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ABSTRACT

The problem-solving strategies used by tenth-grade students in the solution of mathematical problems were investigated. Forty students selected from four high schools were given pretests generating sixteen scores related to mathematics achievement, attitude toward mathematics, and other ability measures. These students were then asked to solve eight mathematics problems in individual sessions. Recorded protocols were coded for three types of variables. Pretest and protocol data were submitted to principal components analysis to reduce the number of variables, regression analysis, and two cluster analyses. The best predictor of problem-solving success was found to be mathematics achievement, which accounted for 50 percent of total score variance. A second predictor, heuristic strategy components, accounted for 13 percent. The cluster analyses were performed in order to detect problem-solving modes. The first clustered students according to the frequency of use of certain heuristic strategies. The second grouped students on their willingness to use trial-and-error and equations. Together the analyses indicated that students using a wide range of strategies and techniques were able to solve more problems. (SD)

Abstract

AN EXPLORATION OF MATHEMATICAL PROBLEM-SOLVING PROCESSES

by

Norman Webb

The problem-solving processes used by forty second year high school algebra students while solving eight problems, the Problem-Solving Inventory (PSI), were studied. Each student was interviewed individually as he worked the PSI doing his thinking aloud. Protocols were recorded on audio tape and coded at a later date. The data collected were scores on sixteen pretests (measures of cognitive and affective variables), frequencies of problem-solving processes, and total scores on the PSI.

Principal component analyses were performed separately on the pretests and processes to reduce the number of variables. A regression with the component scores on the total score of the PSI showed that the Mathematics Achievement component accounted for 50% of the variance in the total scores. The heuristic strategy components, a subset of the process components, accounted for an additional 13% of the variance.

Eight of the ten heuristic strategies tested were found to be used more in solving one or two problems than in solving all the problems and were labeled problem-specific. Two cluster analyses were performed. One distinguished between students who used a range of or did not use a particular type of heuristic strategies. One separated students on their use of trial-and-error and equations.

AN EXPLORATION OF MATHEMATICAL PROBLEM-SOLVING PROCESSES

by

Norman Webb
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Problem solving is a complex process with many factors relating to the ability to solve problems. Our intuition tells us that the more conceptual mathematical knowledge a person knows the more mathematics problems he or she will be able to solve. To a lesser degree, our intuition tells us that the ability to use certain processes (for example, heuristic strategies) will increase our ability to solve problems.

Our intuition is supported by research studies such as Dodson (1970) which show mathematics achievement, a measure of the knowledge, comprehension, and application of mathematical concepts, to be a strong discriminator between good and poor problem solvers. Here the measure of problem-solving ability was insightful problems judged to be at the cognitive level of analysis, a higher level of problem than those used to measure mathematics achievement.

Studies which have focused on problem-solving processes have shown some relationship between problem-solving ability and the use of some processes. What the relationship is between processes and problem-solving ability is less clear than between mathematics achievement and

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This is a report of a dissertation for a Ph.D. from Stanford University.

problem-solving ability. Kilpatrick (1967) using eighth grade students (N = 57) found significant conrelations ($p < .05$) between the total score on twelve problems and the use of the processes successive approximation (.44), trial-and-error (.30), checking (.27), and structural errors (errors in understanding) (-.52).

Lucas (1972) compared the use of heuristic-oriented instruction with instruction which placed strong emphasis on concepts and a minimal emphasis on heuristic strategies in teaching calculus to college students (N = 30). The heuristic-oriented instruction had a positive effect on total problem-solving score (7 problems), kind of notation used, use of the method or the result of a related problem, and reasoning by analogy. The instruction had no effect on the number of diagrams drawn or modified, over-all frequency of checking, number of kinds of executive errors committed, use of trial-and-error, number of times of stopping without a solution, or productivity of equations.

Lipson (1972) compared, in teaching problem solving, the effectiveness of student teachers who had an intensive seminar on heuristic strategies with a group of student teachers who did not have such a seminar. The classes taught by the student teachers who participated in the seminar, on the average, gained more problem-solving scores than the classes taught by the other student teachers.

