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ABSTRACT

This paper proposes two modifications of the Lorenz technique for measuring inequalities in financing education. One is to order expenditures (and tax rates) by district fiscal ability before calculating the cumulative percentages. It is possible, as a result, to get a Lorenz curve that crosses the diagonal line, which gives some useful insights. The other is to calculate Gini coefficients for both expenditures and tax rates and to consider the sum of the two coefficients to be a measure of the total inequality in the system. (Author)

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USE OF THE LORENZ CURVE AND GINI INDEX IN SCHOOL FINANCE RESEARCH

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Those interested in school finance have long been interested in inequalities of educational expenditure and educational effort. Economists have long been interested in inequalities of income. It is unfortunate that there has not been more communication between these two groups of people, for the economists have one standard technique for looking at income inequalities that can be very useful in examining inequalities in educational expenditure and effort. That technique is the use of the Lorenz curve and its associated Gini coefficient. However, the use of the Lorenz curve in the way it is usually used by economists would give misleading results as a measure of inequalities in school finance. It is the purpose of this paper to describe and illustrate a modification of the Lorenz technique that yields some unique insights for those interested in school finance research.

As used in the measurement of income inequality, the Lorenz technique is to array all individuals in order of ascending income. Then at convenient points (for each one percent of the total individuals, say) the cumulative percent of individuals and the cumulative percent of total income is calculated. As an example of how this is done, the information for families in the United States in 1969 is given in Table I. The cumulative percentages are then plotted on a graph called a Lorenz chart. The horizontal axis is the cumulative percentage of families, running from 0 to 100, and the vertical axis is the cumulative percentage of income, also running from 0 to 100. The data given above have been plotted on a Lorenz chart (Figure 1), and are represented by the curved line. The straight diagonal line represents complete equality of incomes. That is, if all families received the same income, the "lowest" ten percent of the families would receive ten percent of the income, the "lowest" fifty percent would receive fifty percent of the income, and so on. The extent to which the curved line sags below the diagonal line, then is a measure of the inequality of incomes. This measure is formalized in the Gini coefficient, which is the ratio of the area bounded by the diagonal line and the curved line, divided by the area bounded by the diagonal line, the horizontal axis, and the right-hand vertical line. A Gini coefficient of zero would represent absolute equality, and a Gini coefficient of 1.00 would represent absolute inequality (the situation in which one individual got all of the income and no one else got any). The Gini coefficient for the family income data displayed above is about 0.36.

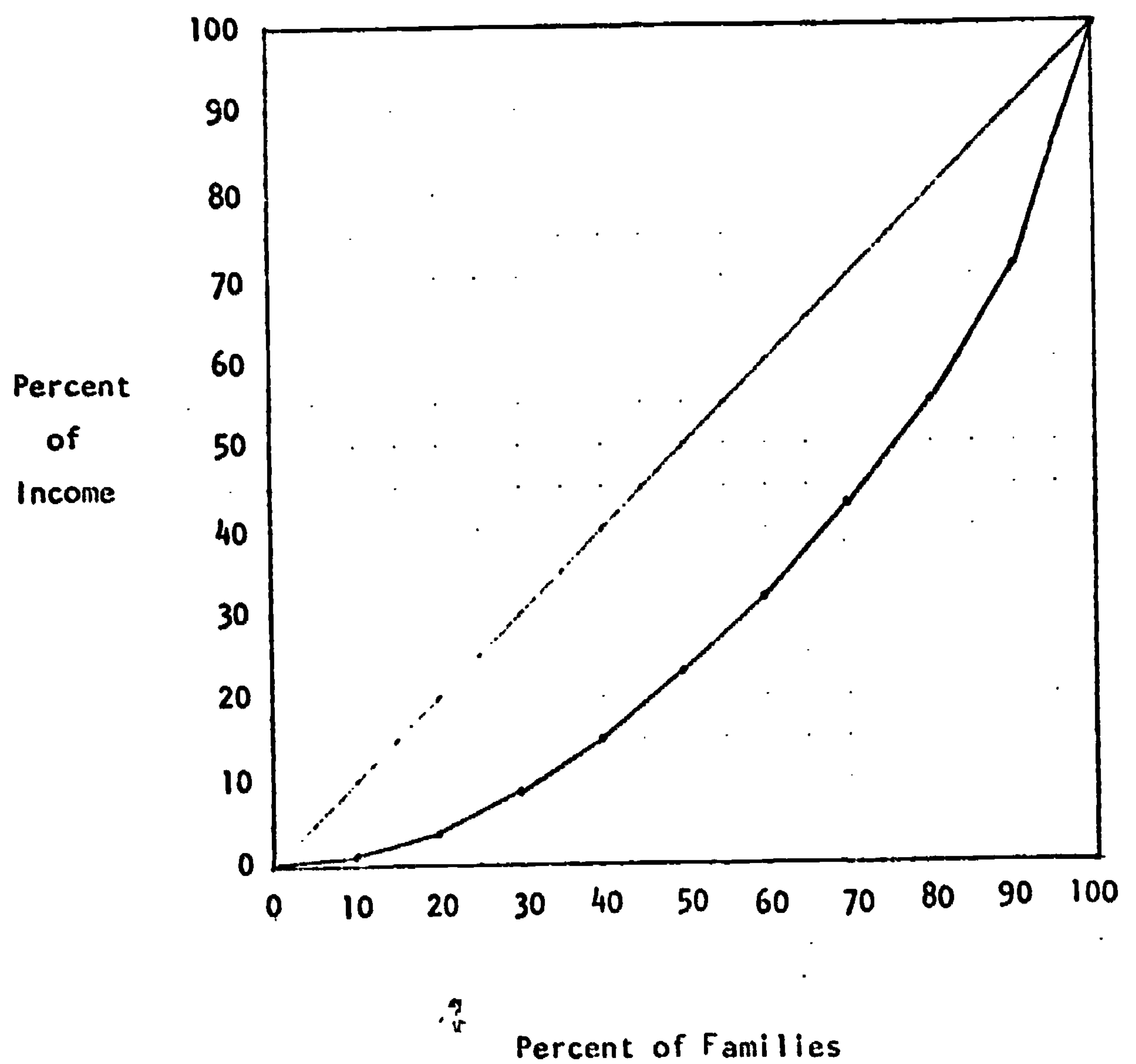
There are, of course, other measures of inequality that can be used. They include the range of incomes (the highest less the lowest), the interquartile range (the difference between income at the third quartile and the first quartile), the interquartile range divided by the mean, the standard deviation, and the coefficient of variation (the standard deviation divided by the mean). Each is a better measure than the previous one, at the cost of increasing difficulty of computation and of explaining to a lay audience. The range takes into account only two incomes, the lowest and the highest.

