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AUTHOR Reid, Christopher J.; Seibert, Warren F.
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ABSTRACT

From the ideas well set forth by Ferguson, Messick and others, a method was sought that would identify different abilities that entered at different stages in a task. The method selected should meet Cronbach's criterion that it would consider individual differences as well as group performance. Tucker's (1960, 1966) method seemed to be able to identify differing abilities while at the same time preserve individual differences. Using Kaiser's Little Jiffy Mark IV to skirt the problem of difficulty factors, a factor of two film tests found two factors operating in each test. Had the task been longer (some authors recommended 1000 items) more factors might have been found; it is equally likely, however, that diminishing returns would soon obtain. It was demonstrated that Tucker's method revealed abilities that a conventional learning curve would mask, and learning curves of several different individuals were illustrated. Some attempt was made to describe the obtained factors with other variables; at least one factor correlated with a variable similar to Gestalt completion or serial integration. Finally the effect of three stimulus characteristics upon item preference was considered. Test part (number of stimulus frames, or number of delay frames) rarely accounted for an important part of item preference, whereas position in the array always did and color did in the two color tests. (Author/MLP)

A Factor Analytic Approach to The Analysis of Learning Curves

J. Christopher Reid
University of Missouri - Columbia
Columbia, Missouri 65201

Warren F. Seibert
Purdue University
Lafayette, Indiana 47907

In his 1957 APA presidential address, Cronbach pointed out that some investigators were concerned with group performance, whereas other investigators preferred to search for individual differences or selected aspects of behavior. The present report seeks to determine both group measures and individual differences in performance on film tests by using a factor analytic method proposed by Tucker (1960; 1966). Tucker's method produces results reminiscent of Fleishman's studies (1967) of the relationship between abilities and performance in successive stages in learning. Thus this report fits into the category of studies inventoried by Messick (1972), who urged the development of sequential models of psychological processes.

The history of film tests of cine-psychometry has been reviewed elsewhere and need not be repeated here. (See Seibert and Snow, 1965; Seibert, 1971a, 1971b.) It will suffice to say that film tests have measured unique and stable abilities such as serial integration and time and motion translation, that are not measurable with paper-and-pencil tests. Film tests have been developed for air force trainees (Gibson, 1947), college students (Seibert and Snow, 1965;

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Seibert, Reid and Snow, 1967), and for normal and dyslexic school children (McDaniel, 1971, 1973). The four film tests used in this study were Short Term Visual Memory II and III (abbreviated STVM II and STVM III) and Short Term Color Memory I and II (abbreviated STCM I and STCM II). These four film tests are described in appendix A. The tests resembled transfer ~~to~~ or learning to learning task since not all stimuli were identical. It was assumed that subjects scores would improve somewhat over items, and that Ferguson's (1954) suggestion that different abilities exert different effects at different stages of learning would be demonstrated.

Tucker's (1960, 1966) method appeared to be a promising solution to the problem of measuring the different traits used at various stages in a task that at the same time preserved individual differences. His method involves the factoring of a trials by subjects data matrix. If the task performed is simple, only one factor might appear, whereas the performance of more challenging (more complex) task might require three or four factors to explain the variance adequately.

Tucker's method is based on the factor analytic equation

$$x_{ji} = a_{j1} y_{1i} + j_{j2} y_{2i} + \cdots$$

where x_{ji} is the score on trial j for individual i, the a's are coefficients depending on trials, and the y's are individual parameters. The a's are constant for all subjects on any given trial, whereas the y's vary across individuals on any given trial. The a's can thus be thought of as a matrix of factor loadings and the y's a matrix of factor scores.

Thus, Tucker's method indicates the complexity of a task by the number of factors required. Second, Tucker's method also indicates how individuals differ on the abilities being used at different stages in the task. If a subject receives a high factor score on the first factor and near zero scores on other factors, he may be said to be a pure exemplar of Factor I. In contrast, subjects with significant factor scores on all factors definin_ a given trial have more complex methods of cognition. Thus, factor scores indicate how one individual's learning differs from another's.

