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ABSTRACT

In an attempt to prepare students for units analysis in the solution of problems in high school chemistry and physics, the author developed learning packets on the factor label method. Thirty-three students used these packets individually during a regular tenth-grade geometry class. There were two packets, each to be used for two days. Prior to the experiment and after instruction, students were tested using parallel forms of an experimenter-developed instrument. Results showed that more than 80 percent of the students achieved at least three of the four learning objectives. Further research is suggested. (SD)

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FACTOR LABEL METHOD OF UNITS ANALYSIS:

A LEARNING PACKAGE FOR SECONDARY MATHEMATICS STUDENTS

by

Margaret Ann Hoile

San Diego State University

May 1, 1974

019 657

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INTRODUCTION AND STATEMENT OF THE PROBLEM

"Unfortunately skills at formulating and translating quantitative statements are given little attention...The language of science ... is usually acquired haphazardly" (Carman, 1972, p. 316). This deficiency in problem solving techniques becomes most apparent in introductory chemistry and physics courses when students are expected to be aware of, and make repeated use of, units analysis in their course work.

Literally, units analysis is setting up problems by analyzing the units of the known quantities and manipulating them so as to obtain the desired units, and hence value, of the unknown quantity. Practically, units analysis involves making any necessary units conversions and then operating with the units as if they were numbers in order to derive the solution to the problem. One form of units analysis used in engineering and physics textbooks is the factor label method. This method involves converting one set of units into another by multiplying by conversion factors written as ratios equal to one, then canceling units as common factors. For an example of the factor label method, see Figure 1.

The purpose of this project was to develop a factor label method learning package and analyze its implementation

in a high school geometry class. The remainder of Chapter 1 contains the rationale and background of the content and approach used.

Problem:

$$60 \frac{\text{miles}}{\text{hour}} = ? \frac{\text{feet}}{\text{second}}$$

Conversion Factors:

$$1 \text{ mi.} = 5280 \text{ ft.} \Rightarrow \frac{5280 \text{ ft.}}{1 \text{ mi.}} = 1$$

$$1 \text{ hr.} = 60 \text{ min.} \Rightarrow \frac{1 \text{ hr.}}{60 \text{ min.}} = 1$$

$$1 \text{ min.} = 60 \text{ sec.} \Rightarrow \frac{1 \text{ min.}}{60 \text{ sec.}} = 1$$

Solution:

$$60 \frac{\text{mi.}}{\text{hr.}} = 60 \frac{\cancel{\text{mi.}}}{\cancel{\text{hr.}}} \times \frac{5280 \text{ ft.}}{1 \cancel{\text{mi.}}} \times \frac{1 \cancel{\text{hr.}}}{60 \cancel{\text{min.}}} \times \frac{1 \cancel{\text{min.}}}{60 \text{ sec.}}$$

$$= 60 \times 5280 \text{ ft.} \times \frac{1}{60} \times \frac{1}{60 \text{ sec.}} = \frac{5280 \text{ ft.}}{60 \text{ sec.}} = 88 \frac{\text{ft.}}{\text{sec.}}$$

Figure 1. The Factor Label Method

Rationale

As previously stated, students' deficiencies in problem solving techniques are most apparent in introductory chemistry and physics courses. The deficiencies may result in the science teacher spending the first three to four weeks of the course introducing the concepts of the factor label method of units analysis. The disconcerted science teacher blames the math teacher for the students' ineptness; the math teacher maintains that units of measure are part of the "physical" realm and if the science teacher wants to use them then it is up to him to teach them. The question then arises, "Why not present the concepts of the factor label method prior to high school

chemistry and physics courses within an integrated mathematics/science lesson, drawing applications from the realm of science and justification from the realm of mathematics?" This question forms the focus of the present project.

Some curriculum groups have made recommendations concerning the inclusion of topics related to the use of units and the factor label method. For example, the Cambridge Conference on School Mathematics (1963) suggests that a piecemeal approach to units analysis, that is, discussing units when the need arises, is all that is necessary. However, the Cambridge Conference on the Correlation of Science and Mathematics in the Schools (1969) was concerned enough about the role of denominate numbers, quantities to which a unit is attached, to devote an entire section of its report concerning elementary schools to them. It noted that denominate numbers are often presented with some sort of "hocus pocus" (p. 46) instead of being treated as natural results in applied mathematics. There is a growing trend toward the integration of mathematics and science at the elementary shool level which will extend into the high school; if this trend continues denominate numbers and units analysis must become vital parts of the mathematics/science curriculum.

How can students obtain a better preparation for units analysis prior to entering their science courses? An obvious answer is to expose the students to some form of units analysis prior to high school chemistry and physics courses. It was decided that learning materials would be developed using the factor-label method for implementation in the high

school geometry class. Most students who take chemistry and physics their junior and senior years take geometry their sophomore year. Besides introducing the students to units analysis, presenting the factor label method at this time serves other purposes. It relates the abstract approach given to length, area, and volume in the geometry course to the units of measure used in daily life, past and present. Also by having to establish equalities involving an unknown quantity, the students review the algebraic skills needed to solve ratio and proportion problems. Thus the factor label method aids in integrating geometry with other mathematics topics as well as with other disciplines.

The first section of this chapter introduced the problem under consideration: students' deficiencies in units analysis, and presented a solution to the problem: factor label method learning materials. This section has presented justification for the work to be done. The following section will present what has been done concerning the factor label method of units analysis in the secondary mathematics curriculum.

Background

Polya (1973, p. 202) states that the "test by dimension is a well-known, quick and efficient means to check geometrical or physical formulas." As well-known as it may be there appears to have been little, if any, research done in the areas of test by dimension or units analysis in recent years. A review of ERIC and the Education Index reveals several articles on the pros and cons of integrating mathematics and science at

the elementary level with the expressed hope that it could someday occur at the high school level. Yet there does not appear any methodology for presenting units analysis in the high school mathematics curriculum.

Space Mathematics: A Resource for Teachers (1972, p.12) notes that "the procedure of writing the units into the computation and then dividing, multiplying, adding, and subtracting units as if they were numbers is not often used in mathematics." The same book states that "some physics and engineering textbooks do use the 'factor label' technique" which at times "offers the best way for one to know what units are involved in the final answer." References to the procedure are made in essentially every high school physics text to the extent of implying the necessity of keeping track of units while working through a problem. Some students pick up the idea as an aid in solving problems but, as most high school chemistry and physics teachers will state, a large percentage never do.

Elementary school mathematics texts may spend a chapter on measurement and conversion from one unit to another but then the technique is dropped upon the administration of the chapter test. Units analysis is generally not mentioned in the secondary mathematics text and even the Cambridge Conference on School Mathematics (1963) only refers to units in a brief appendix.

Since there appears to be no materials or research available on the topic of factor label method in the secondary mathematics curriculum, this project was designed to include

the development and analysis of the implementation of a learning package on the factor label method for use in a secondary math class. The next chapter presents the development of the materials. Chapter III contains an explanation of the implementation. Chapter IV considers the results of the learning package and Chapter V presents the conclusions and recommendations of the project.

Chapter II

DEVELOPMENT OF THE MATERIALS

The purpose of this project was two-fold: 1) to develop a learning package on the factor label method of units analysis; and 2) to analyze its implementation in a high school geometry class. The former will be discussed in this chapter.

A modification of the curriculum development model suggested by Romberg and De Vault (1967) was followed to develop the materials. The major stages of the model (Figure 2) include analysis, pilot, and evaluation. An initial phase of the analysis stage was to establish a task analysis. At that point, the behavioral objectives and the order in which concepts would be taught were determined. With this basis, an instructional analysis took place which resulted in the developed materials and a schedule of presenting the procedures of the factor label method.

Mathematical Analysis

The first stage of the curriculum development model is the analysis stage which involves consideration of the mathematical content to be presented followed by an analysis of how the content should be presented. The remainder of this

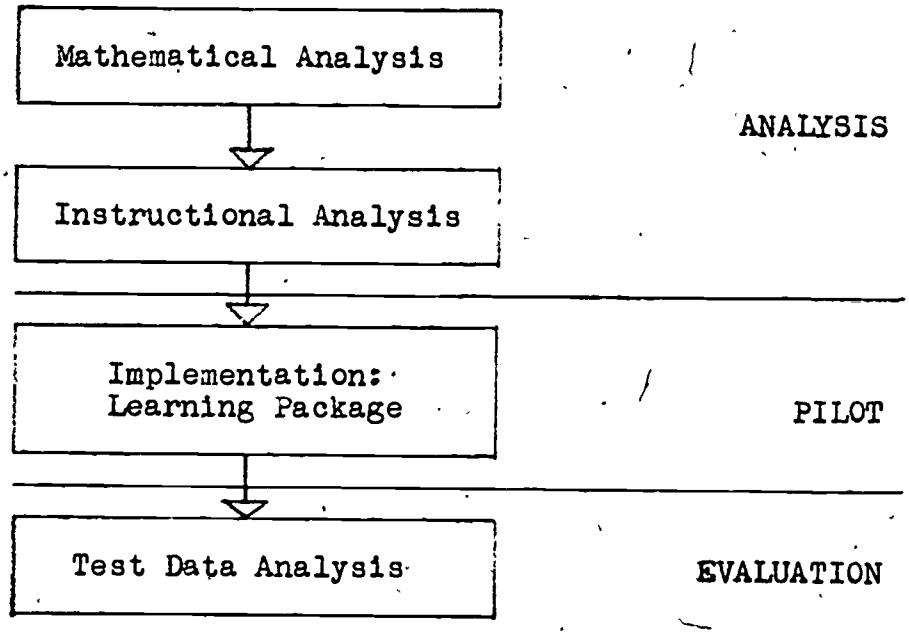


Figure 2. Curriculum Development Model

section contains: 1) an outline of the mathematical content, 2) task analysis, and 3) a list of behavioral objectives.

The content to be presented (Table 1) included definition and application of the factor label method and related mathematical concepts. Since the factor label method relies on the repeated use of ratios, the prerequisite skills identified involve multiplying by fractions and reducing fractions to lowest terms by identifying common factors. Although it was hoped that students at the sophomore level in high school would possess the necessary skills, review of these skills was presented as difficulties arose. The content outline and prerequisite skills were used to establish the task analysis.

