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AUTHOR Burt, Gordon J.
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ABSTRACT

As a part of the two-year process of revising the basic mathematics course at the Open University (Britain), an in-depth survey of students completing the first unit of the course was conducted. A sample of 120 students was divided into three groups of 40; group members received a questionnaire concerning concepts covered, a questionnaire about sections of the unit, or a test on the material. Approximately half of the students in each group returned the questionnaires. The concept questionnaire asked students to rate their prior familiarity, effort needed to understand, and current understanding of each concept identified in a conceptual analysis of the unit. These ratings were submitted to a multivariate analysis of variance. The sections of the unit were rated, and data analyzed similarly. Several relationships were uncovered: difficulty of concepts was predicted by amount of effort, and the time needed per section of a unit was related to level of concepts, number of diagrams, and number of lines in the text. The rating instruments and summaries of responses are included. (SD)

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The Detailed Evaluation of Mathematics Courses at the Open University

Report No. 1: the unit on "Functions" in the
Mathematics Foundation Course

by

Gordon J. Burt

Detailed Evaluation involves obtaining information from students on many detailed aspects of the learning materials, analysing the content of the materials, and carrying out a statistical analysis of both the content and the student-based information. This method is applied to the first unit of the Mathematics Foundation Course.

The detailed evaluation of unit 1, M100

Section

| | |
|----|--|
| 0 | introduction |
| 1 | method |
| 2 | students' reactions to the questionnaire |
| 3 | study patterns |
| 4 | concept and section ratings |
| 5 | the conceptual analysis of unit 1 |
| 6 | the multivariate analysis of concept ratings |
| 7 | the multivariate analysis of section ratings |
| 8 | assessment questions |
| 9 | the validity of the ratings |
| 10 | conclusion |

Introduction

The detailed evaluation of unit 1 involved asking students 158 questions about various aspects of unit 1. A systematic analysis of the quantitative characteristics of the text and of its conceptual structure has been carried out. Multiple regression analysis has been used to establish the relationships which exist between the students' ratings and the content variables. The hierarchical level of a concept or of a section proves to have a powerful relationship with the students' ratings.

Other results of the evaluation include path diagrams illustrating students' study patterns, and the students' comments on the questionnaires themselves.

1. method

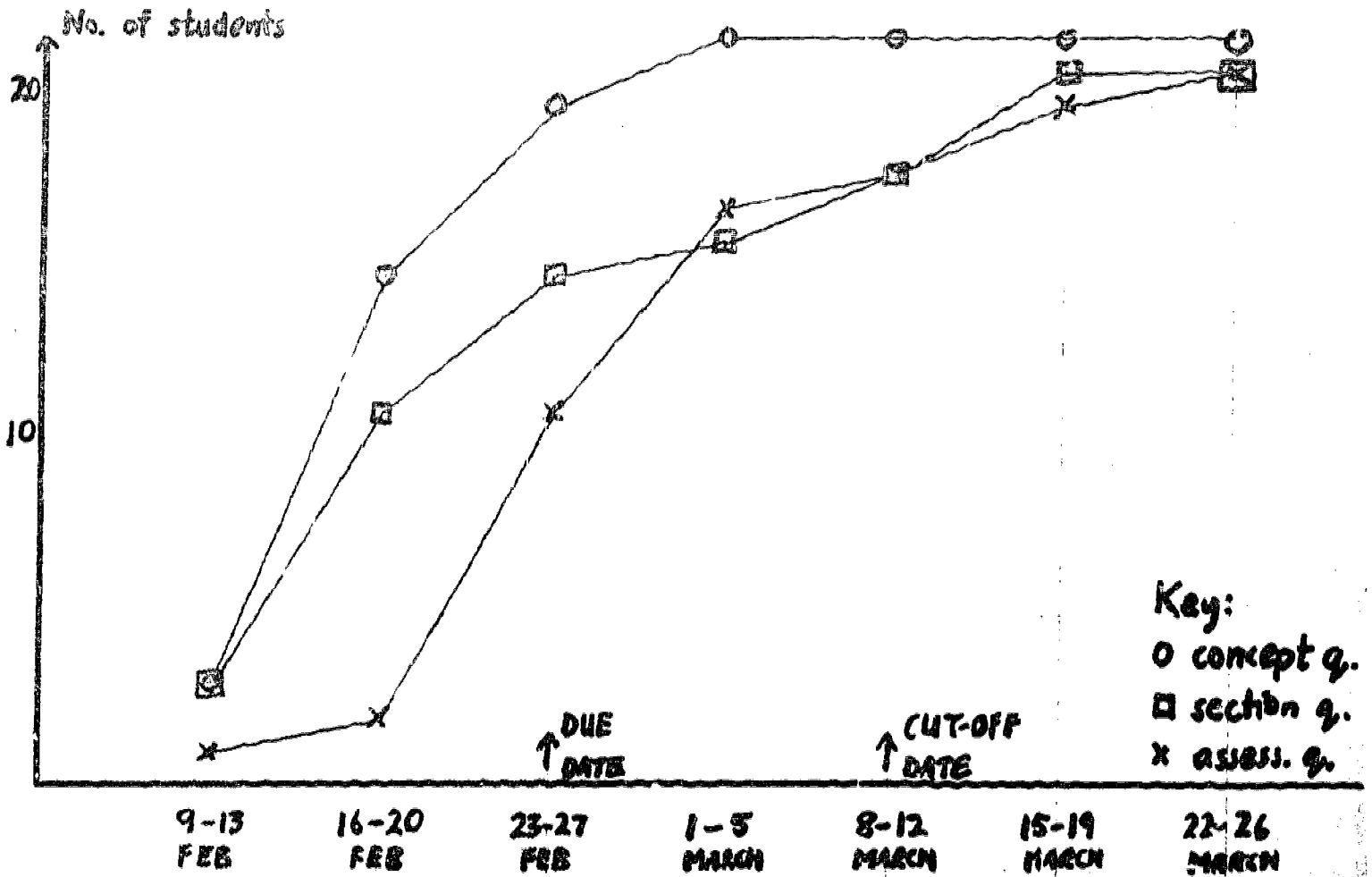
Three questionnaires were designed asking students about the concepts, sections and assessment questions, respectively, for unit 1 of M100, (see Appendix 1). Three similar letters were designed to go with each questionnaire (see Appendix 2). A sample of 120 F-year, M100, students was split into three groups each with 40 students. The first, second and third groups received copies of the concepts, sections, or assessment questionnaire respectively. Table 1 illustrates the time-table associated with all three questionnaires.

Table 1

| Date | Walton Hall | student |
|----------------|--|--------------------------------|
| December, 1975 | | received unit 1 |
| January, 1976 | selected sample designed questionnaires | |
| January 11 | | saw TVO |
| February 6 | sent questionnaires | |
| February 7 | (recommended start to study) | saw TVI |
| February 27 | | due date |
| March 12 | received returns analysed results | cut-off date |
| March 23 & 25 | Mathematics Faculty Workshop | |
| April 5 | wrote Report | received summary of results |

All three questionnaires produced a response from 21 out of the 40 students. Questionnaires started coming back almost immediately. Graph 1 plots the number of students who had responded over a seven-week period.

