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ABSTRACT

This book is a part of a self-study sequence in an engineering certification program. This volume deals with basic measurement related to construction projects; chapters are devoted to stationing, alignment data, curve data, equations, and bench marks. Some knowledge of algebra and trigonometry is assumed. (SD)

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- Horizontal Curves
- Profile Measurements
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Measurements

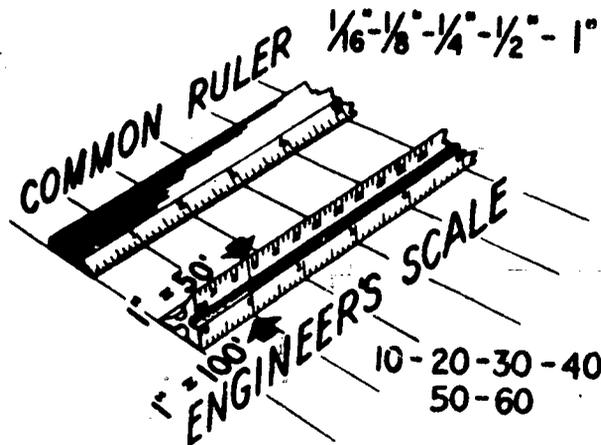
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May 1974

Section 4

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## Measurements - Stationing

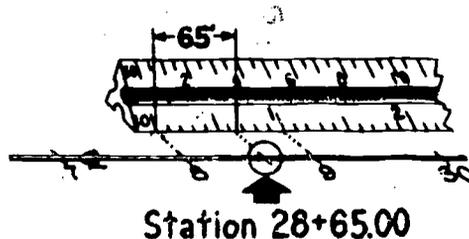


Because engineering surveys and plans are based upon 100-foot measurements, it is necessary to use a scale which will measure directly.

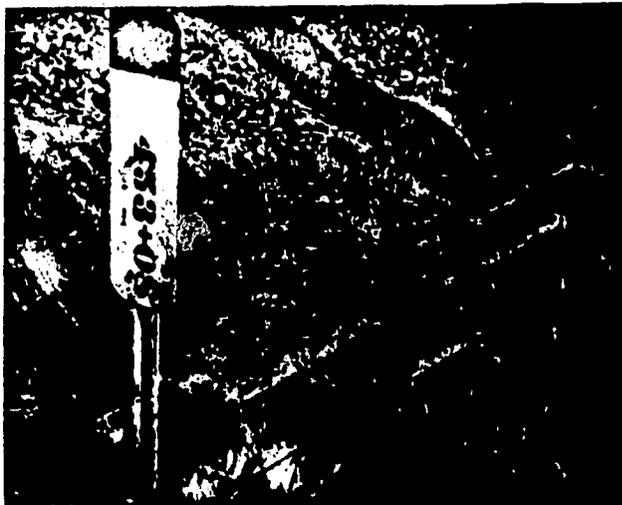
One inch on a common ruler is subdivided by halves, i.e.,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ , etc. These units are not applicable to engineers' measurements because of the necessity for converting each fraction into multiples of 100.

An engineer's scale is based upon 1 inch of the plan being equal to 10 feet, 20 feet, 30, 40, 50, 60 or 100 feet on the ground. The usual scales on Layout Plans are 1 inch of the plan equals 100 feet on the ground or 1 inch of plan equals 50 feet on ground.

The center line of the highway or survey is similar to a tape measure. Every 100 feet along this line is a mark called a "station." These stations are numbered consecutively as the line progresses.

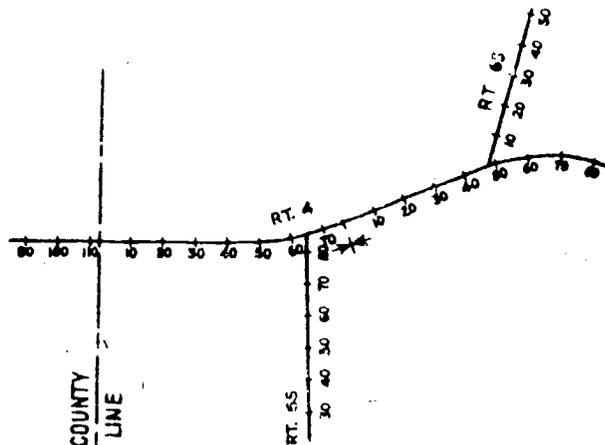


This means 2,865 feet from 0 station.



### Culvert Marker

Everything that is surveyed and measured or proposed within the right of way is referenced to the center line stationing. Like a house number, say, is in the 2800 block, a station number would be in the twenty-eighth 100-foot section, or at station 28+00. These station numbers are found along the road on posts, as in the above picture, which mark most culverts. This marker is 45,308 feet from the beginning station. Normally station numbers progress from south to north and west to east. Each section contains its own set of stations.

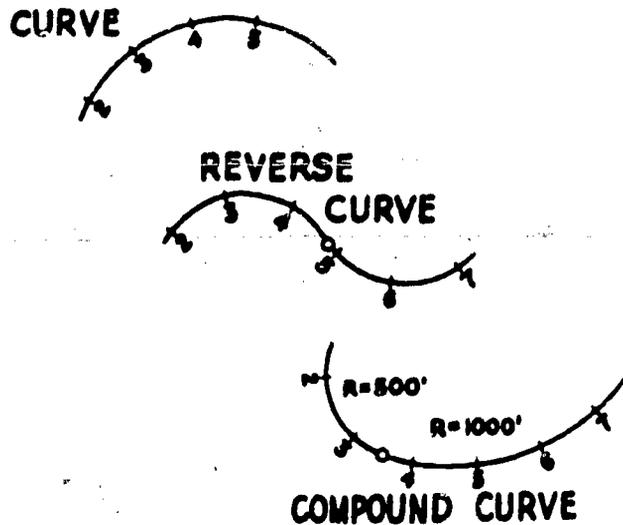


## Alignment Data



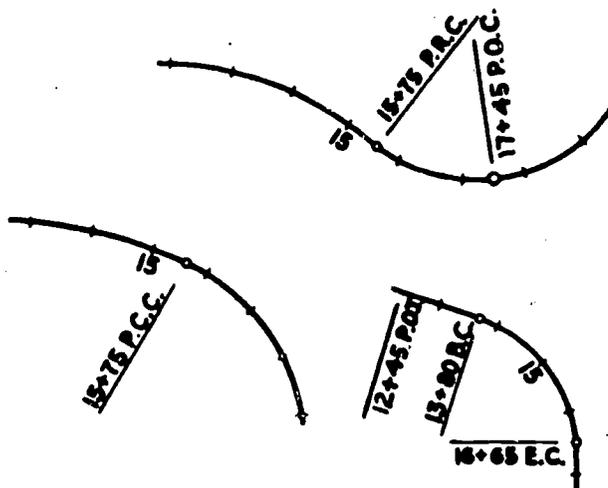
### Center Line Alignment

A highway center line is established by actual surveys and calculations. It is either straight or curved depending in large degree upon the terrain. When speaking of "alignment" no inference is made as to the "grade" or steepness of the highway nor to the slope laterally across the highway.



There are three kinds of curved lines used in California highway work. A simple curve is one which has a single radius continuous throughout the curve. It is a part of a complete circle. A reverse curve is one in which a curve reverses its direction without a tangent separating it from an adjacent curve. This type is usually found only on old

routes and especially in mountainous areas. The third type of curve is a compound curve. In this there are two or more curves of differing radii, such as a sharp curve easing off into a flatter curve. Railroads, certain U. S. Governmental agencies, and some states also use "spiral" curves which, although more closely matching normal driving habits, require special tables and additional calculations to use.

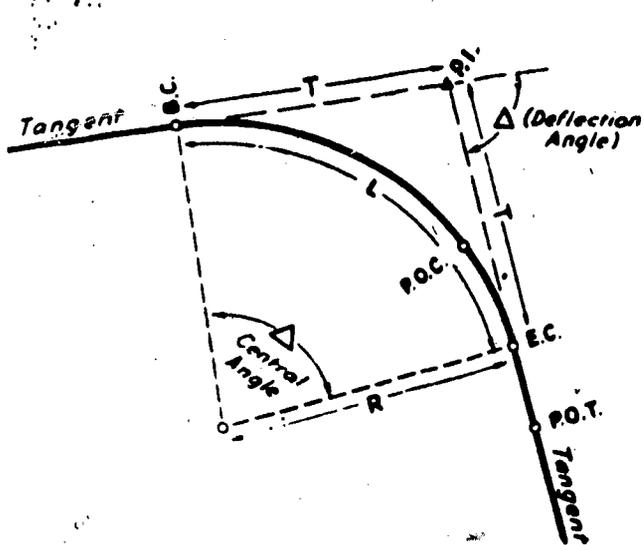


Wherever a center line changes from tangent to curve, or vice versa, or when the radius of curvature changes, an identifying abbreviation is placed opposite the point of change on the line together with its station number. The beginning point of all curves is marked with the abbreviation "B.C." which means Beginning of Curve. The end of the curve is indicated by "E.C." Where one curve compounds into another curve, the point is marked "P.C.C.", Point of Compound Curve. Reversing curves are marked "P.R.C." at the point of reverse.

Other abbreviations used are "P.O.C.", Point on Curve, and "P.O.T.", Point on Tangent.

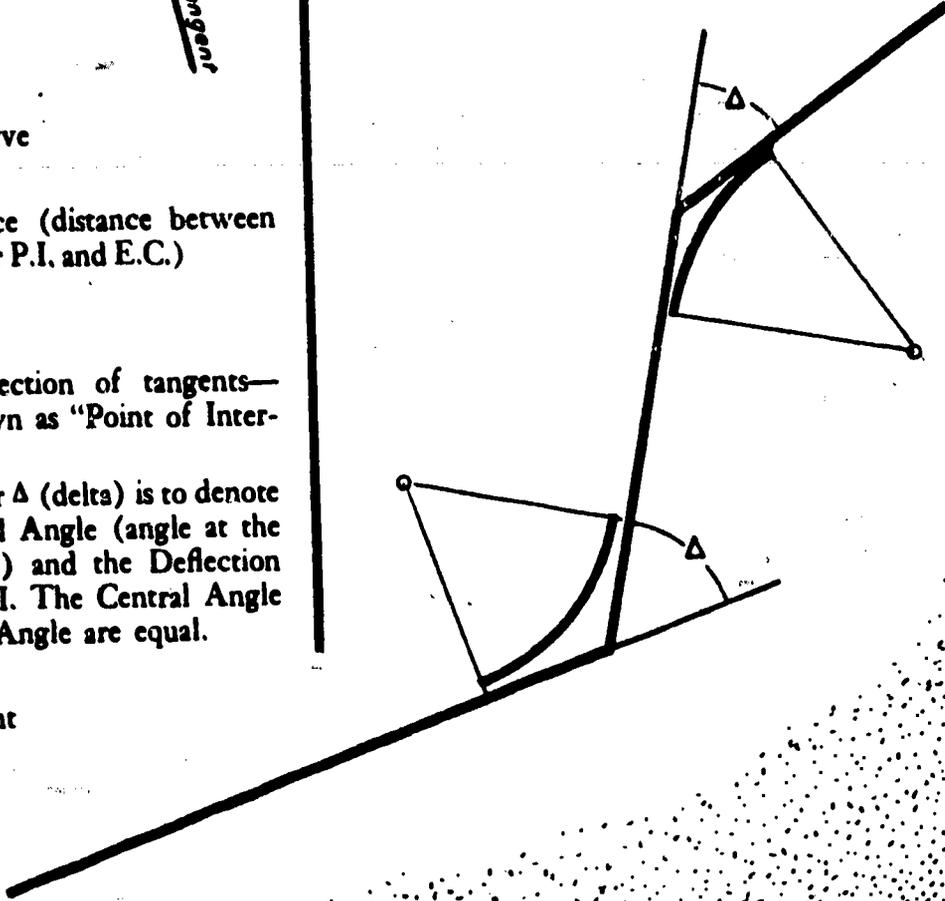
Other states and agencies sometimes use different abbreviations for their curve points. For instance "P.C." and "P.T." for "Point of Curvature" and "Point of Tangency" to denote the beginning and ending of a horizontal curve, or "T.C." and "C.T." to signify "Tangent to Curve" and "Curve to Tangent" points, with both the above illustrations being the same as our "B.C." and "E.C."

# Curve Data



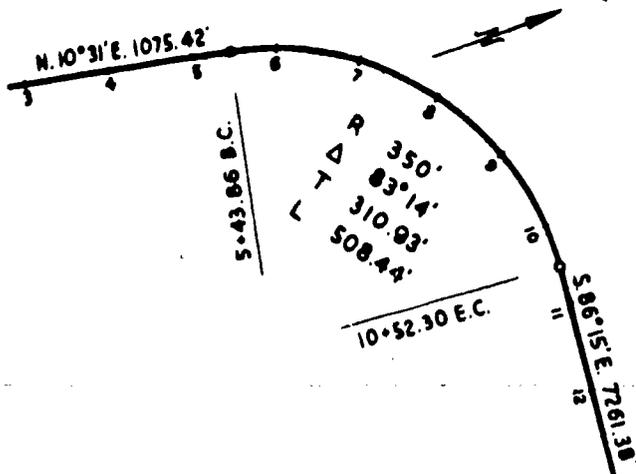
In designing highway alignment every attempt is made to secure a maximum of tangent. Although in many instances other factors may control, the usual design practice is to fit a series of tangents to the topography and connect them with curves of appropriate radii.

- B.C. —Beginning of Curve
- E.C. —End of Curve
- T —Tangent Distance (distance between B.C. and P.I.—or P.I. and E.C.)
- R —Radius of Curve
- L —Length of Curve
- P.I. —Point of Intersection of tangents—commonly known as "Point of Intersection"
- $\Delta$  —The Greek letter  $\Delta$  (delta) is to denote both the Central Angle (angle at the center of curve) and the Deflection Angle at the P.I. The Central Angle and Deflection Angle are equal.
- P.O.C.—Point on Curve
- P.O.T.—Point on Tangent



# Curve Data

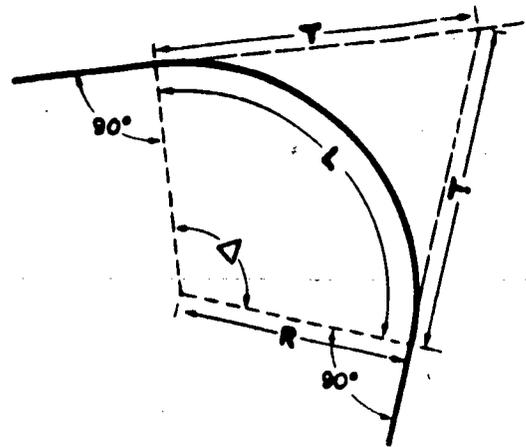
Here is a typical curve as it would appear on the plans. (The tangent distance, the point of intersection, and the center of curve are not drawn on plans.)



The figures on each tangent show the bearing of the tangent and the length of tangent between curves.

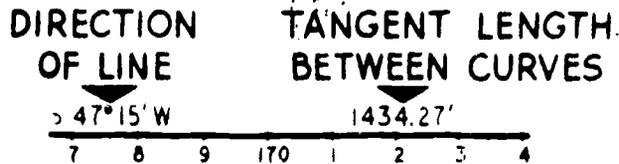
Four elements, all mathematically inter-related, are shown on the plans for each curve:

- R—radius of curve
- $\Delta$ —the central angle
- T—tangent distance (sometimes called the semi-tangent)
- L—length of curve



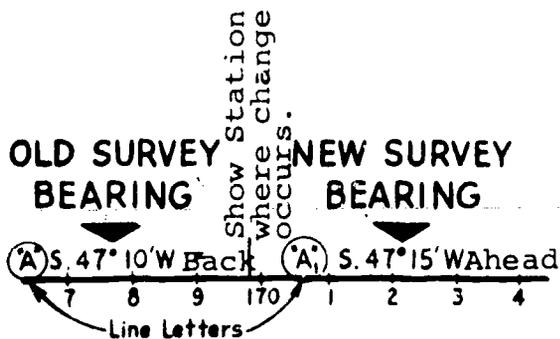
The trigonometric formulas  $T = R \tan \frac{\Delta}{2}$  and  $L = 2\pi R \left( \frac{\Delta \text{ in degrees}}{360} \right)$  shows the relation of these elements.

Equations - Length of Project



Bearing and Length

All tangent lines have two notes which describe the line. One is the bearing or direction in which the line is progressing and the other is the length of the tangent line between curve points.



Show "Back" and "Ahead" for clarification.

Equations

"Equations" are used by surveyors to define lines, points, or elevations which for various reasons have more than one recorded value or description.

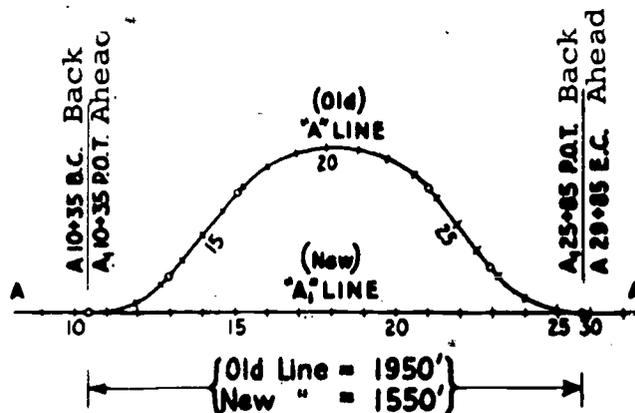
In the case of the "bearing" or direction of line a continuous tangent may, through survey or other discrepancy, have one recorded bearing up to a certain point and from that point on have a different bearing.

A similar situation occurs in regard to elevations of physical points and bench marks. Due to use

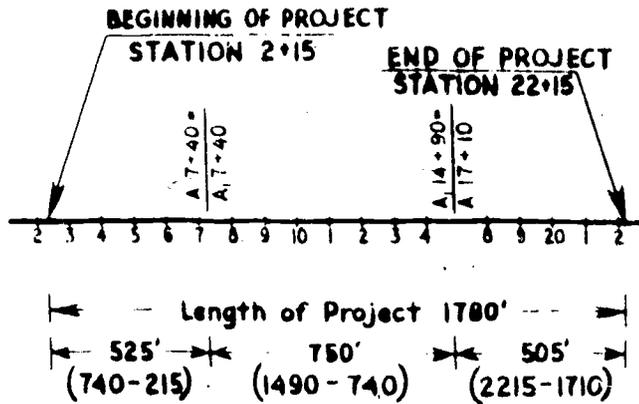
of a different datum or to discrepancies in surveys a point may have more than one recorded elevation. Where these differences occur an "equation" or note is necessary to clarify the figures.

The most common use of "equations" is in "stationing" or records of distance.

### Center Line Alignment



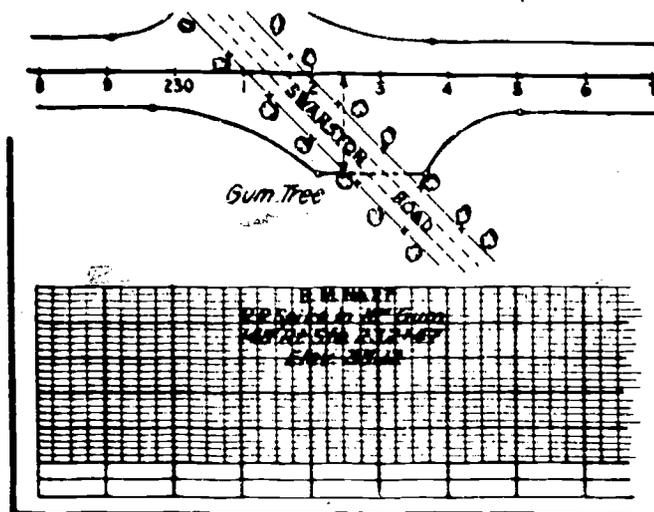
In this example the old "A" line curved around a hill. The new "A<sub>1</sub>" line cuts through the hill and meets the old line on the other side. Where the new line leaves the old at Station 10+35, the only equation is in the line letter A to A<sub>1</sub>. At the other end, however, the stations are different as well as the line letters. The point of departure and the point of returning require an equation to move from one set of data to another.



Calculating the length of a project or even a portion of an alignment requires careful scrutiny of the line between the limits in order to account for the equations. For instance, the above sketch has a beginning station of 2+15 and an ending station of 22+15. At a glance this looks like 2,000 feet. Actually, it is only 1,780 feet because of the difference in the equations.

When writing the equation or referring to it without a map, the use of "Back" and "Ahead" for the appropriate station of equation is recommended. (See top illustration, page 4-11.)

### Bench Marks



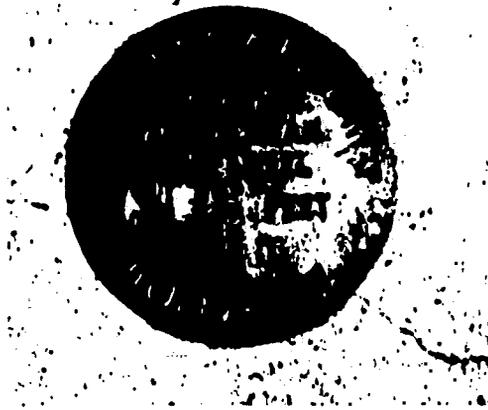
The note shown on the "profile" portion of this layout sheet is a bench mark note. It describes the kind of marker, its location in relation to the center line of the highway, and its elevation.

In the "plan" section of this sheet the location of the bench mark can be measured and found to be in the gum tree shown 148 feet right of Station 232+47. The photograph below is taken at the gum tree looking back toward the highway center line on the overcrossing. The "bench mark" is a railroad spike in the blazed cut in the foot of the tree trunk.



Bench marks are placed by surveyors to serve as permanent reference points. They are elevation markers. Their location and elevation are definitely established and recorded on surveyors' level notes. They are set upon some permanent object in a way which will insure their remaining undisturbed.

Profile levels, or the establishment of elevations along the center line, must be checked against the bench marks as the line progresses. An allowable error of only 0.10 foot per 1,000-foot interval is tolerated.

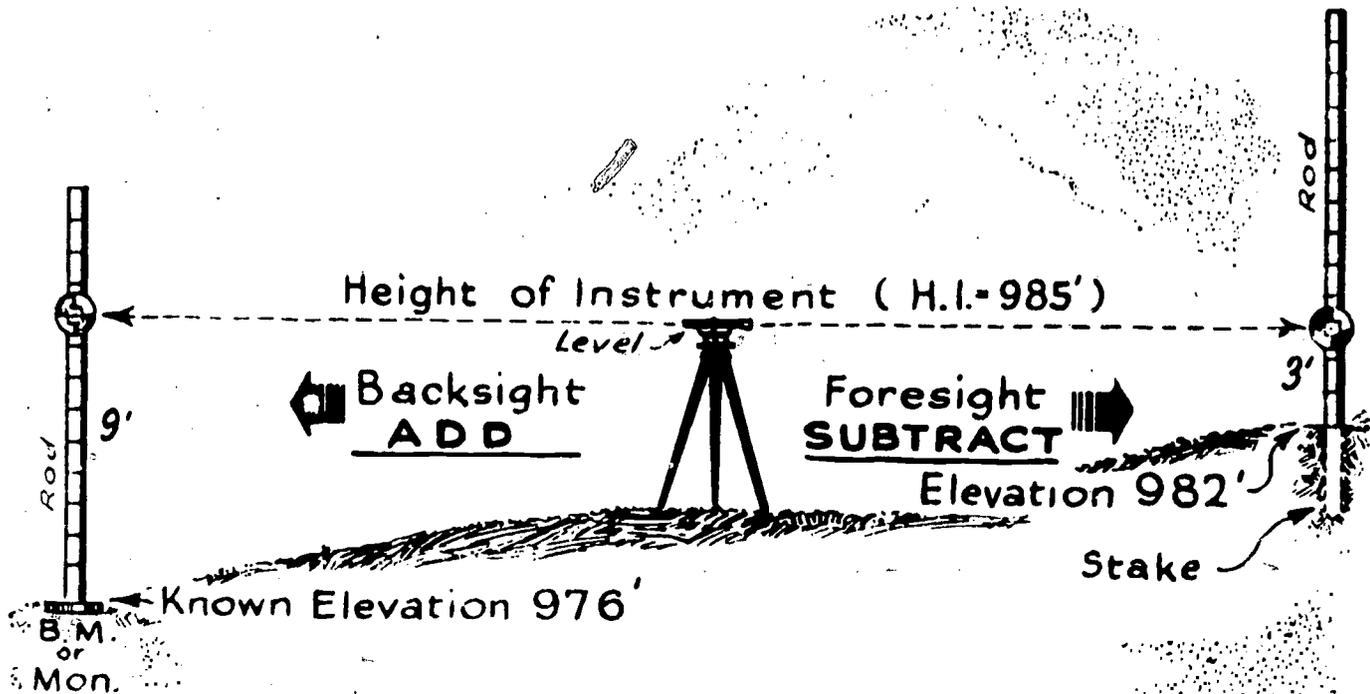


This is a bench mark placed by the Division of Highways. It is a brass plate which lists the bench mark number and the elevation above mean sea level. This particular marker is set in the concrete curb of a bridge, a typical installation.

The bench mark shown below is a "reference mark" located and placed by the U. S. Coast and Geodetic Survey. These monuments are very accurately set and recorded in order to serve as an officially recorded reference point. Any survey, whether it is private or a state highway survey, can be based upon these established control stations.



U. S. Coast and Geodetic monuments identify triangulation stations, traverse stations, azimuth marks, and reference marks and are marked accordingly.



After the survey has been properly referenced and tied into the existing bench marks, the survey may proceed. Reference points may be established by the division anywhere along a proposed route. These markers serve as supplemental ties where distance and topography do not permit sighting on existing monuments. After the alignment of a project has been tied to these markers, subsequent surveys will have a base upon which to work. Topographic, level, and cross section surveys will use these markers as reference points.

In order to establish the elevation of new points it is necessary to begin from an already established reference point such as a bench mark or monument. The level is set up at a convenient location between the reference point and the next objective. The calibrated rod is placed on the reference point and the level is backsighted on the rod. The recorded figure establishes the height (or elevation) of the instrument

in relation to the known elevation. The rod is then moved to the next location and the instrument swung around to foresight on the second point. The difference between the new reading and height of instrument (H.I.) determines the elevation of the second point. Repeating the process will establish elevations of successive points.

○ Care must be taken when establishing new bench marks and elevations from other agencies' vertical control that all the old control bench marks are on the same datum or reference plane. For instance, there is about 2.5 feet difference between the "sea level" used by the U. S. Coast and Geodetic Survey and the U. S. Engineer Department.

## Section 5 - Horizontal Curves

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## Circular Curve--

### A. Terms--

- R = radius of curve
- C = Long chord, Ln. between B.C. and E.C.
- (P.C.)-B.C. = Beginning of curve (Pt. of curve)
- (P.T.)-E.C. = End of curve (Pt. of tangent)
- P.O.C. = Pt. on curve
- L = length of curve
- E = external distance
- M = mid-ordinate distance
- D = degree of curve
- T = semi-tangent dist.
- P.I. = Pt. of intersection of tangent
- (delta)  $\Delta$  = interior ~~angle~~ of curve
- d = subchord  $\Delta$
- c = subchord

### B. Definitions--

- C - straight line from B.C. to E.C.
  - L - arc length B.C. to E.C.
  - E - dist. from mid-pt. on curve (arc) to P.I.
  - M - dist. from mid-pt. on curve (arc) to P.I. of long chord
  - D - angle at the center subtended by a chord of 100'
  - T - dist. from B.C. or E.C. to P.I.
  - $\Delta$  - interior angle of curve - to the deflection angle at the P.I.
  - c - any nominal chord length less than C
  - d - arc subtended by a subchord
- Radian - an arc length equal to R. or  $57.295780^\circ/\text{Rad}$ .

### C. Helpful relationships--

1. Degree curve - most applicable to railroads
  - a.  $\frac{D}{2}$  = deflection per station
  - b.  $L = 100 \frac{\Delta}{D}$
  - c.  $R = \frac{50}{\sin \frac{D}{2}} = \frac{5729.58}{D}$  = arc for 1 degree

Note: The degree curve seems to be favored by the State examiners since it lends itself better to rapid calculations.

2. Radius curve - used by the California Division of Highways. Actual arc measurements used.

Derivation

$$\frac{d}{2} = \frac{360^\circ}{4\pi R} \times \text{arc}$$

a.  $\frac{1718.87338 \times \text{arc}}{R} = \text{deflection in minutes}$

b.  $L = \frac{2\pi R \Delta}{360^\circ} = 0.0174533 \Delta R$

c.  $R = \text{given}$

3. Relationships found in both

$$C = 2 R \sin \frac{\Delta}{2}$$

$$E = R \operatorname{ex} \sec \frac{\Delta}{2}$$

Note:  $\operatorname{ex} \sec = \operatorname{Sec} - 1$

$$\operatorname{Sec} = \frac{1}{\cos}$$

$$M = R \operatorname{vers} \frac{\Delta}{2}$$

$$T = R \tan \frac{\Delta}{2}$$

$$C = 2 R \sin \frac{d}{2}$$

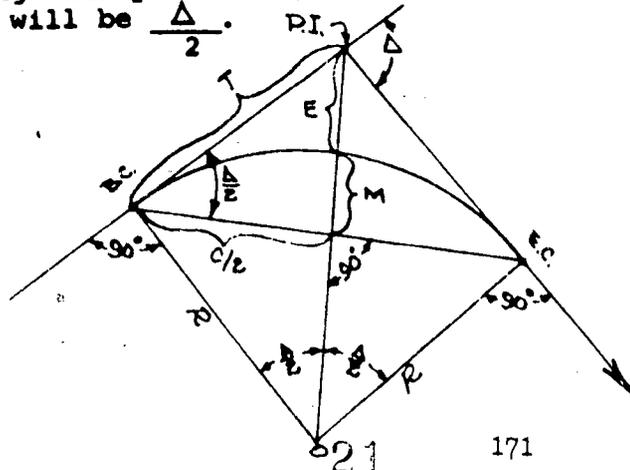
$$d = \frac{360^\circ \times \text{arc}}{2\pi R}$$

deflection  $\Delta$   $C = \frac{\Delta}{2} \times R$  @ B.C. (P.C.) deflecting to E.C. (P.T.)

Note: Best approach for figuring L on a test.

$$\left\{ \begin{array}{l} \frac{L}{R} = \frac{\Delta}{\text{Radian}} \quad \text{or} \quad L = \frac{\Delta}{\text{Rad}} R \quad \text{or} \quad L = \frac{\Delta}{57.295780^\circ} \times R \\ \text{Slide rule accuracy } L = \frac{\Delta}{57.3^\circ} R \end{array} \right.$$

In occupying a P.O.C. with a transit, put the deflection of the back sight in the gun and turn up the fore sight deflection. By following this procedure, the deflection to the E.C. from any P.O.C. will be  $\frac{\Delta}{2}$ .



## SECTION 5.- Horizontal Curves

### I - Introduction

The path followed by a highway is a series of straight and curved lines. This applies to both the top (plan) and side (profile) views.

The straight lines are called tangents because their beginning and end are both tangent to a curve.

Two types of curves are used. Those in the plan are called horizontal curves and the ones in the profile are vertical curves. In California highway work, horizontal curves are portions of a circle. Vertical curves are parabolas. This section will study horizontal curves. Vertical curves are covered in Section 6.

The easiest way to draw a circle is to go to the radius point, measure the length of the radius, and revolve around the radius point. This happens when you use a compass to draw a circle. In surveying, where a radius may be thousands of feet long, it isn't possible to lay out a curve this way. It is necessary to lay out the curve along the circumference line without ever going to the radius point. The understanding of this method and the calculations will use your knowledge of algebra, geometry, and trigonometry.

We describe the basic characteristic of a circular curve by stating its radius in feet. Other states and railroads use the "degree of curve" description. The approximate relationship between degree and radius is  $D = 5730 \div R$ , with R expressed in feet.

A compound curve is two or more circular curves, each having a different radius and each curve adjoining the other. Each curve turns in the same direction.

A broken back curve is two circular curves in the same direction with a short length of tangent between them.

A reversing curve is two circular curves, adjoining or separated by a short length of tangent, with the second curve going in the direction opposite from the first curve.

By definition, in California a "short length of tangent" is one less than 400 feet long.

Compound curves are used only when unavoidable. Reversing and broken back curves are not used in modern practice.

A spiral (or transition) curve is a circular curve with a continuously changing radius throughout its length. We do not use spiral curves on highways in California.

