DOCUMENT RESUME

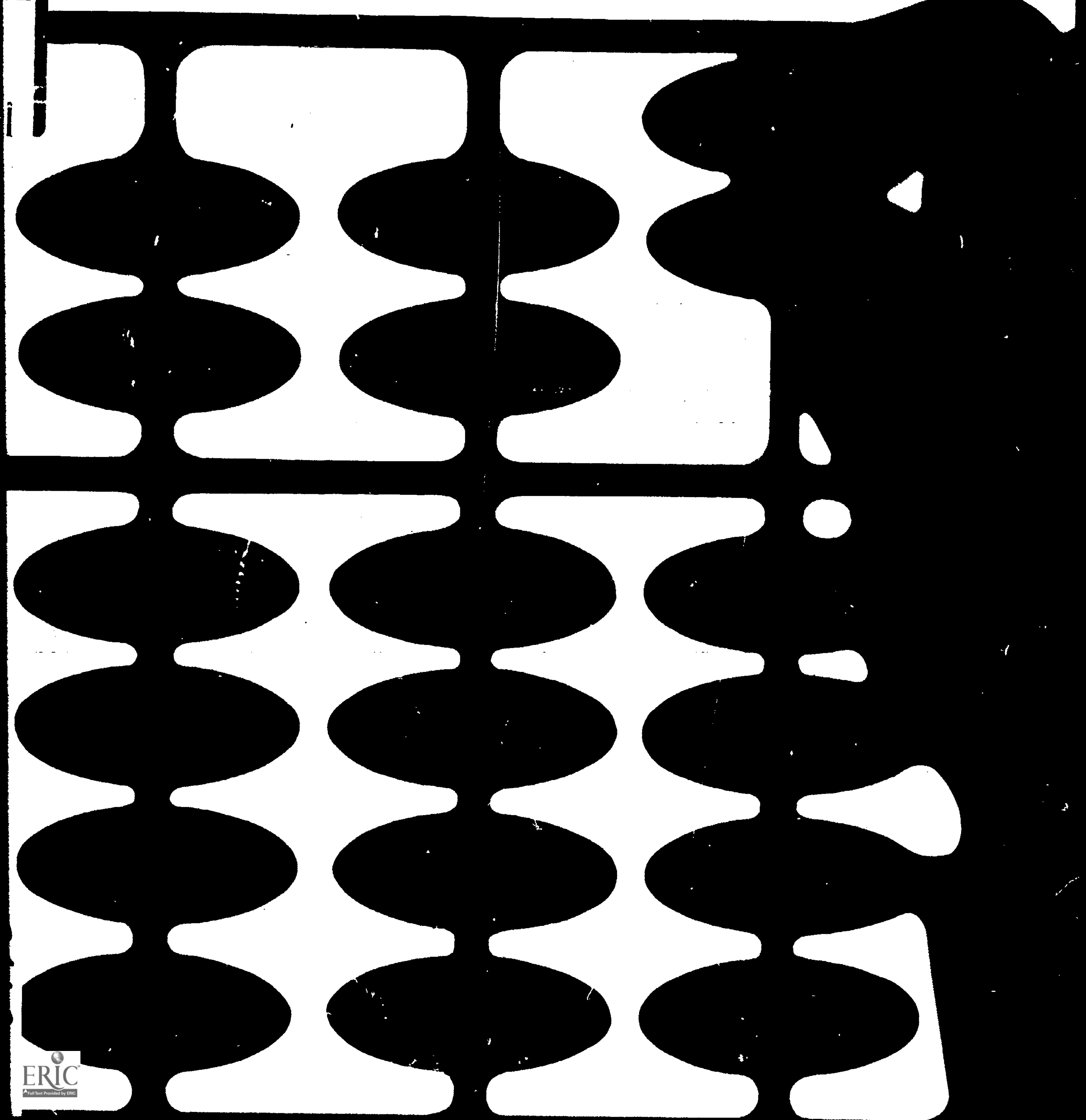
ED 102 029 SE 018 770

TITLE	Math Fundamentals: Selected Results from the First National Assessment of Mathematics.
INSTITUTION	Education Commission of the States, Denver, Colo. National Assessment of Educational Progress.
SPONS AGENCY	Carnegie Corp. of New York, N.Y.; Ford Foundation, New York, N.Y.; National Center for Educational Statistics (DHEW/OE), Washington, D.C.
REPORT NO	R-04-MA-01
PUB DATE	Jan 75
NOTE	56p.
AVAILABLE FROM	Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 (Order Report 04-MA-01; $1.10)
EDRS PRICE	MF-$0.76 HC-$3.32 PLUS POSTAGE
DESCRIPTORS	Community Size; Educational Status Comparison; Elementary Secondary Education; Geographic Regions; *Mathematical Applications; *Mathematics Education; *National Surveys; *Number Concepts; Research; *Testing
IDENTIFIERS	NAEP; *National Assessment of Educational Progress

ABSTRACT

This report, the first of several to be published on the results of the 1972-73 assessment of mathematics, begins with a brief general discussion of the project. The findings with respect to pure computation and computation with translation are then presented in some detail. Data collected from subjects at four age levels (9, 13, 17, and adult) are presented; these data relate to skill at performing the four basic operations, spearately and in combination, ability to compute with fractions, translation (word problems), and approximation. Several sample problems and response patterns are presented and discussed. Group results are summarized; results are also analyzed on the bases of sex, race, geographical region, educational level of parents, and size and type of community. (SD)

MATH FUNDAMENTALS: SELECTED RESULTS FROM THE FIRST NATIONAL ASSESSMENT OF MATHEMATICS



NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

A Project of the Education Commission of the States

Assessment Reports

1969–70 Assessment

Science

1	National Results, July 1970	$1.75
4	Results by sex, region and size of community, April 1971	1.00
7	Results by color, parental education, size and type of community; balanced results, May 1973	2.60

Citizenship

2	National Results, November 1970	1.70
6	Results by sex, region and size of community, July 1971	1.25
9	Results by color, parental education, size and type of community, May 1972	2.85

Writing

3	National Results, November 1970	1.50
5	Results by sex, region and size of community, April 1971	1.25
8	Writing Mechanics, February 1972	1.75
10	Selected Essays and Letters, November 1972	8.65
11	Results by color, parental education, size and type of community, sex and region for objectively scored exercises, May 1973	1.25

1970–71 Assessment

02-GIY	General Information Yearbook—A description of National Assessment's methodology, with special attention given to Reading and Literature, May 1972	1.55

Reading

02-R-01	Understanding Words and Word Relationships, April 1973	1.45
02-R-02	Graphic Materials, June 1973	2.60
02-R-03	Written Directions, May 1973	2.10
02-R-04	Reference Materials, July 1973	2.15
02-R-05	Gleaning Significant Facts From Passages, May 1973	2.60
02-R-06	Main Ideas and Organization, July 1973	2.10
02-R-07	Drawing Inferences, August 1973	2.60
02-R-08	Critical Reading, May 1973	2.35
02-R-09	Reading Rate and Comprehension, December 1972	2.85
02-R-20	Released Exercises, July 1973	3.95
02-R-30	Recipes, Wrappers, Reasoning and Rate: A Digest of the First Reading Assessment, April 1974	1.00
02-R-00	Summary Data, July 1974	.70

(Continued, Inside Back Cover)

NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

MATH FUNDAMENTALS

Selected Results from the First National Assessment of Mathematics

Mathematics Report No. 04-MA-01

January 1975

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402

NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

J. Stanley Ahmann
Director

George H. Johnson
Associate Director

This publication was prepared and produced pursuant to agreements with the National Center for Educational Statistics of the U.S. Office of Education with additional funds from the Carnegie Corporation of New York and the Ford Foundation's Fund for the Advancement of Education. The statements and views expressed herein do not necessarily reflect the position and policy of the U.S. Office of Education or other grantors but are solely the responsibility of the National Assessment of Educational Progress, a project of the Education Commission of the States.

TABLE OF CONTENTS

LIST OF FIGURES AND EXHIBITS v

LIST OF TABLES vii

FOREWORD ix

ACKNOWLEDGMENTS xi

INTRODUCTION xiii
The 1972–73 Mathematics Assessment xiii

CHAPTER 1 Pure Computation 1
Properties of Zero 1
Addition 2
Subtraction 3
Multiplication 7
Division 10
Combined Results 10
Operations with Fractions 13
Computation with Integers 16

CHAPTER 2 Computation with Translation 19
Elementary Translation 19
More Complex Translations 25
Operations with Fractions 27
Approximation 29

CHAPTER 3 Summary of Group Results 31
Nine-Year-Olds: Third Grade vs. Fourth Grade 31
National Assessment Reporting Groups 33
Group Results 35
Overall Group Results 40
Additional Mathematics Assessment Results 41

LIST OF FIGURES AND EXHIBITS

FIGURE 1-1. Percentages of Success for Addition Exercises 4

FIGURE 1-2. Percentages of Success for Subtraction Exercises 6

FIGURE 1-3. Percentages of Success for Multiplication Exercises 9

FIGURE 1-4. Percentages of Success for Division Exercises 11

FIGURE 1-5. Percentages of Success for Fractional Computation Exercises 15

FIGURE 1-6. Percentages of Success for Integer Computation Exercises 18

FIGURE 2-1. Percentages of Success for Simple Translation—9-Year-Old Exercises 23

FIGURE 2-2. Percentages of Success for Simple Translation—9 and 13-Year-Old Exercises 24

FIGURE 2-3. Percentages of Success for Complex Translation Exercises 28

FIGURE 3-1. Differences from National Performance Levels: Third and Fourth Grades 32

EXHIBIT 1. National Assessment Geographic Regions 34

FIGURE 3-2. Median Differences from National Performance Levels by Region 37

FIGURE 3-3. Median Differences from National Performance Levels by Parental Education Level 38

FIGURE 3-4. Median Differences from National Performance Levels by Size and Type of Community (STOC) 39

LIST OF TABLES

TABLE 1. Exercise RB03 1

TABLE 2. Exercise RC02A 2

TABLE 3. Exercise RC01 3

TABLE 4. Exercise RC02B 5

TABLE 5. Exercises RC04 and C10019 5

TABLE 6. Exercise RC17 7

TABLE 7. Exercise RC02C 7

TABLE 8. Exercise RC09 7

TABLE 9. Exercise RC02D 10

TABLE 10. Exercise RC02 12

TABLE 11. Exercise RC02 12

TABLE 12. Exercise C10076 13

TABLE 13. Exercise RA11 13

TABLE 14. Exercise RC15 14

TABLE 15. Exercise RC16 14

TABLE 16. Exercise RC24 16

TABLE 17. Exercise RC25 17

TABLE 18. Exercise RC06 20

TABLE 19. Exercise RC07 20

TABLE 20. Exercise RC03 21

TABLE 21. Exercise RC11 22

TABLE 22. Exercise RC12 25

TABLE 23. Exercise RC13 25

TABLE 24. Exercise RC10 26

TABLE 25. Exercise RC19 26

TABLE 26. Exercise RC08 26

TABLE 27. Exercise RC14 27

TABLE 28. Exercise RC21 29

TABLE 29. Exercise RC23 30

TABLE 30. Exercises RC02A and RC06 33

TABLE 31. Numbers of Exercises on Which Males and Females Were More Successful 35

TABLE 32. Median Differences from National Performance Levels by Race 36

FOREWORD

The National Assessment of Educational Progress (NAEP) is an information-gathering project which surveys the educational attainments of 9-year-olds, 13-year-olds, 17-year-olds and adults (ages 26—35) in 10 learning areas: art, career and occupational development, citizenship, literature, mathematics, music, reading, science, social studies and writing. Different learning areas are assessed every year, and all areas are periodically reassessed in order to measure educational change.

Each assessment is the product of several years work by a great many educators, scholars and lay persons from all over the country. Initially, these people design objectives for each area, proposing specific goals which they feel Americans should be achieving in the course of their education. After careful reviews, these objectives are then given to exercise (item) writers, whose task it is to create measurement tools appropriate to the objectives.

When the exercises have passed extensive reviews by subject-matter specialists and measurement experts, they are administered to probability samples from various age groups. The people who comprise these samples are chosen in such a way that the results of their assessment can be generalized to an entire national population. That is, on the basis of the performance of about 2,500 9-year-olds on a given exercise, we can generalize about the probable performance of all 9-year-olds in the nation.

The National Assessment of Educational Progress also publishes a general information yearbook which describes all major aspects of the Assessment's operation. The reader who desires more detailed information about how NAEP defines its groups, prepares and scores its exercises, designs its samples and analyzes and reports its results should consult the *General Information Yearbook, Report 03/04-GIY.*

ACKNOWLEDGMENTS

Many people have made substantial contributions to the mathematics assessment, from the beginning of the National Assessment of Educational Progress (NAEP) in 1964 to this first report of findings in the area of mathematics. Unfortunately, it is not possible to acknowledge them all here, and an apology is due to those whose names have been omitted.

The original preparation of the objectives and exercises in the area of mathematics was handled by the Educational Testing Service (ETS) and The Psychological Corporation. These materials were reviewed by dozens of consultants, including mathematicians, mathematics educators and interested lay persons, under the general monitoring of the National Assessment staff. Special mention must be made of three individuals and their contributions to the developmental phases: Emil Berger of the St. Paul Public Schools (Minnesota) for his assistance in finalizing the objectives and efforts in developing and field testing exercises, and Dale Foreman of Westinghouse Learning Corporation and Todd Rogers of the University of British Columbia (former NAEP staff members) for their efforts in developing the individually administered mathematics exercises.

The administration of the mathematics assessment was conducted by the Research Triangle Institute (RTI) and the Measurement Research Center (MRC). Scoring and processing were carried out by MRC and by the NAEP staff. Louise Diana of MRC and Fred Schippert of the Detroit Public Schools (Michigan) provided invaluable assistance in developing and refining the categories used to score the exercises. James Wilson of the University of Georgia (Athens, Georgia) and Robert Reys of the University of Missouri (Columbia, Missouri) were extremely helpful in suggesting analysis schemes and in reacting to various drafts of this report.

The actual preparation of this report was a collaborative effort of the National Assessment staff. Special thanks must be given to the following people and departments: the Data Processing Department; Kristi Vaiden and Ava Powell, Research Assistants, Research and Analysis Department; and Marci Reser and Eileen Wollam, Production Assistants, Utilization/Applications Department. Technical analysis for this report was planned and supervised by Wayne Martin; the report was written by Barbara Ward.

J. Stanley Ahmann
J. Stanley Ahmann
Project Director

INTRODUCTION

How much does it cost? How long will it take? Which is the best buy? To find solutions to these and many similar questions, people need to perform some elementary computations. These computations involve deciding which numbers and which mathematical operations (addition, subtraction, multiplication, division) to use and knowing how to perform the required operations. Problems like these confront everyone daily. Unless one has a firm foundation in computational skills, simple problems will require an inordinate amount of time and effort; complex problems will become insurmountable obstacles.

In the early 1960s, mathematics education gained widespread attention with the advent of "modern math." Although the mathematics being taught was not particularly new or "modern," there was an increased emphasis upon understanding the structure and principles of mathematics. In addition, topics such as algebra and geometry were being introduced in the primary grades. These trends produced some fear that traditional topics such as computation might be "shortchanged" in the schools. This report addresses that issue by focussing entirely upon the ability of American school children and young adults to complete basic mathematical computations.

Although the increasing popularity of pocket calculators may reduce the necessity for hand calculation, people will continue to need computational skills. Everyone will not always have a calculator available. Furthermore, it will still be necessary to estimate whether the calculator's answers are reasonable and to determine which operations to use before turning on the machine.

The 1972–73 Mathematics Assessment

How well are Americans able to compute?

To provide data on Americans' skills and knowledge in mathematics, the National Assessment of Educational Progress (NAEP) conducted an assessment in mathematics during the 1972–73 school year. Ability in computation was only one of the many mathematical areas assessed.

The assessment involved students at three age levels—9, 13 and 17-year-olds—and out-of-school 17-year-olds and young adults age 26 through 35.

About three fourths of the 9-year-olds were enrolled in the fourth grade with most of the remainder in the third grade. About three fourths of the 13-year-olds were enrolled in the eighth grade with most of the remainder in the seventh grade.

Two groups of 17-year-olds participated in the assessment: (1) the "in-school" 17-year-olds and (2) the "out-of-school" 17-year-olds who were not enrolled in public or private schools during March 1972 because they had either dropped out or completed high school early. The latter were included to provide a more balanced representation of all 17-year-olds. About 65% of the "in-school" 17-year-olds were enrolled in the eleventh grade; of the remainder, about half were enrolled in the tenth grade and about half in the twelfth. The young adults (ages 26–35) who participated in the assessment were born between April 1, 1937, and March 31, 1947.

The 9-year-olds were assessed in January—February of 1973, the 13-year-olds in

October—December 1972 and the in-school 17-year-olds from March to April 1973. The out-of-school 17-year-old and adult assessments were conducted from October 1972 to July 1973. Out-of-school 17-year-olds and adults were generally interviewed in their homes while students at ages 9, 13 and 17 completed the assessment when attending school.

Over 90,000 people, both in and outside of schools, responded to a wide variety of mathematics exercises. Approximately 25,000 9-year-olds, 30,000 13-year-olds, 34,000 17-year-olds and 4,000 young adults were included in the assessment. National Assessment's probability sampling procedures insure that these people were statistically representative of the total population of the United States at each of the four age levels.

The majority of NAEP exercises are designed to be administered to groups of 8 to 12 people while others are to be given on an individual basis. "Individual" exercises are used to elicit responses which would be difficult to observe in a group situation—for example, performing a science experiment or singing a song. In mathematics, these exercises were often used to observe the process which a person followed to solve a problem.

To prevent reading ability from affecting mathematics performance, the exercises in group packages were played on tape to the respondents. Individually administered exercises were usually read aloud by the interviewer.

