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ABSTRACT

Two pilot studies investigated the effects of using Instructional Math Play (IMP) Kits, pamphlets with which individuals may play the mathematical game EQUATIONS against a computer program. Twenty-nine junior high students in a high-ability mathematics class completed varying numbers of the kits in five 48-minute sessions during a two-week period; ten selected junior high school mathematics teachers worked through the entire series during a two-and-one-half month period. Pre- and post-tests were designed to evaluate subjects' ability (1) to detect the relevance of a particular idea for solving a problem, and (2) to evaluate a mathematical expression involving that idea. Both groups made significant increases. A significant amount of "unencountered learning" was noted. It was suggested that significant effects may be anticipated with less competent subjects than those involved in the pilot study. (Author/JBW)

ABSTRACT

of

INSTRUCTIONAL GAMING AS A MEANS TO ACHIEVE SKILL IN SELECTING IDEAS RELEVANT FOR SOLVING A PROBLEM

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Two pilot studies investigated the effects of using IMP (Instructional Math Play) Kits, pamphlets with which individuals may play the mathematical game EQUATIONS against a computer program. The highly-branched 105-kit series presents a total of 21 mathematical ideas--five versions of each idea. Twenty-nine junior high school students in a high-ability mathematics class completed varying numbers of the kits in five 48-minute sessions during a two-week period; ten junior high school mathematics teachers in a leadership training program worked through the entire series at their individual paces during a two and one-half month period. Pre- and post-tests focusing on the IMP Kit ideas were designed by the investigators to evaluate the subjects' abilities to detect the relevance of a particular idea for purposes of solving a problem and to evaluate a mathematical expression involving that idea.

On the relevance/selecting pretest the students achieved solutions to 3.86 (mean) of the 21 problems; teachers achieved solutions to 8.50. On the posttest students increased their performance by 4.03 solutions to 7.90, and teachers increased theirs by 7.80 to 16.30. Both increases were significant at the .0005 level. Students were also administered straight computation posttests involving the same 21 ideas: they solved 69% of the problems on the computation test, as compared with their solution of 40% of the problems on the relevance/selecting posttest. The magnitude of the changes in performance after exposure to the IMP Kits suggests that significant effects may be anticipated with somewhat less competent subjects than those involved in the pilot studies. A surprising result was the significant amount of "unencountered learning" by students--that is, the number of ideas that a student missed on the pretest, did not encounter in going through the IMP Kits, but did correctly on the posttest. The distribution of the unencountered learning rate suggests the usefulness of research directed to assessing individual learning styles (e.g., specifics-learners as contrasted with generalizers) in order to find more effective means of individualizing learning. The pilot studies point to controlled experiments for further investigating whether effective ways can be devised to improve skill in selecting relevant ideas for solving problems.

INSTRUCTIONAL GAMING AS A MEANS TO ACHIEVE SKILL IN SELECTING IDEAS RELEVANT FOR SOLVING A PROBLEM*

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INTRODUCTION

Any reasonable appraisal of the history of the Twentieth Century would conclude that mankind desperately needs more rational problem-solving. Our very existence is threatened to a degree that it has never been before. A heavy burden must be borne by the term 'rational' if it is to have reference to effective methods of coping with problems of the magnitude and complexity that are faced on earth in the year 1974. Investigations aimed at improving the rationality of problem-solving are of fundamental importance. Studies in this category include, but are not limited to, (1) those that seek to clarify what rational problem-solving must consist of in order to successfully deal with the important contemporary problems of mankind, (2) those that encourage significant decision-makers to employ such rational problem-solving, and (3) those that cast some light on how to educate problem-solvers to focus attention upon those aspects of a problem that are relevant for its solution. As a tentative and preliminary characterization of what we mean by <u>rational</u> problem-solving in the context of the present study, we stipulate the following:

> A rational approach to solving a problem is one that emphasizes the most powerful ideas and skills -- that is, the <u>indispensably relevant</u> ideas and skills -- that will enable a problem-solver to deal with a problem in a way that achieves his goals to a maximum extent.