## II - Horizontal Curves

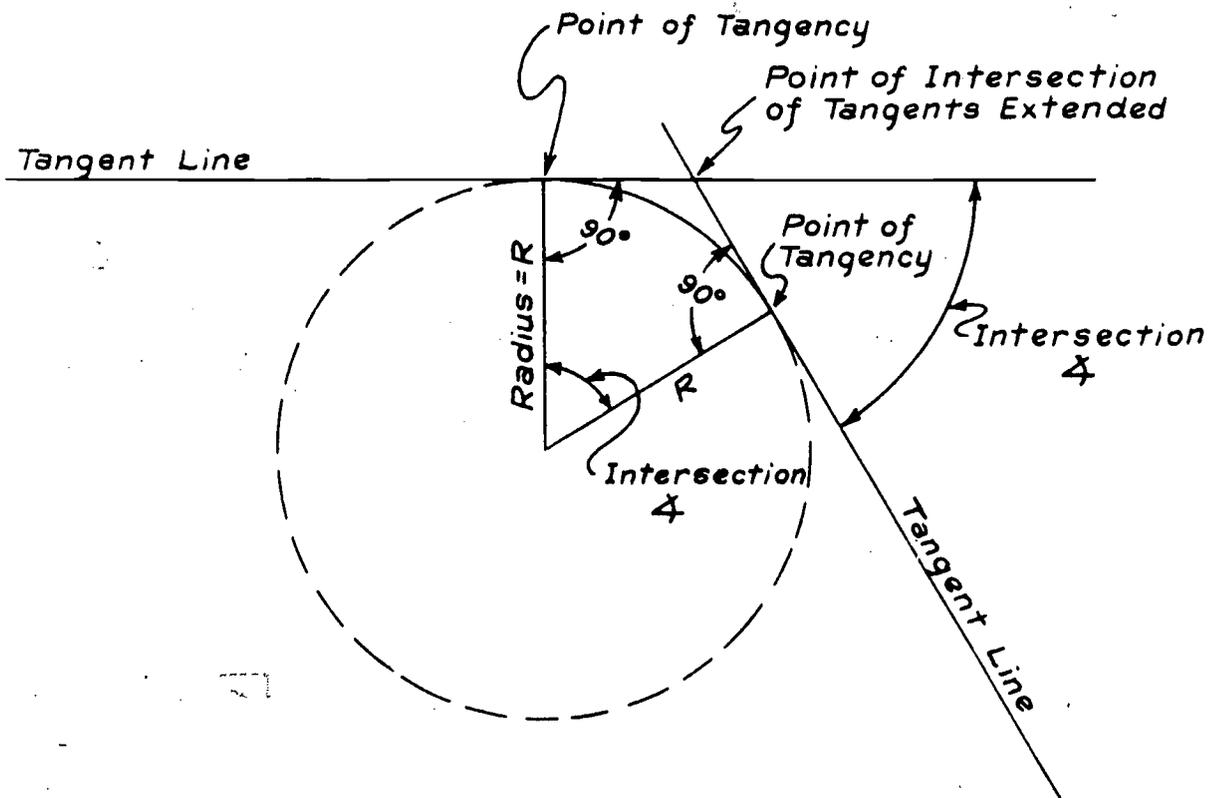


Figure 1

Figure 1 gives a very basic example of the overall picture of a simple horizontal curve. The relations shown will be valid for all simple curve analyses.

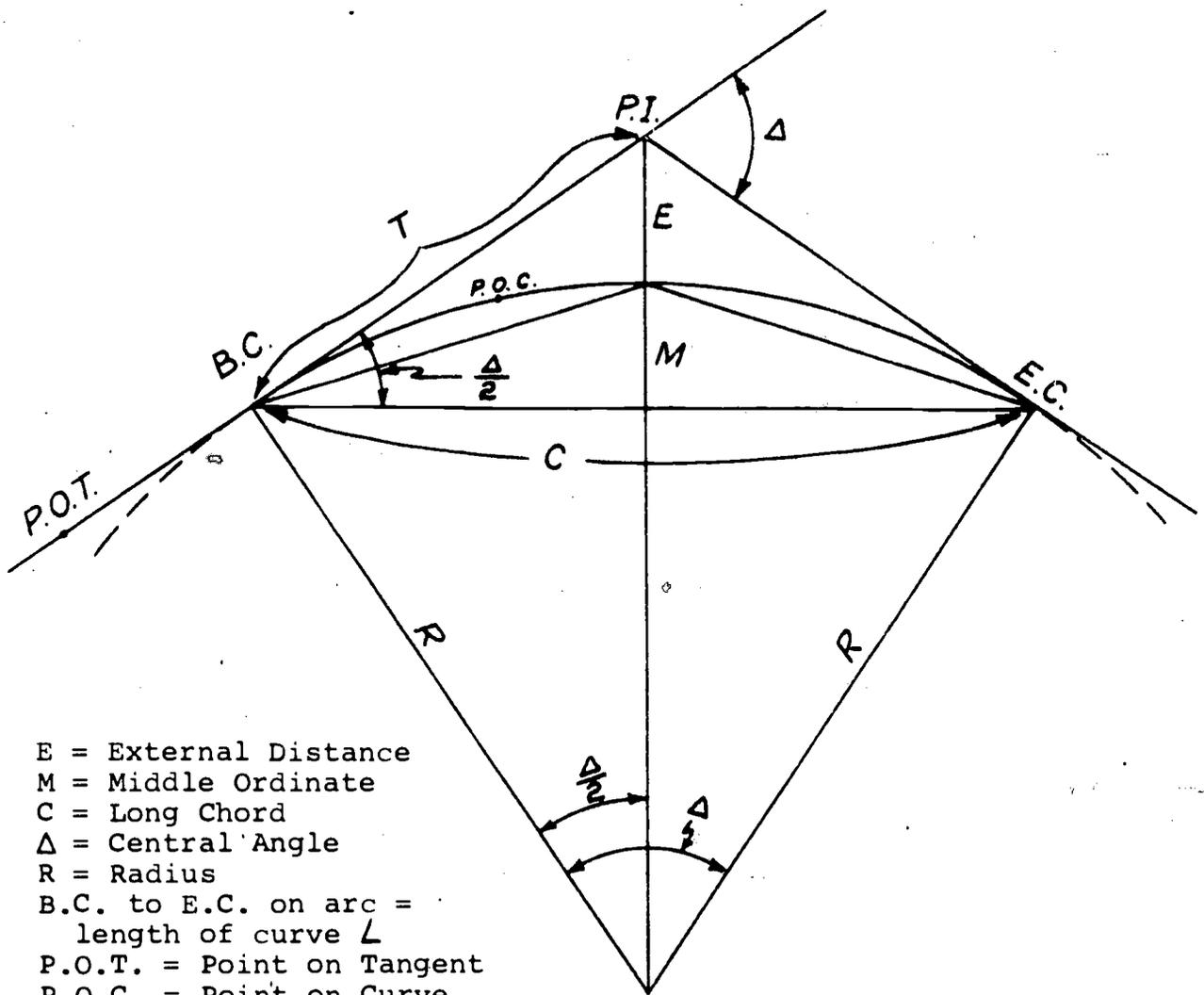


Figure 2

In Figure 2 the "piece of pie" in Figure 1 has been "blown up" and symbols for the various angles, points, and distances have been added.

Before studying the solution of horizontal curves, one should be familiar with the following definitions of terms common to curve work.

1. Arc - path of curve.
2. Arc Distance - length of curve measured along the arc.
3. B.C. - beginning of curve.
4. Central Angle - The angle subtended by the full length of curve. (Also known as delta,  $\Delta$  )
5. Chord - straight line connecting two points on an arc.

6. Chord Distance - straight line distance measured between points on the arc.
7. Deflection Angle - the angle to the right or left from a preceding line produced. d
8. Degree of Curve - the angle at the center of curvature subtended by an arc or chord 100' long. D
9. E.C. - end of curve.
10. Long Chord - the chord connecting the beginning and end of curve. C
11. Middle Ordinate - the distance, along the radius, from the center of the long chord to the arc. M
12. P.C.C. - point of compound curve. This is also the symbol for Portland Cement Concrete, but the context will show which is meant.
13. P.I. - point of intersection (of tangents extended).
14. P.O.C. - point on curve.
15. P.O.T. - point on tangent.
16. P.R.C. - point of reverse curve.
17. Tangent - the straight lines connecting or tangent to circular curves.
18. Tangent Distances - the distance from B.C. or E.C. to the P.I. (Also known as semi-tangent.) = T
19. Tangent Length - the length of tangents between curves.
20. R - radius.
21. External Distance - the distance from the P.I. to the middle of the arc = E.



10. Given D and C to find R:  $R = \frac{1}{2} C + \sin D$
11. Given  $\Delta$ , T, and C to find D:  $\sin D = \frac{(C \times \tan \frac{1}{2} \Delta)}{2T}$
12. Given  $\Delta$ , D, and C to find T:  $T = \frac{(C \tan \frac{1}{2} \Delta) + \sin D}{2}$

The student should study all the properties very thoroughly and then derive the formulas. If you can derive these formulas you will have a much better chance of remembering them.

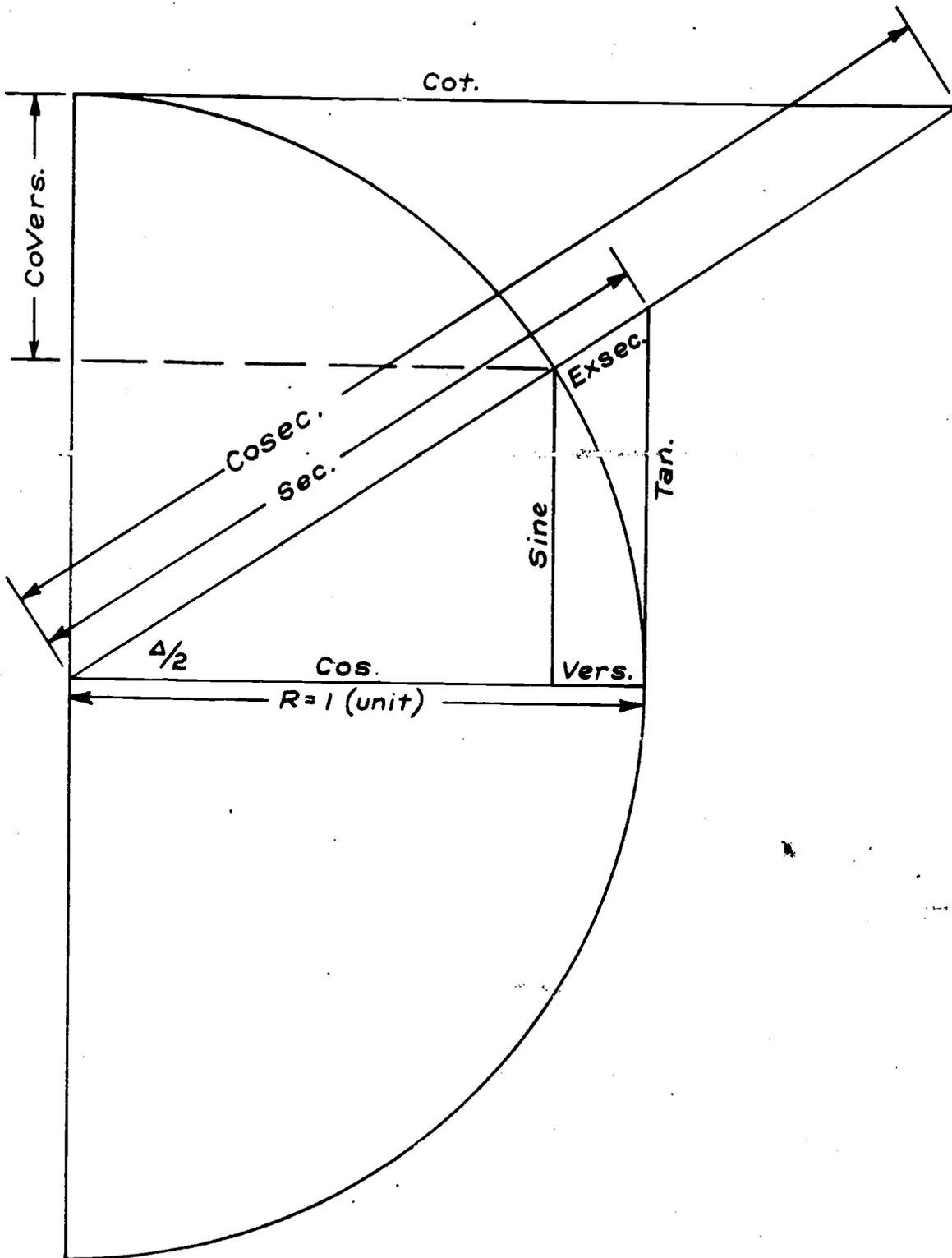


Figure 4

Figure 4 illustrates the trigonometric functions in the unit circle form.

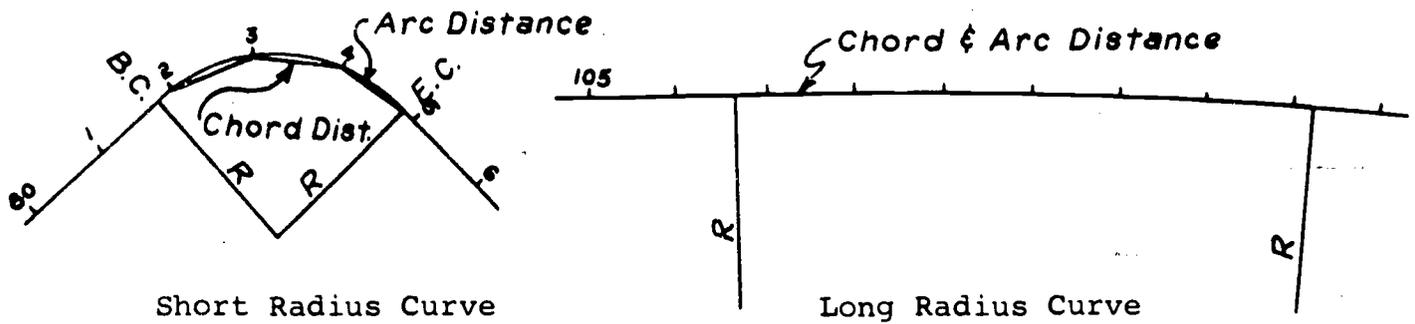


Figure 5

When curves are laid out in the field it is, of course, impractical to measure distances along the arc, so chord distances for 100' arc lengths are computed and these are staked. This gives 100' stations on the arc of the curve. On extremely long radius curves the arc and chord lengths are equal as far as field measurements are concerned.

The sharpness of curve is measured in various ways. Two of the most common are (1) by stating the degree of curve as the angle subtended at the center by an arc 100' long, and (2) by stating the length of radius. The latter is the method used in California. The relationship between degree of curve and radius is

$$R = \frac{(360^\circ)}{D^\circ} \left( \frac{100}{2\pi} \right).$$

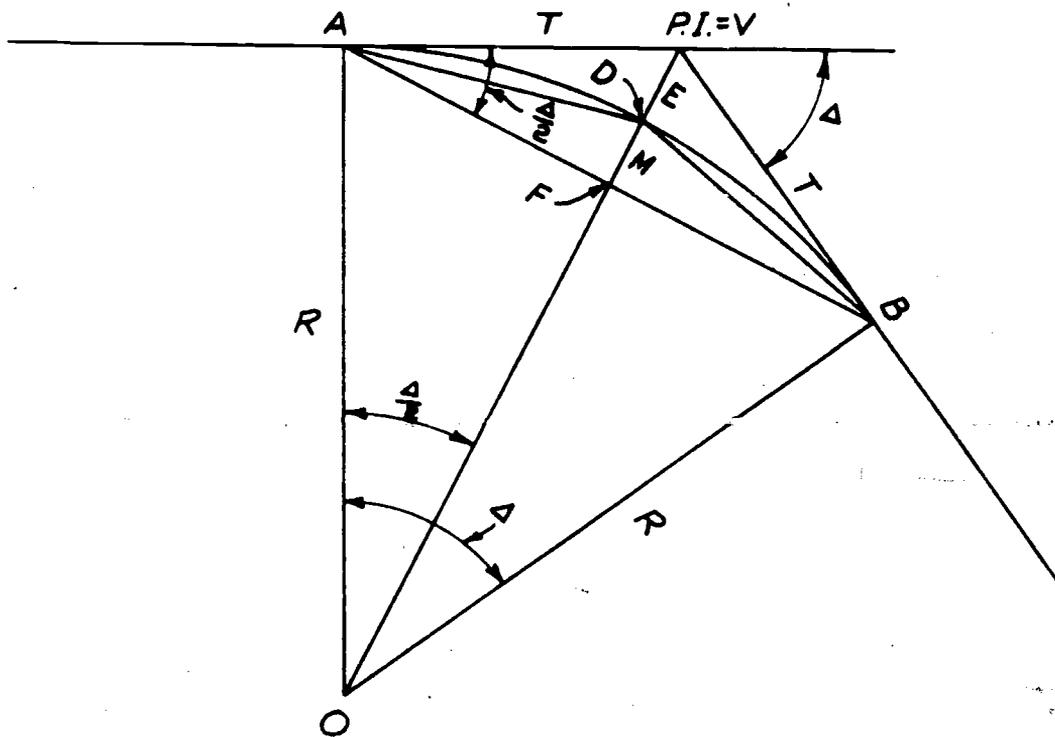


Figure 6

Figure 6 shows two tangents connected by a circular curve. The angle " $\Delta$ " is known from the plans and "R" has been selected from the design speed and topography. The line OV bisects the central angle AOB, the angle AVB, chord AB, and the arc ADB. It is perpendicular to chord AB. Angle " $\Delta$ " equals angle AOB which equals twice the angle VAB or VBA.

AB is the long chord "C". AV equals VB and is the tangent distance "T" or semi-tangent. Distance DF equals "M", or the middle ordinate, while VD equals "E" the external distance.

PROBLEMS AND SOLUTIONS

Problem 1 (Figure 6)

Given	Sought	Formulas
R = 1200', Δ = 22°00'	AV = Tangent Distance	$\frac{T}{R} = \tan \frac{\Delta}{2}$ , T = R $\tan \frac{\Delta}{2}$
In Triangle, AOF, AF = $\frac{C}{2}$	C = Long Chord	C = 2R $\sin \frac{\Delta}{2}$ , C = 2T $\cos \frac{\Delta}{2}$
In Triangle, AVF, $\angle VAF = \frac{\Delta}{2}$	M = Middle Ordinate	M = R - R $\cos \frac{\Delta}{2}$ , M = R(1 - $\cos \frac{\Delta}{2}$ )  M = $\frac{C}{2} \tan \frac{\Delta}{4}$ , M = R $\text{vers } \frac{\Delta}{2}$
In Triangle, ADF, $\angle DAF = \frac{\Delta}{4}$		
VD = E	E = External Distance	E = R $\text{ex. sec. } \frac{\Delta}{2}$
Above Data	L = Length of Curve	L = $\frac{\Delta}{360} 2 \pi R$
Above Data	D = Degree of Curve (Angle at center should be sub- tended by an arc 100' long)	D = $\frac{100 \times 360}{2 \pi R}$

Solutions

$$T = R \tan \frac{\Delta}{2}, T = 1200 \tan 11^{\circ}00' = 1200 (.1943803) = \underline{233.26'}$$

$$C = 2R \sin \frac{\Delta}{2}, C = 2400 \times .1908090 = \underline{457.94'}$$

$$M = R - R \cos \frac{\Delta}{2}, M = 1200 - 1200 \cos 11^{\circ}00', M = 1200 - 1200 (.9816272) = 1200 - 1177.95 = \underline{22.05'}$$

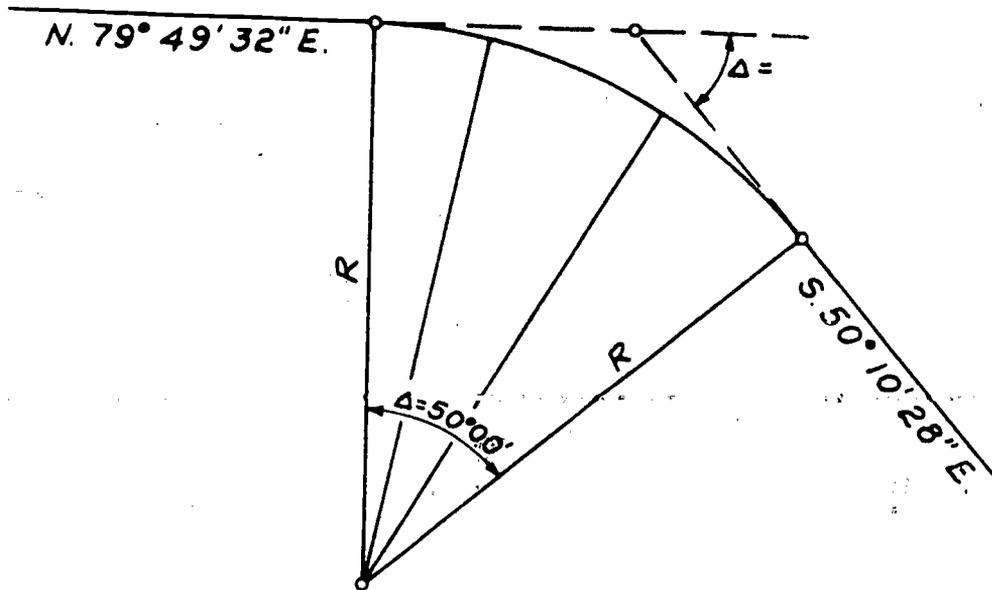
$$E = R \text{ ex. sec. } \frac{\Delta}{2} = 1200 \text{ ex. sec. } 11^{\circ}00' = 1200 (.0187167) = \underline{22.46'}$$

$$L = \frac{\Delta}{360^{\circ}} 2 \pi R = \frac{22^{\circ}00'}{360^{\circ}} 2 \pi 1200 = \underline{460.77'}$$

$$D = \text{Degree of Curve} = \frac{100 \times 360}{2 \pi R} = \frac{36000}{2 \pi 1200} = \frac{36000}{7539.84} = \underline{4.7746^{\circ}} = \underline{4^{\circ}46'29''}$$

# CALCULATING A CURVE

Figure 7



Given  $R = 300'$

Calculate:  $\Delta$ ,  $L$ , and  $T$

Solve for  $\Delta$

$$79^{\circ}49'32'' + 50^{\circ}10'28'' = 130^{\circ}00'00'', \quad 180^{\circ}00'00'' - 130^{\circ}00'00'' = \underline{50^{\circ}00'00''} = \Delta$$

$$L = \frac{\Delta^{\circ}}{360^{\circ}} \times 2\pi R = \frac{50^{\circ}}{360^{\circ}} \times 2\pi \times 300 = \underline{261.80'}$$

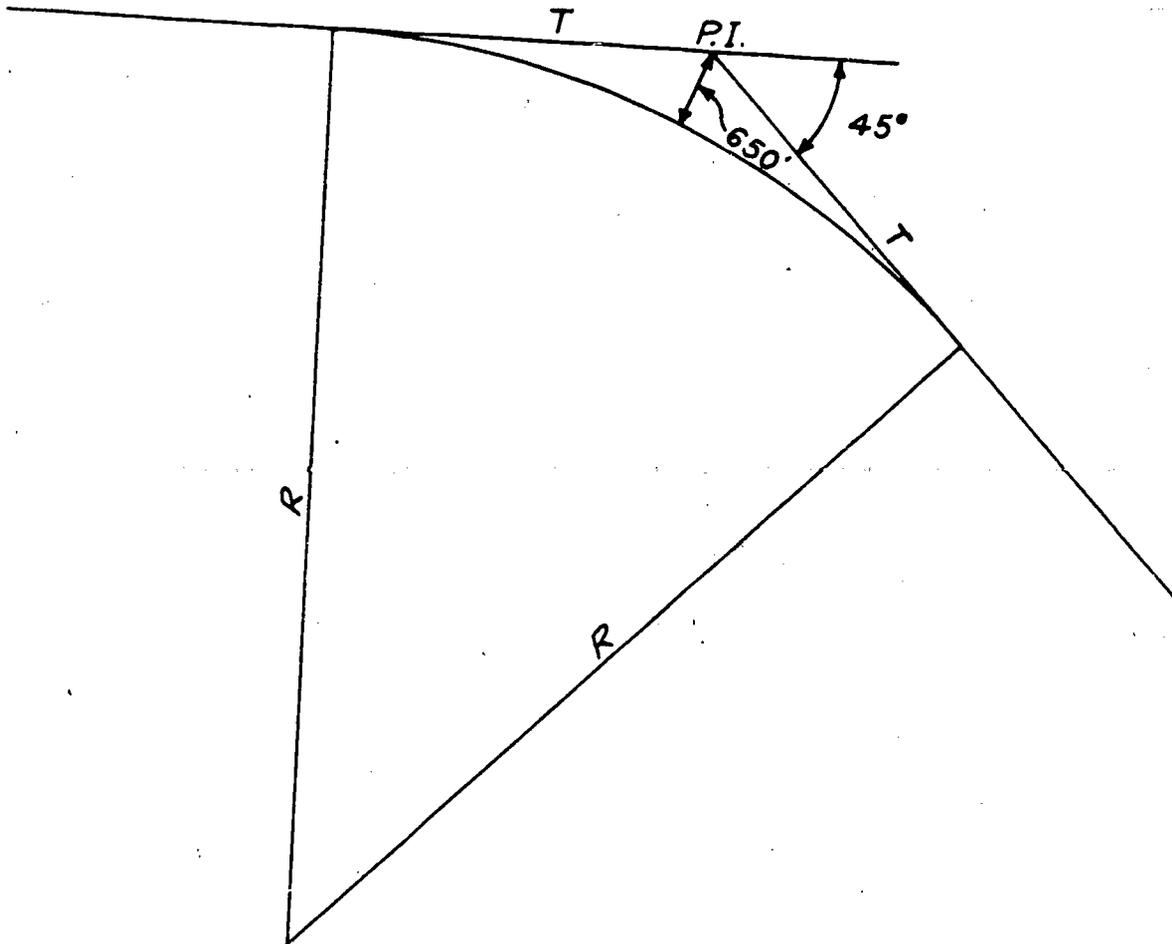
$$T = R \tan \frac{\Delta}{2} = 300 \tan 25^{\circ}00'00'' = 300 \times .46631 = \underline{139.89'}$$

$$\Delta = 50^{\circ}00'00''$$

$$L = 261.80'$$

$$T = 139.89'$$

Figure 8



<u>Given</u>	<u>Sought</u>	<u>Formula</u>
E = 650'	R, T	$E = R \text{ ex. sec. } \frac{\Delta}{2}, T = R \tan \frac{\Delta}{2}$
$\Delta = 45^{\circ}00'$		

SOLUTION

$$R = \frac{E}{\text{ex. sec. } \frac{\Delta}{2}} = \frac{650}{\text{ex. sec. } 22^{\circ}30'} = \frac{650}{.082392} = 7889.10'$$

$$T = R \tan \frac{\Delta}{2} = 7889.10' \tan 22^{\circ}30' = 7889.10' (.4142136) = \underline{3267.77'}$$

Problem 4

Given	Sought	Formulas
<p>R = 1800',  <math>\Delta = 25^{\circ}54'30''</math>                      T = 414.05',                      L = 813.93'</p> <p>Sta. of B.C. =                      294+85.62</p> <p>Sta. of E.C. =                      302+99.55</p>	<p>Curve deflections from                      B.C. to each Station                      and 1/2 Station on the                      curve.</p>	<p>d = deflection                      deflection per                      foot in minutes <math>\frac{\Delta}{2} =</math>  <math>\frac{\Delta}{L}</math></p> <p><math>\frac{\frac{\Delta}{2}}{360} \cdot 60 = \frac{540}{\pi R} =</math>  <math>\frac{\Delta}{2 \pi R}</math></p> <p><u>1718.873 times the arc</u>                      R</p>

Solution

<u>Station</u>	<u>Deflection</u>
E.C. 302+99.55	12 <sup>o</sup> 57.25'
302+50	12 <sup>o</sup> 09.9'
302+00	11 <sup>o</sup> 22.2'
+50	10 <sup>o</sup> 34.4'
301+00	09 <sup>o</sup> 46.7'
+50	08 <sup>o</sup> 58.9'
300+00	08 <sup>o</sup> 11.2'
+50	07 <sup>o</sup> 23.4'
299+00	06 <sup>o</sup> 35.7'
+50	05 <sup>o</sup> 48.0'
298+00	05 <sup>o</sup> 00.2'
+50	04 <sup>o</sup> 12.5'
297+00	03 <sup>o</sup> 24.7'
+50	02 <sup>o</sup> 37.0'
296+00	01 <sup>o</sup> 49.2'
+50	01 <sup>o</sup> 01.5'
295+00	00 <sup>o</sup> 13.7'
B.C. 294+85.62	0 <sup>o</sup> 00.00"

$$\frac{1718.873}{R} = \frac{1718.873}{1800} = 0.954929' / \text{ft.}$$

## CURVE PROBLEMS

Problem 1: In Figure 6, find T when  $R = 2000'$   
and  $\Delta = 20^{\circ}00'$ .

Note:  $\tan 10^{\circ}00' = 0.176327$  and  
 $\tan 20^{\circ}00' = 0.363970$

Answer: 352.65'

Problem 2: In Figure 6, find R to nearest foot when  
 $T = 1100'$  and  $\Delta = 30^{\circ}00'$

Note:  $\cot 15^{\circ}00' = 3.732051$   
 $\cot 30^{\circ}00' = 1.732051$

Answer: 4105.26'

Problem 3: Find length of curve in Problem 2.

Answer: 2149.51'

Problem 4: Find E and M in Problem 2.

<u>Angle</u>	<u>Vers</u>	<u>Ex.Sec.</u>
$15^{\circ}00'$	0.034074	0.035276
$30^{\circ}00'$	0.133975	0.154701

Answers: E = 144.82'  
M = 139.88'

Problem 5:

$$\Delta = 90^{\circ}41'44''$$

$$R = 850'$$

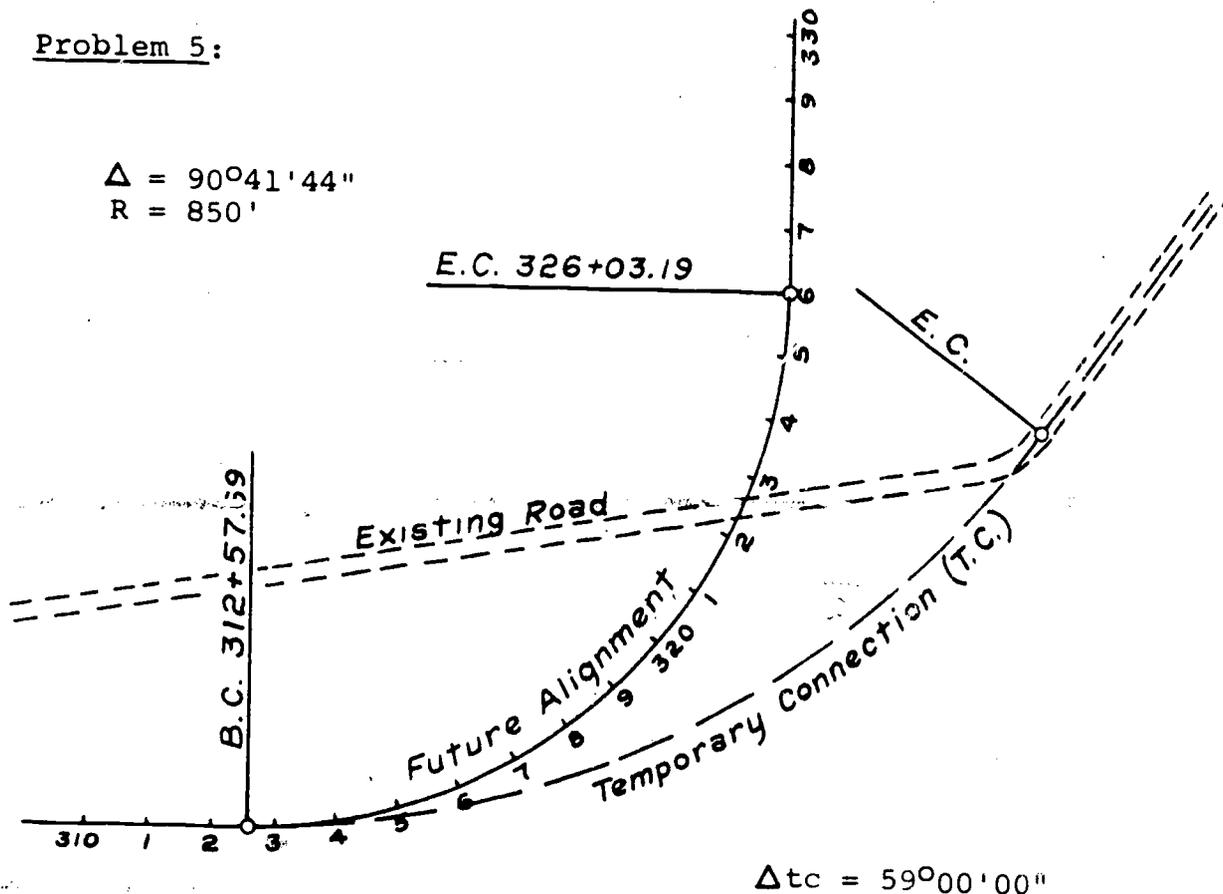


Figure 9

Part A (Future Alignment)

Given  $\Delta = 90^{\circ}41'44''$  and Radius = 850'

Solve for the following: Tangent distance T, long chord C, external E, and mid-ordinate M.O.  
(See functions page 5-16.)