Procedure

The four film tests mentioned above were administered to 159 college students. After each stimulus item, subjects were asked to indicate the letter, or the color, or the correct position of the stimulus in the array depending upon which test was being viewed. K-R 20 reliabilities for STCM I and II and STVM II and III were .84, .69, .31 and .90. For this paper, the reporting of Tucker's method will be limited to STVM II part 1 and STVM III part 1. In STVM II - 1 a circle marker designating the position precedes the array by 52 miliiseconds; in STVM III - 1 not only does the circle marker precede the array but also a bar marker appears at the position with the array. Factor analyses of the 159 x 3 binary matrices were first done using Kaiser's Little Jiffy Mark IV (1970; Kaiser and Rice, 1973) primarily for two reasons: Kaiser's method possibly obviates the concern with artifactual difficulty factors, and it was desired to compare his relatively new technique with more familiar techniques. Factor scores were obtained, and factor loadings and factor scores of selected individuals were plotted.

To further define the obtained factors, and to characterize individuals receiving high or low factor scores, regression analyses were run using a series of other independent variables that had been administered to the 159 subjects at the same time of these film tests.

Finally, to determine the effects of varying stimuli characteristics, regression analyses were run for STCM I and II and STVM II and III item preferences, using part score, location, and name of stimulus as independent variables.

Results

A Little Jiffy Mark IV on STVM II - 1 resulted in an overall root mean square of .14, a measure of sampling adequacy (MSA) of .61, and an index of factorial simplicity (IFS) of .83. The MSA would be characterized as "mediocre" and the IFS as "meritorious." Table 1 shows the rotated factor solution:

Table 1

Little Jiffy Mark IV on STVM II - 1

	Factor		
Item	I	II	III
1	-01	01	26*
13	03	25*	-08
22	-02	26*	03
32	-02	20*	11
39	31*	03	01
48	29*	05	-12
60	3[illegible]*	-02	-01
62	29*	-05	14*

Factor I might be described as late learning, Factor II as early learning only or fatigue and Factor III as being befuddled except at initial and final stages.

Figure 1 shows factor loadings plotted for each of the eight items in STVM II part 1. Figure 2 shows the conventional learning curve based on item preferences. The conventional learning curve completely obscures the fatigue factor.

Figures 3 through 6 show learning curves of four typical subjects with different factor scores. Figure 3 represents the performance of a subject with a low factor I score and a high factor II score. (Factor scores have been rescaled to a mean of 50 and a standard deviation of 10.) The low score on factor I indicates that the subject is not a late learner, and the high score on factor II means that the subject starts out well, which are borne out by the graph. Figure 4 shows the performance of subject with low scores on all factors; the graph shows that he got none of the items right. Figure 5 shows a learning curve for a subject high on both factors; he got most of the items right, as expected. Figure 6 shows a learning curve for a subject scoring high on factor 1 and low on factor 2; his learning curve illustrates his description, based on his factor scores, as a late learner, and not one who has initial success.

A Little Jiffy analysis (a principal components solution followed by rotation of all factors associated with eigenvalues of unity or greater) on binary data having such widely varying item preferences as these data would produce artifactual difficulty factors. It is probable that the Little Jiffy Mark IV does not produce such artifactual factors, since the analysis is of the images, the predicted value of each variable from all others, and the images are multivariate normal. The conservative word "probable" is used intentionally here since the exact constraints, if any, are not yet known. This happy trait of the Mark IV is one of the reasons that the Mark IV was used as the chief technique for this analysis.

