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ABSTRACT

This investigation sought to determine the effect of emphasizing mathematical structure in the acquisition of computational skills by seven- and eight-year-olds. The meaningful development-of-structure approach emphasized closure, commutativity, associativity, and the identity element of addition; the inverse relationship between addition and subtraction; and place value. The control groups essentially used drill-type activities. The subjects were students from a middle class elementary school in Tampa, Florida who had the poorest performances on pretests. Analysis of covariance was used to analyze the data; results favored the experimental group. (Author/LS)

THE EFFECT OF EMPHASIZING MATHEMATICAL STRUCTURE IN THE ACQUISITION OF WHOLE NUMBER COMPUTATION SKILLS (ADDITION AND SUBTRACTION) BY SEVEN- AND EIGHT-YEAR OLDS: A CLINICAL INVESTIGATION

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A characteristic associated with the "new math program" first introduced to elementary schools in the late fifties and presently in use today is the emphasis placed on mathematical structure to facilitate the learning of mathematics principles, concepts, and skills. However, from the beginning there has been considerable skepticism expressed by parents and some educators as to the effectiveness of these programs in developing mathematics skills (computation) in youngsters. A typical comment in reference to "new math" was, "It's no longer important to get the correct answer when computing, rather one must simply understand the process(es) involved." More recently there seems to be a backlash emerging against the modern approach to teaching mathematics in favor of more traditional approaches. This backlash will continue to gain momentum as long as parents believe children can no longer add, subtract, multiply, and divide.

Morris Kline, a much respected mathematician, makes an assault on "new math" in his new book Why Johnny Can't Add; both author and book are receiving wide coverage in the national press (Newsweek, 1973). Many mathematics educators however, dispute Kline's findings with respect to "new math". Edward Begle (1974) in an editorial comment in reference to one of Kline's articles, The New Math: A Passing Aberration states:

> "When he writes about the history of mathematical ideas, Professor Kline is indeed very good. What he has to say is carefully phrased and he demonstrates very sound scholarship.
>
> But when he turns to his favorite obsession, new math, his performance is just the opposite and his scholarship is notable only for its absence. Many of his rather sweeping statements are flatly contradicted by the facts, and for most of the others he offers no substantiation and refers to no evidence....

Whether one is an advocate for Kline or Begle is not important. What is important is recognizing the need for research to examine specific aspects of the modern approach to teaching mathematics to elementary school children. Without specific data differnces of opinion on the effectiveness of "new math" programs will remain unresolved.

The Problem

The present clinical investigation was designed to determine the effect of emphasizing mathematical structure in the acquisition of addition and subtraction computation skills with whole numbers by seven- and eight-year olds. The independent variable was method of instruction. The dependent variable was the child's ability to perform on a test constructed by the investigators to measure computation skills (addition and subtraction), place value and number concepts.

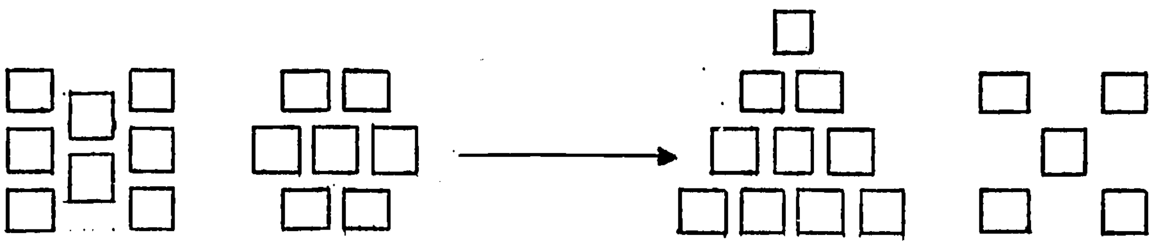
Past research (Greathouse, 1966; Miller, 1957; Shipp and Deer, 1960; Shuster and Pigge, 1965; and Zahn, 1966) seems to support the premise that "meaningful or developmental" instruction yields high achievement in mathematics. In this investigation "meaningful or developmental" instruction was operationally defined in terms of mathematical structure. More specifically, the aspects of mathematical structure emphasized were: (1) Closure propery of addition, (2) Commutative property of addition, (3) Associative property of addition, (4) Identity element of addition, (5) Inverse relationship between addition and subtraction, and (6) Place value. An understanding of the properties mentioned above are within the ability of average seven- and eight-year-olds (Brown, 1969).