Although some processes have been found to be related to solving specific problems, questions remain concerning their use in solving problems in general. Studies by Wilson (1967) and Smith (1973) made the distinction between task-specific and general strategies and indicated some differences in their effectiveness. Wilson compared the teaching of two different levels of heuristics, task-specific and general, to high school students (N = 169). He concluded:

1. When different levels of heuristics are demonstrated in two training tasks, the levels of heuristics may complement each other and lead to superior solving performance.
2. Heuristics demonstrated on one task tend to be used on successive tasks.

Smith compared two groups of college students (N = 176). One group was given advice on the use of task-specific heuristic strategies and the other group was given advice on the use of general heuristic strategies while studying three different subject areas. Students in the task-specific group solved significantly more problems in one subject area and completed the tests in two of the specific areas faster than students in the general heuristic group. The students in the genreal heuristic group did not solve more transfer problems and did not solve them faster.

The major aims of the present study were to consider how cognitive and affective variables and the use of heuristic strategies are related to each other and to the ability of high school students to solve problems; to identify problem-specific strategies from those used in solving problems in general; and to identify problem-solving modes of groups of students.

To reflect the aims of the study, the specific questions that were answered were:

1. What is the separate and cumulative relationship of the pretest

(cognitive and affective) variables and the process variables to the problem-solving scores of second year high school algebra students?

2. Of the heuristic strategies considered, which are problem-specific and which are general strategies?
3. What problem-solving modes can be identified as being used by groups of students?

Design of Study

The paid participants of the study were forty second-year high school algebra students from four high schools which represented different social and economic areas. The students were equally divided by sex and distributed 15, 10, 10 and 5 among the four high schools. The students from each school were given a set of pretests containing measures of mathematics achievement, attitudes towards mathematics, spatial ability, verbal ability, reasoning ability, and problem-solving ability. Each student, in an individual interview, was then asked to solve eight problems in the Problem-Solving Inventory while doing his thinking aloud. The problems in the inventory were selected from thirty problems that were tried in the pilot study. Six of the eight problems were judged to be at the analysis level for the given population. Two of the problems were felt to be at a lower cognitive level. Each student was asked at the end of the interview to complete a questionnaire containing questions about problem-solving habits, the school background, and the interview situation. Three of the interviews did not record completely, giving a total of thirty-seven students for the analyses.

The protocols were coded after all the interviews were completed. The coding system was an adaptation of the systems used by Kilpatrick (1967) and Lucas (1972) and consisted of three types of variables, check list, process sequence, and score. The check list variables represented processes (e.g., draws a representative diagram, recalls the same or related problem), and were checked only once if the process was observed being used while solving the problem. The process sequence variables (e.g., reads the problem, uses deduction by synthesis, uses trial-and-error) were recorded in sequence each time the student performed one of the processes. The result was a sequence of symbols which described the processes the student used to solve the problem and the order in which the processes were observed. Each student was given a total score of zero to five for each problem. The total score was the sum of three subscores: approach (0,1), plan (0,1, or 2), and result (0,1, or 2). Table 1 gives the simple data description for the variables in each of the three main groups.

All of the protocols were coded by the investigator. A sample of forty-five protocols were coded by a second coder for a reliability check of the coding system. Most of the variables with low reliability coefficients were not included in the analyses. Five variables with low reliability coefficients were included for further analyses because they were felt to be of particular interest.

Analyses and Results

Question 1 (Relationship between problem-solving scores and processes). In order to reduce the number of variables, principal component analyses were performed on three sets of variables: pretests, heuristic strategies (a subset of the check list variables), and process sequence

TABLE 1

Simple Data Description
for All Variables
(N = 37)