Graph 1



2. students' reactions to the questionnaires

On two of the questionnaires I asked the student if the instructions were clear ... what difficulties he had completing the questionnaire ... the time he spent on it. Table 2 gives the number of students saying the instructions were clear, the number who indicated difficulty, and the median and range of times spent. Appendix 3 gives the verbatim comments of students. Completing a questionnaire is not an easy task, and I have redesigned later questionnaires, giving fuller instructions which meet some of the difficulties mentioned.

Table 2

| <u>Qaire</u> | <u>Instructions clear?</u> | <u>Difficulty filling it in?</u> | <u>Median time</u> | <u>Range</u> |
|--------------|----------------------------|----------------------------------|--------------------|--------------|
| concepts | Yes = 22/22 | Yes = 11/22 | 20 min | 7-45 |
| | | No = 11/22 | 10 min | 2-15 |
| sections | Yes = 17/21 | Yes = 13/21 | 15 min | 5-35 |
| | | No = 8/21 | 10 min | 3-30 |

3. study patterns

As well as providing detailed information on the assessment questions, the assessment questionnaire provided some information on study patterns. Diagram 1 below gives the students' answers to the question: "when did you start studying unit 1?" These students were new (F-year) students and so received the materials in early December. The main point to notice is that the starting date varied from 8th December to 3rd February. In all cases the actual starting date was earlier than the recommended starting date and earlier than the transmission of TV1.

Diagram 1

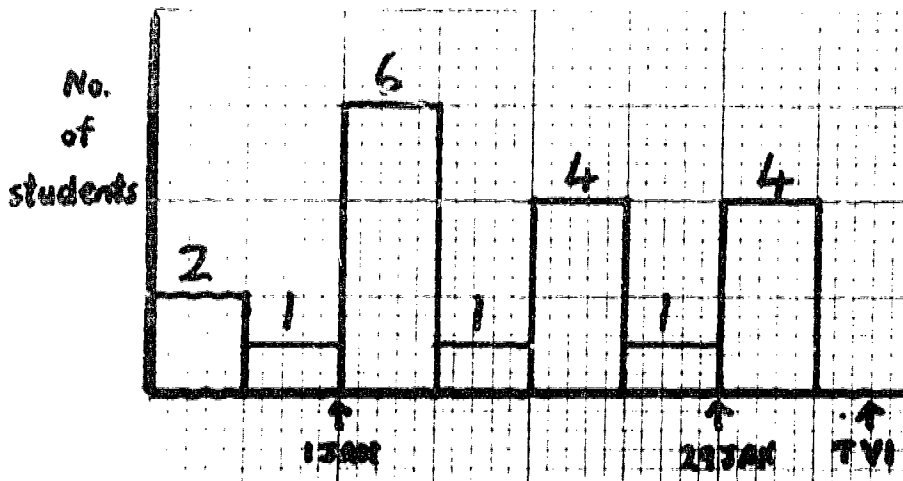


Diagram 2 below gives the time spent by the students on each of the components. The total time of 13 hours was split between text ($8\frac{1}{2}$), assessment (2), tutorials ($1\frac{1}{2}$), and broadcasts (1). These figures should be taken to indicate rough orders of magnitude. Furthermore, in many study situations, it must have been difficult for the student to decide whether to include the study time in the 'text' category or in the 'assessment' category.

Diagram 2

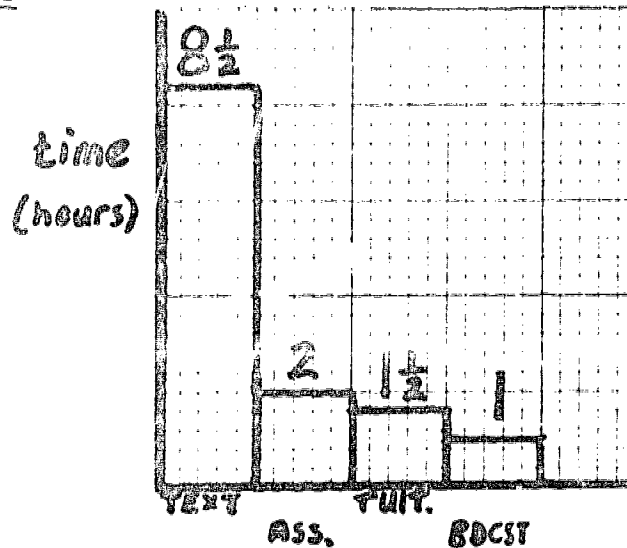


Table 3 shows the number of students who first read the CMA (/TMA) questions at various stages of their study. The pattern is the same for both CMA and TMA questions. The most common time to look at the questions is after the first reading of the text. However a good number of students read the questions before this stage.

Table 3 The number of students who first read the (CMA/TMA) questions:

| | before any study | during first reading | after first reading | during later readings | after later readings |
|-----|------------------|----------------------|---------------------|-----------------------|----------------------|
| CMA | 3 | 4 | 11 | 2 | 1 |
| TMA | 3 | 4 | 11 | 2 | 1 |

N = 21

Table 4 shows the number of students who first attempted the CMA(/TMA) questions at various stages of their study. The pattern is similar for both CMA and TMA questions. The most common time to first attempt the questions is again after the first reading. Very few students attempt the questions before this stage.

Table 4 The number of students who attempt the (CMA/TMA) questions:

| | before any study | during first reading | after first reading | during later readings | after later readings |
|-----|------------------|----------------------|---------------------|-----------------------|----------------------|
| CMA | - | 1 | 11 | 4 | 5 |
| TMA | 1 | 1 | 9 | 5 | 5 |

Table 5 shows the number of students who completed the CMA(/TMA) questions at various stages in their study. Again the pattern is similar for both CMA and TMA questions. The most common time to complete the questions is after subsequent readings - although some students manage to complete the questions after the first reading.

Table 5 The number of students who attempt the (CMA/TMA) questions:

| | after first reading | during later readings | after later readings |
|-----|---------------------|-----------------------|----------------------|
| CMA | 5 | 7 | 9 |
| TMA | 6 | 5 | 10 |

The above tables represent slices through the students' study patterns. Diagram 3 illustrates the patterns themselves for the CMA questions. The numbers in the circles are the numbers of students who followed that pattern. So, for example, 3 students first read the CMA questions before any study (A) of the unit of these 3, 1 attempted the questions during his first reading (B) and completed the questions after his first reading (C) ... of the other 2, they both attempted the questions after their first reading (C), and 1 of them completed the questions after his first reading (C) while the other 1 completed the questions during subsequent readings (D).

Most of the students posted their assignments between 10th February and 1st March. The due date was 27th February and the cut-off date was the 12th March. Graphs 2 and 3 show the number of students who had already posted their assignments by various dates through February. The two graphs show this for TMAs and CMAs respectively. The two curves are at their steepest (ie. the volume of returns per day is greatest) over the period 23rd to 27th February - the latter being the due date. It is surprising that students appear to be working to the due date, when, in fact, it is the cut-off date which is important - perhaps this is something these new students have still to learn.

Diagram 3

A
Before any
study

B
During first
reading

C
After first
reading

D
During later
reading

E
After later
reading

read qns.

③

attempted

①

completed

①

②

①

①

read qns.

④

attempted

②

completed

②

②

①

①

read qns.

⑩

attempted

⑦

completed

③

①

②

②

③

①

③

read qns.

