DOCUMENT RESUME

ED 085 255 SE 017 077

TITLE Three Reports on a Study of 11th Grade Mathematics.
INSTITUTION Chicago Univ., Ill. Graduate School of Education.
PUB DATE 73
NOTE 75p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS Achievement; *Algebra; Attitudes; *Curriculum; Geometric Concepts; *Program Evaluation; *Research; *Secondary School Mathematics; Textbook Research; Transformations (Mathematics)

ABSTRACT

Comparison of two eleventh-grade mathematics programs was made by evaluations of student and teacher attitudes and of student achievement on specific content as well as performance on the ETS Algebra II exam. An experimental program based on Zalman Usiskin's textbook, "Intermediate Algebra," was matched with programs based on traditional texts. Usiskin's program differs in its use of geometrical techniques in the algebraic setting. This feature allows students to handle more application problems. Both student and teacher attitude measures favored the experimental program, but there were no significant differences in performance between groups on the final exam. Part of this document is a description of planned revisions in Usiskin's syllabus based on the analysis of performance on specific content topics. (JP)

THREE REPORTS ON A
STUDY OF 11TH GRADE MATHEMATICS

First Report: Comparison of Attitudes of Eleventh-Grade Students in Mathematics Classes Using Traditional and Experimental Textbooks, by Jeanne Bernhold Hayman, August, 1973.

Second Report: Achievement of Eleventh-Grade Students in Mathematics Classes Using Traditional and Experimental Textbooks, by Zalman Usiskin, September, 1973.

Third Report: Teacher Opinions and Planned Changes in the experimental text Intermediate Mathematics, by Zalman Usiskin, September, 1973.

All studies undertaken at the Graduate School of Education, University of Chicago.

STUDY OF 11th GRADE MATHEMATICS

FIRST REPORT

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER 1

DESCRIPTION OF MATERIALS, SCHOOLS, TEACHERS, AND STUDENTS

1.1 Materials and Types of Schools

This paper is a part of the analysis of the data obtained from the field testing of Intermediate Mathematics, an eleventh-grade experimental textbook by Zalman Usiskin. The motivation for the author to write this book consisted of his observation of 1) the coverage of many disjointed topics in eleventh-grade math, 2) the lack of reliance on geometry, 3) the distorted view of math provided by "Algebra 2" classes that have little to do with algebra, and 4) an equally distorted view of applications.

This textbook was tested during the 1972-1973 school year in four different types of schools as shown in the following table. The classification is based on the use of Geometry: A Transformational Approach (GATA) or a non-transformational geometry in the tenth grade and the Intermediate Mathematics (IM) or a traditional eleventh-grade text the following year. The traditional texts used were

	Tenth-grade math	
Eleventh-grade math	GATA	Traditional
IM	A	B
Traditional	C	D

Modern Algebra and Trig by Dolciani, Berman, Wooton; MSM: Algebra 2 and Trig by Dolciani, et al; and Algebra Two with Trig by Payne and Zamboni.

The teachers in all four types of schools were volunteers to be in this study. Pre- and post-tests on standard algebra content were administered to all of the groups. Opinionnaires consisting of thirty-two questions in September and thirty-one questions in June were given. The first twenty questions on each were to measure feelings of like, dislike, fear, enthusiasm, etc. for math and were from the attitude acale of Aiken and Dreger.[1] The following ten questions were concerned with what the student perceived math to be and had statements on applications, relations of proof to math, etc. One question asked if the student planned on taking math the coming year. The September form contained one question not on the June form which asked for the occupation of the chief-money earner in the family. (See Appendix 1 for a copy of the Opinionnaire.) Opinionnaires were given to the students four times during the year. During the author's visit to the schools, however, a couple of the teachers stated that some of the students appeared to not be marking these reliably. The author, in his comments to the students, then stressed the importance of these opinionnaires. Thus, it was decided to use only the first and last of these opinionnaires for analysis. Only the forms of the students who had completed both of these Opinionnaires were analyzed.

[1]Lewis R. Aiken, "Personality Correlates of Attitude Towards Mathematics," Journal of Educational Research, 56: 476-80 (May, 1963).

1.2 Teachers

A total of thirteen teachers were involved in this study, seven with experimental (AC) classes and six with control (BD) classes. In Table 1.2a, it is seen that each teacher taught one, two, or three classes in this study.

Table 1.2a

DISTRIBUTION OF TEACHERS AMONG CLASSES

	No. with one class	No. with two classes	No. with three classes	Total No. of classes
Experimental	4	3	0	10
Control	1	4	1	12

The teachers were asked various questions about their backgrounds in math and their teaching experience. Table 1.2b indicates that in both the experimental (AC) and the control (BD) groups, the teachers had approximately the same general teaching experience and eleventh-grade teaching experience although the experimental group had one inexperienced teacher. Teachers in both groups had completed approximately the same number of algebra courses. It should be noted that two of the experimental teachers had no college courses on algebra. The two groups of teachers had nearly the same amount of formal exposure to the concepts relating to the experimental text. More of the experimental teachers, however, had actually introduced these into their eleventh-grade curriculum.

TABLE 1.2b

SELECTED CHARACTERISTICS PERTAINING TO THE MATHEMATICAL AND TEACHING BACKGROUND OF EXPERIMENTAL AND CONTROL TEACHERS

	No. of Experimental Teachers	No. of Control Teachers
Number of years teaching high school or college math including present year		
1	1	
2-3		
4-5	2	1
6-7	2	2
8-	2	3
Number of years teaching 11th-grade college-bound math including present year		
1	2	
2-3	1	3
4-6	3	1
7-	1	2
Number of college courses taken with emphasis on algebra		
0	2	1
1-2	1	1
3-4		4
5-8	3	
Number of teachers studying listed topic in a formal course		
Matrices	7	6
Vectors	6	5
Groups	7	6
Homomorphisms	5	5
Linear transformations	6	4
Reflections, rotations, isometries, etc.	4	4

TABLE 1.2b-Continued

SELECTED CHARACTERISTICS PERTAINING TO THE MATHEMATICAL AND TEACHING BACKGROUND OF EXPERIMENTAL AND CONTROL TEACHERS

	No. of Experimental Teachers	No. of Control Teachers
Number of teachers incorporating listed topic into courses taught		
Matrices	6	4
Vectors	4	2
Groups	5	1
Homomorphisms	1	1
Linear Transformations	3	2
Reflections, rotations, isometries, etc.	4	2

1.3 Students

When the study was set up, an attempt was made to match as closely as possible the AC schools with BD schools having students of similar interests and ability. Table 1.3a shows the number of students in each type of school by sex.

TABLE 1.3a

DISTRIBUTION OF STUDENTS BY SEX AND TYPES OF SCHOOLS

	No. of females	No. of males	No. of total students
A	52	62	114
B	0	89	89
C	22	34	56
D	41	45	86
	115	230	345

Unfortunately there were no female students in type B schools which made it very hard to analyze the subgroups by sex. It also can be seen that there are only half as many female students in this study as there are male students. Even after taking into account the fact that there were no type B females, there are still more males.

To check how closely the schools matched, an attempt was made to identify each student by the job classification of the chief money earner in his family. The student's response to the question asking for this information was compared to a table of job types with ratings attached.[2]

[2] Warner, Meeker, and Eells, Social Class in America, (Science Research Associates, Inc., 1949), pp140-141.

To accurately determine the socio-economic class this table is used as only one of the determining factors. Thus, this gives only an approximation for social class for the students becauses it is only a job clsssification. Table 1.3b compares the experimental eleventh-grade text students (AC) with the traditional eleventh-grade text students (BD) by these ratings. (See Appendix 2 for the complete table of job listings).

TABLE 1.3b

OCCUPATION OF CHIEF MONEY-EARNER IN FAMILIES OF EXPERIMENTAL AND CONTROL STUDENTS

Occupational Rating	No. of Experimental Students	No. of Control Students
1	26	27
2	50	49
3	35	38
4	24	28
5	22	21
6	10	11
7	3	1
Totals	170	175

This almost perfect one-to-one correspondence was hoped for but is even closer than was expected.

CHAPTER 2

STUDENT ATTITUDES

2.1 General Attitudes

The general attitude of each student toward math was measured by the first twenty questions on the Opinionnaire. The student had five possible responses: strongly agree, agree, undecided, disagree, strongly disagree. The student's responses are scored by giving one point to the response that indicates the most negative attitude toward math and giving five points to the response that indicates the most positive attitude toward math. Thus, the scores for the twenty questions range from a low attitude of twenty to a high attitude of one-hundred. A score of sixty is neutral. Table 2.1a contains the September and June mean scores, standard deviation and t-test value for different populations.

Using the fact that for these numbers a t-value of 1.960 would be significant at .05 level, we see that there are no significant changes in scores from September to June for any of the groups.