Most of the computational exercises were open-ended, meaning that the respondent had to supply a response rather than select an answer from a number of alternatives. The responses to the open-ended exercises were tabulated in various scoring categories. These scoring categories were defined by examining the types of responses made when the exercises were "tried out" or pretested prior to the assessment. The categories were redefined or modified if the actual assessment results differed from those found in tryouts. Scoring categories included common errors and thus can provide a diagnostic tool to aid in identifying frequently made mistakes.

Responses which followed no discernable pattern were placed in a category called "other unacceptable." For all open-ended exercises, respondents were instructed to write the words "I don't know" on the answer line if they felt they did not know the answer to a problem.

Many of the exercises were administered to more than one age group so that response patterns at different ages could be compared.

National Assessment releases approximately half of the exercises administered in a learning area for a given assessment year. In this report, results for both released and unreleased exercises are discussed; however, the actual exercises are shown only for those items which are released. A brief description of the unreleased exercises is provided.

This report is concerned with computational skills included in the mathematics assessment. Other reports focus upon additional selected areas of American ability in mathematics with complete data on the entire assessment provided in the mathematics technical report.

CHAPTER 1

PURE COMPUTATION

One of the first steps in computation is the ability to perform the four basic operations—addition, subtraction, multiplication and division. The mathematical processes are first learned in work with whole numbers; application of the processes to problem situations usually follows the development of computational skills.

In the exercises discussed in this chapter, the respondents were told whether to add, subtract, multiply or divide. Symbols indicating the appropriate operation were also given in most of these problems. Thus, the skills measured by these exercises were purely computational; the respondents did not have to decide which operation to use.

The computation exercises are ordered as the operations are generally taught—addition, subtraction, multiplication and division. Exercises concerning operations with fractions and with negative numbers are included at the end of the chapter.

Properties of Zero

Exercise RB03 concerned knowledge of the properties of zero in addition, multiplication and subtraction. This exercise was administered at ages 9 and 13 (Table 1).

Percentages of success were extremely high at both age levels on the addition problem. Nine-year-olds had the most difficulty with multiplying a number by zero while 13-year-olds were about equally adept in multiplying a number by zero and subtracting zero from a number. There was a marked increase in performance on each problem from the 9 to the 13-year-old age levels. For example, 1 in 6 of the 9-year-olds said that 3 was the correct answer to 3 x 0 (part B) but only 1 in 20 of the 13-year-olds made this mistake.

TABLE 1. Exercise RB03

		Age 9	Age 13
A.	3 + 0 =		
	3*	94%	98%
	0	5	2
	Other	1	+†
B.	3 x 0 =		
	0*	82	95
	3	16	5
	Other	3**	+
C.	3 - 0 =		
	3*	88	94
	0	11	6
	Other	1	+

**Asterisk indicates correct answer.*
†Plus equals rounded percents less than one.
***Figures may not add to 100% due to rounding error.*

Since all three parts of this exercise were administered to one respondent, it is possible to determine the overall percentage of success for the three parts. At age 9, 71% of the respondents answered all three parts correctly; this percentage was 89% for 13-year-olds. Although this is a 20 percentage point increase between the two age groups, still 1 13-year-old in 10 did not complete all three operations correctly.

Fifteen percent of the 9-year-olds and 5% of the 13-year-olds gave the correct answers to the addition and subtraction problems but missed the multiplication problem. In comparison 7% of the 9-year-olds and 4% of the 13-year-olds successfully solved the addition and multiplication but not the subtraction problem.

Addition

Five exercises dealing solely with addition were included in the assessment. These exercises varied in complexity from adding of one- and two-digit numbers with simple regrouping (carrying), to adding decimals (dollars and cents), to adding large numbers with more difficult regrouping.

Released Exercise RC02 included four problems, one for each of the four basic operations. Results for only the addition problem are discussed here (Table 2); results for the other problems in this exercise will be presented as each basic operation is discussed. This problem, given at all four age levels, asked the respondents to add two two-digit numbers. Simple regrouping (carrying) was necessary since the sum of the digits in the ones column was greater than 10.

TABLE 2. Exercise RC02A

Add:

38 +19	Age 9	Age 13	Age 17	Adult
57*	79%	94%	97%	97%

**Asterisk indicates correct answer.*

There was a 15 percentage point increase from age 9 to 13, indicating that learning of basic operations continues throughout the early grades; however, increases above age 13 were slight. Nine-year-olds regrouped incorrectly in 7% of the cases: Five percent gave the answer as 47, neglecting to add the one into the tens column while about 2% responded with 417, writing the answer to each column separately. The majority of the 9-year-old errors, 12%, were in the category designated "other unacceptable"; that is, the mistakes made followed no discernable pattern. Regrouping errors were considerably fewer at the older ages, accounting for 2% of the 13-year-old errors and less at ages 17 and adult.

An unreleased exercise (C10006) given at ages 9 and 13 asking the respondent to add two two-digit numbers without regrouping showed the following percentages of success: age 9, 69%; age 13, 91%. Results were slightly lower than on Exercise RC02A (shown previously) at age 13 and about 10% lower at age 9 even though no regrouping was involved. This difference may be due to the set up of the problem as the numbers were presented horizontally in a sentence, and the word "sum" rather than the addition symbol was used.

An individually administered unreleased exercise (C10076) involved adding four numbers of one or two digits (for example, 2 + 6 + 11 + 13) with regrouping. It was answered correctly by 72% of the 9-year-olds, 89% of the 13-year-olds, 94% of the 17-year-olds and 93% of the adults.

Addition of four two-digit numbers with regrouping was required in unreleased Exercise C10009. Fifty-three percent of the 9-year-olds, 83% of the 13-year-olds and 93% of the 17-year-olds solved the problem correctly. Failure to regroup accounted for 8% of the errors at age 9, 3% at 13 and 1% at 17.

Released Exercise RC01 involved adding and regrouping with decimals (dollars and cents). The exercise and results are presented in Table 3.

A large number of the 9-year-olds added correctly but made errors in decimal placement. This is understandable as decimals are not generally introduced at the fourth-grade level. If the decimal errors are discounted, 62% of the 9-year-old respondents did the addition correctly, a figure which is close to

TABLE 3. Exercise RC01

Add the following numbers:

$ 3.09
10.00
9.14
5.10

	Age 9	Age 13	Age 17	Adult
27.33* (with or without $ sign)	40%	84%	92%	86%
Dec. plcmt. errors (correct numbers)	22	8	2	6
One or two regrouping errors (may misplace dec.[illegible]al)	5	2	2	2
Other unacceptable	27	6	4	6
I don't know or no response	6	1†	+**	+

Asterisk indicates correct answer.
†Figures may not add to 100% due to rounding errors.
***Plus equals rounded percents less than one.*

results on Exercise C10009 (four two-digit numbers with regrouping: 53%) and C10006 (two two-digit numbers without regrouping: 69%), but below those for RC02A (38 + 19: 79%).

Adult performance on RC01 dropped below that of 17-year-olds although performance of the two groups was quite similar on C10076 (four one- and two-digit numbers) and RC02A (38 + 19).

Judging from the results on this exercise, one adult in seven may well have difficulty in totaling a sales slip or in checking a grocery bill.

Summary of Addition Results

Although the results on these five exercises cannot be aggregated to produce a "score" on addition, they can be placed beside one another to present a general picture of performance (Figure 1-1).

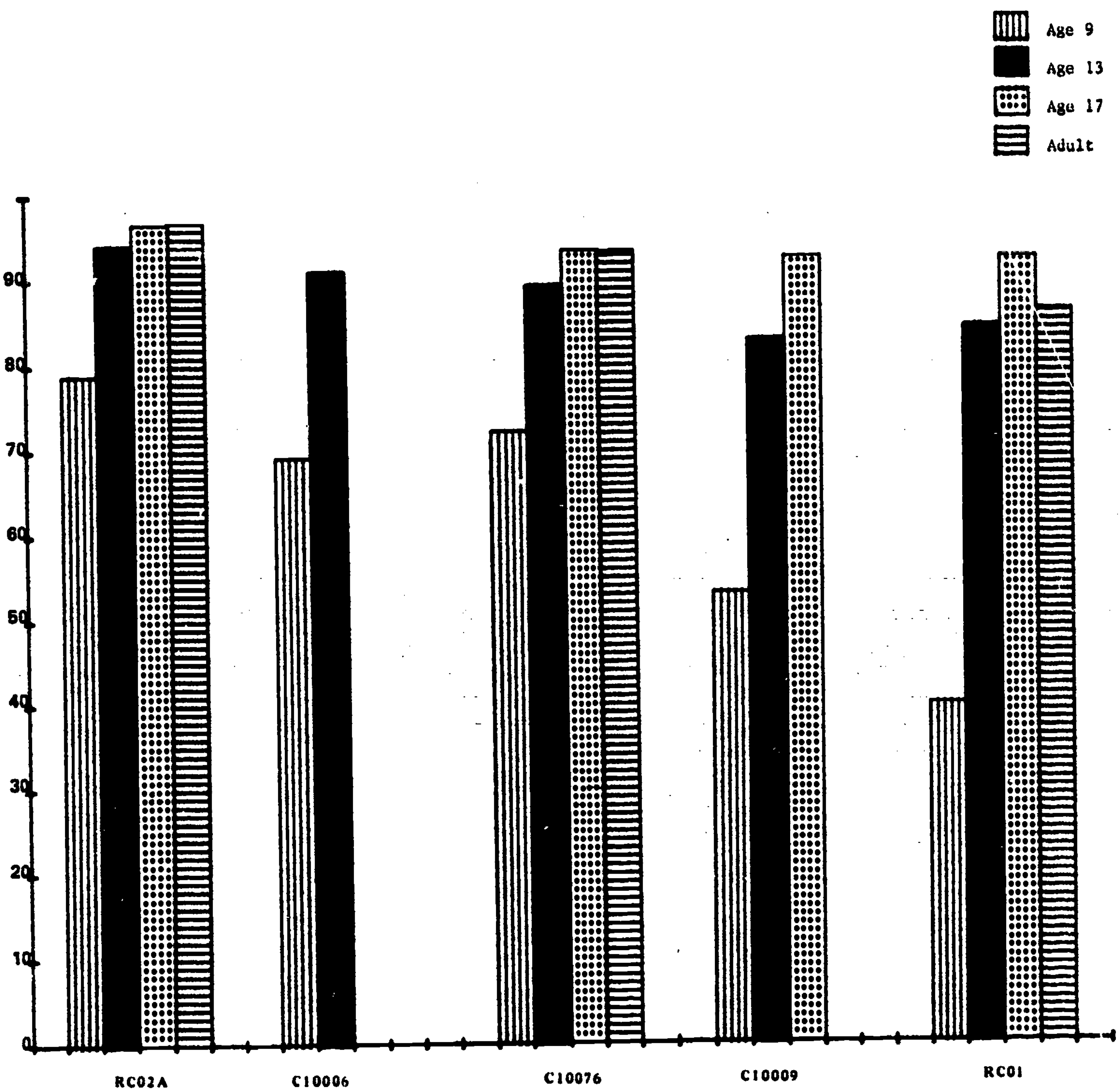
Results for similar exercises fell into similar ranges. Between 70 and 80% of the 9-year-olds, 90 to 95% of the 13-year-olds and about 95% of the 17-year-olds could complete simple addition regrouping problems such as RC02A (38 + 19) and C10076 (adding one- and two-digit numbers with regrouping). On Exercises C10009 (addition of four two-digit numbers) and RC01 (addition of four four-digit numbers), problems requiring more involved addition skills (more numbers to be added and/or more regrouping necessary) 83—84% of the 13-year-olds were successful on the two exercises while 17-year-olds performed at around 92% on both exercises.

The manner in which a problem is presented and the vocabulary used seem to influence performance. The results on C10006—which did not demand regrouping—were lower than those for RC02A—where regrouping was necessary—although, theoretically, C10006 should be the easier exercise. Adults did as well as 17-year-olds on two exercises and fell somewhat below on one, indicating that addition skills do not improve after secondary schooling is completed. From ages 9 through 17, however, performance did improve on all exercises.

Subtraction

Five exercises were devoted to purely computational subtraction. Again, the difficulty of the exercises varied, ranging from exercises requiring only one regrouping, to problems asking more extended regrouping, to decimal subtraction.

FIGURE 1-1. Percentages of Success for Addition Exercises



In Exercise RC02B, respondents were asked to complete subtraction of a two-digit number from a two-digit number with regrouping (borrowing). This exercise was administered at each of the four age levels. The exercise and results are shown in Table 4.

The most frequent mistake at age 9 was subtracting the 6 from the 9, resulting in an answer of 23. Eighteen percent of the 9-year-olds used this technique. This mistake occurred less often at the older age levels (under 2% at 13, 17 and adult); errors at the

TABLE 4. Exercise RC02B

Subtract:

36 -19	Age 9	Age 13	Age 17	Adult
17*	55%	89%	92%	92%

Asterisk indicates correct answer.

older age levels tended to cluster in the miscellaneous "other unacceptable" category.

Two subtraction exercises, one released and one unreleased, called for more extensive regrouping. In RC04, a three-digit number was subtracted from a four-digit number with regrouping in three places. Exercise C10019 required subtraction of a three-digit number from a three-digit number with regrouping in two places. Exercise RC04 and results for both exercises are shown in Table 5.

TABLE 5. Exercises RC04 and C10019

Exercise RC04

Do the following subtraction:

1,054 - 865	Age 9	Age 13	Age 17	Adult
189*	27%	80%	89%	90%
Exercise C10019				
Percent answering correctly	30	75	87	

Asterisk indicates correct answer.

The results are remarkably similar. Data from these two exercises suggest that about 30% of the 9-year-olds, 75—80% of the 13-year-olds and 88% of the 17-year-olds can successfully complete subtraction with regrouping.

For 9-year-olds, the most common mistake (following "other unacceptable") was a reversal error (subtracting some part or parts of the minuend from the subtrahend)—12% on Exercise RC04 and 22% on Exercise C10019. The largest error category at ages 13, 17 and adult, again discounting "other unacceptable," was one error in regrouping. For Exercise RC04, this meant an answer of 199 or 289 rather than 189, the correct solution. On RC04, 6% of the 9-year-olds, 4% of the 13-year-olds, 3% of the 17-year-olds and 3% of the adults made such an error. On C10019, 6% of the 9-year-olds, 5% of the 13-year-olds and 5% of the 17-year-olds made an error in regrouping.

C10076, an unreleased exercise individually administered, also asked for subtraction of a three-digit number from a four-digit number with regrouping in three places (similar to RC04). Percentages of success on this exercise were as follows: age 9, 28%; age 13, 71%; age 17, 80%; and adult, 87%.

Results for 9-year-olds were quite consistent across the three exercises discussed above; performance of the other age groups seems more likely to vary.

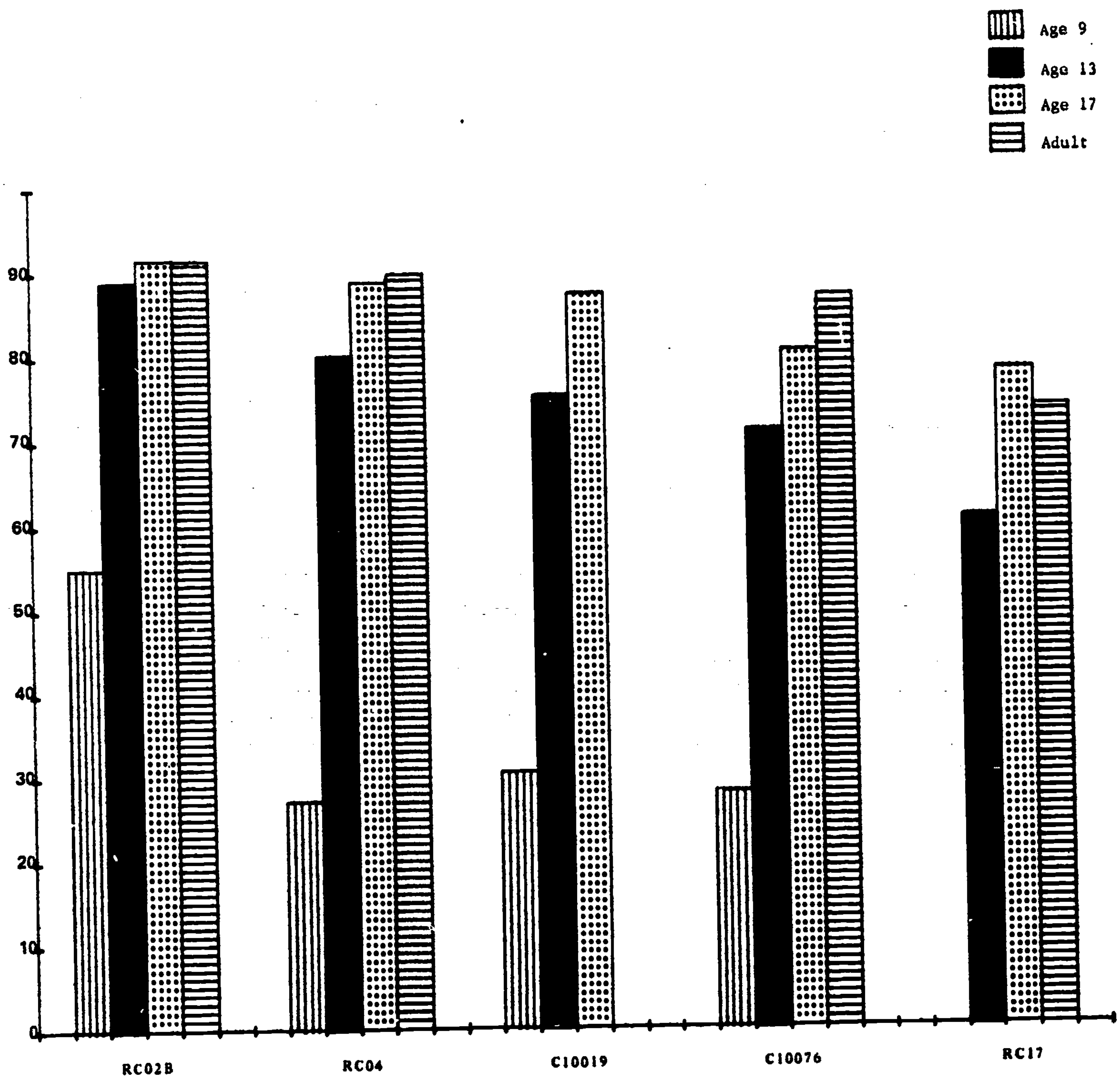
Another released subtraction exercise dealt with decimal subtraction (Table 6). This exercise was not administered at age 9 because decimals are not typically included in the third- or fourth-grade curriculum.

