| AUTHOR | Hoffman，Joseph R．；Tardif，Pobert F． |
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## ABSTRACT

In a project designed to improve elementary school instruction in mathematics，the California Department of Education collected achievement and profile data from 67 elementary schools． Schools were classified according to size，socioeconomic status， minority representation and mobility of students，city size，and type of community．Profile data were related to instructional practice， teacher characteristics，leadership activities，and community involvement．Data were analyzed to identify profile elements which distinguished between high and low achievement schools in the same classification．These data affirmed 60 hypotheses whose implications are discussed in this document．These hypotheses led to plans for workshops in leadership training and for implementation workshops in the schools．The purposes，characteristics，and planning of these workshops are discussed．Appendices provide copies of the survey instrument，information on making school profiles，instruments for evaluating workshops，and transparency masters for use in workshops． （SD）

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# A Plan for Improving Mathematics Instruction In California Elementary Schools 

## Final Report



# A Plan for Improving <br> Mathematics Instruction In California Elementary Schools 

## Final Report oftuc watementis Etucation Taxa foue

Prepared by
JOSEPH R. HOFFMANN
ROBERT F. TARDIF
Consultants in Mathematics Education

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[^1]
## Foreword

Athough the subject of this pablication is the improvement of elementary schoot mathematios. the underlying theme of the document is the achievernert of excellence in education. Those in the Mathematies Education Task Fore were aked to exdentify the features of surcessfal mathematics programs in California. And they found out, as others have also discovered. that more than reform and imovation and $I^{10 n}$ ey athe needed to improve the quality of education. As vital as each of these may be to improved instration, it takes people. It takes a commitment to excellence on the part of scho of administrators. ceachers. parents, students, and the total community. Their willinghess to work for excellence determines whether or not we make lasting improvements $i_{1}$ the quatity of education.

Excellence starts with all of us - with the standards we set for the Performance of our duties and the standards we establish for student achievement.

We must take the lead in maintaining high qualifying standards for the job market and for higher education. We must lead the fight to establish relevant standards: then we must see that every student, minority or majority, rich or poor, goes to the labor market or the university qualified, not simply qualifiable.

Excellence depends upon our acceptance of responsibility for resut for student outcomes.

Excellence depends upon our willingness to put the needs of the child before the needs of the profession or the institution.

If we want to restore excellence as the hallmark of our society, we must restore exeellence as the hallmark of education.

Excellence, in the final measure, depends upon our willingness to be judged by the quality and effectiveness of our work.

If we are to have excellence, we should keep in mind these five actions which fead to improvement in the quality of education:

1. A conscious decision must be made to change - to improve the duality of edacation: but it must not be change simply for change's sake.
2. Responsibility for action must be identified.
3. Priorities must be established.
4. Those procedural and attitudinal changes that make a difference must be found and grouped within an appropriate framework of goals, with provision fiade for monitoring and evaluating the results of our actions.
5. There must be a commitment to excellence on the part of everyone who is involved in education-the taxpayers who provide the resources, the legislative bo dies in matlocate those resources, the policy-making bodies. the administrators, the ${ }^{2} \mathrm{deh}$ ers, the parents, and the students. Everyone of us must care about excellence.
The members of our Mathematics Education Task Force have developed a unique tool that will help you who are involved in mathematics programs bring abo ut in proved quality in your classrooms. I encourage you to take advantage of this plan for improving elementary mathematics programs.

In about 42 B.C… Publilius Syrus sat. "th takes a long time to bring excellence to maturity." But neither we nor our chidere will understamd or appreciate the benclits of that maturity if we never take the time to search fore exellence if we never take the time to give a deeper, higher meaning to our lives.


Suprointendent of Pbhlic Instrucion

## Preface

In 1973 the California Legistatur. responded to public concern about declining seores in mathematics achievement. It instructed the Department of Education to conduct a study identifyiag the salient characteristies of suceessful mathematics programs in California elementary sehools and to disseminate the results to all elementary schools in the state. The cmpirically derived results of the study were to be incorporated into elementary school mathematics programs so that pupil achicvement in mathematios would be improved.

The Department referred the legislative mandate to the Department's Mathematics Education Task Force. The Task Force attempted to identify the characteristics of successful mathematics programs by observing what certain high-acheving elementary schools do that certain low-achieving schoots fail to do and vice versa. Then the Task Force examined the characteristics by type of school, using basic demographic variables that might interact with the characteristics. Finally, it developed an implementation plan that would allow a close examination of the results of the study and provide opportunities for implementation at the school and classroom level.

The members of the Mathematics Education Task Force conducting the study were Robert Tardif (Manager), Joseph Hoffmann, Delmer Lansing, and Fred Lorenzen. Valuable assistance was provided to the Task Force by Leonard Marascuilo, University of California, Berkeley, and several mem' of the Office of Program Evaluation and Research, State Department of Education.

Special commendation is due to the hundreds of teachers and principals in the 67 sample schools in which the Task Force conducted its study. (These schools are listed on page vi.) Without their honest responses, this study would be worthless. Their honesty gives witness to the strength and vigor of the education profession and gives promise that the objectives of this study will soon te accomplished. This publication, A Plan for Improving Mathematics Instruction in California Etementary Schosls, should be a valuable aid in attaining those objectives.

# Elementary Schools Sampled in Mathematies Study 

Ada S. Nelson Elementary School, Whittier
Avalon Gardens Elementary School, Los Angeles Barnard Elementary School, San Diego
Bennell Valley Elementary School, Santa Rosa Broad Avenue Elementary School, Wilmington Buena Vista Elementary School, Walnut Creek
Cajon Park Elementary School, Santec Carlton Hills Elementary School. Santec
Carrisa Plains Flementary School, Santa Margarita
Castle Heights Avenue Elementary School, Los, Angeles
Centralia Elementary School, Anaheim
Coz'swell Elementary School, El Monte
Crescent Elk Elementary School. Crescent City
David Reynolds Elementary School. Fremont
Del Mar Elementary School, El Cerrito
Diamond Heights Elementary School, San Francisco
Fair Oaks Elementary School, Sunnyvale
Fieldbrook Elementary School. Arcata
Florence E. Flanner Elementary School, La Puente
Franiklin Avenue Elementary School. Los Angeles
Franklin Elementary School, Anaheim
Grand Oaks Elementary School, Citrus Heights
Hacienda Elementary School. La Habra
Jackson Elementary School, San Diego
Kathryn Hughes Elementary School. Santa Clara
King Elementary School, Long Beach
Laguna Road Elementary School. Fullerton
Langdon Avenue Elementary School. Sepulveda
La Tijera Elementary School, Inglewood Lichen Elementary School, Citrus Heights Linda Verde Elementary School. Lancaster Lockhurst Drive Elementary School. Woodland Hills Loma Vista Elementary School. Whittier
Los Alamos Elementary School. Los Alamos

Lyndale Elementary School. San Jose
Magnolia Elementary School. Redding
Manchester Elementary School. Fresno
Marion Elementary School, Novato
McCandless Elementary School. Redondo Beach
Mendocino Elementary School, Mendocino
Miller Elementary School, Escondido
Murwood Elementary School, Walnul Creek
Oso Avenue Elementary School. Woodland Hills
Overland Avenue Elementary School, Los Angeles
Palo Verde Elementary Schooi, Palo Alto
Paradise Elementary School, Modesto
Rhoda Street Elementary Seloool, Encino
Richmond Strect Elementary School. El Segundo
Riley Elementary School, San Diego
Rio Vista Elementary School, North Hollywood
Round Valley Elementary School, Bishop
San Pasqual Elementary School, Escondido
Santa Lucia Elementary School, Cambria
Santa Monica Boulevard Elementary School, Los Angeles
Shepherd Elementary School, Hayward
Skycrest Elementary School, Citrus Heights
Spreckels Elementary School. Spreckels
Twin Hills Elementary School, Sebastopol
Vallemar Elementary School. Pacifica
Victorine Klein Elementary School. Mountain View
Vista Mar Elementary School, Daly City
Warner Elementary School, Warner Springs
Washington Elementary School. San Cabriel
Waugh Elementary Scliool. Petaluma
West Hollywood Elementary School. West Hollywood
Westmont Elementary School. Ponoma
Westwood Elemertary School. Santa (lara

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## Chapter 1

## Introduclion to the Study

In the past decade substantial criticism has been directed against mathematics education. Many persons have written and telephoned their complaints about "modern math" programs to members of the $r$ 'ifornia State Department of Education and the
cornic Legislature as well as to editors of newspapers and periodicals. In response to the public concern about the quality of mathematics programs, the California Legislature in 1973 authorized increased funding for the mathematics unit of the State Department of Education and directed the Department to identify those features of school mathematics programs which account for pupil success.

The Department of Education assigned this task to a newly created Mathematics Education Task Force, a Department unit consisting of four consultants. The task force was instructed to design a study that would reveal the information requestel, conduct the study, and report the findings. It wis further stipulated that the report of the study should be suitable for immediate use by individual teachers in developing programs for the improvement of mathematics instruction.

## Rationale for the Study

Some research studies conducted prior to the study described in this report were designed to idertify mathematics programs that had demonstrated pupil success. As a result several attempts were made to transplant "successful" programs into other settings. Unfortunately, the transplanted programs were found to be unsuitable when certain questions were asked:

Do the teachers play an important role in selecting the new program?
Does the program meet the special needs of pupils and teachers in the school?
Do teachers believe the program will succeed?
Do teachers have a personal (or emotional) investment in making the program work?

It was the position of the Mathematics Education Task Force that inattention to such questions was sufficient to jeopardize even the best efforts in program development in mathematics. Therefore, the study was conducted to equip teachers with program development information that teachers could readily manipulate and incorporate into a mathematics program. With the control of program development in the hands of teachers, the responsibility for creating the best possible programs would be theirs also.

## Method of Research

The orientation of this publication and the study reported in it is to provide information useful in the development of mathematies programs that are likely to produce pupil suceess. The definition of an instructionl program as used in the research project is as follows:

An instructional program is the entire set of events that lead up to and inclide the actual learning experience of pupils it the classicom. Thus, an instructional program includes the following: the styles and strategies of the teachers and the school administration; the instructional leadership that influences classroom instruction and management; the concern and planning related to pupil-teacher interactions; the nature of parent/community support for the school program; the quality and use of instructional materials, support materials, and media; and the chmate in the classroom.
The research was conducted in 67 public elementary schools in California during the spring of 1974 by the members of the Mathematics Education Task Force. The sample schools were broadly representative in several demographic variables such as school size, ethnic characteristics, and the mean socioeconomic status of the school population. Half of the target schools had exhibited a pattern of high achievement in mathematics, and the other half had exhibited a pattern of low achievement.
 school site ohserving the oneonge proseram. Buter vicwing teachers and primeipals, and secorin! and arranging for the acpaisition of data on the contire sohool mathematics program. In es ence the ageregatce of data for all schools fabout 500 pieces of data on couch schooh) was utihzed to test an extensive array of hypotheses about the importance of certain program-related variables. Statistical treatment of the data allowed the identitication of of hypotheses most related to pepil
 this report.

Athough the hypotheses maty be refored to as recommendations for progran development. Ilas
 those variahles which secoll hest to aceome for successfa! achievoment pattorns in (alifomia che mentary schools. Nevertheless, in the bands of leathers amd progetm developers. the hypotheses represent a sedontific base for kespmeng or revisung stacessfal schon mathemation prostams.

3

## Chapter <br> 2 Researdh Activilies

The Department of Education stady was designed to identify in cementary mathematios programs those salient program characteristies that decount for consistently low test performance. In a previous stady conducted by the bepartment. grade levels at whieh defieioncies in mathematies achievement tirst appeared were identilied, as were the specific eontent areas that acounted for the deficiencies. In that study the Comprehensine Tests of Basic Skills (CTBS): Arithmetic and the Sequential Tests of liducational Progress ISTI:P II): Computation Subtest and Basic Concepts Subtest were administered to 3000 pupils in classes selected by a stratified random sampling scheme from grade levels four through eight. The findmes of the study indicated that deficiencies in achevement appeared as carly as grade four but were more intensificel in grades five through cight than in grade four.