The present study deals with six ideas of elementary arithmetic and some problems for which these ideas and combinations of these ideas are not only relevant in the sense of being useful, but are also relevant in the sense of being indispensable for purposes of achieving solutions to the problems. Compared to the big problems of today, these problems are precise and simple. But even with relatively simple and precise problems involving only ideas of elementary arithmetic, the skill of a problem-solver in selecting the indispensably relevant idea or ideas from among the repertoire of ideas that he in some sense "understands" is not a trivial skill. This study represents a beginning

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effort in seeking to learn how such skill can be achieved by relatively knowledgeable and competent persons with respect to relatively well-defined problem-solving. The hope, of course, is that what is learned at this simple level can to some significant extent be extended to more complex problems.

Instructional Math Play (IMP) Kits

This preliminary report presents some of the findings of a pair of pilot studies in the use of innovative materials being developed that use instructional gaming and computer-assisted instruction to teach mathematics. These materials are called IMP (Instructional Math Play) Kits. The IMP Kits are 16-page pamphlets that, in effect, are simulations of a computer with which a player can play EQUATIONS: The Game of Creative Mathematics (Allen, 1964). In a highly-branched learning program, the computer is programmed to play, not as a good player, but as a good teacher. Each pamphlet allows the player as many as 2,000 to 3,000 different pathways, depending upon the sequence of moves that he makes, and each pamphlet is designed to present and emphasize one lesson in mathematics. A total of 21 different ideas are presented and emphasized in the series of 105 IMP Kits developed thus far -- five different versions of each idea. As a player goes through an IMP Kit, each move he makes contributes to the generation of a unique code name, which the player looks up in a set of tables in the IMP Kit to discover what play the computer makes in response. Although there are thousands of different pathways that a player may take through a kit, there are only three types of pathways:

1. ones in which the lesson of that kit is not presented (in which case the player is instructed to play the kit again and try some different moves),
2. ones in which the lesson is presented and the play indicates that the player did not understand the lesson (in which case the player will move on to the next kit and encounter later kits that contain the same lesson), and
3. ones in which the lesson is presented and the play indicates that the player did understand the lesson (in which case the player will be instructed to move on to the next kit, but to skip the later kits that contain the same lesson).

METHODS

The use of a series of 105 IMP Kits was pre-piloted with five experienced players of EQUATIONS to determine the appropriate time to allow for administration of the learning program and of the pretests and posttests.

Subjects

Two different groups of subjects participated in these pilot studies: 29 members of an eighth-grade class from Tappan Junior High

School, Ann Arbor, MI., and 10 junior high school mathematics teachers from Detroit, MI. Each group was selected as the most knowledgeable and highest-achieving group of its kind that was conveniently available to test these materials.

The students were in one of the three top-track eighth-grade mathematics classes at Tappan Junior High School where the total eighth-grade enrollment was 254, so all members of the experimental student group were in the top one-third in mathematics achievement in this school. Nearly 90 percent (26 of the 29 students) had parents engaged in professional or academic occupations. Ten of the students had at least one year of experience in playing the EQUATIONS game in elementary school, and the other 19 had at least one month's experience as part of their studies in this eighth-grade class. There were 19 girls and 10 boys in the group.

The 10 teachers were also a specially-selected group, having been chosen by the Detroit Director of Junior High School Mathematics Instruction for a year of academic leave during 1973-74 to enroll in a special leadership training program. Part of their training consisted of learning to teach other teachers how to introduce instructional gaming into the classroom. Two of the 10 had previously completed a two-credit course at the University of Michigan in instructional gaming. All of them had learned to play EQUATIONS to some extent before this study began. There were six women and four men in the group.