Regrouping in two places was required. Decimal placement mistakes accounted for a relatively small percentage of the errors at all age levels. As with the other subtraction exercises, a large percentage of the errors (19%, 11% and 12%, respectively) were scored in the "other unacceptable" category—that is, they did not appear to stem from any particular misconception or confusion.

Summary of Subtraction Results

Performance on the subtraction exercises was generally lower than on the addition exercises (see Figure 1-2). Percentages for 9-year-olds

FIGURE 1-2. Percentages of Success for Subtraction Exercises



appeared to be relatively stable when extended regrouping was required. The widest range of results occurred at the 13-year-old level; this may be due in part to 13-year-old lack of proficiency with decimals. In both addition and subtraction, adults seem to have difficulty working with decimals, falling below the 17-year-olds on both decimal exercises. The most common error at age 9 was a reversal—subtracting the smaller number from the larger number regardless of placement in the problem. Respondents at the

TABLE 6. Exercise RC17

If 23.8 is subtracted from 62.1, the result is

	Age 13	Age 17	Adult
38.3*	61%	78%	74%
Errors in decimal placement	4	3	2
One regrouping error 48.3 or 39.3 (decimal placement did not count)	3	3	5
Subtraction reversal (decimal placement did not count)	6	2	1†

**Asterisk indicates correct answer.*
†Percentages do not total 100% as all response categories are not shown.

three older ages tended to make one regrouping error in the more complex subtraction problems; instances of two regrouping errors in the same problem were rare.

Multiplication

Multiplication, the third computational operation, is generally taught in the third and fourth grades although preparatory work should have begun in the lower grades. Practice with regrouping and with problems in which both factors are greater than 10 continues throughout the upper primary grades. Thus, it is to be expected that 9-year-old performance in multiplication will be below that of the other age levels, as many 9-year-olds are still struggling to memorize the multiplication number facts and have not mastered the multiplication process. One fourth of the 9-year-olds were in the middle of third grade at the time of the assessment and would probably have at best a rudimentary knowledge of the multiplication process. By age 13, most curricula imply that development of the basic computational operations will be complete.

Multiplication computation was measured in six assessment exercises. Released Exercise RC02C asked for multiplication of a two-digit number by a one-digit number with one regrouping step (Table 7).

TABLE 7. Exercise RC02C

38 x9	Age 9	Age 13	Age 17	Adult
342*	25%	83%	88%	81%

**Asterisk indicates correct answer.*

Results are slightly lower than for the addition and subtraction components of this exercise; as expected, the percentage of 9-year-olds answering correctly is considerably lower. Twenty-eight percent of the 9-year-olds simply answered "I don't know." On this particular task, adults performed below both the 13 and 17-year-olds.

The second released multiplication exercise, RC09, is a problem involving exponents in disguise. Although exponents as such are not shown, multiplication of a number by itself a given number of times is simply one way of expressing the exponential relationship. This exercise was more difficult than the first problem for all age levels and especially so for 9-year-olds (Table 8).

TABLE 8. Exercise RC09

10 x 10 x 10 x 10 =

	Age 9	Age 13	Age 17
Respondents answering 10,000	4%	66%	79%
Respondents answering 10^4		1	1

Multiplication by 10s is the basis for our decimal number system and thus is highly important to concepts of place value. However, the low 9-year-old percentage of success is understandable since most textbooks do not present multiples of 10 until close to the end of the fourth grade, if indeed they are introduced that early. Remembering that the assessment was administered to 9-year-olds in January and February when classes would probably have completed only half their textbooks and that one fourth of the 9-year-old respondents were third graders, the results are not surprising. Approximately two thirds of the 13-year-olds and four fifths of the 17-year-olds solved this problem correctly, answering either 10,000 or 10^4.

Common errors at age 9 included giving some answer beginning with 4, such as 40, 400 or 4,000 (34%) or giving an answer of 200 (34%). Respondents may have successfully multiplied the first two 10s and the last two 10s together and then added the results to arrive at the 200 figure.

Thirteen and 17-year-olds were more likely to give 10 with some incorrect number of zeros as the answer (10% and 8%, respectively) although 8% of the 13-year-olds and 5% of the 17-year-olds gave some number involving 4 (40, 400, etc.) as the solution.

Four unreleased exercises were classified as computational multiplication. The first, C10026, was administered only to 9-year-olds and asked multiplication of a two-digit number by a one-digit number with regrouping (carrying). This exercise was similar to Exercise RC02C. Thirty-five percent of the 9-year-olds were able to answer the problem correctly.

Another similar exercise, C10031, was administered at all four age levels. Percentages of success were as follows: age 9, 29%; age 13, 84%; age 17, 90%; and adult, 88%. Approximately one fourth of the 9-year-olds replied "I don't know" to this problem. The largest number of errors at all four age levels were in the "other unacceptable" category, indicating that errors were other than those commonly associated with regrouping.

Multiplication of a three-digit number by a three-digit number with regrouping was called for in Exercise C10032. An example of this type of problem would be 567 x 412. Four percent of the 9-year-olds arrived at the correct answer; 47% gave a response which was an obvious attempt to multiply each column separately. (Using the example shown above, this process would give an answer of 20,614.) At the older age levels, 70% of the 13-year-olds, 81% of the 17-year-olds and 75% of the adults solved the problem correctly. Approximately 6% of the 13-year-olds made errors in regrouping or subtotaling; this figure dropped to a little over 2% at ages 17 and adult.

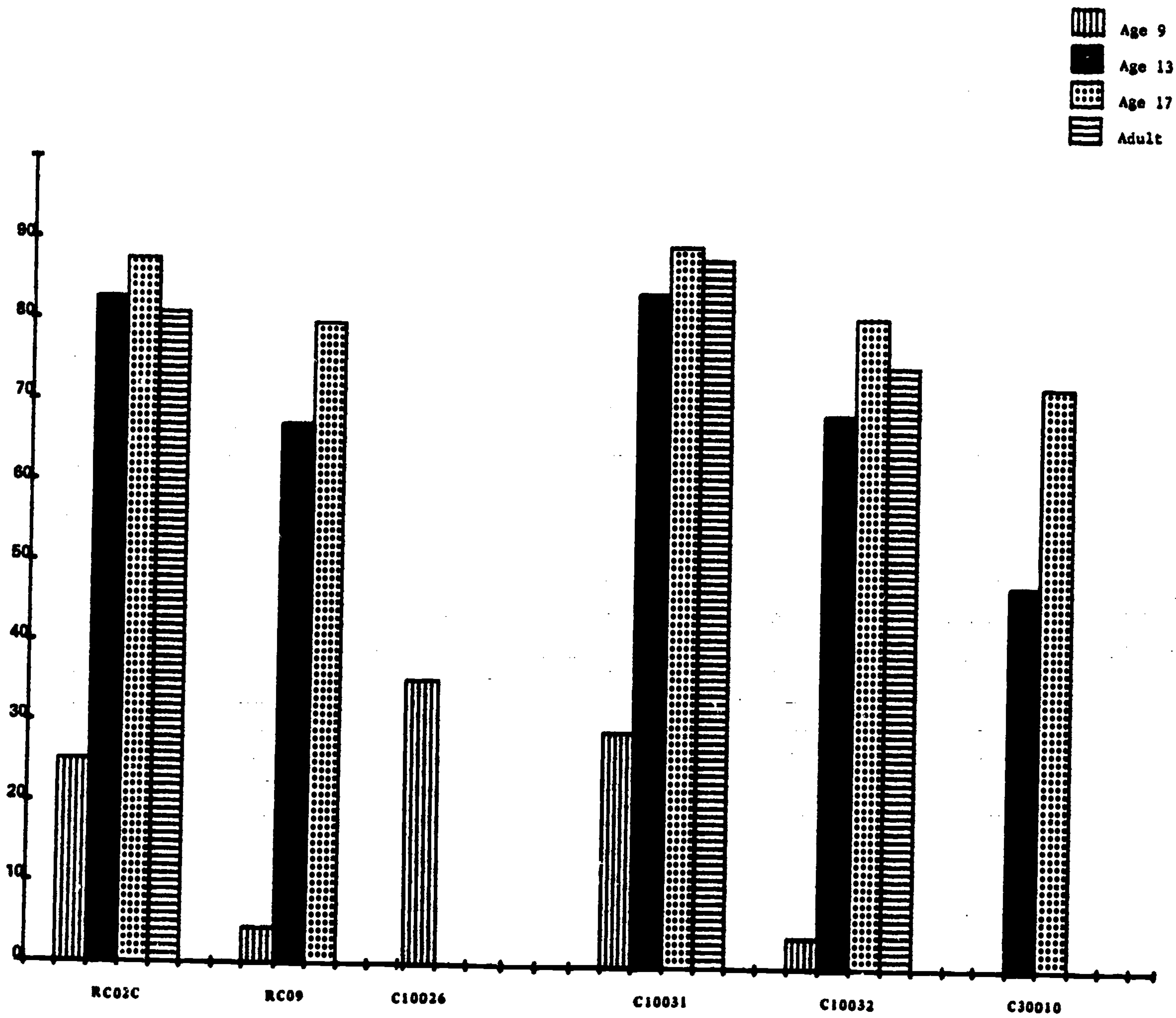
Exercise C30010 concerned multiplication of decimals and was administered only to 13 and 17-year-olds. The correct answer was given by 48% of the 13-year-olds and 73% of the 17-year-olds. Mistakes in decimal placement accounted for 26% of the incorrect answers at age 13 and for 16% of the errors at age 17.

Summary of Multiplication Results

Placing the results from the multiplication exercises side by side illustrates the general range of performance in multiplication existing at the various age levels (Figure 1-3). It must be remembered that, due to the varying difficulty of the problems, the results cannot be combined to arrive at any kind of meaningful overall multiplication "score."

Results were very similar at the three in-school age levels on the two exercises requiring multiplying a two-digit number by a one-digit number with regrouping (RC02C and C10031). Results for 9-year-olds ranged from 25–35% correct on the three simpler multiplication exercises (RC02C, C10026, C10031); only a small proportion of the 9-year-olds could complete the more difficult tasks.

FIGURE 1-3. Percentages of Success for Multiplication Exercises



As with subtraction, 13-year-old results varied more widely than those of any other age group. There is a distinct improvement in several cases from age 13 to 17 (exponential multiplication, 13 percentage points; three-digit by three-digit multiplication, 12 percentage points; and decimal multiplication, 25 percentage points) even though the basic computational operations have supposedly been mastered at age 13. However, these are all skills which are introduced in fourth through sixth grade, and 13-year-olds might well not be very familiar with them. Multiplication of two decimals, which was asked for in the decimal multiplication exercise, is not usually taught until the sixth grade, and about one fourth of the 13-year-olds were beginning the seventh grade at the time of the assessment. This may partially explain the large divergence in 13 and 17-year-old performance on the decimal multiplication exercise.

Results for adults were very close to those for 17-year-olds on C10031, apparently the easiest multiplication problem given at the three older ages. However, adults were 6—7 percentage points below the 17-year-olds on the other two exercises which were administered to them.

The assessment results indicate that 10% of the nation's 17-year-olds and adults cannot complete an elementary multiplication problem with regrouping; approximately 30% of the 17-year-olds did not successfully solve a simple multiplication problem involving decimals.

Division

Division, the inverse of multiplication, is also generally taught in third and fourth grade although some of the underlying concepts may have been explored earlier. Thus, 9-year-olds would not be expected to have great skill in division, and indeed this was evidenced in the assessment results.

The fourth part of Exercise RC02 was a division problem with a one-digit number as the divisor and no remainder (Table 9). Nine-year-olds were rather obviously at a loss as 38% answered "I don't know" and 11% gave no response. Very few at the other age levels did not respond or answered "I don't know"; even those who did not get the right answer attempted to solve the problem.

TABLE 9. Exercise RC02D

Divide:

$5\overline{)125}$	Age 9	Age 13	Age 17	Adult
25*	15%	89%	93%	93%

Asterisk indicates correct answer.

An unreleased division problem, C10076, was also one part of an exercise involving several operations—in this case, addition, subtraction and division. The exercise was administered individually at all four age levels and respondents' methods of working the problems were observed. The division problem asked the division of a five-digit number by a one-digit number with no remainder. Only 5% of the 9-year-olds were able to solve this problem; however, 67% of the 13-year-olds, 78% of the 17-year-olds and 77% of the adults found the correct solution.

A more complex division problem, C10049, was administered at ages 13 and 17. This problem involved dividing a three-digit number by a two-digit number, leaving no remainder. At age 13, 66% correctly solved the problem, and the percentage increased to 85% at age 17. Approximately 26% of the 13-year-olds and 10% of the 17-year-olds were in the "other unacceptable" category, and around 5% of the 13-year-olds and 3% of the 17-year-olds simply answered "I don't know."

Summary of Division Results

Results on the three division exercises are compared in Figure 1-4.

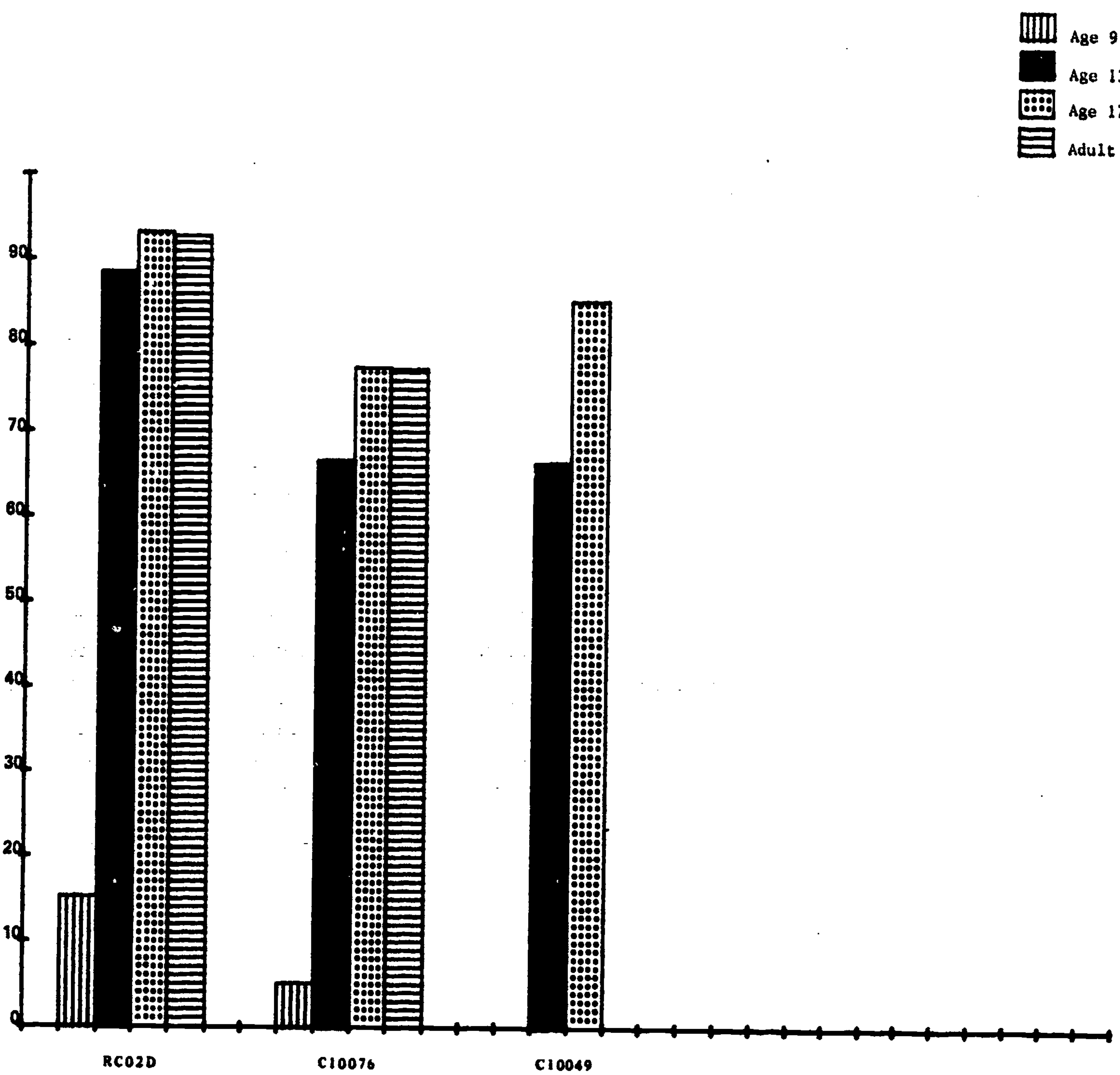
It seems that at least two thirds of the 13-year-olds can do simple long-division problems. Seventeen-year-olds appeared to be more influenced by the content of the problems than 13-year-olds as their performance showed greater variation between Exercises C10076 and C10049. Seventeen-year-old and adult percentages of success on the two exercises administered to both groups were virtually identical.

Combined Results

Two of the exercises discussed above included several parts, each demanding a different computational operation. Since all parts of these exercises were answered by one person, the results for the various parts can be combined and compared.

