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AUTHOR Finch, Harold L.; Tatham, Elaine L.
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ABSTRACT

This document presents a modified cohort survival model which can be of use in making enrollment projections. The model begins by analytically profiling an area's residents. Each person's demographic characteristics--sex, age, place of residence--are recorded in the computer memory. Four major input variables are then incorporated into the model: (1) death rate--deaths are projected for males and females as a function of the number of people for each age; (2) birth rate--three equations are offered, with the most popular projecting the number of births by using the fertility rates of women by age; (3) net migration--total subarea net migration (the number of people moving in minus the number of people moving out of a given area) is assigned by age and sex; (4) land use--designations are based on zoning. Categories employed in this model are: special, agricultural, industrial, commercial, residential (low, medium, and high density), and unzoned. Zoning is specified for the beginning year, and the user has the option of incorporating anticipated zoning changes into the model. Mathematical and computer procedures used in this model include regression analysis and the two-point logistic curve. A hypothetical application of the model is presented, and practice problems are proposed. Sample printouts are appended.

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**DECLINING ENROLLMENT: WHY? WHAT ARE THE
TRENDS AND IMPLICATIONS?**

and

ENROLLMENT FORECASTING TECHNIQUES

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION
WASHINGTON, D. C. 20016

by

Harold L. Finch and Elaine L. Tatham

**Johnson County Community College
College Blvd. at Quivira Road
Overland Park, Kansas**



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PREFACE

This report contains assorted material abstracted from presentations to the AASA-NASE Conference on Declining Enrollments. Dr. Harold L. Finch is Interim Chief Administrative Officer and Vice President of Johnson County Community College; Dr. Elaine L. Tatham is Director of Institutional Research of the college. Dr. Finch made the presentations.

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I. THE EDUCATIONAL NEED FOR DEMOGRAPHIC PLANNING

As the products of the postwar baby boom reached their late teens in the 1960's, it was inevitable that high school enrollments would increase at unprecedented rates. Likewise, the baby bust which followed the boom was destined to have an equally dramatic--but opposite--effect in the 1970's. The impact is already being felt. For those institutions that are unprepared and do not begin to prepare, the worst may yet lie ahead. For the prepared, the next several years will present unprecedented challenges--opportunities for self-renewal and revitalization that will put them in good stead to survive the 70's and to resume vigorous growth in numbers and efficiency in the 80's and on into the twenty-first century.

Some important lessons can be learned from school districts that have been hard hit by declining enrollments. In examining a number of these cases, there seems to be a common pattern of events and circumstances that precede enrollment crises:

- *Failure to Recognize the Problem.* Enrollment forecasts are consistently high. Projections tend to perpetuate the heyday trends of the last decade and do not properly take into account key demographic factors. Missed forecasts are explained away as one-time flukes. Having diagnosed the problem incorrectly, staffing and budgeting plans for the next year then tend to be projected on the basis that enrollments and revenues will be "back to normal."

- *Too Little Too Late.* The seriousness of the problem is recognized too late. The dynamics of the situation are very much like that of quicksand--once an institution is entrapped, corrective efforts tend to add to the problem. For example, at the college level a vicious spiral may develop: decreasing

enrollments bring about reduced revenues; declining income results in cutbacks in programs, staff recruitment, and promotion; economy measures cause the institution to be less attractive to students, which in turn results in additional enrollment reductions.

From this pattern it might be concluded that if an institution is to remain healthy during the 1970's it is essential that effective methods of enrollment analysis and planning be developed--and that this be accomplished and operational before the institution becomes entrapped in the downward spiral.

II. A CASE STUDY IN DEMOGRAPHIC ANALYSIS AND PLANNING

The postsecondary enrollment dilemma and a positive approach to dealing with the problem can be illustrated by examining the situation in suburban Kansas City (Johnson County, Kansas) where school districts are making a concerted effort to generate realistic enrollment forecasts and to formulate practical long-range planning strategies. The enrollment trends, the causes and the associated problems and solutions for Johnson County are different only in degree from those of any other part of the country. Therefore, the approach described herein and the results reported may be useful to other districts which also need plans of action--not reaction--in order to remain, or to become, dynamic, thriving institutions at a time when many are struggling.

During the last decade educational enrollments in Johnson County, like the rest of the country, rose at an unprecedented rate. However, in the 1960's a downward trend began to develop and is currently intensifying. Birth rates were known to be the single most important contributing factor to this decline. Analysis indicates that the genesis of the problem was not low fertility rates in the 50's and 60's, but rather the low birth rates which followed the depression and preceded World War II. This phenomenon, which is not atypical to the rest of the United States, Canada and Europe, is illustrated in Figure 1

below. Although the schematic oversimplifies the complex dynamics of demography, the following conclusions apply to many high schools and colleges throughout the country:

- The current decline in number of high school and college age students will intensify and reach a peak in the late 70's.
- The coming of this decline was predictable in the late 30's-- some 40 years in advance.

Approach: Analytical Modeling.

Johnson County Community College (JCCC) initiated the development of a long-range population planning capability in 1971. The project was jointly funded by the college and the Shawnee Mission K-12 public school district. The first step was the creation of a comprehensive data base for Johnson County. It included such information as births and deaths, population distribution by age and sex, census tract boundaries, housing units, land use zoning and school attendance rates. The accomplishment of this task provided a number of insights into the population profile and past trends. As important as these data were, however, they did not by themselves provide an integrated picture of the interrelationships that exist between and among pertinent

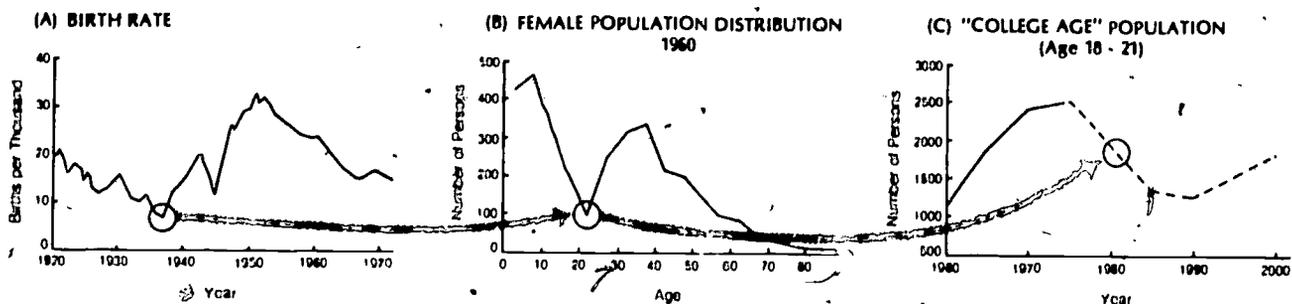


Figure 1. Population trends of a developed subdivision of Johnson County Kansas are representative of much of the nation. (A) Birth rates reached an all-time low during the late 30's; (B) this resulted in a relatively low number of women of child bearing age in 1960; (C) which, in turn, is projected to cause the current downturn in high school and college age population to be at its height during the late 70's.

planning variables, nor did they provide a means of assessing alternatives. To counter this deficiency, the college's research and planning staff developed a computerized planning model designed to translate the demographic data into a form amenable to decision-making.

Output from the demographic model consists of population projections of males and females by age group for each neighborhood or location in the county. Depending on directions specified by the user, a 30-year forecast for one geographic area within the county yields up to 6,000 individual projections. A county forecast for all areas may have as many as 200,000 individual projections.