Experimental Treatment

The learning experience for both the student and teacher groups consisted of working through the set of 105 IMP Kits. However, the method for doing so in the two groups was significantly different. The student group spent six 48-minute classroom sessions spread over a two-week period playing EQUATIONS individually in the IMP Kits. The first session was devoted to instruction on how to use the IMP Kits and to keep the necessary records, and the remaining five sessions were devoted to work in the IMP Kits. Since the students worked at different rates, there were individual differences in the number of kits completed during the two-week period by each student. Only two students were able to finish the set during the allotted time. Since the procedure allowed for branching and skipping a number of kits, even these two did not work through all 105 kits; rather, they did those kits that their performance indicated were appropriate for them to do. Hence, only two of the students encountered in some sense all 21 of the mathematical ideas being presented in the lessons of this series of IMP Kits; the other 27 students encountered only some of the 21 ideas. At the conclusion of the two-week period, most of the students who had not finished asked if arrangements could be made so that they could complete the kits. To gain some indication of how interested they really were in completing the kits, it was made somewhat inconvenient for them by requiring that they come in after school one day to pick up the kits to do over the Thanksgiving holiday. With this procedure 17 of the group picked up the kits, and 12 completed them at home.

The teacher group also had a one-session introduction, but all of their playing through the kits was done on their own time and extended over a period of two and one-half months, including the Christmas vacation. They were receiving university course credits for their participation in the Leadership Program, and the director indicated to them that their performance on these materials would be taken into account in grading them for the course. All ten of the teachers worked through to the final kit in the series, branching and skipping as their performances dictated.

Dependent Variables

The effect of learning experience with the IMP Kits was measured by a specially-constructed series of tests targeted at the 21 mathematical ideas being presented. Each of the tests contained 25 items, four of which did not involve IMP Kit ideas. Answers to these four items were not considered in the evaluation. Versions of these tests were administered as pretests and posttests to determine whether there was any change in scores associated with working through the IMP Kits.

These tests were deliberately designed to be somewhat more difficult than the customary achievement tests which deal with the same ideas. They included items that required subjects to detect the indispensable relevance of an idea for purposes of solving a problem, in addition to knowing how to evaluate a mathematical expression involving that idea. Understanding an idea in the latter sense is all that is required in most of the items on customary achievement tests, but understanding an idea in the former sense is equally important for purposes of practical application of the ideas in real-world problem-solving. To indicate by way of example the kinds of problems included in these specially-designed tests, the instructions to the tests and several of the items illustrating IMP Kit ideas from one of the tests are included here:

INSTRUCTIONS

By writing an X in the Yes or No column, indicate whether or not all of the numbers and operations in Column A can be appropriately ordered and grouped to construct an expression equal to the number listed in Column B.

If your answer is Yes, write that expression in Column C.

	Column A	Column B		Column C	Yes	No
EXAMPLE A	+ x 2 3 4	14	=	2 x (4+3)	X	___

Operations and numbers are grouped by inserting parentheses as in EXAMPLE A.

EXAMPLE B	* 2 3	9	=	3*2	X	___

The * in Column A means "to the power of". Thus 3*2 indicates what is usually written as 3^2.

EXAMPLE C

√ 2 9	3	=	2√9	X	

The √ in Column A means "root of." For purposes of answers here indicate what root you mean by putting a number to the left of the √. Put a 2 if you mean square root; a 3 if you mean cube root, etc.

EXAMPLE D

+ x 2 4 6	11	=			X

There should be no entry in Column C when your answer is No.

	Column A	Column B		Column C	Yes	No
4.	* √ 6 6 7	7	=			
5.	÷ * 1 2 9	3	=			
6.	* * 1 3 6	6	=			
7.	− − 1 3 5	7	=			
8.	÷ ÷ 1 2 4	8	=			
17.	÷ ÷ 1 3 5	19	=			

Readers can gain some impression of the level of difficulty of these items by working the problems themselves. The appropriate answers are included in the Appendix. In scoring these tests, an item was counted correct for YES answers only if an appropriate entry was made in Column C.

For the student group one version of these tests (Test A) was administered as a pretest to 15 members of the group on the Friday preceding the two-week period, and another version (Test B) was administered to the other 14 members. On the Monday following the two-week period, the 15-member group received Test B as a posttest, and the 14-member group received Test A. One week later after the 17 students had worked on the remaining IMP Kits at home, a post-posttest (Test C) was created and administered to students who had not finished the 105-kit series during the five classroom periods. Students were allowed 20 minutes to complete these tests, and it was evident that this was ample time because when given a two-minute warning, practically all of the students immediately handed in their completed tests. In addition, because of some of the experience in this pilot run in the classroom, two special questionnaires were prepared and administered during the week following the two-week period—one by the classroom teacher directed to how the students felt about this kind of learning experience and the other by the experimenters directed to what the students believed accounted for their doing correctly on the posttest some problems which they did incorrectly on the pretest and which involved IMP Kit ideas that they did not encounter in their experience with the IMP Kits. Also a regular computation test involving the IMP ideas but not the relevance feature was prepared and administered to the group 22 days after the two-week experimental period. The classroom teacher plans to administer additional post-posttests during the course of the remainder of the academic year.