BEST COPY AVAILABLE

FIGURE 1-4. Percentages of Success for Division Exercises



Exercise RC02 involved four problems: one in addition, subtraction, multiplication and division. Three of the problems required some simple regrouping. The exercise and results for the four age levels are displayed in Table 10.

At age 9, 7% of the population answered all four parts correctly. Fifty-one percent of the 9-year-olds answered parts A and B (addition and subtraction) correctly, while 9% answered both parts C and D (multiplication and

TABLE 10. Exercise RC02

A. Add:	Age 9	Age 13	Age 17	Adult
38 +19 = 57*	79%	94%	97%	97%
B. Subtract:				
36 -19 = 17*	55	89	92	92
C. Multiply:				
38 x9 = 342*	25	83	88	81
D. Divide:				
5)125 = 25*	15	89	93	93
All four problems correct	7	68	78	72

**Asterisk indicates correct answer.*

division) correctly. In comparison, at age 13, 84% answered parts A and B correctly and 76% answered parts C and D correctly; at age 17, the figures were 89% for parts A and B and 85% for parts C and D. Eighty-nine percent of the adults answered parts A and B correctly and 78% answered parts C and D correctly. Performances on the various sections of this exercise are further displayed in Table 11.

The table shows that while 9-year-olds had the most difficulty with the division problem, the other three age groups found the multiplication problem the hardest. Adults had greater trouble with the multiplication problem than did 13 and 17-year-olds as indicated by the relatively large percentage of adults, 13%, that successfully completed parts A, B and D.

TABLE 11. Exercise RC02

	Age 9	Age 13	Age 17	Adult
Respondents ans. all four parts correctly	7%	68%	78%	72%
Respondents ans. parts A,B,C correctly–D incorrectly	12	5	2	3
Respondents ans. parts A,B,D correctly–C incorrectly	4	10	8	13

About two thirds of the 13-year-olds and about three fourths of the 17-year-olds and adults answered all four problems correctly. Results on the individual problems were fairly high, but respondents had difficulty in completing all four parts of the exercise successfully.

There were three parts in unreleased Exercise C10076—addition, subtraction and division. Results for the exercise are shown in Table 12.

Half of the 13-year-olds solved all three problems as did 63% of the 17-year-olds and 69% of the adults. This is one exercise where adults show a higher percentage of success than 17-year-olds. Their higher overall performance on this exercise is due to their success with the subtraction problem, on which they performed nearly 7 percentage points above the 17-year-olds. The subtraction problem was a fairly difficult one as almost as many 13-year-olds and 17-year-olds solved only parts A and C (addition and division) as solved only parts A and B (addition and subtraction).

Although percentages for the individual parts are reasonably high, respondents again had difficulty in solving all parts correctly.

TABLE 12. Exercise C10076

		Age 9	Age 13	Age 17	Adult
Part A.	Addition (four numbers of one and two digits)	72%	89%	94%	93%
Part B.	Subtraction (four-digit number minus three-digit number)	28	71	80	87
Part C.	Division (five-digit number divided by one-digit number with no remainder)	5	67	78	77
Respondents getting all three parts correct		3	49	63	69
Respondents getting none correct		24	3	3	2
Derived values					
Respondents getting parts A and B correct, part C incorrect		21	15	13	13
Respondents getting parts A and C correct, part B incorrect		1	11	10	5
Respondents getting parts B and C correct, part A incorrect		+*	4	3	3

**Plus equals rounded percents less than one.*

Operations with Fractions

Operations with fractions are usually introduced in the upper primary (intermediate) school grades. The concepts underlying operations with fractions are the same as for whole numbers; often, however, operations with fractions require several steps and the steps must be completed in the proper sequence to obtain a correct answer.

Renaming Fractions

Exercise RA11 asked the respondent to convert an improper fraction to a mixed number. The respondent had to know that fractions are renamed to an equivalent form by dividing the denominator into the numerator and then had to be able to complete the computation correctly. This particular exercise was presented in a multiple-choice format; percentages of respondents choosing a foil (distractor) at each age level are shown beside the foils (see Table 13).

TABLE 13. Exercise RA11

Which one of the following equals $\frac{47}{5}$?

		Age 9	Age 13	Age 17
○	$4\frac{7}{5}$	28%	7%	2%
●	$9\frac{2}{5}$	7	65	81
○	$47\frac{1}{5}$	11	8	4
○	$47 \div \frac{1}{5}$	33	14	9
○	I don't know.	31	6	3
	No response	+*	1†	1

**Plus equals rounded percents less than one.*
†Figures may not add to 100% due to rounding error.

About two thirds of the 13-year-olds and four fifths of the 17-year-olds were able to rename this fraction. Nine-year-olds clearly did not have the skills to deal with this type of problem. About one third of them simply responded "I don't know." The rest tended to choose answers which included the numbers given in the problem. This probably accounts for the number of correct responses being even less than that to be expected by chance.

The fourth foil, 47 ÷ 1/5 was the most popular distractor at all ages. Many respondents appeared to have an idea that division was involved but were not sure how to carry through the operation.

Addition of Fractions

Addition of fractions is a complex process requiring several computational steps. The least common multiple for the denominators must be determined, equivalent fractions found and the sum obtained.

In Exercise RC15, 13-year-olds and 17-year-olds were asked to add two unit fractions (fractions with one as the numerator). The exercise and results are displayed in Table 14.

Performance on this exercise was considerably lower than for the previous exercise (renaming a fraction). Forty-two percent of the 13-year-olds and 66% of the 17-year-olds answered correctly. Thirty percent of the 13-year-olds and 15% of the 17-year-olds responded with 2/5, the fraction obtained by adding the numerators and denominators together without finding the least common multiple. Other frequent answers included 1/5 (9% and 6%, respectively) and 2/6 or 1/3 (5% and 3%).

TABLE 14. Exercise RC15

Do the following addition:

1/2 + 1/3 =

	Age 13	Age 17
5/6*	42%	66%
2/5	30	16
1/5	9	6
1/3 or 2/6	5	3
1/6	2	1
Other unacceptable	10	6
I don't know or no response	2	2

**Asterisk indicates correct answer.*

Multiplication of Fractions

The process for multiplying fractions is simpler than that for adding fractions as equivalent fractions do not have to be found. Thus, it is not surprising that the results were higher on an exercise asking the multiplication of two-unit fractions (see Table 15) than on the previous addition problem.

TABLE 15. Exercise RC16

Do the following multiplication:

1/2 x 1/4 =

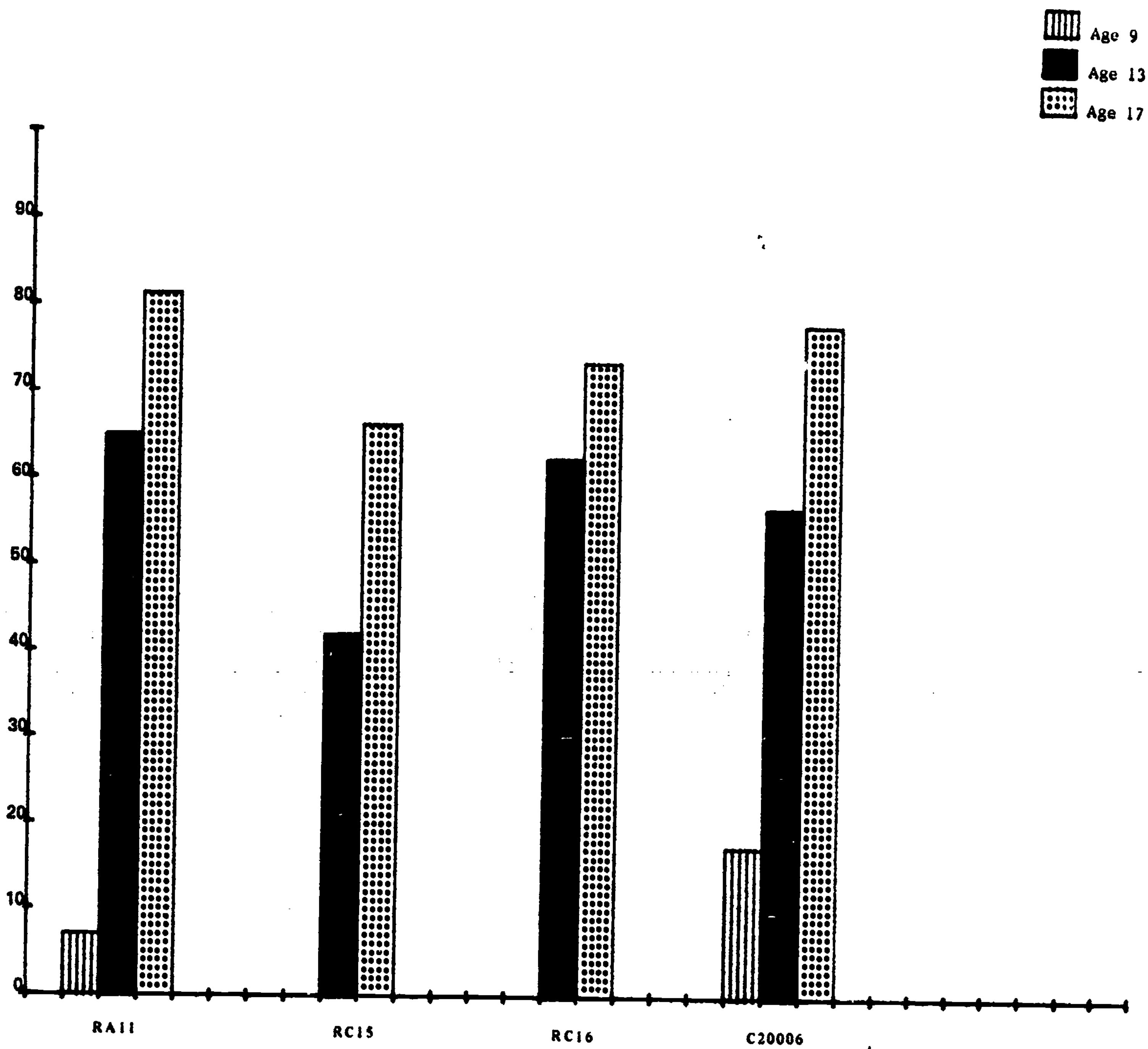
	Age 13	Age 17
1/8*	62%	74%

**Asterisk indicates correct answer.*

Over three fifths of the 13-year-olds and almost three fourths of the 17-year-olds gave the appropriate response. The most common mistake was an answer of 2/4, 1/2 or 4/8 (8% and 6%) closely followed by 2/8 or 1/4 (7% and 5%). Three fourths and 1/6 were also given as answers by a small percentage at each age.

Exercise C20006 asked for the multiplication of a whole number by a fraction. This exercise was administered to 9-year-olds as well as 13 and 17-year-olds. Percentages of success were 17% at age 9, 57% at age 13 and 78% at age 17.

FIGURE 1-5. Percentages of Success for Fractional Computation Exercises



Summary of Fractional Computation Results

No definite pattern of ability in fractional computation emerges after consideration of the results of these exercises. Figure 1-5 illustrates the range of success on these four exercises. Addition of fractions was the most difficult exercise for both 13 and 17-year-olds. Performance on the addition exercise did improve 24 percentage points from age 13 to 17. Twenty percent more 13-year-olds and 8% more 17-year-olds were able to solve the multiplication problem.

Seven percent of the 9-year-olds could rename a given fraction; 17% of them correctly multiplied a fraction by a whole number. Fewer 13-year-olds could multiply a fraction by a whole number than could multiply two-unit fractions; however, these results were reversed at age 17, with more 17-year-olds correctly multiplying a whole number by a fraction. Both 13 and 17-year-olds had the highest percentage of success on the exercise involving renaming of a fraction.

Computation with Integers

In the upper primary grades, consideration of numbers and number systems is generally expanded to include the integers, or the set of positive and negative whole numbers. The number line is used to illustrate the continuity of the number system and to extend the operations to integers. Several exercises in the mathematics assessment involved computation of positive and negative numbers.

Released Exercise RC24 had two parts: The first asked for the addition of two negative numbers and the second, for multiplication of two negative numbers. Results on the exercise are presented in Table 16.

Over half of the 17-year-olds were able to complete both problems successfully while less than one quarter of the 13-year-olds were able to do so. The majority of mistakes were made in determining the correct sign for the answer. For part A, 15% of the 13-year-olds and 10% of the 17-year-olds gave +14, 14 or - -14 as their answer. Some respondents (10% at age 13 and 6% at age 17) attempted to use the difference between the two numbers, resulting in either +4 or -4. Difficulty with signs was much more pronounced on the multiplication problem with 48% of the 13-year-olds and 24% of the 17-year-olds giving a negative sign to their answer. Skill in using positive and negative signs does appear to increase with age since only about half as many 17-year-olds as 13-year-olds used the wrong sign.

TABLE 16. Exercise RC24

	Age 13	Age 17
A. (-5) + (-9) =		
-14*	66%	78%
B. (-2) x (-3) =		
+6 or 6*	39	68
Respondents answering both correct	22	58
Respondents answering A correct and B incorrect	44	20

**Asterisk indicates correct answer.*

Exercise RC25 also involved the addition and multiplication of integers; however, the exercise asked for completion of a statement of a general rule and did not use numerical values. The exercise and results are presented in Table 17.

Half of the 17-year-olds gave the correct answer to both parts of this exercise while only one fourth of the 13-year-olds successfully completed both parts. Percentages of correct responses on this exercise are considerably lower for the addition problem than for the addition part of the integer computation problem (RC24) discussed previously; correct responses on the multiplication problems in the two exercises are fairly close.

Several unreleased exercises also measured skill with operations on integers. The first exercise, B13002, was administered to 9, 13 and 17-year-olds and simply asked the respondent to add a positive and a negative number together. Results were near those for the first integer addition exercise, RC24A—(-5) + (-9)—at age 13 and higher at age 17. Fourteen percent of the 9-year-olds, 63% of the 13-year-olds and 84% of the 17-year-olds gave the correct answer. Over one fourth (29%) of the 9-year-olds added the two numbers together and put a sign, either positive or negative, with the resulting sum. This error was made by 14% of the 13-year-olds and 8% of the 17-year-olds.

TABLE 17. Exercise RC25

	Age 13	Age 17
A. If X and Y are negative numbers, then X + Y		
● is negative.	47%	64%
○ is positive.	19	16
○ may be either positive or negative depending on what X and Y are.	30	19
○ I don't know.	3	2
No response	1	+*
B. If X and Y are both negative numbers, then X x Y		
○ is negative.	29	19
● is positive.	39	64
○ may be either positive or negative depending upon the size of X and Y.	22	11
○ I don't know.	8	6
No response	2	+†
Respondents getting both parts correct	24	51

**Figures may not add to 100% due to rounding error.*
†Plus equals rounded percents less than one.

A second exercise, A22010, demanded completion of two operations with integers, addition and multiplication, and was set up in the following way: x(-x) + y. Numerical values were used in the actual exercise. Twenty-two percent of the 13-year-olds (the only age group to which this exercise was administered) successfully completed the problem. This figure is lower than that for RC24B—(-2) x (-3)—by 17 percentage points. The combination of two operations in one problem and the use of brackets to indicate multiplication may have influenced the results.

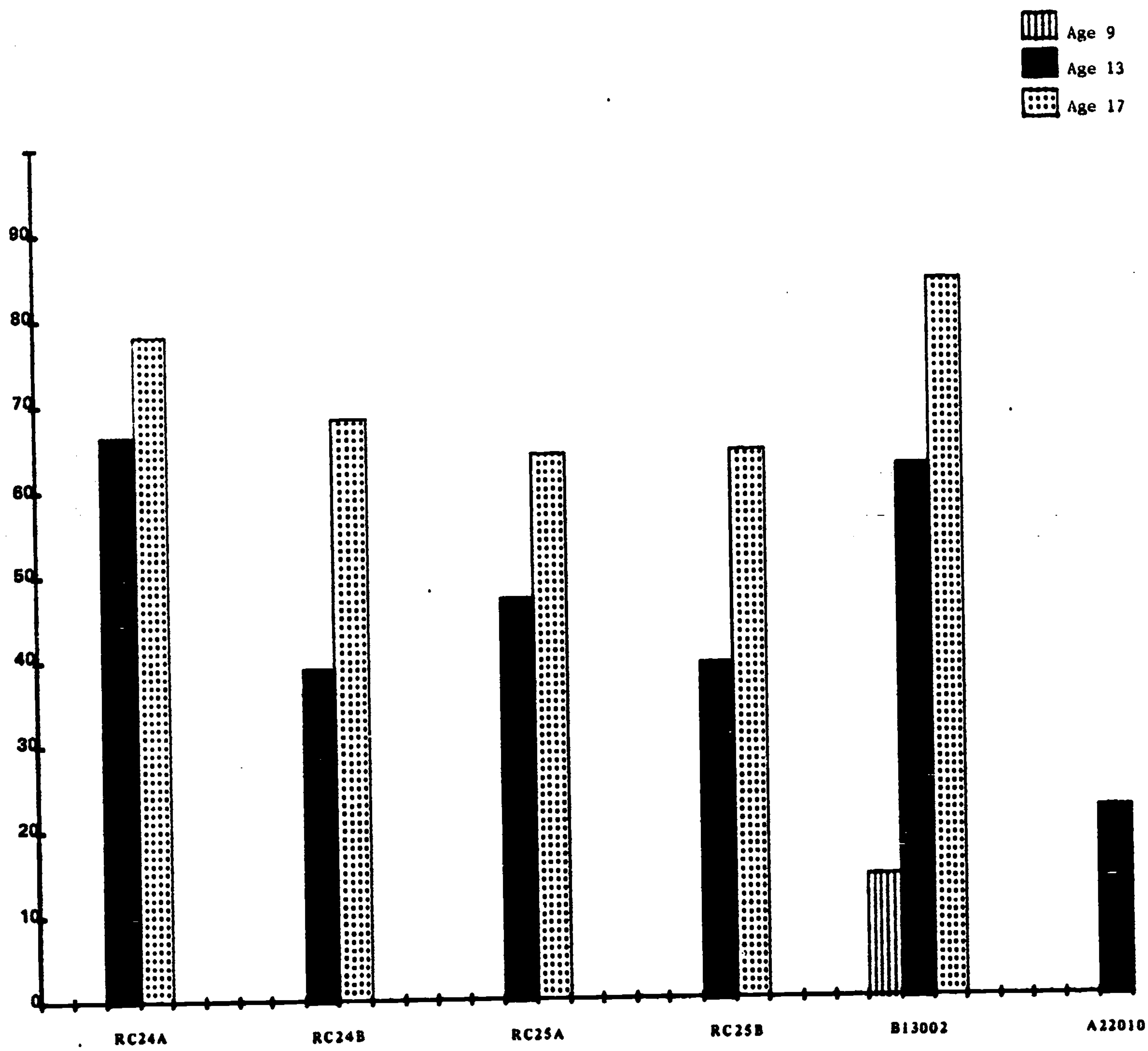
Summary of Skill with Integer Computation

On the integer computation exercises, the percentages of correct responses were somewhat lower than for most other "pure" computation exercises. As the actual computations required in the integer exercises were not difficult—only one-digit numbers were used making regrouping unnecessary—students evidently lacked familiarity with the concepts involved in computing with integers.