TABLE 1

Mathematical Content Outline for the Factor Label Method Learning Package

I. Definition of the factor label method

A. Multiplicative identity

B. Conversion factors

1. Metric system equivalences

2. Other equivalences

II. Use of the factor label method

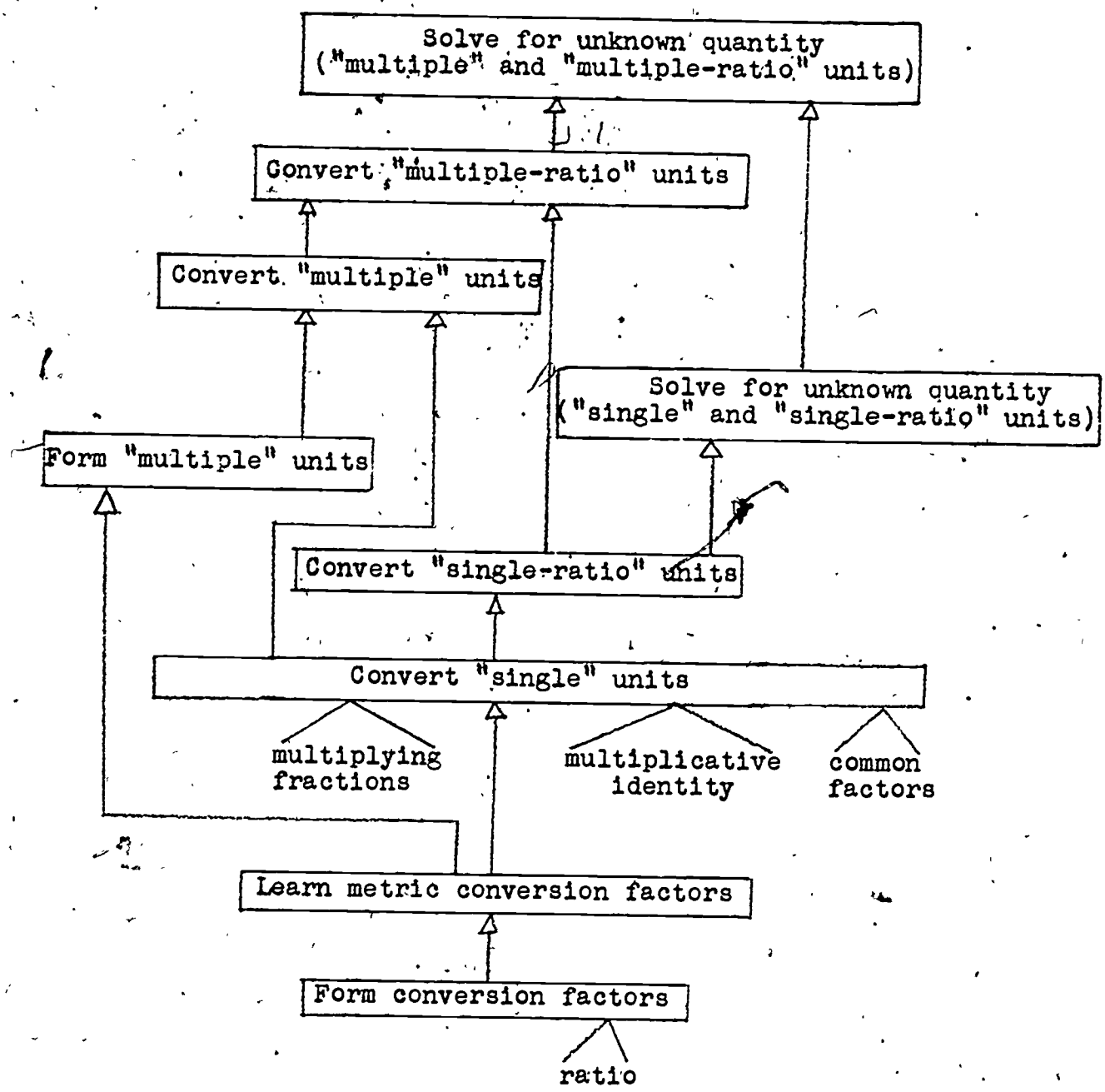
A. Conversion of "singular" units

B. Conversion of "multiple" units

III. Applications: problem solving

Task Analysis

In order to establish a task analysis, reference was made to Developing Individualized Instructional Material by Stuart R. Johnson and Rita B. Johnson (1971). A task analysis involves breaking a learning task down into component tasks, each of which must be mastered prior to mastery of the total task. Therefore a final task, which in this project is using the factor label method of units analysis in problem solving, is mastered by first mastering subtasks. The classification (ordering) of the subtasks suggests a sequence of presentation as does the content outline. The task analysis for this project (Figure 3) was established by considering the complexity of the units involved. Working with, and converting, "single"



Note: ↑ denotes behaviors included in the study.
^ denotes prerequisite behavior or concept.

Figure 3. Task Analysis of Factor Label Method Concepts

units (units involving one unit of measure), and "single-ratio" units (units involving a ratio of two "single" units) was considered prerequisite to working with, and converting, "multiple" (units involving the multiplication of two or more "single" units) and "multiple-ratio" (units involving the ratio of two "multiple" units) units. The desired results of the main subtasks were then used to write the behavioral objectives.

Behavioral Objectives

The overall behavioral objective is that after the implementation of the learning package the students will be able to use the factor label method of units analysis. The learning package was divided into two individualized learning packets with a list of behavioral objectives, achievement of which was considered equivalent to achieving the overall objective. The behavior objectives (Table 2) are in the cognitive domain at the Knowledge, Comprehension, and Application levels of Bloom's Taxonomy (Johnson and Johnson, 1970). Since the second packet relied on mastery of the first packet, the behavioral objectives of the second included the behavioral objectives of the first.

Once the objectives were selected, the criterion measure was established. Criterion measures "allow the collection of evidence of change in behavior, thus giving evidence of instructional effectiveness"(Johnson and Johnson, 1970, p. 31) For the learning packets developed, the criterion measure was performance on a posttest containing eight problems (two

concerning each behavioral objective). Performance on a problem was judged either satisfactory or unsatisfactory depending on whether the student set up the problem correctly using the factor label method or not. The minimum level of acceptable performance for a student to have achieved the overall objective was to score 75 percent or better on the posttest (12½ percent was recorded for each satisfactory use of the factor label method). The percentage of students performing satisfactorily on each problem was considered in determining achievement of the individual behavioral objectives of the learning package.

TABLE 2

Learning Package Behavioral Objectives

1) Given metric units of length and mass, the student should be able to convert to other metric units.
2) Given the conversion factors of any measurement system, the student should be able to convert from one set of "single" or "single-ratio" units to another set.
3) Given the conversion factors of any measurement system, the student should be able to convert from one set of "multiple" or "multiple-ratio" units to another set.
4) The student should be able to solve for "unknown" quantities by working with the units of the "known" quantities.

After determining the mathematical content outline, task analysis, behavioral objectives and criterion measure for the learning package, the minimum level of acceptability of the materials was established. The materials would be considered successful if 75 percent or more of the students scored 75 percent or more on the posttest. This completed the first step of the analysis stage. The next step, instructional analysis, involved the development of the materials needed to present the content according to the task analysis in order to meet the established behavioral objectives.

Instructional Analysis

The previous section dealt with "what" to present, whereas this section deals with "how" to present it. It was decided that an individualized learning packet approach would be used. Two packets were developed: the first deals with the definition of the factor label method and "singular" units, and the second deals with "multiple" units and problem solving. For evaluative purposes, a pretest and a posttest were developed.

The learning packet approach was selected with the philosophy of John F. and Joseph G. Gindele (1972, p.1) in mind. Their philosophy maintains that:

> 1) students will learn better if they know exactly what is expected of them (this is achieved through the utilization of behavioral or measurable objectives), 2) if they are provided with a set of educational experiences to meet those objectives, and 3) if they are able to demonstrate mastery of the material.

Each packet contained a brief introduction on procedure, a list of behavioral objectives, sample problems, in-class assignment and homework assignment. The remainder of this

section presents the details of the development of each packet and the tests to be used to evaluate them.

Packet #1

Packet #1 (Appendix A) was developed to present the concept of the factor label method of units analysis and its use in solving problems involving "single" and "single-ratio" units. The packet was divided into four parts: Packet #1, Day#1 (the material to be completed the first day of class); Packet #1, Day #1, Assignment (homework material); Packet #1, Day #2 (the material to be completed the second day in class); and Packet #1, Day #2, Assignment.

The introduction to Packet #1 includes an explanation of the procedure being used (individualized packets), an explanation of the coding of the pages by packet number and day number, and a list of the desired results (behavioral objectives) that the students are expected to achieve.

The first day's material begins by defining the factor label method. The method used is the factor label method presented in Chemistry: A Modern Course, Merrill Publishing (Smoot, Price and Barrett, 1971): the initial quantity is multiplied by ratios equal to one such that the units can be treated as common factors appearing in the numerators and denominators, and in effect divided out. Since this method involves conversion ratios equal to one, the first problems reviewed the concepts of the multiplicative identity one and conversion factors. Reference was made to Arithmetic for Science Students: A Programmed Discussion by Jay A. Young (1968).

The packet then includes completed sample conversions involving "single" units followed by conversion problems to be completed by the students. The last part of Packet #1, Day #1 and Packet #1, Day #1, Assignment includes discussion of the metric system. The prefixes are defined and sample conversions are demonstrated. This discussion is followed by conversion problems and the assignment. The assignment is to measure parts of the body in inches and then make conversions to the metric sysytem. The figure used in the assignment (fig.1, Packet #1, Day #1, Assignment) and ideas for problems were obtained from Exploring Mathematics On Your Own Series: The World of Measurement by Donovan A. Johnson and William H. Glenn (1961). A reference sheet, All You Need to Know About Metric by the Metric Information Office, National Bureau of Standards, was included for the personal use of the students.

The second day's material introduces the "single-ratio" units. The initial problems involve the comparison between distance traveled (units of length) and units of time. At this point the "chart form" for writing out the conversion problems is introduced (see Packet #1, Day #2, (6), Appendix A). The "chart form" is suggested as an aid in organizing the conversion factors to determine which units may and may not be canceled. Several of the problems presented refer to historic-literary and scientific topics. A main reference for the latter is Space Mathematics: A Resource for Teachers by NASA (1972). The assignment for the second day allows the students to choose between problems of historic-literary and scientific contexts.

As stated, the purpose of Packet #1 is to introduce the factor label method and have the students use it in the conversion of "singular" units. The purpose of Packet #2 is to continue the use of the factor label method to solve for unknown quantities involving "singular" units and to present its use in converting, and solving, problems that involve "multiple" units. How this is accomplished is discussed in the following section.

Packet #2

Like Packet #1, Packet #2 (Appendix B) is divided into four parts: Packet #2, Day #1; Packet #2, Day #1, Assignment; Packet #2, Day #2; and Packet #2, Day #2, Assignment. The introduction presents the desired results the students are expected to achieve upon completion of the packet. The first ideas discussed in Packet #2 are solutions of problems in which the units of the answer are known and are "singular" in form. The approach demonstrated considers the units of the known quantities and the unknown quantity. The units of the known quantities are multiplied or divided so that all units are canceled except the known units of the answer. The students are instructed to use the "chart form" and the factor label method whenever applicable. Sample problems are given which are followed by problems for the students to solve. The Packet #2, Day #1, Assignment continues the lesson by consisting of problems involving solving for unknowns with known "single" and "single-ratio" units of measure.

