DOCUMENT RESUME

ED 092 371 SE 017 806

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TITLE	Comparison of Content Structure and Cognitive Structure in the Learning of Probability.
PUB DATE	Apr 74
NOTE	22p.; Paper presented at the Annual Meeting of the American Educational Research Association (59th, Chicago, Illinois, April 1974)
EDRS PRICE	MF-$0.75 HC-$1.50 PLUS POSTAGE
DESCRIPTORS	*Cognitive Development; Cognitive Processes; *Learning; Learning Processes; *Mathematics Education; *Probability; Programed Instruction; *Research; Secondary School Mathematics; Structural Analysis

ABSTRACT

Digraphs, graphs, and task analysis were used to map out the content structure of a programed text (SMSG) in elementary probability. Mathematical structure was defined as the relationship between concepts within a set of abstract systems. The word association technique was used to measure the existing relations (cognitive structure) in S's memory with respect to the probability theory present in the text. The purpose of this study was to measure the influence of content structure (mathematical structure) of the text on the Ss' cognitive structure. Control and experimental Ss (N=34) were high school (grades 9-12) subjects recruited from study halls and mathematics classes in one high school. Experimental Ss (N=20) studied the probability text while the others studied a programed text on an unrelated mathematical topic. For subjects in the experimental group, a strong similarity between the representation of content structure and cognitive structure was found. The structure methodology used in this study appears to be applicable to many aspects of research on learning mathematical structures and might be a helpful tool in formative evaluation of mathematics curricula. The data on content structure and cognitive structure seem to suggest ways to improve the text to further student learning of structure. (JP)

Comparison of Content Structure and Cognitive Structure in the Learning of Probability*

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During the past decade mathematics curricula have been revised significantly. Curriculum developers have attempted to communicate something more than algorithms and computational skills to the student (cf. Report of the Commission on mathematics, 1959); they have attempted to communicate structures in mathematics (Report of the Cambridge Conference, 1963). Branca (1974) reviews the history of this change in emphasis. In spite of the recent emphasis on structure in mathematics curricula, little empirical work has been done concerning the communication of a mathematical structure to students. This study examined the correspondance between representations of the structure of a programmed probability unit (content structure) and a representation of this structure in high school students' memories (cognitive structures) as a result of studying the probability unit.

In a manner consistent with Shavelson's (1971,1974) definition of structure, Begle (in preparation) defined mathematical structure as "a set of interrelated, abstract, symbolic systems." He stressed the point that mathematical structure is a combination of within system relationships and between system relationships. For the purposes of this study, mathematical structure is defined as the relationships between concepts within a set of abstract systems.

In this study students received instruction in probability (the to-be-learned structure) or in an unrelated topic in mathematics. Before and after instruction, the word association test, a measure of cognitive structure,

*A paper presented at the annual meeting of the American Educational Research Association, Chicago, April, 1974.

was given. The representation of cognitive structure from the word association test was compared with the representation of content structure obtained with the digraph and graph methods. The correspondence between these two representations was interpreted as the correspondence between content structure and cognitive structure. Even though these representations of structure are not comprehensive or error free, they represent an important first step in answering crucial questions about this correspondence.

Method

Subjects

The 34 subjects were middle and upper-middle socio-economic status, Caucasian volunteers recruited from study halls and mathematics classes in one high school (grades 9-12).

Instructional Material

The instructional material was an introductory programmed text on probability.[1] One purpose of this text was to communicate a subject-matter structure, that of elementary probability, to the subjects. Probability was selected because the topic was unfamiliar to most high school students, was easily placed in the normal curriculum sequence, and required few mathematical prerequisites. The programmed text format--small steps, constructed responses, and continual feedback on the correct responses--was used to minimize the chance that proctors would "teach" a structure different from that presented by the text by answering subjects' questions. It also allowed each student to proceed at his own pace. The text, divided into three sections of approximately seventy pages each, covered the following key concepts in probability: PROBABILITY, EXPERIMENT, OUTCOME, EQUALLY LIKELY, EVENT, TRIAL, INTERSECTION, ZERO, INDEPENDENT, and MUTALLY EXCLUSIVE. The key concepts were used in the analysis of content structure and as stimuli on a word association (WA)

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[1] The text, developed by the School Mathematics Study Group, is available from the ERIC Document Reproduction Service, Arlington, Virginia.

test -- a measure of cognitive structure.

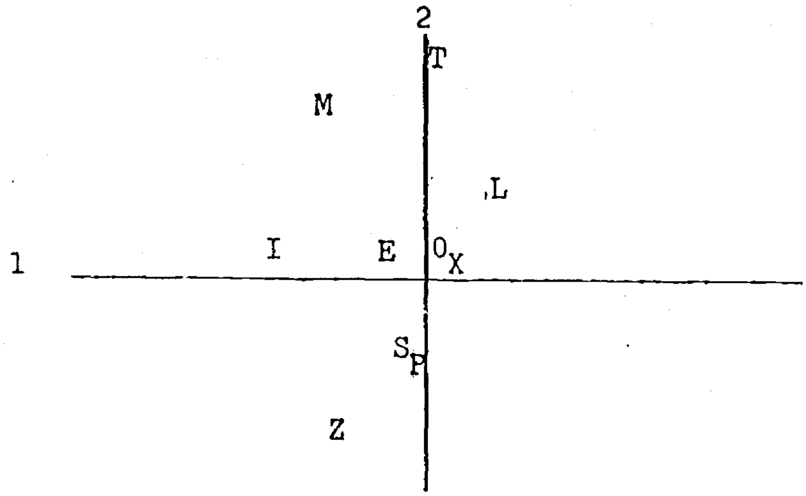
Representation of Content Structure

Following the digraph procedure described by Shavelson (1974), a 10 x 10 digraph distance matrix representing the shortest distances between pairs of the ten key concepts was derived and then converted into a 10 x 10 similarity matrix (cf. Geeslin, 1973). The elements in the similarity matrix indicated the "closeness" of each pair of concepts. Note that this procedure may result in an asymetric matrix. The similarity matrix was examined using Kruskal's (1964) multidimensional scaling procedure. A plot of the results is shown in Figure 1. Figure 1, then, is interpreted as a representation of the structure of the probability text.

