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ABSTRACT

Twenty research reports related to mathematics education are abstracted and analyzed. Four of the reports deal with aspects of learning theory, ten with topics in mathematics instruction (problem solving, number concepts, number operations, and logic), two with teacher education, two with textbook development, one with testing, and one with goals for teaching mathematics. Research related to mathematics education which was reported in RIE and CIJE between October and December 1975 is listed. (DT)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts and Critical Analyses of Recent Research

Center for Science and Mathematics Education
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MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN RESEARCH IN EDUCATION
October - December 1975

ED 107 470 Harris, Margaret L.; Harris, Chester W. A Structure of Concept Attainment Abilities. 394p. Not available from EDRS. Available from CCL Document Service, 1025 West Johnson Street Madison, Wisconsin 53706 ($5.75)

ED 107 487 Kennedy, Keith. The Effectiveness of a Comparative Advance Organizer in the Learning and Retention of Metric System Concepts. 9p. MF and HC available from EDRS.

ED 107 500 Ebeling, David George. The Ability of Sixth Grade Students to Associate Mathematical Terms with Related Algorithms. Available from University Microfilms (74-2646).

ED 107 501 Niemann, Donald Frederick. A Study of the Degree to Which Seventh, Eighth and Ninth Grade Students Have Obtained Minimum Mathematical Competencies and Skills as Recommended by the National Council of Teachers of Mathematics. 153p. Available from University Microfilms (74-13,006).

ED 107 502 Jacobs, Judith Ellen. A Comparison of the Relationship Between the Level of Acceptance of Sex-Role Stereotyping and Achievement and Attitudes Toward Mathematics of Seventh Graders and Eleventh Graders in a Suburban Metropolitan New York Community. 110p. Available from University Microfilms (74-12,844).

ED 107 503 Flake, Janice Louise. The Use of Interactive Computer Simulations for Sensitizing Mathematics Methods Students to Questioning Behaviors. 140p. Available from University Microfilms (74-12,015).

ED 107 504 Freitag, Richard Alan. Case Studies of a Teaching Model: Teaching Through Games. 173p. Available from University Microfilms (74-14,300).

ED 107 505 Murray, Dennis E. Mathematical Readiness of Six-Year-Olds and the Placement of First Grade Mathematical Topics. 126p. Available from University Microfilms (74-16,049).

ED 107 506 Baxter, Marion McComb. Prediction of Error and Error Type in Computation of Sixth Grade Mathematics Students. 168p. Available from University Microfilms (74-15).

ED 107 507 Davison, David Malcolm. A Component for a Methods Course for Preservice Secondary School Mathematics Teachers Involving Computers and Other Laboratory Activities. 149p. Available from University Microfilms (74-14,526).

ED 107 508 Egan, Dennis Edmund. The Structure of Experience Acquired While Learning to Solve a Class of Problems. 106p. Available from University Microfilms (74-15,712).

ED 107 528 Higdon, Danny W. Prospective and Experienced Elementary Teachers: A Comparison of Mathematical Achievement and Attitudes. 21p. MF and HC available from EDRS.

ED 108 006 Anderson, Lorin W. Time to Criterion: An Experimental Study. 19p. MF and HC available from EDRS.

ED 108 010 Michaels, James W. Effects of Differential Rewarding and Inequity on Performance and Satisfaction. 9p. MF and HC available from EDRS.

ED 108 096 Johnson, Mauritz. Some Concommitants of Verbal-Quantitative Aptitude Discrepancy. 66p. MF and HC available from EDRS.

ED 108 748 Walker, Alice A. The Role of Relational Concepts in the Acquisition of Conservation. 8p. MF and HC available from EDRS.

ED 108 864 Dettrick, Graham W. A Study of the Structure of Piagetian Logical and Infralogical Grouping Within the Concrete Operational Period. 26p. MF and HC available from EDRS.

ED 108 895 Schoen, Harold L. Evaluation of the Individualized Approach to Mathematics Instruction at City High School. Final Report. 15p. MF and HC available from EDRS.

ED 108 900 Amster, Harriett; And Others. Effectiveness of Schell's Peer Teaching Program in Mathematics for Junior High School Students. 11p. MF and HC available from EDRS.

ED 108 901 Smith, Gerald John. The Effects on College General Education Mathematics Students of Learning Mathematics Through the Active Manipulation of Materials. 233p. Available from University Microfilms (74-18,908).

ED 108 902 Heller, Marc Stephen. Teacher Approval and Disapproval by Ability Grouping. 142p. Available from University Microfilms (74-17,865).

ED 108 903 Clayton, McLouis. The Differential Effects of Three Types of Structured Reviews on the Learning and Retention of Mathematics. 256p. Available from University Microfilms (74-18,980).

ED 108 904 Mayberry, Claude Antonio, Jr. Training Inner City Elementary School Teachers to Base Mathematics Instruction on Students' Interests. 192p. Available from University Microfilms (74-18,727).

ED 108 905 McGlone, Virginia Elizabeth. Children's Understanding of the Geometric Transformation of Rotation About a Point. 195p. Available from University Microfilms (74-18,729).

ED 108 906 Ford, Shirley. Grouping in Mathematics: Effects on Achievement and Learning Environment. 140p. Available from University Microfilms (74-23,546).

ED 108 907 Jones, William Lane. Comparison of Cognitive and Affective Change of Ninth Grade Students in Open-Space and Closed-Space Classes. 132p. Available from University Microfilms (74-21,534).

ED 108 908 Bien, Ellen Carol. The Relationship of Cognitive Style and Structure of Arithmetic Materials to Performance in Fourth Grade Arithmetic. 177p. Available from University Microfilms (74-22,809).

ED 108 909 Wood, Carolyn M. A Comparison of the Effects of Sequence and Mode Upon the Initial Acquisition, Retention, and Transfer of Elementary Multiplication Concepts. 104p. Available from University Microfilms (74-20,804).

ED 108 910 Moran, Thomas Patrick. Significant Developments in the Use of Computers in School Mathematics: A Sourcebook for Administrators, Teachers, and Teacher Educators. 188p. Available from University Microfilms (74-23,528).

ED 108 911 Blomstedt, Robert Kent. The Effects of Consensus on Verbal Problem Solving in Middle School Mathematics. 181p. Available from University Microfilms (74-24,830).

ED 108 912 Matthews, Larry Allen. An Evaluation of the Effect of Using Activity-Oriented Materials on Student Achievement of Mathematical Competencies for Enlightened Citizens. 261p. Available from University Microfilms (74-23,923).

ED 108 913 Kaiser, Virginia Ruth Stone. An Exploratory Study of Selected Spoken and Unspoken Behaviors in an Inner-City Mathematics Laboratory for Underachieving Students. 355p. Available from University Microfilms (74-24,350).

ED 108 914 Nisbet, Jean Ann. Instructional Sequence for Improving Teacher Question-Asking in Secondary Mathematics. 168p. Available from University Microfilms (74-25,806).

ED 108 915 Raumin, Harley Cyril. Impact of Flexible Class Time on the Achievement and Attitude of High School Students in Plane Geometry. 110p. Available from University Microfilms (74-23,609).

ED 108 916 Newman, Henry Jack. The Comparative Effectiveness of Selected Instructional Strategies for the Teaching of Basic Algebra. 161p. Available from University Microfilms (74-26,608).

ED 108 917 Prigge, Glenn Russell. The Effects of Three Instructional Settings on the Learning of Geometric Concepts by Elementary School Children (Volumes I and II). 386p. Available from University Microfilms (74-26,227).

ED 108 918 Schwieger, Ruben Don. A Component Analysis of Mathematical Problem Solving. 200p. Available from University Microfilms (74-26,777).

ED 108 919 Alessi, Galen James. Effects of Hutchings' "Low Fatigue" Algorithm on Children's Addition Scores Compared Under Varying Conditions of Token Economy Reinforcement and Problem Difficulty. 151p. Available from University Microfilms (74-29,046).

ED 108 920 Kerr, Joseph Jackson, Jr. British and American Arithmetic Devices for the Blind--An Analytical Description. 342p. Available from University Microfilms (74-28,177).

ED 108 921 Mitchell, William Montgomery. The Design of Mathematics Curricula for the Small College. 180p. Available from University Microfilms (74-29,180).

ED 108 922 Wallace, Pearlena. An Investigation of the Relative Effects of Teaching a Mathematical Concept via Multisensory Models in Elementary School Mathematics. 107p. Available from University Microfilms (74-27,501).

ED 108 923 Coleman, Max William. The Relationships of Selected Mental Factors and Thinking Interests to Success in Algebra Using Two Methods of Instruction. 134p. University Microfilms (75-999).

ED 108 924 Malo, George Edward. Differential Treatments in Learning Disjunctive Concepts in Mathematics. 141p. Available from University Microfilms (75-363).

ED 108 925 Moyer, Robert Eugene. Effects of a Unit in Probability on Ninth Grade General Mathematics Students' Arithmetic Computation Skills, Reasoning and Attitude. 178p. Available from University Microfilms (75-377).

ED 108 926 Nelson, Glenn Thomas. The Effects of Diagram Drawing and Translation on Pupils' Mathematics Problem-Solving Performance. 239p. Available from University Microfilms (75-1234).

ED 108 927 Dirkes, Margaret Ann. The Effect of Divergent Thinking Experiences on Creative Production and Transfer Between Mathematical and Nonmathematical Domains. 144p. Available from University Microfilms (74-29,798).

ED 108 928 Pigford, Valma Darleen. A Comparison of an Individual Laboratory Method with a Group Teacher-Demonstration Method in Teaching Measurement and Estimation in Metric Units to Preservice Elementary Teachers. 144p. Available from University Microfilms (75-941).

ED 108 929 Fox, Lynn Hussey. Facilitating the Development of Mathematical Talent in Young Women. 218p. Available from University Microfilms (74-29,027).

ED 108 930 McClintock, Coy Edwin. An Investigation of Transfer of Learning as Mediated by Three Instructional Methods of Teaching Selected Mathematical Generalizations. 221p. Available from University Microfilms (75-2624).

ED 108 931 Barton, Patricia Cole. The Effects of a Piagetian Geometry Model on Beginning School Children. 232p. Available from University Microfilms (75-2051).

ED 108 961 Mayer, Richard E.; Greeno, James G. Restructuring of Meaningful Information in Problem Solving. 9p. MF and HC available from EDRS.

ED 108 962 Bright, George W. Recall of Advance Organizers as Part of Mathematics Instruction. 17p. MF and HC available from EDRS.

ED 108 970 Schall, William; And Others. Developing Mathematical Processes (DMP). Field Test Evaluation, 1973-1974. 52p. MF and HC available from EDRS.

ED 108 972 Uprichard, A. Edward; Phillips, E. Ray. An Intraconcept Analysis of Rational Number Addition: A Validation Study. 17p. MF and HC available from EDRS.

ED 109 149 Hooper, Frank H.; And Others. A Cross Sectional Investigation of Children's Classificatory Abilities. Technical Report No. 295. 99p. MF and HC available from EDRS.

ED 109 235 Villano, Maurice W. The Relationship of Certain Course Characteristics to Student Ratings of Science and Mathematics Teaching at Four-Year and Two-Year Colleges. 19p. MF and HC available from EDRS.

ED 109 240 Burrows, Charles K.; Okey, James R. The Effects of a Mastery Learning Strategy on Achievement. 13p. MF and HC available from EDRS.

ED 110 200 Stallings, Jane. Relationships Between Classroom Instructional Practices and Child Development. 19p. MF and HC available from EDRS.

ED 110 317 Linke, Russell Dean. The Effects of Certain Personal and Situational Variables on the Acquisition Sequence of Graphical Interpretation Skills. Vols. 1, 2, and 3. 962p. MF and HC available from EDRS.

ED 110 328 Maxwell, Ann Alsobrook. An Exploratory Study of Secondary School Geometry Students: Problem Solving Related to Convergent-Divergent Productivity. 133p. Available from University Microfilms (75-3626).

ED 110 329 Tarzaban, Abdulla. Elementary Mathematics Education: Developing a Competency-Based/Field-Centered Program. 131p. Available from University Microfilms (75-3923).

ED 110 330 Bell, Della Pearl Domoneck. Some Characteristics of High- and Low-Achieving Seventh Grade Black Students in Mathematics. 138p. Available from University Microfilms (75-4325).

ED 109 331 Gilbert, Robert Kennedy. A Comparison of Three Instructional Approaches Using Manipulative Devices in Third Grade Mathematics. 298p. Available from University Microfilms (75-2099).

ED 110 332 Bruning, Wayne Allen. Visual Aptitude as it Relates to Student Achievement in Reading and Mathematics. 92p. Available from University Microfilms (75-2963).

ED 110 333 Kellerhouse, Kenneth Douglas, Jr. The Effects of Two Variables on the Problem Solving Abilities of First Grade and Second Grade Children. 85p. Available from University Microfilms (75-5564).

ED 110 334 McMurray, Nancy Ellen. The Effects of Four Instructional Strategies on the Learning of a Geometric Concept by Elementary and Middle School EMR Students. 105p. Available from University Microfilms (74-28,816).

ED 110 335 Burney, Gilbert McCollum. The Construction and Validation of an Objective Formal Reasoning Instrument. 138p. Available from University Microfilms (75-5403).

ED 110 342 Scandura, Joseph M.; And Others. Diagnosis and Instruction of Higher Order Rules for Solving Geometry Construction Problems. 15p. MF and HC available from EDRS.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
October - December 1975

EJ 118 031 Weinreb, Neil; Brainerd, Charles J. A Developmental Study of Piaget's Groupement Model of the Emergence of Speed and Time Concepts. Child Development, v46 n1, pp176-185, March 1975.

EJ 118 060 Johansson, Bo S.; Sjolin, Barbro. Preschool Children's Understanding of the Coordinators "And" and "Or". Journal of Experimental Child Psychology, v19 n2, pp233-240, April 1975.

EJ 118 083 Hughes, Frank G.; Burns, Paul C. Two Methods of Teaching Multidigit Multiplication. Elementary School Journal, v75 n7, pp452-457, April 1975.

EJ 118 366 Rosenthal, Daniel J.; Resnick, Lauren B. Children's Solution Processes in Arithmetic Word Problems. Journal of Educational Psychology, v66 n6, pp817-825, December 1974.

