DOCUMENT RESUME

ED 126 678 FL 007 562

AUTHOR Otani, Yasuteru
TITLE The Influence of the Mother Tongue on Achievement in Mathematics: A Contrastive Study of English and Japanese. Workpapers in Teaching English as a Second Language, Vol. 9.
INSTITUTION California Univ., Los Angeles. Dept. of English.
PUB DATE Jan 75
NOTE 8p.
AVAILABLE FROM English as a Second Language, Department of English, University of California, 405 Hilgard Avenue, Los Angeles, California 90024 ($2.00)

EDRS PRICE MF-$0.83 HC-$1.67 Plus Postage.
DESCRIPTORS Academic Achievement; *Contrastive Linguistics; Cultural Background; Cultural Traits; *English (Second Language); *Japanese; *Mathematical Vocabulary; *Mathematics Education; Native Speakers; Numbers; Sociocultural Patterns; *Sociolinguistics

ABSTRACT

It is commonly held that numbers and number expressions constitute the first universal language independent of any natural language. No doubt has been cast on the notion that mother tongue differences make no difference in learning mathematics. Because of its "universal" character, mathematics was chosen as the subject to be investigated in the IEA International Study of Educational Achievement. A defect in this study, however, is the failure to take into account the cultural background of the participating countries. For example, the mathematical expression "2 x 3," is believed by IEA examiners to express an identical concept universally. However, it does not. In English, it generally means "2 times 3," namely "3 + 3." In Japan, it means "2 no 3 bai," or "2 + 2 + 2." That is, the positions of the multiplier and the multiplicand are reversed, because there is no equivalent way of expressing "A times B" in Japanese. When the number system, multiplication tables and mathematical expressions are thus considered, mathematics remains an artificial language not fully independent of natural languages, rather than the ideal medium modern science is searching for. The results of the IEA test will reflect not only the degree of achievement in mathematics, but also features of the examinee's own language and culture as a background. (Author/CLK)

THE INFLUENCE OF THE MOTHER TONGUE ON ACHIEVEMENT IN MATHEMATICS - A CONTRASTIVE STUDY OF ENGLISH AND JAPANESE -*

Yasuteru Otani**

1 Perhaps it can be said that modern science is gradually shifting its medium from natural languages to an artificial language more suitable for abstract and independent reasoning. Presumably, a typical example of such an artificial language is the system of mathematical symbolism. It is commonly held that numbers and number expressions constitute the first universal language independent of any natural language. In fact, no doubt has been cast on the notion that mother tongue differences make no difference at all in learning mathematics.

2 In 1964, the Council of the International Project for the Evaluation of Educational Achievement (IEA) undertook the first large-scale cross-cultural study of educational achievement in twelve countries.[1)] It was because of its "universal" character that mathematics was chosen as the subject to be investigated.[2)] The primary purpose of the project was to study the relationships between certain variations of educational practice and educational achievement. And the main results of the study were published in 1967.[3)]

These results are of particular interest in that at the 13-year level Japan achieved the highest score under the most unfavorable educational conditions, whereas English-speaking countries obtained much lower scores under far more favorable conditions, and the tables were turned at the level of 17-year-olds. What on earth does this mean?[4)] What are the factors producing or associated with these outcomes? The IEA study mentioned social background, school organization, curriculum, teacher competence, and so on, as principal factors which might have affected students' achievement. Basically, however, it could not supply a satisfactory answer to the above questions.

A grave defect in this study is, it seems to me, the failure to take into account the cultural background of the participating countries. In other words, what prevented the results from being properly interpreted is the common belief that mathematics is "universal." In fact, it seems reasonable to assume that mathematics remains an artificial language not fully independent of natural languages, rather than the ideal medium modern science is searching for. Viewed from this standpoint, the results of the IEA test are bound to reflect not only the degree of achievement in mathematics, but also different features of the examinee's own language and culture as a background. To give a simple illustration, the mathematical expression "2 x 3," which is found in Test No. A. 6, is, of course, believed by all the IEA examiners including representatives from Japan to express an identical concept across the world. The fact is, however, that it expresses no one universal concept: the concept varies from country to

**Visiting scholar in the Department of English 1974-75 from St. Andrew's University, Osaka, Japan.

*I would like to express my deep gratitude to Dr. Clifford H. Prator and Dr. Thomas P. Gorman, for their careful reading of the manuscript and for their valuable comments and suggestions. I should also like to thank Professor Hiromi Ando for his generous help and guidance in the early stages of this study.

country. In the English world, the expression generally means "2 times 3," namely "3 + 3." In Japan, on the contrary, it means "2 no 3 bai,"[5] which signifies "2 + 2 + 2."[6] That is to say, the positions of the multiplier and the multiplicand are reversed, because there is no equivalent way of expressing "A times B" in Japanese. This discrepancy is obviously due to the fact that "universal" mathematics is vitally affected by the particular features of a natural language.