## Research Strategy

The mathematies section of the Comprehensire Tests of Basic Skills was the measurement instrument selected to gauge the pupils' mathematical skills and concepts, as prescribed for instruction in the Mathematics Framework for California Public Schools. ${ }^{2}$ The results of the CTBS tests administered to all sixth graders in each of the previous four years wete assumed to be adequate indicators of achievement strength and weakness. Using three year-to-year difterences in CTBS mean raw scores for schools over a four-year period, the researchers found that all the schools in the study fell into one of the categories which were established to suggest the quality of mathematics programs:

[^2]1. Three increases in mean ratw scores
$\therefore$ Tivo increases and a net increase overall (from the first year to the fourth)
$\therefore$ (Once increase and a net increase overall
2. livo increases and a net decrease overall
3. ()ne increase and a net decrease overall
4. No increase

The target sehools in the study were sampled from two groups of schools:

1. Hish test-performance scheols: (a) the upper 15 percent of all schools for 19091972 (categories I through 6): and (b) the upper 50 percent of all sehools in 1972 that achieved a net increase in raw seores greater than six points (categories 1 through 3 )
2. Low test-performance schools: (a) schools with one increase and a net decrease greater than 14 points (category 5); and (b) schools with no increase and a net decrease greater than 10 points (category 6 )
The use of trends in CTBS scores for identifying target schools is justifiable because a trend over four years could indicate either a concerted effort for improvement or a consistent lack of program success. The use of the results of a single testing of groups of pupils would have provided a less reliable indication of program success.

The study did not attempt to evaluate all of the goals in mathematics that should be achieved by California pupils. For example, it would have been difficult to ase simple test-score trends to assess the ability of pupils to apply mathematical skills in real-life situations, long-term retention, desire to learn independently, or similar goals. However, $C T B S$ scores did provide a reliable indication of the mathematical skill under investigation in this study; namely, computational skill. The definition of high-performance mathematics prograns used in the study must therefore by interpreted in this restricted perspective because 48 of the $98 C T B S$
test items were purely computational. In that sense the $C T B S$ mathematics test measured computational ability well.

## Selection of Sample Schools

A total of 72 schools representing a wide range of demographic characteristics were selected for the researeh sample. The basic demographic characteristies used for eategorizing the sample schools were (a) socioeconomic status; (b) minority representation; and (c) school size (enrollment).

Schools that were determined to be alike for all three of these characteristics were compared. They were judged to be similar for a characteristic if they were in the same third of the distribution of that characteristic value for the sample schools. After the schools had been selected, they were classified further on the basis of three more demographic variables to provide a more complete description of rach school. These three variables were (a) eity size (population); (b) type of community (enviroiment); and (e) pupil mobility.

The first three variables were used in the original sample selection in a way that minimized interdependence among the variables. Specifically, the sehcols were divided into three levels for each of three variables so that 27 categories of schools resulted. For example, one category contained schools in communities that ranked low in socioeconomic status, minority representation, and school size (enrollment). Another eategory contained schools in communities that ranked medium in sociocconomic status and low in minority representation and enrollment. Finally, a third category was composed of schools in communities that ranked high in sociocconomic status and low in minority representation and enrollment. Within each group, two of the schools were randomly selected for the high-performance sample and two for the low-performance sample. It is this pairing of similar schools that is the basis of this study.

The 72 sample schools were selected from the 27 categories, and the last three variables were imposed on these schools after the selection. No guarantee of the independence of these variable groups was present. To have categorized the schools on all six variables prior to selection would have produced $3^{6}$, or 729 categorics, many of which would have contained only one school. Five schools were dropped from the study because of time limitations and scheduling difficulties. A total of 67 schools were examined as a research sample.

## Characteristics of Sample Schools

Each group of schools was represented by a profile based on the six variables, and the majority of California elementary schools could compare with at least one profile. Each profile provided the basis for a separate analysis because the important characteristics that might discriminate between high and low schools in one profile might be insignificant in another. The following are detailed deseriptions of the demographic factors used to describe and categorize the pool of sample schools.

## Socioeconomic Status

An estimate of the socioeconomic status of each sample school community was undertaken by the one individual who was probably the best qualified for the task.- the teacher. The estimates were made when all California public school pupils in grades two and three were being tested in reading as part of the California Assessment Program. ${ }^{3}$ Each pupil's answer sheet was marked by that pupil's teacher for background information, ncluding sociocconomic status. The teacher chose one of the following as indicating the family backgound of tine pupil:

1. Executives, professionals, and managers
2. Semiprofessionals, clerical and sales workers, and technicians
3. Skilled and semiskilled employees
4. Unskilled employees (and velfare recipients)
5. Unknown

The five choices were condensed to three levels for statistical analysis:

Level one-choices 4 and 5
Level two-choice 3
Level three--choiees I and ?
A single statistic for the sociocconomic variable was obtained for each school by computing the mean for the three levels in the item. Therange of the variable had a minimum of 1.00 and a maximum of 3.00 . For the purpose of dividing the total sample into three levels with approximately the same number of schools in each level, the limits of the variable were set as shown in Table 1 (see also Figure 1 ).

[^3]TABLI: 1
Distribution of School Enrollments by Socioeconomic Status

| (iroup | Limits | Sample sehools |  | Responding elementary sel:ouls |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} \text { Nur. } \\ b . \end{gathered}$ | $\begin{aligned} & \text { Per- } \\ & \text { cent } \end{aligned}$ | Number | Percent |
| Low | $1.00 \quad 2.08$ | 23 | 34.3 | 2258 | 46.9 |
| Med:um | $2.09 \cdots 2.40$ | 22 | 32.8 | 861 | 17.9 |
| High | 2.41-3.00 | 22 | 32.8 | 1696 | 35.2 |
| Total. |  | 67 | 99.9 | 4815 | 100.0 |



Mean sociocconomic status (interval: 0.1 )
Fig. 1. Graphic distribution of school enrollments, by socioeconomic status

## Minority Representation

Minority representation was determined for each school from the ethnic school survey which was administered in all California public schools in 1972, one year prior to the mathematies study reported in this publication. In the survey each principal reported the number of the school's pupils from each of the following ethnic groups: Asian, black, Chicano, Native American, and white and other.

By the use of the data provided by school principals, the total percent of nonwhite pupils was ascertained for the schools. The schools were then divided into three groups according to their minority representation as shown in Table 2 (see also Figure 2).

## School Size

The school size (enrollment) for each school was taken from the elementary school questionnaire as reported by school principals to the State

Department of Education in the fall of 1972, one year prior to the study described in this report. An on-site examination of enrollment at the sample schools revealed some discrepancies between the enrollments reported and the enrellments observed. However, the discrepancies were in no case large enough to change the classification of a sample school. The distribution of schools by school size is presented in Table 3 (see also Figure 3 ).

## City Size

City size (population) for each school in the research project was determined at the time of the

TABLI: 2
Distribution of School Enrollments, by Minority (Nonwhite) Composition

| (iroup) | L.mits (percent) | Simple selosols |  | Respending elementary schools |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Numher | Per- <br> cent | Number | Percont |
| L.0w | 11. | 22 | 32.8 | 760 | 25.2 |
| Mcdum | $10 \quad 28$ | 24 | 35.8 | 1145 | 37.9 |
| Hegh | 29100 | 21 | 31.3 | 1110 | 36.9 |
| Total. |  | 67 | 99.9 | 3021 | 100.0 |



Fig. 2. Graphic distribution of school enrollments, by minority (nonwhite) composition

Table: 3
Distribution of Schools, by Size of School

| (iroup | iimits | Sample schools |  | Responding: elementary schools |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Number | Per© Cll | Number | Per. cent |
| Low | 31349 | 23 | 34.3 | 2099 | 29.2 |
| Medium | 350560 | 22 | 32.8 | $21+9$ | 29.9 |
| High | 5611298 | 22 | 32.8 | 2942 | 40.9 |
| rotal. |  | 67 | 99.9 | 7190 | 100.0 |



Fig. 3. Graphic distribution of schools, by size of school
visit to the school. The information was recorded during the interview with the principal and was later modified as necessary after all the data had been examined. The city size categories were small city (or nencity), medium city, and large city. For geographically isolated cities and towns, the category distiactions were based on simple population statistics. When two or more cities were clustered geographically, the observer had to make a subjective choice on the basis of apparent commercial and political spheres of influence. The propulation of the surrounding area was used to determine city size if any one of the following three conditions was applicable:

1. The sehool district boundaries included the surrounding area.
2. Some connecting streets were nonrural: i.c., commercially or residentially developed.
3. The city administrative bodies were interdependent.
Examples of the city size categorics of some of the schools in the study are as follows:

Small city (or noncity) less than 50000
Crescent City
Arcata-Eureka (area)
Santer-El Cajon (arca)
Escondido-San Marcos (area)
Santa Rosa
Pacifica
Mecditum city 50000 to 200000
Citrus Heights-Carmichael-Orangevale (area)
San Gabriel-Alhambra-Monterey Park (area) Fresno
Large city over 200000
Inglewood-Lennox-Havthorne (area)
Anaheim-Fullerton-Buena Park (area)
San Jose
San Diego
San Francisco
Los Angeles
The distribution of sample schools by city size is shown in Table 4.

TiABLI: 4
Distribution of Sample Schools, by Size of City

| (iroup | Limits | Sample sehools |  |
| :---: | :---: | :---: | :---: |
|  |  | Number | Percent |
| Lew | Less than 50) 0 00 | 27 | 40.3 |
| Medium | 50000200000 | 14 | 20.9 |
| High | Over 2000000 | 20 | 29.9 |
| Total |  | $61{ }^{1}$ | 91.1 |

at the 67 sthools in the sample, six were not categorized by city siace because of insufticient information.

## Type of Community

The type of community surrounding each school in the study was recorded by the observer during the interview with the school principal. Two of the following three descriptions could be chosen by the interviewer:

Type I (low) -rural, farming
Type $2(m e d i u m)$-suburban. residential
Type 3 (high) inner city, commercial, industrial
So that the size of each of the three groups in the sample could be made as similar as possible, the definition of Type 2 schools, the largest group, was narrowed. If the school's geographic community contained any rural or farming property, regardless of the extent of residential or industrial development. the community was cateerori\%ed as Type I. If there were no local farming property, then it was classified as Type 2 or Type 3. depending on the
nature of the commercial development in the community. Type 3 was chosen when the following features characterized the comanity:

1. Old apartment, commercial, or industrial buildings (more than 40 years old)
2. Low-income or substandard dwellings
3. Predominantly commercial-zone property

The distribution of sample schools by type of community is shown in Table 5.

TABLE 5
Distribution of Sample Schools, by Type of Community

| Group | Limits | Sample sehools |  |
| :---: | :---: | :---: | :---: |
|  |  | Number | Percent |
| Low | Rural/iarming | 18 | 26.9 |
| Medium | Suburban/resideniel | 29 | 43.3 |
| High | Inner city/ commercial/ industrial | 15 | 22.4 |
| Total |  | $62^{3}$ | 92.6 |

${ }^{3}$ Of the 67 schools in the sample. five were not eategorized by type of community because of insufficient information.

Pupilinion
A pup. mobility index for each school was computed from data collected at the time of the visit by the observer. The principal (or, in most cases, the secretary) of each school gave the number of pupils added to the enrollment between October 1, 1973, and May 1, 1974, and the number deleted during the same period. The mobility index was then derived by dividing the sum of these two numbers by the actual enrollment on October 1. All mobility indices were
divided into three groups of approximately the same size as those shown in Table 6.

TABIE 6
Distribution of Sample Schools by Mobility Index

| Group | Limits | Sample sehools |  |
| :---: | :---: | :---: | :---: |
|  |  | Number | Percent |
| Low | (1)-22 | 17 | 25.4 |
| Medium | 23-37 | 17 | 25.4 |
| High | Over 37 | 16 | 23.9 |
| Total |  | $50^{\text {i }}$ | 74.7 |

af the 67 schools in the sample, 17 were not categorized by mobility index because of insufficient information.

## Research Instruments

Severol instruments were devised to assess the characteristics of the mathematics programs in the sample schools. The instruments included an anonymous teacher-principal opinionnaire, a teacher interview guide, a principal interview guide. and two classrooni observation inventories. The priscipal's interview included factual and policy items pertaining to the curriculum, the school, the staff, the district, and the community. The teacher's interview involved items pertaining to attitude, training, pedagogical tactics, instructional support materials, testing, and classroom climate. (See Appendix A.)