To gain a further appreciation for the factors obtained from the application of Tucker's technique to STVM II, each of the three factor score vectors was used in turn as the dependent variable in a stepwise regression with 13 other tests that had been administered to the subjects at the same time as STVM II. These 13 tests are First and Last Names, Digit Span Visual, Gestalt Completion, Wide Range Vocabulary, Advanced Vocabulary, Color Form Recognition, Film Sequence Memory A and B, Picture Identification, Successive Perception III and IV, and Position Recall II and III. Psychometricians familiar with the French, Ekstrom and Price Kit of Reference Tests will recognize some of these tests; all are described in Seibert, Reid and Snow (1967). Only Picture Identification had a worthwhile correlation (r=.40) with factor scores on factor I. Somewhat similar to a Gestalt completion task, the Picture Identification test is also an important variable in the serial integration factor (Seibert, Reid and Snow, 1967).

Thus subjects who succeeded early in the STVM II task tended to do so by tapping a trait or process somewhat similar to serial integration or Gestalt completion; whether or not subjects also responded correctly to the items in the second half of the test depended largely upon whether or not they made use of the trait or process described by factor II.

The application of Little Jiffy Mark IV to STVM III - 1 resulted in an overall root mean square of .26, a measure of sampling adequacy of .76, and an index of factorial simplicity of .95. The MSA could be described as "middling" and the IFS as "marvelous." Table 2 shows the rotated factor solution:

Table 2

Little Jiffy Mark IV on STVM III - 1

Item	Factor I	Factor II
5	-05	54*
6	-02	43*
24	12	40*
32	32	13
34	43*	-11
48	43*	-06
52	45*	05
55	48*	-02

Factor I might be called late learning and Factor II early learning only or perhaps fatigue.

A Kaiser image analysis (Kaiser, 1963; Reid, 1968) resulted in three factors associated with eigenvalues of unity or greater; the third factor can be tossed. A tendency to overfactor is characteristic of this method. The rotated (varimax) solution is in Table 3.

Table 3

Kaiser Image Analysis on STVM III - 1

Item	Factor I	Factor II	Factor III
5	20	-46	-01
6	17	-37	-00
24	27	-41	-10
32	33	-27	-11
34	32	-12	01
48	34	-17	-09
52	41	-27	01
55	41	-23	-02

This is essentially the same factor pattern as the Little Jiffy Mark IV only less clearly defined.

A Little Jiffy, probably the most common "factor analysis" performed in the literature, resulted in a similar pattern as shown in Table 4:

Table 4

Little Jiffy on STVM III - 1

	Component	
Item	I	II
5	09	81
6	06	73
24	29	66
32	50	35
34	66	-05
48	64	03
52	62	28
55	67	19

A maximum likelihood solution (Jöreskog and van Thillo, 1971) solution was also applied, and produced similar results to the other solutions except that the factors were switched.

Table 5

Jöreskog's Maximum Likelihood on STVM III - 1

	Factor	
Item	I	II
5	99	08
6	39	24
24	40	39
32	17	49
34	12	39
48	07	50
52	24	54
55	17	57

We can be less hesitant about using Little Jiffy and Jöreskog's approach with these data from STVM III - 1 since item preferences are more nearly homogeneous.

Figure 7 shows the factor loadings plotted for each of the eight items in STVM III - 1. Figure 8 shows shows the conventional learning curve based on item preferences. Note that the conventional curve masks the two underlying processes. Figures 9 through 12 show learning curves of four typical subjects with different factor scores.

Figure 9 shows the performance of a subject who is high on factor I and low on factor II; accordingly the graph of his performance indicates that he missed the first two items and got the remaining six items correct. Figure 11 shows the performance of a subject somewhat the opposite of the subject in Figure 9. The subjects in Figure 11 had a low factor I score and a high factor II score.

To get a better understanding of the characteristics of factor I and II, each of the two factor score vectors was in turn used in a stepwise regression with the 13 tests mentioned before as independent variables. Picture Identification was again the only test with a worthwhile correlation; it correlated .39 with both factor I and factor II scores. Thus, success in the STVM III - 1 test seems to have depended somewhat on an ability similar to serial integration or Gestalt completion.