Table I

Family Income in the U.S., 1969

Income Rank	Percent of Total Income	Cumulative Percent of Families	Cumulative Percent of Income
Lowest tenth	1	10	1
Second tenth	3	20	4
Third tenth	5	30	9
Fourth tenth	6	40	15
Fifth tenth	8	50	23
Sixth tenth	9	60	32
Seventh tenth	11	70	43
Eighth tenth	12	80	55
Ninth tenth	16	90	71
Highest tenth	29	100	100

Figure I



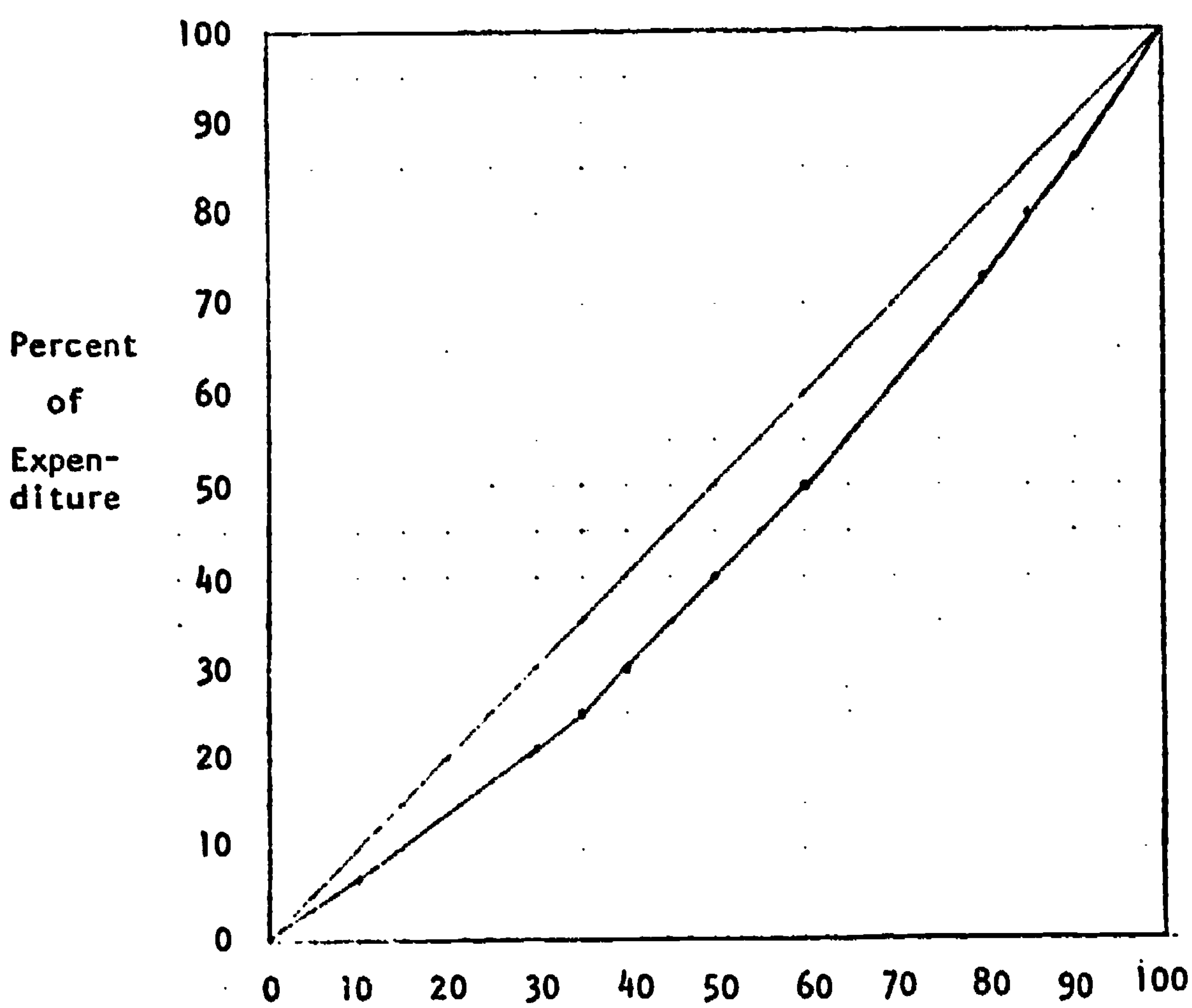
Thus this most widely used measure of inequality is only a measure of two extremes that are almost certain to be unusual cases. The interquartile range ignores these extremes, thus giving a more stable measure at the expense of ignoring half of the data. The interquartile range divided by the mean is simply a way of standardizing the interquartile range so that distributions with different means may be validly compared. The standard deviation (and its standardized version, the coefficient of variation) takes into account all of the data. Its principal drawback is that condensing all of the data into a single statistic does not allow us to look at any other characteristics of the distribution. The Lorenz curve allows visual inspection of the characteristics of the distribution, and the Gini index condenses the information on inequality into a single statistic that is comparable across distributions with different means.

The difficulty with applying the Lorenz curve to expenditures per student in the public schools is that not all of the differences in expenditure are undesirable. At the same time that states are engaged in equalization programs to reduce the differences in expenditure caused by differences in fiscal ability, they are subsidizing programs for special students that result in higher expenditures for them than for normal students. Unfortunately, the results of these two kinds of subsidization are combined in a single statistic called expenditures per student. Part of the difficulty in trying to measure inequalities of educational expenditure is the difficulty of separating expenditures for normal students from expenditures for mentally retarded, physically handicapped, occupational education, and for children in necessary small schools. Measures such as the range, interquartile range, and standard deviation are incapable of doing this. The Lorenz curve, as traditionally used by economists, is also

Table 2

District No.	Students	Expenditures per Student	A.V. per Student	Cumulative Percent of Students	Cumulative Percent of Expenditures
1	1000	800	20,000	10	7
2	2000	900	40,000	30	21
3	500	1000	60,000	35	25
4	500	1100	80,000	40	30
5	1000	1200	100,000	50	40
6	1000	1300	90,000	60	50
7	2000	1400	70,000	80	73
8	500	1500	50,000	85	80
9	500	1600	30,000	90	86
10	1000	1700	10,000	100	100

Figure 2



Gini = .122

Percent of Students

incapable of making this distinction. Table 2 and Figure 2 give fictitious data for a sample of ten districts, and the Lorenz curve based on that data. As with the chart of family incomes, the data have been arrayed in order of increasing expenditure per student, and the cumulative percentage of students and of expenditure is calculated. The data on assessed valuation per student is included for information purposes, but is not used in the calculations. Since there is no way of knowing how much of the expenditure in a particular district is the result of differences in fiscal ability, how much the result of differences in community desires for education, and how much the result of differences in expenditures for special students, the chart is interesting, but not especially informative.