Part B (Temporary Connection)

A temporary connection is to be made to the existing road as shown above. The B.C. at Station 312+57.69 and the P.I. of the 850' radius curve (future alignment) will be the B.C. and P.I. of the temporary connection curve.

Find the radius, length, and E.C. station of the temporary connection curve.

Trig Functions

<u>Angle</u>	<u>Sin</u>	<u>Cos</u>	<u>Tan</u>	<u>Cot</u>
12°10'15"	0.210827	0.977523	0.215675	4.636609
29°30'00"	0.492424	0.870356	0.5657728	1.767494
45°20'52"	0.711386	0.702802	1.012214	0.987933
59°00'00"	0.857167	0.515038	1.664280	0.600861

Also: Sec 45°20'52" = 1.422878  
Vers 45°20'52" = 0.297198  
Ex.Sec. 45°20'52" = 0.422878

Answers: Part I T = 860.38', C = 1209.36', E = 359.45',  
M.O. = 252.62'

Part II R<sub>tc</sub> = 1520.72', L<sub>tc</sub> = 1565.96'  
E.C. = "TC" 328+23.65

Problem 6:

In Figure 9 the deflection angle from the B.C. at Station 312+57.69 to a P.O.C. ahead on the fugure alignment is 20°30'.

Solve for the length of the chord between the B.C. and the P.O.C., and find the station of the P.O.C.

Trig Functions

<u>Angle</u>	<u>Sin</u>	<u>Cos</u>	<u>Tan</u>	<u>Cot</u>
10°15'	0.17794	0.98404	0.18083	5.53007
20°30'	0.35021	0.93667	0.37388	2.67462
41°00'	0.65606	0.75471	0.86929	1.15037
45°20'52"	0.71139	0.70280	1.01221	0.98793

Answers: Chord C = 595.36'  
Station P.O.C. = 318+65.94

Problem 7:

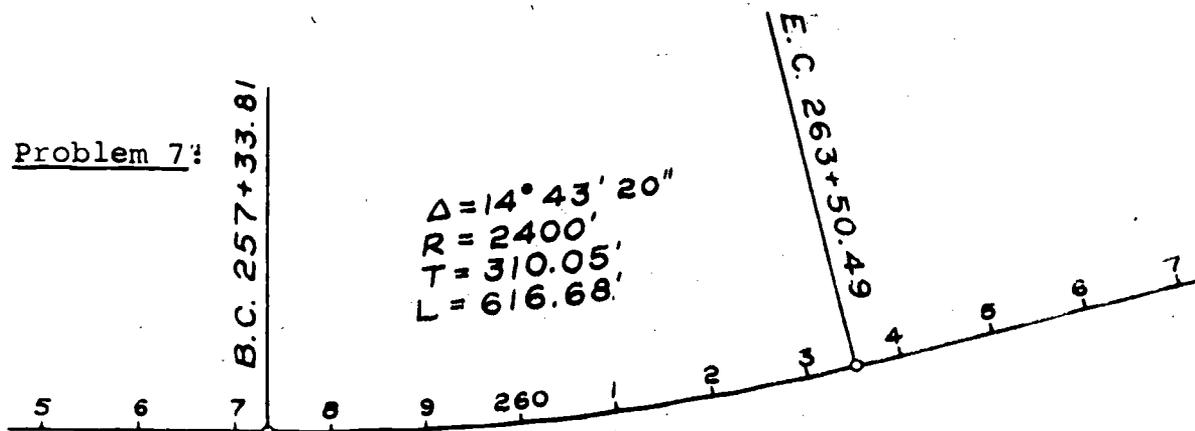


Figure 10

It is desired to increase the middle ordinate on the curve (shown in Figure 10) by moving the  $\mathcal{C}$  to the left without moving the tangents or the P.I.

~~MOVE  $\mathcal{C}$  TO THE RIGHT 2.2 FEET~~

~~If  $\mathcal{C}$  is to be moved 2.2 feet to the left at the center of the curve, what new radius must be used? Find answer to nearest foot.~~

Answer = 2667 feet

Trig Functions

<u>Angle</u>	<u>Sin</u>	<u>Cos</u>	<u>Tan</u>	<u>Cot</u>	<u>Vers</u>
3°40'50"	0.064194	0.997937	0.064326	15.545737	0.002063
7°21'40"	0.128122	0.991758	0.129187	7.740705	0.008242
14°43'20"	0.254133	0.967169	0.262760	3.805759	0.032831
29°26'40"	0.491579	0.870833	0.564493	1.771500	0.129167

Ex. Sec.

0.002063  
 0.008310  
 0.033945  
 0.148326

What will the station of the new B.C. be?  
 (Give your answer to the nearest foot.)

Answer = Station 256+99

Problem 8:

In Figure 10, if the given curve is replaced by a curve with a length of 500.00', find the following:

- a. New R
- b. New T
- c. New stations of B.C. and E.C.
- d. Deflection angles, from B.C.,  
to each station and half station  
on the new curve.

Answers: R = 1945.89  
T = 251.38  
B.C. = Station 257+92.48  
E.C. = Station 262+92.48

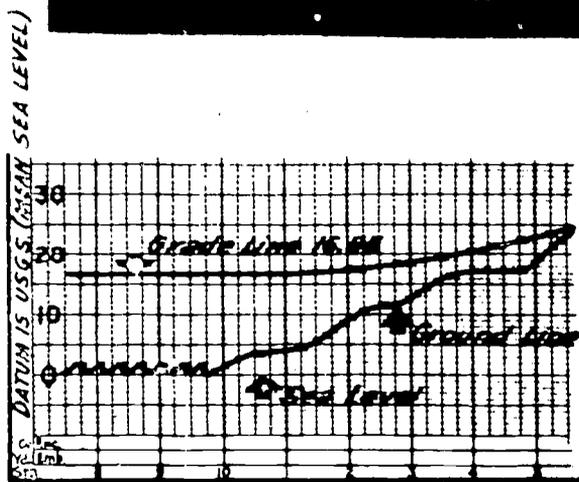
Deflections: Checks at Station 259+00 =  $1^{\circ}34'58.56''$   
at Station 261+00 =  $4^{\circ}31'38.56''$

## Section 6 - Profile Measurements

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Problems	210

# Profile Measurements - Datum



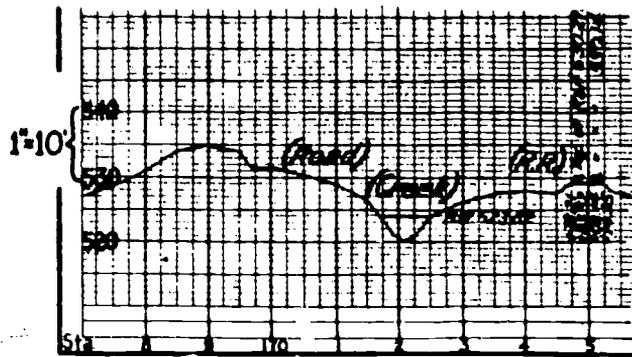
The lower half of a layout plan sheet is called the profile portion. This portion of the sheet represents what the existing ground and proposed improvement would look like if the land was cut along the center line revealing the ground beneath the surface.

The base upon which the data is plotted is a series of vertical lines which correspond to the stations (or scale) of the plan above and a series of horizontal lines which are used to indicate heights or elevations. The station numbers are indicated at the bottom of the sheet.

Figures 0, 10, 20, and 30 represent height in feet above a particular datum. These are elevations. Most elevations are based upon mean sea level, which is the same datum the U. S. Coast and Geodetic Survey uses. The basis of any set of elevations, regardless of whether or not it is mean sea level, is called the "datum."

The ground line, of course, represents the surface of the existing ground along the center line of the highway. It is plotted from level notes or cross section notes made by the surveyors.

The grade line represents the elevations of the proposed roadbed on either the center line, edge of pavement or other convenient reference line. Hills, roads, ditches, creeks, embankments, etc., are especially apparent. Anything that is found by the surveyors as part of the ground details which might be pertinent to construction will be accurately located and noted.

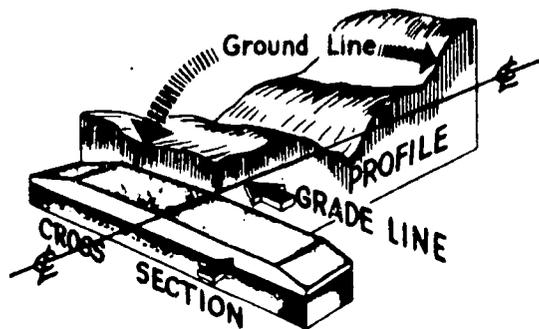


The high water marks of rivers and creeks are necessary to note because of drainage, floods, bridge levels, etc. Railroad rails, existing bridge decks, etc., must be accurately located in elevation in order to smoothly fit the proposed roadbed. Clearances under existing structures or objects such as power lines are necessary for laying grade lines.

As shown above, the vertical scale for elevations (520, 530, 540) is plotted at one inch equals 10 feet. In other words, one vertical inch on the profile drawing equals 10 vertical feet on the ground. One horizontal inch on the drawing equals one station or 100 horizontal feet on the ground. Other scales are also used—such as one inch equals five feet, one inch equals 20 feet, etc.

# Grade Line

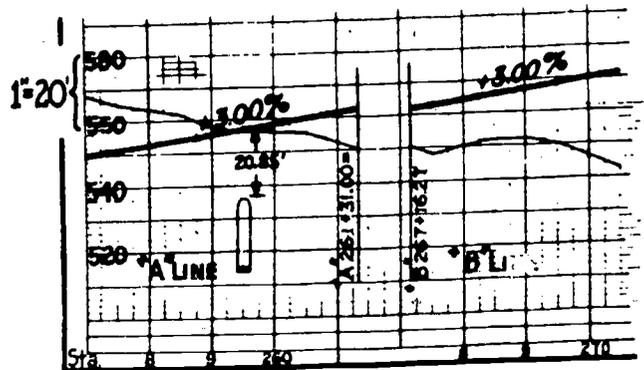
The picture shown below illustrates a block of land cut out of a part of the proposed highway. The irregularly shaped area represents the existing chunk of ground as if it were chopped straight down on the center line. The "profile," which corresponds to the profile as shown on the layout plan, shows the relationship between the ground line and the grade line.



The above sketch also shows the relationship between the profile and the cross section. The heavy line marked  $\mathcal{L}$  is the center line of the highway. It is also the grade line of the profile. This grade line performs the same function on the profile as the center line performs on the plan. In other words, the center line indicates the "horizontal" alignment of the proposed highway, i.e., whether the road is straight or curving to the right or left. The grade line indicates the "vertical" alignment, i.e., whether the road goes uphill, level, or downhill.

Center line stations are duplicated at the bottom of the profile section of the layout sheet. Whenever an equation occurs in the stationing, it must be indicated on the profile. In many instances this may cause a seeming gap in the profile. The reason for this is that the vertical lines

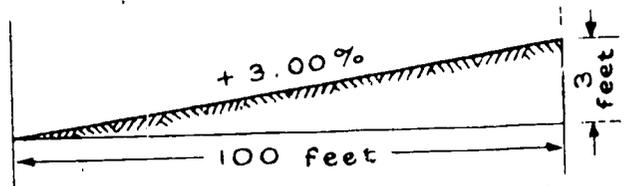
representing 100-foot intervals indicate the progressive stations and when the stations of the equation reduce, then the whole data including the line work must be carried over to the next vertical frame. For example: the equation shown here indicates that the first station A  $261 + 31.00$  equals B  $267 + 16.27$ . If the line work were continuous without a break, it would result in the succeeding stations being placed at an odd location in respect to the vertical lines. Because each vertical frame is measurable to 100 feet, the first station is 31 feet beyond the heavy line, the



equated station is only 16.27 feet beyond the heavy line, so to keep the stations on a heavy line, it is necessary to skip to the next line so the 16.27 feet can be measured from the heavy line. Note that the line letters A and B are noted prominently. These line letters indicate different survey lines or studies.

At station  $259 + 50$  is shown a tunnel and the amount of clearance between the top and the proposed grade or the ground line.

As shown below, a + 3.00 percent grade means that the proposed grade will rise three feet in each 100 feet.



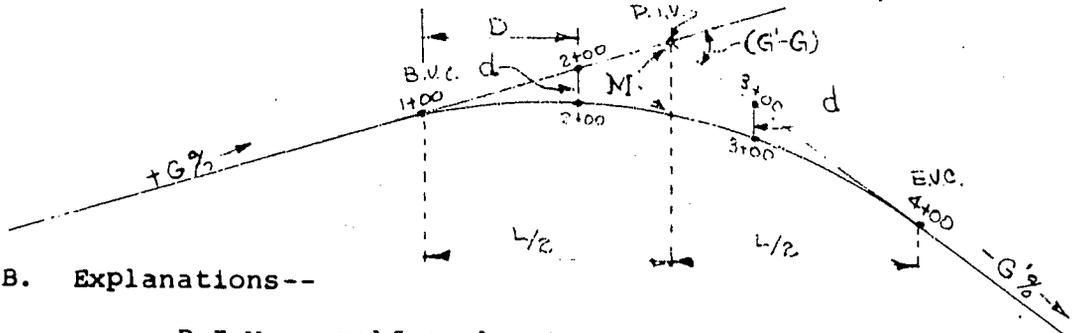
Vertical Curves--

The basic thing to remember is that one is dealing with horizontal and vertical measurements in vertical curves regardless of how steep the grade tangents are. The vertical curve is parabolic and has several reasons for being so. Three (3) of the main reasons are:

1. For a given length of curve, vertical curves allow better (longer) sight distance than a horizontal curve.
2. The transition from the grade tangent is more gradual.
3. For a given sight distance the vertical curve is more economical to construct.

A. Terms--

- P.I.V. or P.I. or V. = Pt. of intersection grade tangent
- B.V.C. = beginning vertical curve
- E.V.C. = end
- M = middle ordinate
- G&G' = grades in percent
- L = (horiz.) length of curve (see explanation)
- d = (vert.) correction from grade tangent to curve
- D = (horiz.) distance from B.V.C. or E.V.C. to any point on curve (see explanation)



B. Explanations--

- P.I.V. - self-explanatory
- B.V.C. - self-explanatory
- E.V.C. - self-explanatory
- M - vertical distance from P.I.V. to mid-point on curve
- (G'-G) = always expressed algebraically, e.i.; if G' = -4% and G = +7% it would be written -4%-(+7%)=-11%
- L = is expressed in 100-foot stations (horiz.), e.i.; a 600-foot vertical curve would be expressed as 6 sta. in the formulas.
- d = (vert.) correction comes out in feet and decimals of a foot. (with the correct sign)
- D - (horiz.) distance expressed in stations as L above. (25', 50', 75', etc., may be expressed in fractions 1/4, 1/2, 3/4, etc., when working longhand or in decimals when using slide rule)

Note: The horizontal lengths of the semi-tangents are equal, and the sum of both the horizontal lengths = L.

C. Helpful relationships--

$$M = \frac{(G' - G)L}{8}$$

$$d = \frac{D^2(G' - G)}{2L}$$

$$M = \frac{1}{2} \left( \text{Elev. P.I.V.} - \frac{\text{Elev. B.V.C} + \text{Elev. E.V.C.}}{2} \right)$$

D. Note: You will like this, as it will prove quite beneficial in working out a vertical curve on an exam not to mention the hundreds of times you will have occasion to use it if you should find yourself faced with computing vertical curves for your own party some day.

Computing Vertical Curve Corrections by Slide Rule

$$\text{Vertical correction } d = \frac{(G' - G) \times D^2}{2L}$$

Method I

- a. Set runner on 2L on "A" scale
- b. Under this set (G' - G) on "B" scale
- c. Move runner to D on "D" scale
- d. read d on "B" scale

NOTE: Check mid-ordinate with  $M = \frac{(G' - G)L}{8}$

Method II

- a. Set runner on computed middle ordinate  $\frac{(G' - G)L}{8}$  on "A" scale
- b. Under this set  $\frac{1}{2}$  V.C. on "C" scale
- c. Read from "C" to "A" for correction

## COOK BOOK VERTICAL CURVE

(for curve with PI at Station +00, +50, +25, or +12.5)

### PROCEDURE

1. Select the desired length of vertical from Design Manuals, etc.
2. After the length is selected, the station of the BVC and EVC is known (center the VC over PI)
3. Divide the curve into the desired number of equal even increments, 100', 50', 25', etc.  
(1000 VC @ 100' = 10; 100' increments, 600' V.C. @ 50' = 12; 50' increments)

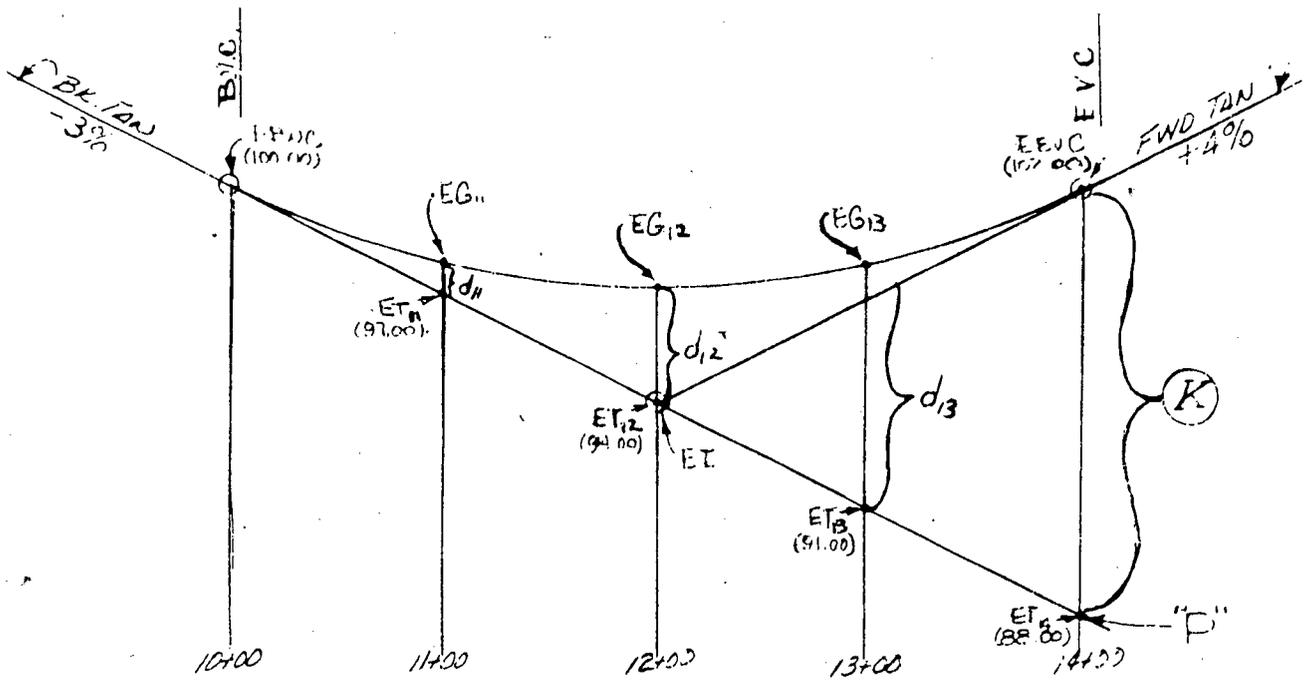
This will give the correct column of curve correction coefficients to use.

4. Extend the back tangent to point "P." ("P" has the same station as EVC). Calculate the elevation of BVC, EVC and Point "P." Calculate the elevation of points on the extended back tangent at the increments selected in Step 3.
5. Find the difference between the elevation of the EVC and the elevation of "P." (the extended back tangent at the station of the EVC.) This difference will become a constant "K" for the curve in question.
6. Multiply this constant "K" by the appropriate curve correction coefficient to obtain correction "d." (1st coefficient  $\sqrt{k}$  = d, 2nd coefficient  $\sqrt{k}$  = d2 etc.)
7. Apply correction "d" to the tangent elevations found in Step 4. This will give the elevations of the desired vertical curve.

EXAMPLE:

400' vertical curve  
 Grade in -3% Grade out +4%  
 Stake at 100' stations (use curve corrections coefficients  
 for increments of 4)

4 increments - 0.0625  
 0.2500  
 0.5625



$$\begin{aligned}
 (K) &= (E \text{ BVC} - "P") = (102.00 - 88.00) = 14.00 \\
 d_{11} &= (K) (C_1) = (14.00) (0.0625) = 0.88 \\
 d_{12} &= (K) (C_2) = (14.00) (0.2500) = 3.50 \\
 d_{13} &= (K) (C_3) = (14.00) (0.5625) = 7.88 \\
 \\ 
 EG_{11} &= (ET_{11} + d_{11}) = 97.00 + 0.88 = 97.88 \\
 EG_{12} &= (ET_{12} + d_{12}) = 94.00 + 3.50 = 97.50 \\
 EG_{13} &= (ET_{13} + d_{13}) = 91.00 + 7.88 = 98.88
 \end{aligned}$$

# VERTICAL CURVE CORRECTION COEFFICIENTS

4
.0625*
.2500*
.5625

6
.0278
.1111
.2500*
.4444
.6944

8
.0156
.0625
.1406
.2500*
.3906
.5625
.7656

10
.0100
.0400
.0900
.1600
.2500*
.3600
.4900
.6400
.8100

12
.0069
.0278
.0625
.1111
.1736
.2500*
.3403
.4444
.5625
.6944
.8403

14
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16
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.0625
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.1406
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.3906
.4727
.5625
.6602
.7656
.8789

18
.0031
.0123
.0278
.0494
.0772
.1111
.1512
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.2500*
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.3735
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.5216
.6049
.6944
.7901
.8920

20
.0025
.0100
.0225
.0400
.0625
.0900
.1225
.1600
.2025
.2500*
.3025
.3600
.4225
.4900
.5625
.6400
.7225
.8100
.9025

22
.0021
.0083
.0186
.0331
.0517
.0744
.1012
.1322
.1674
.2066
.2500*
.2975
.3492
.4050
.4649
.5289
.5971
.6694
.7459
.8264
.9112

- NOTE -  
 LAST CORRECTION COEFFICIENT EQUALS 1.0000  
 $1.000(K) = K$   
 \* ALSO THE M.O. COEFFICIENT EQUALS 0.2500

24
.0017
.0069
.0156
.0278
.0434
.0625
.0851
.1111
.1406
.1736
.2101
.2500*
.2934
.3403
.3906
.4444
.5017
.5625
.6267
.6944
.7656
.8403
.9184

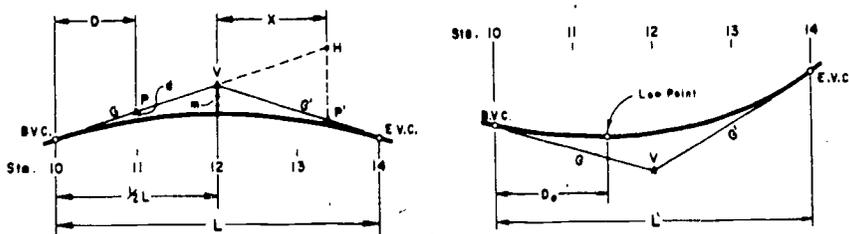
26
.0015
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.0725
.0947
.1198
.1479
.1790
.2130
.2500*
.2899
.3328
.3787
.4275
.4793
.5340
.5917
.6524
.7160
.7825
.8521
.9246

28
.0013
.0051
.0115
.0204
.0319
.0459
.0625
.0816
.1033
.1276
.1543
.1837
.2156
.2500*
.2870
.3265
.3686
.4133
.4605
.5102
.5625
.6173
.6747
.7347
.7972
.8622
.9298

30
.0011
.0044
.0100
.0178
.0278
.0400
.0544
.0711
.0900
.1111
.1344
.1600
.1878
.2178
.2500*
.2844
.3211
.3600
.4011
.4444
.4900
.5378
.5878
.6400
.6944
.7511
.8100
.8711
.9344

FIGURE 7-204.4

## VERTICAL CURVES



## IN ANY VERTICAL CURVE

## WHERE

$$1 \quad m = \frac{(G' - G)L}{8}$$

$$2 \quad m = \frac{1}{2} \left( \frac{\text{Elev. B.V.C.} + \text{Elev. E.V.C.}}{2} - \text{Elev. V} \right)$$

$$3 \quad d = m \left( \frac{D}{L/2} \right)^2 = \frac{4m}{L^2} D^2$$

$$4 \quad d = \frac{D^2(G' - G)}{2L}$$

$$5 \quad X = \frac{100(H - P')}{(G - G')}$$

$$6 \quad S = G - D \left( \frac{G - G'}{L} \right)$$

$$7 \quad D_0 = \frac{L G}{G - G'}$$

L = Length of curve — 100 ft. units or stations.

G and G' = Grade rates — percent.

m = Middle ordinate — ft.

d = Correction from grade line to curve — ft.

D = Distance from B.V.C. or E.V.C. to any point on curve — stations.

S = Slope of the tangent to the curve at any point — percent.

X = Distance, from P' to V — ft.

H = Elevation of grade G produced to station of P'

P and P' = Elevation on respective grades.

D<sub>0</sub> = Distance to low or high point from extremity of curve — stations.

## NOTES

A rising grade carries a plus sign while a falling grade carries a minus sign.

Thus in a crest vertical curve as above, G carries a plus sign and G' a minus sign when progressing in the direction of the stationing. When progressing in the opposite direction, G becomes a minus grade and G' a plus grade.

## 7-204.4 Vertical Curves

Properly designed vertical curves should provide adequate sight distance, safety, comfortable driving, good drainage and pleasing appearance.

A parabolic vertical curve is used. Figure 7-204.4 gives all necessary mathematical relations for computing a vertical curve, either at crests or sags. In lieu of the method shown, the rate of change of grade per station may be used in computations.

Long, flat vertical curves may develop poor drainage at the level section. This difficulty may be overcome by a slight gutter grade adjustment or by short-

ening the vertical curve.

On 2-lane roads, extremely long crest vertical curves (over one-half mile) should be avoided, since many drivers refuse to pass on such verticals despite adequate sight distance. It is sometimes more economical to use 4-lane construction than to obtain passing sight distance by the use of a long vertical curve. (See Index No. 7-207.2.)

Broken-back vertical curves consist of two vertical curves in the same direction separated by a short grade tangent. A profile with such curvature normally should be avoided.

Examination of Equation 2 on page 6-4 shows that the grade line at the mid point of a vertical curve lies exactly halfway between the PI and the mid point of the long chord connecting the BVC and the EVC. Knowledge of this fact sometimes simplifies solution of vertical curve problems.

For example, in problem 4:

$$\text{Elev. PI} = 2010.65 + .03 (150) = 2015.15$$

$$\text{Elev. EC} = 2015.15 + .06 (150) = 2024.15$$

$$\text{Elev. mid point of chord} = \frac{2010.65 + 2024.15}{2} = 2017.40$$

$$\text{Elev. grade line at mid point} = \frac{2015.15 + 2017.40}{2} = 2016.28$$

\*\*\*\*\*

Equation 3 on page 6-4 shows that the vertical curve correction at any point varies as the square of the distance from the BC or EC to the point. Thus at a point one-half way from the BC to the mid point, the correction is  $(1/2)^2$  or  $1/4$  of the middle ordinate. Or at a point  $1/4$  the distance of the mid point, the correction is  $(1/4)^2$  or  $1/16$  the middle ordinate.

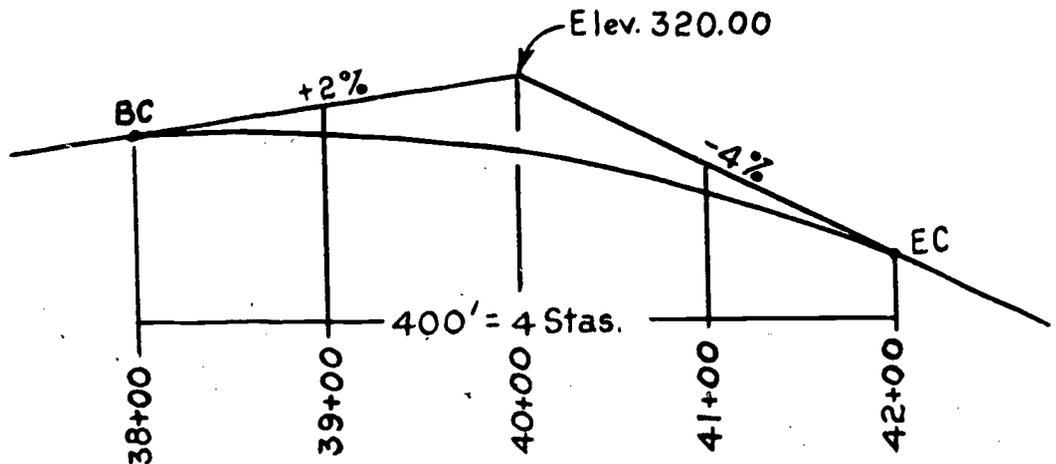
## EXAMPLE PROBLEMS

### Example 1

Given: 400' vertical curve  
G = +2%  
G' = -4%  
V (P.I.) at Station 40+00  
Elevation at V is 320.00

Find: Elevation of grade line at Station 39+00

Solution: It is usually helpful to draw a rough sketch.



First, compute the tangent elevation at Station 39+00.

$$\text{Elev.} = 320 - .02(100') = 318.00$$

Second, calculate the middle ordinate.

$$m = \frac{(G' - G)L}{8} = \frac{[-4 - (+2)]4}{8} = -3'$$

Third, compute the correction from tangent elevation to grade line elevation.

$$\begin{aligned} \text{B.C. is at Station } 38+00 \\ d = \frac{m(D)}{(4/2)^2} = \frac{-3(\frac{1}{2})}{(\frac{4}{2})^2} = -3(\frac{1}{4}) = 0.75' \end{aligned}$$

Therefore: Elevation of grade line at Station 39+00 =  
 $318.00 - 0.75 = 317.25$

### Example 2

Given: Same as Example 1

Find: Elevation of grade line at Station 40+50

First, compute the tangent elevation at Station 40+50.

$$\text{Elev.} = 320 - .04(50') = 320 - 2 = 318$$

Second, calculate middle ordinate.

$$m = \frac{(G' - G)L}{8} = \frac{[-4 - (+2)]}{8} \cdot 4 = -3'$$

Third, compute the correction from tangent elevation to grade line elevation.

