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ABSTRACT

This literature review focuses on determining the psychological variables related to problem solving and presents arguments for self-correcting manipulatives as a media for teaching problem solving. Ten traits are considered: attitude, debilitating anxiety, self-concept, orderliness, set, confidence, impulsive/reflective thinking, concentration and interest span, motivation and interest, and perseverance and patience. Research in each of these areas as related to problem solving is cited. A list of 155 references is included. (DT)

A review of the research relating problem solving and mathematics achievement to psychological variables and relating these variables to methods involving or compatible with self-correcting manipulative mathematics materials.

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Table of Contents

I. Definition of Area of Research

In the very broadest terms the "end" of the investigation is good thinking--the quality of thinking that is necessary to solve a problem (as opposed to an exercise in multiplication requiring a rote response). The focus will be on what hampers and what permits this kind of thinking. This non-routine thinking will be referred to as problem solving. Before establishing the parameters and variables of the investigation some clarifications and delineations are in order.

There are at least two broad categories of skills in problem solving--cognitive skills and psychological skills. By cognitive skills are meant that behavior consciously directed by the problem solver: his application of his heuristics and mental skills, his generating and testing hypothesis, his recalling information, and his applying acquired skills. By psychological skills are meant those indirect factors that provide the context for the above. Some examples are: self-concept, habits, attitudes, frustration threshold, confidence, tenacity.

These traits determine to a large extent one's potential as a successful mathematics student (Dodson, 1972). The educator is faced with a very complex situation. The learning situation and the problem solver's achievement in it, along with his many cognitive and psychological factors, interact and continually change each other. In fact, problem solving proceeds through several stages and a characteristic that helps at one stage can be a hindrance at another. (Williams, 1960)

One approach is three fold: (1) characterize the good problem solver; (2) attempt to promote the development of some of the characteristics of a good problem solver; (3) if successful in promoting these characteristics, determine if the result is a better problem solver.

The focus of this investigation is limited to determining (1) the psychological variables related to problem solving, and (2) the effect of using self-correcting manipulative mathematics materials and compatible teaching situations on these psychological variables.

2. Search of research sources.

The starting points were ERIC and the 1970 Encyclopedia of Educational Research Mathematics. "Mathematical models," its various synonyms and particulars, and "Elementary School Mathematics" were the initial descriptors. Subject indexes were used when possible to review the following journals: The Mathematics Teacher, 1908-1965 (Cumulative Index used), The Arithmetic Teacher 1954-present, Journal for Research in Mathematics. 1970, No. 4--1972, No 4; 1973, No. 4-Present, Educational Studies in Mathematics 1967-Present. The coverage was from Volume I whenever possible. The Journal for Structural Learning which deals with manipulative materials was unavailable locally.

The listings of research on Mathematics education, published annually by The Arithmetic Teacher from 1957 to 1970, then transferred to the newly created Journal for Research in Mathematics Education, were the most useful sources. They include Research Summaries, Journal Published Reports, and Dissertation Abstracts; the entries in these listings for some years include a very brief abstract.

The National Council of Teachers of Mathematics thirty fourth yearbook, Instructional Aids in Mathematics, and its bibliography were another origin. In fact bibliographies continually generated new leads. A letter to the Cuisenaire Company resulted in a prompt reply which included a bibliography. Four of the most important sources came from bibliographies, namely Beougher, Dodson, Williams, and Canadian Council for Research in Education (CCRE).

3. Research findings relating problem solving and mathematics achievement to psychological variables, and relating these variables to methods involving or compatible with self-correcting manipulative mathematics materials.

The author immersed himself in the literature and research of problem solving, manipulatives, and learning theory. Some research (Jeeves, Jeeves and Dienes, J.B. Biggs, Wason and Johnson-Laird) was undertaken to resolve or to substantiate various theoretical questions. Most studies compared the achievement of one or more approaches with manipulatives and the traditional approach. Those advocating heuristics and manipulatives held similar theoretical positions. They favored construction theories such as Bruner and Piaget. The child interacts with the environment and builds and modifies his mental model of the world. This model consists of operations as well as information. The manipulatives form a contrived environment which is supposed to aid and guide the student in this construction.