For the teacher group, Test A and Test B were each administered as pretests to half of them before they began working through the IMP Kits, and after they finished the IMP Kits two and one-half months later each five-person group received the other test as a posttest. The teachers were allowed all the time they wished to complete the tests, and took from 35 to 55 minutes (an average of 46 minutes) to do them.

RESULTS

The data from this pilot study suggest several experiments to design and execute for which the results are likely to show highly significant differences. Among these are:

1. that students do better on the relevance/selecting tests after exposure to the IMP Kits than before,
2. that teachers do better after exposure to the IMP Kits than before,
3. that selected junior high school teachers do better both before and after than selected junior high school students, and improve more in their performance than the students by virtue of exposure to the IMP Kits,
4. that the tests involving the skill of selecting the indispensably relevant ideas for solving the problem are harder than tests involving only computation with the same ideas,
5. that students do better after exposure to some of the IMP Kits than before even on IMP Kit ideas that they have not been exposed to, and
6. that boy students do better than girl students in the extent to which they do better after exposure to the IMP Kits than before on ideas that neither have been exposed to in the IMP Kits, while the girls do better than the boys on ideas which both have been exposed to in the IMP Kits.

The performances on the relevance/selecting test of the students and the teachers both before and after exposure to the IMP Kits are summarized in Table 1. On the pretest the students achieved solutions to just 3.86 (mean) of the 21 problems, while the teachers achieved solutions to 8.50. On the posttest the students increased their performance by 4.03 solutions to 7.90, and the teachers increased theirs by 7.80 to 16.30.

The statistical significance of the increase in performance by both the student group and the teacher group is summarized in Table 2. The mean increase of the students of 4.03 solutions is significant at the .0005 level ($t = 9.11$). The mean increase of the teachers of 7.80 solutions is also significant at the .0005 level ($t = 8.19$). As judged by their performance, after working through the IMP Kits both groups understood about double what they had understood of the IMP Kit ideas beforehand.

Table 1

Pretest, Posttest, and Change Scores on the Relevance/Selecting Test

(21 Problems Involving the 21 IMP Kit Ideas)

	Pretest	Posttest	Change
Students (A) $n_a = 29$			
$\bar{A}$	3.86	7.90	4.03
S_a	1.71	2.99	2.38
S_a^2	2.91	8.95	6.04
Teachers (B) $n_b = 10$			
$\bar{B}$	8.50	16.30	7.80
S_b	2.76	3.23	3.01
S_b^2	7.61	10.46	9.07

Table 2

Change in Performance on Relevance/Selecting Test after Exposure to IMP Kit Ideas

(Difference = Posttest Score - Pretest Score)

	Students	Teachers
n	29	10
$\bar{d}$	4.03	7.80
t_{obs}	9.11	8.19
Significance Level	.0005 $t_{.0005}$ (28) = 3.67	.0005 $t_{.0005}$ (9) = 4.78

Table 3

Difference in Performance Between Students and Teachers on Relevance/Selecting Test

	Pretest	Posttest	Change
F	2.62	1.17	1.50
Significance Level	.05 $F_{.95}(9,28) = 2.24$	not significant	not significant
t_{obs}	not appropriate	7.51	4.03
Significance Level		.0005 $t_{.0005}(37) = 3.58$	.0005 $t_{.0005}(37) = 3.58$
t^*_{obs}	5.00		
f	11.47		
Significance Level	.0005 $t_{.0005}(11) = 4.44$		

The statistical significance of the superior performance of the teachers compared to the students both on the pretest and the posttest and in the amount of improvement is summarized in Table 3. That teachers do better than students is significant at the .0005 level in all three respects (pretest t* = 5.00, posttest t = 7.51, and improvement t = 4.03). Since the assumption of equal variance on the student and teacher pretest scores is disconfirmed at the .05 level of significance (F = 2.62), the t statistic is inappropriate; instead the Satterhwaite approximation of the Behrens-Fisher t* distribution is used to test the hypothesis of equal means for the teachers and students on pretest scores. (See Winer, pp. 41-44.)

