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ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This unit presents to the child the "core" of the program in that most of the topics preceding this one are background material. The operation of addition is presented as an operation on the number line. The content covers inequality (greater than), commutativity, even and odd numbers and introduces the operation of subtraction. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

MINNE Math And Science Teaching project

UNIT VIII
NUMBER LINE

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1965

MATHEMATICS

FOR THE

ELEMENTARY SCHOOL

UNIT VIII

Number Line

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We are deeply indebted to the many teachers who
used earlier versions of this material
and provided suggestions for
this revision

CONTENTS

★ Starring indicates content which is particularly important to the sequential development or evaluation of the program. We ask that all participating teachers try this starred material. It is expected that much of the remaining material will also be used; how much will depend on individual class needs and time available.

Purpose

This unit presents to the child the "core" of the program in that most of the topics preceding this one are background material. The operation of addition is presented as an operation on the number line.

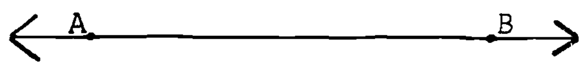
No matter what the operation may be, nor what kind of numbers the operation is performed on, the operation is, essentially, developed in the same manner. For example, the traditional manner of interpreting addition as the union of disjoint sets is limited to counting numbers; this operation has to be relearned or "unlearned" when the child encounters other real numbers such as fractional or negative numbers.

It is recognized that progress will be slow in this unit and mastery should not be expected on this level, for the child is herein presented with a device used for operations extending over many units of the program to follow. The teacher's role in this unit is of paramount importance and the artistry of teaching is challenged to the utmost.

Teacher Background on the Number Line

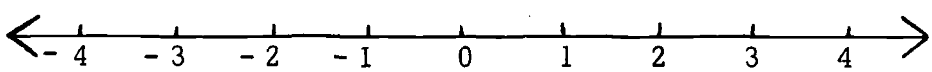
A line segment is a set consisting of two points and all points between them. ("Betweeness" is a relation which mathematicians do not attempt to define. The child will have an intuitive feeling for the concept of betweeness.)

Symbolically, we write $\overline{AB}$ for the line segment AB (as illustrated above). If the line segment AB is extended infinitely in both directions, then the figure becomes a line. (When we speak of lines, of course, we have in mind the familiar straight lines. There is really nothing to be gained by putting in the extra word "straight".)



Symbolically, we write $\overleftrightarrow{AB}$ for the line AB. Observe that we use the arrows to denote the fact that the line extends infinitely to the right of B and infinitely to the left of A. It should be pointed out that the line segment ($\overline{AB}$) is a part of the line ($\overleftrightarrow{AB}$).

If points are marked on a line so that a basic unit of measure partitions the line into equal segments and we correspond each of these points with an integer, we have a number line, which may look like this:



Observe that there is a one-to-one correspondence between the set of integers and a subset of the set of points on the line (remember there are an infinite number of points on a line).

When the number line is introduced in the first grade, only the positive integers and zero will be named. However, if a question is raised as to the numbers to the left of zero on the line, explain to the class that a line does extend infinitely in both directions and there are numbers less than zero. A detailed explanation of the set of negative numbers is not necessary at this time and may only serve to confuse the class.

We are simply constructing a device in which there will be a one-to-one correspondence between the set of non-negative integers and a subset of points on a line. This number line will serve as a basis for the development of the understanding of the operations of addition and, in later grades, for multiplication. Further, it serves as a device for developing competence in these operations. It also serves as a visualization of the ordering of counting numbers (more than, less than).

It is highly recommended that teachers using these materials secure the devices made available to them by MINNEMAST. The classroom number line should be placed in a conspicuous place and each student should have, for his own use, a student number line. When the addition operation is taught, then the addition slide rules ("double" number lines) should be secured for each child.

Suggested Review and Activities on Greater Than

Before teaching this unit, see that the children can identify all numerals from zero through ten. The following pre-activities are designed to review recognition of the numerals.

1. Play the following game, called "Numerals Ready", to review the correct sequence of the numerals 0 through 10.

   Prepare large cards with a numeral from zero through ten on each card.

   Eleven children are chosen to stand at the front of the room, each holding a card with the numeral facing the class. They should be lined up in order, with zero to the left of the class so that numerals read correctly from left to right.

   Choose a child without a numeral card to be the leader. The leader goes to the front of the room and says,

   "Close your eyes and never peek!
   A missing numeral you will seek."

   At this, the rest of the children bury their heads in their arms while the leader chooses one child holding a numeral to go into the hall (or any place where he can't be seen). The rest of the line moves together to fill up the space. Then the leader calls "Numerals Ready" and the children lift up their heads. As soon as they decide which numeral is missing they raise their hands and the leader calls on someone to name it and to walk up to the number line to show where the missing numeral belongs.

If he is right, he then becomes the leader, and the game begins again. If he is wrong, other children are called upon until the correct answer is given.

After three times, each child holding a numeral chooses someone to take his place.

Variation I

With a group of slow learners, it would be best, in first playing this game, to begin with the numerals 1 through 5. Later add 6 through 10 and finally add the zero.

Variation II

Another way to play the same game is to have the leader use this verse:

> "Close your eyes - now be quick,
> To change the numerals is the trick."

This time, instead of removing a child, the leader changes positions of two of them so that two are out of order. At the signal "Numerals Ready", the leader calls on someone to name the two numerals that are out of order, and to go to the number line and change the children back to their original places.

Variation III

In this variation use:

> "Close your eyes and never peek!
> Missing numerals you will seek."

This time the leader may eliminate 1, 2, or 3 children from the number line.

2. "Number Horseshoes"

Have the children put their numeral cards (zero through 10) on their desks. Provide all with pieces of paper to keep their own scores.

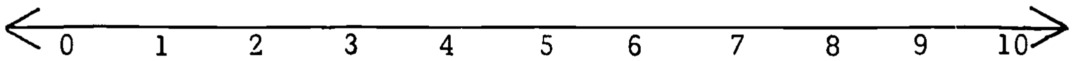
The leader (without being seen) puts any number of counters (Not more than 10) in a bag (or a box). The children place, face down on their desks, the card with the numeral that shows how many counters they think are in the bag. When all children are ready, they turn their cards over. The leader chooses someone to count the counters in the bag while he checks to be sure he counts correctly. The leader then checks to see who has the correct card on his desk. The child who is right makes a "ringer" and gets _2_ points. If no one is right, the ones who are closest get _1_ point. The child with the most points at the end of the game is the winner.

(In this game the same person remains leader for the entire game.)

Variation

Play this as a team game. The leader should be someone on either team. However, if the teams come out even, then alternate leaders - first from one team, then the other. Have a scorekeeper for each team. The scorekeeper does not play. His job is to count the points for his team after each horseshoe toss (each guess as to the number in the bag constitutes a toss). Play 5 times (or ten times if it goes quickly). Then have the scorekeepers add their teams' scores to see which team wins.

3. A number further in the positive direction than another number on the number line is the ★<u>greater</u> one. We frequently chose the positive direction to the right, in which case one number is greater than another if it is to the right of the other.



Ask the children if they know what "greater" means. Have one of them explain what it means. You may clarify the explanation by placing four crayons on the other end of the table. Ask the class which set of crayons has more members. Then indicate that the number of members in that set is greater than the number in the other set. Show that this is correct by first counting the members of the sets and then looking at the number line to see which number is to the right of the other.

Example: 4 is to the right of 2 on this number line,
therefore
4 is greater than 2.
Put it on the board like this: $4 > 2$.

Then explain to the children that when writing that 4 is greater than 2, it is helpful to have a short way of writing it. In mathematics this sign is used: $>$. (The "less than" sign, $<$, will not be introduced in this unit. If the teacher feels that the class is at a point at which it would particularly benefit from the introduction of the "less than" sign, the teacher may introduce it. Much will be accomplished if the children are able to master the use of the "greater than" sign.) When this sign is placed between two numerals, it is easy to see that 4 is greater than 2 because the big end is next to the 4 and the little end is next to the 2, like this: $4 > 2$.

4. Distribute the numeral cards. Cut a card approximately the size of the numeral cards, and print the "greater than" sign $(>)$ on it. Have a child stand in front of the room holding the $\boxed{>}$ card so the class can see it. Ask two other children holding numeral cards to come up and place themselves on either side of the child holding the $\boxed{>}$ sign so that the sentence reads correctly. The arrangement may look like this: $\boxed{4}$ $\boxed{>}$ $\boxed{2}$. Have another child be the checker and check the placement of the cards by demonstrating the sentence on the number line. If it is demonstrated that the mathematical sentence is stated correctly ask each child to give his numeral card to another child and repeat the game.

Variation I

Have each child take a handful of counters from a bag. Then call two children who may count or match their counters in whichever way they can devise. The child holding the number demonstrated as more than the other stand to the right of the $\boxed{>}$ while the other stands to the left of the sign holder. The class may check the sentence by pointing out the numbers used on the number line.

Variation II

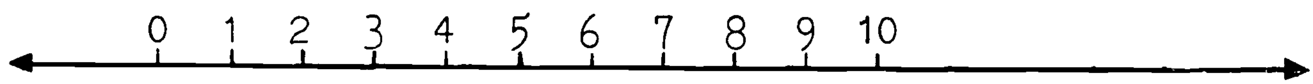
Place the greater than card on the chalk ledge. Let any child volunteer to place his numeral card on the ledge, either to the left or the right of the sign. Have any child who has a numeral card which could be placed at the other side of the $\boxed{>}$ come up and place his card in a correct arrangement. Encourage the children to vary the position of the missing numeral, sometimes to the right and sometimes to the left.

5. Do Worksheets 1 and ★2. Children may insert any numeral in the blank which correctly completes the statement (zero should also be accepted for any answer).

Worksheet 1 Greater Than

1. 4 > ________
2. 9 > ________
3. 1 > ________
4. 5 > ________
5. 10 > ________
6. 8 > ________
7. 2 > ________
8. 3 > ________
9. 6 > ________
10. 7 > ________

1. 3 > ________
2. 8 > ________
3. 10 > ________
4. 4 > ________
5. 5 > ________
6. 9 > ________
7. 7 > ________
8. 6 > ________
9. 8 > ________
10. 1 > ________

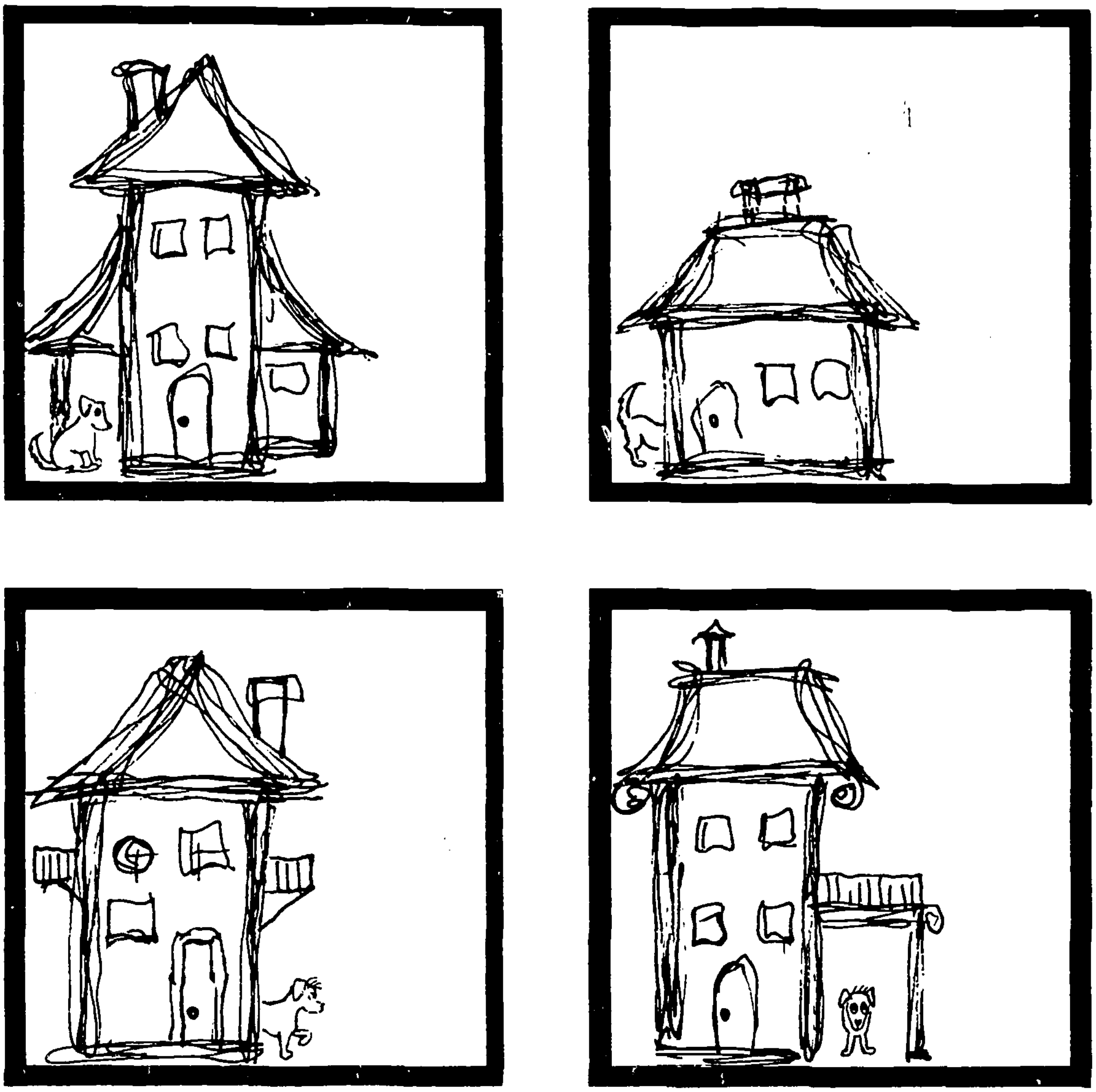


1. 5 > ________
2. 2 > ________
3. ________ > 7
4. 3 > ________
5. 6 > ________
6. ________ > 0
7. 8 > ________
8. 4 > ________
9. ________ > 9
10. ________ > 6

1. 6 > ________
2. ________ > 3
3. ________ > 8
4. ________ > 2
5. 8 > ________
6. 5 > ________
7. ________ > 4
8. 10 > ________
9. ________ > 1
10. 7 > ________

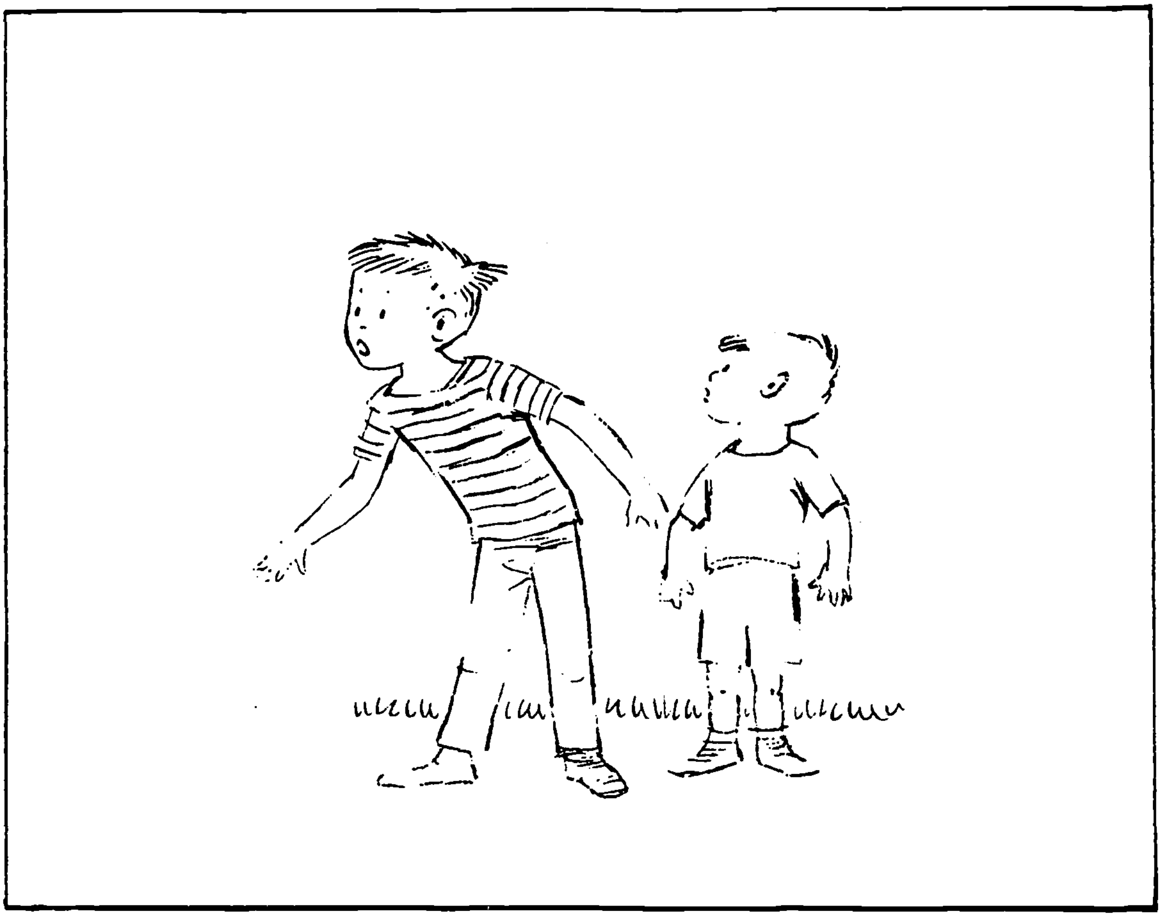
The purpose of "Where is Skip" is to present in the language of the child and in an environmental setting a background for the introduction of the number line. The number line serves as a geometric interpretation for the set of counting numbers, and the operation of addition with these numbers.

Where Is Skip?



WHERE IS SKIP?

"Here Skip. Here, Skip," called Tommy. "Where are you, Skip?"



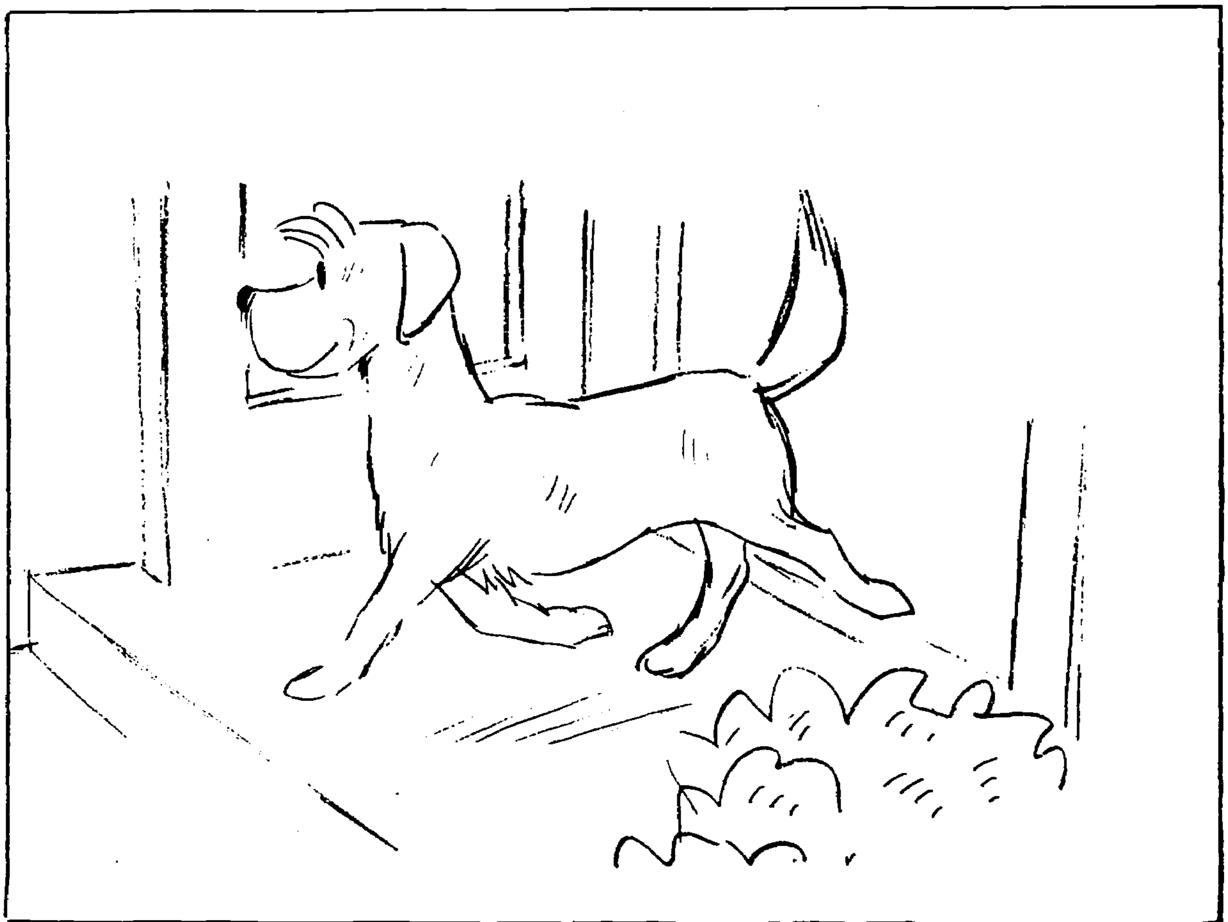
"Is Skip lost, Tommy? Is he lost?" asked Don.