Procedure

The seven- and eight-year olds participating in this investigation attended a middle class elementary school in Tampa, Florida which organizes its students into five units. The students in each unit are assigned to classes according to ability. The four classes comprising Unit II (seven- and eight-year olds) were administered a pretest measuring computational skills, place value and number concepts. On the basis of pretest performance, eight students (those with the lowest scores) were selected from each class (N = 32). Within a class four subjects were randomly assigned to an experimental group and four to a control group. The four experimental groups ($n = 4$, $\Sigma nj = 16$) and the four control groups ($n = 4$, $\Sigma nj = 16$) each received approximately fifty minutes of mathematics instruction per week in addition to regular class work for a period of ten weeks (January 1973 - March 1973). Regular class work in mathematics was not monitored by the investigators.

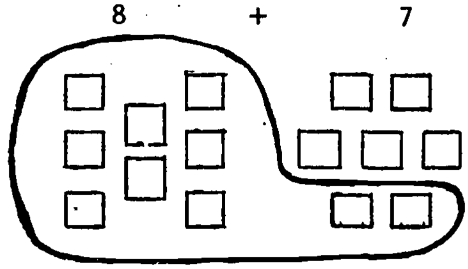
The goal of instruction for both experimental and control groups was to develop computational skills in addition and subtraction. Instruction received by experimental groups emphasized mathematical structure while that administered to the control groups did not and essentially consisted of drill type activities. One technique emphasizing place value and the associative property of addition used with experimental groups to develop basic facts with sums eleven to eighteen was as follows:

Concrete Level - The learner is shown a set of eight blocks and a set of seven blocks arranged in specified patterns. He identifies the number in each set of blocks, determines how many blocks need to be grouped with the set of eight to make a set of ten, removes two blocks from the set of seven, and rearranges the blocks to form a set of ten and a set of five.



Representative Level - The learner is given a sheet of paper with a set of eight "X's" and a set of seven "X's" arranged in specified patterns. He writes the number in each set above the appropriate set, determines how many "X's" need to be grouped with the set of eight to make a set of ten, circles a set of ten "X's" (the set of eight plus two from the set of seven), writes the number in each of the two newly formed sets below each, and then using his knowledge of place value writes the total number of "X's" in both sets.

8 + 7



10 + 5

15

Abstract Level - The learner writes a number story about the process he just completed at the concrete and representative levels.

8 + 7 = □

8 + (2 + 5) = □

(8 + 2)+ 5 = □

10 + 5 = □

15 = □

and

To teach the same skill (basic facts of addition with sums eleven to eighteen) to control groups, games involving only drill material were employed. All groups were instructed separately.

A posttest (same as pretest) was administered to all subjects after five hundred minutes of instruction (ten weeks). Each treatment group was tested separately within a one week period. The test was divided into two parts. Part A measured algorithmic skills in addition and subtraction, Part B measured number concepts and place value.

Twenty algorithms, ten addition and ten subtraction, comprised Part A. Ten algorithms, five addition and five subtraction, required renaming. The simplest addition algorithm involved three single digit addends with a sum less than ten, the most difficult involved three addends, two three digit and one two digit, which required renaming of ones and tens to find the sum. The subtraction algorithms ranged in difficulty from a two digit sum minus a two digit addend requiring no renaming, to a three digit sum minus a three digit addend requiring two renamings. Part B consisted of thirty-five items. Seven items measured place value, eight measured the field properties, ten measured basic facts of addition and subtraction, and ten measured cardinal number and number patterns.

All items on the posttest were abstract in nature in that each required either the reading or writing of abstract symbols (numerals). Further, there was no reliability data on the instrument.

Analysis and Results

Analysis of covariance was used to analyze the data with the pretest acting as a covariate for the posttest. Three scores were reported for each subject on both the pretest and posttest; a score on computation skills (algorithms), one for number concepts and place value, and a total score (sum of first two). Separate analyses were run to compare each experimental and control group within a class on all scores.

