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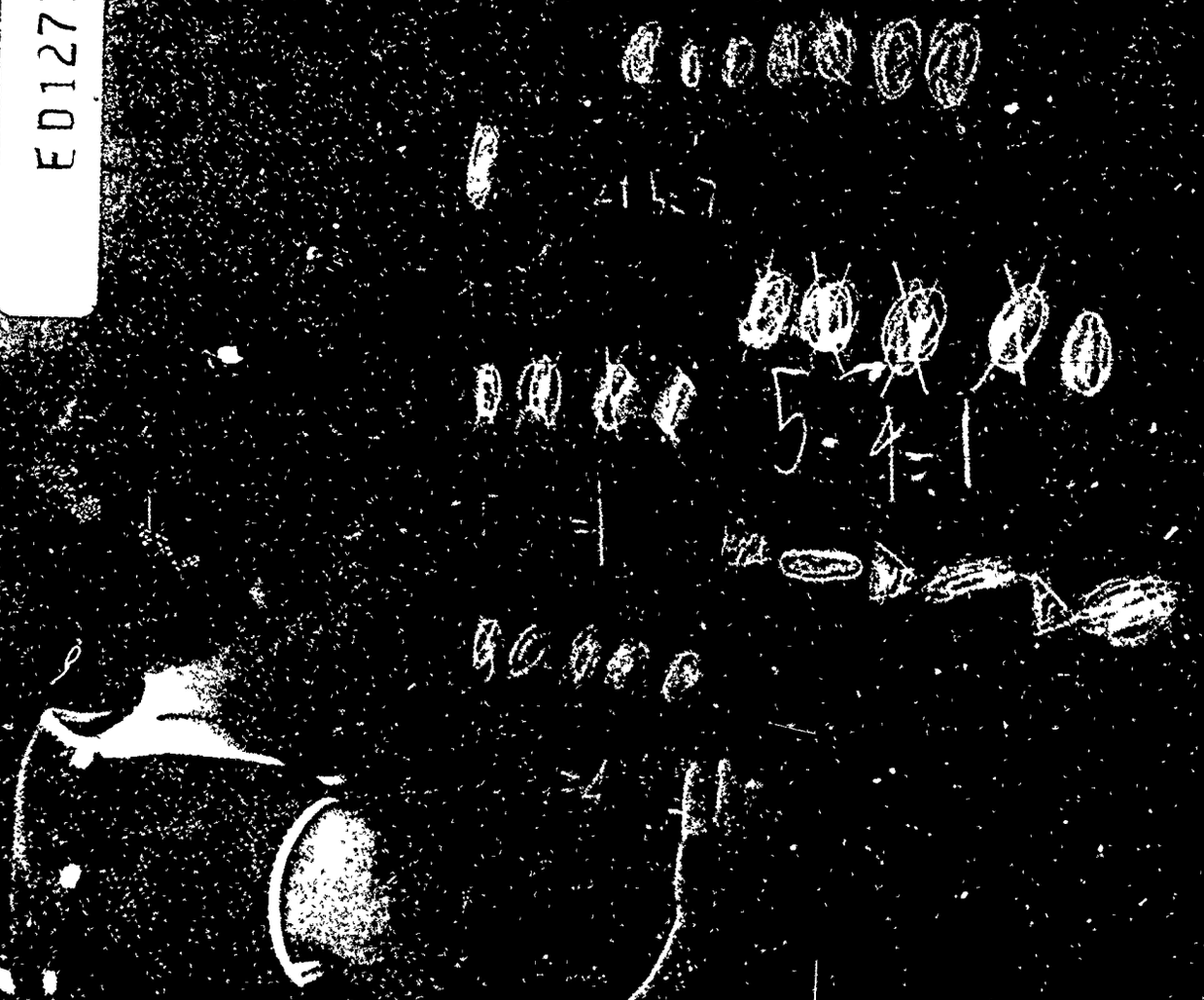
ABSTRACT

This volume is the eleventh in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by first-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of five groups of lessons. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. In this unit, the operations of addition and subtraction of whole numbers are introduced and developed. Section 1 introduces the "a+b" notation and related addition to the joining of sets. In section two, subtraction is developed using missing addend problems. Later sections concern (3) grouping, even and odd numbers, and arrays; (4) introduction to the number line; and (5) numerals through 99. Three supplementary games are also included. (SD)

MINNEMAST

INTRODUCING

MATHEMATICS



UNIT 11

MINNEMAST

2

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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UNIT 11

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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CONTENTS

Complete List of Materials for this Unit

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	** Student Manuals	
1	flannel board	1,3,12,14,17, 18
3-4	sets of objects, for flannel board	1,3,12,14,17, 18,40
1	set of numerals, 1-10, and a + sign, for flannel board	1
30	*** sets of Minnebars	2,3,4,9,28,29, 31,33,Game A, Game B
1500	* counters	5,7,10,11,13,14, 16,21,22,27,34, 35,36,39
30	* rulers	5
100	pieces of 9" x 12" construction paper	7,8,21,Game B
30	sheets of paper	9
21	crayons	14
1	bag	14
30	* pegboards with 20-30 pegs (preferred) or 600 counters	20
1	** picture of Skip	24,25
11	* number cards, black, 0-10	24,25,26
3	** number cards, red, 1-3	24,25,26
1	** house pattern	24,25
14	pieces of 8" x 10" cardboard	24,25
1	roll of masking tape	24,25,28,29,31

2	strips of heavy paper	26
1	long number line on floor	27
30	* number lines with about 1" spacing	27
30	number lines, 0-20, (1/2" spacing) (take from ***slide rule)	28,29,31,Game A
30	scissors	30
30	boxes of crayons	30
150	small paper cups, *souffle (optional)	34,39
1	set of numeral cards, 0,20,30,...to 90	35
30	tagboard "bingo" cards with assorted numerals 0 to 99	35
30	sets of play money: 50 pennies, 10 nickels, 2 quarters	38
10	*shallow trays	39
1	* abacus	40
1	deck of number cards, labeled 1-30	Game A
1	large piece of cardboard, paste	Game B
30	sheets of graph paper, 1/2" squares	Game C

* kit items as well as

** printed materials available from Minnemath Center, 720 Washington Avenue S.E., Minneapolis, Minn. 55455

*** available from The Judy Company, 310 North Second Street, Minneapolis, Minnesota 55401

INTRODUCTION

SUMMARY OF CONTENTS

This unit contains:

- An introduction to addition and subtraction of the counting numbers interpreted in terms of sets of objects.
- An introduction to the number line, a useful model of the real number system, which the children will use throughout the remainder of first grade and in subsequent grades.
- Activities with arrays.
- An introduction to the number $\frac{1}{2}$ and, optionally, to other simple fractions.
- Activities that illustrate place value and the decimal system of numeration from 0 to 100 and, optionally, to 1000.

BACKGROUND

The major topics of this unit are discussed below. More detailed comments are at the start of each section.

Addition and Subtraction

Addition of the counting numbers (i.e., 1, 2, 3, 4, ...) can be introduced in a way that has great intuitive appeal: namely, by interpreting a sum as the number of members in a set of objects that was formed by joining two disjoint sets of objects. (Disjoint sets do not have any members in common.) Similarly, subtraction can be introduced by interpreting a difference as the number of members remaining in a set when some of its members are removed. Teachers have traditionally used these ways of introducing addition and subtraction, although they have not always spoken of sets.

However, we must recognize that a child will be introduced to several extensions of the number system during his elementary schooling. He will first meet the counting numbers (the positive integers such as 2 and 17); then he will successively encounter the number zero (0), the negative integers (such as -3 and -198), the fractions (such as $\frac{3}{4}$ and $-\frac{17}{22}$), and the irrational numbers (such as π and $\sqrt{3}$). Taken together, these types of numbers are called the real numbers.

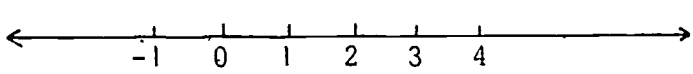
It would be fortunate indeed if the set interpretation could be meaningful for all the real numbers that a student will encounter. But a moment's reflection will show that this is not the case. For example, how could one give a set interpretation of $\frac{1}{2} + \pi$, or $\sqrt{3} - \frac{1}{4}$, or even of 2 - 5?

There is, however, an interpretation of addition and subtraction other than the set interpretation that is valid for all real numbers. This interpretation relies on the number line representation of the real numbers.

Because of the great intuitive appeal of the set interpretation, and because many first grade children are already familiar with it, we will begin the study of addition and subtraction by means of sets. The operations of addition and subtraction will be developed on the number line in Unit 13, Interpretations of Addition and Subtraction. The two interpretations of addition and subtraction using sets and number lines then will be developed in parallel, and the child will be encouraged to use whichever is better for a given purpose.

The Number Line

The number line is a model or representation of the real numbers. You will recall that the word "line" implies an infinitely long straight line. There are an infinite number of points on any segment of the line. Each point on the number line corresponds to one real number, and each real number corresponds to one point on the line. A certain subset of the points on the number line represents the positive and negative integers and zero. These are usually the points that are labeled on a number line:



The number line allows one to visualize the ordering of numbers. If the line is in the customary, but not required, horizontal position, the convention is to have the numbers become greater as one moves to the right. Thus, -8 is to the left of -3 because $-8 < -3$; and 9 is to the right of 5 because $9 > 5$. (Note: $-8 < -3$ is read "-8 is less than -3", and $9 > 5$ is read 9 is greater than 5.)

On all the number lines printed in this unit, the spacing of the integers is made uniform. The uniform spacing is needed when lengths are measured and combined on the number line. However, when numbers are simply being ordered on the line, or when certain points are being counted on the line, the uniform spacing is not necessary. Thus, in certain activities, the children need not be required to make careful measurements to produce uniformly spaced intervals.

There is an analogy between how numbers are related and how points are located on the number line. This is the reason one can add and subtract on the line by manipulating lengths. For example, to add 2 and 3, a length of 2 units and a length of 3 units are placed end to end starting at the 0 on the line and the total length is determined. In exactly the same way, numbers such as $2\frac{1}{2}$ and $3\frac{3}{4}$ can be added.

You may wonder why the number line concept is considered important for young children to have. Four reasons may be given:

1. The use of the number line provides a simple procedure for obtaining correct answers for all addition and subtraction problems that the first grade child will encounter. Even very young or slow children can find answers to problems that might be beyond them if they had to master complicated rules. For example, a subtraction problem such as $27 - 9$, which would require "borrowing" or "regrouping," can be solved by a simple counting procedure.

2. Although only a few children will be able to describe in words the analogy between number relations and relations among points on the number line, we hope that all children will gain an intuitive feeling for it as they use the number line. This feeling should help them to understand how addition and subtraction are related.

3. There are many physical embodiments of number lines with which children work in their daily lives and in their science lessons. Rulers, thermometers, and scales on measuring cups are some examples. Clock faces and instrument dials can be thought of as pieces of a number line bent into circles.

4. The number line will be useful in later grades. It will be used in the development of the concepts of multiplication, division, fractions and negative numbers.

The correspondence between numbers and points on a line will be extended to the correspondence between pairs of numbers and points on a 2-dimensional plane. An example of the extension to the plane is a graph showing a relationship between the time of day and the temperature. Such graphs will be made by the children in later grades and will be encountered throughout their lives.

Fully exploited, the correspondence between number relationships and relationships among points will lead to the ideas in analytic geometry and calculus, which are powerful mathematical tools for the scientist and mathematician. Most children will never study calculus, but those who do may find their study easier because of the concepts they have developed.

NOTES ON TEACHING THIS UNIT

This unit is divided into five sections, each of which emphasizes a particular topic. The sections are divided into lessons. Most lessons are planned to take approximately one class period. It is expected that generally two periods a day will be spent on MINNEMAST materials. You may choose to teach Unit 11 by itself, or you may choose to teach it simultaneously with Unit 10 or Unit 12. It is recommended that one unit be well started before another unit is begun.

Many addition and subtraction combinations are included in the unit. The lessons attempt to provide all children with methods of finding these combinations without memorizing them. There is value in memorizing combinations after the child has a good concept of both the set and number line interpretations of addition and subtraction, but we urge you not to require memorization at this time. Of course, many children will learn certain combinations as they work with them in the unit.

NOTES ON MATERIALS

Counters are often called for in the lessons. Any small, easily available objects may be used for counters. Examples are pebbles, paper clips, pennies, checkers, pegs, paper disks, small plastic figures and marbles. It would be fun for the children if they could sometimes use attractive objects such as flowers, leaves, or shells as counters. It is desirable that the same type of counter should not always be used. Sometimes a combination of different kinds of counters should be used within a single set, e.g., checkers and paper disks.

Minnebars are again used in this unit. The games and activities with the bars expose the children to various mathematical procedures and concepts. Although the children may not often verbalize their thought processes as they manipulate the bars, they are using processes of mathematical logic in addition, subtraction, doubling, and numeration. These concepts are not taught directly but are almost self-taught through discovery as the children use the bars.

In several lessons the colors as well as the number of units of Minnebars are given. The colors are useful in explanations, but your Minnebars may not be similarly colored. If this is the case, use bars with the proper number of units regardless of color.

Nature Worksheets. Certain worksheets in this unit use examples from nature. By looking at the illustrations while doing the arithmetic and in their free times, the children can obtain much information. The drawings should help the children discriminate among the forms of various plants and animals. Examples of concepts indicated by worksheets are: different animals have different food and habitats, different animals move and rest in different ways, and one plant will produce many seeds or fruits. These concepts need not be formally taught; however, if children are interested in a particular idea, discuss it and possibly have them find more about the subject in reference books.

SECTION 1 INTRODUCTION TO ADDITION

This section presents the idea that sums of counting numbers can be interpreted as the numbers of elements obtained by combining disjoint sets, and gives practice in computing the sums of small numbers.

COMMENT

In this section, sets of objects are used to introduce addition concepts. For example, to establish the fact that 2 + 3 = 5, sets containing 2 objects and 3 objects are constructed. These two disjoint (separate) sets are joined and the total number of objects is found to be 5 by counting. Minnebars are also used to help the children see addition relations.

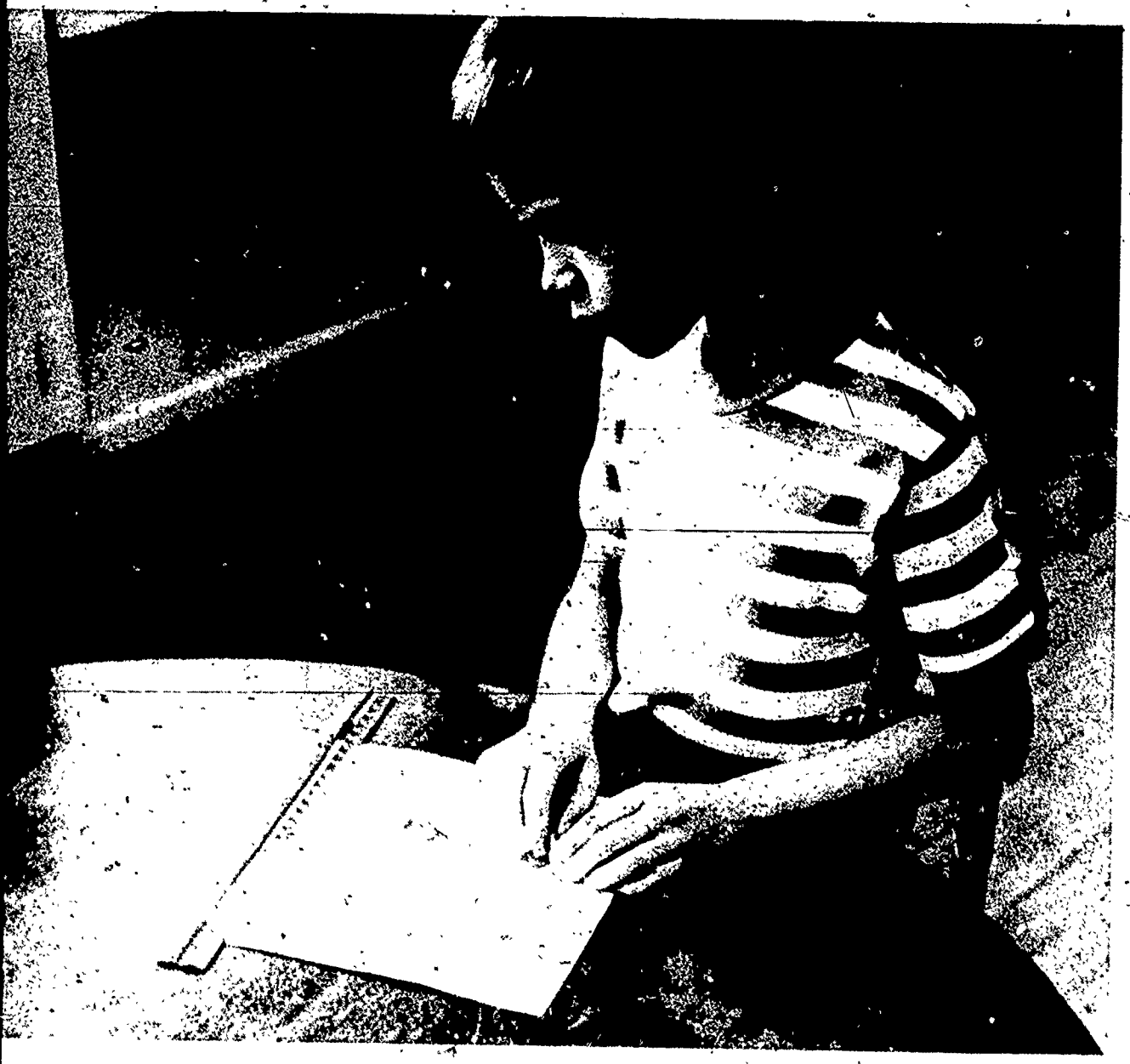
The objective of this section is to have the children understand the method of finding a sum by counting the members of the union of two sets and is not to have them memorize the addition combinations. Although the children are not urged at this time to memorize the addition combinations, the practice with sets of objects should make certain combinations familiar to them.

BACKGROUND

For any numbers a and b, "a + b" indicates a certain number that we call "the sum of a and b." We read "a + b" as "a plus b." Here and in several other places in the unit, a and b are used to represent arbitrary numbers. Hence for a + b one could substitute 3 + 5 or 127 + 29 or any other combination of numbers.

It is important for the children to learn that "a + b" can be considered to be the name for a number, as well as an instruction: "add b to a." One reason for this is that later they will see that different names for the same number may be freely substituted for one another in any mathematical expression without changing the expression. For example, "3 + 5" means "the sum of 3 and 5." They will learn that this number is also referred to as "8." Early recognition by the children that "3 + 5"

designates a certain number will make it easier for them to work with expressions such as "(3 + 5) + 2", or "4 + (3 + 5)." They will also be prepared to understand statements such as "5 + 2 is 7" and, later, "5 + 2 = 7", and still later, "5 + 2 = 3 + 4."



Lesson 1: INTRODUCING THE "A + B" NOTATION

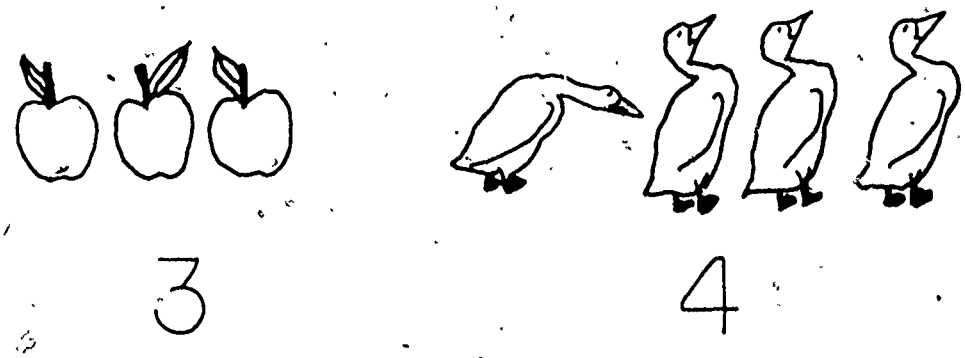
In this lesson the number of members in each of two sets is recorded. The two sets are joined and the number of members in the combined set is recorded in two forms -- for example, as 3 + 4 and 7. The combined set is the union of two disjoint sets, but this terminology will not be used with children at this time. (The sets are disjoint because no object belongs to both sets.)

MATERIALS

- flannel board
- for use on flannel board: two sets of objects, the numerals 1 through 10, and a + sign
- Worksheets 1, 2, 3, and 4

PROCEDURE

A. On one side of the flannel board place a set of 3 apples. Ask someone to put the proper numeral under the set. Place a set of 4 ducks on the other side. Ask someone to put the proper numeral under this set. (Of course, other objects may be used.) It should look something like this:



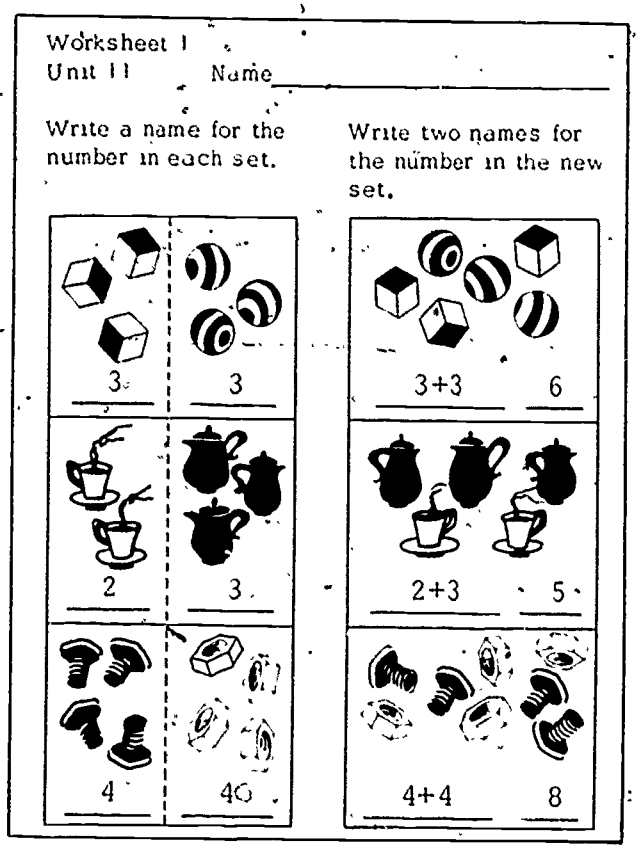
Ask someone to join the sets. Explain that joining two sets means that the two sets are put together to make one new set. Each of the apples and ducks belongs to the new set. Place a + sign between the 3 and 4. Read 3 + 4 as "three plus four," and tell the children that 3 + 4 is a name (or symbol) for the number of members in the new set of apples and ducks.

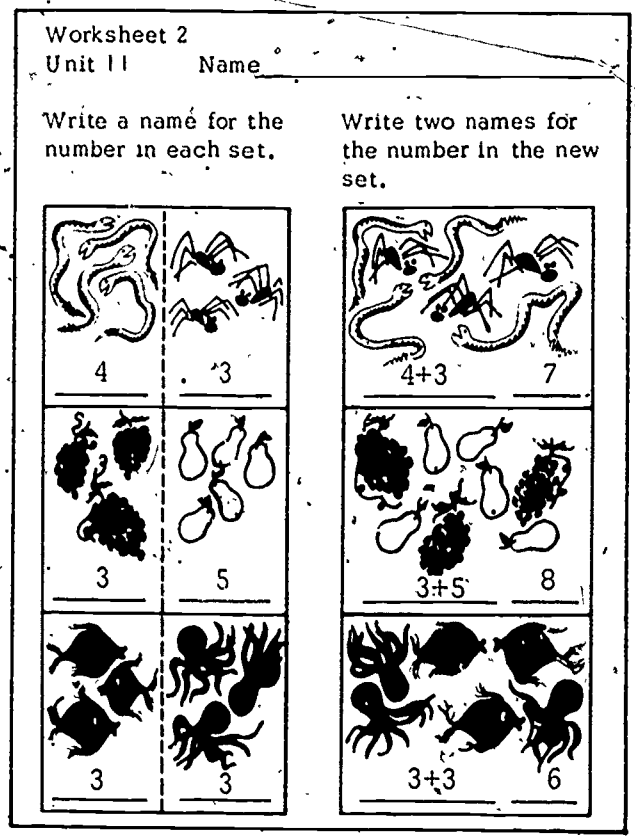
Now suggest that a child count the number of members in the new set. Then you can say, "By counting the members in the new set, we found that it contains 7 members. Now we have two names for the number of members in the new set, 3 + 4 is one name and 7 is another."