②

attempted

①

completed

①

①

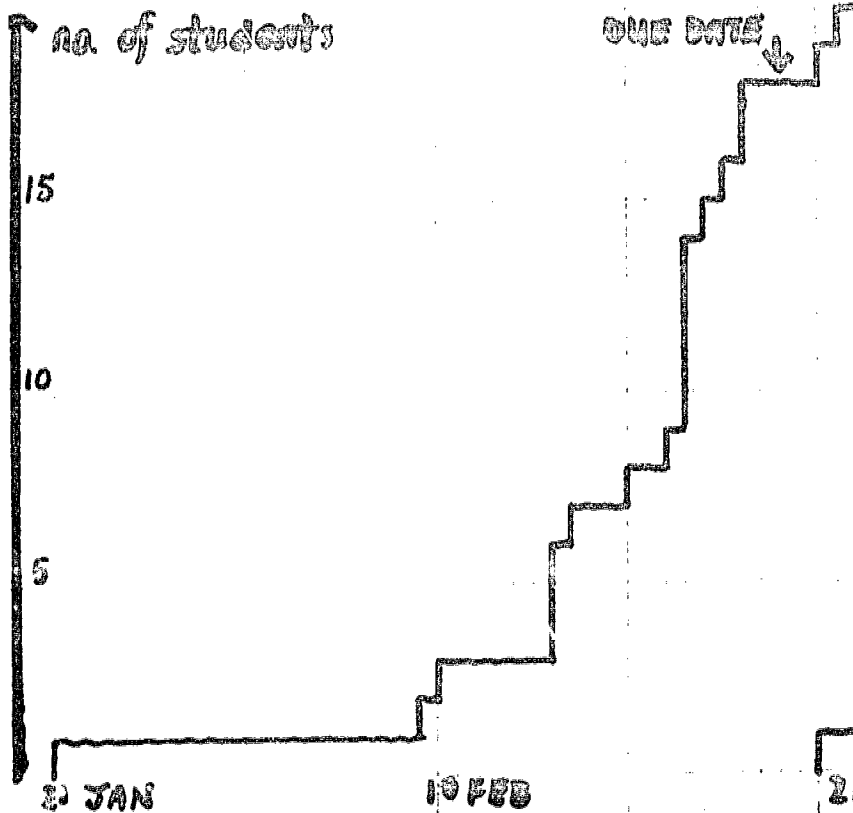
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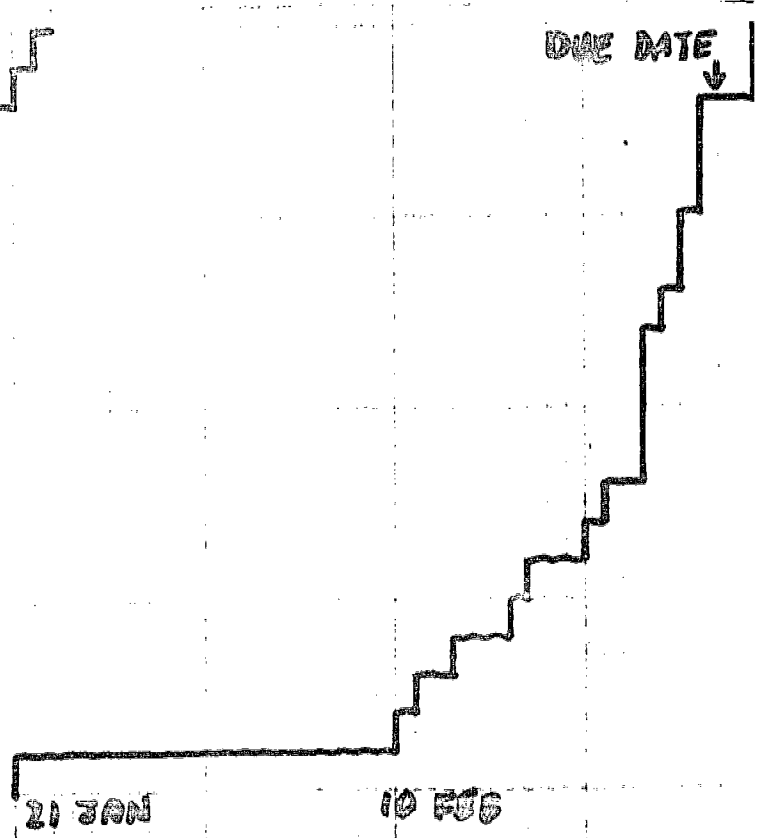
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Graph 2



Graph 3



4. concept and section ratings

Table 6 gives the mean ratings (averaged over the students) for each concept in the glossary of unit 1. The three questions were:

- (1) How familiar were you with the concept before you studied the unit? (*prior*)
- (2) How much effort was required to gain your current understanding of this concept?
- (3) How good is your current understanding of the concept?

The concepts are listed in the order in which they are presented in the unit. Of the first twelve concepts, 'mapping' and 'function' demanded the most effort from the student, and 'function' was the least understood despite the fact that it was relatively familiar to the students beforehand. A study of the text showed that the concept of 'function' was defined without any supporting discussion or examples.

Equality of mappings demands much effort and is poorly understood. The discussion of this concept in the text seems rather confused and no exercise is set on it. The definition of dummy variable is slipped into the solution of an exercise - hence the low understanding rating.

The concept of graph has the greatest prior familiarity, the second lowest effort, and the greatest understanding. This familiarity could well have been capitalised on to teach some of the earlier concepts.

The concepts of "function of 2 variables" and "operator" demand a relatively high amount of effort and are relatively poorly understood. Neither of these concepts are essential to the aims of the unit. The concepts which are mentioned in the objectives are asterisked in table 1.

The arithmetical combination of functions (+, -, \times , \div) caused few problems. However the quotient of two functions required rather more effort and was less well understood - with this concept one needs to be careful about defining the domain.

The remaining concepts demanded much effort and were poorly understood. Simpler examples might aid the understanding of composition of functions. None of the examples on reverse mappings contained real functions and there were no exercises at all. The diagram for classification of mappings is confusing. Finally, inverse function depends on the student understanding reverse mapping.

Table 6 Concepts, Unit 1

| | | prior [†] (1) | effort [†] (2) | understanding [†] (3) |
|----|----------------------------|---------------------------|----------------------------|-----------------------------------|
| 1 | set | 27 | 19 | 45 |
| 2 | element | 32 | 16 | 46 |
| 3 | equality of sets | 26 | 22 | 43 |
| 4 | subset * | 25 | 22 | 44 |
| 5 | proper subset | 17 | 23 | 43 |
| 6 | mapping * | 23 | 25 | 44 |
| 7 | domain * | 23 | 21 | 44 |
| 8 | codomain * | 21 | 23 | 44 |
| 9 | image * | 23 | 22 | 44 |
| 10 | function * | 30 | 28 | 42 |
| 11 | ordered pair | 29 | 23 | 43 |
| 12 | variable | 34 | 22 | 44 |
| 13 | equality of mappings | 16 | 31 | 39 |
| 14 | dummy variable | 16 | 25 | 39 |
| 15 | constant function | 23 | 26 | 44 |
| 16 | graph | 44 | 17 | 47 |
| 17 | function of 2 variables | 26 | 29 | 41 |
| 18 | operator | 18 | 35 | 37 |
| 19 | difference * | 29 | 22 | 45 |
| 20 | sum * | 29 | 22 | 46 |
| 21 | product * | 29 | 22 | 46 |
| 22 | quotient * | 28 | 26 | 44 |
| 23 | composition of functions * | 21 | 34 | 40 |
| 24 | reverse mapping * | 18 | 33 | 39 |
| 25 | one-one mapping * | 19 | 28 | 42 |
| 26 | one-many mapping * | 17 | 31 | 41 |
| 27 | many-one mapping * | 17 | 33 | 40 |
| 28 | many-many mapping * | 17 | 31 | 41 |
| 29 | inverse function | 19 | 36 | 40 |

10 [†]decimal points are omitted

high values denote : very familiar,
very much effort,
very well understood.

