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ABSTRACT

This is the first of six guidebooks on minimum course content for second-year albebra. A survey of the real and complex number systems, solving linear equations and inequalities in one variable, and operations with polynomials are covered in this booklet. Course goals are stated, a course outline is provided, performance objectives are specified, and textbook references keyed to the performance objectives are given. Sample pretest and posttest items are included, along with a bibliography of 16 references. For other booklets in the second-year algebra series, see SE 017 027. (DT)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

Algebra 2P

5216.21

Mathematics

DIVISION OF INSTRUCTION•1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

ALGEBRA 2P

5216.21

(EXPERIMENTAL)

Written by

June Ellis

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

The first of 6 quins which together contain all the concepts and skills usually found in second-year algebra. Includes a survey of the real and complex number system, and operations with polynomials.

Designed for the student who has mastered the skills and concepts of Algebra 1.

TABLE OF CONTENTS

OVERALL GOALS

The student will:

1. Develop, identify, and use the set of complex numbers.

2. Acquire facility in applying algebraic concepts and techniques in solving equations.

3. Write and solve mathematical models that describe given word problems.

TEXT BIBLIOGRAPHY
(*State Adopted)

D_3-Dolciani, Mary P.; Berman, Simon L.; and Wooton, William. Modern Algebra and Trigonometry, Book 2. Boston: Houghton Mifflin Company, 1963.

*D_8-Dolciani, Mary; Wooten, William; Beckenbach, Edwin; Sharron, Sidney. Modern School Mathematics, Algebra II and Trigonometry. Boston: Houghton Mifflin Company, 1968.

N -Nichols, Eugene D.; Heimer, Ralph T.; Garland, Henry C., Modern Intermediate Algebra. New York: Holt, Rinehart and Winston, Inc., 1965.

*PL-Payne, Joseph N.; Zamboni, Floyd F.; and Lankford, Francis G., Jr.. Algebra Two with Trigonometry. New York: Harcourt, Brace, and World, Inc., 1969.

*PA-Pearson, Helen R. and Allen, Frank B. Modern Algebra, A Logical Approach, Book Two. Boston: Ginn and Company, 1966.

COURSE OUTLINE

Related Objective

1-3 I. Complex Number System

A. Develop the system of complex numbers

1. Properties of equivalence relation
2. Order properties of real numbers
3. Field axioms
4. Subsets of the complex numbers

4 B. Study the complex numbers principally as denoted as radicals.

1. Principal square root
2. Simplification and rationalization
3. Multiplication
4. Division (over the reals)
5. Addition and subtraction

5 II. Linear Equations and Inequalities in One Variable

A. Solve mathematical sentences over the complex

B. Graph solutions over the reals

6-14 III. Polynomials and Factoring

A. Identify polynomials

B. Rational operations (ASMD), including synthetic division, with polynomials of degree ≤ 3

C. Binomial Theorem

D. Pascal's Triangle

E. Factor polynomials

1. Binomials

    a. Common monomial factor
    b. Difference of two squares
    c. Sum and difference of two cubes

2. Trinomials

    a. Perfect squares
    b. Product of two linear factors

3. Higher degree polynomials of the form

$$x^{2a} - y^{2c} \qquad \text{or} \qquad x^{3a} \pm y^{3c}$$

F. Write and solve mathematical models of physical problems that can be written as equations of degree ≤ 3 and solved by factoring.

