danger of rote application exists, the procedure is useful in that it at least organizes the material in such a manner that the student may be led to an understanding of the relationships involved.

Whether the process of actually performing the step-by-step procedure itself assists in the conceptualization of the problem as opposed to the efficacy of the concept of dimensional analysis per se remains contentious at this time. In any case it is unlikely that dimensional analysis has any negative effect on student perception of relationships inherent in many problems in introductory chemistry. In spite of the limited basis for these findings dimensional analysis is viewed as a possible means of alleviating or at least reducing some difficulties in proportional reasoning encountered by students who are not yet formal operational in their thinking.

An awareness of the pervasiveness of proportional reasoning in introductory chemistry could be useful to the teacher in devising instructional strategies to reduce difficulties in selected areas. Although non 1:1 molar relationships are undoubtedly mentioned, a larger number of these could perhaps be presented. The tendency of students in the present study to apply 1:1 molar relationships in inappropriate situations is understandable if examples of other ratios are rarely met in the classroom.

The apparent low transferability encountered between general proportional reasoning ability and proportional reasoning in the chemistry context was not altogether unexpected.

The algorithms for dealing with specific types of chemistry problems may overshadow the formal logical requirements of the task. However, one must be cautious in attempting to retrospectively attribute problem solving strategies which may conform to given logical models to students trying to cope with often only partially understood material.

Assessment of students' general cognitive functioning level might be attempted before beginning instruction in chemistry. The type and format of the measures used in the present study, might be appropriate for diagnostic purposes, provided adequate instruction in scoring procedures is made expicit. The sequence of capabilities "conversion-ratio-direct proportionality-inverse proportionality" with respect to proportional reasoning in chemistry could be useful to the teacher as a natural order of succession. Following mastery of chemical conversions, application to balanced chemical equations through the use of mole ratios could be attempted. Direct proportionality applications e.g., "mass to mass" problems in gravimetric stoichiometry and solution chemistry would then follow.

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