EJ 118 465 Mayer, Richard E. Acquisition Processes and Resilience under Varying Testing Conditions for Structurally Different Problem-Solving Procedures. Journal of Educational Psychology, v66 n5, pp644-656, October 1974.

EJ 119 097 Eisenberg, Theodore A. Negation, Disjunctive Syllogisms, and Mathematics Achievement. Journal of Psychology, v90, pp69-74, May 1975.

EJ 119 778 Wylie, Richard E. Mathematics Education for Young Children. Childhood Education, v51 n6, pp343-348, April/May 1975.

EJ 119 969 Richardson, Lloyd I. The Role of Strategies for Teaching Pupils to Solve Verbal Problems. Arithmetic Teacher, v22 n5, pp414-421, May 1975.

EJ 119 988 Webb, Leland F.; Carry, L. Ray. Interaction of Spatial Visualization and General Reasoning Abilities with Instructional Treatment in Quadratic Inequalities: A Follow-Up Study. Journal for Research in Mathematics Education, v6 n3, pp132-141, May 1975.

EJ 119 989 Eastman, Phillip M.; Carry, L. Ray. Interaction of Spatial Visualization and General Reasoning Abilities with Instructional Treatment in Quadratic Inequalities: A Further Investigation. Journal for Research in Mathematics Education, v6 n3, pp142-149, May 1975.

EJ 119 990 Behr, Merlyn J.; Eastman, Phillip M. Interactions Between Structure-of-Intellect Factors and Two Methods of Presenting Concepts of Modulus Seven Arithmetic--A Follow-Up and Refinement Study. Journal for Research in Mathematics Education, v6 n3, pp150-158, May 1975.

EJ 119 992 Bassler, Otto C.; And Others. Comparison of Two Instructional Strategies for Teaching the Solution of Verbal Problems. Journal for Research in Mathematics Education, v6 n3, pp170-178, May 1975.

EJ 119 993 Mpiangu, Benayame Dinzau; Gentile, J. Ronald. Is Conservation of Number a Necessary Condition for Mathematical Understanding? Journal for Research in Mathematics Education, v6 n3, pp179-192, May 1975.

EJ 119 998 Nesher, Perla; Teubal, Eva. Verbal Cues as an Interfering Factor in Verbal Problem Solving. Educational Studies in Mathematics, v6 n1, pp41-51, March 1975.

EJ 120 175 Mason, Emanual J.; And Others. Familiarity with Content and Conditional Reasoning. Journal of Educational Psychology, v67 n2, pp238-242, April 1975.

EJ 120 443 Pendleton, Julienne K. Mathematical Concept Attainment of Sixth Grade Students in Relation to their Cognitive Styles. Southern Journal of Educational Research, v9 n3, pp168-178, Summer 1975.

EJ 121 554 Garigliano, Leonard J. Arithmetic Computation Scores: or Can Children in Modern Mathematics Programs Really Compute? School Science and Mathematics, v75 n5, pp399-412, May/June 1975.

EJ 121 794 Geeslin, William E.; Shavelson, Richard J. An Exploratory Analysis of the Representation of a Mathematical Structure in Students' Cognitive Structures. American Educational Research Journal, v12 n1, pp21-39, Winter 1975.

EJ 121 850 McCarthy, S. Viterbo. Differential V-Q Ability: Twenty Years Later. Review of Educational Research, v45 n2, pp263-282, Spring 1975.

TIME AND ERROR CONSEQUENCES OF IRRELEVANT DATA AND QUESTION PLACEMENT IN ARITHMETIC WORD PROBLEMS II: FOURTH GRADERS. Arter, Judith A.; Clinton, LeRoy. Journal of Educational Research, v68 n1, pp28-31, September 1974.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jerry P. Becker, Northern Illinois University.

1. Purpose

The authors defined mathemagenic behaviors as those actions of the student that aid in attainment of a specified instructional objective. With this definition, and after reviewing a number of studies, the authors took a position that the most mathemagenically favorable placement of questions in arithmetic word problems would be before the listing of data, rather than after it, which, according to the authors, seems to be current practice. Thus, a problem should be stated as (1) "What is the area of our school yard? It is 200 feet long and 180 feet wide", rather than as (2) "Our school yard is 200 feet long and 180 feet wide. What is the area?" The first form is to be preferred since it would appear to focus the attention of the student on relevant data, and therefore reduce computation time.

The authors examined question-placement in a prior study involving 51 educable mentally retarded students at the high school level. No question-placement main effect was found, due presumably to a wide range of student ability and a screening device that correlated more highly with dependent variables in one experimental cell than in the others. Thus, the purpose of the study presently being reviewed was to reexamine the variable of question placement, but in a manner that would avoid difficulties encountered in the earlier study.

The specific hypotheses were:

1. Pre-question problems will result in fewer errors than post-question problems.

2. It will take less time for students to solve pre-question problems than post-question problems.

3. Problems with extraneous data will result in more errors than problems with no extraneous data.

4. It will take more time for students to solve problems with extraneous data.

5. Pre-question problems with extraneous data will result in fewer errors.

6. Pre-question problems with extraneous data will take less time to solve than post-question problems with extraneous data.

2. Rationale

The first study by the authors grew out of earlier research by Frase (1967, 1968), Rothkopf (1966), and Morasky and Willcox (1970). Those studies revealed that question-placement affects inspection activities of students: i.e., pre-questions limit the range of attentive behaviors, while post-questions seemed not to. Also, less time was taken by pre-question groups to process information than for post-question groups. In addition, Goodstein, et al. (1971) and Dunn (1956) found that in arithmetic word problems, the presence of extraneous data reduced the number of problems solved correctly. These two variables were of primary interest in the first Arter and Clinton study, and were examined again in the present study.

3. Research Design and Procedure

All students enrolled in five regular fourth-grade classes in two schools were given the computation and problem-solving subtests of the Metropolitan Achievement Tests. The purpose of the pretesting was twofold. The investigators hoped to reduce the variance attributable to (a) commonalities between the pretest characteristics and the format of the experimental materials and (b) computational ability. Students scoring a minimum of 27 out of 35 on the problem-solving subtest were used as subjects in the experiment (N = 44).

Students were tested individually. First, a paragraph was read by each subject and was timed for reading rate. Then each subject was asked four questions to insure at least a second-grade reading ability. Word problems were presented individually on a small card. Subjects worked out their answers and selected a response from a multiple-choice listing. Timing began when subjects received the card and ended when an answer choice was made.

Each subject received twenty-five word problems. The order of the problems was determined randomly, the same problem order was used for all students, and problems were thought to be typical problems at this level. The problems were presented on a second-grade vocabulary level to minimize the role of reading ability as a factor in problem solving. Extraneous data were placed in the same relative position in all problems involving such data. Examples of problems in the four experimental modes are as follows:

Question before, extraneous data: How much will John spend on candy bars? He wants to buy 8 candy bars for four friends. The candy bars cost 8¢ each.

Question before, no extraneous data: How much will John spend on candy bars? He wants to buy 8 candy bars. The candy bars cost 8¢ each.

Question after, extraneous data: John wants to buy 8 candy bars for four friends. The candy bars cost 8¢ each. How much will John spend on candy bars?

Question after, no extraneous data: John wants to buy 8 candy bars. The candy bars cost 8¢ each. How much will John spend on candy bars?

A 2x2 ANOVA (presence or absence of extraneous data; pre- or post-questions) was used for the dependent variable of errors. A 2x2 ANCOVA was used for the dependent variable of time required to answer problems. The two factors were the same as above and the covariate was reading time on the initial comprehension test.

4. Findings

The 2x2 ANOVA with errors as the dependent variable revealed no main effects nor any interactions. The 2x2 ANCOVA with reading time on the initial paragraph as covariate revealed a significant main effect for extraneous data. No effects were revealed that were due to problem-solving time.

5. Interpretations

The authors suggest that absence of differences in the ANOVA may be explained by observing that better students tended to work longer to get an answer which matched a distractor in a multuple-choice format; thus, they tended more consistently to get correct answers. No other observations were made with respect to the ANOVA. The extraneous data main effect suggested to the authors that irrelevant data require additional processing steps and/or reduces efficiency of the processing system; but, in any case, the main effect is a function of cognitive processing phenomena. Problems with extraneous data resulted in no more errors in problem solving than did problems with no extraneous data. The authors give a tentative explanation for the latter: in future studies, do not use multiple-choice items; rather, ask students to solve the problem showing all work and indicate an answer if one is found. Thus, students cannot use the distractors to find a solution.

Critical Commentary

When the reviewer first skimmed the research report, it seemed that two variables were perhaps more important to examine. These were the prior experience with such problems and the motivation of the students. That is, after a student has solved some problems, placing the question before or after data would seem unimportant. Similarly, for motivated students, placement of the question before or after data would not seem important--students will pick out data and try to solve to answer the question. Frase (1968) has found evidence to support these intuitive feelings.

Another notion that occurred to the reviewer was that more important information related to problem solving might have been sought within the experimental situation. Since the word problems were presented individually on a small card and the experimenter could observe students working,

data might have been collected about how students analyzed the problems, how they reasoned (with or without using the extraneous data), and whether students could check the "reasonableness" of their answers. The experiment could be replicated, with little modification, looking at those variables which seem more important in learning about student mathematical problem-solving processes.

The study was interesting and well designed. The experimental procedures were appropriate and well organized and data were analyzed, interpreted, and reported quite nicely.

Jerry P. Becker
Northern Illinois University

RETROACTIVE INTERFERENCE OF SIMILAR METHODS TO TEACH TRANSLATION OF BASE SYSTEMS IN MATHEMATICS. Barszcz, Edward L.; Gentile, J. Ronald. Journal for Research in Mathematics Education, v7 n3, pp176-183, May 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Eugene D. Nichols, Florida State University.

1. Purpose

To study the effects which learning of a second method for translating numerals from base ten to base three or five has on the proficiency of the use of the first method by seventh- and eighth-grade students.

2. Rationale

It is commonly assumed that the capability to use alternative methods of solving problems implies greater understanding. It is speculated, however, that this does not suggest teaching of new concepts by more than one method. The authors considered the possibility, prior to this study, that an introduction of a second method may indeed interfere with the facility in the use of the previously learned method.

3. Research Design and Procedure

The study consists of two experiments, one using 107 eighth-grade students and the other 102 seventh-grade students, both samples from a suburban Buffalo middle school. Experiment 1 consisted of eight groups of students and Experiment 2 of four groups. Assignment to groups was random. Prior to the instruction, it was ascertained that students were not familiar with the algorithms to be taught in the study.

One half of the students was taught to translate from base ten to the other bases using division by base value (Method B). The article does not provide the details of the methods. All of the instruction was provided by means of a videotaped presentation.

Following this presentation each student in Experiment 1 was required to do successfully either two, four, or eight problems. Each problem was checked by the experimenter before a student proceeded to the next problem. No rationale is provided for the numbers of problems that were required to be solved by students. Following a short break during which the students worked on word puzzles, the experimental group students were taught the second method of translating from base ten to the other bases. The control groups continued working on the word puzzles. At the conclusion of the study, all students were tested by means of five problems which were to be solved using the method which was taught first. For Experiment 1, the criterion measure of success was four correctly solved problems, and for Experiment 2, five correctly solved problems.

4. Findings

To test the effect of the second method on proficiency in the use of the method taught first, the 2x2 factorial ANOVA design was used in Experiment 1. The results show a negative transfer in that the control groups were superior to the experimental groups. To estimate the amount of interference of the second method with the proficiency in the use of the first method, Murdock's formula was used:

$$\text{Percent of transfer} = \frac{E - C}{E + C} \times 100$$

where E is the performance of the experimental group and C the performance of the control group. This formula yielded a transfer of -16.1%.

Since Experiment 2 was a replication of groups 2, 4, 6, and 8 of Experiment 1, one would expect the pattern of results to be the same. Indeed it was. The control groups were superior. Murdock's formula, in this case, yielded transfer of -20.5%.

5. Interpretations

The consistency of the results in both experiments led the authors to conclude that one is not justified in assuming that positive transfer will take place. [Abstractor's Note: It is not clear whether this conclusion is being made for the particular methods of teaching translating from one base to another or if it is presented as a general conclusion.] They remark that positive transfer "probably results from extra care in sequencing and spacing the alternative methods, guided continually by feedback from each student's progress." [Abstractor's Note: No supporting source for this probabilistic statement is indicated.]

The authors then proceed to suggest some possible gymnastics with methods interspaced with rest periods to get some data on proactive effects rather than the retroactive effect, as this study was designed to do.

Critical Commentary

First, two minor points.

The authors see Piaget's concept of conservation and the ability to use alternative methods in solving problems as being analogous when viewed as a potential for success in problem solving. This is a crude analogy at best. The two phenomena are much too different to warrant this analogy.

The authors use 'solutions' and 'problem solving methods' as synonyms--a case of unjustified looseness in terminology. They view "the capability of using alternative solutions [meaning 'problem solving methods'] as evidence of greater understanding." They do not define 'understanding'

and one wonders whether they are guilty of circularity here: understanding implies ability to use alternative methods and ability to use alternative methods implies understanding.

Now, some more significant points.

To be able to assess the meaning of the results, one needs to see just what the two methods of translating from base ten to base five or three are. The authors should have provided examples displaying full details of the algorithms. To make logical assessments and plausible inferences about the meaning of the resulting negative transfer, one would need to know whether

(1) the two methods are such that, due to their nature, they would logically interfere with each other, or

(2) they are such that one would expect them to contribute to a strengthening of the translating skills, or

(3) they are disjoint to the point that each needs to be memorized for its effective use, or . . .

Simply stated, a thorough analysis of the algorithms for translating numerals is essential for a meaningful interpretation of results.

It should come as no surprise to anyone that learning a second method for handling a logarithmic situation may indeed interfere with proficiency in the use of the originally learned method. Most teachers know this. No further research is needed to demonstrate this.

Furthermore, the subject matter on which the authors chose to demonstrate this well-known phenomenon is of dubious value at best. Actually, it is worthless. Why would anyone want to know whether there is transfer when using two methods of translating numerals from base ten to some other base? Why not chose a topic in which transfer might be of significance, such as problem solving in a realistic setting (to the learner)? Or even studying the structure of a numeration system in a base other than ten and its transfer potential to the gaining of insight into the structure of base-ten numeration system?