Repeat the above activity with several other combinations of sets on the flannel board.

B. The joining of sets may be varied by using children instead of flannel board objects. Have two sets of children come to either side of the front of the room. Have a child write the number of children in each set on the board, say 2 and 4. Ask the sets to come together. Have a child write two names for the number in the joined set (2 + 4 and 6) on the board. Repeat this with different groups of children until all have had a chance to be in a set. Some children may need extra practice in writing the + sign.

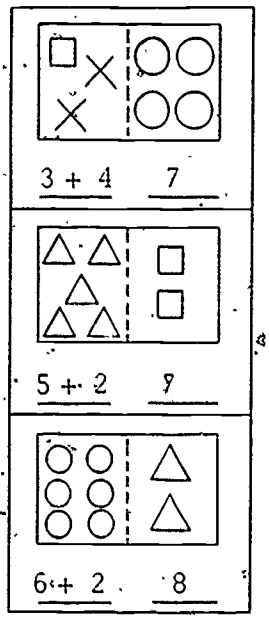
C. Worksheets 1, 2, 3, and 4 should now be used. In the worksheets the two names for a combined set are found by counting. This is not the time for children to memorize combinations.

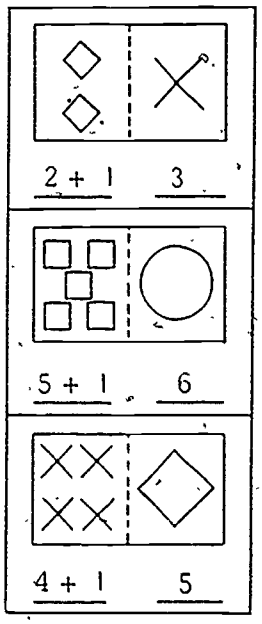




Worksheet 3
Unit 11 Name_____

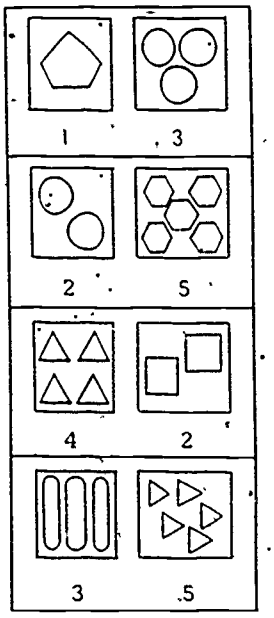
Join the two sets.
Write two names for the number of members in the new set.

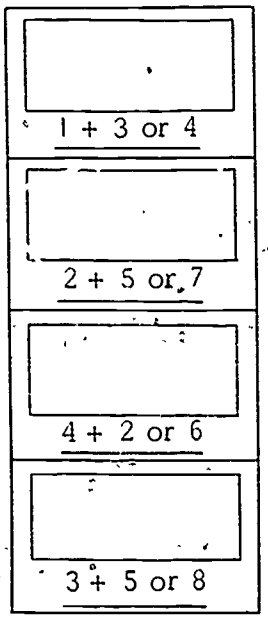




Worksheet 4
Unit 11 Name_____

Draw both sets in the box.
How many members are in the new set?





Lesson 2: MATCHING WITH MINNEBARS

The matching of two bars, one of length "a" and one of length "b," with the single bar of length "a + b" has two different but related interpretations. We can say that the set of a units and the set of b units have been joined to form the set containing a + b units. (The units are the squares marked on the Minnebars.) Or, as a forerunner of the number line, one can say that the length of a units joined to the length of b units gives the length of a + b units. The activities suggested here are intended to give the children a feeling for both these interpretations. The children probably will not express these ideas in words at first, nor should they be urged to do so until they seem ready.

This lesson also reinforces the idea of a number having two names.

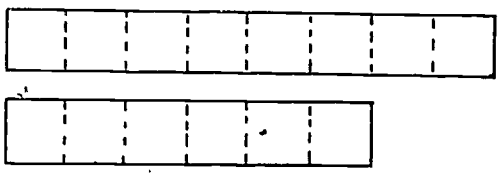
MATERIALS

— a set of Minnebars for each child

PROCEDURE

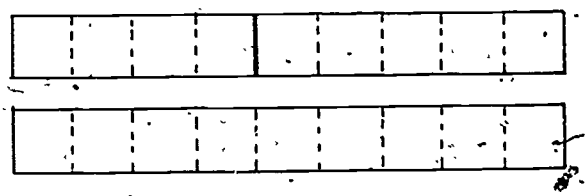
The various activities suggested may be done by an entire class together or by small groups. Feel free to choose certain ones and to vary them as you wish. You will probably demonstrate each activity once or twice. The demonstrations may use an overhead projector or Minnebars taped on the board. Give the children plenty of time for manipulation and experimentation.

A. Place two bars of different lengths side by side.

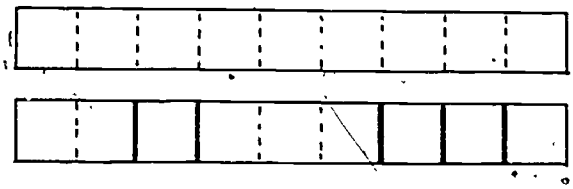


Ask the children to find the third bar that, when combined with the shorter bar, will match the length of the longer bar.

B. Place two bars, each no longer than 5 units, end to end. Ask if the child can "match" this pair with just one bar.



C. Ask the children to match one long bar with several shorter bars. Any combination of shorter bars that match the longer bar may be used. For example:



This could be used as a game for two or more children. They see who can match a long bar in the most ways.

D. Ask the children to match a 10-unit bar, for example, using just one color. Can they make the match with 2 bars of the same color? With 3 bars of the same color?

Try a 9-unit bar and an 8-unit bar.

Most first graders have an idea of "one half", but to many the concept may not be clear. Many will not be familiar with the symbol $\frac{1}{2}$. This is the first of several places in this unit where "one half" can be considered.

When the 10-bar has been matched by two 5-bars, point out that the length of either shorter bar is $\frac{1}{2}$ the length of the longer bar. Write $\frac{1}{2}$ on the board and explain that the length of either short bar is 1 of 2 equal parts of the entire length.

If you feel the children are ready, they may consider other fractions. For example, the length of a 3-unit bar is $\frac{1}{3}$ of the length of a 9-unit bar.

(Note: When you write $\frac{1}{2}$, or any other fraction, place the numerator directly over the denominator, rather than separating the numbers with a diagonal line, i.e., 1/2. Encourage the children to do this, too.)

E. The experience in the above activities can also be obtained in a less formal way by playing the first few sections of the Train Game found in Section 6.

Lesson 3: INTRODUCING THE TERM "SUM"

This lesson introduces the term "sum" and gives practice in giving two names for a sum.

MATERIALS

— several sets of object pictures for flannel board or chalkboard

— Minnebars

— Worksheets 5 and 6

PROCEDURE

A. Lead the children in a discussion something like the following:

MR. SMITH IS THE NAME OF A MAN. (Use the name of your principal.) HE HAS ANOTHER NAME. WHAT IS IT? (Principal of _____ School.) Write both on the board:

Mr. Smith Principal

SINCE THESE ARE BOTH NAMES FOR THE SAME MAN, WE CAN SAY,

MR. SMITH IS PRINCIPAL.

If necessary, repeat this with another person, such as the President of the United States, or Mrs. Jones (Ann's mother).

B. Place a set of 3 objects on one side of the flannel board or chalkboard, and a set of 4 objects on the other side. Have a child join the sets. Have another child write on the board two names for the number of members in the joined sets:

3 + 4 7

SINCE THESE ARE TWO NAMES FOR THE SAME NUMBER, WE CAN WRITE, JUST AS WE DID WITH THE NAMES OF THE PRINCIPAL,

3 + 4 is 7.

Both 3 + 4 and 7 are called the sum of 3 and 4.

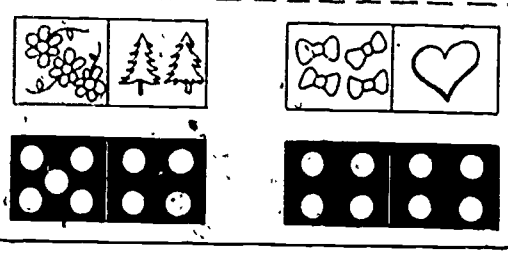
C. Repeat B with sets of Minnebars. Possibly the class could write the names for each sum on paper and check them with the result written on the board by a volunteer.

D. Have the children do Worksheets 5 and 6. It is expected that the children will still be counting to find the number of members in the combined set. If you prefer, you may have the children copy the pictures rather than cut and paste them.

Worksheet 5
Unit 11 Name _____

Look at each picture at the bottom.
Find a number name for each picture.
Paste the picture in that box.
Write another number name on the line.

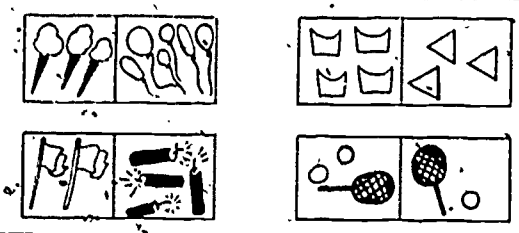
4 + 4 is 8	3 + 2 is 5
5 + 4 is 9	4 + 1 is 5



Worksheet 6
Unit 11 Name _____

Look at each picture at the bottom.
Find a number name for each picture.
Paste the picture in that box.
Write another number name on the line.

2 + 4 is 6	3 + 6 is 9
3 + 2 is 5	4 + 3 is 7



Lesson 4: INTRODUCING THE = SIGN IN RECORDING OF MINNEBAR MATCHING

The = sign is used for the first time in this lesson. The manipulations with Minnebars in Lesson 2 are repeated but in a more formal manner. The children are asked to record the Minnebar matches by writing number sentences. Worksheet 7 shows the exact size of the Minnebar to be matched. On Worksheets 8 and 9 the drawing is reduced in scale requiring the children to find the correct Minnebars by focusing on the number of units rather than the exact length.

MATERIALS

for each child:

— Minnebars

— Worksheets 7, 8, and 9

PROCEDURE

Have each child put a five-bar from his box on his desk, and then put a four-bar in front of the five-bar. Ask them each to find a bar to complete the match. Have a child write the number sentence that describes this match on the board. It might be written as:

4 + 1 is 5 or 5 is 4 + 1.

Tell the children that there is another symbol that is often used in place of the "is" in the number sentence. It is the = sign. If the equal sign is used, the number sentence telling about the matching bars is

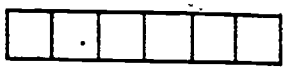
4 + 1 = 5 (read "four plus one equals five") or 5 = 4 + 1.

Challenge the children to use two bars to make as many new matches of the five-unit bar as possible. (1 + 4 = 5, 2 + 3 = 5, 3 + 2 = 5) You may need to help some children by suggesting one of the two bars. Have children write number sentences to describe the matches on the chalkboard.

The children should now be ready for Worksheets 7, 8, and 9, which ask them to use two bars to find matches for a bar of a given number of units. On each worksheet there are exactly enough blanks to record every possible match using two bars. (Thus, the combination 0 + 6 = 6 will not be recorded in this lesson.) Explain that on Worksheets 8 and 9 the picture of the Minnebars to be matched is not the same size as the real bar but has the same number of units.

Worksheet 7
Unit 11 Name_____

Put your 6-unit Minnebar on this paper.
Make as many different matches as you can with two bars.
Write down all the number sentences.



5 + 1 = 6

4 + 2 = 6

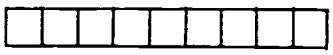
3 + 3 = 6

2 + 4 = 6

1 + 5 = 6

Worksheet 8
Unit 11 Name_____

How can you match a 9-unit bar with two other Minnebars?



8 + 1 = 9

7 + 2 = 9

6 + 3 = 9

5 + 4 = 9

4 + 5 = 9

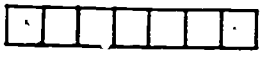
3 + 6 = 9

2 + 7 = 9

1 + 8 = 9

Worksheet 9
Unit 11 Name_____

How can you match a 7-unit bar with two other Minnebars?



6 + 1 = 7

5 + 2 = 7

4 + 3 = 7

3 + 4 = 7

2 + 5 = 7

1 + 6 = 7

Lesson 5: PARTITIONING COUNTERS

This lesson is planned to strengthen understanding of sets and subsets. It provides more practice in finding and recording the sums of small counting numbers.

MATERIALS

for each child:

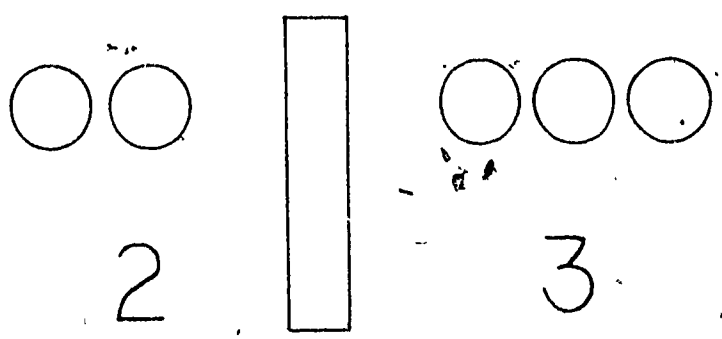
— 9 counters

— ruler

— Worksheets 10, 11, and 12

PROCEDURE

Have each child put 5 counters in a row on his desk. The other counters should be out of his way. Tell the children they will use their rulers to separate the set of counters into subsets. Ask them each to find one of the ways to separate the counters. Have one child draw on the blackboard a picture of the way he divided his counters. It might look like this:

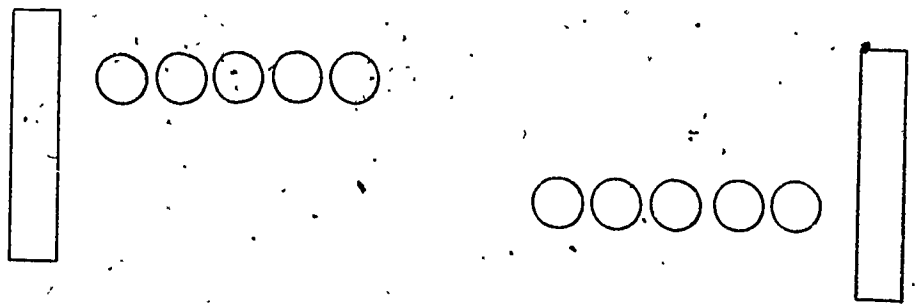


Ask how many counters are on each side of the ruler and have the 2 numerals recorded under the drawing.

Ask a child to complete the number sentence that describes this combination.

$$2 + 3 = 5$$

Now ask other children to record other possible combinations on the chalkboard in the same way. You may have to point out the zero combinations by asking what combinations would result from placing rulers in these positions:

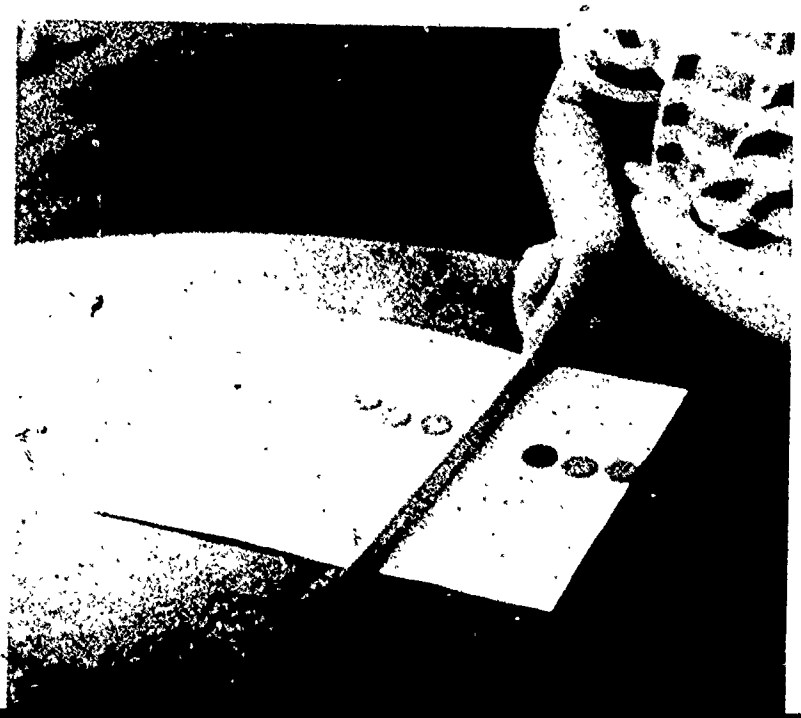


Have the children record all these matches in a column arrangement.

2 + 3 = 5
4 + 1 = 5
3 + 2 = 5
0 + 5 = 5
1 + 4 = 5
5 + 0 = 5

When they are satisfied that they have recorded every one of the sentences, tell them that they have made a "table" which tells all the possible combinations of two subsets totalling five.

When they understand this, have them proceed with Worksheets 10, 11, and 12.



Worksheet 10
Unit 11 Name _____

Make a table for 6 counters.

6	+	0	=	6
5	+	1	=	6
4	+	2	=	6
3	+	3	=	6
2	+	4	=	6
1	+	5	=	6
0	+	6	=	6

Make a table for 4 counters.

4	+	0	=	4
3	+	1	=	4
2	+	2	=	4
1	+	3	=	4
0	+	4	=	4

Worksheet 11
Unit 11 Name _____

Make a table for 2 counters.

0	+	2	=	2
1	+	1	=	2
2	+	0	=	2

Make a table for 8 counters.

0	+	8	=	8
1	+	7	=	8
2	+	6	=	8
3	+	5	=	8
4	+	4	=	8
5	+	3	=	8
6	+	2	=	8
7	+	1	=	8
8	+	0	=	8

Worksheet 12
Unit 11 Name _____

Make a table for 1 counter.

0	+	1	=	1
1	+	0	=	1

Make a table for 9 counters.

0	+	9	=	9
1	+	8	=	9
2	+	7	=	9
3	+	6	=	9
4	+	5	=	9
5	+	4	=	9
6	+	3	=	9
7	+	2	=	9
8	+	1	=	9
9	+	0	=	9

Lesson 6: PRACTICE PROBLEMS

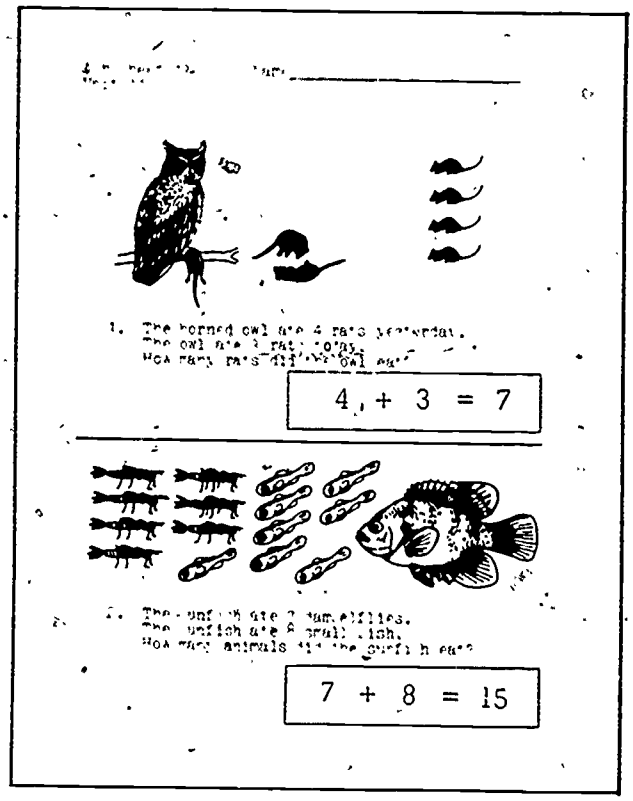
This lesson provides more practice in finding and writing sums.

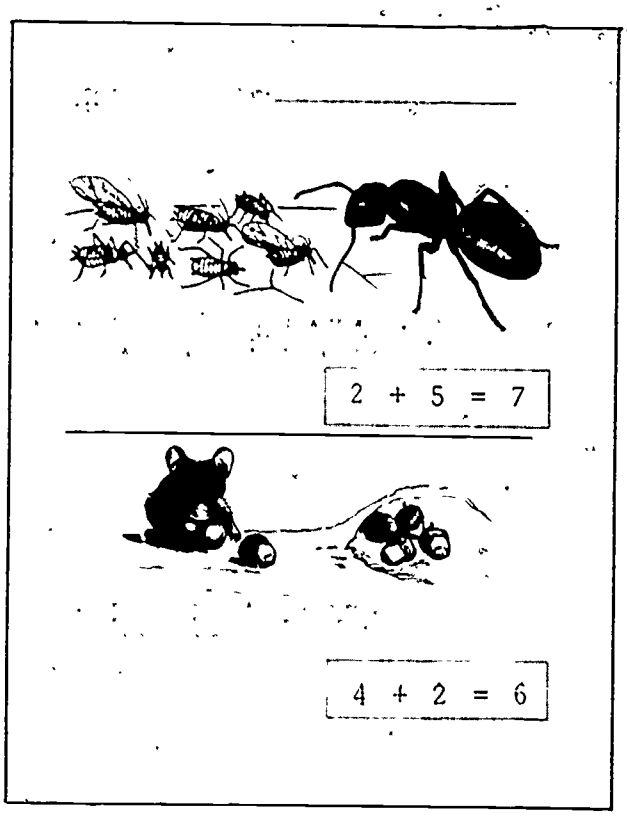
MATERIALS

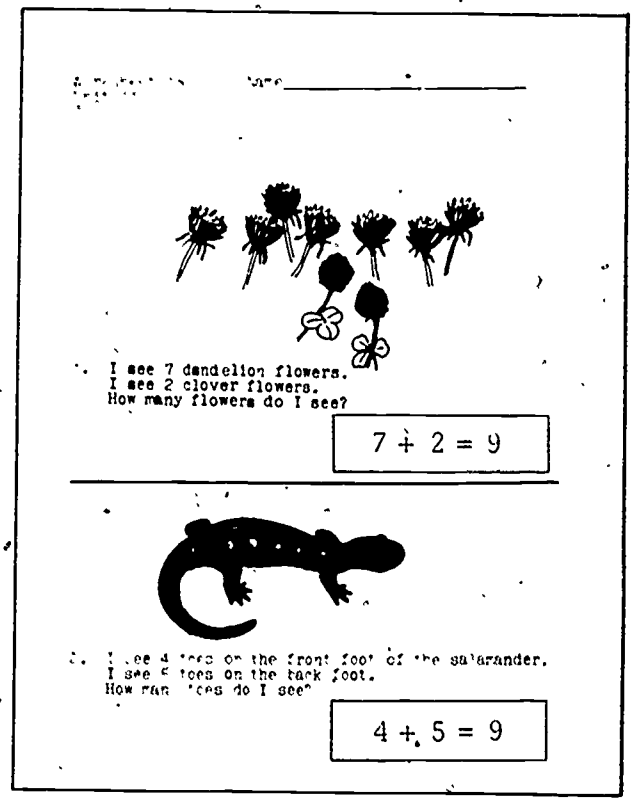
— Worksheets 13, 14, and 15

PROCEDURE

The material of the lesson is completely presented by Worksheets 13, 14, and 15. You will probably need to read the worksheets to the children. Be certain that each child answers the question "How many" with an equation (such as 3 + 2 = 5, or 5 = 3 + 2) and not with a single number (such as 5).







Lesson 7: COMMUTATIVITY OF ADDITION

The addition operation is commutative. This means that two numbers may be added in either order and the sum is the same in both. For example, 5 + 3 = 3 + 5. The idea of commutativity, but not the word, is introduced here. Since this concept will be repeatedly treated in this unit and later units, complete mastery is not required in this lesson.