One of the model's most useful features is its ability to provide for parametric analysis. Many parameters pertinent to enrollment analysis and planning are affected by socioeconomic and political conditions that cannot be known in advance. For example, fertility rate is a function of such factors as international relations, the state of the economy, abortion legislation and worldwide food shortages. Because the user can only speculate as to future fertility rates and because of the importance of this factor in demographic planning, the model makes it possible to analyze a range or series of possible values of this variable. Likewise there is provision for the systematic analysis of the population impact of all other significant parameters. Parametric studies of this type are sometimes referred to as "what if" analyses.

Simulation Process

In making a series of projections, the model begins by analytically profiling the county's quarter million residents. Each person's demographic characteristics--sex, age, place of residence--are recorded in the computer memory. The predicted lives of these persons are then simulated for the coming year. In each neighborhood and age group, some will die. Deaths are analytically simulated using actuarial rates as the basis for forecasting. Others are allowed

to give birth, taking into account prevalent fertility rates as they apply to the age of each of the potential mothers of Johnson County. Other residents will move out of the county and some will move to other neighborhoods within the county. Some will move into Johnson County neighborhoods from outside the county. These actions are simulated by employing current trends in migration propensity factors for each age group and neighborhood. Some areas within the county are allowed to grow to accommodate utilization of undeveloped land. Tracts which are zoned for park or industrial development are not allowed to accept in-migration.

The model uses an enhancement of cohort analysis. A "cohort" is defined as a group of people with a common characteristic. For example, women born in 1950 who live in Johnson County would be a cohort. People can leave this group only through death or through out-migration from Johnson County. For those born outside Johnson County in 1950, they can enter this group only through in-migration to Johnson County. This particular cohort of people can be traced throughout their entire life if death rates and net migration by age and sex are known.

The analytical simulation process is fundamentally straight forward. Referring to Figure 2, next page, assume that 375 women, 24 years of age in 1974 currently live in a specified neighborhood in Johnson County. Also assume that it is known that the following events will take place in the coming year: 15 will move out of the neighborhood, 32 will move in, 1 will die, and 38 will give birth. By simple arithmetic it can be predicted that one year later, the number of women, now 25 years of age, living in the neighborhood will be 391. The portion of the analysis that is not straight forward--the method of estimating the number of births, deaths and migrations--is accomplished through the generation and use of empirically derived mathematical algorithms.

The simulation process is continued one year at a time by allowing each age group to become one year older. The procedure is continued for as many years as the user specifies.

"What If" Variables

To enable the user to conduct parametric analysis, four major input variables are incorporated into the model: death rate, birth rate, net migration and land use.

Death Rates. Deaths are projected separately for males and females as a function of the number of people for each age. The user has two "what if" options. Using the term "nominal" rates to refer to the death rate equations of the model, the percentage of the "nominal" rate to be used in generating the forecast is specified for the beginning year and ending year of the projections. While the initial year multiplier would be 100 percent if the death equations are current, the initial year multiplier permits the use of an equation developed within the last few years. The ending year multiplier allows the user to examine the effect of declining or increasing death rates on the growth of the population.

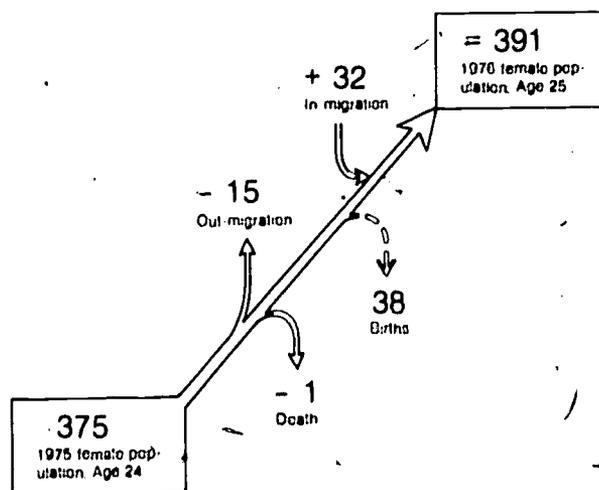


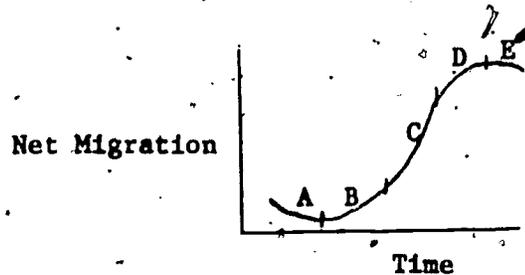
Figure 2. Sample calculation. Birth, death and migration data are obtained via empirically derived algorithms.

Birth Rates. Three options are available for projecting birth rates. The option typically used is the equation which projects the number of births using the fertility rates by age of women. The second option uses the crude birth rate. The third option uses a polynomial equation which can be modified by the user to provide various forecast relationships.

In addition to selecting one of the three birth rate equations, the user has two additional "what if" options. Using the term "nominal" rate to refer to the birth rates of the selected equation, the percentage of the "nominal" rate to be used in generating the forecast is specified for the beginning year and ending year of the projections. While the initial year multiplier would be 100 percent if the selected equation is current, the initial year multiplier is a means to use an equation developed within the last few years. The ending year multiplier permits the user to examine the effect of a declining or increasing birth rate upon the population projections. The multipliers for the intervening years are found by fitting a straight line through the two points determined by the initial year and multiplier together with the ending year and multiplier.

Net Migration. The user specifies the county net migration (number of people moving into minus number of people moving out) for the beginning year and ending year. Given the "what if" county net migration for a specified year, each geographic subdivision of the county (referred to as a subarea) receives a percentage of the county yearly net migration. Subareas are assigned a negative percentage if the net migrations can be expected to be negative. Total subarea net migration is assigned by age and sex. This age and sex distribution is accomplished by combining subareas of Johnson County into five groups on the basis of net migration for the preceding ten years. With net migration plotted as a function of time these five groups represent five stages in the development of land and are designated "A" through "E" on the typical

population S-curve below.



These five stages may be described as follows:

- A. Agricultural - undeveloped in terms of potential population capacity. During this stage there is a net out-migration typical of most rural areas today.
- B. Initial transition from rural to urban/suburban accompanied by a shift from out-migration to in-migration.
- C. Maximum rate of development. During this stage in-migration greatly exceeds out-migration.
- D. Reduced rate of growth. As the area approaches maturity the rate of net in-migration begins to decline.
- E. Maximum development. The available land is essentially fully utilized. During this stage, there is a gradual shift from in-migration to out-migration.

Land Use. Land use designations are based on zoning. Categories employed by the model are special, agriculture, industrial, commercial, low density residential, medium density residential, high density residential and unzoned. The zoning is specified for the beginning year. The user has the option of incorporating anticipated zoning changes into the model. This information can be obtained from city or county zoning boards, regional councils, title insurance companies, etc. Each subarea is divided into smaller subunits. Depending upon the population density of the subarea, these smaller subunits are sections (approximately one square mile), $\frac{1}{2}$ sections (approximately $\frac{1}{2}$ square mile) or $\frac{1}{4}$ sections (approximately $\frac{1}{16}$ square mile). For each of these subunits, zoning is specified. The portion of the subunit which lies within the boundaries of the subarea is

recorded as a fraction. The capacity (maximum number of people who could ultimately reside in the subunit on the basis of current zoning) is calculated and incorporated into the population projections.