Packet #2, Day #2 begins with "multiple" units (units involving the multiplication of "single" units) in which the symbol "-" denotes units multiplication as in "ft.-lbs." After converting, and solving problems involving, "multiple" units, the students start into solving problems with "multiple-ratio" units (units involving the multiplication of "single-ratio" units). The same procedure of operating with the units of the known quantities so as to cancel all but the desired units of the unknown quantity is demonstrated. The final assignment of the packets, Packet #2, Day #2, Assignment includes problems on the material presented in both packets. Ideas for problems used in Packet #2 were obtained from the same references used for Packet #1. Packet #2 concludes the materials developed for introducing the factor label of units analysis in the secondary mathematics curriculum.

Evaluative Tests

To evaluate the need for, and success of, the packets, pre- and post- tests parallel in form were developed. Each test contains two problems designed for testing each of the four objectives (eight problems total). As a midway check for the students and the instructor, a quiz was developed to test the students on the material presented in Packet #1. These tests are included in Appendix C.

The packets and evaluative tests form the learning package developed to present the ideas of the factor label method in the secondary mathematics curriculum. The learning package is the result of the mathematical analysis phase

discussed in the first part of this chapter, and constitutes the instructional analysis phase of the project.

Summary

This chapter has dealt with the "what to present," and the "how to present it," of introducing the factor label method to high school sophomores. In essence, this chapter has discussed the first purpose of the present project: the development of a learning package. The package was constructed by following a modification of the curriculum development model of Romberg and De Vault (1967) through the analysis stage. The second stage of the model is the pilot stage which includes trying out the learning materials. The next chapter will present the implementation of the learning package with the analysis of the results following in Chapter IV.

Chapter III

IMPLEMENTATION OF THE LEARNING PACKETS

The first stated purpose of this project was to develop a learning package on the factor label method of units analysis. The second purpose was to analyze its implementation in a high school geometry class. It is the purpose of this chapter to present the details of the implementation with the analysis of results following in Chapter IV. The items to be discussed include the student population and environment, the teacher and facilities, the schedule of presentation, and the conduct of the study.

Student Population and Environment

The learning packets were implemented at University of San Diego High School from March 18, 1974 to March 28, 1974 in a college preparatory geometry class. The class consisted of 33 students of which one was a freshman, 25 were sophomores, six were juniors, and one was a senior.

University of San Diego High School is on a rotating-drop schedule. Each class is assigned a block letter, A through G. On Monday, A block is first period; blocks B through E are then held during the corresponding second through fifth periods of the school day; F block drops (does not meet); and G block is held sixth period. On Tuesday, blocks B through D rotate with F taking over second period, and E drops. This pattern

continues through the week. The A and G blocks do not rotate; they meet first and sixth periods respectively except on Tuesday when A drops and Friday when G drops. The rotating-drop schedule for blocks B through F results in meeting with a class four days a week at a different time each day. The geometry class involved in the project study was a B block class which met at 9:10 Monday, 10:05 Tuesday, 11:00 Wednesday, and 12:25 Thursday for fifty minutes each day.

Teacher and Facilities

The author of the packets presented the materials to her own geometry class in the classroom regularly used by the class. Each student was given mimeographed copies of the packet materials as they were discussed and assigned. The blackboard was used for explanations and sample problems. Quizzes and tests were typed and administered to the students.

Schedule

The instructional schedule of packet presentation and testing is based on the daily outline displayed in Figure 4. The focus of each day's work is briefly stated. The complete learning package is found in Appendix A and Appendix B. The next section will discuss the actual implementation of the learning package.

Conduct of the Study

The first day's plan included a brief verbal introduction by the teacher on the factor label method of units analysis, and the pretest. The introduction lasted fifteen minutes and

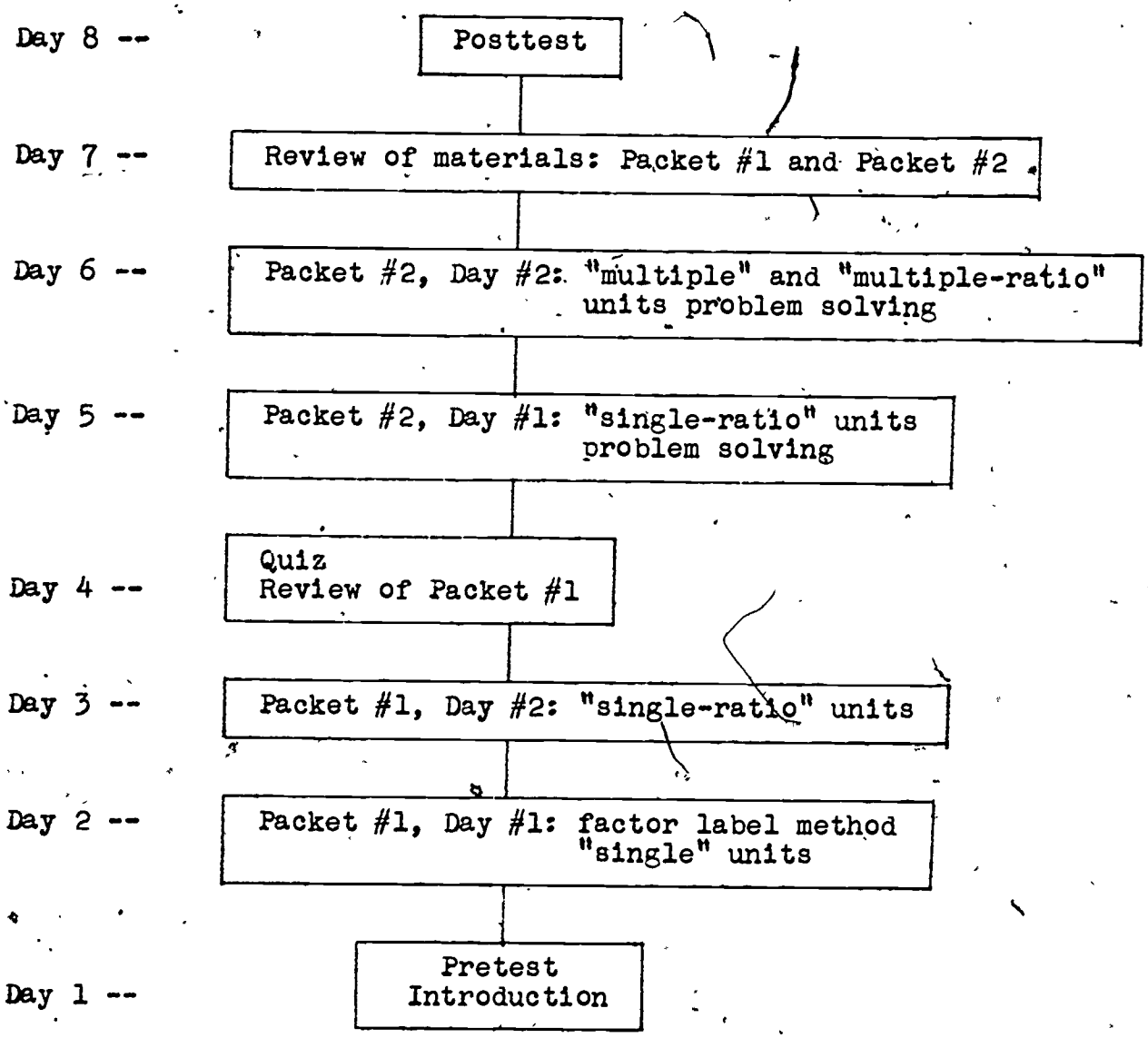


Figure 4. Instructional Plan:
Factor Label Method Materials--
Anticipated Number of Days (8),
Fifty Minutes Per Day

presnted sample "single," "single-ratio," and "multiple-ratio" units problems. Conversion factors (units relationships written as ratios equal to one), cancellation of common unit factors, and setting up problems were demonstrated. The students were attentive but were used to being able to follow along in a textbook during a verbal presentation and at this point none of the materials had been distributed. After the introduction, the pretest (Appendix C) was administered. The students were allowed thirty minutes to complete the eight problems; however, all papers had been turned in within twenty minutes due to the students' not knowing enough about units analysis to complete the problems. Analysis of the pretest results is found in Chapter IV. The main reaction of the students at this point was that it was unfair to test them on topics they had not been taught before; they did not understand the use of a pretest.

The second day's plan of the project included Packet #1, Day #1 materials: factor label method and "single" units. So the students would not feel defeated before starting, the pretests were not returned to them. They were told it would not affect their grades but that it served as an example of the posttest that would count. The materials were distributed and the students were to work on their own asking questions when the need arose.

The main difficulty encountered was that students would not read the directions before attempting the problems and, therefore, they would not include units in the conversion factors. Some students had difficulty with using two conversion

factors in the same problem. Over half of the class finished the in-class assignment. The assignment was distributed but, since it involved measuring parts of the body, only the students with rulers (four students) continued.

The third day's plan included Packet #1, Day #2: "single-ratio" units. Before distributing the packet materials, the teacher answered questions on the previous day's material. The biggest difficulty was using two conversion factors of an unfamiliar measurement system in the same problem. Completed assignments and packets were collected. Due to office requests and replies, the Packet #1, Day #2 lesson was delayed ten minutes. Most students were unable to finish the in-class assignment in the time allowed. The main difficulty was that the students were trying to work the problems directly from the sample problems instead of carefully reading the problem and then developing their own analysis.

The fourth day's plan included a review of the material presented and quiz on Packet #1. The materials collected before were returned to the students with correction comments. As before, questions on the previous day's material were answered. The completed assignments and packets were collected. The teacher presented a brief review of the factor label method of solving conversion problems involving "single" and "single-ratio" units before administering the quiz. The quiz (Appendix C) consisted of two problems testing each of the three objectives of Packet #1. The students were allowed twenty minutes. Several students had trouble operating with the numbers involved in the problems, such as reducing fractions and placing the

decimal point when dividing, and were unable to complete the quiz.

The fifth day's plan included Packet #2, Day #1: "single-ratio" units problem solving. The previously collected papers and quizzes were returned to the students. The quizzes were marked as to which objective the students had not met. For the analysis of the quiz results see Chapter IV. Of the initial 33 students, two were absent the day of the quiz. They were informed of the material included in the review and continued with the rest of the class.

After the quiz was discussed and questions answered, the Packet #2, Day #1 materials were distributed. The main difficulty with this material was inverting given rates. The students were hesitant to accept that saying a car traveled 50 miles/hour was equivalent to saying it took the car 1 hour/50 miles. It was noted that the former is the speed of the car, and the latter is not; however, both statements give the same information concerning the speed of the car. Most of the students completed the in-class assignment. Although they were doubtful concerning density problems in which the unit "g/cc" was unfamiliar. The Packet #2, Day #1, Assignment was distributed at the end of the period.

The sixth day's plan included the Packet #2, Day #2 materials: "multiple" and "multiple-ratio" units problem solving. The same procedure of returning papers, answering questions, and collecting assignments before distributing the packet materials was followed. The students had little difficulty with the "multiple" units and converting "multiple-

ratio" units. Some difficulty was encountered in solving for an unknown quantity when the known quantities involved "multiple-ratio" units. The difficulty resulted from not being sure of canceling common factors. The students, in general, seemed to be able to set up the problem correctly. Several students completed the entire in-class assignment and started the Packet #2, Day #2, Assignment.