Multiplication with negative numbers was more difficult than addition with negative numbers. Respondents did less well, in most cases, on those examples calling for generalization of symbols rather than manipulation of numbers.

A summary of results on these exercises is shown in Figure 1-6.

FIGURE 1-6. Percentages of Success for Integer Computation Exercises



CHAPTER 2

COMPUTATION WITH TRANSLATION

Computation is, of course, an essential part of problem solving. However, computation rarely ever exists in a vacuum. It is of little value to be able to compute correctly if one cannot determine the operation needed in the first place. Similarly, one cannot solve problems without a firm foundation in the computation processes and number facts.

Situations calling for mathematical solutions, whether encountered in the classroom or in everyday life, are generally presented in a verbal form. One must then be able to translate from verbal to mathematical language in order to complete the necessary computations. The exercises presented in this chapter are concerned mainly with the process of translation. The computations to be done after the problem has been structured are, for the most part, not difficult ones.

Problem-solving skills are introduced almost simultaneously with computation skills in the usual mathematics curriculum, and practice in both areas continues throughout the primary grades.

The first group of exercises discussed in this chapter was designed primarily for 9-year-olds although some of the exercises were also administered at age 13. These exercises give an indication of abilities on elementary-level translation problems. The more complex exercises, requiring more difficult computations and/or more involved translations, were administered to the older age levels and are considered in a later section.

Elementary Translation

The elementary translation exercises covered three operations—subtraction, multiplication and division. The first group of exercises considered was administered only to 9-year-olds. Results on these exercises were high in comparison with the results on the pure computation exercises; however, the computations required were not as difficult. Percentages of success were widely varied on these exercises, ranging from 76% to 36%. The highest percentage of success was on a subtraction problem; the lowest, on a division problem.

Seventy-six percent of the 9-year-olds responded correctly to Exercise C10016, an elementary problem requiring subtraction of a two-digit number from a two-digit number with no regrouping. Of this number, less than 1% set up the problem correctly and then failed to find the right answer. Approximately 5% of the respondents made some attempt to add the two numbers given together and another 5% answered "I don't know."

Two exercises assessed 9-year-old multiplication skills. The first was Exercise RC06 (see Table 18). The respondent first had to determine that multiplication was the operation to use and then had to multiply 7 x 3 correctly. A very small percentage of the 9-year-olds, less than 1%, wrote the proper method of solution and failed to multiply correctly. Attempts to add or subtract the two numbers were each made by 2% of the respondents.

Three and 7 were chosen as answers by 2% and 7%, respectively. A large percentage of the mistakes, 24%, were in the "other unacceptable" category.

TABLE 18. Exercise RC06

An astronaut is to orbit the earth in a space capsule for seven days. If he drinks three pints of water each day, how many pints of drinking water will be needed for the trip?

	Age 9
Respondents answering 21 or 21 pints*	46%

*Asterisk indicates correct answer.

Nine-year-olds showed a higher percentage of success (69%) on an unreleased problem, C10028, requiring translation and multiplication. Of this number, however, 21% neglected to include the units (dollars and cents in this case) and 2% included units but identified them incorrectly. The computation required in this instance was multiplication of a two-digit number by a one-digit number with regrouping. Theoretically, this computation should be more difficult than that asked for in Exercise RC06, but 9-year-olds were more likely to succeed with it. There are several possible reasons for this difference. Some number combinations may be inherently more difficult to learn than others. The method of stating a problem and the content of a problem may affect interest in the problem and likewise ability to solve it. For example, students may have been frightened or confused by the word astronaut in problem RC06.

Two simple division problems, one released and one unreleased, were also administered to only 9-year-olds. See Table 19 for results. Four percent attempted to add 2 and 24, 4% subtracted 2 from 24 and 8% used multiplication of 2 and 24 to arrive at their answer.

TABLE 19. Exercise RC07

Betty's dog eats two biscuits every day. How many days will it take the dog to finish a package of 24 biscuits?

	Age 9
Respondents answering 12 or 12 days*	36%
Respondents answering 12 with wrong units—e.g., biscuits	1
Respondents who set up problem correctly—no or wrong answer	+†

*Asterisk indicates correct answer.
†Plus equals rounded percents less than one.

The unreleased exercise, C10044, required division of a two-digit number by a one-digit number. The problem was similar to RC07. On this exercise, 58% of the 9-year-olds assessed gave the correct response. Under 1% successfully wrote down the problem and failed to arrive at the right answer. Around 2% attempted addition, subtraction and multiplication, respectively. A larger portion of the responses to the first exercise, RC07, were in the "I don't know" category, 18% as opposed to 7% on C10044.

It is difficult to account for the 20 percentage point difference in performance on these two exercises. Again, the number combinations used (C10044 involved a number fact which would have been memorized; RC07 did not) or the statement of the problem may be determining factors. Students do not perform tasks in isolation; it is evident that ability in one situation does not necessarily carry over into another slightly different situation.

Three translation problems assessing ability with more complex computations were administered to both 9 and 13-year-olds. Two problems involved subtraction with regrouping; the third required the use of both addition and subtraction.

TABLE 20. Exercise RC03

A rocket was directed at a target 525 miles south of its launching point. It landed 624 miles south of the launching point. By how many miles did it miss its target?

	Age 9	Age 13
99 or 99 miles*	22%	72%
Respondents with correct equation and/or operation with wrong or no numerical answer	10	9

Asterisk indicates correct answer.

On Exercise RC03 (Table 20) the respondent must first "translate" the problem into a number sentence: 525 + N = 624 or 624 - 525 = N. Then the computation must be completed. An answer of 101 was a frequent error—made by 14% of the 9-year-olds and 5% of the 13-year-olds. Subtraction reversal, subtracting the smaller number from the larger number regardless of position in the problem, appears to be a fairly common tendency among 9-year-olds. From 10 to 20% of the 9-year-old respondents made this type of mistake on the subtraction exercises discussed in the first chapter. Some attempt to add the two numbers given was made by 8% of the 9-year-olds and 4% of the 13-year-olds. Nearly 15% of the 9-year-olds did not feel sure enough of themselves to try this problem and simply wrote "I don't know."

The second problem (Exercise C10022) involved the same subtraction operation as in RC03 above: subtraction of a three-digit number from a three-digit number with regrouping in two places. The statement of the problem was also similar to that of the first exercise. Both age levels were more successful on the second problem with 34% of the 9-year-olds finding the right answer and an additional 7% setting up the problem correctly and 80% of the 13-year-olds solving the problem and another 6% translating the problem accurately. About 7% of the 9-year-olds gave an answer which implied reversed subtraction and 4% of the 9-year-olds attempted to add the two numbers together. Under 1% of the 13-year-olds made either of these errors.

Comparing these two subtraction exercises, we find that approximately 22% of the 9-year-olds successfully solved the first problem and 34%, the second—a 12 percentage point difference. There was an 8 percentage point difference at age 13, with 72% correctly answering the first problem and 80% the second.

In reviewing the subtraction results from the first chapter, we can see that 9-year-old results for subtraction with regrouping in more than one place did not go over 30%; thus, the results for subtraction word-problem solving at age 9 were fairly similar to those for the "pure" computation subtraction exercises. Similarly, results for "pure" subtraction computation at age 13 ranged from 70 to 80% which was quite close to the results on the subtraction word problems presented.

A slightly more complex problem asked the 9 and 13-year-old respondents to complete a translation in which two operations were required. The respondents had to both add and subtract, although the process they used to arrive at the solution could vary. The problem and results are displayed in Table 21.

This exercise could be viewed as a trick question since the natural assumption on seeing the few numbers lined up next to one another is to add them all together. However, it can be argued that even at the 9-year-old level, students should begin learning that careful reading and attention to detail are essential elements of mathematics.

Approximately 3 in 20 9-year-olds gave the right answer while 12 in 20 of the 13-year-olds responded correctly. An additional 4% of the 9-year-olds and 6% of the 13-year-olds structured the problem correctly using one of the two possible methods, but failed to find the proper solution. It is evident that 9-year-olds were confused by this exercise as 50% of their responses were in the "other unaccept-

TABLE 21. Exercise RC11

Marie took four spelling tests. Each test had 30 words. On the four tests she spelled correctly the following number of words:

25, 23, 27, and 24.

Altogether, how many words did she MISS on all four tests?

	Age 9	Age 13
21 or 21 words*	16%	60%
Correct process, wrong or no answer	4	6
99 or 99 words (attempted to add 25, 23, 27, 24)	9	10
4 or 4 words	2	1
Other unacceptable	50	18
I don't know	18	4
No response	2†	2†

**Asterisk indicates correct answer.*
†Figures may not add to 100% due to rounding error.

able" category and 18% of their answers were "I don't know." This is not to say that 9-year-olds cannot solve two-step translation problems. The statement of this problem, however, and the way that it was presented probably contributed to the low performance.

Thirteen-year-old performance on this exercise is also lower than for the other elementary translation exercises.

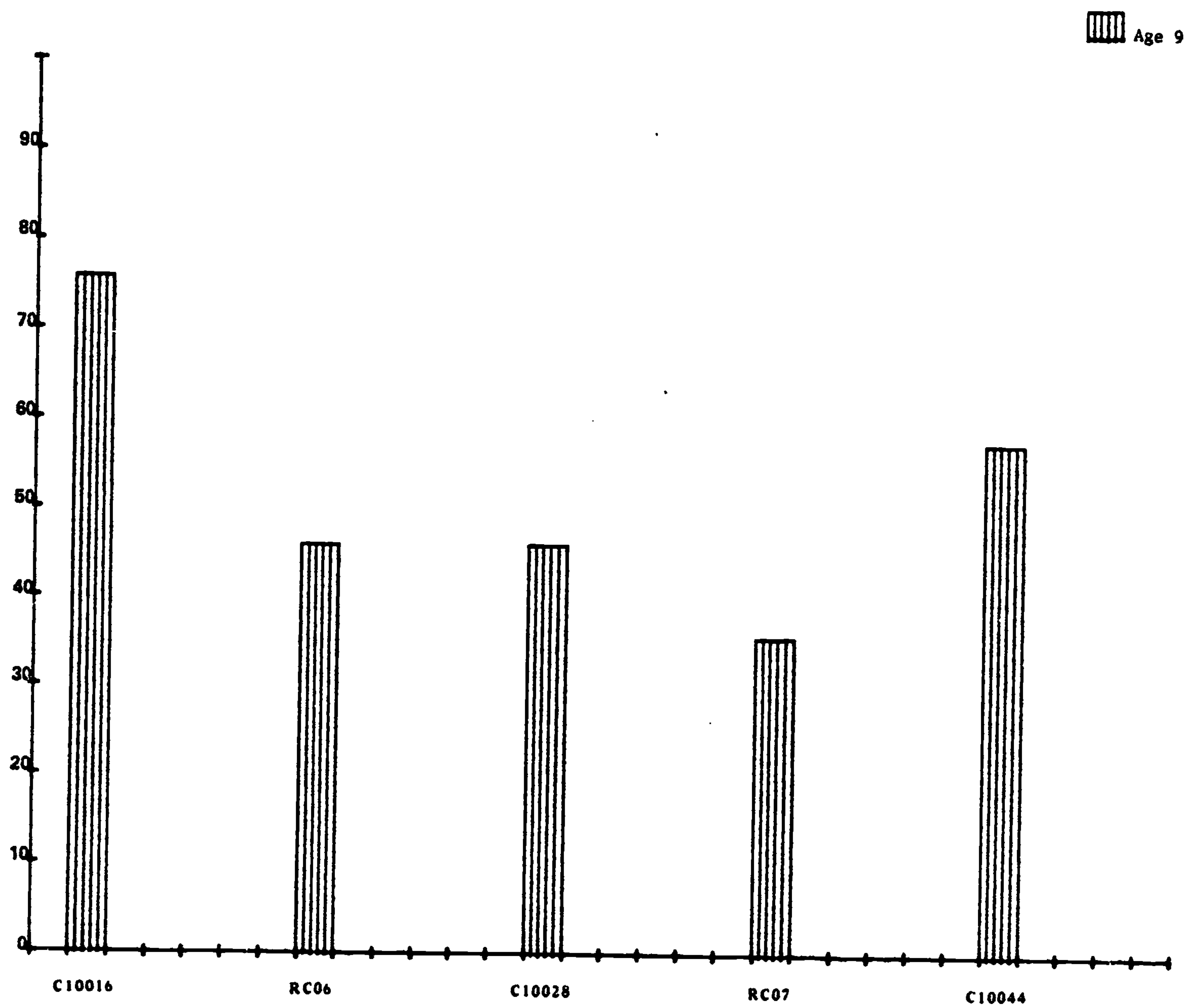
Summary of Results for Elementary Translation

Figure 2-1 shows the results on the elementary translation problems for 9-year-olds. Nine-year-olds exhibited a wide range of ability in solving these problems. They were most successful on the problem requiring simple subtraction and least successful with the division exercise. A very small percentage of the 9-year-olds wrote the proper mathematical expression and then failed to find the answer on these problems, indicating that if they had the skills to write down the problem correctly, they also had the computational ability to solve it.

On the exercises administered to both 9 and 13-year-olds (shown in Figure 2-2), the 13-year-olds were vastly superior with results from 44 to 50 percentage points above those of the 9-year-olds. Nine-year-olds had more difficulty in completing the computations on these exercises; from 4 to 10% of the 9-year-old respondents wrote down the correct method for solving the problem but failed to give the right answer. Six to 9% of the 13-year-olds expressed these problems correctly but gave an incorrect answer.

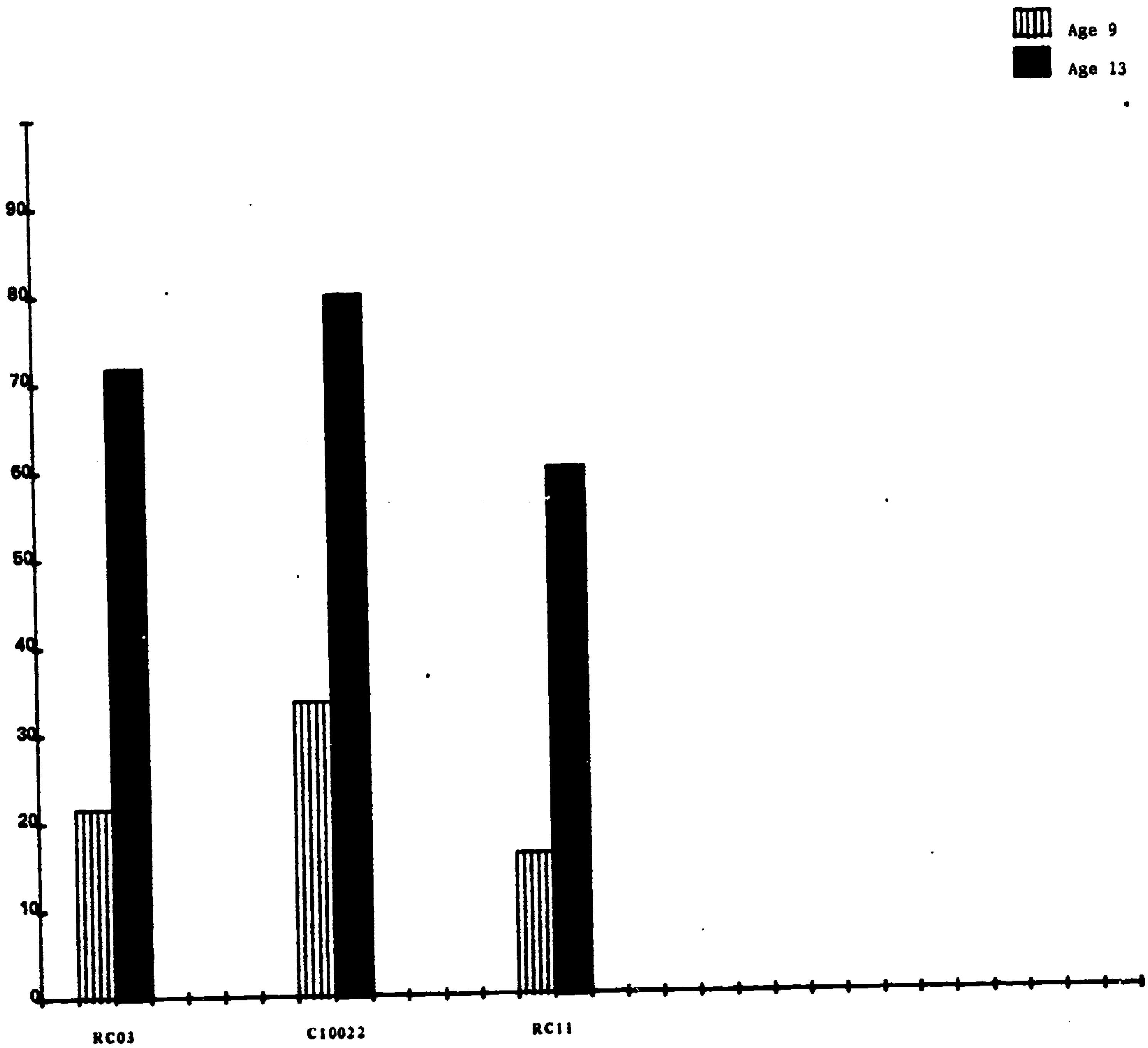
BEST COPY AVAILABLE

FIGURE 2-1. Percentages of Success for Simple Translation—9-Year-Old Exercises



BEST COPY AVAILABLE

FIGURE 2-2. Percentages of Success for Simple Translation—9 and 13-Year-Old Exercises



More Complex Translations

Some of the elementary translations were administered to the 13-year-old respondents as well as the 9-year-olds. In the problems presented in the following section, the computations were slightly more difficult than those discussed above, and the content of the problems was designed to be more appealing to an older audience.