"I guess he is," answered Tommy. "I guess Skip is lost." Tommy and Don went home to tell Mother.

But Skip was not lost. He was out walking. He had been to see many children that morning.

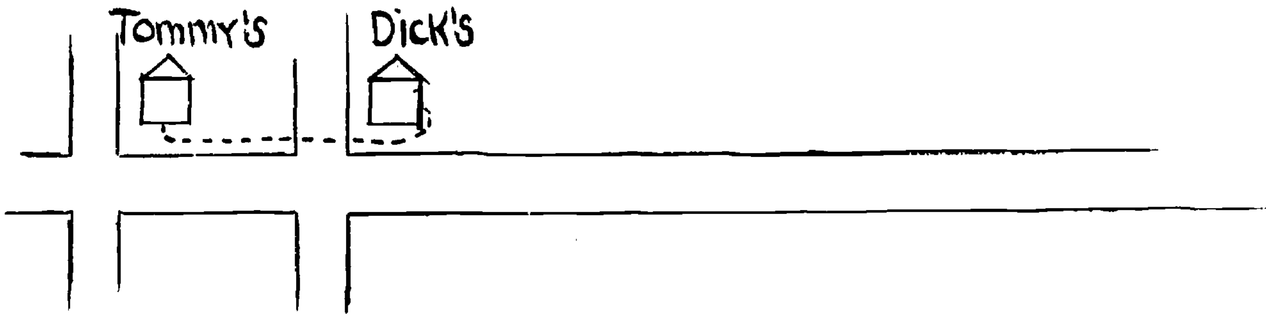
The children were happy. But the mothers and fathers were <u>not</u> happy.

Let's follow Skip and find out what happened.



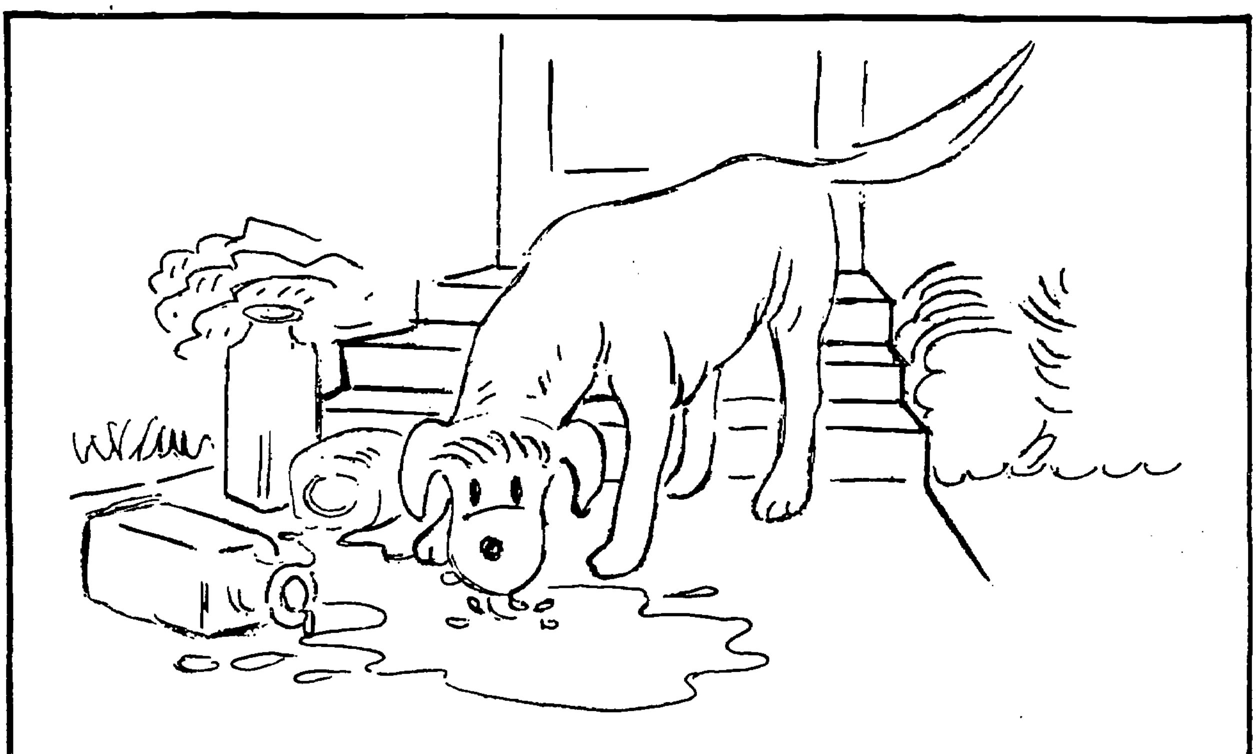
Very early this morning, when Tommy and Don were sleeping, Skip went out of the house.

He went down the street one block to Dick's house. Here is his path.



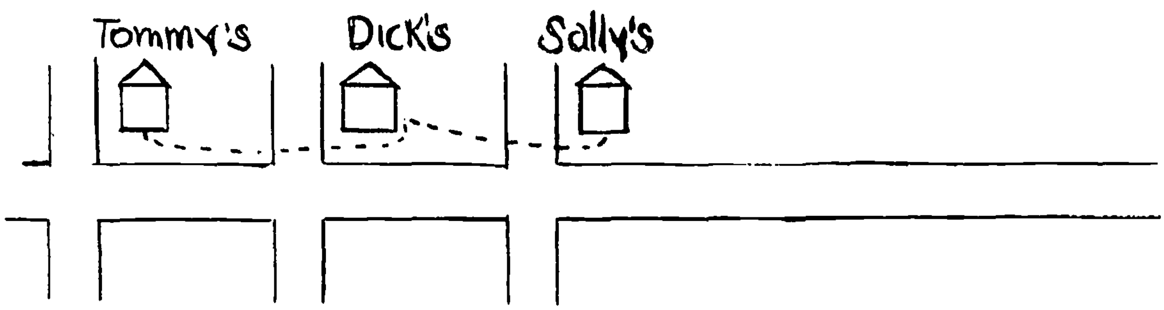
The milkman had been to Dick's house and there were three quarts of milk by the door.

Skip was hungry for his breakfast. Can you guess what happened?



That's why Dick's mother was not happy about Skip's visit.

After his breakfast of milk, Skip went another block to see Sally.



Sally was playing all by herself. She was happy to see Skip.

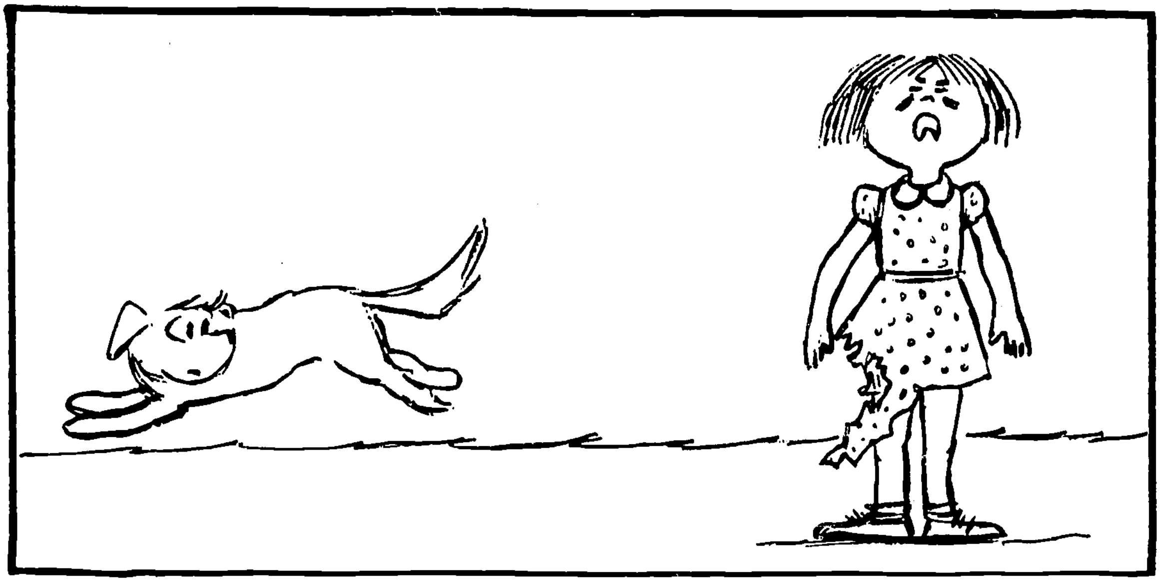
They ran and jumped and played.

They had fun.

But Skip got excited.

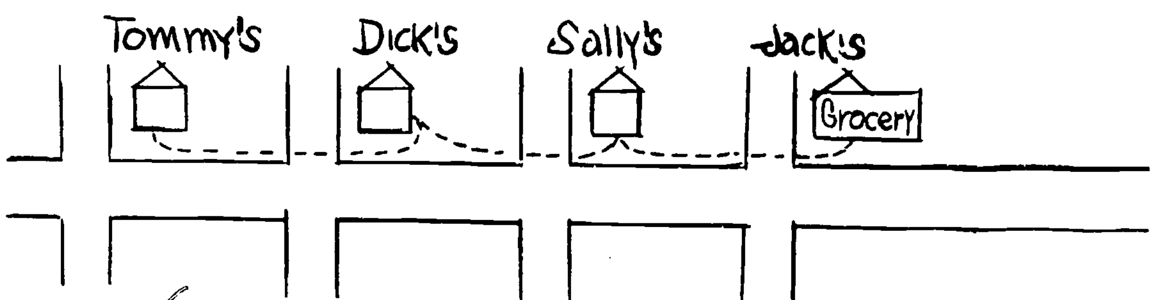
He took Sally's dress in his teeth just as Sally ran away.

Do you know what happened?



That's why Sally's mother was not happy about Skip's visit.

Sally's mother made her go into the house. So Skip walked on for another block.



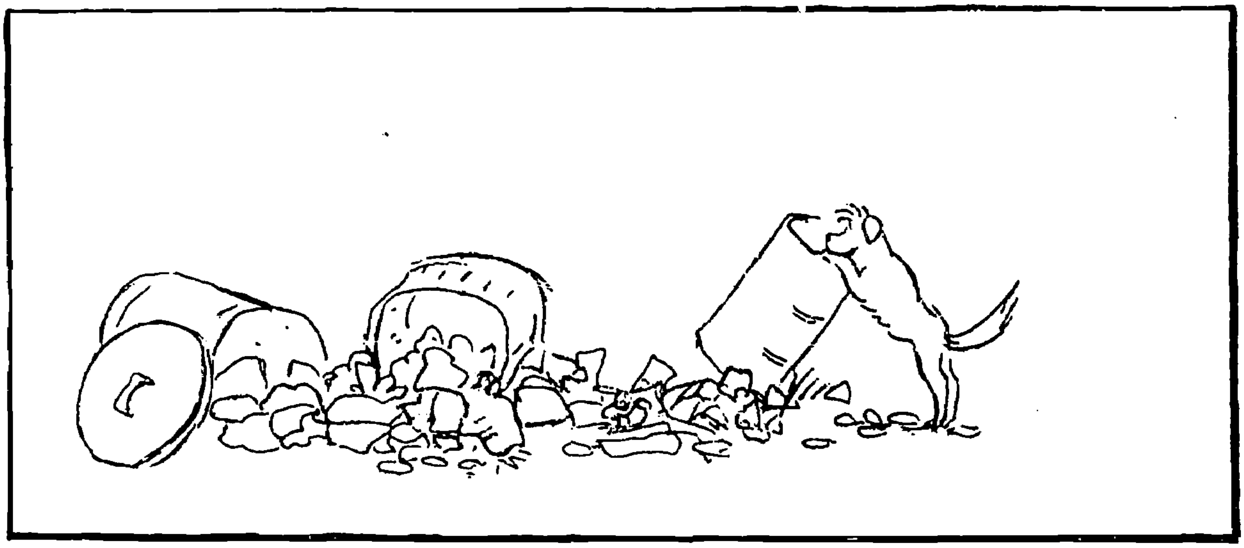
Here was the house of Skip's best friend. Jack lived here. His father had a grocery store. Jack lived up above the store with his mother and father.

Sometimes Jack gave Skip some meat.

Sometimes he gave him cookies or candy. Skip liked to visit Jack.

WHERE IS SKIP?

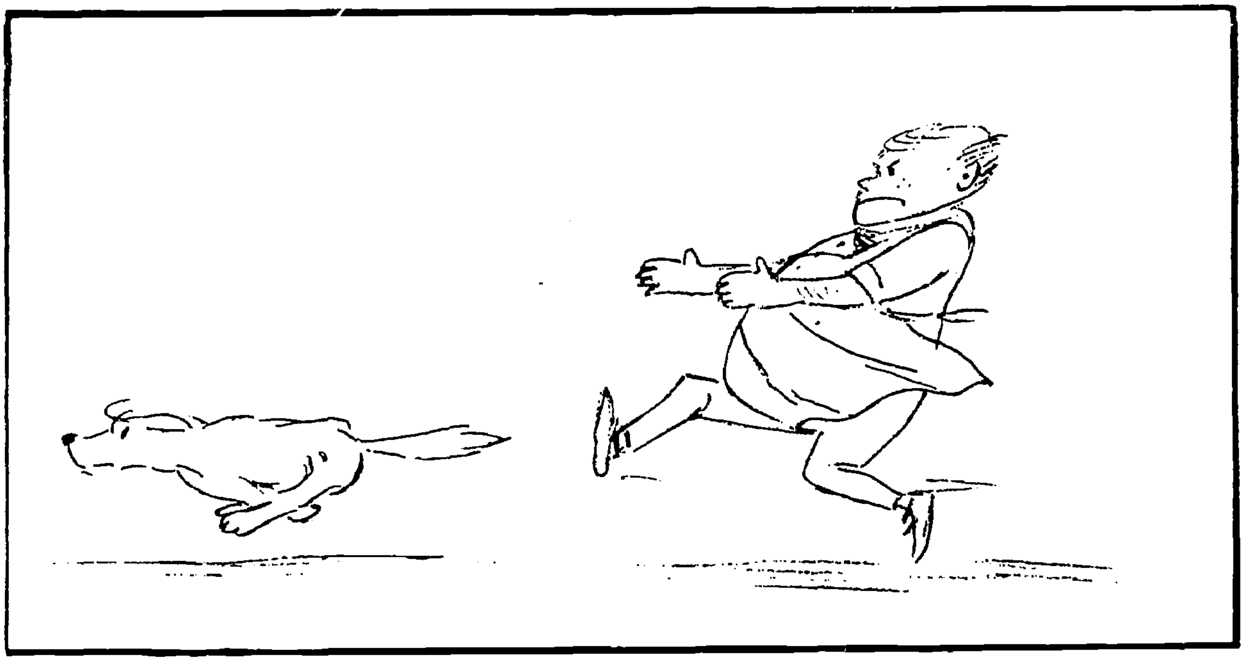
Today Skip looked all around. Jack was not there. But look what Skip found out in the back yard.



What a mess Skip made.

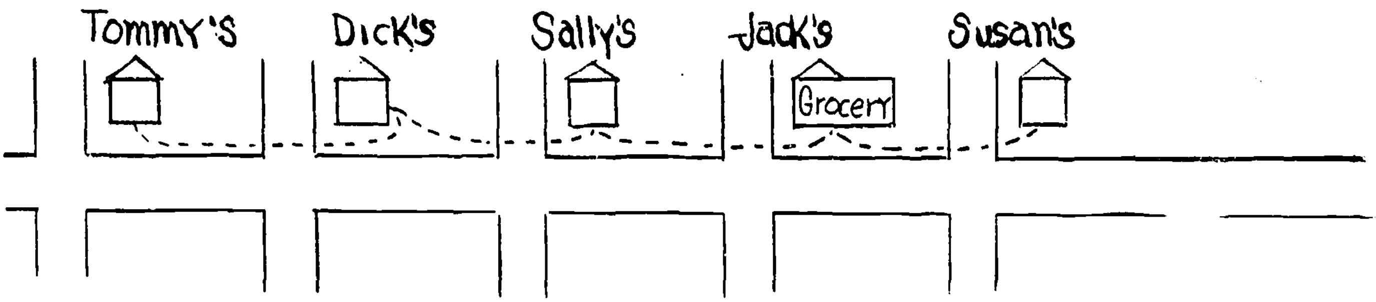
When Jack's father came out he saw Skip.

Jack's father was not happy about Skip's visit.



WHERE IS SKIP?

Then Skip went to see Susan. She lived in the next block.

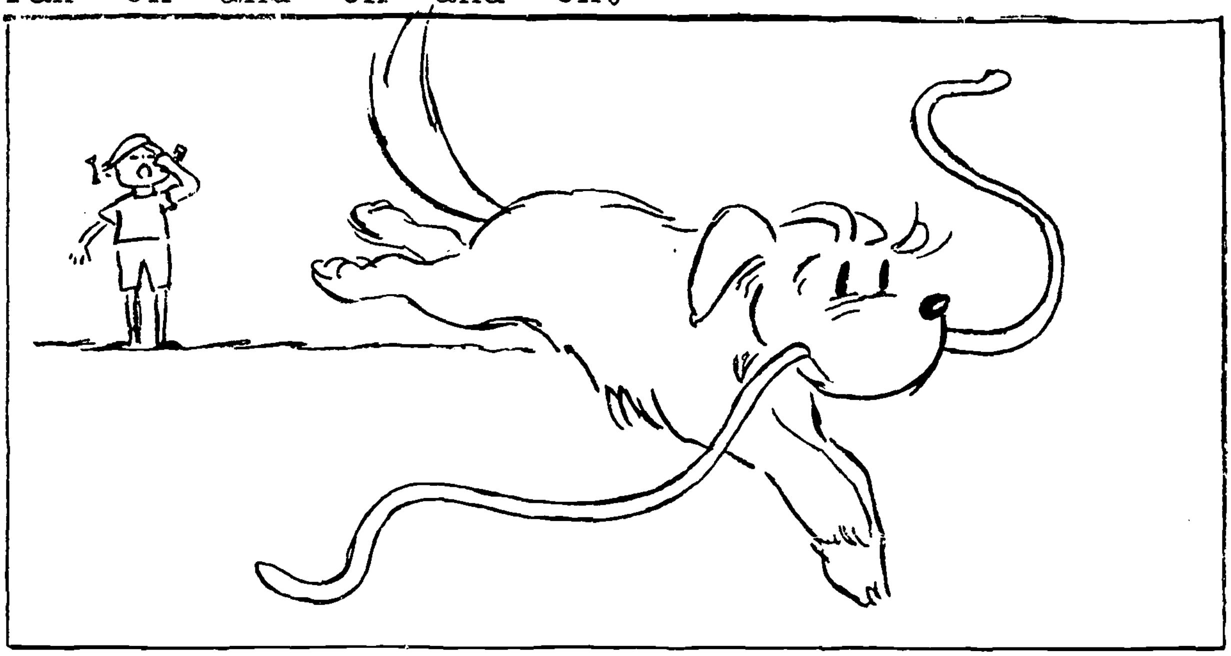


Susan was jumping rope in her yard when Skip came to visit.