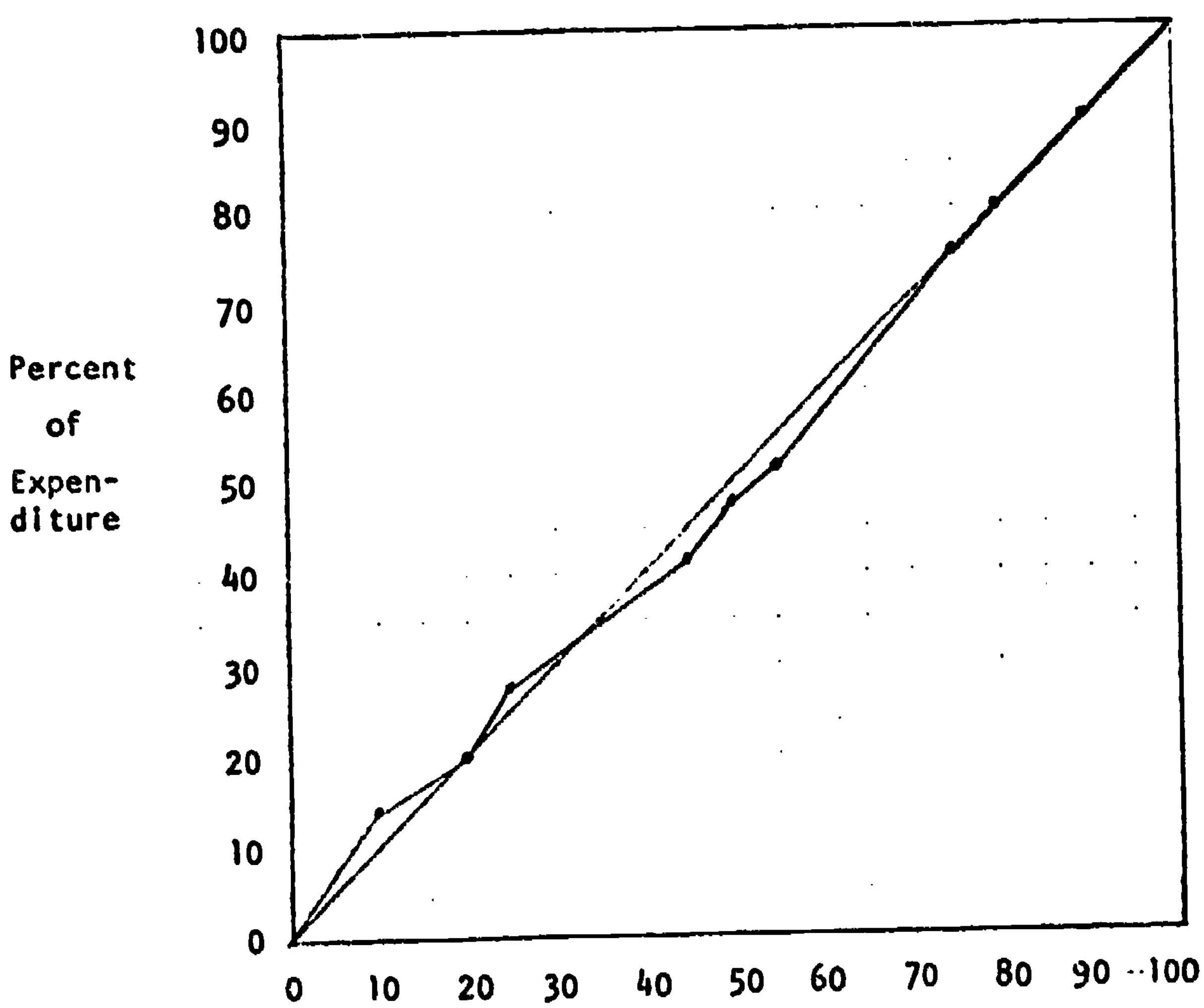
If, on the other hand, we choose to array our districts in order of increasing fiscal ability, the Lorenz chart can then give us information about the extent to which differences in expenditures are correlated with differences in fiscal ability. In Table 3 and Figure 3 this has been done. The data on assessed valuation per student were purposely chosen to be as uncorrelated as possible with expenditures per student. The result shows in the Lorenz chart, where the Lorenz curve is much closer to the diagonal line.

Figure 3 also shows a possible consequence of the use of this technique: part of the Lorenz curve lies above the diagonal line. In the conventional use of the Lorenz curve it is impossible for this to happen. But this is a virtue of this variation of the technique. We can see at a glance, for example, that the ten percent of the districts that are lowest in fiscal ability actually spend 14 percent of the total money spent. It can readily be seen that, with this technique, it is possible to have substantial differences in expenditure, yet have the Lorenz curve lie close to the

Table 3

District No.	Students	Expenditures per Student	A.V. per Student	Cumulative Students	Percent of Expenditures
10	1000	1700	10,000	10	14
1	1000	800	20,000	20	20
9	500	1600	30,000	25	27
2	2000	900	40,000	45	42
8	500	1500	50,000	50	48
3	500	1000	60[illegible]	55	52
7	2000	1400	70[illegible]0	75	75
4	500	1100	80,000	80	80
6	1000	1300	90,000	90	90
5	1000	1200	100,000	100	100

Figure 3



Gini = 0.004

Percent of Students

diagonal line. In calculating the Gini Index, it is appropriate to think of areas between the Lorenz curve and the diagonal line that lie above the line as negative areas. The result is that there could be substantial deviation of the Lorenz curve from the diagonal line, but if this were the result of differences in expenditure that were uncorrelated with differences in fiscal ability, the Gini Index would be close to zero. Indeed, that is so here, where the Gini Index is only 0.004, compared with an index of 0.122 using the same data differently arrayed in Figure 2.

It should be particularly noted that the basic data used in Figures 2 and 3 are identical, and the axes of the Lorenz charts still have the same labels. Only the method of ordering the data before calculating the cumulative percentages is different.

