DOCUMENT RESUME

ED 084 164 SE 017 029

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TITLE	Mathematical Structures 1, Mathematics (Experimental): 5211.21
INSTITUTION	Dade County Public Schools, Miami, Fla.
PUB DATE	72
NOTE	24p.; An Authorized Course of Instruction for the Quinmester Program
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	Behavioral Objectives; *Curriculum; Fractions; Grade 7; Instruction; Mathematics Education; *Number Concepts; *Objectives; Prime Numbers; *Secondary School Mathematics; *Teaching Guides; Tests; Whole Numbers
IDENTIFIERS	*Quinmester Program

ABSTRACT

This is the first of two guidebooks for Grade 7 recommended to build fundamental concepts necessary for success in algebra. Topics covered include numeration systems, operations in non-decimal bases, whole numbers, factors and primes, and fractions. Goals for the course are stated, then, for each topic, performance objectives, a course outline, references, and suggested teaching strategies are provided. Posttest items are included, along with a list of twelve references. For the second guidebook in this series, see ED 067 293. (DT)

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EDUCATION

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

MATHEMATICAL STRUCTURES

5211.21

MATHEMATICS

CE 002 029

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION • 1971

ED 084164

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

MATHEMATICAL STRUCTURES 1

5211.21

(EXPERIMENTAL)

Written by

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for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

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Published by the Dade County School Board

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

One of two quins in Grade 7 recommended to build fundamental concepts necessary for success in algebra. Includes numeration systems, other number bases with operations, properties of rational numbers, factors and primes.

Designed for the highly motivated student in Grade 7.

TABLE OF CONTENTS

GOALS

1. To familiarize the student with the historical background of numeration systems and compare ancient systems with our numeration system.

2. To study bases in order to better understand place value and the structure of the decimal system.

3. To develop student understanding of the closure, commutative, associative, distributive, inverse, and identity properties for addition and multiplication of whole numbers.

4. To introduce the concepts of factors and multiples in order to help in work with fractions.

5. To improve computational skills with fractions.

Key to References
(* State-adopted)

K (1) Keedy, Mervin; Jameson, Richard; and Johnson, Patricia. Exploring Modern Mathematics, Book 1. New York: Holt, Rinehart and Winston, Inc., 1963.

Mc (1) McSwain, E. T.; Brown, K. W.; Gundlack, B. H.; and Cooke, R. J. Mathematics 7. River Forest, Illinois: Laidlaw Brothers, 1965.

*SMS (1) Suppes, Patrick; Meserve, Bruce; Sears, Phyllis. Sets, Numbers, and Systems, Book 1. New York: The L. W. Singer Co., 1969.

PERFORMANCE OBJECTIVES

The student will:

1. Compare three of four numeration systems studied with regard to place value (Egyptian, Babylonian, Roman and Hindu-Arabic).

2. Translate a Hindu-Arabic numeral into a Roman numeral and vice versa.

3. Write a given number in expanded exponential form.

4. Solve addition, subtraction and multiplication problems in a base other than 10.

5. Write the name of a given decimal number in words from its symbol name.

6. Write the name of a given decimal number in symbols from its word name.

COURSE OUTLINE

I. Numeration Systems

A. History

1) Define number and numeral
2) Explain meaning of a numeration system
3) Explain how numbers developed - why primitive man needed numbers
4) Compare Egyptian and Babylonian systems. Stress lack of place value and difficulty in computation.

B. Roman Numerals

1) Read
2) Discuss differences between systems
   a. place value
   b. zero

C. Decimal Numeration System

1) Place value

2) Expanded notation
3) Expanded exponential notation
4) Exponents of zero and one
5) Reading and writing numbers

D. Bases

1) Count in bases 2 thru 12
2) Convert to base 10
3) Convert base 10 to another base
4) Add, subtract and multiply in a base other than 10.

REFERENCES

K (1) Chap. 1, P. 1-30

Good development of numeration systems and bases. Let's Explore exercises lead to discovery of concepts. Exercises on P. 7, 13, and 23 good for practice.