The authors' conclusion that "transfer cannot be taken for granted" has been demonstrated over and over again. No further evidence is necessary, particularly in the context of a narrow and useless piece of subject matter, allowing no generalization.

Eugene D. Nichols
Florida State University

SOME EFFECTS OF REWARD ON THE RESPONSES OF KINDERGARTEN CHILDREN IN MAKING TRANSITIVE INFERENCES. Davis, Edward J. Journal of Educational Research, v69, pp36-39, September 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by J. Dan Knifong, University of Maryland.

1. Purpose

a. Are children's responses to transitive inference questions different for the three relations "as many as," "more than," and "fewer than"?

b. Does the offer of reward affect children's success on these questions?

c. Does the criteria for earning reward affect success?

d. Does the Piagetian stage development (with respect to conservation of number) affect success?

2. Rationale

Children's interest and attention often affect experimental responses. Since Piaget seldom has been concerned about population norms, he has freely ignored interviews (or portions thereof) in which the subject is distracted or fails to concentrate. On the other hand, American behaviorists are interested in population norms, and reject research designs which allow such selective exclusions. For this reason, there is interest in devising reliable experimental situations wherein subjects who can conserve (or whatever) do so consistently.

3. Research Design and Procedure

Forty-two five-year-olds were given instruction on determining the relationship ("as many as," "more than," or "fewer than") between two sets by physically matching the elements of those sets. In addition, each child was classified as a high (or low) conserver of these relations if he/she indicated the correct relation (75% of the time) after one of the sets had been transformed. Following this initial classification and instruction, the children were randomly assigned to one of the three relations for two transitivity tests (spaced two days apart). No child was tested on transitivity for all three relations because of fatigue.

In both transitivity tests the child was shown three sets of items on a table (A, B, and C), and asked to use his/her learned matching skills to compare set A with B. This was followed by a similar request to compare B with C. Set B was then removed from the table and the transitivity question was asked concerning the relation between A and C (which had not been compared previously). In the first test the child was encouraged simply to

work hard on the task; in the second test the same child was offered a reward in one of the two modes. In mode A, half of the children were promised they could have a toy of their choice (one of five) if they "get all of the questions right." In mode B, the other half were promised a toy of their choice (from the same selection) if they "tried real hard for the next few minutes."

4. Findings

ANOVA indicated significant differences:

a. Children classified as high conservers performed better on the transitivity test than did low conservers (response means were 3.9 vs. 2.5 correct out of 6).

b. Children did best on the "as many as" relation, less well on the "more than" relation, and least well on the "fewer than" relation. The means for correct responses for each were 4.2 vs. 3.3 vs. 2.6 respectively.

c. There was a dramatic gain for children assigned to reward mode A ("must get all right"). The mean for the conservers and non-conservers first testing (without a reward) was 2.5 vs. a mean for the second testing (with a reward offer) of 4.3 out of 6. The gain for children assigned to the second reward mode ("try real hard") was a lackluster 3.2 vs. 3.7.

5. Interpretations

a. Children improve more on these tasks when a reward is offered than when they are merely encouraged by an adult interviewer.

b. Mode A ("must get all right for a reward") seems to provide the most dramatic improvement, although this tenet should be held cautiously: (i) the children in mode A had lower initial scores and hence more room for improvement than mode B children; (ii) high conservers tended to improve more than low conservers and there were more high conservers assigned to mode A; and (iii) there is little difference between the final scores of the two modes, i.e., 4.3 vs. 3.7 from above.

Critical Commentary

The principal value of this study is that it reminds us, should it be necessary, that children do not always perform to the best of their ability and, because of this, some of our experimental data may be in error. Piaget has long argued that a child (or an adult) in one stage may often perform at a lower stage. For him, the stage descriptions represent the capacity of the child (or the adult)--not his performance. On the other hand there is considerable American interest and research concerning the nature of rewards in experimental situations. Some of the most interesting research shows the offer of a reward sometimes produces worse, not better, performance (e.g., Cradler and Goodwin, Journal of Educational Psychology, v62,

pp279-285, 1971). The one major fault of the reported study is that it does not build on the American tradition and warn readers that even reward mode A may not encourage every child to always work at his best.

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DIAGNOSIS AND REMEDIATION OF PUPIL ERRORS: AN EXPLORATORY STUDY. Dodd, Carol A.; Jones, Graham A.; Lamb, Charles E. School Science and Mathematics, v75 n3, pp270-276, March 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert E. Reys, University of Missouri-Columbia.

1. Purpose

This study compared the effectiveness of two different programs in preparing preservice elementary-school teachers to identify error patterns and describe remediation for errors occurring in elementary mathematics.

2. Rationale

Diagnosis and remediation of student errors in elementary mathematics would seem to be a valuable tool for teachers, yet it does not receive attention in many special methods classes in teaching elementary-school mathematics. Only three of ten widely used mathematics methods texts contained any material related directly to diagnosis and remediation. This seems to be based on the assumption that diagnostic and remedial skills are acquired indirectly from other experiences. This research provides some evidence to check the validity of this assumption.

3. Research Design and Procedure

One program consisted of 45 students enrolled in the regular elementary mathematics methods class at Indiana University. These students had completed 9 hours of mathematics but did not have any field experience in schools. The other group consisted of 43 students enrolled in an experimental program, the Mathematics-Methods Project at Indiana University sponsored by the National Science Foundation, which integrated mathematics and methods. This program provided school experiences with children via video tapes and classroom contact. It also included a special unit for training students to identify error patterns and possible appropriate remediation.

Twenty students were randomly selected from each group, called "regular" and "project." "Abstract Reasoning" (Form B) of the Differential Aptitude Test (A) and "Multiplication and Place Value Concepts Tests" (M) were used as pretests. Each of these tests served as a covariant in later analysis. Two criterion measures were used. One measure (I) tested the ability to identify error patterns, while the other (R) tested the ability to prescribe appropriate remedial activities. Both criterion measures and the "Multiplication and Place Value Concepts Tests" were constructed by the authors and validated by two experienced mathematics educators. All tests were scored by the authors with the inter-rater reliabilities of .76 for test I and .73 for test R.

4. Findings

There was no significant difference between the regular and project groups in their ability to diagnose errors, but there was a significant difference ($P < .01$) in favor of the project group for the remediation of errors.

As was expected, abstract reasoning ability correlated highly with ability to diagnose errors (.98). Also, conceptual knowledge of multiplication showed a high correlation with diagnostic ability (.98) as well as with ability to prescribe remedial activities (.96).

5. Interpretations

The results suggested that ability to diagnose specific error patterns was more a function of the preservice teachers' reasoning ability and mathematical knowledge than of experiences gained in the methods courses. It did appear that experiences gained in the methods courses significantly affected a preservice teacher's competency in developing suitable remedial procedures. It was not possible, however, to identify which factors were primarily responsible for these differences.

Critical Commentary

This research is directed toward a significant concern in teacher education. Some limitations of the study are acknowledged and the procedures used are clearly presented. It is an exploratory study which hopefully will generate additional studies related to preparing teachers to diagnose and remediate student errors.

Here are some thoughts on this study and additional ideas for any researcher contemplating extending this exploratory investigation.

1. What are typical error patterns in elementary mathematics? How can they be identified? Do they occur with sufficient frequency to justify specific attention in a preservice teacher education program? Are they associated with specific ages or certain cognitive levels of development? These questions are worthy of careful attention.

2. This investigation examined errors related to multiplication of whole numbers. Clearly much research remains to be done on errors related to computation. Yet further research in the area of diagnosis and remediation should also consider the careful examination of non-computational topics in elementary-school mathematics as well.

3. It was not clear to the abstractor at what stage the students in the methods courses were examined, although the assumption is that it would be done at the end of the courses. Change data on these students would be most interesting. That is, how did their diagnosis and remediation competencies upon entering the course compare with their exit competencies? Similar change data might help determine the impact of in-school field experiences on the preservice teachers.

4. It is hypothesized that school experience would make teachers more competent in diagnosis and remediation. This seems reasonable, yet is there research evidence to document these increased competencies? More specifically, is length of experience in the classroom the best predictor or are other factors to be considered? A careful examination of the diagnostic and remediation competencies of experienced teachers would provide some valuable insight.

5. The rating scale used for assessing remedial procedures was helpful, yet without additional specificity it is impossible to replicate this portion of the research or to expand a remediation test beyond the multiplication of whole numbers.

6. This study seems to rest heavily on the tacit assumption that certain procedures are effective in remediating specific errors. This seems reasonable, yet raises some questions. Do identifiable remediation procedures exist for specific errors? Is there research evidence to support the use of any remediation procedure over any other? Is there any identifiable interaction between students, mathematical content, and remediation procedure?

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INTERACTION OF SPATIAL VISUALIZATION AND GENERAL REASONING ABILITIES WITH INSTRUCTIONAL TREATMENT IN QUADRATIC INEQUALITIES: A FURTHER INVESTIGATION. Eastman, Phillip M.; Carry, L. Ray. Journal for Research in Mathematics Education, v6, pp142-149, May 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Gerald Kulm, Purdue University.

1. Purpose

The study sought to determine whether aptitude by treatment interactions exist when using graphical and analytic instructional treatments on quadratic inequalities and the aptitudes of spatial visualization and general reasoning.

2. Rationale

The study was designed to investigate further the results of Carry (1968) and Webb (1971), who obtained contrary and inconclusive evidence of aptitude by treatment interaction. It was conjectured that the deductive nature of the spatial aptitude test (Paper Folding, V_z-2) and the inductive nature of the graphical treatment used in the Carry study confounded the analytic-spatial aptitude question.

3. Research Design and Procedure.

The D.A.T. Abstract Reasoning Test was selected as the measure of spatial visualization, while the Necessary Arithmetic Operations test was retained as a test of general reasoning ability. The former test was substituted for the Paper Folding Test because it is described by Guilford as a "univocal measure of cognition of figural relations." Treatments used by Webb (1971) on quadratic inequalities were revised to make the analytic treatment more deductive and the graphical treatment more inductive than in the original programmed units.

Data were obtained for 80 tenth-grade geometry students, randomly assigned to the two treatment groups. Aptitudes were tested on the first day, programmed units were studied on the second and third days, and a posttest was given on the fourth day.

The aptitude and posttest means of the groups were compared to test for equality of groups and treatments. The aptitude by treatment interaction was tested by an analysis of the heterogeneity of regression to determine whether regression planes for posttest scores onto aptitudes were parallel. Finally, the regression coefficients for prediction of posttest scores from aptitudes were tested for significance.

4. Findings

The means on the aptitude tests and on the posttest for the two groups did not differ significantly, indicating equally effective treatments. The

aptitude by treatment interaction was confirmed ($p < .03$) by the finding that the regression planes were not parallel. General reasoning ability was a significant predictor of posttest scores for the analytical but not the graphical treatment. However, spatial ability was a significant predictor of scores for both treatments.

5. Interpretations

"This study...confirms Carry's original hypothesis that spatial visualization will predict success in a graphical treatment and that general reasoning will predict success in an analytical treatment. This finding emphasizes the importance of a series of research studies devoted to a particular problem. ...the results of this study may be due to combining the figural material with an inductive method and the symbolic materials with a deductive method."

Critical Commentary

The investigators are to be commended for their perseverence in discovering and compensating for the pitfalls that appear to have plagued so many ATI studies, enabling them to produce a significant interaction. It appears quite clear by now that the complexities surrounding accurate aptitude assessment, preparation of correspondingly matched instruction, and precise evaluation of learning make it difficult to obtain ATI results, except in isolated circumstances.

The attempt by these investigators to sharpen aptitude assessment and instruction may have "loaded the dice" somewhat in favor of obtaining significant interaction. For example, the result of structuring the treatments along the inductive-deductive continuum seem to make less clear the existence of a pure analytic-spatial aptitude interaction. One final limitation of the study is that a two-day treatment using an unfamiliar mode of instruction weakens the effectiveness and generalizability of the treatment.

Despite the success of this study, the search for ATI results still appears to be full of dead-ends and complexities that may not be practically resolved with present aptitude assessment methods.

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COMPARISON OF CONTENT STRUCTURE AND COGNITIVE STRUCTURE IN HIGH SCHOOL STUDENTS' LEARNING OF PROBABILITY. Geeslin, W. E.; Shavelson, R. J. Journal for Research in Mathematics Education, v6, pp109-120, March 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Michael C. Mitchelmore, Kingston, Jamaica.

1. Purpose

To determine the extent to which a mathematical structure--that of elementary probability--could be communicated to a group of high school students by a programmed text.

2. Rationale

The content structure of a mathematical topic is defined as the web of concepts and their interrelationships shown in the instructional materials provided for students. The cognitive structure of the topic refers to the organization of the concepts in a subject's long-term memory. It is hypothesized that a topic has no cognitive structure for a student who is unfamiliar with it, but that learning the topic creates a cognitive structure which approximates the content structure and remains relatively constant.

3. Research Design and Procedure

Thirty-four volunteers from grades 9-12 in one high school were assigned randomly to study an SMSG programmed text on elementary probability or a programmed text on an unrelated mathematical topic for one period a day over a three-week period. Subjects were given pretests, posttests, and retention tests (at 11 days delay) of word association (WA) and achievement in probability. A pretest of general attitude to mathematics was also given.

Ten key concepts were isolated for study. The content structure of the probability text was represented by a 10x10 proximity matrix giving the "closeness" of each pair of key concepts; this matrix was then interpreted graphically using multidimensional scaling. The cognitive structure of each subject's memory on each test occasion was represented by a 10x10 relatedness matrix obtained from the WA data; multidimensional scaling was applied to the matrix of median relatedness coefficients within each group of subjects. Content and cognitive structures were compared for individual students using the Euclidean distance between the proximity and relatedness matrices, and for the group as a whole by visual examination of the multidimensional scaling solutions.

4. Findings

Analysis of achievement test data confirmed that students in the experimental group learned to solve probability problems as a result of instruction.

Analysis of content structure led to a two-dimensional scaling solution, the two dimensions being tentatively interpreted as concrete-abstract and subordinate-superordinate concepts.