Comparing the September AC and September BD scores gives a t-value of 3.110 which is a significant difference in attitudes between the two groups with the control (BD) group less positive. The lower mean is caused mainly by

TABLE 2.1a

MEAN ATTITUDE SCORES AND t-VALUES FOR EXPERIMENTAL AND CONTROL STUDENTS IN SEPTEMBER AND JUNE

Population--Month	N	Mean	Standard Deviation	t
A-September	114	76.00	13.8	-0.251
A-June	114	75.54	13.6	
B-September	89	67.07	14.9	-0.297
B-June	89	66.34	17.8	
C-September	56	67.46	14.1	1.437
C-June	56	71.28	14.0	
D-September	86	69.50	15.1	0.307
D-June	86	70.23	16.2	
AC-September	170	73.19	14.4	0.621
AC-June	170	74.14	13.9	
BD-September	175	68.26	15.0	-0.001
BD-June	175	68.25	17.1	
ABCD-September	345	70.69	14.9	0.396
ABCD-June	345	71.15	15.8	

the type B students who are all male and attend a private school. If eleventh-grade math is required of all their students then this could account for why their mean scores are lower. The difference in June between these two has a t-value of 3.512 which is significant and again the control group is less positive. It can be seen that the difference increased by noticing in Table 2.1a that the BD attitudes remained constant while the AC scores went up slightly, although not significantly.

Table 2.1b shows for various groups by sex the mean attitude scores, the standard deviation, and the t-values for September and June. There are no significant differences between September and June scores for any of these subgroups based on sex. The males and females tend to shift in the same directions with the exception of type D schools where the females' attitudes rose and the males' attitudes dropped. It can be noted that none of the mean scores dropped below the neutral value of sixty, so all the groups had somewhat positive attitudes towards math both in September and June.

Another way of representing this material is to use a matrix representation to tabulate the scores. This will show whether the students' attitudes have remained the same or if some, and if so which ones, have greatly changed their views of math. Table 2.1c contains the AC mean scores and the BD mean scores plotted by September against June scores. This frequency distribution table has ten intervals of approximately nine points each in each direction. A

TABLE 2.1b

MEAN ATTITUDE SCORES FOR MALE AND FEMALE EXPERIMENTAL AND CONTROL STUDENTS

Population--Month	N	Mean	Standard Deviation	t
Females-September	115	71.28	15.2	0.744
Females-June	115	72.78	15.3	
Males-September	230	70.40	14.7	-0.036
Males-June	230	70.34	16.1	
A Females-September	52	76.63	13.2	-0.232
A Females-June	52	76.02	13.9	
B Females-September	0			
B Females-June	0			
C Females-September	22	66.78	15.9	0.709
C Females-June	22	70.23	16.4	
D Females-September	41	66.90	15.4	0.903
D Females-June	41	70.02	15.8	
A Males-September	62	75.47	14.4	-0.129
A Males-June	62	75.15	13.5	
B Males-September	89	67.07	14.9	-0.297
B Males-June	89	66.34	17.8	
C Males-September	34	67.91	13.0	1.311
C Males-June	34	71.97	12.5	
D Males-September	45	71.87	14.5	-0.4388
D Males-June	45	70.42	16.6	

TABLE 2.1c

FREQUENCY DISTRIBUTIONS OF SCORES FROM SEPTEMBER AND JUNE OPINIONNAIRES FOR AC AND BD STUDENTS

AC

June	20-27	28-35	36-43	44-51	52-59	60-67	68-75	76-83	84-91	92-100
20-27										
28-35										
36-43				2	2	1				
44-51			2	3	3		2			
52-59					2	2	3	1	1	
60-67				1	6	6	3	6	1	
68-75			1	1		9	12	7	3	2
76-83				2	2	2	13	14	7	3
84-91				1		2	6	6	7	5
92-100				1		1		3	8	4

September

BD

June	20-27	28-35	36-43	44-51	52-59	60-67	68-75	76-83	84-91	92-100
20-27	1	1	2							
28-35			1	1	1		1			
36-43	1		3	7	2	2				
44-51			1	4		2				
52-59		1	1	1	1	3	5	3		
60-67				2	5	9	5	3		
68-75					2	8	10	16	2	
76-83				2	1	2	18	12	4	2
84-91					1	2	1	8	5	2
92-100							2	3	1	2

September

shift from one interval to the next interval indicates a change of from one to seventeen points. The heavy line indicates attitudes that did not change intervals. The dotted lines are those that changed by one interval. Scores above the lines indicate change in the negative direction and scores below the lines indicate change in the positive direction. Approximately thirty percent of the students in each group had attitude changes of over one interval. The changes occur in students who previously had scored at all attitude levels. There is no one group which greatly changed their attitudes as the changes are distributed throughout the attitude range and are both positive and negative.

A chi-square test was made to compare the distribution of attitude scores with social classes as measured by the job classification. It was found that social class and attitude were independent for all groups except the control group which had a chi-square of 90.0 and the control males which had a chi-square of 76.9. A chi-square of 72.2 is significant at the .10 level which indicates only a slight dependence of attitude and social class. These dependences were found only for June as all the September scores are independent of social class.

2.2 Specific Questions

Questions numbers twenty-one to thirty-one on the Opinionnaire are here considered separately, although several of them deal with basically the same topics. Table2.2a contains the mean for the AC and BD students and the t-value comparing these two for September and June for each of these questions.

Questions 21,24, and 27 all are about applications of math. For number 21 a high score is regarded as positve toward math. Using the .05 significance level, where 1.960 would be a significant t-value[3], there was no significant difference between the experimental and control students in their answers in September. In June, however, there was the highly significant difference at the .001 level. The AC students themselves changed their scores significantly higher and the BD students dropped their scores though not significantly. This combination made a very significant difference in their June scores. The only subgroup that changed significantly were the type C females who changed from 3.68 to 4.36.

Question 24 has a low score being regarded as positive. Table 2.2a shows that although there was no difference in scores between BD and AC in September, there was a significant difference in June. The AC mean had dropped which was an action in the positive direction. Comparing the BD males and females shows a significant difference in June. The females' mean had dropped from 2.86 to

[3]All measurements will be at the .05 significance level unless otherwise stated.

TABLE 2.2a

MEANS FOR EXPERIMENTAL AND CONTROL STUDENTS AND t-VALUES FOR SEPTEMBER AND JUNE FOR SPECIFIC QUESTIONS

Question Number	September			June		
	AC mean	BD mean	t	AC mean	BDmean	t
21. Mathematics has a large number of applications to a wide variety of situations.(+)	4.01	3.87	-1.653	4.21	3.75	-4.909
22. Mathematics is a collection of rules which seem to come from nowhere.(-)	2.20	2.65	4.174	2.10	2.49	3.805
23. Most of the mathematics usedby mathematicians today was known to the ancient Greeks. (-)	2.89	2.76	-1.383	2.64	3.03	3.829
24. Almost all of the uses of mathematics are in chemistry or physics.(-)	2.65	2.83	1.489	2.53	2.81	2.471
25. There are postulates (assumptions or axioms) in algebra which have the same role as postulares in geometry. (+)	3.62	3.60	-0.229	3.83	3.64	-2.572
26. Mathematical rules are often closely related to each other.(+)	3.96	3.85	-1.647	4.02	3.84	-2.516

TABLE 2.2a-Continued

MEANS FOR EXPERIMENTAL AND CONTROL STUDENTS AND t-VALUES FOR SEPTEMBER AND JUNE FOR SPECIFIC QUESTIONS

Question Number	September			June		
	AC mean	BD mean	t	AC mean	BD mean	t
27. Other than arithmetic, mathematics does not have too many uses outside of mathematics itself. (-)	1.82	2.15	3.254	1.85	2.25	4.083
28. I usually can understand mathematics from a book without a teacher's explanation. (+)	2.69	2.52	-1.445	2.82	2.49	-2.633
29. New mathematical theorems are discovered every year. (+)	3.27	3.30	0.247	3.43	3.22	-2.585
30. Proof belongs to geometry not algebra. (-)	2.69	2.87	1.520	2.30	2.74	3.610

2.58 while the males' mean rose from 2.82 to 3.01. There seems to be no simple explanation for the change in the females' scores in the desired direction and the change in the males' scores in the opposite direction.

Question 27 has a low score being positive. The table indicates that in September, there existed a significant difference between the two groups. The difference was greater in June because the BD mean increased slightly though not significantly in the negative direction. The subgroup of AC showed no significant difference between sexes in September but in June the difference was significant with the females score changing from 1.78 to 1.68 and the males' scores from 1.86 to 1.96. Although neither change was significant in itself, the combined effect was significant. The bulk of the incorrect change on the part of the males was caused by the type A males who changed their mean from 1.65 to 2.02. Again, no explanation can be given why the males changed their mean in the direction not positively related to math.