"Hello, Skip," said Susan.

"Bow wow," said Skip. Then he took Susan's rope and ran away.

"Skip, you come back here with my rope. Skip -- here Skip," called Susan. But Skip ran on and on and on.



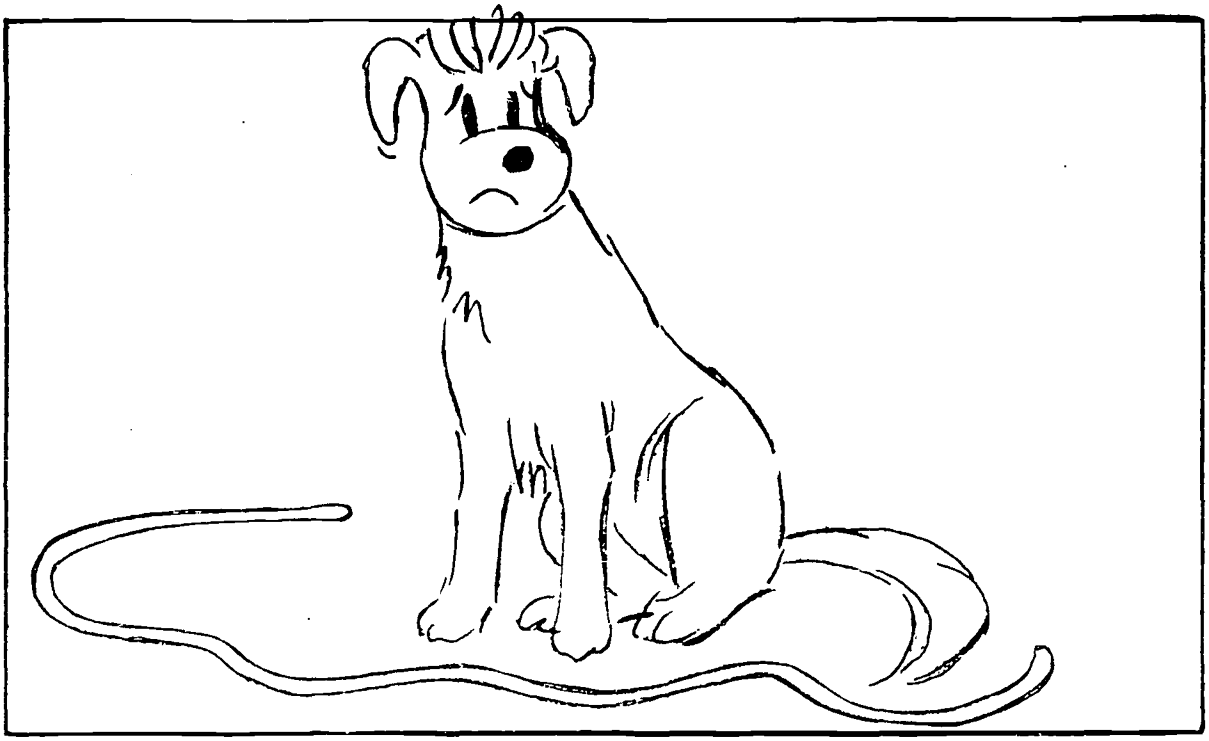
WHERE IS SKIP?

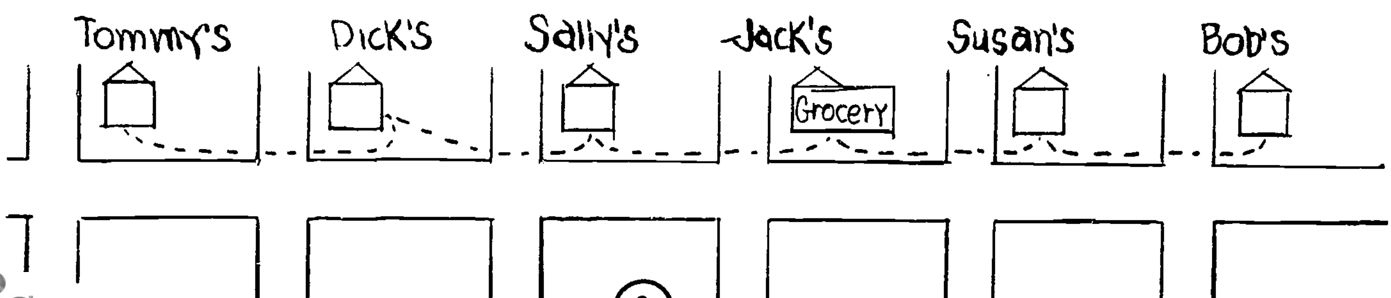
Susan went into the house crying. Her mother gave her another rope, so Susan was happy again.

But Susan's mother was not happy about Skip's visit.

Skip was not so happy either. He knew he had been naughty to take Susan's rope. So he dropped it for her to find, and on Skip went to the next block.

He went to Bob's house.

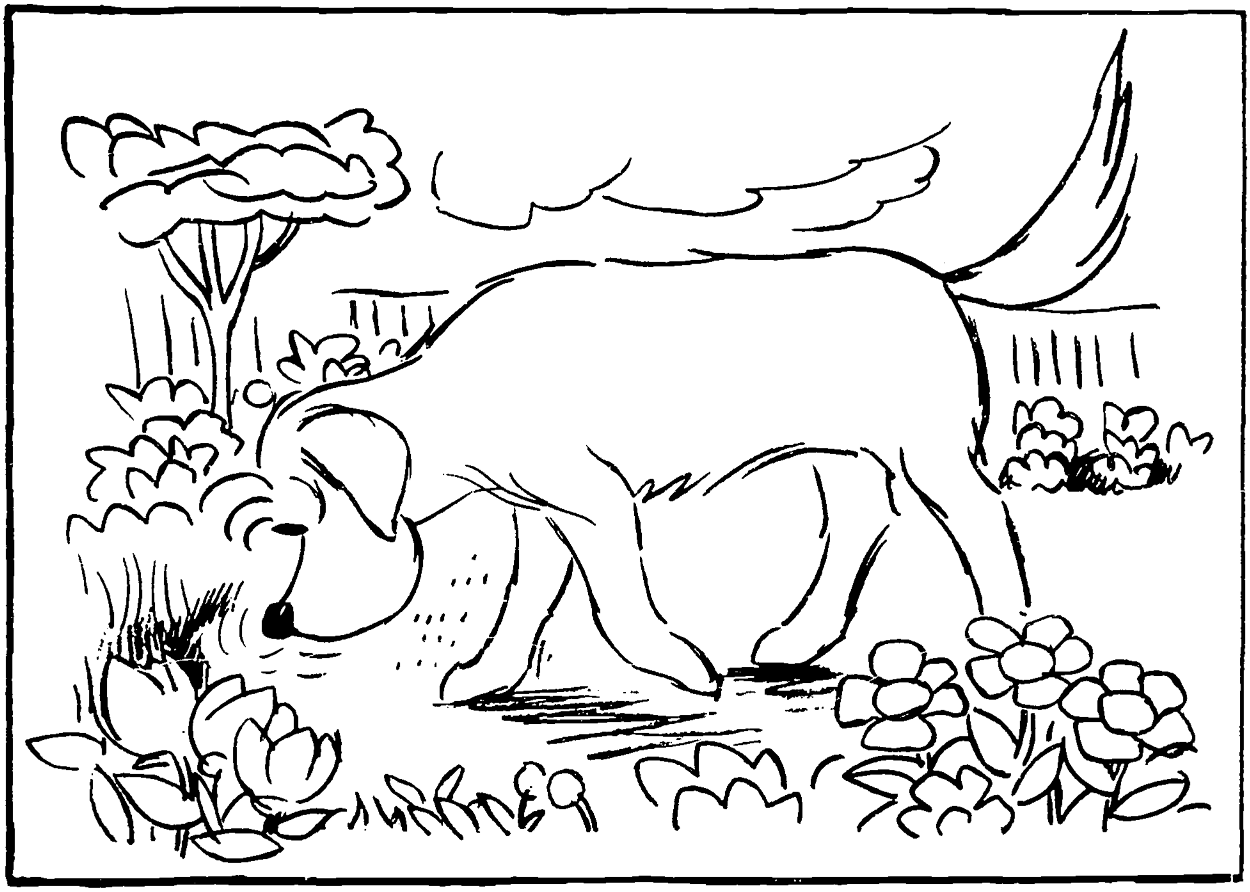




WHERE IS SKIP?

Skip was very hungry now. As he came to Bob's house he was thinking. Then he remembered something. Last week Skip had buried a nice big bone in Bob's yard.

"Where is that bone?" thought Skip. "I can't remember where I put it." He sniffed around the yard.



"Bow wow," cried Skip. "This smells like the right place."

And Skip began to dig.

Do you know where he was looking?

Right where Bob's mother had planted her flower garden.

Skip looked and looked and looked.

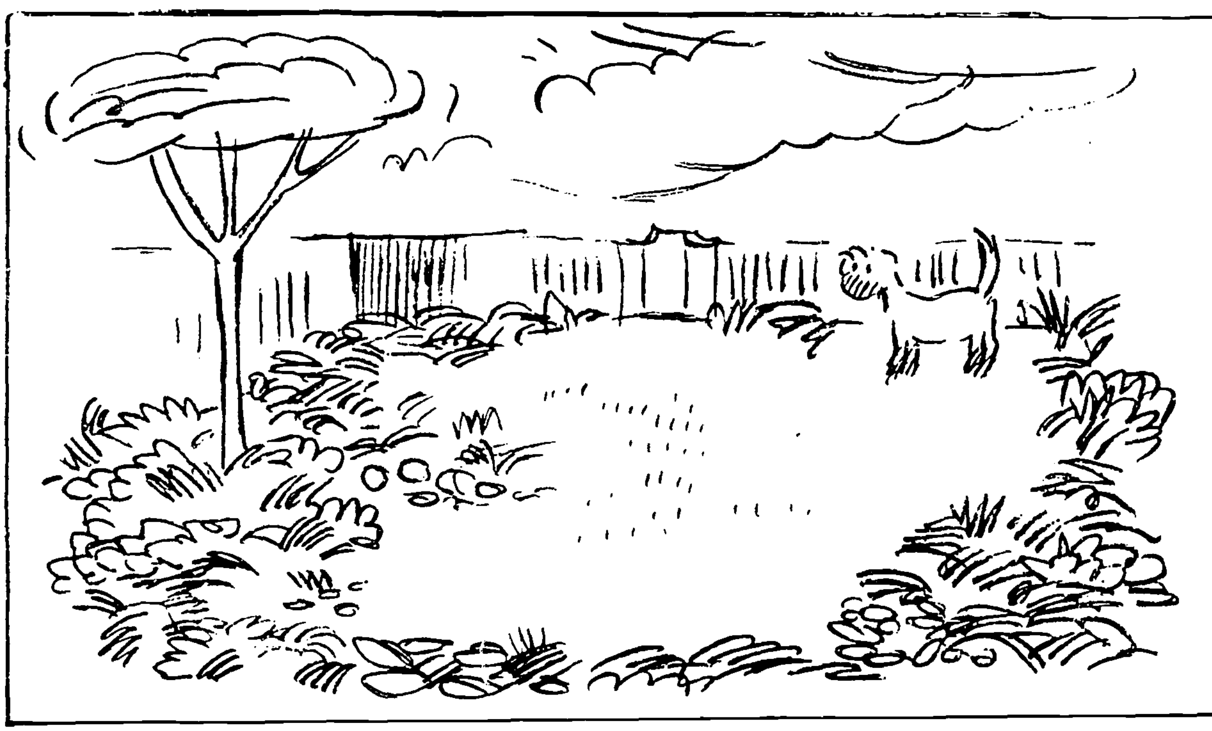
But he did not find his bone.

He sniffed around some more. When he was on the other side of the yard he said to himself, "<u>This</u> must be the place."

So he began to dig again. This time he was looking right where Bob's father had planted the vegetable garden.

Just then Bob's mother came out to hang up her clothes.

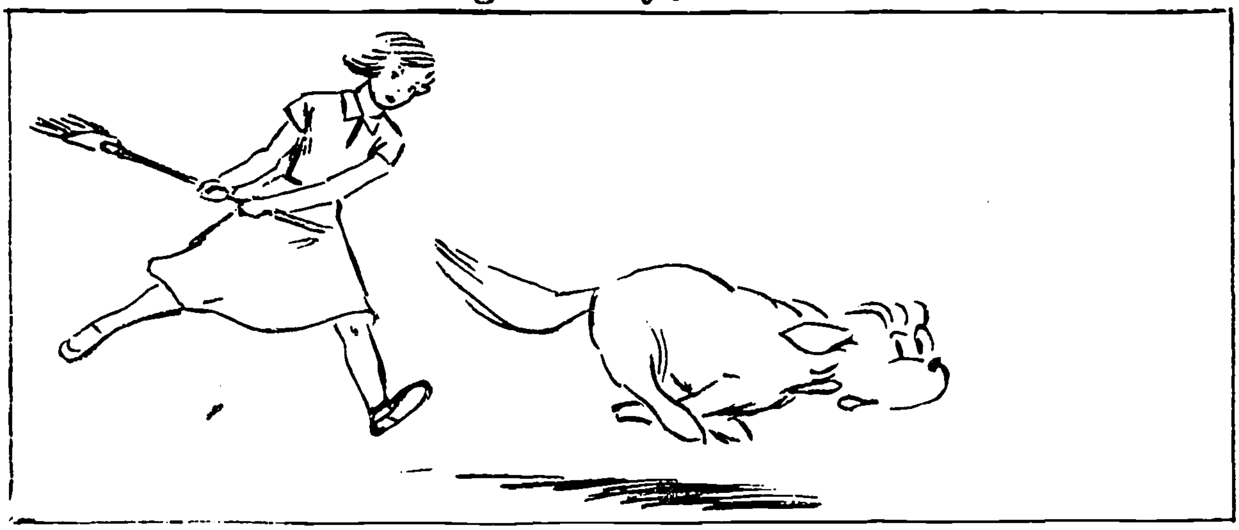
And here is what she saw!



WHERE IS SKIP?

Bob's mother was not happy about Skip's visit.

She made him go away.

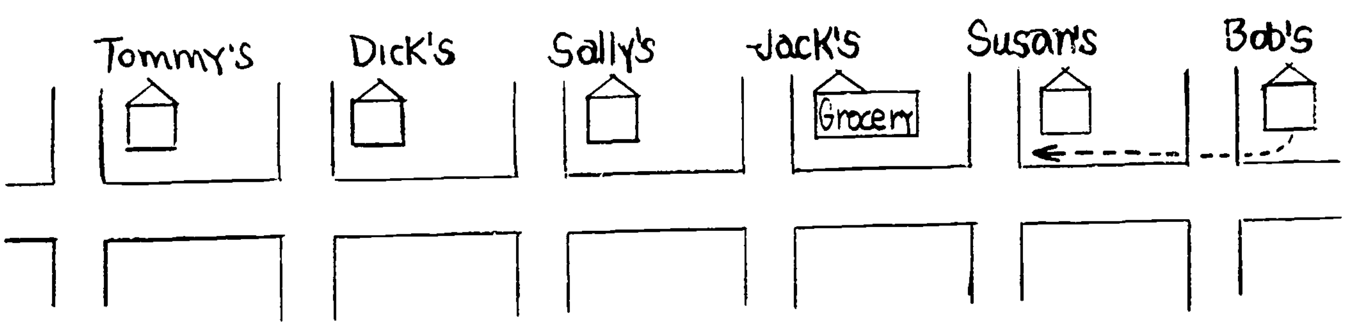


And Skip did not find his bone.

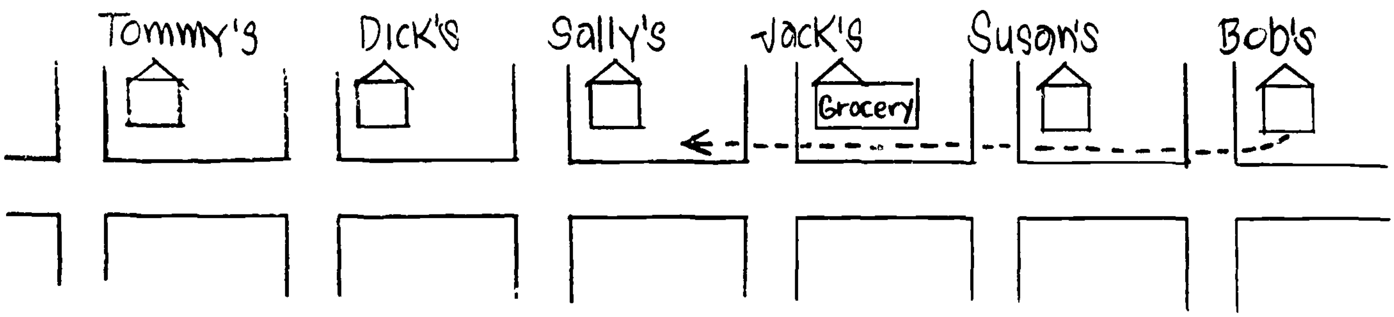
By this time he was very, very, very hungry.

So he thought he would go home for breakfast.

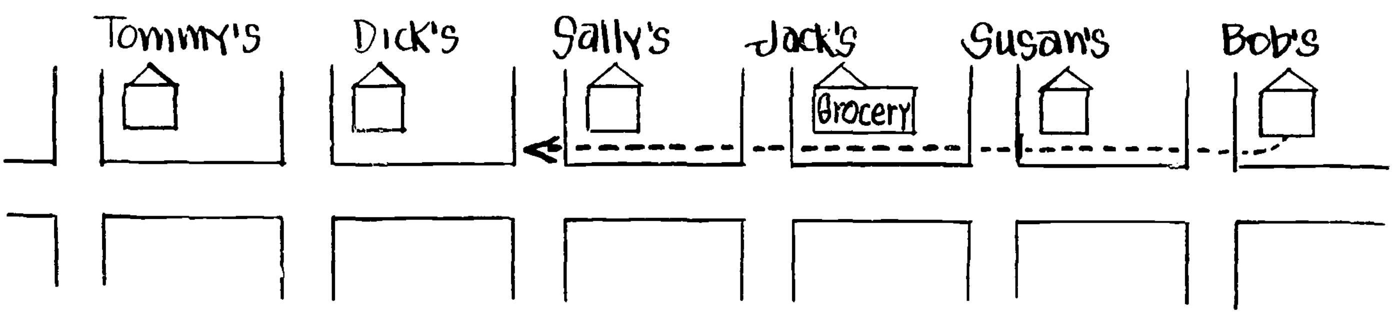
He ran from Bob's house back past Susan's house.



He ran another block past Jack's grocery store.



He ran on for another block past Sally's house.



Faster and faster he ran.

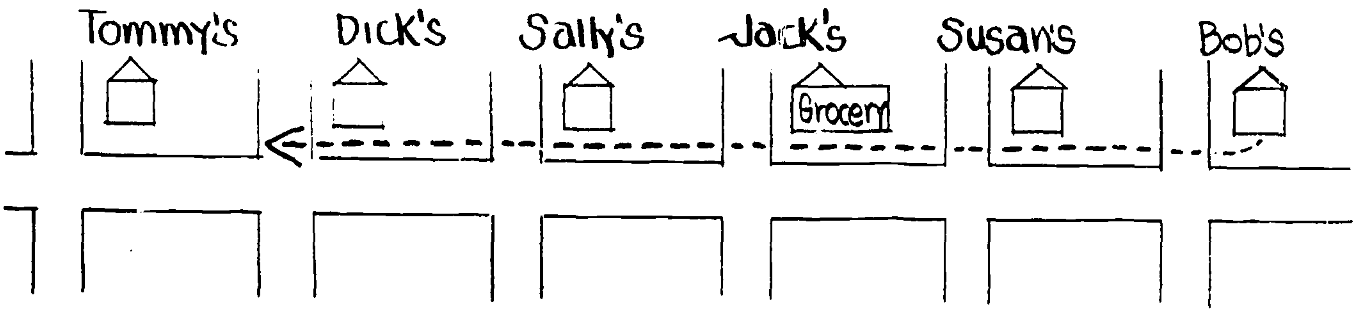
"Where is Tommy?" thought Skip.

"Where is Don?"