E.C. is at Station 42+00

$$d = m \frac{(D/2)^2}{(4/2)^2} = -3 \frac{(1.5)^2}{(2)^2} = -3(.75)^2 = d =$$

$$-3(.5625) = -1.6875, \text{ say } (-1.69)$$

Therefore: Elevation of grade line at Station 40+50 =  
 $318 - 1.69 = 316.31$

### Example 3

Given: Same as Example 1

Find: Slope in % at Station 41+00

Formula is  $S = G - D \frac{(G - G')}{L}$ . D is distance measured from B.C.

$$G = +2\%$$

$$G' = -4\%$$

$$L = 4$$

$$D = 3$$

$$S = +2 - 3 \frac{[+2 - (-4)]}{4}$$

$$S = +2 - 3 \frac{(+6)}{4} = +2 - \frac{18}{4} = +2 - 4\frac{1}{2}$$

$$S = -2\frac{1}{2}\%$$

Example 4

Given: 300' vertical curve

$G = +3\%$

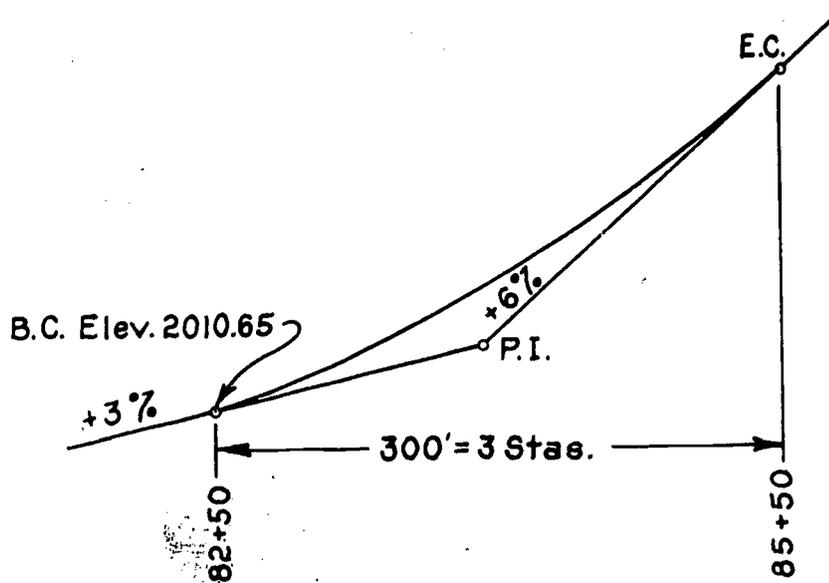
$G' = +6\%$

Elevation at B.C. = 2010.65

B.C. is at Station 82+50

Find: Elevation and Station of mid point of vertical curve.

Solution: Draw a rough sketch.



First, if the B.C. is at Station 82+50 and the vertical curve is 3 Stations long, then the mid point of the curve is at  $82+50 + 150' = \text{Station } 84+00$ .

Second, compute tangent elevation at mid point (V or P.I.)

$$2010.65 + .03(150') =$$

$$2010.65 + 4.5' = 2015.15'$$

Third, calculate M.

$$m = \frac{(G' - G)L}{8} = \frac{[6 - (+3)] 3}{8} = \frac{(+3)3}{8}$$

$$m = \frac{9}{8} = 1.125' \text{ (1.13')}$$

Therefore: Elevation of mid point of vertical curve is  $2015.15' + 1.13' = 2016.28'$ .

Example 4 (alternate solution)

$$\begin{aligned} \text{Elev. of E.C.} &= 2010.65 + 150' (3\%) + 150 (6\%) \\ &= 2010.65 + 4.50 + 9.00 = 2024.15 \end{aligned}$$

$$\begin{aligned} \text{Elev. P.I.} &= 2010.65 + 150 (3\%) = \\ &2010.65 + 4.50 = 2015.15 \end{aligned}$$

$$\begin{aligned} m &= \frac{1}{2} \left( \frac{\text{Elev. B.C.} + \text{Elev. E.C.} - \text{Elev. P.I.}}{2} \right) = \\ &\frac{1}{2} \left[ \frac{(2010.65 + 2024.15) - 2015.15}{2} \right] \\ &= \frac{1}{2} (2017.40 - 2015.15) = +1.13 \end{aligned}$$

$$\begin{aligned} \text{Elev. mid point} &= \text{P.I.} + 1.13 = 2015.15 + 1.13 = \\ &2016.28 \end{aligned}$$

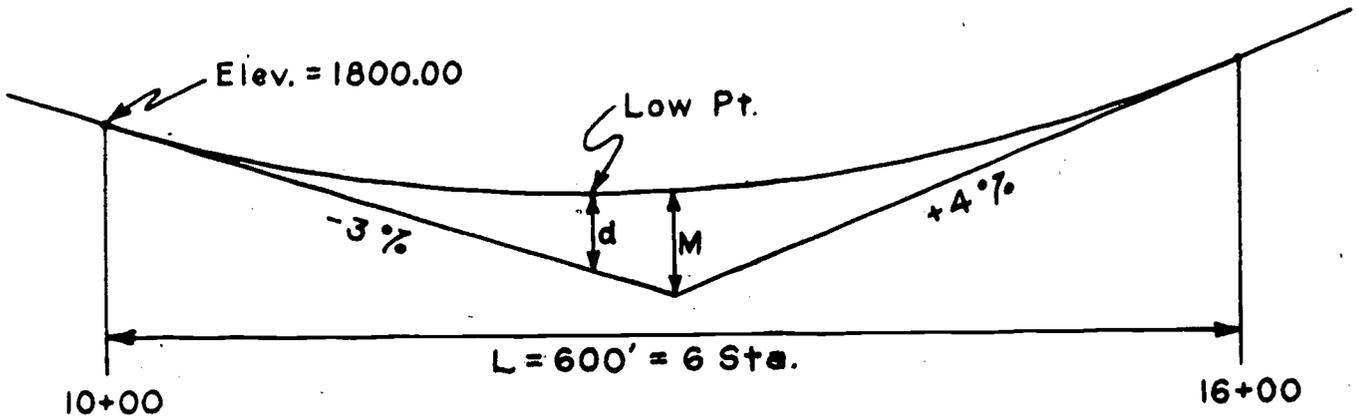
Example 5

Because of drainage considerations, it is many times necessary to determine the low point in "sag" vertical curves.

The following example shows how the low point is found:

Given: 600' vertical curve  
 $G = -3\%$   
 $G' = +4\%$   
 Station of B.C. = 10+00  
 Elevation of B.C. = 1800.00

Solution: First, draw a sketch.



Step 1: Station of low point

The total change of grade =  $G' - G = 4 - (-3) = 7\%$   
 or the change of grade is 7% in 600'.

The change of grade per foot =  $\frac{7\%}{600'} = 0.011667\%/ft.$

At the exact point when a total change of grade equal to 3%; (i.e., where the tangent to the curve has 0% slope,) has been made, the low point will be reached.

Example 5 (by formula)

$$D_0 = \frac{LG}{G - G'} = \frac{6 \text{ Stations} \cdot 3\%}{7\%} = \frac{18 \text{ Stations}}{7}$$

2.5714 Stations

Now let  $x$  = distance in feet from the beginning of the vertical curve to the low point. Then using proportion:

$$x : 3\% :: 1 \text{ ft.} : 0.011667\%$$

$$.011667 x = 3(1)$$

$$x = \frac{3(1)}{.011667} = 257.14 \text{ feet, or } = 2.5714 \text{ Stations}$$

Station of low point:

$$+x = \frac{10+00 \text{ B.C.}}{2+57.14}$$

Station 12+57.14 or location of low point

Step 2: Elevation of low point

$$m = \frac{G' - G}{8} L = \frac{4 - (-3)}{8} 6 = 5.25 \text{ feet}$$

D = 2.5714 Stations from Step 1 (page 6-10)

$$\frac{L}{2} = \frac{6}{2} = 3 \text{ Stations}$$

$$\text{then } d = \frac{D^2(G' - G)}{2L} = \frac{(2.5714)^2 [4 - (-3)]}{2(6)}$$

$$d = 3.86 \text{ feet}$$

$$\begin{aligned} &\text{Tangent grade at Station } 12+57.14 = \\ 1800.00 - 2.5714(3) &= 1792.29 \end{aligned}$$

$$\begin{aligned} &\text{Grade on vertical curve at Station } 12+57.14 = \\ 1792.29 + 3.86 &= 1796.15. \end{aligned}$$

### VERTICAL CURVE PROBLEMS

#### Problem 1

Given: L = 12 Stations  
G = +6  
G' = -3  
Station B.C. = 5+00  
Elev. B.C. = 1500.30

Find: Elevation of vertical curve at Station 10+00  
and at Station 13+00.

Answer: Station 10+00 = 1520.92  
Station 13+00 = 1524.30

#### Problem 2

Given: Same as Problem 1

Problem: It is desired to lengthen the vertical curve to 1600 feet, the P.I. remaining the same. How many feet will the grade line at Station 11+00 be moved vertically?

Problem 3

Given: Same as Problem 1

Find: The station and elevation of the highest point on the vertical curve.

Answer: Station = 13+00  
Elev. = 1524.30'

Problem 4

Given: L = 8 Stations  
G = -4  
G' = +5  
Station B.C. = 4+00  
Elev. B.C. = 500.00'

Find: Elevations of grade line at stations and 1/2 stations throughout the vertical curve. Report your answers on a form similar to the one below.

Station	L	Tangent Grade	Corr.	Finished Grade
3+00		500.00'		
+50				
4+00				
+50				
5+00				
+50				
6+00				
+50				
7+00				
+50				
8+00				
+50				
9+00				
+50				
10+00				
+50				
11+00				
+50				
12+00				
+50				
13+00				
+50				
14+00				

## SUPERELEVATION

According to the laws of mechanics, a vehicle traveling on a curve exerts an outward force called centrifugal force.

On a superelevated highway, this force is resisted by the vehicle weight component parallel to the superelevated surface and side friction between the tires and pavement. It is impossible to balance centrifugal force by superelevation alone, because for any given curve radius a certain superelevation rate is exactly correct for only one driving speed. At all other speeds there will be a side thrust either outward or inward, relative to the curve center, which must be offset by side friction.

If the vehicle is not skidding, these forces are in equilibrium as represented by the following equation:

Where S = Superelevation slope in feet per foot  
F = Side friction factor  
R = Radius of curve in feet  
V = Velocity in miles per hour

$$\text{Centrifugal Factor} = S + F = \frac{0.067V^2}{R} = \frac{V^2}{15R}$$

This formula is used to design a curve for safe operation at a particular speed.

Standard superelevation rates are designed to hold the portion of the centrifugal force that must be taken up by tire friction within allowable limits. The limiting side friction factors related to speed have been found to range from 0.125 foot per foot for curves of 250' radius or less, to 0.015 for curves of radius of 7500' or more.

In some locations, usually above 3,000 feet elevation, where snow and ice conditions prevail, it may be necessary to reduce the maximum superelevation to 0.08 foot per foot.

## Axis of Rotation

For undivided highways the axis of rotation for superelevation is usually the centerline of the roadbed. However, in special cases such as desert roads where curves are preceded by long relatively level tangents, the plane of superelevation may be rotated about the inside edge of the pavement to improve perception of the curve. In flat country, drainage pockets caused by superelevation may be avoided by changing the axis of rotation from the centerline to the inside edge of the pavement.

On divided multilane highways, the two sides are rotated individually about the edge of the pavement nearest the median taking into consideration the ultimate pavement width. If rotation occurs about the center of each pavement, a hazardous and unsightly saw tooth may result. This does not apply to a median that is both narrow and traversable by traffic, in which case the axis of rotation usually lies in the center of the median.

## Superelevation Transition

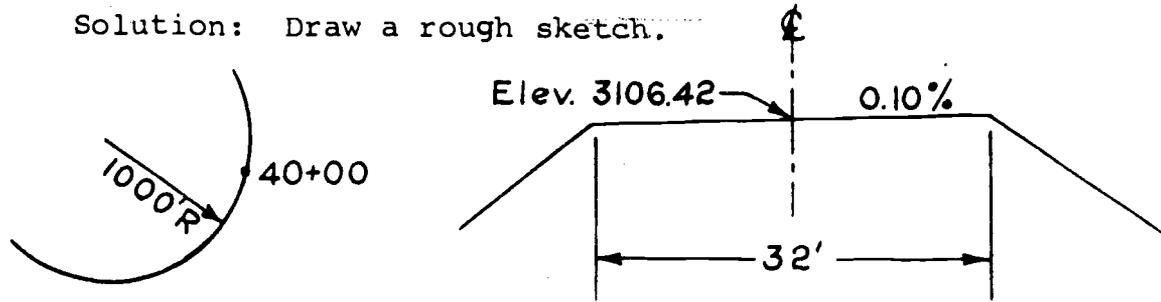
A superelevation transition is variable in length depending upon the amount of superelevation. With respect to the beginning or end of curve, two-thirds of the transition is on the tangent approach and one-third within the curve. This results in two-thirds of the full superelevation at the beginning of the curve.

### Example 1

- Given:
1. Centerline elevation at Station 40+00 is 3106.42
  2. Station 40+00 is located on a 1000' radius curve to the left and the superelevation is 0.10 foot/foot.
  3. The typical cross section is 32' in width - outside of shoulder to outside of shoulder

Find the elevation at the outside edge of the left shoulder.

Solution: Draw a rough sketch.



$$16' \times 0.10 \text{ ft/ft} = 1.60'$$

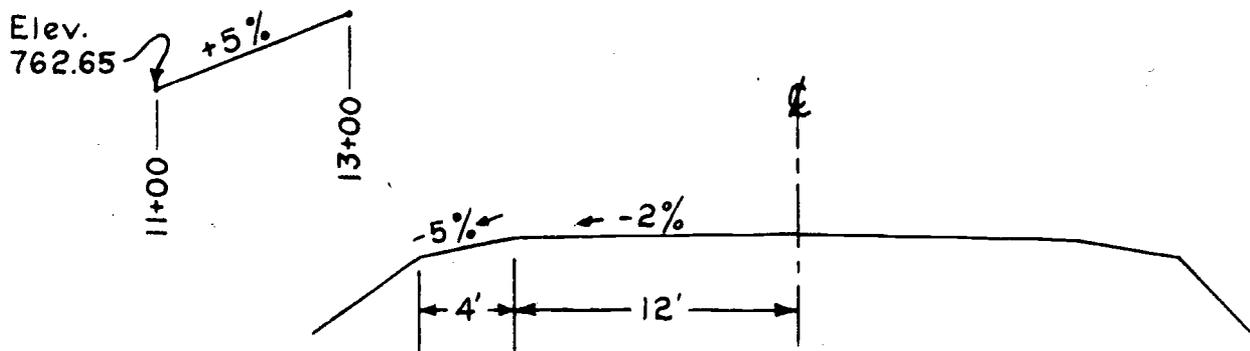
Therefore: Elevation at outside edge of left shoulder is  $3106.42 - 1.60' = 3104.82'$

### Example 2

- Given:
1. Centerline elevation at Station 11+00 is 762.65
  2. Grade is +5%
  3. Typical cross section has crown of -2% for 12' to the shoulder. The 4' shoulder has crown of -5%.

Find: Elevation of outside of shoulder at Station 13+00.

Solution: Draw a rough sketch.



First, find centerline elevation at Station 13+00.

$$762.65 + 0.05(200') = 762.65 + 10.00 = 772.65$$

Second, calculate difference in elevation between outside edge of shoulder and centerline.

$$.02 \times 12' = 0.24$$

$$.05 \times 4' = \frac{0.20}{0.44}$$

Therefore: Elevation at outside edge of shoulder at Station 13+00 is  $772.65 - 0.44 = 772.21'$

### SUPERELEVATION PROBLEMS

#### Problem 1

- Given:
1. Centerline elevation at Station 60+00 = 2402.61
  2. Station 60+00 is located near the center of a long 1200' radius curve to the right and the superelevation rate is 0.07 ft/ft.
  3. Centerline is on a +3% grade.
  4. The width of road, shoulder to shoulder, is 40 feet.

Find: Elevation of finished surfacing at 5 feet left and 10 feet right of Station 58+00.

Answer: 5 ft. left = 2396.96  
10 ft. right = 2395.91

#### Problem 2

Given: Same as Problem 1.

Find: Shoulder elevations (right and left) at Station 61+50.

Answer: Left = 2408.51  
Right = 2405.71

#### Problem 3

Given: Same as Problem 1.

What will the theoretical superelevation rate be at B.C. and E.C. of the curve?

Answer: 0.0467 ft/ft

60

Section 7 - Sections

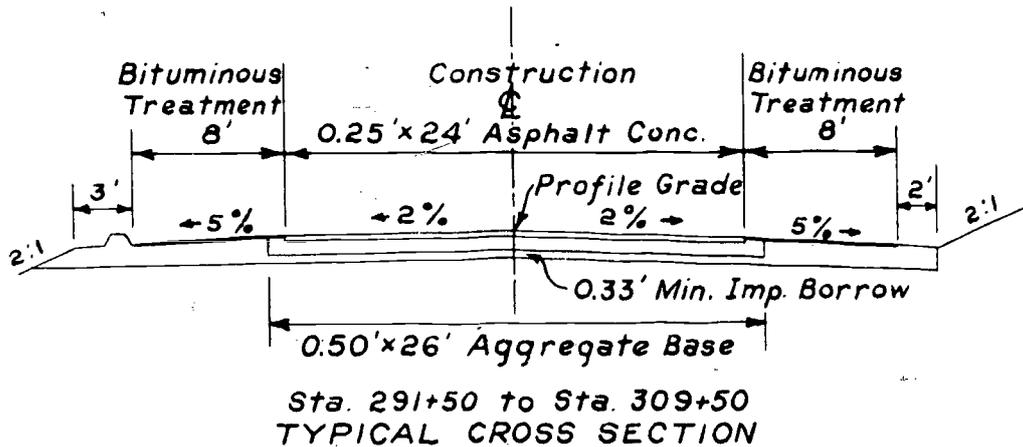
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## SECTION 7

### Sections

Every set of layout plans contains a title sheet, the layout plans and a typical section sheet. This latter sheet, or sheets, indicates how the surfacing and base material is to be placed on the roadbed itself.

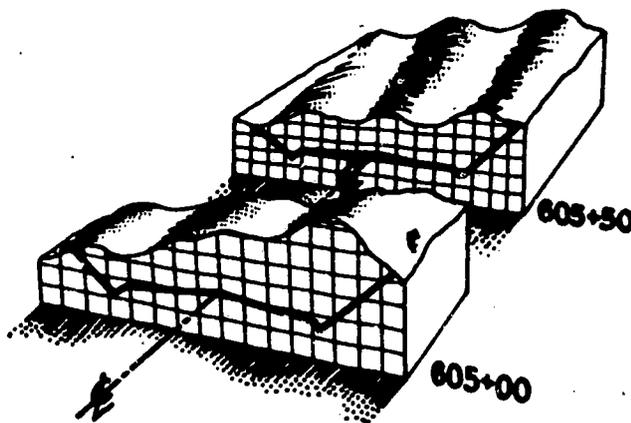


Between the beginning and ending of every project, a roadbed may be composed of many variable widths, materials, thicknesses, slopes, etc. Every section must be shown so that the contractor will have sufficient plans and details from which to work.

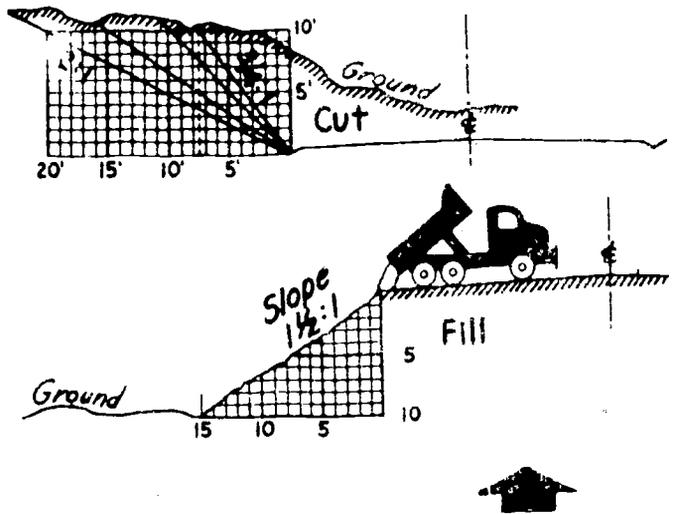
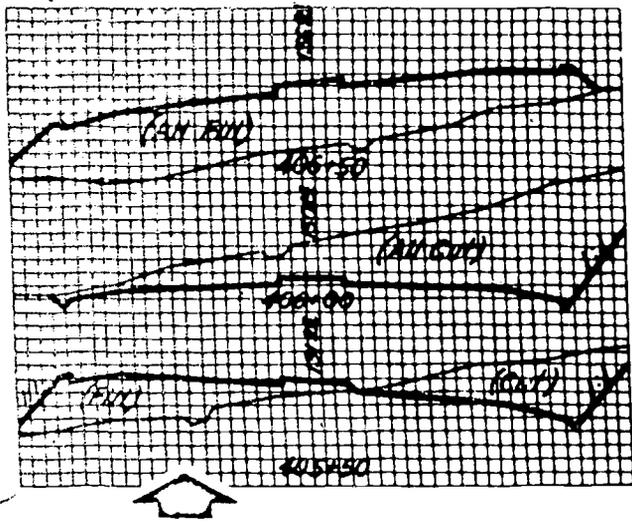
The above section illustrates a typical type of construction on two-lane roadways. The cross section centerline conforms to the alignment centerline. The point where this line meets the grade line is called the "profile grade". On this kind of road, it is the crown of the pavement. Note that the surfacing slopes away from the centerline on both sides. The pavement is 24 feet wide or two 12-foot lanes to be paved with asphalt concrete. The 8-foot shoulders are bituminous treated to provide an all-weather surface. On the right is shown the foot of a cut with a two-to-one slope. On the left is a little hump called a "dike". This is similar to a curb but is formed by a metal shape and pressed into position; it serves as a drainage control and as an edge protection.

A cross section is a slice of the ground or roadway across the line, looking ahead on the line. Cross section surveying establishes the contours of the ground on and adjacent to the roadway. The usual procedure is to take cross section "levels" at right angles to the centerline every 50 feet, extending as far as necessary on both sides. Roads, creeks, and other important ground conditions are cross-sectioned wherever necessary regardless of the station interval.

The sketch below indicates chunks of ground sliced at 50-foot intervals. These cross sections are lined with a grid so that the roadway pattern may be drawn according to the "typical section" dimensions. These sections are required in order to determine the cubic yards of excavation or embankment within the limits of the proposed roadway.



# Cross Sections

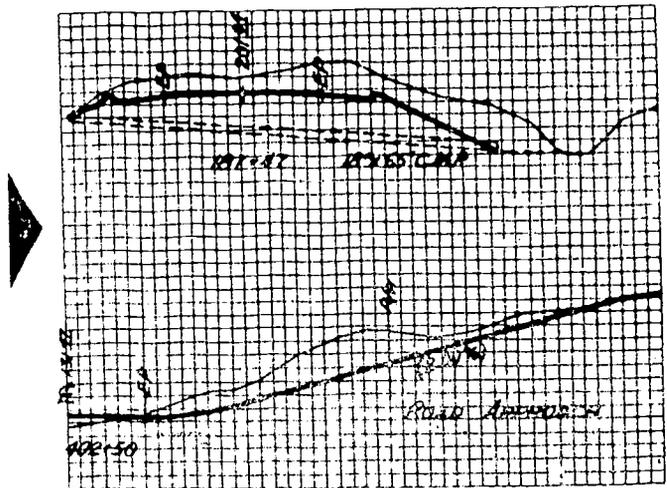


Cross section sheets look like this. Several sections may be shown on each sheet if the ground line is fairly flat. These three sections indicate the three types of sections commonly encountered. At Station 406+50 the entire roadway will be laid above the existing ground; Station 406+00, the roadway will be cut below the existing surface; Station 405+50, the roadway is partially in cut and partially in fill. The lined grid forms a basis upon which to draw the template. One-inch on the grid (both horizontal and vertical) usually represents either five feet or 10 feet on the ground.

## CUT OR FILL SLOPES

Typical sections and cross section templates show a slanting line on each side. This line indicates the proposed surface line of either the cut or fill and it extends to where it meets the existing ground. A 1:1 slope means the slope connecting a point one foot above and one foot outward (45°). The steepness or flatness of a slope is dependent upon the stability of the ground or fill material.

Occasionally, a condition arises which requires a wider cross section than the sheet will hold. The section shown at right indicates two conditions. The top (Station 187+47) shows a proposed culvert location and its relationship to a ditch on the right. The roadbed center line is placed off-center in the sheet in order to contain the whole section. The lower section is so wide that only a half section can be shown. Here the center line is on the left edge. If it were necessary to indicate the left half at this particular location, it could be placed directly above in the normal position.



## Use of Cross-Sections

One of the biggest operations in highway construction is the movement of earth material. Dirt and rocks are excavated from the "cut" areas, hauled and placed on the "fill" areas.

Because this is a major operation, it accounts for a big percentage of the money spent on a highway construction project. The contractor is paid on a volume basis; that is, so much money for each cubic yard excavated and so much for each cubic yard hauled. It is important, therefore, to accurately measure the volume of the material to be dug and hauled. It is also important to know the volume of the cuts and the fills during the design of the highway. When the material from the cuts is just enough to make the desired fills, the grading is said to be "in balance". It is obvious that this is the most economical situation, since there is no wasting or borrowing of material.

Quantities of earthwork are calculated by the use of cross-sections and the "average end area method". The volume of earthwork between two adjacent cross-sections is equal to the average area of the cross-sections (average end area); multiplied by the horizontal distance between the two cross-sections. This may be visualized by referring to the sketch on page 7-2. The volume of excavation between Station 605+00 and Station 605+50 may be calculated by the formula:  $V = \frac{(A_1 + A_2)}{2} \times L$  in which "V" is the volume in cubic feet, "A<sub>1</sub>" is the cross-sectional area at Station 605+00, "A<sub>2</sub>" is the cross-sectional area at Station 605+50, and "L" is the horizontal distance (along centerline) between Station 605+00 and Station 605+50. The horizontal distance is easily measured. In this case "L" is 50 feet.

The areas "A<sub>1</sub>" and "A<sub>2</sub>", being irregular, are not so easily calculated. However, a device known as a planimeter may be used to measure such irregular areas simply by tracing the boundary of the area. The use of planimetered areas for quantities is now usually restricted to preliminary estimates only. With the expanded use of electronic computers, final pay quantities are now usually arrived at by calculations performed by the C.H.P.S. tabulation section from cross-sections taken in the field.

## The Planimeter

Figure 1 shows a sketch of a polar planimeter with an adjustable tracing arm.

The planimeter is supported at three points: the pole (P), the roller (R), and the tracing point (T). If the length of the tracing arm of the planimeter is fixed, as on many planimeters, the usual relation between revolutions of the roller and square inches is 1:10. A corresponding setting at the index (J) on an adjustable arm planimeter would be ten square inches. For simplicity in this discussion, the above relationship of 1:10 is assumed.

Figure 2 shows a large scale sketch of the graduated drum (D) and the vernier (E). Since each revolution of the roller and drum indicates ten square inches and the drum is graduated into 100 equal parts, then each graduation on the drum is equal to 0.01 of a drum revolution or 0.1 of a square inch. The graduated disk (F) is geared to rotate once for every ten revolutions of the drum and is graduated into ten equal divisions, each division indicating one whole drum revolution or ten square inches. A direct reading vernier (E) is provided on the drum for reading to 0.001 of a revolution or 0.01 of a square inch.

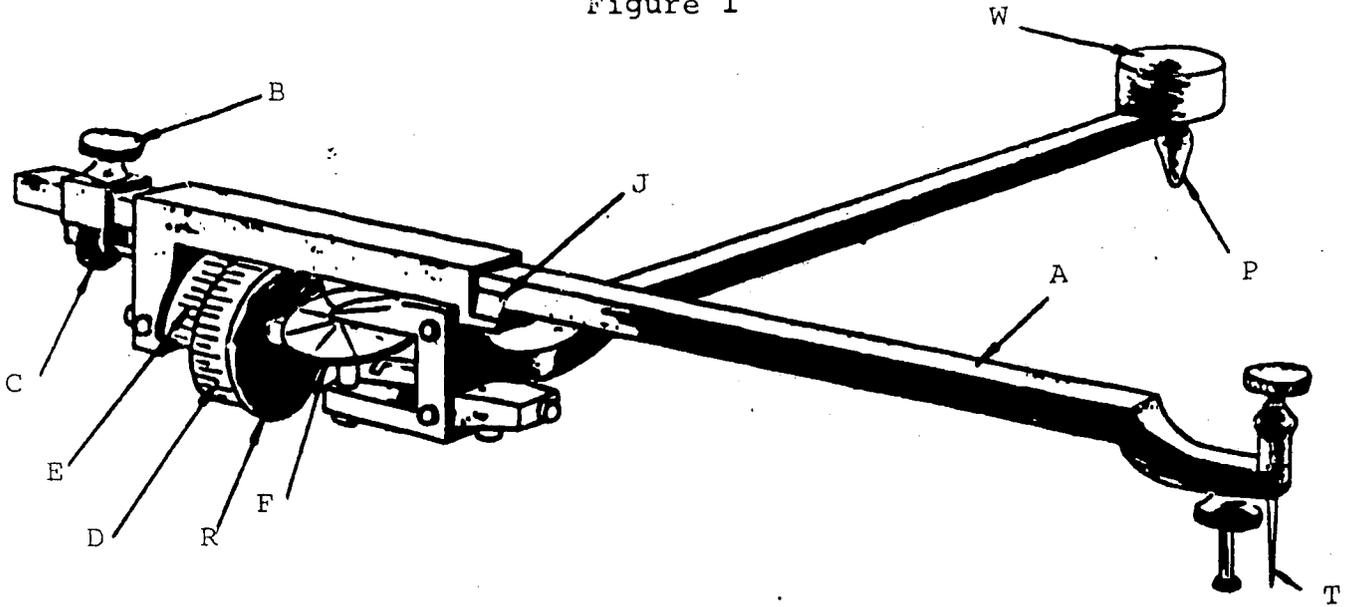
Each space on the vernier in Figure 2 is 0.9 as long as each drum division or ten vernier divisions equal nine drum divisions. The index of the vernier is set at 4.00 square inches. If the vernier were moved upward to indicate an additional 0.01 square inch, its graduation number one would coincide with 4.1 on the drum and the index would be at 4.01 square inches. To record a complete planimeter reading the setting of the disk, drum, and vernier must be considered. Figure 3 indicates a disk reading of one drum revolution or ten square inches, a drum reading of 0.64 of a drum revolution or 6.4 square inches, and a vernier reading of 0.004 of a drum revolution or 0.04 square inches. The total reading is 1.644 drum revolutions or 16.44 square inches.

In using the planimeter slightly different procedure is followed, depending on whether the anchor point is placed inside or outside the area in question.

Only the case where the anchor point is placed outside the area in question will be described here since this is the procedure followed in connection with cross-sections.

When a plotted area is to be determined, the needle of the anchor point (P) is pressed into the paper at a desired location, outside the area in question, being held down by the weight (W). The tracing point (T) is set at a definite point on the perimeter of the figure and an initial reading taken. The perimeter of the figure is then completely traversed in a clockwise direction, until the tracing point is brought to its original position, and a final reading is taken. The difference in the two readings is the area of the figure in square inches. If the traverse has been in a clockwise direction, the resulting difference in readings will be a positive figure.

