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ABSTRACT

A 20-task learning hierarchy for subtraction of fractions was deductively derived using Gagne's task analysis. To test this analysis empirically, composite items were written for each level and administered to students in grades 3-6. Test results were analyzed by the Walbesser Technique and Pattern Analysis; the acceptance levels developed by Phillips and Kane were used as criteria. These analyses yielded ratios for consistency, adequacy, and completeness which were below the acceptance level. The hierarchy was then revised to maximize these ratios, and an index of agreement of .85 was obtained between the expected pattern of responses and the observed level. The investigators note that: (1) results may have been affected by the fact that very few students were able to answer correctly items from the upper levels of the hierarchy; and (2) the study should be replicated using students with a broader range of abilities and ages. (SD)

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DEVELOPMENT OF A LEARNING HIERARCHY FOR THE COMPUTATIONAL SKILLS OF RATIONAL NUMBER SUBTRACTION

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Many researchers have long recognized that sequence is a critical variable in learning (Ausubel, 1963; Bruner, 1964; Gagne, 1965; Glaser, 1964; and Suppes, 1966). The learner begins with simple tasks and progresses to increasingly complex tasks. Thus, the possibility of optimal instructional sequences is suggested. Gagne (1967) and Wang (1973) suggested that it would be pedagogically sound to construct a complete curriculum based on validated learning hierarchies. However, both Gagne (1968) and Pyatte (1969) have pointed out that the determination of this optimal or hierarchical sequence of subtasks from simplest to most complex is not a simple undertaking.

A learning hierarchy constructed by task analysis alone may be incomplete. Empirical evidence must be obtained to verify or refute the hypothesized ordering of the subordinate subtasks in a deductively analyzed

hierarchy (Uprichard, 1970; Cox and Graham, 1966). The purpose of this study was to develop and analyze a learning hierarchy for the computational skills of rational number subtraction.

PROCEDURE

Using Gagne's task analysis, a learning hierarchy for the computational skills of rational number subtraction was constructed. This procedure yielded a hierarchy of 20 subtasks. Based upon the hypothesized ordering of the subordinate tasks, a test was constructed to assess mastery at each level in the hierarchy. In order to minimize chance or careless error a test format similar to the "H-technique" (Stouffer, Borgatta, Hays, and Henry, 1952) was used. The test consisted of composite items for each level. Each composite item consisted of three items testing the same subordinate tasks. Pass was defined as correct responses to two of the three items at each level. The test was administered to 207 elementary school children from grades 3 through 6 in order to obtain a wide range of ability levels. Subjects were selected from schools integrated by busing. Thus, approximately one-third were black; the remaining two-thirds were mostly white with a small number of Spanish-Americans.

Patterns of responses for each transfer in the hierarchy were analyzed using both the Walbesser Technique (1968) and Pattern Analysis (Rimoldi and Grib, 1960). Walbesser has developed a procedure for validating a hierarchically arranged sequence according to numerical criterion using the contingency table of pass-fail responses. The possible outcomes of the subtasks are represented using a binary scale. Ordered pairs designate performance on the subtasks: "1" representing acquisition of a subtask, "0" representing nonacquisition. The ordered pair (0,1) implies that the student failed the more complex task, but passed the simplest task. The ordered pairs (0,0), (1,1) and (1,0) are defined similarly. After scoring, a consistency ratio, an adequacy ratio, and a completeness ratio were calculated for each hypothesized relationship within the hierarchy. The consistency ratio is a measure of how consistent the data are with the hypothesized dependency. The adequacy ratio is a measure of the identified subordinate tasks. The completeness ratio is a measure of the effectiveness of instruction. The level of acceptability used for each of these ratios was that determined by Phillips and Kane (1973) instead of the levels proposed by Walbesser since no instructional sequences were

involved. The Phillips and Kane levels of acceptability are: (1) consistency ratio .85; (2) adequacy ratio .70; and (3) completeness ratio .50.