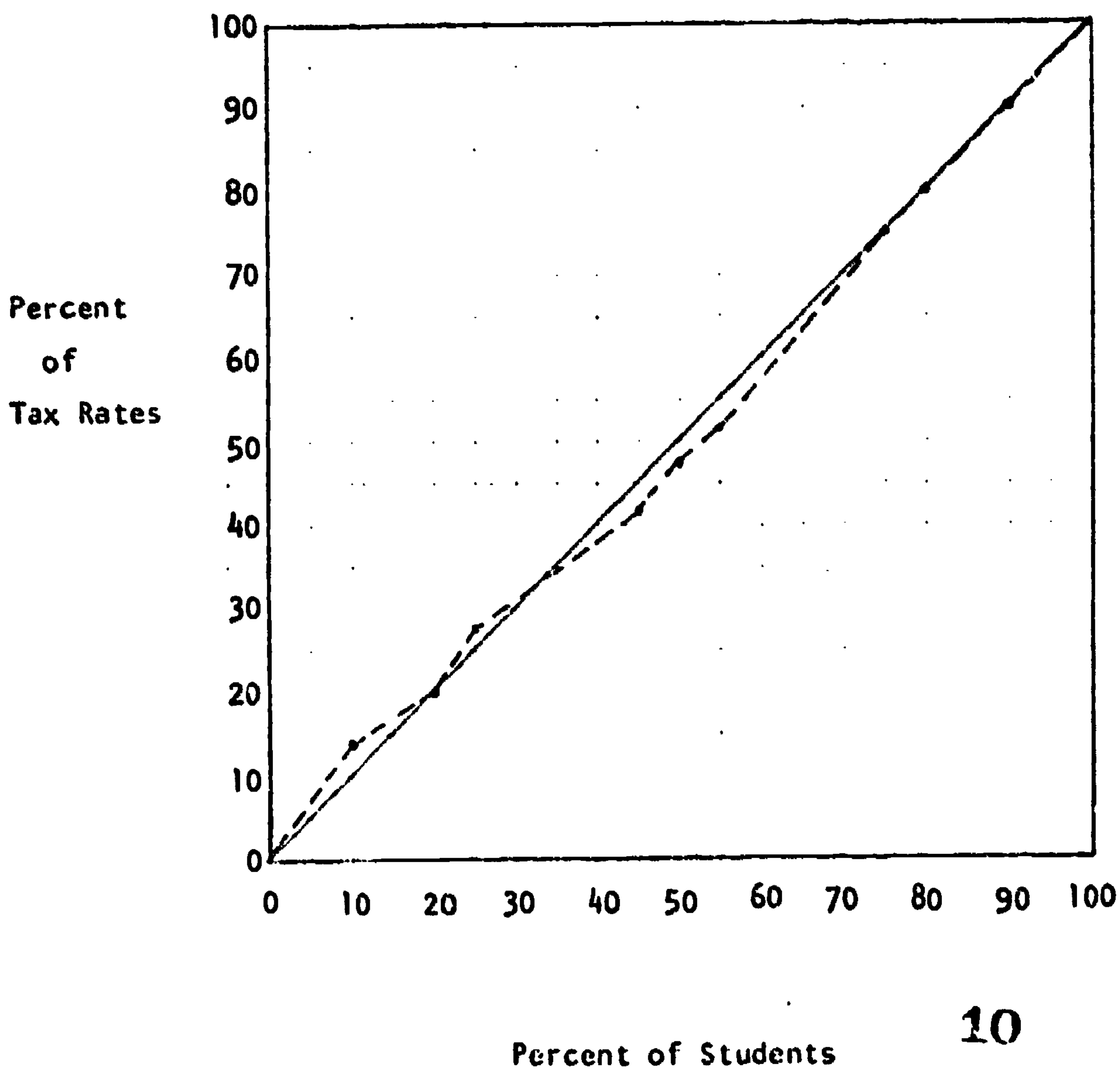
We have been examining one side of the coin of fiscal neutrality, that expenditures should not be a function of community wealth. The other side is that tax effort should not be a function of community wealth. The Lorenz curve can be used to examine this in an analogous fashion, although the meaning of a cumulative percentage of tax rates is not as intuitively obvious. In Table 4 and Figure 4, tax rates have been calculated for the districts in our sample assuming a power equalizing system with full recapture that guarantees an expenditure of $100 per student for each mill of tax rate. It can be seen that the tax rates have a correlation of 1.00 with expenditures, and thus the Lorenz curve for tax rates looks just like the Lorenz curve for expenditures. However, it has an "opposite" meaning. That is, in the same way that high expenditures per student and high family incomes are considered to be "good", high tax rates are considered "bad." Thus, where an invidious discrimination would tend to make the Lorenz curve of expenditures fall below the diagonal, it will make the Lorenz curve of tax rates lie above the

Table 4

Power Equalized System

Dist.	Studs.	Exp./ Stud.	AV per Stud.	Tax Rate	Cumulative Percent of Studs.	Expends.	Tax Rates
10	1000	1700	10,000	17	10	14	14
1	1000	800	20,000	8	20	20	20
9	500	1600	30,000	16	25	27	27
2	2000	900	40,000	9	45	42	42
8	500	1500	50,000	15	50	48	48
3	500	1000	60,000	10	55	52	52
7	2000	1400	70,000	14	75	75	75
4	500	1100	80,000	11	80	80	80
6	1000	1300	90,000	13	90	90	90
5	1000	1200	100,000	12	100	100	100

Figure 4



diagonal. In order to make the Gini coefficients comparable, then, between expenditures and tax rates, the sign of the Gini coefficient for tax rate should be reversed. The Gini coefficient for tax rates in this case is then -0.004. If the two coefficients, for expenditures and tax rate, are added together it should give a measure of the total inequality in the system. In this case the sum of the two coefficients is exactly zero, confirming the idea of a system that is completely neutral fiscally, even though expenditures vary.

In Table 5 and Figure 5 the tax rates have been calculated assuming instead that there is a state flat grant of $500 per student, with no equalization. The tax rates have been calculated on the assumption that all of the rest of the money must be raised by local taxes. Of course it is highly unlikely, in such a situation, that District 10 would continue to spend $1700 per student, taxing itself at 120 mills. But this has been allowed to remain to illustrate the difference that the two approaches make in tax rates when expenditures are not changed. The Gini coefficient for tax rates in this case is 0.435, and the sum of the Gini coefficients for expenditures and tax rate is 0.439.

