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ABSTRACT

This paper reviews a number of past studies in the field of diffusion research, describing the major features of each diffusion model and discussing its value for predicting the spread of educational innovations. Following this review, the author presents a new autocatalytic diffusion model based on the mathematical models of epidemiologists and chemists. This autocatalytic model is adapted to the study of educational innovations, and then the model is applied to historical data and used to predict the life cycle of six different educational innovations. The predicted life cycles matched the actual historical data very well in five of the six cases. The author also uses the model to project the future adoption life cycle of the semester system in Ontario secondary schools, an innovation that is still in the process of adoption. (JG)

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AN AUTOCATALYTIC MODEL FOR
THE DIFFUSION OF EDUCATIONAL INNOVATIONS

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The spread of educational innovations is a topic that has been investigated from a multitude of complementary viewpoints. Some individuals have been interested in dissemination of particular practices or products; others have studied the diffusion process itself; still others have focussed on the adopters, their characteristics and motivations; and most recently, investigators have concentrated on the problems of implementation of innovations. This paper is in the tradition of diffusion research, but is an outgrowth of recent developments in a quantitative approach that has emerged in the field of marketing that, in turn, has its roots in the mathematical models of epidemiologists studying the spread of diseases and chemists investigating the nature of chemical reactions. In particular, a new model to describe the diffusion of new practices and products is presented, similar to that proposed by Bass (1969). The model is then applied to a number of educational and business innovations. The findings are related to the literary tradition of diffusion research (Rogers, 1962, 1971) and to several concerns about the nature of change in education (House, 1974, 1976).

Background

The tradition of research on the diffusion of educational innovations is associated with the work of Paul R. Mort at the Institute for Administrative Studies, where over 150 studies were completed over a period of forty years. In summarizing the overarching findings, Mort (1964) concluded, in part,

- 1) a long time elapses between recognition of a need and creation of an acceptable solution;
- 2) the spread of an educational innovation proceeds at a slow pace, requiring decades to complete;

3) the rate of diffusion for complex innovations is the same as for simple ones, though expensive innovations proceed more slowly than inexpensive ones;

4) during the early stages of its introduction, an innovation is usually "invisible," but its spread is relatively rapid after its recognition;

5) communities vary in the degree to which they take on to new practices.

The Mort studies also discovered, as suggested by point four, that innovations etch out an S-curve when plotted cumulatively through time (Rogers, 1971), and a bell-shaped curve if plotted annually. It is the mathematical modelling of these curves that is the focus of this paper.

Rogers brought the strands of diffusion research in rural sociology, education, anthropology, sociology, medical sociology, and industrial history, economics and engineering together in Diffusion of Innovations (1962). The universality of the S- and bell-shaped curves describing the life cycle of the adoption of an innovative practice or product was established beyond doubt, though attempts at their quantitative description were rather unsuccessful. At the same time, various concepts were presented which give useful meanings to more complex mathematical formulations:

Interaction effect - "the process through which individuals in a social system who have adopted an innovation influence those who have not yet adopted... If the first adopter of the innovation discusses it with two other members of the social system, and these two adopters each pass the new idea along to two peers, the resulting distribution follows a binomial expansion. This mathematical function follows a normal shape when plotted (1962, p. 154)."

Adopter categories - Adopters can be grouped into innovators, early adopters, early majority, late majority and laggards.

Combining the mechanism suggested in his definition of the interaction effect with the idea of adoption of a practice spreading from a group of special innovators to the remaining members of a population, Rogers suggests a two-step flow:

"Most individuals become aware of innovations from mass media and then proceed to discuss these innovations with peers as they evaluate the idea.... Essential to the idea of a two-step flow is a distinction between opinion leaders and their followers....(p. 214)"

".... In other words, when earlier adopters talk to later adopters about a new idea, the rate of adoption proceeds more quickly than when this communication does not occur.

The interaction effect is generally similar to the process by which an infectious disease such as scarlet fever, diphtheria, or measles spreads through a social system. Bailey (1957, pp. 15-22) analyzes the infection process in terms of "infectives" (similar to active adopters of a new idea), "susceptibles" (those who are not yet infected), "removals" by death or isolation (similar to passive adopters), and the "incubation period" (similar to the adoption period).... There is close theoretical similarity of the infection process to the diffusion process. Perhaps some of the complex mathematical equations obtained for different types of epidemics by Bailey (1957) might be fitted to the diffusion of innovations (p. 217)"

Ironically, Rogers suggestion to apply the mathematics of epidemiology to diffusion research was taken up by the one tradition he had omitted: marketing. At the time, he had noted, "Katz and Levin (1959) list of traditions includes marketing research which the present list does not.

There are relatively few research studies available in the field. It is probably true, however, that "... much of interest in the field of marketing may be in the files of private agencies...." (Katz and Levin, 1959)."

Bass (1969) developed a growth model for sales of new products related to the mathematical theory underlying contagion models applied in the field of epidemiology. Bass notes, "Behaviorally, the assumptions are similar in certain respects to the theoretical concepts emerging in the literature on new product adoption and diffusion,... The model differs from models based on the log-normal distribution (e.g., Mansfield, 1961) and other growth models ... in that the behavioral assumptions are explicit (p. 219)." It can also be noted that the normal distribution used to describe the distribution of adopters by year, which Rogers (1962) advocates, is necessarily rejected by the Bass approach since, 1) there is no behavioral rationale for use of the normal or Gaussian distribution for the purpose, and 2) the Bass model does not reduce to a normal distribution.

The Bass model is concisely summarized in Nevers (1972) and Dodds (1973), and the latter notes the model's success not only in modeling past behavior but in predicting future behavior: "One of the advantages of the Bass model is that it permits a forecast of the timing of a turndown in sales during a period in which sales are growing rapidly, where naive forecasting models tend to project indefinite sales growth at rapid rates. Thus Bass in 1966 accurately predicted that a sales peak would occur in 1968 for color television set sales at 6.7 million units, while the industry was building plant capacity for 14 million picture tubes. The overly optimistic industry projections resulted in rather severe economic dislocation which might have been avoided (Dodds, 1973, p. 308)." In addition to the "adoption" of new color

television sets, Bass applied his model to a number of other consumer items.

Nevers (1972) extended the Bass model to the retail service, agriculture and industrial technology sectors, in addition to that of consumer durables, evaluating his results using multiple regression. For the first, he notes, results are good both in terms of estimated time and magnitude of sales peak. A ten year prediction for McDonald's franchises (1955 to 1965) "over which an actual sales peak was realized, yielded very good results (p. 87)." For industrial technology and agricultural sectors, results were reasonably good, though the short period to peak adoption "cast some doubt ... on the significance of predicted times (p. 87)." For the consumer sector, results were excellent for boat trailers, and within four percent of the magnitude of peak year sales for colour television sets, though peak sales were predicted one year earlier than they actually occurred. See Table 1.

Dodds (1973) improved on the methods used by Bass (1969) and Nevers (1972) to estimate adoption curves and parameters by using non-linear estimation techniques in place of their use of linear regression.

INSERT TABLE 1 ABOUT HERE

Following Nevers' lead, it would be possible to test the Bass model with educational data. How well would it fit, say, the data reported by Carlson (1965) for the adoption of Modern Math in Allegheny County, Pennsylvania, and the State of West Virginia? In what year would it predict the peak number of adoptions of the semester system in secondary schools of Ontario, Canada, given the numbers having adopted semestering in the past five years?

These questions are answered in this paper, but using a model developed independently of Bass by Lawton (1974). In many respects, this model is similar to Bass's in that it uses formulations developed in epidemiology and, incidentally, chemistry. Terminology from the latter field is used to describe the model: an autocatalytic model for adoption of innovations. In chemistry, a reaction in which the molecules of a substance formed as the reaction proceeds acts as a catalyst to hasten the formation of additional molecules is termed "autocatalytic." This terminology is particularly appropriate to the adoption of innovations since this type of reaction is analogous to Roger's interaction effect, described previously, which recounts the influence of one adopter on others who have not yet adopted a new idea, practice or product.

The autocatalytic model described below has at least two notable differences from the Bass model. First, there is a single rate constant p for all adopters, rather than two rates--one for innovators and one for imitators--as proposed by Bass. The concept of a rate constant which is determined experimentally is fundamental to the study of the velocity of chemical reactions (Sienko and Plane, 1961). Typically, the rapidity with which two chemicals react is proportional to their concentration and chemical nature, to temperature, and to the presence of a catalyst. Any automobile owner living in an area of severe winters or in an oceanside community is

well aware that iron rusts more quickly in humid weather than dry (i.e., it depends on the amount of moisture in the air), and that this reaction is speeded by the presence of salt. While a car in Phoenix might take 30 years to rust, the same damage might occur in Toronto in just 3 years; that is, rusting might occur 10 times as fast in the latter city. In theory, the rate formula for this reaction might look something like this:

Rate = p (concentration of moisture, salt, iron), where p is the so-called "rate constant," which must be determined by experiment. Clearly, if stainless steel were substituted for iron in the same equation, p would be much smaller since the rate of corrosion would be much less. This concept of the rate constant can be developed using the (disease analogy, as well: a highly contagious disease has a far larger constant than one which is not easily spread. In both cases, the rate constant does not depend on the extent of the materials or populations involved. Hence, rates for adoption of a given innovation in different jurisdictions can be compared directly, as can rates computed from either raw data or percentage distributions. In addition, the rate constant for the autocatalytic model differs from that defined by Rogers (1962, p. 139). The latter gives a distinct value from year to year, making comparisons even between the same innovation at different times and different places impossible. Similar limitations apply to the use of rates for different innovations using Rogers' definition. The second important departure from Bass (1969) is the use of the notion of a "seed" which starts the adoption process, much as a seed sets off a chemical reaction. Using the disease analogy, this seed is a group of "carriers" that introduce a disease to a new population of "susceptibles", thus starting an epidemic.