The pattern analysis technique was used to analyze the responses for the complete hierarchy on a subject by subject basis. The index of agreement given by the pattern analysis indicates the amount of agreement or correlation between two patterns. In this case, the index of agreement indicates the agreement between the observed and expected patterns. If the tasks were truly hierarchical, where each subtask was a necessary prerequisite to the next, once a learner failed a given level he would be expected to fail all subsequent levels. Thus, the expected pattern was defined as one where no correct responses followed an incorrect response.

RESULTS

The initial hypothesized hierarchy developed by task analysis is given in Appendix A. A computer program based on the Walbesser Technique was used to give the pass-fail response patterns between all relationships. That is, item 1 was paired with all 20 items; item 2 with all items; etc.; until all possible pairs of items were considered.

In order to empirically validate this hypothesized hierarchical sequence, the consistency, adequacy, and completeness ratios for each relationship within the hierarchy were examined. No ordering of the 20 tasks yielded acceptable levels an all three ratios among all relationships in the hierarchy. The ordering which yielded the best fit to the data is shown in Table 1.

TABLE 1

Ratios for the Empirical Ordering (N=207)

Level	Consistency	Adequacy	Completeness
1-3	.97	.84	.98
3-5	.92	.71	.83
5-7	.89	.69	.59
7-2	.95	.57	.77
2-8	.92	.44	.73
8-4	.92	.52	.58
4-9	.92	.61	.62
9-6	.70	.82	.51
6-11	.90	.59	.43
11-10	.75	.68	.30
10-13	.67	.54	.23
13-17	.88	.45	.14
17-14	.28	.77	.13
14-15	.77	.70	.20
15-12	.47	.86	.25
12-19	1.00	.22	.13
19-18	.46	.65	.06
18-20	.62	.33	.05
20-16	.21	.77	.07

The test based upon the hypothesized ordering of the subordinate tasks yielded an index of agreement of .81. After final revision, the empirical sequence yielded an index of agreement of .85 which indicates a higher agreement between the expected and observed response patterns. No statistical test of significance for the index of agreement has been developed.

The internal consistency of the test based on the initial hierarchy was determined using the Kuder-Richardson Formula 20 (Nunnally, 1967). The value of this coefficient was .89. The pattern of responses of the empirical sequence was analyzed to determine if the ordering exhibited a hierarchical structure based on item difficulty. These results are given in Table 2.

The percentage of subjects passing each of the 20 items were arranged in descending order from .99 representing test item number 1 to .08 representing item number 20. Inspection of these test scores indicated that items 9 and 12 were out of order.

TABLE 2

Item Difficulty

Item	Difficulty
1	.99
3	.95
5	.91
7	.86
2	.75
8	.67
4	.60
9	.48
6	.59
11	.40
10	.43
13	.50
17	.33
14	.30
15	.28
12	.31
19	.16
18	.15
20	.11
16	.08

CONCLUSIONS AND DISCUSSION

In interpreting the results of this study due consideration must be given to two sources of artifact. (1) The response patterns of students are affected by prior educational experiences. That is, in testing, if the student knows many of the items he will answer these correctly regardless of the order in which they appear. (2) It appears from the results of the item analysis that very few Ss responded correctly to items from the upper levels of the hierarchy. When dealing with contigency data, it is essential that a sufficient number of Ss respond correctly at all levels of the hierarchy. Thus, this study should be replicated with a larger sample over a wider range of ability and achievement levels.

Levels in the hierarchy were rearranged based on Ss response patterns on the test items. Two major revisions involved items in which: (1) The answers required rewriting in simplest form. These items were found to be more difficult than those problems with mixed numerals which required finding a common denominator before the subtraction could be perfomred. However, this could simply be a problem with reading and following directions. (2) It was found that writing equivalent fractions and expressing whole numbers with mixed numerals were

simpler tasks than subtraction items which did not involve the processes. That is, writing 2/3 as 4/6 or 7 = 6 4/4 were simpler tasks than $3/4 - 1/4 = \square$. Also writing 7 as 6 4/4 was very simple by itself but became a much more complex task when encountered in a situation as $7 - 3\ 1/4 = \square$.