The teacher observation procedure involved a meeting with the teacher or team leader before a scheduled class period to ascertain general information about the lesson to be taught and then a series of five-minute observations of the class. A fiveminute interval occurred between observation sessions. A meeting after the class with the teacher concluded the series of interviews and observations.

## Chapter <br> 3

## Verification of Hypotheses

No absolute rules exist for successful teach-ing-only hypotheses about what works best. Educators must form hundreds of hypotheses daily in planning lessons and teaching: these hypotheses involve all aspects of the instructional program. In the research study analyzed in this report, pedagogical hypotheses were devised and categorized as follows:

1. General classroom teaching styles
2. Specific teaching techniques
3. Instructional materials
4. Teacher/pupil interactions
5. Instructional leadership
6. Parent/community involvement

Most teachers' hypotheses are never articulated explicitly; nevertheless, educators hope that their
assumption that an hypothesis is verified and generalized if it is used more frequently in schocls with high pupil achievement than in schools with low pupil achievement. ${ }^{4}$ It was further supposed that the hypotheses would be verified for some types of schools and not for others. Each hypothesis accordingly has been identified for the categories of schools for which it has been verified. Each hypothesis has also been identified to indicate whether it has been verified for all schools in general. The analyses of all the hypotheses led to verification of 60 hypotheses for three or more school demographic characteristics.

The absence of a black square in one of the cells does not imply that the characteristic was unimportant in a particular type of school. Rather, there was apparently so little difference in the use


This black square indicates that the program feature described by this hypothesis was a significantly more frequent element of programs in high-achieving scheols than in low-achieving schools regardless of the background characteristics of the school.

This black square indicates that in sehools in which pupils from minority groups constitute between 10 and 28 percent of the school population, high-achieving schools exhibited the program feature deseribed by the hypothesis signifieantly more than did low-achieving sehools.
hypothetically based actions will result in pupil achievement. The implicit form of each hypothesis used in the study is that pupils tend to exhibit higher achievement patterns if action $X$ occurs.

Hundreds of hypotheses were considered by the researchers and were exposed to verifications in the sampling of schools. The study was based on the
of this characteristic by high- and low-achieving schools that it had no effect on achievement.

[^4]
## Ise of the School Profile Key

A printed card is provided with this report so that the results can be easily analyzed and hypotheses applied to each school. You will need a paper punch to make the key operational.

Each school will have a unique "profile," and one key should serve the entire staff in a school. However, additional keys can be produced from 3 $\times 5$-inch cards.

A. Punch holes in the appropriate boxes for your school according to the definitions of the demographic variables. (In borderline cases punch two holes.)
B. Place the key over the corresponding part of any item identified in the study.
C. Note how many holes in the key are superimposed over a black square. A black square indicates that the item is a significant discriminator between high and low test performance schools for schools similar to yours. The more black squares revealed, the more important that item is to the improvement of the mathematics program in your school.

General Classroom Teaching Styles

1. Discovery and inquiry are emphasized in teaching: limited use is made of "traditional" methods.

2. Learning activities make provisions for participation by all pupils; i.e., activities which appeal to a variety of interests and abilities.

3. The improvement of pupil attitudes is a major cmphasis in classroom teaching.

4. The faculty recognizes, discusses, and attempts to remedy disruptive pupil behavior.


## General Classroom Teaching Sty ${ }^{\text {mos }}$ (Continued)

7. Teachers perceive mathematies as a useful and important learning activity for pupils.

8. The teaching focuses on the content of mathematics.

9. The learning of computational skills and concepts is valued highly and is emphasized as an important goal.

10. The teaching activities allow pupils to refine their understanding of concepts, and a convergent mode of thinking is used.

11. The classroom activities are managed in a way that promotes orderly and quiet pupil behavior.

12. A classroom esprit de corps is promoted so that pupils take pride in their behavior and achievement.

13. Instruction is activity-centered, and techniques such as games, learning centers, and active learning situations are used.

14. Learning assignments are differentiated for the particular needs of individual pupils.

15. The teacher diagnoses. understands, and acts upon each pupil's learning difficulties.

16. Pupils are involved in plaming their own activities.

17. Teachers use audiovisual materials or visual demonstrations in a formal classroom atmosphere.

18. The teacher gives clear verbal instructions in a wellcontrolled classroom atmosphere.

19. The instruction is based on intensive, responsive verbal presentations in which pupils are told specifically what they are expected to know.

20. The chalkboard is used frequently.

21. The overhead projector is used frequently.

22. The teacher and aides (where available) supervise and tutor the pupils as they study.


Instructional Materials
25. Teachers are given special training in effective techniques for presenting instructional materials.

26. Assignments and directions are clear.

27. Additional instructional materials are acquired and used in the classroom.

28. A broad range of audiovisual instructional materials are acquired and utilized in the classroom.

29. Teachers are encouraged to reduce dependence on textbook presentations.

30. Manipulative aids are used in the learning of skills and concepts at all levels of ability.


Instructional Materials (Continued)
31. Mathematics is integrated with other subject areas.

32. Pupils are allowed to accelerate individually or together to the next higher grade level of instructional materials.
33. Frequent enrichment instruction is provided tor capable pupils.

34. Mathematically gifted pupils explore abstract concepts. logic, and mathematical applications as part of their classroom learning.

35. A special mathematics program is offered to gifted pupils.


Teacher/Pupil Interactions


## Teacher/Pupil Interactions (Continued)

37. Teachers have a positive feeling toward pupils and a feeling of being wellitied by them in return.

38. Teachers avoid abusive criticism of pupils and refrain from scolding or threatening pupils or arguing with them.
39. Teachers consistently charify pupil misconceptions. correct pupil errors, and resolve difficulties promptly.

40. Teachers allow pupils to discover their own errors and be self-directed.

41. Teachers are sympathetic and tolerant and allow pupils to express their own opinions.

42. Teachers praise and encourage pupils by providing support. optimism, or an appreciation of pupil responses.

Teacher/Pupil Interactions (Continued)
43. Teachers accep and use the ideas of their pupils.

44. The instructionai program is focused on pupils and their needs.

45. The instruction is restricted to presentations, demonstrations, and selection of topies by the teacher alone.

46. The teacher controls the chassroom seene with interesting teacher-directed activities.

47. Teachers hold informal. relaxed conferences with their pupils to allow free exchange of opinions and discussion of needs.


Instructional Leadership
48. The school's mathematics program is made a matter of high priority.


Instructional Leadership (Continued)
49. The school's mathematics program is updated to reflect new trends in mathematics education.

50. Schoolwide instructional objectives are developed for all subject areas.

51. Instructional objectives are developed for the mathematics programs.

52. A consultant assists in the development and maintenance of a coherent mathematics program.

53. Teachers feel confident in their ability to teach mathematics.


56. The school principal supports teachers who find mathematics textbooks inadequate.


## Parent/Community Involvemant

57. The mathematics program supports parental views of what should be taught.

58. The school staff is involved in a community relations campaign to develop parental interest in the school's mathematics program.

59. Teach rs confer with the parents of each pupil at least three times annually regarding the pupil's learning progress and problems.

60. The mathematics program is appropriate for the cultural character of the community and realistically reflects the community's needs, goals, and interests.


## Chapter 4

## Implications of the Hypoiheses

Each of the 60 hypotheses was used in the study in the following context: Pupils tend to exhibit higher achievement patterns if . . . (hypothesis). For types of schools in which a particular hypothesis was not found to be an important contributor to pupil success, the reader should not assume that pupils tend to exhibit lower achievement patterns if ... (hypothesis). Therefore, this study provides no basis for discontinuing an activity which has been designed to enhance the mathematics program in a school. Rather, the study provides a basis for establishing priorities for future program planning and program analysis.

The study indicated, for example, that except for schools in a wealthy or suburban community or in a large city, the use of stimulating learning activities (hypothesis 18) is not a characieristic which distinguishes high-achievement schools from low-achievement schools. Nevertheless, the use of stimulating learning activities should not be discontinued; the study does not assert that the use of stimulating learning activities produces low achievement.

The same interpretation applies to hypotheses for which orly one or two black squares are shown for a given type of school. On the other hand, when three or four black squares are shown for a given type of school, that hypothesis deserves careful consideration for implementation. The hypothesis is especially important if the black squares shown for a given school type are the only ones given for that hypothesis. When five or six black squares are shown for a given school type, that hypothesis is an essential ingredient for successful mathematics program planning, and its implementation should receive the highest priority.

The following discussions of the implications of the hypotheses should provide the reader with useful ideas and directions for developing implementation plans. When a set of hypotheres has been identified for implementation in a tiven school, all of the implications should be considered
and a general implementation pattern sought. It must be remembered that the hypotheses are designed as the basis for future program development and as such serve as focuses for the planning activities of teachers, administrators, and members of the community. Those who create the program will believe in its ultimate success.

Discovery and inquiry are emphasized in 1 teaching: limited use is made of "traditional" methods.
Encouragement of pupil discovery and inquiry in mathematics represents one of the most universally accepted aspects of the mathematics revolution of the 1960s. Most educators agree that pupils should think about what they are learning; that pupils should relate what they are learning to what they have already learned; and that mathematical concepts deserve exploration by learners of all abilities. The research study led to some revealing observations about discovery and inquiry activities. To begin with, such activities must be carefully planned and executed; they tend to lose their effectiveness if overused as a motivational device. Furthermore, effective discovery activity need not involve the entire class at one time, and discovery should not always be used to introduce every topic. In most cases effective discovery and inquiry learning activities are created by inspiration. These activities-and their inspiration-should be shared liberally by teachers.

## ) The teaching reflects "traditional" con2 cerns; limited use is made of discovery and inquiry techniques.

A "traditional" mathematics program may mean many things to many people; but for most teachers it probably means using a program which preceded "new math." One of the characteristics of that earlier curriculum was deliberate attention to the development of computational skills. In the "new
math" this attention was often diverted to developing an understanding of "modern" topics in early grades. Another characteristic of "new math" was the unfortunate requirement that teachers present textbook topics with which they were not familiar. The implications of the study are that more attention should be given to computational skills and that unfamiliar topics should not be taught until the teacher understands them thoroughly.

## 3 Learning activities make prorisions for par3 ticipation by all pupils; i.e., activities which appeal to a variety of interests and abilities.

Teachers must recognize the importance of participation by all pupils. Learning activities are simpler to plan if the activities involve only one ability level or one interest area; but clearly such an approach is not adequate for meeting all of the pupils' needs. If selection and purchase of instructional materials are based on the needs of all types of pupils, an improved instructional approach can be undertaken. Then, for every lesson, methods for involving all pupils in the activities must be developed.

## 4 Mathematics lessons are prepared thoroughly.

Planning and preparing mathematics lessons are not accomplished to the same degree in all schools. Sometimes, in those schools of low mathematical skill performance, extenuating circumstances may have prohibited adequate lesson preparation, but educators must realize that thorough lesson preparation will lead to improved performance in all types of schools. Lesson development may require eliminating the prohibiting circumstances of the school and developing creative plans to be shared among teachers, possibly including team teaching opportunities.

## 5 The improvement of pupil attitudes is a major emphasis in classroom teaching.

During the interviews the researchers asked teachers which instructional emphasis was considered more important: how pupils felt toward learning (attitudinal) or what pupils learned (cognitive). Teachers' responses were classified according to whether emphasis was all attitudinal, all cognitive, or somewhere in between. Although an emphasis on attitude to the exclusion of cognitive learning is not to be recommended, schools in the identified categories should consider increasing the emphasis on attitude. Most teachers indicated that
they were concerned about both areas of learning and that both were essential.

## 6 The faculty recognizes, discusses, and attempts to remedy disruptive pupil behavior.

In some locations teachers and principals ranked pupil attitude toward school as a major impediment to the success of their mat ematics programs, even though the pupils may have attained high achievement in mathematics. The result indicated that in those schools the faculty had recognized the problem, openly discussed it, and prepared a plan for resolving it. Most schools had the problem of poor pupil attitude, and more teachers needed to be aware of the problem and concerned about it. The research study affirmed that taking steps toward resolving the problem can inprove mathematics instruction.