Figure 1

Two Dimensional Scaling Solution

For the Digraph Analysis of Content Structure



Key

P = Probability
I = Independent
E = Event
Z = Zero
L = Equally Likely

S = Intersection
T = Trial
X = Experiment
M = Mutually Exclusive
O = Outcome

The multidimensional scaling results of the digraph analysis of content structure are shown in Figure 1. The results are consistent with our interpretation of the subject-matter. Although interpretation of the multidimensional scaling solution is somewhat subjective, dimension 1 seems to reflect the notion of using mathematics as a model of "real-world" experience. That is, EXPERIMENT, OUTCOME, TRIAL, and EQUALLY LIKELY are (concrete) concepts that can be observed in the physical world; the other concepts (moving from right to left) are (abstract) mathematical concepts that we use to build a mathematical model of the physical world.

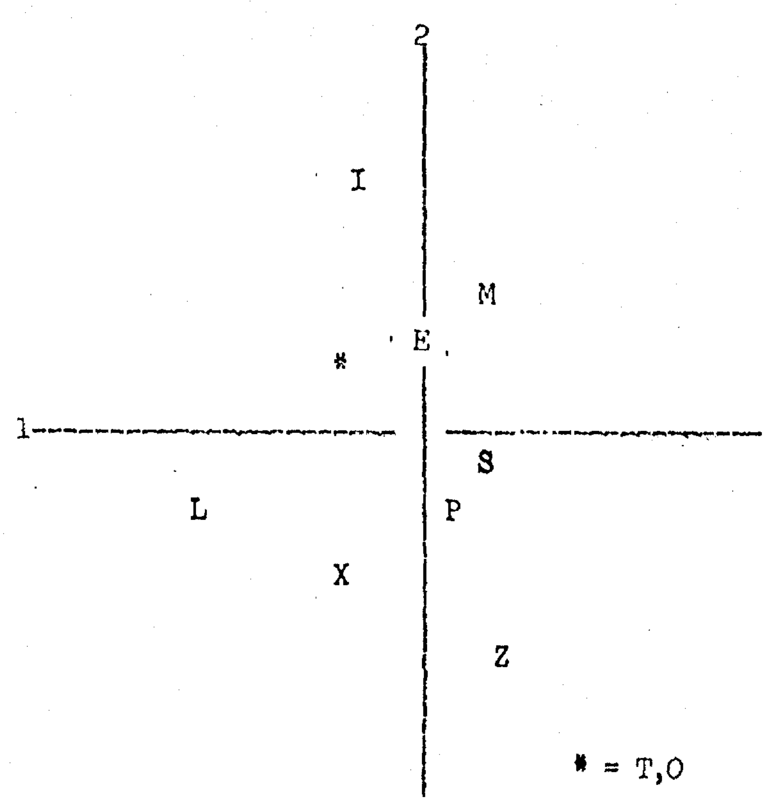
As for the second dimension, three clusters of key concepts may be identified. Cluster 1 includes the concepts of MUTALLY EXCLUSIVE (M), INDEPENDENT (I), and EVENT (E). Cluster 2 contains the concepts of TRIAL (T), OUTCOME (O), EQUALLY LIKELY (L), and EXPERIMENT (X). Cluster 3 groups together the concepts of PROBABILITY (P), INTERSECTION (S), and ZERO (Z). These clusters may form a hierarchy of mathematical concepts with clusters 1 and 2 at one level and cluster 3 at the next superordinate level. Cluster 1 (M,I,E,) represents mathematical concepts modeling cluster 2 (T,O,L,X), the physical concepts, and concepts in cluster 3 (P,S,Z) are mathematical concepts that tie together the model and the physical world.

A second method for representing content structure is graph theory (Harary and Norman, 1953). Graph theory may be distinguished from digraph theory in that the former ignores the direction of lines while the latter places an emphasis on directed lines. The same key concepts were used in this analysis as were used in the digraph analysis. The only change made in Shavelson's digraph procedures was to replace directed lines with non-directed lines. Thus the elements in the graph distance matrix are equal to the smallest element in each pair of corresponding cells in the digraph distance matrix. Note that the graph distance matrix will always be symmetric while this is not necessarily true of the digraph distance matrix. Obviously if a symmetric digraph results from the digraph analysis, the structure representations by graph and digraph will be equivalent. The graph distance matrix was converted into a 10 x 10 similarity matrix and examined using multidimensional scaling. The plot of the results, shown in Figure 2, is interpreted as a second representation of the structure presented by the probability text.