No cognitive structure could be obtained from WA data on the pretest or for the control group on the posttest. A one-dimensional solution was obtained for the experimental group posttest, and a two-dimensional solution for the experimental group retention test; in both cases, cognitive structure shared some features with content structure but subjects appeared not to make all the distinctions present in the text. A two-dimensional scaling solution was obtained for the control group retention test, indicating some verbal learning; but the cognitive structure bore no relationship to the content structure.

Analysis of the Euclidean distance between the proximity and relatedness matrices indicated that cognitive structure moved significantly closer to the content structure as a result of instruction.

Examination of the correlations of the Euclidean distances on each test occasion with attitude and achievement test data revealed a weak relationship between learning to solve probability problems and learning the structure of probability concepts.

5. Interpretations

It is concluded that the methods described for the analysis of content and cognitive structure can be applied to the study of mathematics curriculum and learning. Subjects in the experimental group learned a significant portion of the structure of probability as a result of instruction while the control group learned almost nothing of the structure. The structure methodology might be a helpful tool in formative evaluation and in the design of text materials.

Critical Commentary

It is necessary to modify the authors' conclusions by pointing out three forms of bias in their report.

Firstly, the authors show a strong tendency to make statements which simply do not fit the data. Experimental subjects are said to have "retained that portion of the structure they learned" when rho = .09 on the common dimension. Cognitive structures are said to correspond "much more closely" to content structure as a result of instruction, but the fall is only from .039 to .032, no standard deviation is quoted, and the data were obtained from different numbers of students; furthermore, the data are illustrated on a graph with a false origin. A change from .040 to .039 to .034 is somehow regarded as "of a magnitude and rate ... not that of" .039 to .032 to .033. Examples could be multiplied.

Bias is also shown by selective reporting. The close correlation (rho = .79) of content structure on dimension 1 with posttest cognitive structure in the experimental group is noted, but the insignificant correlation (rho = -.23) with corresponding retention test data is

ignored. An embarrassing similarity between the control and experimental groups on retention test WA data is labelled as spurious, although the control group data had been analyzed closely earlier in the report. Again, one isolated non-significant correlation slightly larger than average is siezed upon as supportive evidence, whereas several higher but clearly incidental correlations are passed by.

A third, less systematic, bias is introduced by retaining data from all the subjects tested on each occasion despite sizeable attrition. From pretest to posttest, the experimental group dropped from 19 to 11 and the control group from 14 to 6 subjects. In these circumstances, changes in mean scores are as likely to be due to changes in group composition as to treatment effect. Although the repeated-measures analysis of variance used only data on subjects present for all three tests, descriptive statistics are presented and interpreted for all students present on each occasion. Perhaps the authors felt that analyzing the data for the small number of students left after attrition would have made the results too tenuous; but that is as it should be.

Now to examine the validity of the structure methodology. The authors do not state whether the "proximity of key concepts in the text material" (from which content structure is obtained) is physical or conceptual. A longer discussion in another paper (American Educational Research Journal, v12, pp21-39, Winter 1975) suggests that the measure might be better described as "semantic priority". The measure is unsymmetrical, so it is doubtful whether a Cartesian graph is a suitable method of representation; indeed, the reported multidimensional scaling solution does have some rather peculiar features (e.g., intersection closer to probability than independent is to event). More evidence is required before one could accept that content structure, as derived by this method, accurately represents the conceptual structure of a mathematical topic.

The major problem with the procedure for analyzing cognitive structure (assuming the WA technique is appropriate) is in measuring the degree of similarity between two such structures. The authors offer only visual inspection (which is easily biased) and the Euclidean distance (which they admit has shortcomings); but they give no indication of the likely effects of measurement error nor even of the variability in structure from student to student. This makes it very difficult to judge the reliability of the WA technique, although the low posttest-retention test correlation for Euclidean distance data (tau = .54), and the complete dissimilarity of the scaling solutions, suggest rather poor reliability. Since also the proximity matrix is in general unsymmetrical whereas the relatedness matrix is necessarily symmetrical, comparison between content and cognitive structure would seem to be difficult, if not pointless.

In short, this investigation is little more than an exploratory pilot-study. Unfortunately, it does not seem to have established the validity of the procedures under scrutiny. The authors are to be congratulated for pioneering an interesting approach to structural analysis, but there are still many questions to be answered before the methodology can be used with confidence in curriculum and learning research.

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FURTHER INVESTIGATIONS OF THE YOUNG CHILD'S CONCEPTION OF NUMBER. Gelman, Rochel; Tucker, Marsha F. Child Development, v46, pp167-175, March 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Richard Lesh, Northwestern University.

1. Purpose

The article reports two experiments about children's number judgments concerning small sets (i.e., 2-5 items). The first study involves two questions: (a) To what extent do children's number judgments ignore number-irrelevant properties of a set (i.e., the color of the objects, the identity or shape of the objects)? (b) To what extent do children aged 3 to 5 count covertly--even when estimating the size of small sets? The second study investigated whether, when children deal with set sizes they can easily estimate, they realize that surreptitious changes in the color or identity of the objects does not affect the numerosity of the set.

2. Rationale

The article reviews several studies showing that 3-year-old children can, under certain circumstances, accurately estimate the size of small sets of objects (i.e, 3-5 objects). In fact, although the authors do not mention it, pigeons have been taught to identify accurately small sets of objects under some conditions. The authors state that such studies have "generally assumed that the young children rely primarily on perceptual processes (for example, subitizing) to estimate small numbers." However, other studies demonstrate that there are conditions under which young children can and will count in order to estimate. In fact, some researchers claim that children prefer a counting process when estimating. These claims raise the question, "Under what conditions do children use (a) counting processes, (b) perceptual processes, or (c) perceptual and counting processes in concert?"

The authors review research (e.g., von Gast) claiming that preschoolers who accurately assign cardinal numbers to small sets of homogeneous items sometimes fail to do so when confronted with sets whose items vary in color, size, shape, and identity. However, other studies have shown that 3- and 4-year-olds consistently estimated small numbers (1 to 4) accurately, even when the sets were comprised of heterogeneous materials. The authors hypothesize that the negative results occurred because an expectancy for homogeneous arrays had masked children's ability to estimate heterogeneous arrays. To follow up this hypothesis would require a modified replication of the von Gast study. However, such a replication study was not the purpose of this article. Instead, the experiments were extensions of previous studies conducted by Gelman.

Gelman (1972a, 1972b) had shown that, under some circumstances, children as young as age 3 can correctly classify addition and subtraction as operations which increase or decrease number, and that they can classify length and density of configuration as irrelevant to number. The present article focused on the question of whether young children know that

substitution transformations which alter the color or identity of elements are irrelevant to number.

3. Research Design and Procedure

(a) First Study

The experiment involved four main variables: (a) age (3-, 4-, or 5-year-old groups); (b) array type (homogeneous or heterogeneous); (c) set size (linear arrays of 2, 3, 4, 5, 7, 11, or 19 objects);[1] and (d) exposure to time (1 sec, 5 sec, 1 min).

One-hundred forty-four children (76 boys, 68 girls; median ages of the 3-, 4-, and 5-year-old groups were 3.7, 4.5, and 5.3, respectively) were assigned to either a homogeneous or heterogeneous condition. Each child was tested on each set size at each exposure time; the order was varied. The items forming the arrays were also varied (red or blue, stars or circles, .6 cm diameter). The arrays were displayed on 14 x 35.5 cm white posterboard cards. Just before presenting each display, E asked, "How many things are on this card?"

Anecdotal information was recorded indicating children who counted overtly when making their estimates. No attempt was made to record non-overt counting procedures (e.g., eye gestures, etc.).

(b) Second Study

A two-phase "magic" paradigm was used. The first phase established an expectancy for a given numerosity (.e., three items), and the second assessed the child's reaction to variations in the expected set. Variations involved changing from a homogeneous set (e.g., three green mice) to a heterogeneous set (e.g., two green mice and one toy soldier), or changing from a heterogeneous set to a homogeneous set.

Children were not explicitly asked to make number judgments. Instead 48 3- and 4-year-olds were given a brief training procedure in which examples and reinforcement were used to teach them to select correctly three item sets. The primary intent of the study was to discover whether children would believe that the concept they had learned was invariant despite changes in the color and identity of the objects. For 17 of the children, special care was taken to ensure that the children understood that the sets could involve heterogeneous items as well as items that were all alike. However, there was no real guarantee that the concept the children learned in the training procedure was in fact a concept of number. In fact, the "conclusion" section of the article admits that some children abstracted non-number concepts (e.g., identity) from the examples that were given in the training procedure.

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[1]The larger arrays were included as part of another project, and were not considered in this article.

4. Results

(a) First Study

Chi squares (or Fisher exact probabilities when expectancies were low) were computed for each set size, at each exposure time, and at each age level. Of the 45 comparisons, only one was reported to have attained a .05 level of significance. No statistics were reported for the other 44 comparisons; so it is not known if they were all close to .05. However, on the basis of these data, the authors concluded that homogeneity did not significantly affect accurate extimation at any age level. So, data from homogeneous and heterogeneous conditions were combined to analyze the effects of set size and exposure time on accuracy.

A table showing the effects of set size, age, and exposure time revealed: (a) an overall interaction between exposure time and accuracy, with highest accuracy for longer exposure times; (b) almost all ages were able to assign correct values to 2-item arrays, most were correct for 3-item arrays, and accuracy began to fall off for 4-item arrays--especially for younger children; (c) an age x set size x exposure time interaction revealed that the effect of exposure time on accuracy was greatest for larger sets and for younger children.

In each cell of the above table, data were reported showing the number of children who responded correctly and the number of children who counted overtly. These data revealed that: (i) young children were more likely to count overtly than older children; (ii) younger children tend to count even for small sets and brief exposure times, whereas older children were unlikely to count for smaller sets or for brief exposure times; (iii) longer exposure times encouraged more overt counting.

(b) Second Study

Evidence from the training procedure indicated that the children formed expectancies for both the number and identity of objects in the 3-item sets.

When the "magic" paradigm involved changing from a homogeneous to a heterogeneous set, 11 of 29 children indicated that the numerosity of the sets was unaffected.

These results indicate that, under some conditions, children are able to treat "identity-change" as a number-irrelevant transformation--at least for the small sets and relatively small identity changes that were involved in this study.

Critical Commentary

The results of this study are unexciting to say the least. Consequently, an elaborate discussion of methodological ambiguities seems unjustified. Teachers have known for years that young children often count when they estimate the number of items in a set--even small sets (i.e., 2-5 items). However, with the exception of some recent work by Steffe,

little has been done to trace the development of children's counting skills, or the effect of these skills on number concepts.

Both in the psychological literature and in educational practice, it is well established that: (a) children sometimes have difficulty estimating the numerosity of heterogeneous sets; (b) children sometimes do not realize that the numerosity of a set is invariant under number-irrelevant transformations of length density, or identity, of the objects. However, far less is known about the conditions under which the above factors alter the difficulty of a task.

The two experiments reported in this article did little to shed light on either of the above two issues.

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THE EFFECT OF EXTENDED IN-SERVICE TRAINING CURRICULA UPON THE MATHEMATICS ACHIEVEMENT AND ATTITUDES OF ELEMENTARY TEACHERS. Greene, John F.; Archambault, Francis; Noland, William. Paper presented at the Annual Meeting of the American Educational Research Association (AERA), San Francisco, April 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Donald J. Dessart, The University of Tennessee-Knoxville.

1. Purpose

The primary purpose of this study was to ascertain the effects of two in-service training approaches on the mathematical achievement and attitudes of primary and intermediate grade teachers. The two approaches included: the Full Group training model, in which selected teachers experienced an intensive summer workshop and a series of implementation workshops during the academic year, and the Restricted Group training model, in which selected teachers experienced only workshop training during the academic year.

2. Rationale

Curricula materials used in the workshops were developed by the Mathematics-Methods Project (MMP) of Indiana University. These materials, designed initially for pre-service teachers, consist of a series of instructional units. Each unit is a topical package comprising materials, activities, and teaching procedures related to the topic; related elementary school content material; and teaching techniques appropriate for elementary school children. The units were used by the participants in an activity-oriented laboratory setting in which discovery learning and small-group activities were stressed.

3. Research Design and Procedure

Teachers who participated in the study were selected from four rural school districts near Portland, Maine. The Full and Restricted Group teachers had volunteered from three of the districts; the Control Group, on the other hand, had volunteered from a fourth district. The total sample included 31 teachers in the Full Group, 33 in the Restricted Group, and 23 in the Control Group. The teachers were evenly distributed from grades one through six.

The teachers were tested with both an achievement test in mathematics and attitude scales prior to the summer workshops (May 1974), at the beginning of the academic-year workshops (October 1974), and at the conclusion of the academic year (May 1975).

Mathematical achievement was measured by a 37-item instrument previously developed for the MMP. The internal consistency (as determined by Kuder-Richardson Formula 20) was .74, and the coefficient of stability over a five-month period was determined to be .84.

Teacher attitudes were measured by the Aiken Mathematics Attitude Scale, which is composed of two subscales: the first, consisting of eleven items, assesses Enjoyment of Mathematics; and the second, made up of ten items, measured the Value of Mathematics. The instrument was subjected to a factor analysis using pre-treatment data. The results supported a three-factor rather than a two-factor solution. The three factors were named "Enjoyment of Mathematics," "Value of Mathematics," and "Future Value of Mathematics." Estimates of internal consistency determining the alpha coefficient were .93, .83, and .78, respectively.

Teacher attitudes toward the teaching of mathematics were assessed with a modification of the McCallon-Brown Scale. This semantic-differential-type instrument was first developed to measure attitudes toward mathematics. The instrument was modified to assess attitudes toward teaching mathematics, and it was also subjected to item analytic and factor analytic techniques. As a consequence, a single factor solution was adopted with final scores being a summation of 13-item scores.

A 3x2 ANCOVA with three treatment levels (Full, Restricted, and Control) and two teaching levels (primary, grades 1-3; and intermediate, grades 4-6) was employed with the pre-test measures as covariates. To determine only the effects of the summer workshops on attitudes and achievement, the May 1974 results were used as covariates with the October 1974 results as criteria measures; whereas, to determine the effects of summer and academic-year workshops combined, the May 1974 results were used as the covariates and the May 1975 data were used as the criteria.

4. Findings

It was found that the summer workshops alone had no significant effects ($p = .05$) upon teacher achievement nor upon the Enjoyment or the Future Value factors of the Aiken Scale. However, significant main effects were determined for the Value Factor of the Aiken Scale and the Attitude Toward Teaching Mathematics Scale.