Whereas Bass and Lawton have, in essence, proceeded from Rogers' concept of interaction to the macroscopic, Carlson (1965) proceeded to the microscopic in Adoption of Educational Innovations. His analysis of social interaction (p. 19) suggests the internal processes of an autocatalytic reaction, which the mathematical models describe only in terms of cumulative effects. Carlson does present cumulative data for six educational innovations (Modern Math, Language Laboratories, Team Teaching, Programmed Instruction, Accelerated Programs, and Foreign Language in elementary schools), and he relates rate of diffusion--defined as the area under the histogram which gives the percentage of adoptions over time--with characteristics of individual innovations. As an empirical definition of rate of diffusion, Carlson's choice may be sound if an innovation's period of adoption is over. However, the definition is useless if one wishes to predict the course of adoption of a new practice, such as semestering, when its spread has just begun. Of most relevance to this study is the data Carlson presents; its reanalysis using the autocatalytic model appears later.

More recently, House's The Politics of Educational Innovations (1974) has elaborated on the social process of implementing innovative changes in education. Much of his analysis concerns the distinction between innovations adopted by two different types of entities--the individual for what he terms "household innovations," or the organization for what he terms "entrepreneurial innovations" which are meant, in the end, to affect people other than those directly involved in the adopting agency. Examples of the first are television sets and automobiles, examples of the latter are computer assisted instruction and McDonalds' franchises.

In general, House is critical of diffusion models, such as the information flow model proposed by Pederson (1970), because they tend to view all adopters as equal, equally aware and equally likely to adopt. House notes this assumption is false, and emphasizes that social structure is the dominating force, particularly in the case of entrepreneurial innovations. This latter point is graphically illustrated by points plotted on maps of Illinois showing the state-wide spread of programs from the educationally gifted. Clearly, urban centres located along major highways adopted first, while small, remote rural communities adopted last.

In fact, however, House has not actually rejected diffusion models; he has simply refined the target population into ever smaller groups on the basis of various factors affecting the likelihood of adoption. In marketing terminology, he has segmented the market according to size and geographic proximity to transportation corridors. Since in practice diffusion models generally assume the similarity of adopting units, market segmentation clearly makes them more applicable by making the populations to which they are applied more homogeneous in fact. Recognition that detailed analysis of both population structures and the adoption/implementation process is possible, and indeed necessary, does not contradict the validity of macroscopic modeling of the diffusion process. The situation is not unlike that which occurs in chemistry:

"For most reactions, it is only the disappearance of starting materials and the appearance of final products that can be detected; i.e., only the net reaction is observable. In general, however, the net reaction is not the whole story but simply represents a summation of all the changes that occur. The net change may actually consist of several consecutive reactions, each of which constitutes a step in the formation of final products....

It is important to keep clear the distinction between net reaction and one step in that reaction (Sienko and Plane, 1961, p. 237)."

So, too, macroscopic diffusion models such as that presented here focus on net effects. Study of individual steps, in all their (linear and non-linear) complexity and with their host of intermediate results, is an important complementary area of investigation.

Overview of the Autocatalytic Model

Immediately after the appearance of a new practice, product or idea in the educational arena, its adoption by educators, schools or school boards unfolds year by year. A plot of the adoption of such an "innovation" against time yields a graph like that in Figure 1 for NEWSTYLE teaching. This plot of adopting units (in this case teachers) as a function of time is called the life cycle of the innovation.

Insert Figure 1 about here

The life cycle function contains some of the essential information needed for planning and dissemination decisions. The sum of all the numbers of adopters for each year (the area under the "curve") represents the total population of adopters for the innovation to whom information must be disseminated or training provided; the peak height provides information about the capacity of the training or disseminating unit (if any); and the width of the curve gives time elapsed between first acceptance by innovators and late acceptance by laggards and, hence, the date at which support services can end.

If this life cycle curve were known prior to the emergence of an innovation, then the planning and dissemination decisions concerning the innovation would be greatly simplified. Of course, the exact life cycle cannot be known until after the end of the cycle, too late to be of assistance in planning and dissemination decisions.

The difficulty of making reasonable predictions about life cycles is perhaps most severe when one is dealing with truly "new" ideas, practices, and products which are drastically different from those that preceded. Open plan schools, programmed instruction, semestered high schools (radically different for Ontario, at least), and two-tiered metropolitan governance of schools are but a few examples. Thus, the potential value of applying some of the results of studies which have focussed on the time evolution of adoption of new products (e.g., Bass, 1969) is obvious.

Most of the models that have been developed are aimed at a broad range of distinctive "new" practices and products, as opposed to minor changes in those currently in use, which have the characteristics that the innovation can be adopted but once during its life cycle. Hence, if a school has adopted programmed instruction for mathematics and later replaces worn programmed texts, only the first adoption is counted. Further, the model ignores the exceedingly important possibility that the innovation might be dropped later, as would occur if programmed texts were replaced by Computer Assisted Instruction for teaching mathematics.

Both the Bass(1969) and Lawton (1974) models put forth the theory that the timing of an individual's initial adoption of an innovation is strongly related to the number of previous adopters of that innovation.

Essentially, Rogers' interaction effect, characterised by increased familiarity with the innovation as it is being applied in the field,

and the desire to emulate opinion leaders in high status positions (or in layman's terms, the desire to "keep up with the Jones") are thought to provide, in terms of social dynamics, the autocatalytic force.

Other possible factors, advanced by House (1974) include economic motivation for profit and career advancement, might be of particular importance in the case of entrepreneurial innovations.

Ultimately, the thorough understanding of this autocatalytic force and the factors involved in its creation is of critical importance. As House (1974) notes, the "tempo of economic development depends on the speed of the innovation's diffusion and implementation (p. 259)." Certainly, control of economic development is a critical factor in all societies. Nevertheless, a thorough understanding of the drive behind adoption of innovations is not necessary for applying the autocatalytic model if one can assume that, for a given population at a given time, the net force is set. That is, one must assume that among school administrators in given jurisdiction, the force would not change during an innovation's adoption life cycle.

Though this assumption limits the validity of the model in theory, it may be reasonable in practice. Only extreme changes in external factors are likely to effect the process of adoption, judging from results reported later, and when these occur, they are unlikely to pass unobserved. Indeed, a quasi-experimental situation is created when projecting the future path of adoption for an innovation which facilitates the identification of variables affecting the innovation process. By analogy, the spread of an epidemic can be projected early in its course, even if the mechanism by which it is spread is unknown. Further, a sudden cessation of the epidemic might provide a clue which would lead to understanding the mechanism. In

short, modeling the macroscopic is not dependent on understanding the micro-processes.

Mathematical models, such as that presented here, cannot be trusted or considered useful until they can be shown consistent with, and provide an explanation for, a large portion of actual life cycles of innovations. As noted previously, the Bass model has proven quite successful; so too has the autocatalytic model for the adoption of innovations. It has been applied to more than thirty sets of data, and given consistently good results. In particular, its predictions for the timing of and number of adoptions in the peak year of the life cycle when applied to historical data are excellent for the following. A detailed examination will be provided for the educational innovations in a later section.

Education

Modern Math
Foreign Language (elementary)
Accelerated Programs
Programmed Instruction
Language Laboratories
Semestering

Retail Service

McDonald's franchises

Industrial Technology

Rapid Bleach Process licenses

Consumer Products

Power Lawnmowers
Boat Trailers
Television Sets
Clothes Dryers
Air Conditioner
Freezers
Record Players
Tape Recorders
Cable TV Subscribers

The autocatalytic model describes the entire life cycle for an innovative idea, practice or product in terms of three simple numbers: the size of the population of adopters, denoted by N , which is the total number of units that would adopt the innovation if no other innovations

or disturbances occurred; the number of first year adopters, denoted by s_1 , and the rate constant, denoted by p , which is a measure of how fast the innovation spreads through the population of adopters. As is the case in chemistry, the rate constant for a given innovation or class of innovations must be determined by examining historical data which, in essence, represents previous "experiments."

The examination of some thirty sets of historical data led to the surprising conclusion that p is relatively stable for consumer products, (i.e., House's "household innovations") and may be stable though somewhat larger in magnitude for commercial and educational innovations, (i.e., House's "entrepreneurial innovations"). If ^{this} "discovery" is true, then one needs only two parameters to estimate the total life cycle of an innovation: the size of the population of adopters N and the number of 'first year adopters s_1 . The effects of differences in either the number of first year adopters or the sizes of populations of adopters on the subsequent evolution of the life cycle curve can easily be examined by use of the model.

Use of the Autocatalytic Model

The model can be used in four distinctive ways: 1) analysis of historical data, 2) extrapolation of life cycle curve from early data on the adoption of an innovation, 3) prediction of life cycle curve before the introduction of an innovation, and 4) estimating the "value" of a totally new innovation from early adoption data.

Analysis of historical data using the autocatalytic model amounts to validating the model. Had the model not been able to reproduce the characteristics of actual life cycles, then it would have been discarded. In practice, the best fit of the model to an actual life cycle is deter-

mined by computer analysis,⁴ using an approach similar to that of Dodds (1973). An example of such an analysis is shown in Figure 2 for Cable TV subscribers, a typical household innovation.

Insert Figure 2 about here

Analysis of historical data provided strong evidence that the autocatalytic model is quite consistent with the time evolution actually observed, Figure 2 being a typical example. In all, over thirty life cycles were reduced to the three basic parameters, N , s_1 , and p . At this point it was discovered that the rate constant p tended to take on values between 0.4 and 0.6 for consumer products, 0.6 and 0.7 for commercial products, and 0.6 and 1.4 for educational innovations. Over the range from 0.4 to 0.6, differences in the parameter p have only a very minor effect on the life cycle curve. From these "experimental" data, it would appear ^{an} a priori estimate of $p = 0.5$ should be adequate for most analyses of the adoption of consumer products; of $p = 0.65$ for commercial products; and $p = 0.8$ for educational innovations of an entrepreneurial kind. The high value of the rate constant for educational innovations is particularly striking in view of Mort's findings that educational innovations were slow to spread; discussion of this finding appears in a ^{later} section.

Extrapolation or forecasting of future adoptions of an innovation from early adoptions is a particularly practical use of the autocatalytic model. As noted earlier, Bass correctly predicted the peak year of colour television sales, and Dodds (1973) demonstrated the capability of the Bass model for extrapolating the life cycle of Cable TV operations. Using the autocatalytic model to analyze data presented in Table 2 on the conversion

of Ontario secondary schools to the semester system of scheduling courses (Hill, 1974) produces the results graphed in Figure 3, given the assumption that all 470 high schools responding in Hill's study eventually convert. Note that 1974-75 and 1975-76 are predicted as peak years, with conversions tapering off at the end of the decade.