So far we have applied Tucker's method of analyzing learning curves by factor analysis to two different film tests to support the idea that different abilities exert different effects at different stages of learning. We have also attempted to describe these abilities by a set of 13 aptitude tests.

Finally, we report how the three specific stimulus characteristics were related to item preference. One stimulus characteristic was number of delay frames, each set of which defined a part. Although this report has concentrated on one part of STVM II and one part of STVM III, each test had eight parts. A second characteristic was that different letters were randomly selected for each item. Third, the correct answer could be in any of 8 positions in the 2 x 4 array of letters.

Accordingly, regression analyses were run on item preferences of STVM II and III with parts, positions, and letters as dependent variables to measure the proportion of variance accounted for by these three stimulus characteristics.

A multiple regression equation with 26 letter vectors, 3 position vectors, and 1 part vector, with item preferences of STVM II as the dependent variable produced an $R^2 = .71$, or $\hat{R}^2 = .37$. A similar equation but with the part vector dropped gave an $R^2 = .70$, or $\hat{R}^2 = .37$. An F test between these two models of course was not significant, which meant that part (that is, number of delay frames) played no important role in accounting for any of the variance in item preference.

An equation with the 26 letter vectors alone gave an R^2 of .33 with item preference, but the unbiased estimate, $\hat{R}^2$, shrunk to zero. Interesting differences occurred in the partial sums of squares among the letters, but this discussion can be saved for another time.

An equation with the 3 position vectors alone gave an R^2 of .50, $\hat{R}^2 = .41$. Thus, position in the array accounted for 41% of the variance of item preferences.

A multiple regression equation with 26 letter vectors, 3 position vectors, and 1 part vector, with item preferences of STVM III as the dependent variable gave an R^2 of .73, or $\hat{R}^2 = .42$. A similar equation but without the part vector gave an R^2 of .71, or $\hat{R}^2 = .38$. An F ratio between these two models was 2.68 (1, 29) which is not significant. Thus, as in STVM II, part again played no important role in accounting for variance of item preferences.

An equation with the 26 letter vectors alone gave an R^2 of 42, but $\hat{R}^2$ shrunk to zero. A regression equation with the 3 position vectors alone produced $R^2 = .45$ or $\hat{R}^2 = .37$. As in STVM II, position in the array accounted for 37% of the item preference variance.

It is interesting to compare the influence position has in these two tests with two other film tests, Short Term Color Memory (STCM) I and II which used a 2 x 3 stimulus array presenting 6 colors from a pool of 9.

A multiple regression equation with 9 color vectors, 6 position vectors, and one part vector with item preference for STCM I as the dependent vector produced an $R^2 = .72$, or $\hat{R}^2 = .61$. A similar equation only without the part variable produced an R^2 = .71 or $\hat{R}^2$ = .61. Thus the variable of part (the number of frames the stimulus remains on the screen) had no predictive value to item preference at least in the 4 to 12 frames range.

A model using only the 9 colors as independent variables produced an R^2 = .57 or $\hat{R}^2$ = .48. F (5, 40) 2.60 is significant at the .05 level. Thus, both color and position made significant contributions to the variance in item preference in STCM I.

A multiple regression equation with 9 color vectors, 6 position vectors, and one part vector with item preference for STCM II as the dependent vector resulted in an R^2 = .73, or $\hat{R}^2$ = .63. When the part vector was dropped from the model, the R^2 was .66, or $\hat{R}^2$ = .54. The two regression models produced an *F* ratio = 10.86 (df = 1, 39) which is significant at the .01 level.

When the regression model contained the part and color vectors, the R^2 = .64, or $\hat{R}^2$ = .55. The full model (R^2 = .73) compared with this model yielded an *F* (5, 39) of 2.56, significant at the .05 level.

When the regression model contained both part and position vectors, the R^2 was .16, or $\hat{R}^2$ = .03. Thus color makes the most contribution to item preference variance, but part and position still make some contribution. In none of the other three tests in this report did part make a significant contribution to item preference variance.