Mc (7) Chap. 1, P. 5-12
Chap. 17, P. 313-330

Chap. 1 - develops idea of a numeration system. Omit part of Chap. 1 which begins decimals which is in the following quin.

Chap. 17 - many good exercises on bases. This is too long and bases should not be covered in such depth.

SMS (1) Chap. 4, P. 66-80

Brief exercise on ancient systems of numeration on P. 80.
P. 66-79 excellent work on exponents and bases. Omit P. 76 on commutative and associative properties.

SUGGESTED STRATEGIES

1. Use idea of decreasing value of exponents to explain meaning of zero and one as exponents.

2. Demonstrate with fingers "one hand arithmetic" to show how to write numerals in base 5.

3. Discuss the use of base 2 in computers. An activity would be to take the class to a computer center.

4. Be careful not to spend too much time on bases. Bases are presented only to enhance the understanding of the decimal system.

PERFORMANCE OBJECTIVES

The student will:

1. Use set notation to define the natural and whole numbers.

2. Perform the four operations with whole numbers.

3. Identify or give examples of the commutative, associative, distributive, and identity properties.

COURSE OUTLINE

II. Whole Numbers

A. Definitions

1) Natural numbers
2) Whole numbers

B. Review Computation

C. Properties

1) Commutative for addition and multiplication
2) Associative for addition and multiplication
3) Distributive
4) Identities for addition and multiplication

REFERENCES

K (1) Chap. 2, P. 37-49, 57-84
Good development of the properties of 0 and 1, with emphasis on the fact that division by 0 is undefined. Discovery approach to the properties.

Mc (7) Chap. 1, P. 21-32
Addition properties on page 21, multiplication properties on page 22, distributive property on page 25. Shows on pages 23 and 24 that the properties do not hold for subtraction and division. Good review work in computation.

SMS (1) Chap. 1, P. 17-18
Review of whole number computation
Chap. 4, P. 76
Commutative and associative properties developed using other bases. The other properties are introduced and explained in Chapter 5 using integers.

SUGGESTED STRATEGIES

1. A pretest of computation skills can determine how much time should be spent on review. Be sure to include on the pretest the most often missed types of problems such as division with zeros in the quotient.

2. Enough set notation must be introduced to allow students to understand the definition of natural and whole numbers given in set notation.

3. Having students demonstrate the truth of the properties by doing computation in two ways not only develops an understanding of the properties but also provides practice in skills.

PERFORMANCE OBJECTIVES

The student will:

1. Identify prime and composite numbers.
2. Write the prime factors of a number.
3. Write the GCF when given a pair of numbers (neither one greater than 100).
4. Write the LCM when given a pair of numbers (neither one greater than 100).

COURSE OUTLINE

III. Factors and Primes

A. Prime Factoring

1) Sieve of Eratosthenes
2) Definitions
   a) factor
   b) smallest factor
   c) prime number
   d) composite number
3) Rules for divisibility by 2, 3, 4, 5, 9, 10
4) (Optional) Goldbach's Conjecture

B. Greatest Common Factor

1) Common factors
2) Relatively Prime
3) GCF

C. Least Common Multiple

1) Define multiple
2) LCM

REFERENCES

K (1) Chap. 4, P. 134-179
Develops factors and multiples, primes and composite numbers, even and odd numbers. There are 2 proofs on P. 160 and 163.
Greatest Common Factor P. 155
Least Common Multiple P. 179

Mc (7) Chap. 5, P. 79-92
Good development and practice exercises on factors, multiples, GCF and LCM.
Reviews exponents on P. 86.

SMS (1) Chap. 5 - middle of chapter
P. 105 - 110. The beginning of Chap. on integers to be omitted. Not enough drill on these pages.
P. 36-37 Divisibility and Multiples

SUGGESTED STRATEGIES

1. Have students work on a factor tree bulletin board using fall leaves or some other original composition of its use.

2. Give the students a ditto with the numbers 1-100. Have them use all the divisibility rules, definitions, etc., and see who can come up with all the prime numbers.