Exercise RC12 required multiplication and was administered only at the two upper age levels (Table 22).

TABLE 22. Exercise RC12

A sports car owner says that he gets 22 miles per gallon of gasoline. How many miles could he go on seven gallons of gasoline?

	Age 17	Adult
154 or 154 miles*	89%	90%
Respondents setting up problem correctly but giving wrong answer	1	1

**Asterisk indicates correct answer.*

This was a relatively simple problem in which the respondent multiplied 22 x 7 to find the answer. Although the computation required one regrouping, it was not difficult. Both 17-year-olds and adults performed well on this exercise with only about 10% at each age level unable to complete it correctly.

Exercise RC13 also was concerned with cars (Table 23). Here the respondents had to divide to decide the time required to cover a certain distance. This exercise was administered at age 9 as well as to the three older ages; as might be expected, the 9-year-old percentage of success was not high. A larger number of 13-year-olds are still having problems with division, as judging from the number who set up the problem and did not solve it. Some of the 13-year-olds probably had

TABLE 23. Exercise RC13

If John drives at an average speed of 50 miles an hour, how many hours will it take him to drive 275 miles?

	Age 9	Age 13	Age 17	Adult
5½, 5 hrs. 30 min., 5½ hrs., 5.5, etc.*	6%	33%	64%	67%
Wrote down problem right, no or incorrect answer	1	15	13	8
Answering 5 and 25, 5 hrs. and 25	1	11	4	3

**Asterisk indicates correct answer.*

difficulty with the concept of hours and minutes as they did not appear to be sure what to do with the remainder. Although 18% of the 9-year-olds tried to either add or subtract the two numbers, the majority of the 9-year-old mistakes (42%) fell into the "other unacceptable" category. Errors at the upper age levels also tended to be in the "other" category, although 6% of the 13-year-olds attempted to add or subtract the two numbers while 7% of the 13-year-olds made a try at multiplying them together.

Percentages of respondents finding the correct answer were lower on Exercise RC10, shown in Table 24. If we think of the situation as being 9 times as much snow as rain, the number sentence becomes $9 \times N = 1{,}602$ or $1{,}602 \div 9 = N$. Approximately 44% of the 13-year-olds, 70% of the 17-year-olds and 72% of the adults were able to determine the appropriate operation, although only 31%, 53% and 58%, respectively, of these age groups were then able to complete the operation properly. An error made by about 5% of the 13-year-olds and 17-year-olds was attempting to multiply the numbers; the frequency of this error dropped to about 4%

TABLE 24. Exercise RC10

Weathermen estimate that the amount of water in nine inches of snow is the same as one inch of rainfall. A certain Arctic island has an annual snowfall of 1,602 inches. Its annual snowfall is the same as an annual rainfall of how many inches?

	Age 13	Age 17	Adult
178 or 178 inches*	31%	53%	58%
Attempted at solution by division—wrong or no answer	14	19	15

Asterisk indicates correct answer.

at adult. Twenty-three percent of the 13-year-olds, 11% of the 17-year-olds and 15% of the adults responded "I don't know" to this exercise.

Several other problems involving translation from verbal to mathematical forms were assessed. One of these, RC19, could be solved by setting up a proportion (Table 25). Two steps would then be needed to solve the proportion. This exercise was administered only to 17-year-olds and adults. There was a 10 percentage point jump in performance from age 17 to adult. This is quite unusual since adults tend to perform at about the 17-year-old level or below it on the other computation exercises. Slightly more adults than 17-year-olds answered "I don't know"—6% at age 17 compared to 9% at adult.

TABLE 25. Exercise RC19

If there are 300 calories in nine ounces of a certain food, how many calories are there in a three ounce portion of that food?

	Age 17	Adult
100 or 100 calories*	70%	80%
Right process, wrong answer	3	2

Asterisk indicates correct answer.

An unreleased exercise (C10051) also involved proportion, in this case, determining what constituted a simple plurality using the numbers given. Percentages of success on this exercise were as follows: at age 13, 39%; at age 17, 64%. The most common error was in assuming that 51% was needed to obtain a plurality (14% for 13-year-olds, 10% for 17-year-olds). This exercise did not demand a great deal of computation but mainly an understanding of the proportion concepts involved and of the terminology used. Thus it is not really possible to tell from the scoring categories whether the respondent lacked the basic concepts or whether his computational skills were at fault.

Exercise RC08, administered at the three upper age levels, asked for the difference between a negative number and a positive number (shown in Table 26). This problem was put in a familiar context, that of temperatures above and below zero. Although it was certainly possible to complete this problem without formal knowledge of the operations on positive and negative numbers, some concept of the number line, as represented on the thermometer, was needed.

TABLE 26. Exercise RC08

The air temperature on the ground is 31 degrees. On top of a nearby mountain, the temperature is -7 degrees. How many degrees difference is there between these two temperatures?

	Age 13	Age 17	Adult
-38, 38 or 38°*	39%	65%	67%
24, 24°, -24 or -24°	33	21	17
39	1	2	2
Other	17	10	10
I don't know	4	3	4
No response	7	+†	+**

Asterisk indicates correct answer.
†Figures may not add to 100% due to rounding error.
***Plus equals rounded percents less than one.*

Over one third of the 13-year-olds answered this problem correctly; however, another third of the 13-year-olds evidently interpreted difference to mean subtraction of the numbers provided without regard to the signs of the numbers. Approximately two thirds of the 17-year-olds and adults were able to give the correct solution, a 100% improvement over the 13-year-old results. Thirteen-year-olds were more successful with the problems on adding two negative numbers described in the first chapter (RC24A, 66%; B13002, 63%).

Summary of Results of More Complex Translations

One striking fact from these exercises was that in all of the five exercises administered to both 17-year-olds and adults, the adults did as well or better than the 17-year-olds. Results for the more complex translation exercises are displayed in Figure 2-3.

Performances were higher for adults on two exercises and the two groups performed at about the same level on the remaining three exercises. In comparison, there was only one exercise in pure computation on which the adults performed significantly better than 17-year-olds. For other pure computation problems administered to 17-year-olds and adults, on seven exercises the two groups were within 1 percentage point of each other and on four exercises adult performance was below that of the 17-year-olds.

There was a fairly consistent increase in ability to solve word problems from ages 13 to 17. For three of the exercises, RC10 (snowfall, rainfall, 1,602 ÷ 9), C10051 (percentage problem) and RC08 (temperature difference), the difference in performance at the two age levels was from 21 to 26 percentage points. On one exercise, RC13 (gasoline mileage, 275 ÷ 50), 31 percentage points separated the two groups.

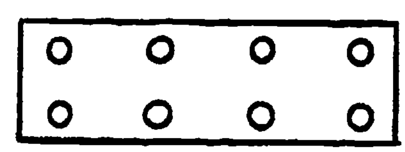
How do the above results compare with those given for "pure computation" in Chapter 1?

As might be expected, the percentages are lower on most of the translation exercises for each age level. The exercises used to measure the two skills, pure computation and computation with translation, are not directly comparable as the exercises vary in degree of computational ability required as well as in context of problems. Consequently, comparisons between these two areas should be made very carefully, if at all.

Operations with Fractions

Operations with fractions were also included in the translation exercises. The first of these was administered only at age 9 (Table 27). This problem involved more than one operation: The 9-year-olds had to determine that there were eight dots in the box, then had to take one fourth of eight (1/4 x 8) and finally had to subtract the number obtained from the total eight dots.

TABLE 27. Exercise RC14



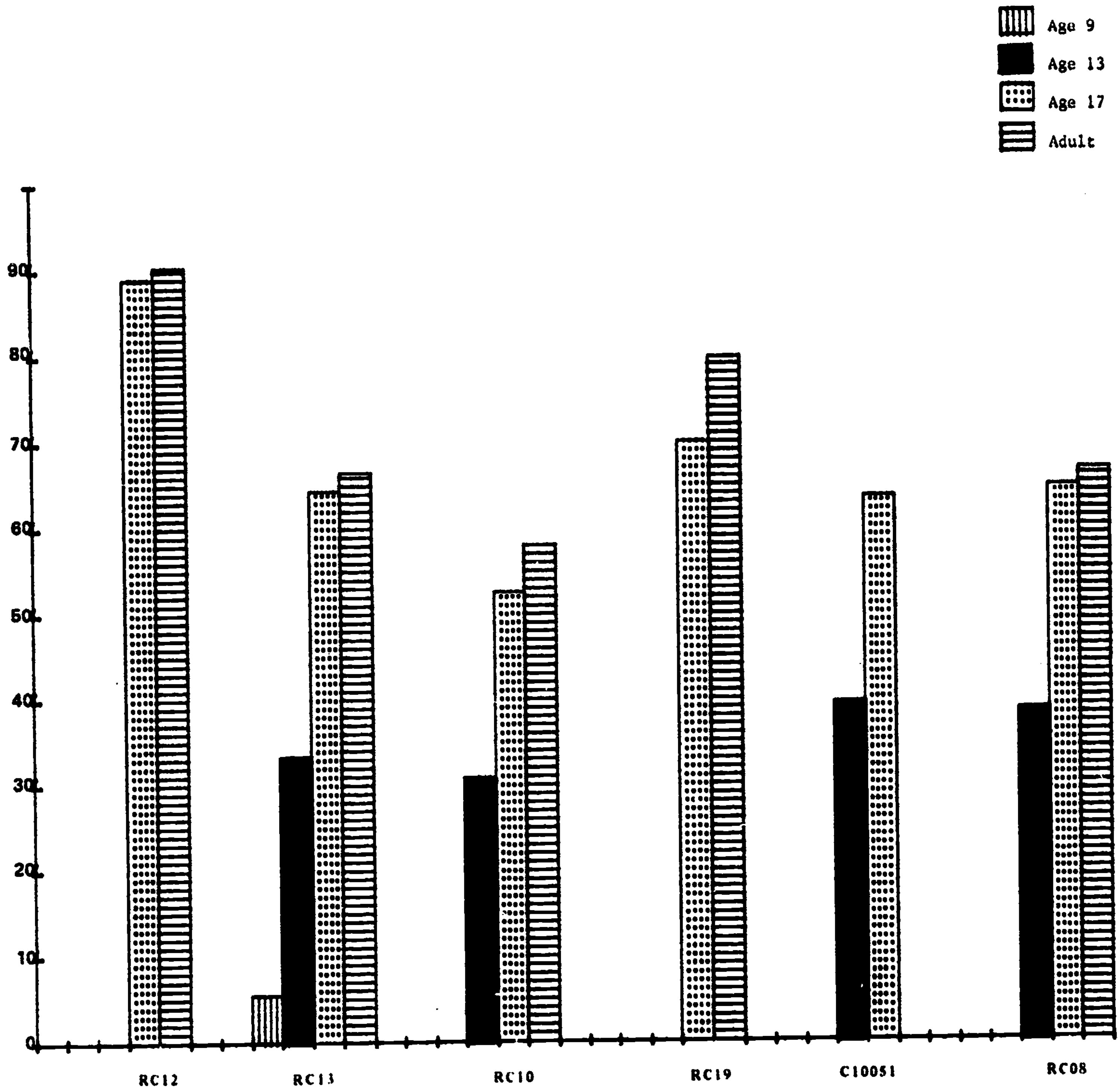
If one fourth of the dots on the above figure are removed, how many dots will be left?

	Age 9
6 or 6 dots*	21%
3/4	1
4, 4 units (dots, circles, etc.) or 1/2, 1/2 units	48†

**Asterisk indicates correct answer.*
†Percentages do not total 100% as all response categories are not shown.

Answers of either 6 or 3/4, with or without units (dots, circles, etc.), were counted as correct. A large group of the 9-year-olds, almost 50%, gave an answer of 4, 4 units or

FIGURE 2-3. Percentages of Success for Complex Translation Exercises



1/2, 1/2 units. Approximately 3% of the 9-year-olds responded with 2, 2 units, 1/4 or 1/4 units.

Another exercise (C20001) given at ages 9, 13 and 17 required adding fractions that were shown in pictorial form. Results showed a definite improvement as the age of the respondents increased. While only 14% of the 9-year-olds successfully completed the exercise, over two thirds (69%) of the 13-year-olds and over four fifths of the 17-year-olds (84%)

translated the exercise into mathematical terms and performed the computation correctly. Adding numerators and denominators without finding a common multiple was a popular technique at age 9 with 9% answering in this manner. The percentage using this method dropped to 5% at age 13 and 1% at age 17. A large number of errors were categorized as "other unacceptable" at all age levels. Thirteen-year-olds and 17-year-olds were more willing to attempt the problem than 9-year-olds since nearly 20% of the 9-year-olds responded "I don't know." Less than 2% of the 13 and 17-year-olds stated that they did not know the answer.

The results on this exercise for 13 and 17-year-olds are much higher than for Exercise RC15 involving the addition of 1/2 and 1/3 as discussed in Chapter 1. On Exercise RC15, approximately 42% of the 13-year-olds and 66% of the 17-year-olds gave the right answer as compared to 69% and 84%, respectively, on C20001. This is a 27 percentage point difference at age 13 and an 18 point difference at age 17. There are several possible explanations for these differences. The computational problem with no translation involved only unit fractions (fractions with one in the numerator) while the translation problem had different numbers in the numerators and denominators. The pictorial representation in the unreleased exercise may have been a major factor, allowing respondents to visualize the problem and estimate a reasonable solution.

Approximation

Approximation is an important element in computational skill. It is a means of quickly estimating whether an answer is reasonable and also is essential in mental arithmetic. The advent of computers and hand calculators will not eliminate the need for skill in approximation. The computer operator needs some idea of whether the computer's answer is reasonable; the average citizen may not have a calculator available for simple transactions.

TABLE 28. Exercise RC21

John has 382 stamps in his stamp collection, Greg has 224, Pete has 310 and Bob has 175. The number of stamps the boys have altogether is CLOSEST to which one of the following numbers?

	Age 9
900	18%
1,000	24
1,100	31
1,200	22
I don't know.	5
No response	+*

**Plus equals rounded percents less than one.*

The problem displayed in Table 28, requiring adding and approximation, was given only to 9-year-olds. This exercise was administered in a multiple-choice format so it was not possible to determine the method used to work the problem. Some of the respondents may have carefully added the numbers given and then rounded their answer while others may have rounded the numbers given before attempting to add. At any rate, the number of respondents giving the correct answer is not much larger than that which would be expected by chance. In fact the responses are fairly evenly distributed across the four foils (distractors) which seems to indicate that the chance factor is fairly strong.

A similar exercise (RC23) was administered to 17-year-olds and adults. The numerical values were larger and the context of the question was intended to be more relevant to the concerns of the older age groups. Here respondents filled in their own answer rather than making a choice among several alternatives (Table 29).

If the two acceptable categories are added together, and given the wording of the problem it is appropriate that they should be, the adults outperformed the 17-year-olds by 10

TABLE 29. Exercise RC23

In one year, a government department spent the following sums on four projects:

Project A:	$11,954,164
Project B:	1,126,055
Project C:	4,170,522
Project D:	750,572

Approximately how many MILLIONS of dollars were spent on these four projects? Give your answer to the nearest MILLION dollars.

	Age 17	Adult
18 with some indication of million	48%	44%
18 only - no units indicated	5	18
Total of above two categories	53	63*

Percentages do not total 100% as all response categories are not shown.

percentage points. A number of the respondents evidently misunderstood the directions and gave the exact sum of the numbers shown rather than round their answer to the nearest million—13% of the 17-year-olds and 11% of the adults made this mistake. Another frequently scored response was 17 or 17,000,000; this answer was given by 7% of the 17-year-olds and 6% of the adults. Approximately 15% of the 17-year-old responses and 12% of the adult responses were scored in the "other unacceptable" category.

The percentages of acceptable responses on this exercise are below those for most other exercises in this report. They are lower for age 17 and adult results than on any of the "pure computational" exercises. The only results which approach them at these age levels are the exercises involving translation with long division (RC10—snowfall to rain) and with negative numbers (RC08—temperature). It seems that approximating the sum of large numbers is a difficult task which perhaps deserves more emphasis in the school curriculum.

CHAPTER 3

SUMMARY OF GROUP RESULTS

The preceding chapters described the performance of the nation as a whole for each of the four age levels. National Assessment (NAEP) also reports results for the following groups within the American population: region of the country, sex, race, level of parental education and size and type of community (STOC). The differences in achievement among these groups provide an indication of areas of strength and weakness in American education.

Nine-Year-Olds: Third Grade vs. Fourth Grade

For the mathematics assessment, in addition to the groups listed above, the differences in performance between 9-year-olds in the third grade and in the fourth grade were studied. In the elementary mathematics curriculum, each new step depends upon the skills learned previously. A similar developmental sequence is followed in schools throughout the United States, so one would expect third graders in New York to have encountered approximately the same material as third graders in California. This is much more true in mathematics than in learning areas such as social studies and language arts. The distinct grade-to-grade progression in mathematics thus makes it reasonable to consider differences in performance between the third and fourth grade. By age 13, most of the computational skills discussed in this report should have been mastered, and significant differences in 13-year-old grade-level performance would not be anticipated.