PERFORMANCE OBJECTIVES REFERENCES

The student will:

Performance Objectives	D_3	D_8	N	PA	PL
1. Outline the complete number system and define each of its subsets, $(i = \sqrt{-1})$ If Dolciani text is used, take note in Teacher Manual for suggested approach when trigonometry is <u>not</u> included. i. Discuss a pure imaginary number as either of the square roots of a negative real number. ii. Express an imaginary number in the form $a \pm bi$, a and b real numbers $b \neq 0$ and $i = \sqrt{-1}$ iii. Emphasize: $\{\text{complex numbers}\} = \{\text{real numbers}\} \cup \{\text{imaginary numbers}\}$	2,6,11, 32,399	2,6,39, 364	1-4,21, 33,34, 251,252	1,5,16, 21,103, 589	24,29, r-2, r-9, 2-4
2a. Identify the properties of an equivalence relation. reflexive $a = a$ symmetric if $a = b$, then $b = a$ transitive if $a = b$ and $b = c$, then $a = c$	11,32	39	4	21,103	
2b. Identify the order properties for the real numbers.	48-50	76,77	6,18, 24-26	47	42
3. Identify the field axions of the complex numbers illustrated by a given example.	11-19, 27,32 410	20,38, 372	17,23, 34,38 252,267	22,31, 103, 108	14,31 520-594

	D_3	D_8	N	PA	PL
4. Simplify, rationalize, and perform the basic operations of addition, subtraction and multiplication of radical expressions including complex numbers of the form a + bi. Division of radical expressions over the reals only. Stress rational denominators in radical expressions. Emphasize factoring as an aid in simplifying. Discuss rules for divisibility. Example: $\sqrt{459} = \sqrt{3.3.3.17}$ $= 3\sqrt{51}$ Example: $\sqrt[3]{a^5b^3 + a^7b^5} = \sqrt[3]{a^3a^2b^3 + a^3a^4b^3b^2}$ $= \sqrt[3]{a^3b^3(a^2 + a^4b^2)}$ $= ab\sqrt[3]{a^2(1 + a^2b^2)}$	247, 258, 267	313,325	33-35, 47-50	122-130	5-19
5. Solve, over the reals, and graph simple linear equations and inequalities in one unknown. Solve similar equations over the complex. Facility in solving and graphing linear equations and inequalities in one unknown is an asset when working with equations and inequalities of greater degree, although we find little reference in Algebra II. Consult Algebra I texts.	2,46-54	75-84	not covered	41	18-22

Objective	D_3	D_8	N	PA	PL
6. Classify a given polynomial, state the degree of each of its terms, and the degree of the polynomial.	40	50	59-62	148-153	50
7. Apply the definitions for the basic operations to add, subtract, multiply, and divide polynomials over the complex.	41-43 121-124 140-143 398-405	58-61 256-259 273-276	64-78 253-259 264-279	154-163 594-605	31-43 50-59
8a. Use the binomial expansion theorem to expand an expression of the form $(a + b)^n$, and relate this expansion to Pascal's Triangle. Work with the binomial expansion theorem can develop an appreciation of the notion of paterns in algebra.	511-513 581-589	612-616	510-515	741-745	60-63
8b. Find a specified term in the expansion of a given binomial.	587-589	612-616	513-515	745	64-65
9. Factor polynomials of degree two and special polynomials of higher degree. To factor a polynomial of the form $Ax^2 + Bx + C$ find two factors of the product AC whose sum (or difference) equals B. Rewrite Bx as a sum or difference using these factors.	124-143	259-271	78-90, 283	172-183	65-72, 80-82

Example: $3x^2 + 17x + 10$
$3x^2 + [15x + 2x] + 10$
$3x\ (x + 5) + 2\ (x + 5)$
$(3x + 2)\ (x + 5)$

Example: $2a^2 - 13a - 7$
$2a^2 - [14a - a] - 7$
$2a\ (a - 7) + (a - 7)$
$(2a + 1)\ (a - 7)$