Figure 2

Two Dimensional Scaling Solution

For the Graph Analysis of Content Structure



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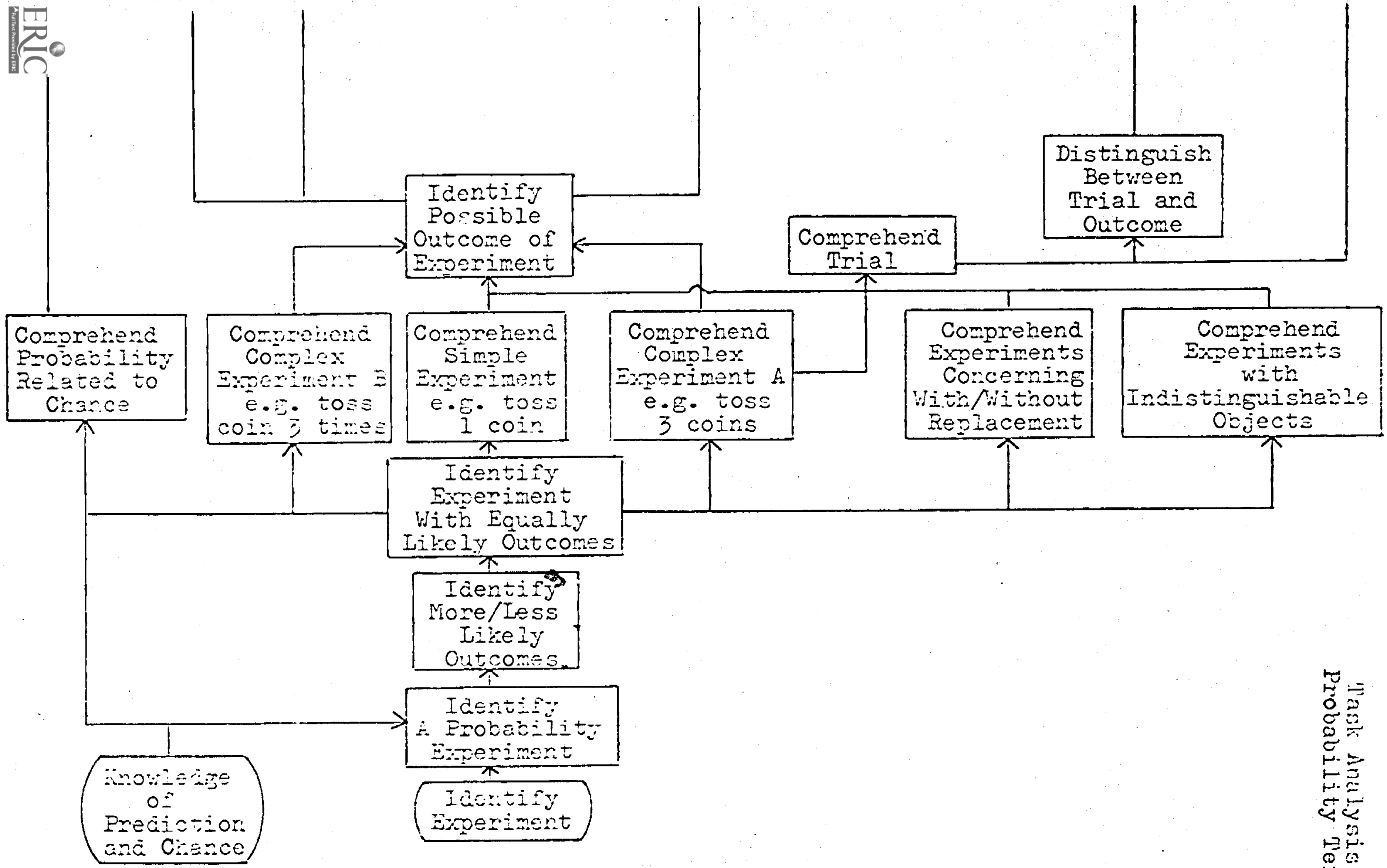
Examining Figure 2, the graph representation of content structure, we see no essential differences from the digraph representation. TRIAL (T) and OUTCOME (O) are not distinguishable in the graph analysis indicating that we lose some information in this analysis as expected. Since no major differences were observed between the digraph and graph analysis of content structure, we will refer only to the digraph representation in the remainder of this discussion.

Finally, task analysis was used to map the structure of the instructional material (Gagne, 1965, 1970). Task analysis produces an alternate (to the digraph/graph analyses) structural representation. Points represent competencies and lines represent relationships between competencies. This is a psychological definition of structure and therefore different from what subject-matter experts mean when they use the term structure. However, we use task analysis in the present study to link the digraph/ graph representations to a more traditional approach.