Modified Cohort Analysis

The model has two major differences from the typical cohort survival model. Cohort analysis for a population living within a fixed geographic area does not directly consider the population affects of zoning or non-constant rates of migration. For example, suppose a subarea of the county has a 1970 population of 20,000. The typical cohort analysis might project a population of 30,000 in 1980 and a population of 60,000 in 2000. However, if the land is primarily zoned for low density residential, a maximum realistic population might be 50,000. The basic problem is related to estimates of net migration. To alleviate the problem and prevent a geographic area from exceeding its capacity, a two-point logistic curve (see Section III) is used to develop sub-area net migration percentages for the ending year. The logistic curve is also used to distribute the population of each subarea to smaller geographic units such as sections.

In summary, the model can be described in general terms as being a modified cohort survival model. Regression analysis is used to develop equations for birth rates and death rates. The two-point logistic curve is used to develop net migration subarea percentages and for distributing the population of each subarea to smaller units such as sections.

Output

A computer printout mock-up is shown in Appendix A. For each identified subarea, the user specifies the year increment for printout. For example, data can be printed for each year, every three years, every five years, or in general every n years where n is an integer. If the starting year is 1973 and n is four, printouts will be for 1977, 1981, etc. The user also specifies the

age increment for printout. If the selected increment is five, the printout will give the forecast for ages 0-4, 5-9, 10-14, etc.

For each year of printout, the population is presented using the following age code. When the printout is by five-year age groups, the "age 5" denotes children less than five years old, while "age 10" denotes children five through nine, and "15" denotes children age ten through fourteen. When the printout is by one-year ages, the "age 1" denotes births, while "age 2" denotes children age one, "age 3" denotes children age two, etc. The printed ages are, therefore, used as upper limits to the actual age interval. Corresponding migration and death output are applicable to the preceding year.

In addition, for each year of printout, population may be optionally presented by $\frac{1}{4}$ section (or other subunit) for the following age groups:

- Under five
- Five through eight
- Nine through eleven
- Twelve through fourteen
- Fifteen through seventeen
- Eighteen and nineteen
- Twenty through twenty-four
- Twenty-five through forty
- Forty-one through fifty-nine
- Sixty and over

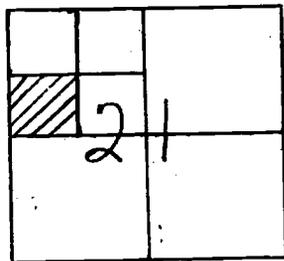
The population for these $\frac{1}{4}$ sections (sixteen $\frac{1}{4}$ sections = one section) should be considered in aggregation with surrounding $\frac{1}{4}$ sections. This aggregation is especially important in areas which currently are sparsely populated or where zoning changes seem likely. These $\frac{1}{4}$ section populations should be used individually only as guides showing future population trends.

The approximate 1960 and 1973 populations are shown in the mock-up printout on page 12 for each $\frac{1}{4}$ section along with their capacities. The relationships of the 1973 population to the capacity is presented as a decimal "FC" (1973 population/capacity). Projections by $\frac{1}{4}$ section continue to present the relationships of projected population to capacity as a decimal under the heading "Pop./Cap."

The $\frac{1}{4}$ sections are coded as follows:

- The first and second digits refer to the township
- The third and fourth digits refer to the range
- The fifth and sixth digits refer to the section
- The seventh digit is the $\frac{1}{4}$ section (beginning in the upper right hand quadrant with the number 1 and moving counter clockwise to the upper left hand quadrant denoted 2, the lower left quadrant denoted 3 and lower right quadrant denoted 4)
- The eighth digit is the $\frac{1}{4}$ section (designated by the same procedure used for $\frac{1}{4}$ sections).

For example, the digits 12252123 designate township 12, range 25, section 21, upper left hand quadrant, lower left $\frac{1}{4}$ section. This $\frac{1}{4}$ section is shaded in the sample section 21 below.



The mock-up sample on the following page has a 0 as the eighth digit since the level of detail is only $\frac{1}{4}$ section. The program as applied to Johnson County analyzed three major sets of assumptions. They are summarized below:

- Low Growth Assumption. For net migration of 1200 per year, the printout is by five-year age increments for males and females for 1978, 1983, 1988, 1993 and 1998. The fertility rates by age of women were permitted to decline so that by the year 2001 they will be 80 percent of the 1973 fertility rates.
- Nominal Growth Assumption. For net migration of 3000 per year, the printout is by one-year age increments for males and females for 1974-1984. Fertility rates by age of women were assumed to continue at the current rates.
- High Growth Assumption. For net migration of 5000 per year, the printout is by five-year age increments for males and females for 1978, 1984, 1988, 1993 and 1998. Fertility rates were assumed to gradually increase to the 1970 fertility rates.

SAMPLE PRINTOUT INCORPORATING SUB-UNIT PROJECTIONS

CAPACITY/AREA = 0. 256. 0. 1200. 2400. 4000.
 SUB AREA = TEST STARTING YR. = 1973. TOT. POP. = 3391. DFC/DT = -.005000 FC = 0.820500
 TCUR = 1973. TBASE = 1960. SUB A = -0.19610739E 01 SUB B = 0.33946969E-01
 BIRTH RATE WHAT IFR (FRAC1,FRAC2) = 0.70 0.70 BIRTH RATE TYPE USED = 1 DEATH RATE WHAT IFR (FR1,FR2) = 1.00 1.00

ID	PO P60	PO P73	CAP.	FC	DFC/DT	PF	A	B
12251530	7.0	5.0	5.0	0.9999990	0.0000000	0.00160	-0.14243654E 02	0.33946969E-01
12251630	602.0	741.0	1176.0	0.6343538	0.0094192	1.16980	-0.99227750E 00	0.33946969E-01
12251640	983.5	971.0	1223.0	0.7971941	0.0000000	0.00000	-0.18101845E 01	0.33946969E-01
12252110	2058.0	1937.0	1992.0	0.9723896	0.0000000	0.00000	-0.58240337E 01	0.17403603E 00
12252120	2058.0	1728.0	2352.0	0.7346939	0.0000000	0.00000	-0.14599056E 01	0.33946969E-01

YEAR = 1978 SUB AREA = TEST

AGE	MALE POP.	MALE MIG.	MALE DEA.	FEM. POP.	FEM. MIG.	FEM. DEA.
5	205.1	-0.9	0.9	194.8	0.5	0.6
10	243.6	-4.3	0.1	245.1	-5.2	0.1
15	251.2	-9.7	0.1	211.0	-8.6	0.1
20	250.6	-9.2	0.1	246.4	-5.9	0.1
25	238.0	6.1	0.1	227.5	9.1	0.1
30	243.1	9.4	0.2	257.2	6.5	0.1
35	276.2	0.3	0.3	300.0	-2.9	0.2
40	190.7	-7.6	0.3	166.1	-7.6	0.2
45	104.3	-2.4	0.3	116.5	-2.1	0.2
50	101.8	-1.7	0.5	124.1	-1.9	0.3
55	119.2	-2.7	0.9	138.9	-1.2	0.6
60	112.5	-0.7	1.4	140.3	-0.1	1.1
65	106.1	-0.8	2.1	137.6	-1.0	1.7
70	85.7	-0.7	2.7	103.9	-0.6	2.0
75	64.2	-0.2	3.0	63.7	0.0	2.1
80	27.5	0.0	2.0	42.7	-0.2	2.3
85	17.1	0.5	1.8	23.7	0.7	1.9
90	7.7	0.0	1.0	10.8	-0.5	1.3
95	1.3	0.0	0.2	5.4	0.0	0.9
100	0.4	0.0	0.1	2.5	0.0	0.4
105	0.0	0.0	0.0	0.0	0.0	0.0
110	0.0	0.0	0.0	0.0	0.0	0.0
0	2657.3	-22.8	18.1	2766.1	-20.8	16.3

T R S I / 4	POP.	POP./CAP.	CAPACITY
12251530	5.21	1.04120	5.00
12251630	737.51	0.62714	1176.00
12251640	986.08	0.80626	1223.04
12252110	1962.31	0.98509	1992.00
12252120	1732.35	0.73654	2352.00
0	5423.46	0.80371	6748.04

Hypothetical Illustration

Assume a fictitious county called Pirate County had a 1970 population of 300,000. The age and sex distribution is known for 1970. Equations have been developed for estimating births, deaths and net migration. The number of two year old males in 1971 is obtained by adding the number of one year old males in 1971 to the estimated net number of one year old males who moved into the county during 1971 and subtracting the estimated number of one year old male deaths for 1971. The procedure is similar for all ages and each year. The sample printouts in Appendix A illustrate the impact of some alternative user assumptions for population growth in Pirate County.