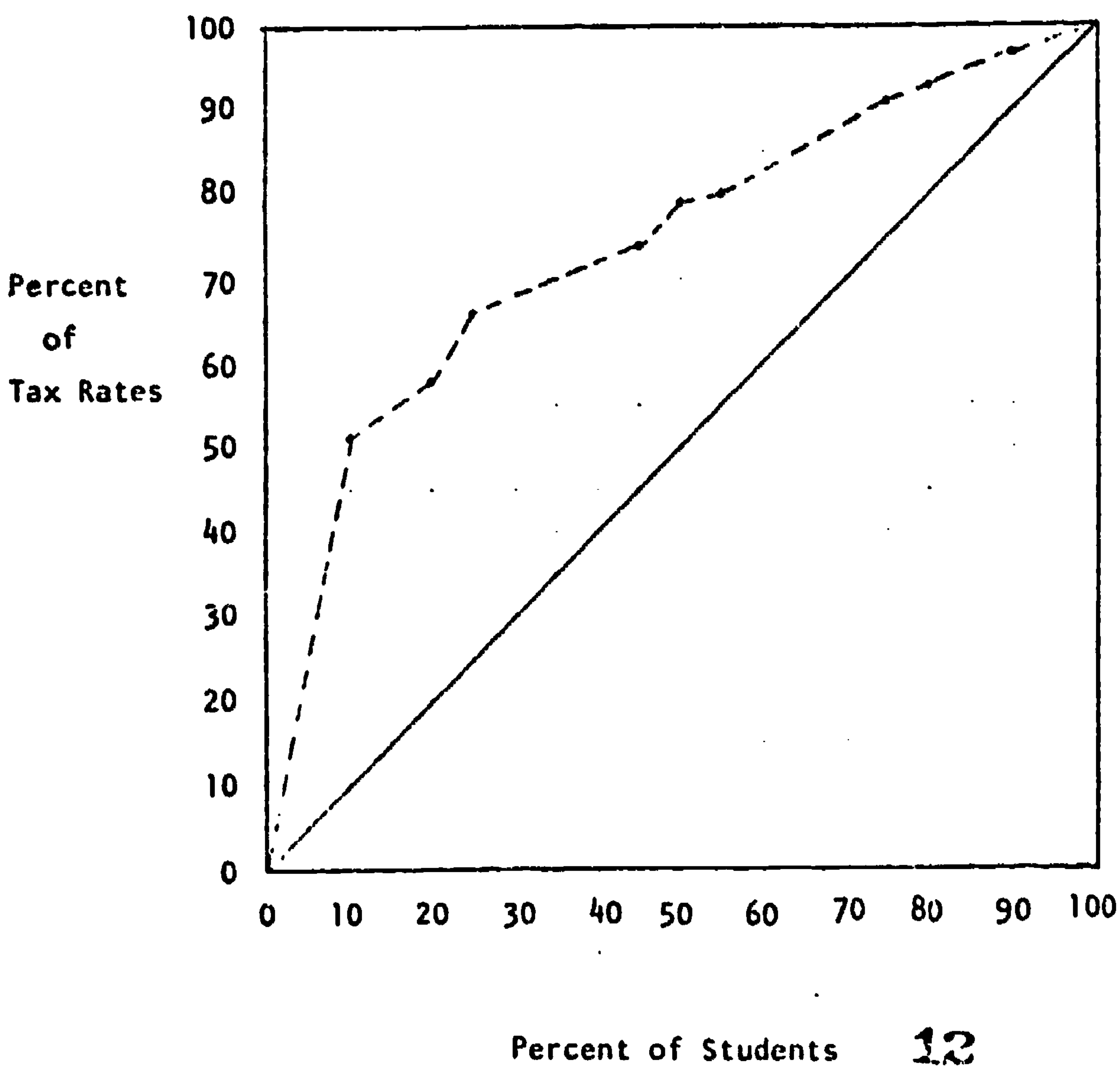
(please see page 11)

Table 5

$500 Flat Grant

Dist.	Studs.	Exp./ Stud.	AV per Stud.	Tax Rate	Cumulative Percent of Studs.	Expends.	Tax Rates
10	1000	1700	10,000	120	10	14	51
1	1000	800	20,000	15	20	20	58
9	500	1600	30,000	37	25	27	66
2	2000	900	40,000	10	45	42	74
8	500	1500	50,000	20	50	48	79
3	500	1000	60,000	8	55	52	80
7	2000	1400	70,000	13	75	75	91
4	500	1100	80,000	8	80	80	93
6	1000	1300	90,000	9	90	90	97
5	1000	1200	100,000	7	100	100	100

Figure 5



It seems possible that the aggregate of decisions of the separate school districts might be in a direction that would tend to minimize the sum of the Gini coefficients for expenditures and tax rates. Continuing with the assumption of a $500 flat grant per student, and using the same type of table calculation as previously used, the following Gini coefficients were obtained:

	Expenditures	Tax Rates	Expenditures plus Tax Rates
Assuming equal expenditures of $1200 (tax rates ranged from 7 to 70 mills)	0.000	0.373	0.373
Assuming equal tax rates of 13 mills (expenditures ranged from $630 to $1800)	0.158	0.000	0.158
Assuming an in-between position (tax rates ranged from 10 to 20 mills, and expenditures from $700 to $1500)	0.104	0.127	0.231

The same thing was tried assuming that there was no state aid, and that all income was raised through local taxes, with the following results:

	Expenditures	Tax Rates	Expenditures plus Tax Rates
Assuming equal expenditures of $1200 (tax rates ranged from 12 to 120 mills)	0.000	0.378	0.378
Assuming equal tax rates of 20 mills (expenditures ranged from $200 to $2000)	0.267	0.000	0.267
Assuming an in-between position (tax rates ranged from 16 to 40 mills, and expenditures from $400 to $1600)	0.149	0.164	0.313

It appears from this very preliminary investigation that in an unequalized system the sum of district decisions that results in least variation in tax rates results in the smallest combined Gini coefficient.

In applying this technique, it is only approp:'ate to use assessed valuation per student as an ordering device in states where there has been almost complete equalization of assessment ratios statewide. Otherwise, the appropriate measure to use is full value, or equalized value, per student; the appropriate measure of tax effort, then, is tax rate on equalized value.