In summation for the three questions on applications, all three questions showed significant differences in June between the experimental and control groups and two of these originally had no significant differences in September. All the differences in mean scores were in favor of the experimental students.

Questions twenty-two and twenty-six have to do with the internal consistency of math. Question 22 has a low score being positive. In September there is a highly significant difference between the two groups and there still is in June because both groups change their scores in the direction desired. In the control group, there is no significant difference between sexes in September, but in June there is a significant difference. The females have dropped their mean from 2.60 to 2.31 while the males have only dropped their mean from 2.69 to 2.63.

On question 26 a high score is desired. Table 2.2a indicates that there was no significant difference between the experimental and control students in September and that in June the difference was significant because of an increase in mean by the AC students. None of the subgroups had a significant difference in scores.

Thus on the questions of internal relationships in math, there are significant differences between the groups in June for both questions while September had only one group with a difference. The changes on both questions were in the positive direction for the experimental students.

Questions 23 and 29 have to do with whether math is a purely historical subject or is still growing today. On question 23 a low score is desired. In September there is no significant difference between AC and BD while in June there is significant difference at the .001 level. This is caused by the fact that there is significant change in the BD mean from 2.76 to 3.03 which is in the negative direction. A large part of the AC change is due to the fact that A changes significantly from 2.82 to 2.55. A large part of the BD change is due to the fact that B changes significantly from 2.79 to 3.11. Another subgroup that changes significantly is the type C females who drop their mean from 3.14 to 2.50. There are significant differences between the sexes in September for the control schools with the females scoring more positively than the males. They both change their means in the negative direction and thus are significantly different in June also, the females changing from 2.60 to 2.87 and the males changing from 2.89 to 3.17.

Question 29 has a high score being positive. Table 2.2a shows that there was no significant difference in means between AC and BD students for September but that by June, the difference was significant. The AC mean score had changed in the positive direction. Of the possible subgroups, type B students, which were all male, had significantly changed their mean in the negative direction from 3.37 to 3.11.

Thus for the questions concerning the contemporary growth of math, both showed no significance difference between the groups in September and significant differences in June with the experimental group moving in the positive direction. Type B students, who had the transformational geometry and the traditional eleventh-grade texts, changed significantly in the negative direction for both questions.

Questions 25 and 30 are concerned with proofs and postulates and their relation to algebra. On question 25 a high score is desired. Table 2.2a indicates that though there is no significant difference in September between the experimental and control students, there is a significant difference in June. This is caused mainly by the AC group which changes significantly in the positive direction from 3.62 to 3.83. Type C changes significantly from 3.45 to 3.75. Type D changes significantly from 3.44 to 3.69. The group of all females also changes significantly from 3.51 to 3.80. The type C females change significantly from 3.27 to 3.86. All the students changed their mean significantly in the positive direction from 3.61 to 3.73.

Question 30 has a low score being a positive answer. In September there was no significant difference between the AC and BD groups but in June the difference had become significant. This was largely due to the AC group changing significantly in the positive direction. Type C students changed their score significantly from 3.04 to 2.34. The group of all males in the study changed their mean scores significantly from 2.87 to 2.57 which brought their mean closer to that of the females. Type C males changed significantly from 3.03 to 2.30. All the students changed their mean from 2.79 to 2.52 significantly in the positive direction.

For both of the questions about the relationship of postulates and proofs to algebra, there was no significant difference in mean scores for experimental and control groups in September, but there were significant differences in June in favor of the the experimental students. There were significant changes in AC and specifically in C, the group which had not used the transformational geometry, in the positive direction on both questions. Also these two were the only questions that for the whole study population showed significant changes in score and these changes were in the positive direction.

Question 28 is intended to measure the reading level difficulty of the text. Table 2.2a indicates that though there was no significant difference in September, there was a significant difference in June with AC moving toward an easier understanding of the text and BD toward having more difficulty with reading the text although neither of these changes is significant.

The students were asked in Question 31 whether they intended to take math the following year. The results of this question were tested against attitude and social class to see which, if either, had an effect on the student pursuing more math. In September when the attitudes were plotted against the intentions to pursue math, a chi-square test [4] indicated that these two were not independent for the experimental students with chi-square of 50.8 for the AC and are just barely independent for BD with a chi-square of 30.3 which is significant at .10 level. In June when

[4] A chi-square value of 31.5 is significant for 18 degrees of freedom at .05 level.

the attitudes were again plotted against the intentions to pursue math, it was found that the two were not independent for either group by a chi-square of 42.7 for AC and of 47.3 for BD. In the subgroups of the males for BD, a chi-square value of 58.7 showed they weren't independent for June.

The social classes were also plotted against the intention to continue math. The chi-square of this tabulation for each group shows these to be independent. Table 2.2b compares the figures for AC and BD for both September and June.

In comparing the September and June responses on continuing math, each group and subgroup were analyzed. The June scores were dependent on the September. Table 2.2c contains these data. The number of experimental yes responses stays the same while the maybe responses decrease and the no responses increase. For the control group the no responses stay the same while the maybe responses decrease and the yeses increase. To possibly analyze why the BD group increased the number of yes responses while the AC group increased the number of no responses, it was noticed that the BD increase appeared to be in the males so the type B students (all males) were hand checked. In September 25 of the 89 had responded maybe or no while in June only 6 of the 89 did not say yes. Recalling that attitude and the intention to pursue math are not independent and that the B group had the lowest attitude, it is possible that a fourth year of math is required or strongly urged for these students.

TABLE 2.2b

NUMBERS OF STUDENTS PLANNING TO CONTINUE IN MATHEMATICS IN THE EXPERIMENTAL AND CONTROL POPULATIONS BY THE OCCUPATION OF THE CHIEF MONEY EARNER IN THE FAMILY

Occupation	Experimental No. of Students	Experimental No. Continuing	Control No. of Students	Control No. Continuing
September				
1	26	19	27	13
2	50	33	49	25
3	35	25	38	27
4	24	14	28	23
5	22	14	21	13
6	10	6	11	5
7	3	1	1	0
June				
1	26	17	27	17
2	50	31	49	33
3	35	25	38	31
4	24	17	28	25
5	22	13	21	20
6	10	7	11	10
7	3	1	1	0

TABLE 2.2c

NUMBERS OF STUDENTS PLANNING TO CONTINUE IN MATHEMATICS IN THE EXPERIMENTAL AND CONTROL POPULATION BY SEX

Population-Month	Responses yes	no	maybe
AC			
Females-September	42	10	22
Females-June	41	29	4
Males-September	70	8	18
Males-June	70	15	11
Total-September	112	18	40
Total-June	111	44	15
BD			
Females-September	18	12	11
Females-June	21	16	4
Males-September	88	15	31
Males-June	115	15	4
Total-September	106	27	42
Total-June	136	31	8

CHAPTER 3

TEACHER REMARKS ABOUT THE TEXTBOOK

At the end of the year a question form was given to both sets of teachers. Some of the questions on the experimental form asked specifically about certain sections of the IM text. Both forms asked where difficulties in subject matter occurred, use of text aids, etc. Only the questions that had any bearing on attitudes are considered here in Table 3a.

The AC teachers felt the text was for average to above-average students and thought the reading level was easy. They also approved of the number of applications while control teachers wanted more applications. The AC teachers felt stronger about recommending their text than did the control teachers. Two experimental teachers felt they would have trouble with the math but said that they did not.

The two experimental teachers who would not recommend the book are both type C. One said he would not recommend it for students who had not studied the GATA. The other teacher was indifferent. The only teacher who felt that the reading level was hard was a type C teacher. On the questions that asked specifically about sections of the IM book there is a general consistency in response with the exception of two teachers, one type C and one type A. These are the two teachers who did not list having any college algebra courses. This means that the teachers who have no background will probably not like the IM text, although they stated that they had no difficulty with the math in the text.

TABLE 3a

TEACHER RESPONSES TO QUESTIONS

Question	No. of Experimental Teachers	No. of Control Teachers
In general, I feel that the book is written for		
a) the above-average 11th grade college-bound math student	4	1
b) the average 11th grade college-bound math student	3	3
c) the below-average 11th grade college-bound math student		2
in my school.		
Compared with other mathematics textbooks I have taught from, this book is		
a) easier to read and understand	4	2
b) at about the same level	1	2
c) harder to read and understand	1	1
What do you think about the number of applications in this book?		
a) too much		
b) too little		4
c) just about right	7	2
I		
a) would strongly recommend	4	
b) would recommend	1	3
c) am indifferent to	1	2
d) would not recommend	1	1
this text to other 11th grade math teachers.		