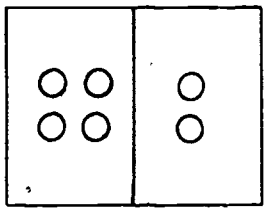
MATERIALS

for each child:

- 6 counters, or small objects
- 9" x 12" piece of construction paper
- Worksheets 16 and 17

PROCEDURE

Give each child a piece of construction paper. Have him fold it down the middle, unfold it, and make a line on the fold:



Ask each child to place 4 counters on the left side and 2 on the right side. Have someone write the names of the combined set on the board:

4 + 2 is 6

Ask each child to turn his paper so the set of 2 is on the left side and the set of 4 on the right. Have another child write the new number sentence:

2 + 4 is 6

Help them to see that the number of counters in both cases is the same and, therefore,

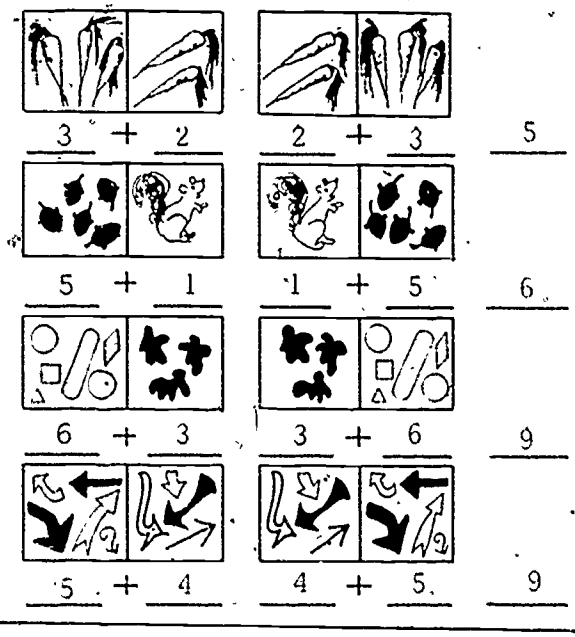
4 + 2 is 2 + 4

The children could also leave the paper and counters on their desks and walk around to view them from the opposite side.

Repeat until the class is ready for Worksheets 16 and 17.

Worksheet 16
Unit 11 Name_____

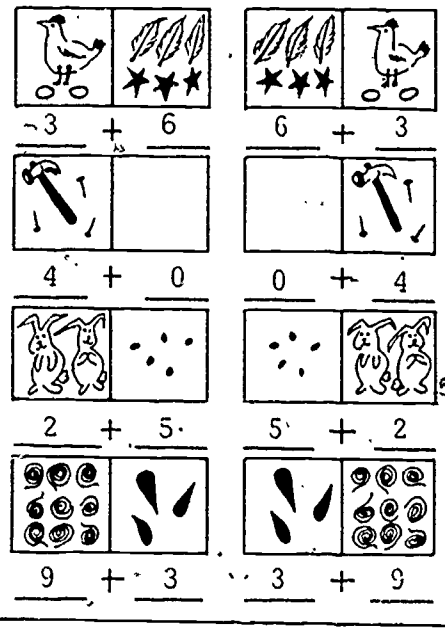
Write a name for the number in each picture. Then write a new name for the number in both sets of pictures.



3 + 2	2 + 3	5
5 + 1	1 + 5	6
6 + 3	3 + 6	9
5 + 4	4 + 5	9

Worksheet 17
Unit 11 Name_____

Write a name for the number in each picture. Then write a new name for the number in both sets of pictures.



3 + 6	6 + 3	9
4 + 0	0 + 4	4
2 + 5	5 + 2	7
9 + 3	3 + 9	12

Lesson 8: JOINING THREE SETS AND ASSOCIATIVITY

This lesson introduces the associative principle that states $(a + b) + c = a + (b + c)$, where a, b, and c represent any numbers. The expression $(a + b) + c$ means that the sum $a + b$ is first found and then c is added to this sum, whereas in $a + (b + c)$ the sum $b + c$ is found first and then the sum is added to a. For example, $(3 + 4) + 2 = 7 + 2 = 9$ and $3 + (4 + 2) = 3 + 6 = 9$.

It is this principle that gives meaning to the expression $a + b + c$, for it says that we may add any three numbers by grouping and adding them as we wish. Although it is desirable to present this lesson to the children, some may not master the concept at this time.

MATERIALS

- to stand on chalkboard ledge:
  - set of 6 objects cut from construction paper
  - set of 7 objects cut from construction paper

PROCEDURE

- Separate the set of 6 objects into subsets of 3, 2, and 1. Stand the subset of 3 on the chalkboard ledge. The subsets of 2 and 1 should be placed on the window sill and your desk (or any convenient location so they are easily seen).

- Have one child bring the set of 2 from the window sill and place it to the left of the set of 3 on the chalkboard ledge. Have the name of the number of objects in the joined set written as 2 + 3 over the objects. Tell the children that before the third set is joined with the others, parentheses will be placed around 2 + 3 to show that the sets containing 2 and 3 were put together first.

(2 + 3)

●● ●●●

Ask another child to bring the set of 1 and place it to the right of the other sets. Have a child write the number of this set above it. (1) Have him place a + sign between the sum (2 + 3), which is a single number, and the 1.

(2 + 3) + 1

●● ●●● ●

(2 + 3) + 1 is one name for the number of objects in the joined set. Ask for another name. (6) Have the sentence (2 + 3) + 1 = 6 written somewhere else on the board.

Erase the board except for (2 + 3) + 1 = 6. Place the subsets as they were at the beginning of the lesson. Repeat the above procedure changing the order of joining the sets. First have the set containing 1 joined to the set containing 3, and then have the set of 2 placed to the left of these two. This will yield:

2 + (3 + 1)

●● ●●● ●

2 + (3 + 1) is one name for the number of objects in the joined set. Ask for another name. (6) Have the sentence 2 + (3 + 1) = 6 written under the previously recorded (2 + 3) + 1 = 6.

From these two number sentences we see that here it didn't matter which sum was found first. Because this is always the case, we often don't write any parentheses.

Repeat the entire lesson with a set of 7 objects divided into subsets of 1, 4, and 2.

Lesson 9: COMMUTATIVITY AND ASSOCIATIVITY WITH MINNEBARS

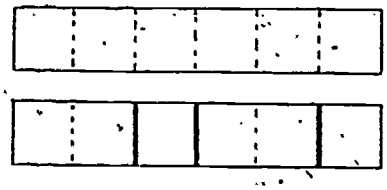
This lesson provides informal experiences with the commutative and associative principles of addition. These principles are used, but not specifically mentioned, in activities with Minnebars. If children wish to use parentheses to show which sums were found first, they may. However, the last lesson showed that this is not necessary.

MATERIALS

- set of Minnebars for each child
- paper

PROCEDURE

A. Have each child place a dark green 6-bar on his desk. Ask the children to match the 6-bar in as many ways as possible, using any number of bars. Have them record the matches on a record sheet. For example, one match might be:



This is recorded as $6 = 2 + 1 + 2 + 1$.

Ask questions similar to these.

DOES IT MAKE ANY DIFFERENCE WHICH OF THE SHORTER BARS IS PUT DOWN FIRST? (no)

CAN THE ORDER OF TWO OF THE SHORTER BARS BE CHANGED AND STILL HAVE THE CORRECT MATCH? (yes)

B. Repeat with other bars such as 4, 3, and 5.

Game: Parts 11 and 12 of the "Train Game" from Section 6 would be appropriate here.

SECTION 2. INTRODUCTION TO SUBTRACTION

This section contains an introduction to subtraction in terms of sets and presents activities showing the relation between addition and subtraction of counting numbers.

BACKGROUND

Subtraction can be interpreted in terms of the number of members in various sets; it is appropriate for the children to work with collections of actual objects in this introduction. When one counting number is subtracted from another, the resulting number is the difference between the two. When a set with a certain number of members is removed from a set with at least as many members, the number of members in the remaining set is equal to the difference between the number of members in the first two sets.

Another interpretation of the difference, a - b, of any two numbers is that the difference is the number to be added to b to obtain a. For example, 7 - 3 is the number that must be added to 3 to obtain 7. This is the concept of the "missing addend" problems, for example 3 + ____ = 7.

The expression a - b, which is read "a minus b," shows that b is subtracted from a but also is the name of a unique single number. It is important for the children to realize this. For example, 5 - 3 is a name for a unique number, 2, and also says that 3 is subtracted from 5.

The inverse relation between addition and subtraction is considered in this section. Suppose we start with 5, add 2 to it, and then subtract 2 from the sum. The result is the original number 5. This may be written as (5 + 2) - 2 = 5. In language suitable for a child, this may be stated as "Subtraction undoes addition, and addition undoes subtraction."

Lesson 10: FINDING THE MISSING ADDEND

This lesson presents the problem of finding a missing addend. This problem is related to subtraction, and is used to introduce subtraction. The child should simply find the missing addend by counting procedures.

MATERIALS

- 10 counters of any kind for each child
- Worksheets 18, 19, 20, 21.

COMMENT

The term "addend" can be introduced here by discussing an example with the children. In the number sentence 2 + 1 = 3, 2 is one addend, 1 is the other addend. Recall that both 3 and 2 + 1 are called the sum of 2 and 1.

PROCEDURE

Use the following story to show the class how to use counters to find a missing addend.

(1) Mary had 4 kittens.
She fed 2 kittens.
How many more does she have to feed?

On the board write 2 + □ is 4.

Say, "Place the two kittens that she fed on your desk using counters. Now, if these are the two she has fed and she has four kittens, how many more must she feed? Use your counters and find the others."

Use the same procedure with problems such as the following to give the children additional practice.

(2) I have 3 books.
I need 5 books in all.
How many more books must I get?

3 + □ is 5.

Lesson 11: STORY PROBLEMS IN SUBTRACTION

This lesson provides activities in which a child can see the need for subtraction. He will remove a subset from a set and count the members of the remaining set. No technical words or symbols are introduced until the next lesson.

MATERIALS

— 10 counters for each child

PROCEDURE

Each child should have 10 counters. Tell the children the following "story."

Paul had 8 pennies.
He spent 5 pennies for candy.
How many pennies did he have left?

Ask the children to suggest how to show what happened by using counters. If necessary, guide them through the following steps.

PLACE 8 COUNTERS FOR THE 8 PENNIES ON YOUR DESK.
TAKE AWAY 5 COUNTERS FOR THE 5 PENNIES HE SPENT.
HOW MANY PENNIES ARE LEFT? (3)

Repeat with problems similar to the following ones. Ask the children to make up story problems of their own. Be sure to use one or two problems where the resulting difference is 0.

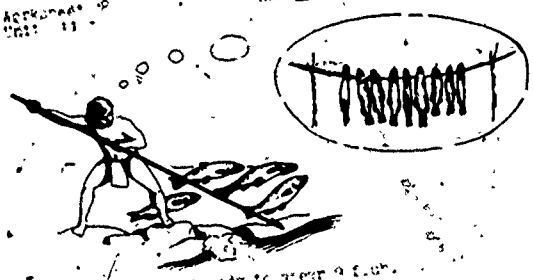
Doug had 9 airplanes.
He gave 3 airplanes to his friends.
How many airplanes did Doug have left?

Mother baked 3 cakes.
Ann's club ate 3 cakes.
How many cakes were left for Mother?

Bob caught 10 fish.
He dropped 4 of them into the lake.
How many fish did he have left to take home?

Worksheet 18
Unit 11

Name_____



1. The fishing boy wants to catch 9 fish.
He has caught 4 fish.
How many fish does he still need to catch?

4 + 5 = 9

2. There were 13 turtle eggs in the ground.
3 have hatched.
How many should still hatch?

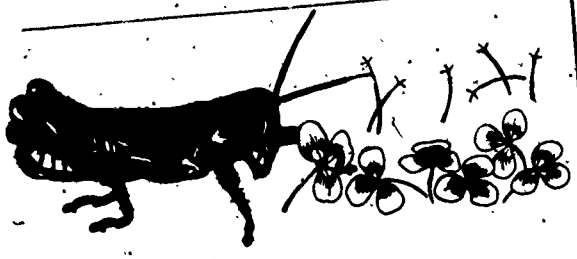
3 + 10 = 13

Worksheet 19
Unit 11

Name_____

1. The grouse laid 9 eggs.
4 have already hatched.
How many should still hatch?

4 + 5 = 9



2. The grasshopper will eat 12 clovers.
He eats 6 of them.
How many will he eat later?

6 + 6 = 12

Worksheet 20
Unit 11

Name_____

Find the missing addend.

1 + [6] is 7	[3] + 3 = 6
2 + [2] is 4.	[1] + 3 = 4
4 + [2] is 6	[0] + 2 = 2
2 + [3] is 5	[3] + 5 = 8
4 + [0] is 4	[2] + 3 = 5
5 + [2] is 7	[5] + 3 = 8
7 + [1] is 8	[1] + 8 = 9
1 + [2] is 3	[1] + 1 = 2
9 + [0] is 9	[0] + 3 = 3

Worksheet 21
Unit 11

Name_____

Find the missing addend.

4 + [3] = 7	1 + [9] = 10
[2] + 6 = 8	[1] + 9 = 10
[1] + 7 = 8	8 + [0] = 8
2 + [1] = 3	[6] + 2 = 8
2 + [4] = 6	[0] + 6 = 6
6 + [3] = 9	[3] + 4 = 7
2 + [5] = 7	9 + [0] = 9
[2] + 7 = 9	5 + [4] = 9
[6] + 3 = 9	[1] + 8 = 9

Lesson 11: STORY PROBLEMS IN SUBTRACTION

This lesson provides activities in which a child can see the need for subtraction. He will remove a subset from a set and count the members of the remaining set. No technical words or symbols are introduced until the next lesson.

MATERIALS

— 10 counters for each child

PROCEDURE

Each child should have 10 counters. Tell the children the following "story."

Paul had 8 pennies.
He spent 5 pennies for candy.
How many pennies did he have left?

Ask the children to suggest how to show what happened by using counters. If necessary, guide them through the following steps.

PLACE 8 COUNTERS FOR THE 8 PENNIES ON YOUR DESK.
TAKE AWAY 5 COUNTERS FOR THE 5 PENNIES HE SPENT.
HOW MANY PENNIES ARE LEFT? (3)

Repeat with problems similar to the following ones. Ask the children to make up story problems of their own. Be sure to use one or two problems where the resulting difference is 0.

Doug had 9 airplanes.
He gave 3 airplanes to his friends.
How many airplanes did Doug have left?

Mother baked 3 cakes.
Ann's club ate 3 cakes.
How many cakes were left for Mother?

Bob caught 10 fish.
He dropped 4 of them into the lake.
How many fish did he have left to take home?

Lesson 12: INTRODUCING THE "A - B" NOTATION

In this lesson the children find the number of members in a difference set and write an appropriate subtraction equation.

MATERIALS

- flannel board with several sets of up to ten objects each
- Worksheets 22, 23, and 24

PROCEDURE

Repeat each of the activities A and B several times with different number combinations until you feel the children are ready to use the worksheets.

A. Ask five children by name to come to the front of the room. Ask how many there are. Write the correct numeral on the board. Ask three of them by name to go back to their seats. Ask how many went away from the front of the room. Record this next to the previous numeral. Explain that the set of children remaining at the front is the difference set. Put a minus sign between the 5 and 3 on the board. Tell the children that 5 - 3 is one name for the number of members in the difference set. Ask if someone can give another name for the difference. (2) Ask if someone can write the number sentence showing these two names on the board. (5 - 3 = 2 or 5 - 3 is 2)

B. Place seven objects on the flannel board. Ask the children questions similar to the following:

HOW MANY OBJECTS ARE IN THE SET ON THE FLANNEL BOARD?

CAN YOU WRITE THE NUMBER ON THE BOARD?

WILL SOMEONE REMOVE (OR TAKE AWAY) A SUBSET FROM THE SET ON THE FLANNEL BOARD?

HOW MANY DID HE REMOVE?

CAN SOMEONE WRITE THIS NUMBER ON THE BOARD?

HOW MANY ARE IN THE DIFFERENCE SET?

CAN SOMEONE WRITE THE NUMBER SENTENCE THAT GIVES TWO NAMES FOR THE DIFFERENCE?

C. More subtraction practice is provided by Worksheets 22, 23, and 24. Read the directions on Worksheet 22 with the children. Guide them through the first problem in a manner similar to this:

WRITE THE NUMERAL FOR THE TOTAL NUMBER IN THE SET UNDER THE PICTURE TO THE LEFT. (4)

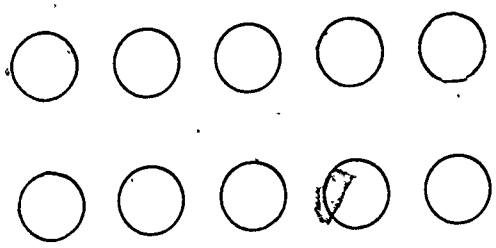
THE MINUS SIGN TELLS THAT A SUBSET IS TAKEN AWAY. AFTER THE MINUS SIGN, WRITE THE NUMERAL THAT TELLS HOW MANY WERE TAKEN AWAY. (4 - 3)

THE NUMBER LEFT IS GIVEN BY 4 - 3. TO THE RIGHT OF THE EQUAL SIGN, WRITE ANOTHER NAME FOR THE NUMBER REMAINING. (4 - 3 = 1)

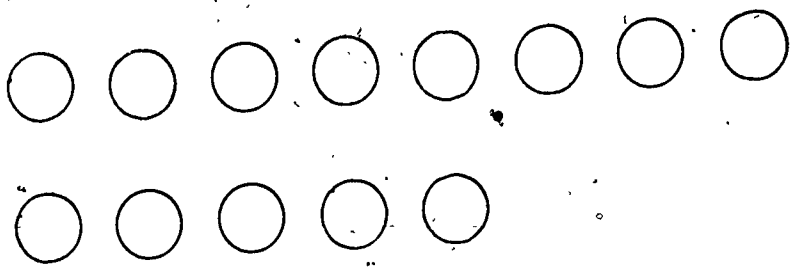
As soon as the children are ready, let them finish by themselves. On Worksheets 23 and 24 the children will need to write in the minus sign.

The children may like to make up stories to go with the pictures.

A. Have the children set out two rows of five counters on their desks.



Ask each child to add three counters to one row.



HOW MANY COUNTERS MUST YOU TAKE AWAY TO HAVE THE SAME NUMBER IN EACH ROW AGAIN? (3)

This activity could also be done with Minnebars.

B. Show the children a bag in which you have put 13 crayons and tell them how many are in the bag. Have a child put 8 more crayons in the bag. Ask someone to take out enough crayons so that 13 crayons are in the bag once again.

Then ask a child to remove 4 crayons from the bag. Ask how many crayons must be added to have 13 crayons in the bag again.

Repeat these activities, having the children write number sentences on the chalkboard that describe the two situations.

$$(13 + 8) - 8 = 13$$

$$(13 - 4) + 4 = 13$$

Lesson 13: WRITING SUBTRACTION SENTENCES

This lesson reinforces the concepts of subtraction presented in the previous lesson.

MATERIALS

— 10 counters for each child

— Worksheets 25, 26, and 27

PROCEDURE

Each child should have 10 counters, paper and pencil on his desk. Ask each child to spread out the counters in some arrangement on his desk.

HOW MANY COUNTERS DO YOU HAVE ON YOUR DESK? (10) WRITE THE NUMBER DOWN.

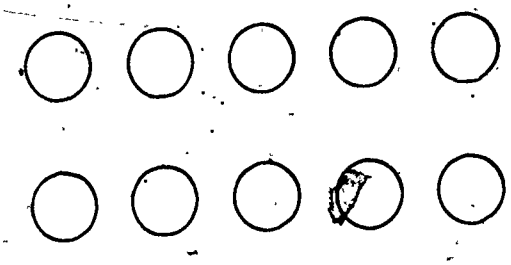
Ask each child to remove (or take away) three counters and write that down.

CAN YOU WRITE A NUMBER SENTENCE THAT TELLS HOW MANY YOU STARTED WITH, HOW MANY YOU TOOK AWAY, AND HOW MANY ARE LEFT? (10 - 3 = 7)

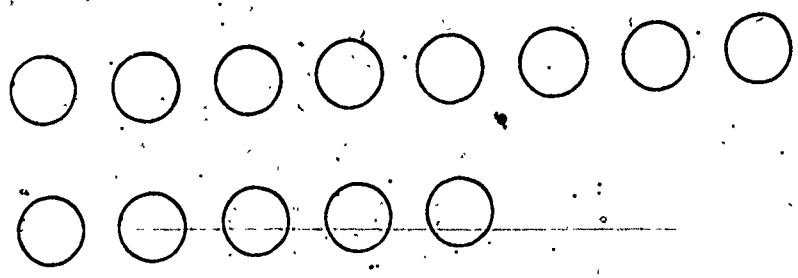
Step-by-step guidance of some children may be necessary here.

Repeat the procedure with different number combinations until you wish to use the worksheets. The children should be allowed to count the difference sets whenever they wish. From numerous activities of this sort, the children will become familiar with many subtraction number facts while developing an understanding of the operation involved. Formal memorization of the facts should be delayed until the process is very well mastered. By that time, many of the subtraction relations will have been learned unconsciously.

A. Have the children set out two rows of five counters on their desks.



Ask each child to add three counters to one row.



HOW MANY COUNTERS MUST YOU TAKE AWAY TO HAVE THE SAME NUMBER IN EACH ROW AGAIN? (3)

This activity could also be done with Minnebars.

B. Show the children a bag in which you have put 13 crayons and tell them how many are in the bag. Have a child put 8 more crayons in the bag. Ask someone to take out enough crayons so that 13 crayons are in the bag once again.

Then ask a child to remove 4 crayons from the bag. Ask how many crayons must be added to have 13 crayons in the bag again.

Repeat these activities, having the children write number sentences on the chalkboard that describe the two situations.

$$(13 + 8) - 8 = 13$$

$$(13 - 4) + 4 = 13$$

Lesson 14: THE INVERSE RELATIONSHIP OF ADDITION AND SUBTRACTION

These activities use sets of objects to give the children an idea of the inverse relationship between addition and subtraction. If one adds a number of members to a set and then takes away the same number of members from the set, one reverts to the original number of members in the set. In the same way, if one takes away a certain number of members from a set, adding the same number of members will return the set to its original number of members.

For example, $(8 + 3) - 3 = 8$.

MATERIALS

for each child:

— 13 counters

for the class:

— 21 crayons

— bag

to use on flannel board, figures similar to these:

— 5 apples

— 2 oranges

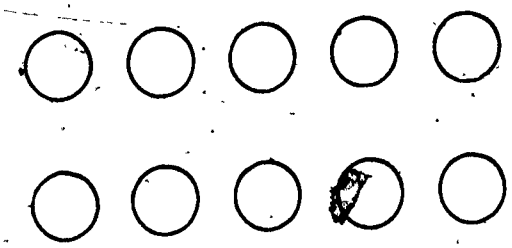
— 4 ducks

— 3 geese

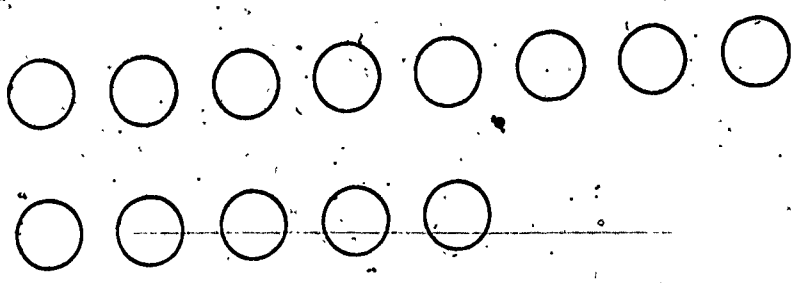
PROCEDURE

Give the children experience with this idea in several different situations. Some possibilities are on the next page. Of course, different figures than those suggested can be used.

A. Have the children set out two rows of five counters on their desks.



Ask each child to add three counters to one row.



HOW MANY COUNTERS MUST YOU TAKE AWAY TO HAVE THE SAME NUMBER IN EACH ROW AGAIN? (3)

This activity could also be done with Minnebars.

B. Show the children a bag in which you have put 13 crayons and tell them how many are in the bag. Have a child put 8 more crayons in the bag. Ask someone to take out enough crayons so that 13 crayons are in the bag once again.