Table 7 gives the mean rating for understanding and the mean time for each subsection of the text. This information confirms the above discussion of the concept ratings.

Table 7

Sections, Unit 1

| Unit | | time (minutes) | understanding |
|------|-------------------|-------------------|---------------|
| 1.0 | Intro. | | |
| 1.1 | Map. function | | |
| 1.2 | Combine fn. | | |
| 110 | Intro. | | |
| 111 | → Defn. | 11 | 44 |
| 112 | Set | 13 | 45 |
| 113 | Map. | 18 | 42 |
| 114 | More def. | 21 | 42 |
| 115 | Specify map. | 25 | 42 |
| 116 | Graph | 32 | 40 |
| 117 | More def. | 22 | 43 |
| 120 | Intro. | | |
| 121 | Arithmetic | 14 | 45 |
| 122 | Composition | 29 | 38 |
| 123 | Decomposition | 15 | 41 |
| 124 | Reverse | 22 | 39 |
| 125 | Classify | 33 | 39 |
| 126 | Inverse | 30 | 36 |
| 127 | Inverse Composite | 24 | 39 |

5. the conceptual analysis of unit 1

Each concept is defined in the glossary. With the exception of the definition of set, these definitions refer to other concepts in the glossary. The level of a concept is taken to be one higher than the level of the highest-level concept in the definition.

Example:

| | | |
|----------------|---|-------------|
| SET: | the definition does not contain any unit 1 concepts | ... LEVEL 1 |
| ELEMENT: | the definition mentions SET (level 1) | ... LEVEL 2 |
| SUBSET: | the definition mentions SET (level 1) and ELEMENT (level 2) | ... LEVEL 3 |
| EQUAL SETS: | the definition mentions SET (level 1) and ELEMENT (level 2) | ... LEVEL 3 |
| PROPER SUBSET: | the definition mentions SET (level 1), ELEMENT (level 2), SUBSET (level 3), EQUAL SETS (level 3) | ... LEVEL 4 |

Although the analysis does not require this, we may illustrate the hierarchical levels of concepts in a diagram. Diagram 4 shows this for the example diagram 5 for the entire unit.

Diagram 4

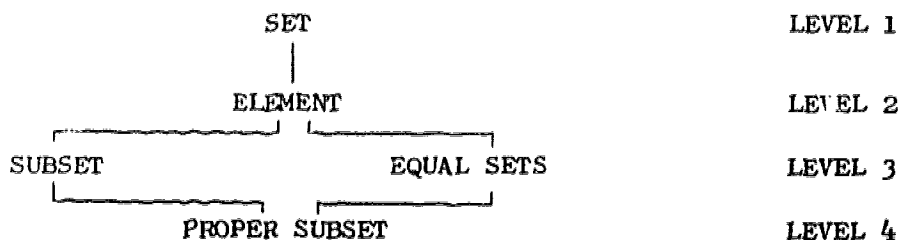
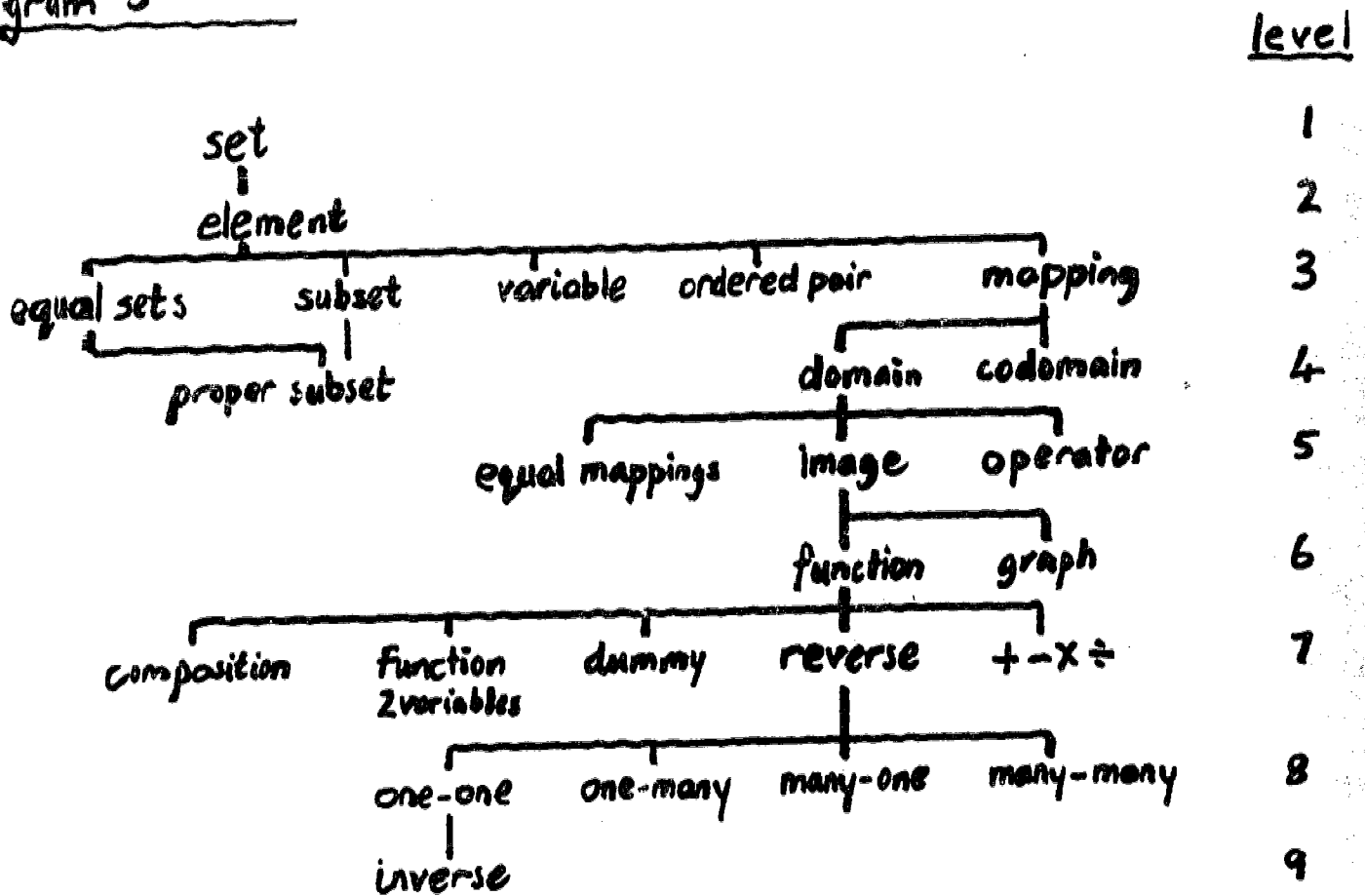


Diagram 5



6- the multivariate analysis of concept ratings

Regression analyses were carried out on five variables:

- 1- LEVEL the level of the concept in the hierarchy (see Diagram 5),
- 2- FREQUENCY the number of times the concept was mentioned in the text,
- 3- PRIOR the mean rating for each concept on 'prior familiarity',
- 4- EFFORT the mean rating for each concept on 'effort',
- 5- UNDERSTANDING the mean rating for each concept on 'current understanding'.