Illustration of Results

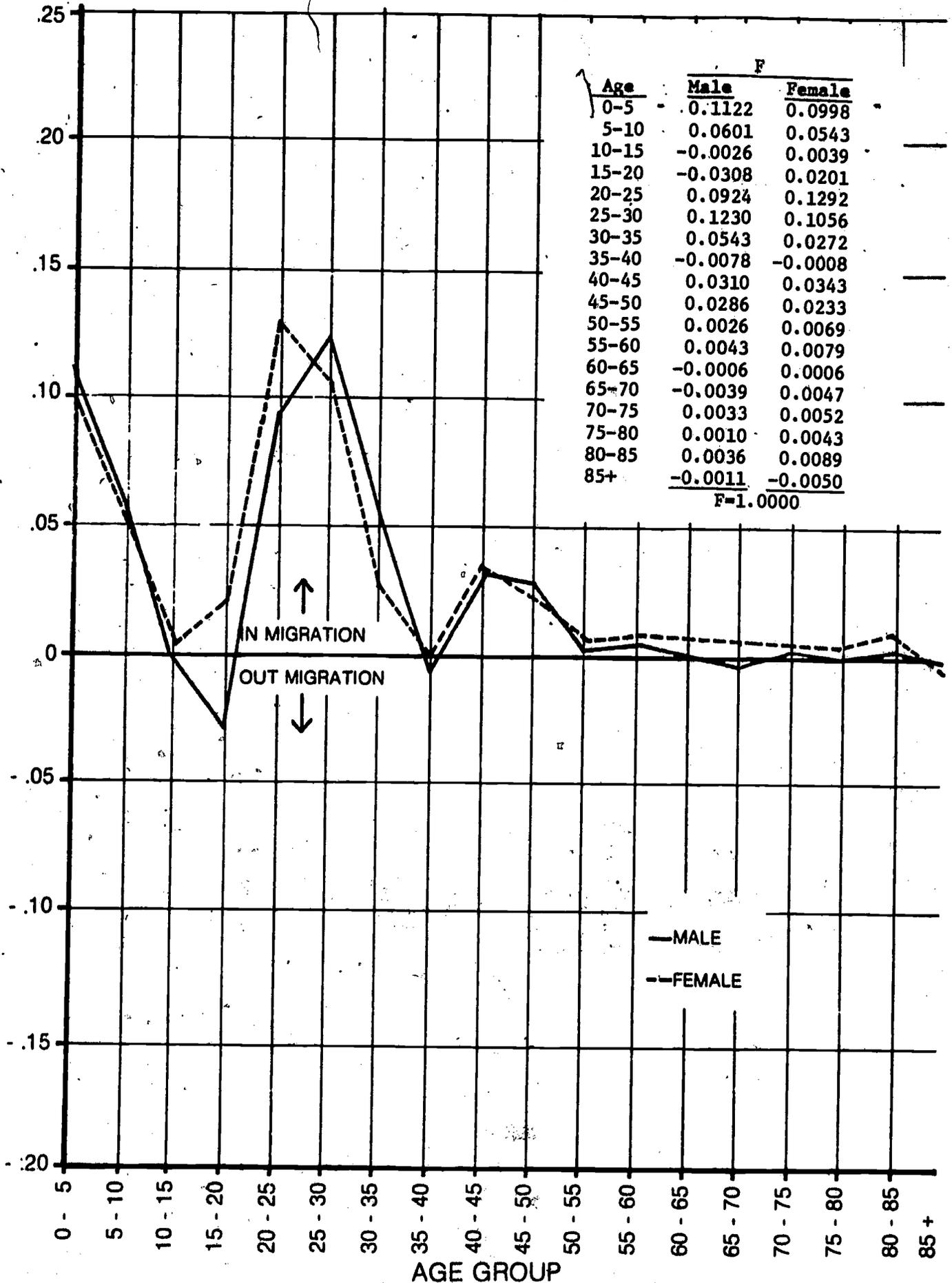
The county can be characterized as a growing community with a positive net migration and with the number of births approximately three times the number of deaths. The table below summarizes the population projections for the year 1985 and the year 2000 under each of eight user alternative assumptions. The graph on the following page characterizes net migration by age group and sex. The printouts in Appendix A present the sample assumptions together with the corresponding projections for 1985 and 2000.

PIRATE COUNTY POPULATION PROJECTIONS
1970 Population was 300,000

<u>User Alternative Assumptions</u>	<u>1985 Population</u>	<u>2000 Population</u>
Zero Net Migration	363,616	414,249
Low Birth Rate and Net Migration	376,851	433,345
Low Net Migration	381,678	454,718
Low Birth Rate	412,561	511,139
Nominal	417,800	535,655
High Birth Rate	423,040	560,399
High Net Migration	453,924	616,594
High Birth Rate and Net Migration	459,576	644,495

PIRATE COUNTY

F = NET MIGRATION BY AGE AND SEX
NET MIGRATION



III. MISCELLANEOUS THEORY

TWO-POINT LINEAR REGRESSION

$$Y = bX + c$$

Given two points (x_1, y_1) and (x_2, y_2) the equation is

$$Y = \frac{y_2 - y_1}{x_2 - x_1} (X - x_1) + y_1$$

Example

Using 1960 and 1970 Pirate County population

Y = Projected population for year X

y_2 = 1970 population

y_1 = 1960 population

$$Y = \frac{300,000 - 180,000}{1970 - 1960} (X - 1960) + 180,000$$

$$Y = 12,000 (X - 1960) + 180,000$$

If $X = 1980$, $Y = 420,000$

$X = 2000$, $Y = 660,000$

MULTIPLE-POINT LINEAR REGRESSION

$$Y = bX + a$$

where Y is criterion

X is predictor variable

Given n pairs of observations for X and Y,

$$b = \frac{n \sum_{i=1}^n XY - \sum_{i=1}^n X \sum_{i=1}^n Y}{n \sum_{i=1}^n X^2 - (\sum_{i=1}^n X)^2}, \quad a = \bar{Y} - b \bar{X} = \frac{\sum_{i=1}^n Y}{n} - b \left(\frac{\sum_{i=1}^n X}{n} \right)$$

Example

Using the population data for Pirate County, 1920-1970, the criterion Y is population while the predictor variable X is year,

$$n = 6, \quad \sum_{i=1}^6 X = 1920+1930+1940+1950+1960+1970=11670$$

$$\sum_{i=1}^6 Y = 18000 + 30000 + 39000 + 75000 + 180000 + 300000 = 642000$$

$$\sum_{i=1}^6 X^2 = (1920)^2 + (1930)^2 + (1940)^2 + (1950)^2 + (1960)^2 + (1970)^2 = 22,699,900$$

$$\sum_{i=1}^6 XY = (1920)(18,000) + (1930)(30,000) + (1940)(39,000) + (1950)(75,000) + (1960)(180,000) + (1970)(300,000) = 1,258,170,000$$

$$b = \frac{6(1,258,170,000) - (11670)(642,000)}{6(22,699,900) - (11670)^2} = \frac{5688}{1.05} = 5417.14$$

$$a = \frac{642,000}{6} - 5417.14 \left(\frac{11670}{6} \right) = -10,429,337.30$$

$$Y = 5417.14(X) - 10,429,337.30$$

For X = 1980, Y = 296,600 which is less than actual 1970 population.