Then ask a child to remove 4 crayons from the bag. Ask how many crayons must be added to have 13 crayons in the bag again.

Repeat these activities, having the children write number sentences on the chalkboard that describe the two situations.

$$(13 + 8) - 8 = 13$$

$$(13 - 4) + 4 = 13$$

C. - Put 5 apples on the flannel board. Ask how many apples there are, then add 2 oranges. Ask the children to think of two names for the new set of fruit and to write them underneath each other on the chalkboard.

7

(5 + 2)

Remove the two oranges and ask someone to write number sentences telling what had happened to the number of apples and oranges. Have them use the names they wrote for the original set of fruit.

7 - 2 = 5

(5 + 2) - 2 = 5

D. Put 4 ducks on the flannel board. Add 3 geese. Have the children again give two names for the new set.

(4 + 3)

7

Take 3 geese away. Have the children complete the number sentences.

(4 + 3) - 3 = 4

7 - 3 = 4

Leaving these number sentences on the board, add the 3 geese again and write the two names.

4 + 3

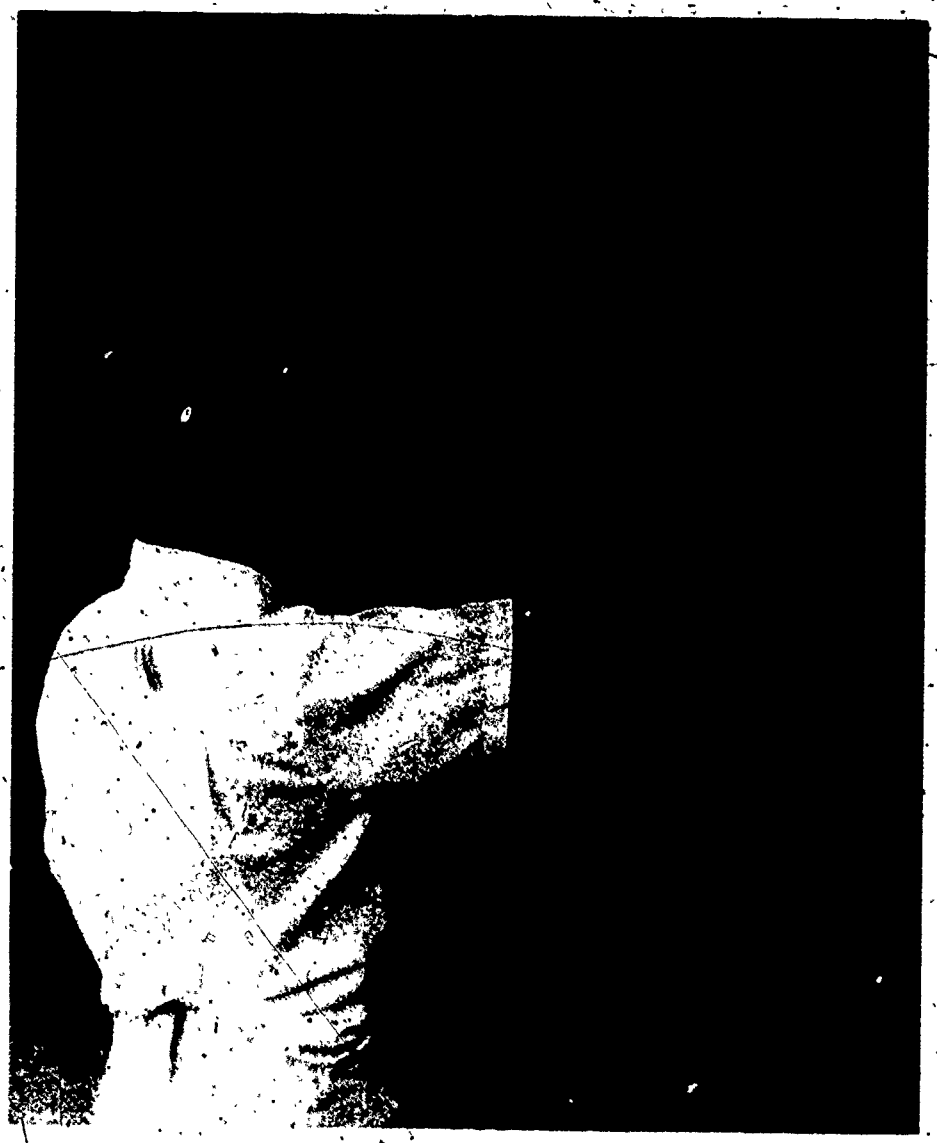
7

Now remove 1 of the ducks and 2 of the geese and have a child complete the number sentences.

$(4 + 3) - 3 = 4$

$7 - 3 = 4$

Tell the children that the number sentences are the same whether 3 geese or 2 geese and 1 duck are removed. In each case the subset removed contained 3 members.



SECTION 3 GROUPING, EVEN AND ODD NUMBERS, ARRAYS

This section is designed to give experiences in grouping members of a set into subsets containing equal numbers of members, to introduce even and odd numbers, and to give experiences in forming arrays of objects and symbols.

VOCABULARY

The following words are used in this section. Although the children should understand and be able to use the words, they should not be expected to define them.

Row: a horizontal line of objects or symbols.

Column: a vertical line of objects or symbols.

Array: a regular arrangement or pattern of objects or symbols.

Rectangular array: an array made up of rows and columns in which each row (or column) contains the same number of members. For example:

```
x x x x
x x x x
x x x x
```

BACKGROUND

Previously the children have counted or constructed sets with 2, 3, 4, ... members, and have compared the number of members of two sets by pairing members of the two sets. At the beginning of this section, the children will separate sets of objects into subsets containing equal numbers of objects, i.e., into equivalent subsets. This experience with grouping will provide a background for the study of the decimal number system, which we ordinarily use.

Even and odd numbers are conveniently introduced by attempting to separate a set into two equivalent subsets. If the number of members in the initial set is even, it is possible to divide the set into two equivalent subsets so that every member of one subset can be paired with one member of the other subset. If the number of members in the initial set is odd, it will not be possible to separate the set into two equivalent subsets. Without discussing division, the idea is introduced that even numbers are divisible by two and odd numbers are not.

While it is sometimes all right to have the subsets arranged in a random pattern, for some activities it is useful or necessary to have them in a regular pattern. Such a regular arrangement is called an array.

There are various special arrays. Two examples of them are the triangular array and the rectangular array: The elements of a triangular array might be arranged in these ways:

```
x x x
 x x
  x
```

```
x
x x
x x x
x x x x
x x x x x
x x x x x x
x x x x x x x
```

The elements of a rectangular array might be arranged this way:

```
o o o o o
o o o o o
o o o o o
o o o o o
o o o o o
o o o o o
o o o o o
o o o o o
```

Because only the rectangular array is used in the first grade MINNEMAST units, the children will use the single word "array" to mean "rectangular array." However, the teacher should use the term "rectangular array" often enough so that the children learn its meaning.

A child will count the total number of objects in an array, count the number in each row (soon seeing that he needs to count the objects in only one row, as all rows contain an equal number), and count the number of rows. For example, he will discover by counting that 3 rows of 5 each contain 15 objects and that a set of 15 objects can be separated into 3 subsets containing 5 each.

Experiences with many different arrays will provide a background for an interpretation of multiplication developed in second grade. In this unit, words such as "multiply" and "factor" are not used, and the children should definitely not be required to memorize multiplication facts.

In this section the children will also have direct experience with the commutative law of multiplication. This law states that it does not matter in what order two numbers are multiplied -- i.e., $2 \times 6 = 6 \times 2$. The term "commutative law of multiplication" will not be used with the children. They will manipulate arrays and observe that, for example, the following arrays contain the same number of elements.

```
x x        x x x x x x
x x        x x x x x x
x x
x x
x x
x x
```

The children's understanding of the properties of arrays will be valuable when they do encounter formal multiplication.

Lesson 15: FORMING GROUPS OF CHILDREN

After the activities in this lesson, the children should be able to form groups of specified size. They should get a feeling for, but probably not verbalize, the fact that one can form fewer large groups than small groups from a given number of children. Much counting practice is also provided.

PROCEDURE

Have each child find a partner. If one child is without a partner, appoint him to count and report the number of pairs. If all have partners, you do the counting. Write the results on the board in the form:

12 sets of two and 1 leftover

Have the children arrange themselves in groups of three. Again have the groups of three counted and recorded in the form:

8 sets of three and 1 leftover

Repeat with groups of 4 and 5. Direct the grouping enough so that the same child is not left out of a complete group more than once.

Lesson 16: GROUPING A SET OF COUNTERS INTO EQUIVALENT SUBSETS

This lesson provides more experiences in forming groups of specified size. It also introduces the idea of one-half of the members of a set.

MATERIALS

— 20 counters for each child

— Worksheets 28, 29, 30, and 31

PROCEDURE

A. Each child should leave 20 counters on his desk. Ask the children to separate these 20 counters into 4 subsets that have the same number of members (i.e., find 4 equivalent subsets). Let the children discover their own ways of doing this. When they have finished, ask:

HOW MANY ARE THERE IN EACH SUBSET? (5)

HOW MANY SUBSETS OF FIVE ARE THERE? (4)

Follow the same procedure for separating the initial set into 2 and 5 subsets.

Propose finding 3 equivalent subsets using all the counters. Let the children discover that this is impossible.

Although arrays have not been introduced yet, it is likely that some children will develop strategies that involve arranging the counters into regular patterns or arrays.

B. Now ask the children to separate the 20 counters into subsets each containing two counters. Tell each child to count the subsets and to report how many groups of two he has found. (10) Ask how many counters are in the whole set. (20)

Continue by having the counters grouped into subsets containing 3, 4, and 5 each. Each time have the number of subsets counted and reported. When subsets of three are used, the results should be reported as "6 subsets (or groups) of three with 2 leftovers."

C. Ask the children to separate the 20 counters on their desks into subsets containing 10 each. Ask questions similar to this:

HOW MANY COUNTERS ARE LEFTOVER? (none)

HOW MANY SUBSETS DO YOU HAVE? (two)

Have each child place one subset of 10 counters at the right of his desk and the other near the left.

IF YOU PICK UP THE LEFT SUBSET OF COUNTERS, WHAT PART OF YOUR COUNTERS WILL YOU HAVE IN YOUR HAND? ("one-half" or "half")

If no one volunteers the above answer, give the answer. In any case write $\frac{1}{2}$ on the board and explain that the 10 counters picked up are 1 of 2 subsets containing equal numbers of counters.

Repeat the activity starting with 10 counters and separating them into 2 subsets of 5 each. This time ask:

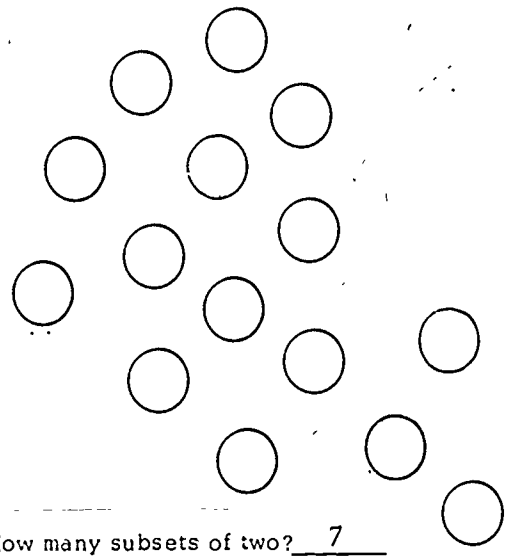
IS 5 ONE-HALF OF 10? (yes)

WHY? (10 counters can be separated into two subsets each containing 5 counters.)

D. Worksheets 28, 29, 30, and 31 give practice in grouping and finding $\frac{1}{2}$ of the members in a set, and also will show whether any children will need additional work.

Worksheet 28
Unit 11 Name_____

Draw curves around subsets of two



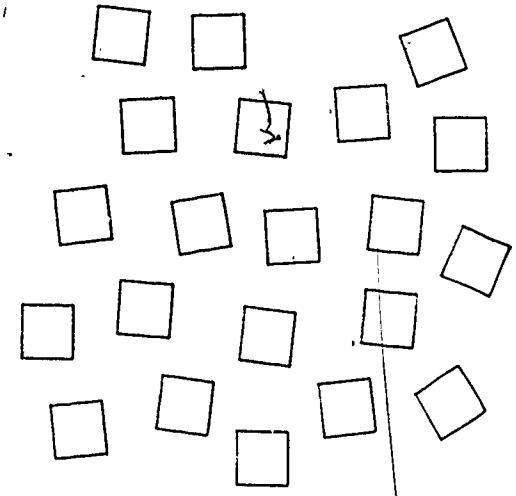
How many subsets of two? 7

How many leftover? 1

How many in the whole set? 15

Worksheet 29
Unit 11 Name_____

Draw curves around subsets of three.



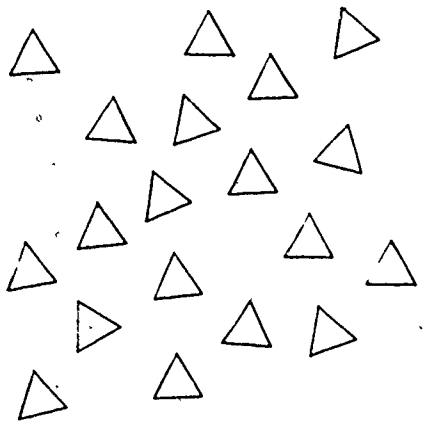
How many subsets of three? 7

How many leftover? 0

How many in the whole set? 21

Worksheet 30
Unit 11 Name_____

Draw curves around subsets of four.



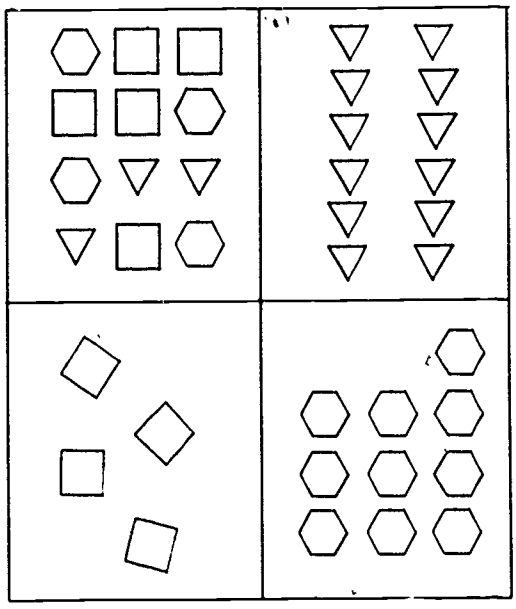
How many subsets of four? 4

How many leftover? 3

How many in the whole set? 19

Worksheet 31
Unit 11 Name_____

Draw a curve around $\frac{1}{2}$ of the objects in each set.



Lesson 17: INTRODUCING "EVEN" AND "ODD"

This lesson introduces even and odd numbers by pairing members of a set. In Unit 13 there will be activities with even and odd numbers represented on the number line.

An even number is divisible by 2, an odd number is not. This is not explicitly stated to the children but is presented through activities with sets. If a set can be separated into two equivalent subsets, it contains an even number of members. If a set cannot be separated into two equivalent subsets, the set contains an odd number of members. (Recall that equivalent sets contain equal numbers of members. Equivalent sets of objects are not equal sets because they contain different objects. That is, a set of four apples is equivalent -- but not equal -- to another set of four apples. These sets have an equal number of members, but the apples in the two sets are not the same apples.)

MATERIALS

— flannel board with about 15 figures, mostly different

— 16 counters for each child

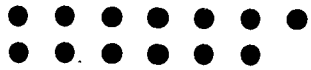
— Worksheets 32A, 32B, 33A, and 33B

PROCEDURE

A. Each child should have 13 counters on his desk. Suggest that the class pretend the counters are children getting ready to play a game.

CAN TWO TEAMS OF THE SAME NUMBER OF PLAYERS BE CHOSEN? (no)

Let each child arrange his counters to find out. He may discover that a convenient method of attempting to construct two equivalent sets is to form a row (or column) of pairs of counters, and then to take all on the top as one set and all on the bottom as the other. Of course, with 13 counters, not all can be paired. For example:



The leftover one is the "odd" counter. Because of this, 13 is called an odd number.

Repeat the above procedure with 16 counters. If your class is ready, drop the "team" reference and ask them if they can separate a set of 16 counters into two equivalent subsets. This time they should see that all the counters can be paired. The subsets "come out even." For this reason 16 is called an even number. It should be noted that each of the two subsets contains one-half of the counters.

B. Have a child put 6 objects on the flannel board. (For example, 3 ducks, 2 apples, and a flag.) Write the numeral 6 on the chalkboard. Have another child move the figures so that they are obviously paired, but not necessarily in a regular arrangement. Say:

> WHEN MARY PAIRED THESE 6 OBJECTS, THEY "CAME OUT EVEN." THERE WAS NONE LEFTOVER. A SET OF 6 OBJECTS CAN BE SEPARATED INTO TWO SUBSETS EACH CONTAINING THE SAME NUMBER OF OBJECTS. 6 IS AN EVEN NUMBER.

Have a child pair the 6 objects another way. The children should realize that the result will be the same.

Have one child clear the flannel board and another one put up 9 objects. (Avoid using a set with just one different object or it will be called the "odd" one.) Write the numeral 9 on the board. Have another child pair these objects.

> WHEN BILL PAIRED THESE 9 OBJECTS, ONE WAS LEFT WITHOUT A PARTNER. THIS LEFTOVER ONE IS CALLED THE "ODD" OBJECT. A SET OF 9 OBJECTS CANNOT BE SEPARATED INTO TWO SUBSETS BECAUSE 9 IS AN ODD NUMBER.

You might point out that each subset does not contain half of the objects.

Have another child pair the 9 objects another way. The children should realize that the result will be the same.

C. Have each child spread 6 counters on his desk and put the others aside. Ask:

IS 6 EVEN OR ODD? (6 is even because each counter can be paired with another with no leftover counter.)

Without moving the 6 counters, have each child put a seventh counter on his desk. Ask:

IS 7 EVEN OR ODD? WHY? (Odd, because the seventh counter can't be paired with another counter.)

From this experience (repeated as you wish), some children may make the correct generalization that an even number plus one has an odd-numbered sum.

Repeat the activity by starting with an odd number of counters, say 7. Have each child add one counter to the set and determine that 8 is even. Again, after varying numbers of trials, most children will understand the rule that an odd number plus one has an even-numbered sum.

The idea that the sequence of counting numbers alternates odd, even, odd, even, ... will be seen by many children at this time. The concept will be considered again in Unit 13.

As an enrichment activity for interested children, you can encourage the discovery in a similar manner of rules such as those given in an abbreviated form below.

even + even is even
odd + odd is even
even + odd is odd
even - even is even
even - odd is odd

Lesson 20: CONSTRUCTING ARRAYS

In this lesson the children construct rectangular arrays of objects. They will find that with a given initial set only certain rectangular arrays can be made without "leftovers." For example, if the initial set has 15 members, an array containing 3 rows can be made, but not one containing 4 rows. (A particular rectangular array can be constructed if the number of the rows, or columns, is a divisor of the number of members in the set. This explanation should not be given to the children.)

MATERIALS

for each child:

— pegboard with 20 or 30 pegs (preferable) or 20 or 30 counters (alternative)
(One suitable type of pegboard is a 12" ceiling tile with golf tees for pegs.)

PROCEDURE

In the description of the procedure, it is assumed that each child has a pegboard. If sufficient pegboards are not available, the children may work in small groups, or substitute similar activities with counters.

A. A pegboard and 15 pegs should be on each child's desk. The other pegs should be in a container or out of sight. Ask each child to make an array of 3 rows with the 15 pegs. Ask how many pegs are in each row of the completed array. Such an array is called a 3 x 5 array (read as "three by five"), where the first number gives the number of rows and the second number may be thought of either as the number of columns or the number of members in each row.

```
x x x x x
x x x x x
x x x x x
```

Worksheet 32A
Unit 11 Name_____

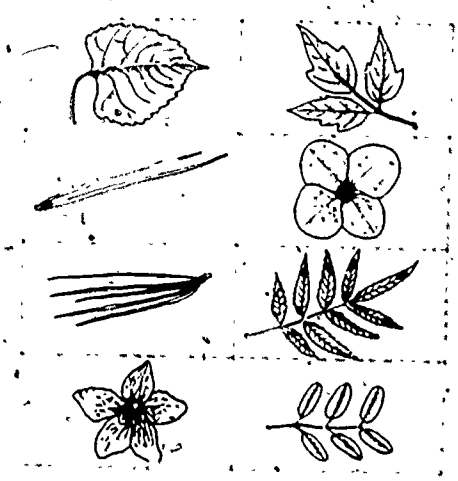
ODD	EVEN

Worksheet 32B
Unit 11 Name_____

ODD	EVEN

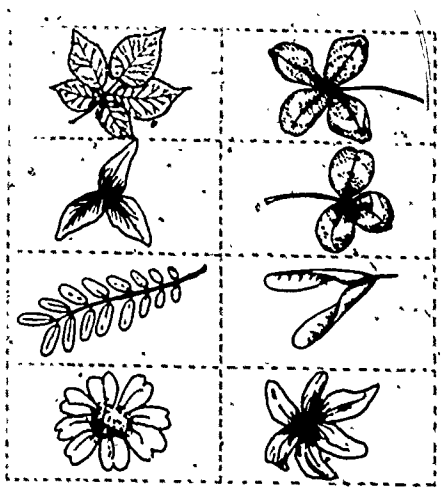
Worksheet 33A
Unit 11 Name_____

Cutouts for Worksheets 32A and 32B



Worksheet 33B
Unit 11 Name_____

Cutouts for Worksheets 32A and 32B



Lesson 18: INTRODUCING ARRAYS

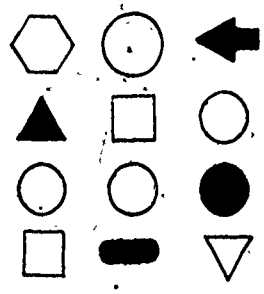
Until this lesson the children arranged subsets of a set in any way they chose. Now they are required to put the subsets into patterns of regular rows and columns, i.e., into rectangular arrays. It is useful to remind the children at intervals that an array is a pattern.

MATERIALS

— flannel board with about 20 objects

PROCEDURE

A. Have a child place 3 objects side by side on the flannel board. Ask another child to put 3 objects right under the first three. Continue until there are 4 rows of 3 objects each.



Explain that this pattern is an array. Show a horizontal line of objects and say that this is called a row. Use the word "horizontal," telling the children it means sideways.

HOW MANY ROWS ARE THERE? (4)

Show a vertical line of objects and say that this is called a column. Use the word "vertical," telling the children it means straight up-and-down.

HOW MANY COLUMNS ARE THERE? (3)

HOW MANY MEMBERS ARE THERE IN THE ARRAY? (12)

Of course, this last answer should be found by counting. Repeat the same procedure with arrays having rows of 4 and 6 members.

B. Have the children look around the room for arrays. When an array is found, have the child who found it count the rows, columns, and total number of members.

Some possibilities are:

desks
windows (perhaps 1 row of 6 windows)
floor tiles
egg carton placed in the room for this lesson
charts placed in the room for this lesson
books on a table arranged for this lesson
fabric patterns

The children may like to tell about arrays they have seen outside of the classroom. Examples are an eye chart, cars in parking lot, shoe rack, food on store shelves, wallpaper.

Lesson 19: COUNTING ARRAYS

In counting the number of set members in each row, the child should soon see that he needs to count only those in one row, as all rows contain the same number.

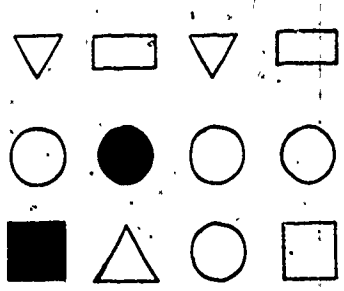
It is expected that the children will count the total number of members of the array; however, if a child can get the right total without counting every member, he should certainly be allowed to use his method.