Variables	Mean	s.d.	Max.	Min.	Student Distribution 0	1,2	3,4	5,6	7,8<
Check List									
Mnemonic notation	1.22	1.11	4.00	0.00	12	20	5	0	0
Rep. diagram - yes	3.49	1.17	6.00	1.00	0	7	23	7	0
- no	0.24	0.49	2.00	0.00	29	8	0	0	0
Auxiliary line	2.62	0.79	4.00	1.00	0	15	22	0	0
Recall a concept	1.54	1.39	5.00	0.00	11	16	9	1	0
Related problem	1.16	1.14	4.00	0.00	12	21	4	0	0
Meth. related prob.	0.49	0.69	2.00	0.00	24	13	0	0	0
Inductive reas.	0.22	0.48	2.00	0.00	30	7	0	0	0
Generalization	0.05	0.23	1.00	0.00	35	2	0	0	0
Specialization	0.70	1.00	3.00	0.00	21	12	4	0	0
Successive approx.	0.46	0.77	3.00	0.00	25	11	1	0	0
Misintrepret prob.	0.59	0.72	2.00	0.00	20	17	0	0	0
Irrevelant basis	0.51	0.84	3.00	0.00	24	11	2	0	0
Bright idea	0.78	0.79	3.00	0.00	15	21	1	0	0
Checks by equation	0.38	0.64	2.00	0.00	26	11	0	0	0
Checks condition	1.05	0.71	2.00	0.00	8	29	0	0	0
Reasonable result	0.95	0.85	3.00	0.00	14	22	1	0	0
Checks by special.	0.46	0.56	2.00	0.00	20	17	0	0	0
Der. by anth. meth.[a]	0.22	0.42	1.00	0.00	29	8	0	0	0
Doesn't know how	1.27	1.37	5.00	0.00	15	16	5	1	0
Counting error	0.95	2.36	14.00	0.00	22	13	1	0	1
Algebraic error	4.14	2.32	9.00	0.00	2	6	14	10	5
Other error	0.73	0.99	4.00	0.00	20	15	2	0	0
Process Sequence									
Reads	16.03	4.85	32.00	11.00					
Draws figure	4.97	3.59	15.00	1.00					
Deduct. by Synthesis	23.05	6.57	34.00	8.00					
Deduct. by Analysis	9.38	4.73	21.00	3.00					
Trial and error	11.32	10.21	47.00	0.00					
Recall	4.30	3.12	12.00	0.00					
Equation	11.30	6.36	24.00	0.00					
Algorithm	29.65	10.63	62.00	11.00					
Checks	5.30	2.91	14.00	1.00					
Structural Error	7.30	5.01	23.00	1.00					
Stops w/o Solution	4.22	2.79	13.00	0.00					
Scores									
Time	71.76	17.26	107.25	35.25					
Plan	9.32	4.26	16.00	2.00					
Result	6.76	4.11	14.00	0.00					
Total Score[b]	23.19	8.88	38.00	6.00					

[a] Derives solution by another method

[b] The total score on the PSI has a Cronbach alpha of 0.78

variables. The names given the main components for each set of variables were:

Pretest Components	Heuristic Strategy Components	Process Sequence Components
Mathematics Achievement	Pictorial Representation	Deductive Production
Verbal Reasoning	Productive Checking	Random Production
Field-Dependence	Concrete Representation	Non-production
Negative Anxiety	Recall	Recall Production
	Sudden Insight	

Tables 2, 3 and 4 give the loadings of the variables in each set on the respective components.

TABLE 2

Principal Components - Pretests

Pretest	Principal Components			
	Math Achieve.	Verbal Reasoning	Field-Dependence	Neg. Anx.
Arithmetic Reas.	0.597	0.369	0.060	-0.053
Math Inventory	0.792	0.378	-0.104	0.131
Math vs Non Math	0.602	-0.522	0.201	0.261
Math Fun vs Dull	0.814	-0.314	-0.164	0.296
Pro Math Self Cn.	0.804	-0.385	-0.080	0.197
Math Easy vs Hard	0.864	-0.138	-0.168	0.155
Ideal Math Self	-0.661	-0.092	0.343	0.475
Facilitating Anx.	0.593	-0.315	0.140	-0.314
Debilitating Anx.	-0.655	0.043	-0.239	0.549
Actual Math Self	0.737	-0.273	-0.101	-0.196
Vocabulary	0.307	0.661	-0.164	0.326
Speed Log. Reas.	0.261	0.504	0.162	-0.242
Hidden Figures	0.128	0.045	0.819	-0.039
Transitive Reas.	0.445	0.501	0.379	0.126
Punched Holes	0.665	0.066	0.442	0.263
Insightful Prob.	0.591	0.408	-0.378	-0.020
Cumulative % of Total Variance	40	53	63	70