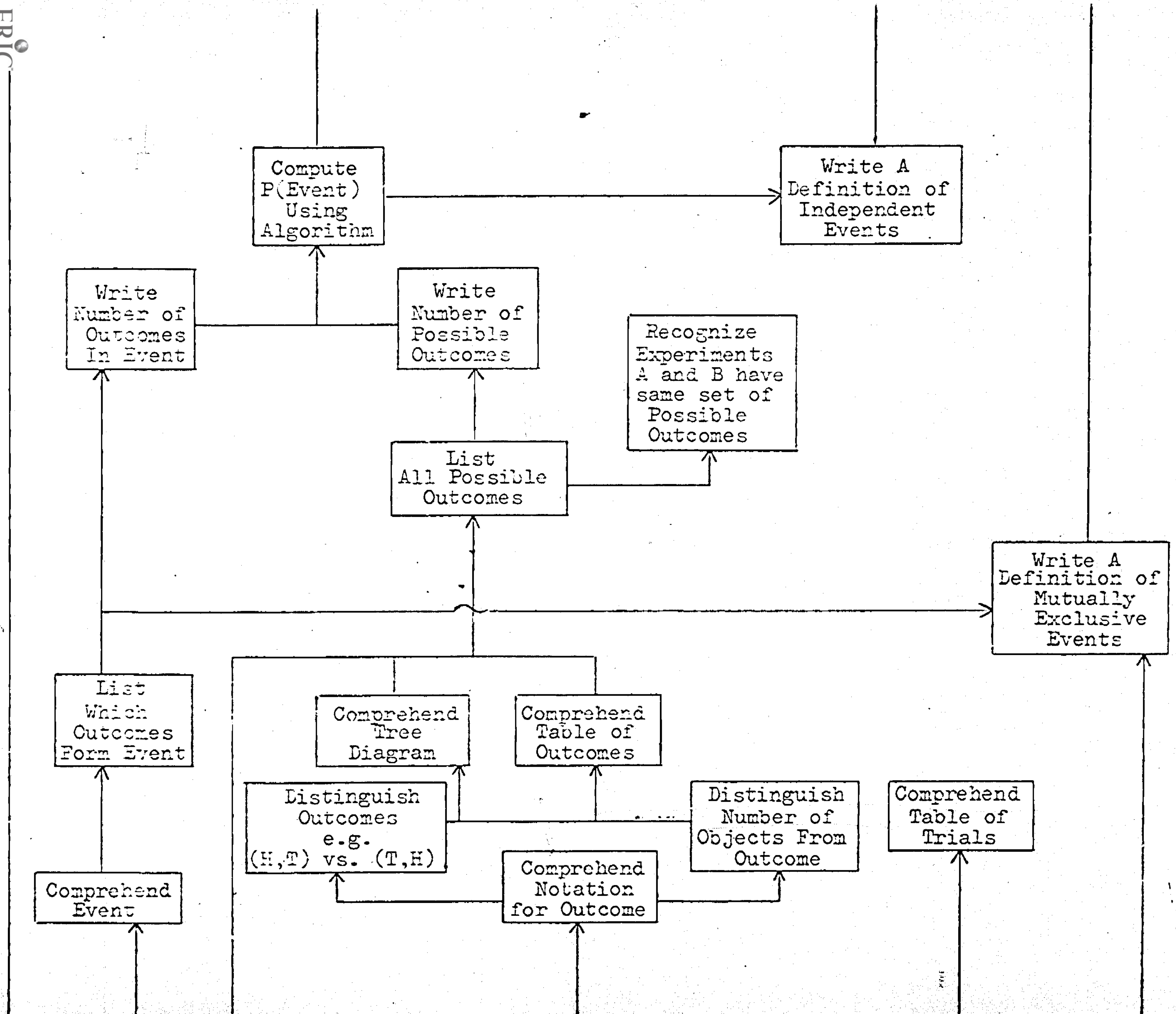
The resultant hierarchy is presented in Figure 3. The investigator was