Insert Table 2 and Figure 3 about here

If one assumes instead that the population of adopters for semestering is only 250, then 1973-74 was the peak year for conversions, and that the process will be completed by 1978-79. Regardless of which value of N is used, p is quite large in comparison to consumer and commercial rate values, equalling .74 in the first case and .85 in the second. Note that from historical data of a complete life cycle, all three parameters N , s_1 , and p can be estimated statistically (in this case using a nonlinear estimation procedure). In forecasting from very early data, however, it is generally useful to fix N based upon "outside" information. Otherwise, an exponential curve would be calculated with N being infinite. In this example, the fact $N = 470$ yields a p with a smaller standard error than is true for $N = 250$ (.04 vs. .07) suggests that $N = 470$ is the more reasonable assumption.

Forecasting the entire life cycle of the adoption of an innovation is perhaps the most difficult, yet most important, use of the model. In this instance, opinion surveys might be used to estimate the potential number of adopters (be they boards, schools, or individual teachers) and the number most likely to adopt the innovation in the first year. The rate constant p would normally be taken as 0.8 for educational innovations, 0.65 for commercial products, and 0.5 for consumer products, based on the historical data

noted earlier. One cannot expect perfect accuracy, of course, since forecasting is inherently difficult. If the size of the population of adopters is grossly over- or under- estimated, or the estimate of first year adopters is very poor, then the projected life cycle estimate will be very poor. In education, sizes of adopter populations are in most cases reasonably easy to estimate; e.g., there are about 650 high schools in Ontario, 100,000 teachers, etc. Admittedly, these are maximums which may be applicable in some cases and not in others, but at least estimates can be made on some basis. Perhaps the best approach is to use probability distributions for population size, number of first year adopters, and p in order to produce a band of functions which contain the "true life cycle" with a certain degree of probability.

Using the autocatalytic model to study the "value" of an innovation to the population of potential adopters is the final application that can be made. If an innovation is so novel that the adopting population is unable to appreciate its "true value", there will be no catalytic forces to impel the reaction. What would happen if demonstration projects were initiated in a few schools, or if a concerted effort was made to publicize those already having adopted the innovation? In essence, these steps amount to increasing the number of first year adopters, s_1 , which is an index of the perceived value of the innovation in the population as a whole. Once one can effect s_1 , one can learn the effects on the life cycle of doubling or tripling s_1 . In fact, it would be seen that considerable leverage is supplied by s_1 . If subsequent adoptions were below that predicted on the basis of the artificially stimulated s_1 , it would be concluded the innovation was not highly valued by the adopting population.

Intuitive Development of the Autocatalytic Model

Underlying the autocatalytic model for adoption of innovations is the assumption that the innovations involved either provide an entirely new function (e.g., public day-care for pre-school children) or provide a significantly altered method or capability with respect to established functions (e.g., open-plan elementary schools). The basic problem under consideration is the dynamic aspect of the adoption (or diffusion) process. The process of unfolding adoptions may be viewed as the conversion of a population of actual adopters. The nature and dynamics of this conversion process determines the life cycle of an innovation.

Chemical kinetic models are concerned with the conversion of a population of molecules of one type into a population of molecules of another type. Based on some simple mechanistic assumptions, these kinetic models predict the dynamic (i.e., time evolution) nature of the conversion process. These same models have been adopted to the study of the dynamic behaviour of animal populations and epidemiology.

Consider for the moment, that we have an experimental method for teaching a subject, codename NEWSTYLE, that represents a radical departure from previous methods (much as immersion French differs from the traditional, grammatical approach to language instruction). Given its cost, attractiveness, practicability, etc., there are some numbers of teachers (or schools or boards, depending on the appropriate unit of analysis) who would ultimately adopt NEWSTYLE teaching if it were disseminated unchanged. This number, N , is the population of adopters and is generally unknown--and may in fact change if, for example, a change in regulations make NEWSTYLE acceptable in another jurisdiction.

At the instant NEWSTYLE teaching is announced to the educational community, there are no adopters, only N potential adopters. However, with each passing

year, some number of these N potential adopters decide to adopt--and are thus converted into actual adopters. The time evolution of this conversion generates the life cycle of NEWSTYLE teaching.

The nature of the conversion process, though not determined by the model itself, is suggested by the research of Carlson (1965), Rogers (1962; 1971), Havelock (1970), Mort (1964), House (1974; 1976), and others. At first, since there are no NEWSTYLE teachers, other teachers feel no professional drive to adopt it. Also, they have no opportunity to see other teachers using it, no opportunity to discover if they "need" it. In this situation, diffusion will be inhibited by the lack of any social forces encouraging its adoption. But then, a few innovators, individuals who have wide contacts and are attracted toward new methods, learn of NEWSTYLE, decide to try it, and find it to their liking. As more and more of their associates adopt NEWSTYLE, the inhibiting effect of isolation from NEWSTYLE decreases and ultimately vanishes. Thus, prior adoptions catalyze (stimulate) later adoptions. Indeed, once the conversion process begins, it is doubtful that it can be stopped except by extreme action.

Although it is somewhat unflattering to educational innovators, the analogy to the spread of a contagious disease, as suggested by Rogers, is very helpful. Let adopting NEWSTYLE teaching be equated to catching Disease X . In this framework, some number of people in Canada are "susceptible" to disease X ; let this number be N , the number of "adopters" of the disease. The rest of the population is "immune," for reasons that are themselves open to investigation. Nothing occurs until some number of "carriers", denoted by n_0 , enter the population. These carriers are not themselves "victims" (i.e., adopters); they merely carry the disease and can communicate it to "susceptibles" on contact. In the educational community, these n_0 carriers represent the effects of dissemination by field agents, professors of

education, Ministry personnel, leaders of professional organizations, salesmen, advertising, demonstrations, etc. They are not themselves real adoptions (i.e., teachers using NEWSTYLE in the classroom), but they do provide social impetus by communicating the value of NEWSTYLE to the population of potential adopters and by creating the appropriate imagery that makes NEWSTYLE appealing (e.g., NEWSTYLE teachers are leaders). That is, they catalyze susceptibles into adopting NEWSTYLE.

If $S(t)$ is used to denote the cumulative number of people who become "ill" (adopt) by year t , then the number who become "ill" (adopt) in the first year ought to be some fraction of the N "susceptibles" who come into contact with the n_0 "carriers". If one assumes that the likelihood of a "susceptible" contacting a carrier is proportional to the number of "carriers", then

$$S(1) = f \times n_0 \times N \quad (1)$$

where f is a proportionality constant, n_0 is the number of "carriers" and N is the number of "susceptibles". Once others among the "susceptibles" have become "ill" (adopt NEWSTYLE), they too can communicate the "disease" to their fellow "susceptibles" (cause other teachers to adopt NEWSTYLE); once "ill", an individual becomes "immune" and cannot become "ill" (adopt) again. Hence, the number becoming "ill" (adopting) during the second year alone, $S(2) - S(1)$, ought to be proportional to the number of remaining "susceptibles", $N - S(1)$, times the number of effective "carriers", which now equals $n_0 + S(1)$ since it includes those catching the "disease" in the first year. That is,

$$S(2) - S(1) = f \times [n_0 + S(1)] \times [N - S(1)]. \quad (2)$$

Generalizing to year t , the number who become "ill" (adopt) in year t should be given by

$$S(t) - S(t-1) = f \times [n_0 + S(t-1)] \times [N - S(t-1)]. \quad (3)$$

In the limit, the difference equation in (3) corresponds to the differential equation for a second order autocatalytic reaction in chemistry. The differential equation is

$$dS(t)/d(t) = f*[n_0 + S(t)] [N - S(t)] \quad (4)$$

and its solution, which indicates the number of people who will become "ill" (adopt) by time t , is

$$S(t) = (N + n_0) / [1 + (N/n_0)e^{-pt}] - n_0, \quad (5)$$

where $p = f*(N + n_0)$ is defined to be the rate constant. It can be shown (Lawton, 1974) that the number of "carriers" n_0 can be determined from the size of the population of adopters N , the rate constant p , and the number of first year adopters $S(1) = s_1$. In fact,

$$n_0 = Ns_1 e^{-p} / [N(1 - e^{-p}) - s_1]. \quad (6)$$

Combining (2) and (3) yields a theoretical model for cumulative adoptions of any new innovation in which repeated adoption is not a significant factor or can be discounted. This model depends on the three basic parameters N , s_1 and p . Given these three numbers, the entire unfolding of both cumulative and annual adoptions can be computed.

The life cycle for such an innovation is simply the derivative (rate of change) of the cumulative number of adoptions $S(t)$. Thus, (2) and (3) provide a theoretical model for life cycles of innovations--the lapse between introduction and total diffusion of an innovation. In particular, the life cycle, or rate of adoption by "susceptibles", as a function of time, is given by

$$S'(t) = p(N + n_0)Q(t) / [1 + Q(t)]^2, \quad (7)$$

where n_0 is given by (6) and $Q(t) = (N/n_0)e^{-pt}$.

Analysis Using the Autocatalytic Model.

Next, the nature and characteristics of the theoretical life cycle model will be examined, followed by comparison of theoretical results with those of actual life cycles.

First, consider the case of a very innovative new practice which will eventually have a very large number of adopters (i.e., a very large N), but which has only a very small number of adopters in the first year, relative to the number of "susceptibles". That is, $s_1 \ll N$. For example, N might be 50,000 and s_1 only 500. In this case, the theoretical life cycle has the bell-shaped curve shown in Figure 4. Doubling N simply doubles the height of the curve without increasing its width, while doubling p simply halves the width of the curve, making it more sharply peaked, thereby forcing all adoptions to occur in half the time.

Insert Figure 4 about here

Next, consider the case where the number of first year adopters, s_1 , is only about one-tenth the total number of adopters N (i.e., is an order of magnitude less than N). This might occur, for example, when a new series of learning kits is introduced by a publisher and a large number of the company's regular clients order the first kit in the series. In such cases, one might have 5,000 kits adopted in the first year out of a total of 50,000 that would eventually be adopted. In this case, the theoretical life cycle has the appearance of a truncated bell-shaped curve as seen in Figure 5.

Insert Figure 5 about here

Doubling p still has the effect of halving the length of the life cycle; that is, it doubles the rate of adoption, with twice as many adopting in a given year. Doubling the size of N , however, is no longer a simple doubling of the height of the life cycle curve.