MATERIALS

— Worksheets 34 and 35

PROCEDURE

A. Draw a few arrays on the board and ask questions about them similar to those on Worksheets 34 and 35. For example, you might use:



HOW MANY OBJECTS IN THE FIRST ROW? (4)

HOW MANY OBJECTS IN THE SECOND ROW? (4)

HOW MANY OBJECTS IN THE THIRD ROW? (4)

HOW MANY OBJECTS IN EACH ROW? (4)

HOW MANY OBJECTS IN THE ARRAY? (12)

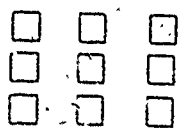
Have one child at a time go to the board, count, and record his results to demonstrate his method to the class.

When you feel the children are sufficiently prepared, have them do Worksheets 34 and 35.

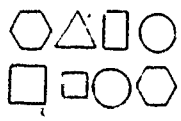
B. (Optional) When the children have counted the arrays, you may give further practice in finding fractional parts of sets. For example, ask the children to draw a curve around $\frac{1}{2}$ of the members in a set. The set chosen should contain an even number of members.

Children who have the ability may enjoy drawing curves around, say, $\frac{1}{4}$ of the members of a set. This set should be one with four rows or four columns.

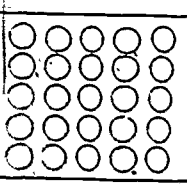
Worksheet 34
Unit II Name_____



How many rows? 3
How many in each row? 3
How many in the array? 9



How many rows? 2
How many in each row? 4
How many in the array? 8

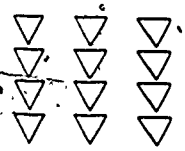
* 

How many rows? 5
How many in each row? 5
How many in the array? 25

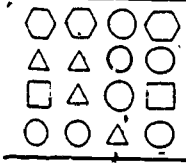
CAT
RAN

How many rows? 2
How many in each row? 3 (1) **
How many in the array? 6 (2) **

Worksheet 35
Unit II. Name_____



How many rows? 4
How many in each row? 3
How many in the array? 12



How many rows? 4
How many in each row? 4
How many in the array? 16

HELLO
HELLO
HELLO

How many rows? 3
How many in each row? 5 (1) **
How many in the array? 15 (3) **

How many rows? 5
How many in each row? 2
How many in the array? 10

*For children who can work with numbers over 20.

**Answers in parentheses apply if the child considers words rather than letters.

Lesson 20: CONSTRUCTING ARRAYS

In this lesson the children construct rectangular arrays of objects. They will find that with a given initial set only certain rectangular arrays can be made without "leftovers." For example, if the initial set has 15 members, an array containing 3 rows can be made, but not one containing 4 rows. (A particular rectangular array can be constructed if the number of the rows, or columns, is a divisor of the number of members in the set. This explanation should not be given to the children.)

MATERIALS

for each child:

— pegboard with 20 or 30 pegs (preferable) or 20 or 30 counters (alternative)
(One suitable type of pegboard is a 12" ceiling tile with golf tees for pegs.)

PROCEDURE

In the description of the procedure, it is assumed that each child has a pegboard. If sufficient pegboards are not available, the children may work in small groups or substitute similar activities with counters.

A. A pegboard and 15 pegs should be on each child's desk. The other pegs should be in a container or out of sight. Ask each child to make an array of 3 rows with the 15 pegs. Ask how many pegs are in each row of the completed array. Such an array is called a 3 x 5 array (read as "three by five"), where the first number gives the number of rows and the second number may be thought of either as the number of columns or the number of members in each row.

```
x x x x x
x x x x x
x x x x x
```

Although the form 3 x 5 is a convenient way to specify an array, it is not required that you use it in this unit. It is always possible to say instead "an array with 3 rows and 5 columns" or "an array with 3 rows of 5 members each."

Ask each child to make a rectangular array containing 3 columns from the 15 pegs.

HOW MANY ARE IN EACH ROW? (3)

HOW MANY ROWS ARE THERE? (5)

WHAT IS THE NAME OF THIS ARRAY? (a 5 x 3 array)

B. Each child should have a pegboard, 12 pegs, paper and pencil on his desk. Explain that the pegs this time represent a group of children that are going to march in a parade. How many ways can they be arranged in rows and columns? After a child has produced an array, say, of 3 rows of 4 each, ask him:

HOW MANY ARE IN EACH ROW? (4)

HOW MANY COLUMNS ARE THERE? (4)

CAN YOU WRITE A NAME FOR THIS ARRAY ON YOUR PAPER? (3 x 4)

CAN YOU MAKE ANOTHER RECTANGULAR ARRAY USING ALL 12 PEGS? (yes)

HOW MANY ARRAYS CAN YOU MAKE USING ALL 12 PEGS? (six: 1 x 12, 2 x 6, 3 x 4, 4 x 3, 6 x 2, 12 x 1)

The children should record in some fashion all those that they find.

Repeat the above with various numbers of pegs in the initial set: 15, 14, 7, and 13 would be interesting ones to try.

Lesson 21: COMMUTATIVITY WITH ARRAYS

By physically turning arrays the children will have direct experience with the commutative law of multiplication, although the words "commutative" and "multiplication" will not be used with the children.

MATERIALS

for each child:

- 20 counters or 20 pegs
- 9" x 12" construction paper or pegboard
- Worksheets 36 and 37

PROCEDURE

This activity is described for children using counters and construction paper, but pegs and pegboards could also be used in a similar way.

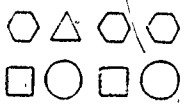
Each child should have counters and one piece of construction paper. Have each child make an array of 2 rows of 6 counters (a 2 x 6 array) on the paper and count the total number of counters. (12)

Have the children carefully turn their papers one quarter turn. Ask someone to describe the array now. (6 rows of 2 counters or a 6 x 2 array) Help them discover, without counting, that 6 rows of 2 counters is also 12. Have them check this by counting. (This shows that 2 x 6 = 6 x 2, but since multiplication has not been introduced, this won't be stressed here.)

Repeat this activity with other arrays such as those having 3 rows of 4, 2 rows of 5, and 3 rows of 3. Note that when the 3 x 3 array is rotated one quarter turn, it is unchanged.

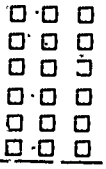
Have the children complete Worksheets 36 and 37.

Worksheet 36
Unit 11 Name_____

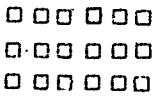


How many rows? 2
How many in each row? 4
How many in the array? 8

How many rows? 4
How many in each row? 2
How many in the array? 8



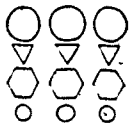
How many rows? 6
How many in each row? 3
How many in the array? 18



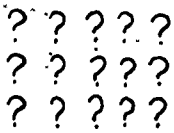
How many rows? 3
How many in each row? 6
How many in the array? 18

Worksheet 37
Unit 11 Name_____

How many rows? 3
How many in each row? 4
How many in the array? 12

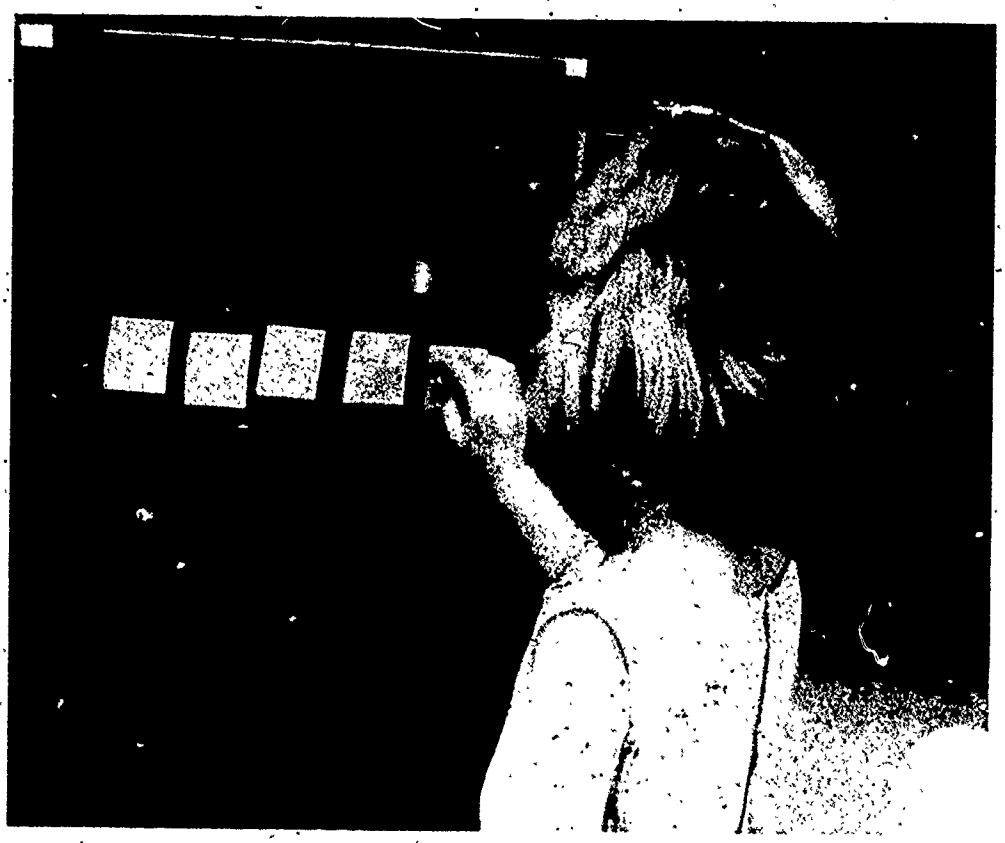


How many rows? 4
How many in each row? 3
How many in the array? 12



How many rows? 3
How many in each row? 5
How many in the array? 15

How many rows? 5
How many in each row? 3
How many in the array? 15



Lesson 22: BUILDING NUMBERS BY REPEATED ADDITION

Counting by 2's, 3's, etc., is given as a use of arrays and as practice in addition. A similar lesson using the number line will be included in a later unit.

MATERIALS

— 20 counters for each child

— Worksheets 38, 39, 40, 41, 42, and 43

PROCEDURE

Have each child follow a procedure similar to the following.

Place 2 counters side by side near the top of a piece of paper. Write the numeral 2 beside them. Place 2 more counters under them.

HOW MANY NOW? (4)

Record this beside the second row. The paper should now look like:

● ● 2
● ● 4

Continue placing pairs of counters and recording until all 20 counters are used.

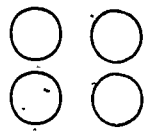
● ● 2
● ● 4
● ● 6
.
.
.
● ● 20

The children can see that, when they read the recorded numbers, they are "counting by 2's", i.e., they are saying a sequence of numbers where each number is 2 more than the preceding number. If practice is needed, have them count aloud "1, 2, 3, 4, 5, 6, ... clapping on the underlined numbers.

Repeat, placing counters in sets of 3's and 5's.

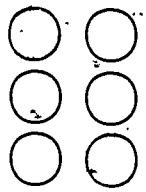
The children should now be prepared for independent work with Worksheets 38 through 43. (The starred problems are for those children who can work with numbers over 20.) They may write numerals in the circles to help their counting.

Worksheet 38
Unit II Name_____



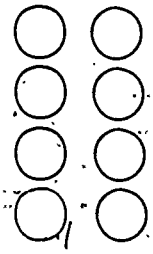
2 + 2 = 4

2 twos are 4



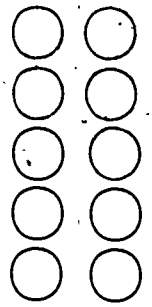
2 + 2 + 2 = 6

3 twos are 6



2+2+2+2 = 8

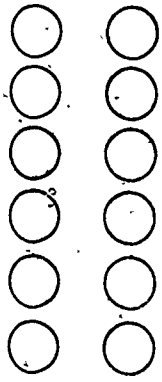
4 twos are 8



2+2+2+2+2 = 10

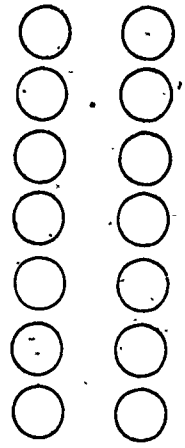
5 twos are 10

Worksheet 39
Unit II Name_____



2+2+2+2+2+2 = 12

6 twos are 12



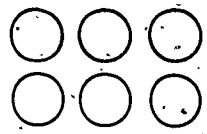
2+2+2+2+2+2+2 = 14

7 twos are 14

Count by twos.

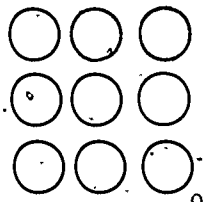
2 4 6 8 10 12 14

Worksheet 40
Unit 11 Name _____



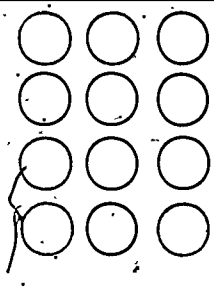
3 + 3 = 6

2 threes are 6



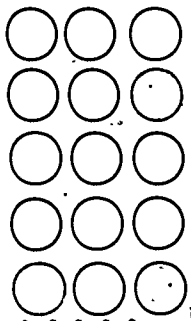
3 + 3 + 3 = 9

3 threes are 9



3+3+3+3 = 12

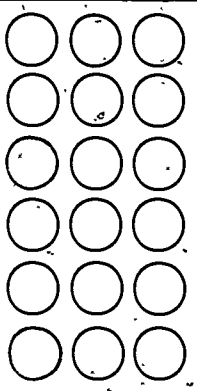
4 threes are 12



3+3+3+3+3 = 15

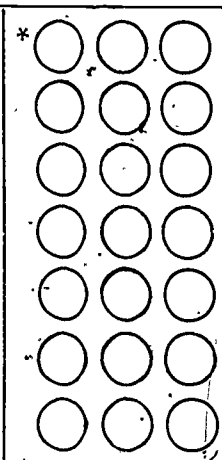
5 threes are 15

Worksheet 41
Unit 11 Name _____



3+3+3+3+3+3 = 18

6 threes are 18



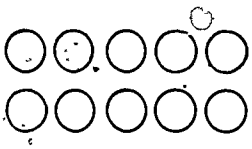
3+3+3+3+3+3+3 = 21

7 threes are 21

Count by threes.

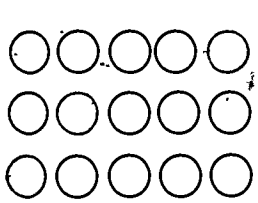
3 6 9 12 15 18 21

Worksheet 42
Unit 11. Name _____



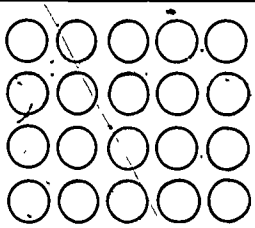
5 + 5 = 10

2 fives are 10



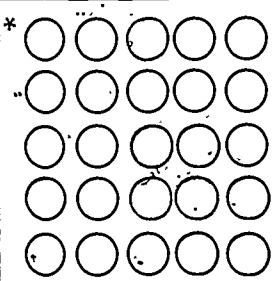
5 + 5 + 5 = 15

3 fives are 15



5 + 5 + 5 + 5 = 20

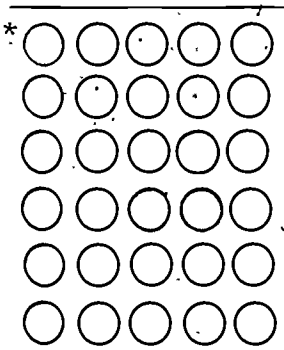
4 fives are 20



5+5+5+5+5 = 25

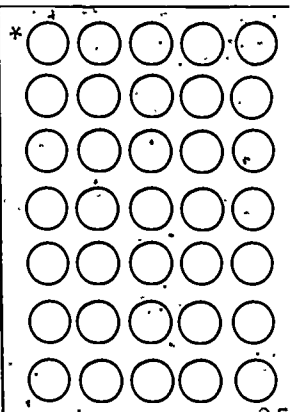
5 fives are 25

Worksheet 43
Unit 11. Name _____



5+5+5+5+5+5 = 30

6 fives are 30



5+5+5+5+5+5+5 = 35

7 fives are 35

Count by fives.

5 10 15 20 25 30 35

Lesson 23: SECRET CODES WITH ARRAYS (OPTIONAL)

This is an optional activity for enrichment. It may be considered a "challenge" activity.

PROCEDURE

The coded message is prepared by printing the words in a message one below the other, so that each letter is an element in a column and row. Add X's to make the array rectangular. For example:

```
M A R Y X X
H A D X X X
A X X X X X
L I T T L E
L A M B X X
```

Record the number of rows. (5) This is the clue. Now write the letters from each column in a line, starting with the left column and going from top to bottom:

M H A L L A A X I A R D X T M Y X X T B X X X L X X X X E X

Give the line of letters and the clue (the number of rows) to the class. Can a child decode the message?

You may have to decode the first message. After seeing the method demonstrated once or twice, pairs of children can code and decode messages together.

SECTION 4 INTRODUCTION TO THE NUMBER LINE

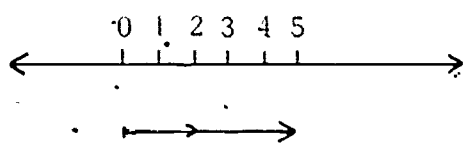
This section introduces the number line, which is a model of the system of real numbers, and previews addition and subtraction on the number line.

BACKGROUND

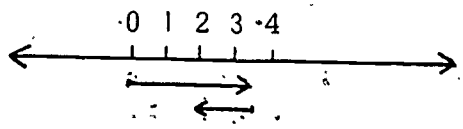
This section contains a story, "Skip's Trip," and related activities that introduce the number line. Skip, the mischievous dog, takes a trip along his street, stopping at various houses located at intervals along the street. The street is visualized as a line and the houses where he stops are numbered in order from his starting point, giving a representation of the number line.

Lessons 24, 25, and 27 involve locating numbers (integers) and counting spaces on a number line. These activities do not require that the integers be evenly spaced. Therefore, although the number lines given to the children will have uniform intervals, the children may correctly draw their own number lines without using rulers to mark the intervals.

Lessons 26, 28, 29, 30 and 31 preview addition and subtraction of numbers represented by specific lengths on the number line. For these activities a number line with uniform intervals must be used. With reference to a particular number line, any length represents a number. Thus, to add two numbers, two lengths may be put together. The figure below illustrates finding the sum $2 + 3 = 5$ by putting a 2-unit length with a 3-unit length.



The operations with lengths apply to all real numbers, not only integers (which are only a subset of all real numbers). For example, in the illustration below, a number close to 1.5 is subtracted from a number close to 3.3. The numbers 1.5 and 3.3 are real numbers but not integers.



To emphasize the important idea that the numbers being added and subtracted are represented by distance on the number line, the children in this unit use either Minnebars or strips of paper to show the lengths. There will be much more addition and subtraction on the number line in Unit 13, Interpretations of Addition and Subtraction.

Numbers to the left of 0 are used in this section. These are the negative numbers conventionally written as, for example, -4. It seems better to introduce the conventional notation later when the children have a feeling for order on the number line and have experience with the - sign as a symbol for the subtraction operation. At this stage we have chosen to denote positive numbers by black numerals and negative numbers by red numerals. There were two reasons for this color choice. First, the children will use positive numbers most of the time and, thus, usually will use ordinary black pencils. Second, when negative numbers are used later, one interpretation of them will be in terms of debits (negative) in contrast to credits (positives). For this it is natural to use the bookkeeper's "red ink" numerals for debits.

Lesson 24: INTRODUCING THE NUMBER LINE

This lesson introduces the ordering of integers on the number line. (The integers are whole numbers ... -2, -1, 0, 1, 2, 3 ...)

MATERIALS

- figure of Skip
- numeral cards (black) 0 - 10
- numeral cards (red) 1 - 3
- house pattern
- masking tape or stick-tite

PROCEDURE

Before reading the story, "Skip's Trip," draw two long horizontal lines on the chalkboard to serve as a street map. The card showing Skip's house, the numeral 0, and the figure of Skip should be in place as shown.

Read the story, "Skip's Trip," pausing at the indicated places to ask questions and have the cards posted. The story introduces the concept of the number line although it does not call it by name.

After the story present the following.

A. Show how Skip moved from his house to the store (3).

WHICH DIRECTION IS THAT ON THE STREET MAP?

AS HE RAN TOWARD THE STORE, DID THE HOUSE NUMBERS BECOME GREATER OR LESSER?

B. Now move Skip past the store.

ARE THE HOUSE NUMBERS GETTING GREATER OR LESSER? (greater)

WHICH WAY IS HE GOING? (away from home)

Have the children post the cards for a few more houses.

C. IF SKIP KEPT ON GOING ALONG THE STREET A LONG WAY, WOULD HE BE FARTHER FROM HIS HOUSE? (perhaps)

It should be brought out here -- perhaps by your suggestion -- that the street might curve back toward Skip's house, as real streets sometimes do. In this case, as Skip continued along the street he could get nearer his home. We wouldn't know whether, for example, House 21 or House 32 was farther from Skip's house. To avoid this confusion, we will from now on say that the street goes on forever in opposite directions, always straight. Then the street can be called a line.

D. Place three houses on the other side of Skip's house -- to the left as you face the map -- and label the left one "School.".

LET'S PRETEND SKIP HAS STARTED OFF TOWARD THE SCHOOL INSTEAD OF TOWARD THE STORE. MOVE SKIP TO THE FIRST HOUSE HE WOULD COME TO. WHAT NUMBER SHOULD WE PUT ON THIS HOUSE?

Guide the children to see it should be a 1 because it is one space from 0, Skip's home, but it should be different from the 1 already used so that two houses won't have the same numbers. Suggest that a different color be used. Place a red numeral 1 card. Continue with the second and third houses to the left of 0.

Illustrated by Virginia Knight.

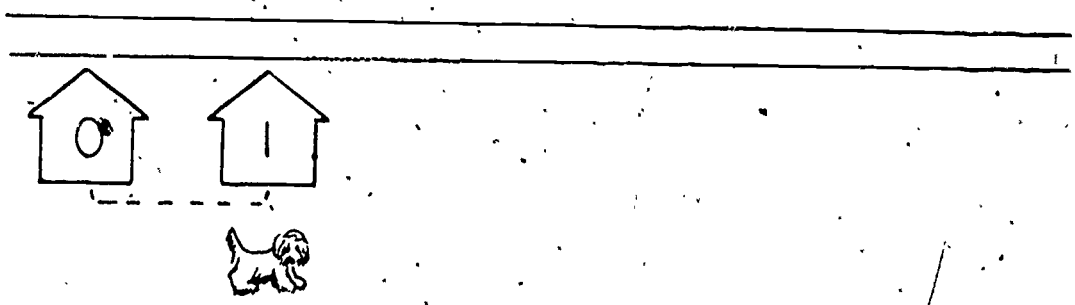
SKIP'S TRIP

Hello. My name's Skip. I live in this corner house with Tommy and Don. I have a lot of fun playing with them. Sometimes I even try to go to school with them, but then they tell me, "Go home, Skip. Go home, boy." Usually when they say that I do go home, but sometimes I don't. I kinda like to go around and see what's happening in the rest of the neighborhood.

One Saturday morning I woke up early and since the door upstairs was closed so I couldn't go wake up Tommy and Don, I thought I'd go outside and see if I could find anything interesting to do.

After pushing the backdoor open and running outside, I trotted up the street one space to the very next house where Dick lives.

> Show Skip's path on the number line (street map). Place Dick's house one space away from Skip's house, but without any number on it. CAN ANYONE PUT THE NUMBER ON DICK'S HOUSE? Have a child place the numeral 1 on the house and move Skip to that house.



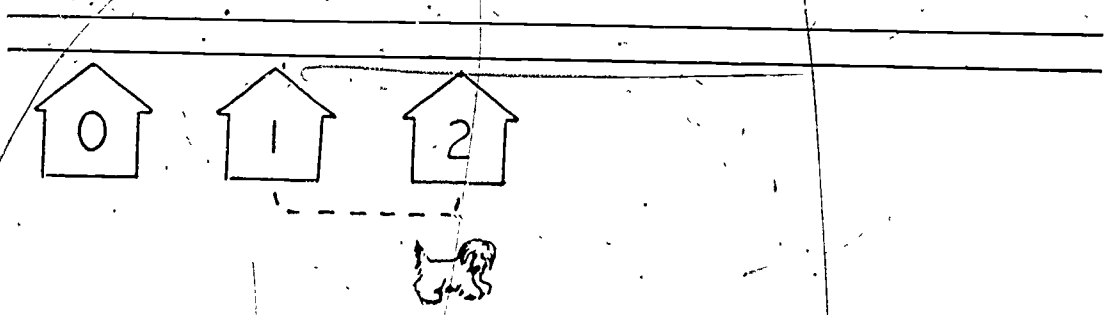

MILK
MILK

When I got to Dick's house, I didn't find anything interesting in front, so I went around back. The first thing that I noticed was that the milkman had been by and had left two quarts of milk by the backdoor. Since I hadn't had my breakfast yet, I was very hungry. So ... well, maybe you can guess what I did? (Show picture)

Just as I got the milk carton open, Dick's mother came out the door. She was pretty unhappy about my visit so I ran away as fast as I could.