## 7

 Teachers perceive mathematics as a useful and important learning activity for pupils.An understanding of the purposefulness of mathematics learning is a characteristic of highperforming pupils and teachers. Ti, , hypothesis was corroborated by the questionnaire responses from teachers and principals, who apparently felt that their programs were designed to serve useful purposes and that pupil time was not being wasted. The hypothesis might, however, merely reflect teacher attitudes toward mathematics in general. In either case improvement in a sehool's mathematics program could be expected if purposeless and time-wasting exercises were eliminated.

## 8 The teaching focuses on the content of mathematics.

The research study indicated that some schools could expect improvement in their mathematics program if they would provide more structure, including well-written objectives, mastery-level learning criteria, pupil achievement profiles, and minimum learning requirements. Pupils in a structured program would have progress charts or a similar record of achievement. In general, emphasis in a structured program would be placed on learning the subject matter rather than on pursuing the interests of individual pupils.

The learning of computational skills and concepts is valued highly and is emphasized as an important goal.
Computational skills may be mastered by the use of manipulatives, games, and other interesting
activities and need not involve heavy drill exercises. Some schools may make effective use of pupil goal setting by emphasizing computational skills as an important goal or by associating rewards with achievement of these skills.

The teaching activities allow pupils to refine their understanding of concepts, and a convergent mode of thinking is used.
A convergent mode of thinking is used in teaching when pupils may be easily confused by ambiguity or distraction. For example, the convergent mode of thinking is illustrated by the use of a single model for the addition concept; i.e., $x+$ $y$ means taking $x$ things and $y$ other things all together. Ambiguities are avoided in the convergent mode of thinking, such as when the number line model or "block" arithmetic is used. Distractions are avoided also, such as in determining whether the model works when some of the $x$ things are the same as some of the $y$ things. In general, in the convergent mode pupils look fo: a single correct response; discovery and inquiry techniques arc not emphasized.

## 11 <br> The classroom activities are managed in a way that promotes orderly and quiet pupil behavior.

Orde. 'iness and quiet are characteristic of the type of classroom management in which discussion is carefully controlled by the teacher, pupil assignments are completed quietly and intently, and pupils understand and accept the ground rules for acceptable behavior.

12A classroom esprit de corps is promoted so that pupils take pride in their behavior and achievement.
A formal atmosphere was associated with high performance in several types of schools, a result that must be interpreted as to how pupils respond when visitors are present. The formalism could have been based on a desire to perform well and please the visitor, or it could have demonstrated an appreciation for the visitor taking time to watch the class. In either case the behavior indicated an esprit de corps reflecting pride in the class accomplishments and comprehension by the pupils of standards of classroom behavior. The results of the research study should not be interpreted as encouraging continuous formal classroom standards. Instruction is activity-centered, and techniques such as games, learning centers, and active learning situations are used.
The teacher may be in a dominant role in activity-centered instruction, but the pupils are not passive. The pupils are challenged as individuals. They participate actively and learn as a result of what they are doing. The results of the study indicated that activity-centered instruction is effective when lecturing, recitation, drill, and discussion are not effective. Activity-centered instruction is different from the independent learning project, which does not involve the teacher to the extent that activity-centered instruction does.

## 14. Learning assignments are differentiated for the particular needs of individual pupils.

Differentiated pupil assignments can be used in any classroom to provide an opportunity for some individualized instruction. Assignments can be planned for two or more groups of pupils in a class so that each group's assignment provides the type of learning activity beneficial to the members of that group. The assignments may differ according to the interests of the pupils, the mathematics achieverment levels, the class projects, or the rewards given for performance. Differentiated assignments should allow some pupils to explore mathematical concepts in depth and others to spend more time improving retention of basic skills. So that equality of achievement can be maintained, different amounts of study may be required.

15 The teacher diagnoses, understands, and acts upon each pupil's learning difficulties.
For schools with a medium or large minority representation, it is particularly important that teachers know their pupils' learning difficulties. Ethnic barriers can be overcome by a conscious program of diagnosing learning difficulties. The mode of diagnosing can be either formal (making use of special testing programs) or castal (teacherperceived). In either case the teacher takes time to prescribe learning activities that will assist the pupil to overcome the difficulties.

## $16 \begin{aligned} & \text { Pupils are involved in planning their own } \\ & \text { activities. }\end{aligned}$

Teachers and principals in many types of schools find that involving pupils in the selection of learning activities tends to produce higher perfor-
mance. Arrangements vary greatly, Tivo whes are differentiated assigmments and individu rning contracts. The common characteristic of these plans is the provision of a choice of several alternative learning activities from which pupils (individually or collectively) may select what appears to them most beneficial. That some pupils will try to take advantage of this instructional method is understood. But the many advantages of the method merit its serious consideration. First of all, the opportunity to select what one does increases the interest in the selected activity. Furthermore, one learns to be responsible for making decisions. Teachers find the creativity needed in plaming for alternatives to be exhilarating and rewarding. Sharing by teachers becomes a necessity and opens new avenues of teaching. Finally, selection of learning activities can be used as a reward for achievement and can be initiated on a small scale in any classroom.

17Pupils hare opportunities to engage in independent learaing projects.
The use of independent kearning projects as a teaching technique is difficult for the teacher but is often successful in improving mathematies learning for the pupils. The project approach has been proposed by leading educators, particularly by those in the Nuffield Project in England ${ }^{5}$ The technique often succeeds because pupils engage in appealing and meaningful activities that require a thorough comprehension of mathematical concepts.

## 18 <br> Stimulating learming activities are planned for pupils: e.g. . game-like actirities. fascimating materials, and lively discoussions.

In some classrooms the observers noted very lively, stimulating learning activities. Teachers were playing games with the pupils, presenting fascinating materials, or encouraging discussion. In contrast, in some classrooms the presentations appeared to be dull and uninteresting, most of the pupils seemed bored, and the teacher did not seem interested. If teachers are enthusiastic, are not too dependent on textbooks, and are flexible toward novel learning activities, they will make the classroom an exciting place for leazning.

[^5]Teachers use audioristal materials or visual demonstrations in a formal classroom atmosphere.
Teachers were observed using audiovisual materials or visual demonstrations more often in highperformance schools than in low-performance schools. These learning activities sometimes involved pupils actively and sometimes passively. The study revealed, however, that the use of audiovisual materials and demonstrations depended for its effectiveness on a formal classoom atmosphere in contrast to a casual atmosphere. Constraints must be placed on the way in which demonstrations and audiovisual materials may be used to improve performance. That is, pupils must understand that, athough the material presented is attractive they are expected to consider the material as matter for study.

20The teacher gives clear verbal instructions in a well-controlled classroom atmosphere.
Classroom observations of teachers disclosed frequent use of the lecturing technique-logical, clear. and matter-of-fact expository teaching. This style of teaching was effective in classrooms in which teachers tended to dominate the learning enviromatent.

## The instruction is based on intensive, 21 responsire verbal presentations in which pupils are told specifically what the . are expected to know.

In chassroom observations of teachers, a technique was observed that differed from lecturing in several ways. The technique was characterized by intensity and succinctness and amounted to the issuance of athoritative statements of fact. What was to be learned and remembered was made clear. Teachers using this technique emphasized verbalization in their teaching and in evaluating learning.

## 22 The chalkboard is used frequently.

According to the researeh study, teachers and principals considered the chalkboard as an effective teaching tool. The value of having pupils work at chalkboards is based partially on providing an immediate check of answers and on the erosspollination of ideas that occur as pupils watch each other work. Teachers found that exampies and directions written on the chalkboard helped pupils organize and clarify their understanding.

## 23

The use of overhed projectors wals found to be valuable for the same reasons chalkboards were found valuable. Other valuable features were the convenience of facing the class: the saving of material for later review: and the "magie-like" projection from table to sereen which tended to fascinate learners.

24The teacher and aides (where arailable) supervise and tutor the pupils as the? studs.
A great deal of class time was used for seatwork. Pupils completed assignments independently or received individual tutoring. As the teacher moved about the chassoom, a personal contact was made with each pupil who needed assistance. If the teacher could not keep up with the demands, aides and other pupils were directed to assist in the tutoring.

## 25 <br> Teachers are given special training in effective techniques for presenting instructional materials.

The extent of inservice training in mathematics was a factor which was observed to differentiate between high-performance and low-performance schools regardless of whether the training was in content or in methods. Nearly all teachers indicated that inservice training had been beneficial and should be designed to meet specific teaching needs. Some of the topics suggested by chementary teachers were the following:

| Motivation | Development of |
| :--- | :--- |
| Metrics | materials |
| Laboratory | Problem solving |
| Slow learners | Calculators in the |
| Different learning | classoom |
| styles | Applications |
| Mathematics games | Mathematics for |
| Enrichment materials | consumers |

## 26

Assignments and directions are ckear.
Clear explanations and directions for mathematies assignments were especially critical in some types of schools. New teachers often have difficulty in explaining and giving directions. The ability to explain and give directions clearly should, therefore, be a prerequisite for teachers hired to teach in these schools.

Additional instructional materials are acquired and used in the classroom.
Teachers in some sehools attributed their pupils' high performance to acquisition of additional instructional materiats. One may assume that the materials were carefully selected to enhance the total mathematics program. The actual amount of expenditure was unavailable, but it should be noted that the schools involved were not wealthy. Improvement in a sehool's mathematies program can, therefore, often be expected to accompany an increase in budget priority.

## 28 <br> A broad range of andion'isual instructional matcrials are ucquired and witized in the classroom.

The high rating, by teachers and principals, of the use of audiovisual instructional media in mathematies programs was observed in several parts of the research study. This supportive evidence makes consideration of such media essential for certain types of schools. Audiovisual media included a broad range of materials and a broad range of modes of presentation. from passive to active pupil participation.

## 29 <br> Teachers are encouraged to reduce depende:ceo on textbook presentations.

The responses by teachers and principals demonstrated that much support existed for the use of nontextbook, nonworkbook learning activities and teacher-designed follow-up activities after completion of textbook assignments. Independence from textbook learning calls for a creative approach to teaching.

## 30 Hanipulative aids are used in the learning of skills and concepts at all levels of ability.

The study revealed that the use of manipulative aids in mathematics instruction was receiving new, widespread interest. Their use was noted particularly in schools in which the staff was actively pursuing a program of improvement. Furthermore, their application appeared at all levels of pupil ability: they were not limited to remediation. The manipulative aids used were often inexpensive teacher-made materials created from objects common in the pupil's environment.

## 3 Mathematic: is integrated with other subject areas.

Integration of mathematics instruction into other stibject areas has been given increased atten-
tion recently. One way this integration may be accomplished is through individual pupil projects. which may be undertake'n sequentially as courses evolve and as mathematical concepts arise in the various course subjects. Numerous ideas for pupil projects are suggested in the teacher guides for some mathematics texthooks.

## 32 <br> Pupils are allowed to aicelerate individually or together to the next higher grade level of instructional materials.

Accelerated programs may refer to opportunities for pupils to complete the materials assigned to their grade level at a faster pace and continue with materials normally assigned to the next higher grade level. Becaluse of complications with crossing over grade lines, accelerated learning may also mean probing deeper into an assigned topic or exploring an independent project. Acceleration should not mean giving a pupil more practice in topies already mastered.

## 33 Frequc'nt enrichment instruction is prorided for capable pupils.

In the researeh study the term entichment instruction was used to refer to the offering of follow-up activities for all pupils who were capable of performing them. Enrichment instruction can help in narrowing the gap between high- and low-ability pupils: enrichment stimulates teachers and high-ability pupils intellectually and motivates other pupils to learn mathematies.

Mathermatically gificed pupils cxplore ab34 stract concepts. logic: and mathematical applications as part of thair classroom learning.
Mathematically talented pupils can be found in nearly every school. The complete mathematics program will provide opportunities for them. Those opportunities should go beyond accelerated and enrichment instruction (see hypotheses 32 and 33) by providing greater challenge and difficulty. However, such opportunities should never burden pupils with repetitive practice in topies they have already mastered.

## 35 A special mathematic:s program is offered to gifted pupils.

A startling result that appeared in the study that provided nearly perfect discrimimation between high- and low-performance schools was a stlecessful program of special instruction for gifted and
high-achieving pupils. Low performance was found to accompany inadequate instruction for gilted and high-achieving pupils.