Mean scores and standard deviations of treatment groups on the pretest and posttest are presented in Table 1. The descriptive data on groups in all four classes indicates experimental subjects superior to control subjects in (a) addition and subtraction algorithms and (b) number concepts and place value. In classes I and II mean scores of control subjects on number concepts and place value actually declined. Analysis of covariance yielded significant differences ($\alpha < .05$ or $\alpha < .01$) between experimental and control groups on (a) addition and sbutraction algorithms, (b) number concepts and place value, and (c) total score in three of four classes (see Tables 2-5). In class III no significant differences were obtained between experimental and control groups.

Overall, the results seem to indicate that emphasizing mathematical structure in instruction facilitates the development of number concepts, place value, and computational skills (addition and subtraction) in seven- and eight-year olds. However, it must be remembered that the subjects participating in this investigation were those who scored the lowest on the pretest in each of their respective ability groups.

Educational Significance

As previously stated there is a backlash emerging against new math primarily due to a lack of development of computational skills in youngsters. An analysis of the data from the present clinical investigation suggests

Table 1: Mean Scores and Standard Deviations Of

Treatment Groups on Pretest and Posttest

Class	Groups	PRETEST				POSTTEST			
		No. Concepts & P. V.		Computation		No. Concepts & P. V.		Computation	
		Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
I	Experimental	24.50	.57	10.50	.57	30.75	.95	15.00	.81
	Control	28.00	2.7	10.00	1.82	26.25	2.06	10.75	.95
II	Experimental	24.25	.95	8.00	.81	28.75	1.50	15.50	.57
	Control	23.00	2.16	9.00	1.41	20.50	3.69	7.75	1.70
III	Experimental	21.00	1.41	7.00	2.44	26.50	3.87	13.75	2.62
	Control	22.75	4.03	7.75	3.30	23.66	3.78	9.00	.32
IV	Experimental	15.25	2.87	3.25	2.21	29.00	2.16	13.00	2.16
	Control	15.50	1.29	3.50	1.91	18.75	1.50	7.25	2.50

Table 2: Analysis of Covariance on

Class I Scores

Computation (Addition and Subtraction)

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	37.57	1.00	37.57	57.00**
Residual or Error	3.29	5.00	.66	
Total	40.86	6.00		

Number Concepts and Place Value

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	32.50	1.00	32.50	13.15*
Residual or Error	12.36	5.00	2.47	
Total	44.86	6.00		

Total Score

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	57.80	1.00	57.80	13.58*
Residual or Error	21.28	5.00	4.26	
Total	79.08	6.00		

$*p < .05$
$**p < .01$

Table 3: Analysis of Covariance on Class II Scores

Computation (Addition and Subtraction)

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	84.09	1.00	84.09	54.26**
Residual or Error	7.75	5.00	1.55	
Total	91.84	6.00		

Number Concepts and Place Value

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	73.19	1.00	73.19	20.11**
Residual or Error	18.19	5.00	3.64	
Total	91.38	6.00		

Total Score

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	352.88	1.00	352.88	259.88**
Residual or Error	6.79	5.00	1.36	
Total	359.67	6.00		

** $p < .01$

Table 4: Analysis of Covariance on Class III Scores

Computation (Addition and Subtraction)

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	29.62	1.00	29.62	5.53
Residual or Error	21.39	5.00	5.34	
Total	51.01	6.00		

Number Concepts and Place Value

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	17.82	1.00	17.82	1.24
Residual or Error	57.46	5.00	14.37	
Total	75.28	6.00		

Total Score

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	140.83	1.00	140.83	4.98
Residual or Error	113.17	5.00	33.56	
Total	254.00	6.00		

Table 5: Analysis of Covariance on

Class IV Scores

Computation (Addition and Subtraction)

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	65.47	1.00	65.47	10.00*
Residual or Error	32.66	5.00	6.53	
Total	98.13	6.00		

Number Concepts and Place Value

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	204.22	1.00	204.21	82.100**
Residual or Error	12.44	5.00	2.49	
Total	216.66	6.00		

Total Score

Source of Variation	Adjusted Sum of Squares	d.f.	Mean Squares	F
Between Treatments	511.37	1.00	511.37	72.94**
Residual or Error	35.05	5.00	7.01	
Total	546.42	6.00		

*p < .05
**p < .01

that at least one aspect of "new math", the emphasis on mathematical structure, needs to be researched further. If the advocates against "new math" do not exercise caution they may very well end up "throwing out the baby with the bath water."

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