TWO-POINT LOGISTIC CURVE

S - shaped 

$$P_{t_1+\theta} = \frac{K}{1+e^{a+b\theta}}$$

Where P is population

t_1 is starting year

θ is number of years past starting year

K is holding capacity

Given K and P_{t_1} for year t_1

P_{t_2} for year t_2 , where $t_2 > t_1$

$$a = \ln\left(\frac{K-P_{t_1}}{P_{t_1}}\right) \quad b = \left[\frac{\ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right) - a}{t_2 - t_1} \right]$$

Example

$$K = 1,500,000$$

$$t_1 = 1960 \quad , \quad P_{1960} = 180,000$$

$$t_2 = 1970 \quad , \quad P_{1970} = P_{1960+10} = 300,000$$

$$a = \ln\left(\frac{1,500,000-180,000}{180,000}\right) = \ln\left(\frac{1,320,000}{180,000}\right) = \ln\left(\frac{132}{18}\right)$$

$$= \ln\left(\frac{22}{3}\right) = \ln(22) - \ln(3) = 3.09104 - 1.09861 = 1.99243$$

$$b = \frac{\ln\left(\frac{1,500,000-300,000}{300,000}\right) - 1.99243}{1970-1960} = \frac{\ln\left(\frac{1,200,000}{300,000}\right) - 1.99243}{10}$$

$$= \frac{\ln(4) - 1.99243}{10} = \frac{1.38629 - 1.99243}{10} = \frac{-.60614}{10} = -.060614$$

$$\text{So } P_{1960+\theta} = \frac{1,500,000}{1+e^{1.99243-\theta.060614}}$$

Proof that a and b are obtained as specified on preceding page for the logistic curve.

$$P_{t_1+\theta} = \frac{K}{1+e^{a+b\theta}}$$

At time t_1 , $\theta = 0$, $P_{t_1} = \frac{K}{1+e^a}$ so $P_{t_1}(1+e^a) = K$

$$P_{t_1} + P_{t_1}(e^a) = K$$

$$P_{t_1}e^a = K - P_{t_1}$$

$$e^a = \frac{K-P_{t_1}}{P_{t_1}}$$

$$a = \ln\left(\frac{K-P_{t_1}}{P_{t_1}}\right)$$

At time t_2 , $\theta = t_2 - t_1$ where $t_2 > t_1$

$$P_{t_2} = \frac{K}{1+e^{a+b(t_2-t_1)}}$$

$$P_{t_2} \left(1+e^{a+b(t_2-t_1)}\right) = K$$

$$P_{t_2} + P_{t_2}e^{a+b(t_2-t_1)} = K$$

$$P_{t_2}e^{a+b(t_2-t_1)} = K - P_{t_2}$$

$$e^{a+b(t_2-t_1)} = \frac{K-P_{t_2}}{P_{t_2}}$$

$$a+b(t_2-t_1) = \ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right)$$

$$\text{So } b(t_2-t_1) = \ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right) - a$$

$$\text{And } b = \frac{\ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right) - a}{t_2-t_1}$$

MULTIPLE-POINT LOGISTIC CURVE

$$P_{t_1+\theta} = \frac{K}{1+e^{a+b\theta}}$$

Given year t_1 , capacity K and n observations, a and b are found so that the "best fit" is found. The procedure utilizes a transformation as follows:

$$P_{t_1+\theta}(1+e^{a+b\theta}) = K$$

$$P_{t_1+\theta} + P_{t_1+\theta}e^{a+b\theta} = K$$

$$P_{t_1+\theta}e^{a+b\theta} = K - P_{t_1+\theta}$$

$$e^{a+b\theta} = (K - P_{t_1+\theta}) \frac{1}{P_{t_1+\theta}}$$

$$a+b\theta = \ln \left(\frac{K - P_{t_1+\theta}}{P_{t_1+\theta}} \right)$$

$$\text{Let } Z = \ln \left(\frac{K - P_{t_1+\theta}}{P_{t_1+\theta}} \right)$$

$$\text{Then } Z = a + b\theta$$

Next, find the best linear fit for $Z = a + b\theta$, where $\theta = t_2 - t_1$

Example

Use the 1950, 1960 and 1970 Pirate County population data* together with a capacity of 1,500,000 people and a base year $t_1=1960$

$$\text{For } 1950, \theta = -10, Z = \ln \left(\frac{1,500,000 - 75,000}{75,000} \right) = 2.94444$$

$$1960, \theta = 0, Z = \ln \left(\frac{1,500,000 - 180,000}{180,000} \right) = 1.99243$$

$$1970, \theta = 10, Z = \ln \left(\frac{1,500,000 - 300,000}{300,000} \right) = 1.38629$$

$$* P_{1950} = 75,000$$

$$P_{1960} = 180,000$$

$$P_{1970} = 300,000$$

$$\sum_{i=1}^3 \theta = -10 + 0 + 10 = 0$$

$$\sum_{i=1}^3 EZ = 2.94444 + 1.99243 + 1.38629 = 6.32316$$

$$\sum_{i=1}^3 \theta^2 = 100 + 0 + 100 = 200$$

$$\sum_{i=1}^3 \theta Z = (-10)(2.94444) + 0(1.99243) + 10(1.38629) = -15.5815$$

$$b) \frac{3(-15.5815) - (0)(6.32316)}{3(200) - (0)^2} = -.07791$$

$$a = \frac{6.32316}{3} - (-.07791)\left(\frac{0}{3}\right) = 2.10772$$

$$\text{So } z = 2.10772 - .07791\theta$$

$$\text{Or } P_{t_1+\theta} = \frac{K}{1+e^{2.10772-.07791\theta}} \text{ for } K = 1,500,000$$

For 1980, $\theta = 20$

$$P_{1980} = \frac{1,500,000}{1+e^{2.10772-1.55820}} = \frac{1,500,000}{1+e^{.54952}}$$

$$= \frac{1,500,000}{1+1.7324} = \frac{1,500,000}{2.7324} = 548,968$$

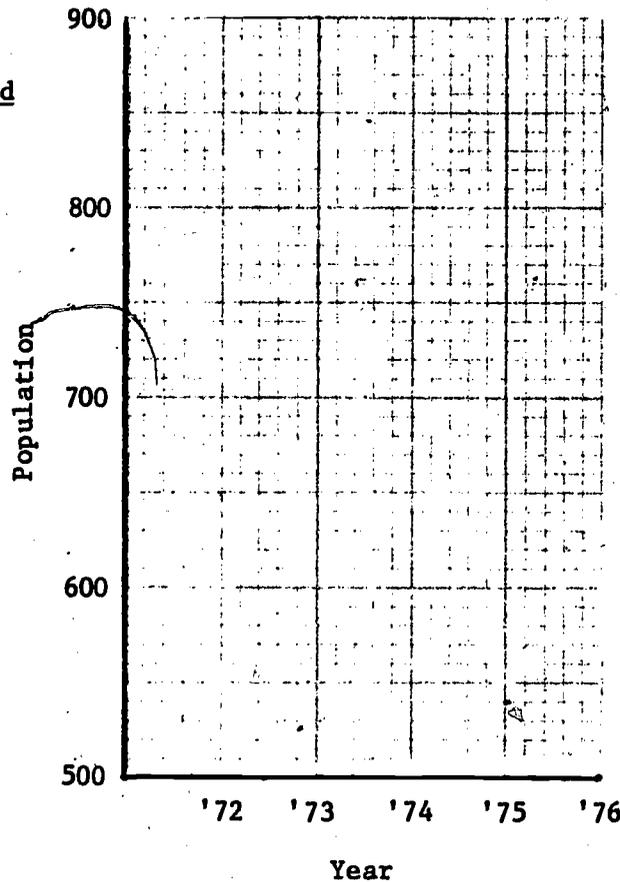
IV. PRACTICE PROBLEMS

**PRACTICE PROBLEM: REGRESSION AND
COHORT SURVIVAL METHODOLOGY**

Problem: Estimate the 1976 population by age group using regression and cohort survival methods of analysis. The population data base for the years 1972-75 is given in the table below:

Age	Actual				Regress. '76	Cohort '76
	'72	'73	'74	'75		
6	690	660	600	560		
7	720	718	693	636		
8	747	734	740	721		
9	760	755	749	762		
10	703	775	778	779		
11	656	724	806	817		
12	560	663	738	830		

Regression Method



Sample Cohort Survival Projection #1

$$a = 6, a + 1 = 7$$

$$\frac{P(a+1, '73)}{P(a, '72)} = \frac{P(\quad, '73)}{P(\quad, '72)} = \frac{(\quad)}{(\quad)} = (\quad) \quad \textcircled{1}$$

$$\frac{P(a+1, '74)}{P(a, '73)} = \frac{P(\quad, '74)}{P(\quad, '73)} = \frac{(\quad)}{(\quad)} = (\quad) \quad \textcircled{2}$$

$$\frac{P(a+1, '75)}{P(a, '74)} = \frac{P(\quad, '75)}{P(\quad, '74)} = \frac{(\quad)}{(\quad)} = (\quad) \quad \textcircled{3}$$

$$\frac{\textcircled{1} + \textcircled{2} + \textcircled{3}}{3} = \frac{(\quad) + (\quad) + (\quad)}{3} = \frac{(\quad)}{3} = (\quad) \quad \textcircled{4}$$

$$\text{So } P(\quad, '76) = \textcircled{4} \times P(\quad, '75) = (\quad) \times (\quad) = \boxed{\quad}$$

Sample Cohort Survival Projection #2

$$a = 6, a + 1 = 7$$

$$\frac{P(a+1, '73)}{P(a, '72)} = \frac{P(\quad, '73)}{P(\quad, '72)} = \frac{(\quad)}{(\quad)} = (\quad) \quad \textcircled{1}$$

$$\frac{P(a+1, '74)}{P(a, '73)} = \frac{P(\quad, '74)}{P(\quad, '73)} = \frac{(\quad)}{(\quad)} = (\quad) \quad \textcircled{2}$$

$$\frac{P(a+1, '75)}{P(a, '74)} = \frac{P(\quad, '75)}{P(\quad, '74)} = \frac{(\quad)}{(\quad)} = (\quad) \quad \textcircled{3}$$

$$\frac{\textcircled{1} + \textcircled{2} + \textcircled{3}}{3} = \frac{(\quad) + (\quad) + (\quad)}{3} = \frac{(\quad)}{3} = (\quad) \quad \textcircled{4}$$

$$\text{So } P(\quad, '76) = \textcircled{4} \times P(\quad, '75) = (\quad) \times (\quad) = \boxed{\quad}$$

PRACTICE PROBLEM: TWO-POINT LOGISTIC METHOD

Problem: Forecast the population living within the boundaries of a urban-suburban school district for various land use assumptions using the two-point logistic method of analysis for the years 1980, 1990, 2000:

$P_{1960} = 180,000$ $P_{1970} = 300,000$ Area = 400 sq. mi.

Land Use Assumption:

Zoning	Sq.Miles	Ultimate Density Pop/Sq.Mile	Holding Capacity
R ₁ : Low Density Residential		4,795	_____
R ₂ : Medium Density Residential		12,787	_____
R ₃ : High Density Residential		25,574	_____
Agricultural		1,066	_____
Parks, Commercial, Industrial		0	=====

K = Total Holding Capacity =

$$P_{1960+\theta} = \frac{K}{1 + e^{\left\{ \ln\left(\frac{K-P_{t1}}{P_{t1}}\right) + \left[\frac{\ln\left(\frac{K-P_{t2}}{P_{t2}}\right) - \ln\left(\frac{K-P_{t1}}{P_{t1}}\right)}{t_2-t_1} \right] \theta \right\}}$$

Where $\theta = 1980 - 1960 = 20$

$P_{t1} = 180,000$

$P_{t2} = 300,000$

$t_2 - t_1 = 1970 - 1960 = 10$

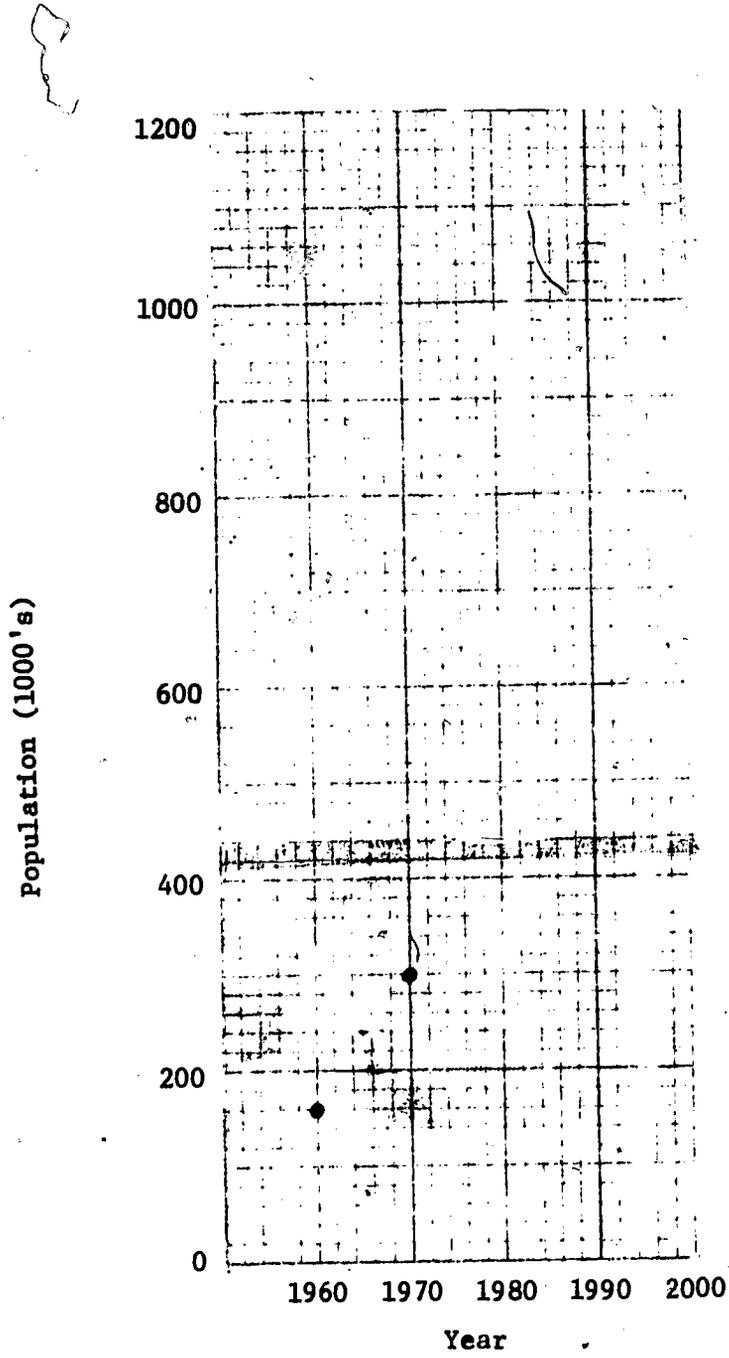
$K =$ _____

Solving the above equation yields the following results:

Land Use	K	P ₁₉₈₀
	350,000	340,000
	600,000	420,000
	4,100,000	490,000

So $P_{1980} \approx$

Two-Point Logistic Method



APPENDIX A

SAMPLE PRINTOUTS

ASSUMPTION:
ZERO NET MIGRATION

10/22/74. 10.18.37.
PROGRAM COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS, & PRT. AGE INTERVALS
? 1970,2000,15,10

ENTER YR, Z(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
? 2000,1,1

ENTER YR & Z(FRACTION) TO BE APPLIED TO DEATH RATES
? 2000,1

ENTER MIG. & START P FL. NAME, YR1, NET MIG1, YR2, NET MIG2
? COHRPIR, 1970, .001, 2000, .001

YEAR = 1985

	1985	1984	1984	1985	1984	1984
AGE	M POP	M MIG	M DEA	F POP	F MIG	F DEA
10	33579	0	85	32032	0	59
20	27381	-0	14	26076	0	10
30	34348	0	23	33272	0	19
40	21604	0	27	24103	0	25
50	20232	0	67	22214	0	55
60	18330	0	173	19935	0	137
70	15256	-0	351	16197	0	287
80	6528	0	358	7780	0	368
90	1557	0	170	2504	0	277
100	214	0	36	489	0	93
110	22	0	4	66	0	12
0	179049	0	1307	184567	0	1342

YEAR = 2000

	2000	1999	1999	2000	1999	1999
AGE	M POP	M MIG	M DEA	F POP	F MIG	F DEA
10	33624	0	78	32058	0	60
20	36020	-0	18	34389	0	18
30	29274	0	19	27978	0	22
40	20249	0	42	28775	0	49
50	20174	0	87	29660	0	102
60	17178	0	157	24103	0	211
70	15740	-0	363	16735	0	465
80	11439	0	628	11841	0	865
90	3699	0	370	3385	0	529
100	514	0	76	348	0	95
110	42	0	6	24	0	7
0	207953	0	1844	206296	0	2424

STJP.

ASSUMPTION:
 NET MIGRATION DECREASES 67 PERCENT
 BIRTH RATES DECREASE 20 PERCENT BY 2000

10/22/74. 09.54.01.
 PROGRAM COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS, & PRT. AGE INTERVALS
 ? 1970, 2000, 15, 10
 ENTER YR, & Z(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000, .9, 1
 ENTER YR & Z(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000, 1
 ENTER MIG. & START P FL. NAME, YR1, NET MIGI, YR2, NET MIG2
 ? COHRPIR, 1970, 1000, 2000, 1000
 YEAR = 1985

AGE	1985 M POP	1984 M MIG	1984 M DEA	1985 F POP	1984 F MIG	1984 F DEA
10	37715	172	82	32108	154	57
20	28802	-33	15	27490	24	10
30	35091	215	24	34779	235	20
40	23829	47	30	26429	26	28
50	21215	60	70	23018	58	56
60	18843	7	178	20387	15	140
70	15376	-5	353	16397	5	290
80	6526	4	358	7879	10	373
90	1583	3	173	2592	4	286
100	217	0	36	490	0	93
110	21	0	4	64	0	12
0	185219	470	1321	191632	530	1366

YEAR = 2000

AGE	2000 M POP	1999 M MIG	1999 M DEA	2000 F POP	1999 F MIG	1999 F DEA
10	31627	172	71	30098	154	54
20	35962	-33	18	34375	24	19
30	31403	215	20	30729	235	25
40	33223	47	46	32249	26	54
50	32747	60	96	32650	58	113
60	19378	7	173	23114	15	227
70	16367	-5	376	17418	5	483
80	11666	4	637	12183	10	886
90	3732	3	374	3488	4	545
100	518	0	76	352	0	97
110	42	0	6	24	0	7
0	216666	470	1893	216679	530	2511

STJP.

ASSUMPTION:
NET MIGRATION DECREASES 67 PERCENT

10/22/74. 09.49.23.
PROGRAM COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS, & PRT. AGE INTERVALS
? 1970,2000,15,10

ENTER YR, % (FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
? 2000,1,1

ENTER YR & % (FRACTION) TO BE APPLIED TO DEATH RATES
? 2000,1

ENTER MIG. & START P FI. NAME, YP1, NET MIG1, YP2, NET MIG2
? COWPPIP,1970,1000,2000,1000

YEAR = 1985

AGE	M POP	M MIG	M DEA	F POP	F MIG	F DEA
10	35994	172	89	34280	154	62
20	28994	-33	15	27673	24	10
30	35091	215	24	34779	235	27
40	28229	47	37	27229	26	28
50	21215	60	70	23018	56	56
60	18843	7	178	20357	15	140
70	15376	-5	353	16397	5	290
80	6526	4	358	7879	10	373
90	1583	3	173	2592	4	286
100	217	0	36	490	0	93
110	21	0	4	64	0	12
0	187690	470	1328	193988	530	1372

YEAR = 2000

AGE	M POP	M MIG	M DEA	F POP	F MIG	F DEA
10	37920	172	88	36797	154	67
20	39657	-33	19	37902	24	20
30	32354	215	21	31637	235	25
40	33223	47	46	32249	26	54
50	32747	60	96	32650	52	113
60	19378	7	173	23114	15	227
70	16367	-5	376	17418	5	483
80	11666	4	637	12193	10	886
90	3732	3	374	3488	4	545
100	518	0	76	352	0	97
110	42	0	6	24	0	7
0	227604	470	1912	227114	530	2526

STOP.

**ASSUMPTION:
BIRTH RATES DECREASE 20 PERCENT BY 2000**

10/29/74. 09.35.23.
PROGRAM COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS, & PRJ. AGE INTERVALS
? 1970,2000,15,10

ENTER YR, &(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
? 2000,.8,1

ENTER YR & &(FRACTION) TO BE APPLIED TO DEATH RATES
? 2000,1

ENTER MIG.& START P FL.NAME, YR1, NET MIG1, YR2, NET MIG2
? COHRP1R,1970,3000,2000,3000

YEAR = 1985

AGE	M PJP	M MIG	M DFA	F PJP	F MIG	F DFA
10	33344	517	91	36410	462	63
20	32020	-100	16	30676	72	11
30	36576	646	25	37794	704	21
40	28280	140	35	31079	79	32
50	23181	179	75	24624	173	59
60	19869	21	186	21490	44	147
70	15617	-14	357	16798	16	296
80	8522	13	357	8077	29	383
90	1637	8	180	2768	12	306
100	222	0	37	493	0	93
110	21	0	4	61	0	11
0	202289	1409	1362	210272	1591	1424

YEAR = 2000

AGE	M PJP	M MIG	M DFA	F PJP	F MIG	F DFA
10	39075	517	87	37086	462	66
20	42838	-100	21	41021	72	22
30	37496	646	24	37982	704	30
40	39171	140	53	39196	79	64
50	37893	179	113	38632	173	135
60	23777	21	205	27134	44	259
70	17620	-14	402	18783	16	520
80	12120	13	657	12866	29	930
90	3798	8	381	3696	12	579
100	527	0	78	360	0	99
110	44	0	7	25	0	8
0	254359	1409	2026	256780	1591	2713

STOP.