Some charts of expenditures per student and of tax rates for Oregon districts are appended (Figures 7 to 9) as an illustration of the use of the technique with real data. Assessments are carefully equalized in Oregon, so that assessed valuation can confidently be used as a measure of fiscal ability. One of the things that may immediately be noted is the nearness of the Lorenz curves to the diagonal line, compared with the curve in Figure I for family income in the United States. This is only partially a result of the state equalization program in Oregon. It is also an artifact of our method of measurement of educational expenditures which makes the implicit assumption that all students in a particular school district have exactly the same amount spent on them. The result of the aggregation is a vast leveling of expenditure discrepancies. The analogous case is that of constructing a Lorenz curve of income inequalities by region of the U.S. If this is done for the four regions into which the country is divided by the U.S. Statistical Abstract, with the assumption made that all families within each region receive the same income, the results are as shown in Table 6 and Figure 6. This Lorenz chart and Gini coefficient should be compared with Figure I. Of course this aggregation error is not peculiar to the Lorenz technique, but will affect any measure of inequality one might choose to use.

Table 6

Family Income in the U.S., 1969, by Region

	Families (000)	Median Income	Cumulative Percent of Families	Cumulative Percent of Income
South	19,247	$ 8,105	30	26
Northeast	15,461	10,018	55	52
North Central	17,537	10,020	82	81
West	11,172	10,037	100	100