The seventh day's plan included a review of the materials. Collected papers were returned and questions were answered. After student questions were answered (15 minutes of the period), the teacher presented a summary of the material from both packets. A verbal drill on the meanings of the prefixes used in the metric system was followed by sample problems. Conversion problems and solving for an unknown quantity were discussed. The remaining ten minutes of the period was left for individual study and aid.

The eighth day's plan included the posttest. The first 15 minutes of the period was allotted for answering student questions. The questions mainly dealt with problem solving that included "multiple-ratio" units of an unfamiliar system of measurement. Within the 15 minutes, all questions posed by the students were answered. The posttest was then distributed. The students had thirty minutes to complete the eight problems (two problems on each of the four objectives stated for Packet #2). Of the 33 students in the class, two were absent the day of the posttest. They took the same test the next week and were included in the analysis of the results discussed in Chapter IV. All students were able to complete what they knew on the posttest in the time allowed.

General Comments

While working on the packet materials, the students were allowed to work together with the understanding that each student was expected to work alone on the quiz and tests. It would have been beneficial to have had a teacher's aid present to assist in distributing and collecting materials, and answering individual questions.

Throughout the implementation, evaluations of the collected packets and assignments were used to plan what would be presented in, or added to, the next day's question and answer period which usually lasted ten to fifteen minutes. The packets and assignments were checked and corrected daily to encourage the students to continue with the materials on schedule, and not fall behind. The main difficulties the students had resulted from not reading the directions carefully, and keeping track of the units. At the beginning of the project the students were starting with the problems instead of reading the directions. By the end of the project most of the students were reading and following directions and therefore they knew how to use the units involved in the problems.

This chapter has presented the implementation of the learning package on the factor label method of units analysis in a high school geometry class. Background information on the student population, teacher and facilities, and schedule of presentation involved in the project were included. These items were followed by a discussion of the daily lessons and testing. The materials developed were supplemented by verbal presentations by the teacher based on the analysis of the

collected papers. This chapter then has presented the second stage of the curriculum development model, pilot stage. The evaluation stage of analyzing the results of the testing will be discussed in Chapter IV.

Chapter IV

RESULTS

The stated purpose of this project was: 1) to develop a learning package on the factor label method of units analysis, and 2) to analyze its implementation in a high school geometry class. The first step was presented in Chapter II followed by a discussion of the implementation in Chapter III. The purpose of Chapter IV is to discuss the second step of the project purpose, that is, to present the analysis of the results of the learning package. This chapter constitutes the third stage of the curriculum development model, evaluation.

The criterion measure was performance on a posttest containing two problems on each of the four behavioral objectives of the learning package. A pretest parallel in form to the posttest was used as a comparative measure. The goal of the learning materials was to present and teach a technique of solving problems. The problems on the pretest and posttest were marked satisfactory (S) or unsatisfactory (U) according to whether the student correctly set up the problem or not. A mark of "Sc" was given on problems 1a) and 1b) if the student correctly set up the problem but had used an incorrect conversion factor. It was assumed for this study that after setting up a problem correctly the student would be able to

solve the problem correctly. Often times this was not the case due to errors in multiplying and dividing. A complete table of the data obtained is found in Appendix D. The remainder of this chapter will present a summary of the data.

Overall Results

Concerning overall results, the 33 students had a pretest mean of 1.5 and a posttest mean of 6.4 out of a possible raw score of 8. An "S" mark received one point and a "U" mark used zero points. A modification in the scoring was made for problems 1a) and 1b). Since the behavioral objective stated (Packet #2, Day #1): "Given metric units of length and mass, you should be able to convert to other metric units," the students were tested on their ability to use the factor label method and their knowledge of the metric conversion factors. Therefore, if the set-up was correct the student received ½ point; if the set-up and conversion factors used were correct he received one point. That is, a student received ½ point for an "Sc" mark and one point for an "S" mark. In percentages, the mean pretest score was 18.8 percent and the mean posttest score was 80 percent. These results are given in Table 3.

Behavioral Objectives

The minimum level of acceptable performance for the student to have achieved the overall objective was to score 75 percent or better on the posttest. The minimum level of acceptability for the learning materials was that 75 percent or more of the students would score 75 percent or better on

TABLE 3

Overall Results of the Pretest and Posttest

	Pretest	Posttest
Mean (out of 8 items)	1.5	6.4
Mean in terms of percentages	18.2%	80.0%

the posttest. Of the 33 students, 25 or 75.8 percent of the students scored 75 percent or better on the posttest. Therefore the materials were considered successful.

To analyze the results of the individual behavioral objectives, the percentage of students who received an "S" mark (accomplished mastery) was computed for each problem on the pretest and posttest (Figure 5). The average of the percentages assigned to the problems testing each objective was then computed (Table 4). The results of the posttest are that 56.1 percent mastered objective (1), 87.9 percent mastered objective (2), 87.9 percent mastered objective (3), and 81.8 percent mastered objective (4).

Of the four objectives, objective (1) had the lowest percentage of student mastery. Mastery of objective (1) relied on not only using the factor label method of problem conversion (77.3 percent of the students correctly set up the problems), but also knowing the metric conversion factors (only 56.1 percent of the students used the correct metric conversions). Mastery of the other objectives relied on the correct use of the factor label method with known conversion factors.

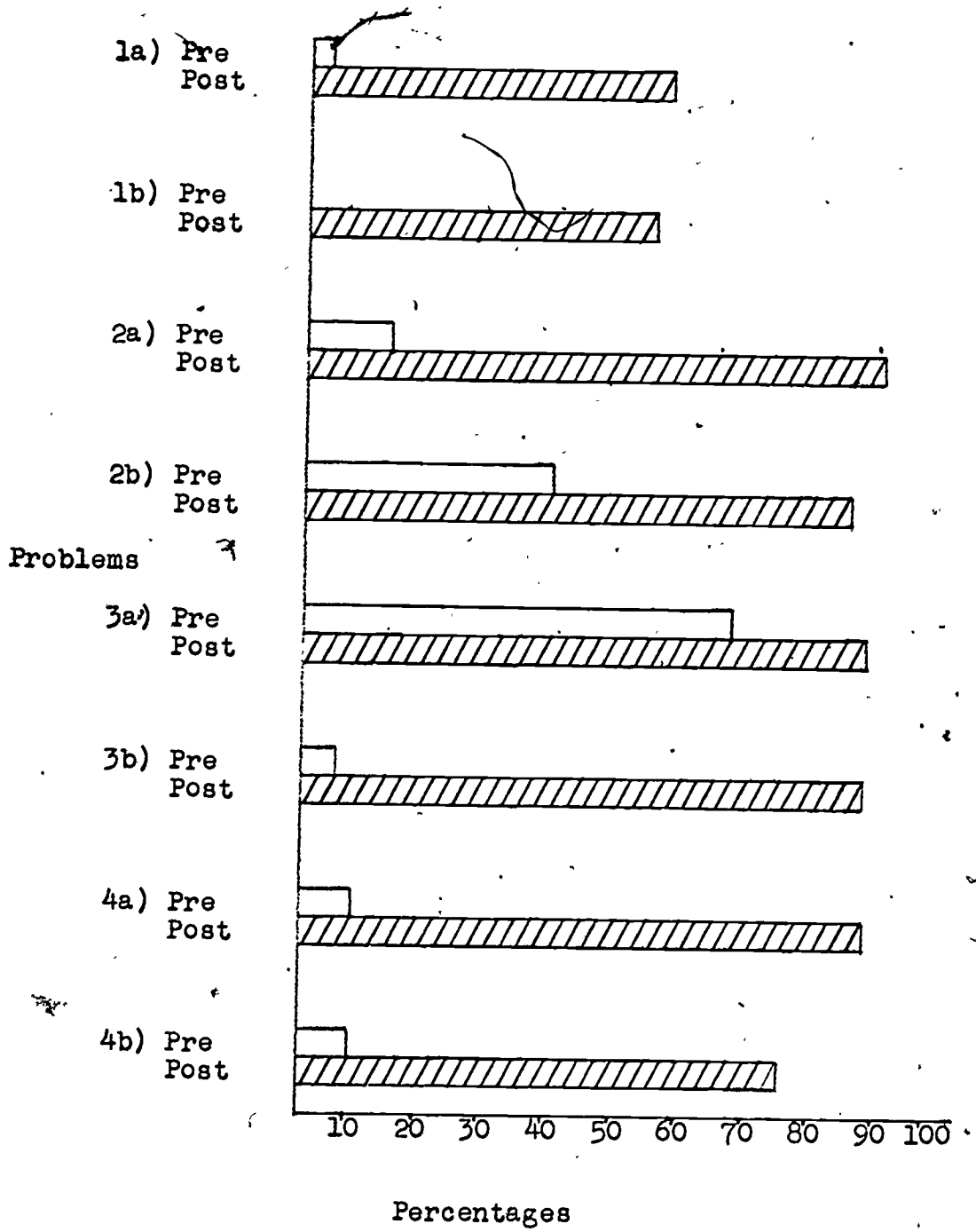


Figure 5. Percentage Graph of Students Satisfactorily Setting Up Each Problem of the Pretest and the Posttest

TABLE 4

Mastery of the Learning Package Behavioral Objectives

Objective #	Student Mastery Percentages	
	Pretest	Posttest
1	1.5%	56.1%
2	25.8%	87.9%
3	36.4%	87.9%
4	9.1%	81.8%

The results of the quiz administered after the implementation of Packet #1 were not included in the previous statements of results but are now considered for completeness. The quiz was marked as to which objectives the students had achieved. Therefore the results included marks of "none," "1," "2," "3," "1 & 2," "1 & 3," "2 & 3," and "1, 2, & 3." A complete table of the data is found in Appendix D. A summary of the data appears in Table 5. It should be noted that the objectives considered are those in the introduction to Packet #1 (Appendix A). It is again evident that the students had not learned the metric conversion factors since only 42.4% of the students met objective (1).

Chapter Summary

This chapter has presented the results of the evaluation stage of the project study. Since 75.8 percent of the students scored 75 percent or better on the posttest as compared to

TABLE 5

Mastery of Objectives of Packet #1:

Summary of Quiz Results

OBJECTIVE #	NUMBER OF STUDENTS MASTERING THE OBJECTIVE	PERCENTAGE OF STUDENTS MASTERING THE OBJECTIVE
1	12	42.4%
2	18	60.6%
3	21	69.7%

96.7 percent scoring 37.5 percent or below on the pretest, the learning package developed to present the factor label method of units analysis was considered successful. The evaluation stage completes the curriculum development model used in this study and presented in Chapter II. The next chapter will conclude the report of the project by discussing general conclusions and recommendations.

Chapter V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The factor label method of units analysis is a problem solving technique used in the fields of chemistry and physics. It consists of treating units as numbers and writing conversion factors as ratios equal to one. Although it is based on the mathematical concepts of common factors and the multiplicative identity, there appears to be no research, or materials, available on its implementation in the secondary mathematics curriculum. Therefore, it was the purpose of this project to develop a learning package on the factor label method and analyze its implementation in a high school geometry class.

