DOCUMENT RESUME

ED 136 962 PS 009 216

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TITLE The Relationship Between Piaget's Concept of Reversibility and Arithmetic Performance Among Second Graders.
PUB DATE Apr 77
NOTE 20p.; Paper presented at the Annual Meeting of the American Educational Research Association (New York, New York, April 4-8, 1977)

EDRS PRICE MF-$0.83 HC-$1.67 Plus Postage.
DESCRIPTORS *Arithmetic; *Cognitive Development; Elementary Education; Grade 2; *Problem Solving; Research; *Sex Differences; *Thought Processes
IDENTIFIERS *Piaget (Jean)

ABSTRACT

The relationship between the development of reversible thought and performance in arithmetic equations among children was investigated. Subjects were 86 second grade boys and girls. Two reversibility tasks and 20 addition and subtraction equations were administered. Results indicated significant correlations between the two variables only among the female subjects. This sex difference is discussed in the context of studies which obtained conflicting findings on sex differences in mathematics learning among early grades. The results were interpreted to provide qualified support for the educators' assumption that reversible thought is related to children's performance at arithmetic equations. (Author/SB)

13.12 3-77

THE RELATIONSHIP BETWEEN PIAGET'S CONCEPT OF REVERSIBILITY AND ARITHMETIC PERFORMANCE AMONG SECOND GRADERS

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The Relationship between Piaget's Concept of Reversibility and Arithmetic Performance among Second Graders

The relationship between the development of reversible thought and performance at arithmetic equations among children was investigated with eighty-six boys and girls in the second grade. Two reversibility tasks and twenty addition and subtraction equations comprised the experimental tasks. The results showed significant correlations between the two variables only among the female subjects. This sex difference was discussed in the context of studies which obtained conflicting findings on sex differences in mathematics learning among early grades. The results were interpreted to provide qualified support for the educators' assumption that reversible thought is related to children's performance at arithmetic equations.

The phenomen of number conservation is manifested when a child recognizes that two numerically equivalent sets remain equivalent despite physical rearrangement of the sets (Piaget, 1968, p. 978). The study of number conservation originated with Piaget (1952) and was pursued because Piaget views it as the chief index of children's natural number competence. Instead of studying the development of the number concept by examining the manner in which children perform arithmetic computations, Piaget studied his subjects in a number conservation task. He believes that the stages of number conservation are synonymous with the ontogenesis of the child's conception of natural number.

Because educators typically employ arithmetic computations to study children's development of number concepts, they became very interested in the relation between children's performance in number conservation tasks and in arithmetic performance. This interest led to numerous empirical investigations. (Williams, 1958; Dodwell, 1960; Hood, 1962; Almy, 1966; Steffe, 1966; Wheatley, 1970). In all these studie . the investigators found that children's number conserving ability was positively and significantly correlated with their performance in arithmetic. Such empirical findings enable educators to suggest that number conservation is an important factor in learning arithmetic in the first grade (cf. Copeland, 1974, pp. 90-92; Wheatley, 1970, pp. 294, 299).

The educator's continual interest in the practical implications of Piaget's theory of number development in mathematics education evolves round the functional contributions of central Piagetian concepts to the child's acquisition of certain arithmetic skills. One such central Piagetian concept is reversible thought, which refers to the child's ability to cancel mentally any physical action on objects he has performed in order to picture in his mind's eye the original state of the objects. (Piaget, 1952; 1967; pp. 79-91; Flavell, 1963; Ginsburg and Opper, 1969, pp. 151-152, 167-168). Basically Piaget's concept of reversible thought means mental flexibility, i.e. the child's ability to anticipate and his ability to look back analytically (hindsight). (Inhelder and Piaget, 1969). There is a twofold importance of the concept of reversible thought: (1) Piaget emphasizes that it is the psychological mechanism underlying the child's development of conservation concepts. (2) More importantly, because flexibility of hindsight and foresight transforms and elaborates the development of cognitive structures, the concept of reversible thought assumes central significance in the wider context of Piaget's theory of intellectual development. (Inhelder and Piaget, 1969). The educator's focus on reversible thought leads to the following question: What functional relevance has such an abstract concept of reversible thought to children's acquisition of arithmetic skills? It appears that some educators do attribute an important role to the development of reversible thought in children's acquisition of arithmetic skills. This is evidenced in the following quotations:

"The youngster has achieved reversibility in that if one of two equal sets is re-arranged such as from a row of objects to a pile or heap and the other from a heap to a row, he realizes that the number of each set has not changed, that is heap to row is the same as from row to heap as far as the number of the set is concerned.