I went one space and stopped at the next house, which was Sally's. That made Sally's house the second house from mine.

CAN ANYONE MOVE SKIP TO SALLY'S HOUSE AND PUT THE RIGHT NUMBER ON HER HOUSE? (2)



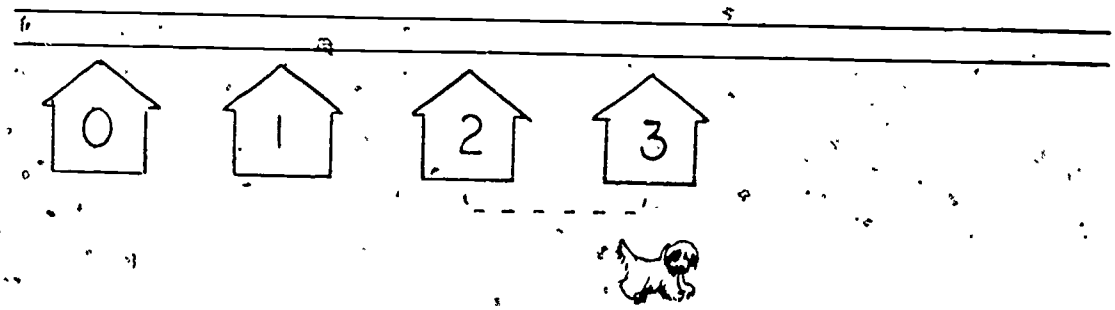
Sally, who is a good friend of mine, was playing by herself in the backyard. She was happy to see me, so we started running, jumping and playing together. Unfortunately, I didn't notice that the lawn sprinkler was on and got my feet all wet and muddy. When I jumped up, I got Sally's dress muddy, too. She didn't seem to mind, but just then her mother came out. Sally's mother was pretty mad: "Sally, are your clean clothes dirty already? Skip, you go on home."

But I didn't go home. I scampered to the next house one space beyond Sally's.

Place the next house on the map.

My favorite friend, Jack, lived in the third house from me. Jack's father owned our neighborhood grocery store, and Jack lived above the store with his father and mother.

CAN ANYONE PLACE THE RIGHT NUMBER ON THE STORE? (3) AND MOVE SKIP TO THE STORE?



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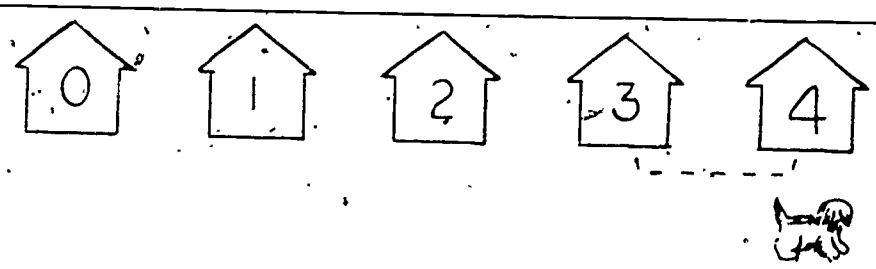
Sometimes Jack would give me a bone or a cookie, but I couldn't find him anywhere. He wasn't around at the back of the store either. But as I passed the garbage cans, I could smell something delicious. It smelled like nice, juicy meat scraps. I thought that I would try to get some and - oohh, what a mess I made! But Jack's father heard the garbage cans tip over, so he came out and chased me away. He was pretty mad.

One space from the store was the house where Susan lived. Susan's house was the fourth house from mine.

Place the next house on the map. CAN ANYONE PLACE THE RIGHT NUMBER ON SUSAN'S HOUSE? (4) AND MOVE SKIP TO SUSAN'S HOUSE?



Susan was jumping rope in her yard when I came up. She was my friend too, so she stopped jumping rope and patted me on the head for a while.

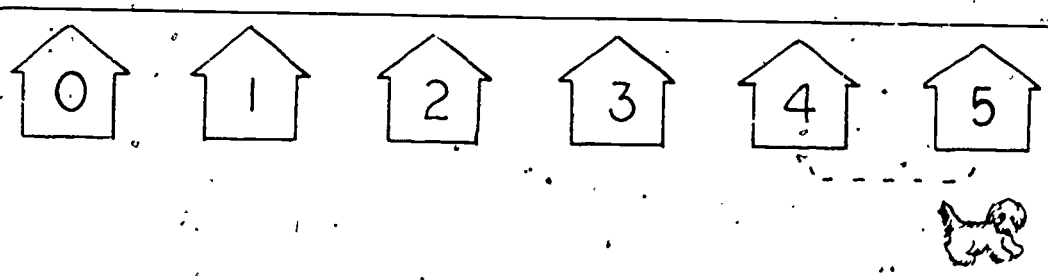
But I didn't want to be petted, I wanted to play games. So I grabbed Susan's rope in my mouth and ran away, hoping that Susan would chase me.

"Skip, come back here with my rope," Susan called. "Here, Skip. Here, Skip." When I wouldn't take her rope back to her, rather than chase me, Susan started to go into her house, calling, "Mother, Skip's got my jump rope."

Well, that didn't sound like very much fun to me. So I dropped her rope and went on one space to the next house.

Bob's house is next to Susan's. I was now five spaces away from my own house.

Place the next house on the map. Have a child move Skip and place the right number on the house. (5)



By now I was very hungry, but I remembered that I had buried a big bone someplace in Bob's yard. Unfortunately, I couldn't remember where I had buried it. Even after sniffing around the flowers, around the tree, and all around the yard, I couldn't remember where the bone was. So I started digging to see if I could find it. I dug around the flowers. I dug around the tree. And I dug in the yard. But I couldn't find that bone.

Just then Bob's mother came out to hang up her clothes. You can imagine what she saw. (Show picture)

She was very upset by my visit, so I ran for home as fast as I could go.

Have a child move Skip rapidly as you read the remainder of the story, "counting down" with the house numbers as he moves the figure of Skip.

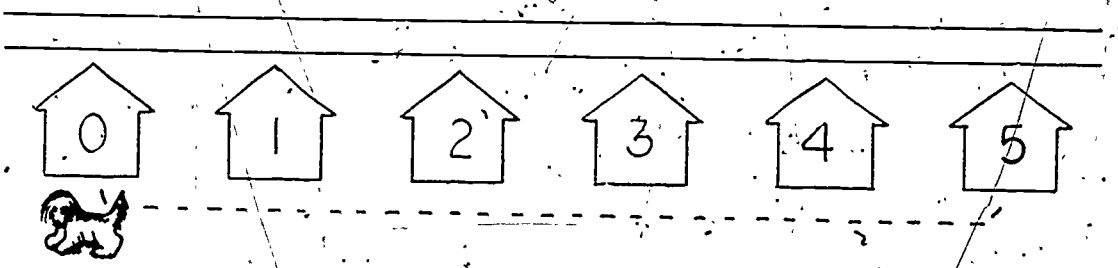
I ran from Bob's house (5) past Susan's house (4).

I ran another space to the store where Jack lives (3).

I ran on another space past Sally's house (2).

Faster and faster I ran until I passed Dick's house (1).

With only one space to go, I was soon back at my own home (0).



Tommy and Don, who had been watching me run home, ran out to greet me.

"Here, Skip. Here, Skip," Tommy called.

"Where have you been, Skip?" Don asked, as he patted me on my head and rubbed by ears.

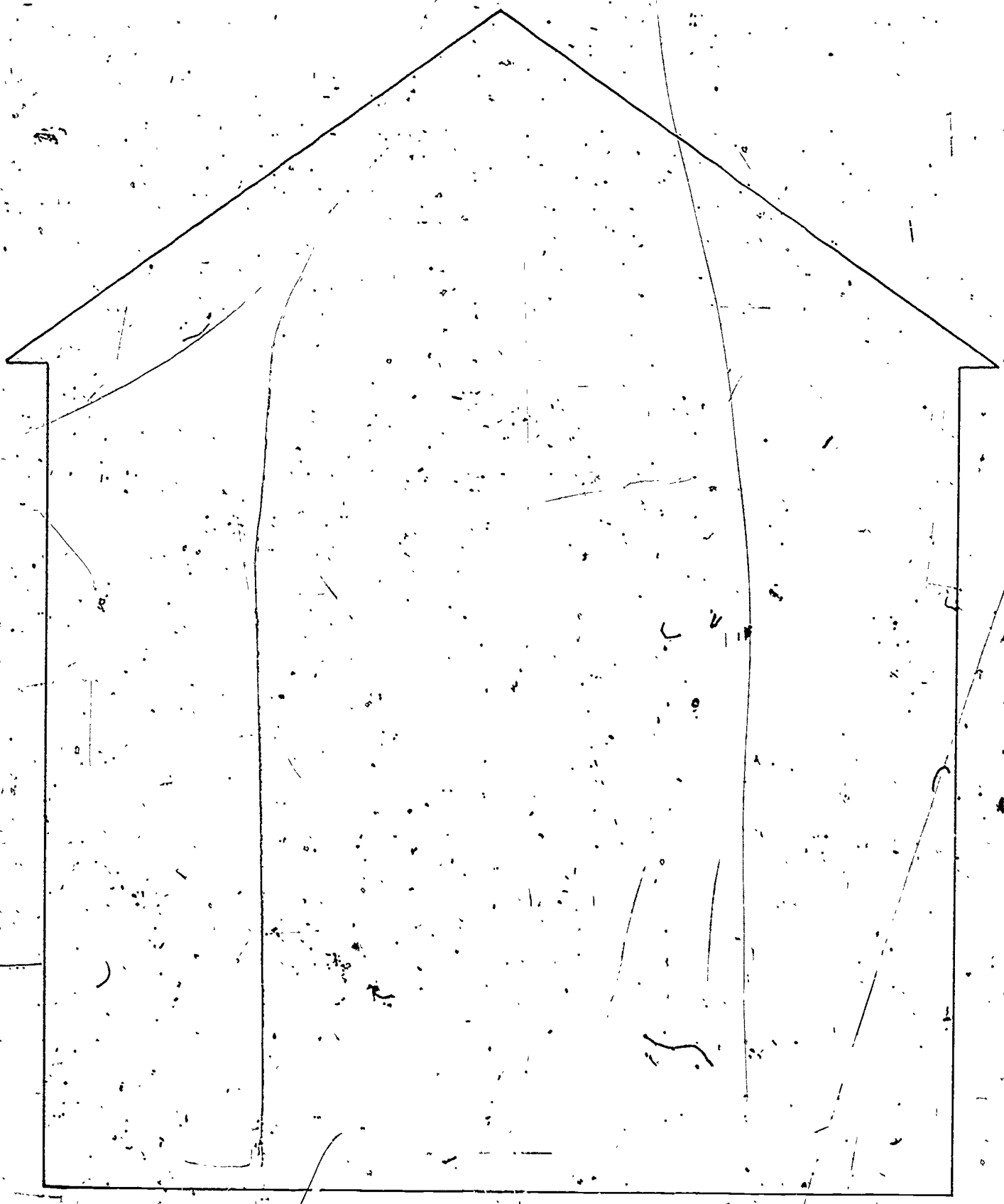
"Arf, arf," was all I answered, wagging my tail.

I was glad to be home, but I couldn't tell the boys where I had been. And after thinking about it a little, I wouldn't have wanted to tell them even if I could have.

"Good boy," Tommy said as he patted me on the head too. I kinda hung my head and slowly wagged my tail. The boys didn't know about Dick's milk carton, Sally's dress, Jack's garbage cans, Susan's jump rope, and Bob's yard.

At this point, return to the questions suggested at the beginning of the lesson.

Use this pattern to make 14 cardboard houses.



Sample of Skip figures to be pasted on two sides of a piece of cardboard.

Three red numeral cards -- 1, 2, 3 -- are provided with this unit. The regular numeral cards provided by MINNEMAST should be used when black numeral cards are called for.

Lesson 25: LOCATING NUMBERS AND COUNTING SPACES ON THE NUMBER LINE

MATERIALS (from Lesson 24)

- figure of Skip
- houses
- numeral cards (black) 0 - 10 and (red) 1 - 3

PROCEDURE

Redraw the line for the street map. Mount the houses and the 0. Put the remaining numeral cards in view, but not in order. "Skip" should also be in the tray. Tell the children that Skip is hiding and that they must try to find him. You will give clues about where he is. After you have given a clue, pick a volunteer to come up and post Skip where he thinks the dog is hiding.

At first, give clues such as "Skip is at house black 2" or "Skip is at house red 2." As each child counts spaces and correctly posts the dog figure, he may also stick in place the correct numeral for the house where Skip is hiding. Vary the instructions occasionally; e.g., "Skip is at the fifth house to the right of his own." (Emphasize that right and left are determined when you are facing the map.) Sometimes give "number" clues without mentioning houses at all. "Skip is at black 2." Encourage the children to post the Skip figure as quickly as possible. This may prompt them to use more efficient methods such as starting from an already posted number (not 0) to find a new one. Continue the game until all the black and red numerals have been posted.

Point out that the game is now too easy. Everyone can tell where Skip is hiding. Ask if Skip can sometimes be between two houses. Give clues such as "Skip is between houses 4 and 5" or "between houses 0 and black 1." Then begin saying just "between black 3 and black 2" or "between 0 and red 1."

After a few clues of this type have been given, ask:

DO YOU KNOW EXACTLY WHERE SKIP IS HIDING? (no)

WHY NOT? (He could be anywhere in the space between the numbers.)

Then give some clues with longer intervals; e.g., "Skip is between black 3, black 8" or "black 5, black 2." Here it is even harder to know just where Skip is hiding since he could be anywhere between the two clue numbers.

When the children have had enough practice, choose a child to be Skip. He should choose a hiding place on the story map and whisper it to you. He then must give the class some clue as to Skip's hiding place. He should make it as hard as possible for the other players to find Skip. The other players can ask questions about Skip's location but these questions must be answerable by yes or no. Suppose "Skip's" clue is "I am between red 3 and black 10." The dogcatchers could ask questions such as "Are you hiding between red 3 and black 7?" or "Are you hiding at black 5?" etc., until they find Skip. Many children should discover efficient types of questions to use to find Skip quickly, questions that narrow the location down by stages: "Is Skip hiding among the red houses?" (no) "Is he between 0 and 5?" (yes) "Is he between 0 and 3?" etc. Because the main purpose of the game is to give practice in locating numbers on the line, the strategy of finding efficient questions is just an additional benefit for those children who may work it out themselves.

Lesson 26: MOVES ON THE NUMBER LINE

Activities in this lesson should help the children to learn to locate numbers and to count spaces on the number line with ease. Combining moves on the number line is similar to addition with sets in Section 1. For example, when we start at 3 and move 2 spaces in the positive (greater than) direction, we have combined the set of 3 unit-spaces (between 0 and 3) and the set of 2 unit-spaces (between 3 and 5) to obtain the entire set of 5 unit-spaces (between 0 and 5).

The ideas of $\frac{1}{2}$ space moves and the existence of numbers halfway between the counting numbers are introduced.

Some children will be able to give the correct responses to Worksheets 44 and 45 immediately. They may go to Lesson 28. Other children who need more practice may benefit from Lesson 27.

MATERIALS

— numerals 0 - 10 (black) and 1 - 3 (red) from Lesson 24

— Worksheets 44 and 45

— strips of heavy paper in lengths of 1 and 2 number line units

PROCEDURE —

A. Post or draw a number line with units marked on it similar in length to those used for "Skip's Trip." Arrange the numerals in order after choosing a position of 0. The location chosen for the starting point 0 should vary from activity to activity to show that it is an arbitrary decision.

It is preferable to stop using the figure of Skip now in order to avoid too much dependence on Skip, and to guide the child to respond to the conventional number line.

Ask the class how the number line has changed. (No Skip or houses. The 0 may be shifted, which makes no difference.) Tell them that they can now use this line of numbers to do interesting things, just as they did interesting things with Skip and his street.

Directions and questions similar to the following should now be used:

CAN SOMEONE POINT TO BLACK 3? HOW MANY SPACES FROM 0 IS IT? (3 in the "greater than" direction)

MOVE 2 MORE SPACES TO THE RIGHT -- IN THE GREATER THAN DIRECTION. AT WHICH NUMBER ARE YOU NOW? (5)

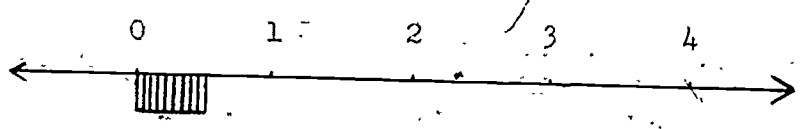
HOW MANY UNIT SPACES FROM 0 IS THAT? (5)

Repeat this procedure with other starting points and numbers of unit spaces. Sometimes move to the left -- in the "less than" direction. Sometimes start on a red numeral. The children can take turns giving starting points, spaces to be moved, and direction.

B. After the class understands moves of whole spaces, introduce some half-unit moves. Show the class the 1-unit strip of paper. Have a child hold it on the number line to show it is just as long as one unit space. Fold it so it is half as long.

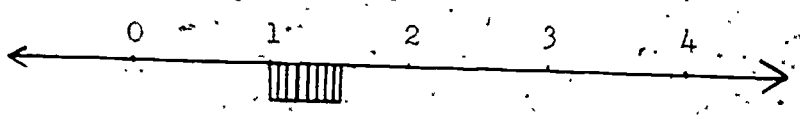
HOW LONG IS THE FOLDED STRIP? ($\frac{1}{2}$ unit space)

Cut the strip on the fold. Place the left end of the cut strip at 0 on the number line.



WHAT NUMBER IS AT THE RIGHT END OF THE STRIP? ($\frac{1}{2}$)

Mark the end point $\frac{1}{2}$. Place the left end of the strip at 1.



WHAT NUMBER IS AT THE RIGHT END OF THE STRIP? ($1\frac{1}{2}$)

Place the left end of the cut strip at $\frac{1}{2}$.

WHAT NUMBER IS AT THE RIGHT END OF THE STRIP? (1)

Lesson 27: PHYSICAL ACTIVITIES WITH THE NUMBER LINE (OPTIONAL)

The activities in this lesson are designed to reinforce the child's concept of the number line. They should be especially helpful to the child who has trouble ordering the numbers on the line. Some children will not need this lesson.

MATERIALS

- number line on the floor with step-long intervals (marked with chalk or masking tape)
- individual number lines and counters

PROCEDURE

A. Mark a number line on the floor with chalk or masking tape. Have the children close their eyes, and tell them that you are standing on the zero. After explicitly saying that you will move one unit space between marks with each step, take "loud" steps on the line. Tell the children which direction you walk. When you stop, have the children predict which numeral you are on. They then open their eyes to check their predictions.

B. Follow the procedure of A except for starting at numbers other than 0.

C. As a variation of either A or B, have children do the walking. They can pretend to be grasshoppers, rabbits, kangeroos, etc.

D. Commercial "track" games, such as "Chutes and Ladders," provide practice in counting spaces.

E. Partners play, with each having a small (about 1" units) number line and counter. One says, for example, "I put my counter on 3 and move 4 in the 'greater than' direction. What number do I land on? How many spaces am I away from 0?" The other child may use his counter and number line to find the answer. The partners alternate questioning. For each correct answer the child can make one tally mark.

Lesson 28: LENGTH OF MINNEBARS ON THE NUMBER LINE

In Lessons 29 and 31 the children will add and subtract with Minnebars on a number line. Before starting these lessons, they should be able to find out how many units are on the side of the bar (without counting) by placing it on a number line that has units the same length as those on the bars. Naturally they could count the units directly on the bar, but reading the length on a number line emphasizes the relation between number and length on a number line.

MATERIALS

for each child:

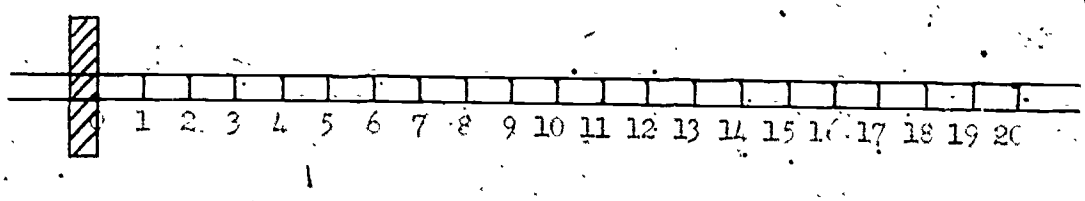
— 1 set of Minnebars

— 1 number line having numerals from 0 to 20 with ½" spacing (The number line tapes designed for the addition slide rules may be used, or number lines may be drawn on strips of paper or on masking tape.)

— masking tape

PROCEDURE

Each child should have his number line taped at the top of his desk. One edge of the masking tape should be exactly at the 0, as in this sketch:



The first twenty numerals should be visible.

Have each child place a 5-unit Minnebar along his number line. The left end of the bar should be at the 0 mark of the numberline. Have the children notice that the 5 unit-squares on the bar correspond exactly to the first 5-unit length on this number line. Lead them to see that the length of one unit-square on the bar is the same as one unit length on this number line. (Other number lines may have different length units.)

Ask the children to turn the Minnebars so the colored sides are underneath. Ask each child to find a Minnebar whose length is 6, without looking at the colored side. You can guide them by asking questions such as, "How can the number line help you?" and "Where must you place the left edge of the bar?" When each child seems to have found his 6 bar, ask each to hold up the bar.

ARE ALL THE BARS THE SAME COLOR?

If a child has difficulty in aligning the end of the bar with the 0, a paper clip or similar object can be taped just to the left of the 0. Then the bar can be pushed against the barrier at 0.

Repeat the lesson with different lengths as often as seems necessary. You will probably want to leave the number lines in place for the next lesson.

Lesson 29: ADDING WITH MINNEBARS ON THE NUMBER LINE

In this lesson concrete objects (Minnebars) are used to show the addition of numbers represented by lengths on the number line. The children should not be required to memorize the combinations.

MATERIALS

for each child:

— 1 set of Minnebars

— 1 number line

— Worksheet 46

PROCEDURE

Have the number lines taped on the desks as in Lesson 28. Have the Minnebars turned with the colored sides up. Since all sets of Minnebars are not colored the same, your bars may not agree with those referred to here. Simply use the colors you have.

Introduce the worksheet by going through similar problems together. Typically you might say the following:

FIND A RED BAR AND A LIGHT GREEN BAR. PLACE THEM END-TO-END ON THE NUMBER LINE. HOW MANY UNITS LONG ARE THEY TOGETHER? (7)

HOW MANY UNITS LONG IS THE RED BAR? (2)

HOW MANY UNITS LONG IS THE LIGHT GREEN BAR? (5)

CAN ANYONE WRITE A NUMBER SENTENCE ON THE BOARD THAT TELLS TWO NAMES FOR THE SUM OF 2 AND 5 ? (2 + 5 = 7)

When you feel the children are ready, use Worksheet 46. Worksheet 46 specifies the colors of the Minnebars to be added. If the children cannot read the color names, show the colors with crayon marks. If some children particularly enjoy this activity, they could combine Minnebars of their own choice and write the appropriate number sentences.

The Train Games 4 through 8 from Section 6 should be useful at this time.

Worksheet 46
Unit H Name_____

Red Blue ___ + ___ = ___	Light Green Red ___ + ___ = ___
Red Black ___ + ___ = ___	Orange Red ___ + ___ = ___
Dark Green Orange ___ + ___ = ___	Dark Green Red ___ + ___ = ___
Blue Black ___ + ___ = ___	Yellow Light Green ___ + ___ = ___
Light Blue Orange ___ + ___ = ___	Light Blue Yellow ___ + ___ = ___

Lesson 30: JOINING STRIPS OF DIFFERENT LENGTHS ON THE NUMBER LINE

Every point on the number line, not only those points marked with integers, represents a real number. Thus, any two real numbers can be added on the number line if the appropriate lengths can be located. For example, the sum $1.23 + 1.79 = 3.02$ could be found, although the procedure would be almost impossible on a small scale number line.