If s_1 is less than an order of magnitude less than N (e.g., if s_1 is only one-third of N), the change in the life cycle curve is still more striking. In this case, as seen in Figure 6, the life cycle takes on the character of an exponential decay with the largest number of adopters in the first year. This situation might arise when the next issue ^{in the} ~~of a~~ series of learning kits is introduced, and 20,000 kits are purchased by previous users who are familiar with and pleased with the kits. Doubling p still has the effect of halving the life cycle, while doubling N again simply doubles the height of the curve.

Insert Figure 6 about here

It turns out, then, that the theoretical life cycle of an innovation has the shape of a bell-shaped curve truncated from the left. The exponential life cycle in Figure 6 is just an extreme case in which only the right tail of the bell-shaped curve remains. The degree of truncation depends only on the number of first year adopters, s_1 , relative to the total number of eventual adopters N .

Besides the number of first year adoptions, there are two other prominent features of an innovation's life cycle: 1) the time to reach maximum (peak) adoptions, and 2) the maximum number of annual adoptions. These two properties of the life cycle are easily predicted from the three basic parameters, N , s_1 and p . The relationships are derived in Lawton (1974), and are summarized below.

Time to peak adoptions:

$$t_m = \frac{1}{p} \ln(N/n_0) = \frac{1}{p} [\ln(N) - \ln(n_0)] \quad (8)$$

Peak adoptions:

$$s(t_m) = p(N+n_0)/4, \quad (9)$$

where $n_0 = Ns_1 e^{-p} / [N(1-e^{-p}) - s_1]$.

Application to Historical Data

As noted earlier, the autocatalytic life cycle model is only useful since it displays the characteristics of actual historical life cycles of the adoption of innovations. This reasonable nature of the model has been demonstrated in its application to sets of data selected from education, business and industry. Values of N , s_1 and p were estimated using nonlinear estimation techniques which gave values resulting in the closest agreement between theoretical and historical life cycles. Presented below are the life cycles for six educational innovations; that of a seventh, semestering in Ontario secondary schools, appears in Figure 3 presented earlier. In each case, actual data are plotted as a histogram while the theoretical life cycle is given as a continuous curve.

The data for Figures 7 through 14 are derived from Carlson (1965, p. 68) using an Autorol curve reader to obtain values from the graphical data he presents. The data, in percentages, are reported in Table 3. A cross check using Modern Math data (Carlson, 1965, p. 16) indicates any errors are negligible.

Insert Table 3 about here

Figure 7 is concerned with the percentage of school boards adopting Modern Mathematics in Allegheny County, Pennsylvania, and the State of West

Virginia. Separate results from the two jurisdictions are reported in Figures 8 and 9. The rate constants for Allegheny County is considerably larger than that for West Virginia, suggesting that there is a distribution of rates under different "experimental conditions". Discovery of the nature of the differing conditions in the two contexts affecting the rate of diffusion is, at this point, a matter of speculation. In all cases, there is an excellent fit between the historical data and theoretical model.

Insert Figures 7, 8, and 9
about here

Results for the teaching of Foreign Language in elementary schools (Figure 10) are also excellent, as are those for Accelerated Programs (Figure 11), Programmed Instruction (Figure 12), and Language Laboratories (Figure 13). Only the fit between the historical data for Team Teaching and the theoretical model for those data is poor (Figure 14). Indeed, one must conclude that, in this one case, the autocatalytic model does not hold. Whether failure of the model is due to the setting, the particular innovation, measurement error or whatever, cannot be determined.

Table 4 summarizes the estimates of parameters and their standard errors for six educational innovations, including Semestering but excluding Team Teaching. Standard errors, shown in parentheses, are approximate but give an indication of how well the parameters are determined. Since N was set for Semestering, its standard error was not estimated. In addition to the parameters and their errors, the table indicates the number of years to peak adoption. Note that the shortest times--for modern math and programmed texts--corresponds to the situations with the highest rate constants.

The rates, p , for these educational innovations are plotted on a continuous scale along side those for the business and industrial sectors in Figure 15. The contrast, to say the least, is startling. The educational innovations themselves fall into two groupings. Accelerated

Insert Table 4 about here

Insert Figures 10, 11, 12, 13, 14
and 15 about here

Programs, Language Laboratories, Semestering, and Foreign Language for elementary schools all have rates ranging from 0.7 to 0.9, much as do industrial products. Programmed Instruction and Modern Mathematics, on the other hand, have much higher rates, averaging about 1.3. All educational rates are far higher than those for consumer products, and all entrepreneurial innovations (which includes all of the educational examples reported here as well as those in the commercial sector), exceed those for household innovations (limited to consumer products).

Discussion

Perhaps the most interesting insight the autocatalytic model gives concerning the adoption of educational innovations is the variation in rates of adoption for different innovations, the different rates for different sectors outside education, and the different rates for household and entrepreneurial innovations. Indeed, this ability to measure rates meaningfully is perhaps its most useful feature for theoretical purposes, though at this point in time one can do little more than speculate about the differing rates displayed in Figure 15. Answers are needed to the

following questions.

Why are educational rates of adoption so high in comparison to those in other sectors? One possibility might be the nature of the adopting unit. In education, as in industry which also displayed higher rates, the school or school board is often the unit studied. Rogers (1971) notes the need for study of "authority decisions" to adopt innovations within formal organizations. Organizations are smaller in number than are individual consumers, and presumably their members' knowledge about innovations is better, especially those individuals whose job it is to ensure that the organization adapts to changes in the environment.

This view is supported by House's (1974) classification of innovations as entrepreneurial and household. With stronger social and economic factors affecting the entrepreneur located in an organization, one would expect rates for the former to exceed those for the latter. Though no data for household innovations in education are reported here (i.e., those for which the teacher alone is the appropriate adopting unit), one might surmise such rates would be lower than for those innovations adopted at the school or board level. Of course, if the rewards accruing to teachers who innovate are as low as House suggests, examples of genuine household innovations in education may be too rare for sufficient data to be collected to test the hypothesis.

Yet, how is it that the Mort studies concluded that innovations were slow to be adopted in the schools? His finding is in direct contradiction to our finding that rate constants are higher for schools than for any consumer product, and are higher than for most industrial innovations. There are no clear answers to this paradox, but we would suggest three explanations. First, the Mort studies were not comparative in nature.

While they concluded innovation in education was slow to take hold, they may have ignored the fact it was slower elsewhere. Second, Mort tended to concentrate on a national perspective, whereas the data analysed here are for much smaller populations. Using the disease analogy, a virus does not spread within a given population until it has first invaded it--though it may be ravaging other groups. Thus, the spread of a disease may be much slower within a whole nation, due to internal barriers, than in a ^{state,} province or county. So too might an innovation spread more slowly from the national perspective. Finally, an explanation may be found in the secular changes between the time when most of the Mort studies were completed and the dates of the data studied here. The past fifteen years have seen far greater commitment of public funds to education than was true between 1930 and 1960--and money tends to stimulate change and innovation (House, 1976). Thus, delays in accepting new educational practices may actually have decreased since the Mort studies began.

To many, this last interpretation might be encouraging since innovation and change in education have often been treated as "good" while maintenance of traditional methods have been viewed as "bad". But, underneath the sermons advocating new practices, has always been the spectre of fadism. Was Modern Mathematics a thoughtful reform of curriculum, or a fad adopted to please critics? Is semestering in Ontario a fad which will die out in a few years, or a genuine reform in the scheduling of classes and courses? And, what is more, does it really matter, one way or the other? Is there a morality of innovation?

The rate constant of the autocatalytic model may prove to be one index useful in distinguishing fads--or reforms adopted in fad-like fashion--from more substantial changes introduced after adequate planning. It is our suspicion that any product or practice which is introduced with such

rapidity that its rate constant exceeds one can only be thought of as a fad. The history of modern math and the difficulties that followed its premature introduction into schools which lacked teachers trained in the subject and adequately prepared pupils, tend to confirm this view, as does the brief life of programmed texts in most schools. This is not to say that neither of these two innovations has its merits; it merely suggests that they were superficially adopted for their symbolic value to prove modernity, while the changes their genuine use would require were ignored or left to be worked out. In a sense, they were "adopted" but not "implemented". The danger of such fadism is apparent, for it clearly endangers the reputation of valuable innovations which may be discredited by too hasty adoption for the wrong reasons. To be sure, some amount of imagery is necessary in education; but education is too fundamental to society to survive on imagery alone. Thus, introduction of new practices requires a depth of preparation probably precludes extremely short life cycles--and extremely large rate constants. "[T]he vocational structure itself becomes a major impediment to innovation," House (1974, p. 260) notes in describing Junossey's (1966) analysis of the relationship between technological innovation and economic development. "[T]he vocational structure changes by the training or retraining of those already in the vocation or by entry of newcomers into it (House, 1974, p. 260)." But this is a slow process, and may extract a heavy human toll by requiring retraining of individuals. The individual's "skills become increasingly obsolescent.... The process of innovation requires that workers not use acquired knowledge and skills, but continually learn new ones (p. 261)."

It appears, then, that exceedingly high rate constants for innovations reveal fads or superficial adoption of innovations, but fail to assist in assessing the true worth, in educational terms, of the innovations involved.

Further, investigation of the same entrepreneurial innovation in different settings would reveal a distribution of rate constants which, when related to various sociological, psychological and economic variables, might provide insight into the effect of contextual factors on the rate of adoption of educational innovations. Similarly, by studying the rates for different innovations in the same setting, it may be possible to distinguish classes of innovations. In short, comparative analysis of rate constants may prove a useful starting point for further investigations of the diffusion/adoption/implementation process itself. Finally, one must beware of the "cult of modernism" which tends to assume that all innovations are good; one must look at the human costs and learn to weigh these, so that beneficial change can be distinguished from changes made for the sake of change, or for the advantage of those who are not directly affected.