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               0                 0
0 0 0 0 0 0 = 0 0     and       0 0 = 0 0 0 0 0 0
             0 0 0             0 0 0
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Reversibility is also necessary for the 'additive' concept.

If a child knows 3 + 2 = 5, can he also solve 5 = $\square$ + 2 or 3 + $\square$ = 5? Many children are 'taught' addition when they have not yet reached the stage of reversibility of thought necessary for the conservation of number concepts involved in such problems. It isn't surprising that first grade teachers find it difficult to teach these ideas."

Copeland (1974, p. 90)

The preceding quotations indicate that Copeland believes reversible thought to underlie number conservation and that he has shown examples of arithmetic skills where the concept of reversible that appears indicated. Because research has shown that inversion reversibility training does accelerate children's development of number conservation, (cf. Brainerd, 1973), Copeland's practical applications of the reversible thought concept appears legitimate. Moreover, because reversible thought involves anticipation and hindsight, (cf. Inhelder and Piaget, 1969), Copeland's specific application of reversible thought to the child's successful solution of arithmetic equations of the type 3 + $\square$ = 5 and 5 = $\square$ + 2, appears intuitively sound. However, there has been no empirical study substantiating Copeland's intuitive link between the child's development of reversible thought and his successful solution of the specific type of equation problems he cited. The purpose of the present study is to redress the lack of empirical research on the relation between reversible thought and the solution of simple arithmetic equations. If it is the case that the child's development of reversible thought is associated with ~~plays a crucial part to~~ his success in solving simple arithmetic equations, we would obtain a significantly positive relation. We would then have provided the necessary empirical base for Copeland's application of Piaget's concept of reversible thought to specific equation problems in addition and subtraction.

METHOD

Subjects

Forty-five boys and forty-five girls in the second grade were randomly selected from three elementary schools in North Vancouver, B.C. to participate in this study (15 boys and 15 girls per school). The schools were located in a middle class section of North Vancouver. The subjects ranged from 7 years 4 months to 9 years 1 month, with a mean of 7 years 10 months. Half the male and female subjects were given the reversibility tasks first and then 20 addition and subtraction equations. For the remaining forty-five subjects, the order of task performance was counter-balanced. There was a one-week interval between presentations of the reversibility tasks and the arithmetic equations.

Four boys failed to complete the reversibility tasks because of interruptions by sports days and field trips which are common activities at the end of the school year. This resulted in a total of 86 subjects in the study: forty-one boys and forty-five girls.

The present study was conducted in May and June, 1976 and specifically with second graders. The reason was that in the North Vancouver School District, arithmetic equations in addition and subtraction were being introduced and taught in January in the second grade. Common sense suggests that the second graders would need time to assimilate and master their new skills in the arithmetic equations. Thus the present author chose to run the study in May and June in order to obtain a valid measure of the relation, if any, between second graders' performance in arithmetic equations and their performance in the reversibility tasks.

Stimuli and procedure

The reversibility tasks. There were two reversibility tasks, the administration of which lasted approximately ten minutes. The subject was taken out of the classroom individually to a separate room. He/She was seated comfortably at a typical school desk, opposite the experimenter. After rapport with the subject was established, the experimenter began the experiment.