Example: $x^6 + y^6 = (x^2)^3 + (y^2)^3$
$(x^2+y^2)\ (x^4-x^2y^2+y^4)$

	D_3	D_8	N	PA	PL
10. Factor the sum and the difference of two cubes.	128	261	89	178	83-34
11. Use synthetic division to divide a linear binomial into a polynomial of greater degree, **over** the complex.	522	382	344-347, 365	not covered	90-92
It will be helpful to teach the Rational Root Theorem (see Dolciani '63, page 250 and Dolciani '68, page 316) as it applies to factoring.					
12. Apply the remainder theorem to evaluate a polynomial.	521-524	381	343-347	not covered	77-80
13. Apply the factor theorem to polynomials of degree four or less to find the factors of a polynomial.	143-145, 521-525	382	348-349	not covered	77-80
14. Use the results of the fundamental theorem of algebra.	525	385	358	not covered	not covered
Solving of polynomials is not an objective of this quin. However, you will want students to understand that every polynomial with complex coefficients and degree n, a positive integer, has n linear factors not necessarily distinct.					
15. Write and solve mathematical models for word problems which can be solved by the algebraic skills developed in the quins.	62, 137, 173	68,82, 269	not covered	not covered	22,120
Include appropriate word problem experiences periodically as students progress through the quin.					

SAMPLE PRETEST ITEMS

In each of the following exercises, 1-4, use the symbols $>, <, \in, \notin, \subset$, or $\not\subset$ to make a true statement.

1. 9 ____ 15

2. $\sqrt{3}$ ____ {the real numbers}

3. -6 ____ -1

4. $\{3\}$ ____ R

5. Use a quantifier to make the sentence $x + 7 = 7 + x$ into a true statement about real numbers.

6. Write the conjunction of the statements "$9 + 12 = 12$" and "$7 \times 4 = 28$" and state whether the conjunction is true or false.

7. Write the disjunction of the statements in Problem 6. and state whether it is true or false.

In each of exercises 8-15 justify the given statement by citing an axiom.

8. $9 + 0 = 9$

9. $7 \times \frac{1}{7} = 1$

10. $2 \times 3 = 3 \times 2$

11. $(4 + 5) + 6 = 4 + (5 + 6)$

12. $20\ (9 + 4) = 20 \times 9 + 20 \times 4$

13. $(4 \times 2) \times 7 = 7 \times (4 \times 2)$

14. If $m = 2$, then $2 = m$

15. If $m - n$ and $m = 2$, then $n = 2$

Find the value of each expression in exercises 16 — 21

16. $-27 \div (+3) =$ ____

17. $(-8)\cdot(4) + (-6) =$ ____

18. $(1 \div [2 - (-3)]) \times 5 =$ ____

19. $2 + [34 \div (-2)] - 2 =$ ____

20. $\dfrac{(3 \times 10^{-4})\ (3 \times 10^{9})}{6 \times 10^{2}} =$ ____

21. $\frac{(2.4)\ (-3.9)}{(0.13)} =$ ____

Find a polynomial in simple form equivalent to the given expression in exercises 22 through 28.

22. $3a - b + 2(a - 4b)$

23. $4(x - y) - 2y$

24. $(x^3 - 6x^2 + 3x - 2) + (2x^3 - 3x^2 - 2x + 4)$

25. $(2x^3 - 4x^2 + 2x + 1) - (2x^3 - 6x^2 - 2x - 7)$

26. $4(x^2 - 2x + 1) + 2(x^2 + 3x - 1)$

27. $3(x^2 + 3x - 2) - x(2x^2 + 5x - 9)$

28. $5y - [2y + (y - 3)] - (y - 2)$

Solve each equation in 29—33 over the real numbers.

29. $5(z - 4) - 7 = 8$

30. $11 - 5(3x - 2) = 3(x - 5)$

31. $7y - 6(3y + 5) = 5(3 - y)$

32. $\frac{x}{2} - 5 = \frac{x}{7}$

33. $\frac{3}{y} + 1 = 4 - \frac{2}{y}$

34. The sum of two consecutive odd integers is 68. What are these integers?

Solve for x in exercises 35—39. (Note: $a, b, c \neq o$)

35. $ax = b$

36. $3x - a = b$

37. $\frac{ax}{b} = c$

38. $\frac{b}{x} = \frac{a}{c}$

39. $ax = bx + c$

Evaluate:

40. $|-5| =$ ___

41. $|8 - 10| =$ ___

42. $-|5 - 3| =$ ___

Determine and graph the solution set over the real numbers in exercises 43 through 46.