A modification of the curriculum development model of Romberg and De Vault (1967) was used to develop the materials. The mathematical analysis phase of the analysis stage resulted in a content outline, a task analysis, and a list of behavioral objectives. These were then used in the instructional analysis phase to develop the two packets, the pretest and the posttest. The materials were implemented in a college preparatory geometry class at University of San Diego High School for the pilot stage.

The goal of the project was to have students learn the factor label method of units analysis prior to their enrollment in high school chemistry and physics. The criterion measure of

the study was performance on a posttest containing eight problems, two on each of the four learning package objectives, with the minimum level of acceptable performance set as a score of 75 percent or better on the posttest. The minimum level of acceptability of the learning materials was that 75 percent or more of the students would score 75 percent or better on the posttest.

Conclusions

The project focused on the question "Why not present the concepts (of the factor label method) prior to high school chemistry and physics courses within an integrated mathematics/science lesson, drawing applications from the realm of science and justification from the realm of mathematics?" Since 75.8 percent of the students scored at least 75 percent on the posttest, the materials were considered successful and the implementation of the factor label method in a secondary mathematics class is considered possible and worthwhile.

Over 80 percent of the students demonstrated ability to use the factor label method by mastering objectives (2), (3), and (4) of the packets. The weak point of the project was that only 56.1 percent of the students mastered objective (1) of the packets. This low level of performance resulted from inadequate knowledge of the metric conversion factors needed to solve the problems testing the achievement of objective (1). However, 77.3 percent used the factor label method correctly in setting up the problems. Therefore, the overall objective of using the factor label method was also achieved for each packet objective.

Recommendations

This project resulted from a desire to present some form of units analysis to students prior to their enrollment in high school chemistry and physics. The real success of the materials developed relies on the students' ability to use the concepts presented during their sophomore year in the fall of their junior year. Therefore, a follow-up study should take place during the students' junior year. A comparison should be made between the achievement of students who had the factor label method materials during their sophomore year and the achievement of students who did not.

The materials developed do not exhaust all of the approaches that could be used to teach the factor label method. The content of the packets included problems from historic-literary and scientific contexts to allow the students some choice, recognizing the various interests of the students. A teacher could adapt the format to other disciplines as well. Depending on the makeup of the class, the problems could introduce quantities used in chemistry, physics, business or economics.

The class participating in this study was unfamiliar with the individualized packet approach. It was proved beneficial for the teacher to include brief verbal explanations when new concepts were introduced in the materials. The amount of teacher input needed depends on the students' previous experience with learning packets. At all times the teacher should be receptive to the need for clarification of any of the ideas supporting the factor label method. Weak prerequisite

skills can be detected by analyzing the performance on a pretest. The skills can then be reviewed before or during the implementation of the factor label method materials as needed.

If the individualized learning packet approach is used to present the concepts of the factor label method, a student/teacher ratio of not more than 20/1 is strongly suggested. For this study, it would have been useful to have had a teacher's aid available to assist in the distribution and collection of materials. When students work on individualized packets throughout a class period, they have specific questions requiring specific answers. A student/teacher ratio of 33/1, as was the situation in this study, is too large to give the students the individual attention necessary.

The implementation of the factor label method materials in a secondary mathematics class helped to connect mathematical concepts with their scientific applications. If the trend of integrating mathematics and science at the elementary school level expands into the high school, materials such as those used in this study will become vital parts of the integrated curriculum. With the cooperation of mathematics and science teachers, lessons can be developed to connect the two disciplines even before integration takes place. More research should be done in the area of the factor label method of units analysis to demonstrate its need and usefulness in an integrated mathematics/science curriculum.

APPENDIX A

PACKET 1

LEARNING PACKET: PROBLEM SOLVING I

WHAT: This is an individualized packet containing a method of solving problems (factor label method)that can be used in many situations.

WHO: You! It is up to you to read and complete the packet and complete the assignments. If at any time you have a question: ASK for an explanation.

WHEN: Each page has a Packet #, Day #, and Page #. These serve as a guide. For instance today you should complete the pages denoted by Packet # 1, Day # 1 and then after you have checked your work continue with Packet # 1, Day # 1, Assignment. Tomorrow would be Packet # 1, Day # 2 ...

WHERE: In class! You may be able to start into the Assignment sheets in class also.

HOW: READ ALL DIRECTIONS! This packet is about a method of solving problems. Therefore you must show your work as directed at all times.

RESULTS: You should be able to score 75% or better on a test on the following:

1) Given metric units of length or mass you should be able to convert to other metric units (therefore you are to know the prefixes used in the metric system).

2) Given the conversion factors of any measurement system, you should be able to convert from one unit to another.

3) You should be able to solve for "unknown" quantities by looking at the units.

All of the above are to be done by the factor label method.

GOOD LUCK!

PACKET # 1, DAY # 1

1. Throughout this packet you will be solving problems by using the factor label method. This involves converting measurements from one set of units to another by multiplying by a conversion factor equal to 1 and "canceling" out units.

   CONFUSED? Read on before you give up!

2. Recall that any number divided by itself is equal to _____.

3. Therefore, $\frac{6}{6}$ or $\frac{5}{5}$ is equal to _____.

4. Now 12 inches = 1 foot.

5. Does $\frac{12 \text{ inches}}{1 \text{ foot}} = 1$? _____

6. Yes, but be careful: $\frac{12}{1} \neq 1$; yet $\frac{12 \text{ inches}}{1 \text{ foot}} = 1$, since 12 in. = 1 ft.

   Remember to LOOK AT THE UNITS! $\frac{12 \text{ in.}}{1 \text{ ft.}} = 1$ is a conversion factor.

7. Suppose you wanted to find the number of inches in 5 feet. You know the answer is 60 inches, but how did you decide that?

   Here's the approach you'll be using:

   5 ft. = ? in.

   Now 5 ft. = 5 ft. x 1 = 5 ft. x $\frac{12 \text{ in.}}{1 \text{ ft.}}$ = ? inches

   The units can be canceled as common factors in the same way numbers can be:

   So $5 \cancel{\text{ft.}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft.}}} = 5 \times 12 \text{ in.} = 60 \text{ in.}$

8. Why use $1 = \frac{12 \text{ in}}{1 \text{ ft.}}$, and not $1 = \frac{1 \text{ ft.}}{12 \text{ in.}}$?

   Let's use $1 = \frac{1 \text{ ft.}}{12 \text{ in.}}$ and see what happens:

   $5 \text{ ft.} \times \frac{1 \text{ ft.}}{12 \text{ in.}} = ?$

   Now you cannot cancel "ft.," they must be multiplied: ft. x ft. = ft.^2.

   So $5 \text{ ft.} \times \frac{1 \text{ ft.}}{12 \text{ in.}} = \frac{5 \text{ ft.}^2}{12 \text{ in.}}$

   This is "correct" but not useful since you were asked to find the number of inches in 5 feet.

   THE POINT? -- Be sure to use the conversion factor in the way that units can be canceled so as to have the answer asked for in the problem!

PACKET # 1, DAY # 1

SO WHAT? You already knew that 5 ft. = 60 in. without this, right?

9. Well what about unfamiliar measurements?
For instance, how many centimeters are there in 5 ft. if you know that 1 in. = 2.54 centimeters?

10. You know from Exercise 7 that 5 ft. = 60 in., so you want to find:

60 in. = ? centimeters

60 in. x 1 = ? centimeters What conversion factor can be used for 1? 1 = _____

So 60 in. x _____ = ? centimeters

Canceling out "in." and multiplying, you have _____ centimeters.

11. "Practice Makes Perfect"--

Fill in the proper conversion factors, cancel units, and solve:

a) 3 yd. x _____ = ___ ft.

b) 24 ft. x _____ = ___ yd.

(5280 ft. = 1 mi.)

c) 3 mi. x _____ = ___ ft.

d) 10560 ft. x _____ = ___ mi.

12. Suppose you had the following measurement system: 1 tic = 3 tacs
1 tac = 4 toes
1 toe = 5 wins

Find the following:

a) 6 tacs = ? tics
6 tacs x _____ = _____ tics

b) 6 tacs = ? toes
6 tacs x _____ = _____ toes

c) 4 toes = ? wins
4 toes x _____ = _____ wins

Show units! Keep track of units: after you have canceled units, make sure you have the units you want remaining.

PACKET # 1, DAY # 1

13. Given: 1 bay = 5 mays
1 may = 5 rays
1 ray = 5 says

Find: a) 6 mays = ? rays

6 mays x _____ = ____ rays

b) 10 rays = ? mays

10 rays x _____ = _____

c) 42 bays = ? mays

_____ x _____ = _____

14. In Exercise 13 the measurement system is based on the multiples of 5. The metric system is based on multiples of 10 and uses the Greek and Latin prefixes that denote the multiples.

The following are the most common prefixes and their meanings:

kilo (k)	thousand	$1000 = 10^3$
hecto (h)	hundred	$100 = 10^2$
deka (da)	ten	$10 = 10^1$
deci (d)	tenth	$\frac{1}{10} = 10^{-1}$
centi (c)	hundredth	$\frac{1}{100} = 10^{-2}$
milli (m)	thousandth	$\frac{1}{1000} = 10^{-3}$

15. In the metric system, the basic unit of length is the meter (m) and the basic unit of mass is the gram (g).

Therefore, 1 kilometer (1 km) = 1000 meters
1 centimeter (1 cm) = $\frac{1}{100}$ meters or 100 cm = 1 m

Likewise, 1 kilogram (1 kg) = 1000 grams
1 centigram (1 cg) = $\frac{1}{100}$ gram or 100 cg = 1 g

16. How many centimeters (cm) are there in 1 kilometer (1 km)?

1 km = ? cm Hint: change "km" to "m" then change "m" to "cm" using two conversion factors

$1\ \cancel{km} \times \frac{1000\ \cancel{m}}{1\ \cancel{km}} \times \frac{100\ cm}{1\ \cancel{m}} = ?\ cm$

$1 \times 1000 \times 100\ cm = \underline{100000 cm} = \underline{10^5\ cm}$

PACKET # 1, DAY # 1

17. How many centigrams (cg) are equal to 1 kilogram (1 kg)?

1 kg = ? cg

1 kg x _____ x _____ = ___cg

18. Does more practice mean "more perfect"? Oh well, it can't hurt!

a) 1 hectometer = ? cm

1 hm x _____ x _____ = ____cm

b) 1 dekameter = ? cm

1 dekameter x _____ x _____ = _____cm

c) 1 decigram = ? milligram

1 dg x _____ x _____ = _____mg

Can you determine how many centigrams are equal to 1 centimeter?

No! This would involve a conversion factor between length and mass which you haven't been given.

When you reach this point, check over what you have done so far; then, continue with Packet # 1, Day # 1, Assignment.