First of all I used variables 1, 2 and 3 to predict the effort demanded by a concept. Diagram 6 shows the correlations between the three predictor variables. The variable 'prior familiarity' enters the regression first with a correlation of 0.68 (n = 29). Next, 'level' enters the regression and increases the multiple correlation to 0.81. The variable "frequency" does not increase the multiple correlation significantly. The regression equation is therefore:

$$\text{EFFORT} = 3 - 0.45 \text{ PRIOR} + 0.12 \text{ LEVEL}$$

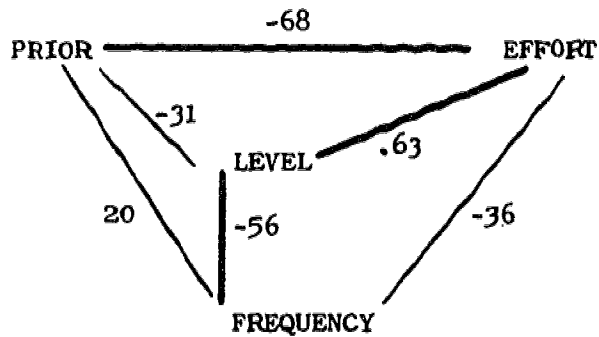
R = .81

↑ T = 4.5

↑ T = 3.8

P < 0.1%

Diagram 6



The residuals are interesting: 'operation', 'composition', 'function', and 'inverse' require much more effort than predicted ... 'dummy variable', 'proper subset', 'image', 'domain', and the arithmetic of functions require much less effort than predicted. These residuals suggest the unmeasured effects of presentation, inherent difficulty (given conceptual level), and new terminology.

The next regression used variables 2 and 3 to predict the 'current understanding' of each concept. In fact, only 'prior familiarity' was significant, and the equation was:

$$\text{UNDERSTANDING} = 3.6 + 0.29 \text{ PRIOR}$$

$r = .75$
↑ T = 5.9
P < 0.1%

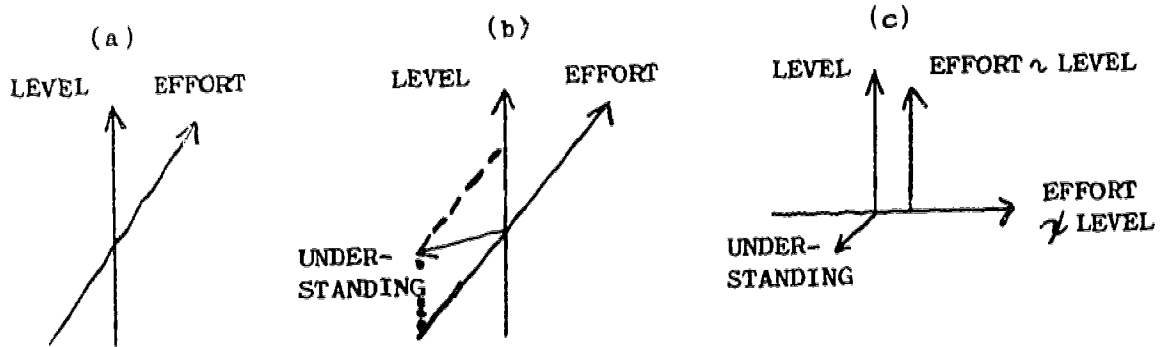
The next regression used variables 2, 3 and 4 to predict the 'current understanding' of each concept. 'Effort' had the highest correlation (.87) with understanding. Next to enter the regression was 'level' increasing the multiple correlation to 0.90. The equations were:

$$\text{UNDERSTANDING} = 5.3 - 0.49 \text{ EFFORT} + 0.03 \text{ LEVEL}$$

$R = .90$
↑ T = 9.5
↑ T = 2.5

Studying the above three equations, it seems clear that EFFORT can be regarded as a measure of conceptual difficulty - hence the paradoxical result that greater effort leads to less understanding! Then we have the paradoxical result that an increase in conceptual level is apparently associated with an increase in understanding. This is an artifact of the correlation between effort and level. Diagram 7 (a) shows the positively correlated vectors 'effort' and 'level', (b) shows the understanding projection on this plane, and (c) shows the resolution of the effort vector into a component in the level direction and a component orthogonal to the level direction.

Diagram 7



7. the multivariate analysis of section ratings

Regression analyses were carried out on 11 variables:

1. UNDERSTANDING the mean rating for each section on 'understanding'
2. TIME the mean rating for each section on 'time'
3. LINES the number of lines of print in each section
4. SYMBOLS the number of lines of print with symbols in each section
5. REPCONCEPTS the number of times mathematical concepts are mentioned in each section
6. DIFCONCEPTS the number of different mathematical concepts mentioned in each section
7. EFFORT the mean effort rating for concepts mentioned in each section
8. EXERCISES the number of exercises in each section
9. DIAGRAMS the number of diagrams in each section
10. PRIOR the prior familiarity of the concept with lowest prior familiarity in each section
11. LEVEL the level of the concept with the highest level in each section.

First of all, I ignored the data on introductions and summaries. Some preliminary analyses suggested that these short sections behaved very differently from the basic teaching sections and that their presence emphasised a general 'length' effect. However, their omission meant an even smaller sample size of 14.

In the first analysis, I used all the variables 3 to 12 to predict understanding. In fact, only 'level' entered the equation significantly:

$$\text{UNDERSTANDING} = 4.8 - .11 \text{ LEVEL}$$

$R = - .70$

$$\uparrow T = -3.4$$

In the next analysis I used all the variables 3 to 12 to predict time. An examination of the T values for the regression coefficients suggested that 'lines', 'diagrams', 'prior' and 'level' were the variables which gave the greatest contribution to the prediction.

A stepwise regression was carried out using these four variables as predictors. "Level" entered the equation first, its correlation with time being 0.64. "Diagrams" entered next, increasing the multiple correlation to 0.84. "Lines" entered next, giving R = 0.95. "Prior" did not add significantly to the equation. So the equation is:

$$\text{TIME} = -8.2 + 3 \text{ LEVEL} + \text{DIAGRAMS} + 0.08 \text{ LINES}$$

| | | | | | | |
|---------|---|---------|---|---------|---|---------|
| R = .95 | ↑ | T = 7.0 | ↑ | T = 5.8 | ↑ | T = 4.4 |
|---------|---|---------|---|---------|---|---------|

It is interesting to compare this equation with the previous one. The variable 'level' is the most powerful single predictor of both understanding and time. However time is also dependent on 'diagrams' and 'lines'. These latter two variables can be interpreted as measures of "length". Table 8 shows that all measures of length (ie. variables giving the number of times something occurs in the section) have higher correlations with time than with understanding.