Figure 1



- A) Tracing Arm
- B) Screw Clamp
- C) Tangent Screw
- D) Graduated Drum
- E) Drum Vernier
- F) Graduated Disk
- J) Tracing Arm Index
- P) Pole
- R) Roller
- T) Tracing Point
- W) Weight

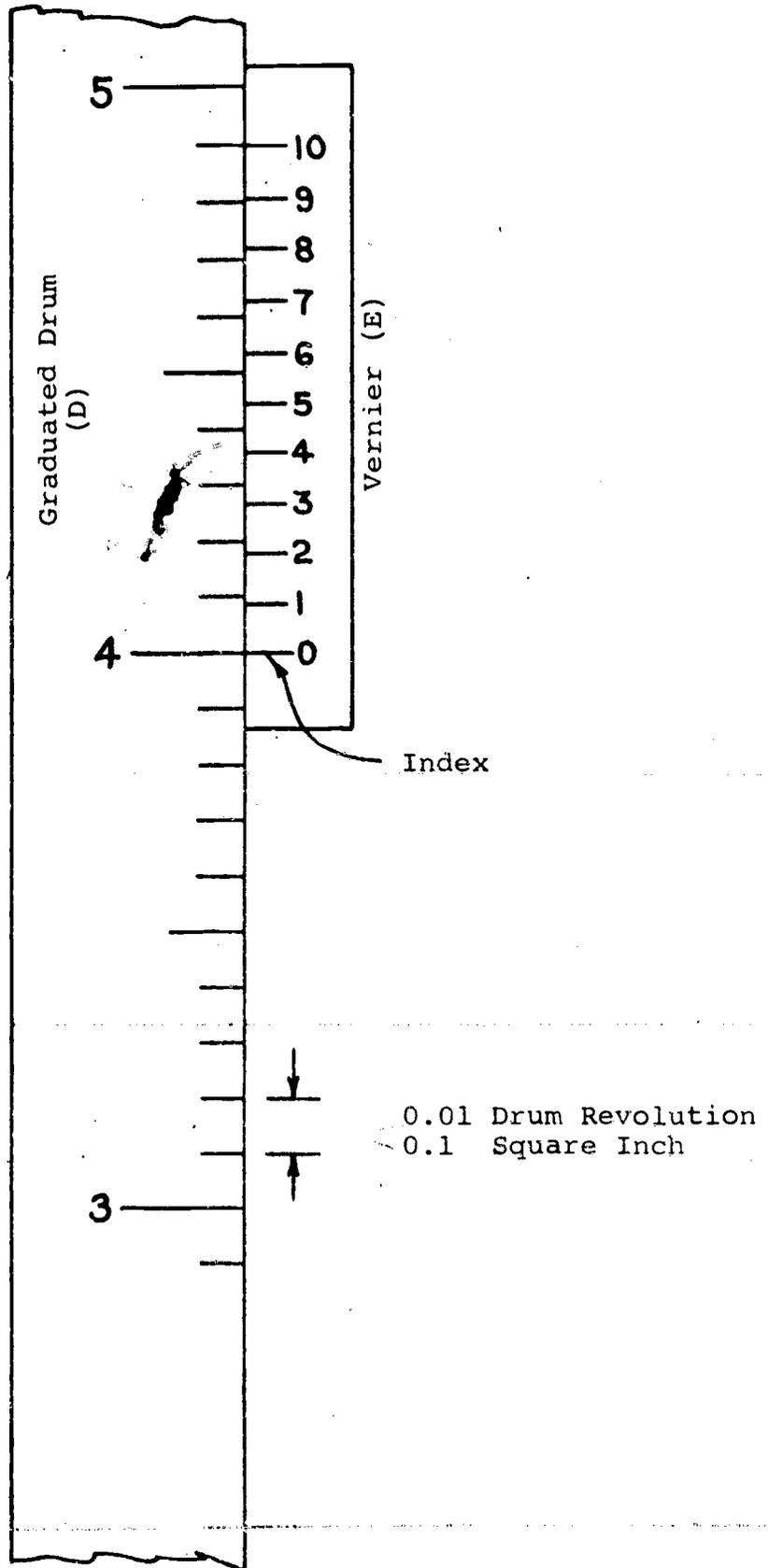


Figure 2

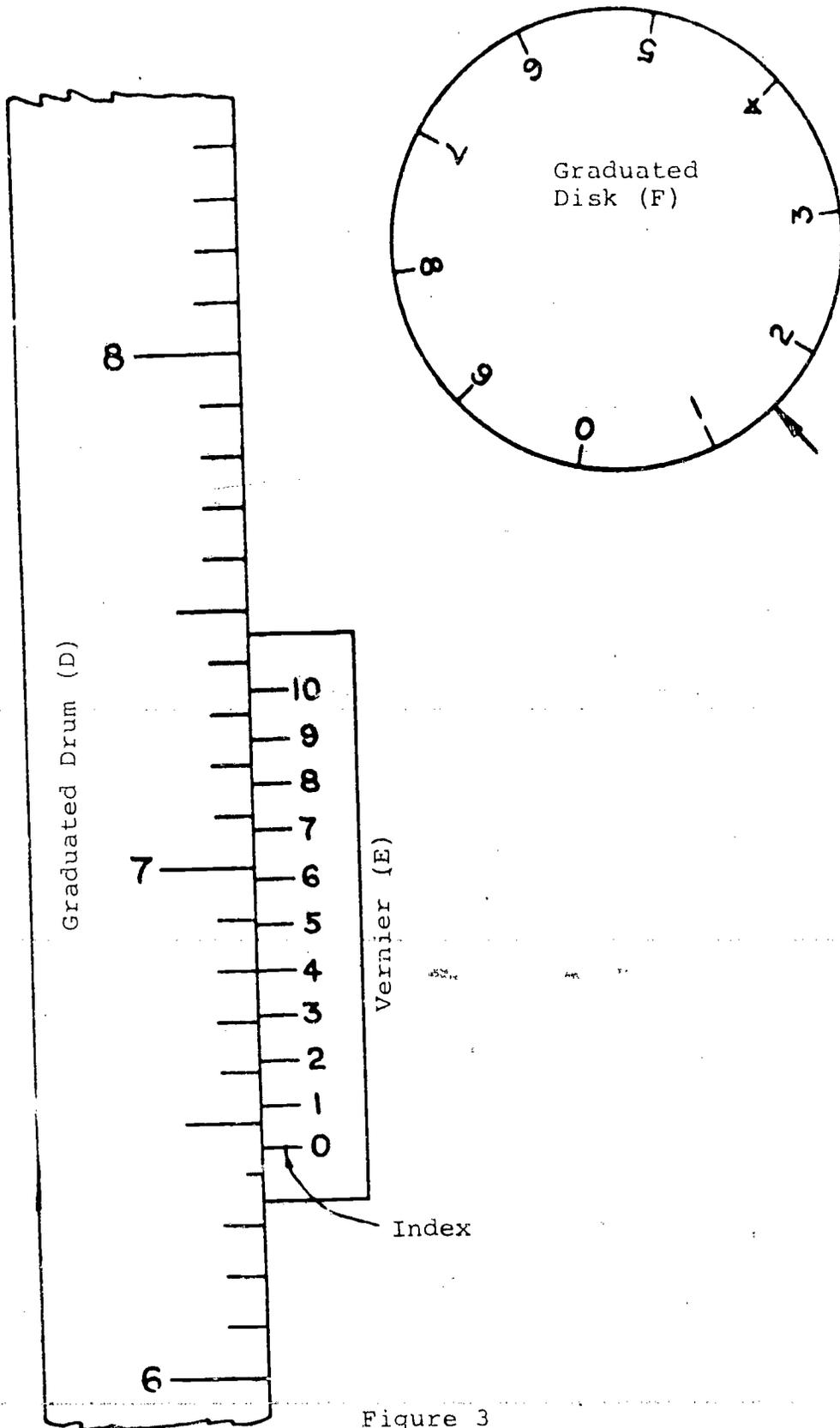


Figure 3

## Roadbed Notes

Before staking a job for construction, the resident engineer will make up a set of roadbed notes. These will show for each station and half-station, as well as certain odd "control" stations, such information as the roadbed width, the profile grade, relationship between the profile grade and  $\ell$  of construction where they are not the same line, the cross slopes of the roadbed, information about any ditches either on the sides of the roadbed or in the median where there are divided roadbeds, and the cut or fill slope to be used in construction.

These notes may be compiled from information shown in the layouts, the typical section sheets, the "super" charts, or charts of the superelevations given to the roadbeds when on a horizontal curve, etc. They will be used by the construction survey crews for setting the necessary survey stakes for the contractor to perform his work, and will be used for calculating the quantities involved in certain phases of the work.

The width of roadbed is considered to be the usable or traversable portion between the outer edge of the shoulders exclusive of the drainage ditches. "Roadway" differs from "roadbed" in that "roadway" applies to the outer extremities of the cut line or fill line.

Profile grade usually indicates the crown or top of the centerline of the pavement. This is the grade to which the pavement is built. In the case of a divided roadbed in which there are actually two separate pavements, there may be two separate profile grades. Or, in the case of a symmetrically divided roadbed, the profile grade may be on centerline in the division strip. Some projects are planned to be done in stages. In other words, the grading contract may be accomplished as one project and paving contract as another. In this case, the roadbed note would indicate that the grading profile would be a certain distance below finished grade of the ultimate pavement.

In rolling country where differing kinds of ground strata would be encountered, it may be advisable to change the slope lines on cuts and fills from one slope to another as conditions vary. The points where these changes in slope are desired are indicated on the layout sheets by appropriate symbols. These notes are put on the layout sheets in conformance to the templates as shown on the cross sections.

221/222

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## Construction Staking of Roadway Sections

The marking of construction survey stakes does not follow any exact set of rules. Different party chiefs or resident engineers may have preferences in methods of placing data on the stakes. Most of the information contained in the following notes will be basic and, it is hoped, will help the newer Aids interpret the various methods of stake marking which they may encounter. On any given project it is best to go over the marking method used with someone familiar with it, if any questions occur.

Much of the information needed by the survey party to place stakes for the various phases of grading work comes from the typical section page(s) or the construction detail page(s) referring to the various types of ditches on the project. Therefore these pages should be studied thoroughly and referred to frequently.

Some of the terms and abbreviations frequently found on these pages or used in connection with this work are:

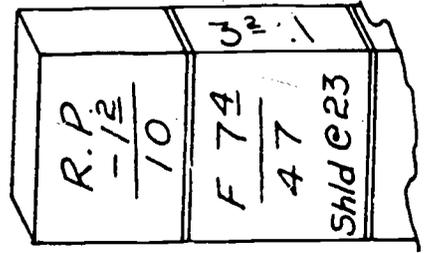
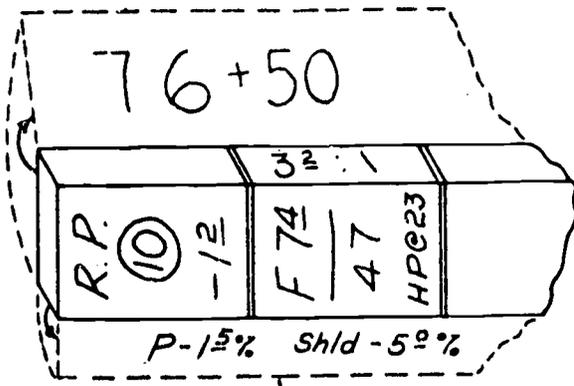
- H.P. Hinge point - The point at the outer edge of the shoulder in fill. In cut section it is on the cut slope at a point level across from the outer edge of the shoulder. The elevation on the slope stake refers to the hinge point.
- O.G. Original ground
- R.P. Reference point
- F.L. Flowline - (Referring to the elevation of a ditch or curb flowline.)
- 2:1 or Flatter - The slope (or slope ratio) is to be 2' horizontal to each vertical foot unless the slope needs to be flattened to reach some planned minimum distance from centerline. Minimum distance slopes automatically blend the transitions between cuts and fills presenting a more pleasing appearance. Variable slopes are used to fit conditions where a constant distance from centerline is maintained for toe of fill or top of cut slopes.

Sh. or Shld. - Shoulder

P. Pavement

Every effort should be made to field check the various stakes the inspector may be using. If a slope stake doesn't seem right its elevation can be checked by hand leveling to an adjacent slope stake. If the distance from centerline is under suspicion, measure in or out from it and the adjacent stake on each side to establish a common distance from centerline at all three points and see if they line up. Check the fill or cut against the slope ratio as shown in the first example on the following pages.

Remember that a big part of the job is to detect errors in staking before they cause costly repair work. After a little work with a cloth tape, folding rule, hand level, or pencil and paper, most errors can be detected. If the inspector lets the constructor build a slope with an obvious bulge, a ditch with an obvious dip in its flowline, a curb with an obvious swale or hump, or lay a pipe that drains backward it is no excuse to say, "The survey party made a mistake", or "Well, that's the way they staked it".



ALTERNATE

FIGURE A

## FIGURE A

This is a reference slope stake at right of Station 76+50. It is located  $47' + 10' = 57'$  from centerline.

Stake is referenced (or offset) 1.2' higher than toe of slope point and is 10' farther out. (El. 82.7 - El. 81.5 = 1.2') ( $57' - 47' = 10'$ ).

The slope stake is not usually set at the actual toe of slope because Section 16 of the Standard Specifications requires the contractor to clear an area at least 5' outside of slope limits, and during that operation it would be difficult to avoid knocking out the stakes.

In this example, the toe of slope is 1.2' lower than the R.P. and -1.2 is noted on the R.P. marker stake. If the toe of slope was higher than the R.P. the stake would be marked +1.2. To keep this straight remember that in almost all cases you will be resetting the slope stake from the R.P. To do so you would either go up (+) or down (-) from the R.P. stake as indicated.

In Figure A, the R.P. stake data tells you to go in 10 feet toward centerline and down 1.2 feet to relocate the toe of slope. The slope stake information is also repeated just below the reference data so that you have all of the information needed to reset the slope stake from the R.P.

The back of the stake is marked with the Station. One edge has the slope ratio (3.2 to 1). The other edge has the planned pavement slope (-1.5%) and shoulder slope (-5%).

This is one method of referencing slope stakes presently being used in District II.

To check the slope ratio (3.2 to 1) against the fill (7.4') note that this slope will have a horizontal distance of  $47 - 23 = 24'$ ; i.e., total distance out from  $\mathcal{C}$  minus distance out to H.P. = horizontal distance of slope.

$$\text{Slope Ratio} = \frac{\text{Hor. Dist. of Slope}}{\text{Height of Fill}} = \frac{24}{7.4} = 3.24 \text{ to } 1.$$

This is recorded on the stake to the nearest 0.1 (3.2 to 1). If this type of calculation doesn't check out, the staking notes should be rechecked. There is an error in either the distance out (47') or the slope ratio. Note that you may not get an exact check if you multiply the fill by the slope ratio to get the horizontal distance:  $7.4 \times 3.2 = 23.68'$  (should be 24').

This is because the slope ratio has been rounded off to 3.2 (from 3.24).

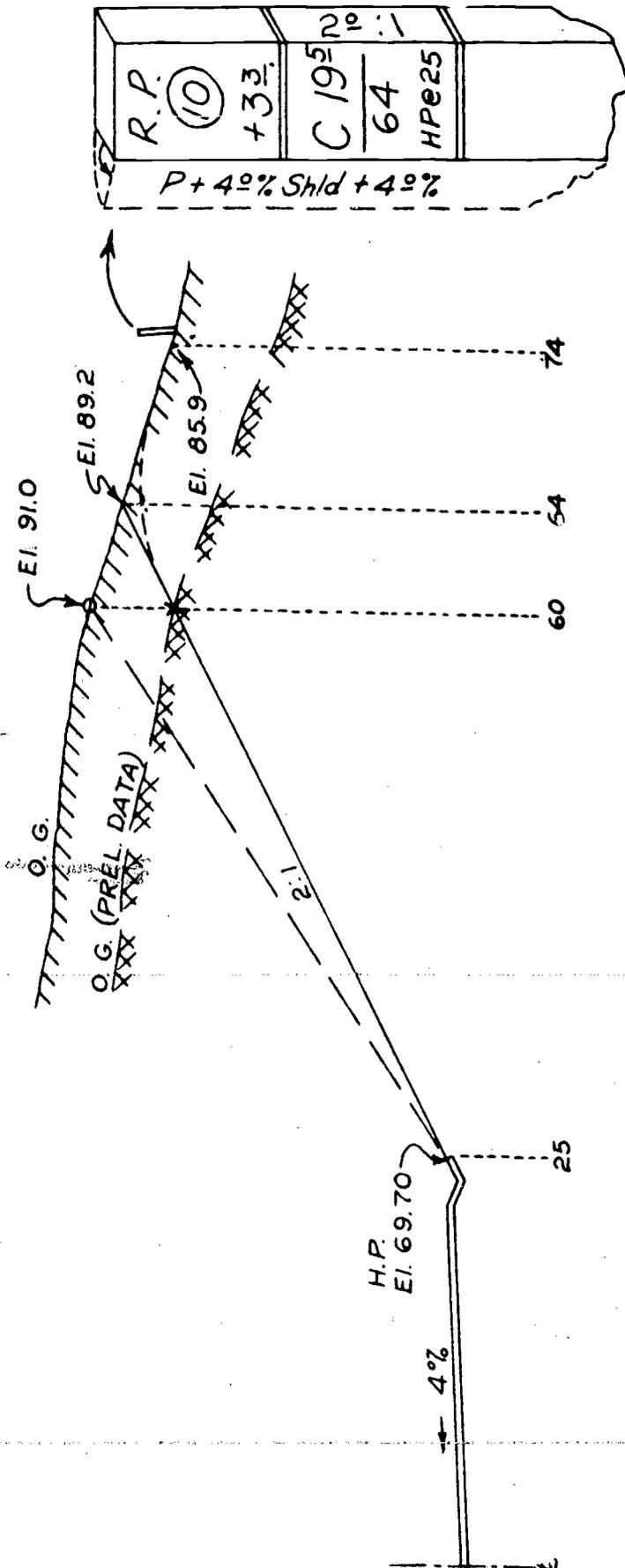


FIGURE B

FIGURE B

In this example the reference stake is located  $64' + 10' = 74'$  from centerline.

The R.P. is 3.3' lower than the top of cut and 10' farther out. (El. 89.2 - El. 85.9 = 3.3')

Another reason for referencing slope stakes is apparent here. Making the slope rounding at the top of cut obviously would destroy the stake location.

Checking the stake:  $64 - 25 = 39$ .

$$\frac{39}{19.5} = 2.0 \text{ to } 1 \quad \text{OK.}$$

You will note that the elevation of the hinge point is the same as that of the edge of shoulder in cut sections, which provide for a side ditch for drainage.

## Slope Staking Survey Methods

1. In country where the terrain is relatively flat, the engineer's level with tripod, level rod and rag tape are most commonly used. Self-reading level rods that can be adjusted so that the elevation of the point is read directly are used to advantage.

Since fill and cut sections are uniform, stakes are normally set at 100 foot stations (say 2+00, 3+00, 4+00, etc.)

Centerline profile can be run in concurrently with slope staking operations.

2. In rolling mountain country Rhodes Arc, level rod, and rag tape are used quite commonly. This method is fast and produces good results if care is exercised in setting right angles for the section and plumbing the rod on steep side hills. The prism right angle mirror is suitable for setting right angles, except on steep side hills.

Slope stakes are set at 100-foot stations and at closer intervals where there are frequent changes from cuts to fills and quantities of earthwork may vary.

The centerline profile is run in advance by differential leveling methods with engineer's level and rod.

3. In rugged steep mountainous country the transit, chain, and level rod are most commonly used to set slope stakes from centerline on accurate right angles. A right angle prism mirror is not satisfactory for this purpose on steep side hills. Elevations and distances are determined from trigonometric functions of vertical angles and slope distances. This method is slow, but it will provide required results in steep country where slopes extend for long distances.

Where there are large fills and cuts involved, it may be advantageous to run a transit-chain traverse and differential levels circuit around on the toe of fill or top of cut slopes, to be used as a control for setting slope stakes and reference points.

In the transit and chain slope staking method the profile levels on centerline are run in advance with engineer's level and rod.

### How to Find the Catch Point

The slope stake or catch point is the term used for the point of intersection of the slope line and the original ground. The approximate location is shown either in the preliminary cross sections for the project or Earthwork Slope Stake Data sheets from IBM calculations. However, this only provides a trial location as the actual ground elevations may vary due to discrepancies in the preliminary cross section survey, contour map elevations or physical changes in the ground.

For example, in Figure B the catch point from preliminary data is 60 feet. The actual elevation found at 60 feet out is 91.0. The cut would be  $91.0 - 69.7 = 21.3'$ ; using 2:1 cut slope the slope distance would be  $21.3 \times 2 = 42.6'$ ; adding 25' (distance from  $\mathcal{C}$  to H.P.) the total calculated distance is  $42.6 + 25 = 67.6'$ . The catch point must be farther out. It is apparent that the natural ground is higher than that shown by preliminary data, possibly due to the skew of angle on which the original cross section was taken.

A second trial is made at a point farther out taking into consideration that the ground elevation is decreasing, thereby reducing the height of cut. This is the trial and error method. The number of trials needed to find the catch point will be reduced with experience.

On the final trial at 64 feet out the elevation is found to be 89.2. The cut would be  $89.2 - 69.7 = 19.5'$ ; using 2:1 cut slope the slope distance is  $19.5 \times 2 = 39.0'$ ; adding 25' (distance from  $\mathcal{C}$  to H.P.) the total calculated distance is  $39.0 + 25.0 = 64.0'$ . This checks to be the catch point for the slope stake, which is to be referenced out as described in the paragraph following Figure B.

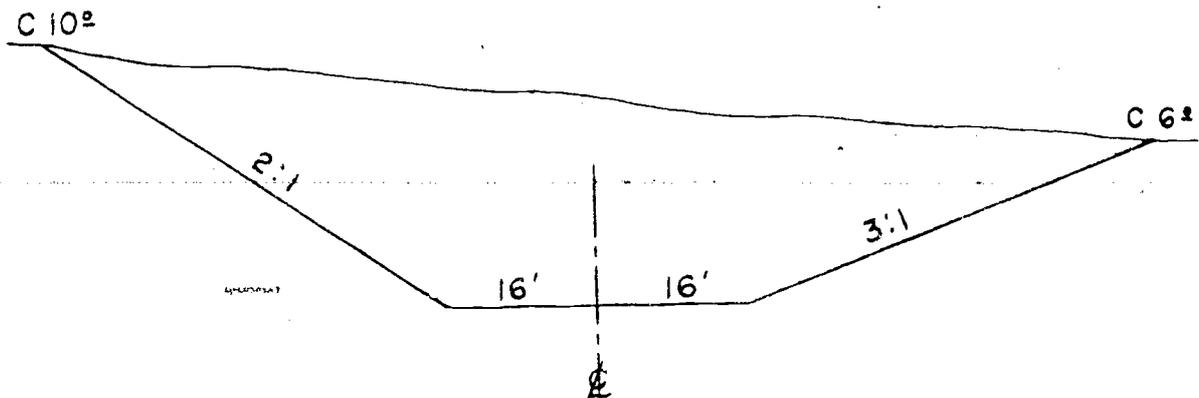
With the use of aerial photographs to obtain design cross sections, and electronic computers to

calculate quantities, modifications in the actual field work during construction have occurred. Actual cross sections are now taken in the field by the construction survey parties and stakes are set for later use as slope stake reference points. These stakes are usually set 15 to 20 feet outside the preliminary catch points. The stations of the points are usually shown on the back of the stakes with the elevations, and the distances the stakes are from  $\ell$  are shown on one edge.

The terrain notes (original ground elevations) and template notes (roadbed elevations), together with necessary fill and cut slopes, minimum catch distances, etc., are then sent to the computer section in Sacramento. Tabulations of ground and roadbed elevations, slope stake catch points, and quantities of earthwork, are computed and returned to the field. With these tabulations the necessary slope stake information is obtained and placed on the stakes set during the cross sectioning operation.

### Slope Stake Problems

1. At what distance out from centerline would the slope stakes be placed in the following Typical Section?

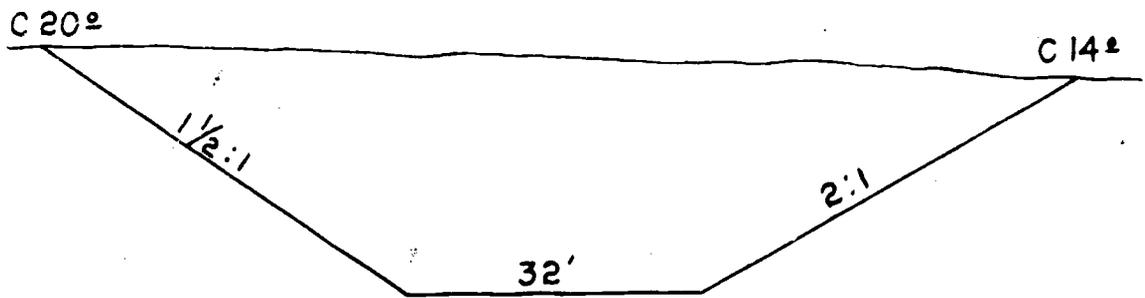


$$10 (2) + 16 =$$

$$6 (3) + 16 =$$

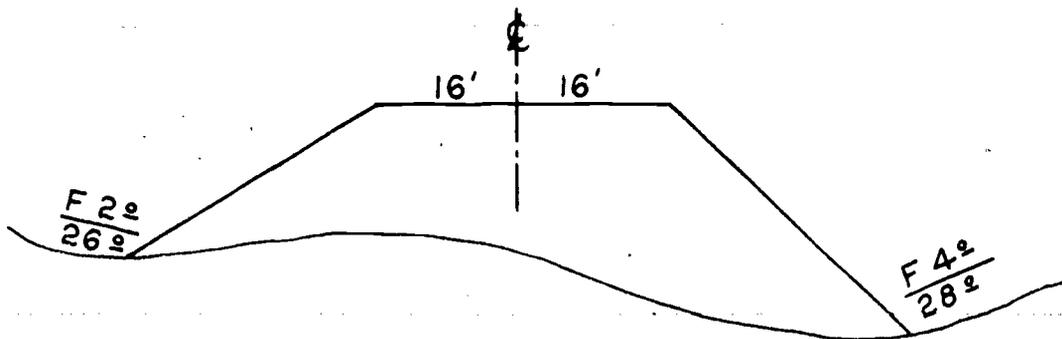
Answer  
 36' Left  
 34' Right

2. What is the horizontal distance across the top of the roadway cut section shown?



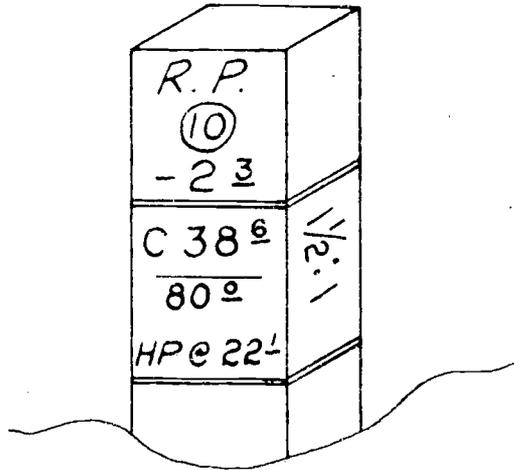
Answer: 90'

3. Determine the side slopes of the shallow embankment section shown below.



Answer: 5:1 Left  
3:1 Right

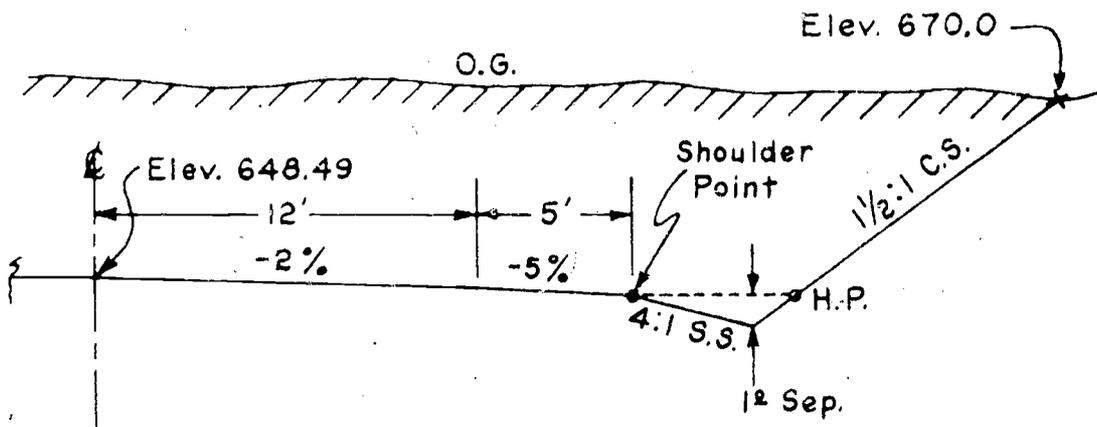
4. A slope stake reference point marker reads as follows:



- How far out from  $\mathcal{C}$  is the top of cut slope located?
- What is the horizontal distance of the slope?
- How far out from  $\mathcal{C}$  is this R.P. stake located?
- If elevation at slope stake or catch point is 655.7, what is the elevation at the R.P. stake?
- Using elevation given above, what is the elevation at the hinge point?

Answers: (a) 80.0' (b) 57.9' (c) 90.0' (d) 658.0' (e) 617.1'

5. Given the following information:



- Find elevation at shoulder point.
- Find distance out to hinge point and elevation.

- (c) Determine distance out from centerline to the slope stake at top of cut. Disregard cut slope rounding.

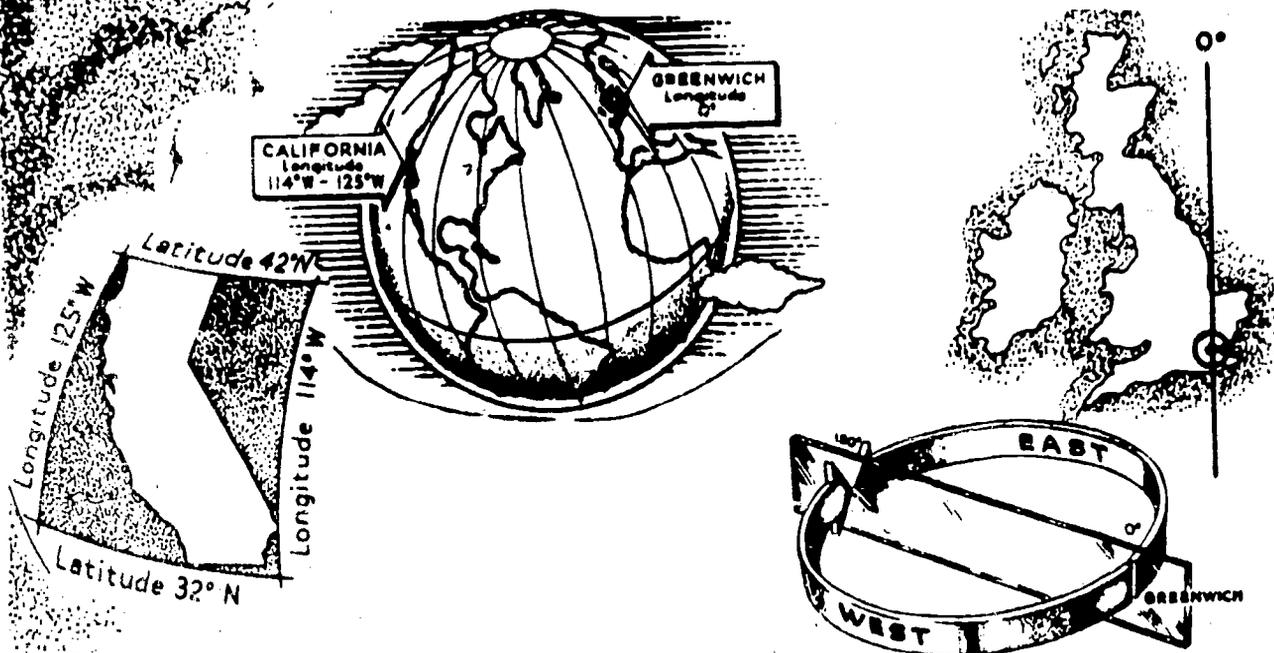
Answers: (a) 648.00 (b) 22.5', 648.00 (c) 55.5'

Section 8

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# Latitudes - Longitudes



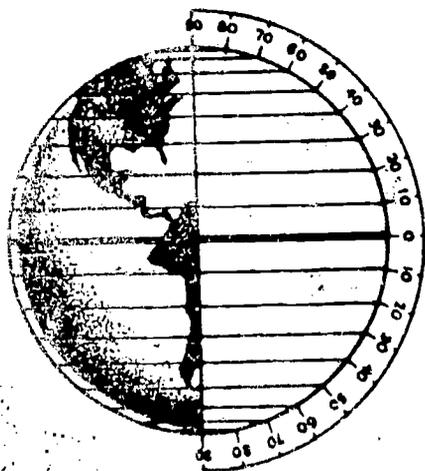
Latitudes and Longitudes are measuring lines used to identify geographical positions anywhere in the world. Any point on the earth's surface can be identified precisely by reference to these lines. They cover the earth, by degrees, with a gridwork of north-south and east-west lines. Pin-point geographical measurements are made possible by the use of degrees and their smaller fractions. A full circle contains 360 degrees. Each degree contains 60 minutes, and each minute contains 60 seconds.

## • LATITUDES

Latitude lines circle the world parallel with the equator. These lines are identified by their position either north or south of the equator. The equator is 0 degree latitude. The degrees of latitude increase as one proceeds from the equator toward either north or south poles where the latitude is 90 degrees. California lies between latitude 32 degrees and latitude 42 degrees north of the equator.

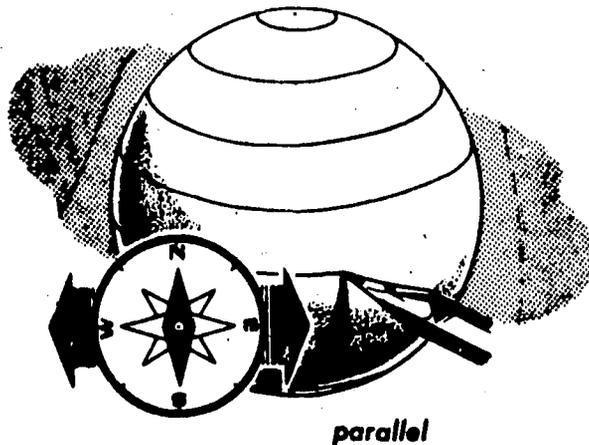
## • LONGITUDES

Longitude lines run north and south. Each line is identified by its number of degrees east or west from the starting point at Greenwich, England. Thus, there are 180 degrees west and 180 degrees east of "zero" meridian at Greenwich. Each degree represents 69.17 statute miles at the equator. This distance applies only at the equator as the distances between longitudes gradually diminish approaching the poles, like orange peel segments. The International Date Line (for most of its length) is at the 180-degree longitude, or directly opposite Greenwich.