ASSUMPTION:
NOMINAL ASSUMPTIONS

10/22/74. 09.27.47.
PROGRAM COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS, & PRT. AGE INTERVALS
? 1970,2000,15,10

ENTER YR, % (FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
? 2000,1,1

ENTER YR & % (FRACTION) TO BE APPLIED TO DEATH RATES
? 2000,1

ENTER MIG. & START P FL. NAME, YR1, NET MIG1, YR2, NET MIG2
? COHPRIR,1970,3000,2000,3000

YEAR = 1985

	1985	1984	1984	1985	1984	1984
AGE	M POP	M MIG	M DFA	F POP	F MIG	F DFA
10	40826	517	98	38777	462	69
20	32219	-100	16	30867	72	11
30	36576	646	25	37794	704	21
40	28280	140	35	31079	79	32
50	23181	179	75	24626	173	59
60	19869	21	186	21490	44	147
70	15617	-14	357	16798	16	296
80	6522	13	357	8077	29	383
90	1637	8	180	2768	12	306
100	222	0	37	493	0	93
110	21	0	4	61	0	11
0	204971	1409	1370	212829	1591	1430

YEAR = 2000

	2000	1999	1999	2000	1999	1999
AGE	M POP	M MIG	M DFA	F POP	F MIG	F DFA
10	46511	517	107	44175	462	82
20	46932	-100	23	44929	72	24
30	38514	646	25	38954	704	31
40	39171	140	53	39196	79	64
50	37893	179	113	38632	173	135
60	23777	21	205	27134	44	259
70	17620	-14	402	18783	16	520
80	12120	13	657	12866	29	930
90	3798	8	381	3696	12	579
100	527	0	78	360	0	99
110	42	0	7	25	0	8
0	266906	1409	2049	268749	1591	2730

STOP.

ASSUMPTION:
 BIRTH RATES INCREASE 20 PERCENT BY 2000

10/22/74. 09.58.20.
 PROGRAM COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS, & PRT. AGE INTERVALS
 ? 1970,2000,15,10

ENTER YR, Z(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000,1.2,1

ENTER YR & Z(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000,1

ENTER MIG.& START P FL.NAME, YR1, NET MIG1, YR2, NET MIG2
 ? COHRPIR,1970,3000,2000,3000

YEAR = 1985

AGE	1985 M POP	1984 M MIG	1984 M DEA	1985 F POP	1984 F MIG	1984 F DEA
10	43308	517	106	41144	462	74
20	32419	-100	16	31058	72	11
30	36576	646	25	37794	704	21
40	28280	140	35	31079	79	32
50	23181	179	75	24626	173	59
60	19869	21	186	21490	44	147
70	15617	-14	357	16798	16	296
80	6522	13	357	8077	29	383
90	1637	R	180	2768	12	306
100	222	0	37	493	0	93
110	21	0	4	61	0	11
0	207653	1409	1378	215387	1591	1425

YEAR = 2000

AGE	2000 M POP	1999 M MIG	1999 M DEA	2000 F POP	1999 F MIG	1999 F DEA
10	54064	517	127	51375	462	97
20	51025	-100	25	48837	72	26
30	39531	646	25	39927	704	31
40	39171	140	53	39196	79	64
50	37893	179	113	38632	173	135
60	23777	21	205	27134	44	259
70	17620	-14	402	18783	16	520
80	12120	13	657	12866	29	930
90	3798	R	381	3696	12	579
100	527	0	78	360	0	99
110	44	0	7	25	0	R
0	279570	1409	2072	280829	1591	2748

STOP.

ASSUMPTION:

NET MIGRATION INCREASES 67 PERCENT

10/22/74. 10.03.57.

PROGRAM COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS, & PRT. AGE INTERVALS

? 1970,2000,15,10

ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE

? 2000,1,1

ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES

? 2000,1

ENTER MIG. & START P FL.NAME, YR1, NET MIG1, YR2, NET MIG2

? COHRPIR, 1970, 5000, 2000, 5000

YEAR = 1985

AGE	1985 M POP	1984 M MIG	1984 M DEA	1985 F POP	1984 F MIG	1984 F DEA
10	45658	862	108	43274	771	75
20	35445	-167	18	34060	120	12
30	38062	1077	26	40810	1174	23
40	32731	233	41	35729	132	37
50	25148	298	80	26234	288	63
60	20895	35	195	22593	74	154
70	15858	-23	360	17198	27	302
80	6518	22	357	8275	48	393
90	1690	13	187	2944	20	325
100	227	0	37	495	0	92
110	20	0	3	59	0	11
0	222253	2348	1412	231671	2652	1488

YEAR = 2000

AGE	2000 M POP	1999 M MIG	1999 M DEA	2000 F POP	1999 F MIG	1999 F DEA
10	55103	862	126	52254	771	96
20	54206	-167	26	51955	120	28
30	44673	1077	28	46272	1174	37
40	45119	233	60	46143	132	75
50	43038	298	130	44613	288	158
60	28176	35	237	31154	74	292
70	18874	-23	428	20147	27	556
80	12574	22	676	13550	48	973
90	3864	13	388	3903	20	612
100	536	0	79	368	0	102
110	45	0	7	25	0	8
0	306209	2348	2186	310385	2652	2935

STOP.

ASSUMPTION:
 BIRTH RATES INCREASE 20 PERCENT BY 2000
 NET MIGRATION INCREASES 67 PERCENT

10/2/74. 10.11.56.
 PROGRAM COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS. & PRT. AGE INTERVALS

? 1970,2000,15,10

ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE

? 2000,1.2,1

ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES

? 2000,1

ENTER MIG. & START P FL. NAME, YR1, NET MIG1, YR2, NET MIG2

? COHRPIR,1970,5000,2000,5000

YEAR = 1985

AGE	1985 M POP	1984 M MIG	1984 M DEA	1985 F POP	1984 F MIG	1984 F DEA
10	48344	862	116	45835	771	81
20	35653	-167	18	34259	120	12
30	38062	1077	26	40810	1174	23
40	32731	233	41	35729	132	37
50	25148	298	80	26234	288	63
60	20895	35	195	22593	74	154
70	15858	-23	360	17198	27	302
80	6518	22	357	8275	48	393
90	1690	13	187	2944	20	325
100	227	0	37	495	0	92
110	20	0	3	59	0	11
0	225146	2348	1421	234430	2652	1494

YEAR = 2000

AGE	2000 M POP	1999 M MIG	1999 M DEA	2000 F POP	1999 F MIG	1999 F DEA
10	63806	862	150	60550	771	114
20	58697	-167	28	56243	120	30
30	45758	1077	29	47309	1174	37
40	45119	233	60	46143	132	75
50	43038	298	130	44613	288	158
60	28176	35	237	31154	74	292
70	18874	-23	428	20147	27	556
80	12574	22	676	13550	48	973
90	3864	13	388	3903	20	612
100	536	0	79	368	0	102
110	45	0	7	25	0	8
0	320489	2348	2212	324006	2652	2956

STOP.

UNIVERSITY OF CALIF.
 LOS ANGELES

JAN 30 1976

CLEARINGHOUSE FOR
 JUNIOR COLLEGES