Table 8 Correlations of other variables with "understanding" and with "time"

* 'length' variables

| | * | * | * | * | * | * | * | * | * | | |
|--------------------|-------|---------|-----|----------|-----|----------|--------|-----------|----------|-------|-----|
| | LINES | SYMBOLS | REP | CONCEPTS | DIF | CONCEPTS | EFFORT | EXERCISES | DIAGRAMS | PRIOR | LEV |
| UNDER- STANDING | 02 | -22 | -05 | -38 | -51 | -40 | -27 | 27 | -71 | | |
| TIME | 27 | 35 | 39 | 70 | 30 | 57 | 57 | -40 | 61 | | |

The residuals for the last equation are interesting: the sections on composition, reverse and inverse mappings have greater times than predicted, while the sections on specifying mappings, arithmetic of functions, and decomposition have lower times than predicted. These residuals suggest the effect of the unmeasured variable: inherent difficulty (given conceptual level).

8. assessment questions

Table 9 sets out the mean time for each of the assessment questions. The time for the TMA was 81 minutes while the time for the CMA was 43 minutes.

Table 9

| | <u>minutes</u> | <u>individual questions</u> | <u>minutes</u> |
|-------------------------------|----------------|-----------------------------|----------------|
| TMA question 1 (assessed) | 45 | U1 | 2.5 |
| TMA question 2 (non-assessed) | 36 | 2 | 2.4 |
| CMA | 43 | 3 | 3.3 |
| | | 4 | 3.4 |
| | | 5 | 3.3 |
| | | 6 | 3.6 |
| | | 7 | 3.5 |
| | | V1 | 3.9 |
| | | 2 | 3.2 |
| | | 3 | 3.6 |
| | | 4 | 4.8 |
| | | 5 | 5.2 |

The above table also gives the times for individual CMA items. Items which are bracketed together test much the same skill. The time for individual items do not vary much. The two shortest questions (U1 and U2) required recognising whether two simple functions were one-one or not. The longest question involved the concept of modulus function and of composition, (V5).

9. the validity of the ratings

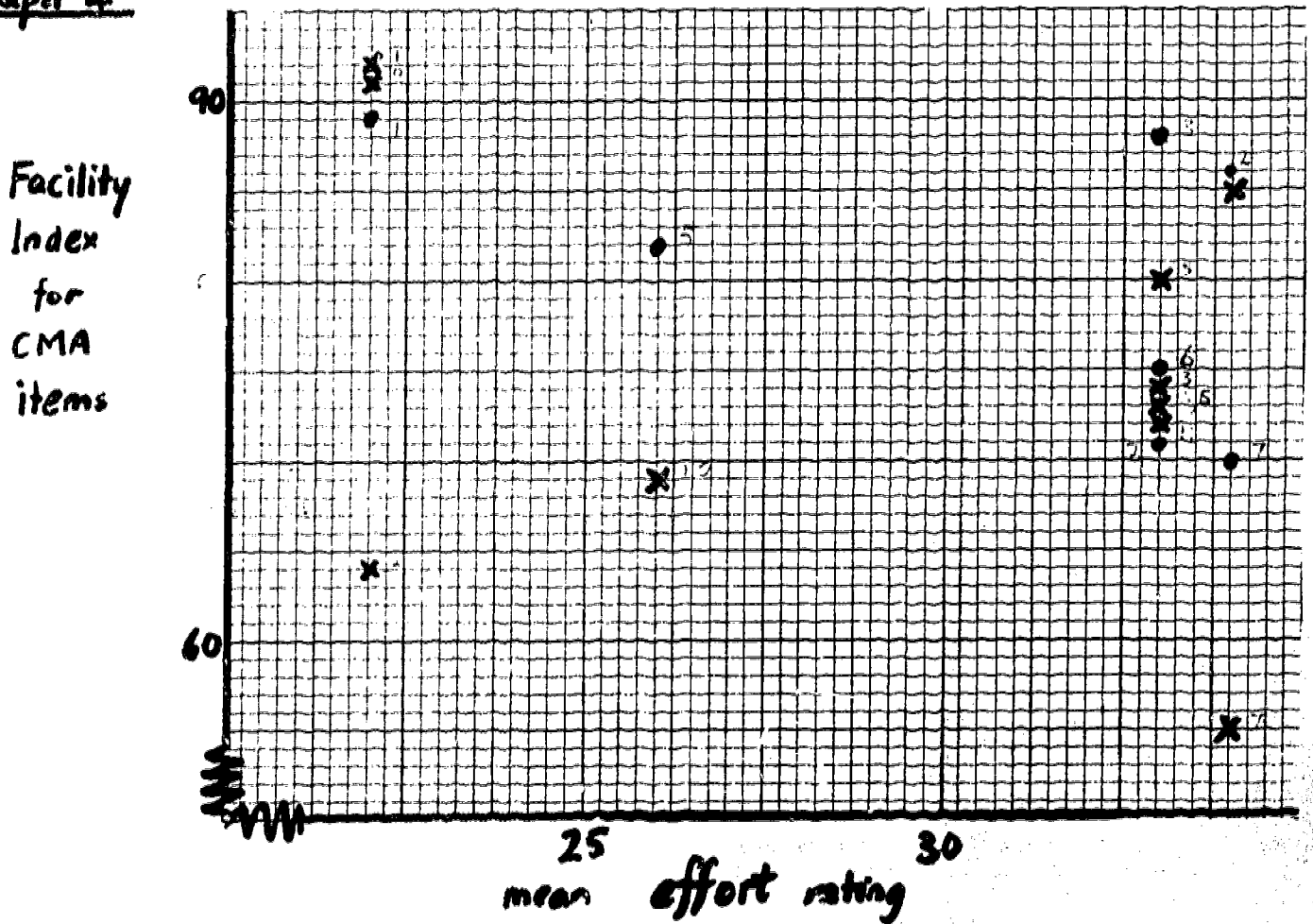
Many people ask me why I use ratings as opposed to looking at the students' actual performance. My answer follows. I do intend to set performance items on one of the units later in the course. I do not believe that it is feasible to use only performance items:

- (1) Students have to spend a greater amount of time on performance items as opposed to ratings (cf. Tables 2 and 9).
- (2) So the cover of traditional CMAs is less than the cover of my ratings.
- (3) I can ask rating questions about the presentation directly, whereas some doubtful reasoning needs to be applied if we are to use CMA results to diagnose faults in the presentation.
- (4) Often traditional CMAs test a number of different skills at once - eg. is the item V5 time-consuming because of the modulus function or because of composition?
- (5) It is possible to construct easy or difficult items on the same skill - e.g. items U1 and U2 required applying the one-one concept to very simple functions.

My investigations have already demonstrated to some extent the validity of the ratings:

- (a) at the Mathematics Faculty Seminar, people's guesses of the "top 3 concepts" and the "bottom 3 concepts" were 'right' about 2 out of 3 times;
- (b) in sections 6 and 7, I have demonstrated significant relationships between measures of content and mean ratings;
- (c) the ratings for concepts were consistent with the ratings for sections - but these two types of ratings were given by two completely separate groups of students;
- (d) graph 4 plots the facility index of CMA items (1974 x 1975) against the mean effort rating for that concept in the question requiring most effort.