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Table 1

PREDICTED VERSUS ACTUAL TIME AND
MAGNITUDE OF SALES PEAK*

Product / Technology	Predicted Time of Peak (no. of periods)	Actual Time of Peak (no. of periods)	Predicted Magnitude of Peak (Units)	Actual Magnitude of Peak (Units)
Boat Trailers	9.8	10	205,240	206,000
Color TV, Retail	6.0	8	5,733,400	5,490,000
Color TV, Manuf.	5.8	7	6,637,800	5,981,000
Holiday Inns	10.9	11	131.6	141
Howard Johnson Mot.	9.0	11	38.6	48
Howard Johnson. Hol. Inn, & Ramada Inn	9.8	11	202.6	216
McDonald's '55-'65	6.1	6	119.7	113
Cont. Bleach Range	3.2	4	16.7	18
Rapid Bleach	4.1	4	7.2	7
Conversion, 70 percent H ₂ O ₂ delivery system	3.3	4	48.5	50
Hybrid Corn	3.1	4	24.5	23

* Nevers (1972, p. 88)

Table 2

ADOPTION OF SEMESTER SYSTEM IN ONTARIO*

Year	Number Adopting Each Year	Cumulative** Percentage
1968	1	.20
1969	6	1.50
1970	9	3.40
1971	11	5.74
1972	23	10.64
1973	52	22.13

* Hill (1973, p. 8)

** Assuming 470 of 650 secondary schools
will eventually convert

Table 3

CUMULATIVE PERCENTAGE ADOPTION OF INNOVATIONS

Year	Modern Math	Foreign Language	Accelerated Program	Programmed Instruction	Language Labs	Team Teaching
1952		1.25	.67			
1953		2.02	1.15			
1954		2.88	3.26			
1955		3.36	3.55			
1956		4.70	6.14		1.06	
1957		8.26	8.35		1.82	
1958	.77	11.62	25.15	1.92	2.78	2.88
1959	5.95	17.86	36.19	3.26	7.10	4.80
1960	20.54	23.81	50.59	12.29	18.14	6.43
1961	44.93	30.05	57.02	25.24	24.38	9.79
1962	65.66	34.94	63.17	42.82	36.19	18.05
1963	74.11	36.19	64.32	49.25	45.12	20.45

Carlson (1963, p.68)

Table 4

ESTIMATES OF PARAMETERS AND STANDARD ERRORS FOR
EDUCATIONAL INNOVATION LIFE CYCLES

Innovation	N *	S.E. (N)	p	S.E. (p)	s_1 *	S.E. (s_1)	Years to Peak
Modern Math	77.6%	0.6	1.36	0.03	1.34%	0.09	3.8
Modern Math (W. Virginia)	17	0.33	1.02	0.06	1.23	0.10	---
Modern Math (Allegheny)	39	0.34	1.31	0.04	1.23	0.08	---
Foreign Language	39.4%	1.2	0.64	0.04	0.30%	0.05	7.5
Accelerated Program	65.0%	1.7	0.91	0.07	0.21%	0.07	6.7
Programmed Instruction	54.0%	2.6	1.25	0.13	0.86%	0.26	4.0
Language Labs	56.3%	6.5	0.77	0.11	0.54%	0.09	6.2
Semester System	470	---	0.74	0.04	1.70	0.29	7.7

* Parameter estimates with percentage signs (%) are reported in terms of the percentage of the total possible adopters who would eventually adopt. Other parameter values for N and s_1 are in actual frequencies.

Captions for Figures

Figure Number	Caption
1	Life Cycle of NEWSTYLE Teaching
2	Cable TV Subscribers in United States
3	Semester System in Ontario
4	Life Cycle when s_1 Is Several Orders of Magnitude Less than N
5	Life Cycle when s_1 Is Only an Order of Magnitude Less than N
6	Life Cycle when s_1 Is the Same Order of Magnitude as N
7	Modern Mathematics (Combined Groups)
8	Allegheny County Modern Mathematics
9	West Virginia Modern Mathematics
10	Foreign Language in Elementary Schools Accelerated Program
11	Programmed Instruction
12	Language Laboratories
13	Team Teaching
14	Comparison of Rate Constants