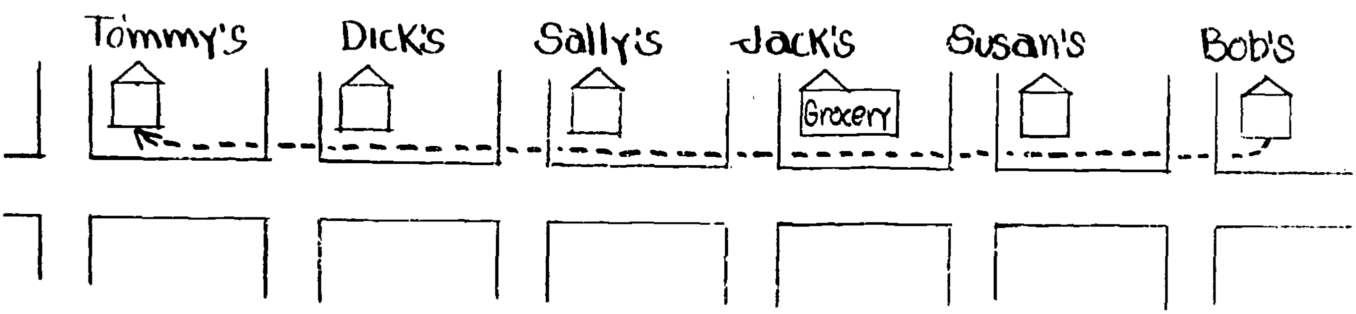
"Bow wow, bow wow," he said as he ran faster and faster and faster.

WHERE IS SKIP?

After another block he ran past Dick's house.



One more block to go. Then Skip was home, at Tommy's house.



And there were Tommy and Don still looking for Skip.

"Here, Skip! Here, Skip!" called Tommy.

"Where are you, Skip?" called Don.

"Bow wow, bow wow," said Skip as he ran to the boys.

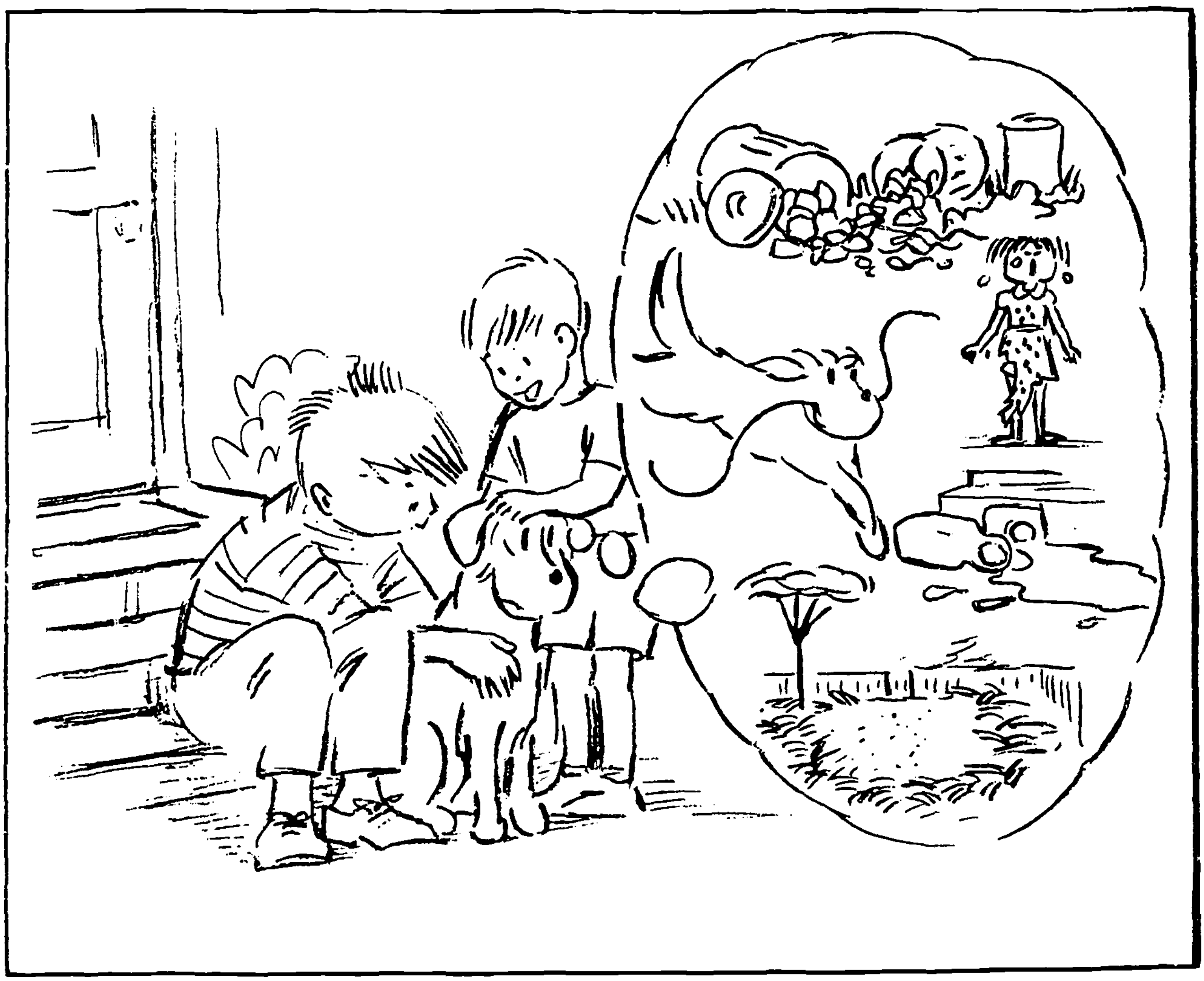
"Oh, here you are," laughed Tommy. "We thought you were lost."

"Where have you been, Skip?" asked Don.

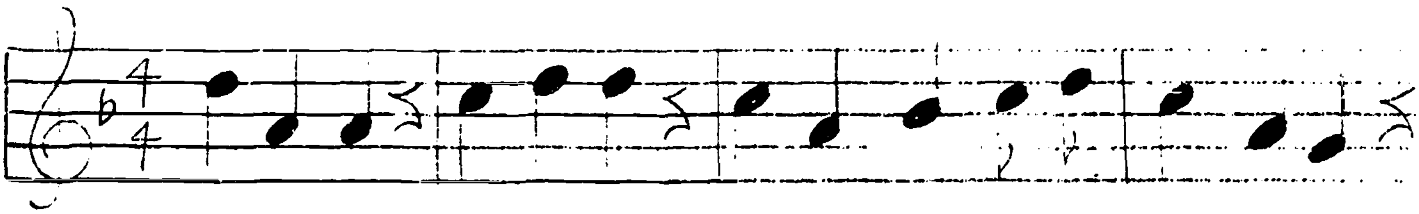
But Skip did not say a word. He looked at the boys, hung his head, and slowly wagged his tail.

He thought about Dick's milk bottle and Sally's dress and Jack's grocery store and Susan's rope and Bob's garden.

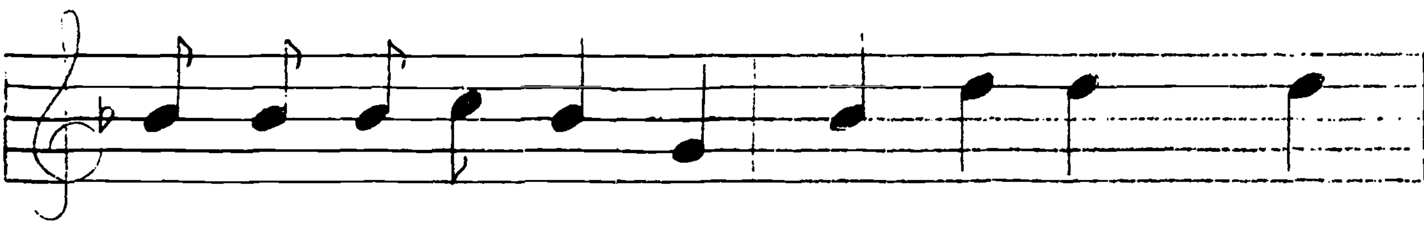
But Skip did not say a word.



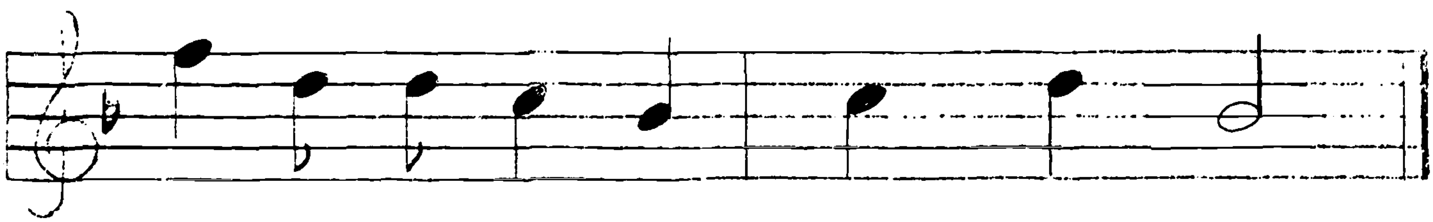
Where Is Skip?



1. Where is Skip? Where is Skip? He's gone off on a morn-ing trip.
2. Where is Skip? Where is he? Two blocks down un-der Sall-y's tree.
3. Where is Skip? Where is he? Three blocks down at Jack's gro-cer- y.
4. Where is Skip? Where'd he run? Four blocks down Su-san saw him come.
5. Where is Skip? Far from home. At Bob's house loo-king for his bone.
6. Where is Skip? Where is Skip? Head-ing home from his morn-ing trip.



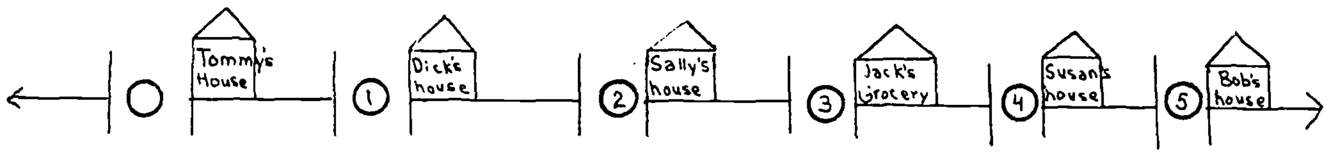
1. He went off to Dick's house first of all -- and
2. Sall-y played with Skip and didn't get hurt, but
3. In the yard he found a barrel of scrap, and
4. Su - san jumped her rope first slow then quick, then
5. He dug up the gar - den in Bob's yard, where
6. There he found his play-mates Tom and Don, a-



1. Bang! went the milk on Skip's first call.
2. Rip! went a piece of Sall --- y's skirt.
3. Crash! went the trash right in Skip's lap.
4. Skip took the rope, my what a trick.
5. Bob's Mom and Dad had worked so hard.
6. Won - der - ing where their dog had gone.

Additional Activities

1. At the end of the story, put on the chalkboard the diagram showing Skip's complete trip. (Houses should be placed right at the corners.)



Then have the children extend Skip's trip to cover 10 visits by making up more of the story.

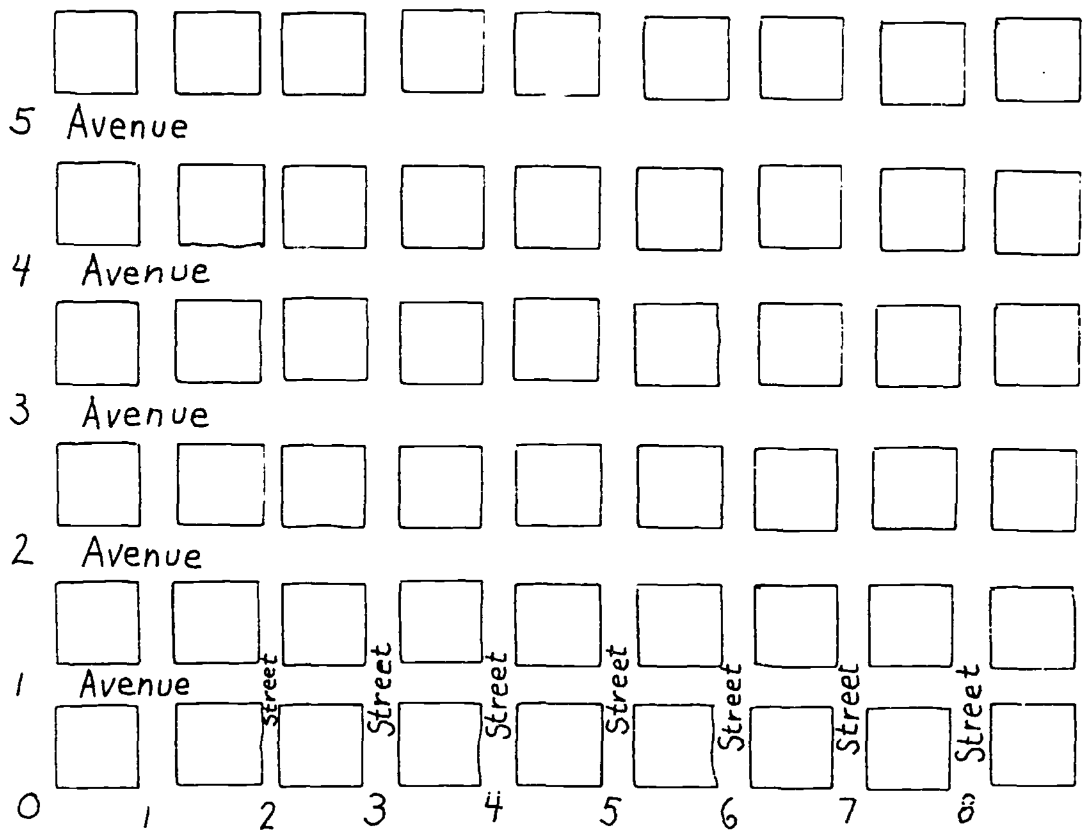
Have children discover how many blocks Skip went from Tommy's house to Susan's.

2. Use the tune of "Ten Little Indians:"

One little, two little, three little pennies,
Four little, five little, six little pennies,
Seven little, eight little, nine little pennies,
Ten pennies in my pocket.
Ten little, nine little, eight little pennies,
Seven little, six little, five little pennies,
Four little, three little, two little pennies,
One penny in my pocket.

For additional music experience with "Ten Little Indians" have children make up other items.
Rabbits (10 rabbits eating lettuce)
Marbles (10 marbles in my pocket)
Jump Ropes (10 jump ropes to play with)

3. Let's see what Skip's up to now. (Use a large magic slate to locate spots named. If teacher has no large slate, use chalkboard.)



Skip ran away.

Say, "The postman saw Skip at 7th Street and 4th Avenue."
Have a child put an X at 7th Street and 4th Avenue.

Say, "Skip chased a black cat from 1st Avenue to 3rd Avenue on 5th Street."
Have a child draw a line to show where Skip chased the cat.

Say, "The milkman saw Skip at 5th Street and 3rd Avenue."
Have a child put a closed curve where the milkman saw Skip.

Say, "Skip met Jimmy at 3rd Street and 1st Avenue."
Have a child put a triangle where Skip met Jimmy.

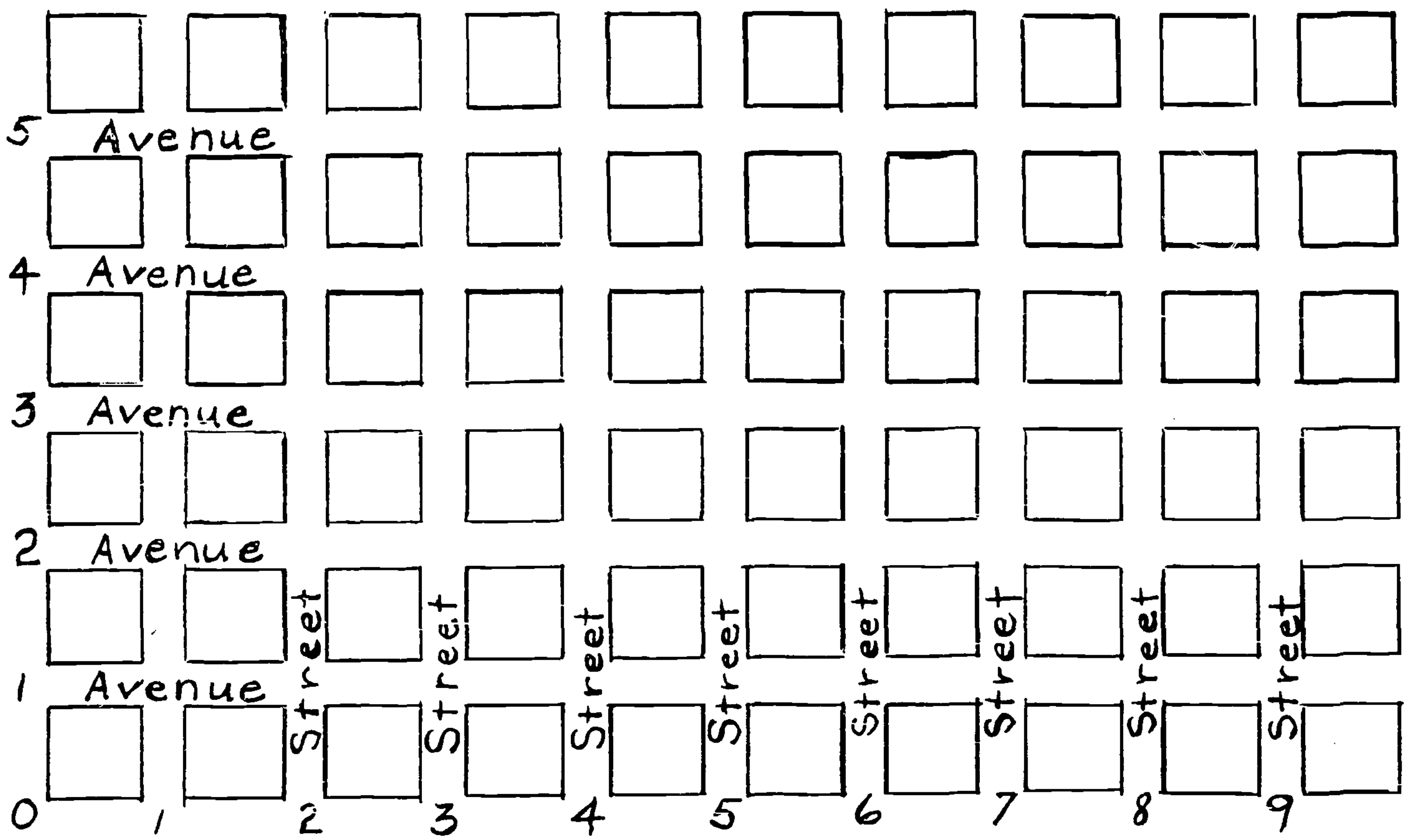
Practice should be given on a map (such as the one illustrated above) to provide sufficient review of the terms first (1st), second (2nd), third (3rd), and so on.