Graph 4



10. conclusion

This evaluation of Unit 1 has not been a full-scale "Detailed Evaluation" - I have asked only 158 questions as opposed to the 589 I am asking about Unit 7. Even so, this report demonstrates the importance of many of the features of Detailed Evaluation:

- (a) the detail provided by information on individual concepts and sections enabled me to consider principles of presentation. This would not have been possible with information at a grosser level - indeed, I felt a need for information on specific examples and exercises.
- (b) the large number of questions meant that this detail was available for a number of different objects (concepts/sections) and for a number of dimensions (prior familiarity, etc.). By making the appropriate comparisons, I could reach important interpretations of the information.
- (c) the content analysis of the unit made explicit one very important variable (i.e. level) and enabled the testing of hypotheses about the relationship between content and ratings.
- (d) the multivariate analysis enabled me to select the most powerful predictors of various criterion variables - in doing so, to set up plausible models of student learning.
- (e) although I have not interviewed any of the students, the request for comments on the questionnaire has given useful information about how the students have approached this task.
- (f) the report on the Mathematics Faculty workshop has indicated how the results may be applied to the design of new courses. (*Appendix 4*).

Looking ahead, I want future evaluations to have more questions about more detailed aspects of the unit. I would like to see relationships established between ratings and other content measures besides 'level'. Finally I would like to see much stronger applications of the results.

Appendix 1

When you have finished your study of unit 1, please complete this rating sheet by putting a circle round the appropriate number for each of the three questions and each of the concepts.

| concepts (page in text) | prior familiarity | | | | | effort | | | | | current understanding | | | | | |
|--------------------------------|-------------------|---|---|---|---|--------|---|---|---|---|-----------------------|---|---|---|---|------|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | |
| set (4) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (1) |
| element (4) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (2) |
| equality of sets (4) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (3) |
| subset (5) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (4) |
| proper subset (5) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (5) |
| mapping (12) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (6) |
| domain (12) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (7) |
| codomain (12) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (8) |
| image (12) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (9) |
| function (12) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (10) |
| ordered pair (15) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (11) |
| variable (16) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (12) |
| equality of mappings (17) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (13) |
| dummy variable (18) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (14) |
| constant function (18) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (15) |
| graph (23) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (16) |
| function of two variables (27) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (17) |
| operator (29) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (18) |
| difference (30) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (19) |
| sum (30) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (20) |
| product (30) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (21) |
| quotient (30) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (22) |
| composition of functions (33) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (23) |
| reverse mapping (38) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (24) |
| one-one mapping (41) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (25) |
| one-many mapping (41) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (26) |
| many-one mapping (41) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (27) |
| many-many mapping (41) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (28) |
| inverse function (44) | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | (29) |

Your reactions to this method of feedback:

- Were the instructions clear?
- How long did it take to fill in the rating sheet?
- Did you have any difficulty giving the ratings.....if so, what were they?



Sections, Unit 1

Please complete this sheet by putting a circle round the appropriate number to indicate your understanding of the general development of ideas in the sections and by writing in how many minutes you spent on each section. You can either note your study times as you work through the unit or you can just try and estimate them after you have completed your study.

| Section (page in text) | understanding of general development of ideas. | | | | | time (mins) (approx.) | | |
|------------------------------------|--|---|---|---|---|-----------------------|--|---------|
| | 1 | 2 | 3 | 4 | 5 | | | |
| the unit as a whole(4) | | | | | | | | (4-7) |
| section 1.0 (p. viii) (5) | | | | | | | | (8-10) |
| 1.1 (p.1) (6) | | | | | | | | (11-13) |
| 1.2 (p.30) (7) | | | | | | | | (14-16) |
| section 1.1.0 (p.1) (8) | | | | | | | | (17-19) |
| 1.1.1 (p.1) (9) | | | | | | | | (20-22) |
| 1.1.2 (p.4) (10) | | | | | | | | (23-25) |
| 1.1.3 (p.6) (11) | | | | | | | | (26-28) |
| 1.1.4 (p.11) (12) | | | | | | | | (29-31) |
| 1.1.5 (p.14) (13) | | | | | | | | (32-34) |
| 1.1.6 (p.19) (14) | | | | | | | | (35-37) |
| 1.1.7 (p.27) (15) | | | | | | | | (38-40) |
| 1.2.0 (p.30) (16) | | | | | | | | (41-43) |
| 1.2.1 (p.30) (17) | | | | | | | | (44-46) |
| 1.2.2 (p.32) (18) | | | | | | | | (47-49) |
| 1.2.3 (p.37) (19) | | | | | | | | (50-52) |
| 1.2.4 (p.38) (20) | | | | | | | | (53-55) |
| 1.2.5 (p.40) (21) | | | | | | | | (56-58) |
| 1.2.6 (p.44) (22) | | | | | | | | (59-61) |
| 1.2.7 (p.47) (23) | | | | | | | | (62-64) |
| Summary and conclusions (p49) (24) | | | | | | | | (65-67) |

not at all good very good

Tick box (68) if you noted your study times as you went along (68)

Tick box (69) if you just tried to estimate them after your study (69)

Your reactions to this method of feedback:

- (1) Were the instructions clear?
- (2) How long did it take to fill in the rating sheet?
- (3) Did you have any difficulty giving the ratings .. if so, what were they?

Unit 1 assignment questionnaire

Try to give as accurate an answer to the questions as possible - but don't worry if you can't remember the exact dates (questions 1 to 3) or the exact times (questions 10-15) - just give the best approximation!

1. I started my study of unit 1 on
2. I posted TMA01 to my tutor on
3. I posted CMA41 to Walton Hall on

| Day | Month | |
|-----|-------|-------|
| | | (1-3) |
| | | (4-6) |
| | | (7-9) |

For each of questions 4 to 9 circle the option (A/B/C/D/E) which applies:

- | | | | | | | |
|---|---|---|---|---|---|------|
| 4. I first read the CMA questions for unit 1 ... | A | B | C | D | E | (10) |
| 5. I first attempted the CMA questions for unit 1 ... | A | B | C | D | E | (11) |
| 6. I completed the CMA questions for unit 1 ... | A | B | C | D | E | (12) |
| 7. I first read the TMA question for unit 1 ... | A | B | C | D | E | (13) |
| 8. I first attempted the TMA question for unit 1 ... | A | B | C | D | E | (14) |
| 9. I completed the TMA question for unit 1 ... | A | B | C | D | E | (15) |

Options

- A. before any serious study of the unit 1 text
- B. during my first serious reading of the text
- C. after my first serious reading
- D. during subsequent readings
- E. after subsequent readings

Please indicate roughly how many minutes you spent on the following components of this unit:

| | time (mins) | |
|--|-------------|---------|
| 10. T.V. broadcast and associated materials... | | (18-20) |
| 11. Radio broadcast and associated materials.. | | (21-23) |
| 12. Tutorial (excluding travel)..... | | (24-26) |
| 13. Correspondence text..... | | (27-29) |
| 14. CMA questions U1..... | | (30-32) |
| U2..... | | (33-35) |
| U3..... | | (36-38) |
| U4..... | | (39-41) |
| U5..... | | (42-44) |
| U6..... | | (45-47) |
| U7..... | | (48-50) |
| V1..... | | (51-53) |
| V2..... | | (54-56) |
| V3..... | | (57-59) |
| V4..... | | (60-62) |
| V5..... | | (63-65) |
| 15. TMA assessment question 1..... | | (64-68) |
| 22 non-assessment question 2..... | | (69-71) |



THE OPEN UNIVERSITY

INSTITUTE OF
EDUCATIONAL TECHNOLOGY

7th February, 1976

Dear

You may be aware that over the next two years we are going to rewrite the Mathematics Foundation Course. Obviously we shall need to make a number of assumptions about how students learn from correspondence materials. In the past we have tested our assumptions by asking "students" to study draft materials. This method has not been a complete success. So we are trying an alternative: asking M100 students like yourself about different aspects of their study. In this way we hope to establish some guiding principles about how to design the materials for the new Foundation Course.