When determining the effect of the total program (both summer and academic-year workshops combined), no significant effects were found for teacher achievement or for any of the three factors of the Aiken Scale. With regard to the attitudes toward teaching mathematics, a statistically significant effect for treatment was found. Comparison of adjusted treatment means revealed that the Full Group participants were more positive (but not significantly) toward teaching mathematics. A further analysis showed that most of the gains of the Full Group could be attributed to the summer workshops rather than the academic-year program.

5. Interpretations

The lack of significant effects upon mathematical achievement of teachers was attributed to one or more of the following: (1) the major focus of the achievement test is content whereas methodology was emphasized in the workshops; (2) mathematical content is not emphasized sufficiently in the MMP materials; and (3) too much was covered during the workshop sessions.

In analyzing the effects of the workshops on attitudes toward teaching mathematics, it was concluded that the summer workshop had a positive impact. This conclusion could not be reached for the academic-year workshop alone nor the additional school-year training for summer participants.

In light of some of the positive effects found in the affective domain, it was concluded that an approach of coordinating content materials with teaching methodology has promise for in-service work.

Critical Commentary

The need to document the effectiveness of workshops and training sessions is highly important as we move into a period in which newer teachers bringing fresh ideas to the classroom will be less numerous, so that renewal through workshops will become necessary. Consequently, additional studies of this type are most desirable.

This study illustrates once again the acute problem of instrumentation in research. Obviously, something changed in the participants as a result of the training, but the testing revealed very little of what had changed. The researchers offered explanations of the lack of effects upon mathematical achievement of the teachers. It is too bad that these explanations could not have been anticipated earlier and incorporated in the study.

The glimmer of change in the teachers in the affective domain was not reassuring and, perhaps, represents a major finding of this study. One could speculate that this positive change could have affected pupil achievement. Consequently, the study might have concentrated upon pupil achievement in mathematics rather than teacher achievement with more fruitful results.

The lack of statistically significant findings raises the question of the determination of Type II error and the possibility of wringing more information from the results of the study. Unfortunately, most educational studies do not deal with this additional analysis.

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HEMISPHERIC PROCESSING AND COGNITIVE STYLES IN LEARNING-DISABLED AND NORMAL CHILDREN. Guyer, B. La Rue; Friedman, Morton P. Child Development, v46, pp658-668, September 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Grayson H. Wheatley and Robert L. Frankland, Purdue University.

1. Purpose

The purpose of this study was to determine the relationship among scores on cognitive abilities tests for learning-disabled and normal children. The tests were chosen for their association with specialized brain hemisphere processing. In short, the study compared the cognitive styles and hemispheric processing of learning-disabled and normal children.

2. Rationale

It is well documented that the two hemispheres of the brain process stimuli differentially, the left hemisphere in a verbal-analytic manner, the right hemisphere in a viso-spatial manner. This study attempted to use hemispheric processing theory as a basis for comparing learning-disabled and normal children's cognitive skills and cognitive style. Of the numerous cognitive style dimensions, the authors select Witkin's field articulation theory (as measured by the Rod and Frame) as most related to hemispheric processing. The research in hemispheric processing and cognitive style would seem to suggest that learning-disabled children may have deficits in one of the two cognitive systems.

3. Research Design and Procedure

The S's were 41 boys from a school for children with learning disabilities and 41 boys from an elementary school located in the same area and serving the same socioeconomic population. The boys, ages 7.7 to 12.7 years, were matched on age and IQ. The scores on the Peabody Picture Vocabulary Test ranged from 90 to 146. Achievement scores were obtained for reading vocabulary, reading comprehension, mathematics calculation, and mathematics reasoning. A battery of cognitive tests were selected to tap selected hemispheric processing and cognitive styles.

Cognitive Abilities Tests. The authors selected the following tests as measures of left hemisphere processing: (1) Hand-awareness test, assessing the ability to report how many and which fingers are touched by the E; (2) Auditory sequential memory, measuring short-term retention and repetition of a series of digits; (3) Verbal recognition, identifying words previously presented; (4) Verbal closure, a cloze test; and (5) Equivalence, the ability to find a common invariant among a list of words.

The following tests were selected as measures of right hemisphere processing: (1) Visual sequential memory, measuring short-term memory for symbolic nonsense forms; (2) Visual recognition, measuring long-term memory of line drawing; and (3) Visual closure, measuring ability to perceive a "gestalt" upon presentation of random bits of a picture.

Cognitive style was assessed with the Portable Rod-and-Frame Test (Nickel, 1971). Field dependence as measured by this test has been associated with left hemisphere function. Because crossed dominance has been postulated as a factor in reading disability, the Edinburgh Inventory (Oldfield, 1971) was administered as a measure of lateralization.

4. Findings

As expected, the normal children performed significantly better ($p < .001$) than the learning-disabled children on all achievement measures. More importantly, the differences on the cognitive abilities tests were found for those which assess left (dominant) hemisphere processing. Specifically, t-tests for matched samples revealed differences in favor of the normal children for field articulation ($p < .01$), hand awareness ($p < .02$), verbal closure ($p < .05$), and verbal recognition ($p < .10$). Using discriminant-function analysis, age and reading comprehension considered together correctly classified 80% of the learning-disabled children and 73% of the normal group. When just the cognitive abilities tests were used in discriminant-function analysis, the Rod-and-Frame, verbal recognition, and verbal closure tests predicted group membership (normal, learning-disabled). There was evidence from several statistical tests that visual closure (right hemisphere ability) was a predictor of academic achievement in learning-disabled children.

5. Interpretations

The major findings of this study can be stated as follows:

1) The cognitive processing abilities that were deficient in learning-disabled children can all be theoretically related to left (dominant) hemisphere functioning.

2) Learning-disabled children did not display a greater degree of crossed dominance than normal.

3) Body awareness and field independence were related.

4) On most tests of cognitive processing, learning-disabled children were found to perform as well as normal children when age and IQ were controlled.

Since some evidence was found for a nonverbal (right hemisphere) approach to school tasks by learning-disabled children, the authors suggest nonverbal representation be explored in developing verbal representational systems with LD children.

Learning-disabled children are definitely more field dependent. The authors discuss the problems a field dependent (not always learning-disabled) child faces as he copes with school tasks, that more often than not assume field independence and require left hemisphere processing.

Critical Commentary

The authors are to be commended for relating two important theories, field articulation and hemispheric specialization, which may provide the framework for new insights into personalizing instruction. The research design was basically sound and the data anaylsis insightful. Multivariate techniques were appropriately applied.

However, the authors failed to indicate how field articulation is associated with hemispheric specialization. In the Rod-and-Frame test, subjects are required to position a rod vertically independent of a surrounding frame. The field dependent person is one who relies on spatial cues, namely the relationship between the rod and the frame, while the field independent person positions the rod by ignoring the perceptual cues. Analogously, in our laboratory it has been observed, using EEG measures, that young children show right hemisphere processing of a conservation task when perceptual cues are present but show left hemisphere processing during a presentation with perceptual cues screened out (Wheatley, Frankland, Mitchell, in preparation). Thus right hemisphere processing would be characteristic of a field dependent child. This argument is strengthened by the relationships the authors report between learning-disabled children's (mostly field dependent) performance on the visual closure task and academic achievement.

Two aspects of the procedure are questionable. In light of the evidence that left-handed persons differ in their lateralization from right-handed persons, the sample should have been restricted to right-handed persons. Additionally, the practice of matching on IQ and age raises questions about the interpretation of the results.

The implications of this study for mathematics educators, beyond the factors already discussed, lie in the relationship established between visospatial ability and mathematics achievement. Further studies are needed to explore the usefulness of this information for curriculum development and instructional procedures.

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Robert L. Frankland
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Reference

Wheatley, G., Frankland, R., and Mitchell, R. Hemispheric Specialization and Cognitive Development. Purdue University, West Lafayette, Indiana. In preparation.

MATHEMATICS GOALS: WHAT DOES THE PUBLIC WANT? Hershkowitz, Martin; Shami, Mohammad A. A.; Rowan, Thomas E. School Science and Mathematics, v75, pp723-728, December 1975.

Also reported in:

RELATIVE IMPORTANCE OF "KNOWLEDGE OF MATHEMATICAL CONCEPTS" AND "MASTERY OF CONCEPTUAL SKILLS" AS GOALS OF PUBLIC EDUCATION. Shami, Mohammad A. A.; Hershkowitz, Martin. Operations Research, Inc., Silver Springs, MD. 1973.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jane Gawronski, Department of Education, San Diego County.

1. Purpose

To identify valid school goals which are related to needs as determined by a wide variety of respondent groups.

2. Rationale

Goals and objectives have been of concern to mathematics educators for some time. However, there does not appear to be agreement for the basis upon which objectives are identified. Mathematicians select objectives based on the content and structure of mathematics while educators place considerably more emphasis on learning theory. An alternate way of establishing a framework within which objectives could be designed is by determining what is important to most people in the general public since schools exist because of and for that group. Goals which are important to the general public could then be used to give direction to the more specific level of goal development which would be carried out by mathematics educators.

3. Research Design and Procedure

The study sought the opinions of people from the general public and various private and educational groups on 37 goal statements. The goal statements directly related to mathematics were

- knowledge of mathematical concepts

- mastery of computational skills.

The respondent groups were: students, school staff, central staff, parents, Boards of Education, business/industry, general public, elected and appointed officials, Maryland State Department of Education staff, and post-secondary educators. Random selection procedures were used to identify respondents in each group. Each respondent was asked to indicate an opinion about the importance of each goal on a 5-point saale from "very important" (5) to "not at all important" (1). Sample sizes ranged from 8,432 for students to 129 for MSDE staff. The mean importance score and the rank of each goal was calculated for each respondent group.

4. Findings

The concepts goal was ranked between 16th (Boards of Education) and 27th (Central staff and MSDE staff) by all respondent groups while the skills goal fell even lower, between 27th (central staff) and 35th (general public). Thus, "mastery of computational skills" fell in the cardinal rank "least important" and "knowledge of mathematical concepts" fell in the rank "less important."

5. Interpretations

It appears that the respondents generally considered mathematical goals to be of rather low importance. However, the mean importance score for the goal "ability to apply knowledge and skills to the solution of real-life problems" ranged from 4.4 to 4.8. This places it in the "most important" category for all respondent groups. This may be interpreted to mean that people want to see more relevance built into the curriculum. Mathematics in a public school organization should serve the needs of the individual, the community, and society. One might examine the present curriculum to determine the extent to which it meets these needs and to make revisions to the curriculum which would bring it closer to meeting them. This could probably best be accomplished by interdisciplinary teams working closely with students, parents and other members of the community. In any case, more emphasis can be placed on mathematical applications.

Critical Commentary

This study arose out of a recognition that since schools exist because of and for the public, curriculum should reflect the needs of that public. In this case, the public was defined by the ten respondent groups. It would be informative if descriptive data such as socioeconomic status of some of those groups were given since the student, parent, and general public views may differ with age, socioeconomic status, and even urban, suburban, rural locales.

The study does, however, represent an attempt to survey and determine the relative importance of school goals for the general public. That is not to say that any of the 37 goals are unimportant, but rather to place the goals in a rank order as perceived by the general public. This provides a basis for curriculum development that is both responsive to community needs and community based. This study also raises a question concerning the responsibility for school goals and hence curriculum development related to those goals. Is this responsibility to be borne by mathematics educators, the community the school serves, or is it a responsibility to be shared by both "publics"?

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TWO METHODS OF TEACHING MULTIDIGIT MULTIPLICATION. Hughes, Frank G.; Burns, Paul C. Elementary School Journal, v75, pp452-457, April 1975

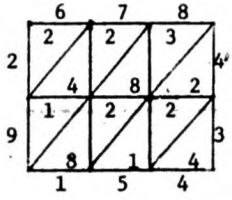
Also reported in:

A COMPARISON OF TWO METHODS OF TEACHING MULTIDIGIT MULTIPLICATION. Hughes, Frank George, The University of Tennessee, 1973. Dissertation Abstracts International, v34A, pp2460-2461, November 1973.

Expanded Abstract and Analysis Prepared Espceially for I.M.E. by Douglas A. Grouws, University of Missouri-Columbia.

1. Purpose

To compare the effectiveness of two strategies for teaching multidigit multiplication to fourth-grade students. One strategy involved the lattice method and the other was the distributive method, taught in most contemporary textbooks. Each method is illustrated below.



678
x43
2034 = (3 x 678)
2712 = (40 x 678)
29154

2. Rationale

The authors point out that few studies have been done on the distributive method and none on the lattice algorithm. Based on a logical analysis they also state that the lattice method seems to have much to recommend it.

3. Research Design and Procedure

The experimental design was essentially a Solomon Four Group design with a retention measure administered three weeks after the posttest.

	Pretest	Treatment	Posttest
Group A	O_1	X	O_3
Group B	O_2	Y	O_4
Group C		X	O_5
Group D		Y	O_6

The sample consisted of twelve fourth-grade classes from four school districts. The twelve classes were randomly assigned to either Treatment X, the distributive method, or Treatment Y, the lattice method, such that each treatment group included two urban classes, two rural classes, and two inner-city classes. Groups pretested were also determined randomly.

A series of 15 lessons and a teacher's manual were constructed for each treatment. Meetings were held with the classroom teachers to discuss the plan of study, the material to be used, and the nature of the testing program. Instruction during the first three days of the study was devoted to readiness games and activities that required using the basic multiplication facts. This was followed by instruction on multidigit multiplication using the assigned method. Student practice exercises to be completed in class were given in the teacher's manual. No mathematics homework was assigned during the twenty-one-day study.

Four variables were measured in the study: achievement in multidigit multiplication, rate at which multiplication exercises were completed, understanding of the multiplication process, and attitude toward multidigit multiplication. Achievement was measured by a 25-item instrument which was experimenter-constructed and had a test-retest reliability of .93. Rate was measured in minutes required to complete the achievement test. "The instrument used to measure understanding [italics added] of the multiplication process was a composite test, using the appropriate twenty-two items from the Stanford Achievement Test, Form X, Intermediate I, and the Metropolitan Achievement Tests Form G." The reliability coefficient for this set of items was .87. Attitude was measured by the Dutton Attitude Scale.