TABLE 3

Principal Components - Heuristic Strategies

Heuristic Strategy	Principal Components				
	Pictorial Repre.	Product. Checking	Concrete Repre.	Recall	Sudden Insight
Mnemonic not.	0.069	0.240	0.635	-0.446	0.255
Rep. diag.- yes	0.712	-0.292	-0.152	-0.373	0.016
- no	-0.677	0.180	0.099	0.025	-0.327
Auxiliary line	0.590	0.091	0.165	0.111	-0.037
Related prob.	0.183	0.490	0.078	0.656	0.192
Meth. rel. prob.	0.488	0.320	-0.180	0.631	0.201
Inductive reas.	0.026	-0.378	-0.307	-0.205	0.103
Generalization	0.584	-0.137	-0.030	0.187	-0.521
Specialization	0.445	0.301	0.605	-0.183	-0.089
Successive appx.	0.146	-0.191	0.324	0.295	-0.453
Bright idea	0.255	0.112	-0.021	-0.082	0.647
Checks by eq.	0.323	0.597	-0.283	-0.375	-0.206
Checks condition	0.449	-0.498	0.415	-0.046	0.054
Reasonable result	-0.415	0.262	0.597	0.092	0.103
Checks by spec.	0.096	0.501	0.082	-0.219	-0.330
Der. anth. meth.	0.083	0.696	-0.388	-0.265	-0.042
Cumulative % of Total Variance	17	31	42	53	61

TABLE 4

Principal Components - Process Sequence

Process Sequence	Principal Components			
	Deductive Production	Random Production	Non-Production	Recall Production
Count. error	-0.375	0.305	-0.008	-0.155
Alg. error	-0.360	0.242	0.543	-0.409
Other error	-0.050	-0.232	0.469	-0.367
Reads	-0.256	0.535	0.345	-0.026
Draws figure	0.479	0.584	-0.098	0.113
Deduct. by syn.	0.763	0.011	0.349	-0.010
Deduct. by anal.	0.184	-0.335	0.625	0.450
Trial-and-error	-0.132	0.723	-0.127	0.492
Recall	0.209	-0.302	0.261	0.738
Equation	0.835	-0.240	0.060	-0.200
Algorithm	0.399	0.614	-0.043	0.263
Check	0.677	0.065	0.005	-0.405
Structural error	0.390	0.607	0.124	-0.252
Stops w/o soln.	-0.252	0.261	0.742	0.112
Cumulative % of Total Variance	20	37	50	62

Different regression analyses were performed using the total score of the Problem-Solving Inventory as the dependent variable and changing the order in which the independent variables, the three sets of component scores, were introduced into the equation. Table 5 gives the results of the regression analysis, when the four pretest components were introduced into the regression equation first, followed stepwise by the heuristic strategy components. Table 6 gives the results of the regression analysis when the heuristic strategy components were introduced first followed by the pretest components.

TABLE 5

Regression Analysis on Total Score

with Pretests entered before Heuristic Strategies

Component	Multiple R	Multiple R^2	Increase in R^2	F Value to Enter/Remove	P
Pretests					
Math Achieve.	0.7047	0.4966	0.4966	34.5228	0.0000
Verbal Reas.	0.7422	0.5509	0.0543	4.1130	0.0504
Neg. Anx.	0.7604	0.5782	0.0273	2.1389	0.1530
Field-Depend.	0.7735	0.5983	0.0201	1.5974	0.2152
Heuristic Strategies					
Pict. Repr.	0.8235	0.6781	0.0798	7.6832	0.0093
Sudden Insight	0.8438	0.7121	0.0340	3.5402	0.0696
Prod. Checking	0.8518	0.7255	0.0134	1.4205	0.2428
Concrete Rep.	0.8523	0.7265	0.0010	0.0985	0.7559
Recall	0.8523	0.7265	0.0000	0.0032	0.9553

TABLE 6

Regression Analysis on Total Score with Heuristic Strategies entered before Pretests

Component	Multiple R	Multiple R^2	Increase in R^2	F Value to Enter/Remove	p
Heuristic Strategies					
Pict. Repr.	0.5510	0.3037	0.3037	15.2625	0.0004
Prod. Checking	0.6487	0.4209	0.1172	6.8812	0.0129
Sudden Insight	0.7009	0.4913	0.0704	4.5698	0.0400
Concrete Rep.	0.7287	0.5310	0.0397	2.7082	0.1096
Recall	0.7290	0.5314	0.0004	0.0283	0.8675
Pretests					
Math Achieve.	0.8166	0.6668	0.1353	12.1830	0.0015
Verbal Reas.	0.8407	0.7068	0.0401	3.9655	0.0559
Field-Depend.	0.8511	0.7243	0.0175	1.7781	0.1930
Neg. Anx.	0.8523	0.7265	0.0021	0.2118	0.6491

Question 2 (Problem-specific versus general strategies). Cochran Q was calculated for each of ten heuristic strategies to test if the strategy was used more in solving one problem than in solving all of the problems. Eight of the ten strategies were used significantly ($p < .05$) more on one or two problems than on all eight problems. Table 7 lists the ten heuristic strategies and the results of calculating the Cochran's Q for each strategy.