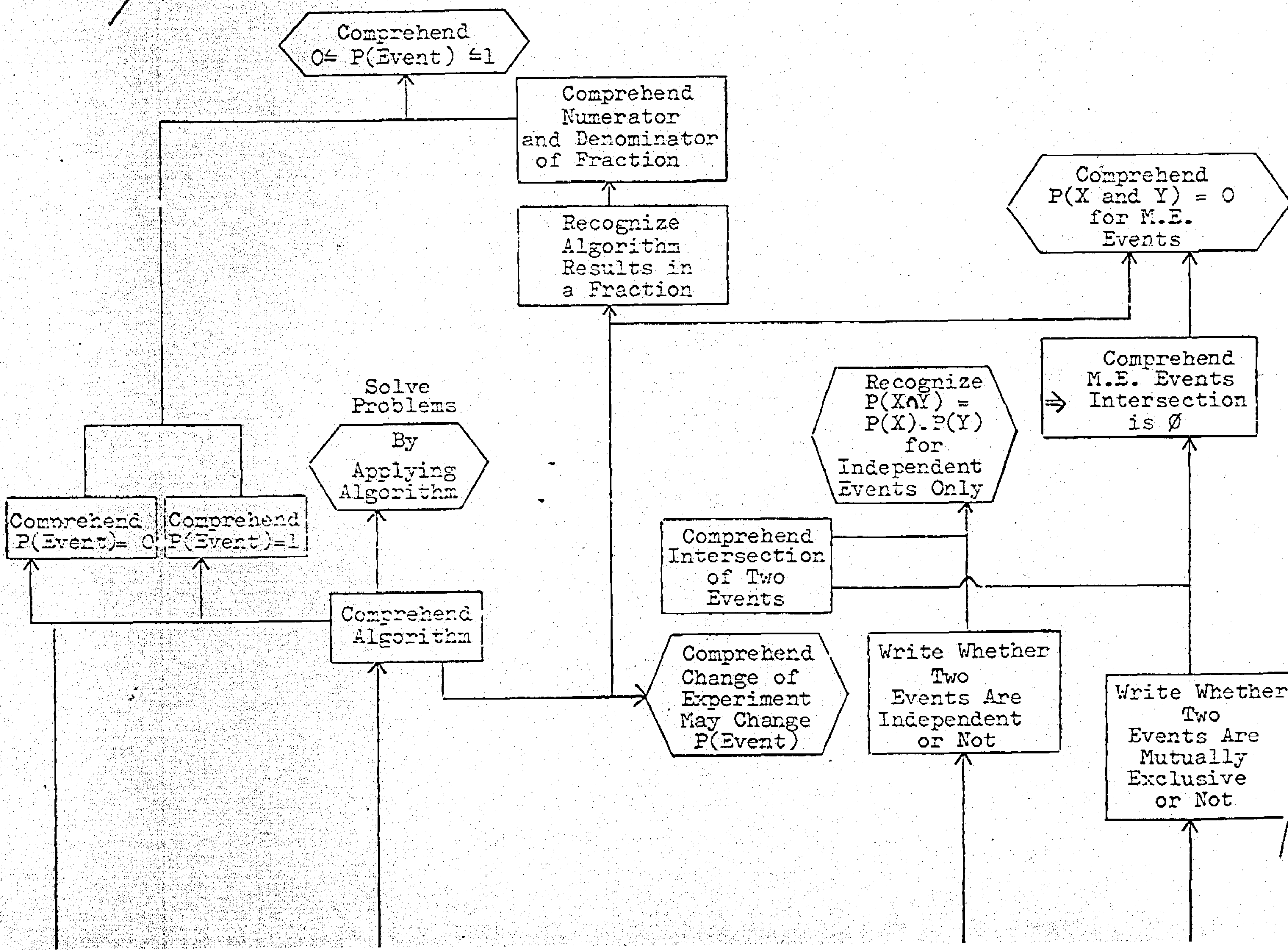
Figure 3

Task Analysis of Probability Text*



*The use of the word "comprehend" in the task analysis deviates somewhat from Gagne's (1970) terminology. A definition of the word comprehend and a discussion of its use is given by Romberg and Wilson (1969).


Compute
P(Event)
Using
Algorithm
Write A
Definition of
Independent
Events
Write
Number of
Outcomes
In Event
Write
Number of
Possible
Outcomes
Recognize
Experiments
A and B have
same set of
Possible
Outcomes
List
All Possible
Outcomes
Write A
Definition of
Mutually
Exclusive
Events
List
Which
Outcomes
Form Event
Comprehend
Tree
Diagram
Comprehend
Table of
Outcomes
Distinguish
Outcomes
e.g.
(H,T) vs. (T,H)
Distinguish
Number of
Objects From
Outcome
Comprehend
Table of
Trials
Comprehend
Event
Comprehend
Notation
for Outcome


Comprehend
0≤ P(Event) ≤1
Comprehend
Numerator
and Denominator
of Fraction
Recognize
Algorithm
Results in
a Fraction
Comprehend
P(X and Y) = 0
for M.E.
Events
Comprehend
M.E. Events
⇒ Intersection
is Ø
Recognize
P(X∩Y) =
P(X).P(Y)
for
Independent
Events Only
Solve
Problems
By
Applying
Algorithm
Comprehend
P(Event)= 0
Comprehend
P(Event)=1
Comprehend
Intersection
of Two
Events
Comprehend
Algorithm
Comprehend
Change of
Experiment
May Change
P(Event)
Write Whether
Two
Events Are
Independent
or Not
Write Whether
Two
Events Are
Mutually
Exclusive
or Not

not able to determine a satisfactory method for obtaining a "distance" matrix from the resultant heirarchy. One could count "boxes" between concepts, but the boxes do not represent concepts alone but rather they represent manipulations or performances with concepts. Thus, for example, the concepts OUTCOME and EVENT appear in several boxes and one could arrive at several distances between these concepts depending on the boxes selected. Additionally, the boxes are derived in a somewhat subjective manner. A logical analysis by one author may not be the same as a logical analysis for a second author; thus causing the two authors to arrive at different distance matrices. The task analysis should be useful in interpreting the other content analyses and the analyses of the WA data, but does not appear to be a satisfactory representation of structure as we have defined it.

Representation of Cognitive Structure

Cognitive structure was investigated using a word association technique (Geeslin, 1973; Johnson, 1967, 1969; Shavelson, 1971, 1972, 1973). Empirical evidence in support of the cognitive structure interpretation of WA data has been provided by Shavelson (1974) and can be found in a number of other sources (Deese 1962; Johnson 1967; and Shavelson 1971, 1972, 1973).

The WA test consisted of one page of instructions and one page for each set of responses to each of the ten key concepts, respectively. Subjects were instructed to write as many other mathematical concepts related to the key concept as they could in one minute. Four random sequences of the stimulus words were used to prevent a possible sequence effect. A particular sequence was assigned randomly to subjects at each test administration.

The word association (WA) data were converted into a matrix of similarities between concepts by means of the relatedness coefficient as described by

Shavelson (1974). (See also (Garskof & Houston, 1963).

Instrumentation

In addition to the WA test, two other measures were used to provide further information about the subjects: an attitude questionnaire and an achievement test on probability. The attitude questionnaire was the "Pro-Math Composite" scale (PY011; see Wilson, Cahen, & Begle, 1968) developed by the National Longitudinal Study of Mathematical Abilities (NLSMA). The scale was designed to measure general attitude toward mathematics. Internal consistency coefficient alpha for this scale was 0.76 for the subjects used in this study.

The main achievement test consisted of twenty-eight free response and seven multiple choice items. The first thirty items tested comprehension of the material presented in the probability text. The last five items presented problems on probability in a perspective different from that used in the programmed text. Internal consistency coefficients alpha calculated from experimental subjects' data in the present study were 0.780 and 0.794 at posttest and retention test, respectively.

In addition to the thirty-five item achievement test, two ten item tests were given to the experimental subjects at the end of Sections 1 and 2 of the probability text, respectively. These tests were used only to give experimental subjects a progress check and to help insure that subjects did not proceed so quickly through the programmed material that little or no learning took place.