Problem solving: Problem solving by its very nature calls upon many, many talents. Success is influenced by both cognitive and psychological factors. It is multi-staged and requires the higher order cognitive skills of anyalysis, synthesis, and evaluation. To solve a problem the problem solver must understand the problem, seek relationships, generate hypotheses, and evaluate them. For instance, to understand the problem and be able to state it often is the hardest part whereas in most mathematics problems the problem is already formulated. Williams (1960) discusses three categories

that determine success in problem solving: (1) factors of the situation, (2) factors of the problem solver's experience, and (3) characteristics of the problem solver.

Polya in How To Solve It classifies heuristics under four stages. Problem solving requires not only following a set sequence but also judging what heuristic is most appropriate for "this problem" at "this stage". Achievement and problem solving are different. Achievement is generally a measure of rote learning. Teaching procedures that give the best results in terms of rote learning do not work well in promoting problem solving ability. O'Brien (1973) suggests that emphasis on rote learning hampers the child in developing relational thinking by reinforcing his natural atomism.

Definitions of a problem. The definition used in Dodson's (1972, page vii) Characteristics of Successful Insightful Problem Solvers is: "By insightful, the Panel meant non-routine, challenging mathematics problems. Such problems require the student to transfer knowledge, skill, and background to unpracticed contexts or to use their mathematics in novel ways." In discussing earlier research Dodson notes that in almost all the prior studies, problem solving in mathematics meant solving word problems. This is particularly true of studies at the elementary level.

Arguments for self-correcting manipulatives as a media for teaching problem solving. Manipulatives are compatible with many learning theories. They are concrete and sensorial. They are compatible with what is known about problem solving. Problems with structural materials easily present situations that promote generalizing, self-checking, inductive and deductive reasoning, and independent work.

They give feedback so the student can modify, clarify, and retest his position. They can be used privately and allow the student to set his own pacing. Self-pacing is usually held most efficient for teaching complicated techniques (Williams, 1960). Trial and error, a good problem solving approach, is made easier by manipulatives. It is more convenient, easier, and quicker to manipulate than to draw. Manipulatives are more accurate, particularly for three dimensional problems, and they develop spatial ability, which is closely related to problem solving. (Dodson, 1972)

Manipulatives embody many mathematical concepts. The terminology of mathematics such as "square numbers" shows the close tie between geometric forms and algebraic group structures. The Arabic decimal notation is clearly embodied in Dienes's Multibased Arithmetic Blocks and the Montessori bead material.

Problem solving can be improved by separating the production of hypotheses from their evaluation. A too critical frame of mind impedes production of hypotheses (William, 1960). John Bates's (talk at Belleville Area Teacher's Center, 1973) "divergent tasks" lend themselves to hypothetical production as does Gattegno's "pedagogy of situations" referred to by Madame Frederique Papy. A quote from the Comprehensive School Mathematics Program overview clarifies this term: "The humanistic philosophy and the functional-relational approach combine especially well with a pedagogy of situations, in which the imagination of the children is captured by an easily-imagined situation presented to the children. In this environment the children eagerly make suggestions to solve problems presented them; they are motivated by their own freedom to think and the intellectual inter-

action with their peers". (For a more extensive discussion see Frederique's Creative Freedom, chapter 4.) Other teaching methods that lend themselves to using manipulatives are the "math lab" and the "discovery approach".

Traits Reviewed. These traits are not disjoint. Some will be discussed together if they are close in meaning or are at opposite ends of a continuum. Characteristics that cannot be manipulated in an educational setting such as sex and social economic class will be omitted. The traits included in this review are: attitudes, anxiety, self-concept, orderliness, set, confidence, impulsive/reflexive thinking, concentration and interest span, motivation and interest, and perseverence and patience.