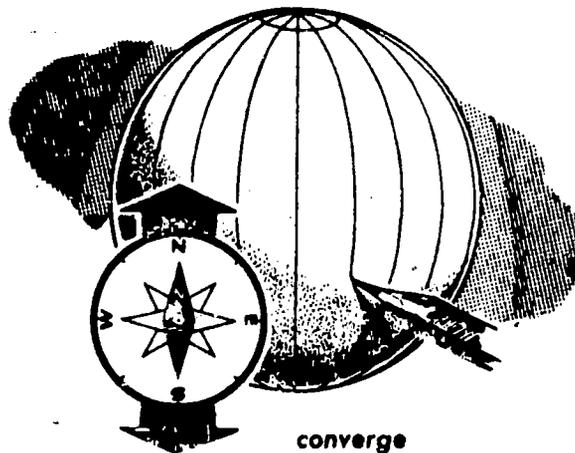
## BASE AND MERIDIAN

### Base Lines



Base and meridian lines are similar to latitude and longitude lines except they are established by geographical points. They form the basis for establishing land areas.

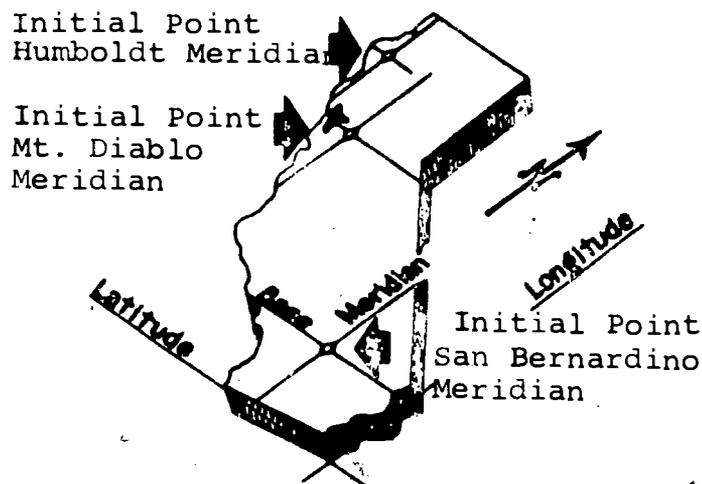
### Meridian Lines



Although latitude and longitude provide the primary control for geographical locations and identification, by act of the Federal congress the U. S. Land Survey System was established to regulate cadastral

surveys and establish localized systems for land descriptions. The latitude and longitude of the "Initial Point" were set and these points then had meridians and bases projected from them to establish a rectangular network of surveys and descriptions. Each of these systems is known by the name given to its principle meridian, usually named after a prominent feature of the area.

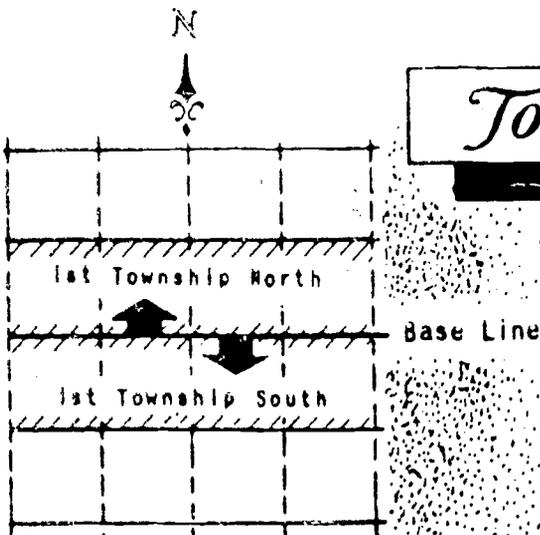
There are three of these grid systems in California. The Humboldt Meridian, with its origin centered near Eureka; the Mt. Diablo Meridian, centered on Mt. Diablo; and the San Bernardino Meridian, centered near San Bernardino.



	Latitude	Longitude
H.M.	-40°25'04"N.	124°07'11"W.
M.D.M.	-37°51'30"N.	121°54'48"W.
S.B.M.	-34°07'10"N.	116°58'15"W.

The principle meridian of each system is run north and south on a true meridian of longitude from the initial point, and the principle base line is extended east and west of the initial point on a true parallel of latitude. Standard parallels are established at intervals of 24 miles north and south of the principle base line, and guide meridians at 24-mile intervals east and west of the principle meridian. At 6-mile intervals between these, similar N-S and E-W lines are run; the N-S lines being "range" lines and the E-W lines "township" lines.

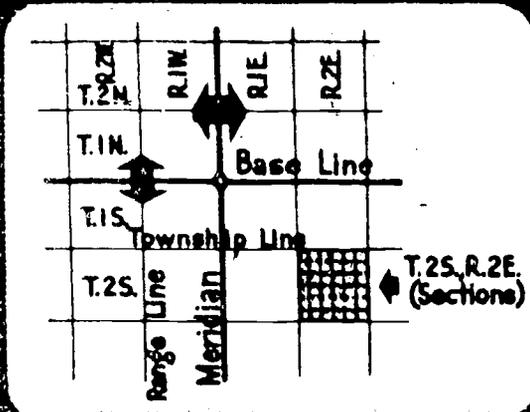
# Township and Range



## • TOWNSHIP

Townships are rectangular blocks of land theoretically six miles square. They have no connection with townsites or towns. Starting from a base line there are tiers of townships paralleling the base line. These tiers are numbered according to their position north or south of the base line.

Example: *The second tier of townships south of a base line is identified by the abbreviation T. 2 S. (Township 2 South).*



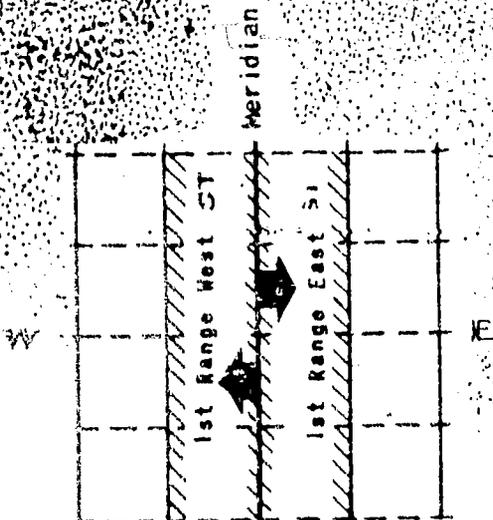
## • RANGE

Starting from the meridian through the point of origin of each system there are columns of townships called ranges, east and west of the meridian, numbered progressively away from the meridian.

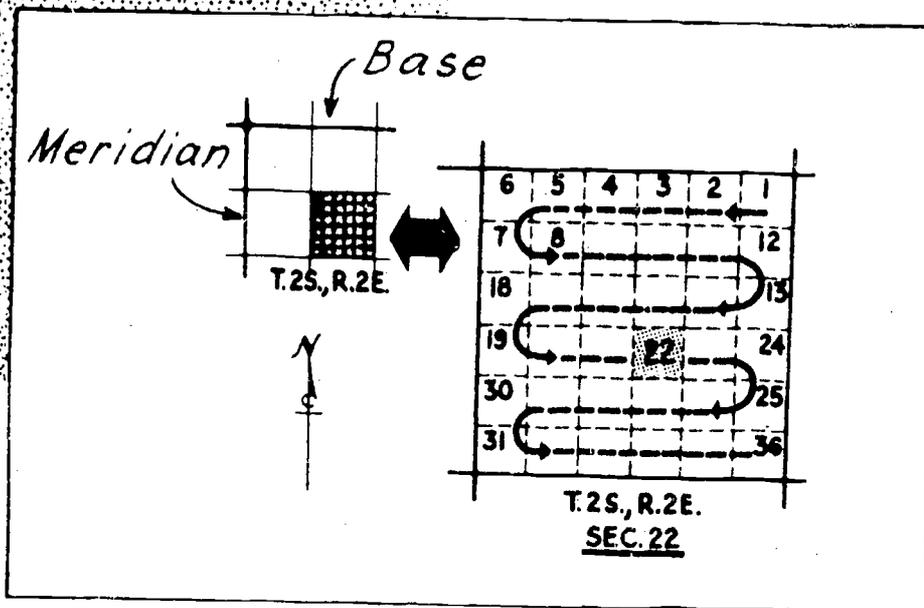
Example: *The second range of townships east of the meridian line is identified by the abbreviation R. 2 E. (Range 2 East).*

Any particular township may be found by identifying first its township number, second its range number, and third its base and meridian system.

Example: *T. 2 S., R. 2 E., M. D. M. (Township 2 South, Range 2 East, Mt. Diablo Meridian).*



# Sections



## Sections

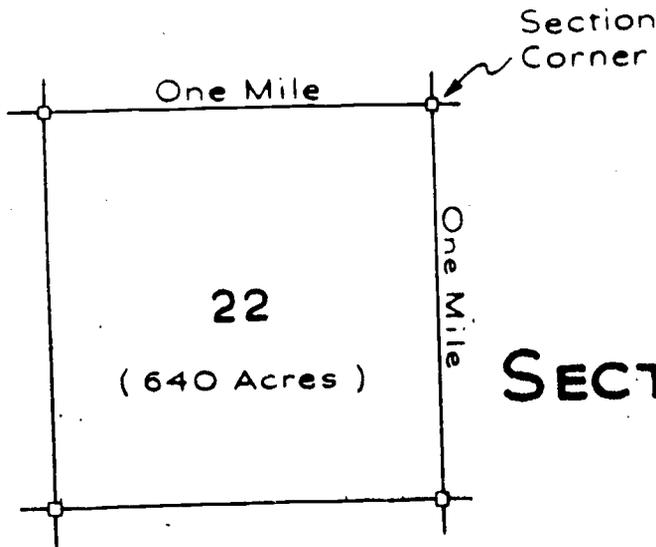
A township is further divided into 36 sections. Theoretically each section contains one square mile. Sections are numbered consecutively starting with 1 at the top right or northeast section, thence progressing alternately westerly and easterly through successive tiers of sections ending at number 36 in the southeast corner of the township.

There are thousands upon thousands of sections numbered 22. But there is only one Section 22, for instance, in Township 2 South, Range 2 East of the Mt. Diablo Meridian system. No section can be identified without referring to its number, township, range, and base and meridian.

## Irregularities of Land Surveys

Due to the convergence of meridians toward the north and south poles, and the resulting narrower widths of townships, the areas established cannot be truly rectangular. Errors in the original U. S. land surveys and the special conditions posed by the old Spanish land grant boundaries cause many irregular sections and townships in California.

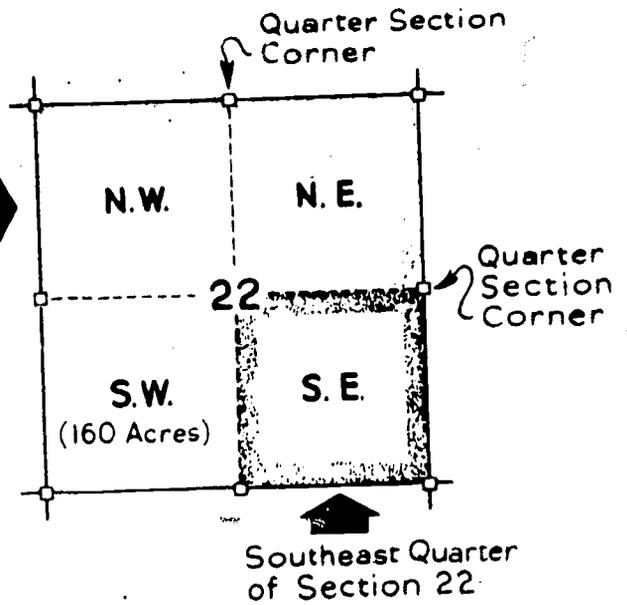
For a full explanation of these matters the "Manual of Instructions for the Survey of the Public Lands of the United States," issued by the Superintendent of Documents, Washington, D. C., should be consulted.



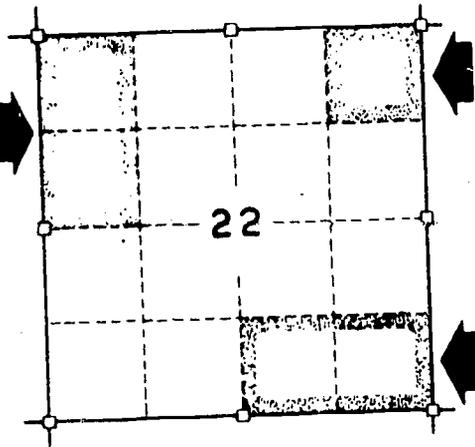
## SECTIONS

North Half of Section 22

## QUARTER SECTIONS



West Half of Northwest Quarter Section

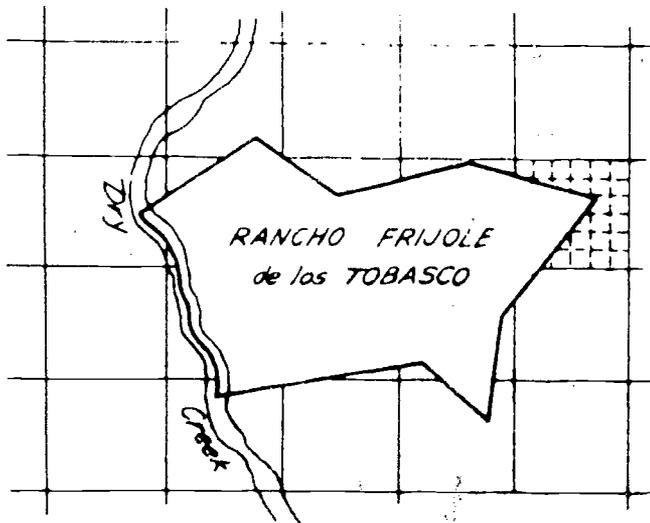


Northeast Quarter of Northeast Quarter Section

## QUARTER QUARTER SECTIONS (1/16 SECTION)

South Half of Southeast Quarter Section

# Ranchos

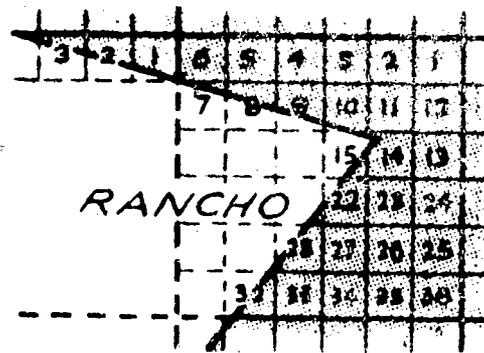


During the early development of California there were no established base lines or reference lines to identify particular areas or tracts of land.

When certain areas of land were granted to early settlers, they were called ranchos and given an identifying name. In order to define the limits of the rancho, it was necessary to identify the boundaries by certain landmarks or by measurements. Many of the old landmarks have since been destroyed or changed.

Measurements were made by the most practical means at hand. Linear measurements were usually either in varas (33 in.) or in leagues (3 miles). To measure distances from a landmark, for instance, two men on horseback might drag a cord or rawhide thong line of perhaps 100 varas length, one standing still while the other rode ahead and then passing each other, repeating the "leap-frogging" while counting the 100 vara lengths. Most of these boundaries have since been re-established by accurate surveys and the lines are now identified by bearings and lengths in feet.

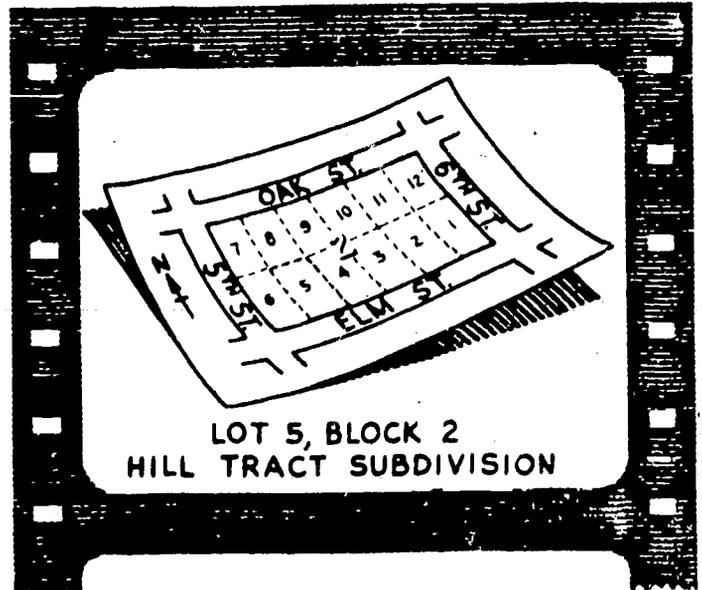
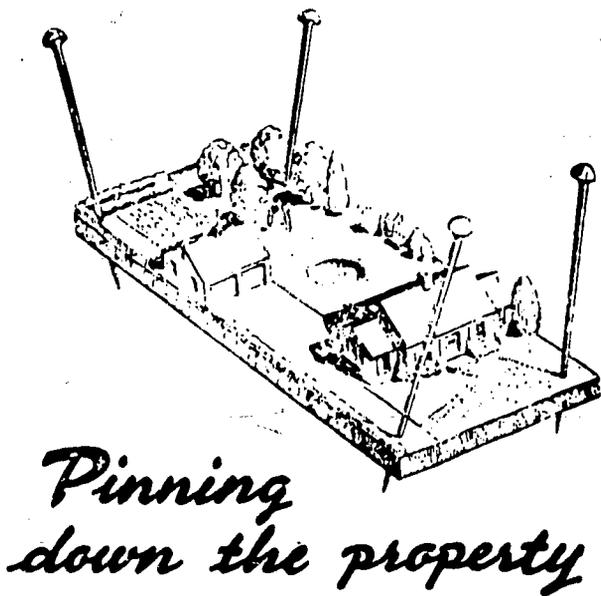
Because rancho boundaries were legally accepted as defining certain areas of land in California, they have been maintained through the years. When the State was subdivided into a grid-iron of townships, ranges, and sections, the lines were stopped at the rancho boundaries.



Where sections are not complete within a township because of irregular rancho lines, there is no change in the numbering of the sections. The same rule applies to township numbers.



# Subdivisions



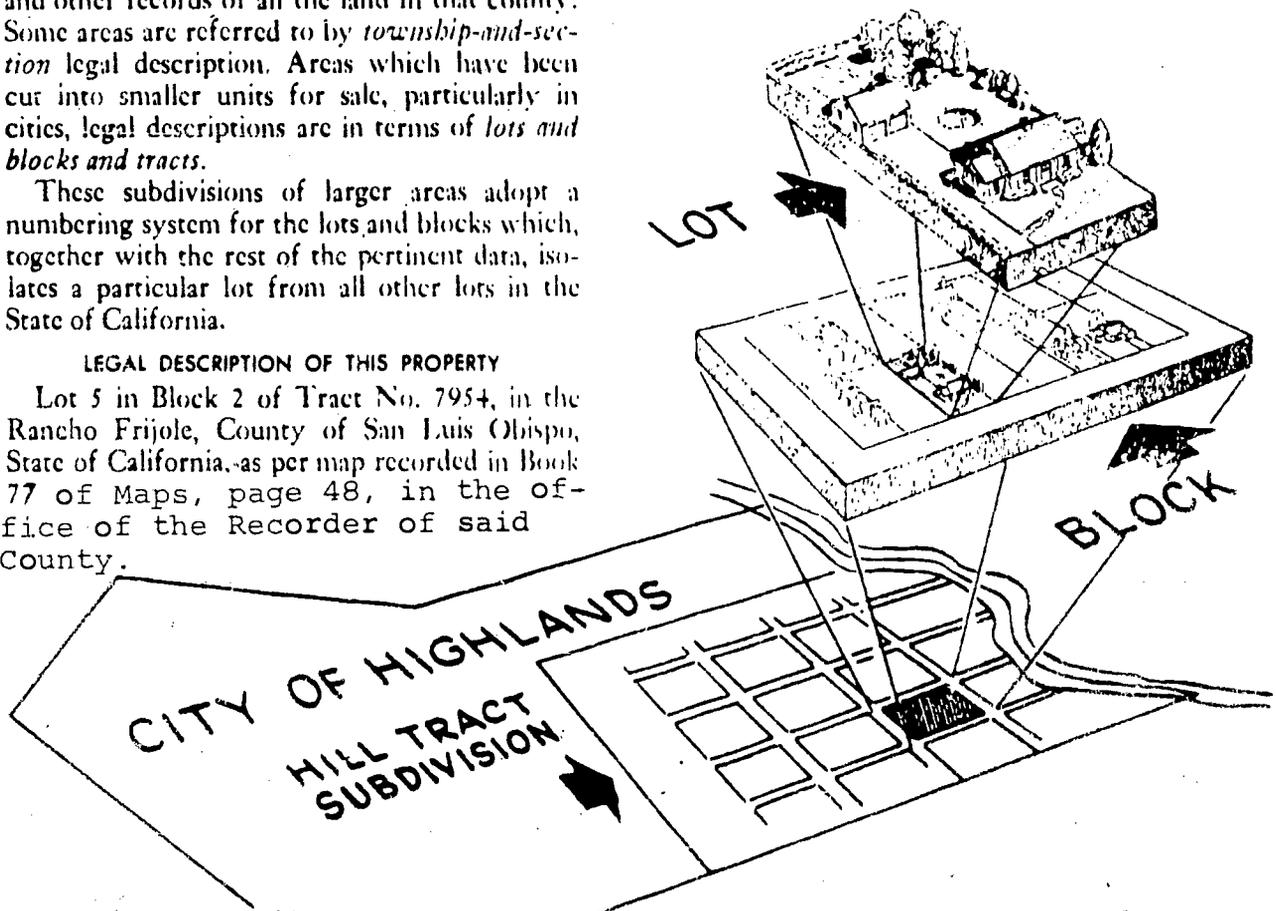
## EVERY LOT HAS ITS PLACE ON SOME MAP

At the office of the county recorder at each county seat in California, are filed official maps and other records of all the land in that county. Some areas are referred to by *township-and-section* legal description. Areas which have been cut into smaller units for sale, particularly in cities, legal descriptions are in terms of *lots and blocks and tracts*.

These subdivisions of larger areas adopt a numbering system for the lots and blocks which, together with the rest of the pertinent data, isolates a particular lot from all other lots in the State of California.

## LEGAL DESCRIPTION OF THIS PROPERTY

Lot 5 in Block 2 of Tract No. 7954, in the Rancho Frijole, County of San Luis Obispo, State of California, as per map recorded in Book 77 of Maps, page 48, in the office of the Recorder of said County.



## PROBLEMS

1. What do the initials M.D.M. stand for?
  - a. Measured distance, and mean
  - b. Mt. Diablo Meridian
  - c. Medical Doctor, and meat
  
2. Calculate the bearing and distance in miles of a line joining the S.W. corner of the N.E.  $\frac{1}{4}$  of the N.E.  $\frac{1}{4}$  of Section 29 and the S.W. corner of the N.E.  $\frac{1}{4}$  of the S.W.  $\frac{1}{4}$  of Section 11, both sections in the same township.

Answer: Bearing = N 45° 00' E  
Distance = 3.54 miles

3. The center of the City of Susanville is approximately the center of Section 32, T. 30 N., R. 12 E., M.D.M. Neglecting survey errors and the curvature of the earth, what is the straight line distance between Susanville and Mt. Diablo?

- \*a. 187 miles
- b. 195 miles
- c. 194 miles

4. What is the length of one side of a square area of ground having an area of one acre?

A = 208.47'

5. What is the distance between

- |                       |                  |
|-----------------------|------------------|
| a. Township lines     | Answer: 6 miles  |
| b. Standard parallels | Answer: 24 miles |
| c. Guide meridians    | Answer: 24 miles |

6. Theoretically, range lines are

- a. Parallel
- b. Converge from south to north (answer)
- c. Converge from north to south

7. Three of the following pairs of sections do not have a common side. Identify them:

- Sec. 7, T. 18 N., R. 18 W. and Sec. 12, T. 18 N., R. 19 W.  
\*Sec. 15, T. 18 N., R. 18 W. and Sec. 21, T. 18 N., R. 18 W.  
Sec. 33, T. 19 N., R. 19 W. and Sec. 4, T. 18 N., R. 19 W.  
\*Sec. 33, T. 19 N., R. 19 W. and Sec. 27, T. 19 N., R. 19 W.  
Sec. 1, T. 19 N., R. 19 W. and Sec. 6, T. 19 N., R. 18 W.  
\*Sec. 24, T. 19 N., R. 19 W. and Sec. 20, T. 19 N., R. 18 W.

Answer indicated by asterisk.

Section 9 - Bearings

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## BEARINGS

The "bearing" of a line is the direction of the line with respect to a given "meridian", and is indicated by the quadrant in which the line falls and the acute angle which the line makes with the meridian in that quadrant. The reference meridian may be either "true", magnetic, or assumed, and the bearings will likewise be "true", magnetic, or assumed in keeping with the reference meridian.

"Observed" bearings are those for which the actual bearing angles are directly obtained by survey work, and "calculated" bearings are those for which the bearing angles are indirectly obtained by calculations.

A "true" bearing is one whose reference meridian is a true meridian of the earth passing through the earth's poles (geographic north and south), and the projection of which passes through the celestial pole (geodetic north); the position of which is defined by its angular relationship to the North Star, or Polaris. For any given point on earth the true meridian is always the same, and hence directions referred to the true meridian will remain the same regardless of time.

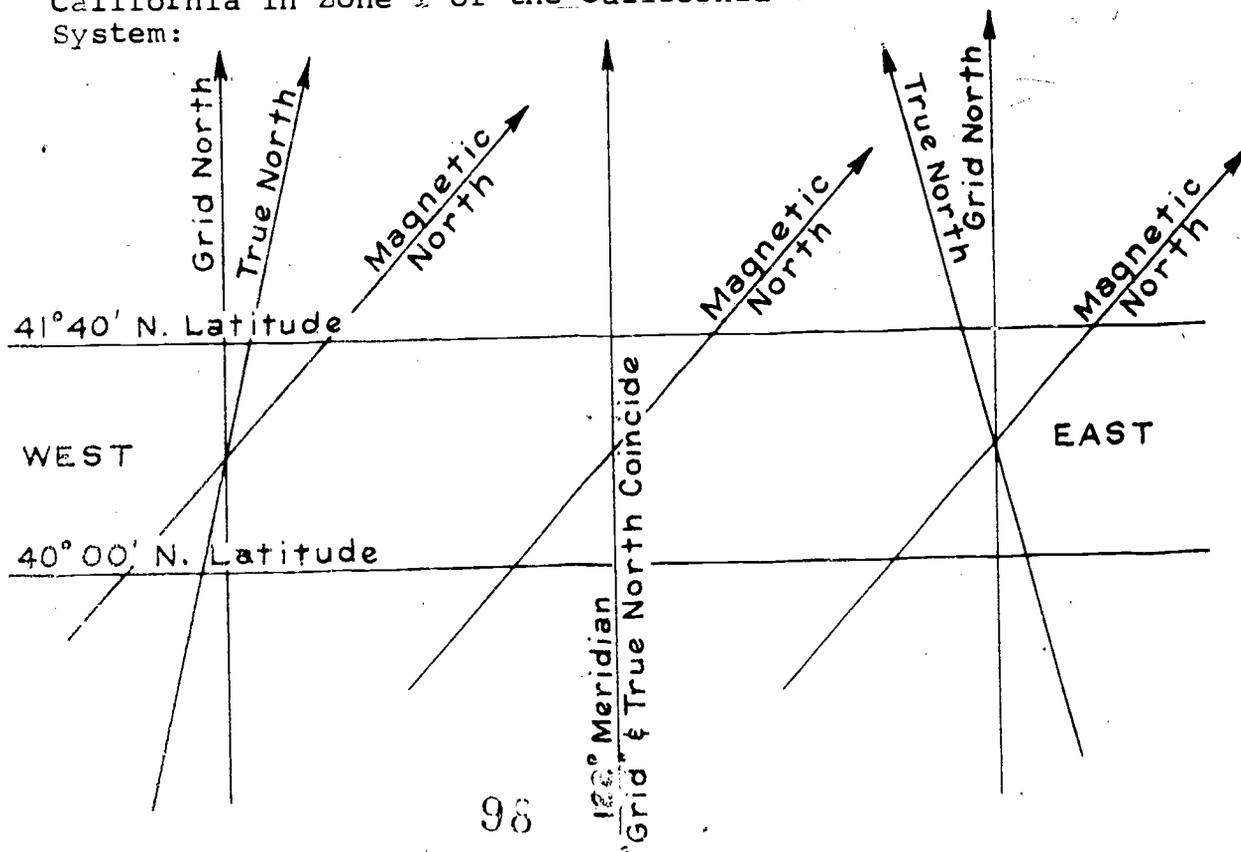
A magnetic bearing is one whose reference meridian is the direction taken by a freely suspended magnetic needle, i.e., a compass. The magnetic poles are at some distance from the true poles, so the magnetic meridian is generally not parallel to the true meridian. The location of the magnetic poles is constantly changing, hence the magnetic bearing between two points on the earth's surface does not remain a constant. Although used quite extensively in the past, magnetic bearings are now used mainly for reconnaissance work and for rough surveys.

The angle between a true meridian and the magnetic meridian at the same point is called the magnetic declination. A line on the earth's surface which has the same magnetic declination throughout its length is called an "isogonic" line, and the line where the magnetic and true meridians coincide is called an "agonal" line. Throughout California the

magnetic declination is east, i.e., the magnetic pole lies east of the true pole from any position in California.

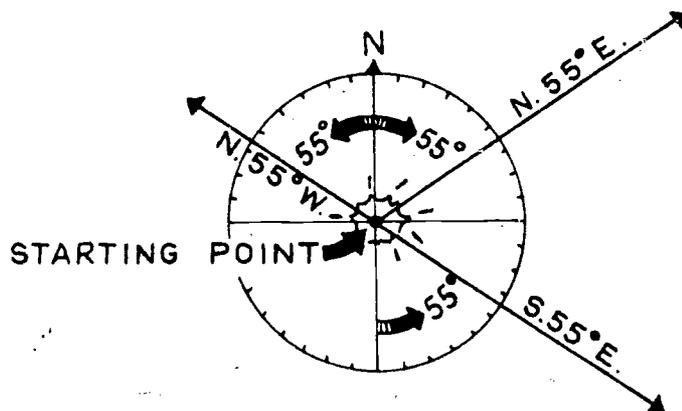
An "assumed" bearing is a bearing whose prime meridian is assumed. In some cases the relationship between it and the true meridian is given a definite calculable relationship. This is the case with the "grid", or California Coordinate System. The use of this system by professional engineers and surveyors for making maps has been approved by the California Legislature. It is based on the "Lambert conformal conic projection" which, theoretically, projects a cone through the earth's surface and uses a small portion of this projection, spread flat, to form a plane grid for locating points. This system is being used more and more by the Division of Highways for their survey and calculation work. One of the many reasons it is recommended is that the meridians on this projection remain parallel and do not converge as the true and magnetic meridians do.

The approximate relationship of the various meridians is shown below as they relate in northern California in Zone 1 of the California Coordinate System:



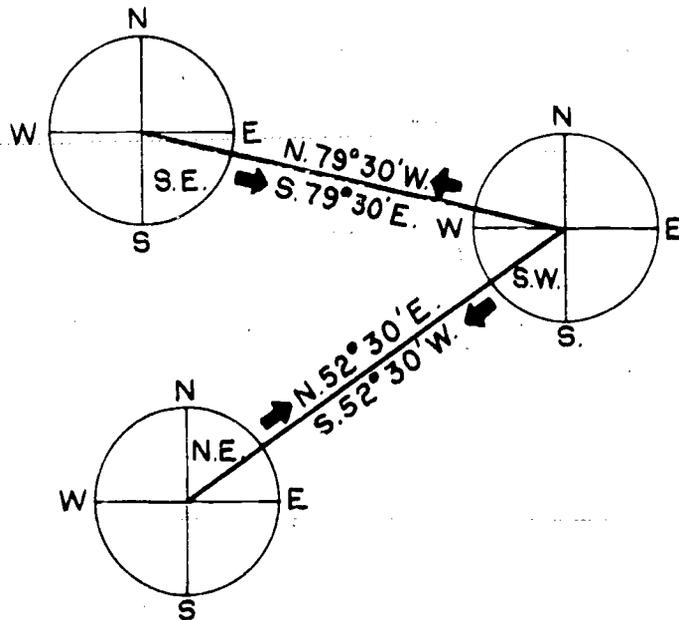
All bearings, in highway engineering, must be definitely and minutely described. It is not sufficient to describe a line or bearing as simply northeast or southwest. It must be described as so many degrees, minutes, and seconds in the direction in which the line is progressing. The accuracy of calculations is dependent on exact measurements of distances and bearings.