4. Distribute Worksheet ★3.

5. Distribute Worksheet 4 and put your own directions on the chalkboard. Children could give you directions to print.

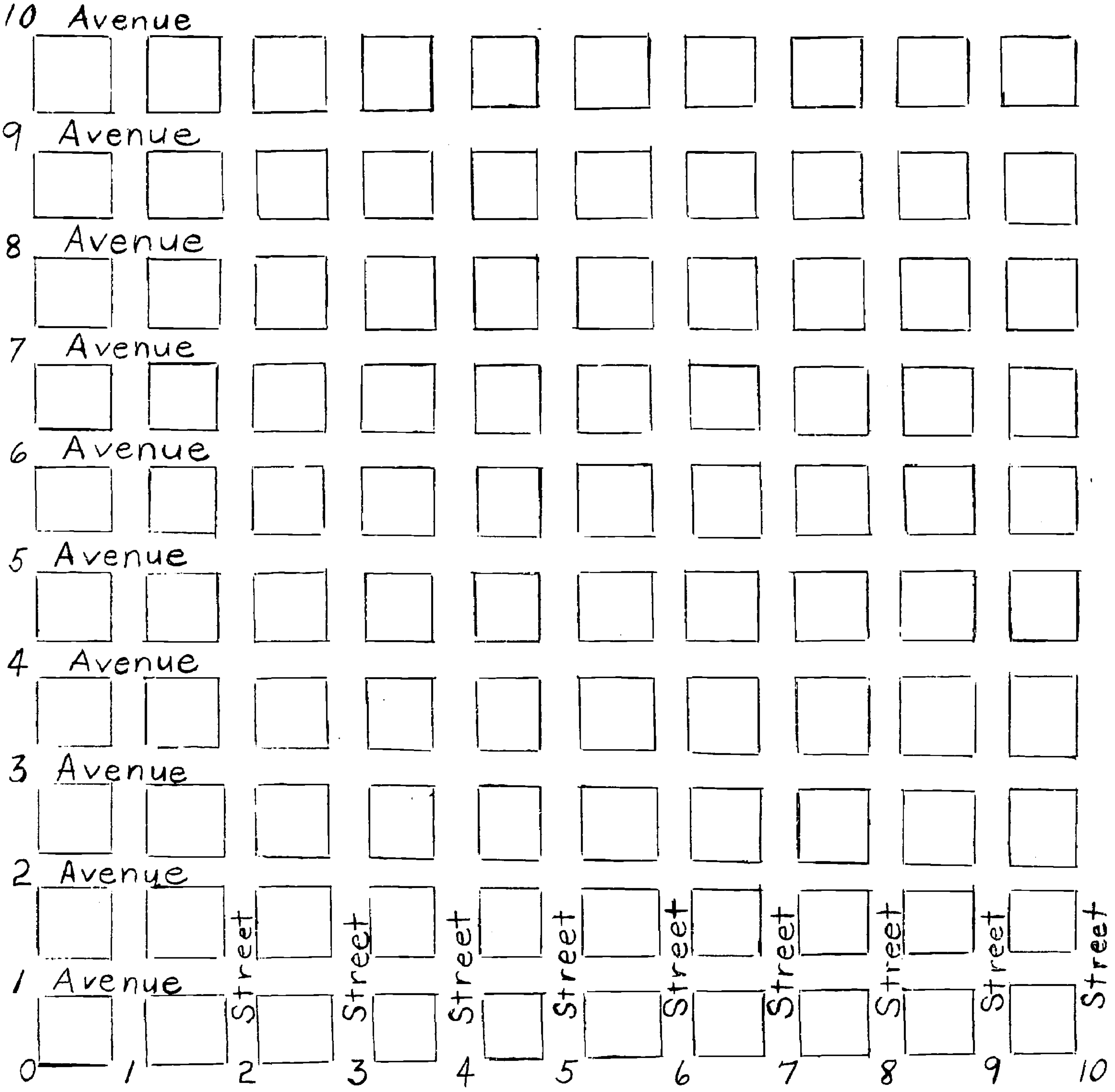
6. At this point the children may enjoy again playing the <u>Let's Go</u> game which was presented in Unit V. This game will strengthen the skill of accurately locating points on a map.

Worksheet 3 The Number Line



1. Put a blue X at 7th Street and 4th Avenue.
2. Put a red X at 2nd Street and 5th Avenue.
3. Draw a path from the corner of 1st Street and 2nd Avenue to 1st Street and 5th Avenue. Then continue the path from where it is. Go along 5th Avenue to 4th Street.
4. Put a green X at 1st Street and 5th Avenue.

Worksheet 4 The Number Line

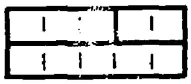


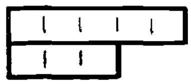
Suggested Activities Using Minnebars

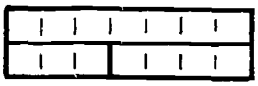
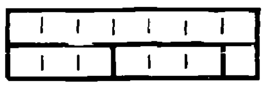
Activities with the Minnebars are first and most extensively described in Unit IV. Those activities which are most important for the number line are repeated here.

Materials: The teacher should have a set of Minnebars for each child. A set consists of 10 one-unit bars, 5 two-unit bars, 4 three-unit bars, 3 four-unit bars, and 2 each of the lengths five through ten units.

1. Provide opportunities for free play with the bars.

★2. Place two bars (each no larger than the five unit bar) end to end, and ask the children if they can find just one bar that will "match" these two bars in length. (In other words, the correct bar would be just as long as the two bars put together.) The teacher may need to demonstrate this: 

★3. Place two bars, one shorter than the other, side by side  and ask the children to find the bar which will complete the match.

4. Ask the children to match one long bar with several shorter bars. Any combination or number of shorter bars may be used to match this one large bar. For example, a seven-unit bar may be matched in this way:  or, perhaps like this: 

Ask the children to see how many different ways they can match a long bar. With the nine-unit bar, for example, there are enough ways for each child in the class to match the bar in a different way.

5. Hold up, with the partition marks showing, a five-unit bar and a ten-unit bar. Say, "What do you notice about the long bar and the short bar?" Try to get the following responses: "They are both marked off into parts." "Both are colored." "Every space in the long bar is the same size as every space in the short bar." "It would take two of the short ones to match the long bar." If no one suggests using two short bars to make one long bar, ask the question, "What is the smallest number of the short bars that we need to put with the long bar so as to match it?"

★6. Next, ask the children to match a ten-unit bar, for example, using just 2 bars. (This particular bar could be matched using a three-unit bar and a seven-unit bar.)

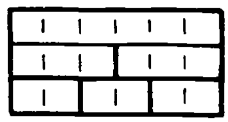
Following this activity, ask the children which bars can be matched using just 2 shorter bars. (All of the bars except the one-unit bar can be matched in this way.)

7. Have the children discover which bars can be matched using two bars of the same length (color), and which cannot be matched in this way.

8. Ask the children to match the six-unit bar, for example, using only the three-unit bars. Next, ask them to match the same bar using only the two-unit bars. Point out that in both of these matches, the children matched the "target" using only bars of the same length.

Now select another even unit bar and ask the children if they can match it using all bars of the same length. After one solution has been found ask for another solution. For the children who are slow to find a second solution, ask how many bars are needed to make the match, and how long each bar is.

Help the children to discover that if a bar can be matched with, for example, 2 three-unit bars, it can also be matched with 3 two-unit bars. To aid in illustrating this, it may be helpful to have the children place their solutions directly under each other as shown.

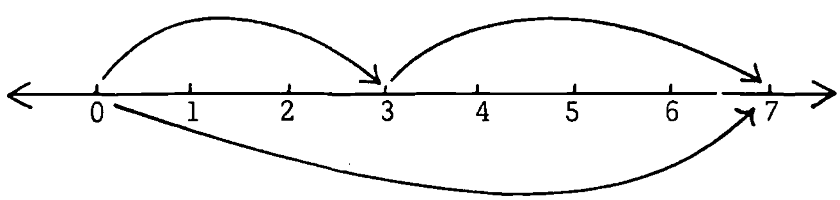


9. Have the children classify the bars by how many ways they can be matched using only shorter bars of equal length. (The two, five, and seven-unit bars can be matched in the fewest ways, and the six, eight, and ten-unit bars can be matched in the most ways.)

Teacher Background on Addition on the Number Line

Addition operation can be performed by starting at zero and laying off the segment indicated by the first addend. Then starting at the point corresponding to the first addend and by laying off the segment indicated by the second addend, the point which corresponds to the sum is reached. This is illustrated as follows:

$$3 + 4 = 7$$

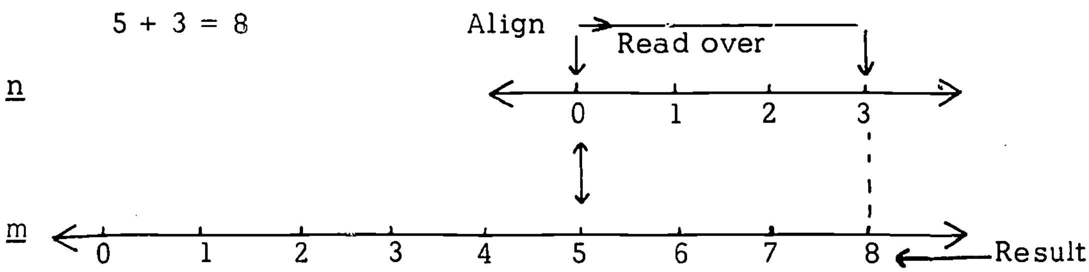


If necessary, use the story "Where is Skip?" as a point of reference.

It is suggested that in the earlier uses of the number line in the addition operation the child be encouraged to draw the lines indicating the addends and the sum. However, the teacher should avoid rejecting the performance of the "advanced child" who knows the addition combinations and, therefore, arrives at the sum before indicating the addends. Actually, this response tends to become somewhat automatic as the child practices in the use of the number line. The teacher, with little effort, can provide the students with sufficient practice in the addition of two numbers whose sum is less than or equal to ten.

A variation of the above should be introduced at this time which requires the use of the double number line. A teacher could demonstrate this with two yardsticks or rulers. For example, if we call one number line m and the other n, to add:

$$5 + 3 = 8$$



Steps:

a. Locate "5" on line m.
b. Place line n as shown in sketch with "0" above "5".
c. Read over to "3" on line n.
d. Read the sum on line m below "3".

It is "8", and, therefore, on the double number line we have shown that

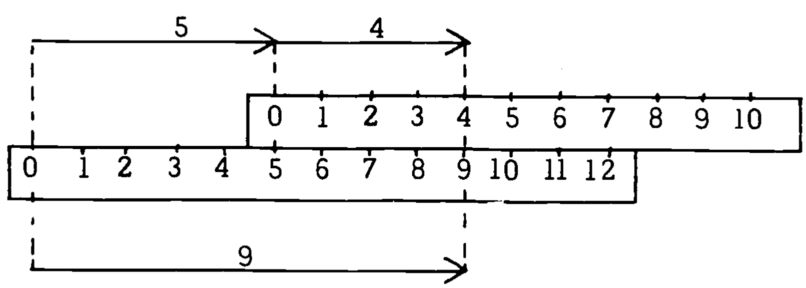
$$5 + 3 = 8$$

One of the most vital teaching aids furnished by Minnemast is the addition slide rule. Each student will be furnished one of these and discretion should be used as to whether the children be allowed to take them home. There is an advantage in allowing the parents to inspect the slide rules in that it gives them an insight into the concept of addition interpreted as "translation of origin" of one of the number lines.

The classroom slide rule, also furnished by Minnemast, is a replica of the small, plastic slide rules which the children use. It should be placed in a conspicuous place and hung from the top of the chalkboard. The teacher should certainly make frequent use of the classroom slide rule as the students are learning how to use their individual slide rules. In advanced units a third number line (Nomogram) will be used in the slide rule to perform additions.

The slide rule in use will appear as follows:

$$5 + 4 = 9$$



One of the most important relations of equality is that of symmetry. The symmetric property states that if a - b, then b = a. Therefore, it is quite natural to the child to accept the validity of the following:

If a + b = c,	then	c = a + b.
If 3 + 4 = 7,	"	7 = 3 + 4.
If 5 = 3 + 2,	"	3 + 2 = 5.
If 6 + 0 = 6,	"	6 = 6 + 0, etc.

The teacher can give many similar examples of the symmetric relation in the addition operation. Stating this in words, of course, we have

addend + addend = sum

has the same meaning as

sum = addend + addend.

This will accustom the child to seeing and stating the addition operation in both ways. In some number sentences the sum will be on the right side and in others it will be on the left side. It is hoped that the teacher will be able to eventually use both forms interchangeably without confusing the student. The child should reach the point where he reacts with equal ease and without frustration to

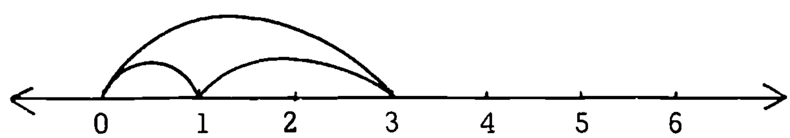
$3 + 4 = \_\_\_\_\_$ and $\_\_\_\_\_ = 3 + 4$.

Suggested Activities on Addition on the Number Line

1. Present the children with a problem to be solved similar to the following:

One afternoon Skip ran one block to Dick's house and stopped. Then he ran two more blocks to the grocery store. How many blocks did he run?

Have the children place a marker to represent Skip at 0 on the number line. Then ask a child to show the path that he ran by moving the token along the number line. Write the mathematical sentence 1 + 2 = _____, and have the child write in the sum.



Introduce the words "addend" and "sum". It is not necessary for the teacher to expect the children to learn specific verbal definitions of these words. If she uses them correctly over a period of time and demonstrates their meaning when using them, the children will intuitively grasp their meanings.

Present several problems such as that listed above and have the children first show the addends on the number line, then write in the sums. Gradually, as they become accustomed to the pattern, have them write the whole mathematical sentence themselves.

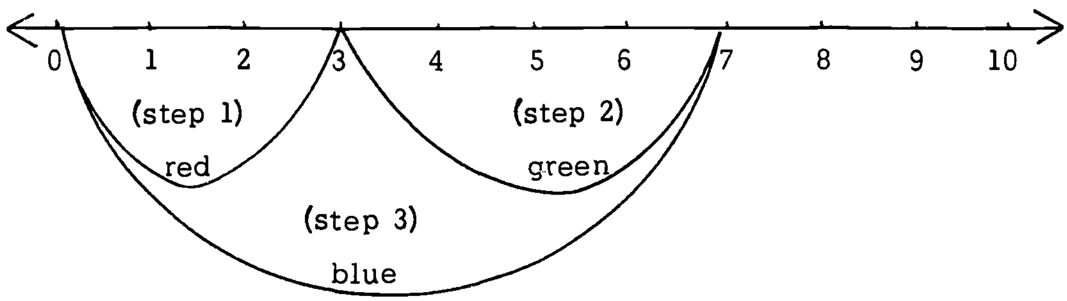
2. Do Worksheets 5 and ★6.

Put an example on the board before letting children do the worksheets.

Example:

$$3 + 4 = \underline{\hspace{4cm}}$$

(put sum here)



Worksheet 5 Addition and the Number Line

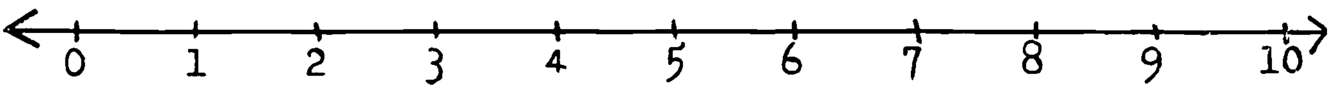
Directions: Mark the first addend with a red crayon.

Mark the second addend with a green crayon.

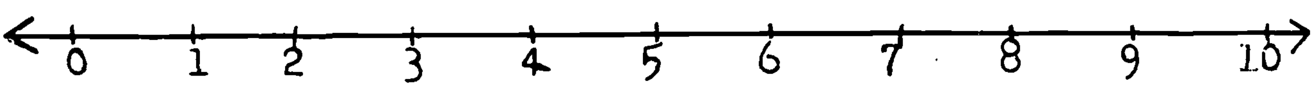
Mark the sum with a blue crayon.

Put the sum in the blank space.

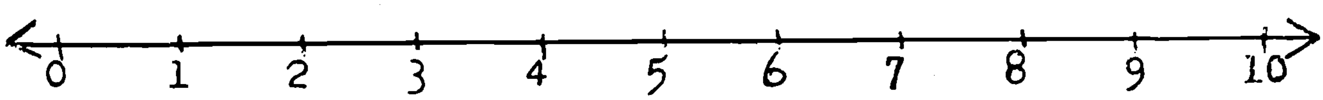
1. $2 + 3 =$ _____



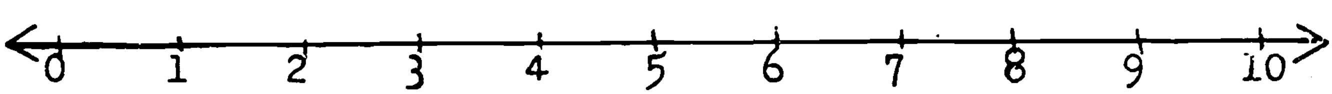
2. $5 + 3 =$ _____



3. _____ $= 6 + 3$



4. _____ $= 3 + 0$



Worksheet 6 Addition and the Number Line

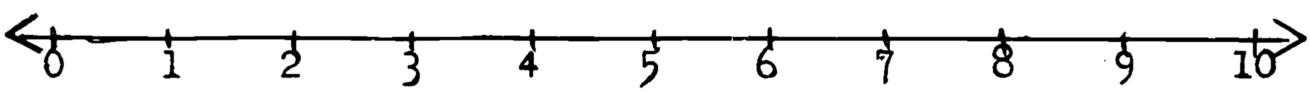
Directions: Mark the first addend with a red crayon.

Mark the second addend with a green crayon.

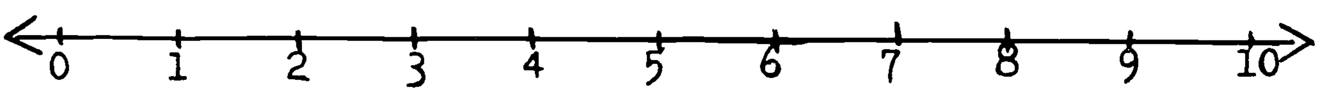
Mark the sum with a blue crayon.

Put the sum in the blank space.

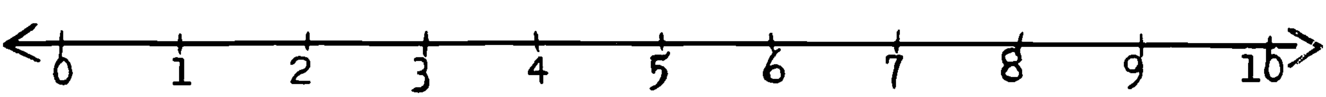
1. 4 + 5 = _____



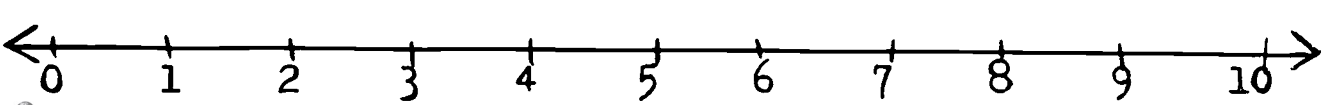
2. _____ = 2 + 6



3. 5 + 3 = _____



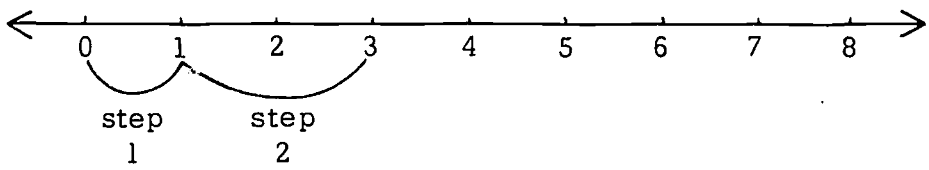
4. _____ = 5 + 4



Additional Activities on Addition on the Number Line

1. Have the number line where the children can see it. Give them this problem:

(Step 1) If you walk one block to your friend's house (show it on the chalkboard under or over the number line),



(Step 2) and 2 blocks to another firend's house, how many blocks have you walked?

(Be sure the children understand that zero is the starting point.)

2. If Skip has a set of 6 bones and finds a set of 4 more, how many bones does he have in the union of the two sets?

3. If a grasshopper hops 5 units on the number line and then, after resting a while, hops 4 more, how many units has he hopped?

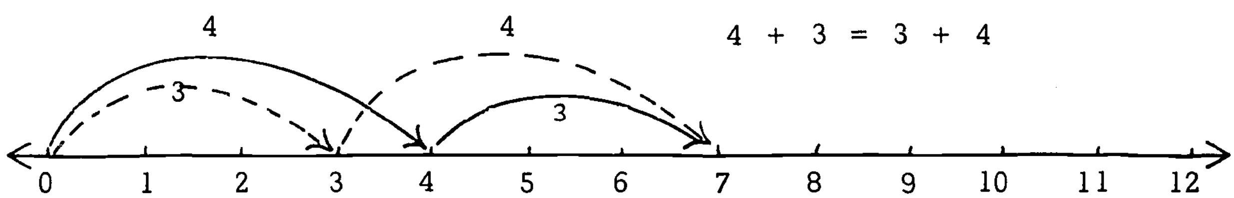
Teacher Background on The Commutative Property of Addition

It is expedient to point out at this time the importance of distinguishing between the operation "a + b" and "b + a". In the child's language, "3 + 2" and "2 + 3" are two different ways of <u>naming the same number</u>. As a matter of fact, there are many ways to represent the same number, such as: "4 + 1", "7 - 2", "5 · 1", etc.