In order to measure the first four traits above the National Longitudinal Study of Mathematics Achievement (NLSMA) staff developed the NLSMA Attitude Inventory. Dodson's (1972) report is the most comprehensive study of problem solving. He started with a 10% stratified random sample of the NLSMA Z-Population (23,645 when in 10th grade). Students not taking 11th grade mathematics were deleted. This deletion combined with attrition resulted in a final sample size of about 900 that completed the NLSMA attitude inventory.

In the table (page 8) subscales are shown along with a summary of the results:

NLSMA Attitude Inventory Results for Insightful Problem Solvers

	Level of significance—1 way ANOV				
	Total Criterion Test	Algebra Subtest	Geometry Subtest	Number Subtest	Relative strength compared with all variables. Range 1-8 with 1 highest.[b]
I. Attitude Toward Mathematics					
A. Math vs. Non Math	.001	.01	.001	.001	4
B. Math Fun vs. Dull	.001	.05	.001	.001	
C. Pro-Math Composite	.001	.05	.001	.001	
D. Math Easy vs. Hard	.001	.01	.001	.001	
II. Mathematics Test Anxiety					4
A. Facilitating Anxiety	.001	.01	.001	.01	
B. Debilitating Anxiety	.001	.001	.001	.001	
III. Self-Concept					4
A. Ideal Math. Self-Concept[a]	.001	.001	.01	.001	
B. Actual Math. Self-Concept	.001	.05	.001	.001	
IV. Orderliness					
A. Orderliness[a]	.01	not sign. at .05	not sign. at .05	.01	6
B. Messiness	not sign. at .01	not sign. at .05	not sign. at .05	not sign. at .05	8

a. negative relationship

b. Categories by relative strength are: Mathematics Achievement, Cognitive Reasoning, Other Cognitive, Personality, Teacher Background, Teacher Attitude, School and Community.

Attitude. The author found no research directly relating problem solving and attitude toward mathematics. The expectations of the math community that the "new math" and the "discovery method" would improve attitudes toward and about mathematics have not been fulfilled. Possibly the increased difficulty of the new programs counteracted any positive effects. Also most of the teachers taught the "new math" just like they had the old.

In his review, "Current Research in Mathematics Education", Romberg (1969) gives three reasons why attitude studies have not been more fruitful. Namely the shortcomings are the tests and the lack of a sound theory, the common use of a single, global measure of a set of predispositions or feelings, and the ignorance of what procedures might modify attitudes.

Even so the research on attitudes and mathematics has been enormous. Romberg commends Aiken's (1969) careful and critical review. The findings have not been very encouraging. This is clear from Neale's 1969 article, "The Role of Attitudes in Learning Mathematics." Neale also does a good job teasing out the various aspects of attitudes by analyzing the instruments. Presently students' attitudes toward mathematics become increasingly negative and there is little or no effect on mathematics achievement except in the cases where the attitude is either extremely positive or negative.

If one wishes to review attitudes they should refer to the research listings and summaries by Riedesel, Suydam, and Weaver.

As mentioned earlier achievement and problem solving are different. Out of fourteen attitude factors tested, Cattell and Butcher (Neale, 1966) found "submissiveness" to be the strongest with a +.50 correlation with achievement! This characteristic would surely impede hypothesis production! Furthermore curiosity was found to have a negative correlation (-.20) with achievement.

The methods involving or compatible with self-correcting manipulative mathematics materials fared much better than the "new math" in promoting favorable attitudes. Canadian and English teachers have overwhelmingly agreed that children ejoy mathematics more with Cuisenaire rods than with traditional methods and have a better attitude toward school in general (OCRE, 1964; Howard, 1957). The OCRE study consisted of over 50 comparisons employing at least 15,000 students plus eight teacher surveys involving some 600 teachers who have used the Cuisenaire rods. Howard visited 21 classes talking to 31 British teachers. Surely the judgement of such a large number of teachers directly involved cannot be discounted. J.B. Biggs (1965) reports that preliminary findings in an investigation by the National Foundation for Educational Research in England and Wales (NFER) indicates that use of uni-models such as Cuisenaire rods improves attitude only slightly over the traditional approach except for highly intelligent boys, but that multimodels such as Dienes's MBA yield a marked improvement in attitudes. On the other hand (Beougher, 1967) reports that Mott, Schott, Jamison, Anderson, and Swick generally found no significant effect either way by manipulatives and Green (1970) reports that a diagram approach to fractions was