Summary

From the ideas well set forth by Ferguson (1954), Messick (1972) and others, we sought a method that would identify different abilities that entered at different stages in a task. The method selected should meet Cronbach's (1957) criterion that it would consider individual differences as well as group performance. Tucker's (1960, 1966) method seemed to be able to identify differing abilities while at the same time preserve individual differences.

Using Kaiser's (1973) Little Jiffy Mark IV to skirt the problem of difficulty factors, we factor analyzed two film tests and found two factors operating in each test. Had the task been longer (some authors recommend 1000 items) we likely might have found more factors; it is equally likely, however, that diminishing returns would soon obtain.

We demonstrated that Tucker's method revealed abilities that a conventional learning curve would mask, and we illustrated learning curves of several different individuals. We made some attempt to describe the obtained factors with other variables; at least one factor correlated with a variable similar to Gestalt completion or serial integration.

Finally we considered the effect of three stimulus characteristics upon item preference. Test part (number of stimulus frames, or number of delay frames) rarely accounted for an important part of item preference, whereas position in the array always did and color did in the two color tests.

Appendix A

Short Term Color Memory I: Ss are first shown a stimulus pool of nine hexagonal color chips (a red, green, purple, yellow, orange, brown, gray, pink and blue). After these are presented and named, the 54 items of the test then present the color chips in a 2 by 3 array, holding the array on the screen either for the duration of 4, 8, or 12 frames. There are thus three 18-item subtests, each with a different stimulus exposure duration. In each item, after the array disappears, there are two blank film frames, then an empty hexagonal marker appears in one of the array positions. The Ss are to indicate the color which occupied the marked array position.

Short Term Color Memory II: This film test is similar to STCM I. It differs in that it marks the color to be remembered by following the array with a horizontal colored bar in the center of the screen. The Ss are then to indicate the position occupied by the color.

Short Term Visual Memory II: A film test of 64 items which includes eight eight-item subtests. In each item an eight letter, 2 by 4 array is presented tachistoscopically (i.e., for about 31 milliseconds) in screen center. A black circle marker appears to mark one of the eight array positions and Ss are to record the letter occupying the designated position. The circle marker may precede the array by 52 milliseconds or may follow it by 10, 94, 177, 260, 344, 428, or 510 milliseconds.

Short Term Visual Memory III: This film test is highly similar to STVM II, except that it employs two markers to designate the same array position in each test item. Not only does a circle marker appear as in STVM II, but also simultaneously with each 2 by 4 array, a black vertical bar marker appears.

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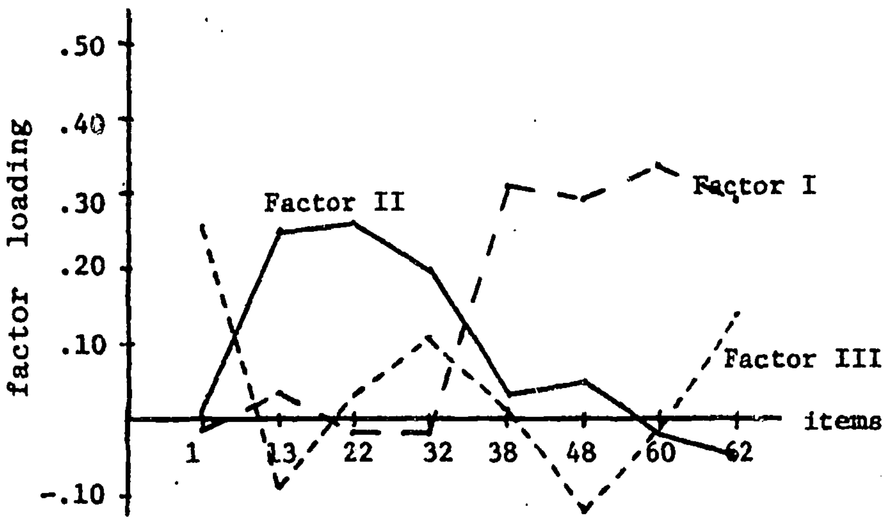