Programs for the gifted are usually designed to provide special instruction for pupils with abilities that meet particular criteria. Special funds are often available for stach programs. In some programs for the gifted. the pupils were excused from their regular mathematics class and permitted to attend one with other gifted pupils. Eivery school should consider a program for pupils in the upper 10 percent of the pupil population. The abilities of that group may, however, vary greatly from school to school, and each program should be tailored acoordingly.

Mathematies programs for gifteal pupils entail exploring abstract concepts, learning to play mathematical games that involve logic, and applying mathematical understanding to other subject areas such as science: the social sciencess art (patterns and geometric models): and (eronomics (sec hypothesis 34). The gilted program may inchade fiede trips that are of particutar interest to the pupils.

## - 3 Teachers are anare of and are semsitioce to abilities. needs. and interests of individlal ptuils.

The researel study findines supported the basic principle of education that the suceessful tedeher is sensitive to the individual pupil's abilities. needs. and interests. Suggested plans for implementing that principle are as follows:
a. A program of intraschool visitation might be a starting point to help teaciers become aware of which of their classroom behavior reflect sensitivity and which do not. The visitations must be nonthreatening and nonevaluative: they must be considered exploratory and thempentic. Moreover, although a teacher might be considered quite sensitive to pupil needs. the experience of observing another teacher can help observers consider their own teaching and can lead to an understanding of professional strengths and weaknesses. Alter the visitation each teacher-observer should be able to prepare a list of specific ways in which he/she will try to be more sensitive to pupil needs. A "buddy system" whereby teachers observe and record attempts to implement the new teather behaviors would help improve teaching through repetition and reinforcement. Educational research has shown such a plan to be effective in modifying teaching behavior.
b. $A$ second phan miget be considered as an alternative or as an extension of the previous one In the second plan each teacher assesses each pupil in terms of objective tests, subjective reports, interviews, and parent conterences. From these observations the teacher prescribes a program of instruction for cach pupil (possibly with the assistance of a team including other teachers) and implements that progeani for a prearranged length of time. At the end of the program. the gains are assessed for each pupil, and the program for that pupil is rated by the teacher on its effectiveness and is modified accordingly.

## 37 Teachors have a peositive feeling toward mupils and a feeling of being well-liked b. them in: return.

Teacher evaluations of the quality of their interactions with pupils may be used as a measure of how well they relate to their pupils or, more simply, how well pupils seemi to like them. Improvement in this area might be accomplished by a "buddy system" in which two teachers observe each other in all contacts with pupils. The evaluation tool could be a printed form on which to record the number of positive and the number of negative communications from teacher to pupil. The Flanders interaction analysis technigue is a useful. effective model for this type of ohserving. ${ }^{\circ}$

## 38 Teachers anooid abusince criticism of pupils and rejrain from scolding or threatining pupils or arguing with them.

In some sehools avoidance of abusive treatment of pupils is critical. Although pupils may be responsible for certain classroom problems abusive criticism by the teacher increases frustration and harms the learning processes. Alternatives to angry reactions to pupils are expressions of disappointment, redirection of activiticis, discriminatory approval of properly behaving pupils. $6 r$, when possible, removal of pupils causing problems (see discussion of hypothesis 37).

## 3 Teachers consistently clarify pupil miscon- <br> 3 ceptions, correct pupil errors, and resolve difficulties promptly.

In some schools teachers should be constantly alert to pupil difficulties and respond promptly. Lessons need to be planned for maximum immedi-

[^6]ate assessment of pupil understanding (contrary to hypothesis 40).

40
Teachers allow mupils to diseoncor their own crrors and be self-directed.
In some schools prompt attention to pupil difficulties was found in the study not to be effective in improving performance (see hypothesis 39). In these schools pupils should be encouraged to check their own answers. During discossions the teacher should encourage pupils to respond to the contributions of others.

T T Teachors are swmothetic and tolerant and allow pupils to cexpess their ow'n opinions.
The leaching styk deseribed in hypotinesis 41 reflects a confident, self-assured teacher who aceepts the feelings of pupils. That type of teachor is often referred to as a "good listener" who never lesitates to draw out a pupil's feelings and opinions in order to understand that pupil's problems.
42 Teachers praise and encourage pupils bur proriding sumport, optimism, or an appre'ciation of pupil response's.
Praise and encouragement can be casily seen in a nod, a smile, a touch, a word or two, or an claborate reward system. In their classroom observations the researchers were particularly attentive to the subtle expressions rather than the elaborate cowards. Teachers can improve their skills in this area by using one of the plans described in hypothesis 37.

## Teachers accept and use the ideas of their pupils.

Possibly the greatest praise that pupils can receive is to have their ideas accepted by the teacher and used to help their classmates. Teachers need to have flexible lesson plans and be alert to useful pupil ideas. Sometimes the idea can be used in a simple way, perhaps as a clarification of a statement. At other times the idea may be used to rebuild an entire lesson or learning activity. The source of an idea should be broadly publicized within the elass, to parents, and to other teachers.

## 44

The instructional program is focused on pupils and their needs.
Hypothesis 44 is contrary to hypothesis 8 . which focuses on content. When the mathematies program focuses on the learner, more time will be spent in considering the pupil's interests and
ambitions. Furthermore, leaching will be flexible to atcommodate pupil suggestions. Progress will be evaluated on the basis of objectives which were mutuatly selected rather than preselected by the teacher or textbook.

> 45
> The instruction is restricted to presentations, demonstrations, and the selection of topics by the teacher alone.

In teacher-dominated chassroons, the pupils make little contribution to the lesson. The teacher selects the lesson topic and the time schedule for it. The pupils seldom interrupt with ideas and suggestions, and their questions are answered by the teacher with authoritative finality.

46The teacher controls the classroom with interesting teacher-directed activities.
In this type of classroom control, the teacher manipulates pupils to provide learning activities. Pupils perform on cue so that concepts can be illustrated and learned by classmates. The lesson may be lively and interesting. but the teacher is the primary center of attention. Misbehavior is suppressed by intensive focus on activities.

## Teachers hold informal, relaxed confer47 ences with their pupils to allow free exchange of opinions and discussion of needs.

Most teachers talk frequently with their pupils on an individual basis, but making appointments to confer with pupils is quite a different form of teacher-pupil interaction. The primary advantage of prearranged conferences is the free exchange of needs and opinions under nonstressful circumstances. That is, when a conference is held at a prearranged time, there is more of a positive client-counselor relationship and less of a negative chiid-adult relationship. When the reason for conferring is immed ate (i.e.. not prearranged), the conference may serve as incitement for an undesirable behavior pattern (namely, misbehavior, disapproval, embarrassment). In a prearranged conference, however, this pattern need not occur, as both pupil and teacher can share goals and concerns from positions of mutual acceptance.

48
The school's mathematics program is made a matter of high priority.
Many schools have a cycle of priority setting for subject areas so that the mathematies program is periodically given specific attention. For some
schools, however, the mathematies program should receive immediate and continuing attention.

## 49 <br> The school's mathematic:s program is updated to reflect new trends in mathematic:s ellucation.

High pupil performance in mathematies may in some schools result from teacher belief that their mathematics program has been updated. Therefore to attain higher pupil performance, a school should examine and consider alopting a new mathematios program. The addition of a mathematies specialist or mathematics resource materials may also be effective. Teachers should be made to feel that their mathematics program is being improved in conformity with new trends.
50
Schoolvide instructional objectives are developed for all subject areas.
Sehoolwide instractional objectives serve a broader purpose than providing uniform guidelines for learning: they stimulate communication between and within grade levels: they refresh the memories of experienced teachers and curb the excesses of the novices: they allay some of the fears of the parents; and they provide a finite tandle on an infinite task. Instructional objectives are not meant to be a description of all the learning expected of all the pupils. Rather, they serve as a map, providing enough detail to help travelers see where they are and how to get where they would like to be. The development of instructional objectives is a major effort that should not be undertaken without careful preparation. The entire school staff, interested parents, and community representatives should participate.

## 51 <br> Instructional objectives are developed for the mathematics programs.

Instructional objectives should be developed for coordination of the mathematies program between and within grade levels. The objectives serve as orderly guidelines for teachers to use when selecting topics. Without guidelines, umecessary repetition or serious gaps are more likely to occur in pupils` mathematics education (sec the discussion for hypothesis 50).

## 52 <br> A consultant assists in the development and maintenance of a coherent mathematics program.

A surprisingly large number of schools operate without the services of a mathematies specialist. In some types of sehools. the use of a consultant
could detinitely improve mathematies pertormance, Schools should consider the following methods of acquiring consultant help: hiring a district or interdistrict consultant: hiring a parttime consultant to serve several neighboring schools: releasing a key teacher to provide leadership: using the services of a gaduate assistant from a nearby college: discovering a volunteer in the community with expertise: and soliciting help from the offiee of the county superintendent of schools.

### 5.3 Teachers feel conficlent in their ability to teach mathematics.

Teacher confidence in relation to the teaching of mathematios is a factor which should be considered carefully in designing inservice training. Developing confidence in onc's teaching ability is particularly difficult when pupils perform poorly. As a starting point to overcome lack of confidence, teachers should seareh out and identify all the good aspeets and successes of their programs. These positive clements should then be expanded. Thereafter. causes of failures should be identified and remedied.

$$
\begin{aligned}
& \text { Teachors attend mathematics workshous } \\
& \text { and courses. }
\end{aligned}
$$

Implicit in this hypothesis is the assumption that teachers are given opportunities and encouregement to study mathematics. Moreover. opportunities should be convenient and attractive and workshop leaders should be both knowledgeable in mathematies and responsive to kearners.

Courses should be designed to accommodate the wide range of background and ability of the teachers. The teachers should be shown relationships between the mathematical concepts they teach and the broader structure of mathematical systems. Encouragement to attend such special mathematies courses is given real eredibility by the school's arranging for substitutes, staging spectal assemblies for the pupils of released teachers, and having the principal teach classes for the teachers. Encouragement calls for real effort on the part of the principal.

## 55 A district-le'el emphasis on mathematios education is initiated and publicized.

The degree of emphasis given mathematies by the district administration is an important consideration in pupil performance. It has been found, for example, to be in the best interests of the pupils in one school of a district to increase the emphasis on mathematics in the other schoots of the district. Specific proposals delineating mathe-
maties program goals and phans should be carefally designed for cach district.

## (1) The mincipal sumports teachers whof find mathe'matic: le'vothooks inaderquate.

It was found in the study that in those schools in which principals felt that the mathematies textbooks were inadequate. the teaching was sach llat pupils performed well. This Gading reveals the worth of mathematies programs which are not dependent on textbook teaching and further reveals the important role of the principal in schools with such programs.

## 57

The mathomatic:s program supperts parental views of what shomid be tanght.
In the schools in which the teachers felt that parents views were rethected in the mathematies program. pupil performance was better. The statf in those schools may have sensed parental satisfaction with the program and may have actually promoted that satisfaction in every contact with parents. Teachers in such sehools may use parental evaluation as a souree of program plaming for individual pupils as well as for the entire school.

The school staff is inwolved in a commmenit. 58 relations campaign to develop parental imrolvement in the school's mathematic's program.
Parental involvement is a way to improve a school's mathematies program. Developing the interest and involvement of the parents is a frequently recurring theme in teachers' meetings because it is recognized as an important factor in pupil performance. Parents usually become involved not as the result of the efforts of teathers to do something individually but rather as the result of a schoolwide community relations program similar to a political campaign. Strategies must involve current community issues, grass-roots support and work, and advertisment of the school's potential role in helping to produce a better life in the commenity. Parent-teacher associations. the school board, and parent advisory groups are valuable instruments in stach a campaign.

Teachers confer with the parents of each pupil at least three times anmually regarding the pupil's learning progress and problems.
Many teachers in high-performance schools indicated that they conferred with parents of each pupil at least three times per shool year. The
twatherparent conderences included in the researeh study were only those in which a pupil's progress was diseussed. The setting for most conferences was after school in the chassroom, but many mectings were hold in the homes of pupils or sometimes in mplanned meeting places.

The mathematies program is appropriate' 60 for the cultural character of the commme. mity and realistically reflects the commumity's mededs, gools, and interests.
An important relationship between the success of a school's mathematics program and its cultural
rekeamce in the commmaty was disclosed by the rescarch study. In schools where teathers and principals felt that their program addressed itself to the cultural chamater of the commonity, pupils performed better. The research study was bised on the assumption that diferent progranas must be designed for different commenties to maximize pupil performance and the results corroborated that position. Clearly, a program to improve the performance of pupils in a given school must be based on the application of allermatives aceording to the needs and goals relevant to the community.