TABLE 3a-Continued

TEACHER RESPONSES TO QUESTIONS

Question	No. of Experimental Teachers	No. of Control Teachers
When you first began this course did you feel that you would have troubles with the mathematics?		
a)definitely		
b)somewhat	2	
c)not really	5	6
Did you have troubles with the mathematics of this course?		
a)definitely		
b)somewhat		
c)not really	7	
Before this year, were you dissatisfied with the 11th grade course as outlined in most commercial texts?		
a)very much	1	
b)only slightly	3	2
c)no	2	4
Has teaching this course changed your attitude towards 11th grade math and teaching 11th grade math?		
a)yes	3	
b)no	4	

CHAPTER 4

SUMMARY AND CONCLUSIONS

The general attitudes of the students were the same in September and June with no significant difference in means for any of the possible subgroups. The attitudes appeared to not be independent of the occupation of the chief money earner in the family for all subgroups.

On the ten specific questions of the Opinionnaire in all the cases where there were no significant differences in September, there were significant differences in June which favored the experimental students. On the questions that had a significant difference in September, there was still a significant difference in June and the experimental students had moved their mean in the positive direction. Generally, where there is a significant difference between male and female means within some group, the females always score more positively than the males. A conjecture might be that this is because the females who take eleventh-grade math are more interested in math.

Specifically about applications of math, all the experimental teachers had felt that there were a sufficient number of these and their students' means had become more positive on these questions about applications. Only one-third of the control teachers felt their texts had enough applications while the means of their students stayed the same or became more negative, though not significantly.

Two-thirds of the experimental teachers felt that the reading level of the IM text was easier than most other texts while only one-third of the control teachers felt this way about their texts. Though there were no differences in the students' means in September on the question whether they could understand the text without the teacher's help, in June the difference was significant with the experimental students feeling they could understand the book better than the control the students did.

The intentions of the students to pursue math as recorded in June are dependent on their responses in September. Although in September the attitudes and intent to continue math are independent, they are slightly dependent in June. The control group had an increase in the number of students continuing math while there was no such increase in the experimental group.

The experimental teachers were more positive on their text than the control teachers were on theirs with the exception of the experimental teachers who had no college algebra.

The only specific questions that had the whole study population change significantly their responses were the two on the relation of proof and postulate to algebra. Thus, the eleventh-grade math appears to show this relationship to all the students, regardless of text.

APPENDIX 1

OPINIONNAIRE

Name ____________________ Period ________ Do not write inside box

School ________________ Teacher ____________

DIRECTIONS: Each of the statements on this opinionnaire expresses a feeling or belief which a person might have towards mathematics. You are to express the extent of agreement between the belief or feeling given in each statement and your own personal feeling or belief. The five choices are: Strongly disagree (SD), Disagree (D), Undecided (U), Agree (A), Strongly agree (SA). Circle the letter which best indicates how closely you agree or disagree with the feeling or belief expressed in each statement as it concerns you.

1.	I do not like mathematics. I am always under a terrible strain in a math class.	SD	D	U	A	SA
2.	I do not like mathematics, and it scares me to have to take it.	SD	D	U	A	SA
3.	Mathematics is very interesting to me. I enjoy math courses.	SD	D	U	A	SA
4.	Mathematics is fascinating and fun.	SD	D	U	A	SA
5.	Mathematics makes me feel secure, and at the same time it is stimulating.	SD	D	U	A	SA
6.	I do not like mathematics. My mind goes blank and I am unable to think clearly when working math.	SD	D	U	A	SA
7.	I feel a sense of insecurity when attempting mathematics.	SD	D	U	A	SA
8.	Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	SD	D	U	A	SA
9.	The feeling that I have toward mathematics is a good feeling.	SD	D	U	A	SA
10.	Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.	SD	D	U	A	SA

11.	Mathematics is something which I enjoy a great deal.	SD	D	U	A	SA
12.	When I hear the word math, I have a feeling of dislike	SD	D	U	A	SA
13.	I approach math with a feeling of hesitation -- hesitation resulting from a fear of not being able to do math.	SD	D	U	A	SA
14.	I really like mathematics.	SD	D	U	A	SA
15.	Mathematics is a course in school which I have always liked and enjoyed studying.	SD	D	U	A	SA
16.	I don't like mathematics. It makes me nervous to even think about having to do a math problem.	SD	D	U	A	SA
17.	I have never liked math and it is my most dreaded subject.	SD	D	U	A	SA
18.	I love mathematics. I am happier in a math class than in any other class.	SD	D	U	A	SA
19.	I feel at ease in mathematics, and I like it very much.	SD	D	U	A	SA
20.	I feel a definite positive reaction to mathematics; it's enjoyable.	SD	D	U	A	SA
21.	Mathematics has a large number of applications to a wide variety of situations.	SD	D	U	A	SA
22.	Mathematics is a collection of rules which seem to come from nowhere.	SD	D	U	A	SA
23.	Most of the mathematics used by mathematicians today was known to the ancient Greeks.	SD	D	U	A	SA
24.	Almost all of the uses of mathematics are in chemistry or physics.	SD	D	U	A	SA
25.	There are postulates (assumptions or axioms) in algebra which have the same role as postulates in geometry.	SD	D	U	A	SA

26.	Mathematical rules are often closely related to each other.	SD	D	U	A	SA
27.	Other than arithmetic, mathematics does not have too many uses outside of mathematics itself.	SD	D	U	A	SA
28.	I usually can understand mathematics from a book without a teacher's explanation.	SD	D	U	A	SA
29.	New mathematical theorems are discovered every year.	SD	D	U	A	SA
30.	Proof belongs to geometry, not algebra.	SD	D	U	A	SA

31. What exactly does the chief money-earner (father, mother, stepfather, guardian, etc.) in your family do for a living? Please try to give the job title (or job description) as best as you know it.

____

32. At the present time, do you think you will take mathematics during the next school year (73-74) either in high school or college?

Yes ____________ No ____________ Undecided ____________

APPENDIX 2--SCALE FOR RATING OCCUPATION

Rating Assigned to Occupation	Professionals	Proprietors and Managers	Business Men
1	Lawyers, doctors, dentists, engineers, judges, high-school superintendetnts, veterinarians, ministers, (graduated from divinity school), chemists, etc. with post-graduate training, architects,	Business valued at $75,000 and over	Regional and divisional managers of large financial and industrial enterprises
2	High-school teachers, trained nurses, chiropodists, chiropractors, undertaker, ministers(some training), newspaper editors, librarians (graduate)	Business valued at $20,000 to $75,000	Asst. Managers and office and department managers of large businesses, assist. to executives, etc.
3	Social workers, grade-school teachers, optometrists, librarians (not graduate) undertakers asst. minister(no training)	Business valued at $5,000 to $20,000	All minor officials of businesses
4		Business valued at $2,000 to $5,000	
5		Business valued at $500 to $2,000	
6		Business valued at less than $500	
7			

APPENDIX 2--SCALE FOR RATING OCCUPATION-Continued

Rating for Occupation	Clerks and Kindred Workers	Manual Workers	Protective and Service Workers	Farmers
1	Certified Public Accountants			Gentleman Farmers
2	Accountants, salesmen of real estate, of insurance, post-masters			Large farm owners, farm owners
3	Auto salesmen, bank clerks and cashiers, postal clerks, secretaries to executives, supervisors of railroad, telephone, etc., justices of the peace	Contractors		
4	Stenographers, bookkeepers, rural mail clerks, rail-road ticket agents, sales people in dry goods store, etc.	Factory foremen, electricians plumbers, carpenters watchmakers own business	Dry cleaner butcher, sheriffs, railroad engineers and conductors	
5	Dime store clerks, hardware salesmen, beauty operators, telephone operators	Carpenters, plumbers, electricians, (apprentice) timekeepers, lineman, telephone or telegraph, radio repairmen, medium-skill workers	Barbers, firemen, butchers apprentice, practical nurse, policemen, seamstress, cooks, bartenders	Tenant farmer
6		Moulders, semiskilled workers, assistants to carptenters, etc.	Baggage men, night policemen and watchmen, taxi and truck drivers, gas station attendants, waitresses	Small tenant farmers
7		Heavy labor, migrant work, odd-job men, miners	Janitors, scrubwomen, newsboys	Migrant farm laborers

APPENDIX 3--PARTICIPATING SCHOOLS AND TEACHERS

Schools	Teachers
Adams High School Rochester, Michigan	John Guerrieri Joyce Krull
Adrian High School Adrian, Michigan	Sam Ewing
Fennville High School Fennville, Michigan	S. D. Hoffman
Gonzales United High School Gonzales, California	Pamela Bailey
New Trier West High School Northfield, Illinois	Joanne Oravec Herbert Wolf
Niles West High School Skokie, Illinois	Donald Field Robert Murphy
Notre Dame High School Harper Woods, Michigan	James A. Berch Mark Recor
Wauwatosa East High School Wauwatosa, Wisconsin	Thomas H. Duffy
Wauwatosa West High School Wauwatosa, Wisconsin	Tom Fitzpatrick

BIBLIOGRAPHY

Aiken, Lewis R. "Personality Correlates of Attitude Towards Mathematics." Journal of Educational Research, 56:476-80 (May, 1963)

Coxford, Arthur F., and Usiskin, Zalman P. Geometry: A Transformational Approach. River Forest, Illinois: Laidlaw, 1971.