43. $7x - 3 < 4x + 9$

44. $7 - 3x \geq x + 13$

45. $|x - 5| = 9$

46. $|2 - x| < 5$

47. In making an indirect proof of the assertion "If $x \in R$ and $2x + 5 = 1$, then $x \neq 2$," you would make the assumption that ________.

ANSWER KEY

1. $<$
2. $\in$
3. $<$
4. $<$
5. $\forall x \in R \quad x + 7 = 7 + x$
6. $9 + 12 = 12 \wedge 7 \times 4 = 28$, False
7. $9 + 12 = 12 \vee 7 \times 4 = 28$, True
8. Additive identity
9. Multiplicative inverse
10. Commutative, multiplication
11. Associative, addition
12. Distributive
13. Commutative, multiplication
14. Symmetric
15. Transitive
16. -9
17. -38
18. 1
19. -17
20. 1.5×10^3
21. -72
22. $5a - 9b$
23. $4x - 6y$
24. $3x^3 - 9x^2 + x + 2$
25. $2x^2 + 4x + 8$

26. $6x^2 - 2x + 2$

27. $-2x^3 - 2x^2 + 18x - 6$

28. $y + 5$

29. $z = 7$

30. $x = 2$

31. $y = -\frac{15}{2}$

32. $x = 14$

33. $y = \frac{5}{3}$

34. 33, 35

35. $x = \frac{b}{a}$

36. $x = \frac{a + b}{3}$

37 $x = \frac{bc}{a}$

38. $x = \frac{bc}{a}$

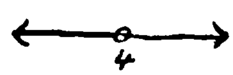
39. $x = \frac{c}{a-b}$

40. 5

41. 2

42. -2

43. $\{x \mid x < 4\}$



44. $\{x \mid x \leq -\frac{3}{2}\}$

45. $\{-4, 14\}$

46. $\{x \mid -3 < x < 7$

47. $x = 2$

SAMPLE POSTTEST ITEMS

For a final test, choose an appropriate number of test questions from the following collection.

1. Use suitable set notation to identify each set.

   a. The set of real numbers less than 15.

   b. The set of positive odd integers.

   c. The set of whole numbers.

   d. The set of all odd whole numbers whose squares are greater than 30.

   e. The set of rational numbers.

   Write a verbal description of each of the following:

   f. $N \subset I$

   g. $\{x : x < -5, x \in R\}$

   h. $I \subseteq Q$

   i. $Q \cup Ir = R$

   j. $Ir \cap N = \emptyset$

   Use the symbols of set notation to write each of the following:

   K. The set of all x such that x is an element of the real numbers and x + 12 = 15.

   l. The set of irrational numbers is a subset of the real numbers

   m. 3 is an element of the set of rational numbers.

   Identify each of the following complex numbers as

   (a) pure imaginary (b) imaginary (c) real

   n. $\sqrt{-25}$

   o. $(\sqrt{-3})^2$

   p. $(\sqrt{-6})(\sqrt{-7})$

   q. $-3 - 3i$

   r. $-\sqrt{36} + \sqrt[3]{-27}$

s. i^{4n}; $n > 0$

t. $\frac{8 + 4i}{4}$

2. Identify the property (properties) which justifies the statement.

   a. For all real numbers r, s, and t, if r = s and t = r, then r = t

   b. State and illustrate each of the Axioms of Inequality (order properties) for a field.