Data were analyzed using analysis of variance, except the retention data which were analyzed using t-tests.

4. Findings

Students in the lattice treatment group performed significantly better than those in the distributive treatment on the achievement and time measures (i.e., the lattice group's mean time was less). There were no differences in the understanding and attitude scores.

Pretesting significantly increased speed on the posttest time measure but produced no significant effects on achievement, "understanding," or attitude.

The urban, rural, and inner-city children performed significantly better on the computation than did their counterparts using the distributive algorithm. The rural group using the distributive method had significantly higher scores on the understanding measure than the other rural group.

5. Interpretations

The authors conclude that the lattice method of teaching multidigit multiplication "might well be given consideration." They point out that

the lattice method may be of value for the slower pupil and helpful for the teacher in diagnostic work, but quickly add that these variables were not tested in the study. It is also pointed out that topic-specific tests of "understanding" and attitude need to be developed.

Critical Commentary

In the realm of alternate treatment studies, this study was interesting. It examined two multiplication algorithms which seem to have merit on the basis of logical analysis. The treatments were carefully developed and replicable. The computation achievement test was valid and reliable and the significant differences in favor of the lattice method on this instrument suggest that further study is warranted.

The authors' consideration of multiple criteria for effectiveness is commendable. The implementation, however, was very poorly done. Using the Dutton instrument to measure "which [group] had a more favorable attitude toward multidigit multiplication" was certainly not a specific enough measure and thus inappropriate.

Similarly, choosing 22 items from two standardized achievement tests implies that the instrument did not suit the author's avowed purpose of measuring "which [group] had a better understanding of the multiplication process." In future work "understanding" must be more carefully defined and more attention given to producing a valid instrument to measure it.

Finally, the authors' judgment that it was appropriate to provide the lattice grids on the posttest and retention computation tests for the lattice group is questionable. Perhaps constructing the grids should be a part of the computation process when the lattice method is used, especially if time is one of the variables being measured.

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SEX BIAS IN ELEMENTARY SCHOOL MATHEMATICS TEXTS. Jay, Winifred T.; Schminke, Clarence W. Arithmetic Teacher, v22, pp242-246, March 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Martin L. Johnson, University of Maryland.

1. Purpose

To analyze selected elementary mathematics textbooks to determine if if there are:

(a) more masculine, feminine, or neutral situations in word problems and illustrations,

(b) more famous men than famous women featured,

(c) more male occupations than female occupations featured,

(d) men and women featured in nonstereotyped roles.

2. Rationale

Sex stereotyping is currently under attack in all aspects of American society. A careful look at the instructional materials used in the elementary school may reveal that sex stereotyping exists, even in our mathematics textbooks. Jay (1973) found definite evidence of inequality of treatment of the sexes in content of 12 selected elementary mathematics textbooks. Concern was expressed that educational opportunity may not exist equally for boys and girls and that students' perceptions as they relate to mathematics may be influenced in the early grades by such subtle forces.

3. Research Design and Procedure

To answer question (a), eighty-one children from grades 2, 4, and 6 and 30 parents were asked to classify situations according to gender. The situations were generated by taking each word problem, illustration, general setting or activity and asking the raters to classify it as masculine, feminine, or either. A total of 160 "settings" were presented for classification.

Two adult female coders analyzed approximately 4100 pages of text material to answer questions (b), (c), and (d). The coders recorded famous people and occupations featured in "quantitative settings".

4. Findings

(a) From the 160 "settings" presented for classification, 113 were judged to be neutral, 23 were judged masculine, 9 feminine, with no concensus on 15. The great number of "neutral" settings notwithstanding, there were more than twice as many settings judged masculine as compared to feminine.

(b) Of 49 "famous people" cited in the context of the material, 46 were male, indicating a degree of sex stereotyping. The three famous females were found in two of the four sixth-grade textbooks. No famous females were found in ten of the 12 textbooks and none were found in other than sixth-grade texts.

(c) Of 104 occupations recorded, 86 were associated with males and 18 with females. A wide range of differences was found in occupations listed for men and women. Only four jobs on the female list required professional training: teacher, journalist, librarian, and nurse.

(d) No genuine nonstereotyped male-female roles were found, although four borderline cases were found. These included male teachers in elementary school, fathers doing the marketing, girls painting a room, and girls swimming across a pool.

5. Interpretations

1. A degree of sex stereotyping exists in the mathematics textbooks studied.

2. A functional interpretation of the material analyzed suggests boys and men are shown in typically masculine-related roles and and girls and women are shown in typical feminine-related roles.

3. Educators and school personnel at all levels must carefully consider balance in the presentation of sex roles when selecting textbooks and other instructional material.

4. Classroom teachers must supplement text material with examples, exercises, and activities that feature boys and girls in unusual, nonstereotyped roles.

Critical Commentary

This study makes a significant contribution to the mathematics education literature. Textbook writers and those responsible for selection of materials must be aware of such subtle stereotyping and possible effect on young students, both male and female.

The description of the classification "setting" is insufficient for those wishing to replicate the study. Specific examples of actual settings and instructions are needed.

Although the researchers were able to answer the specific questions asked, other questions related to this research can be raised:

(1) What was the sex of the coders for the classification setting? Would a different pattern have emerged if the coders had been all male or all female?

(2) Was there any relationship between the number of stereotypic examples and sex of the writers of the textbooks studied? Were any of the texts written by women?

(3) What was the copyright date of the books selected? While there is no logical excuse for sex stereotyping, hopefully current editions have eliminated such situations.

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EFFECT OF INTERSPERSED QUESTIONS ON LEARNING FROM MATHEMATICAL TEXT. Kuehls, Ernest A. Journal for Research in Mathematics Education, v7 n3, pp172-176, May 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E by William E. Geeslin, University of New Hampshire.

1. Purpose

To determine the effect on achievement of adding questions at the end of every paragraph or two of mathematical text. Also, the possible interaction between student ability level and achievement was investigated.

2. Rationale

One goal of instruction is to make students capable of learning well by reading on their own. Research concerning programmed materials has implied that while students learn from such materials, they do not necessarily learn more or more efficiently. Thus text that combines both straight exposition and an element of programming might prove to be superior in terms of student achievement to other types of text.

3. Research Design and Procedure

A treatment-by-ability level design was used. Sixty college students (30 honors students and 30 nonhonors students) in first-year calculus were assigned randomly to two treatments (text without questions versus text containing periodic multiple choice questions). Students read the 4500-word text on elementary matrix algebra in one session. Students had not been exposed to the material prior to treatment and were tested immediately following the reading session. The test consisted of 20 multiple-choice skill questions and four proof questions. Test scores were analyzed using ANOVA and t-tests.

4. Findings

A significant ability-level effect and a significant treatment-by-ability-level interaction were found by ANOVA ($p < .05$). A significant t statistic was found in comparing the two treatments for nonhonors students ($p < .05$). Honors students scored higher in the non-question treatment while nonhonors students scored higher in the question treatment. Honors students scored higher than nonhonors students when treatments were combined.

5. Interpretations

The most effective learning materials for a particular group of students may involve assigning students a text based on their ability and achievement levels. That is, a particular course might use more than one text-type. Additionally, neither type of text appeared sufficient in providing students with competency in construction of proofs.

Critical Commentary

The study suggests that text selection may be a way to take advantage of a possible aptitude and treatment interaction. Since textbooks can be entirely controlled in the classroom, the results of this study could be put to practical use with controlled and predictable results. Naturally, the study should be replicated and should be conducted over a longer instructional period to ascertain if similar results occur in other situations. Given the failure to show consistent ATI's in educational research, one must remain skeptical. Nonetheless, in terms of an immediate payoff for classroom practice, studies of this nature could be quite significant. The practitioner would need more information on the criterion test such as examples of questions and reliability of the test.

The study was reported in a manner that may prevent it from having much impact on educational theory or learning theory. Almost no theoretical framework was provided in the article. Three of the four papers listed in the reference section were apparently not mentioned in the article. (One of these refers to the ANOVA computer program used.) Two of these articles would appear to have important implications for the present study. What type of questions should be interspersed in text? What is the optimal spacing of questions for a given ability level? What type of student information processing is assumed in these studies? What information about learning theory was gained from the study? Theoretical questions should be discussed briefly in an article since they can lead to new hypothesis and more complete understanding of learning behavior. The author's viewpoint on these questions is important to the reader.

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University of New Hampshire

THE RELATIONSHIP OF ACHIEVEMENT ON A TEACHER-MADE MATHEMATICS TEST OF COMPUTATIONAL SKILLS TO TWO WAYS OF RECORDING ANSWERS AND TO TWO WORKSPACE ARRANGMENTS. Majors, Gene W.; Michael, Joan J. Educational and Psychological Measurement, v35, pp1005-1009, Winter 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by F. Richard Kidder, Longwood College.

1. Purpose

This study examines the effects of varying the test format on student test scores. Is the average level of achievement on an arithmetic computation test related to (a) how the answers are recorded--directly on the test paper or on a separate sheet, and (b) the workspace arrangement--performing the computation on the test paper itself or performing the computation on separate scratch paper?

2. Rationale

The validity of test results is of utmost importance. If scores are affected by a change in test format, then conclusions drawn from these scores are suspect. Since standardized tests are being used more and more widely for both evaluative and predictive purposes and since these tests, in general, use the separate workspace and answer-sheet format, Majors and Michael's study becomes particularly relevant to mathematics educators.

3. Research Design and Procedure

The sample consisted of 4 seventh- and 4 eighth-grade mathematics classes (30 students per class) from a junior high school in a mixed socio-economic community of Orange County, California. The results of a pretest, Comprehensive Test of Basic Skills (CTBS)--Mathematics Computation, Level 3, Form Q, indicated no significant differences in computational skills for the four classes within a grade level.

Each class within grade was randomly assigned to one of four treatments (test format) as follows: (1) workspace provided on the test paper and a detached answer column, (2) workspace provided on the test paper and no detached answer column (necessitating that answers be written on the test paper), (3) no workspace on the test paper (scratch paper provided) and a detached answer column, and (4) no workspace on the test paper and no detached answer column. The same teacher-made, 30-item test on addition, subtraction, multiplication, and division of integers and fractions was then given to all subjects.

Test scores were analyzed using a two-way analysis of variance and a descriptive comparison of group means.

4. Findings

The main effect pertaining to presence or absence of a detached answer column was significant at the .05 level for the seventh grade and at the .001 level for the eighth grade. The main effect of having or not having workspace on the test paper was only significant ($F < .05$) for the seventh-grade subjects. Interaction of these two effects was not found to be significant at either grade level.

5. Interpretations

Majors and Michael feel that varying the test format as to having/not having a detached answer sheet and the presence/absence of workspace on the test paper may very well affect the validity of the scores of students taking examinations which emphasize computational skills. From a descriptive interpretation of subgroup mean scores, they further suggest that having no record answers on a separate answer sheet may result in lower level of test performance and that providing workspace on the test may improve students' computational skills.

Critical Commentary

At first glance it appears that Majors and Michael have opened a Pandora's Box. If their findings and conclusions are valid, does it not open to question the findings and conclusions being drawn from large-scale standardized testing programs? With what confidence can the findings be accepted? Majors and Michael cast the first doubt. They describe their study as being a 2x2 quasi-experimental design. Why? Which, if any, of the assumptions underlying the two-way analysis of variance was not met? No indication is given in either the findings (ANOVA table) or the authors' discussion. Even so, the study should not be ignored. A replication is suggested using a standardized test with a separate answer sheet contrasted with the same test items written in teacher-made form with workspace and answers on the test.

F. Richard Kidder
Longwood College

AN ANALYSIS OF THE FRACTION CONCEPT INTO A HIERARCHY OF SELECTED SUB-CONCEPTS AND THE TESTING OF THE HIERARCHICAL DEPENDENCIES. Novallis, Carol F. Journal for Research in Mathematics Education, v7 n3, pp131-144, May 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas E. Kieren, University of Alberta.

1. Purpose

The (clearly) stated purpose of this research was the following question: Given a particular proposed hierarchy of subconcepts of the fraction concept, is this a sequence supportable by empirical data?

2. Rationale

Two basic theoretical frameworks were used in this study. The first dealt with the construct of the concept of fraction and was based upon the work of Bruner. The fraction concept was defined as the "inferred common characteristics of the class of all models and situations that can describe a fraction" (p. 137). Specifically, these models always contain a reference to a unit, and the parts counted and compared are equivalent in a manner related to that unit and involving the ratio notion.

This concept of fraction is applied in two ways to the study. The behaviors which constitute each subconcept conform to the constraints of this definition. Further, the test for each subconcept forced a student to relate models and fractions in four ways which are implied by the definition.

The second framework used is based upon Gagné's writings. This framework pictures a concept broken down via task analysis into prerequisite subconcepts. A major theorem of this framework, used in the study, is that if a student knows a concept, he will know the prerequisite concepts. This theorem gave rise to the major research hypothesis:

> If a student has attained criterion on the sub-test associated with a supreordinate subconcept, then he has also attained criteria on the subtest associated with each immediate subordinate subconcept (p. 138).

The possible acquisition patterns are illustrated in Figure 1.

Super-ordinate	Sub-ordinate	
	Acquisition	Non-acquisition
Acquisition	(1,1)	(0,1)
Non-acquisition	(1,0)	(0,0)

Figure 1

3. Research Design and Procedure

To test the above research hypothesis first demanded the construction of a hierarchy. Each subconceptual element used one of three fraction models: a geometric region [part-whole (PW)], a set [part-group (PG)] or a unit segment of a number line (NL). The hierarchy was based upon prior research, mathematical analysis, and text-use patterns, and consisted of 16 sub-elements with 23 testable dependencies.

The investigator constructed a fraction concepts test with a subtest (normally of four items) for each of the 16 subconcepts. This test and subtests were subjected to reliability and validity analysis. A level of 75 percent was set as the criterion level for each subconcept.

The test was administered to 279 students from one school--77 from grade 4, 79 from grade 5, and 123 from grade 6. The test was administered in two parts on two consecutive days by the investigator.

To test the hypothesis above, two ratios from the work of Gagné and Walbesser were used:

$$\text{Ratio 1} = \frac{n(1,0)}{n(1,0) + n(0,1)}$$

$$\text{Ratio 2} = \frac{n(1,1) + n(1,0)}{n(1,1) + n(1,0) + n(0,1)}$$

It was argued that only situation (0,1) contradicted the hypothesis. Hence it was decided if Ratio 1 exceeded .75 [n(1,0) was at least 3 times greater than n(0,1)], then Ratio 2 would be computed and deemed supportive if it exceeded .9.