3. Before presenting GCF and LCM, review exponential notation and then use exponential notation in finding GCF and LCM.

4. Have students do a bulletin board comparing the methods of finding the GCF and LCM.

PERFORMANCE OBJECTIVES

The student will:

1. Simplify fractions.
2. Express a given fraction as an equivalent fraction is higher terms.
3. Express a mixed number as a fraction.
4. Express an improper fraction as a mixed number.
5. Order two or more fractions.
6. Perform the four operations with fractions, whole, and mixed numbers.
7. Identify the commutative, associative, distributive, identity, and inverse properties from examples.
8. Determine whether a given set is closed for a given operation.

COURSE OUTLINE

IV. Fractions

- A. Vocabulary
- B. Equivalent forms
  - 1) Higher terms
  - 2) Lower terms
  - 3) Fraction to mixed number
  - 4) Mixed number to fraction
- C. Ordering
  - 1) Expressing with a common denominator
  - 2) Cross products method
  - 3) The property of denseness
  - 4) Averaging

D. Computation

1) With fractions
2) With mixed numbers
3) With combinations of whole numbers, fractions, and mixed numbers

E. Properties

1) Review commutative, associative, distributive, identity, and closure.
2) Introduce inverse

F. Word Problems

REFERENCES

K (1) Chap. 6, P. 231 -261
Covers both computation and properties using the discovery approach

Mc (7) Chap. 6, P. 97-120
Covers skills and word problems but omits properties except for closure and inverse, which they call reciprocal.

SMS (1) Chap. 3, P. 46-52
A very brief treatment which could be used only as a review.

SUGGESTED STRATEGIES

1. A pretest would be useful to determine the ability levels of the student in computing with fractions.

2. The vocabulary of fractions should be used by both students and teacher so it becomes familiar to the students.

3. A review of averaging can be used when discussing finding a fraction between two given fractions.

4. Working with the properties is a good way to practice computation skills.

5. Since students are normally weak in problem solving, time should be alloted to this area whenever possible throughout the quin.

SAMPLE POSTTEST ITEMS

I. Numeration Systems

1. Compare three of the four numeration systems studied. In your discussion, tell whether or not the system has place value.

2. Write the following in Roman numerals:

   a) 562 b) 1969 c) 1,000,486

   Write as a Hindu-Arabic numeral:

   d) LXIII e) MMXCV f) DLXV

   g) MCMLXXI

3. Write in expanded form using exponential notation:

   62,975

4. Work the following problems in the base indicated in the problem:

   a) $21_5 + 34_5$ b) $55_7 + 43_7$ c) $201_5 - 43_5$

   d) $43_7 \times 5_7$ e) $101_2 + 111_2$ f) $64_{12} + 66_{12}$

5. Write in words:

   a) 3,000,024 b) 72,906,421 c) 2,003,020,000

6. Match Column I with Column II

I	II
a) Forty-nine billion	1. 349,000,000
b) Three hundred forty-nine million	2. 67,000,004
c) Twenty-eight thousand ninety-nine	3. 28,099

d) Six thousand six

e) Sixty-seven million four

4. 49,000,000,000

5. 6,006

II. 1. a) Define the whole numbers using set notation.

b) Define the natural numbers using set notation.

2. Perform the indicated operations.

a) 2607 + 432 + 1856 +29

b) 4000 - 3247

c) 24872 - 6794

d) 2067 x 49

e) 971 x 607

f) 14003 ÷ 67

g) 14694 ÷ 237

3. Which property of whole numbers is being illustrated?

a) (4 + 3) + 7 = 4 + (3+7)

b) a (b + c) = ab + ac

c) 6 + 0 = 6

d) 3 (4 • 5) = 3 • (5 • 4)

e) d • 1 = d

III. 1. a) Which numbers are prime?

12, 17, 51, 31, 49

b) Which numbers are composite?