Although the hierarchy was not validated according to criterion established, the index of agreement for the empirical sequence was greater than for the hypothesized sequence. Thus, one might conclude that for an entire sample of learners the empirical sequence would be the more optimal.

Validated learning hierarchies can serve as useful models for instructional design, basis for development of diagnostic instruments, and a means for prescribing individualized instruction. Thus, further research should attempt to perfect techniques of hierarchy validation and determine the effects of instructional sequences based on learning hierarchies upon acquisition and long term retention.

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Appendix A

Identification of Hierarchy Cells

Level	Identification	Example
1	Subtracting with fractions having like denominators where the difference is expressed in simplest form.	$\frac{5}{7} - \frac{2}{7} = \square$
2	Subtracting with fractions having like denominators where the difference is equal to zero.	$\frac{1}{4} - \frac{1}{4} = \square$
3	Writing equivalent fractions by dividing the numerator and the denominator by the greatest common factor.	$\frac{3}{6} = \frac{\square}{2}$
4	Subtracting with fractions having like denominators, where the difference must be expressed in simplest form.	$\frac{3}{4} - \frac{1}{4} = \square$
5	Subtracting a whole number from a rational number named by a mixed numeral.	$5\frac{1}{2} - 2 = \square$
6	Subtracting with mixed numerals having like denominators where the difference may or may not require writing in simplest form.	$5\frac{4}{5} - 3\frac{1}{5} = \square$
7	Subtracting with mixed numerals where the difference is a whole number.	$9\frac{2}{3} - 3\frac{2}{3} = \square$
8	Writing equivalent fractions by multiplying the numerator and denominator by the same number.	$\frac{2}{3} = \frac{\square}{15}$
9	Subtracting with fractions where the lowest common denominator is the larger of the two given denominators and the difference may or may not require rewriting in simplest form.	$\frac{3}{10} - \frac{1}{5} = \square$
10	Subtracting with mixed numerals where the lowest common denominator is the larger of the given denominators.	$8\frac{3}{4} - 3\frac{1}{8} = \square$
11	Changing the name of a whole number to a mixed numeral.	$7 = 6\frac{\square}{5}$
12	Subtracting a number named by a mixed numeral from a whole number.	$5 - 2\frac{1}{2} = \square$

Level	Identification	Example
13	Subtracting with fractions where the lowest common denominator is the product of the given denominators and the difference may or may not require rewriting in simplest form.	$\frac{1}{2} - \frac{1}{5} = \square$
14	Subtracting with mixed numerals where the lowest common denominator is the product of the two given denominators.	$5\frac{3}{8} - 2\frac{1}{7} = \square$
15	Subtracting with mixed numerals where the lowest common denominator is different from given denominators and the difference is expressed in simplest form.	$12\frac{1}{4} - 6\frac{1}{6} = \square$
16	Subtracting with mixed numerals where the lowest common denominator is different from the given denominator and the difference must be expressed in simplest form.	$12\frac{7}{9} - 2\frac{1}{6} = \square$
17	Changing the name of a mixed numeral in standard form to one which has an improper fractional part (equivalent mixed numerals).	$5\frac{1}{3} = 4\frac{\square}{3}$
18	Subtracting with mixed numerals or with a fraction from a mixed numeral where the lowest common denominator is a multiple of the two denominators and renaming an equivalent mixed numeral is required.	$4\frac{2}{4} - \frac{5}{8} = \square$
19	Subtracting with mixed numerals or with a fraction from a mixed numeral where the lowest common denominator is the product of the two denominators and renaming an equivalent mixed numeral is required.	$11\frac{1}{3} - 7\frac{1}{2} = \square$
20	Subtracting with mixed numerals or with a fraction from a mixed numeral where the lowest common denominator is neither a multiple nor the product of the given denominators, and renaming an equivalent mixed numeral is required.	$13\frac{1}{6} - 2\frac{7}{8} = \square$