superior to a manipulative approach with respect to attitudes. A math lab with manipulatives and a Piagetian curriculum involving active manipulation resulted in significantly more positive attitudes than other methods with which they were compared, so Whippel (1972) and Johnson (Suydam and Weaver, 1973) reported. Hollis (ED, December, 1972), Vance (1971), Wasylyk (Vance, 1971), Vance and Kieren (Suydam and Weaver, 1973) report more positive attitudes with math labs while Wilkinson (Suydam and Weaver, 1973) found no change. Burgerr (Suydam and Weaver, 1971) found the regular use of mathematical games improved attitudes. Robertson (Suydam and Weaver, 1972) reports that the discovery approach gave significantly better attitudes than the expository approach.

More than anything this research shows the possibilities inherent in this approach. It also cautions us that it is not a magic method. The teacher is the key variable. Vance and Kieren (1971) report on three math labs, each reflecting a different way os using laboratory activities. The integrated way reflected the best organization and planning. The results were superior to the other two ways on every count. Enough said?

Debilitating anxiety has a strong negative relationship to insightful problem solving (Dodson, 1972). It was the strongest predictor of all the personality variables being significantly related to the total criterion test and all the subtests. Sowder (1974) using the NLSMA Y-population found it to have "very strong discriminating strength" with respect to geometry achievement. Williams (1960) reports that subjects scoring high on the Taylor

Manifest Anxiety scale are more rigid than others and that normal subjects become more rigid when they are made to feel anxious or too ego involved in the problem solving activity. Forhetz (1970) found test anxiety to vary with respect to ranked difficulty of subject as perceived by the student. Johnson (Kilpatrick, 1969) found an interaction of test anxiety and test difficulty on arithmetic reasoning. Both Beeson and Beavers (Suydam and Weaver, 1971) found test anxiety affected scores. Szetela (Suydam and Weaver, 1972) agreed that even with high IQ students test anxiety appeared to interfere with mathematics learning, but Flynn and Mauser (Suydam and Weaver, 1972) found no significant differences between gifted students at any anxiety level.

Natkin (Aiken, 1969) demonstrated that it is possible to affect anxiety and attitude in a short time by associating mathematics with something pleasant. Both Williams and Biggs (Kieren, 1971) report that a traditional approach produces higher number anxiety than a unimodel or multimodel approach. Davis, Sutton, and Smith (Kieren, May 1971) report that manipulatives and play-like activities "can provide an information seeking, non-authortarian environment." Anxiety is related to rigidity, reports Williams (1960), who recommends that intrinsic rather than extrinsic motivation be used in order not to reduce flexibility. Montessori (1969) agrees. Biggs (1965) claims extrinsic motivation is effective only for assimilation learning and that intrinsic motivation is needed for constructive accomodation type learning.

The facilitating and debilitating aspects of anxiety are illustrated in these remarks by Williams (1960). The anxious student will probably strive hard and be successful if only acquisition of knowledge is important. But where originality of response is required as in problem solving, the inflexibility of his thought and inability to suspend judgement might be a severe handicap.

Clark (1971) found no interaction between anxiety and feedback whereas Beeson (personal conversation, 1973) found anxious students did progressively worse whereas confident students were able to utilize the fact that certain previous statements were wrong in deducing answers to later questions.

In his dissertation Forhetz (1970) discusses those aspects of anxiety that are generally agreed upon by all theorists. This section will be concluded by mentioning three that seem to apply. (1) Uncertainty is the key to the occurrence of anxiety. (2) Anxiety usually occurs in conjunction with other effects, such as defensiveness. (3) Anxiety is usually debilitating especially with respect to complex tasks.

Self-concept is not well defined. Many diverse measures are used. Hopefully in the future its various components will be varied separately so their effects and interactions can be accurately determined. It is for this reason conflicting results are expected.