3. From the list at the right, select the properties of the real numbers that best justify the statements in items a-l, and write the number of your choice in the appropriate blank. A number can be used more than once. You may need to use more than one number for a given statement.

a.	$(a + b)c = (ac) + (bc)$	1.	Distributive axiom
b.	$(x + y) + (-(x + y)) = 0$	2.	Commutative axiom of addition
c.	$-1\left(\frac{1}{a}\right) = -\frac{1}{a}$	3.	Associative axiom of addition
d.	$a + (-b) = -b + a$	4.	Substitution principle
e.	$(-5)(-3) = 15$	5.	Closure
f.	$z(xy) = (zx)y$	6.	Axiom of zero
g.	$a + [b + (-b)] = a + 0$	7.	Axiom of inverses
h.	$[4 + (-8)]5 = 4 \cdot 5 + (-8)5$	8.	Axiom of one
i.	$(0 + 0)n = 0 \cdot n$	9.	Commutative axiom of multiplication
j.	$7 \cdot 3 = 21$		
k.	$a[(b + c) + d] = a[b + (d + c)]$		
		10.	Associative axiom of multiplication
		11.	Multiplicative property of 0
		12.	Multiplicative property of -1

4. Perform the indicated operations, and express each answer in simplest form.

a. $-\sqrt{x^5}$

b. $(2 + bi) - (3 - 6i)$

c. $\sqrt{28\ (x + y)}$

d. $\dfrac{1}{4\sqrt{2} - 3\sqrt{3}}$

e. $(-2 + \sqrt{5}\ i\ )^2$

f. $\dfrac{3 + \sqrt{4}\ i}{6 - \sqrt{4}\ i}$

g. $\dfrac{i}{5 + i}$

h. $\dfrac{5\sqrt{24} - 3\sqrt{18}}{\sqrt{6}}$

i. $a\sqrt[5]{-b^7} + b\sqrt[5]{-a^5 b^2}$

j. $\sqrt[3]{-81y^5}\quad (8\sqrt[3]{9y} - \sqrt[3]{16y^3}\ )$

k. $\dfrac{7}{5 - 2i} - \dfrac{5i}{-5 - 2i}$

5. Solve and graph

$-4 < y - 1$

6. State which of the following algebraic expressions are polynomials. If the expression is a polynomial, classify it

(1) as monomial, binomial, trinomial
(2) according to the nature of the coefficients

i. complex numbers
ii. rational numbers
iii. real numbers

(3) by degree

a. $\sqrt{5}x^2 + 4xy - 10y^2$

b. $2x^2 + 7\sqrt{y}$

c. $\dfrac{a}{15} + 9a$

d. $\sqrt[5]{-32}$

e. $5ix^5 y^5 - 6ix^5 y^2 + 2$

f. $\frac{2y - 14}{2y + 6}$

g. $1.7a^2 b^2 + 5.9a^2 b - 17 a^2 b^3$

7. Perform each of the indicated operations

a. $(4ix^2 y^5) + (3ix^2 y^5) - (-6ix^2 y^5)$

b. $(2x - 3)\ (2x + 3)\ \ 4x^2 + 9)$

c. $[(3 + 2i) + x][(3 + 2i) - x]$

8. Using the binomial expansion theorem

a. Expand $(3x^2 + y)^5$

b. Find the 4th term of $(x + i)^{10}$

9,10. Factor over the complex

a. $a^2 + b^2$

b. $rs^8 - r$

c. $m^3 - 27n^6$

d. $4x^2 - 12x - 7$

11. $(2x^3 - 7x^2 - 17x + 10) \div (x - 5)$

12. Apply the remainder theorem to evaluate

$P(3)$ if $P = x^5 - 3x^4 + x^2 - 2$

13. Use the factor theorem to factor

$x^4 - 27x^2 + 14x + 120$

14. Use the results of the Fundamental Theorem of Algebra to determine the number of linear factors in the following expression.

$x^3 + x^2 - x - 1$

15. Suitable word problems can be found under <u>References</u> on page 9 of this quin.

Other sources are included in the Bibliography.