When it is stated that

$$4 + 3 = 3 + 4$$

the very important <u>commutative</u> property of addition is illustrated. This property reduces by half the number of addition combinations the child usually learns up to $9 + 9 = 18$. The use of the number line makes this quite clear to the child.



Suggested Activities on The Commutative Property of Addition

1. Print problems such as 2 + 3 = _____ on the board. Each child holds up a numeral card that shows how many.

2. Use colored discs on flannelboard to show 4 + 2 = 6. A child may be chosen to show one "story" about 6, as above. A second child may go to the chalkboard and write what is shown on the flannelboard, using numerals. A third child is chosen to tell if the equation affirms the flannelboard facts. Get three children involved in each turn of play. All choose a successor for next turn.

3. Another activity to demonstrate the commutative property of addition.

   Equipment: Beads, or other small objects to use as counters; chalkboard or flannelboard; numerals to use on flannelboard; cutouts of + and = signs for use on flannelboard. Each child should have a set of cards bearing numerals 0 - 10; one card for each numeral.

   Procedure:
   "Put 2 counters on your desk."
   "Put 4 more counters on your desk."
   "Hold up the card that shows how many you have when you count all of these together." (Children should hold up the card bearing numeral 6.)
   "Right. There are six." (Teacher records 2 + 4 = 6 on board.)
   "Put your counters back in your boxes."
   "This time place 4 counters on your desk."

"Put two more counters with these four."

"How many have you altogether?" (Again children should hold up card bearing 6.)

"That is right, we have six."

"Did it make any difference in the result whether we placed the 2 counters on our desks first and added the 4 to them, or if we placed the 4 counters on our desks first, and added 2 counters to them?" **NO**

"That is right. In both cases, the result is 6."

4. This exercise should be repeated with all the addition combinations, bringing out the idea each time that the order in which the addends are used does not affect the sum.

   Both the horizontal and vertical methods of recording should be used; the children should become familiar with both:

$$2 + 4 = 6 \qquad\qquad \begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array} \quad \begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array}$$

$$4 + 2 = 6$$

5. Also use the following examples: (Remember to use the terms "addend" and "sum".)

① $\begin{array}{r} 4 \\ +2 \\ \hline \boxed{} \end{array}$ ② $\begin{array}{r} 4 \\ +\boxed{} \\ \hline 6 \end{array}$ ③ $\begin{array}{r} \boxed{} \\ +2 \\ \hline 6 \end{array}$

   Each time have the children hold up the card with the answer.

6. Set up teams and have children choose team names. Have the first member of each team turn his back to the board as the

example is written. At a signal from the teacher they turn, see the example, and hold up their numeral cards with the correct answers. The person who has the correct answer first gets a point for his team. In case of a tie, each team should get a point.

Then the next player on each team stands, etc., until all have had a turn (if one team is short a player, the first one on that team takes an extra turn). When everyone has had one turn (or 2 - as you decide beforehand), then the team with the most points wins. It is wise to have four teams so the game moves quickly.

7. To strengthen the meaning of sets and subsets and to introduce the idea of the "open sentence", the following activity may be developed. Call attention to the fact that the sum is listed first sometimes and last at other times.

Start this game with one open spot.

$$6 = 3 + \underline{\qquad}$$

Then use

$$6 = \underline{\qquad} + \underline{\qquad}$$

Do this for number 1, 2, 3, 4, 5 as well as 6. Later in the year do 7, 8, 9.

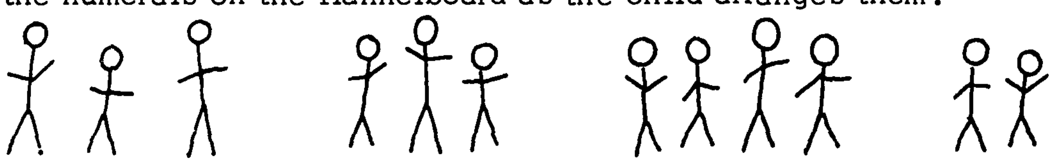
Equipment:

Flannelboard, the numerals 1 - 10 (T), and the equal sign (=). Chalkboard could be used in place of a flannelboard.

Procedure:

The teacher places the numeral six on the flannelboard (or chalkboard). She then calls on a child to arrange a set of six children in some designated area of the room. After the six on the flannelboard she places an equal sign, and asks if someone can arrange two sets of children that will, together, match the set of six. If the children are slow in responding, the teacher will ask four children to make a set, at the same time placing the numeral four after the equal sign. Call the children's attention to the fact that the numeral four tells how many children are in this set. As a plus (+) sign is placed after the four the teacher asks, "What set can we put with this set to make the sets, together, match the six?" When two is decided upon, the numeral two is placed after the plus (+) sign. (6 = 4 + 2)

If a child, or children, suggests the sets, the teacher will place the numerals on the flannelboard as the child arranges them.



The mathematical sentences should be written as 4 + 2 = 6, and as 6 = 4 + 2. The children should become familiar with both.

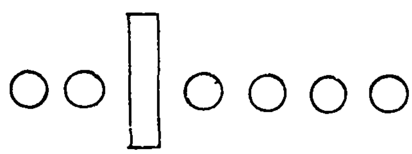
8. All of the possible combinations of numbers, up to and including those making the number ten, should be used and reused in this way. It might be best to use all possible combinations of a certain number at one time, moving to the combinations making another number at the next session.

2 = 1 + 1	3 = 1 + 2	4 = 1 + 3	5 = 4 + 1
	3 = 2 + 1	4 = 3 + 1	5 = 1 + 4
		4 = 2 + 2	5 = 3 + 2

6 = 4 + 2	7 = 4 + 3	8 = 4 + 4	9 = 5 + 4
6 = 2 + 4	7 = 3 + 4	8 = 5 + 3	9 = 4 + 5
6 = 3 + 3	7 = 5 + 2	8 = 3 + 5	9 = 3 + 6
6 = 5 + 1	7 = 2 + 5	8 = 6 + 2	9 = 6 + 3
6 = 1 + 5	7 = 6 + 1	8 = 2 + 6	9 = 2 + 7
	7 = 1 + 6	8 = 1 + 7	9 = 7 + 2
		8 = 7 + 1	9 = 1 + 8
			9 = 8 + 1

10 = 5 + 5	10 = 2 + 8
10 = 4 + 6	10 = 8 + 2
10 = 6 + 4	10 = 9 + 1
10 = 3 + 7	10 = 1 + 9
10 = 7 + 3	

9. Have children put 6 counters (or blocks, or 6 of any stationary item) on the corner of their desks. Have them place their rulers as shown: (draw on the board). Then say to them, "If we place the



ruler here, we separate the set of 6 things into 2 subsets. How many are to the left of the ruler?" (Wait for answer.) "How many are to the right of the ruler?" (Answer.) Then say, "You have discovered that

<u>Left</u>	plus	<u>Right</u>	equals	Sum
2	+	4	=	6

Be sure they understand this. Then give them Worksheet 7. Read the directions with them. Then let them do the worksheet independently.

10. Use Worksheets 7 through 12, as the children are ready.

Worksheet 7 Addition

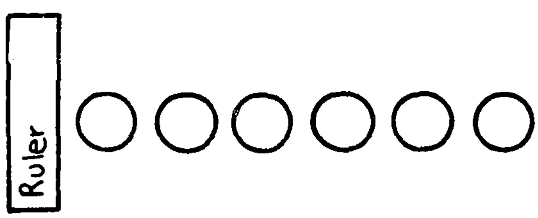
You have done this with 6 counters.

You discovered that $2 + 4 = 6$.

Now place your ruler in all other ways to separate this set. Write the results here.

Left		Right		
___	+	___	=	6
___	+	___	=	6
___	+	___	=	6
___	+	___	=	6
___	+	___	=	6
___	+	___	=	6

We hope you remember <u>this</u> way of placing the ruler.



There is another way which is also easy to forget.

Worksheet 8 Addition

Put 5 counters on your desk.

Use your ruler to separate them.

Make the same kind of table as you did for 6 counters.

Left		Right	
___	+	___	= 5
___	+	___	= 5
___	+	___	= 5
___	+	___	= 5
___	+	___	= 5
___	+	___	= 5

Do the same with 4 counters.

Left		Right	
___	+	___	= 4
___	+	___	= 4
___	+	___	= 4
___	+	___	= 4
___	+	___	= 4
___	+	___	= 4

Worksheet 9 Addition

Put 2 counters on your desk.

Use your ruler to separate them.

Make the same kind of table as you did for 6 counters.

Left		Right	
___	+	___	= 2
___	+	___	= 2
___	+	___	= 2

Put 7 counters on your desk.

Use your ruler to separate them.

Make the same kind of table as you did for 6 counters.

Left		Right	
___	+	___	= 7
___	+	___	= 7
___	+	___	= 7
___	+	___	= 7
___	+	___	= 7
___	+	___	= 7
___	+	___	= 7
___	+	___	= 7

Worksheet 10 Addition

Put 1 counter on your desk.

Use your ruler to make the same kind of table as you did for 6 counters.

Left		Right	
___	+	___	= 1
___	+	___	= 1

Put 8 counters on your desk.

Use your ruler to separate them.

Make the same kind of table as you did for 6 counters.

Left		Right	
___	+	___	= 8
___	+	___	= 8
___	+	___	= 8
___	+	___	= 8
___	+	___	= 8
___	+	___	= 8
___	+	___	= 8
___	+	___	= 8

Worksheet 11 Addition

Put 9 counters on your desk.

Use your ruler to separate them.

Make the same kind of table as you did for 6 counters.

Left		Right	
___	+	___	= 9
___	+	___	= 9
___	+	___	= 9
___	+	___	= 9
___	+	___	= 9
___	+	___	= 9
___	+	___	= 9
___	+	___	= 9
___	+	___	= 9
___	+	___	= 9

Put 0 counters on your desk.

What kind of table can you make now?

___ + ___ = 0

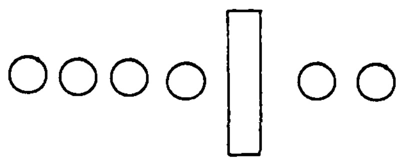
Worksheet 12 Addition

$2+4=$ _____	$2+2=$ _____
$4+2=$ _____	$0+3=$ _____
$3+3=$ _____	$2+3=$ _____
$5+1=$ _____	$4+0=$ _____
$1+5=$ _____	$2+5=$ _____
_____ $=4+2$	$6+0=$ _____
_____ $=2+4$	$1+6=$ _____
_____ $=3+3$	_____ $=4+3$
_____ $=5+1$	_____ $=0+5$
_____ $=1+5$	_____ $=7+0$

Teacher Background on Subtraction

The Additive Concept

The preceding activities of partitioning into subsets should provide the readiness necessary to develop ability to find the missing addend in the addition operation. It is a very natural step to proceed from:



$$4 + 2 = 6$$

to any of the forms of the number sentence:

4 + _____ = 6	6 = 4 + _____
_____ + 4 = 6	6 = _____ + 4
2 + _____ = 6	6 = 2 + _____
_____ + 2 = 6	6 = _____ + 2

Again, the basic teaching device should be the number line. For example, in the number sentence:

3 + _____ = 7,

the teacher locates the points associated with the given addend and sum. The completion of the number sentence requires the answer to the question: "If you are at '3' on the number line, how many segments would you have to move to get to '7'?" The number sentence:

8 = _____ + 5,

suggests the question: "If a person is at '5', how many spaces would that person move to get to '8'?" It is imperative that the child understand that

first addend + second addend = sum

is the same number sentence as

sum = first addend + second addend

Caution: Since this can be readily recognized as subtraction, a teacher should be extremely careful not to generalize two sweepingly by making statements such as: in addition we always count in a certain direction, etc.

The Minus Symbol

The teacher should observe that no mention has been made of the operation "subtraction". The activity of dividing into subsets, and finding the missing addend has served as a background for this operation. The student should have very naturally and almost unconsciously entered into supplying the missing addend in number sentences. It is suggested that the teacher not "make a big issue" of this by detailing it as a new operation. It is only after the students become quite adept in this activity that a distinction is made; i.e., that the problem of finding the missing addend is called "subtraction" and the "minus" symbol (-) can be used.

From the number sentence:

$$3 + \_\_\_\_\_ = 7$$

or

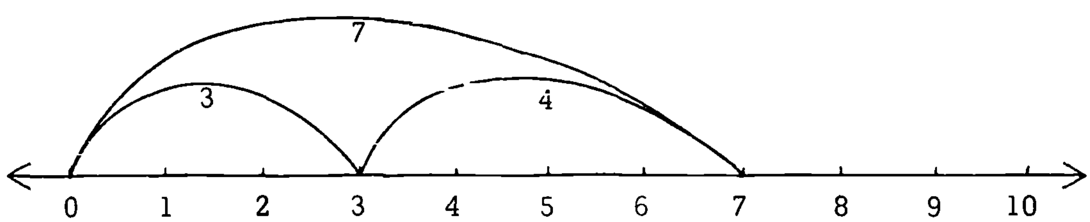
$$7 = 3 + \_\_\_\_\_,$$

the "subtraction form" of the number sentence is introduced:

$$7 - 3 = \_\_\_\_\_$$
$$\_\_\_\_\_ = 7 - 3.$$

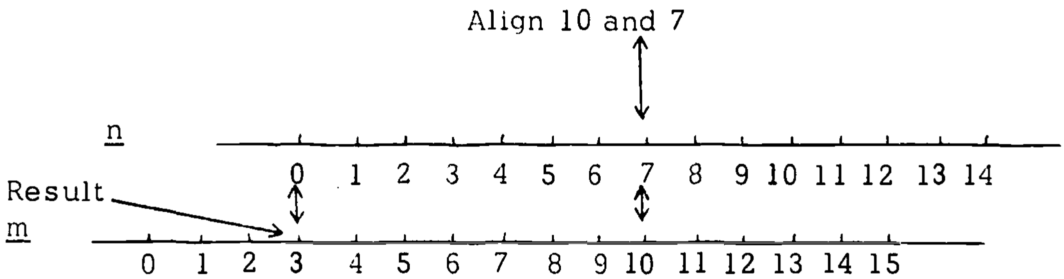
The teacher, and later, the student, can illustrate this on the number line. As a matter of fact, they will have been doing this with the missing addend on the number line.

$$4 + \_\_\_\_\_ = 7$$



A variation of the above can be introduced at this time which requires the use of the double number line. For example, if we call one number line m and the other n, to subtract:

$$10 - 7 = 3$$



a. Locate "10" on the m line.
b. Find "7" on the n line.
c. Place the "7" above the "10".
d. Now read the point on m which falls below the "0" on n. It is "3".

The children have their own slide rules which they used in addition. The teacher may demonstrate with the classroom slide rule and the children should be furnished sufficient exercises for practice in subtraction with their rules.

Many examples can be introduced by the teacher before the student undertakes any of the worksheets. One of the difficulties experienced by the student lies in the different forms in which the operation of subtraction is expressed. This is illustrated in the number sentences:

$$5 - 3 = \_\_\_\_$$

$$\_\_\_\_ = 5 - 3$$

$$5 - \_\_\_\_ = 3$$

$$3 = 5 - \_\_\_\_$$

Obviously, the numeral 2 will make each a true statement originating from the operation

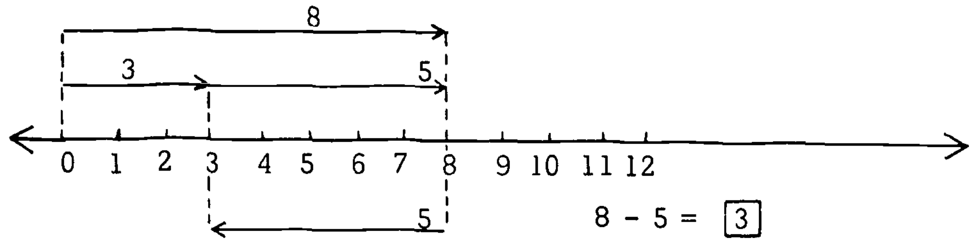
$$2 + 3 = 5.$$

Inverse of Addition

The statement that the subtraction is the inverse of the addition operation may be stated in the child's language as: "Subtraction undoes addition and addition undoes subtraction." It is very important that the child recognize this inverse relationship.

This may be illustrated in the use of the number line,

$$3 + 5 = 8$$



in the number sentence,

$$(3 + 5) - 5 = 3$$

or conversely,

$$(8 - 3) + 3 = 8$$

Numerous examples of this should be given because this type of number sentence is simply a special case of the generalized principles

$$(a + b) - c = a + b - c$$

and

$$(a - b) + c = a - b + c,$$

which are so important in the structure of arithmetic.

It is strongly recommended that the terminology "inverse operation" not be used in the first grade. The child is ordinarily well-satisfied with the expression, "One operation undoes the other". Many examples of the following type may be used before giving out Worksheet 19:

$(5 - 3) + \boxed{3} =$	$(9 + \square) - 3 = 9$
$(\square - 4) + 4 = 9$	*$(8 + \square) - \square = 8$

*The last sample exercise can be completed by filling in the frame with any counting number less than 9.

If this is the child's first contact with parentheses, it may be simply stated that "parentheses are mathematical punctuation used when we have two operations to perform and the parentheses simply show which one is to be done first. For example, in the exercise

$$(4 + 5) - 6 = 3,$$

the parentheses indicate that the addition is to be performed before the subtraction."

Suggested Activities on The Additive Concept of Subtraction

1. Draw a number line with chalk on the floor. The number line should be large enough for the children to step out paces. (Different types of floors will be helpful. A strip of contact paper on the floor with the numerals written with felt pen is another suggestion. This strip may be easily removed.)

   Present a problem such as the following to the class: If you are standing on the numeral three on the number line and you want to move to eight, how many units would you have to move?

   The child stands on three on the number line, and steps each unit to land on eight, counting the steps as he moves. Have someone write on the board the number sentence which was demonstrated, e.g., 3 + _____ = 8. Have him write in the missing addend. Allow as many children as possible to perform this activity with different number sentences. Refer, whenever possible, to the terms "addend" and "sum".

   When the children have grasped the concept, present a number sentence on the board such as the following: _____ + 5 = 6. Let the children suggest ways in which this sentence might be paced off. They may come to the correct interpretation that, if standing on five, they would move one unit on the line to land at six.

   Variation I

   The children may work in teams. The teacher may give each team, gathered around a number line on the floor, a list of number sentences to work out. The children may take turns stepping out the sentences. After they have demonstrated each number sentence, another team member should check the sentences.

Variation II

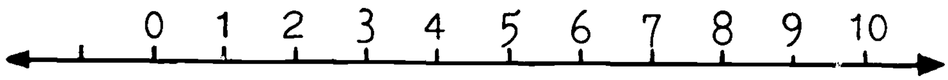
The children may make their own examples, pace them on the large number line, and record them on paper.

2. Do Worksheets 13, 14, and ★15.

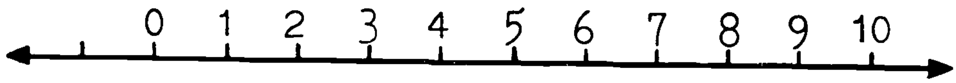
Worksheet 13 Addition

1 - Mark the moves on the number line using a pencil.

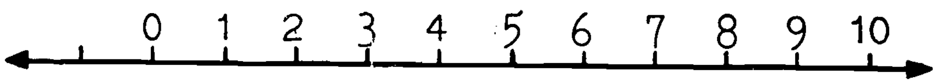
2 - Make a closed curve around the <u>sum</u> in each number sentence.

1. 

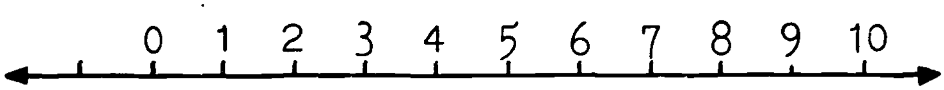
$$4 + \_\_\_ = 6$$

2. 