## Chapter 5 <br> Implementation of a Plan for Improvememt

So that the results of this study ean be put into the hands of teachers for the greatest effect. a two-stage dissemination plan has been included as an integral part of an overall implementation plan. In the first stage of the plan. Whe results of the study will be presented to school teams consisting of a prineipal or an administrator and a chassroom teacher. In the second stage each school team will conduct a program for implementing the results of the study in the sehool's mathematies program. Both stages of the dissemination plan are based on a workshop format. The workshop for the first stage is designed for leadership training, and the workshop for the second stage is designed for classroom implementation.

## Workshops in Leadership Training

The first stage of the plan for improving clementary mathematies programs will consist of leadership training workshops. The suggestions given here are direeted to a district, county, or state supervisor, administrator, or consultant responsible for the elementary mathematies curriculum in more than one school. The workshops can. however, be integrated into any inservice training program concerned with mathematics teaching at the elementary level. The steps to be taken in conducting a leadership training workshop are the following:

1. Arrange for the participation of school teams from 20 to 40 local schools approximately six weeks in advance of the workshop. Reserve the use of a suitable, centrally located meeting room on a convenient day for a three-hour workshop. An assistant will be useful at the workshop for providing hospitality and for helping participants analyze their school's mathematics program profile. The motivation for attending the leadership workshop should be that participants will identify ways of improving their mathematics programs.
2. Send a coply of Appendix 13 of this puhlication to cach participating school approximately two weeks in advance of the workshop, including the cover letter explaining the importance of collecting the requested information prior to the workshop.
3. Collect the following materials. one eopy for each participating school (The materials may be duplicated from this publication, or additional copies may be purchased from Publications Sales, Cahfornia State Department of İducation. P.O. Box 271, Sacramento 95802. The price is $\$ 1.25$ plas 6 percent sales tax for California residents.):
a. Printed materials Implementation workshop outline Hypotheses for improving elementary mathematies programs
Implications of hypotheses
Sehool profile key (card)
Evaluation of workshop in leadership training (see Appendix (')
b. Transparencies for use on overhead projector (see Appendix D)

Selection of sample schools for this study
Instruments used in study of schools
Sample hypothesis
Categories of hypotheses
School profile key
4. Conduct the workshop, using the following guidelines (total time- 180 minutes):
a. Part Onte ( 50 minutes)

Registration--Premeeting
Distribution of name tags
Workshop introduction
Overview, charge, and instructions
List of main topics in these guidelines
Explanation of purpose and responsibility
Suggestion: note-taking

Disctasion of the exsermed
Selection of sample schonds (tataspatemew) all typers of sehools identilied defonition af high-acheving and low-achioving solooh: amd sampling procedure
Instruments used in study (tramparency). on-site ohservation ot labling: interviews of teachers amd primeipals: and anoilymons guestionmaires
Hypotheses (tramparency): 500 hypotheses terted: of hypotheses veritied statistically: and hypotheses cheched by type of shool
School protile key (tamsparemey): medhod of recording school intormation on key and using the key to select applicable bypofleses famomstration of sample hypetheses tramsaremey)
(ategories of hypotheses (tramparemey): examples of cach cateonty
Qucstions about the research medhod (tomminute maximum)

## 1. Part Two ( 40 minutes)

School profile key preparation
Tramsparency
Individual assistamce
(iompletion by every tam
Gentification of applicable hypothexes
Use of school protile key
Recording of number of hack sumaten shown for cacla hypothess (mark onty those hypotheses with thee or more hack spuares)
Carefal reading of applicahle hopothese
Worishop break dinc!aded in ahe to-minute period)
C. Part Threc ( 60 ) minutes)

Implications of hypotherses
Hypotheses with most hach sy" acc to he first items
Hypotheses to the read amd corresponding implication and illustrative remarts tos he made as time permits
Discossion of one callegory of hypothex:s at a time
Ixplamation of hypotheses a likely but not guaranteed ways of improving prosrams
Application of hypotheses to known sita: tions in participating schools
Not all hepotheses to be diselused
(d. Part $F \dot{\circ}$ (ar (.30) minutes)

Disenssion of implementation workshom
Outline of workshops to be reviewed
 Wechs
Advance poblicity to hersim immediately (bucstions and problems: conclatin! rex marks
lealnation of leadership traming worhshops Purpose: 10 assist State Department of - ducation in plaming fature workisops ('ompletion of sugecoted form (sce Apperndix (')
5. Inclade in the follow-ap:

Fiopphone calls to aid in atranging wortilops ()Nery made to f̈and whether evalaation forms were tamed in
levahalion sont from lealership traming workshops

## Implementation Workshops:

Immediately after a leadershap taming workshop has been completed, pamming shombl be begun for implementation workshops at the shool kev. The persons invoived in the plaming will be the members of the schooi teams that participated in line leadershap taming worksop. The seps to he tatern in comducting an implementation workshop are the following:
i. Aet promptly i fwo-hon planimg mecting for all teachers, administrators. resource persomact.
 shleduled two to four weens in adrance of the workshop. (Auch of the vatue of the leadersinip traning wortshop could be lost if the maphe mentation workshop is leha more that form Weeks after the leadershap worksop.) ti possibe. the workshop should be selochuled on the afternoon of a shortemed div. Time should be allowed for teachers to take care of keson plams. reports, and clerical duties prior to the work-


The productivity of the workshop will he enhanced if the meeting phac i : away from school and atministralive affecs. A ncarby busimess office. church, home restatarant, or other facilaty can provide a pleasant and relaxed atmosphere with few distractions. inturaptions. and reminders of frusiations.
-. Publicize the workshof. Publecity will help
 teacher on the school leadership team should take primary responsibility for publicizing atal persenting the work shop. (ireater productivity amsi program improvemont often result when reform is mitiated by classroom teachers.

Posters in the teachers' room, announcements in the daily bulletin, and other media should reflect the team's creativity and the sehool's special needs. Some questions that might be considered are: (a) What are we doing about mathematics? (b) How effective is the mathematics program in our school? (c) Is our mathematics program up-to-date") and (d Why can't Johnny add?
3. Prepare the workshop training materials. Preparation requires listing the hypotheses (without the black squares) in the order of importance to the improvement of the mathematics program. The list for each of the six categories of hypotheses should be on a separate page and one copy should be made for each teacher. An explanatory sheet should explain briefly that the hypotheses apply specifically to this type of school and that if teachers can find ways to implement the hypotheses. improvement in the pupils' mathematics achievement can be expected. One copy of Appendix E should be prepared for each participant.
4. Conduct the workshop according to specific guidelines. The workshop should provide ample opportunity for open discussion about the applicable hypotheses. Teams of three to five persons should then prepare implementation plans and ensure that the plans are carried out.
a. Introduction and explanation ( 20 minutes)

Explain the need for improvement in the school's mathematics program.
Explain the hypotheses verification procedure used in the research.
b. Discussion of hypotheses ( 60 minutes)

Seek ideas for implementing the hypotheses.
Discourage negative, pessimistic opinions. Use implications printed in the report to stimulate discussion.
Take notes on each idea for implementation, including who made it.
Look for pians that implement more than one hypothesis.
c. Assignments on implementation teams (30 minutes)

Select plans with high potential or interest. Name the plans and start a sign-up sheet for each one.
Allow time for each teacher to select two teams on which to participate.
Set an initial meeting time for each team. See that the leader serves on all teams.
d. Evaluation of implementation workshop (10 minutes)
Purpose: Assist State Department of Education persomel in planning future workshops. Complete the form in Appendix E.
5. Carry out follow-up activities:
a. Remind teachers of their team meeting.
b. Seek a leader for each team.
c. Publicize the plans developed by each team.
d. Provide liaison betwen teams.
e. Seek cooperation between teachers and administrators.
f. Send evaluations to the coordinator of the leadership training workshop.

## Recommendations for Further Research

A number of ideas emerged during the planning stages of the study that deserve further examination. The most obvious additional researeh needed in comnection with the study is follow-up in the following areas:

- Which hypotheses created the most profound changes in mathematics programs?
- Which types of schools were helped the most?
- How much improvement in pupil achievement occurred as a result of implementation of the various hypotheses?
Furthermore, a school-by-school survey of implementation plans developed by sehool staff could be a valuable resource. A comparison of teacher-pereeived effectiveness and the actual effectiveness of the approach to program improvement suggested in this study might also be made.

Replication of this study for another sample of schools in California or for schools in another state would be useful. Verification could be accomplished by the use of different observation instruments, interview procedures, and questionnaires. In partieular, more subtle interview questions might provide better insight into such teacher characteristics as sense of mission, empathy, rapport, perception of individual pupil needs, ability to listen. and innovativeness. Interview questions with structured response choices might be designed to provide more uniformity of interpretation. The questionnaires might also be modified to ensure uniformity of interpretation, and opentended question techniques might be utilized. The observation instruments could be simplified and checked for observer reliability.

As to the demographic variables used in the study for categorizing sample schools. data are available for a different basis for measuring each school's sociocconomic status and puria mobility. Other variables might be considered that statistically or logically provide better categories than those selected in this study. The additional variables could include mean family income, variance of the distribution of family incomes, educational expenditure per pupil, educational achievement for parents, pupil-teacher ratio, age and type of shool building, and variables that affect learning but lic beyond the direct control of a school's staff.

The study might be improved by excluding those schools from the sample for which certain major changes in demographic variables have occurred during the period used for establishing the trend of mathematies achievement. Likewise. some assurance should be present that the mathematics program scrutinized in the study at cach sample school is basically the same one which produced the trend in achievement at that school. At least the factors that acoount for the trends should be identified.

Despite the evide range of program characteristics selected prior to visits at the sample sehools. many more characteristics exist, some of which may be significant for proeram improvement: eg. the kind of homework assignments and the amount of emphasis placed on them; the type of report cards: and the quality of interaction between the principal and the pupils. Other charatereristics might be revealed by researchers concentrating on one type of school, comparing fe:tures of highachieving school programs with features of lowachieving school programs. Characteristics thus found might be quantified so that their oceurrence could be recorded for one type of school in the sample. After verifying or rejecting the various hypotheses regarding features of mathematios programs for that type of school. the researchers would move on to another type of school and repeat the process.

Many of the recommendations for mathematics programs made in this study can also be applied to
the teaching of other subjects, particularly reading, spelling. and grammar: that is, subjects that utilize mental processes and skills similar to those used in the study of mathematics. Furthermore, achievement scores for these subjects are available from the same source as the mathematies achievement scores.

This research study has provided useful information for organizing and implementing changes in the mathematies program of a given school. The hypotheses are verified for different types of schools on the basis of total school seores. It is unlikely that a single teacher could implement all the hypotheses that apply to that teacher's school, nor could a single teacher expect all the applicable hypotheses to be effective in improving achievement in one chassroom. An additional useful study might be undertaken in which the chassroom is made the basic unit.

The number of control variables would have to be expanded consederably in a chassroom-level study beyond the demographic variables used in this school-level study. Descriptors of the sehoolwide mathematics programs would become variables orer which the individual teacher would have no control. but these variables would nevertheless affect the mathematics program in each classroom. Such a classroom-level study would aid individual teachers in a given school setting independently of the suceess or failure of other teachers in the same school. This strategy would be particularly advantageous in a school with weak internal communication channels, uncooperative factions, or an influential group of unadaptive teachers. A classrocomlevel study might also be used effectively in the various departments and divisions of secondary sehools, grades seven through fourteen.

In summary, the model for educational research proposed in this report has many possible variations and implications for improvement in instructional programs at all levels in all subject areas. Hopefully, educationall researchers will use this new mathematies program model to provide frontline teachers with guidelines for "what to do when the students arrive on Monday morning."