4. Findings

Eighteen of 23 dependencies were supported. In fact, Ratio 2 was 1 in 10 cases. In four cases, n(0,1) = 0. These were:

- PG, congruent parts → NL
- PW, congruent parts → NL
- PG, congruent parts → PG, visual equivalence
- PW, congruent parts → PW, visual equivalence.

Although partly supported, the development of PG and PW models were non-parallel. For example, *PG, comparison → PG, comparison with "as many as" terminology* was not supported, but the parallel dependency *PW, comparison → PW, comparison with "as large as" terminology* was. Also, although *PG, congruent parts → PG non-congruent parts*, and *PW, congruent parts → PW, non-congruent parts* were both supported, mean scores would indicate that *PW, non-congruent parts* is more difficult than *PG, non-congruent parts*.

5. Interpretations

Four main conclusions were stated:

(1) associating fractions with PG or PW models is prerequisite to associating them with points on a number line;

(2) associating fractions with PG or PW models is prerequisite to using them in comparison situations with these models;

(3) associating fractions with PG or PW models is prerequisite to associating fractions with the respective model involving an equivalent fraction with parts arranged to show this;

(4) associating fractions with PG or PW models is prerequisite to associating fractions with the respective model having non-congruent parts, where the parts are equal in area (PW).

The investigator also makes a small number of speculative conclusions based upon observation of student work. An important one is that students are not exposed to a sufficient variety of fractional situations to encourage generalizations, nor do they come in contact with sufficient negative instances of the concept.

Critical Commentary

1. This is a very carefully done study. One is especially impressed with the care taken in describing the subconcept elements in the hierarchy and the corresponding test items. Other researchers should find these aspects of the work useful.

2. Along with the investigator, the abstractor wonders if results would be different if other hierarchies of the same concepts were tried or if instruction was involved. Indeed one wonders about the value of this kind of conceptual analysis.

3. The analysis did not involve algebraic aspects of the rational number concept, particularly operations. While this was not the obvious intent of the study, it should be noted that the ratio notion of equivalence does not seem to be well covered.

4. It would be interesting to add other models to the study: for example, other measurement models or operator models (exchanges, "stretcher-shrinker").

5. The results of the study are clouded by the performance of students. On 14 of the dependencies, over half the group fell in the (0,0) non-achievement class and on 7 dependencies, over 250 of the 279 students fell in this class. Supportive data on these dependencies seem suspect.

6. The observational work of the investigator was excellent. It seems a shame that more of this was not reported as it seems to be the potentially most significant aspect of the study.

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COMPREHENSION OF LANGUAGE CONNECTIVES AND PROPOSITIONAL LOGICAL RELATIONSHIPS. Paris, Scott G. Journal of Experimental Child Psychology, v16, pp278-291, October 1973.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Mary Ann Byrne, University of Georgia.

1. Purpose

The study was designed to determine developmental changes in comprehension of propositional relationships implied by language connectives and to identify "strategies underlying comprehension of the abstract relationships." A verbal assessment was made of eight linguistic connectives in all truth-functional forms.

2. Rationale

"Research on children's understanding of language connectives is important for analysis of developmental changes in comprehension of abstract relationships." Bourne and O'Banion (1971) found that propositional rules could be used by six-year-olds to classify successfully stimuli in a card-sorting task. But propositional rules appear to be more difficult to comprehend when expressed verbally than nonverbally. Thus a verbal assessment task structurally similar to nonverbal assessment tasks is important to allow comparison with other research results.

Research has shown that comprehension of connectives varies depending on age and type of connective. The study provides a uniform assessment across age and connectives.

3. Research Design and Procedures

Subjects were 40 students (20 male, 20 female) at each of the following levels: grade 2, 5, 8, 11, and college. The pre-college students were from predominantly white, middle-class schools in the same geographical area. The college students were in an introductory psychology course. The subjects were randomly chosen at each level from three classes.

The task was a group-administered series of slides, each accompanied by the reading of a sentence. The subject was to decide if the description was true or false for the slide. Control items were used to assure that the data represented how a subject comprehended a connective.

Each of the 32 test items consisted of a sentence having one of eight linguistic connectives and a slide that would represent one of four truth forms: TT, TF, FT, or FF. The following connectives were used: and, but, both-and, neither-nor (conjunction); or, either-or (disjunction); if-then (conditional); and if and only if-then (biconditional).

The data were reported in tables giving the percent of error on each connective according to truth form and grade level. The data were analyzed in a five-factor analysis of variance (Connective x Truth Form x Grade Level x Sex x Ss).

4. Findings

Conjunction: Error patterns for the four connectives were similar across grade levels and truth forms with the exception of second graders and the connective "neither-nor." In this case, the error rates for the TT and FF were noticeably higher than for any other conjunctive case.

Disjunction: Both disjunctive connectives were scored as inclusive disjunction. The tendency to treat disjunction as exclusive was more evident in older Ss. The difficulty of mixed truth forms decreased with age. An F test indicated that performance on "either-or" sentences was superior to that on "or" sentences.

Conditional: The error rates for the FT and FF forms were noticeably higher than the TT and TF forms.

Biconditional: The error rate for the FF form was noticeably higher than the other forms. The conditional FT case had a noticeably higher error rate than the biconditional FT case.

The analysis of variance showed a significant main effect for Connectives, Truth Forms and Grade Level at the .001 level. The conjunctive connective was easiest to understand followed by biconditional, disjunctive, and conditional in that order. The grade levels were ordered as expected. There were significant interactions at the .001 level between Grades and Connectives, Grades and Truth Forms, Connectives and Truth Forms, and among Grades, Connectives, and Truth Forms.

5. Interpretations

a) The high error rate of second graders on the "neither-nor" connective suggests that these Ss responded to the features of the elements in a proposition without regard for the verbal connective.

b) The disjunctive connective "either-or" seemed to be an aid in differentiating the components which facilitated correct processing of TF and FT items and promoted exclusive processing of disjunctions.

c) The errors on the FT and FF forms of the conditional suggest that "Ss consistently employed erroneous, nonlogical processing strategies."

d) The results for the FT form of the conditional and biconditional suggest that Ss did not differentiate these two logical relationships.

e) A strategy that responds "True" to TT instances and "False" to any other instance is called a conjunctive set since it is appropriate for the conjunctive connectives with the exception of "neither-nor." Second graders seemed to use this strategy for all connectives, indicating little regard for the logical relationships implied by the verbal connectives.

f) There are two developmental patterns suggested by the data. One concerns "the differentiation of conjunctive from disjunctive propositions with increasing age." The other concerns the tendency, with increasing age, to set up a cause-and-effect relationship between the components in a conditional or biconditional proposition.

Critical Commentary

The researcher provided a clear and detailed statement of purpose and rationale. He concluded with a discussion of his insights, based on the data, as they pertained to the questions he raised. These sections were probably more developed than is often the case in reports of research studies.

The tables with error rates are valuable. Many data are also described which provide a closer look at a trend noticeable in the tables of error rates.

The description of the study did not make clear how the slide was designed to represent both components of the sentence.

The least developed aspect of this study was the report of statistical procedures. The statistical analyses were not described prior to references to the results of specific tests in the discussion of results.

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A DIFFERENT LOOK AT WORD PROBLEMS. Rogers, Margaret Anne. Mathematics Teacher, v68, pp285-288, April 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Nicholas A. Branca, San Diego State University.

1. Purpose

The purpose of this paper was to answer the following question: What impression would one get regarding the activities of contemporary American men and women solely on the basis of the word problems appearing in current second-year algebra texts?

2. Rationale

No contextual framework or model is given for the study nor are there any stated assumptions. No previous research is cited.

3. Procedure

The verbal problems from each of eight contemporary second-year algebra textbooks were studied and catalogued according to the sex and activities of the characters involved.

4. Findings

Problems concerning men's activities outnumbered those concerning women's activities by at least 5 to 1 in 7 of the 8 textbooks examined. It was observed that 282 problems involve men exclusively, 42 deal solely with women and 82 include both. Of the 282 problems dealing with men, 85 deal with a large variety of occupations, 47 deal with monetary transactions, 33 center on sports, and a number deal with various modes of transportation. In contrast, only 7 careers are mentioned for women, no monetary transactions are recorded, 2 women appear alone in problems dealing with sports, and no women travels on her own initiative. In the area of age problems, it was found that men average 27.5 years while women average 21.5. Men are also found to outdo women in being more alert and in being more scientific. Only in problems dealing with social occasions do women hold their own. It was found that although women are not even given credit for casting shadows, there is one activity in which they excel--sitting. In the 10 problems concerning this activity, 44 women were engaged as compared to 33 men.

5. Interpretations

The major conclusion of the author is that 7 of the 8 texts investigated create the impression that women and their activities are comparatively quite dull and insignificant. She states also that the same impression would result if texts from other mathematics courses or even other disciplines were considered.

Critical Commentary

Although the author does not answer the question posed at the beginning of her article in general, she does present a description of the problems appearing in contemporary second-year algebra texts and gives her impressions regarding the activities of contemporary American men and women from them. The data are presented in paragraph form and it is therefore difficult to make comparisons for many of the categories described. The question addressed is a significant one. However, the author does little to indicate what the implications of the findings are or what effects they are having or may have. The article is important in that it does make one aware of the situation as it presently exists. By backing up her findings with related research and by describing the conceptual framework in which she is working, the author could have made a much stronger case for the implications that are merely alluded to.

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THE EFFECT OF MATHEMATICS INSTRUCTION USING MANIPULATIVE MODELS ON ATTITUDE AND ACHIEVEMENT OF PROSPECTIVE TEACHERS. Warkentin, Gary. Journal for Research in Mathematics Education, v6 n2, pp88-94, March 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Randall I. Charles and Frank K. Lester, Jr., Indiana University.

1. Purpose

This study investigated the effect of mathematics instruction using a laboratory/models approach on attitudes and achievement of prospective elementary teachers.

2. Rationale

The professional opinion of mathematics educators and the results of various research studies indicate a need to improve the mathematical preparation and attitudes toward mathematics of prospective elementary school teachers. In addition, there is evidence that many elementary teachers fail to see much relationship between the mathematics courses they take in college and the mathematics they will teach. Efforts to remedy these conditions have included integrating the mathematics content and methods courses, and teaching the mathematics courses using a laboratory approach or some other approach having an activity orientation.

From a learning theory perspective, Bruner suggests that even for adults the learning of mathematical ideas is often enhanced by including concrete and iconic experiences.

3. Research Design and Procedure

The sample consisted of students enrolled in a theory of arithmetic course. Six sections (N = 149) were taught using a laboratory/manipulative models approach (experimental treatment). Nine sections (N = 197) were taught using a conventional lecture/textbook approach (control treatment). The mathematical competence of the students was measured prior to instruction using the arithmetic section of the Stanford Achievement Test (SAT). Their attitudes toward mathematics were assessed at the beginning of the course using a 20-item attitude questionnaire. All sections met for the same amount of time for an entire semester. The same subject matter was considered by both treatments. Each of the instructors of the experimental sections used the same manipulative models and written materials. The students in these sections worked in small groups in a laboratory setting. The same textbook was used by the instructors of all control sections. A comprehensive examination was developed to cover the content of the textbook used by the control group and was administered to both groups at the end of the semester. The attitude instrument used as a premeasure was also used as a postmeasure.

Statistical analyses were performed to determine if there were any differences between the experimental and control groups on: (a) the SAT,

(b) the attitude premeasure, (c) the comprehensive final exam, and (d) the attitude postmeasure. In addition, the extent of attitude change during the semester was analyzed for both groups. Finally, the correlation between Ss' performance on the SAT and their final grades was determined in an attempt to identify a reasonable predictor of success in the course. In each statistical test the student was used as the statistical unit of analysis.

4. Findings

The two treatment groups differed significantly ($p < .01$) on attitudes toward mathematics and achievement in the course. The experimental group was favored on the attitude measure and the control group was favored on the achievement measure. The experimental group showed a significant gain ($p < .01$) in attitude while the control group did not. A significant correlation was found for both groups between the SAT scores and the final grades for the course.

5. Interpretations

The author considers the significant gain in attitude toward mathematics of the experimental group to be important since "...the attitudes that were changed had probably been formulated over a considerable period of time." He attributed the significant difference in achievement in favor of the control group to the fact that the control sections were able to cover more content than the experimental sections. Since the comprehensive test was based on the content of the textbook, there were some items on the final examination that were covered by the control sections but not covered by the experimental sections.

Although there was a significant correlation between the SAT scores and final grades, scores on the SAT accounted for only 40% of the variance in final grades. This suggested that the level of mathematical competency of the Ss enrolling in the course was quite varied.

It was recommended that this study be replicated in a way which would insure that the content taught would be the same for the two groups. Future studies might also compare the two groups with respect to content retention and success in teaching mathematics. It would also be appropriate to investigate the optimum amount of time preservice teachers should be allowed to use manipulative models in learning mathematics.

Critical Commentary

Research which attempts to identify more effective ways to teach mathematics to elementary teachers is tremendously important. For this reason the author is to be commended for choosing such a significant issue for investigation. However, there are several serious weaknesses which severely limit the value of the research reported in this study.

1. The procedures used to identify the two treatment groups are not discussed. It is apparent that students were not randomly assigned to

treatments but rather that sections were randomly assigned to treatments. Consequently, it appears that the unit of statistical analysis should have been class sections instead of students. If this had been done, it is doubtful that any of the group differences would be statistically significant at even the .05 level.

2. The author claims an interest in identifying a predictor of student success in mathematics yet he provides no rationale for his choice of the Stanford Achievement Test as the potential predictor.

3. An adequate description of the treatments is not provided. The method of instruction used in the laboratory/models approach is somewhat vague and the textbook used by the lecture/discussion sections is not identified.

4. Evidently, all classes were not taught by the same instructor. The reviewers wonder if any attempt was made to standardize instruction within treatments and otherwise control the teacher variable. Also, the number of different instructors is not mentioned. Could it be that there were 15 different instructors?