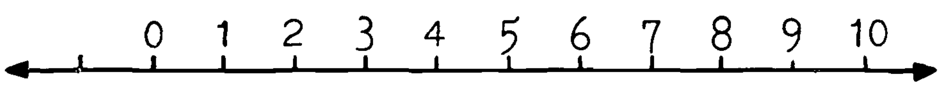
$$2 + \_\_\_ = 5$$

3. 

$$\_\_\_ = 8 + 1$$

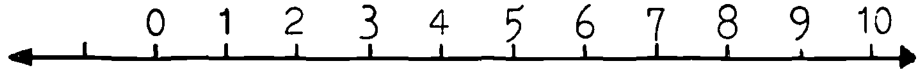
4. 

$$\_\_\_ = 3 + 2$$

5. 

$$\_\_\_ + 4 = 5$$

Worksheet 14 Addition



$1 + 1 = \underline{\quad}$

$\underline{\quad} = 1 + 1$

$\underline{\quad} + 1 = 2$

$1 + \underline{\quad} = 2$

$2 = \underline{\quad} + 1$

$2 = 1 + \underline{\quad}$

$1 + 2 = \underline{\quad}$

$2 + 1 = \underline{\quad}$

$1 + \underline{\quad} = 3$

$2 + \underline{\quad} = 3$

$3 = \underline{\quad} + 2$

$3 = \underline{\quad} + 1$

$1 + 3 = \underline{\quad}$

$3 + 1 = \underline{\quad}$

$2 + 2 = \underline{\quad}$

$\underline{\quad} = 1 + 3$

$\underline{\quad} = 3 + 1$

$\underline{\quad} = 2 + 2$

$1 + \underline{\quad} = 4$

$3 + \underline{\quad} = 4$

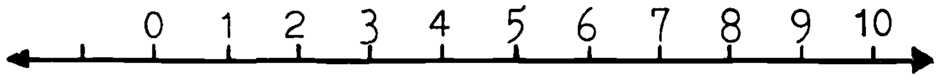
$2 + \underline{\quad} = 4$

$4 = 1 + \underline{\quad}$

$4 = 3 + \underline{\quad}$

$4 = 2 + \underline{\quad}$

Worksheet 15 Addition



1 + ___ = 3

___ + 5 = 6

0 + 4 = ___

2 + ___ = 5

___ + 1 = 4

1 + 1 = ___

0 + 0 = ___

___ + 2 = 2

3 + ___ = 6

2 + 2 = ___

1 + ___ = 6

4 + ___ = 5

___ + 1 = 2

5 = ___ + 1

___ = 6 + 0

0 = 0 + ___

3 = 1 + ___

___ = 4 + 1

2 = 1 + ___

4 = ___ + 2

___ = 5 + 1

5 = 3 + ___

___ = 0 + 5

___ = 3 + 1

2 = ___ + 2

4 = 1 + ___

Suggested Activities on the Minus Symbol

1. If the teacher would like to provide the students with learning situations to "condition" them for the different forms of the number sentence, the following game is suggested.

   Let a student state a simple addition number that he is familiar with, such as:

   $$4 + 5 = 9.$$

   Call this the title to the story, "Four, Five, and Nine". Then suppose this story can be divided into "chapters". Each chapter would express the same fact in a different form. Let the students give the names of the chapters. For example, there are 16 in all.

   1) 5 + _____ = 9
   2) _____ + 5 = 9
   3) 9 = 5 + _____
   4) 9 = _____ + 5
   5) 4 + _____ = 9
   6) _____ + 4 = 9
   7) 9 = 4 + _____
   8) 9 = _____ + 4
   9) 9 - 5 = _____
   10) _____ = 9 - 5
   11) 9 - 4 = _____
   12) _____ = 9 - 4
   *13) _____ - 5 = 4
   *14) 4 = _____ - 5
   *15) _____ - 4 = 5
   *16) 5 = _____ - 4

   *It is evident that these last four number sentences are addition operations using the subtraction language.

   This game, if continued at length, could develop into meaningless and tiresome repetition. However, the challenge to the students is presented in being able to name a chapter, which was not previously mentioned, when called upon to respond. As the "story" proceeds into the "final chapters", it becomes increasingly difficult to mention a "new chapter".

The variation, of course, is provided by continual changing of the "title of the story", such as:

$6 + 3 = 9$	$3 + 5 = 8$	$4 + 3 = 7$
$2 + 7 = 9$	$2 + 6 = 8$	$5 + 2 = 7$, etc.

It is possible that the teacher will find it necessary to provide more practice than is found in the three worksheets on the following pages.

2. Introduce the children to the following problem situation:

   a. Let's say that the grasshopper hops 5 units but then decides to hop back 3 units to visit his cousin. How many more units must he hop to be back at home at 0?

      Ask some child to space out the problem on the floor number line. From the action demonstrated, encourage the class to discuss and suggest ways of writing a number sentence for the problem.

      The children may suggest the sentence 5 = 3 + ______, or 3 + ______ = 5. Explain that there is another way of writing the story as 5 - 3 = ______, which we may call the subtraction form more by demonstrating problems on the number line than by verbal explanation. Suggested problems:

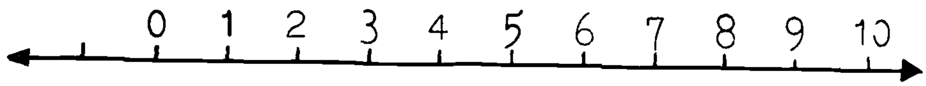
   b. How far must he hop back if he hops 5 units and wants to hop back home?

   c. If I have 4 apples and I give you 3, how many apples are left? (Use the number line to figure it.)

3. As soon as the children have gained some insight into the use of the subtraction form, encourage them to make up their own problem situations and demonstrate them to the class using the number line.

4. Do Worksheets 16, 17, ★18, ★19, ★20. It is not necessary for every child to do each of these worksheets. The teacher should assign them according to the needs of the individual children.

Worksheet 16 Subtraction



1 - 1 = ___

___ = 1 - 1

___ - 1 = 2

4 - ___ = 2

2 = ___ - 1

2 = 4 - ___

5 - 2 = ___

2 - 1 = ___

4 - ___ = 3

3 - ___ = 3

3 = ___ - 2

3 = ___ - 1

4 - 3 = ___

3 - 1 = ___

2 - 2 = ___

___ = 4 - 3

___ = 3 - 1

___ = 2 - 2

5 - ___ = 4

4 - ___ = 4

2 - ___ = 0

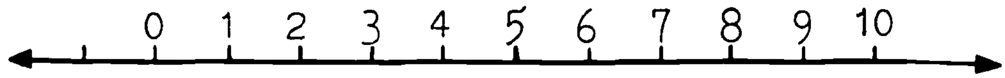
4 = 4 - ___

4 = 5 - ___

0 = 2 - ___

Worksheet 17

Subtraction



$4-2=\underline{\qquad}$

$4-4=\underline{\qquad}$

$3-3=\underline{\qquad}$

$5-1=\underline{\qquad}$

$6-5=\underline{\qquad}$

$\underline{\qquad}=4-2$

$\underline{\qquad}=6-4$

$\underline{\qquad}=3-3$

$\underline{\qquad}=5-1$

$\underline{\qquad}=6-5$

$6-\underline{\qquad}=6$

$2-\underline{\qquad}=0$

$3-\underline{\qquad}=2$

$5-\underline{\qquad}=4$

$4-\underline{\qquad}=1$

$0=\underline{\qquad}-2$

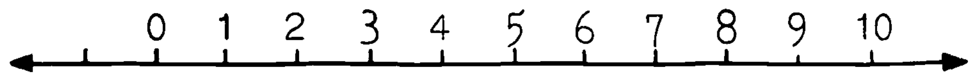
$2=\underline{\qquad}-4$

$3=\underline{\qquad}-3$

$4=\underline{\qquad}-1$

$1=\underline{\qquad}-5$

Worksheet 18 Subtraction



4 − ___ = 3

6 − ___ = 6

5 − 4 = ___

2 − ___ = 0

___ − 1 = 4

1 − 1 = ___

0 − 0 = ___

___ − 2 = 2

3 − ___ = 1

2 − 2 = ___

1 − ___ = 1

5 − ___ = 5

___ − 1 = 2

5 = ___ − 1

___ = 6 − 0

0 = 0 − ___

3 = 4 − ___

___ = 4 − 1

2 = 4 − ___

4 = ___ − 2

___ = 5 − 1

2 = 3 − ___

___ = 6 − 5

___ = 3 − 1

2 = ___ − 2

4 = 6 − ___

Worksheet 19 Subtraction

BIG SALE

BOOKS 10¢ each 3¢ off ____ sale price	PENCILS 5¢ each 1¢ off ____ sale price
TOYS Balls 10¢ each Sale 4¢ off Price ____	SALE! Dolls 10¢ each Sale 2¢ off Price ____
TOY CARS 5¢ each 2¢ off ____ sale price	AIRPLANES 10¢ each not on sale ____ price

Worksheet 20 Subtraction

Fill in the frame to make these true number sentences.

1. $(7-3)+3=\square$
2. $(8+2)-2=\square$
3. $(\square-3)+3=9$
4. $(7+\square)-2=7$
5. $(0+6)-6=\square$
6. $(5+4)-\square=5$
7. $(8-0)+\square=8$
8. $(9-\square)+\square=8$

Make up your own number sentences for 9 and 10.

9. $(\square-\triangle)+\triangle=\square$
10. $(\bigcirc+\square)+\square=\bigcirc$

Teacher Background on the Associative Property of Addition

In the preceding worksheet the students were introduced to "parentheses" in demonstrating inverse operations. The student should be receptive to the presence of parentheses in number sentences as necessary mathematical punctuation. They can develop an intuitive understanding of the "associative principle of addition" without which we would have to use many more parentheses. When they discover in the expression

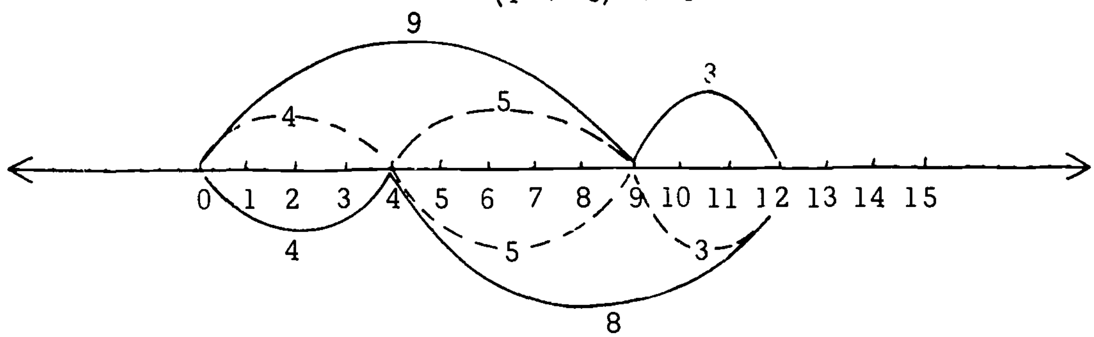
$$(2 + 3) + 4 = 2 + (3 + 4)$$

that $5 + 4$ and $2 + 7$ name the same number, they realize they do not need parentheses. The teacher also will realize the importance of the associative principle of addition as it relates to what is commonly referred to as "carrying" in the addition operation covered in a later grade. The first worksheet contains number sentences illustrating "associativity".

The associative principle of addition can be illustrated by the use of the number line; for example

$$(4 + 5) + 3 = 4 + (5 + 3)$$

$$(4 + 5) + 3$$



$$4 + (5 + 3)$$

In essence, we are illustrating on the number line that "$9 + 3$" and "$4 + 8$" are names for the same number, 12.

The next two worksheets contain number sentences using parentheses in both addition and subtraction operations. The final two worksheets supply number sentences in which the form is varied and is somewhat cumulative. Caution should be exercised in the administrating of these two worksheets as they are usually difficult for the average student and mastery in performance is not expected at this level. If frustration is evident, these last two worksheets could be considered as optional or only for the superior students.

Worksheet 21 Addition

1. $(5+1)+2=\square$ $\quad 5+(1+2)=\square$

2. $(6+0)+3=\square$ $\quad 6+(0+3)=\square$

3. $(2+5)+3=\square$ $\quad 2+(5+3)=\square$

4. $4+(3+2)=\square$ $\quad (4+3)+2=\square$

5. $3+(2+5)=\square$ $\quad (3+2)+5=\square$

6. $\square=2+(3+1)$ $\quad \square=(2+3)+1$

7. $9=\square+(2+3)$ $\quad 9=(\square+2)+3$

8. $(3+4)+5=3+(\square+5)$

9. $(6+0)+\square=6+(0+4)$

*10. $9-(5-4)=\square-8$

*Careful!

Worksheet 22 Addition

$(2 + 2) + 2 =$ ___

$(1 + 2) + 3 =$ ___

$(2 + 3) + 1 =$ ___

$(3 + 1) + 2 =$ ___

$(2 + 1) + 3 =$ ___

$(3 + 2) + 1 =$ ___

___ $+ (2 + 2) = 6$

___ $+ (2 + 3) = 6$

___ $+ (3 + 1) = 6$

___ $+ (1 + 2) = 6$

___ $+ (1 + 3) = 6$

___ $+ (2 + 1) = 6$

___ $+ (1 + 1) = 6$

___ $+ (1 + 4) = 6$

___ $+ (4 + 1) = 6$

___ $+ (1 + 0) = 6$

___ $+ (0 + 1) = 6$

___ $+ (5 + 1) = 6$

___ $+ (1 + 5) = 6$

___ $+ (5 + 0) = 6$

Worksheet 23 Addition and Subtraction

$(6-2)-3=$ ___	$(2+2)-$ ___ $=1$
$(5-3)-1=$ ___	$(2+3)-$ ___ $=4$
$(3-1)-2=$ ___	$(1+1)-$ ___ $=0$
$(6-1)-3=$ ___	$(4+1)-$ ___ $=2$
$(3-2)-1=$ ___	$(2+2)-$ ___ $=3$
$(4-1)-1=$ ___	$(0+1)-$ ___ $=0$
$(5-1)-4=$ ___	$(3+1)-$ ___ $=0$
$(6-4)-1=$ ___	$(3+4)-$ ___ $=6$
$(5-1)-0=$ ___	$(3+1)-$ ___ $=4$
$(5-0)-1=$ ___	$(6+0)-$ ___ $=5$
$(6-5)-1=$ ___	$(1+5)-$ ___ $=0$
$(6-1)-5=$ ___	$(4+1)-$ ___ $=0$
$(6-5)-0=$ ___	$(1+5)-$ ___ $=6$
$(6-0)-5=$ ___	$(6+0)-$ ___ $=1$

Worksheet 24 Subtraction

$4 - 1 = \_\_\_$	$5 = \_\_\_ - 1$
$4 - 4 = \_\_\_$	$5 = \_\_\_ - 4$
$5 - 3 = \_\_\_$	$5 = \_\_\_ - 3$
$5 - 2 = \_\_\_$	$5 = \_\_\_ - 2$
$\_\_\_ = 4 - 1$	$(3 - 2) - 1 = \_\_\_$
$\_\_\_ = 4 - 4$	$(4 - 1) - 2 = \_\_\_$
$\_\_\_ = 5 - 3$	$2 - 2 = \_\_\_$
$\_\_\_ = 3 - 2$	$\_\_\_ = 2 - 2$
$4 - \_\_\_ = 3$	$\_\_\_ = 2 - 1$
$1 - \_\_\_ = 0$	$\_\_\_ = (5 - 2) - 2$
$5 - \_\_\_ = 5$	$5 = \_\_\_ - 2$
$3 - \_\_\_ = 2$	$0 = 2 - \_\_\_$

$1 = (5 - 2) - \_\_\_$

Worksheet 25 Addition

$4 + 1 = \_\_\_$	$5 = \_\_\_ + 1$
$1 + 4 = \_\_\_$	$5 = \_\_\_ + 4$
$2 + 3 = \_\_\_$	$5 = \_\_\_ + 3$
$3 + 2 = \_\_\_$	$5 = \_\_\_ + 2$
$\_\_\_ = 4 + 1$	$(2 + 2) + 1 = \_\_\_$
$\_\_\_ = 1 + 4$	$(2 + 1) + 2 = \_\_\_$
$\_\_\_ = 2 + 3$	$(1 + 2) + 2 = \_\_\_$
$\_\_\_ = 3 + 2$	$\_\_\_ = (2 + 2) + 1$
$4 + \_\_\_ = 5$	$\_\_\_ = (2 + 1) + 2$
$1 + \_\_\_ = 5$	$\_\_\_ = (1 + 2) + 2$
$2 + \_\_\_ = 5$	$5 = (\_\_\_ + 2) + 1$
$3 + \_\_\_ = 5$	$5 = (2 + \_\_\_) + 2$

Suggested Activities on Evens and Odds

The following activities are designed to teach the children the difference between "odd" and "even" numbers.

The technique for beginners to use in determining whether a number is odd or even is to pair the members of a set.

If each member can be paired with exactly one other member, then the number is even. If one member is left alone, the number is odd.

Example A A set with 3 members

ball block chair

Put any two members together to form a pair, e.g., ball-block. That leaves chair. Since there is no other member of this set to pair with chair, then it is evident that a set with 3 members has an "odd" number of members. Therefore, 3 is an odd number.

Example B A set with 4 members

Ball block chair eraser

The members of this set can be paired and no member is left alone. Therefore, 4 is an even number.

1. Provide several examples, such as those given above, to allow opportunity for children to discover which numbers are odd and which are even. Use concrete objects for this discovery.

   Be sure the children understand that the result is the same regardless of how the members are paired.

2. Remind children of the map of the school district. Say, "You put your houses on it, didn't you? Does everyone's house have the same address? Of course not; the addresses are all different. Did you know that houses have what we call 'odd' or 'even' numbered addresses?"

"A house number like 7 is odd, for if we make 7 marks, and try to pair them, what happens?" (Have a child demonstrate on chalkboard or flannelboard.) "The one left over cannot be paired with another mark, for we have no more; therefore, the mark is 'odd' - it is different from the others. If a house number is 8, this is an 'even' number because we don't have 1 left over." (Have a child demonstrate on board.) "Can you think of some other numbers that are odd or even?" (Have some children list them on the board as they think of them.)

3. Have the children list even-numbered sets and odd-numbered sets. For example, even-numbered: the set of eyes; odd-numbered: the set of fingers on one hand.

4. Magazine pictures containing odd or even numbers of objects may be put into "odd" or "even" piles. The children should tell how the odd numbers can be made even and how the even numbers can be made odd (the new resulting number should also be told).

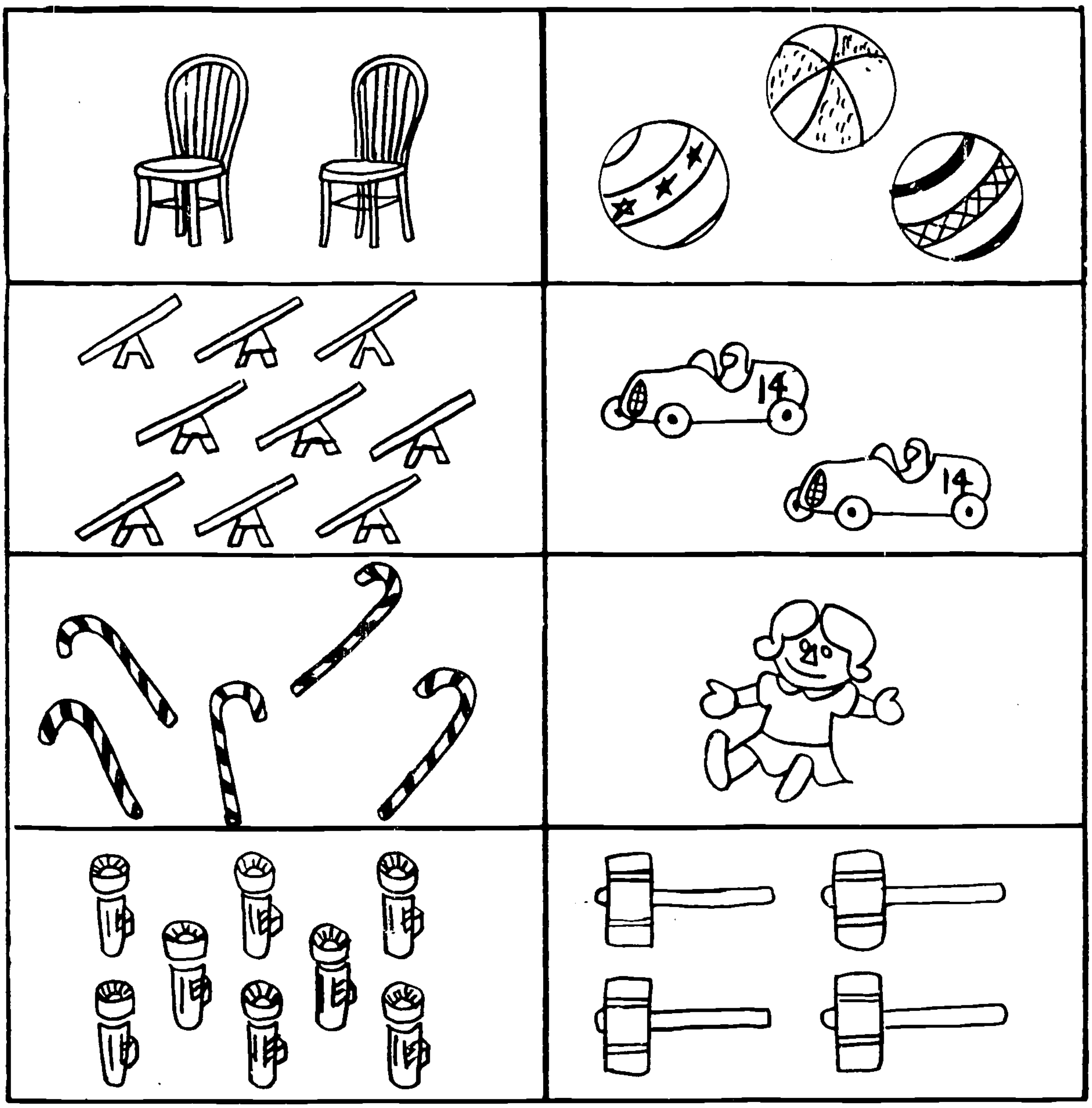
5. Give the children Worksheets 26 through 29. After the directions have been read, and are understood, these worksheets can be worked independently.