PACKET # 1, DAY # 1, ASSIGNMENT

1. Early measures of length were based on parts of the human body:

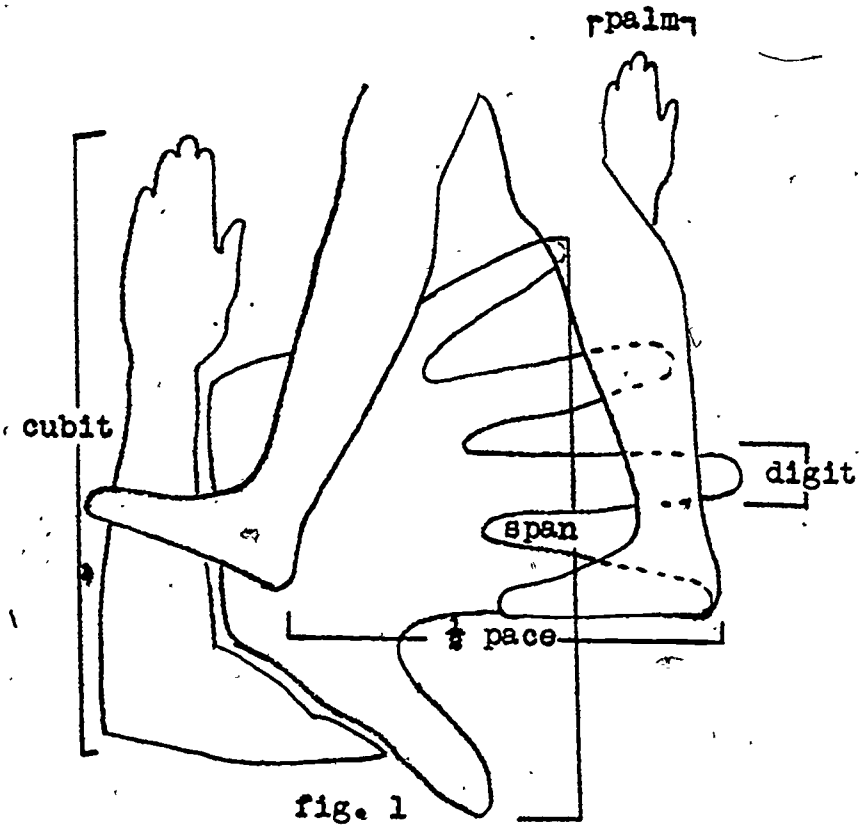


fig. 1

2. Measure the following parts of <u>your</u> body, then make the conversions using the factor label method (2.54 cm = 1 in.):

a) cubit = ___ in. = _____ cm = _____ m

b) span = ___ in. = _____ cm = _____ dm

c) ½ pace = _____ in. = _____ cm = _____ m

d) palm = ____ in. = _____ cm = _____ dm

3. Learn the meanings and abbreviations for the Latin and Greek prefixes used in the metric system.

4. The early measures varied from person to person. The rulers of some countries established uniform units. King Henry I of England (1100-1135 A.D.) proclaimed that a yard be the distance from <u>his</u> nose to the end of the thumb of <u>his</u> outstretched hand.

PACKET # 1, DAY # 1, ASSIGNMENT

5. The Egyptians used stones for measures of weight. Today, stone is still used as a unit of measure. (1 stone = 14 lbs.) Calculate your weight in stones;

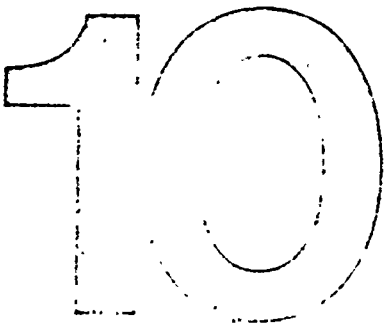
weight = _____lbs. = ___stones

6. Given that 2.2 lbs. = 1 kg, find the following:

weight = _____lbs. = _____kg = _____g

All You Need to Know About Metric

(For Your Everyday Life)



Metric is based on Decimal system

The metric system is simple to learn. For use in your everyday life you will need to know only ten units. You will also need to get used to a few new temperatures. Of course, there are other units which most persons will not need to learn. There are even some metric units with which you are already familiar: those for time and electricity are the same as you use now.

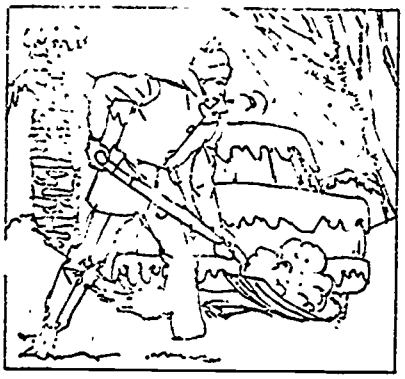
BASIC UNITS

METER: a little longer than a yard (about 1.1 yards)
LITER: a little larger than a quart (about 1.06 quarts)
GRAM: about the weight of a paper clip

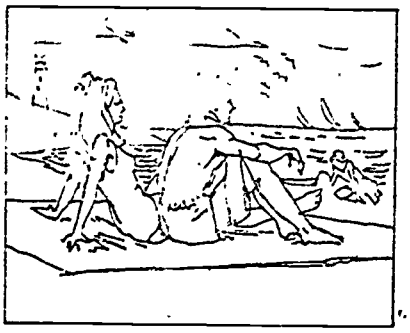
(comparative sizes are shown)

1 METER

1 YARD



25 DEGREES FAHRENHEIT

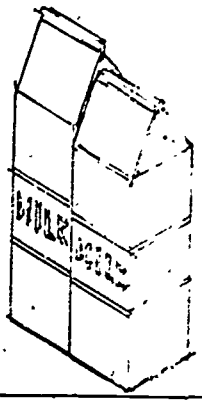


25 DEGREES CELSIUS

COMMON PREFIXES
(to be used with basic units)

Milli: one-thousandth (0.001)
Centi: one-hundredth (0.01)
Kilo: one-thousand times (1000)

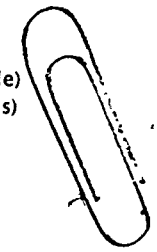
For example:
1000 millimeters = 1 meter
100 centimeters = 1 meter
1000 meters = 1 kilometer



1 LITER 1 QUART

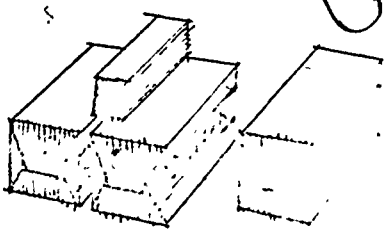
OTHER COMMONLY USED UNITS

Millimeter:	0.001 meter	diameter of paper clip wire
Centimeter:	0.01 meter	width of a paper clip (about 0.4 inch)
Kilometer:	1000 meters,	somewhat further than ½ mile (about 0.6 mile)
Kilogram:	1000 grams	a little more than 2 pounds (about 2.2 pounds)
Milliliter:	0.001 liter	five of them make a teaspoon



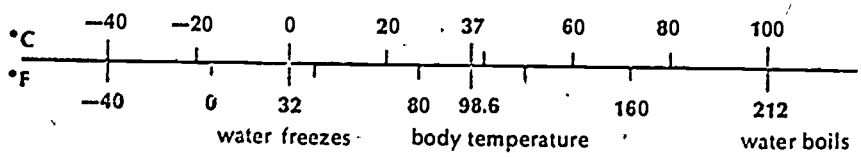
OTHER USEFUL UNITS

Hectare: about 2½ acres
Tonne: about one ton



1 KILOGRAM 1 POUND

TEMPERATURE
degrees Celsius are used



For more information, write to: Metric Information Office, National Bureau of Standards
Washington, D.C. 20234

PACKET # 1, DAY # 2

19. Thus far the conversion problems have involved only one property (mass or length) of an object. Situations arise when more than one property are considered at the same time.

20. Consider the speed of an object. Speed is the comparison between distance traveled and a unit of time. If you travel 240 miles in 4 hours then your average speed in mi./hrs. will be $\frac{240 \text{ mi.}}{4 \text{ hrs.}} = 60 \frac{\text{mi.}}{\text{hr.}}$.

What is this speed in $\frac{\text{mi.}}{\text{min.}}$?

$$\frac{60 \text{ miles}}{1 \text{ hr.}} \times \frac{?}{} = ? \frac{\text{mi.}}{\text{min.}}$$

$$\frac{60 \text{ mi.}}{1 \cancel{\text{hr.}}} \times \frac{1 \cancel{\text{hr.}}}{60 \text{ min.}} = \frac{60 \text{ mi.}}{60 \text{ min.}} = 1 \frac{\text{mi.}}{\text{min.}}$$

21. Another way of writing out a problem using the factor label method is to make a "chart" of the conversion factors used:

Example: $\frac{30 \text{ meters}}{\text{minute}} = ? \frac{\text{cm}}{\text{sec.}}$ (Note: must change "m" to "cm" and "min." to "sec.")

Draw the following: $\frac{}{} \Big| \frac{}{} = \frac{}{}$

Insert the problem: $\frac{30 \text{ m}}{1 \text{ min.}} \Big| \frac{}{} = \frac{? \text{ cm}}{\text{sec.}}$

Now insert the conversion factors (after each draw a vertical line):

$$\frac{30 \cancel{\text{m}}}{1 \cancel{\text{min.}}} \Big| \frac{100 \text{ cm}}{1 \cancel{\text{m}}} \Big| \frac{1 \cancel{\text{min.}}}{60 \text{ sec.}} = \frac{? \text{ cm}}{\text{sec.}}$$

$$\frac{30}{1} \Big| \frac{100 \text{ cm}}{1} \Big| \frac{1}{60 \text{ sec.}} = \frac{50 \text{ cm}}{\text{sec.}}$$

Always check that you have canceled correctly !

22. Average walking speed is 4 mi./hr.? Use the chart form to find average walking speed in ft./min. (5280 ft. = 1 mi.)

$$\frac{4 \text{ mi.}}{1 \text{ hr.}} \Big| \frac{}{} = \frac{}{} \frac{\text{ft.}}{\text{min.}}$$

23. $\frac{60 \text{ mi.}}{\text{hr.}} = ? \frac{\text{ft.}}{\text{sec.}}$ Hint: change "hr." to "min." then change "min." to "sec."

$$\frac{60 \text{ mi.}}{\text{hr.}} \Big| \frac{}{} = \frac{}{} \frac{\text{ft.}}{\text{sec.}}$$

PACKET # 1, DAY # 2

Use the chart method:

24. $\frac{90 \text{ km}}{\text{hr.}} = \frac{? \text{ m}}{\text{sec.}}$ _____ = _____

25. Prior to Queen Elizabeth I, the mile (Roman mile) was equal to 5,000 feet which was not a integer multiple of the furlong (unit of measure = 220 ft.). Queen Elizabeth I changed the mile to 5,280 ft. How many furlongs are there in 1 mile?

$\frac{1 \text{ furlong}}{220 \text{ ft.}}$ _____ = $\frac{\text{furlongs}}{\text{mi.}}$

26. When the United States changes over to the metric system, kilometers will replace miles. What will be the equivalent of the 55 mi./hr. speed limit sign in km/hr.? (3.28 ft. = 1 m)

27. The speed of light is 186,300 mi./sec.

a) What is its speed in mi./hr.?

b) From a) calculate the speed in mi./yr. (use 365 days = 1 yr.)

c) The number of miles light travels in one year is called a <u>light year</u>.