Dolciani, Berman, Wooton. Modern Algebra and Trig. Boston: Houghton Mifflin, 1963.

Dolciani, et.al. MSM: Algebra 2 and Trig. New York: Houghton Mifflin, 1968.

Payne and Zamboni. Algebra Two with Trig. New York: Harcourt Brace and World, 1969.

Usiskin, Zalman P. Intermediate Mathematics. Volumes One and Two. Chicago, Illinois: the author, 1972.

Warner, Meeker, and Eells. Social Class in America Chicago, Illinois: Science Research Associates, Inc., 1949.

STUDY OF 11th GRADE MATHEMATICS

SECOND REPORT

There are three parts to this report. The first part involves only students who used Intermediate Mathematics and deals with their ability to understand concepts unique to this approach, as based upon performance on a special exam given to these students. The second and third parts compare experimental and control students on the ETS Algebra II exams. In the second part, we ask the question: Which populations performed relatively more strongly on which items, accounting for differences in the populations in June? In the third part, we ask: Which populations performed better on the entire tests, accounting for differences in the populations in September?

Concepts Unique to Intermediate Mathematics

Following this report, a copy of the Intermediate Mathematics Final Exam is given. This exam was designed to test concepts found in IM which are not in most standard 11th grade texts. The items were chosen to reflect the general and specific areas as described in Table 1 on the next page, where an item analysis is given.

Two features are immediately evident from the table. First, the exam was hard, a mean score being only half correct. Second, the "A" group performed much better than the "C" group (here "C" does not mean "control") - this is due more to the nature of the classes than treatment, in my opinion. Last September, a couple of "A" classes were shifted from average to above-average students too late for me to make any compensation. The "A" group had studied GATA, the "C" group had not.

An average A student answered 53.7% of the questions, an average C student 41.0%. One way to equate the groups is thus to add about 13% to the scores of C students. Having added this percentage (modified C in the table), we subtract modified C from A and multiply by 100 to get an adjusted difference in percentages.

These adjusted differences indicate that study of the geometry seems to make no relative difference in learning

Table 1: IM Final Exam Item Analysis

No.	Content	Total	A	C	Adj. C	A-Adj. C
		Percentages Correct				
	Transformation ideas in GATA:					
1	Congruence	79	87	65	78	9
6	Preservation properties	91	93	86	99	-6
11	Size transformations	89	93	82	95	-2
19	Parallelism	55	57	51	64	-7
25	Composition	60	65	49	62	3
	Groups					
2	Definition	62	65	57	70	-5
7	Identification	38	45	25	38	7
12	Negative identification	32	32	32	45	-13
20	Isomorphic groups	44	52	29	42	10
26	Isomorphism comp. nos.	28	32	22	35	-3
	Matrices					
3	Finding images	67	74	54	67	7
8	Non-matrix for transf.	35	40	25	38	2
13	Matrix for transformation	65	72	52	65	7
21	Transf. composition	50	55	42	55	0
27	Scale transformation	51	58	39	52	6
	Transf. in algebra					
4	Similarity in conics	27	36	11	24	12
9	Scale transf. of graphs	25	26	25	38	-12
14	Symmetry in conics	52	60	35	48	12
22	Translation of graphs	57	59	54	67	-8
30	Scale transformation	26	31	15	28	3
	Special developments					
5	Periodicity	34	40	23	36	4
10	Slope	50	48	52	65	-17
18	Cosine and sine	34	38	28	41	-3
23	Relations and functions	31	32	28	41	-9
28	Complex numbers	26	32	14	27	5
	Miscellaneous					
15	Corresponding properties	30	28	34	47	-19
16	Estimation and abs. value	60	64	52	65	-1
17	Intervals and approximation	60	66	49	62	4
24	Logarithms	58	66	43	55	11
29	Complex number mult.	63	67	55	68	-1

	Total	A	C
Number of students	186	121	65
Mean	14.8	16.1	12.3
Standard deviation	4.85	4.72	4.00
Reliability	.76	.75	.64
Mean % correct	.49	.54	.41

of the geometric ideas found in IM but does make a difference in the learning of matrices. This may be because transformations are learned before matrices. Since the learning of matrices comes quickly after transformations, it is best to have assimilated the transformations.

Special developments using transformations were relatively better learned by those not having the geometry, due to performance on Question 10.

Although there are differences in individual problems, the learning of groups, transformations in an algebraic setting, and various miscellaneous special developments are not affected by having studied GATA.

Questions 4, 14, 20, and 24 relatively favor those who studied GATA. The first two are very geometric in character. The latter two deal with isomorphic groups and it is possible that this is due to classes in population C not having enough time to study isomorphism in as much detail as population A.

Questions 9,10, 12, and 15 most relatively favor those who did not study GATA. Except for 12, the questions are the most identified with manipulative skills. They suggest possibly that schools which did not use the geometry spent more time on manipulative skills in the algebra, consistent with the standard courses in both cases.

But one must caution any overall interpretation of these scores. The use of relative performance forces nearly equal numbers of questions to favor one group or the other. It can determine no overall trends.

It is most difficult to judge overall performance on individual questions; 30% correct on a difficult question may be better than 70% on an easy question. These questions have never been standardized so there is no relative standard of performance. With these factors in mind, let us look at each of the six categories.

Transformation ideas also in GATA: Question 25, on composition, is tricky. Only the results of Question 19 are disappointing. Combined with Question 14 in a different category, they seem to indicate that visualization of transformations is not very good at this level. The other questions about transformations show a level of competence equal to any in any of the traditional questions on the ETS tests. (Average competence on traditional skills on the ETS tests for these students was 65%.)

Groups: Two experimental classes skipped (did not study) the material on groups. This affects the scores. Question 7 may have had higher scores if choices A and C had been switched; more students thought choice A than choice C. This is pretty tricky and in conjunction with 2 we might say that students knew pretty well what the definition of group is. Questions 12 and 20 indicate that the idea of corresponding groups of powers and multiples leading to logarithms was either not discussed or not understood. Question 26 is poorly worded so that choice F is not totally incorrect. 62% answered either F or G.

Matrices: 3 and 8 are disappointing; 8 does not seem to be difficult, and one would want a higher percentage on the easy Question 3.

Transformations in algebra: The most disappointing question of the entire test is 4, dealing with the similarity of all parabolas. I am at a loss to explain this result as there is a section devoted to the idea in IM. Question 9 is tricky; the easier question 22 shows that the idea was reasonably well understood. Questions 14 and 30 show that IM classes did not do well on those questions related to transformations and conic sections. Perhaps some material was skipped: later we note that the ETS exam only covers parabolas and circles and IM classes performed strongly.

Special developments: Question 5 was not studied by many classes, as it comes late in the book. Question 18 is difficult, being like a C exercise in the text. Question 23 is quite easy and shows that the writing is misleading; students believe transformations are more general than relations. Question 38 is also not difficult, but one would not know this from the performance.

Miscellaneous: Results are pleasing with the exception of the difficult Question 15; 16, 17, 24, and 29 are not easy questions.

In summary, these items seem to indicate that more work is needed with corresponding properties and revision is needed with functions and perhaps with some conics. They also seem to indicate that GATA is not necessary for IM except as it might give more time to discuss later topics in IM and would help skills with matrices.

ETS Final Exam Item Analysis

The reason for having half the students in each class take form A in June (form B in September) and the other half take the forms in reverse was to have twice as many questions to analyze, 80 instead of 40.

Here are the total score results on the two exams. June scores were analyzed for all students who had taken the correct September exam.

	BA Exp.	BA Cont.	AB Exp.	AB Cont.
Population (number)	88	94	94	95
Mean score	26.9	23.3	25.2	22.8
Standard deviation	7.74	5.75	7.18	6.80
Reliability of test	0.91	0.78	0.88	0.86
Average % correct		Exp (65.0)	Cont (57.5)	

Table 2 (next page) gives percentages correct for each of the 80 items on these exams. Scores of the lower control population are adjusted up 7% so that one can compare <u>relative performance</u>. This adjustment does not say anything about which group is better - this is done in the next section by adjusting for scores in September. This adjustment only enables one to see which items were strong or weak for the particular populations.