Worksheet 26 Evens and Odds

Cut out the pictures on the next page and paste the sets with an even number of members in the column under the word "even". Paste the sets with an odd number of members in the column under the word "odd".

even	odd

Cut-outs for Worksheet 26   Evens and Odds

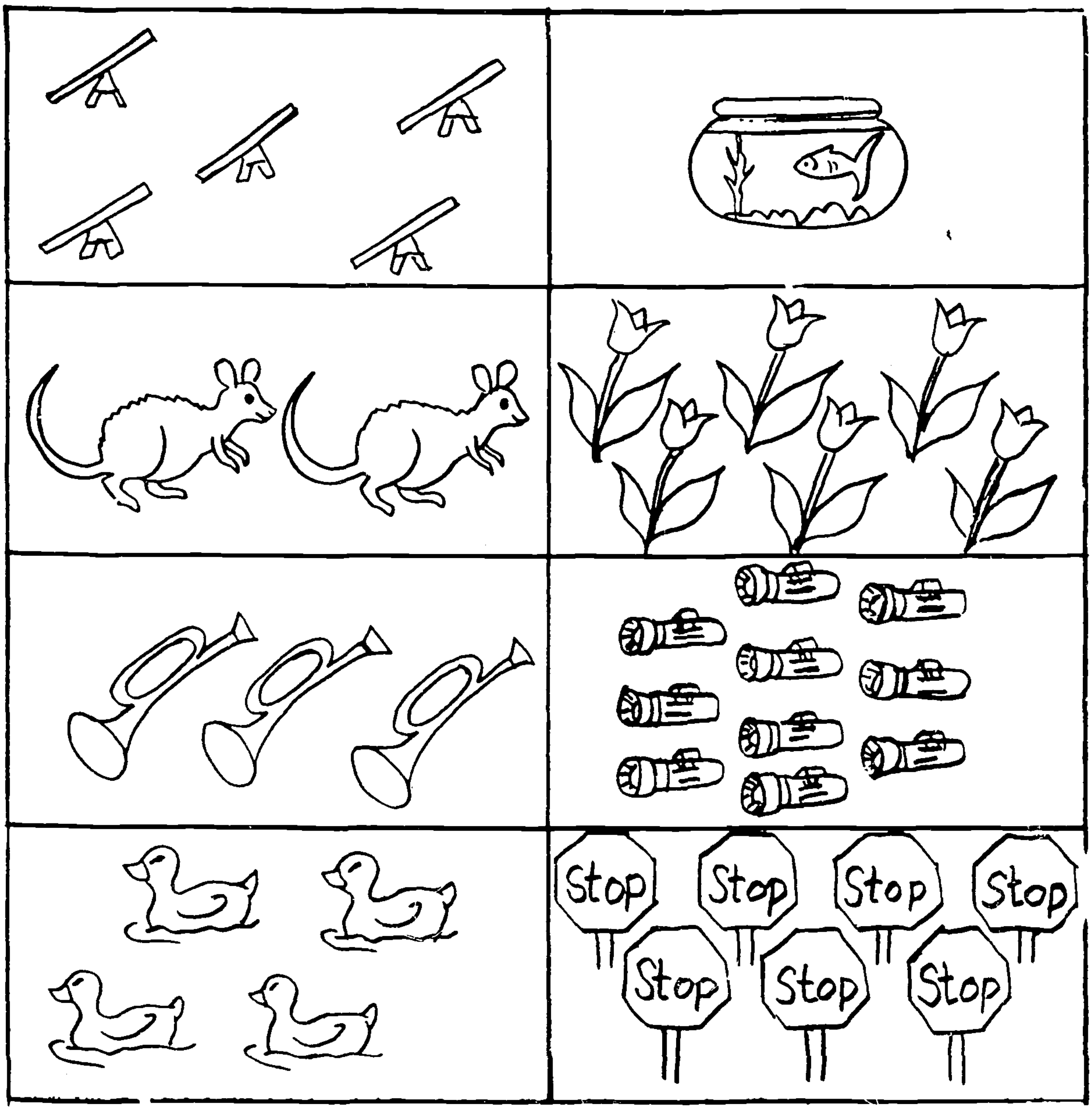


Worksheet 27 Evens and Odds

Cut out the pictures on the next page and paste them in the column where they belong.

odd	even

Cut-outs for Worksheet 27 Evens and Odds



Worksheet 28 Evens and Odds

Cut out the boxes of numerals that are on the next page. Paste them in the odd or even columns where they belong.

odd		even	

Cut-outs for Worksheet 28 Evens and Odds

1	2	5	9
6	3	10	4
9	8	1	7
0	7	2	10

Worksheet 29 Evens and Odds

Draw an odd number of objects in each box in the odd column. Draw an even number of objects in each box in the even column. Draw what the word in each box tells you to draw.

odd	even
trees	balls
houses	airplanes
boys	cars

Suggested Activities on An Addition Machine

This is a follow-up activity to the use of the "double number line" (addition slide rule). In a certain sense, an addition slide rule is an "adding machine". There is a great advantage in having the student practice selecting the sum from two given addends. Patterns of numbers appearing in the sums should be recognized by the students and accepted by the teacher.

Working with the "Addition Maçhine" should be a discovery experience for the child. He has previously been introduced to many operations which he will not be able to discover in a different context and in depth. In presenting this activity to the children, the teacher should be conscious of the fact that there is much more in this chart than first graders will be expected to recognize. The patterns they do not see at this time should not be "told" to them, although the teacher may give cues if she feels a child is on the verge of a discovery. Let the children know that there are so many interesting patterns on the addition machine that they may even find some that the teacher has not discovered.

Build an Addition Machine on the chalkboard. Begin by writing the numerals in the headings of the first row and the first column as in the figure below.

Column

	+	0	1	2	3	4	5	6	7	8
Row --->	0									
	1									
	2									
	3									
	4									

Print the 0 in the row heading and the 0 in the column heading and write the sum of 0 + 0 in the box where the 0 row and 0 column intersect. Skip some boxes and point to the 3 in the row heading and the 1 in the column heading and write the sum of 3 + 1 in the box where the row and column intersect. Do this a few times without telling the class how you are getting your answers. Tell the children that you would like to give them a chance to show how clever they are by how quickly they notice the number pattern you are making on the chart. Point to any empty box on the addition machine and ask if anyone could tell what numeral should be written there. If no answer is given, write in the correct numeral and proceed at random to any other empty box. As soon as the child notices the process you are performing and a pattern in the numerals you are writing, he should tell which numeral comes next. You will be able to judge from this whether the child actually sees a pattern developing. When a child does catch on to the pattern, make a game of not telling the "secret" to the others. (Placing several numerals in consecutive order may be a help to the children who are slow in noticing a pattern.

Continue in this fashion, with the teacher or the child writing in the numerals until the entire chart has been completed. (See Teacher's Copy.)

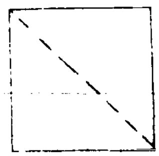
Discuss with the children how they were able to tell which numerals were to be placed on the chart. Ask them what patterns they noticed. (It may be fun to keep a tally count of the many different patterns which may be suggested.) Have them demonstrate with the chart how the numerals fit into their individual patterns. (Strips of paper, the length of the chart, may be used to cover up the other rows and columns to help direct attention to a certain portion of the chart.) When opportune, give clues which may give further direction to the children's thinking. The teacher ought to lead the class to discuss and investigate the patterns as they are demonstrated by the children. Some of the patterns which are meaningful, because of the material the child has just covered in the Minnemast units, are listed below. The class may see some or all of these patterns at this time. The important thing is that they come from the child's own discovery.

An Addition Machine

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

1. Mirror Symmetry

If the addition machine were folded on the diagonal, the numerals on one half would be the mirror reflection of the numerals on the other half.

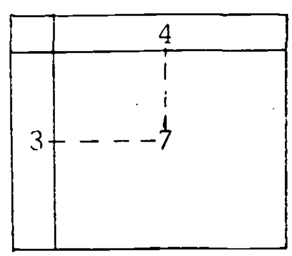


This would imply that if you were constructing the addition machine you would have a short cut for your thinking. You would need to think through just one-half of the answers.

2. Commutativity

Since this is an addition machine you can think of the numerals in the top row and first side column as addends. If you trace your finger down the column and the row, the numeral at the intersection would be the sum. The sum would be the same if you began with either the row or the column first.

3 + 4 or 4 + 3



The same form could be used to demonstrate subtraction, which is, actually, the process of finding the missing addend.

3. Diagonal Relationship of Identical Sums

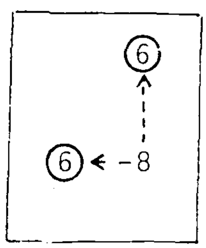
An obvious pattern to notice is that the same numeral appears on the diagonal connecting a row with its corresponding column.

+	0	1	2	3	4	
0			2		4	
1		2		4		
2	2		4			
3		4				
4	4					

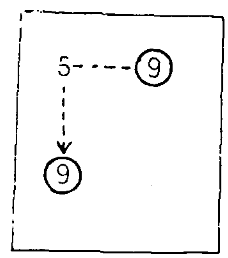
4. Addition Undoes Subtraction

Point to a numeral on the machine. Add two and subtract two. With your finger trace down two spaces for the addition of two and to the left two spaces for the subtraction of two. Note how the subtraction undoes the addition. It could also be shown in this way that addition undoes subtraction.

6
add 2
subtract 2



9
subtract 4
add 4



5. Odds and Evens

Look at the diagonal pattern. If the first numeral in the row or column is odd or even the other numerals on that diagonal will be odd or even also.

+	0	1	2	3	4	5
0		1				
1			3			
2				5		
3					7	
4						

+	0	1	2	3	4	5	6
0							
1							
2	2						
3		4					
4			6				

The diagonals from upper left to lower right also contain alternate sets of evens and odds.

Looking down a column it may be discovered that adding 1 to every odd number gives an even number, and adding 1 to every even number gives an odd.

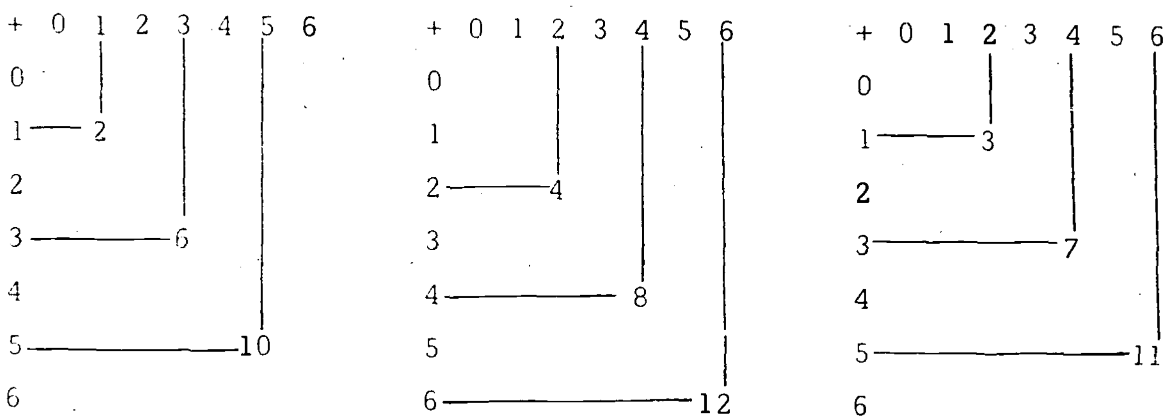
+	0	1	2	3	4	5	6	7
0	0	1						
1	1	2						
2	2	3						
3	3	4						
4	4	5						
5	5	6						

Note the pattern in the addition of odds and evens:

When adding 2 odd numbers the sum is an even number.

When adding 2 even numbers the sum is an even number.

When adding an odd and an even, the sum is an odd number.



6. After a general discussion of the Addition Machine the children may be given Worksheet 30, and asked to fill in the numerals, being conscious of looking for new patterns. (The large chart on the chalkboard should be erased before the children begin their individual

sheets.) At a later time each child may choose a partner with whom he may share and discuss the pattern discoveries he has made, and the ones he remembers from the class discussion.

7. After the children have their own "addition machines", do several exercises with the class such as the following:

   Put index finger of the right hand on the 5 column, and the index finger of the left hand on the 3 row.

   Slide the fingers down the column and across the row until they meet.

   The box that forms the intersection of row 3 and column 5 is the sum of the two numbers.

   Continue using other addends until children are familiar with the use of this "addition machine" to find the sum.

8. Choose a sum.

   Have children find all of the different addends that make that sum.

MY ADDING MACHINE

ROW →

+	0	1	2	3	4	5	6	7	8	9	10
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											

Suggested Review Activities

The following activities may be used for review and reinforcement whenever the teacher feels a need for it.

1. Play games with flash cards.

   Use a pack of flash cards with all number combinations from 0 through 10. (Answers should not be provided on either side of the card.) If there are children in the class who are capable of adding numbers with sums up to 20, include these cards in the pack. If this is not the case, remove them. Shuffle the cards and pass 2, face down, to each child. Tell the class that they may not look at their cards until you say "Add". Have the children work with partners. Each child places his two cards on his desk and, at the word "Add", turns them both over and finds the correct numeral card to express the answer to each problem. When both children are finished, each child checks the other's answers. If there is a disagreement, the teacher should be consulted. The number line should be used to illustrate the correct answer.

   Another day, do the same thing, but let each couple keep their scores. (The next time pair off the best players as partners, etc.)

   Variation I
   The same game may be played as a team game. Divide the class into two teams. The teacher is the referee who checks answers for correctness. This time only one card is passed out to each child. The teams alternate turns. At his turn, each child turns over his card, and then finds the numeral card that shows the sum. If his answer is correct, he scores 2 points. If not, he must wait

his turn again and then try another answer. If this answer is correct he scores 1 point. Encourage the use of the number line for those who must try again.

Variation II

In this variation, two children from each team turn over their cards simultaneously. The first one to hold the correct answer above his head scores a point. If the first child gets it wrong and the second child gets it right, the second child gets the point. (Their examples will probably be different, so their sums will not agree. The teacher will need to check both the addend card and the numeral card that indicates the sum.)

Variation III

Provide each child with paper and distribute ten cards to each. Instruct the children to copy their examples on their papers arranging the addends as they are on the cards, but have them add the + sign, i.e.,

$$\begin{array}{r} 4 \\ +\ 2 \\ \hline \end{array} \qquad \text{(or he may wish)} \quad 4 + 2 =$$

Then have them print the sum on their papers. When they have done this with all of their cards, they should turn in their papers for marking.

Another day mix the cards and do it again.

Variation IV

The first few times this is done, the papers should be corrected by the teacher. Later this could be a race. The first child to finish all ten problems will read them aloud to the class and the other members of the class will check his answers. If all ten are correct, he wins the race. The other class members can go on to finish their problems either at that time, or during activity time and check each others work. If there is disagreement the teacher should help.

2. Play the game "Magic Box" to provide practice in addition. Provide 1 set of cards with numerals 1 through 10 and a large cardboard box in which a child can hide. Put a slip in the top or side of the box, large enough to slide the numeral cards in and out. Provide 2 sets of cards with numerals 1 through 5. One child hides in the "Magic Box" with the numeral cards 1 through 10.

   Another child selects two numeral cards, from the packs of 1 - 5, and puts them in the Magic Box slot. He then taps the box and says,

   "Hocus, pocus,
   Magic Box,
   Give me an answer
   To my knocks".

   The child in the box is to select the card that tells the answer and put it back through the slot with the 2 "problem" cards.

   Example: One child puts a 3 card and a 2 card into the Magic Box slot, knocks, and says the little rhyme. The child in the box pushes back the 3 card and 2 card and then the card with the answer - 5.

   The child remains in the box until he makes an error or has three turns. If he makes an error, the person who put the cards in the box takes his place. If he has three turns without an error, he may choose his replacement.

Children putting the cards into the box should hold them high to let the rest of the class see them before they go into the Magic Box. As the cards come out, the class should be shown them again and they should read aloud "3 plus 2 equals _____ (Whatever card has been pushed out as the answer.) The person showing the cards should then ask the class, "Is it right?" If it is, he chooses another child for a turn to put cards into the Magic Box. If the answer is wrong, he becomes the Magician in the Magic Box.

(If a cardboard box isn't available, a desk can be used by covering the sides with paper so that the "Magician" cannot be seen.)

3. The following activity should be stimulating to the children. However, it is not placed in this unit because of its appeal as a motivated, real life situation. It is a readiness activity for topics in later units, such as predictions and probability. The number of different ways an event may take place (in this case, the purchase of the notebooks and rulers) is basic to the understanding or probability.

Put on the chalkboard:

	BIG SALE	
<u>Notebooks</u>		<u>Rulers</u>
5¢ each		5¢ each
	SALE	
1¢ off		2¢ off
	NOW ONLY	
4¢		3¢
Not more than <u>3</u> of each to a customer		

a. Choose a child to be Tom. Give him a dime and let him decide how to spend it.

b. Do this with different amounts of money and different children.

c. Have the class decide how many different combinations are possible if a child has 25¢.

Answer: (16 combinations)

Notebooks	Rulers
1	0
1	1
1	2
1	3
0	0
0	1
0	2
0	3
2	0
2	1
2	2
2	3
3	0
3	1
3	2
3	3

d. Have the children list the different combinations possible if a child has 25¢ to spend and the limit is 5 of each to a customer. (Note: The number of combinations is a very large number.)

Worksheet 31 Number Line

1. The books are 10¢ each. Sally wants 2 books.
   How much money will she need? ☐

2. The pencils are 5¢ each. Dick wants 3 pencils.
   How much money will he need? ☐

3. The balls are 6¢ each. Jane wants 2 of them.
   How much money will she need? ☐

4. The toy cars are 3¢ each. The 3 children each want one. How many cars will they get? ☐

   How much money will they need? ☐

5. The dolls are 8¢ each. Sally wants 2 of them.
   How much money will she need? ☐

6. The ice cream cones are 10¢ each.
   How much money will 3 children need? ☐

7. The toy airplanes are 6¢ each.
   What will 2 airplanes cost? ☐

Worksheet 32 Number Line

1. Tom had 5 marbles and lost 2.
   How many marbles did he have left? [ ]

2. Susan got a pencil for 4¢ and an eraser for 1¢. How much money did she spend? [ ]

3. Dick, Jane, and Sally each had a ball. How many did they have all together? [ ]

4. Nancy and Alice each had 2 dolls. When they put their dolls together how many did they have? [ ]

5. Jerry and Dick play with airplanes. Jerry has 2 and Dick has 1. How many do they have together? [ ]

6. Dick has 7 rocks. Tom has 5 rocks.
   How many have they together? [ ]

7. Dick and Tom give 4 of their rocks to Jack.
   How many do Dick and Tom have left? [ ]