Thus, it is probably only in the light of further contrastive analysis of these languages that the results of the IEA study can be more properly interpreted.

3.1 The Number System

It is assumed that the "number sense" is something innate and number is a universal concept not necessarily related to language. We must not, however, lose sight of the fact that the number system we use today is one of a great many systems, and that in one sense Chinese and Japanese have the most concise numerals and the most rigid decimal system in the world.[7] And there is no question that these constitute the distinct advantage both peoples enjoy in understanding numeral concepts and mastering number skills. This is why they do not need to learn addition and subtraction tables as European and American children do.

The number systems of European languages are far more complex than their Oriental counterparts. As is well known, many languages exist which employ a strange mixture of the duodecimal and the vigesimal systems, together with the decimal system. For example, the numeral representing 91 is "4 x 20 +11 (quatre-vingt-onze)" in French, and "1 + 4 1/2 x 20 (en og halvfemsindstyve)" in Danish. And it was the unfortunate results of this complexity that brought the French some years ago to undertake a drive to adopt a concise "octante" and "nonante" for 80 and 90. In English, too, the vigesimal expressions like "two score" and "three score and ten" are not obsolete yet. In addition, the duodecimal system exemplified in such expressions as "dozen," "gross," "inch," "foot" is still influential. The system of weights and measures followed especially in English-speaking countries is generally very different from the metric system. Needless to say, these different and unrelated units cannot fit in with modern mathematics, which is based on the rigid decimal system.

Another disadvantage English has in the number system is irregularity in the numerals between 11 and 19. They do not follow the same system as other numbers: "eleven," "twelve" instead of "ten-one," "ten-two"; "thirteen" (3 + 10), "fourteen" (4 + 10) instead of "ten-three," "ten-four."[8] These present a marked contrast to Japanese which uses the same system as the Arabic figures: "ju-ichi" (10 + 1), "ju-ni" (10 + 2) . . . .[9]

Number is essentially an organized system of thinking. And we cannot underestimate the influence of inconsistency in the number system on beginners in mathematics, especially on their learning operations of addition and subtraction.[10]

3.2 Multiplication Tables

According to my private questionnaire to over five hundred American students at UCLA, as many as thirty-eight percent of them cannot recall the multiplication tables. In Japan, however, more than ninety-nine percent of college students have a good command of them.[11] Contrary to the popular view, this is primarily due to the difference between the students' languages, not to differences in their abilities.

In short, Japanese numerals are, firstly, simpler in their construction (generally one consonant + one vowel), secondly, able to be further simplified according to the context, and thirdly, able to be read making a variety of puns, owing to the character of Japanese which abounds in homonyms. This is why telephone numbers and even big figures of national budgets are so easily memorized. European visitors are often astonished to see Japanese secondary school students answer automatically the long figures for $\sqrt{2}, \sqrt{3}, \pi$, etc.[12]

Likewise, Japanese multiplication tables are far easier to memorize and recite than English ones.[13] Japanese primary school pupils learn the complete multiplication tables in four or five weeks in their second grade. To meet the needs of the duodecimal or vigesimal system, European and American pupils need to master up to the 12's or even to the 20's, whereas their Japanese counterparts do not need to learn beyond the 9's. It is needless to say that mathematics is better learned by understanding than by mere memorization. But it is also true that effective skill in multiplication and division rests on memorizing basic multiplication facts.

Thus, we must recognize that the learning of elementary mathematics, especially the four fundamental operations of rational numbers, is affected by the mother tongue far more extensively than we have imagined. From this, we can safely predict that China, which has similar cultural characteristics to Japan, would have achieved one of the highest scores, if she had joined the test.

4 Mathematical Expressions

Those are the advantages Japanese has in mathematical operations. And this is clearly proved by Table 1 of the IEA results, especially the extremely high scores in arithmetic skills such as "Basic Arithmetic" and "Advanced Arithmetic." But, in the matter of certain mathematical expressions, Japanese is on the other hand seriously disadvantaged. Compare Japanese with English in the following expressions.