Figure 1

LIFE CYCLE OF NEWSTYLE TEACHING

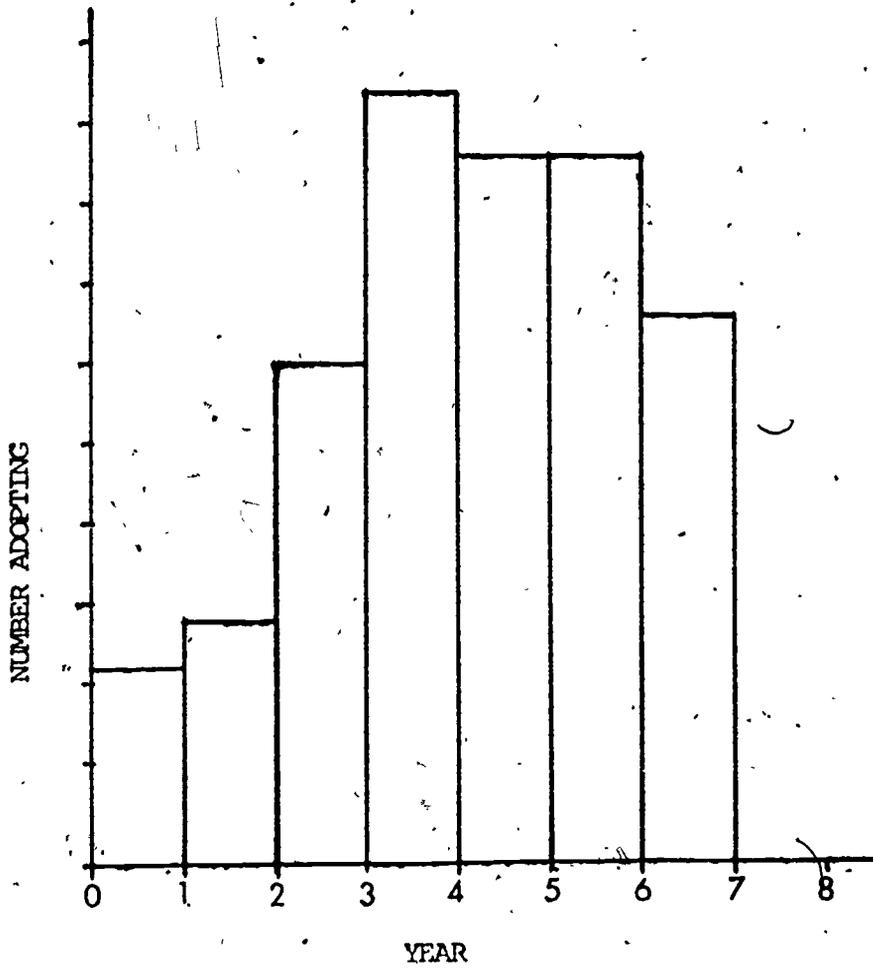


Figure 2
Cable TV Subscribers in United States

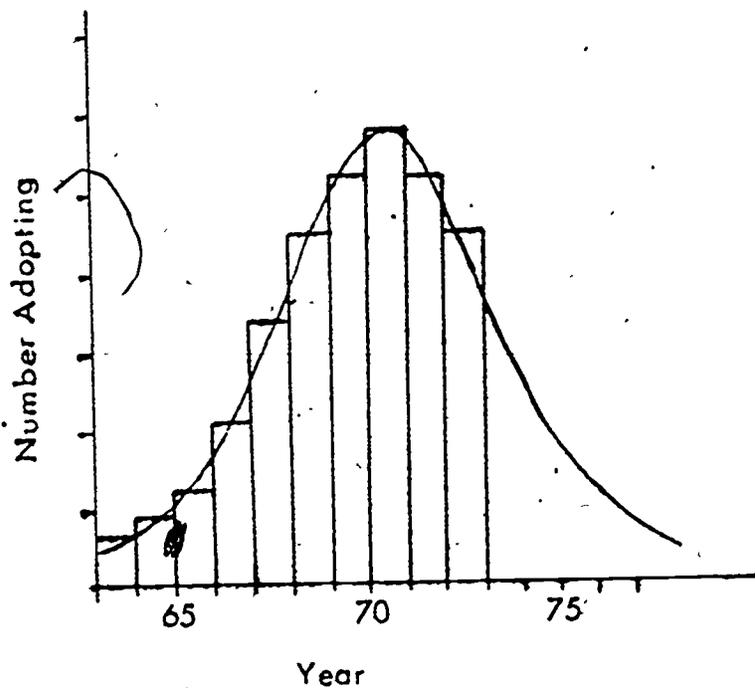


Figure 3
Semester System in Ontario

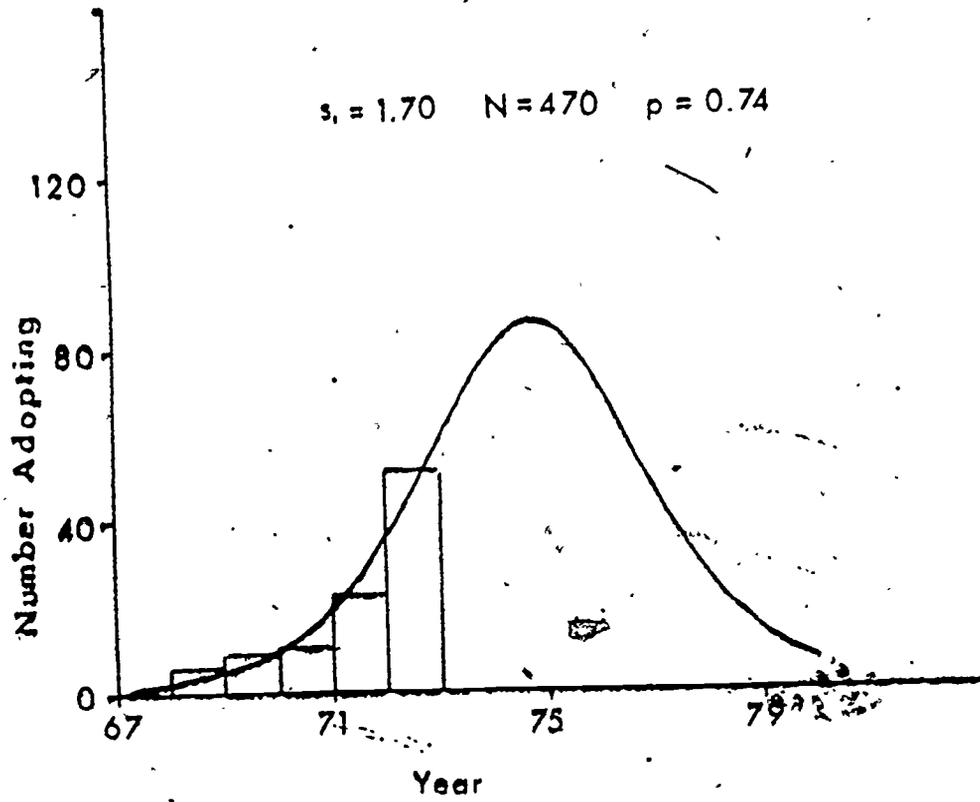


Figure 4

Life Cycle when s_1 Is Several Orders of
Magnitude Less Than N

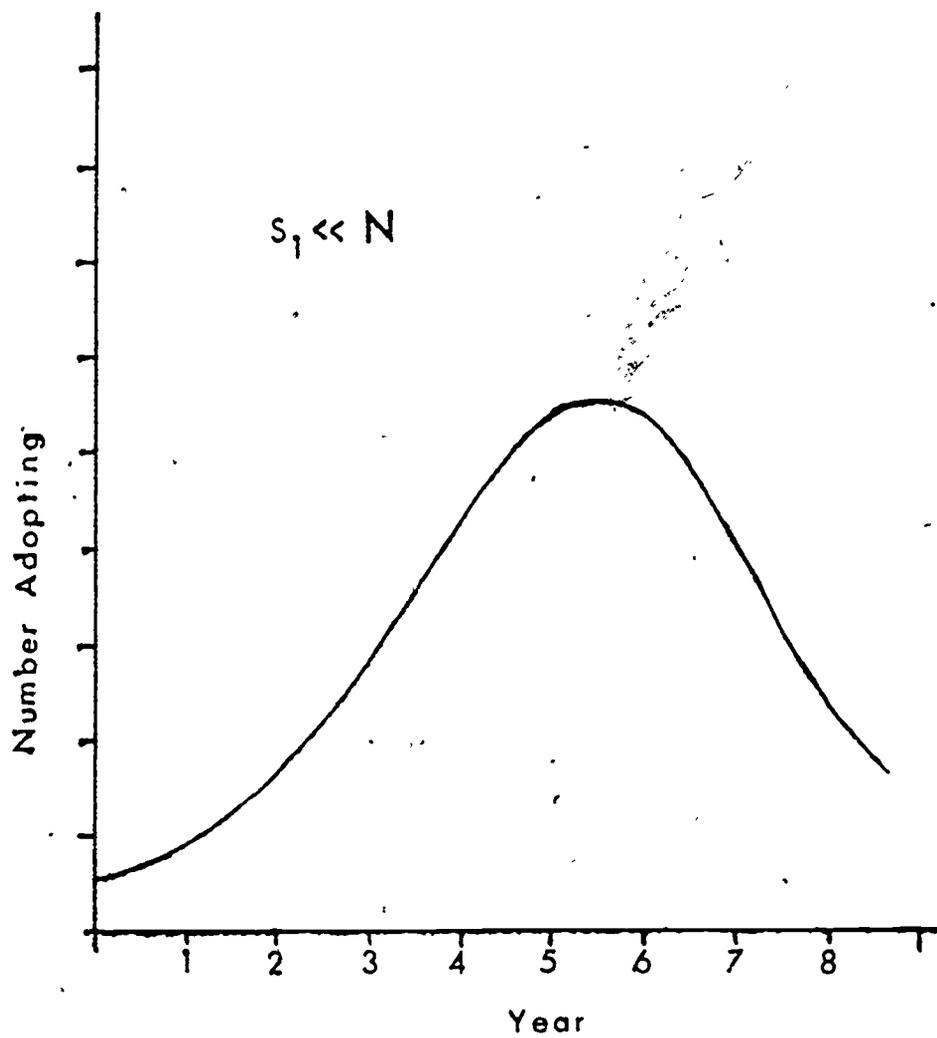


Figure 5

Life Cycle when s_1 Is Only an Order of
Magnitude Less Than N

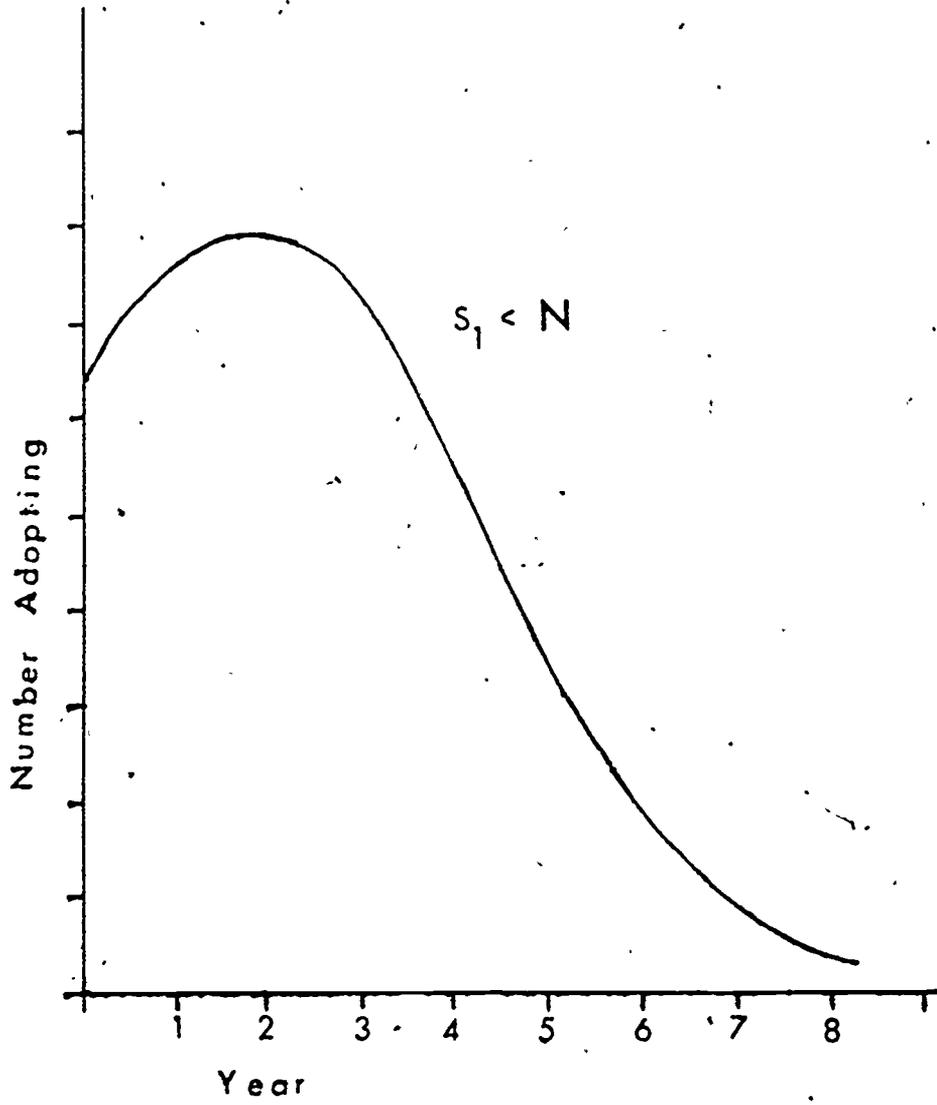


Figure 6
Life Cycle when s_1 Is the Same Order of
Magnitude as N

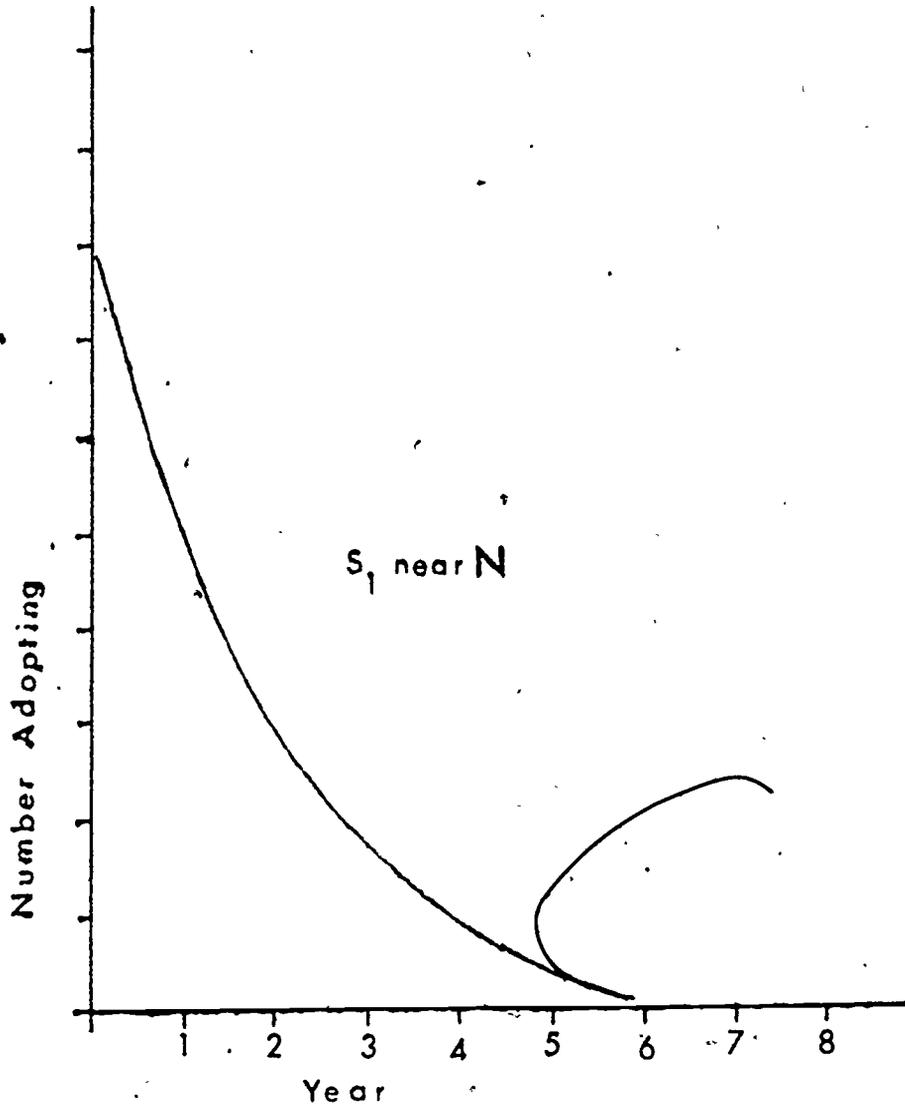


Figure 7

Modern Mathematics

(Combined Groups)