Treatment and Procedures

Subjects were assigned randomly to experimental and control treatments. Subjects in the experimental treatment read and studied the programmed text on probability theory. Subjects in the control group read and studied a programmed text on an unrelated mathematical topic, negative number bases. Experimental subjects (N = 20) were never separated from control subjects

(N = 14) during the experiment but subjects knew that two different programmed texts were being used.

The study was conducted over 22 calendar days in the subjects' classrooms during normal school hours near the end of the 1971-1972 academic year With the exception of the retention test, the study was carried out during consecutive, 53 minute meetings of the classes which met every school day. The first class meeting was devoted to orientation and pretesting. The orientation informed the subjects that they were participating in a study to find out how students learn mathematics. The attitude questionnaire, the WA test, and the achievement test on probability were administered, in the order listed, to all subjects prior to instruction. A brief discussion on using programmed instruction effectively followed the pretesting. Each subject then read the text assigned to him. At the end of each text section, each subject in both treatments received a short review test over the section he had just completed. (The probability text did not have a test for Section 3, the final section). Since instruction was self-paced not all Ss needed the entire instructional period to complete the text material; conversely, not all Ss read the entire text. However, all experimental subjects completed the second text section and most of the third section. Subjects who finished early were allowed to read, draw, or study material of their choosing as long as the material was non-mathematical.

After instruction, all subjects were given the WA test and achievement test, in the order listed. All subjects in a class were posttested at the same time and then 11 calendar days afterwards, the WA test and achievement test were readministered as retention tests to subjects in the sequence listed. The design of the study, then, was a 2 x 3 (treatment by test occasion) design with repeated measures on the latter factor.

Results

Cognitive Structure

Achievement test data. Achievement test scores were used as a methodological check on students' learning of probability. If differences between treatment groups in traditional measures of learning mathematics are observed, confidence is increased in interpretations of differences observed in cognitive structure. Descriptive data from the achievement test are presented in Table 1. These data were analyzed by a 2 x 3 (treatment by test occasion) analysis of variance with repeated measures on the second factor. Results obtained were: a) a significant treatment effect ($F = 22.55$, $df = 1/14$, $p < .01$); b) a significant test occasion effect ($F = 25.19$, $df = 2/13$, $P < .01$); and c) a significant interaction between effects ($F = 24.59$, $df = 2/13$, $p < .01$).

At pretest there was little difference between experimental and control subjects, but large differences occur between groups at posttest and retention test. The experimental group learned to solve significantly more problems in probability as a result of instruction than did subjects in the control group.

Table 1

Means and Standard Deviations of Scores on the Achievement Test for each Treatment and Test Occasion.

	Pretest	Posttest	Retention Test
Experimental Subjects	$\bar{x} = 4.53$ $\sigma = 3.50$ $n = 19$	$\bar{x} = 21.00$ $\sigma = 5.01$ $n = 15$	$\bar{x} = 24.64$ $\sigma = 5.26$ $n = 11$
Control Subjects	$\bar{x} = 5.50$ $\sigma = 4.83$ $n = 14$	$\bar{x} = 6.45$ $\sigma = 3.93$ $n = 11$	$\bar{x} = 7.83$ $\sigma = 5.64$ $n = 6$

Word association data. The results of the multidimensional scaling (Kruskal, 1964) of median RC matrices at posttest and at retention test are presented in Figures 4 and 5, respectively. Pretest WA data and control group WA data, with the exception of control group retention test WA data presented in Figure 6, consisted of mostly zero elements and thus scaling solutions could not be obtained. These results are consistent with our assumptions that : a) high school students were unfamilar with the concepts of probability and b) instruction in probability would change cognitive structure concerning concepts in probability. These assumptions are supported by the analysis of achievement test scores.

Comparison of Content Structure and Cognitive Structure

One way to compare content structure and cognitive structure is to visually examine the correspondence, or lack of it, between the multidimensional scaling solutions for the digraph and RC matrices. For subjects in the experimental group, a strong similarity between the representations of content structure and cognitive structure was found. At posttest (Figure 4) experimental subjects distinguished concepts on only one dimension. With the exception of PROBABILITY the concepts are ordered almost exactly the same as in the content structure representation. However, the subjects appeared not to learn the distinctions in relationships that require a second dimension. At retention test (Figure 5), experimental subjects not only retained that portion of the structure they learned, but also reorganized their cognitive structures in a manner more consistent with content structure. Again the subjects appeared not to make all the distinctions present in the content. For example, INDEPENDENT, EVENT, MUTUALLY EXCLUSIVE, OUTCOME, EXPERIMENT, and TRIAL are grouped too closely with no strong distinction between abstract versus concrete concepts noted in the content structure. These results are consistent with the results of a study of eighth grade students (Geeslin, 1973, 1974), although not as striking as the eighth grade results.

It was possible to obtain a scaling solution (Figure 6) of control subjects' retention test WA data. However, the clusters of concepts we noted in the content structure (Figure 1) do not seem to be present in control subjects' cognitive structures. Thus, although control subjects' became familiar with the concepts through the testing procedure, they did not organize these concepts in an interpretable manner. This indicates that although the verbal environment (e.g. the testing procedures) may increase the RC coefficient, the presence of concepts in the verbal environment is not sufficient to provide the knowledge of structure provided by instruction. That is, the instructional material provides something more than just familiarity with concepts, it provides something concerning the relationships between these concepts.