TABLE 7

Frequencies of Heuristic Strategies by Problems

Heuristic Strategy	Problem								Cochran's Q	p
	1	2	3	4	5	6	7	8		
Mnemonic not.	0	2	17	10	0	14	2	0	77.73	.001
Rep. diag.- yes	9	33	11	33	0	2	33	8	153.05	.001
Related prob.	3	6	0	11	7	12	2	2	30.88	.001
Inductive reas.	0	0	1	2	1	4	0	0	14.39	.05
Specialization	1	8	0	9	2	2	4	0	31.08	.001
Successive appx.	0	7	4	1	3	2	0	1	16.02	.025
Bright Idea	5	4	4	1	7	4	2	2	7.76	.50
Checks by eq.	0	3	5	0	4	1	1	0	16.89	.02
Checks condition	1	3	21	8	2	5	0	0	69.21	.001
Der. anth. meth.	3	0	1	1	0	0	2	1	7.99	.50

Friedman Statistic

$\underline{x_r^2}$ = 11.79 (p = .20)

A Friedman two-way analysis of variance by ranks (x_r^2) was used to test if any problem evoked a larger number of different strategies compared to the other problems. This analysis was performed to test if the problem-specific strategies were all used in solving the same problem. The Friedman statistic was not significant at the .05 level (p = .20). For the different strategies, the one or two problems each problem-specific strategy was used on varied over the eight problems.

<u>Question 3 (Problem-solving modes)</u>. Two cluster analyses were performed on the students to try and identify problem-solving modes. One clustered the students by using their frequency of use of heuristic strategies (Table 8) and one by using their scores on the process sequence variables (Table 9). The distinguishing characteristics between the three heuristic strategy clusters, called H1, H2a and H2b, were the use of a range of different strategies by one group (H2b) and the complete non-use of production strategies by another group (H1). The group H2b, who used a range of different strategies, had a significantly higher mean on the total score for the inventory. The distinguishing characteristics between the process sequence clusters (named Q1, Q2a, Q2b) were the use of trial-and-error, equations, and structural errors. The group which used a medium amount of trial-and-error and equations (Q1) and which made very few structural errors had the highest mean score on the Problem-Solving Inventory. Combining the two cluster analyses (Table 10), the trend was for those students who used a wide range of heuristic strategies and a medium amount of trial-and-error and equations or a high amount of deduction to score higher on the inventory.

<u>Other Results</u>. No significant sex difference were found in student's use of problem-solving strategies or in their ability to solve the problems correctly.

Some differences between schools in the use of processes were found to be statistically significant. Most of these differences were attributed to differences between schools in mathematics achievement. Differences found between schools that could not be accounted for by differences in mathematics achievement were in the use of "checking that the solution satisfies the conditions" and "equations."

TABLE 8

Means of each Heuristic Strategy for Heuristic Strategy Clusters

Heuristic Strategy	Cluster H1 N = 12	Cluster H2a N = 7	Cluster H2b N = 18	Grand Mean
Inductive reas.	0.00	.71	.17	.22
Specialization	0.00	.57	1.22	.70
Successive appx.	0.00	.43	.78	.46
Generalization	0.00	0.00	.11	.05
Der. anth. meth.	.08	0.00	.39	.22
Checks by eq.	.25	0.00	.61	.38
Checks by spec.	.58	0.00	.56	.46
Meth. rel. prob.	.08	0.00	.94	.49
Related prob.	1.00	.14	1.67	1.16
Bright idea	1.08	.29	.78	.78
Reasonable resul	1.17	.57	.94	.95
Checks condition	.83	1.57	1.00	1.05
Mnemonic not.	1.17	1.14	1.28	1.22
Auxiliary line	2.33	2.71	2.78	2.62
Rep. diag.-yes	3.00	4.14	3.56	3.49
-no	.58	.14	.06	.24