$2/3$
- two over three or two-thirds
- 3 bun no 2

$a - b \div c \geq d$
- a minus b divided by c is greater than or equal to d
- a kara, b o c de watta ($\div$) mono o hiku ($-$) to, d yori okii ($>$) ka hitoshii ($=$)

$\int_a^b f(x)dx$
- integral from a to b of f of x
- f(x) o a kara b made sekibun (integral) suru

We can read them in English in exact accordance with the order of the symbols, but not in Japanese. To read them observing the Japanese word order, the order of the symbols must be reversed: e.g., a, b, c, $\div$ - d $\geq$.[14]

In other words, the structure of mathematical expressions is utterly foreign to Japanese, and it can be inferred from this that mathematical expressions are not independent of nor neutral among natural languages. This is easily understood if we recall the fact that current mathematical expressions were modeled in Europe on European languages in the sixteenth to the eighteenth centuries. As the expressions become more complicated, this discrepancy between Japanese and the expressions is aggravated, and it constitutes a serious obstacle for Japanese students in the way of learning advanced mathematics. This results in a definite gap between attainment in elementary and advanced mathematics which cannot be observed in European and American students. The IEA results show up the gap by the fact that the Japanese students' score on calculus is one of the lowest among the twelve countries.

Furthermore, in view of the above, it is also hardly surprising that achievement in mathematics often shows a significant correlation with that in English at the advanced level of Japanese secondary schools.

5. As can be seen from the above, it is imperative for the objective evaluation of the results to get beyond the limitation of the IEA study, and seek rather to tap underlying cultural factors. Considered in this light, the results of the IEA test will be sure to betray their deeper implications. And these findings prompt the conjecture that mathematics, which enjoys universality as the medium of modern science and technology, does not deserve to claim a full exterritoriality against natural languages.

FOOTNOTES

1. The coordination of the project was carried out from the UNESCO Institute in Hamburg.
2. Torsten Husén, ed., International Study of Achievement in Mathematics: A Comparison of Twelve Countries (Stockholm: Almqvist & Wiksell, 1967), I, 34.
3. Some of the main results of the study are as follows:

Table 1. 13-Year-Olds

	Aus.	Bel.	Eng.	Fin.	Fra.	Jap.	Neth	Sco.	Swe.	U.S.	Total
Mean (70 items)	20.2	27.7	19.3	24.1	18.3	31.2	23.9	19.1	15.7	16.2	19.8
Hours of mathematics instruction per week	5.1	4.6	4.0	3.0	4.4	4.5	4.0	4.6	3.8	4.6	
Size of class	36	24	30	36	29	41	25	30	26	29	
Public expenditure per pupil ($)	240	288	348	130		81	191	361	483	545	

Table 2. Mathematics Students in Final Secondary Year (17-Year-Olds)

	Aus.	Bel.	Eng.	Fin.	Fra.	W. Ger.	Isr.	Jap.	Neth.	Sco.	Swe.	U.S.	Total
Mean (69 items)	21.6	34.6	35.2	25.3	33.4	28.8	36.4	31.4	31.9	25.5	27.3	13.8	26.1
Hours of mathematics instruction per week	6.9	7.4	4.4	4.0	8.9	4.2	5.0	5.4	5.1	6.2	4.6	5.0	
Size of class	22	19	12	23	26	14	20	41	19	21	21	21	

Table 3. Estimate of Increase in Mathematics Performance from 13-Year Olds to Terminal Mathematics Students

Level of performance of 13-year-olds *	Aus.	Bel.	Eng.	Fin.	Fra.	Jap.	Neth.	Sco.	Swe.	U.S.	Total
	-1.52	-1.22	-1.47	-1.33	-1.53	-0.90	-1.33	-1.53	-1.64	-1.69	-1.49
Level of Performance of terminal math. students *	0.86	1.66	1.61	1.28	1.63	1.55	1.53	1.18	1.18	0.14	1.11
Increase in level	2.38	2.88	3.08	2.61	3.16	2.45	2.86	2.71	2.82	1.83	2.60

*Relative to grand mean and standard deviation of terminal nonmathematics students.

- Ibid., I, 254, 277, & 279; II, 22, 24 & 30

4. The statement of Time reporting the results is interesting.

"The tests also undermined the conviction of American education that better teaching lies in smaller classes." --Time, 17 March 1967.

Did Japan achieve the high score because of, or, in spite of, its large classes?

5. "3 bai" means "3 times."
6. Thus, the horizontal form "2 x 3 = 6" is expressed in the vertical form as "3" x2 / 6 in English and "2" x3 / 6 in Japanese, respectively.
7. Hisanosuke Izui, "Sushi no Sekai (Numerals)," Gengo Seikatsu (Language and Life) (Tokyo: Chikuma Shobo), Nov. 1973, p. 17.
8. In German, this irregularity continues endlessly beyond 20: einundzwanzig (1 + 20) . . . neunhundertneunundneunzig (900 + 9 + 90) . . . .
9. This is why English-speaking peoples do not denote the numbers of years and pages as (1914 - 8) or pp. 112 -5, which are common in Japanese.
10. An obstacle Japanese beginners encounter, on the other hand, concerns the procedure for separating figures. The figure 280000000, for example, is read

in Japanese as "2 oku 3000 man," and a reasonable separation would be 2,3000,0000. The figure, however, is usually separated based on European languages as 230,000,000, marking off thousands and millions, which is utterly foreign to Japanese.