A bearing will always state first the primary direction, north or south, and next the angle east or west. In other words, a line would be described as heading north and deflected so many degrees toward the east or west. The same would apply if the line was headed south. A bearing therefore will never have a value of over  $90^{\circ}$  as this would put the line in a different quadrant, and consequently the primary cardinal direction would shift  $180^{\circ}$ .



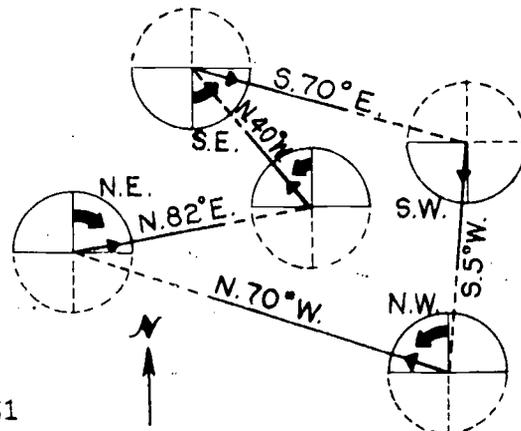
A straight line may have two bearings. It sounds contradictory, but isn't. It all depends on how you look at it. If you were standing at the starting point, as in the above sketch, you could look back, or southerly, along the straight line and you would be looking south 55 degrees toward the east. Now turn directly opposite. You are looking north 55 degrees toward the west. It all depends on where you start and in what direction you are going.

In the lower sketch you can follow a series of lines but the bearing depends on what direction you're going on each course. Starting at the lower circle you would be going N. 52° 30' E. on the first course and then N. 79° 30' W. on the second. Reversing the direction you would start at the upper circle and proceed S. 79° 30' E. and then S. 52° 30' W.



In determining the bearings for all the lines surrounding a tract of land, it is necessary to assume a logical starting point and then follow the courses continuously around the tract and return to the starting point. This is called a closed traverse.

Note in the lower sketch how the traverse is followed clockwise. If the direction was followed counter-clockwise at any point, the bearing letters would change or reverse but not the numbers.



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## BEARINGS AND MAGNETIC DECLINATION

1. The magnetic bearing of a line is  $N. 75^{\circ} 26' E.$  If the magnetic declination is  $19^{\circ} E.$ , what is the true bearing of the line?

Answer:  $S. 85^{\circ} 34' E.$

2. Two correct bearings have been taken on a line, one magnetic, one a true bearing. If the true bearing is  $N. 14^{\circ} 00' W.$  and the magnetic bearing is  $S. 36^{\circ} 00' E.$ , what would the magnetic declination in the area be?

Answer:  $22^{\circ} E.$

3. What is the included angle between two lines, one having a true bearing of  $N. 43^{\circ} 10' E.$ , the other a magnetic bearing of  $S. 37^{\circ} 50' W.$  Assume magnetic declination  $19^{\circ} E.$

Answer:  $13^{\circ} 40'$  included angle

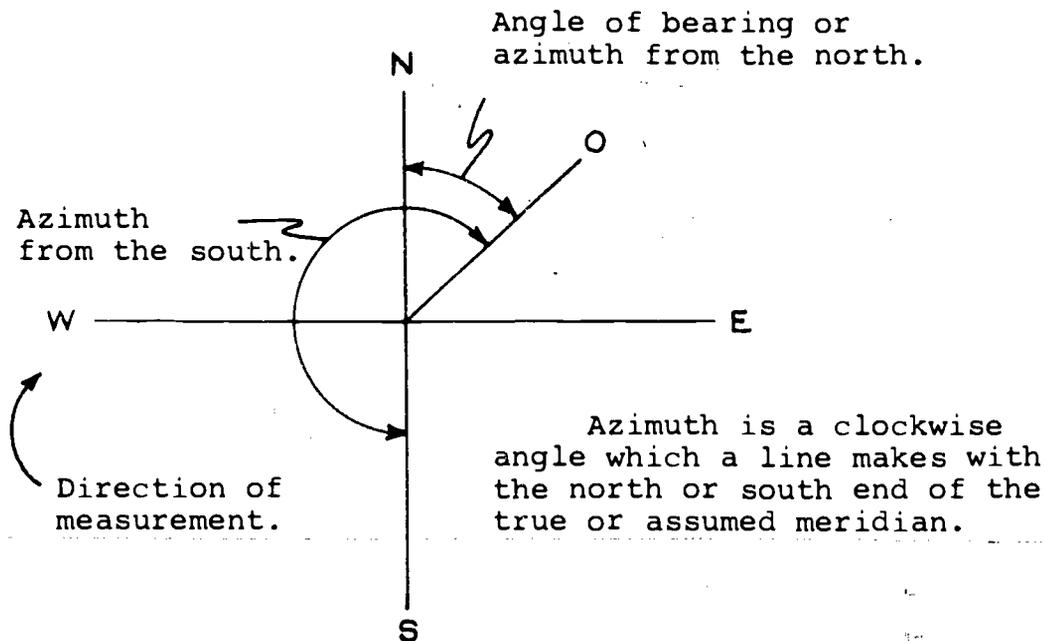
## AZIMUTHS

The "azimuth" of a line is its direction as given by the angle between the meridian and the line, measured in a clockwise direction. As with bearings, azimuths can be either "true", "magnetic", or "assumed", depending on the meridian to which they are referenced. Azimuths can be indicated from either the south point or the north point of a meridian, but they are always measured in a clockwise direction. U. S. Coast and Geodetic Survey always uses south as the zero direction for its work.

In our survey work we use assumed azimuths in making contour and topography maps by the transit method. When the instrumentman and plotter orient on a line and read angles and distances to plot various points on a hard copy or other map, the angles are always in a clockwise direction and thus they are azimuth angles from an assumed meridian.

Azimuths as well as bearings can be either "observed" or "calculated". An observed azimuth is simply one which is read by use of the transit or

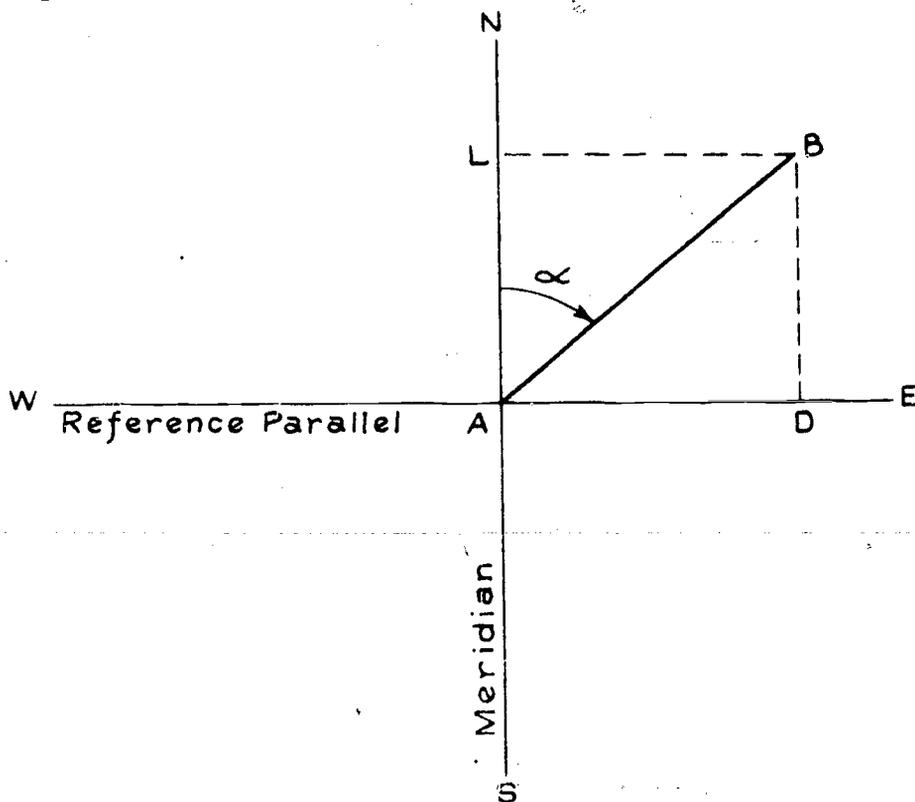
compass in the field. Calculated azimuths are those obtained by computation, such as adding to or subtracting from known bearings or azimuths the field observed angles to determine other bearings or azimuths.



### LATITUDES AND DEPARTURES

Possibly the most useful of all methods for plotting maps from field data is that of latitudes and departures. It is certainly the most practical for extensive systems of horizontal control and triangulation figures, and is among the most accurate for traverse plotting. The California Coordinate System uses this method for positioning in its grid system.

The basis of this method is a system of rectangular coordinate axes which are oriented according to the field notes, which may be either assumed, magnetic, or true north. The north-south axis is called the meridian, the east-west axis is the reference parallel. The projection of a line in the traverse on the meridian is known as the latitude and the projection on the reference parallel is the departure.



$\alpha$  = Bearing of AB

AL = Latitude of AB = AB cos  $\alpha$

AD = Departure of AB = AB sin  $\alpha$

$\tan \alpha = \frac{AD}{AL} = \frac{\text{Departure}}{\text{Latitude}}$

These projections are readily found, as the bearing and length of each line in the traverse has been noted in the field, and since the traverse and the coordinate axes are referenced to the same north, the bearing of a line in the traverse gives its angle with the meridian. Thus, each line becomes,

in effect, a right triangle with its projections on the axes being the legs. The latitude will then equal length (x) cos bearing, and the departure will equal length (x) sin bearing.

The latitude corresponds to the "y" axis, or meridian, and the departure corresponds to the "x" axis, reference parallel, and from this it would follow that  $\frac{\text{departure}}{\text{latitude}} = \text{tangent of the bearing angle}$ .

Any latitude of a line with a northerly bearing is positive, while latitude from a line with a southerly bearing is negative. Any departure of a line with an easterly bearing is positive, and any departure from a line with a westerly bearing is negative. Since a closed traverse must end at the same point (an exception to this is when it is closed on a triangulation point instead of the point of beginning), then the sum of the northerly or positive latitudes must equal the sum of the southerly or negative latitudes, and the sum of the easterly or positive departures must equal the sum of the westerly or negative departures. In other words,  $\sum L=0$  and  $\sum D=0$ . Similarly, for the exception noted above, if the point of beginning be joined to the triangulation station on which the traverse was closed and this line be added to the traverse, then  $\sum L=0$  and  $\sum D=0$ . In highway work, an open traverse is always joined to another known line or point or made a closed traverse so that the accuracy of the survey can be checked.

#### Latitudes and Departures

1. If the bearing of a line 400 feet in length is S.  $30^{\circ} 00'$  W., what is its latitude and departure?

Answer: Latitude = -346 feet  
Departure = -200 feet

2. The latitude of a line is +284 feet and the departure of the same line is -203 feet. What is the length and bearing of this line?

Answer: Length = 350 feet  
Bearing = N.  $35^{\circ} 34'$  W.

3. The latitude of a line is +125 feet and the bearing is N.  $75^{\circ} 00'$  E. What is the length of the line?

Answer: Length = 483 feet

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Section 10 - Contours

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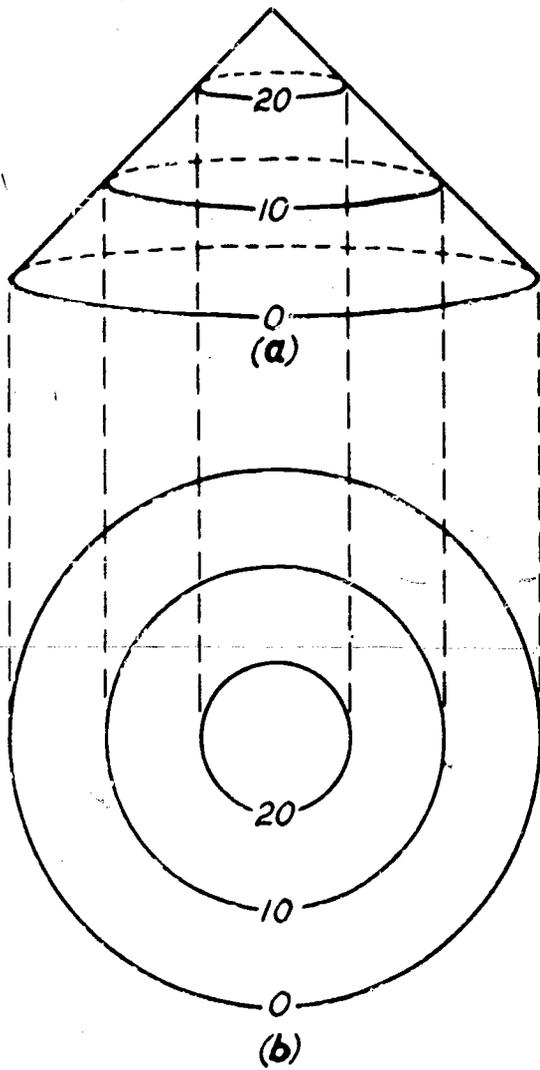
## Contours

A contour, or a contour line, is a line that would be formed if the surface of the ground were intersected by a level surface. The term "contour" is also generally applied to a line that is drawn on a map to represent a contour on the ground. In order to illustrate the principle underlying the use of contours for representing relief, two simple geometric solids are considered first; namely, a cone and a hemisphere in Figure 1. In each of these illustrations the object is shown in perspective in view (a), and by means of contours in view (b).

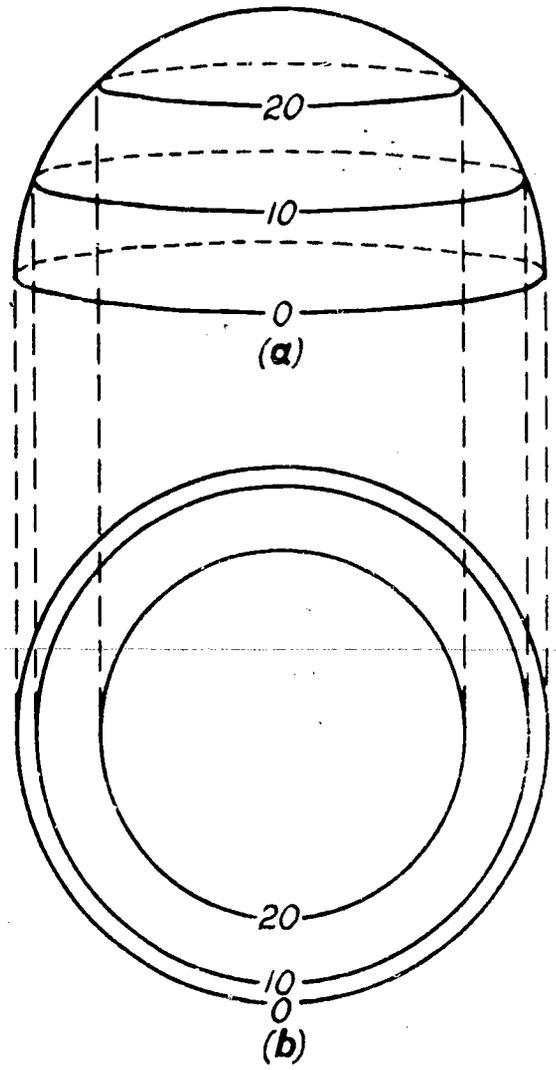
The outline of the base of a cone is a circle, and the intersection of the curved surface of a cone with any plane parallel to the base is also a circle. In the perspective view (a), each such circle is represented as an ellipse or oval; but in the plan view (b), each circle is shown in its true shape. For the purpose of identification, the ellipse and the circle that are at the same level are given the same number. If it is assumed that the ground at a certain place has the form of a cone with a level base, the ellipse in view (a) and the circles in view (b) represent contours. Also, if the elevation of the base of the cone is taken as zero and the vertical distances from the base to the contours marked 10 and 20 are, respectively, 10 and 20 feet, the number written on each contour indicates the elevation of every point on that contour or the elevation of the contour.

The outline of the base of the hemisphere in Figure 1 is a circle, and the outline of the intersection of the surface of the hemisphere with any plane parallel to the base is likewise a circle. The contours on ground of hemispherical form would therefore be ellipses in a perspective, as in view (a), and circles in a plan, as in view (b).

Properly located contours indicate elevations with a relatively high degree of accuracy. Also, contours can be drawn on a topographic map easily and rapidly. Therefore, they are used much more commonly than other symbols for showing relief. The United States Coast and Geodetic Survey, The United States Geological Survey, and many engineers engaged in private enterprises show practically all relief by means of contours. A map whose primary purpose is to indicate the positions of contours is known as a "contour map". See Figure 2(A).



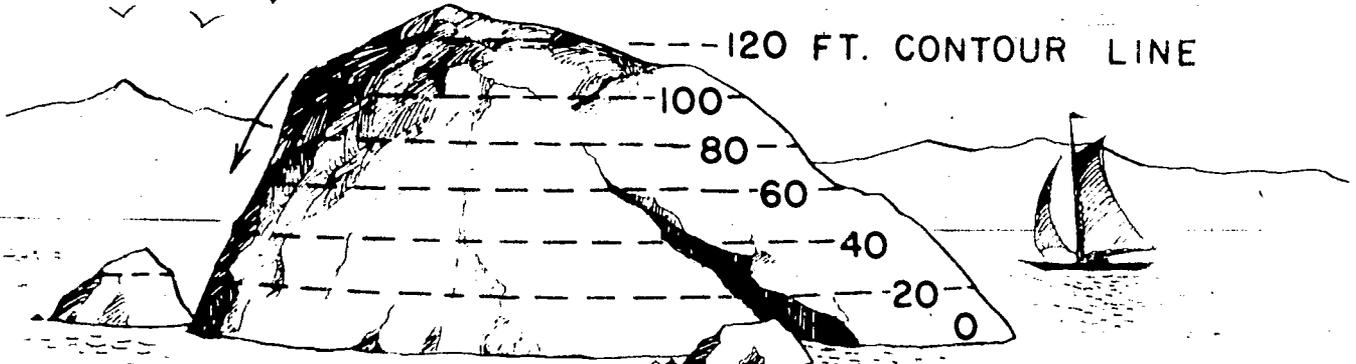
Cone



Hemisphere

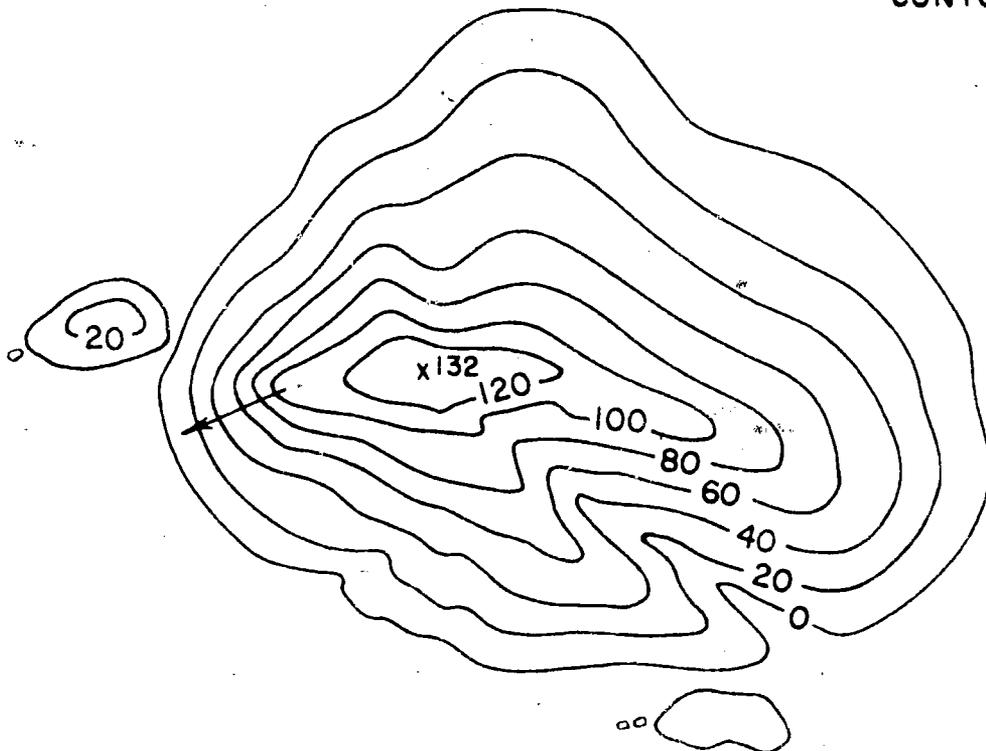
Figure 1

TOP 132 FT. ABOVE SEA LEVEL



( B )

NOTE IF WATER SHOULD RISE 20 FT., WATER LINE (0) WOULD BE AT 20 FT. CONTOUR LINE.



( A )

Figure 2

# CONTOURS

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## 1. Elevations of Contours

It follows from the definition of a contour that all points on a given contour have the same elevation above an assumed datum, or surface of zero elevation. Elevations are shown on a contour map by marking the numerical values of the elevations in feet on some of the contours and also at summits and low points.

In order that the configuration of the ground may be readily visualized from a contour map, the vertical interval between adjacent contours, or the difference between the elevations of such contours, is made the same for all parts of the map. This vertical interval on a particular map is called the "contour interval" for that map. In Figure 1 the difference between the elevations of two adjacent contours is shown as 10 feet, and the contour interval of these cases is therefore 10 feet. The best contour interval depends on the degree of accuracy desired, the slope of the ground, and the scale of the map.

## 2. Representation of Typical Formations by Means of Contours

In Figures 2 to 6, inclusive, are represented the five formations that are of most common occurrence on the earth's surface. In each of the five illustrations the surface formation is represented in view (a) by means of contours on a plan, and in view (b) by means of a perspective. Arrows are inserted to indicate the direction in which water would flow, that is, the direction in which the ground slopes downward, but such arrows would not be shown on an actual contour map.

In the case of a hill or an island, each contour has the general form of a loop as shown in Figure 2 (a). Obviously, the higher contours are smaller than the lower contours and are entirely enclosed by them. The contour lines are drawn freehand and are made more or less wavy to conform more nearly to the usual irregularities in the ground surface. The ground that is represented in this case is comparatively regular, but the contours for more rugged territory would differ only in the size of the indentations in the loops.

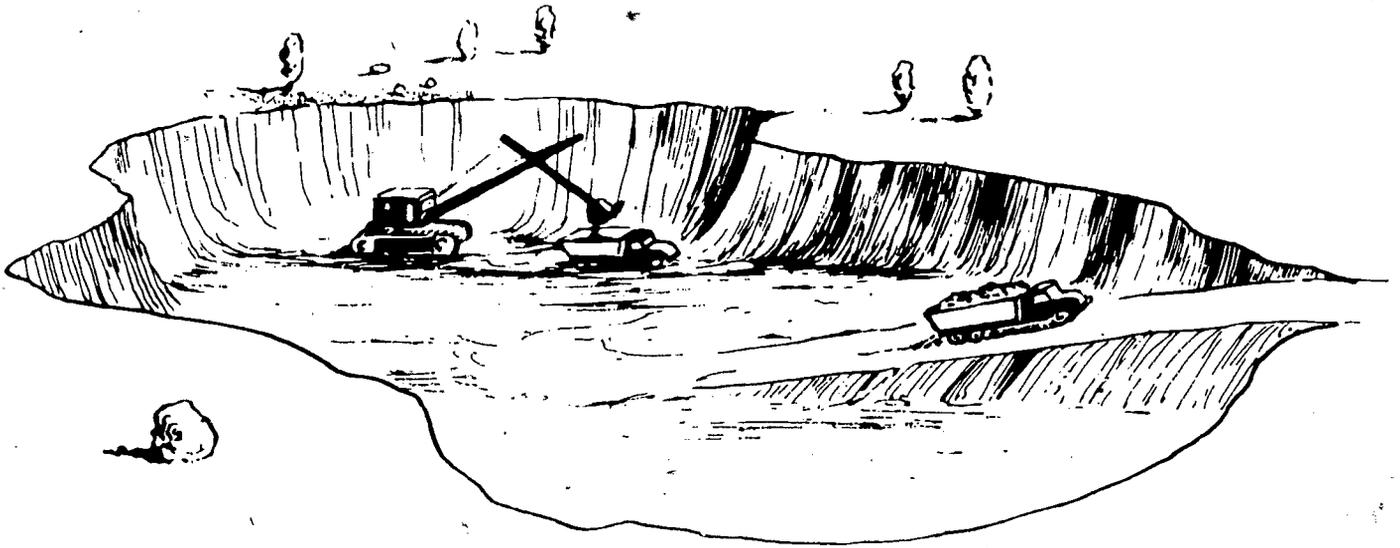
Contours at a depression or hollow in the ground have the same general appearance as contours at a hill. Such formations of fairly irregular shape are represented in Figure 3. In the case of a depression, the lower contours are smaller than, and enclosed by, the higher ones

If both a hill and a depression are shown on a contour map, it is customary to make an obvious distinction between the hill and the depression by drawing short lines perpendicular to the contours at the depression as in Figure 3 (a).

Figure 4 represents a highway cut. Here a portion of a hill has been excavated to accommodate a highway. Notice that where the ground is steep, like the cut near the road, the contours are close together. Where the ground is flatter the contours are farther apart.

Figure 5 represents a highway fill. Here a depression has been filled with dirt from a highway cut. Figure 5(a) illustrates a contour map of a fill section.

Figure 6 illustrates various topographic features. Notice, for example, how a mountain saddle, a ridge, a creek, and flat area are shown with contour lines.



(b)

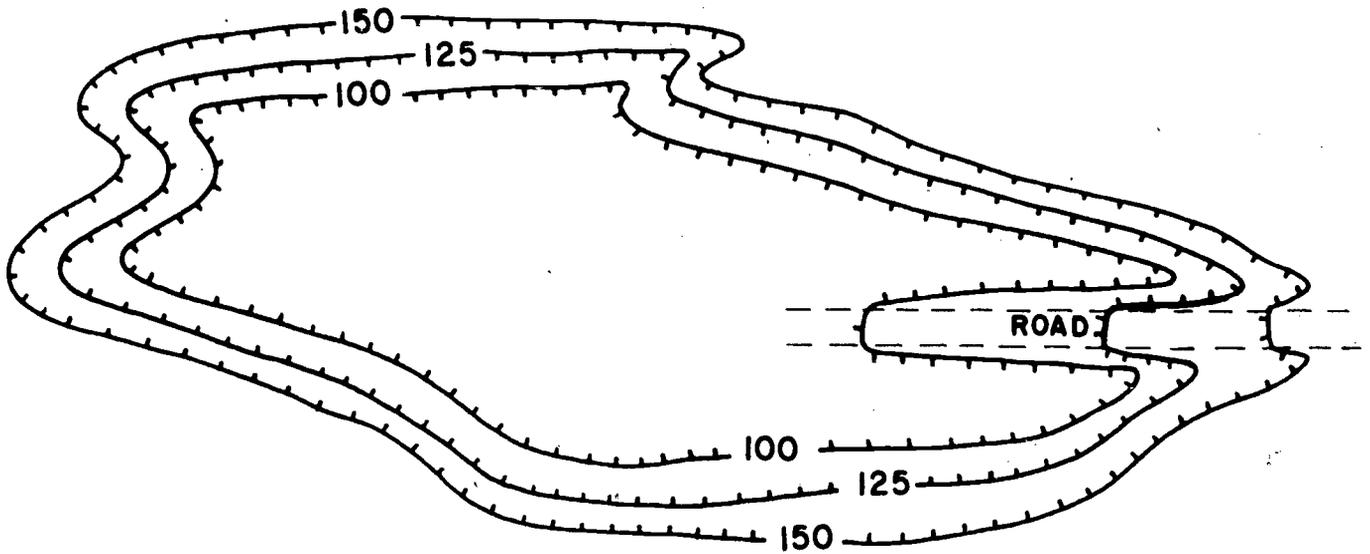
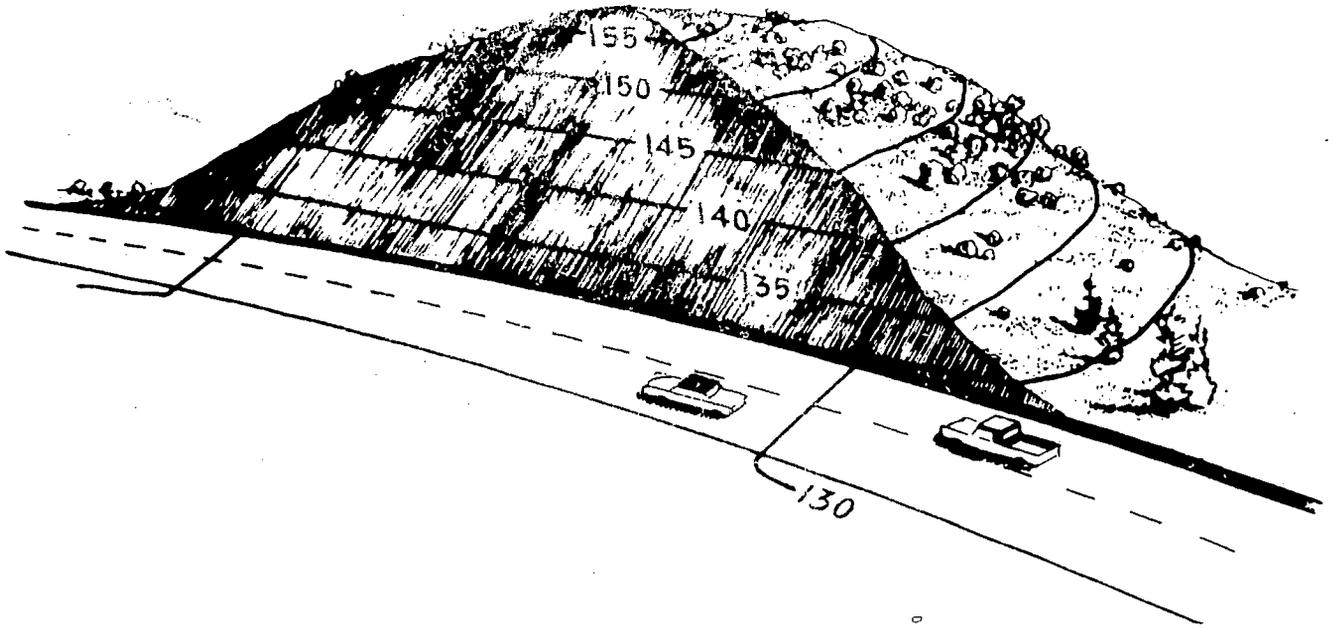


