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ABSTRACT

This study examined the relationships between spatial ability, mathematics achievement, and sex for students in grades 2-7. Four spatial tests (embedded figures, serial integration, coordination of viewpoints, and rotation and development of surfaces) were used to measure spatial ability; sex and mathematics achievement, as measured by the Iowa Test of Basic Skills, were determined from school records. Data for the 90 subjects were submitted to four three factor (grade x sex x mathematics achievement) analyses of variance. All analyses yielded results indicating a relationship between spatial and mathematical ability. For two of the tests, sex differences favoring males were revealed. (SD)

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SPATIAL ABILITIES, MATHEMATICS ACHIEVEMENT, AND THE SEXES

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Spatial Abilities, Mathematics Achievement, and the Sexes

Abstract

The literature suggests a relationship between spatial and mathematical thinking and a sex difference favoring males regarding spatial abilities. This study examined these notions regarding specific types and levels of spatial abilities. Four spatial tests, embedded figures, serial integration, coordination of viewpoints, and rotation and development of surfaces, representing various levels of spatial thinking complexity were administered to 90 children enrolled in grades two through seven. The scores from the four tests were analyzed in separate three-factor analyses of variance (Grades x Sex x Mathematics Achievement). All four data analyses revealed that high mathematics achievers appear to have greater spatial ability than low mathematics achievers; that older children seem to have greater spatial ability than younger children. Data analyses for the two tests measuring higher level spatial ability, coordination of viewpoints, and rotation and development of surfaces, indicated sex differences favoring males.

Spatial Abilities, Mathematics Achievement,

and the Sexes

Recent reviews by Fennema (1974) and Maccoby and Jacklin (1974) establish male superiority in spatial abilities and in mathematical abilities both appearing during early adolescence and continuing throughout adulthood. The concurrent developmental trend in these two abilities leads to the suggestion that less adequate spatial abilities among girls might help explain less adequate achievement in mathematics. Fennema's excellent discussion of this topic identifies the key problems which cloud adequate synthesis of past work and serve as focal points for the present study.

1. There is no uniform agreement on the critical factors which specify "spatial ability."
2. While fairly good evidence exists for a relationship between spatial abilities and mathematical achievement at the secondary and college levels, little is known about such relationships at earlier developmental levels.

Each of these issues merits a brief elaboration.

<u>Spatial Ability and Its Subdivisions</u>

Smith (1964) cites early studies by Kelly, Guilford, and Thurston establishing two or more factors entering into tests purporting to measure spatial abilities. In reviewing the studies related to spatial ability and sex differences, Maccoby and Jacklin (1974) divided the studies into two

categories. Thirty studies seem to involve nonanalytic spatial processes (e.g., mazes, form boards, or block counting) and 47 studies seemed to involve analytic visual spatial processes (e.g., embedded figures or rod and frame). The analytic tasks were tentatively viewed by Maccoby and Jacklin as requiring "decontextualization", i.e., the process of disembedding the stimulus or figure from its surrounding context. Both types of spatial tasks showed a similar developmental pattern with respect to sex differences, a male advantage appearing in adolescence.

Smith (1964) appears to have synthesized the research literature on spatial abilities about as thoroughly as any investigator in this area. Based on a review of the factor studies available, and his own research, Smith concluded that the critical factor in tests of spatial ability is the ability to grasp, as a whole, the configurational aspects of a pattern, i.e., to visualize, hold in mind, and perform operations with patterns perceived as integrated wholes. Given this critical feature of spatial ability, Smith points out that part of the difficulty in detecting this ability in factor studies is that spatial tasks may be accomplished using different ways of perceiving and solving the problems.

One can appreciate Smith's notion of grasping visual configurations as opposed to analyzing distinctive features of figures by noting his comparison of ways in which designs can be reproduced from memory.

> Those subjects who can form and retain an image of the design will be best served by a "fixative" mode of attention. Having fixated the design, they will retain the figure "in mind" as a complete gestalt and will have no difficulty in reproducing the figure in its correct proportions. On the other hand, subjects who form visual images with difficulty will be best served by a "diffusive" mode of attention. They will tend to glance at different parts of the figure during the exposure and will attempt to remember as many details as possible. They will produce a design which may include most of the necessary details, but may well be incorrect as an organized structure or whole. The proportions of the figure may be quite incorrect. (pp. 207-208).