POSTTEST ANSWER KEY

1. a. $\{x \mid x < 15\}$

b. $\{x \mid x = 2K + 1, \forall K \in I, K \geq 0\}$

c. $\{0, 1, 2, 3, \ldots\}$

d. $\{x \mid x = 2K + 1, \forall K \in W, K > 2\}$

e. $\{x \mid x = \frac{p}{q} \quad p \in I, \quad q \in N\}$

f. Natural numbers are a subset of the integers.

g. All real numbers less than -5.

h. Integers are a subset of the rationals.

i. Union of the rationals and irrationals equals the reals.

j. The irrationals and the naturals have no elements in common.

k. $\{x \mid x \in R \wedge x + 12 = 15\}$

l. $Ir \subset R$

m. $3 \notin Q$

n. A

o. C

p. C

Q. B

r. C

s. C

t. B

2. a. Transitive Property of Equality

b. $\forall a, b \in R, a = b, \quad a < b, \vee a > b.$

$\forall c \in R$, if $a < b$ then $a + c < b + c$.

$a > b$ then $a + c > b + c$.

$\forall c \in R$, $c > 0$ if $a < b$ then $ac < bc$.

$a > b$ then $ac > bc$.

$\forall c \in R$, $c < 0$ if $a < b$ then $ac > bc$.

$a > b$ then $ac < bc$.

$\forall a,b,c \in R$ if $a < b \wedge b < c$ then $a < c$

3. a. 1

   b. 7

   c. 8, 12

   d. 2

   e. 4, 12, 5

   f. 10

   g. 7

   h. 1

   i. 6

   j. 5

   k. 3

4. a. $-x^2 \sqrt{x}$

   b. $-1 + (b + 6)i$

   c. $2\sqrt{7x + 7y}$

   d. $\dfrac{4\sqrt{2} + 3\sqrt{3}}{5}$

   e. $-1 - 4\sqrt{5}\, i$

   f. $\dfrac{7 + 9i}{20}$

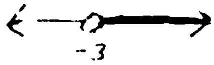
   g. $\dfrac{1 + 5i}{6}$

   h. $5\sqrt{4} - 3\sqrt{3}$

   i. $-2ab \sqrt[5]{b^2}$

   j. $6y^2 \sqrt[3]{6y^2} - 72y^2$

   k. $\dfrac{45 + 39i}{29}$

5. $y > -3$ 

6. a. trinomial, real, 2nd. degree

b. not a polynomial

c. binomial, rational, 1st. degree

d. monomial, rational, zero degree

e. trinomial, complex, degree 10

f. not a polynomial

g. trinomial, rational, degree 5

7. a. $13i \quad x^2y^5$

b. $16x^4 - 81$

c. $5 + 12i - x^2$

8. a. $(3x^2)^5 + 5(3x^2)^4 y + 10(3x^2)^3 y^2 + 10(3x^2)^2 y^3 + 5(3x^2)y^4 + y^5 =$

$243x^{10} + 405x^8 y + 270x^6 y^2 + 90x^4 y^3 + 15x^2 y^4 + y^5$

b. $-120ix^7$

9. a. $(a + bi)\ (a - bi)$

10. b. $r(s + i)\ (s - i)\ (s + 1)\ (s - 1)\ (s^2 + i)\ (s^2 - i)$

c. $(m - 3n^2)\ (m^2 + 3mn^2 + 9n^4)$

d. $(2x + 1)\ (2x - 7)$

11. $2x^2 + 3x - 2$

12. $P(3) = 7$

13. $F(-2) = 16 - 108 - 28 + 120 = 0 \Rightarrow (x + 2)$

$F(3) = 81 - 243 + 42 + 120 = 0 \Rightarrow (x - 3)$

$F(+4) = 256 - 432 + 56 + 120 = 0 \Rightarrow (x - 4)$

$F(-5) = 625 - 675 - 70 + 120 = 0 \Rightarrow (x + 5)$

14. 3

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