Figure 4

One Dimensional Scaling Solution
Experimental Subjects Posttest
Median Relatedness Coefficient Matrix

```
                 2
                 |
                 |
                 |
1  ---ZI--------M |S-OXETPL------
                 |
                 |
                 |
```

Key

P = Probability	S = Intersection
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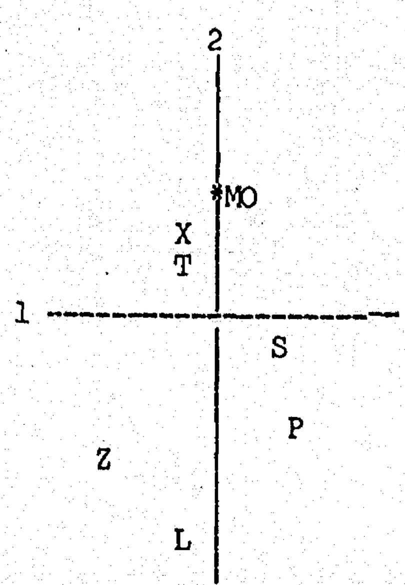
E = Event	X = Experiment
Z = Zero	M = Mutually Exclusive
L = Equally Likely	O = Outcome

Figure 5

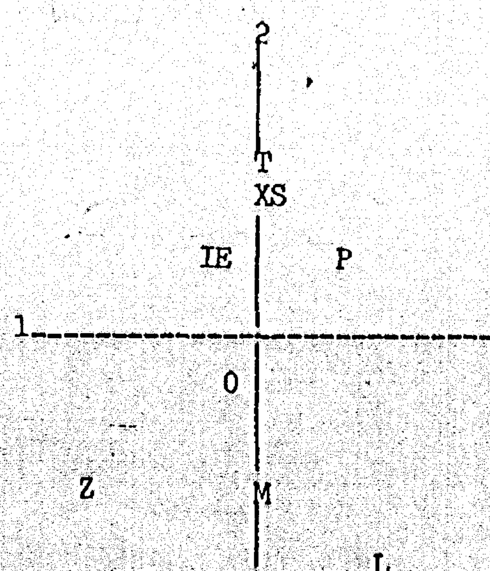
Two Dimensional Scaling Solution
Experimental Subjects Retention Test
Median Relatedness Coefficient Matrix



* = I,E

Figure 6

Two Dimensional Scaling Solution
Control Subjects Retention Test
Median Relatedness Coefficient Matrix



Key

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I = Intersection	T = Trial
E = Event	X = Experiment
Z = Zero	M = Mutually Exclusive
L = Equally Likely	O = Outcome

A second method of comparing structures is to calculate the Euclidean distance [2] between the digraph similarity matrix and the RC matrix. This provided an indication of the similarity between the two matrices. (The smaller the distance, the closer the match between the RC matrix and the digraph similarity matrix.[3]) For each subject, at each testing time, the Euclidean distance between his RC matrix and the digraph matrix was calculated. The correspondence between content structure and cognitive structure is shown in Figure 7 for the experimental and control groups. These data indicate that experimental subjects' cognitive structures corresponded much more closely to the content structure following instruction than prior to instruction. Some change in control subjects' cognitive structures is noted also, but the magnitude and rate of decline were not of the magnitude and rate found in the experimental group. The control group retention test data was obtained in the last week of school and the number of subjects dropped significantly (N=6). It appeared that only the brighter control subjects appeared for this test session and thus the similarity to experimental data is probably spurious and was not consistent with other results (Geeslin 1973, 1974).

A nonparametric analysis of variance (Bradley, 1968) was performed on the Euclidean distance data at pretest and posttest. The cognitive structure of subjects in the experimental group corresponded more closely to content structure than did the cognitive structure of subjects in the control group (pl <01).

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[2] The Euclidean distance is obtained by squaring each difference between corresponding elements of two matrices (e.g. a subject's RC matrix and the digraph similarity matrix), summing the squares, taking the square root of this sum, and dividing by ninety (the number of off-diagonal elements in each matrix).

[3] Since the smallest value of a RC is zero, some RC matrices consist of only off-diagonal elements that are zero. This may cause a Euclidean distance to be smaller than it should be, since it is possible to be further away from the content structure.

Scores from the attitude, achievement, and cognitive structure measures were intercorrelated to explore possible relationships among the variables. These correlations, Kendall's Tau, are presented in Table 2 for the experimental group. The correspondence variable refers to the Euclidean distance between an individual's RC matrix (cognitive structure) and the digraph similarity matrix (content structure). Perfect correspondence between achievement data and WA data would be indicated by a correlation of -1.0 since a smaller Euclidean distance score implies a closer relationship between content structure and cognitive structure.

Figure 7

Median Euclidean Distances Between
Content Structure and Cognitive Structure

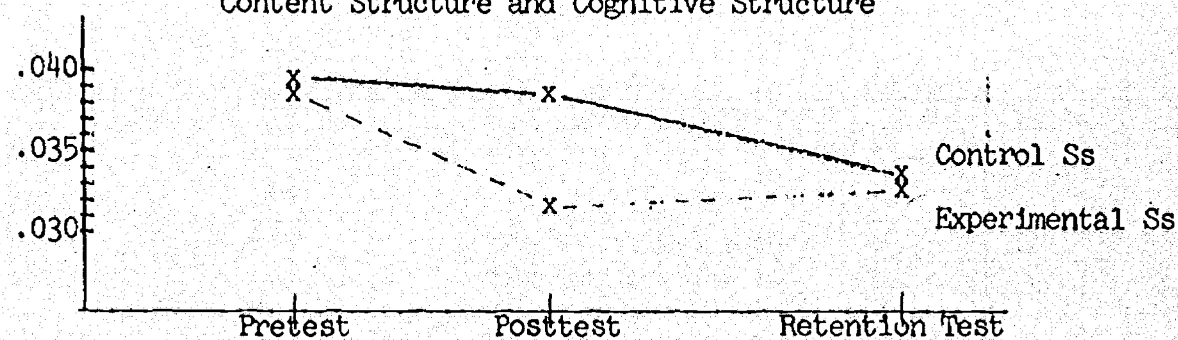

Table 2

Rank Order Correlations (Tau) Between All
Measures for Subjects in the Experimental Group