Bernstein, Alpert (Aiken, 1969) contend that feelings, expectations, performance, and self-concept form a self-perpetuating cycle. As early as the third grade academic success is related to the way a child perceives his world and his relation to it. This relationship is somewhat independent of ability, Haggard (Cleveland et al, 1967)

found. The most striking results are in problem solving and concept development, which are closely related to good personality adjustment (Cleveland et al 1967). See Suydam and Weaver (1971, 1972) for studies by Alberti, Bachman, Loguidice, and Schneider.

Ruedi and West's (Trimmer, 1973) hypothesis that an open school would produce a better self-concept than a traditional school was not supported. Pagni et al (1973) in his paper, "The Brookhurst Project—A Mathematics laboratory that works", found a significant psoitive change in self-concept with respect to mathematics and English. Two other math labs in the same district revealed no such change. Pagni attributed the difference to extensive planning, curricula tailored to student's needs, and hard work. In listing the chief benefits of Cuisenaire rods Ellis (CCRE, 1964) says: "The child is unrestricted. He is free to make "new" discoveries for himself. ...Check results. He soon learns to rely on his own criteria for correcting mistakes... [Children need to experience success.] ... colored rods effectively minimize failure..."

Orderliness. It is interesting to note that even though Dodson (1972) found little relation of orderliness or messiness to insightful problem solving that both the best and the worst groups were indifferent to disorder and messiness in their environment.

Set. Set, rigidity, functional fixness are all terms referring to a lack of flexibility. This is the tendency to keep applying the same method as used in solving previous problems when it is no longer appropriate or when easier methods exist, as in the Luchins Water Jar Test. (Wason, 1968)

Gilford (Williams, 1960) found flexibility important during hypothesis production. Rigidity was found related to self-confidence, intelligence, and anxiety, but not to creativity by Kempler (Kilpatrick, 1969), Wodtke (Aiden, 1969) and Williams, (1960).

The author recommends the reader read P.C. Wason's "On The Failure to Eliminate Hypothesis--A Second Look", and O'Brien and Budde's "Hypothesis Testing--A Classroom Activity". Wason used a guess-my-rule to a three number sequence activity to study subjects thinking as they generated and tested hypothesis. The subjects, overwhelmed by their hypothesis, tried to confirm it rather than to deny or modify it. They were either unable or unwilling to discard a hypohtesis even after getting contradictory evidence. One student was carried from the room in a catatonic state. His psychological history was not known but it demonstrates an extreme case of the interplay of rigidity and emotions. The implications for teaching of heuristics through group problem solving is clear from Budde's replication of the experiment. In administering the task to a group rather than to an individual, entirely different results were obtained.

Generating hypothesis is of couse a necessary but not sufficient condition in problem solving. Raaheim's (Kilpatrick, 1969) research clarified this issue. He partitioned problems into two types: (1) problems in which the goal is clear; and (2) problems in which the goal is clear but difficult to obtain. Raaheim found success in the first type to be related to the ability to <u>find many functions</u> for a given object while success in the second type is related to the ability of <u>finding many objects</u> that serve a given function.

Evidence that the teaching method can promote such needed flexibility is found in the following. Brownell (Beougher, 1967) found seven year olds using Cuisenaire rods "exhibited a high degree of flexible thinking," giving "28 ways of writing 20 without any clues, including 10 + 10, (3 x 5) + (5 x 1), 1/2 of 40, 200-180, 4 + 5 + 8 + 1 + (2 x 1), etc." Trout (CCRE, 1964) concurred. Vance (Trimmer, 1973) and Sutton-Smith (Kieren, 1969) found play-like settings to promote novel responses. It is appropriate to conclude this section by reporting that Luchins who is largely responsible for the interest in this variable noticed that children from permissive and active schools suffered less from rigid method set than children from authoritarian schbols (Williams, 1960).