Figure 1

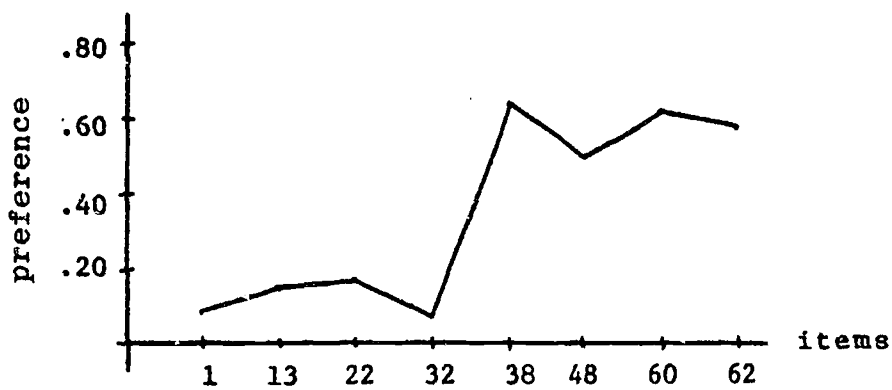
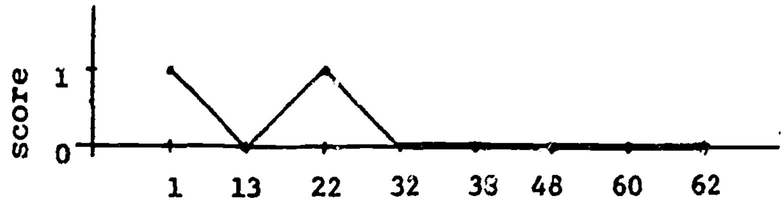
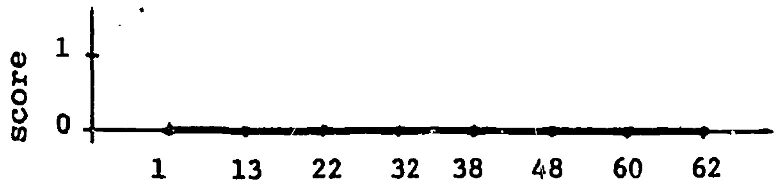