With respect to sex difference in this particular ability, Smith cites a study by Taylor (1960) involving 100 boys and 100 girls having an average age of 14 years, 4 months. When a memory for designs test was scored conventionally, a difference in means favoring the boys by about 3 score points was shown. When scored for accuracy of details, the mean scores favored the girls by about 1 score point. When marked for correctness of proportions, the mean difference swung back to favor the boys by a striking 12 points (rounded). Thus, Smith sees the ability to visualize and retain patterns as the critical spatial factor, and when

this factor is isolated by scoring techniques, there is evidence that the difference between boys and girls is further accentuated.

Need for Developmental Studies

Most studies showing a relationship between spatial abilities and mathematical achievement have been conducted at the secondary and college level. Smith maintains that spatial abilities are most directly related to the mathematical thinking required for higher level conceptualization and may have little to do with the numerical facility involved in ordinary arithmetic. Fennema (1974) in direct reaction to Smith's position writes, "In order to learn new ideas, learners are dependent upon prerequisite ideas in their cognitive structure. Little is known concerning the impact of spatial ability on the acquiring of these prerequisite mathematical ideas on which all later mathematical knowledge is based." (p. 8) There is a need, as Fennema points out, to study the effect of spatial ability on mathematical learning at various developmental levels.

Design of the Study

Our discussion of some of the critical issues involved in spatial ability points to an assumption which seems sufficiently warranted to provide a working hypothesis for further research. Smith's contention that the critical skill in spatial tasks is the ability to grasp, retain, and

manipulate visual configurations seems reasonable. Given this assumption, one might ask whether this ability might not have a hierarchical structure ranging, perhaps, from an ability to conceptualize patterns to the ability to mentally transform those patterns into different forms. Recognizing that the first iteration of such a hierarchy is likely to be crude in many ways, we would like to propose four ascending steps or tasks in which the ability to visualize configurations and perform mental operations with them will be manifest. An appropriate test for each of these abilities is shown in parentheses.

1. The ability to form a simple pattern from a limited series of stimuli seen one at a time. (Serial Integration)
2. The ability to perceive a configuration and to retain that configuration in mind despite distracting elements. (Embedded Figures)
3. The ability to perceive a three-dimensional object and conceptualize that object sufficiently well to describe portions of it not immediately visible. (Coordination of Viewpoints)
4. The ability to conceptualize a three-dimensional object and to mentally transform this object into a two-dimensional representation. (Rotation and Development of Surfaces)

It seems important to determine if tests of the four separate types of spatial abilities differentiate between students classified as to grade, sex, and mathematics achievement. The current study is designed with these variables in mind and makes provisions for four measures (levels) of spatial ability, a standardized measure of arithmetic achievement, six developmental levels (grades 2 through 7), and a representation of both sexes at each grade level.

Method

Instruments

Four tests of spatial ability were developed by the authors, each test designed to measure a more complex type of spatial ability:

Serial Integration (SI) (McDaniel, 1974) This is a group test designed to measure the ability to integrate into a pattern visual stimuli seen one at a time. Subjects watch a screen while single lines are projected one at a time. Then four figures are shown on the screen. The children must select the pattern formed if all lines were presented simultaneously.

Embedded Figures (EF) (McDaniel, 1974) This is a group test designed to measure the ability to hold a visual gestalt in spite of distracting elements. Subjects watch a screen while a simple pattern is projected for five seconds. Following this, a response display is shown containing four

more complex designs, one of which has included in it the more simple pattern. The children must select the design containing the embedded figure.

Coordination of Viewpoints (CV) (Guay, 1975) This is an individually administered test designed to measure the ability to visualize the shape of objects from various viewpoints. Subjects are seated at a round table and observe a particular three-dimensional geometric object (cube, pyramid, etc.). Three line drawings are then projected onto a screen. The children must select the one that best represents the object's appearance as seen from specified viewpoints other than their own.

Rotation and Development of Surfaces (RD) (Guay, 1975) This is an individually administered test utilizing the same objects as the CV Test. The purpose of this test is to measure the ability to anticipate the rotation and development of object surfaces. While inspecting the object, the children must select from among three drawings projected on a screen the one which represents the object with its surfaces rotated and developed into a single plane.