Confidence. Self-confidence reflects past successes and failures. Kempler (Aiken, 1969) remarks that pupils who constantly fail mathematics lose confidence and develop hostility and dislike for the subject. Self-confidence is related to persistence. Burron (1972), using ability and cognitive level of mathematical tasks as variables, reports, "a marked difference in behavior related to self-confidence. Pupils in the low group seemed hesitant, threatened, or reluctant to respond to divergent questions. High group pupils displayed little of this behavior."

When asked about the advantages of Cuisenaire rods British teachers frequently replied (Howard, 1957) that their use resulted in a gain of confidence especially with bright students. Banta (Trimmer, 1973) found Montessori children that went into a nongraded primary school to be more assertive than two comparison groups. Vance, 1971 reports"...students [in a math lab] gain a feeling of

being able to achieve objectives in mathematics independently and come to view mathematics as having an experimental basis". Williams (1960) says that a teacher can reduce anxiety, increase persistence, preclude frustration, inspire confidence by beginning with easy problems and progressing to the difficult. He says "it is claimed" that structural methods present problems so students can cope with them, thus avoiding the development of harmful attitudes and fears that inhibit problem solving. Fennenia (May, 1973) describes manipulatives as tools for problem solving. She says their availability increases the student's self-confidence since he doesn't need to depend on the fallibility of his memory. But Williams (1960) found over-confidence correlates with the length of time wasted on unsuccessful problem solving attempts.

Impulsive/Reflective Thinking. A pupil who is impulsive, active, and heedless might be expected to do well at the production stage of problem solving but not at the evaluation stage (Williams, 1960). Using the impulsive/reflective variable defined by Jerome Kagan Cathcart et al (1969) found second and third grade students who were reflective and took longer were better in problem solving and mathematics achievement. Banta (Trimmer, 1973) found Montessori students to have more impulse control.

Concentration and interest span. Even though no attempt was found to study the effect of concentration or interest span on problem solving or learning most educators will agree that they are important.

The Montessori method for increasing concentration is to allow the student to select from the lessons to which he has already been introduced, and then to work uninterrupted. Banta (Trimmer, 1973)

confirmed the concentration of Montessori students.

Howard (1957) and Trout (CCRG, 1964) report overwhelming teacher belief that Cuisenaire students have increased attention spans. Hildebrand and Johnson (Nasca, 1966) report that second graders working with the Cuisenaire rods have an extreme interest in abstraction. They state that interest could be maintained for an hour sometimes.

Motivation and Interest. Motivation is essential to teaching problem solving (Riedesel, 1969). Suydam and Riedesel (1969) indicate that research shows greater achievement in problem solving is promoted by finding problems of interest to the pupils, and that math games increase motivation.

Children who discover their own rule are so eager to use it that they even ask for problems so they can apply their rule (Sanders, 1964). Manipulatives attract attention and stimulate curiosity, which is important in arousing intrinsic motivation (Fennema, 1973).

Sudduty's (1963) dissertation reviewed five journals from their origin to 1962. One of the three purposes most often recorded for the use of aids in teaching mathematics was for stimulation and motivation. J.B. Biggs (1965) found multimodel approaches provided the best motivation. Lamon et al (1971) states, "Most of the students [6th grade] who participated in the experiment were highly motivated to manipulate the structural embodiment [of a vector space]. These mathematics experiences generated interest and excitement during the whole experimental period [6 weeks]". Trout and Thomson (CCRE, 1964) report similar experience with Cuisenaire rods as does May (1968) for a Math Lab. Burron (1972) reports both high and low ability students preferred manipulative activities and that a change to

non-manipulative tasks evoked a drop in enthusiasm in the low ability students.

Neale (1969) using the findings of Cattell and Butcher and observations of Jackson as a basis sketches the following argument. What makes Sammy learn is not that he enjoys learning all these great things about the beauty and order of mathematics but that he wants to be an obedient person and do his duty. This is due to "the hidden curriculum which promotes the virtues of patience, compliance, and obedience." Pupils have little opportunity to pursue their interests. Learning is a job that must be done like it or not. Intrinsic motivation will play little role in learning until the institutions of learning are radically changed.