Alpha Centauri is the closest star to our solar system, and is 4.3 light years away. (Which means the light the Earth receives today from Alpha Centauri left there <u>4.3 years ago</u>.)

How far away, in miles, is Alpha Centauri?

PACKET # 1, DAY # 2, ASSIGNMENT

Use factor label. SHOW YOUR WORK!

1. Do problem L) or S):

L) The Jack and Jill nursery rhyme (17th century) was originally a song of protest against a local tavern owner who cheated on the size of his drinking glasses.

1 jill = 2 jacks, and 1 jack = 4 ro (Egyptian measurement for a mouthful)

How many ro's are there in one jill?

$$\_\_\_\_\_ = \frac{\text{ro's}}{1 \text{ jill}}$$

S) The astronomical unit (AU) is the average distance of the Earth from the sun (1 AU = 9.3×10^7 miles approximately).

How many kilometers are there in one AU? (1 km = .6 mi.)

$$\_\_\_\_\_ = \frac{\text{km}}{1 \text{ AU}}$$

2. Do problem L) or S):

S) Pluto is the most distant planet in our solar system. Its maximum distance from the sun is about 4.6×10^9 miles. (1 AU = 9.3×10^7 miles; 1 light year = 5.88×10^{12} miles)

Find its distance in AU's:

Find its distance in light years:

L) "The length of the ark shall be 300 cubits, the breadth of it 50 cubits, and the height of it 30 cubits." (Genesis 6:15) What were the dimensions of the ark in feet? length:_____

(1 cubit = 18 in.)

breadth:_____

height:_____

PACKET # 1, DAY # 2, ASSIGNMENT

3. Do All:

Look up the following and give the number indicated by each expression:

a) score of years:

b) gross of boxes:

c) ream of paper:

Look up the following and express each in a more common unit of measure:

a) peck:

b) barrel

c) hogshead:

4. When a spacecraft returns from the Moon, lunar gravity will slow it down until it enters the Earth's gravitational influence. Earth's gravity will cause it to accelerate until it reaches a speed of nearly 25,000 mi./hr. Convert this to ft./sec.

5. 4 mi./hr. = ? m/min. (1 km = .6 mi.)

APPENDIX B

PACKET 2

LEARNING PACKET: PROBLEM SOLVING II

At this point, ask yourself whether or not you met the desired results of Packet # 1:

Yes? Congratulations. Keep going and you should be able to meet the desired results of Packet # 2.

No? Go over what you missed before you continue. This Packet is a continuation of Packet # 1 so review Packet # 1 before continuing. Study the material and READ and FOLLOW ALL DIRECTIONS!

RESULTS: You should be able to score 75% or better on a test on the following:

1) Given metric units of length and mass, you should be able to convert to other metric units.(Do you know the metric conversion factors ?)

2) Given the conversion factors of any measurement system, you should be able to convert from one set of "singular" (will be explained) or "single ratio" units to another set.

3) Given the conversion factors of any measurement system, you should be able to convert from one set of "multiple" or "multiple ratio" units to another set.

4) You should be able to solve for "unknown" quantities by looking at the units.

GOOD LUCK!

PACKET # 2, DAY # 1

1. Thus far you have used the factor label method to convert from one set of measurement units to another.

   Examples: in. → cm; kg → g; mi./hr. → ft./sec.

   The factor label method of units conversion can also be used to solve problems in which the unit of measure of the answer is known.

   Don't stop now, you've gotten this far!

2. The average speed of a car was 50 mi./hr. It took the driver $1\frac{1}{2}$ hrs. to drive home. How far was he from home (in mi.) originally?

   Look at the Units: you are to find "mi."
   you are given "mi./hr." and "hr."

   $$\frac{\text{mi.}}{\cancel{\text{hr.}}} \Big| \cancel{\text{hr.}} = \text{mi.}$$

   Therefore $$\frac{50 \text{ mi.}}{\cancel{\text{hr.}}} \Big| 1\tfrac{1}{2}\ \cancel{\text{hr.}} = 50 \text{ mi.} \Big| 1\tfrac{1}{2} = 75\text{mi.}$$

   Therefore the driver was 75 mi. from home.

3. The speed of a manned spacecraft in a high circular orbit about the Earth is about 17,500 mi./hr. How many miles could the spacecraft go:

   a) in 6 hrs.?

   b) in 1 day? (Note: change "day" into "hrs.")

   c) in 1 week?

PACKET # 2, DAY # 1

4. At an average speed of 50 mi./hr., how long would it take to drive non-stop from San Diego to Dayton (2700 miles):

a) in hrs.? Now you are to find "hrs."
you are given "mi./hr." and "mi."

$\frac{\text{hr.}}{\cancel{\text{mi.}}} \Big| \cancel{\text{mi.}} = \text{hr.}$ therefore $\frac{1 \text{ hr.}}{50 \cancel{\text{mi.}}} \Big| 2700 \cancel{\text{mi.}} = 54 \text{ hrs.}$

Same procedure as in #2.

b) in days?

c) Suppose you can drive only 10 hrs./day then how many days would it take?

Show your work. Use factor label method!

5. The factor label method can be used to solve problems involving units other than distance and time.

6. Set up and solve the following:

a) Averaging 18 miles per gallon of gasoline, how much gasoline is needed to drive from San Diego to Dayton? (2700 miles)

b) At 55¢/gallon, how much would you spend on gasoline to drive to Dayton? (Use the result from #6a)

7. Polyester knit is selling for $3.98/yard. How many yards can you buy for $17.91?

8. Suppose, on Planet Ogeidnas you can go 16 san/id.

a) How far could you go in 6 ids?

PACKET # 2, DAY # 1

b) How long would it take to go 32 sans?

9. Density is the comparison between mass (amount of a substance) and volume (amount of space the substance occupies).

Density is mass per unit of volume. Example: $\frac{\text{grams}}{\text{cubic centimeter}} = \frac{g}{cc}$

a) The mass of a golf ball is 45.6 g.
The volume of a golf ball is 42.1 cc.
What is the density of a golf ball in g/cc?

b) The volume of the Earth is 1.08×10^{27} cc.
The mass of the Earth is 5.97×10^{27} g.
What is the density of the Earth in g/cc?

PACKET # 2, DAY # 1, ASSIGNMENT

1. Walking is good exercise:

   a) At 4 mi./hr., how far could you walk in $2\frac{1}{2}$ hrs.?

   b) At 4 mi./hr., how long would it take to walk 14 miles?

2. At the Museum of Natural History in Balboa Park the Foucault Pendulum has a $\frac{7.16 \text{ sec.}}{\text{period}}$ swing.

   a) How many periods of swing will it complete in 42.96 sec.?

   b) How long would it take to complete 5 periods of swing?

3. Finish the following table:

Substance	Volume (cc)	Mass (g)	Density (g/cc)
water at 4 C	1.00 cc	1.00 g	_____
calcium chloride	_____	4.30 g	2.15 g/cc
ammonium magnesium	5.52 cc	_____	1.84 g/cc

PACKET # 2, DAY # 2

10. Up to now, the unit of measure associated with a number has been "singular" (ft., sec., g,...) or "single ratio" (ft./sec., mi./hr., g/cc,...) in form.

Just as units can be canceled (divided out) like numbers, units can be multiplied resulting in "multiple" units of measurement (ft.-lb., ft.-lb. per sec.,...). Note that in "ft.-lb." the symbol (-) denotes multiplication.

11. WORK

What is "work"?

Scientifically "work" is done when a force moves an object some distance. When you lift an object 1 ft. that weighs 1 lb. you have done 1 ft.-lb. of work.

When you lift 5 lbs. 2 ft., you have done 2 ft. x 5 lbs. or 2 x 5 ft.-lbs. = 10 ft.-lbs. of work.

12. Figure the work done:

a) 24 lbs. lifted 3 ft.: _____ ft.-lbs.

b) 100 lbs. lifted $\frac{1}{2}$ ft.: _____ ft.-lbs.

c) 5 kg lifted 2 m: _____ kg-m

13. Lifting 4 kg, $\frac{1}{2}$ m results in 2 kg-m of work. What is the amount in terms of ft.-lbs.? (1 kg = 2.2 lbs. 1 m = 3.3 ft.)

$$\frac{2\ \cancel{kg\text{-}m}}{} \cdot \frac{3.3\ ft}{1\ \cancel{m}} \cdot \frac{2.2\ lbs}{1\ \cancel{kg}} = \underline{?\ ft.\text{-}lbs.}$$

$$\frac{2}{1} \cdot \frac{3.3\ ft.}{} \cdot \frac{2.2\ lbs.}{} = \underline{1452\ ft.\text{-}lbs.}$$

14. Find the equivalent of lifting a 48 oz. book, 9 in.: (16 oz. = 1 lb.)

a) in ft.-lbs.

PACKET # 2, DAY # 2

b) use answer from a) to find the work done in kg-m:

15. Suppose on Planet Ogeidnas, "krow" is measured in "tf.-bl."

a) How much krow is done in lifting a 2 bl. object $4\frac{1}{2}$ tf.?

b) How much krow is done, in tf.-bl., when you lift 7 bl., 9 ni.? (assume 1 tf. = 21 ni.)

16. POWER

Power is work per unit of time (example: $\frac{\text{ft.-lb.}}{\text{sec.}}$)

James Watt, an English inventor, figured the average horse could lift 550 lbs. 1 ft. in 1 sec.

Therefore 1 horsepower = $\frac{550 \text{ ft.-lbs.}}{1 \text{ sec.}}$

a) Of what horsepower is an engine that is capable of lifting 1100 lbs. 4 ft. in 2 sec.? Look at the Units!

The engine is capable of $\frac{1100 \text{ lbs.} \times 4 \text{ ft.}}{2 \text{ sec.}} = \frac{1100 \times 4 \text{ ft.-lbs.}}{2 \text{ sec.}}$

or $\frac{2200 \text{ ft.-lbs.}}{\text{sec.}}$

Therefore $\frac{2200 \cancel{\text{ft.}}\text{-}\cancel{\text{lbs.}}}{\cancel{\text{sec.}}} \Big| \frac{1 \text{ horsepower}}{\frac{550 \cancel{\text{ft.}}\text{-}\cancel{\text{lbs.}}}{\cancel{\text{sec.}}}} = \frac{2200}{550} \Big| 1 \text{ horsepower}$

$= 4$ horsepower

PACKET # 2, DAY # 2

b) Of what horsepower is an engine that is capable of lifting 1650 lbs. 3 ft. in 3 sec.?