The table speaks for itself and there is no need to look at each question. Positive differences indicate control relative strength; negative differences indicate experimental

Table 2: ETS EXAM ITEM ANALYSIS

Item No.	Content Area	Percentage Correct Cont.	Cont-Adj.	Exp.	Cont Adj -Exp.
	Radicals				
A1	Cube root	95	102	96	6
B1	" "	88	95	82	13
A6	Addition of square roots	76	83	75	8
B7	" " " "	62	69	63	6
A26	Equations	50	57	67	-10
B36	"	59	66	62	4
	Exponents				
A2	Fundamental property	92	99	92	7
B2	" "	93	100	87	13
A8	Zero exponent	87	94	89	5
B10	" "	86	93	88	5
A11	Negative exponent	71	78	85	-7
B13	" "	60	67	78	-11
A34	Fractional exponent	23	30	18	12
B37	" "	45	52	43	11
A21	Equation	83	90	85	5
B5	"	98	105	97	8
	Quadratic Sentences				
A4	Solving	71	78	76	2
B6	"	77	84	81	3
A27	Find equation given roots	48	56	53	3
B21	" " " "	60	67	58	9
A31	Condition for equal roots	55	62	57	5
B27	" " " "	60	67	57	10
A36	Nature of roots	63	70	59	11
B20	" " "	60	67	66	1
A20	System with inequality	69	76	84	-8
B25	Inequality	50	57	51	6
	Polynomial Operations				
A5	Linear	76	83	84	-1
B4	"	86	93	86	7
A9	Applying distributivity	61	68	77	-9
B9	" "	73	80	73	7
A22	Multiplication	47	54	70	-16
B26	"	44	51	57	-6
	Systems				
A7	Simple 2 by 2	72	79	77	2
B11	" " " "	69	76	73	3

Item No.	Content Area	Cont.	Cont-Adj.	Exp.	Cont Adj -Exp.
		Percentage Correct			
	Systems (cont.)				
A13	2 by 2 involving variable	55	62	72	-10
B8	" " " " "	32	39	57	-18
A25	Linear-quadratic	46	53	54	-1
B31	"	28	36	38	-2
	Word Problems				
A30	Numerical	68	75	79	-4
B18	"	78	85	90	-5
A10	Coin	68	75	81	-6
B12	"	75	82	87	-5
	Lines				
A12	Slope	65	72	66	6
B16	"	70	77	70	7
A17	Equation from graph	42	49	49	0
B34	" " "	33	40	40	0
A28	No. of solutions from graph	81	88	78	10
B19	" " " " "	74	81	78	3
	Progressions (Sequences)				
A14	Recognition of geometric	39	46	56	-10
B23	Recognition of arithmetic	46	53	48	5
A24	nth term	38	45	76	-31
B14	" "	62	69	72	-3
	Factoring				
A15	Fraction simplification	18	25	36	-11
B15	" "	40	47	40	7
A18	Difference of two squares	50	57	76	-19
B24	" " " "	21	28	21	7
A40	Coefficient from factor	49	56	56	0
B33	" " "	50	57	56	1
	Logs				
A16	Traditional definition	61	68	51	17
B38	" "	28	35	40	-5
A39	Multiplication property	32	39	52	-13
B32	Division property	24	31	34	-3
	Conic Sections				
A19	Circles	49	56	54	2
B29	"	52	59	90	-31
A23	Parabola equation from graph	52	59	51	8
B28	" " " "	12	19	33	-14

Item No.	Content Area	Cont.	Cont-Adj.	Exp.	Cont Adj -Exp.
	Complex Numbers				
A29	Multiplication	39	46	73	-27
B30	"	37	44	64	-20
A32	Powers of i	31	38	40	-2
B22	" " "	25	32	23	9
A38	Condition for imaginary nos.	16	23	38	-15
B40	" " " "	50	57	55	2
A35	f( ) notation	70	77	73	4
B17	" "	86	93	73	20
A37	Absolute value - 2 dim.	58	65	52	13
B39	" " - 1 dim.	65	72	61	11
A33	Linear inequalities - 2 dim.	66	73	75	-2
B35	" " - 1 dim.	42	49	50	-1
A3	Fractional equation	86	93	91	2
B3	" "	80	87	86	1

Percentage Correct (spans Cont., Cont-Adj., Exp., Cont Adj -Exp.)

strength. The reader should be careful not to consider any one question too seriously. With so many items, some variance must be expected. For example, Item 9 on Form A (polynomial operations) favors control, while the almost identical item 9 on Form B favors experimental.

Here we list average adjusted differences by content area:

Table 3: ETS EXAM DIFFERENCES BY CONTENT AREA

Area	No. of questions	Ave. Diff. (Adj Cont - Exp)
Radicals	6	4
Exponents	10	5
Quad. sentences	10	4
Poly. ops.	6	-3
Systems	6	-4
Word problems	4	-5
Lines	6	4
Progressions	4	-10
Factoring	6	-3
Logs	4	-1
Conic sections	4	-9
Complex nos.	6	-9
f( ) notation	2	12
Abs. value	2	12
Lin. ineq.	2	-2
Fract. eqns.	2	2

One can only make significant conclusions about differences when they do not follow content patterns in different texts. Conic sections, function notation, and absolute value seem most surprising. I would not have surmised any differences in these. The other differences seem explainable either by random fluctuations or by the fact that some of the content comes late in either IM or the control texts and so is likely to be skipped.

The reader may wonder what implications there are for IM, given these general and specific data. Revisions will rewrite some of the material on functions. It was already planned to simplify some of the sbsolute value material but relegate other ideas to a later chapter. The extra work with corresponding properties mentioned in the first part of this report may include earlier work with fractional exponents. The other large differences in exponents (except for negative exponents) are misleading; the percentages of correct answers for IM students are very high and could not be improved much.

In general, the ETS exams stress skills. Whether IM generally helps or hampers skills cannot be judged from the type of analysis given above. A more global analysis, taking into account knowledge known at the beginning of the year, is needed.

ETS September-June Comparison

In pre-test post-test studies such as this one, a common way of analyzing data is to use analysis of covariance. One of the assumptions made in such analyses is that the scores on tests are normally distributed (i.e., distributed like the familiar bell-shape curve). Table 4 shows the frequency distributions for the two ETS June tests for Experimental and Control populations.

Table 4: FREQUENCIES OF SCORES ON ETS JUNE TESTS

Form A			Form B	
Cont.	Exp.	Score	Cont.	Exp.
2	1	7	2	
		8		
		9	1	
		10	2	3
1	2	11	2	1
1		12	1	1
2	2	13	3	3
3		14	2	
2	1	15	3	1
3	3	16	2	4
4	2	17	1	4
1	3	18	6	2
4	5	19	6	6
5	2	20	6	2
5	6	21	2	1
6	2	22	4	7
2	3	23	3	4
8	4	24	2	2
7	3	25	8	6
8	2	26	7	4
8	3	27	8	1
7	4	28	4	4
6	3	29	5	4
1	4	30	4	5
3	4	31	4	7
2	5	32	1	6
1	1	33	2	5
2	4	34	1	5
	3	35	2	2
	7	36		2
	4	37	1	2
	3	38		
	2	39		

The table shows a curious result: with the exception of the Form A Control distribution, the frequencies are distributed bimodally - they peak twice. Form A Experimental peaks around 21 and then around 35; Form B Control peaks around 19 and 26; Form B Experimental peaks around 22 and 31. Thus one cannot assume normal distributions for this population. But one can assume that if the test had been given to a larger number of students, the distribution would have been normal.

All this makes any conclusions from covariance to be fraught with possible error, but it does not make covariance analyses impossible. Here they are with some key statistics given. The idea is to calculate an adjusted June mean for each group on each test which depends upon the scores of the students on the test they took in September.

Table 5: MEANS AND ADJUSTED MEANS FOR ETS JUNE TESTS, BASED UPON ETS SEPTEMBER TESTS

		Sept. mean	June mean	June adjusted mean
Sept. A, June B:	Exp:	16.37	25.17	24.05
	Cont:	13.57	22.75	23.85
Sept. B, June A:	Exp:	16.21	26.87	25.26
	Cont:	12.36	23.26	24.77

The difference in adjusted means is not significant (either educationally or statistically), suggesting that the reason experimental students outscored control students in June was entirely due to the fact that they knew more in September. This would seem to indicate little difference on standard content performance between IM and control students.

Some may wonder why students in a nonstandard book would score as well on standard content. There seem to be the following contributing factors: (1) The 11th grade course differs quite a bit in different schools, so there really is not a standard course even when students study from the same text. If this is so, IM may not really be so nonstandard. (2) Teachers in a new book may feel guilty about not teaching traditional material and so may actually emphasize traditional stuff more than they normally would. (There is evidence that at least one experimental teacher omitted all nonstandard content after around December of the school year.)

The following are probably not factors. Teachers in experimental materials try harder. There is no evidence to support this. Indeed, all teachers in a study usually try a little harder. Hawthorne effects? The newness of materials often makes things more difficult for a teacher, counterbalancing any Hawthorne effect. Hawthorne effects on students do not last for an entire year, as anyone knows who has tried something new but lousy.