In view of the overwhelming current use of "stars" etc. as extrinsic motivation, please consider this argument taken from J.B. Biggs's article, "Towards a psychology of educative learning." Only concurring arguments from other sources will be cited.

Structural learning, learning that requires some accommodation, is intrinsically motivated, whereas extrinsic motivation is needed to motivate assimilation. If the reorganization is too drastic for the individual to assimilate, he will withdraw from the situation or resort to rote memorization. When a classroom activity ceases to become an end in itself and is no longer self-motivating and self-reinforcing, it needs external motivation.

O'Brien (personal conversations, 1972-74) senses this when he talks about classroom activities having "grabbing power" and being "self-sustaining". Maria Montessori (1969) before 1912 discouraged extrinsic motivation [also see Trimmer 1973]. Her son, Mario

Montessori (1961-62), states that the sensitive period of the child determines the interest: "What left the older children more or less indifferent aroused intense interest in the younger ones." Johntz, Davis, and Papy have made similar claims that support the earlier statement of Biggs. "...there seems to be built into cognitive activity a principle of optimal development such that these activities that the growing individual finds most pleasurable and rewarding at a given stage of development are those that will help to bring about the progression to the next higher stage."

He advises the teacher to key (my term) on the motivation level of the student for an indication of how well the prescribed educational activity meets the student's need.

Extrinsic motivation is negatively related to the breath of coding (or structured learning) for it "inhibits accommodation while generally facilitating assimilation, thus disturbing the balance between the two." Strong incentives such as competition and punishments appear to actively discourage conceptual learning.

Perserverence and patience. John Holt in How Children Fail maintains that many children cannot tolerate uncertainty and the resultant frustration. They give up and will put down anything just to escape the situation.

William J. Wright supplied the author with a thirty-three page abstract of his dissertation, The Determinants of Peristence at a Learning Task. Wright took premeasures of generalized confidence and task relevance two weeks prior to a problem-solving experiment. Data was collected during the experiment on (1) how many problems they expected to get right from the next group of five problems

and (2) how many they thought they got right.

4. Summary

Intrinsic motivation and giving the student enough time to complete a complex task will facilitate the learning of problem solving. Kilpatrick studied under Polya and is intensively interested in problem solving. In his article, "Problem Solving and Creative Behavior in Mathematics", he says: "We need to know much more about using problems to stimulate independent and creative thinking." The author feels that problems, which students of a wide range of ability and background can solve by using a method that fits their level of sophistication, would be especially fruitful.

Some progress has been made, but researchers must now concentrate more on the stages and skills of problem solving and identifying which traits are moderators for each skill at each stage. The teacher can utilize this increased understanding by giving practice of each skill involved such as finding relationships and generating hypotheses and can give the students a knowledge of the moderator traits. Polya (1957) sees "looking back" reflecting on the problem solving as essential to transfer and the development of a general skill in problem solving. Thus by helping students with introspection a teacher can help them understand the theory of heuristics and the interplay of these psychological variables so they can make an effort to manipulate them to their advantage rather than to be manipulated by them.

One last look at the variables is in order. The impulsive state is probably useful in producing hypotheses and the reflective state

is probably best for evaluating hypotheses. When the going gets tough it is concentration and persistence that pay off. It is then that confidence, lack of anxiety, lack of rigidity, flexibility, and an ability to cope with uncertainty help.

The research reviewed indicates that manipulatives can provide a medium for fostering the growth of problem solving. There are three statements in Kilpatrick's article that could be construed to support this point of view. They are: (1) "Since the solution of a problem--a mathematics problem in particular--is typically a poor index of the process used at that solution, the problem-solving process must be studied by getting the subjects to generate observable sequences of behavior." (2) Good problem solvers can find contradictions when they exist (e.g. linear equations will have no unique solution if their slopes are equal). Concrete presentation promotes the finding of contradictions. (3) Dienes and Jeeves, studying how subjects reorganize stimulus structures, found that children can particularize better than adults and postulated some kind of structural learning. The question does not seem to be whether to use manipulatives or not but how and when to use them best.

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