Procedure

The SI, CV, and RD Tests were administered to a total of 90 children, approximately 15 children selected at random from each of six grade levels, two through seven. The EF Test was administered only to the children enrolled in

grades five through seven. The children were selected from one school located in Lafayette, Indiana.

Children at each grade level were classified according to two factors: (1) math achievement and (2) sex. The children were divided at the median of each grade level into high and low math achievers on the basis of their total mathematics achievement scores on the Iowa Tests of Basic Skills. These scores were available from school records.

Results

The SI, CV, and RD scores were analyzed in separate 6x2x2 (Grades x Sex x Mathematics Achievement) analyses of variance. The EF score was analyzed employing a 3x2x2 (Grades x Sex x Mathematics Achievement) analysis of variance. Although none of the four analyses revealed either second- or third-order interaction effects among the three factors, significant main effects of the individual factors were found.

Significant grade main effects were found for SI scores, $\underline{F}(5,66) = 4.19$, $\underline{p} < .005$; EF scores, $\underline{F}(2,34) = 6.41$, $\underline{p} < .005$; CV scores, $\underline{F}(5,66) = 4.58$, $\underline{p} < .005$; RD scores, $\underline{F}(5,66) = 6.16$, $\underline{p} < .001$. Since only interactions of grade with sex or math achievement were of primary interest to this study, no statistical probes were conducted on the significant grade main effects. However, an inspection of the grade marginal means indicated a trend toward an increase in SI, EF, CV, and RD scores with an increase in grade.

Significant mathematics achievement main effects were found for SI scores, $\underline{F}(1,66) = 12.98$, $\underline{p} < .001$; EF scores, $\underline{F}(1,34) = 19.61$, $\underline{p} < .001$; CV scores, $\underline{F}(1,66) = 9.88$, $\underline{p} < .005$; RD scores, $\underline{F}(1,66) = 5.72$, $\underline{p} < .05$. The data revealed that the mean scores for all four tests were significantly higher for the high mathematics achievers than for the low mathematics achievers. The mean scores for the SI were 21.40 and 18.08; for the EF, 21.44 and 17.70; for the CV, 9.42 and 7.49; for the RD, 9.24 and 7.91.

Significant sex main effects were found for CV scores, $\underline{F}(1,66) = 7.21$, $\underline{p} < .01$, and RD scores, $\underline{F}(1,66) = 4.62$, $\underline{p} < .05$. The data indicated that males had a significantly higher CV mean score (9.30) than females (7.85); males had an RD mean score (9.20) that was significantly higher than females (7.98). No significant sex differences were found for the SI or EF scores.

Discussion

These findings reflect on the issues raised initially. All four spatial tests differentiated between high and low achievers in mathematics. High mathematics achievers were found to have greater spatial ability than low mathematics achievers. The findings suggest that spatial factors are related to numerical facility starting at a relatively early point in the students' school experience and continuing through the elementary and junior high school grades. Thus,

these results indicate that the relationship between spatial abilities and mathematical achievement, which is fairly well established at the secondary school level (Fennema, 1974), extend downward to the earlier grade levels. Since the contents of the Iowa test at the elementary levels stress calculation skills, these findings do not support Smith's (1964) contention that spatial abilities are related only to abstract abilities required in higher mathematics.

The findings also suggest that females and males often have different aptitudes for spatial thinking. Although the scores on the spatial tests requiring the more elementary spatial abilities (SI and EF) were independent of the subjects' sex, males' performance on the higher level spatial tests (CV and RD) was significantly better than females'. The sex differences were not found to be a function of any variation in grade level. These observations are consistent with literature reviews (e.g., Maccoby & Jacklin, 1974) indicating sex differences favoring males, but inconsistent with that portion of the reviews suggesting that sex differences become evident only during early adolescence.

Taken as a whole, the study suggests that there may be value in following Smith's notion that the ability to grasp and manipulate gestalts is the significant factor in spatial ability. Accepting this position opens the way to attempts to dimensionalize this factor along a continuum ranging from

simple recognition and retention of patterns to more elaborate mental manipulations of visual images. This study also points the way to the more precise analysis of the spatial abilities which seem to be associated with mathematical competence. The findings support the contention that the relationship between spatial and mathematical thinking appears to exist for lower level as well as higher level spatial abilities. And finally, this study provides a more precise specification of the types of spatial abilities which seem to be associated with sex differences. The results support the notion that sex differences in favor of males may exist only for higher level spatial abilities.

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