Figure 2



S 15; factor scores = 35, 57, 66

Figure 3



S 19; factor scores = 31, 41, 41

Figure 4

S 21; factor scores = 66, 63, 76

Figure 5

S 56; factor scores = 56, 44, 54

Figure 6

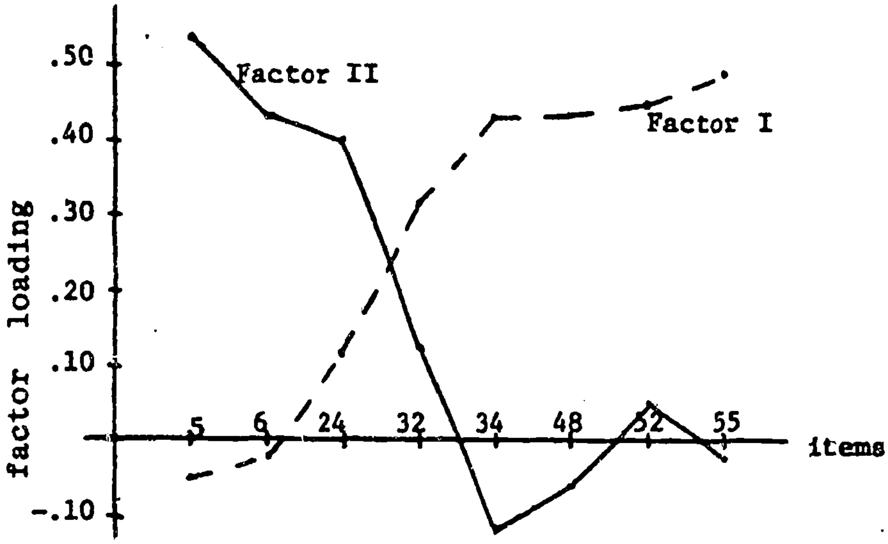


Figure 7

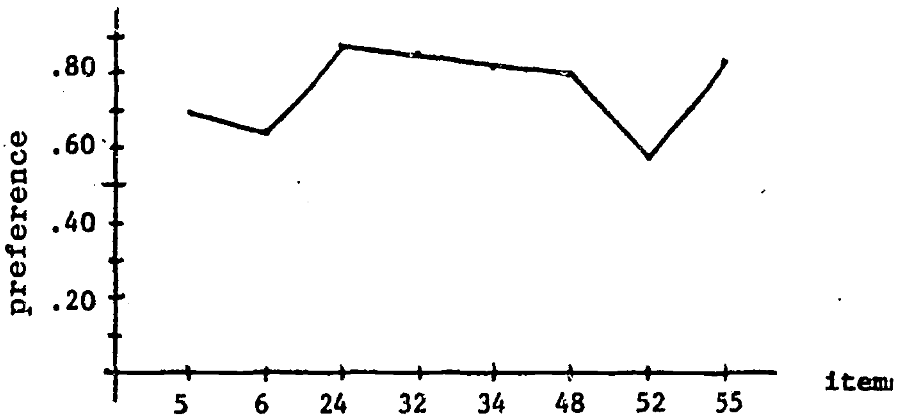
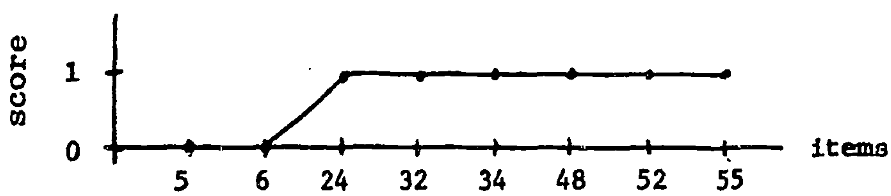
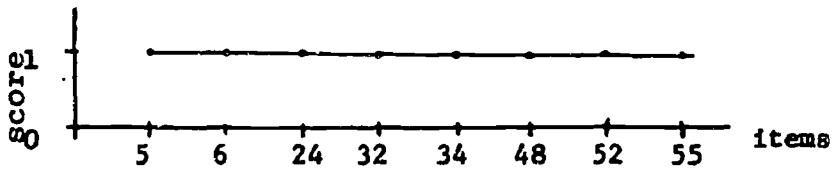


Figure 8



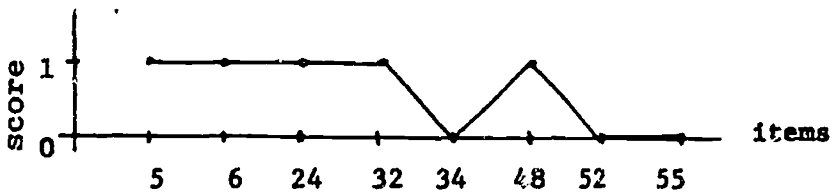
<u>S</u> 30; factor scores = 54, 44

Figure 9



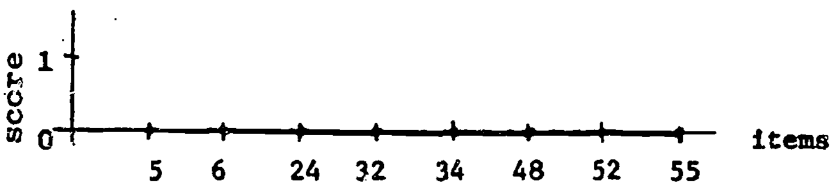
S 111; factor scores = 59, 59

Figure 10



S 152; factor scores = 39, 52

Figure 11



S 155; factor scores = 17, 20

Figure 12