	Achievement			Correspondance of Structures		
	Pre	Post	Retention	Pre	Post	Retention
Attitude	183	214	-058	-065	205	-038
Achievement						
Pre		031	071	080	162	372
Post			907	010	-237	046
Retention				038	-023	0
Correspondence of Structures						
Pre					007	333
Post						539

The correlations indicate that scores on the attitude scale have a low correlation with scores on other variables. Scores on achievement at pretest have a low correlation with achievement posttest scores. This is consistent with the findings that subjects knew little about probability before the study but learned about probability as a result of instruction. A high correlation between scores on achievement at posttest and retention test was obtained; this indicates subjects retained knowledge in accordance with their immediate learning. The correlations between variables representing the correspondence of cognitive structure with content structure showed a pattern similar to that of the achievement test scores. Low correlations between achievement and correspondence variables were obtained with the exception of the posttest data. This may indicate that learning to solve problems and learning of mathematical structure represent different aspects of learning. Although this finding is consistent with past studies (Shavelson, 1971, 1972), a stronger relationship was expected. The posttest correlation between achievement and the correspondance variable was of moderate size and in the expected direction indicating some connection between learning of structure and achievement in this sample.

Conclusions

This study indicated that the analysis of content structure using digraph theory could be applied to a mathematics curriculum. The results of the analysis -- a map of content structure-- agreed with our understanding of the structure of the subject matter in probability.

The achievement test data indicated that the programmed text on probability was effective in teaching probability to high school students. Compared to subjects in a control group, subjects in the experimental group learned how to solve significantly more problems as a result of instruction and retained this learning at retention test. Furthermore, subjects in the experimental group learned a significant portion of the structure of probability as a result of instruction while the control group learned almost nothing of the structure. This learning of structure was retained until retention test time (a factor not investigated in prior studies). However, learning structure and learning to calculate solutions to problems in probability may develop independently of each other.

The structure methodology used in this study appears to be applicable to many aspects of research on learning mathematical structures and might be a helpful tool in formative evaluation of mathematics curricula. That is, the data on content structure and cognitive structure seem to suggest ways to improve the text to further student learning of structure.

References

Begle, E.G. A program of research in mathematics education. School Mathematics Study Group, Stanford University, Stanford, California, in preparation.

Bradley, J.V. Distribution-Free Satistical Tests. Englewood Cliffs, New Jersey: Prentice-Hall, 1968.

Branca, N.A. Learning Mathematical Structures. Paper presented at the meeting of the American Educational Research Association, Chicago, 1974.

Deese, J. On the structure of associative meaning. Psychological Review, 1962, 69 161-175.

Garskof, B. E. and Houston, J.P. Measurement of verbal relatedness: An idiograph approach . Psychological Review, 1963, 70, 277-288.

Gagne, R. M. The Conditions of Learning. New York: Holt, Rinehart, and Winston, 1965.

Gagne, R. M. The Conditions of Learning. (2nd ed.) New York: Holt, Rinehart and Winston, 1970.

Geeslin, W.E. An exploratory analysis of content structure and cognitive structure in the context of a mathematics instructional unit. (Doctoral dissertation, Stanford University) Ann Arbor, Michigan: University Microfilms, 1973.

Geeslin, W. E. An analysis of content structure and cognitive structure in the context of a probability unit. Paper presented at the annual meeting of the National Council of Teachers of Mathematics, Atlantic City, 1974.

Harary, F. and Norman, R.Z. Graph theory as a mathematical model in social science. RCGD NO. 2., University of Michigan Institute for Social Research, Ann Arbor, Michigan, 1953.

Johnson, P.E. Some psychological aspects of subject-matter structure. Journal of Educational Psychology, 1969, 58, 75-83.

Kruskal, J.B. Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. Psychometrika, 1964, 29, 1-27.

Report of the Cambridge Conference on School Mathematics. Goals for School Mathematics. Boston: Houghton Mifflin, 1963.

Report of the Commission on Mathematics. Program for College Preparatory Mathematics. New York College Entrance Examination Board, 1959.

Shavelson, R. J. Some aspects of the relationship between content structure and cognitive structure in physics instruction. (Doctoral dissertation, Stanford University) Ann Arbor, Michigan: University Microfilms, 1971. No. 71-19,759.

Shavelson, R.J. Some aspects of the correspondence between content structure and cognitive structure in physics instruction. Journal of Educational Psychology, 1972, 63, 225-234.

Shavelson, R.J. Learning from physics instruction. Journal of Research in Science Teaching, 1973, 10, 101-111.

Shavelson, R.J. Some methods for examining a subject-matter structure in prose material and a student's memory. Paper presented at the meeting of the American Educational Research Association, Chicago, 1974.

Wilson, J.W., Cahen, L.S. and Begle, E.G. (Eds.). Description and statistical properties of Y-population scales. NLSMA Report No. 5, School Mathematics Study Group, Stanford University, Stanford, California, 1968.