A laminated strip of bristolboard 38.10 cm long and 5.08 cm wide was placed in front of the subject. A red line was marked vertically 5.08 cm from each end of the bristolboard, and eight blue chips were spaced evenly between these two red lines, with a spacing of about 1.27 cm between chips. The subject was asked to study the row of chips, noting the length of the row and how far apart the chips were from each other. The experimenter allowed the subject 15 seconds. At the end of this period of time, the experimenter shortened the distance between the chips such that they were almost touching one another. The subject was then shown three strips of cardboard, each 15.24 cm long and 5.08 cm wide. On each strip was pasted a row of 4 blue chips. The first strip showed the original spacing, namely 1.27 cm interval between chips. The second strip showed a narrow spacing, .635 cm interval between chips and the third strip showed a spacing 1½ times wider than the original, namely 1.905 cm between chips. The subject was asked to pick the strip with the chips which showed the original spacing, with the accompanying instruction of: "If I want to change this row of chips back to look like the first row I showed you, point to the strip which shows how far apart the chips will have to be".

The subject was given a maximum of 20 seconds to pick his/her response choice. The rationale for using 4 chips on each response choice pertains to the experimental focus which was centered on the subjects' ability to judge the necessary and appropriate spacing between the chips in order to retrieve the original lay-out of the chips. For this purpose, 4 chips sufficed.

The second reversibility task involves the same procedure. However a different set of stimuli was used. A second piece of bristol-board was used which had the same measurements as the first. Again two red vertical lines were drawn from each end of this bristolboard. A row of eight blue chips were placed in the middle of the bristol-board. The spacing was about .635 cm between chips.

The subject was instructed and given 15 seconds to study the length of the row of chips and the spacing. After that, the experimenter spaced out the chips evenly such that the end chips sat squarely on the two red vertical lines. Then the subject was shown three strips of cardboard on each of which was pasted four chips. The first showed a spacing much closer than the original spacing, the chips were almost touching each other. The second strip showed a spacing $1\frac{1}{2}$ times wider than the original spacing, .953 cm. The last showed the original spacing. The subject was given 20 seconds to choose his/her response.

<u>The Arithmetic Equations</u>

The 10 addition equations and the 10 subtraction equations used in the study, are shown in Table 1. These arithmetic equations consisted of variations of two themes: $X + \square = Y$ or $X - \square = Y$; $\square + X = Y$ or $\square - X = Y$.

Because Piaget's concept of reversible thought is exemplified by the child's ability to anticipate and to look back (hindsight), the use of these arithmetic equations appeared to provide opportunity for such cognitive activities. In the equations $\begin{matrix}(X + \square = Y,) \\ (\square - X = Y)\end{matrix}$ the child had to look back and forth in anticipation and hindsight, in computing the correct answer. Thus these equations could be considered empirical analogies to Piaget's abstract concept of reversible thought. The form of $X + Y = \square$? was not included even though it constitutes one form of arithmetic equations. This is because it appears to involve more anticipation rather than both anticipation and hindsight. The form $\square + \square = 9$ was omitted because there were too many possible answers. Compared to X + $\square$ = Y and $\square$ + X = Y, it does not focus as directly on the missing component of the equation, in that there can be only one numeral possible for the missing cell which would fit the equation.

The subject was taken out of the classroom individually for this phase of the experiment to a separate room. He/She was seated comfortably at a typical school desk opposite the experimenter. The experiment began after the experimenter struck up rapport with the subject. The latter was presented with a pencil and a sheet upon which the addition and subtraction equations were clearly shown. The subject was asked to complete the equations. The experimenter occupied herself with some drawing task while the subject worked at the equations. All subjects were given a maximum of thirty minutes to complete the arithmetic equations. The experimenter gave no feedback whatsoever to the subjects.

Insert Table 1 about here

Results

The mean scores and standard deviations appear in Table 2. Analyses of the differences between the means using one-way analysis of variance (ANOVA) revealed no sex differences (F (1, 84) $<$ 1).

Insert Table 2 about here

Because either positive or negative relations can result from the pooling of two distinct subgroups of subjects, e.g., boys and girls, separate correlations within each subgroup were computed (cf. Glass and Stanley, 1970, p.123). The results are shown in Table 3. These results show that significant relationships between reversible thought and arithmetic equations were obtained for only one subgroup of subjects, namely, the girls. However, the same did not hold for the remaining subgroup of subjects, the boys.