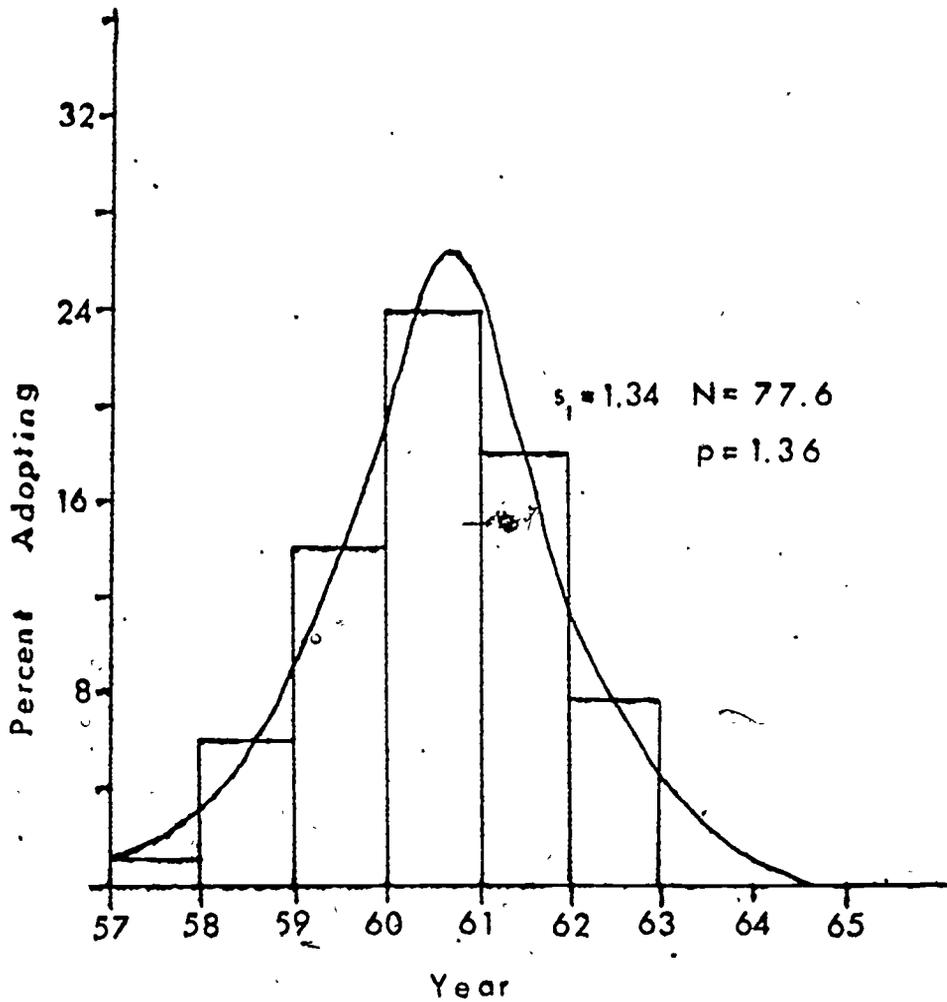


Figure 8

Allegheny County Modern Mathematics

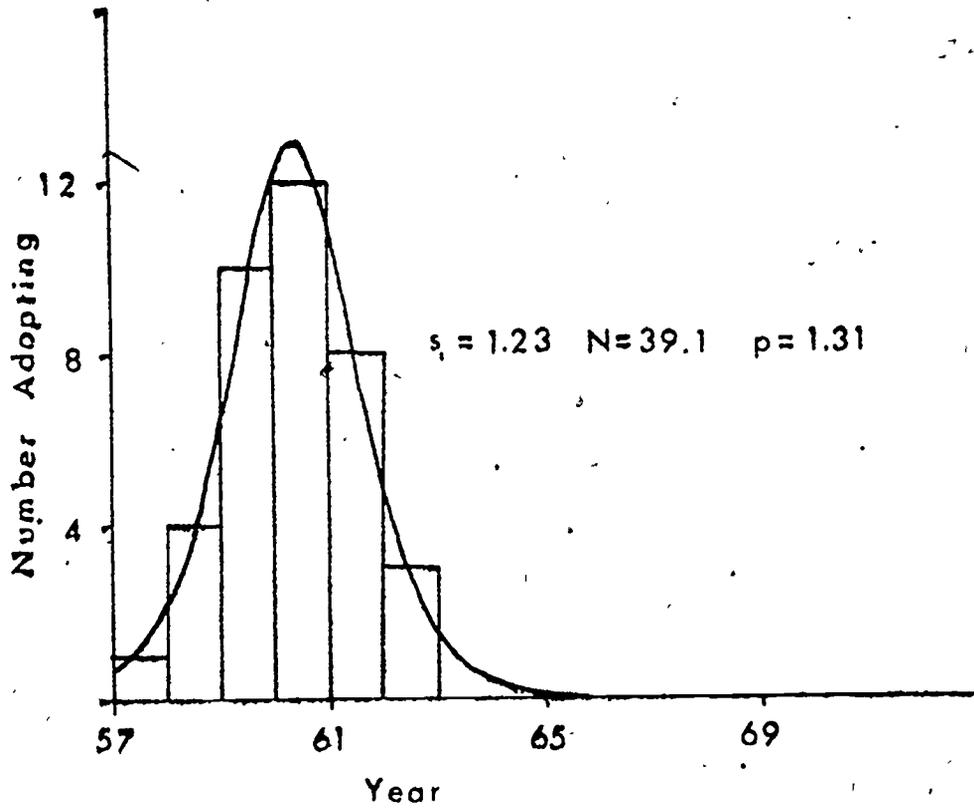


Figure 9

West Virginia Modern Mathematics

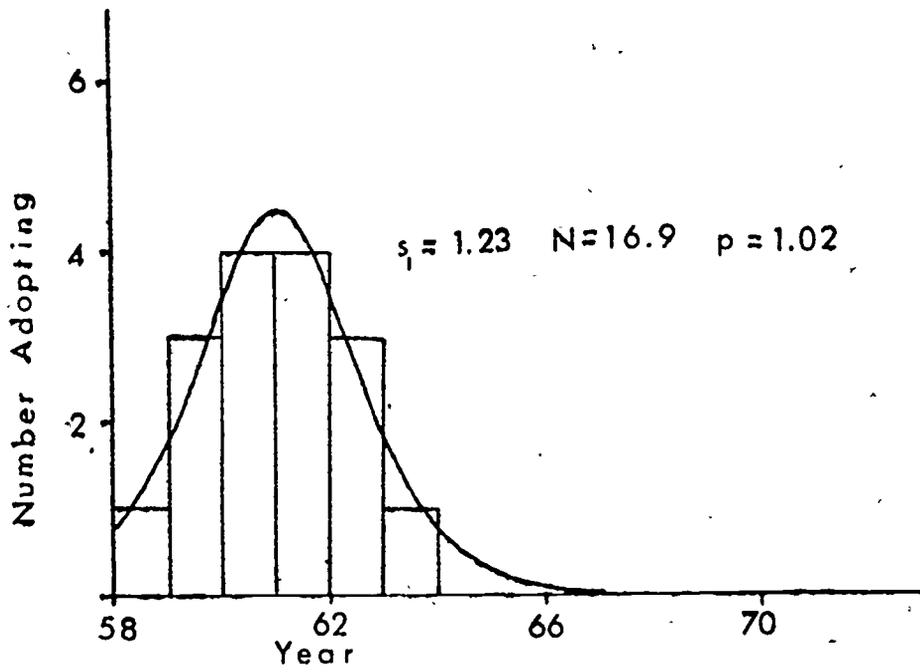


Figure 10

Foreign Language in Elementary Schools

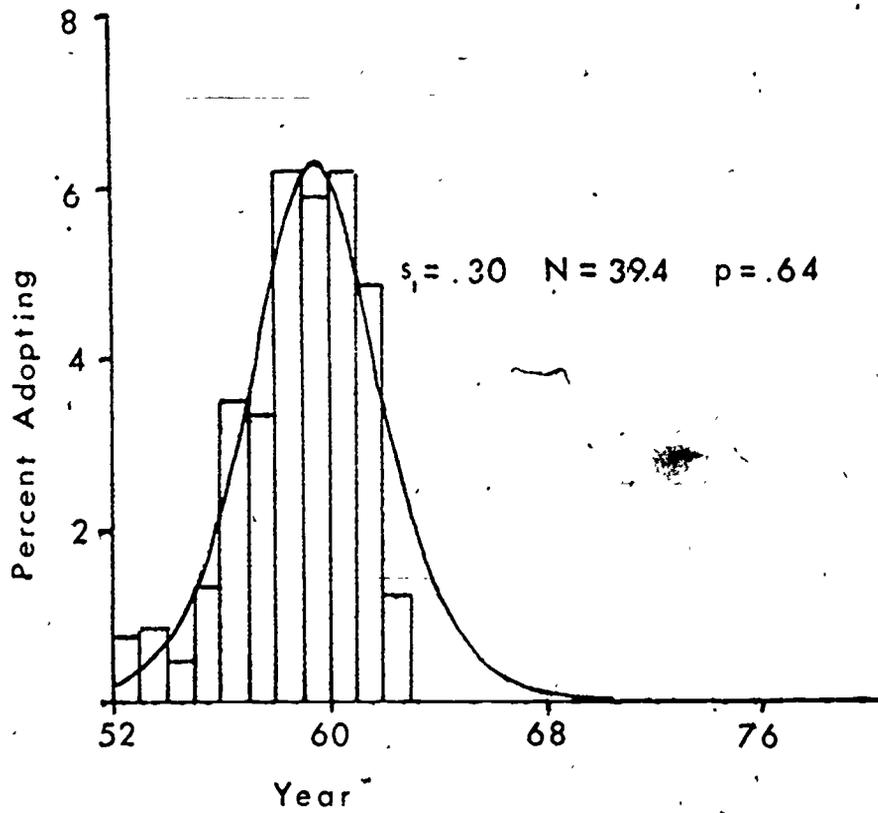


Figure 11