As you may imagine, we already have a lot of information about students' assessment performance and their overall reactions to the units. But we are still rather ignorant about students' reactions to more detailed aspects of the units.

I would like to ask you the following questions about your study of unit 1 (Functions):

- (1) On what dates did you start and finish your work on unit 1?
- (2) At what stage did you work on the CMA and TMA questions?
- (3) How much time did you spend on the various components of unit 1?

NOTE: Your answers to these questions will NOT affect your assessment grades in any way; information about individual students will NOT be passed on to the Mathematics Faculty.

I am asking only a limited number of students to answer these particular questions. In this way I can cover many different aspects without asking any student to complete more than one questionnaire in the year.

I would be very grateful if you would give the answers to these questions on page 2. Please return this sheet in the reply-paid envelope. When I have analysed the results I shall send a summary of my conclusions to all those who have taken part.

Thank you for your co-operation.

Yours sincerely,

Gordon Burt

Gordon Burt
Lecturer in Educational Technology
attached to the Mathematics Faculty

Appendix 3

"Did you have any difficulty giving the (concept) ratings ... if so, what were they?"

Having completed unit 1, I tended to base my Prior Familiarity rating on the difficulty I experienced.

Prior Familiarity dubious^{due} to alien notation.

Having done some of the work before (about 9 years ago), I had met many of the concepts before, but had forgotten many of them. Therefore I could not say I was familiar with the concepts just before opening the book, but on meeting the concepts I found I knew something about them.

It's difficult to say how familiar one was before with a concept, unless of course one knew nothing of it.

Many of my problems in the text stem from adopting a different notation. It is therefore difficult to assess familiarity as I had met most concepts but often not in the same forms.

The prior familiarity column posed a problem. If you have never met the item before does it mean you are not at all familiar with the concept? For instance the term proper subset was new to me, but I knew about subsets. This same dilemma was posed in No. 14, 12, etc. On the other hand did very familiar mean you had met and studied it before or were very familiar with the text of Unit 1 and the terminology used in it? I interpreted this column as very familiar if I had a good prior knowledge, or had met similar work, and the other grades accordingly.

The effort column was difficult to judge in isolation. I would have answered more assuredly if I was answering if the concept was grasped easily etc.

The current understanding column was fairly straight forward.

Assessing effort required.

I do not feel sure what my current understanding is after only just completing one unit.

It is difficult to rate one's own understanding of a situation. The result can only be judged by self examination.

One can only give the ratings using the material (text) read as evidence of understanding. Further material (i.e. Questions) could devalue one idea of current understanding.

Very difficult to be objective. Difficult to assess degree of knowledge required. Assumed current knowledge to be concerned with unit content.

In some cases the idea was very quickly dealt with for example reverse mapping and inverse functions when I felt there was a lot of special cases. Perhaps a six-sense but sufficient to make one a little concerned about ones own ability.

Appendix 3 (contd.)

"Did you have any difficulty giving the (section) ratings ... if so, what were they?"

No! I took the average of each section, excluding the introductions and summary, for 1.1 and 1.2.

3 - Q. good? 4 - good?

Having done quite a lot of preparatory reading, familiarity with notions made it easier.

Yes - I have done the work in Unit 1 before so it was just revision therefore I am not representative.

Over simplification caused boredom, as with lack of explanation, lack of continuity. Deciding which region a rating was generated from.

I spent a lot of time going back in the unit and re-reading parts whose definitions I was not clear about.

Yes already worked through unit 1 two or three times .'. difficult to recall time spent.

Timing estimates very difficult - kept picking up and re-reading.

It would have been easier if the form had been received earlier so that (68) could be done. I presume these results are analysed by computer, could not boxes 4, 6 & 7 be programmed.

Had gone through unit before receiving this form and so had to estimate, would have preferred to note information while first studying unit. Consider estimates, done approx four weeks later, unreliable.

estimating study time at a later date.

The data I am supplying is very inaccurate, due to the length of time after study. I would have preferred to tick box 68.

Not in ratings. but timings impossible to calculate since this questionnaire was received after I had finished the unit.

Trying to think back after such a long time. Unit first read in December.

Appendix 4

Internal Memorandum

From Gordon Burt

To M101 Course Team and others

Subject A report on the Detailed Evaluation workshops

Date 26th March 1976

The first workshop concentrated on the results for the concepts in Unit 1, discussing the interpretation of these results, and their implications. The second workshop studied each sub-section of the unit in turn, giving a critique of the presentation, and then relating the critique to the results on that section and on the concepts in that section. Drawing on the discussions at these two workshops, I have written the section "implications for M101" below. The section following that gives a brief account of the results (I can give the full report to anyone who wants it).

I think we should hold other Detailed Evaluation workshops - in a number of guises. These are described in "suggested activities". Both workshops ran into time trouble! This might be avoided in future, if I give a quick overview of the results, using them to point to specific problems which we can then concentrate on (see the final section on "improved plan for Detailed Evaluation workshops").

Implications for M101

1. The presentation of concepts in the unit:

- how many different concepts do you use in the unit?
- how familiar will the students be with each concept before they study the unit?
- how many concept levels have you stacked on top of one another?
- what is the inherent difficulty of each concept?
- have you provided the right sort of examples and exercises to enable the student to understand and apply each concept?
- are your explanations clear and simple?
- has your presentation become confused and bogged down in unhelpful qualifications?
- have you structured the concepts in the sections in a clear and simple manner?
- is each concept essential to your aims - "if in doubt, throw it out!"

Unless we allocate some time to tackling these specific questions, they will tend to be ignored.

2. How much time do you think the students will spend on each section?

I shall provide an estimate based on an analysis of your unit and the application of the statistical results for units 1, 2, and 7. But you should have your own estimate too.

The results

I have obtained equations illustrating the following relationships:

- the effort required to understand a concept is a function of how familiar the students are with the concept before they start and of the hierarchical level of the concept.
- students have less understanding of concepts which are at a higher level and which have required greater effort.
- the average study time for each section is a function of the conceptual level of each section and of the length of each section.

Much greater discussion centred on the text itself and on the results for individual concepts. For example:

- the concept "graph" was not introduced till half way through the unit - despite the fact that it was the concept which students were most familiar with.
- students were relatively familiar with the concept "function" before studying the unit but had relatively little understanding of the concept after the unit - we noticed that Unit 1 (Functions) had no teaching about functions!
- the concept "operator" required much effort and was little understood - yet this concept is in no way essential to unit 1.

Suggested activities

- M101 Blocks which have overlap with M100 units 2 to 7 might hold similar workshops to relate a textual criticism of these units to the Detailed Evaluation of these units.
- M101 Blocks which have overlap with M100 units 9 onwards might carry out a textual criticism of these units and use that as a basis for designing a Detailed Evaluation of the relevant unit.
- textual criticism workshops might be run for M101 units - addressing the questions on page 1.
- we might carry out Detailed Evaluation of those M100 units (e.g. 22) which have implications for the remake of the second-level courses.

Improved plan for Detailed Evaluation workshops

- quick five-minute presentation of general findings.
- hand out results for individual concepts (or sections etc.) - for reference.
- select just a few items for closer examination.
- carry out a textual criticism on these few items.