## Appendixes

Appendix A. Observation instruments Used in the Study
Appendix B. Cover Letter and Information Needed to Make a School Protile
Note: The material contained in Appendix B should be sent to those invited to attend a leadership training workshop. The material should be sent well in advance of the workshop. and those invited should be reminded of the location and time of the workshop. (See instructions for workshops in leadership training on page 30.)
Appendix C. Evaluation of Workshop in Leadership Training
Note: The evaluation form is to be completed at the conclusion of the workshop. Each participant should be given an evaluation form. (See instructions for the evaluation of workshops in leadership training on page 31.)
Appendix D. Transparency Masters
Note: A complete set of transparencies should be made from the masters provided. (See the outline of the presentation of the transparencies on page 31.)
Appendix E. Evaluation of Implementation Workshops
Note: The evaluation form is to be used at the conclusion of the implementation workshop. Copies may be provided to workshop leaders for their use when they conduct implementation workshops in their own schools. Or a single copy may be given to each leader with instructions to make copies for the teachers at his/her school. (See reference to implementation workshops on page 32.)

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## Appendix A

# Observation Instruments Used in the Study 

Mathematics Program Inventory (School Level)
A. Description of Your School's Mathematics Program

Check the items which best describe your school's mathernatics program:

| $\square$ | Exemplary | $\square$ | Traditional | $\square$ | Modular scheduling |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ | Experimental | $\square$ | Contemporary | $\square$ | Homogeneous grouping |
| $\square$ | Relevant | $\square$ | Creative | $\square$ | Heterogeneous grouping |
| $\square$ | Content-centered | $\square$ | New program | $\square$ | Reflects teachers' vicws |
| $\square$ | Pupil-centered | $\square$ | Same old program | $\square$ | Reflects administration views |
| $\square$ | Teacher-centered | $\square$ | High priority | $\square$ | Reflects parental views |
| $\square$ | Accelerated | $\square$ | Medium priority | $\square$ | Reflects societal needs |
| $\square$ | Remedial | $\square$ | Low priority | $\square$ | Based on modern curriculum |
| $\square$ | Team effort | $\square$ | Differentiated stafting |  | program |
| $\square$ | Innovalive | $\square$ | Differentiated pupil assignments | $\square$ | Schoolwide performance obj |

Other $\qquad$

## B. Teaching Methodology Used in Your School's Mathematics Program

Check the items which best describe the teaching methodology used in your school's mathematics program:

| $\square$ | Much individualization | $\square$ | Multimedia approach | $\square$ | Emphasis on computational skills |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ | Some individualization | $\square$ | Performance contracts |  | and concepts |
| $\square$ | Little/no individualization | $\square$ | Learning packages | $\square$ | Textbook-oriented |
| $\square$ | Discovery approach | $\square$ | Task cards | $\square$ | Workbook-oriented |
| $\square$ | Individually prescribed instruction | $\square$ | Role playing | $\square$ | Worksheet-oriented |
| $\square$ | Diagnosis of learning difficulties | $\square$ | Mathematics games | $\square$ | Chalkboard |
| $\square$ | Independent projects/inquiry | $\square$ | Peer tutoring | $\square$ | Overhead projector |
| $\square$ | Some enrichment instruction | $\square$ | Teacher-pupil conterences | $\square$ | Classroom learning centers |
| $\square$ | Some gifted instruction | $\square$ | Frequent practice/drill | $\square$ | Resource center outside of room |
| [ | Manipulative aids |  |  | $\square$ | Small-group projects |

Other $\qquad$
C. Description of the Teaching Style in Your School's Mathematics Program

Rank numerically (1,2,3. and so forth) the teaching modes in the order of usage:

| —— Lecture/demonstration | - Recitation/drill | - Demonstration |
| :--- | :--- | :--- |
| - Discussion | - Individualized instruction | - Tutoring |
| - Question/answer (not | - Small-group instruction | Programmed instruction |

Other

## D. Appraisal of the Pupil Levels in Your School's Mathematics Program

Indicate the approximate number of pupils in your school (or room) nathematics program for each characteristic:


What is the grouping criterion? $\qquad$

## E. Improvement of Your School's Mathematics Program

Rank numerically (1, 2, 3, and so forth) the ten items which you feel describe the most effective ways which have been used to improve your school's mathematics program:


Other

## F. Identification of Factors Limiting Program Success

Rank numerically (1,2,3, and so forth) the characteristics which you perceive as limiting the success of your school's mathematics program:


Faculty abilities in mathematics
Ethnic composition
Faculty attitudes toward De facto segregation mathematics Class size
Pupil attitudes toward school
Pupil attitudes toward mathematics
Pupil motivation levels
School's traditionalism

Condition of buildings
Program irrelevancy Lack of concern for individual pupil needs

Diluted program
__ Shortage of aides
_ _ Ability grouping scheme
__ Mobility/absentecism rate
__ Resistance to change and innovation
_ Lack of program objectives
__- Inappropriate textbook
___ Ineffective use of existing resources

Other $\qquad$

## G. Your Opinion of Your School's Mathematics Program

Check the ap,ropriate box of the scale which best describes your opinion of your school's mathematics program:

1. Sensitivity to individual abilities. needs, and interests
2. Purposefulness of mathematics tasks and assignments
3. Encouragement of pupil discovery and inquiry in mathematics
4. Quality of teacher/pupil interaction
5. Inservice training in mathematics content
6. Inservice trairing in mathematics methods
7. Planning and organization of mathematics program
8. Instructional objectives for mathematics program
9. Planning and preparation of mathematics lessons
10. Provision for participation by all students
11. Diagnostic and prescriptive methods for learning
12. Extent that pupils are involved in selecting mathematics activities
13. Special instruction for gifted and high achievers

14. Extent mathematics program is integrated with other subjects
15. Relevancy of mathematics program for cultural character of the school
16. Extent mathematics program has been updated
17. Adequacy of basic mathematics textbook
18. Exient of independence from textbook to workbook
19. Clarity of explanations and directions in mathe matics
20. Use of audiovisual media in mathematics
21. Use of manipulative aids in mathematics
22. Use of small-group working situations
23. Opportunitios for independent mathe. matics projects and experiments
24. Use of activity-centered mathematics instruction
25. Teacher's confidence with mathematics


## Mathematics Teacher Inventory

## Personal

Sex__ Age level (decade)___ Ethnicity____ Sociocconomic status
Daily teaching load (in hours)____
$\qquad$ Daily teaching load (in hours)

## Education

College major $\qquad$ College minor $\qquad$
Recency of formal mathematies training (year)
Number of college courses in mathematics methods $\qquad$ Number of college courses in mathematics content Number of workshops in mathematics $\qquad$

## Experience



Choice of teaching carcer again?

## School Goals

Does the teacher have a perception of school goals?
View of school goals, purposes $\qquad$
View of school priorities
View of current school objectives/priorities attained
View of school problems
View of textbook $\qquad$ Supplementary materials
Adherence to textbook $\qquad$ Page by page?
Variance of mathematics program/strategy from day to day (or week to week) $\qquad$
Encourages alternate methods in pupil problem solving
Extended assignments (hour/week) $\qquad$ Number of monthly use of media nurees: Overhead projector__ 16 mm film_ ITV _Tape Filmstrip $\qquad$ Other
Awareness of pupil weaknesses/strengths
Radical changes in teaching methods/organization (past four years) $\qquad$ In what area?
Individualized instruction Ability grouping $\qquad$ Manipulative aids
$\qquad$
Less emphasis on textbooks $\qquad$ Activity centers $\qquad$ Other changes
Preference/avoidance of certain mathematics topics
Flexibility in developing/using new strategies
Preference/avoidance of certain mathematics topics
Source of new podagogical ideas
Techniques used tostimulate interest and motivate pupils $\qquad$

## Relationships/Committees/Improvements

View of effectiveness of teacher/parent communication
View of parent support of mathematics program
View of parent support of teacher innovation/experimentation
Actual participation on committees/curriculum planning $\qquad$
Opportunity for participation on committees
Number of teacher/parent conferences (this year)
Standard mathematics achievement tests administered and frequency

Are test data available to teacher?
Do classroom physical facilities, equipment, furniture appear to be adequate?
Does teacher experiment with new strategies?
Does textbook restrict the way the teacier teache: mathematics?
Is class time efficiently used?


## Mathematies Srhool Inventory

## Principal Variables

Sex___ Age level (decade)___ Ethnic origin
Who plans mathematics inservice workshops?
Greatest parent contribution to mathematics program in past year
Prediction of greatest contribution parents might make to mathematics program in next year
Number of visitations to each mathematics classroom per year
Support/encouragement of mathematics program (nonvisitation)
View of the need for evaluation of mathematics program
Actions/programs to improve mathe matics achievenent test scores
Understanding of thr potentials/limitations of teacher.
Encouragement of teacher self-improvement
Extent of encouragement of teacher innovation/experimentation in mathematics program
Faculty ability to idertify with school/conmunity
Involvement with school/community/parent committee-

## School Variables

Mathematics consultant in district?
Other curriculum consultant working in mathematics?
Consultant role in planning of the mathematics curriculum/progran
Rating of mathematics progran in district (low, medium, high).
Number of mathematics workshops this year___ Last year
Workshop topics
Priority level of mathematics in district $\qquad$ Number of teachers with M.A. degree
Number of teachers in probationary status $\qquad$
School reputation in district (low, medium, high)
School priorities
$\qquad$
School problems
Faculty strengths/weaknesses
Is mathematics program relevant to social needs? To pupil nceds?
Which needs of pupils are not met by program?
Textbook publisher
View of textbook
Number of disadvantaged pupils $\qquad$

Workshops critically needed


## Ohserved Classroom Behaviors and Organization



Notes:

## Classroom Observation Scale

Pupil Behavior

| 1. Apathetic | 234 | Alert |
| :---: | :---: | :---: |
| 2. Obstructive | 1234 | Responsible |
| 3. Uncertain | 1234 | Confident |

5. Lacking in enthusiasm 1234 Enthusiastic
o. Compliant 1234 Self-directed
6. Lacking in participation 1234 Full participation

## Teacher Behavior

| 8. Partial | 1 | 2 | 3 | 4 | Fair (impartial) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 9. Autocratic | 1 | 2 | 3 | 4 | Democratic |
| 10. Aloof | 1 | 2 | 3 | 4 | Responsive |
| 11. Restricted | 1 | 2 | 3 | 4 | Understanding |
| 12. Harsh (cold) | 1 | 2 | 3 | 4 | Kindly |
| 13. Dull | 1 | 2 | 3 | 4 | Stimulating |
| 14. Stereotyped | 1 | 2 | 3 | 4 | Original |
| 15. Apathetic | 1 | 2 | 3 | 4 | Alert |
| 16. Unimpressive | 1 | 2 | 3 | 4 | Attractive |
| 17. Evasive | 1 | 2 | 3 | 4 | Responsible |
| 18. Erratic | 1 | 2 | 34 | Steady |  |
| 19. Exeitable | 1 | 2 | 3 | 4 | Poised |
| 20. Uncertain | 1 | 2 | 3 | 4 | Confident |
| 21. Disorganized | 1 | 2 | 34 | Systenatic |  |
| 22. Inflexible | 1 | 2 | 3 | 4 | Adaptable |
| 23. Pessimistic | 1 | 2 | 3 | 4 | Optimistic |
| 24. Immature | 1 | 2 | 3 | 4 | Integrated (mature) |
| 25. Narrow (rigid) | 1 | 23 | 4 | Broad (accepting) |  |


| 26. Unenthusiastic | 1 | 2 | 3 | 4 | Enthriastic |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 27. Impatient | 1 | 2 | 3 | 4 | Patient |
| 28. Uncontrolling | 1 | 2 | 3 | 4 | Controlling |
| 29. Inhibited | 1 | 2 | 3 | 4 | Uninhibited |
| 30. Unprepared | 1 | 2 | 3 | 4 | Prepared |
| 31. Inefficient | 1 | 2 | 3 | 4 | Efficient |
| 32. Lacking in humor | 1 | 2.3 | 4 | Sense of humor |  |
| 33. Ignores pupils | 1 | 2 | 3 | 4 | Listening ability |
| 34. Frustrated | 1 | 2 | 3 | 4 | Happy |
| 35. Unimaginative | 1 | 2 | 3 | 4 | Resourceful |
| 36. Subjective | 1 | 2 | 3 | 4 | Objective |
| 37. Coarse | 1 | 2 | 3 | 4 | Refined |
| 38. Uncooperative | 1 | 2 | 3 | 4 | Cooperative |
| 39. Lacks ambition | 1 | 2 | 3 | 4 | Ambitious |
| 40. Inconsiderate | 1 | 2 | 3 | 4 | Considerate |
| 41. Worried | 1 | 2.3 | 4 | Buoyant |  |
| 42. Unreliable |  | 3 | 4 | Reliable |  |
| 43. Dependent |  | $\cdots$ | Self-reliant |  |  |

## Classroom Environment

| 44. Teacher-centered | 1 | 2 | 3 | 4 | Pupil-centered |  | 54. Disorderly |  | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | Orderly

Notes:

# Appendix $\mid$ Cover Letter and Information Needed B To Make a School Profile 

## (COVER LETTER)

## Dear Friend:

You recently received an invitation to attend a special workshop to be presented in your area. In anticipation of your attendance, we would remind you to make all the necessary arrangements and to ob tain the information requested below.