This lesson provides a first step towards working with numbers that are not counting numbers. Paper strips of lengths near an integral number of units (when measured) are given to the children. They place the strips end to end on the number line and note that the total length is near an integral number of units. This is all that is desired at this stage.

MATERIALS

— scissors

— crayons

— Worksheets 47 and 48

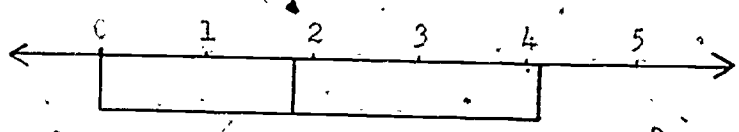
Worksheet 47 contains a larger-scale number line to be cut out and used by the children in place of the number line used in Lessons 28 and 29. This is done so that the length of a strip can be "near" 5, for example, and still have the length obviously different from 5.

PROCEDURE

The children should color and then cut out the strips on Worksheet 47. Slight inaccuracies in the lengths due to variable cutting ability will not matter. The number lines should be cut on the dotted lines. Slippage may be prevented by taping the number lines to the desks.

Have the children complete Worksheet 48. They are to place individual strips and combinations of strips on the number line and note the lengths are near integral numbers of units.

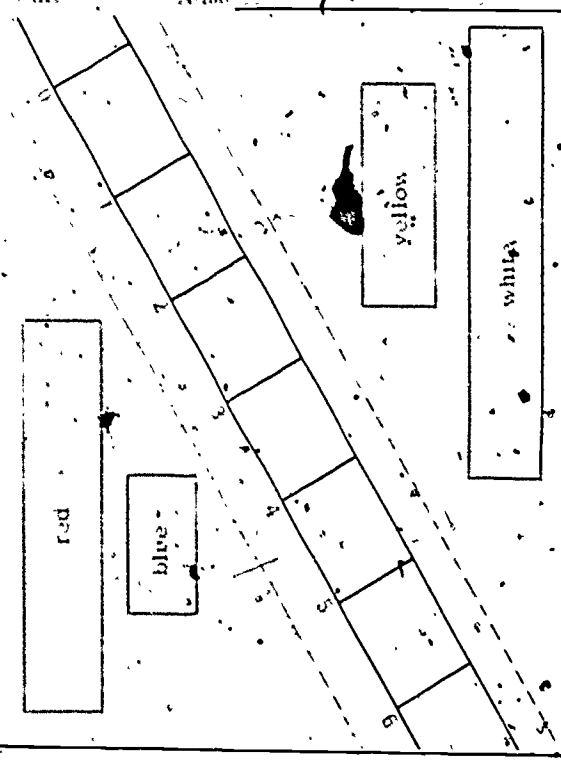
For example, in the sketch below the combined length of the strips is "near 4 units."



You will need to guide the children through the exercises to an extent depending on their reading and motor skills.

If certain children know combinations such as $3 + 2 = 5$ (which is not required), they may like to predict that the sum of the red length and the yellow length will be near 5 units. They should then lay the strips on the number line and observe that the total length is, indeed, near 5 units.

Worksheet 47
Unit 11 Name _____



Worksheet 48
Unit 11 Name _____

1. Length of blue strip is near 1 units.
   Length of red strip is near 3 units.
   Their length together is near 4 units.

2. Length of yellow strip is near 2 units.
   Length of white strip is near 4 units.
   Their length together is near 6 units.

3. Length of red strip is near 3 units.
   Length of yellow strip is near 2 units.
   Their length together is near 5 units.

4. Length of white strip is near 4 units.
   Length of blue strip is near 1 units.
   Their length together is near 5 units.

Lesson 31: SUBTRACTING WITH MINNEBARS ON THE NUMBER LINE

This lesson provides an introduction to subtraction on the number line by the manipulation of actual objects (Minnebars) on the line. Subtraction will be considered more in Unit 13, Interpretations of Addition and Subtraction.

MATERIALS

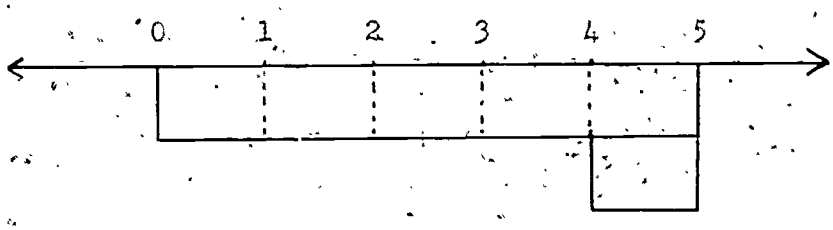
for each child:

- 1 set of Minnebars
- 1 number line
- masking tape
- Worksheet 49

PROCEDURE

Have the number lines taped to the desks as in Lessons 28 and 29. The Minnebars should be placed with the colored sides up.

Tell the children that they are now going to subtract on the number line. Ask for suggestions on how to find the difference between the length of a black bar and the length of a light green bar. Probably the clearest arrangement is to place the longer bar on the number line and to lay the shorter one beside it with the right edges matching. Then the difference is read from the number line at the left edge of the short bar. This is illustrated below.



The black bar is 4 units less than the light green bar and the light green bar is 4 units greater than the black bar.

Ask someone to write the appropriate number sentence on the board. (5 - 1 = 4)

When the children understand the procedure, have them find the differences in length of Minnebars of the colors suggested on Worksheet 49.

Worksheet 49
Unit 11 Name_____

Yellow Orange ___ - ___ = ___	Dark Green Light Green ___ - ___ = ___
Orange Black ___ - ___ = ___	Purple Red ___ - ___ = ___
Yellow Red ___ - ___ = ___	Dark Blue Dark Green ___ - ___ = ___
Dark Blue Orange ___ - ___ = ___	Dark Blue Yellow ___ - ___ = ___
Purple Dark Green ___ - ___ = ___	Purple Orange ___ - ___ = ___

SECTION 5 INTRODUCTION OF NUMERALS THROUGH 99

The lessons in this section give practice in repeated grouping into subsets according to a given rule, give an explanation of our decimal positional number system, and introduce the written and spoken numerals through 99 (optional through 999).

BACKGROUND

Preceding MINNEMAST units have only used the numbers from 0 to 20. Place value notation has not been introduced: for example, 14 has been written as a symbol for "fourteen" without commenting about the two separate digits in the numeral. Now the concept of place value will be introduced. The Hindu-Arabic decimal system, the number system we generally use, is called a positional number system because the numerical value of a string of symbols depends on the relative position of the symbols in the string. For example, 325, 235, and 523 indicate different numbers. (This is not true in all systems of numeration that have been used in the world.)

Writing numbers greater than 9 is probably so familiar to you that you hardly realize what a wonderful invention place value notation was. Suppose our number system had no place value notation. A different symbol would have to be invented for each number (as was done for the numbers from 1 to 9). At most, we could memorize a few thousand different number symbols. Imagine the expressions for astronomical distances in such a system! Numeration systems like that of the Romans do not require as many different symbols as there are numbers to represent, but new symbols must be continually introduced as the numbers represented grow larger. Addition and subtraction operations are less efficient with Roman numerals and multiplication and division operations become very complicated. The power and efficiency of place value notation can be appreciated by comparison with other notations.

When numerals using the decimal place value system are read or written, the positions of the digits tell what they stand for. When a real number is represented, the digit to the extreme right (before the decimal point) represents the number of ones, the next digit to the left represents the number of tens, and so on. This system uses a zero to show the absence of certain groupings. For example, in the number 3027, there are 3 thousands, no hundreds, 2 tens, and 7 ones. Our number system is said to be a base 10 system because each position of a digit stands for a power of 10. For example, in the numeral 9785, we have 5 ones (or 5×10^0; 10^0 is equal to 1), 8 tens (or $8 \times 10^1 = 8 \times 10$), 7 hundreds (or $7 \times 10^2 = 7 \times 10 \times 10$), and 9 thousands (or $9 \times 10^3 = 9 \times 10 \times 10 \times 10$).

The children should begin to appreciate the power and usefulness of the place value notation, not merely to learn (by rote) how to write multi-digit Arabic numerals. This is why the MINNEMAST approach may appear indirect. We begin by providing background experience in repeated grouping (say, grouping a set into subsets of 5, then grouping five of the subsets of five, etc.). Grouping and recording groups of ten are stressed. These groups are used in the explanation of place value. Then the ordinary number words for 20 to 99 are introduced. Finally, the abacus is introduced as a device to represent our positional number system. The abacus will be used again in Unit 13.

Lesson 32: GROUPING GAMES

In this lesson, the children will form groups according to a given rule (e.g., form into groups of 2; or, by another rule, form into groups of 3). The groups formed are counted. The children regroup several times choosing different partners each time. After a few trials the children should discover the important fact that the number of groups formed (from the same number of children) depends only on the rule used and not on the particular composition of the groups. This idea is basic to the grouping activities in the remainder of the unit.

PROCEDURE

Have the children play A and B each three times.

A. Game: The Rule of Two

Have the children move around the room. Tell the children that at a signal (such as a bell) they should find partners to form pairs and stop moving. If an odd number of children are playing, a single child will be left without a partner. He should stand alone. Have this child (or another one if there is no child without a partner) count the number of pairs and record it on the chalkboard. The record might look like the following:

Rule of Two

12 pairs . . . 1 leftover

Begin the game again with each child moving about alone. At the signal, have the children form pairs with different children than in the first playing. Again record the number of groups. After a third playing, the children will probably arrive at the correct conclusion that if the rules and the total number of players are left unchanged, the same number of groups will always be formed.

B. Game: The Rule of Three

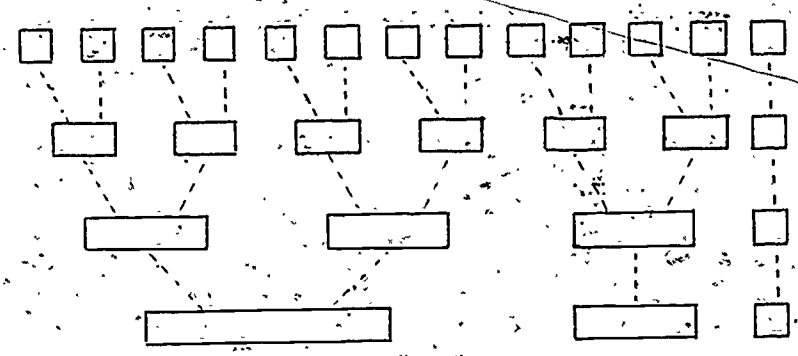
Play this game as Game A, except that new groups are formed by three children joining hands. The record might look something like the following:

Rule of Three

8 groups of three 1 leftover

Lesson 33: GROUPING MINNEBARS BY TWOS AND THREES

Here the children make repeated groups with Minnebars according to a given rule and count the groups formed. The first rule given for grouping is that any 2 pieces of the same size can be exchanged for one longer piece that is equivalent in total number of units to the two smaller pieces. When starting with thirteen 1-unit bars, the process can be represented by this diagram:



The process ends with one 8-unit bar, one 4-unit and one 1-unit, a total of 13 units. The second rule given calls for grouping any 3 pieces of the same size.

These activities have value in providing background for certain science topics and mathematical concepts such as multiplication and probability. However, the immediate application is to provide background for Lessons 35 and 39 in which grouping by tens leads directly to the decimal system of numeration. Groups of smaller size are easier to manipulate and perceive than the groups of 10. Experience with them helps prepare the child for grouping by tens.

MATERIALS

for each child:

— 1 box or bag of Minnebars

— Worksheets 50 and 51

PROCEDURE

A. Have each child place Worksheet 50 and a box of Minne-bars on his desk. Ask him to remove the black 1-unit bars from the box and put them on his desk. (There are 13 black bars.)

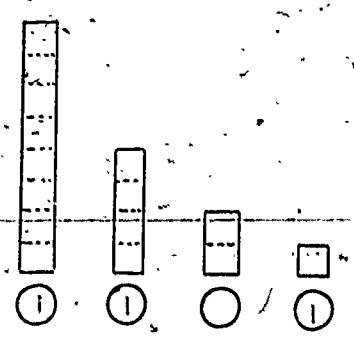
Tell the children that the rule for this worksheet is that any two small pieces of the same size can be traded for one longer piece, equivalent in total number of units to the two smaller pieces (i.e., [ ] [ ] can be traded for [:] and [:] [:] can be traded for [: : :].

Each child then starts trading his unit pieces for 2-unit bars. He should put a pair of 1-unit bars back into the box, take out a 2-unit bar, and place the 2-bar in a column under the picture of the 2-bar on the worksheet. He continues as long as he can exchange pairs of 1-bars for 2-bars.

Then pairs of 2-bars are traded for 4-bars. The 4-bars should be placed in a column on the worksheet under the picture of the 4-bar.

When the 2-bars have been paired, he trades a pair of 4-bars for an 8-bar, and the 8-bar is placed on the worksheet.

He should now remove the bars from the worksheet and record with a tally mark in the proper column the number of bars he had of that type. The record sheet will look like this if 13 black bars were used:



(One 8-unit bar and one 4-unit bar and one 1-unit bar are equivalent to thirteen 1-unit bars.)

Repeat this activity as often as you wish, starting with different numbers of 1-unit bars.

B. Practice counting by tens with the class. When they can count 0, 10, 20, etc., start with a number such as 2. Ask the name for 10 and 2, 20 and 2, etc. Illustrate on the board that

$$0 + 2 = 2$$
$$10 + 2 = 12$$
$$20 + 2 = 22$$

etc.

Then have them count aloud 2, 12, 22, 32, 42, and so on.

C. Number Bingo. - Prepare tagboard cards similar to the following:

69	24	73	92	71
58	9	17	89	26
16	0	X	61	35
12	21	84	44	1
33	52	13	76	92

Each card should have a different combination of numerals from 0 to 99. Each child should have a supply of counters. Call out numerals one at a time. If a child locates the given numeral on his card, he covers it with a counter. The winner calls out BINGO when any row, column or diagonal is covered.

D. Counting lunch tickets, dental cards, book reports, books, coats, children, etc., are all possible situations where the number names may be reinforced.

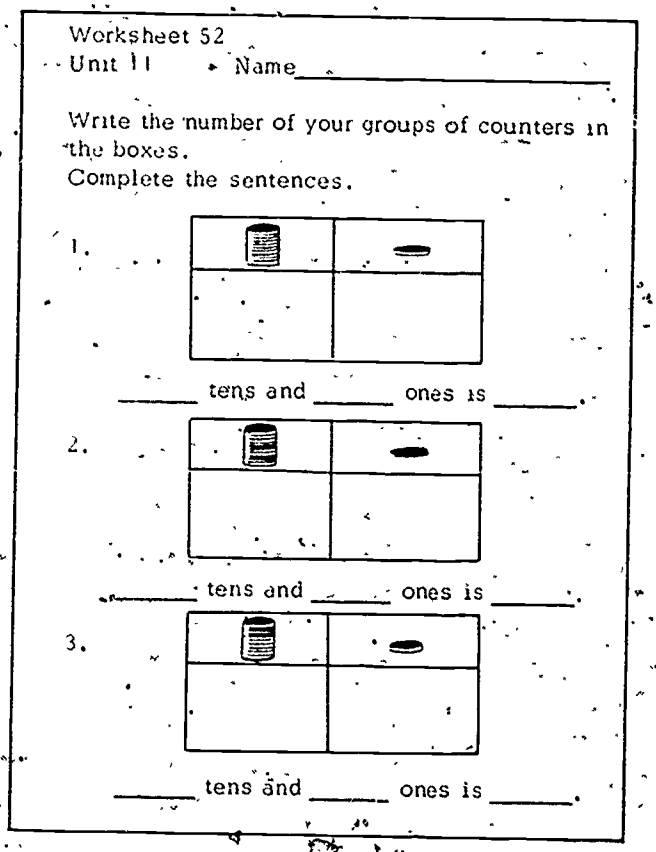
Lesson 34: THE ONES PLACE AND TENS PLACE

This lesson introduces the essential concept of place value in our decimal system of numeration; i.e., it will show that, when 36 is written, it means 3 groups of ten and 6 ones.

MATERIALS

- for each child:
  - 50 disk counters
  - 5 small paper cups
  - Worksheet 52

The small paper cups are not necessary, but are suggested to cut down on the number of dropped counters.


Worksheet 52
Unit 11 Name _____

Write the number of your groups of counters in the boxes.
Complete the sentences.

1. _____ tens and _____ ones is _____.

2. _____ tens and _____ ones is _____.

3. _____ tens and _____ ones is _____.

PROCEDURE

A. Have each child place almost all (about 45) of his 50 counters on his desk along with his paper cups and Worksheet 52.

Have the children group their counters by tens. You can tell them to use the little cups to hold the groups of ten counters since stacks of ten fall over easily. Have the cups containing the groups of ten placed to the left of the ungrouped counters. When all the groups are made, have the number of groups of ten and the number of ungrouped single counters recorded in the table on Worksheet 52. The answers will vary because the children will have different numbers of counters. It is the process that is important, not the "answer." Do not have them complete the sentence under the table yet.

Lesson 37: FINDING GROUPS OF TEN ON WORKSHEETS

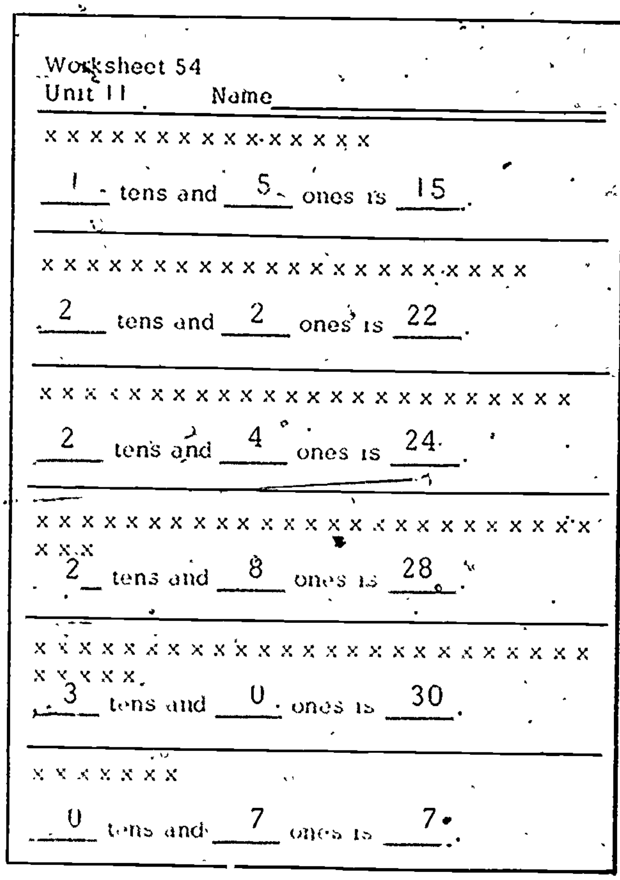
In this lesson the grouping activities are more abstract. Here the children are to find sets of 10 symbols on a printed page. They are, of course, unable to physically group these symbols, but they can draw a line enclosing them to indicate the groups.

MATERIALS

- Worksheets 54, 55, and 56

PROCEDURE

Have the children look at Worksheet 54. Tell the children that they are to show groups (or sets) of ten by drawing a curve around each group of 10 x's, and afterwards record the number of tens and the number of ones leftover on lines on the worksheet. They should then write a "shorter name" for the number.



Worksheet 54
Unit II Name

x x x x x x x x x x x x x x x

1 tens and 5 ones is 15.

x x x x x x x x x x x x x x x x x x x x x x

2 tens and 2 ones is 22.

x x x x x x x x x x x x x x x x x x x x x x x x

2 tens and 4 ones is 24.

x x x x x x x x x x x x x x x x x x x x x x x x x
x x x

2 tens and 8 ones is 28.

x x x x x x x x x x x x x x x x x x x x x x x x x
x x x x x

3 tens and 0 ones is 30.

x x x x x x x

0 tens and 7 ones is 7.

Lesson 35: NUMBER WORDS FROM 20 TO 99

It is strongly recommended that Game B of Section 6 be started now.

If the children do not know the words for 20, 30, 40, 50, 60, 70, 80, and 90, they should learn them. If they have understood Lesson 34, they should now be able to count with understanding to 99.

MATERIALS

— numeral cards 0, 20, 30, ... to 90

for each child:

— tagboard "bingo" cards with assorted numerals 0 to 99

— counters

PROCEDURE

The procedure to be followed here depends greatly on the previous achievement of the class. If they are able to do Activity B easily, they can go to Lesson 36 directly. Perhaps they will need the practice indicated in all activities.

A. Give one of the numeral cards to each of ten children. Have these ten children order themselves and then have the class identify the numerals. You may have to introduce the names of some numbers. Then stand behind various card-holders at random and have the children give the appropriate name.

B. Practice counting by tens with the class. When they can count 0, 10, 20, etc., start with a number such as 2. Ask the name for 10 and 2, 20 and 2, etc. Illustrate on the board that

$$0 + 2 = 2$$
$$10 + 2 = 12$$
$$20 + 2 = 22$$

etc.

Then have them count aloud 2, 12, 22, 32, 42, and so on.

C. Number Bingo. - Prepare tagboard cards similar to the following:

69	24	73	92	71
58	9	17	89	26
16	0	X	61	35
12	21	84	44	1
33	52	13	76	92

Each card should have a different combination of numerals from 0 to 99. Each child should have a supply of counters. Call out numerals one at a time. If a child locates the given numeral on his card, he covers it with a counter. The winner calls out BINGO when any row, column or diagonal is covered.

D. Counting lunch tickets, dental cards, book reports, books, coats, children, etc., are all possible situations where the number names may be reinforced.

Lesson 36: TWO-DIGIT NUMBERS AND COUNTERS

Again in this lesson the children form subsets of ten counters. They then exchange a subset of ten for an equivalent counter marked with a T. This exchange illustrates the idea of tens in two digit numbers.

MATERIALS

for each child:

— 50 paper disk counters

— Worksheet 53

PROCEDURE

Each child takes 4 of his counters and marks both sides of each with a "T". Tell them that the T mark stands for ten. Have the marked counters in boxes (or other containers) and the unmarked counters on the desks. Tell the children that this time when a set of 10 counters is formed, it should be exchanged for one T-counter, which is worth 10 of the unmarked 1-counters. Have them make sets of 10 and exchange them as long as they are able. Ask one or two of the children to report the final number of counters of each type that he has. (4 T-counters and 6 1-counters, if none of his original 50 has been lost.) Write 4 T's and 6 1's on the board.

Ask for suggestions of another way to write the number of counters represented by 4 T-counters. (40) Write 40 under the 4 T's on the board. Ask for suggestions of a symbol to use for the "and" in the T expression. (+) Write + under the "and" and write in the 6. Ask for and write another name for this sum. (46) They may need to recall how they wrote similar numerals in Lesson 35; e.g., 4 tens and 6 ones is 46. The form on the board should now be:

4 T's and 6 1's
40 + 6 = 46

Each child should put the T-counters back in the box along with a few of the unmarked counters. The other unmarked counters should be on his desk. He should then repeat the procedure three times with different numbers, recording his results on Worksheet 53.

Once again, different numbers will be recorded on the worksheets. You should see that the children understand the process, but it is not necessary to check the accuracy of the counting of the groups of ten.