12, 17, 51, 31, 49

2. Write the prime factorization of each number.

a) 20 b) 72 c) 98

3. Find the GCF.

a) 20, 30 b) 17, 51

4. Find the LCM.

a) 15, 18 b) 6, 10, 16

IV. 1. Simplify

a) $\frac{10}{15}$ b) $\frac{12}{36}$ c) $\frac{40}{90}$

2. Express each fraction as an equivalent fraction with denominator 96.

a) $\frac{5}{8}$ b) $\frac{19}{24}$

3. Express as fractions

a) $7\frac{2}{3}$ b) $2\frac{5}{9}$

4. Express as mixed numbers

a) $\frac{20}{3}$ b) $\frac{82}{7}$

5. Make a true statement by inserting $>$, $<$, or = between the fractions

a) $\frac{2}{5}$ $\frac{9}{24}$ b) $\frac{7}{9}$ $\frac{84}{108}$

6. Perform the indicated operations.

a) $\frac{6}{7} + \frac{1}{3}$ b) $\frac{11}{12} - \frac{2}{3}$ c) $2\frac{1}{3}$
$+ 4\frac{1}{4}$

d) $10\frac{2}{5} - 8\frac{2}{3}$

e) $\frac{4}{5} \cdot \frac{15}{16}$

f) $3\frac{1}{2} \cdot 2\frac{1}{5}$

g) $\frac{2}{5} \div 7$

h) $5\frac{1}{4} \div 2\frac{1}{7}$

7. Which property of fractions is being illustrated?

a) $\frac{1}{2} \cdot 2 = 1$

b) $\frac{1}{2}(2 + \frac{2}{3}) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot \frac{2}{3}$

c) $\frac{1}{2} \cdot (2 \cdot \frac{2}{3}) = (\frac{1}{2} \cdot 2) \cdot \frac{2}{3}$

d) $\frac{1}{2} + 2 + \frac{2}{3} = 2 + \frac{1}{2} + \frac{2}{3}$

e) $\frac{2}{3} \cdot \frac{5}{5} = \frac{2}{3}$

8. a) Is $\{0, 1, 2\}$ closed for addition?

b) Is $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \ldots\}$ closed for multiplication?

KEY TO POSTTEST

I. 1. Egyptian - no place value. System based on powers of 10. It took too long to write large numbers.

Babylonian - too few symbols and too long to write numbers. No place value.

Roman - seven basic symbols. Place value consisted of whether number was to be subtracted or added (example - IX or XI)

Hindu - Arabic - our system.

2. a) DLX II b) MCMLXIX c) MCDLXXXVI

d) 63 e) 2,095 f) 500,065

g) 1,971

3. $6 \cdot 10^4 + 2 \cdot 10^3 + 9 \cdot 10^2 + 7 \cdot 10^1 + 5 \cdot 10^0$

4. a) 110_5 b) 131_7 c) 103_5

d) 311_7 e) 1100_2 f) 10_{12}

5. a) Three million twenty-four

b) Seventy-two million, nine hundred six thousand, four hundred twenty-one.

c) Two billion, three million, twenty-thousand.

6. a) 4 b) 1 c) 3

d) 5 e) 2

II. 1. a) $\{0, 1, 2, 3...\}$ b) $\{1, 2, 3, 4...\}$

2. a) 4924 b) 753 c) 18,078

d) 101,283 e) 589,397 f) 209

g) 62

3. a) Associative, addition

b) Distributive

c) Identity, addition

d) Commutative, mulitiplication

e) Identity, multiplication

III. 1. a) 17, 31 b) 12, 51, 49

2. a) $2^2 \cdot 5$ b) $2^3 \cdot 3^2$ c) $2 \cdot 7^2$

3. a) 10 b) 17

4. a) 90 b) 240

IV. 1. a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{4}{9}$

2. a) $\frac{60}{96}$ b) $\frac{76}{96}$

3. a) $\frac{23}{3}$ b) $\frac{23}{9}$

4. a) $6\frac{2}{3}$ b) $11\frac{5}{7}$

5. a) $>$ b) $=$

6. a) $\frac{25}{21}$ b) $\frac{1}{4}$ c) $6\frac{7}{12}$

d) $1\frac{11}{15}$ e) $\frac{3}{4}$ f) $7\frac{7}{10}$

g) $\frac{2}{35}$ h) $2\frac{9}{20}$

7. a) Inverse, mulitplication

   b) Distributive

   c) Associative, mulitplication

   d) Commutative, addition

   e) Identity, multiplication

8. a) No. b) Yes.

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