The workshop is based on a study completed by the Mathematics Education Task Force of the California Btate Department of Education. To apply the results to your school. you must classify your school according to six variables:

- Socioeconomic status
- Minority representation
- School size (enrollment)
- City size (population)
- Type of community
- Pupil mobility

Each of these variables has three ctassifications: low, medium, and high.
The information on the attached sheets describes the variables and detines the timits of the three classifications for each variable. Before the workshop presentation, current information and statistics should be collected for your school. On the basis of this information, a profile key card will be punched at the workshop and used to determine which recommendations apply to your school. In the case of incomplete or inconclusive information, decisions about the classifications will be made at the workshop before the profile key card is punched.

Sincerely yours.

## Information Needed to Make a School Profile

Six demographic variables are used to analyze the results of this study. Every school ranks in one of the three levels of each variable: low, medium, or high. The limits of those levels, along with instructions, are as follows:

## Socioeconomic Status

The level of a school's socioeconomic status is determined by the following computation:
Three (3) points for each percent of all pupils whose supporting parent is an executive, a professional or semiprofessional, a manager, a clerical worker, or a college-trained technician
Two (2) points for each percent of all pupils whose supporting parent is a skilled or semiskilled employec
One (1) point for each percent of all pupils whose supporting parent is an unskilled employee, is unemployed, or is receiving welfare assistance

| Scale: | High | 241 | 300 |
| :--- | :--- | :--- | :--- |
|  | Medium | 209 | 240 |
|  | Low | 100 | 208 |

Example: $\quad 20$ percent professional 60 points
70 percent skilled 140 points
10 percent unskilled 10 points
Totals: $\overline{100}$ percent $\overline{210}$ points
Socioeconomic status level: Medium (between 209 and 240)

## Minority Representation

The minority representation level of a school is determined by combining the percent of Asian, black, Chicano, and Native American pupils enrolled:

| Scale: | High | 29 |
| :--- | :--- | ---: |
|  | Medium | 10 |
|  | Low percent |  |
|  | 08 | 0 |
|  | 9 percent |  |
|  | perent |  |

## School Size (Enrollment)

The enrollment igures should be taken from the most recent records available. If significant changes are anticipated, the projected enrollmen: figure should be used.

| Scale: | High | Over 560 |
| :--- | :--- | :--- |
|  | Medium | $350-560$ |
|  | Low | Under 350 |

## City Size (Population)

City size is determined by the combining of population figures for residentially or commercially connected communities.

| Scale: | High | Over 200000 |
| :---: | :---: | :---: |
|  | Medium | 50000 200 000 |
|  | Low | Under 50000 |
| Example: | High | Anaheim-.Fullerton-Buena Park (area) Los Angeles |
|  | Medium | Citrus Heights-Carmichael (area) |
|  | Low | Santee - El Cajon (area) |
|  |  | Santa Rosa |
|  |  | Pacifica |

## Type of Community

The classifications for the community environnent are not quantified, but the "high," "medium," and "low" labels are retained for convenience. If a school's geographic commurity contains any rural or farming industry, regardless of the extent of residential or industrial development, it falls in the "low" category. If there is no farming but there are old commercial or industrial buildings, substandard dwellings, or predominantly commercial zoned property, then the community falls into the
"high" category. Other communities are suburban residential and are labeled "medium." The communities from which bused pupils come are not considered in this classification.

Scale: High Old commercial or industrial buildings, substandard divellings, commercially zoned (no farming) Medium Suburban, residential (no farming, no industry) Low Rural, farming

## Pupil Mobility

The pupil mobility index used in this study is the sum of the pupils added to or deleted from the school's enrollment between October 1 and May 1 divided by the actual enrollment on October 1 and expressed in percent. A projected sum for the seven-month period may be used if a trend has been establisned.

| Scale: | High | Over 37 percent |
| :--- | :--- | :--- |
|  | Medium | $23-37$ percent |
|  | Low | Under 23 percen |

## Appendix C <br> Evaluation of Workshop in Leadership Training

Name (optional) $\qquad$ Title of position
School $\qquad$ Telephone number $\qquad$
School address $\qquad$ City $\qquad$
$\qquad$
County $\qquad$ ZIP code
Workshop leade $\qquad$ Date $\qquad$

Record of school profile key:


Please mark all items which indicate your opinion.

1. Discussion of the Research:
$\square$ Clear
Interesting
Too short
$\square$ Too longImportantUnimportant $\square$ UnclearBorin:g
2. Marting of Applicable Hypotheses:Interesting $\square$ Boring
Too shortToolungImportantUnclearUnimportant
3. Implications of Hypotheses:
$\square$ Clear
$\square$ Interesting
$\square$ Boring
Too short
$\square$ Too longImportant
$\square$ UncleirUnimportant
Comments:
$\qquad$

## 4. Discussion of Workshop in Leadership Training:

$\square$ Clear
$\square$ Unclear
$\square$ Interesting
$\square$ Too short
$\square$ Important
Comments:BoringToo longUnimsportant
5. General Rating of the Workshop Experience:
$\square$ HelpfulUselessNeeded $\square$ Unneeded
$\square$ Interesting
$\square$ UninterestingPleasingDisappointing
$\square$ Disapponing
Comments: $\qquad$ $\square$
$\qquad$
6. Plans for Implementation Workshop in Your School:
$\square$ Already scheduled (date $\qquad$ ).Intend to set date promptly; no problems.Intend to set date, but there are problems. Explanation: $\qquad$Not likely. Explanation:
7. Anticipated Attitude of Teachers to Implementation of the Study in Your School:
$\square$ Very helpful $\square$ Somewhat helpfulProbably helpfulHarmful Comments: $\qquad$
$\qquad$ Palpul
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Please hand this evaluation to the leader before leaving. Thank you.

| Appendix | Transparency Masters |
| :---: | :---: |
| D | Trent |

Selection of Sample Schools for Mathematics Study


Instruments Used in Mathematics Study at the School Level

1. Teachers' and principals' selection of descriptors for their matt matics program*
2. Teachers' and principals' ranking of ten descriptors for thi mathematics program*
3. Teachers' and principals' ranking of ten methods used to impro their mathematics program*
4. Teachers' and principals' ranking of ten problems that affected $t$ success of their mathematics program*
5. Teachers' and principals' rating of their success in attaining certa objectives in their mathematics program*
6. Observers' rating of classroom attitudes, behaviors, and atmosphe। on the Likert Scale of Opposites
7. Observers' count of types of classroom interactions and technique
8. Information about schools obtained in interviews with the princip:
9. Information about teachers obtained in personal interviews
[^7]
## Sample Hypothesis

Pupils tend to exhibit higher achievement patterns if . . .


Pupils are involved in planning their own activities.


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## Categories of Hypotheses

- General classroom teaching styles
- Specific teaching techniques
- Instructional materials
- Teacher/pupil interactions
- Instructional leadership
- Parent/community involvement


## School Profile Key

|  | Low | Medium | High |
| :--- | :---: | :---: | :---: |
| Socioeconomic status | $100-208$ | $209-240$ | $241-300$ |
| Minority representation | $0-9$ | $10-28$ | $29-100$ |
| School size (enrollment) | Under 350 | $350-560$ | Over 560 |
| City size (popuiation) | Under 50M | $50 \mathrm{M}-200 \mathrm{M}$ | Over 200M |
| Type of community | Rural | Suburban | Industrial |
| Pupil mobility | Under 23 | $23-37$ | Over 37 |



## Appendix <br> E <br> Evaluation of Implementation Workshop

Name (optional) $\qquad$ Title of position $\qquad$ School $\qquad$
School address $\qquad$ City $\qquad$
County $\qquad$ ZIP code
$\qquad$

Workshop leader $\qquad$ Date $\qquad$
Please mark all items which indicate your opinion:

1. Explanation of Workshop Purpose and Plan:
ClearInteresting
$\square$ Tooshort
Important
UnclearBoringToo longUnimportant
2. List of Hypotheses:
$\square$ Clear
$\square$ Interesting
$\square$ Too short
$\square$ Tou longImportant
$\square$ UnclearBoringUnimportant
3. Discussion of Implementation of Hypotheses:
$\square$ Clear
interesting
Unclear
Boring
$\square$ Too shortToo longImportant

- Unimportant
Comments: $\qquad$

4. General Rating of Workshop Experience:

| $\square$ Helpful | $\square$ Needed | $\square$ Interesting | $\square$ Pleasing |
| :--- | :--- | :--- | :--- |
| $\square$ Useless | $\square$ Unneeded | $\square$ Uninteresting | $\square$ Disappointing |
| Comments: |  |  |  |

5. Anticipated Effect of This Plan on Improvement of the Mathematics Program in Your School:
$\square$ Very helpfulSomewhat helpfulProbably helpfulHarmful

## 6. Additional Information

List the hypotheses by number which seem to you to be most important for improving : :e mathematics program in your school:

List the hypotheses which you will be most concerned with on your implementation teams:

If you are the leader of a team, (a) circle the numbers of the hypotheses listed above which your team will implement; and (b) briefly describe the plan for implementation as you see it. Use the other sude of this sheet.

Please return this evaluation form to the workshop leader.

## SCHOOL PROFILE KEYS

Issued as a supplement to A Plan for Improving Mathematics Instruction in California Elementary Schools: Final Report of the Mathematics $\quad$ 'tucatlon Task Force (Sacramento: California State Department of Education. 1976). The use of the school profile key is explained on page 9 of the publuqtion.



[^0]:    
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    $*$ materials not available from other sources．ERIC makes every effort＊ to obtain the best copy available．Nevertheless，items of marginal＊ reproducibility are often encountered and this affects the quality＊ of the microfiche and hardcopy reproductions ERIC makes available＊ via the FRIC Document Reproduction Service（EDFS）．EDRS is not＊ responsible for the quality of the original document．Reproductions＊ ＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊＊

[^1]:    Copies of this publication are available for $\$ 1.25$ each, plus 6 percent sales tax for Culifornia residents, from Publications Sales. California State Department of Education, P.D. Box 271. Sacramento, CA 95802. A list of other publications that are available from the Department, Selected Publications of the California State Department of Education, may be obtained by writing to the same address. Two other publications of particular interest to mathematics educators are Mathematics Framework for California Public Schools (1975) and Inservice Guide for Teaching Measurement, Kindergarten Through Grade Eight: An Introduction to the SI Metric System (1975). Both are avaitable at $\$ 1.25$, plus 6 percent sales tax for California residents.

[^2]:    ${ }^{1}$ Final Report of the Special Mathematics Arficvement Study. Prepared by the Office of Program Livaluation and Research. Sacramento: California State Department of Education, 1973.
    ${ }^{\mathbf{2}}$ Mathematics Framework for California Public Schools: Kinder. garten Through Grade Twelve. Sacramento: California State Department of Education, 1975.

[^3]:    ${ }^{3}$ Profile's of Scheol District Performance, 1973.74. Prepared by the Otfice of Program livahation and Rescarch. Sacramento: California State Department of Fducation. 1974.

[^4]:    ${ }^{4}$ A 90 pereent confidence interval was used for significance testing. The researchers wished to include all hypotheses that held promise for inproving programs and to have hypotheses submitted for experimental testing later.

[^5]:    ${ }^{5}$ Beginnings. Nuffeld F'oundation Mathematics Pioject. New York: John Wiley \& Sons, Inc.. 1967. See alo Hathematics: The First Three Years. Nuffield Foundation Niathemancs Proiect. New York: John Wiley \& Sons, Inc., 1972.

[^6]:    ${ }^{6}$ Ned A. Flanders, A nalyzing F'aching Behavior. Reading, Mass.: Addison-Wesley Publishing Co.. Inc., 1970.

[^7]:    *Anonymous questionnaires