Insert Table 3 about here

Discussion

The purpose of the present study was to investigate whether reversible thought is in any way related to performance in arithmetic equations among children. Significant positive results would furnish empirical justification to the educator's assumption of an underlying relationship between the two kinds of abilities. Conversely, negative results would cast grave doubts on the same assumption. The obtained results support the educator's assumption with an important qualification. It appears that significant relationships between the two variables under study held for the female subjects only. The appearance of a sex difference here is of interest. In a scholarly review on sex differences in mathematics learning, Fennemar (1974) discussed nine studies that used early elementary school children (grades 1 - 3) as subjects. Two studies (Hervey, 1966; Lesser, Fifer and Clark, 1965) reported that boys performed better than girls. Hervey (1966) measured the ability to solve verbal problems before instruction in the specific mathematical operation that would enhance the solution, while Lesser, Fifer and Clark (1965) measured numerical scale and space scale. Interestingly two studies found the reverse. (Lowery and Allen, 1970; Wozencraft, 1963). Lowery and Allen (1970) measured the ability to categorize items differing on one, two, or three attributes. Wozencraft (1963) used standard achievement test-scores. However in five other studies involving a variety of measures of mathematical learning, no significant differences were found, i.e. boys and girls performed similarly. In view of these findings, Fennema concluded that there are no consistent significant differences in the learning of mathematics by boys and girls in the early elementary years, (cf. Fennema, 1974, p. 128). Although the present finding of a sex difference does not upset the general scale of Fennema's conclusion, it does suggest that perhaps the issue of sex differences in mathematics learning among young children in the early grades, may not be closed. However it must be remembered that the purpose, the cognitive and arithmetic tasks used in this study may impede comparisons with previous studies yielding sex differences among subjects in early grades.

Despite the hackneyed caution among statisticians (cf. Glass and Stanley, 1970, pp. 121-123) that correlations do not imply causation, the point bears repeating. The present obtained significant relationship between reversible thought and arithmetic equation performance among female second graders does not imply that reversible thought contribute in a causal manner to the development of arithmetic computational skill, or vice versa for girls. Moreover, while the obtained correlations were significant, the portion of variance attributable to, or predictable from one variable to the other remains small. This fact underscores the importance of judiciously interpreting the obtained significant correlations. A feasible way of interpreting obtained relationships between any tasks embodying

Piagetian concepts, such as conservation or reversibility and tasks testing arithmetic performance among young children may be that they all measure or tap different aspects of the same cognitive processes involved in the child's development of number concepts.

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TABLE 1

Arithmetic Equations used in this Study

Additions

$2 + \square = 9$	$\square + 3 = 11$
$\square + 4 = 7$	$5 + \square = 11$
$6 + \square = 8$	$\square + 8 = 15$
$\square + 9 = 10$	$2 + \square = 11$
$5 + \square = 9$	$\square + 8 = 12$

Subtractions

$11 - \square = 3$	$\square - 1 = 4$
$\square - 6 = 7$	$9 - \square = 2$
$9 - \square = 7$	$\square - 3 = 9$
$\square - 4 = 2$	$14 - \square = 7$
$8 - \square = 2$	$\square - 7 = 5$

TABLE 2

Means and standard deviations in subjects' task performances.

	Boys	Girls
Reversibility	1.585 (0.706)	1.667 (0.564)
Addition	9.342 (1.087)	9.156 (2.393)
Subtraction	7.024 (2.937)	6.913 (3.238)
Addition and Subtraction	16.366 (3.455)	15.861 (5.139)

TABLE 3

Correlations between reversibility and arithmetic performance

	REVERSIBILITY TASKS	
	Girls	Boys
Addition	.377**	.059
Subtraction	.357**	- .067
Addition and Subtraction	.318*	- .039

** $p < .01$
* $p < .05$