Figure 6

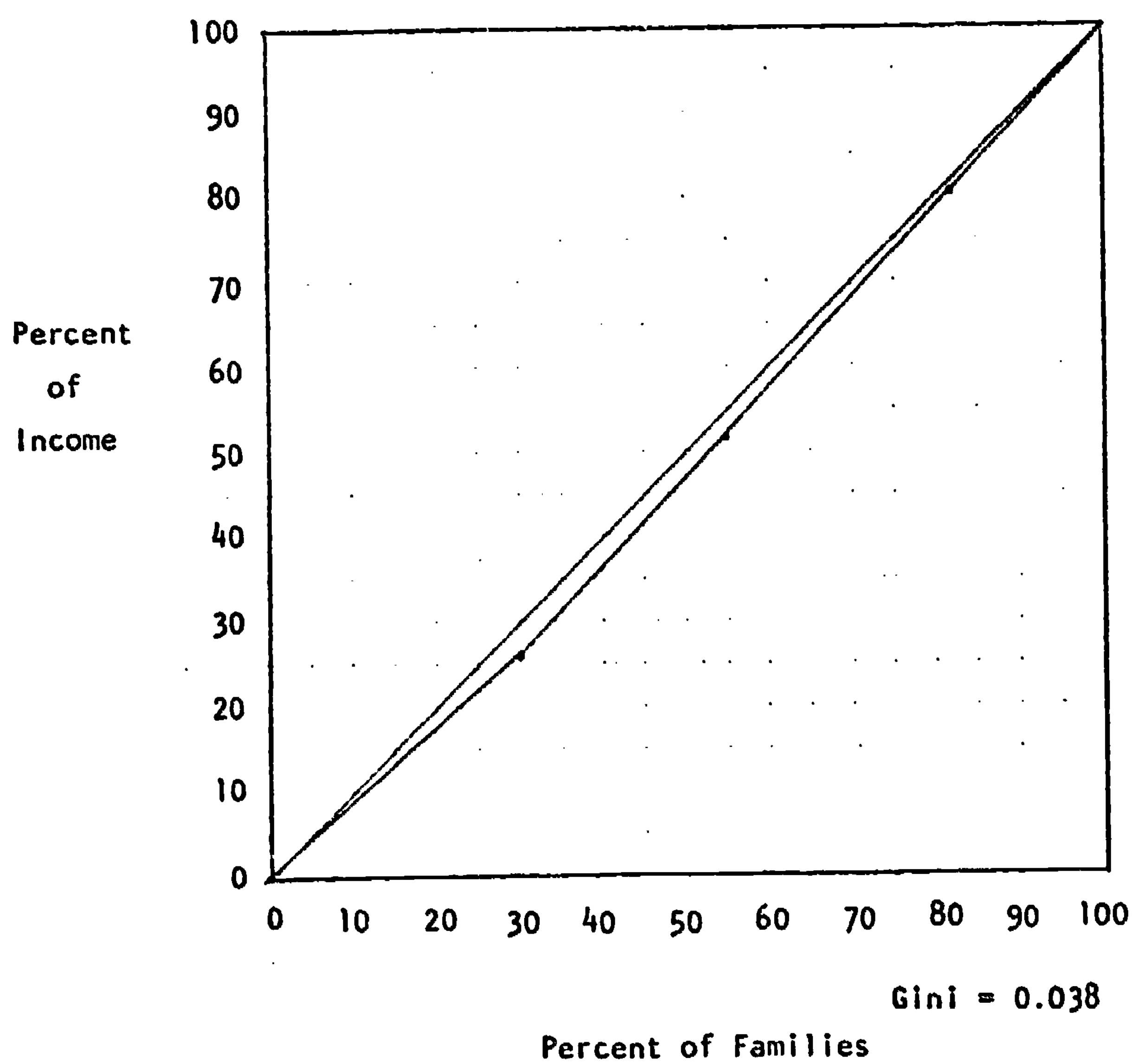


FIGURE 7

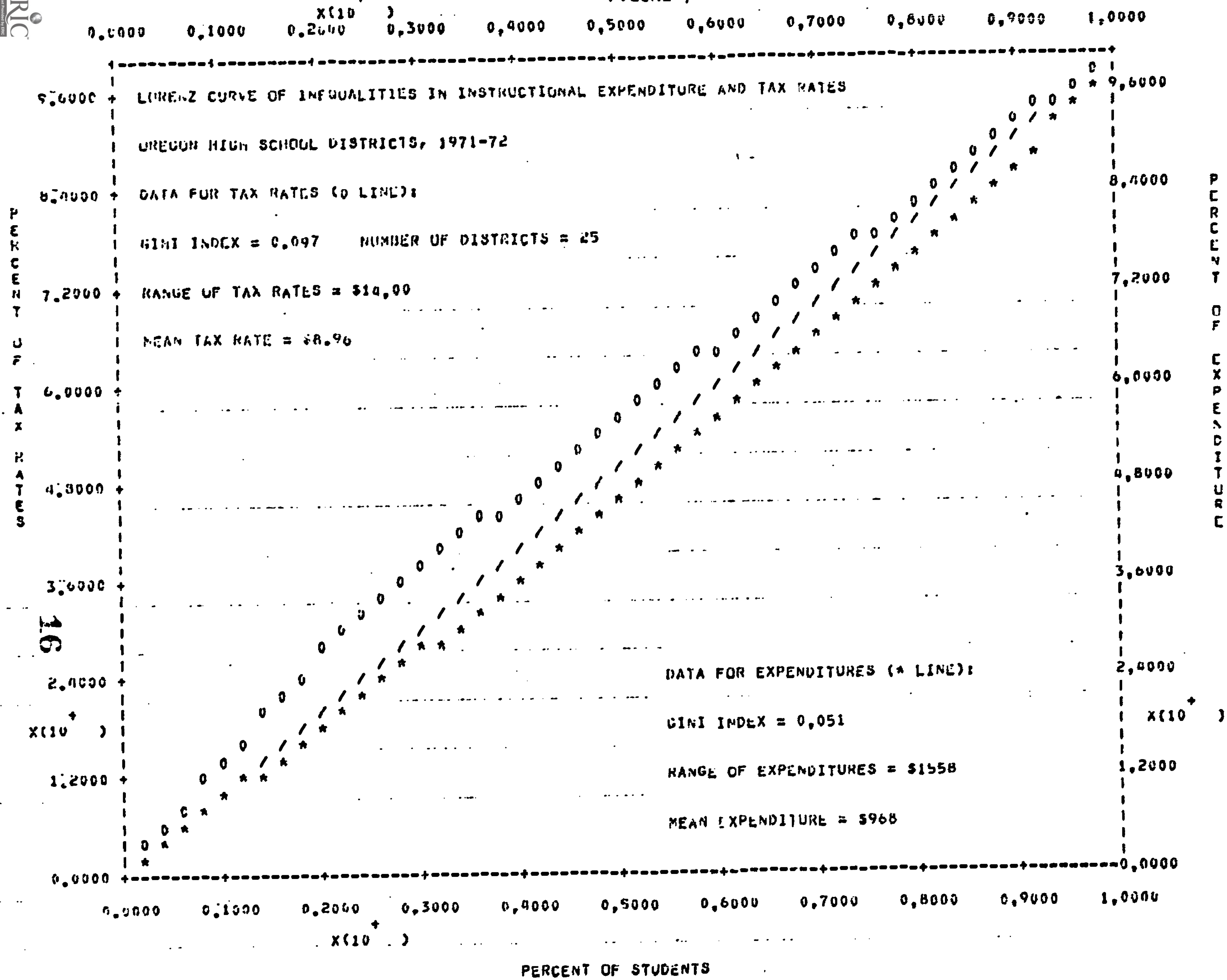

LORENZ CURVE OF INEQUALITIES IN INSTRUCTIONAL EXPENDITURE AND TAX RATES
OREGON HIGH SCHOOL DISTRICTS, 1971-72
DATA FOR TAX RATES (0 LINE):
GINI INDEX = 0.097 NUMBER OF DISTRICTS = 25
RANGE OF TAX RATES = $14.00
MEAN TAX RATE = $8.96
DATA FOR EXPENDITURES (* LINE):
GINI INDEX = 0.051
RANGE OF EXPENDITURES = $1558
MEAN EXPENDITURE = $968
PERCENT OF TAX RATES
PERCENT OF EXPENDITURE
PERCENT OF STUDENTS
0.0000
0.1000
0.2000
0.3000
0.4000
0.5000
0.6000
0.7000
0.8000
0.9000
1.0000
1.2000
2.4000
3.6000
4.8000
6.0000
7.2000
8.4000
9.6000