The comparative performances of the students on the relevance/selecting test and the computation test along with the statistical significance of the difference in scores is summarized in Table 4. One of the students had transferred from the class by the time that the computation test was administered, so a total of 28 students was involved in the comparison on this pair of tests. For the computation test, 19.4 (mean) of the students were correct over the 20 IMP Kit ideas compared. (One IMP Kit idea that did not lend itself to presentation in the computation test was not used in the comparison.) On the relevance/selecting test, the number of students correct was only 11.3 (mean) per problem. Another way of stating it is that the students got 69% of the problems correct on the computation test, but only 40% on the relevance/selecting test. The difference is significant at the .0005 level (t = 5.17).

Table 4

Comparison of Number of Students Who Responded Correctly to Problems Involving 20 IMP Kit Ideas Presented in (a) a Relevance/Selecting Test and (b) a Computation Test

(28 Students)

	Relevance/ Selecting Test	Computation Test	Difference
n = 20			
$\overline{X}$	11.30	19.40	8.10
S_x	8.24	8.25	7.01
S_x^2	69.61	68.02	49.12
t			5.17
Significance Level			.0005 $t_{.9995}(19)=3.88$

The comparative performances of the boy students and the girl students in learning ideas from the IMP Kits, both those ideas encountered in their exposure to the kits and also those not encountered, are summarized in Table 5. The rate at which students learned ideas that they did not encounter in going through the IMP Kits is here referred to as their "unencountered learning rate." One of the most surprising results of this pilot study was a significant amount (at the .0005 level, t = 6.444) of such unencountered learning; the mean unencountered learning rate for the entire group of students was .132. The unencountered learning rate for each student is derived by dividing (a) the number of ideas such that the student both missed those ideas on the pretest and also failed to encounter them in going through the IMP Kits into (b) the number of those ideas that were correctly done on the posttest. The encountered and overall learning rates are derived similarly. These definitions of the learning rates assure, of course, that all effects will be in the positive direction. However, by an even more stringent definition of unencountered learning that would allow for effects in the negative direction, there is still a significant amount (at the .01 level, t = 2.841) of such unencountered learning. See Table 6.

As would be expected the encountered learning rate for the entire group was significantly higher (.0005 level, t = 6.685) than the unencountered learning rate. The mean encountered learning rate was .404 and the overall was .287.

Table 5

Comparisons of Learning Rates (Encountered, Unencountered, and Overall) of Boy Students and Girl Students

	Encountered Learning Rates			Unencountered Learning Rates			Overall Learning Rates		
	Boys	Girls	Total	Boys	Girls	Total	Boys	Girls	Total
n	9	20	29	9	20	29	9	20	29
$\overline{X}$	.371	.420	.404	.193	.105	.132	.290	.285	.287
S_x	.243	.211	.218	.108	.097	.107	.128	.144	.137
S_x^2	.059	.044	.048	.012	.009	.012	.016	.021	.019
t_{obs} (Total pre-post)			9.990			6.444			11.264
Significance Level			.0005			.0005			.0005
	$t_{.9995}(27)=3.690$			$t_{.9995}(27)=3.690$			$t_{.9995}(27)=3.690$		
F (Boy-Girl)			1.333			1.219			1.260
Significance Level			NS			NS			NS
	$F_{.75}(8,19)=1.43$			$F_{.75}(8,19)=1.43$			$F_{.75}(8,19)=1.43$		
t_{obs} (Boy-Girl)			2.035			2.254			.067
Significance Level			.05			.025			NS
	$t_{.95}(27) = 1.703$			$t_{.975}(27)=2.052$			$t_{.60}(27) = .256$		
t_{obs} (Total Encountered-Unencountered)	6.685								
Significance Level	.0005								
	$t_{.9995}(27) = 3.690$								

There were also interesting differences in performance between boys and girls. The girls had a significantly (.05 level, borderline .025 level, t = 2.035) higher encountered learning rate (.420) than the boys (.371), while the boys had a significantly (.025 level, t = 2.254) higher unencountered learning rate (.193) than the girls (.105).