Experimental students were brighter and perhaps covariance is not telling the whole picture. The difference in June and September means (amount of learning?) is higher for control than for experimental students on each test.

In final summary, this analysis agrees with many other analyses done with the tenth grade GATA - i.e., standard content performance does not seem to be much affected by the use of simple nonstandard approaches to it.

STUDY OF 11th GRADE MATHEMATICS

Third Report

This third and last report has three parts. First is a summary of teacher opinions, second is a summary of planned changes in the text in light of all the data collected, and third is a tentative revised section sequence.

Summary of Teacher Opinions

In the first report of this study, Jeanne Hayman summarized the opinions of teachers in the formal study. Here we summarize the opinions of all teachers who took the time to fill out and return the long opinionnaire. Nine teachers did so, so we have two more opinions here than in the first report. This report also attempts to be a little more explicit and is able to give connections with possible changes in the materials.

Opinions were asked about the ease, niceness, difficulty, and unclarity of some sections or chapters, and whether some were skipped that teachers would have liked to cover. Here these opinions are summarized by section. Where opinions seem to cover an entire chapter, these are also noted. Parentheses indicate numbers when more than one person felt a particular way. When there are no parentheses, the opinion is that of one teacher.

CHAPTER 1: (Addition and Multiplication of Reals) Easy

2: (Further Properties of Real Nos.) Easy
- 2.5 easy
- 2.7 unclear (3), hard (2), delete
- 2.9 hard
- 2.10 unclear

3: (Transformations) Nice, hard, easy (6)
- 3.8 hard
- 3.9 nice

4: (Matrices) Nice (3), easy (8)
- 4.5 nice
- 4.7 unclear

5: (Lines and Circles)
- 5.4 unclear, nice
- 5.6 unclear

6: (Systems and Applications) Delete, hard, systems easy
- 6.1 hard
- 6.4 hard (2), skipped
- 6.5 hard (3), skipped
- 6.6 hard (4), skipped
- 6.7 hard (4), skipped
- 6.8 hard (2)

7: (Rotations, Sines, and Cosines) Nice (2), hard, easy
- 7.3 hard
- 7.4 nice (3)

7.5 nice (3), hard
7.6 nice (4), hard, easy
7.7 nice (2)
7.8 nice (2), unclear, delete

8: (Groups) Delete, nice (2), hard (2)
8.2 delete
8.3 delete
8.4 nice
8.5 delete
8.6 nice
8.7 hard
8.8 delete (2), nice, unclear
8.9 delete (2), hard, unclear
8.10 delete (2), nice, hard, unclear (2)

9: (Complex Numbers) Unclear, nice (2), easy (5)
9.6 nice
9.7 nice

10: (Quadratic Sentences) Nice
10.2 hard (2)
10.4 nice
10.6 unclear, hard
10.7 unclear (2)

11: (Quadratic Relations) Nice (2), hard, skipped parts (2)
11.4 unclear (2)
11.5 delete, nice
11.6 delete (2)
11.7 delete (2), nice
11.8 delete

12: (Functions) Easy (2), hard
12.3 unclear
12.4 nice
12.5 unclear
12.6 hard
12.8 nice, delete
12.9 delete

13: (Exponents and Logs) Nice, easy
13.6 hard, nice
13.7 nice (2), hard
13.8 unclear, hard (2), nice
13.9 hard (2), nice
13.10 nice (2), hard

14: (Trigonometry and Circular Functions) Nice, hard (2), easy, skipped

14.3 nice

15: (Advanced Manipulative Techniques) Nice, skipped (2), hard
- 15.2 hard
- 15.4 unclear
- 15.5 hard
- 15.6 hard (2)

16: (Sums, Sequences, and Statistics) Hard, skipped
- 16.5 nice
- 16.7 nice
- 16.8 nice, skipped
- 16.9 hard
- 16.10 hard
- 16.11 hard

17: (Summary) Nice

It is obvious that teachers were not of one mind when it came to these judgments. It is nice that most sections are represented in some way. This makes it easy to consider revisions. All those where more than one person found it hard will certainly be rewritten.

Only 5 people returned the textbook use form. This is too bad, for as one can see in the following table, there was incredible variance as to the use of the text. (GATA means that the students had studied that geometry.)

NO. OF DAYS SPENT ON EACH CHAPTER (INCLUDING EXAMS)

	Teacher 1	2	3 GATA	4 GATA	5 GATA
1	10	13	9	10	4
2	14	17	10	10	6
3	15	30	10	7	4
4	17	13	15	7	5
5	12	16	10	10	9
6	12	19	20	12	8
7	10		19	10	10
8	11		10	12	11
9	9	18	11	15	10
10	9	*	15	15	11
11	12	*	8	15	13
12	11		5	15	10
13	14		8	15	14
14	20		5	5	10#
15	14		5		8
16			5	5	11
17				5	5

*supplemented with material like that found in these chapters.
#supplemented with more trigonometry.

It is recognized that in later chapters, the small numbers of days usually represent skimming or choosing selected topics.

What does this data show? Only that there are teachers or classes which can cover the book and they do this by going through the first part quickly. There are other classes or teachers who have great difficulty covering even half the book. Four out of the five could get through Chapter 13, and this is what I would call the general minimum for a course of this type.

Before teaching this book, five teachers were only slightly dissatisfied with contemporary texts, 2 were satisfied, and 1 was greatly dissatisfied. The course changed the attitudes of 3 of 7 who taught the course. People supplemented with word problems, trig, logs, and conics.

All teachers used the notes to the teacher, chapter tests, and answers to exercises, in varying amounts. Five teachers wanted students to have answers to odd exercises, two wanted them to have all answers, and 1 for easy exercises. The summary chapter 17 was liked and thought helpful and effective by those who used it. One person thought it was difficult to understand.

Specific questions were asked about a variety of topics and developments. With respect to groups, only 3 of 8 delayed the definition until Chapter 8; the others had given it earlier. Two of the 3 liked the delay, the other did not. (I see no reason to change even with this data.) The delaying of functions did not bother any of the teachers, all of whom delayed. (It is curious that this was delayed whereas groups were not.)

The usefulness of the groups chapter as a unifier was concurred by 5 teachers, but 1 teacher felt that the material forgotten made the chapter hard, and 2 teachers wondered if the unification was appreciated as much by the students as by the teachers. (A subtle and probably accurate point, but I'd rather have more than meets the eye than less.)

Delaying factoring until after the quadratic formula bothered only one teacher. Three teachers thought they'd have trouble with the material, but none said they did. Only one teacher mentioned a particular topic (statistics) that was hard to understand.

Everyone felt that the number of applications was "just about right," neither too little nor too much. Four of 8 teachers wanted the application sections 10.6 and 13.2 integrated into the text. I disagree with this on the grounds that the applications are integrated throughout the rest of the book and it is a needed change. There is also the pedagogical double-bonus in that some teachers will not skip the section because they don't skip anything, while at the same time it points out that teaching some applications is as important as teaching mathematics.

All but one teacher felt there were not too many problems in Sections 6.5 - 6.8. All but two teachers felt that there were enough problems in Chapters 1, 2, and 5. In certain cases problems will be added or deleted in these chapters, but there will be no drastic changes in numbers of problems.

Specific feelings about difficulty and opinions about niceness of development compared with others known to the teacher were sought for the following topics:

slopes of perpendicular lines - easy (4), average (2), hard (1)
nicest (1), about as nice (6)

operations with complex numbers - easy (3), average (4)
nicest (6), about as nice (1), not as nice (1)

definition of sine and cosine - easy (3), average (4)
nicest (4), about as nice (3)

formulas for $\cos(x+y)$ and $\sin(x+y)$ - easy (2), average (4), hard (1)
nicest (6), about as nice (1)

logarithms - easy (2), average (1), hard (2)
nicest (3), about as nice (3)

laws of exponents and corresponding properties - average (6), hard (1)
nicest (2), about as nice (3), not as nice (2)

graphing of conics - easy (1), average (2), hard (2)
nicest (3), about as nice (1), not as nice (1)

functions - easy (3), average (4)
nicest (3), about as nice (4)

The above are rather positive responses. As indicated elsewhere, the material on corresponding properties will be modified to make it more clear (and hopefully bring out the niceness of the idea!).

Teachers were asked whether they agree or disagree with certain possible changes in the text. Responses to all were mixed, and almost all in equal amounts! Here is what is now planned (number of teachers agreeing or disagreeing with the change).

Sections 6.5 to 6.7 will be moved back (4 agree, 3 disagree).

Development of complex numbers will be kept as it is (5 agree, 3 disagree).

Difficult material from 6.1 will be deleted (3 agree, 3 disagree).

Section 2.9 will be about where it is (5 agree, 3 disagree).