FIGURE 8

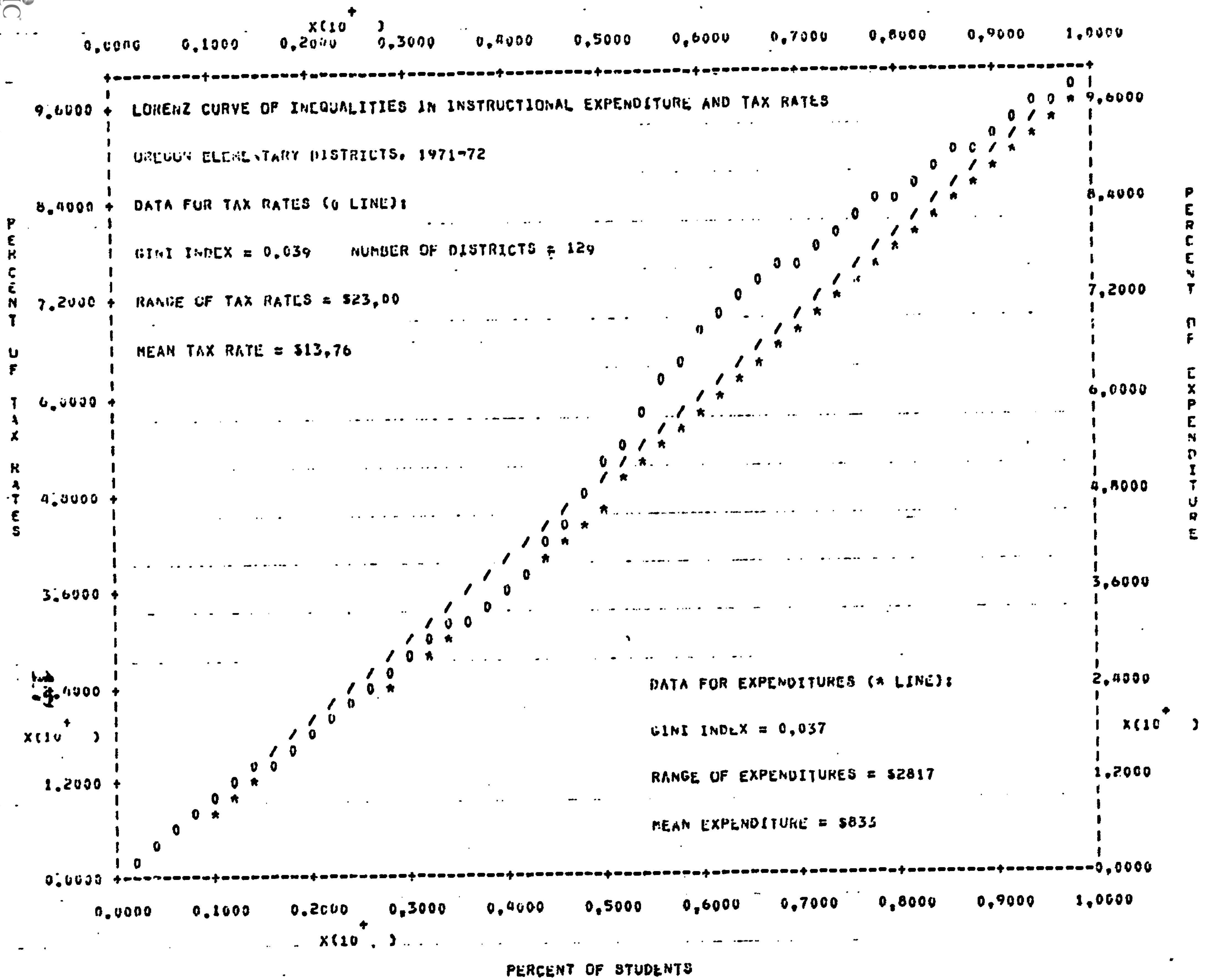
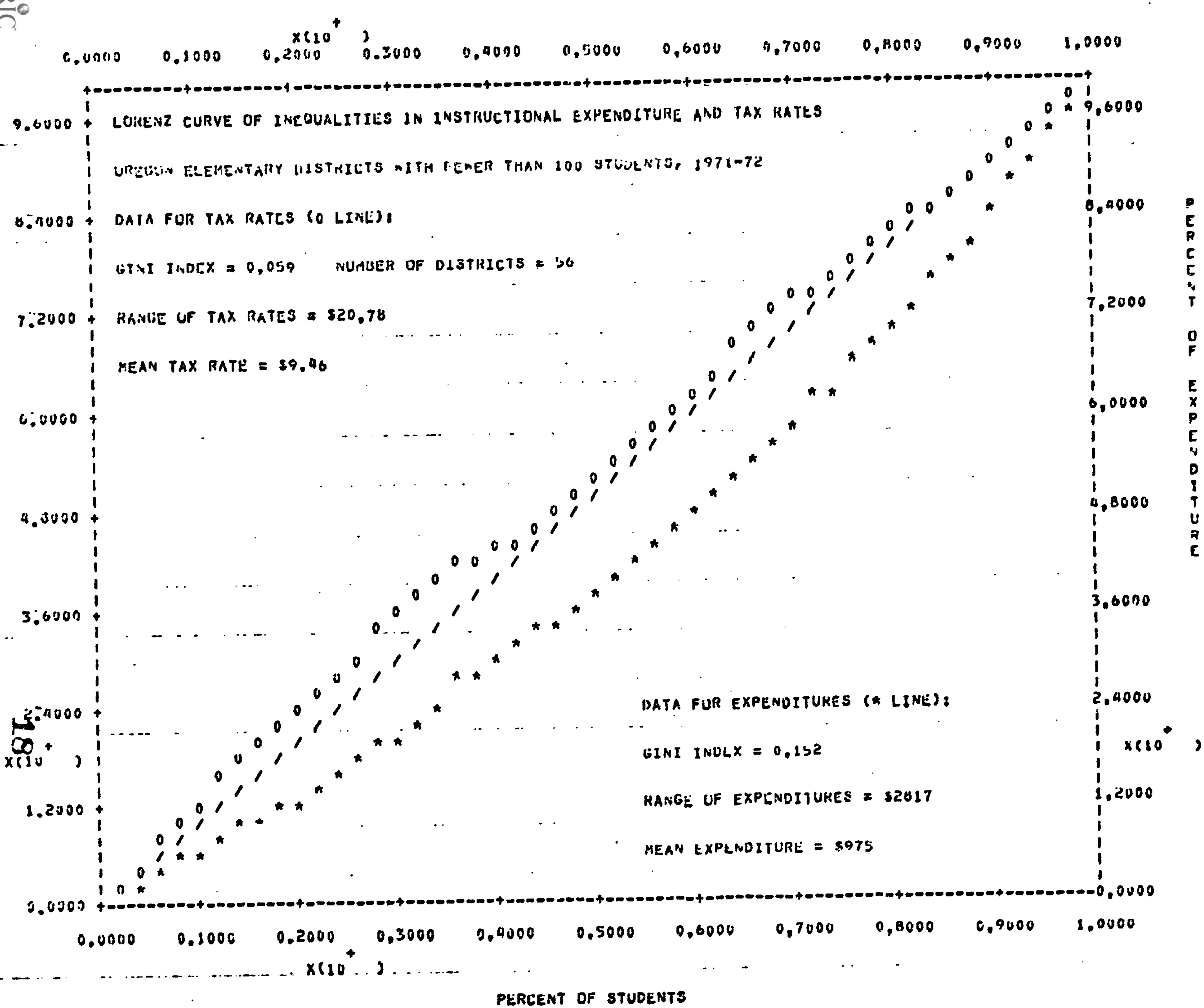

LORENZ CURVE OF INEQUALITIES IN INSTRUCTIONAL EXPENDITURE AND TAX RATES
OREGON ELEMENTARY DISTRICTS, 1971-72
DATA FOR TAX RATES (0 LINE):
GINI INDEX = 0.039
NUMBER OF DISTRICTS = 129
RANGE OF TAX RATES = $23.00
MEAN TAX RATE = $13.76
DATA FOR EXPENDITURES (* LINE):
GINI INDEX = 0.037
RANGE OF EXPENDITURES = $2817
MEAN EXPENDITURE = $835
PERCENT OF TAX RATES
PERCENT OF EXPENDITURE
PERCENT OF STUDENTS
0.0000
0.1000
0.2000
0.3000
0.4000
0.5000
0.6000
0.7000
0.8000
0.9000
1.0000
1.2000
2.4000
3.6000
4.8000
6.0000
7.2000
8.4000
9.6000

FIGURE 9



LORENZ CURVE OF INEQUALITIES IN INSTRUCTIONAL EXPENDITURE AND TAX RATES
OREGON ELEMENTARY DISTRICTS WITH FEWER THAN 100 STUDENTS, 1971-72
DATA FOR TAX RATES (0 LINE):
GINI INDEX = 0,059
NUMBER OF DISTRICTS = 56
RANGE OF TAX RATES = $20,78
MEAN TAX RATE = $9.46
DATA FOR EXPENDITURES (* LINE):
GINI INDEX = 0,152
RANGE OF EXPENDITURES = $2817
MEAN EXPENDITURE = $975
PERCENT OF TAX RATES
PERCENT OF EXPENDITURE
PERCENT OF STUDENTS
0,0000
0,1000
0,2000
0,3000
0,4000
0,5000
0,6000
0,7000
0,8000
0,9000
1,0000
9,6000
8,4000
7,2000
6,0000
4,8000
3,6000
2,4000
1,2000

In summary, this paper proposes two modifications of the Lorenz technique for measuring inequalities in financing education. One is to order expenditures (and tax rates) by district fiscal ability before calculating the cumulative percentages. It is possible, as a result, to get a Lorenz curve that crosses the diagonal line, which gives some useful insights. The other is to calculate Gini coefficients for both expenditures and tax rates, and consider the sum of the two coefficients to be a measure of the total inequality in the system.