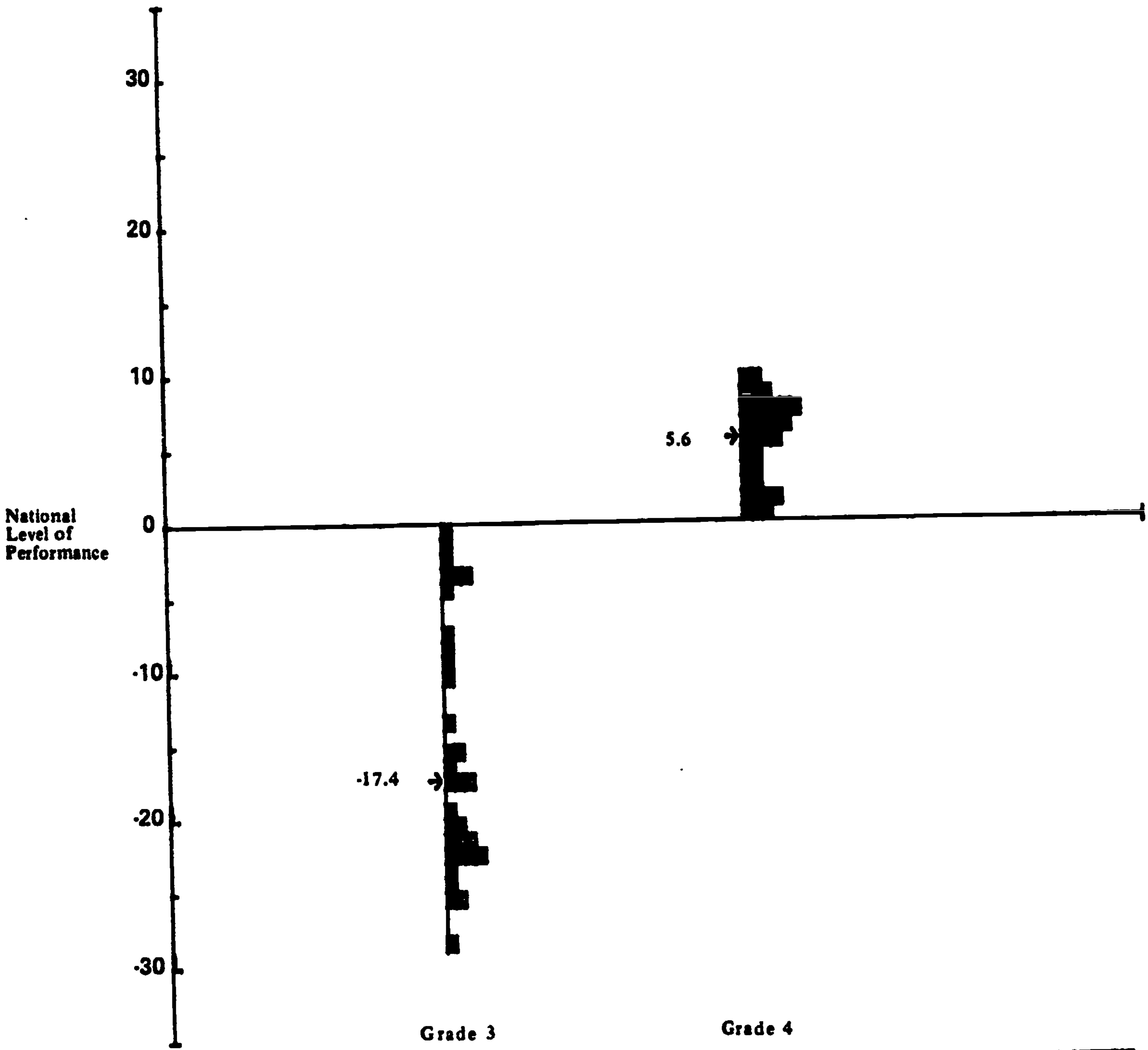
Multiplication and division are generally introduced in the third and fourth grades; therefore, differences in ability in these two skills at the two grade levels would be expected. Practice in addition and subtraction with regrouping continues during these grades; one would expect proficiency on simpler problems at both grade levels with the third graders less adept at problems demanding more extensive regrouping.

For the 9-year-olds, only those students born between January 1, and December 31, 1963, were assessed so the chronological age range of both the third and fourth graders was within one year. Approximately three fourths of the 9-year-olds were in the fourth grade and one fourth in the third grade. The 9-year-old assessment took place in January and February when students were about halfway through the year's course work.

Figure 3-1 provides a summary of results for third and fourth graders on all 9-year-old exercises discussed in this report. The graphs indicate the difference in achievement levels between third- or fourth-grade students and 9-year-olds nationally.

The horizontal line represents the national level of success for each exercise. Each square stands for one exercise; the distance from the horizontal line is the difference between group and national performance on an exercise. At the fourth-grade level, this difference was similar for a number of exercises so that the squares are quite close to one another. Differences from national performance were more dissimilar at the third-grade level and thus the squares have a wider spread. The arrow on each graph indicates the median group difference, that is, the midpoint in the particular distribution shown.

FIGURE 3-1. Differences from National Performance Levels: Third and Fourth Grades



The difference from the national percentage of success ranged from 1 to 28 percentage points below the national level for third graders. For fourth graders, no results fell below the national levels. The median difference for the third graders was -17.4; for the fourth graders, +5.6.

How do these summaries relate to results on specific exercises? Two examples are shown in Table 30.

For the addition problem with regrouping, a skill which most third graders would have been exposed to, 62% of the third graders and

TABLE 30. Exercises RC02A and RC06

Exercise RC02A

Add:

$$\begin{array}{r} 38 \\ +19 \\ \hline \end{array}$$

(57)	National	79%	
	Third	62	(-17)*
	Fourth	86	(+7)

Exercise RC06

An astronaut is to orbit the earth in a space capsule for seven days. If he drinks three pints of water each day, how many pints of drinking water will be needed for the trip? (21)

National	46%	
Third	21	(-25)*
Fourth	55	(+9)

**Asterisk indicates the difference from the national percentage.*

86% of the fourth graders responded correctly. The word problem requiring knowledge of a multiplication fact was much more difficult for third graders than for fourth graders. Only 21% of the third graders, compared to 55% of the fourth graders, solved this problem correctly.

A closer investigation of results provides an indication of types of errors which third graders made more often than fourth graders. Third graders have slightly more difficulty with regrouping in addition. Around 5% more third graders made this error on one exercise (RC02A) and 1% more on another (C10009). They also showed a greater tendency toward subtraction reversal—subtracting the smaller number from the larger number regardless of position in the problem. In an exercise requiring subtraction with one-place regrouping (RC02B), 18% of all 9-year-olds made a subtraction reversal while 27% of the third graders did so. On another, more difficult, subtraction problem (C10019), 22% of all 9-year-olds and 26% of the third graders made a reversal error. In two subtraction word problems given to 9-year-olds (RC03 and C10022), grade level did not make a large difference in the number of subtraction reversal errors. If third graders failed to solve the problem, their answers were more likely to be scored as "no response," "I don't know" or "other unacceptable."

The error patterns at the two grade levels were not significantly different on the multiplication and division problems. Third-grade responses again were more likely to fall into the "no response," "I don't know" and "other unacceptable" categories.

The summary results indicate that there is a large increase in learning and ability in the interval between third and fourth grade. The difference in results was greatest on addition and subtraction items requiring regrouping and on the multiplication problems with simple regrouping. Subtraction reversal errors decreased somewhat by the fourth grade; however, identifiable regrouping errors in subtraction and multiplication became more prevalent at the higher grade level.

It is evident that formal education makes a great impact in the area of mathematics. Questions remaining to be answered are whether teaching of some mathematics skills should be moved into lower grades and whether higher degrees of mastery than have been reported here would be desirable for fourth graders. The elementary mathematics programs in American schools appear to have been effective; however, continued research may provide data which can be used to make these programs even more effective in the future.

National Assessment Reporting Groups

The National Assessment reporting groups are defined as follows:

Sex

Results are presented for males and females.

Race

Currently, results are reported for Blacks and Whites.

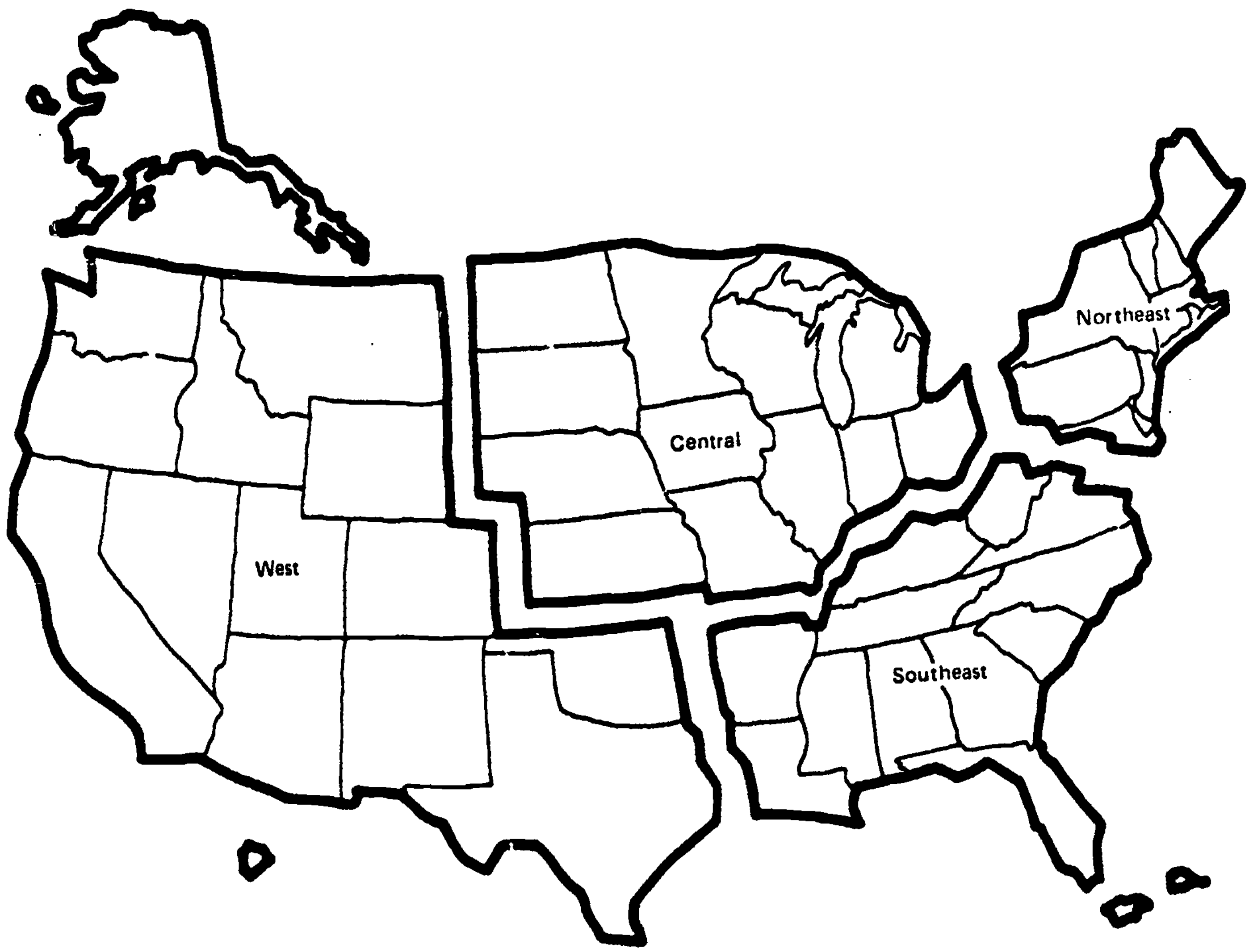
Region

The country has been divided into four regions—Southeast, West, Central and Northeast. The states that are included in each region are shown in Exhibit 1.

Parental Education

Four categories of parental education are defined by the Assessment. These categories include (1) those whose parents have had no high school education, (2) those who have at least one parent with some high school education, (3) those who have at least one parent who graduated from high school and (4) those who have at least one parent who has had some post high school education.

EXHIBIT 1. National Assessment Geographic Regions



Size and Type of Community (STOC)

Community types are identified both by the size of the community and by the type of employment of the majority of people in the community.

High metro. Areas in cities with a population greater than 150,000 where a high proportion of the residents are in professional or managerial positions.

Low metro. Areas in cities with a population greater than 150,000 where a high proportion of the residents are on welfare or are not regularly employed.

Extreme rural. Communities with a population under 3,500 where most of the residents are farmers or farm workers.

Urban fringe. Communities within the metropolitan area of a city with a population greater than 200,000, outside the city limits and not in the high or low metro groups.

Main big city. Communities within the city limits of a city with a population over 200,000 and not included in the high or low metro groups.

Medium city. Cities with populations between 25,000 and 200,000.

Small places. Communities with a population of less than 25,000.

Group Results

Sex

Neither sex has a clear advantage in computational ability since results for males and females varied at the different age levels. Male and female overall performance differed by only 1 percentage point at ages 9 and 17; girls had approximately a 3 percentage point advantage at age 13, while for adults, males outperformed females by about 4 percentage points.

TABLE 31. Numbers of Exercises on Which Males and Females Were More Successful

	Age 9	Age 13	Age 17	Adult
Number of exercises administered	33	37	33	18
Number of exercises on which more males answered correctly	13	6	18	14
Number of exercises on which more females answered correctly	20	31	15	4

Table 31 shows the number of exercises on which each sex had a higher percentage of success. Females performed noticeably better than males at age 13 while males had a similar advantage at the adult level. Differences in performance for 9 and 17-year-olds are not nearly so marked; females show a slight advantage at age 9 while males have a small edge at age 17.

At all ages, males generally did better than females on the more difficult exercises and on word problems. Females tended to do better on "pure computation" exercises demanding the application of a specific mathematical process. For example, 9-year-old males did better than females on such exercises as the difference between a rocket target and the actual landing point, apportioning an equal number of dog biscuits over a number of days and determining how many words a girl missed on four spelling tests. Nine-year-old females, on the other hand, were more successful with exercises on adding, subtracting and multiplying with one-place regrouping. Males at the three older age levels had a higher performance level on the exercises involving difference in air temperatures and time required for a car to travel a certain distance. Females had an advantage in decimal subtraction and multiplication at ages 13 and 17 and in decimal addition at all three of the older age levels. There were, of course, some exceptions to these generalizations at every age level.

TABLE 32. Median Differences from National Performance Levels by Race

	Age 9	Age 13	Age 17	Adult
White	3*	4	4	3
Black	-14	-21	-21	-19

Numbers indicate median difference from national percentages. Negative numbers indicate performance below that of the nation; positive numbers indicate performance above national levels.

Color

Blacks appeared to have difficulty with computation, their performance being generally below that of the nation as a whole. The differences from the national levels for Whites and Blacks for the various age levels are shown in Table 32.

As the table indicates, White performance was above that of the nation and was virtually constant at all age levels. The difference in performance between Whites and Blacks was smallest at age 9 and increased for 13 and 17-year-olds, with no appreciable change in relative performance between ages 13 and 17.

At age 9, Blacks showed the greatest difference from national performance on the simple translation problems and on addition and multiplication with simple regrouping. Performance on these exercises was as much as 20 to 25 percentage points below the national level. The divergence was not so marked on exercises that almost all 9-year-olds could do—for example, the properties of zero exercise—or exercises that almost no 9-year-olds could do—for example, long-division problems. Thus, at age 9, Black children appear to be having difficulty in mastering skills which they should be acquiring at this age level.

Blacks at the three older age levels were closest to the national levels of performance on the exercises where results were generally high, such as the properties of zero and the addition exercises. Some of the greatest differences in performance between Blacks and the nation occurred on the complex word problems and on problems involving the renaming and addition of fractions. The complex word problems were read on tape. However, in most of these problems, the computations required (such as long division) were not easy, and the difficulty of the computation coupled with the necessity for comprehending the verbal element involved may well have accounted for low performance on these items.