A definition of unencountered learning that would allow for an indication of "unlearning" is reported in Table 6. The test results show that some students do problems involving the same idea correctly on the pretests and incorrectly on the posttests. Interpreted as "noise" and assuming that there is the same amount of noise in the data on students that do problems incorrectly on the pretest and correctly on the posttest, there is a question as to what extent the indicated gains on the unencountered items can be attributed to noise, rather than to "learning" in some sense. The more stringent definition of unencountered learning (ULR_s) discounts the first definition by the amount of noise.

$$ULR_s = \frac{B-A}{A+B+C}$$

, where A = number of items correct on pretest and incorrect on posttest,

B = number of items incorrect on pretest and correct on posttest,

C = number of items incorrect on both tests.

Table 6

Significance of Stringent Unencountered Learning Rate (ULR_s) and Comparison of Performances of Boy Students and Girl Students

	ULR_s		
	Boys	Girls	Total
n	9	20	29
$\overline{X}$	.193	.061	.102
S_x	.108	.108	.123
S_x^2	.0116	.0116	.0151
t_{obs} (Total Pre-Post)			2.841
Significance Level			.01 $t_{.99}(27) = 2.473$
F (Boy-Girl)	1.004		
Significance Level	not significant $F_{.75}(8,19) = 1.42$		
t_{obs} (Boy-Girl)	3.057		
Significance Level	.005 $t_{.995}(27) = 2.771$		

In addition to there still being significant unencountered learning as measured by ULR_s, there was also significantly more of it by boys than girls. Since none of the boys were incorrect on posttest items that they were correct on in the pretest, while some of the girls were, the differences in favor of the boys were even more significant as measured by ULR_s (.005 level, $t = 3.057$).

To provide some further information about the surprising existence of this "unencountered learning," the 21 students who experienced it completed the following opinionnaire:

Name ______________________

On the test you took before you played the IMP Kits, you did not show that you could use this idea:

(Example inserted here of the idea that this student missed on pretest, did not encounter, but did correctly on posttest)

On the test you took after you played some of the IMP Kits, you showed that you could use it.

We are interested in the fact that you seem to have learned this idea during this time. However, the kits you played in class did not present this idea. Please check the reason (or reasons) that best apply:

0 I learned it from another student.

1 I learned it in math class between the two tests, but not from the kits.

3 I guessed at it on the second test, and it turned out to be right.

11 I learned something from what I saw in the kits that helped me to do it on the second test.

8 Other ______________________

The numbers in the blanks on the left side indicate the number of responses to the right that were given by students.

DISCUSSION

The experimental treatment of working through the IMP Kits has a pronounced effect upon the arithmetical problem-solving skills of highly competent junior high school students and teachers on problems that require both knowledge of how to compute and also the skill of selecting the relevant ideas for solving the problem. The magnitude of the effect suggests that significant effects may also be anticipated with somewhat less competent subjects than those in the pilot studies reported here.

The magnitude of the change in performance after exposure to the IMP Kits, in combination with the large difference in performance on the computation test as compared with performance on the relevance/selecting test, indicates that the experimental treatment and dependent variables used in these studies can provide an effective test of whether skill in selecting relevant ideas for solving a problem can be improved. The particular null hypothesis that can be tested is:

> Working through IMP Kits is not an effective method for improving skills in selecting relevant ideas for solving arithmetical problems.