11. My data reveal that American-born Japanese whose mother tongue is practically English do not master the multiplication facts as easily as native Japanese do.

12. $\sqrt{2} = 1.41421356 \ldots$
(hito yo hito yo ni . . . = Night after night . . .).

$\sqrt{3} = 1.7320508 \ldots$
(hito nami ni ogore ya . . . = Treat me to a drink as others do . . .)

$\pi = 3.14159265 \ldots$
(san-ishi ikoku ni mukau . . . = A doctor goes abroad . . .)

And the Japanese alphabet (i ro ha ni ho he to . . .) itself is a fine poem!

13. This is also true of Chinese and Korean multiplication tables. The first complete Japanese multiplication tables published in 970 were called "Kuchi Zusami" which means "reciting." And this presents a marked contrast to the "Pythagorean Tables" which were supposed to be just looked at.

14. In fact, this kind of notation, in which operation symbols are placed, not between, but after quantities, was advocated by J. Lukasiewicz, a Polish mathematician, in 1951. It is called the "reverse Polish notation" and has in practice been adopted for use with electronic computing machines.

REFERENCES

Allen, Harold B., and Russell N. Campbell, eds. Teaching English as a Second Language. New York: McGraw-Hill, 1972.
Carroll, John B. Language and Thought. Englewood Cliffs: Prentice-Hall, 1964.
Celce-Murcia, Marianne, and Lois McIntosh, eds. An Introduction to the Teaching of English as a Second Language. Los Angeles : UCLA, 1974 (prepublication edition).
Greenberg, Joseph H., ed. Universals of Language. Cambridge: M.I.T. Press, 1963.
Hymes, Dell, ed. Language in Culture and Society. New York: Harper and Row, 1964.
Institute of Languages and Linguistics, Georgetown University. Report on a Conference on the Meaning and Role of Culture in Foreign Language Teaching. Washington: U.S. Office of Education, 1961.
Lado, Robert. Language Teaching. New York: McGraw-Hill, 1964.
Macnamara, John. "The Effects of Instruction in a Weaker Language." The Journal of Social Issues, 23, No. 2 (1967).
Mandelbaum, David G., ed. Selected Writings of Edward Sapir in Language, Culture and Personality. Los Angeles: University of California Press, 1963.
Nida, Eugene A. Customs and Cultures. New York: Harper, 1954.
Takeuchi, Yoshimi. Chugoku o Shirutameni (To Understand China). Tokyo: Keiso Shobo, 1970.
Toyama, Shigehiko. Gengo to Shiko (Language and Thought). Tokyo: Shogakukan, 1968.

X X X

Dantzig, Tobias. Number: The Language of Science. New York: Macmillan, 1933.
Fehr, Howard F., ed. Mathematical Education in the Americas. 2 vols. New York: Columbia University, 1962 & 1966.
Husén, Torsten, ed. International Study of Achievement in Mathematics: A Comparison of Twelve Countries. 2 vols. Stockholm: Almqvist and Wiksell, 1967.

Izui, Hisanosuke. "Sushi no Sekai (Numerals)." Gengo Seikatsu (Language and Life), Nov. 1973.

Keeves, J. P., and W.C. Radford. Some Aspects of Performance in Mathematics in Australian Schools. Hawthorn: Australian Council for Educational Research, 1969.

Land, Frank. The Language of Mathematics. New York: Doubleday, 1963.

Mizutani, Shizuo. Gengo to Sugaku (The Language of Mathematics). Tokyo: Kitamori Shuppan, 1970

Munroe, M. Evans. The Language of Mathematics. Ann Arbor: University of Michigan, 1963.

National Institute for Educational Research. International Study of Achievement in Mathematics. Tokyo: NIER, 1967.

Organisation for European Economic Co-operation. School Mathematics in OEEC Countries. n.p.: OEEC Office for Scientific and Technical Personnel, n.d.

Pidgeon, Douglas A., ed. Achievement in Mathematics. London: National Foundation for Educational Research in England and Wales, 1967.

Postlethwaite, Neville. School Organization and Student Achievement: A Study Based on Achievement in Mathematics in Twelve Countries. New York: John Wiley, 1967.

Toyama, Hiraku. Sugaku no Manabikata · Oshiekata (How to Teach and Learn Mathematics). Tokyo: Iwanami Shoten, 1972.