Score Variables

Score Variable	H1	H2a	H2b	Grand Mean	$F_{2,34}$	P less than
Time	72.35	67.82	72.96	71.76		
Plan	6.58	8.0	11.67	9.32		
Result	4.00	5.43	9.11	6.76		
Total Score	17.33	20.00	28.33	23.19	8.66	.005

TABLE 9

Means of Process Sequence Variables
for Process Sequence Clusters

Variable	Cluster			Grand Mean
	Q1 N = 17	Q2a N = 14	Q2b N = 6	
Count. error	.35	.79	3.00[a]	.95
Alg. error	3.70	3.93	5.83	4.14
Other error	.47	1.07	.67	.73
Reads	15.00	15.92	19.17	16.03
Draws figure	4.00	5.86	5.67	4.97
Ded. by syn.	23.11	25.70	16.67	23.05
Ded. by anal.	10.88	9.79	4.17	9.38
Trial & Error	13.50	4.57	20.83	11.32
Recall	4.82	4.64	2.00	4.30
Equation	12.00	14.14	2.67	11.30
Algorithm	31.47	26.79	31.17	29.65
Check	4.47	7.14	3.33	5.30
Structural error	5.00	9.21	9.33	7.30
Stops w/o soln.	3.53	4.93	4.50	4.22

Scores

	Q1	Q2a	Q2b	Grand Mean	$F_{2,34}$	P Less Than
Time	69.56	74.23	71.92	71.76		
Plan	10.94	8.86	5.83	9.32		
Result	8.05	6.50	3.67	6.76		
Total Score[b]	26.29	22.43	16.17	23.19	3.35	.05

[a] One student had 14 counting errors. Without this student, the mean of counting errors for Q2b would be 0.80.

TABLE 10

Distribution between Heuristic Strategy Clusters and Process Sequence Clusters

Heuristic Strategy Clusters	Process Sequence Clusters		
	Q1	Q2a	Q2b
H1	5 (22.00)[a]	5 (13.00)	2 (16.50)
H2a	3 (24.33)	1 (16.00)	3 (17.00)
H2b	9 (29.33)	8 (29.13)	1 (13.00)

[a] The mean total score on the PSI for each cell is given in parenthesis.

The self-reporting information obtained from the questionnaire indicated that those students who reported having a hard time concentrating on their homework scored lower on the inventory and made less use of "deduction by synthesis" and "equations." "Drawing a representative diagram" was positively correlated with agreement to the statement "drawing a figure helps to solve math problems." Female students, on the average, felt more strongly than male students that setting up a problem as neatly as possible helps in solving the problem.

Conclusions and Implications

Mathematics achievement was the variable with the highest relation to mathematical problem-solving ability. The use of heuristic strategies had some relation to mathematical problem-solving ability not accounted for by mathematics achievement. In particular, the component Pictorial Representation accounted for a significant amount of the variance. Thus, the processes used by students in this study added to their ability to solve problems beyond their mathematical conceptual knowledge (mathematics achievement).

Students who used a wide range of heuristic strategies, on the average, were better problem solvers. Most of these heuristic strategies were found to be problem-specific. This implies that in order to solve several different problems, a range of problem-specific strategies needs to be employed.

Strategies such as "specialization" and "successive approximations" were used at least once on six of the eight problems, but were used by more students on one or two of the problems. One possible reason for the restricted use of these strategies is that the students only used the strategies in obvious ways and did not employ the strategies where they could be strategically used. One direction for research would be to examine whether students can be taught to use such strategies to solve a wide range of problems.

Students who used a moderate amount of trial-and-error and a moderate amount of equations or who used equations often and trial-and-error seldom performed about the same on the Problem-Solving Inventory. Students who used a high frequency of trial-and-error and had a low use of equations did not do as well. For these high school students and for the problems in the inventory, it appeared that trial-and-error as an approach to problem solving had a value as a supplementary process to the use of equations, but not as a replacement for the use of equations.

This study did not include an instructional phase because it was desired to observe what heuristic strategies were being used by students without any special instruction. It was found that the students did use some heuristic strategies and that these strategies did have an effect on their ability to solve the problems. Research is needed which will include an instructional phase on the use of heuristic strategies to determine if such instruction increases the effectiveness of heuristic strategies beyond that accounted for by mathematics achievement.

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