Work with corresponding properties will be increased (6 agree, 2 disagree).

The most general questions on the form concerned overall difficulty, readability, and recommendation. Teachers were split as to whether the book was written for the above-average or average 11th grade college-bound math student (4 for above-average, 5 for average). Yet 5 of 8 thought the book was easier to read and understand than other math books they had taught from (2 thought about the same, 1 thought more difficult). This either means that standard books are very difficult to read or that the content makes teachers feel the book is for above-average students. Finally, 5 teachers felt they would strongly recommend the text to other 11th grade math teachers (the question was not worded well enough to tell if this meant they would recommend it for students, but it seems from other responses that this generalization can be made), 2 would recommend, 1 was indifferent, and 1 would not recommend.

Summary of Planned Changes in the Text and the Reasons for Them

There were seven goals in the writing of Intermediate Mathematics, that the materials should (1) cover the standard content, (2) be at the level of the average 11th grade college-bound mathematics student (i.e., be no more difficult than other texts at this level), (3) give a reasonable picture of mathematics with particular attention to some fundamental concepts of algebra and linear algebra, (4) give a picture of the multitude and variety and nature of applications of mathematics, (5) make use of previous learnings in geometry to a greater extent than is now done, (6) make the 11th grade course more continuous and unified instead of flitting from topic to topic, and (7) be implementable without teacher re-training.

Quite ambitious! We now examine each goal and see to what extent (if any) it has been reached.

1. Coverage of standard content. On the ETS tests, comparative weaknesses of IM students were found on fractional exponents, f( ) notation, and graphing of absolute value sentences in one and two dimensions. The material on fractional exponents will be presented earlier in the text, so that more students will have an opportunity to study it. (See (3) below.) More questions on f( ) notation will be given. An attempt

will be made to simplify the copy on absolute value now in sections 2.9, 6.1, and 12.7, but no additional work is planned.

The ETS tests in general seem to indicate that standard content was learned as well by IM students as by the controls. The above weaknesses are minor and so it seems that this goal has been reasonably reached.

2. Difficulty level. Teachers judged the book as for average or as for above average students in about equal numbers, yet most felt the book was easier than other texts at this level. It is difficult from these reactions to determine whether the book has met this goal. Right now I would say that it seems that the book is about even with others in difficulty and perhaps easier in readability - this paradox is probably caused by a feeling that the content is more strange to these students (and teachers?) or (wrongly, in my opinion) that the work with groups and matrices and transformations may be more appropriate for better students. What mathematics is about we should not keep from average students. There will be no compromise with the attempt to make this book for average students, for it is those students who need an improved mathematics course more than any others.

3. Picture of mathematics. There are several important concepts in mathematics which are missing from this book: vector, limit, continuity, neighborhood, etc. It is impossible to do all of this in a single year. But in line with

the other goals of the book, I have tried to make the book give as accurate a picture of mathematics as it can, including major ideas from algebra (group, isomorphism) and linear algebra (transformation, matrix) not normally applied at this level. The attitude questions show that IM students better realize that mathematics has not been the same since ancient times and is still growing. I can only hope that this goal has been reached; at least the data does not contradict that hope.

Of all the newer concepts, the idea of corresponding properties and isomorphism is the one that seems to need most improvements. This will be introduced earlier in the book and used to develop fractional and real exponents. There will be an attempt to make the material on groups a little easier. Matrices and transformations seem to be at the proper difficulty level as almost all teachers judged these chapters to be among the easiest in the book.

4. Picture of applications. From my questions asked informally to classes, in spite of the performance on both the opinionnaire and on the ETS exams, I feel that students still identify applications with word problems and get psyched out before they start. That is, the data seems to support the fact that the students do have a pretty good picture of the enormity of applications, but my emotions tell me that

there still is almost no comfort with these.

What will be done? Since the traditional word problems have few real applications, these will be moved back in the book and treated as what they are - puzzle problems. The simpler necessary translations (e.g., howmuch do m items at 30¢ cost?) wil be moved even earlier in a chapter devoted to modeling. Distance formulas will be treated as attempts to model so that emphasis will be given to the modeling idea. Things like reflections as modeling mirror images, rotations as modeling physical turns, and symmetry as modeling balance will be mentioned. All this can be done by rephrasing things and not adding content so will not add to the length of the book.

5. Use of geometry. It is hoped that teachers were aware that, with the exception of Chapters 1 (real numbers), 13 (exponents and logs), 15 (advanced manipulation), and 16 (sums, sequences, statistics), the material is approached in a very geometric manner. The use of transformations enables this course to be tied in with geometry to an extent virtually impossible by any other known approach. Again I can only hope that the student sees that geometry need not be distinct from algebra. No changes are anticipated.

6. Continuity. The book was designed so that, beginning with Chapter 3, each chapter is needed at least a little for the next, until Chapters 14-16, which can almost be taken

in any order. Some changes will be made and it is hoped that this continuity will not be lessened.

The movement of applications and modeling to Chapter 3 will connect with Chapter 2 because the properties of real numbers and distance were chosen with applications in mind. Modeling will naturally lead in to the geometry (modeling the physical world) of Chapter 4. There still will be a discontinuous lead-in to the material on sines and cosines (Chapter 7 is only slightly needed) but this cannot be avoided. Extending the material on isomorphism now makes nth roots a mathematical motivation for complex numbers in addition to the existing geometric motivation. Other chapters are left nearly the same, except that the always discontinuous advanced manipulative techniques will be lengthened.

It is hoped that the changes might increase the continuity and integrative nature of the book and continue the natural "spiralling" which is always one of the features of a unified approach.

7. Implementation. Having come through 6 goals in reasonably good shape, I am worried about this one. The problem here is not that teachers felt inadequate - unanimously, teachers felt that they had no problem with the material. But those who did not have solid mathematics background misjudged difficulty and skipped many of the sections which are keys to really understanding what is and has been going on. For exam-

ple, it was common in my visits to schools to note that section 11.5 on scale transformations was being skipped. Here is the extension of the graph translation theorem and an extension which can ease so much of what follows.

Skipping the material on groups (as two teachers did) nullifies the developments of complex numbers and logarithms, each of which is designed to give students more of a feeling for what is going on. And if these things are skipped, it is natural to ask: For what reasons did we study all that stuff on transformations and matrices at the beginning of the book?

It seems to me that the problem is not so much getting people to teach IM as getting teachers to realize the power of some of the ideas therein so that if they ever teach this content (from whatever source), they will realize what is useful and not pass it over. Six years ago, few mathematics teachers knew anything at all about transformations and certainly little about their power in geometry. In some areas of the country, this situation has been turned around. But teachers seem to fear groups a little more than transformations, and I don't know if this fear can easily be overcome. The 8th (and most subtle) goal of IM is to turn this fear around.

INTERMEDIATE MATHEMATICS

Tentative Revised Section Sequence

CHAPTER 1: ADDITION AND MULTIPLICATION OF REAL NUMBERS

1.1	Sentences and Solutions
1.2	Equations Involving Only Addition
1.3	Equations Involving Only Multiplication
1.4	The Group Properties
1.5	Repeated Addition and the Distributive Property
1.6	Repeated Multiplication and the Power Property
1.7	Halves and Square Roots
1.8	What Is a Real Number?

CHAPTER 2: FURTHER PROPERTIES OF REAL NUMBERS

2.1	Order and Inequality
2.2	Linear Sentences
2.3	Solving Linear Sentences
2.4	Sentence-Solving Involving Two or More Variables
2.5	The FOIL Theorem
2.6	Real Number Lines and One-to-one Correspondence
2.7	The Nested Interval Property

CHAPTER 3: APPLICATIONS AND MODELING

		(old)	
3.1	The Variety of Applications of Mathematics	6.4	
3.2	Modeling	6.8	
3.3	Modeling Distance On a Line	2.8	
3.4	Modeling "Nearness"	2.9	
3.5	Modeling Distance In a Plane	2.10	
3.6	Translating Real Situations Into Mathematics		new

CHAPTER 4: TRANSFORMATIONS

4.1	Reflections	3.1
4.2	Notation for Transformations	3.2
4.3	Translations	3.3
4.4	Rotations	3.4
4.5	Composites of Transformations	3.5
4.6	Composites of Reflections	3.6
4.7	Congruence	3.7
4.8	Size Transformations and Similarity	3.8
4.9	Symmetry	3.9

CHAPTER 5: MATRICES

5.1	Introduction	4.1
5.2	Matrix Multiplication	4.2
5.3	Applications of Matrix Multiplication to Transformations	4.3
5.4	Matrices for Certain Reflections	4.4
5.5	2 x 2 Matrices and Transformations	4.5
5.6	Matrices for Certain Rotations	4.6
5.7	Which Transformations Have 2 x 2 Matrices?	4.7