17. An average man can lift 90 lbs. 1 ft. in 1 sec.
Therefore 1 manpower = $\frac{\text{90 ft.-lbs.}}{\text{1 sec.}}$

a) Calculate manpower in terms of $\frac{\text{m-kg}}{\text{sec.}}$: (1 m = 3.3 ft.
1 kg = 2.2 lbs.)

b) How many lbs. can a 54 manpower engine lift 18 ft. in 2 sec.?
(First figure 54 manpower in terms of $\frac{\text{ft.-lbs.}}{\text{sec.}}$)

18. Suppose on Planet Z-49 they measure "nodpower."

1 nodpower = $\frac{\text{3 nerf-zoot}}{\text{dot}}$. Calculate "nodpower" in $\frac{\text{nuf-koot}}{\text{spot}}$:

(Assume: 1 nerf = 2 nuf
2 zoot = 1 koot
1 dot = 3 spot)

PACKET # 2, DAY # 2

19. Suppose on Planet Z-49 they also measure "doodle":

$$1 \text{ doodle} = \frac{1 \text{ nodpower}}{\text{scriggle}} = \frac{3 \text{ nerf-zoot}}{\text{dot-scriggle}}$$

Calculate 1 doodle in $\frac{\text{nuf-koot}}{\text{spot-wiggle}}$: (Assume 1 wiggle = 4 scriggle)

PACKET # 2, DAY # 2, ASSIGNMENT

Use factor label method (chart form). Show work in space provided.

1. 1 km = _____ dekameter

2. 1 kg = _____ centigram

3. $\frac{90\ km}{hr} = \frac{\quad\quad mile}{hr.}$ (1 km = .6 mi.)

4. $\frac{90\ km}{hr.} = \frac{\quad\quad ft.}{sec.}$ (5280 ft. = 1 mi.)

5. What is the volume, in cc's, of an object whose density is $\frac{2.8\ g}{cc}$ and mass is 322 g?

6. What is the horsepower of an engine which is capable of lifting 3305 lbs. 2 ft. per second?
(Recall 1 horsepower = $\frac{550\ ft.\text{-}lb}{sec.}$)

7. Suppose you had the following situation:

a) .3 moles, 288 °K, 8 liters, ? atm, and the constant $\frac{.082\ atm\text{-}liter}{1\ moles\text{-}°K}$

Solve for ? atm:

Hint: Set up chart so as to cancel all units but "atm"!

b) ? moles, 283 °K, 4.94 liters, 1 atm and the constant $\frac{.082\ atm\text{-}liter}{1\ moles\text{-}°K}$

Solve for ? moles:

APPENDIX C

PRETEST-POSTTEST

QUIZ

PRE-TEST NAME:

Use factor label method as demonstrated. Your set-up and answers will be graded so show all work.

1. a) 2 kilometers = _____ centimeters

b) 360 centigrams = _____ decigrams

2. a) $3\frac{1}{2}$ cubit = _____ inches (1 cubit = 18 inches)

b) 60 mile/hour = _____ feet/second (5280 ft. = 1 mile)

3. a) How many $\frac{\text{ft.-lbs.}}{\text{sec.}}$ is equal to lifting 100 lbs. 3 ft. in 4 sec.?

b) Convert 3 $\frac{\text{wiggle-spot}}{\text{zoot}}$ into $\frac{\text{scriggle-dot}}{\text{koot}}$. (2 wiggle = 1 scriggle
3 spot = 1 dot
1 zoot = 2 koot)

4. a) At 90 kilometers/hour how long will it take to go 225 kilometers?

b) 1 horsepower = $\frac{550 \text{ ft.-lbs.}}{\text{sec.}}$ How many lbs. can a 1 horsepower engine lift 10 ft. in 5 seconds?

POST-TEST NAME:

Use factor label method as demonstrated. Your set-up and answers will be graded so show all work.

1. a) 4 kilograms = _____centigrams

b) 240 centimeters = _____decimeters

2. a) 20 spots = _____tots (8 spots = 1 tot)

b) 6 bills/day = _____ spills/ray (1 bill = 2 mills
1 mill = 3 spills
1 day = 4 rays)

3. a) How many $\frac{\text{kg.-m}}{\text{sec.}}$ is equal to lifting 32 kg 2 m in 4 sec.?

b) Convert 2 $\frac{\text{mop-mill}}{\text{mingle}}$ to $\frac{\text{top-till}}{\text{tingle}}$. (1 mop = 2 tops
2 mills = 1 till
3 mingles = 1 tingle)

4. a) At 88 ft./sec. how long will it take to travel 1 mile? (1 mile = 5280 feet)

b) 1 manpower = $\frac{\text{90 ft.-lbs.}}{\text{sec.}}$ How much could an average man lift 4 ft. in 6 sec.?

QUIZ: PACKET #1 NAME:

Use factor label method as demonstrated. Your set-up and answers will be graded so show all work.

1. a) 1 km = _____ dm

b) 1 kg = _____ cg

2. 1 mile = 5280 feet

a) 2 mile = _____ inches

b) Given: 1 boat = 2 coat
1 coat = 4 loads
3 loads = 1 toad

5 boats = _____ toad

3. a) The speed limit in a school zone is 25 miles/hour. In the metric system, what is the speed limit in km/hr.? (1 km = .6 mi.)

b) 4 mi./hr. = ___ ft./sec.

APPENDIX D

SUMMARY OF TEST RESULTS

TABLE A

Test Results: Pretest

STUDENT #	PERFORMANCE BY PROBLEM								TOTAL SCORES	
	1a	1b	2a	2b	3a	3b	4a	4b	Raw Score	%-Score
1	U	U	U	S	U	U	U	U	1	12.5
2	U	U	U	S	S	U	S	U	3	37.5
3	U	U	U	S	S	S	U	U	3	37.5
4	U	U	U	U	U	U	U	U	0	0.0
5	U	U	U	U	S	U	U	U	1	12.5
6	Sc	U	S	S	S	S	U	S	5.5	68.75
7	U	U	U	U	U	U	U	U	0	0.0
8	U	U	U	S	S	U	U	U	2	25.0
9	U	U	U	S	S	U	U	U	2	25.0
10	U	U	S	U	S	U	U	U	2	25.0
11	U	U	U	U	U	U	U	U	0	0.0
12	U	U	U	U	S	U	U	U	1	12.5
13	U	U	U	U	S	U	U	U	1	12.5
14	U	U	U	S	U	U	U	U	1	12.5
15	U	U	U	U	U	U	U	S	1	12.5
16	U	U	U	U	U	U	U	U	0	0.0
17	U	U	U	S	U	U	U	U	1	12.5
18	U	U	U	U	S	U	U	U	1	12.5
19	U	U	S	S	U	U	U	U	2	25.0
20	U	U	U	U	U	U	U	U	0	0.0
21	U	U	U	U	S	U	U	U	1	12.5
22	U	U	U	U	S	U	U	U	1	12.5
23	U	U	U	U	S	U	U	U	1	12.5

TABLE A (Con't.)

Test Results: Pretest

STUDENT #	PERFORMANCE BY PROBLEM								TOTAL SCORES	
	1a	1b	2a	2b	3a	3b	4a	4b	Raw Score	%-Score
24	U	U	U	S	S	U	U	U	2	25.0
25	U	U	U	S	S	U	U	U	2	25.0
26	U	U	U	U	S	U	S	S	3	37.5
27	S	U	U	S	S	U	U	U	3	37.5
28	U	U	U	U	U	U	U	U	0	0.0
29	U	U	S	U	S	U	S	U	3	37.5
30	U	U	U	U	S	U	U	U	1	12.5
31	U	U	U	U	S	U	U	U	1	12.5
32	U	U	U	U	S	U	U	U	1	12.5
33	U	U	U	S	S	U	U	U	2	25.0

Note: "S" represents a satisfactory factor label set-up.
"Sc" represents a correct factor label set-up with an incorrect conversion factor.
"U" represents an unsatisfactory factor label set-up.

TABLE B

Test Results: Posttest

STUDENT #	PERFORMANCE BY PROBLEM								TOTAL SCORES	
	1a	1b	2a	2b	3a	3b	4a	4b	Raw Score	%-Score
1	Sc	S	U	S	S	S	S	S	6.5	81.25
2	S	S	S	S	S	S	S	S	8	100.0
3	Sc	Sc	S	S	S	S	S	S	7	87.5
4	S	S	S	S	S	S	S	S	8	100.0
5	Sc	Sc	S	S	S	S	S	S	7	87.5
6	S	S	S	S	S	S	S	S	8	100.0
7	Sc	U	U	U	U	U	U	U	.5	6.25
8	S	S	S	S	S	S	S	S	8	100.0
9	Sc	S	S	S	S	S	S	S	7.5	93.75
10	S	S	S	S	S	S	S	S	8	100.0
11	S	S	S	U	S	S	S	S	7	87.5
12	Sc	Sc	S	S	S	S	S	S	7	87.5
13	S	Sc	S	S	S	S	S	U	6.5	81.25
14	S	S	S	S	S	S	S	S	8	100.0
15	U	U	S	S	S	S	S	S	6	75.0
16	S	S	S	S	S	S	S	S	8	100.0
17	U	U	S	S	S	S	S	S	6	75.0
18	U	U	U	S	U	S	S	U	3	37.5
19	S	S	S	S	S	S	S	S	8	100.0
20	S	Sc	S	U	S	S	S	U	5.5	68.75
21	U	U	S	S	S	U	U	U	3	37.5
22	S	Sc	S	S	U	S	S	U	5.5	68.5
23	S	S	S	S	S	S	S	S	8	100.0

TABLE B (Con't.)

Test Results: Posttest

STUDENT #	PERFORMANCE BY PROBLEM								TOTAL SCORES	
	1a	1b	2a	2b	3a	3b	4a	4b	Raw Score	%-Score
24	S	S	S	S	S	S	S	S	8	100.0
25	Sc	S	S	S	S	S	S	S	7.5	93.75
26	U	U	S	U	S	U	S	S	4	50.0
27	S	S	S	S	S	S	S	S	8	100.0
28	Sc	U	S	S	S	S	U	S	5.5	68.75
29	S	S	S	S	S	S	S	U	7	87.5
30	S	U	S	S	S	S	S	S	7	87.5
31	U	U	S	U	U	U	U	U	1	12.5
32	S	S	S	S	S	S	S	S	8	100.0
33	S	S	S	S	S	S	S	S	8	100.0

Note: "S" represents a satisfactory factor label set-up.
"Sc" represents a correct factor label set-up with an incorrect conversion factor.
"U" represents an unsatisfactory factor label set-up.

TABLE C

Mastery of Objectives of Packet #1:

Quiz Results

STUDENT #	OBJECTIVES MASTERED	STUDENT #	OBJECTIVES MASTERED
1	none	18	none
2	1, 2, 3	19	2, 3
3	2, 3	20	1, 3
4	2, 3	21	absent
5	3	22	2, 3
6	1, 2, 3	23	1, 2, 3
7	none	24	1
8	2, 3	25	1
9	2, 3	26	none
10	none	27	1, 2, 3
11	2, 3	28	2, 3
12	3	29	1, 2, 3
13	2, 3	30	none
14	1, 2, 3	31	none
15	none	32	1, 2, 3
16	2, 3	33	1, 2, 3
17	absent		

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