Accelerated Programs

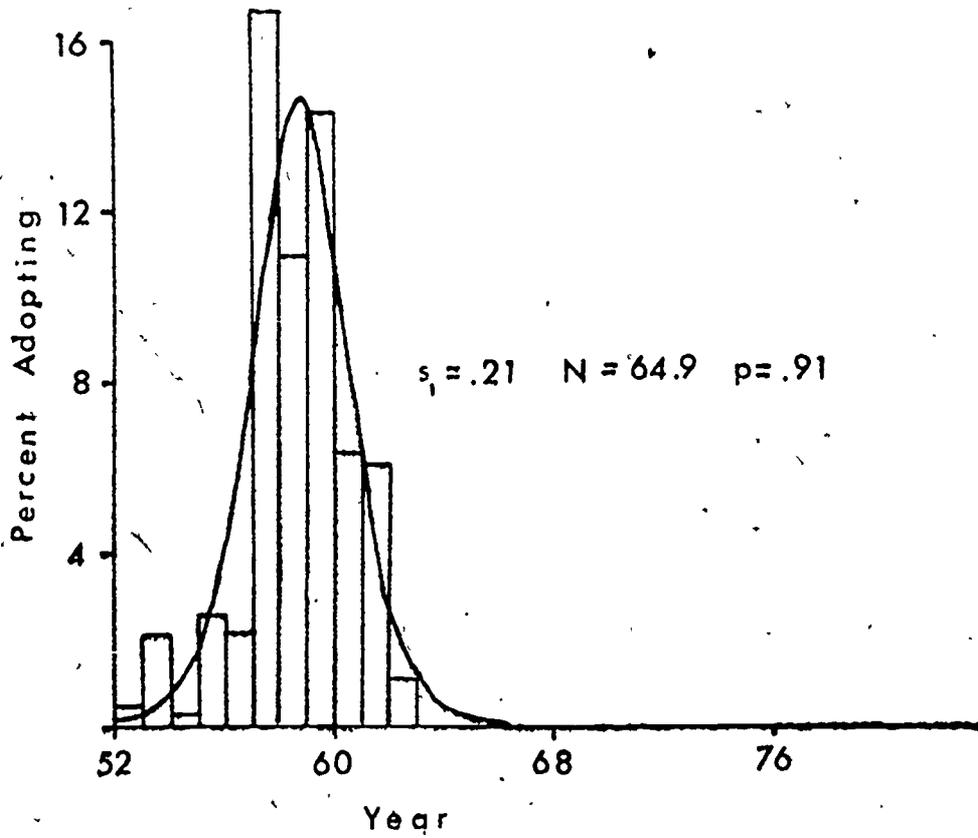


Figure 12

Programmed Instruction

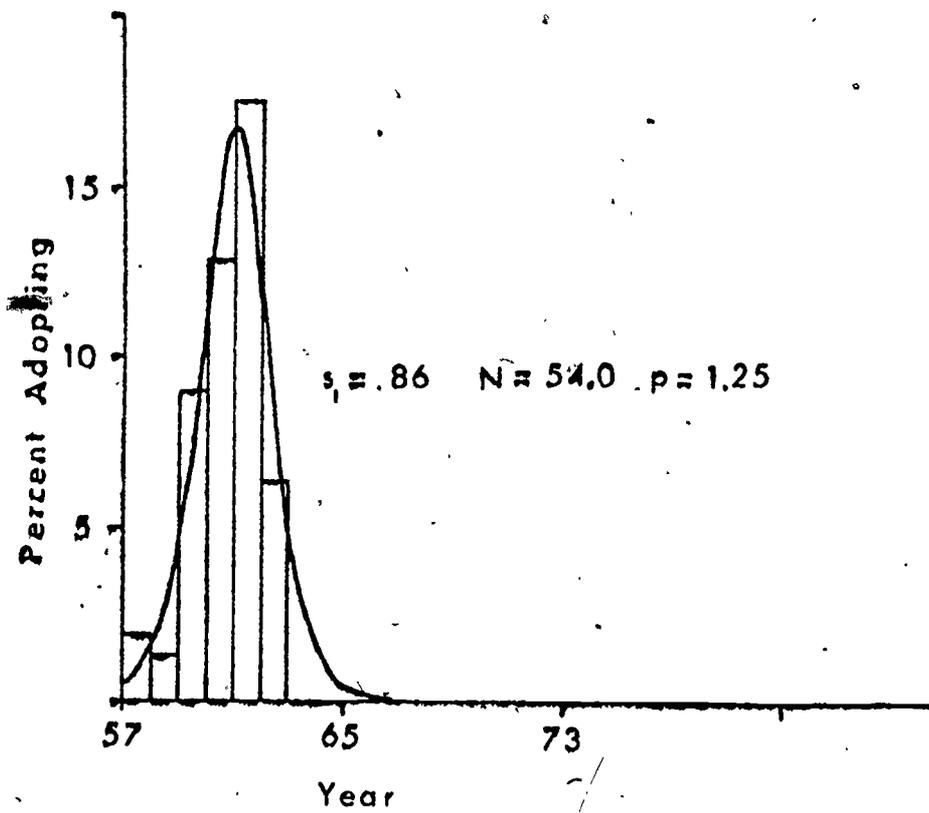


Figure 13

Language Laboratories

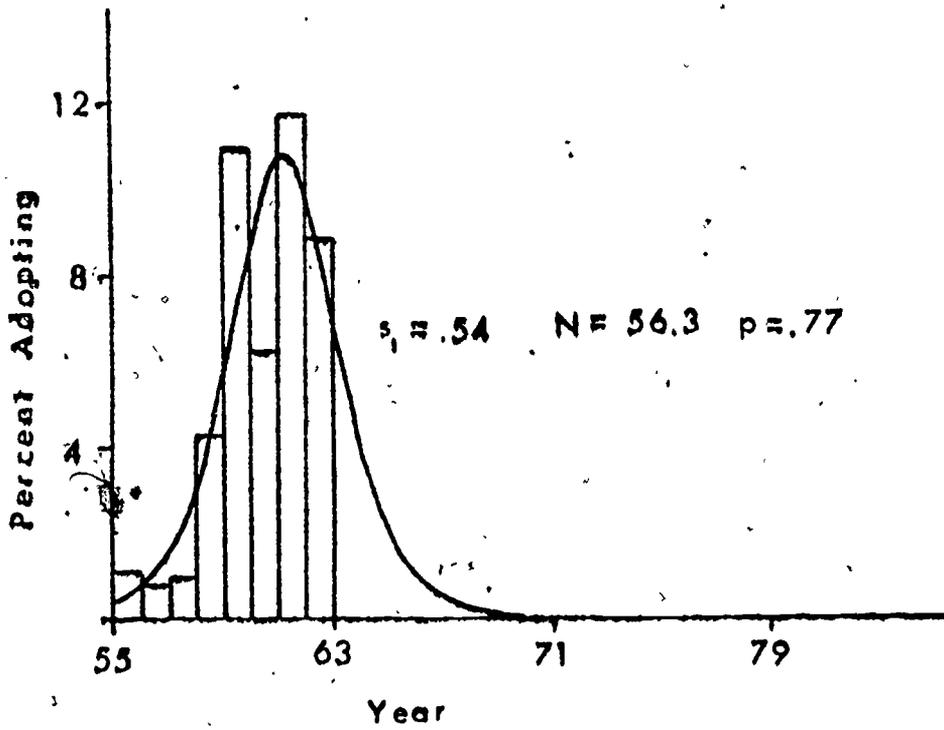


Figure 14
Team Teaching

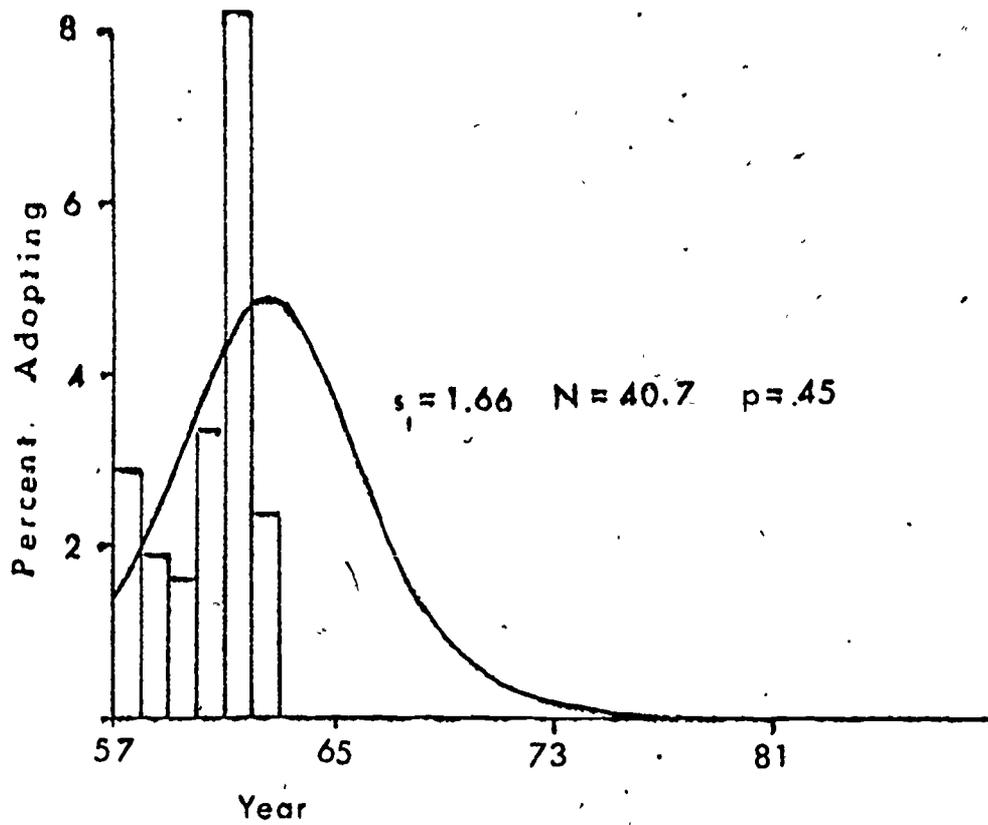


Figure 15

COMPARISON OF RATE CONSTANTS