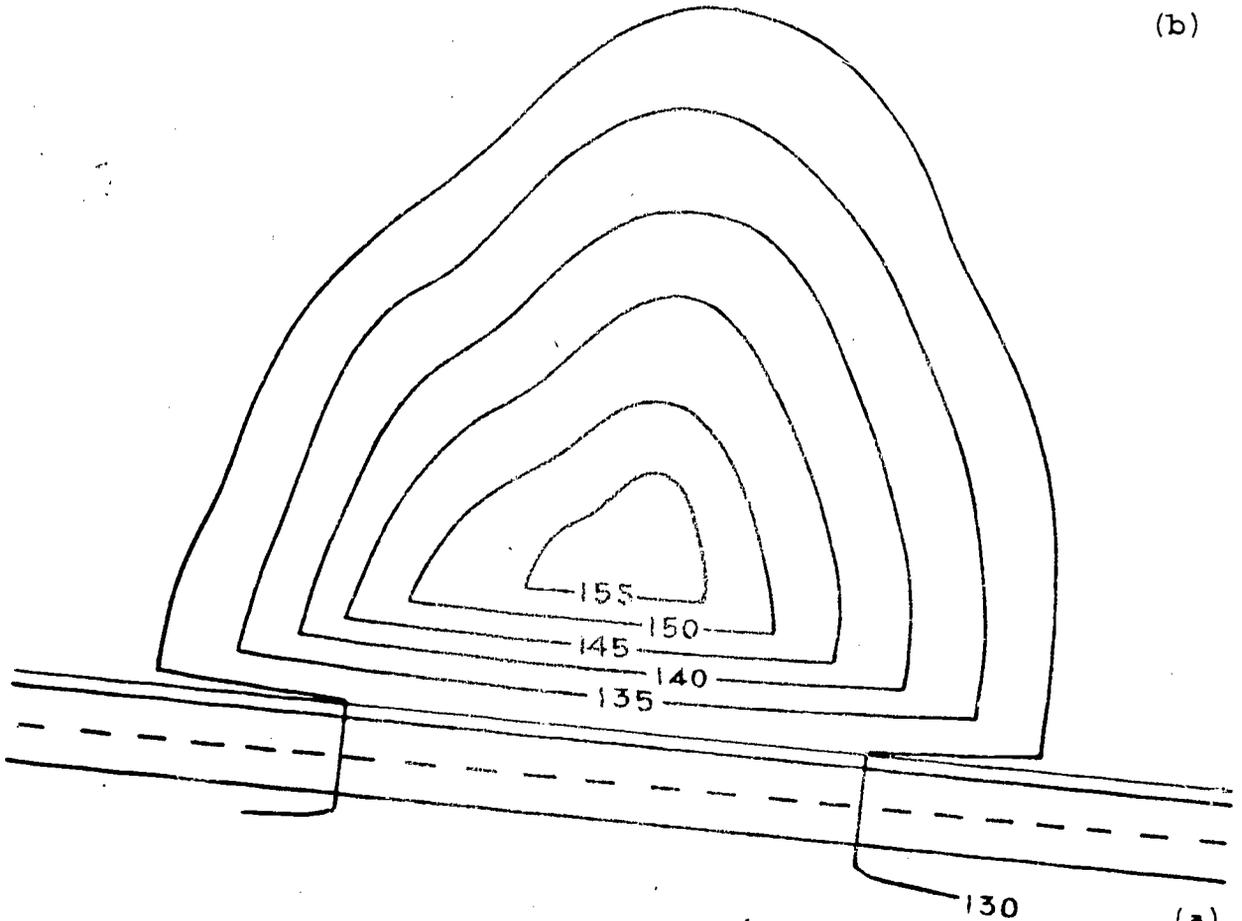
Figure 3

(a)

## DEPRESSION CONTOURS



(b)



(a)

Figure 4

**CUT SECTION**

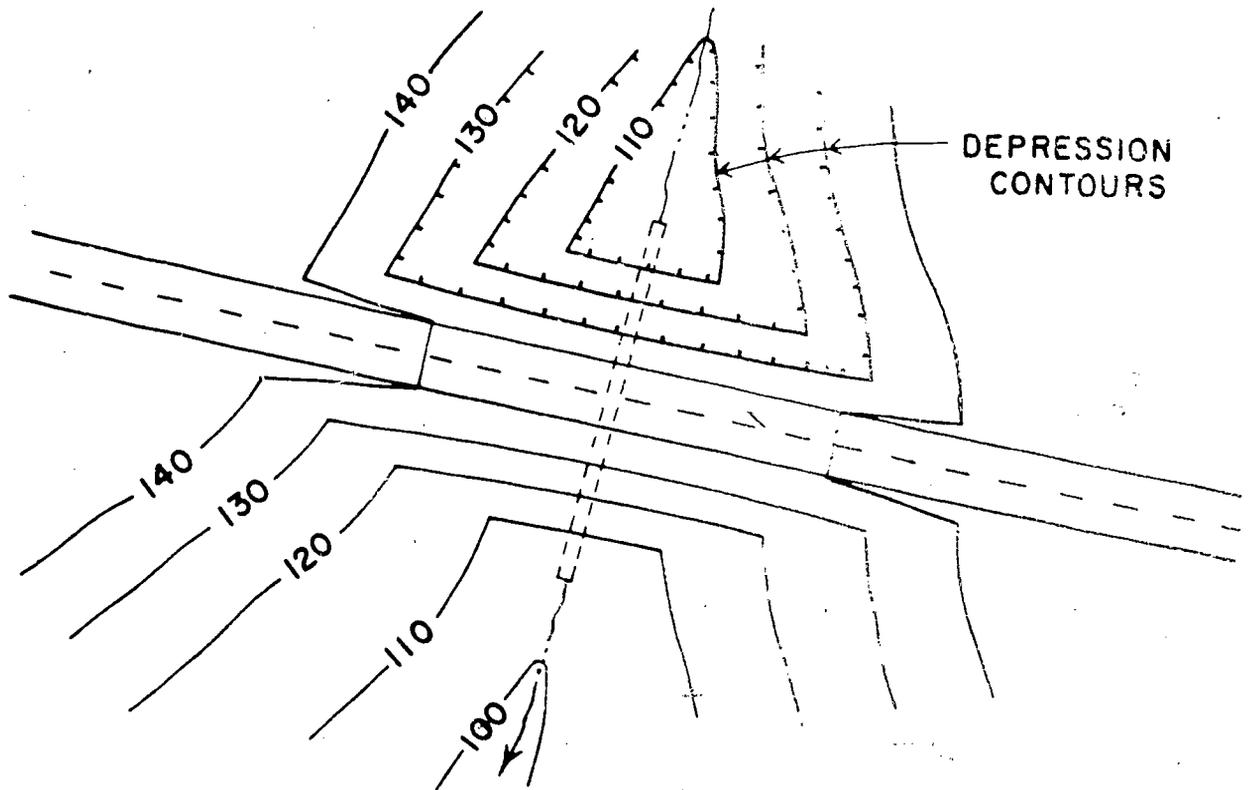
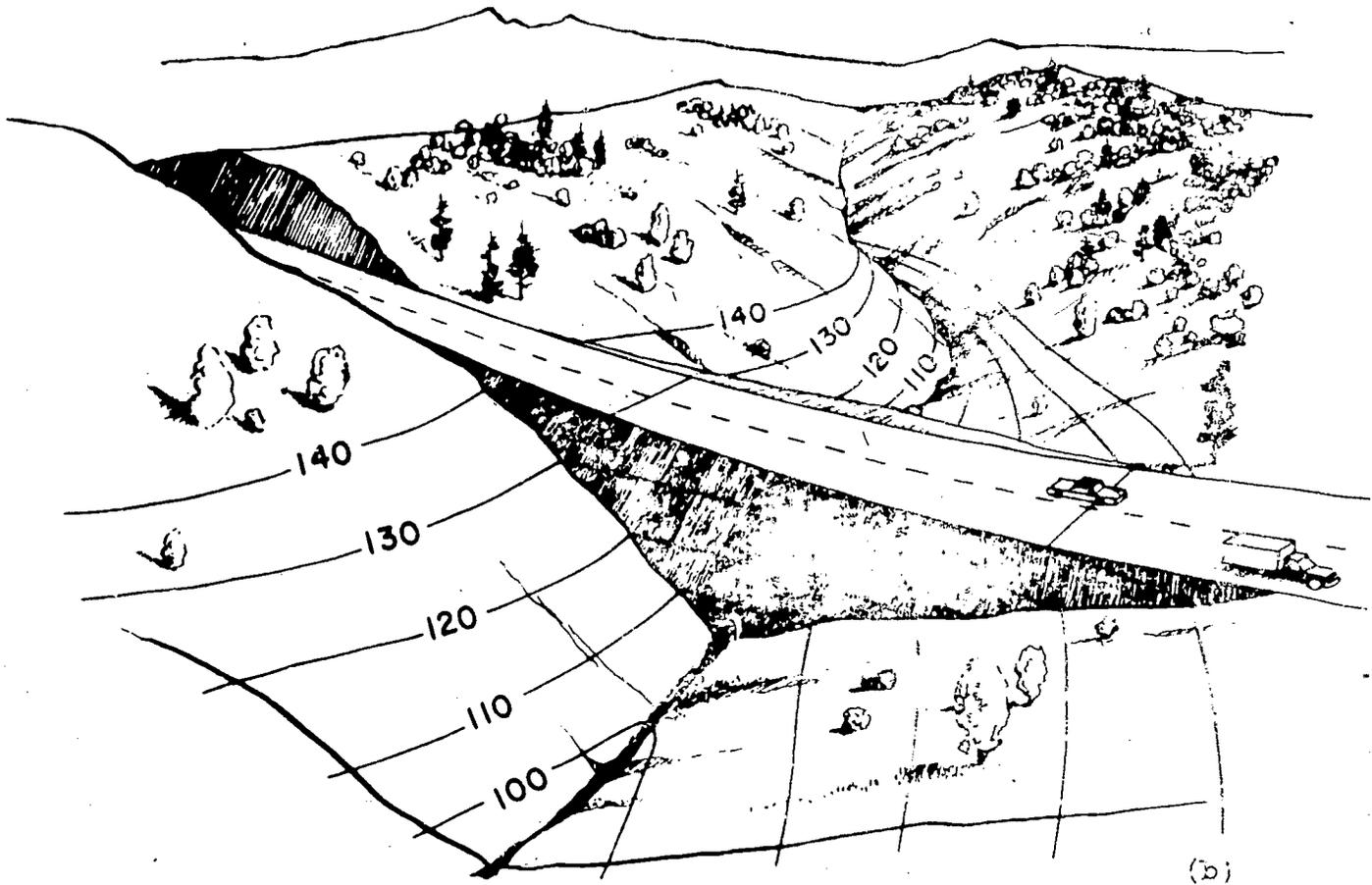
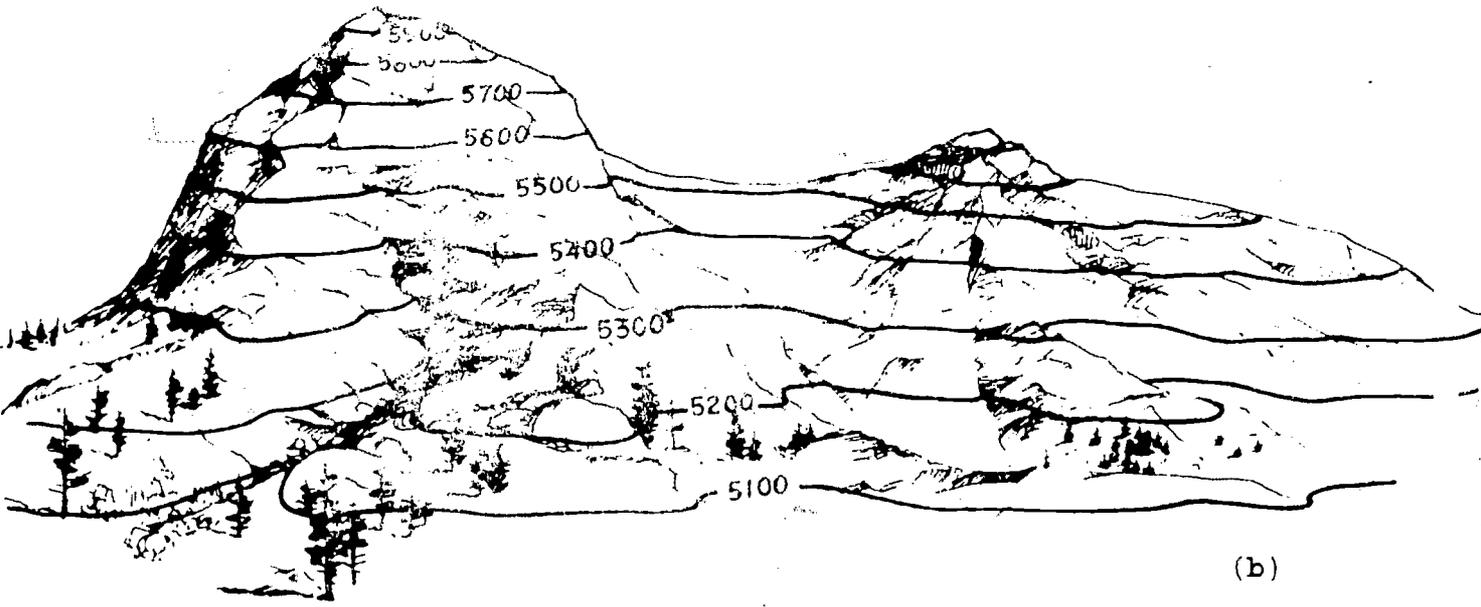


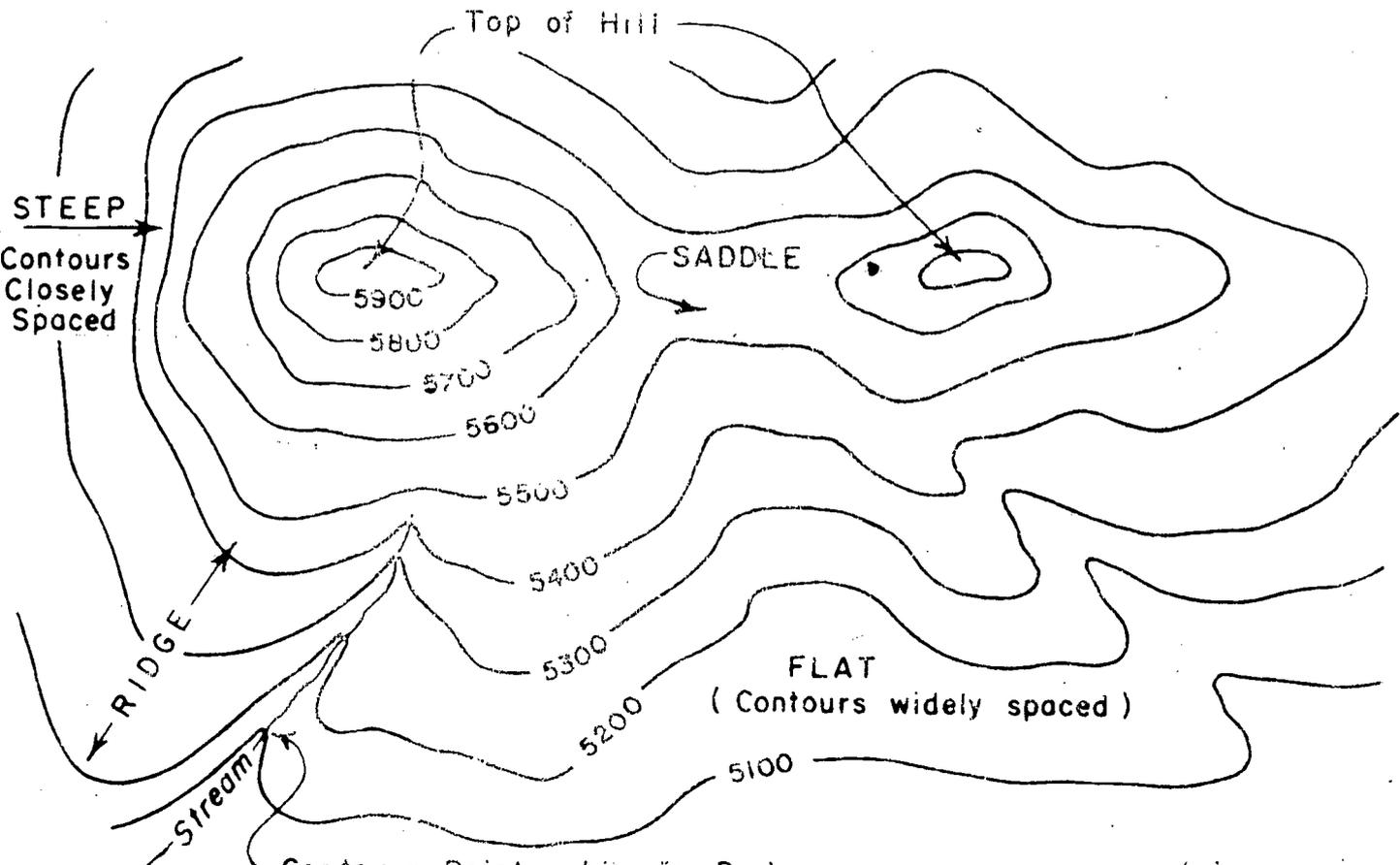
Figure 5

**FILL SECTION.**

(a)



(b)



STEEP  
Contours  
Closely  
Spaced

SADDLE

FLAT  
(Contours widely spaced)

Contours Point uphill for Drainage

(a)

Figure 6

CONTOURS

### 3. Characteristics of Contours

A study of the significance of contours reveals that such lines have the following characteristics:

a. All points on a given contour have the same elevation.

b. A contour on the ground closes on itself. In other words, if a person starts at any point on a contour and follows the path of the contour, he will eventually return to the starting point. A contour may close on a map as indicated in Figures 2 and 3, or it may be discontinued at any two points at the borders of the map as indicated in Figures 5 and 6. Such points mark the limits of the contour on the map, but the contour does not end at those points.

c. The direction of a contour at any point is at right angles to the direction of the steepest slope of the ground at that point.

d. Contours on the ground cannot cross one another, nor can contours having different elevations come together and continue as one line. However, where an overhanging cliff or a cave is represented on a map, contours on the map may cross. The lower contour or contours must then be shown dotted. At a vertical ledge or wall, two or more contours may merge. A final check should be made of a newly completed contour map to avoid crossing or branching contours.

e. The horizontal distance between adjacent contours indicates the steepness of the slope of the ground between the contours. Where the contours are relatively close together, the slope is comparatively steep, and where the contours are far apart the slope is gentle. For example, the surface of the hemisphere in Figure 1(a) slopes more steeply between elevations 0 and 10 than between elevations 10 and 20, and in view (b) the horizontal distance between contours 0 and 10 is less than the distance between contours 10 and 20. Also, where the spaces between contours are equal the slope is uniform, and where the spaces are unequal the slope is not uniform. Thus, in the case of the cone in Figure 1(a), the curved surface slopes uniformly from the base to the apex and the contours in view (b) are spaced equally. In Figures 4 and 5 variable slopes are illustrated.

f. As a contour approaches a stream, the contour turns upstream until it intersects the shore line. It then crosses the stream and turns back along the opposite bank of the stream. If the stream has an appreciable width on the map, the contour is not drawn across the stream but is discontinued at the shore with which it merges.

#### 4. Classification of Methods for Locating Points for Contour Maps

A survey that is made for the purpose of obtaining data for a topographic map consists in running a traverse line and in locating details from points on that line, with additional points obtained by the cross-section method.

##### a. Cross-Section Method

The cross-section method is preferable where the area to be mapped is limited in extent and the map is desired primarily to show the relief. If the area is comparatively long and narrow, as in the case of a survey for the location of a highway or railroad, it is usually advisable first to run an open traverse and to establish stations on the traverse line at 100-foot intervals and at intermediate points where necessary. The elevation at each station is also established. Then, at each station on the traverse line, a perpendicular to the line is extended in each direction from the line, the details of topography are located along the perpendiculars by their distances from the traverse line, and their elevations are established from those of the stations. In other words, cross sections are taken at the various stations.

Another common procedure for locating contours by the cross-section method is illustrated in Figure 7. First the area to be mapped is divided into a series of equal squares by running lines on the ground parallel and perpendicular to a convenient base line. This is called a grid. The elevations of all the corners of these squares and of intermediate high and low points are then determined, the points are plotted, and the contours are located on the plot from the observed elevations by means of interpolation. For convenience each elevation is written on the plot so that the decimal point also marks the position of the point at which the elevation is determined.

In order to simplify the identification of the various points in the notes, it is customary to designate by letters the division lines that extend in one direction, and by figures the lines that run in the perpendicular direction. The point at the intersection of any two lines is then designated by the letter and figure of the respective intersecting lines. In the case illustrated in Figure 7, the line AI is selected as a base; the division lines that are perpendicular to the base are designated as lines A, B, C, etc., and the lines parallel to the base are designated as lines 1, 2, 3, etc. The intersection of the line D and the line 5 is designated D5. If the low point between C5 and C6, whose elevation is 730.8, is 75 feet from C5, it is designated as C5+75. Also, the high point at elevation 746.5, which is located between lines 4 and 5 and A and B, is called A+80, 4+40. In this method the intersection points are located at definite horizontal intervals and, consequently, those points do not generally lie on the contours. The points on the contours are then plotted by assuming that the ground slopes uniformly between the points whose elevations have been established.

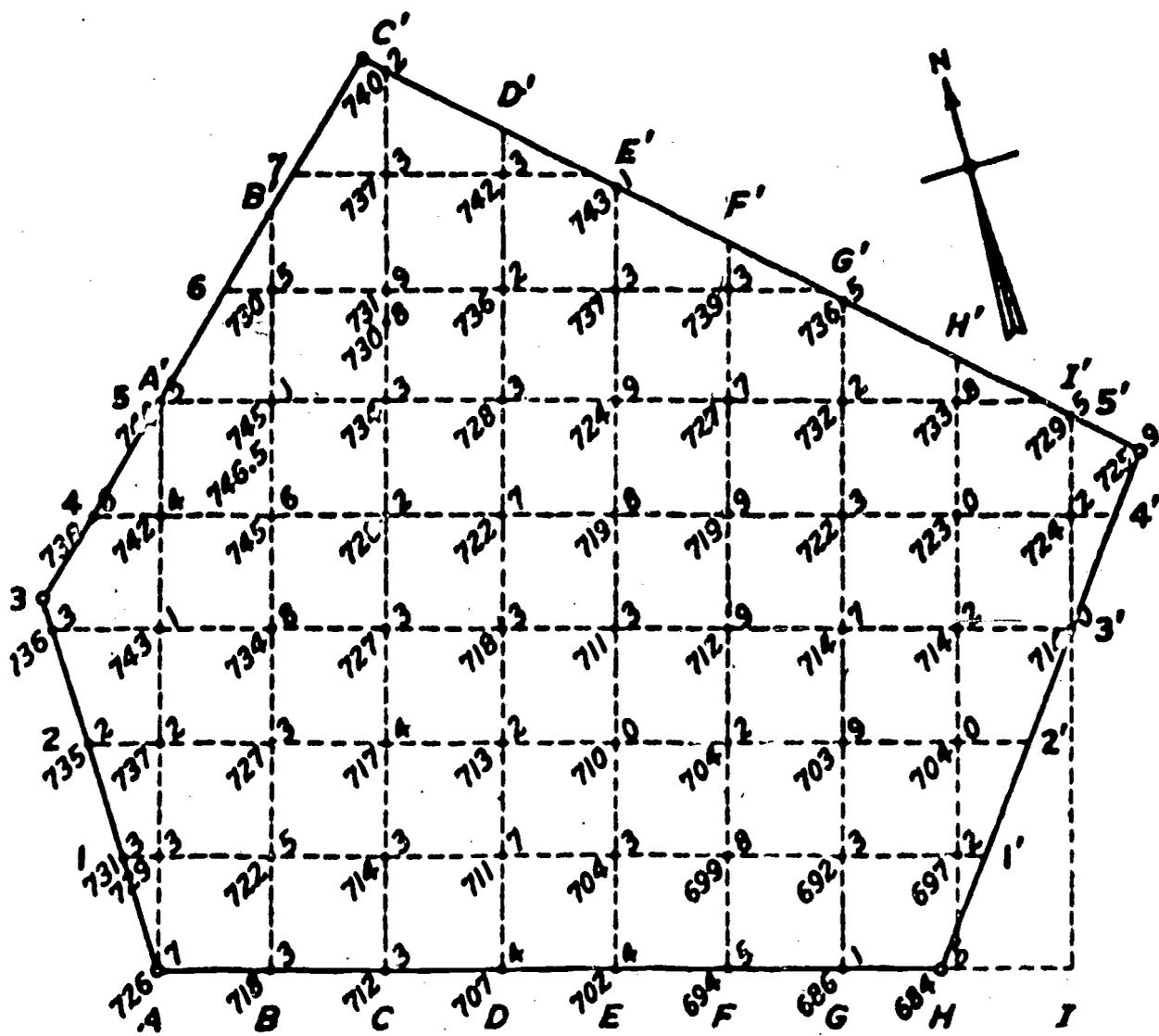


Figure 7

Grid method for location of Contours

b. Other Methods of Locating Points for Contour Maps

In addition to the cross-section method there are several other ways of obtaining data for the preparation of contour or topographic maps. These depend on the use of the maps and the character of the area to be mapped. Some are simple in plan and execution and cover but a few areas; others are complex and difficult and may cover hundreds of square miles. Aerial photogrammetry, for example, may be used for the latter.

The principal instruments used in ground surveys are the engineer's transit, the plane table, the engineer's level, the hand level, and the clinometer.

A common method of locating points for plotting contours is running a traverse and locating the transit at each corner of the traverse and obtaining radial "shots" measuring the distance, horizontal, and vertical angles to locate these points and main topographic features.

Problem:

The upper portion of Figure 8 illustrates a map prepared by a survey crew showing the location of all points and their elevations necessary to prepare a contour map of the area shown at the bottom of the page.

- a. Plot a 5-foot interval contour map.
- b. Compare the 10-foot contours with those shown in Figure 9.

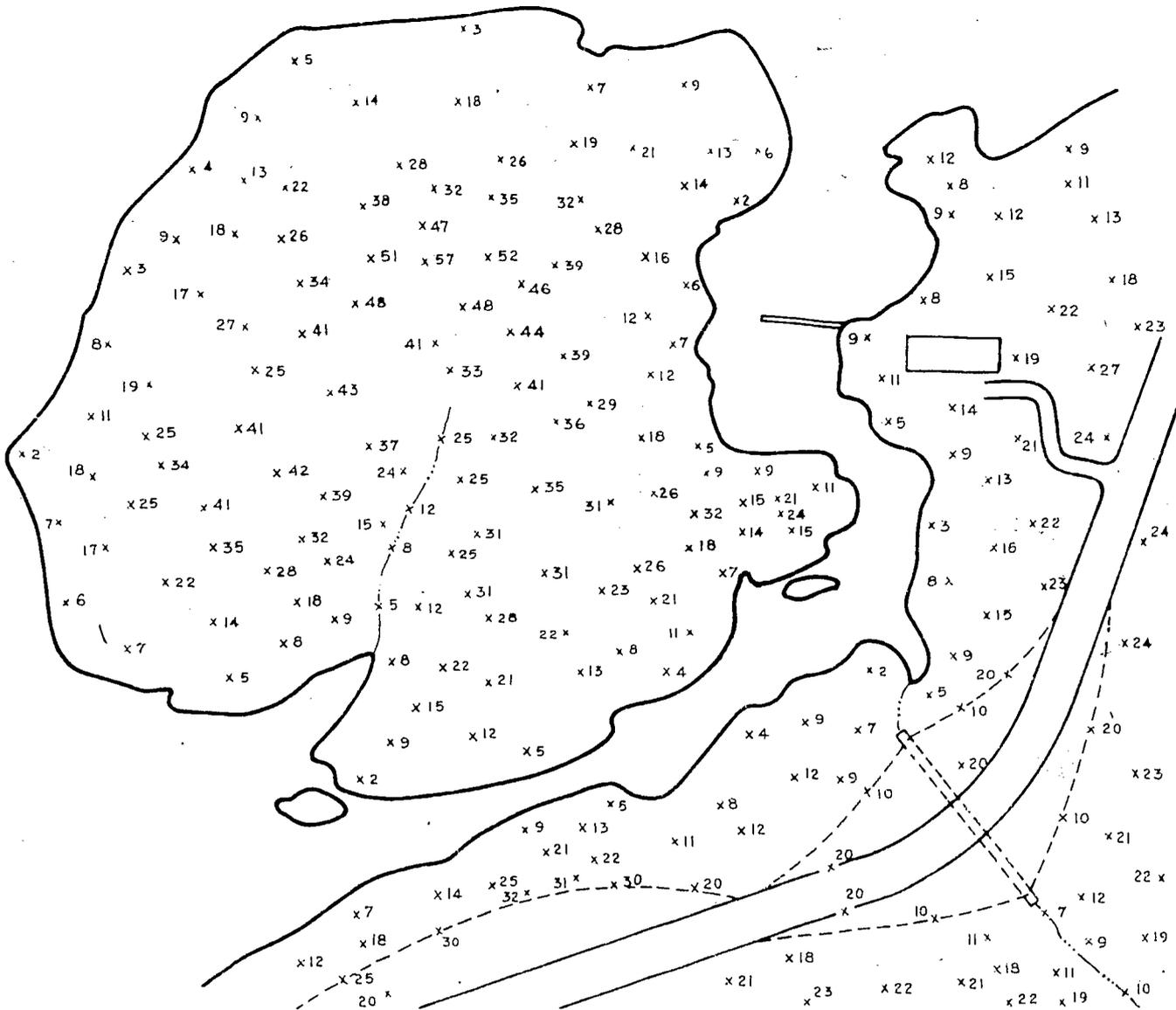
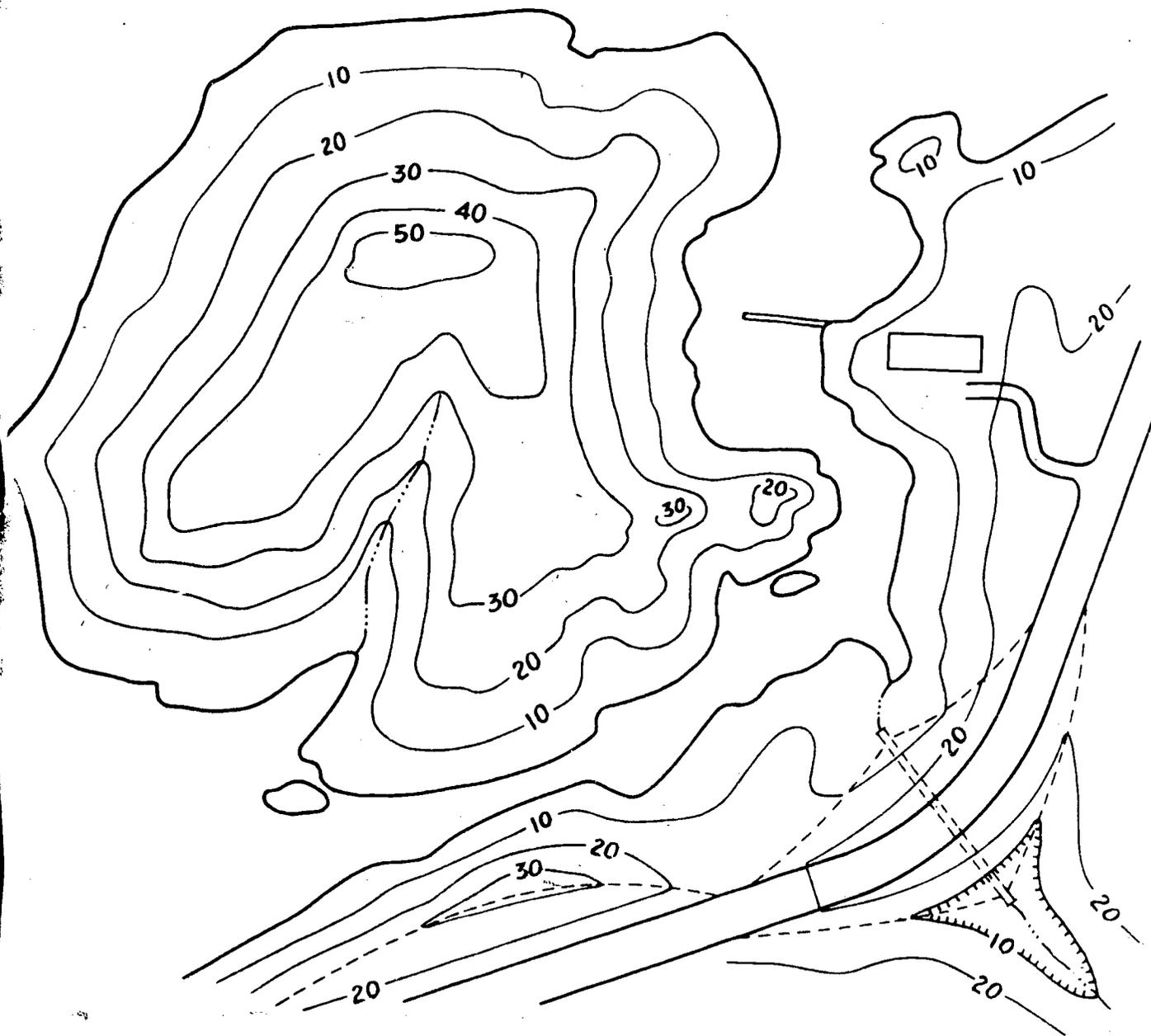


Figure 8



**Figure 9**

273 (Pages 274-341 have been omitted)

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