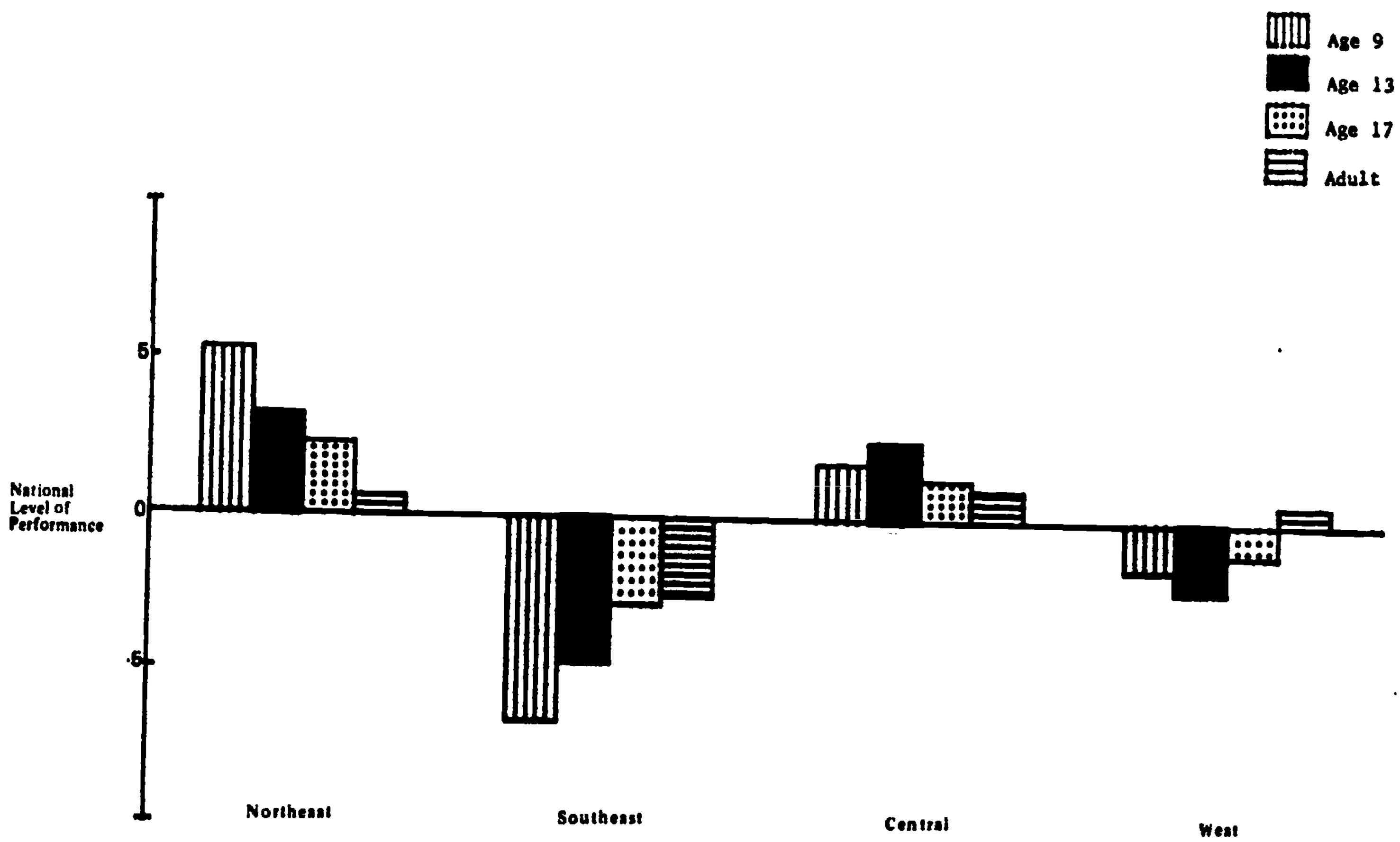
Blacks at all age levels had higher percentages in the "other unacceptable" category, the category for unacceptable answers which followed no discernable error patterns. For the most part, percentages of responses in the "I don't know" and "no response" categories were similar for both Blacks and Whites.

Region

National Assessment reports results for four regions of the country: the Northeast, Southeast, Central and Western regions. The Northeastern region performed above national levels at all ages although this tendency decreased with age. The Southeast was approximately 6 percentage points below the nation in overall performance at age 9; however, performance in relation to the nation steadily improved from ages 13 to adult.

At the three school-age levels, individuals in the Central region tended to attain percentages a point or two above that of the nation while those in the West tended to be slightly below the nation. Adults in both the Central and West regions performed very slightly above national levels. Figure 3-2 displays regional performance relative to that of the nation at each age level. The graph shows the median differences of group performance compared to the national performance.

FIGURE 3-2. Median Differences from National Performance Levels by Region



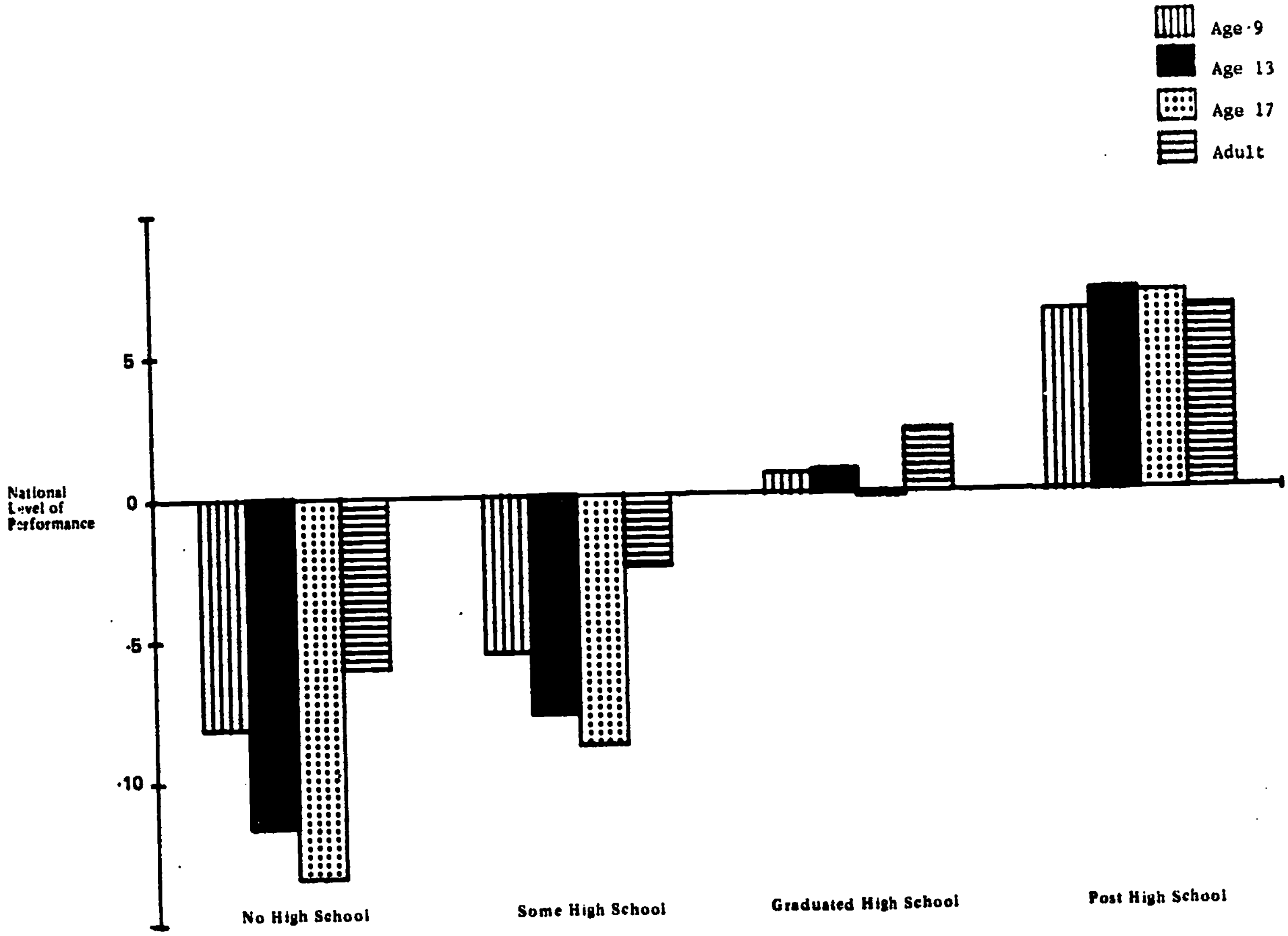
Parental Education

Level of parental education had a considerable influence upon performance on the assessment exercises. Percentages of success increased with higher levels of parental education. Those whose parents had no high school education were from 8 to 13 percentage points below the nation as a whole while those with at least one parent having some post high school education were 6 to 7 percentage points above the national level.

Performance across age levels was also affected by level of parental education. For the no high school and some high school groups at the three school-age levels, the 9-year-olds were closest to the national level of performance, and 13 and 17-year-old performance dropped progressively farther away. The difference across 9, 13 and 17-year-old performance for the groups with parents who graduated from high school or had some post high school education was relatively minor. Differences of group performance from national performance are shown in Figure 3-3. The graph depicts the median difference from national performance for each group at each parental education age level.

Parental education level had less effect on the adult results. Adults whose parents had no or some high school education were not as far below the national levels as their school-age counterparts. Adults with at least one parent who had graduated from high school performed about 1 percentage point above respondents still in school; adults with at least one parent having some post high school education had about the same advantage as respondents with a similar background at the younger ages. In comparing the adult results

FIGURE 3-3. Median Differences from National Performance Levels by Parental Education Level



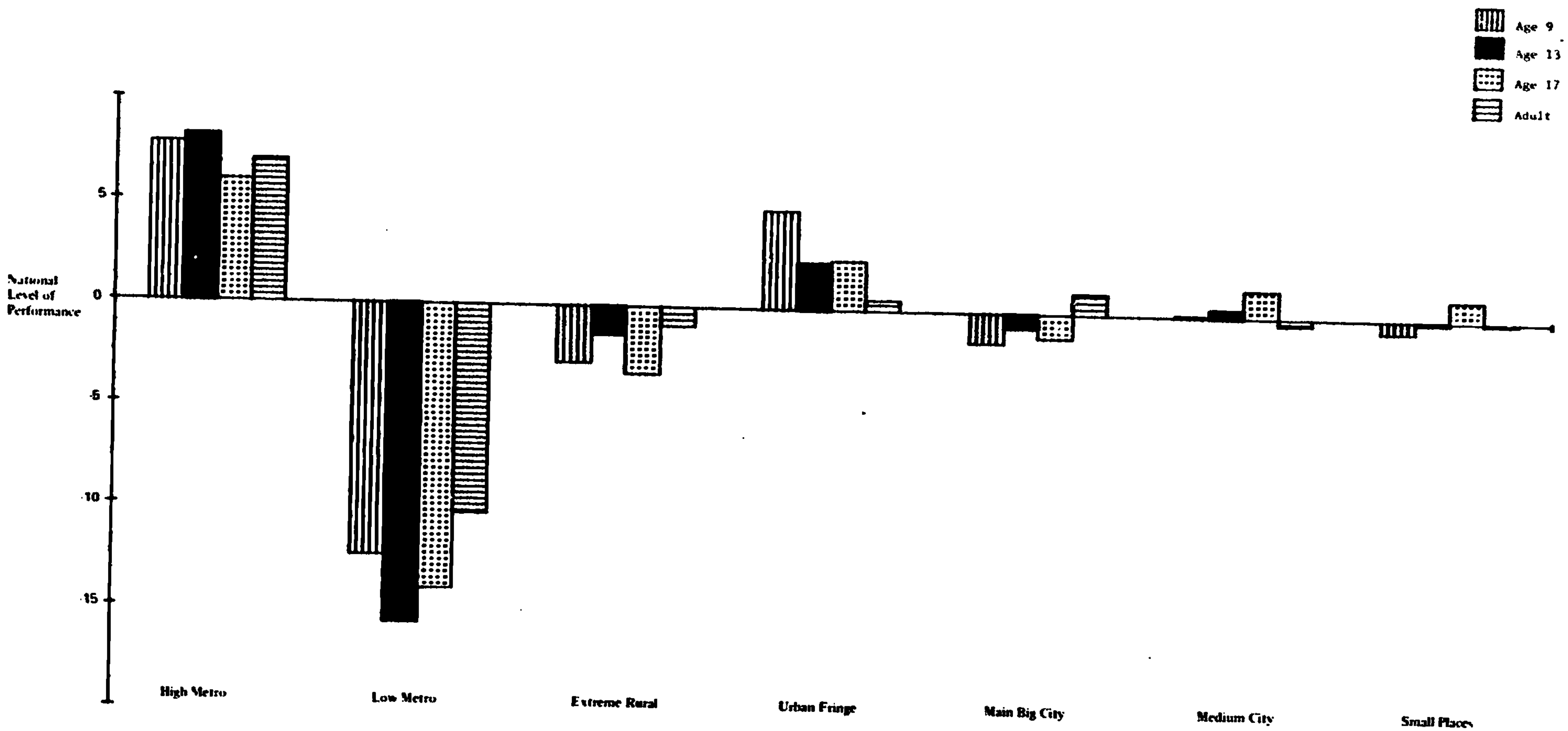
to those of the school-age respondents, it must be remembered that the adults answered only about half as many exercises, and some of the more difficult exercises were not administered to adults. The adult exercises were intended to measure ability in skills which adults use in their daily life, and the more "academic" skills, such as computation with integers and with fractions, were omitted from the adult assessment.

Size and Type of Community (STOC)

NAEP identifies seven community types, basing the groupings both on the size of the community and on type of employment of the majority of people in the community. The seven community types are high metropolitan, low metropolitan, extreme rural, urban fringe, main big city, medium city and small places. Median differences from national performance for each community type at the four age levels are displayed in Figure 3-4.

Results for two types of communities—high metropolitan and low metropolitan—differed appreciably from national percentages. High metro areas are in or near large cities, and most of the adults in the community are in managerial or professional positions. Low metro areas are also in or near a large city, but a high proportion of the adults are on welfare or are not regularly employed. The high metro performed consistently above the nation on almost all exercises at every age

FIGURE 3-4. Median Differences from National Performance Levels by Size and Type of Community (STOC)



level with overall results approximately 6 to 8 percentage points above national levels. Overall results for the low metro group were 10 to 16 percentage points below the national levels, showing the greatest difference from the nation at age 13.

The high metro group performed consistently well on almost all the exercises at every age level. Overall results for the low metro group were below those for the nation on most exercises and were furthest below at age 13.

High metro 9-year-olds showed greatest advantage relative to the nation on the simple word problems, while age 13, 17 and adult high metro respondents had the greatest advantage on those word problems in which long division was required.

Low metro individuals generally showed the least difference from the nation on the simple computation problems on which national percentages were high. Although patterns were not consistent on every problem, 17-year-olds and adults were usually closer to the nation on simple computation exercises than the younger age levels. In contrast to their overall performance, 13-year-olds held a slight advantage over the older groups (when compared to national performance) on several addition problems.

Two other types of communities, the extreme rural and the urban fringe, showed differences from the nation although not as substantial as for the above two community types. At ages 9, 13 and 17, the extreme rural group's performance was 1 to 3 percentage points below that of the nation and the urban fringe performance was 2 to 5 percentage points above. Results for adults in these two groups were quite close to national results.

Performances of main big city, medium city and small places respondents were very similar to that of the nation for all ages.

Percentages of success on specific exercises for groups other than high and low metro did not follow any meaningful pattern of divergence from national percentages.

Overall Group Results

Results for individuals living in low metro communities, for individuals whose parents had some or no high school education, for Blacks and for those living in the Southeast showed the greatest difference from national levels of performance. Similar disparities have been found, to a greater or lesser degree, in other learning areas assessed by National Assessment.

The differences from national levels of performance are not necessarily diminished by additional years of schooling. Low metro individuals, Blacks and those whose parents had some or no high school education showed greater differences from the national level at ages 13 and 17 than at age 9. Results for the Southeast region of the country did come closer to national levels as the age of the respondents increased.

The lowest percentage of success for all exercises given was 58% at age 17 and 53% for adults—on the complex word problem requiring long division to translate snowfall into rainfall. In computational mathematics at least half of the population at or near the completion of schooling could accomplish the basic computational skills measured by this assessment.

It is expected that future assessments in mathematics will provide even more precise indicators of areas of strength and weakness in mathematics knowledge and skills. Information on current levels of performance and also on changes in these levels over time can provide an additional source of empirical information to be used as a tool in educational decision making.

Additional Mathematics Assessment Results

This report has presented results for mathematical computation. Results for other selected areas of the mathematics assessment will be found in topical reports similar to this one. Complete documentation of exercises and data on exercise results will be available in the mathematics technical reports.

ADDITIONAL MATHEMATICS PUBLICATIONS FROM NATIONAL ASSESSMENT

The Mathematics Assessment: A Statistical Report will be a two-volume presentation of all released information on the mathematics assessment. Included will be complete exercise texts, exercise documentation, scoring guides, data on national performance levels and data on performance of the National Assessment reporting groups: age, region, sex, color, level of parental education and size and type of community.

Consumer Mathematics: Selected Results from the First National Assessment of Mathematics will describe performance of 13-year-olds, 17-year-olds and adults in solving problems in a consumer-related setting. Skills in using averages, percents, proportions, measurement and geometric concepts and in reading and interpreting graphs will be discussed.

The Mathematics Assessment: An Overview will summarize results from various content areas of the mathematics assessment. Major content areas include number and numeration concepts, properties of numbers and operations, arithmetic computation, estimation and measurement, algebra, probability and statistics and geometry.

Mathematics Objectives details the objectives of mathematics education at the four assessment age levels as compiled by mathematicians and mathematics educators. The exercises contained in the mathematics assessment are designed to measure achievement of these objectives.

U.S. GOVERNMENT PRINTING OFFICE 1974 678 985/82 REG. 8

Official National Assessment Reports
(Continued from Inside Front Cover)

Literature

02-L-01	Understanding Imaginative Language, March 1973	$2.65
02-L-02	Responding to Literature, April 1973	2.85
02-L-03	Recognizing Literary Works and Characters, April 1973	2.60
02-L-04	A Survey of Reading Habits, May 1973	2.85
02-L-20	Released Exercises, April 1973	3.45
02-L-00	Summary Data, June 1973	1.30

1971–72 Assessment

03/04-GIY	General Information Yearbook—A description of National Assessment's methodology, with special attention given to Music, Social Studies, Science and Mathematics, December 1974	1.20

Social Studies

03-SS-01	Political Knowledge and Attitudes, December 1973	1.05
03-SS-02	Contemporary Social Issues, July 1974	.65
03-SS-00	The First Social Studies Assessment: An Overview, June 1974	.90

Music

03-MU-01	The First National Assessment of Musical Performance, February 1974	.55
03-MU-02	A Perspective on the First Music Assessment, April 1974	.45
03-MU-03	An Assessment of Attitudes Toward Music, September 1974	.85
03-MU-00	The First Music Assessment: An Overview, August 1974	.60

1972–73 Assessment

Mathematics

04-MA-01	Math Fundamentals: Selected Results from the First National Assessment of Mathematics, January 1975	1.10

NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS
A Project of the Education Commission of the States
Suite 700
1860 Lincoln Street
Denver, Colorado 80203

Education Commission of the States

NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

MATHEMATICS
REPORT 04-MA-01
1972-73 ASSESSMENT

A Project of The Education Commission of the States