5. There are several questions regarding instrumentation which the report does not answer. How were the attitude and achievement instruments selected? Was the attitude questionnaire developed by the author? If not, what criteria were used to select an attitude measure? How valid and reliable were the instruments? Why was no achievement test developed which covered the content of both treatments instead of only the lecture/discussion treatment? Why wasn't the comparison of treatments performed using only test items related to the content which was common to both treatments? These questions and the fact that there is no description given of the attitude questionnaire or the comprehensive examination make it difficult to draw any reasonable conclusions from the results obtained.

6. More content was covered in the lecture/discussion treatment than in the laboratory/models treatment. Why weren't parallel instructional materials developed?

7. The author offers no explanation for the significant attitude change among Ss in the experimental treatment. Was the change due to the presence and use of manipulative models or was it due to the informality fostered by small group interaction among the Ss? Of course it may have been either or both of these factors or there may have been some entirely different contributing set of factors. It may be that Ss in the experimental treatment simply became aware of how the instructors wanted them to respond to the attitude questionnaire.

8. The significant differences between the groups in course achievement is practically meaningless since there were some items on the final examination that were covered by the control sections that were not covered by the experimental sections. This source of invalidity could have been eliminated by tossing out those items on the examination which dealt with content covered only in the control sections.

The weaknesses cited and questions raised in this commentary make it impossible to draw valid conclusions concerning the effectiveness of a

laboratory/models approach to teaching mathematics to prospective elementary teachers. The reader is cautioned against using the results of this study to make decisions regarding feasible alternatives to the conventional lecture/discussion method of instruction.

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THE EFFECTS OF DIFFERING PRESENTATIONS OF MATHEMATICAL WORD PROBLEMS UPON THE ACHIEVEMENT OF PRESERVICE ELEMENTARY TEACHERS. Webb, Leland F.; Sherrill, James M. School Science and Mathematics, v74, pp559-565, November 1974.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James E. Schultz, The Ohio State University.

1. Purpose

The purpose of the study was to determine how achievement of preservice elementary-school teachers in solving printed mathematical word problems is affected by

1. the presence of accurate pictorial representations,
2. the presence of inaccurate pictorial representations, and
3. the absence of pictorial representations.

2. Rationale

Literature regarding the inclusion of pictures with verbal mathematical problems was cited as being sparse and inconclusive. The few writers mentioned who addressed this issue were found to be in conflict as to whether pictures should be accurately drawn.

A 1970 study by Sherrill (1973) indicated a possible negative effect on achievement of tenth-grade Ss when distorted pictures were included with a mathematical word problem. This follow-up research probed a possible difficulty in the 1970 study; namely, that the Ss had not been informed that the pictures presented in the test possibly were inaccurate.

3. Research Design and Procedure

Ten mathematical word problems were constructed by modifying the ten most appropriate items from the twenty-item test used by Sherrill. Three versions of a multiple-choice test were constructed by including with the same tem items either (1) accurate pictures, (2) inaccurate pictures, or (3) no pictures. One of the distractors for each question was the answer that was implied if the subject assumed an inaccurate picture was correct.

Eighty preservice elementary-school teachers were randomly classified into three groups. The three different versions were administered one each to the three groups with cell sizes of 32, 26, and 22 respectively. Procedural questions were answered first, and the Ss were explicitly told to read all of the directions before beginning the test, which included a statement that pictures (if any) "may or may not be accurately drawn." The Ss were allowed 50 minutes to complete the test, which was judged to be sufficient time for all individuals in the sample.

4. Findings

The three group means were compared using analysis of variance. It yielded a significant F-ratio ($p = .0000$). Kramer's Modification of Duncan's New Multiple Range Test for unequal cell sizes was applied to rank the means of the samples. The results indicated that both hypotheses were supported:

1. The group receiving accurately drawn pictures performed significantly better than either of the other groups.

2. The group receiving no pictures performed significantly better than the group receiving inaccurately drawn pictures.

5. Interpretations

The following conclusions were drawn within the context of the study:

1. A subject's ability to solve mathematical word problems is affected by the method of presentation.

2. The presentation of accurate pictures aids student achievement with mathematical word problems.

3. The presentation of inaccurate pictures hinders the solution of mathematical word problems.

It was also observed that the data seemed to support the 1970 findings of Sherrill even with the added caveat regarding distorted pictures.

The investigators recommended the following modifications be considered in future studies:

1. A mixture of problems of all three types could be presented together on each test. Subscales could be formed to determine the effects including possible interactions.

2. A classroom presentation could be developed to test the effects of lecture or discussion.

3. Intervening variables could be selected to determine if there are interactions between the treatments and such factors as age, sex, visual ability, and IQ.

4. The experiment could be replicated except that the directions could be verbally administered.

Critical Commentary

The article did little to remedy what the authors cited as a lack of evidence to refute or sustain the use of accurate pictures or drawings when presenting mathematical word problems. If this was to be a follow-up of an earlier study, why was the sample changed from tenth-grade students

in the 1970 study to preservice elementary-school teachers who were enrolled in mathematics courses? The changing of the age level for the experiment decreases the probability for an additive effect on our knowledge of learning.

Was it appropriate to modify a 20-item test for tenth graders to make it a 10-item test for preservice teachers? The investigators argued that a .75 reliability was "reasonable" considering the length of the test. Should the test have been longer instead? Would it have been better to form randomly groups of equal cell sizes?

Several flaws appear in the article, enough to make some readers question, perhaps, the credibility of the entire experiment. Following are two examples of such flaws.

The authors cite a single source in support of making deliberate errors in pictorial representations. The source given was page 9 of a 1966 article by Trimble. Yet the Trimble article appears on pages 6 through 8 of the journal and the quotation given does not appear either in the Trimble article or on page 9. (One wonders why the writers ignored Polya's discussion of accurately drawn pictures vs. inaccurately drawn pictues in his book on problem solving, How to Solve It.)

Another flaw in the article appeared in the table which was given to rank the means of the three groups. The table was incorrect in that means of the groups were ranked 1, 3, 2 yet the contrast reported for $X_1 - X_2$ was smaller than either $X_1 - X_3$ or $X_3 - X_2$. A closer check reveals that the results were associated with the wrong contrasts.

Contrast	Reported Value	Correct Value
$X_1 - X_3$	18.566	12.969
$X_1 - X_2$	4.523	18.566
$X_3 - X_2$	12.969	4.523

(The first two columns appeared in the article. The third is supplied here by the abstractor.) It is also the case that the other two columns of the same table (not shown here) listed the group numbers and cell sizes, so that entries in the same row of the table were unrelated; namely the given table was in fact two separate tables.

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Sherrill, James M. The Effects of Different Presentations of Mathematical Word Problems upon the Achievement of Tenth Grade Students. School Science and Mathematics, v73, pp277-283, 1973.

A COMPARISON OF TWO METHODS OF COLUMN ADDITION. Wheatley, Grayson H. Journal for Research in Mathematics Education, v7 n3, pp145-155, May 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Karen Fuson, Northwestern University.

1. Purpose

This study compared the effects of training in two methods of column addition. The methods were direct (adding straight down a column of figures) and tens (adding numbers which make ten). Aptitude-treatment interactions, the effects both of initial and of pre- and post-training preference of method, and the effects of using the fingers when adding were also examined.

2. Rationale

The author reviewed the recommendation of several methods books for elementary teachers concerning how to teach elementary students to add a column of figures. He concluded that there is a wide disagreement about the preferred method. He reports two previous studies in this area. Lankford (1972) concluded that good computers are more likely than poor computers to add digits in order. Chesin and Quast (1970) reported that 58% of the students enrolled in an elementary education course on methods added down a column, 14% added up, and 28% added by tens.

3. Research Design and Procedure

The subjects were all of the fourth-grade pupils (4 classes totaling 92 pupils) in a suburban area Wisconsin school.

The design was a 2x3 counterbalanced design. There was one within-subjects factor (treatment: direct and tens) and one between-subjects factor (ability: three levels determined by the teachers for mathematics instruction). Two classes received the direct training followed by the tens training, and two classes received the tens training followed by the direct training.

The training was done by the homeroom teachers in the heterogeneous homeroom classes. The teachers followed written protocols in presenting the lessons. Training on each method consisted of fifteen-minute periods on four consecutive days. Each training session entailed five minutes of explanation by the teacher and ten minutes of practice time on prepared worksheets. A pretest was given during one week. The first training period was on Monday through Thursday of the following week. Posttest 1 was given on that Friday. The second period of training was on the following Monday through Thursday, and Posttest 2 was given on Friday.

The pretest consisted of 50 single-digit column addition problems with the number of addends varying from two to seven. The numbers in the problems were generated from a table of random numbers. Posttest 1 was a form parallel to the pretest in numbers of addends and containing numbers also

generated from a random number table. Posttest 2 was identical to the pretest. Five minutes were allowed for each test (less than 3% of the subjects finished any test).

Following each posttest the pupils were asked, "Did you use the method you practiced this week in working the problems on this test?" If the student answered "No", his scores were dropped from the analysis on that posttest. Pupils were also asked to indicate whether they used their fingers in adding.

For each posttest, three 2 (Method x 3 (Ability) analyses of variance were performed on the number correct, number attempted, and percentage correct. A 2 (Method) x 2 (Fingers) analysis of variance was performed on the same three measures from each posttest. A 2 (Method) x 2 (Sex) analysis of variance was also done for the three measures from each posttest.

4. Findings

Students using the direct method were able to answer correctly 15% more problems on posttest 1 and 18% more problems on posttest 2 than students using the tens method. Both differences were significant at the .01 level. Similar results were obtained for the number of problems attempted. There were no differences in accuracy between the two methods; the means for both methods on both posttests were around 90% correct.

The differences in the three ability levels were significant for the number correct, the number attempted, and the percentage correct. These differences ranged from 3% to 30% with most differences moderately large (4 to 6 problems out of 40).

Students perferred the tens method. Using the tens method was reported by 52.5% on the pretest, and 59% reported using it on posttest 2. This shift came from students trained in each method. After being trained on the direct method, 20% of the students still used the tens method on the posttest following direct training. Only 1% of those trained on the tens method failed to use it on the following posttest. The number of subjects changing from direct to the tens method was nonsignificant, however (Chi square = 1.25).

Forty-six percent of the sample answered "Yes" when asked if they had used their fingers when adding on each posttest. There was 88% agreement of those using their fingers on the two posttests. A 2 (Method) x 2 (Fingers) analysis of variance for each posttest revealed that pupils who used their fingers had significantly fewer problems correct and significantly fewer problems attempted than students who did not use their fingers. The size of the difference was not reported. There was no difference in the accuracy (percentage correct) for those who used or did not use their fingers. There was no interaction with the method and finger use. Seventy-six percent of the low-ability subjects, 60% of the average-ability group, and 14% of the high-ability group used fingers.

Analysis of the data revealed no significant sex differences.

5. Interpretations

The author interprets the findings as suggesting some advantages for the direct method. It is 15% to 18% faster and just as accurate for students at all ability levels.

Though the number of students changing to the tens method was not significant, there was a steady increase after each week regardless of the method taught. Twenty percent of those trained on the direct method used the tens method immediately following the training, while virtually all of those trained on the tens method used it immediately afterwards. These data indicate the strong appeal of the tens method to students.

The subjects in this study placed considerably above the national norm in mathematics on the Iowa Test of Basic Skills. Thus it is somewhat surprising that 46% of them reported using their fingers on the posttests. The percentage of students "counting on" with their fingers may be higher than is commonly supposed.

By the end of the study each student had worked approximately 350 problems. But accuracy (percentage correct) began and ended at about 90%. Evidently performance on column addition exists at some base level and is not improved much by practice.

Critical Commentary

This study suffers from several design and analysis problems. First, the design involved four different teachers, two doing each order of treatment. Although the teachers worked from the same written protocols, no attempt was made to see that they did in fact teach the method suggested and no control was done of the teaching ability of an individual teacher. Thus the teacher was an uncontrolled variable. No data are reported by teacher, so no judgment can be made about the confounding resulting from this variable. Some attempt should have been made to show that pooling the students from different teachers did not matter. The teaching task was a simple one, so this may not have been as critical as it seems, but this problem whould have at least been acknowledged.

Perhaps the most serious oversight is that the pretest data were never analyzed to determine whether the two treatment groups were different in their initial ability to add. The table of means for "percent correct of those attempted" indicates that these groups were different. All three ability levels of the direct-tens groups had means above 90%, while the low-ability group in the tens-direct group had a mean of 77%.

The author's argument concerning why separate analyses of variance were done on each posttest is unconvincing. Separate analyses assume that the measures and the treatments were unrelated. This is in fact not true. One would expect the results on posttest 1 to be related to the results on posttest 2 and would want to examine this relationship. This design would have been more properly analyzed and interpreted if a repeated measures analysis of variance had been done using the pretest, posttest 1, and posttest 2. The author's failure to use this method of analysis is particularly strange because he was so careful to make parallel, and thus easily interpretable, forms of the test.

In fact, the order of presentation of the methods did produce different results. When the direct method was first, the mean for the percentage correct dropped for each ability level. When the direct method was second, it raised the mean for the percentage correct. How this interacted with the initial accuracy of the two groups is unclear, for the direct-first group is the one with the high pretest means.

Ability level was a key variable in this study, but the author reports only that this was defined by "using the groupings determined by the teachers for mathematics instruction." Some indication of how they made this distinction would make the data more interpretable.

A final problem not addressed by the analysis of the study is that using the tens method gives practice in the direct method, but using the direct method gives no practice in the tens problem. On almost any tens problem, only some of the numbers will add up to ten. The other numbers must be added directly. In addition, ten of the fifty problems were two-addend problems. These would have been done in the same direct way no matter which method one used on other problems. Thus, the two types of training and of testing did not measure equal amounts of each method.

The author was quite careful about certain things: the generation of the tests and the examination of possible bias in dropping data from subjects who did not use the trained method. This makes even more incomprehensible his very serious oversight in not checking the equivalence of the groups on the pretest, his erroneous choice of separate analyses of the posttests, and his total lack of control of the teacher variable.

In summary, one must be quite careful about accepting the results of this study. Even with its limitations, however, it does seem to indicate that four short practice sessions on the direct method of column addition will increase the speed, though not the accuracy, of fourth graders generally above the national mean. The same students seem to prefer the tens method, although it does not increase their speed.

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