A pair of interesting side issues also emerged: one dealing with generalization and the other with possible differences in learning styles of the two sexes. Through exposure to one set of ideas, some students apparently learn something about a related set of ideas. For example, perusal of performance on various test items suggests that for some students learning something about exponentiation by working through the IMP Kits also increases performance on problems involving the root operation--even though the learner does not encounter the latter idea in the kits. If it turns out to be more generally the case that some groups of learners tend to generalize better in certain kinds of learning environments than other groups, it will be of interest to study the attributes of those groups and those learning environments that contribute to this ability. The association between sex and generalization detected in this pilot study with the students suggests the usefulness of research devoted to determining attributes of learners that will detect such strengths and differences in learning style more effectively. Knowledge of such properties may well lead to more effective ways of individualizing learning. For example, with respect to individualizing learning environments for generalizers and specifics-learners, perhaps some grouping according to performance measured by the unencountered learning rate--e.g., upper quartile and lower quartile--would serve adequately. Such individualizing should certainly be accompanied by efforts to learn more about effective means of enabling all learners to become strong generalizers. Hopefully, achieving greater skill in selecting relevant ideas for solving problems will help learners to generalize their experiences more effectively.

The main results of these pilot studies point rather directly to experiments for investigating the initial question raised in this report: Can effective ways for improving skill in selecting relevant ideas for solving problems be devised? Instructional gaming--or, to be more precise, simulation of instructional gaming in the form of IMP Kits--may be an example of one effective way to improve such skill. In terms of the kind of data generated in these studies, one measure of the lack of such skill would be a discrepancy between a subject's performance on the computation test and his performance on the relevance/selecting test. The comparison of performances for each subject on such tests would identify ideas that the subject does understand in the sense of being able to use those ideas in making computations, but does not understand in the sense of having sufficient skill to detect the relevance of those ideas in solving a somewhat broader problem and selecting them to solve it. A pair of such tests administered before an experimental treatment would provide a difference score indicating the extent of the subjects' lack of skill in selecting relevant ideas; a pair of posttests, another difference score, indicating the extent of any change in such skill associated with the experimental treatment. Such data were unfortunately not generated in these pilot studies; the agenda for future investigation will give high priority to remedying this deficiency.

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Appendix

A. Some Suggested Responses to Sample Items from Relevance/Selecting Test

	Column A	Column B	Column C	Yes	No
4.	* √ 6 6 7	7 =	6 √7 * 6	X	
5.	÷ * 1 2 9	3 =	9 * (1÷2)	X	
6.	* * 1 3 6	6 =	6 * (1*3)	X	
7.	- - 1 3 5	7 =	3 - (1-5)	X	
8.	÷ ÷ 1 2 4	8 =	2 ÷ (1÷4)	X	
17.	- ÷ 1 3 5	19 =			X

B. Ideas of the IMP Kits

1. A + B - C = A - (C - B); especially A + B = A - (0 - B).
2. A x B' ÷ C' = A ÷ (C' ÷ B'); especially A x B' = A ÷ (1 ÷ B').
3. A = A * 1.
4. 1 = 1 * A.
5. 0 = 0 * A'.
6. A * (B + C) = (A * B) x (A * C).
7. A * (B x C) = (A * B) * C.
8. 1 ÷ [A' * (B - C)] = A' * (C-B); especially 1 ÷ A' = A' * (0-1).
9. 1 = A' * 0.
10. A x [B' * (C-D)] = A ÷ [B' * (D - C)]; especially A x B' = A ÷ [B' * (0 - 1)].
11. A √ B = C if and only if C * A = B.
12. A' √ B = B * (1 ÷ A').
13. 0 = A' √ 0.
14. (A ÷ B') * C = (A * C) ÷ (B' * C).
15. A = 1 √ A.
16. 1 = A' √ 1.
17. A = B' √ A * B'.

18. $1 \div [(A - B) \surd C'] = (B - A) \surd C'$; especially $1 \div C' = (0 - 1) \surd C'$.

19. $(A' \times B') \surd C = A' \surd (B' \surd C)$.

20. $A * (B' \div C') = (C' \div B') \surd A$; especially $A * B' = (1 \div B') \surd A$.

21. $[A' \surd (B' * C')] \surd D = [(A' \div C') \surd B'] \surd D$.

Notes: In standard notation,

a. $A * B$ is A^B,

b. $A \surd B$ is $\sqrt[A]{B}$, and

c. $2 \surd B$ is $\sqrt{B}$.

Variables with primes stand for non-zero numbers.