Worksheet 53
Unit 11 Name_____

1. I had _____ T's and _____ ones.

or _____ + _____ = _____

2. I had _____ T's and _____ ones.

or _____ + _____ = _____

3. I had _____ T's and _____ ones.

or _____ + _____ = _____

Lesson 37: FINDING GROUPS OF TEN ON WORKSHEETS

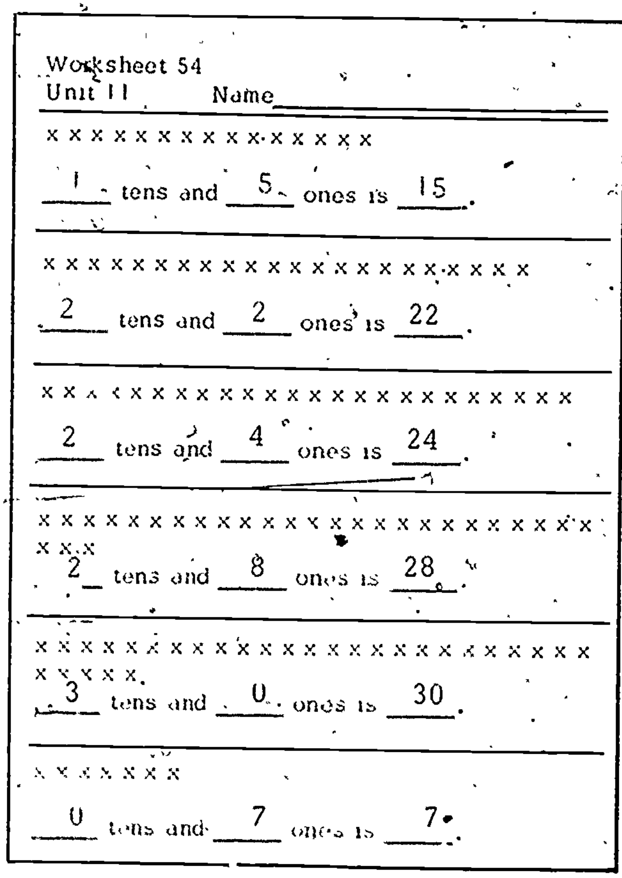
In this lesson the grouping activities are more abstract. Here the children are to find sets of 10 symbols on a printed page. They are, of course, unable to physically group these symbols, but they can draw a line enclosing them to indicate the groups.

MATERIALS

- Worksheets 54, 55, and 56

PROCEDURE

Have the children look at Worksheet 54. Tell the children that they are to show groups (or sets) of ten by drawing a curve around each group of 10 x's, and afterwards record the number of tens and the number of ones leftover on lines on the worksheet. They should then write a "shorter name" for the number.



Worksheet 54
Unit II   Name

x x x x x x x x x x x x x x x

1 tens and 5 ones is 15.

x x x x x x x x x x x x x x x x x x x x x x

2 tens and 2 ones is 22.

x x x x x x x x x x x x x x x x x x x x x x x x

2 tens and 4 ones is 24.

x x x x x x x x x x x x x x x x x x x x x x x x x
x x x

2 tens and 8 ones is 28.

x x x x x x x x x x x x x x x x x x x x x x x x x
x x x x x

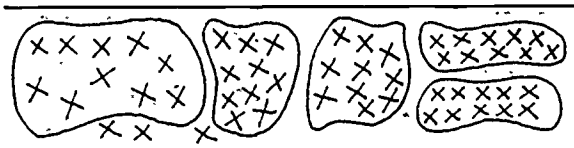
3 tens and 0 ones is 30.

x x x x x x x

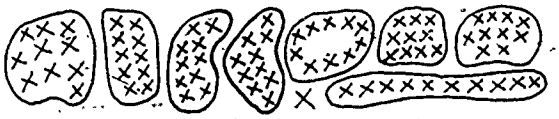
0 tens and 7 ones is 7.

On Worksheets 55 and 56 the objects have already been grouped by tens to avoid confusing the children in counting. Here they also are to record the number of groups of tens and the ones on the lines and write the "shorter name."

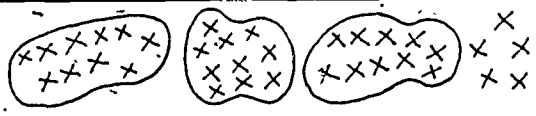
Worksheet 55
Unit 11 Name_____



5 tens and 3 ones is 53



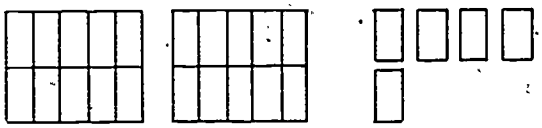
8 tens and 1 ones is 81



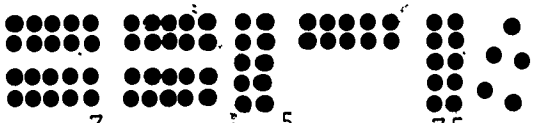
3 tens and 5 ones is 35

4 tens and 0 ones is 40

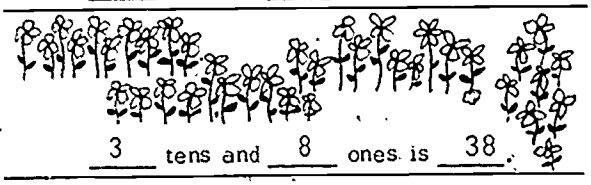
Worksheet 56
Unit 11 Name_____



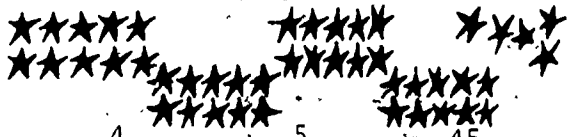
2 tens and 5 ones is 25.



7 tens and 5 ones is 75.



3 tens and 8 ones is 38.



4 tens and 5 ones is 45.

Lesson 38: PENNY, NICKEL, AND QUARTER GAME

In this lesson the children use play money. They form sets of 5 pennies and exchange for equivalent nickels. Sets of 5 nickels are formed and exchanged for equivalent quarters.

In addition to the experiences with common units of money, the activities here provide practice in grouping and exchanging equivalent sets.

MATERIALS

Play money for each child:

— 50 pennies

— 10 nickels

— 2 quarters

For play pennies the paper counters can be used. Play nickels or quarters can be heavy disks of a different size or color, poker chips, commercial play money, etc.

PROCEDURE

Divide your class into groups of 6 to 10 children. Choose two children to be bankers for each group. Each has a desk or table as a bank. Banker #1 has two boxes at his bank. One is initially empty and one contains all the nickels. Banker #2 also has two boxes, one containing all the quarters and the other empty.

Give each of the other children fifty pennies and a slip of paper with a numeral on it. (Use various numerals from 41 to 49.) Each child places beside the paper the number of cents (or pennies) indicated on the paper. The rule is that the child should exchange his pennies until he obtains the fewest possible number of coins. Tell the children that they may exchange 5 pennies for 1 nickel at the first bank and 5 nickels for 1 quarter at the second bank. Let them devise their own procedures.

When they have finished, check the results. The children then exchange their nickels and quarters so that at the end of the game each child gets back his original number of paper disk counters.

To give more interest to the game, you may suggest that the slip of paper is a bill that the child must pay to receive a prize (such as a star), a snack, or a treat. The bill and money can be presented to you at the "store" (your desk).

Another variation is to have the slips of paper represent tickets that must be paid for before the child can go on a trip (to the playground or lunch room, for example).

Lesson 39: THE HUNDREDS PLACE (OPTIONAL)

This lesson extends the concept of place value to the hundreds place. You may use the lesson if you feel that your children are ready for it. If a child masters the ideas in this lesson, he will be able to use with understanding numbers to 999. Some children will independently extend the concept to thousands, ten thousands, etc.

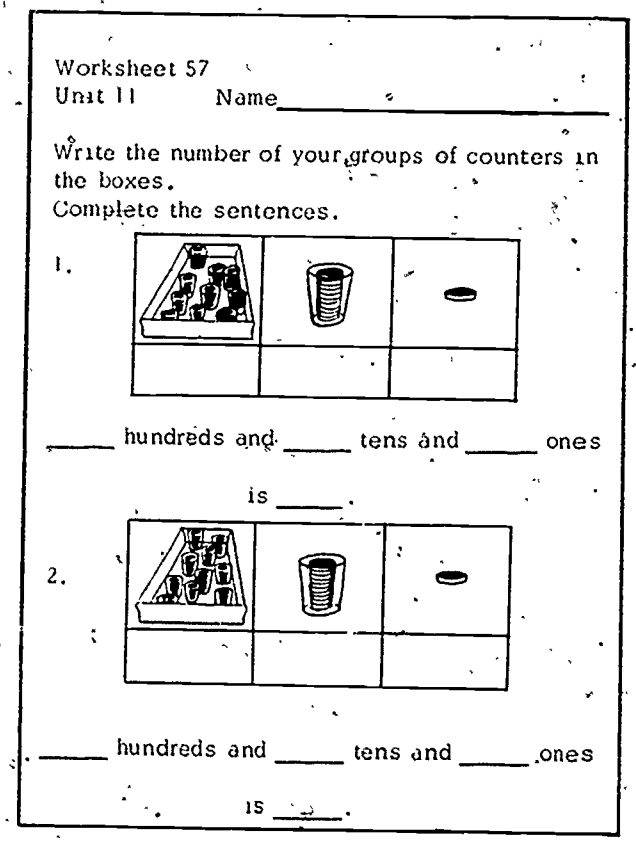
MATERIALS

for each child:

- 50 or more counters (paper disks)
- 5 or more small paper cups

for each group of 3 or 4 children:

- a shallow tray or box top, as from a Minnebar box
- Worksheet 57



Worksheet 57
Unit 11 Name_____

Write the number of your groups of counters in the boxes.
Complete the sentences.

1.

_____ hundreds and _____ tens and _____ ones

is _____.

2.

_____ hundreds and _____ tens and _____ ones

is _____.

PROCEDURE

The children will work in groups of 3 or 4.

Have each child place almost all of his counters (about 45) on his desk along with his paper cups and Worksheet 57. Each child should begin by grouping his own counters by tens, placing each set of ten in a paper cup. After this has been done, each small group of children should combine their counters. First they should see if there are enough single counters to make more groups of ten. Next, they should make a second grouping by tens, i.e., form groups of 10 tens. A convenient way to make this grouping is to place 10 cups each containing 10 counters in a box lid, or similar tray, to keep them together.

Each child should record on Worksheet 57, #1, the number of ungrouped counters, the number of groups of ten, and the number of groups of 10 tens that his group of children has formed.

Ask for a volunteer to tell another name for "10 tens." If the idea of 10 tens being one hundred is new or difficult for the children, you should take time here to reinforce this idea. For example, you could draw an array of 10 rows of 10 objects each. The children could then count the one hundred objects.

The sentence under the table in #1 should now be completed. Explain that in writing numerals for numbers over one hundred we use three digits. The hundreds place is to the left of the ten's place.

Repeat this activity with different numbers of counters for #2 on the worksheet.

Lesson 40: REPRESENTING NUMBERS ON AN ABACUS

The abacus has been used in various forms for centuries as a physical device for recording numbers. It is still a convenient and useful tool and can be used for many quite complicated mathematical tasks. It will be used in later MINNEMAST materials to illustrate addition and subtraction operations. Here the children will use it primarily as another concrete representation of numbers up to one hundred.

An abacus should be available for the children to experiment with before any formal presentation is made.

MATERIALS

- abacus
- about 15 objects for the flannel board
- Worksheets 58 and 59

PROCEDURE

A. Show the children 5 or 6 of the objects, and tell them that they will be counting them. Place the cleared abacus where the class can see it. Have one child move one bead into the column on his right for each object as it is counted. Have the child moving the beads report the number of beads he moved into the column.

Display 14 objects. Repeat the above procedure. Ask how they should proceed after the ninth object. Lead them to suggest using the next column to show the groups of ten. They should see this easily after their other work with grouping. (If you have a 20 bead abacus, it is possible to have 14 beads in the first column. This is correct, as an intermediate step. However, it is wise to have the children realize that to agree with conventional notation only nine beads can be in any column. Perhaps explain that if there are many more than nine beads in any one column, it becomes confusing to tell the number of beads.)

Repeat with different numbers of objects for practice.

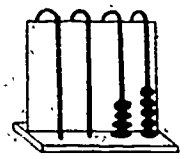
B. The children should now be ready for reading a number from an abacus. Show the children an abacus with 3 beads in the right hand column and 2 beads in the next column. Have one child write the number of tens and the number of ones on the board (2 tens and 3 ones). Have another child write the numeral in the decimal system (23).

Repeat this activity with other numbers as often as you feel is necessary.

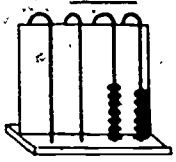
C. Have the class do Worksheets 58 and 59.

Worksheet 58
Unit 11 Name_____

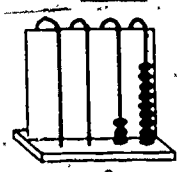
Name the number shown by the beads on the abacus in each picture.



4 tens + 5 ones = 45



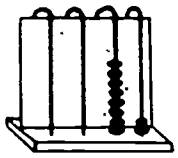
7 tens + 6 ones = 76



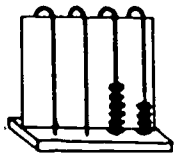
3 tens + 9 ones = 39

Worksheet 59
Unit 11 Name_____

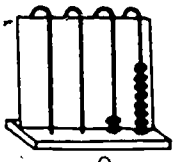
Name the number shown by the beads on the abacus in each picture.



8 tens + 2 ones = 82



6 tens + 4 ones = 64



2 tens + 8 ones = 28

SECTION 6 SUPPLEMENTARY GAMES

There are many mathematical concepts to which young children can and should be exposed. Some of these include properties of numbers, commutativity of addition, and principles of numeration. They should meet these concepts in many contexts. One is the more formal, structured situation described in the preceding lessons. Another very fruitful one is the game situation. Games can be introduced at the appropriate time in the formal presentation and then played and expanded (preferably by the children) throughout the year.

The games described here should be regarded as skeletons for similar games. They should probably be introduced to small groups of children at a time. Use the games according to the children's needs and interests.

Game A: TRAINS

The various train games described here are designed to develop spatial relationships, develop an intuitive concept of length, and present addition concepts within a new embodiment.

MATERIALS

— Minnebars

— deck of cards with numerals 1 to 30

PROCEDURE

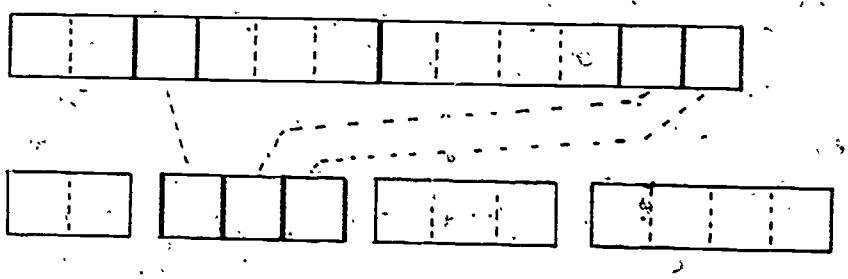
1. The children will have had many opportunities through the year to play with the Minnebars. They will match the bars; build towers, trains, buildings, roads or patterns; and develop their own activities. Let them make up stories to go with their designs. The Minnebars should be left in a place easily accessible to the children for use during free time. Encourage free activity as well as the following structured situations.

2. Have children work in groups of three or four. Each group should have three or four sets of Minnebars. Tell each child to take a handful of Minnebars and to build a train with it. Ask questions of each group concerning such things as which train is the longest and how they can be sure, and what kinds of cars each train has. The purpose here is not only to broaden the child's concept of length and addition but also to widen his investigation of the train he built.

3. With the children working in pairs, ask one child to pick up a set of Minnebars. The other is then to take a different set of bars that have the same number of units altogether. There are a variety of ways for the child to find out if he has the same number of units, including matching colors by one-to-one correspondence, or laying out one set of Minnebars end to end and constructing another train of the same length.

4. Tape a number line with the same spacing as the Minnebars to the floor or a counter top. Ask the children to build trains along this number line. First, have them line up the rear end of the caboose at 0 and build different length trains. Have them give the train the name of the point on the number line reached by the front of the engine.

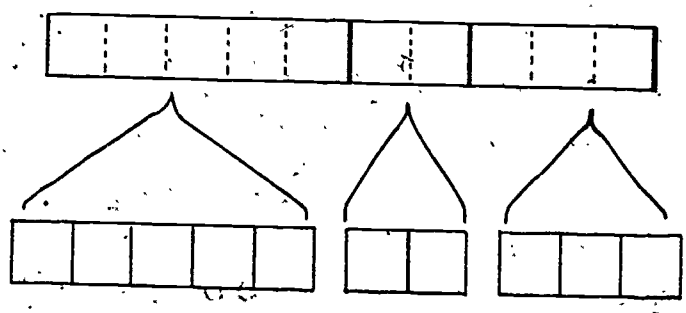
5. After a child has built a train, ask him to rearrange the pieces so that the Minnebars of equivalent size are grouped together. This set could be rearranged this way:



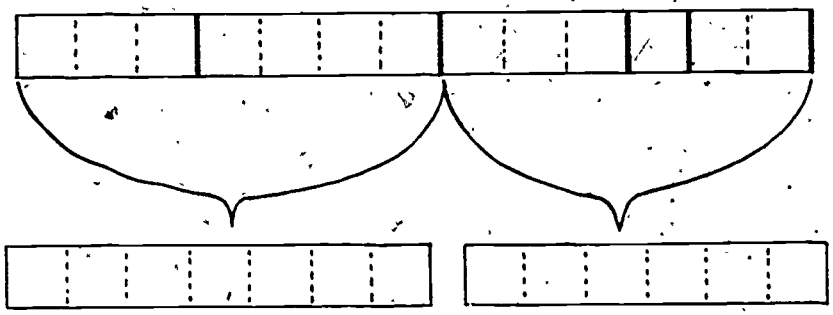
6. Each child should start with one set of Minnebars. Have the child build a train with any combination of Minnebars. After his partner has removed some of the cars, he is to put other cars in their place so that the train doesn't become longer or shorter. A car cannot be replaced with the same type of car. For example, a 5-unit car might be replaced by a 3-unit car and a 2-unit car.

Some possible expansions and variations of this game are:

A. The "changer" might try to use as many bars as possible in each substitution. For example:



B. The "changer" might try to replace the cars in such a way that the total train has as few pieces as possible.



7. Prepare a deck of index cards numbered 1-30. For individual play, the child would draw a card and build a train of that length. For a pair of children, have one child draw a card and both children build trains of that length. The two children can race to see who can build his train first.

8. Have the children line up a train on a number line and then move it so that the rear end of the caboose is at any numeral (e.g., 3). They should notice at which numeral the front end of the engine stops. Have them move the same train several times, noting the positions of the caboose and the engine after each move. Have them describe their train in this way:

Rear of Caboose moves to	Front of Engine moves to
3	12
8	17
6	15
1	10
0	9

9. One child builds a train while his partner is not looking. The builder makes a record of his train and takes it apart. He then gives the record to his partner who rebuilds the train.

The record could be a series of appropriately colored crayon marks or a series of numerals to designate the length, e.g., orange + yellow + dark green + light blue + dark blue, or 3 + 4 + 6 + 7 + 8.

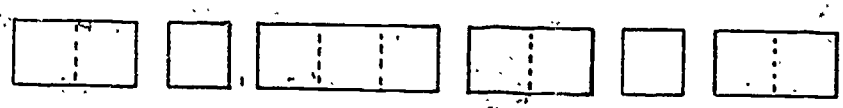
10. Encourage a child to see how many trains of different lengths he can build using only blocks of one color. He should compare the lengths of the trains by noting the number of units in each train. He might record these trains in the following manner:

Blue Trains (using 4-unit Minnebars)

Number of Cars	Length of Train
1	4
2	8
3	12
4	16
5	20

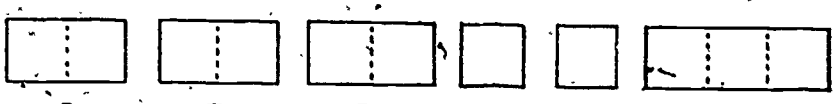
This recording may be done in any similar manner.

11. Have a child build a train and record the number names of the cars in his train.



2 + 1 + 3 + 2 + 1 + 2

Then he is to rearrange the train with equivalent cars together.



2 + 2 + 2 + 1 + 1 + 3

Ask how this could be written in a shorter form, and, if necessary, suggest the way by saying that there are 3 cars of 2 units each, etc.

12. Encourage the children to find various names for describing the number of units in the same length train, as shown below. Have them build trains, rearrange them and record.

long name	short name
3+2+1+4+5+2+2	19
2+2+2+3+4+5+1	19
3(2)+3+4+5+1	19

13. After activity 12, have children substitute longer cars for two or more smaller pieces so that the train remains the same length. They should record these in the same way.

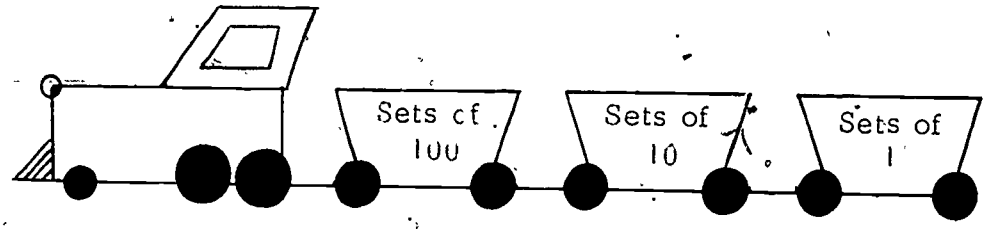
Child substitutes:	Cars in Train	Long Name	Short Name
—	6	3+2+1+4+6+1	17
5 for 3 + 2	5	5+1+4+6+1	17
10 for 5+1+4	3	10+6+1	17

Game B: EXCHANGING GAME

This game reinforces the idea that multi-digit numerals represent a great many ones or units. The game should be started soon after Lesson 34. It will continue for up to three months.

MATERIALS

- many small slips of paper
- locomotive and 3 cars cut from colored construction paper The last car should be labeled "Sets of 1's," the next car "Sets of 10's," and the car just behind the locomotive "Sets of 100's." The cars should be pasted around three edges to a sturdy cardboard backing, and left open at the tops so that they form pockets.



PROCEDURE

There are many desirable activities that you might want to keep track of for each child. Among these might be: reading a book, being on time for school, doing a homework assignment, bringing back a note sent home. Choose one or a combination of these that you expect each child to have done at least 50 times in three months.

Each time a child does the activity you are recording, give him a small slip of paper with his name on it and have him put his slip in the one's car (caboose) of the train. He should fasten all his slips together with a rubber band or paper clip. When he has 10 "passengers" in the caboose, he can exchange the 10 slips for a slip of a different color, the 10-slip. This different colored slip is put into the tens car. The child continues to collect slips for the ones car until he has 10 more that he can exchange for a 10-slip. Ten of the 10-slips can be exchanged, when they accumulate, for a slip of a third color, the 100-slip, which goes in the hundreds car.

Meanwhile, you keep all the bundles of slips that the children have exchanged. At the end of the playing time (perhaps three months), reverse the procedure. Call in all 100-slips and provide 10-slips in their stead. Finally, have the children exchange the 10-slips for slips of the original color (10 for each 10-slip).

Before the final exchange, the children should predict how many slips they will have altogether after the exchange. Have them check their prediction by counting slips that they hold at last.

Game C: BUILDINGS

These games are designed to develop spatial relationships, present addition concepts within a new embodiment, and introduce the concept of scale drawing.

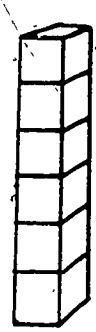
MATERIALS

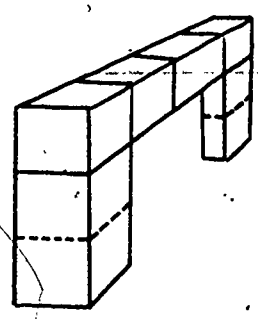
— Minnebars

— graph paper

PROCEDURE

A. For the first day or two, allow the children to construct "buildings" (vertical arrangements) by stacking Minnebars. They may build one-legged buildings, two-legged buildings,





or multi-legged ones. It will be a challenge to make multi-legged buildings stand up. The children will soon discover that the cross pieces may not always be horizontal and that, in order to make them so, wood must be added to the shorter leg.

The children will find they can build more varied buildings by constructing them flat on their desks and pretending they are standing. The problem of making them stand limits the patterns. Encourage flat building for the next activity.

B. After a child has leveled a building, ask him to keep a record of his building on graph paper.

Give each child a sheet of graph paper and ask him to make a picture of his building by using one square on the paper to represent one unit on the Minnebars. The record might look like this: