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ABSTRACT

This monograph attempts to provide the teacher with examples, techniques, and tips for developing order in number. Topics covered include pre-number order and classification; beginning number activities; number lines; the 100-square counting chart; ordering the integers; fractional numbers; and games, counting rhymes, and other activities. A vocabulary list, a resource list with names and addresses of suppliers, and a short bibliography are included. (DT)

THE MICHIGAN COUNCIL OF TEACHERS OF MATHEMATICS



MCTM

# Guidelines for Quality Mathematics Teaching

*A Monograph Series*

# ORDER

IN

# NUMBER

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Monograph No. 3
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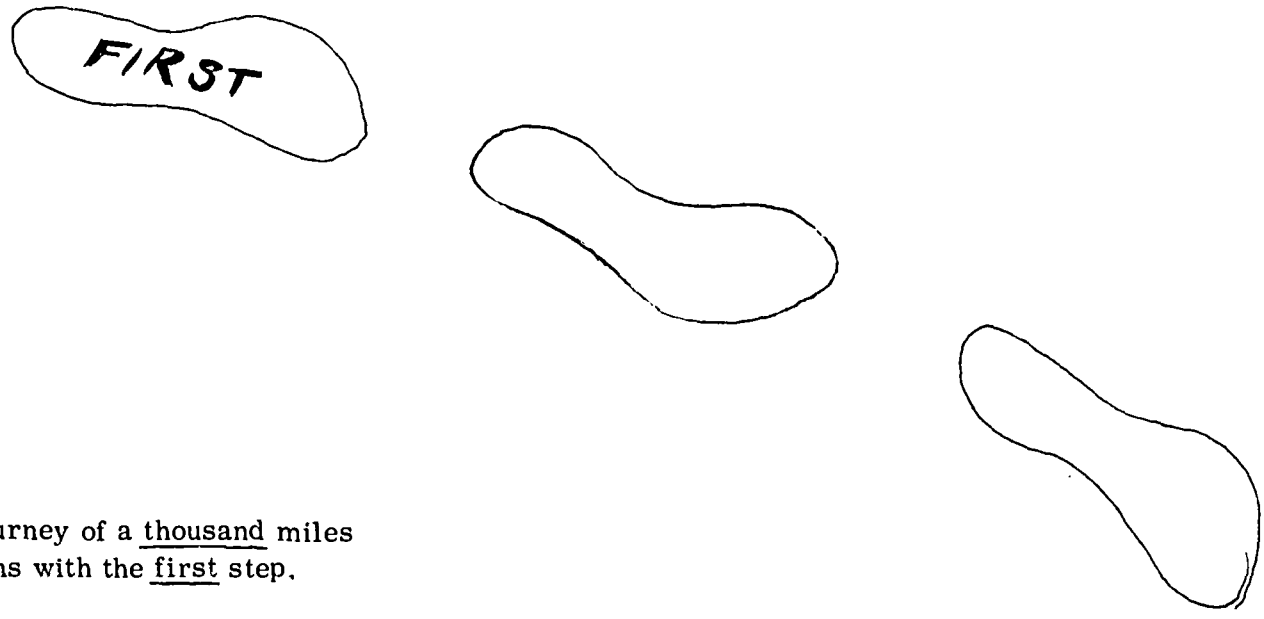
# PREFACE

This monograph is the third in a series to be published by the Guidelines Committee of the Michigan Council of Teachers of Mathematics. The Michigan Education Association provides some financial support for the committee.

This monograph has been designed to provide the teacher with many examples, techniques, and tips for developing order in number.

Alert teachers will constantly discover new and ingenious ways to pursue their important order sense which adds so much smoothness and strength to the entire mathematics program.

Special materials referred to in the monograph are asterisked. A resource list for these materials is included at the end of the monograph.




journey of a thousand miles
begins with the first step.

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## PREVIEW AND OBJECTIVES

Order is the heart of mathematics. Relationships exist such that each new concept derives from an earlier idea, locks into, and helps to unify the entire program. Before the first step is taken, the teacher should be well aware of this continuity and should constantly keep in mind the purpose, the direction, and the order necessary to give meaning to the long journey.

The foundation upon which mathematics rests is the numeration system. Simply stated, our decimal numeration system consists of: (1) a set of ordered symbols $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and (2) an orderly rule, known as "place value", which allows the formation of an infinite number of numerals greater than 9 by placing two or more of the original ten symbols in determined positions. A primary aim is that each child have a thorough understanding of the concept of number and of decimal place value. Underestimating the importance of this objective is a serious error.

Our mathematical system basically consists of four binary operations on the numeration system. Pupils who have developed a sense of number order are capable of discovering the interrelations between the fundamental binary operations of addition, subtraction, multiplication, and division. Clearly, a second objective is that of providing many opportunities to investigate, discover, and achieve the ability to recognize order patterns and relations in mathematical situations; to do creative rational thinking; and to apply an ordered and logical approach to solving problems. These experiences teach "how to learn".

Because a child should be able to express the ideas that he perceives or conceives, a sufficient vocabulary is essential. A third objective is to help the pupil

develop and build such a vocabulary.

The following pages offer suggestions, techniques, and approaches for the teaching of these ideas, emphasizing a background of order.

The procedure begins with the use of concrete or physical objects, continues to semi-concrete pictures and other representations, to semi-abstract number lines, diagrams and charts, and finally to abstract number and algebraic symbols.

## PRE-NUMBER ORDER AND CLASSIFICATION

### Early Experiences

Parents informally introduce the child to order and classification. Mother puts the knives, forks, and spoons in order in the drawer. She takes her newly purchased groceries from the shopping bag, puts the meat in the refrigerator, the bread in the breadbox, and the canned goods in the cupboard. Children are asked to put their rooms in order - toys in the toybox, clothes in the closet, and books on the shelf.

### Kindergarten Class Activities

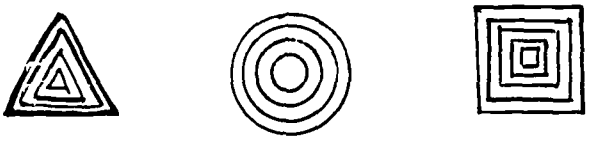
The kindergarten teacher should use every natural opportunity to enlarge on these early experiences. Activities may include classifying objects according to:

(a) color - putting red crayons and yellow crayons into separate baskets.

(b) size - putting large, medium-sized, and small similarly shaped blocks on the top, middle, and bottom shelves, respectively; fitting graduated stacked toys inside each other.

(c) shape - finding circular, triangular, or square-shaped objects in the room; arranging graduated shapes* inside each other (See Fig. 1) to give practice in both series and shape discrimination.

FIG. 1



Other basic classifications include:

(a) texture - metal, plastics, wood, straw, glass, paper, cloth.

(b) measure units - length, liquid, weight, including fractional parts of these units, such as 1/2. metric units (measure)

(c) animal, mineral, vegetable.

(d) hard or soft things.

(e) things that move or don't move.

(f) pairs that go together, things that come in pairs, in sets of three, four, five, or ten.

Concrete Aids

Concrete aids include bath scales, balance scales, different-sized cartons. various-sized steel washers, nuts and bolts, calendar, thermometer, puzzles, play money and toy cash register, blocks, clocks, attribute blocks.

Introducing the Fundamentals of Order

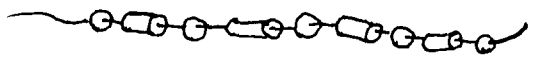
Rote counting activities help children learn number names in the proper order. (Sample games and rhymes are included on a later page.) When children count in order, they are working with both cardinal and ordinal number concepts.

Provide practice:

(a) having them count: the boys, the girls, goldfish, pennies, bounces of a ball, taps on drum, hand claps, jumps, bends, days of the week, rope skips, days until Halloween.

(b) have them tell: which child is first, second, or last in line; reverse the line, now who is first, second, and last.

(c) ask them to: arrange themselves in line as, boy, girl, boy, girl, etc.; stand third in line; take a second book; take the first seat; string wooden beads in patterns according to color, red, green, red, green, etc.; or by shape.



Suggested approach for introducing the concepts of order and classification:

The teacher should prepare a kit of:

(1) nine forms, ○ , △ , □ , each about 4" in width, made of construction paper,* each in three different colors as red, yellow, and blue.

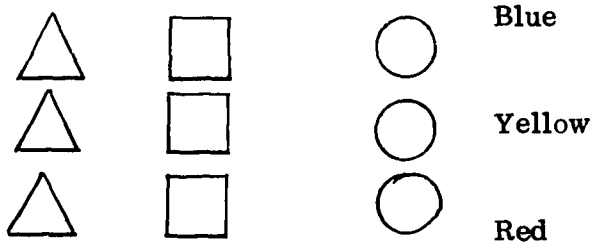
(2) three loops of yarn in the same three colors, and large enough for containing three of the forms.

(3) a triangle, a square, and a circle cut from manilla oaktag and large enough to frame three of the forms when placed inside. Glue magnetic strips* on all items.

Activity 1

Introduce the lesson by having children identify each form by shape and color. Next, mix up the forms on the magnetic board and ask children to arrange the forms in order. Some will arrange by shape, some by color, others will make a pattern, and though none are incorrect, an arrangement by both color and shape shows a high level of ordering ability. (See Figure 2)

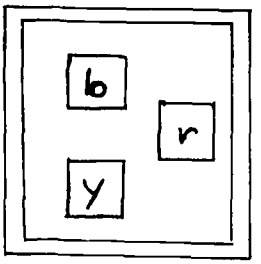
FIG. 2



Activity 2

Place the large square manilla* frame on the magnetic board. Have individual children select the forms to go inside this frame. Each should tell why he made this selection. (Squares go in a square shaped frame, See Fig. 3.) Repeat the activity using the other frames.

FIG. 3




Activity 3

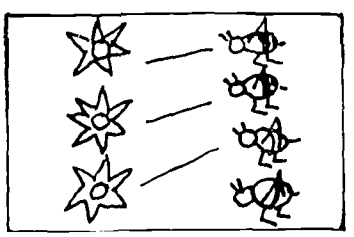
Place a loop of blue yarn on the magnetic board. Have individual children select the forms to go inside this loop. Repeat the activity using the other yarn loops.

## BEGINNING NUMBER ACTIVITIES

### Equivalent and Nonequivalent Sets

Practice with nonequivalent sets: have individual children make sets with many members, fewer members, more members, three members, one more member, etc. Specifically, try the following set comparison:

FIG. 4



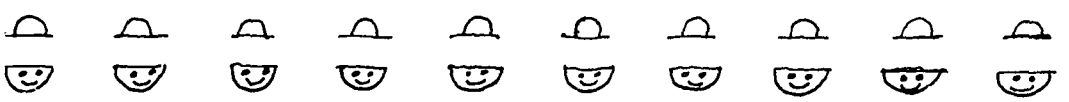
Use Figure 4 and ask the following:

Does every flower have a bee?
Does every bee have a flower?
Are there more flowers than bees?
Are there more bees than flowers?
How many bees are left over?

These sets are not equivalent because 3 is less than 4 (when counting, say 3 first then 4), and 4 is greater than 3 (count 4 after 3).

Young children often believe that a set has more members than another if it occupies more space. This simple test, from Piaget, has been tried out on many first graders with the following results:

Equivalent sets of hats and heads are placed on the magnetic board. (About 10 of each.)

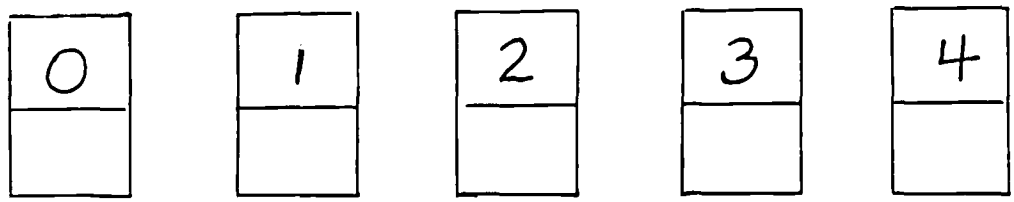


A child places a hat on each head and all the children agree that the heads and hats just match. However, if the heads are pushed closer together, the children will change their minds and say there are more hats, and if now the hats are pushed closer together, they will all agree that there are more heads. Usually it is not until the end of the second primary year that most children will agree that the sets, no matter how arranged, are equivalent.

Cardinal (Counting) Number Sense

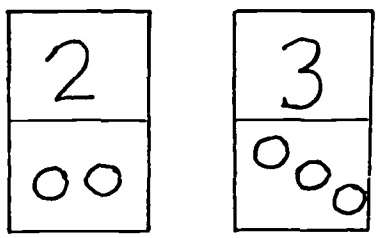
"Show Me Cards" are suggested for use in developing cardinal number concepts.

(a) Teacher preparation: Cut Show Me Cards, 7" by 4" from strips of oaktag.* Draw a line across the middle of each card. As each number concept and the resultant numeral is developed, each child receives a card and writes the newly learned numeral on the upper half.




(b) Uses:

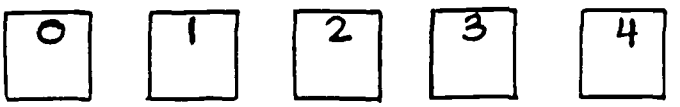
(1) Counters may be placed on the spaces at the bottoms of the cards to to show corresponding sets. Notice that the set of three counters has one more than the set of two counters, and this shows the order relationship between the numbers 2 and 3. It is this relationship which enables a person to count, and which gives meaning to the words he uses.

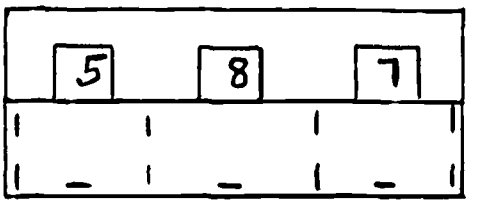



(2) The child holds his set of cards in a pile, moving the correct card to the front position in order to show answers to questions about cardinal number: How many feet does a kitten have? How many kangaroos are in the room? The child can show answers to questions about number order: What number comes after two? What number is one greater than three? One less than three? What number is between two and four?

Three-Pocket "Show Me Cards" can also be used.

(a) Directions; Staple a 7" by 1 1/2" strip of oaktag* across the bottom half of a 7" by 3" strip, and form three pockets.




(b) Uses: Provide each child with a show me card and a set of digits on 2 1/2" squares. These squares are placed in the pockets to show the answers to problems given by the teacher. Directions can be given to illustrate counting number (a-f), place value (g-i), or number order (j-k) at various levels of difficulty.

Show me:

(a) nine

(b) nineteen

(c) four hundred nine

(d) ninety-one

(e) one more than ninety-one

(f) sixteen; ten more than sixteen

(g) a number that has 7 tens and 4 ones

(h) the first two place number

(i) the greatest two place number

(j) three in the first pocket and five in the last pocket; the numeral that comes between them

(k) eight in the middle pocket; the neighbors of eight in the other pockets

Number Order

Children should be given many opportunities to use number order by

(a) telling whether a number is greater than another.

(b) arranging numbers in order of size.

(c) finding dates on a calendar.

(d) locating pages in a book.

(e) telling the order of objects or people in a line.

(f) practicing exercises by writing what comes before, between, or after certain given numbers.

| Before | Between | After |
|---|---|---|
| _____, 5 | 4, _____, 6 | 5, _____ |
| _____, 9 | 20, _____, 22 | 8, _____ |

Ordinality

Chairs, books, encyclopedia, and the footprints on the cover, can be ordered from left or right; animals and people in a line can be ordered from the front or back. The order depends on the established reference point.

Using ordinal words, first, second, etc. and 1st, 2nd, etc. and a reference point the child can be directed to:

(a) find the eightth chair in the row.

(b) find the 7th book on the shelf.

(c) find the thrid frog in the picture.

(d) find the 4th turtle.

(e) find the 3rd volume in the set of encyclopedia.

(f) which child is ninth in line. Turn the line. Who is ninth now?

Another activity would be to arrange the members of these sets in their usual order:

Set A = {G, A, F, E, D, C, H, B}

Set B = {fifth, first, eighth, tenth, third, ninth}

Set C = {6th, 4th, 10th, 7th, 5th, 8 th, 3rd, 2nd, 1st, 9th}

Calendar order can be introduced by having children.

(a) Write months in order.

(b) Write days of the week in order.

(c) Write ordinal numerals under these days to show their order in the week:

| Tue. | Sat. | Sur | Wed. | Mon. | Fri. | Thu. |
|---|---|---|---|---|---|---|
| 3rd | | 1st | | | | |

## NUMBER LINES

A number is one of the most effective ways of showing the order relationship of numbers. A number line is formed naturally and meaningfully when numerals are placed in increasing or decreasing order along a line.

### Building Number Line Readiness

The following activities done in sequence aid in developing the number line concept.

(a) Strings of beads are hung above a pocket chart. (concrete)

(b) Children place perception (or picture) cards in increasing order below the beads. (semi-concrete)

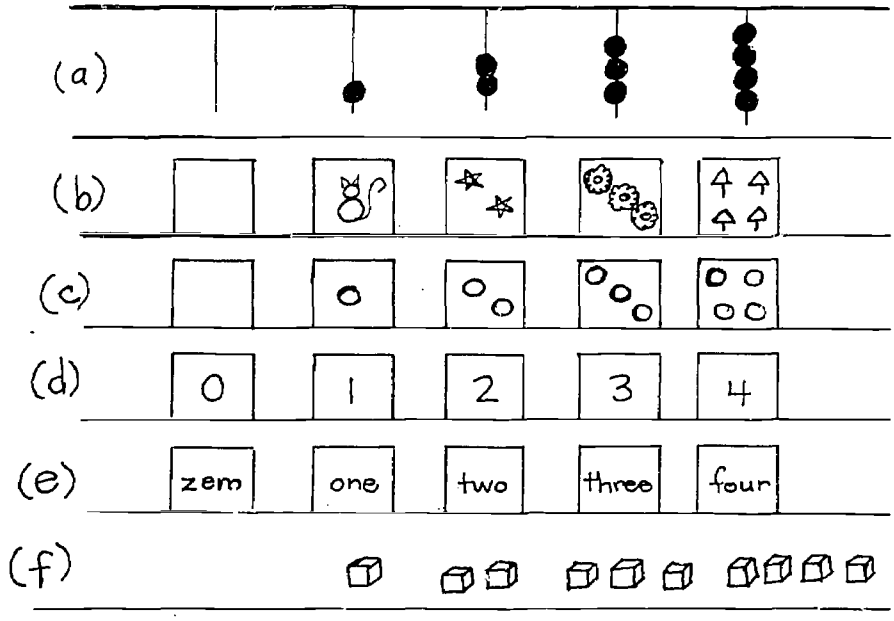
(c) Correspondingly, they place dot cards below each of the perception cards. (semi-abstract)

(d) Numeral cards are placed below the dot cards. (abstract)

(e) Number words are placed to match. (abstract)

(f) Small blocks* are placed on ledge below the chart. (concrete)

The figure below illustrates the completed activity.



As an additional activity omit one of the strings of beads,* for example, the three string, and have the children tell, but not show, where it belongs. Thus, practice is given in the use of order words, such as, "between", "before", or "after", which are needed to describe where the string belongs. This procedure helps build a mathematical vocabulary.

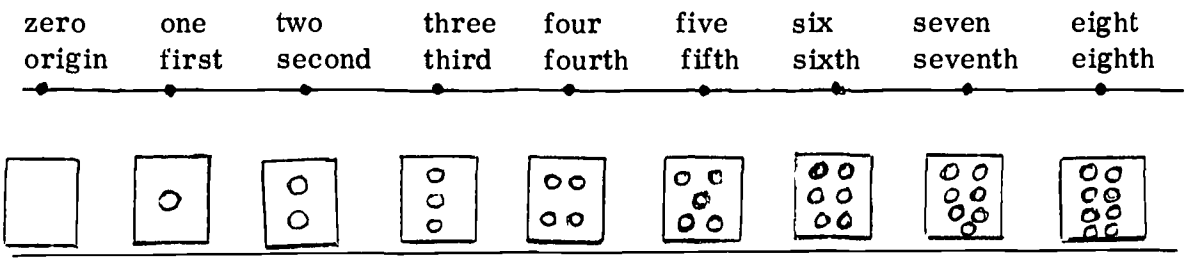
If blocks are tapped, children can count taps and discover that each increased set has one more tap. Using all their senses has learning value for children and also teaches them to listen.

Suggestions for Developing a Number Line With Primary Children

Teacher preparation: Cut ten cards (7" x 11") from dark green railroad board* or rigid cardboard. Glue on large white dots (milk-bottle tops*) to make sets having from 0 to 9 members. The 10 card is constructed from bright red railroad board.*

Mix up and distribute the cards (1 to 9) to the class. Ask, "Who thinks he has the card that should be first?" Have the child place the "one" card on the chalkboard ledge. Write "one" and "first" high above the card and have the children say the words together. Call for the second card, then the third, etc. until all the cards have been arranged in order. Draw a line above the cards, place dots for particular points and have these numbered from 1 to 9. Have the "ten" card placed and the point numbered. The red card calls attention to "ten" as the key number in the decimal numeration system. A look at their hands explains the reason for ten's importance. Finally the empty card is placed and the 0 point is numbered. This point will be designated as the origin or starting point. (If desired, the original words may now be erased.)

TENS SYSTEM




Using the Dot-set Number Line

Place a cardboard frog on 0. No counting is done at this point and if a child insists on counting here, he must say, "Zero". As the frog jumps, count only when he finishes his first jump and lands on the 1 point and again when he lands on the 2 point, etc. It is the interval between two numbers that is the important part of the concept.

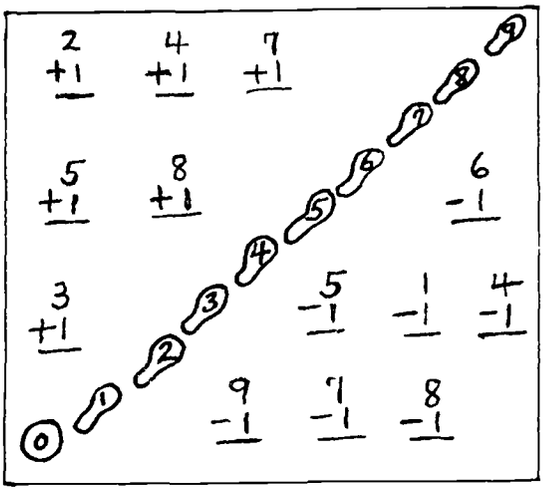
Addition and subtraction follow automatically from counting on the number line. To illustrate addition, start at zero and take one step. Moving from 0 to 1 is 1 step. Continue: 1 step and 1 step are two steps; 2 steps and 1 step are 3 steps; etc. This is the "successor concept" (each number after the first is one more than the preceding number) and is the intrinsic order of the natural numbers. Substraction is merely the inverse of addition and is shown by a downward, or to the left, ordered movement along the line. Arcs or arrows may be drawn to represent both operations.

Variations for Number Line Use

(1) Footprints made front construction paper* (laminated or covered with clear contact paper) can be taped to the chalkboard in a vertical line, so a finger can walk

"up" or "down" to prove problems. Or these little footprints may be drawn on a ditto practice sheet where a child must solve equations involving addition and subtraction of 1. Intuitively, the child recognizes that (a) adding 1 to a number produces a greater number; (b) subtracting 1 from a number gives a number that is less than the beginning number; (c) counting one more is the basis of adding. Adding or subtracting 0 produces no change - the fingers cannot move. Emphasize that the number of steps is the number added.

FIG. 5

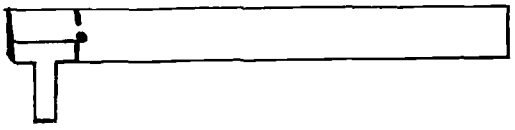
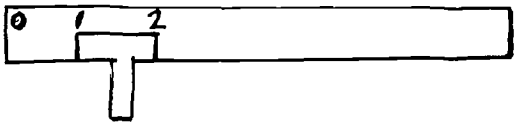



(2) Floor number lines can be made with (a) paper footprints taped to the floor; (b) a strip of wide masking tape numbered with a felt dri-marking pen; or (c) red or yellow oilcloth marked with black numerals. Begin at zero and listen for the footsteps before counting each number. To emphasize number order have one child walk to 4 and another child to 7. Compare the steps each child took: Joe took 3 more steps. Where is he? How many steps did each child take? Which is greater 4 or 7? When you walk along a number line, do you come to the 4 point first? When you walk from point 8 to point 3 what points did you step on along the way? Is the point for 4 between 3 and 5?

(3) Have children form a number line by holding numeral cards from 1 to 10 and ordering themselves accordingly. Turn several so their backs are to the class and have class tell which numerals are missing from the line. Substraction of 1 may be illustrated by having children leave the line one at a time, the ninth child going first, eighth next, etc.

(4) Individual number lines may be drawn on paper. If these are laminated a grease pencil or crayon may be used for drawing arcs. Say, "Let's start at 0 and draw a big loop to 5". Count to the right or up the line; count to the left or down the line; count by 2's; by 3's; by 5's. (It is to be noted that parts of the number line may be needed for certain problems and counting does not always start at zero.)

Child-constructed Number Lines

Primary children may construct their own first number line on a 30cm x 5cm strip of oaktag.* Each child is given a small  (cut from oaktag by the teacher). The top of the T measures 3cm across. The child uses the T to mark off his 30cm strip. He holds the T by the stem and places the top at the left (or 0) edge of his strip.  He marks a point and numbers it 1. He places the T at 1 and marks off 2, 

and continues to mark and number points until the end of the strip is reached. Older children use similar cardboard rulers marked off in 1cm units (constructed by themselves) until they are thoroughly familiar with their use. When these are mastered

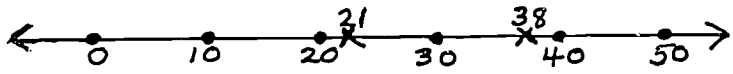
they can construct other metric rulers.

Addition as illustrated on any number line, such as a ruler, is an ordered process since the line segments, representing the addends, must be placed end to end. When addition is illustrated by sets of objects, the process is not an ordered one since sets may be joined together to form one set in a helter-skelter dumping manner.

Using the Number Line to Round Numbers

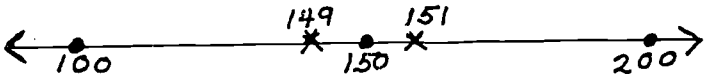
Knowing the order of a number in the number scale is necessary when rounding to the nearest 10, 100, or 1000.

Nearest 10:



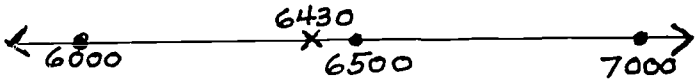

38 rounds to 40
21 rounds to 20

Nearest 100:




151 rounds to 200
149 rounds to 100

Nearest 1000:




6430 rounds to 6000

Using the Number Line to Compare Numbers

Number lines may be used to work these examples:

(a) Circle the greatest number: 631 136 361 ;or, 1002 1020 1200

(b) Arrange from least to greatest: 4806 4860 4006 4800 4880

(c) Arrange the scores on an arithmetic test in descending order (high to low) 94, 88, 70 96, 90, 99, 89, 75, 69, 97.

(d) Fill in the blanks in each number sequence:

1. 1, 2, 3, 4, 5, __, __, __

2. 1, 3, 5, 7, 9, __, __, __

3. 100, 90, 80, 70, 60, ___, ___, ___

4. 7, 14, 21, 28, 35, ___, ___, ___

5. 1, 2, 4, 8, 16, ___, ___, ___

6. 72, 61, 50, 39, 28, ___, ___, ___

## THE 100-SQUARE COUNTING CHART

Learning to write all the numbers in order from 1 to 100 is sometimes a difficult job for a third grade child. A large 100-square counting chart shows the pattern of the decimal number system. The chart can be used as follows:

(1) Cover all but the first row; children learn to say and write these numbers in order.

(2) Expose the second row, and have children compare these numbers with the first row. Recover this row and have them write these "teen" numbers.

(3) Proceed by uncovering each row in turn, have children study the pattern, say and write the numbers.

(4) As soon as the pattern is apparent to the children they should try to write the next row without uncovering it. It is then uncovered for checking only.

The counting chart can be also used to

1) Find all numbers (a) greater than 95, (b) less than 10, (c) having 4 in the ones place. (d) having 9 in the tens place. (e) that come between 6 and 10, (f) having 4 tens and 9 ones, (g) having 7 tens and 4 ones.

2) Count by tens starting (a) at 10, (b) at 6, (c) at 27.

3) Find the neighbors of (a) 46, (b) 99.

4) Count backwards from (a) 20 to 10, (b) 26 to 16.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

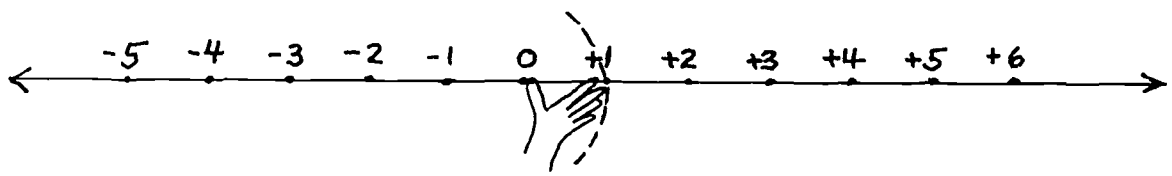
FIG. 6

## ORDERING THE INTEGERS

The set of integers is. $\{. . ., -4, -3, -2, -1, 0, +1, +2, +3, . . .\}$. A number line is an effective aid for introducing the negative numbers and their order in the set of integers.

A number line can be constructed simply and easily by using a span (stretched space between thumb and index finger) as a unit of measure. (See Fig. 7). Mark and number a zero point; measure off the first interval by placing the thumb on 0 and by using the index finger to describe an arc on the line; number this point +1 and continue to measure the right ray of the line using this span. (Notice that using the fingers in this manner suggests the invention of a compass for line measurement.) In like manner, a blackboard eraser may be used as a unit of measure instead of the thumb-finger span.

FIG. 7




After the right ray has been measured and numbered, measure one interval on the left ray. Ask the pupils to suggest how this point might be numbered to distinguish it from the positive one (+1) point on the right. An agreement should be made that since the point is one interval below the zero point that it may by numbered the opposite of 1, more commonly termed "negative 1", and symbolized as -1. The other intervals are similarly measured and the points numbered as -2, -3, etc. Signs are given to the positive numbers as +1, +2, +3, etc., and indicate that 2 and +2 are names for the same number. 0 is neither positive nor negative. Signed numbers indicate both magnitude and direction. +3 shows a magnitude of 3 units in a positive direction; -7 shows a magnitude of 7 in a negative direction.

An integer number line can be used in the following ways:

(1) the set of negative numbers below zero are used to describe (a) locations on a line - to the left of 0 on a horizontal line, below 0 on a vertical line; (b) below or south of the equator or to the west of a 0 point on a map; (c) below 0 on a thermometer; (d) "in the hole" when playing a game; (e) in debt; or charging an item, (f) below sea level; (g) to give closure to the substraction operation so that subtraction may always be performed.

(2) the numbers below 0 follow the same order as those above 0 but in an opposite direction.

(3) zero itself is not thought of as a "nothing quantity", for example, a temperature recording of $0^{o}$ is of a higher temperature order than $-1^{o}$, or $1^{o}$ below 0.

(4) counting in either direction on the line distinguishes the order of the integers. 0 is less than any positive integer, and 0 is greater than any negative integer; on a horizontal line, any number to the right of another number is the greater of the two. On a vertical line <u>above</u> is greater, <u>below</u> less.

(5) when marked with negative (-) or positive (+) signs, the same number may be used to name two points on the number line, each the same distance from zero but in opposite directions.

(6) the set of integers extends to infinity in either direction.

(7) the integers are ordered as: . . . -3 -2 -1 0 +1 +2 +3 . . .

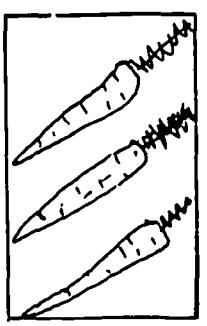
(8) the set of all integers greater than -3 is $\{-2, -1, 0, +1, +2, +3, \ldots\}$.

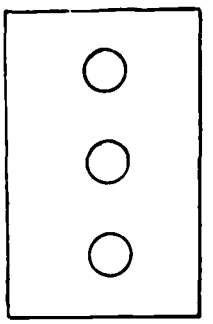
(9) the set of all integers less than -3 is $\{-4, -5, -6, -7, \ldots\}$.

(10) the set of all integers greater than -2 and less than +2 is $\{-1, 0, +1\}$.

## INTRODUCING FRACTIONAL NUMBERS

The set of dark green dot cards that were previously used to introduce the number line may be used to introduce the fractional number 1/2. The cards are placed in order on the chalkboard ledge with a card-width of space between them. Other cards of railroad board* with pictures of fruit or vegetables in sets of one to nine are placed to correspondingly cover the proper dot set. (Primary children may draw these pictures.)





After four or more of the cards have been placed ask a child to place a card that shows one-half grapefruit. If he has trouble doing this ask him to keep his card while other cards are being placed. Next, try 2 1/2 bananas. This picture shows more than two bananas but less than three, and seems to be the easiest fractional set for the primary child to place correctly. 2 1/2 is recorded at the proper point on the line, and is read, "two and one-half". Other cards such as 4 1/2 tomatoes or 1 1/2 watermelons are similarly placed before the 1/2 grapefruit is again attempted and correctly placed. Discussion of its fractional value brings out that it is more than 0, or no grapefruit, but less than 1, or a whole grapefruit. The point on the number line for 1/2 is marked, numbered, and described as: 1/2 is less than 1 but more than 0; 1/2 comes between 0 and 1.

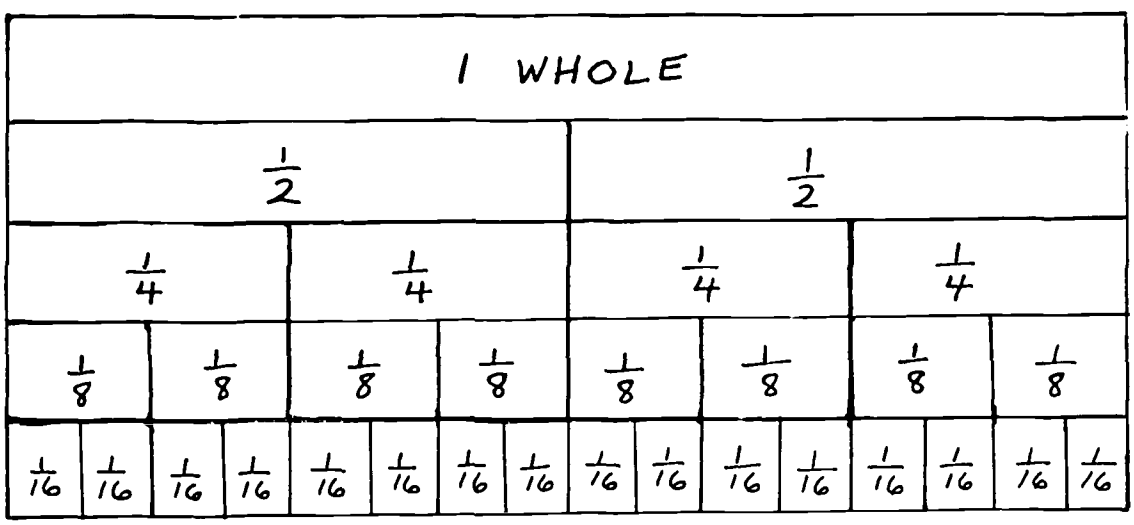
Other uses of the fractional number line:

(1) Count by halves from 0 to 10.

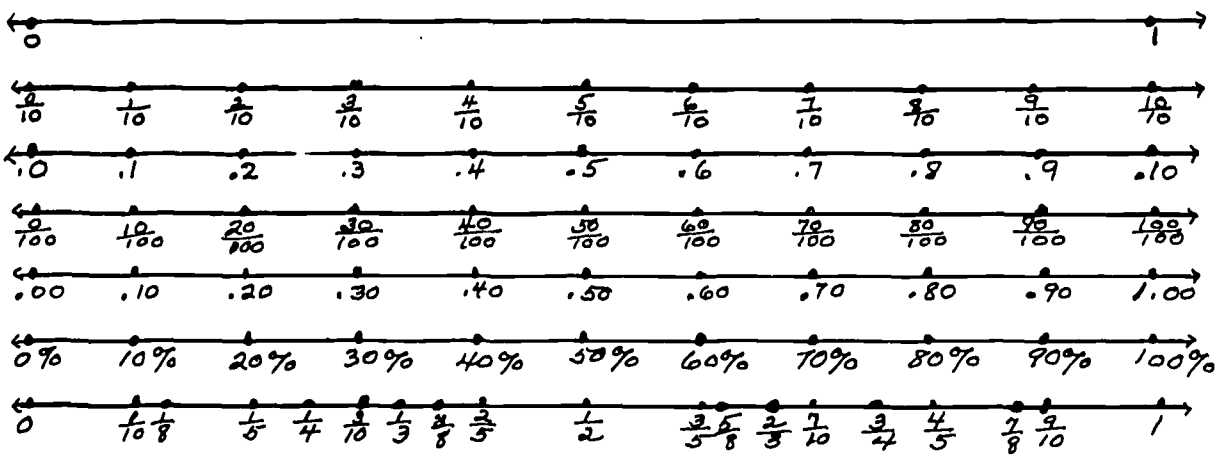
(2) Determine which is greater, 2 1/2 or 2; 6 or 6 1/2; etc.

(3) Discuss the denseness of the fractional numerals on the number line: Between which two whole numbers points does 1/2 belong? Are there any other points between 0 and 1? How many points? Do they represent whole numbers or fractions? Are they in a certain order? Where will the fractions less than 1/2 come? Imagine cutting a pizza pie as more and more people come to supper and note the relationship of 1/2 to 1/4, etc.

(4) To show the equivalent fractions for 1/2's, 1/4's, 1/8's, and 1/16's, cut strips of paper all the same length (wide adding machine tape* is fine), label the first strip "1 whole". Fold the next into halves and label each part "1/2". Fold the next into four equivalent parts and label each part "1/4". Continue for eights and sixteenths. Paste the strips under each other on a background of oaktag to form a chart. Use for finding equivalent fractions. Do the same thirds, sixths, and twelfths; fifths and tenths.

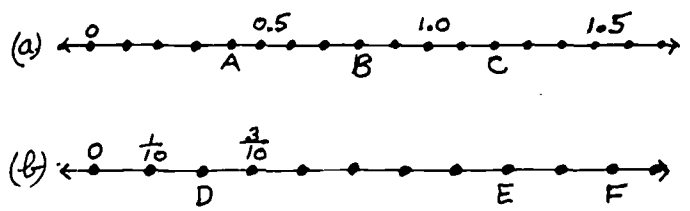


To show advanced students the relationship between common fractions, decimals, and percents, divide and mark number lines, and glue in a corresponding order on a large piece of background paper.




Here are some exercises for advanced students to reinforce the ability to order fractions:

(1) What number does each letter represent?




(2) Arrange these fractions in ascending order (least to greatest):

(a) 1/3, 1/5, 1/2, 1/4, 1/6

(b) 1.0, .01, .1, .05, .15

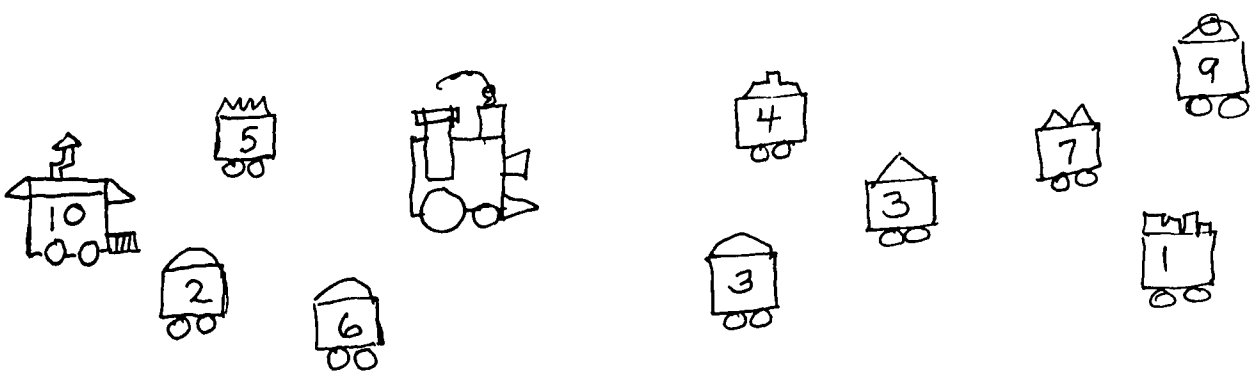
GAMES, COUNTING RHYMES AND ADDITIONAL ACTIVITIES

1. Clothes Pin Hang

Hang a piece of clothesline where children can reach it. A box of snap clothespins with numerals marked on them is placed nearby. Children are asked to hang the clothespins on the line in order When they are not looking mix up the pins and let them discover the mix-up.

2. Order Train

Prepare a cardboard engine and cars and number them. Tape magnets on back of each, so they may be arranged in the correct order. A train without magnets may be laid on a table or arranged along a ledge. Numbered blocks make an easy train to assemble.




3. Hands

Cut a pair of hands from oaktag and laminate. The fingers are to be numbered orally reading from left to right. Directions for the children to follow:

(a) Put a bandaid on the third finger.

(b) Put this ring on the fourth finger.

(c) Put fingernail polish (use red crayon) on the second finger; the ninth finger.

(d) Mark a cut on the sixth finger.

4. Poison Number

Children stand. They count around and around the room from 1 to 100. Any child who must say, "Five", or any number that is poison for that particular game, must sit. Last child standing wins the game.

5. Buzz

Similar to Poison Number, Instead of saying, "Five", the child says, "Buzz".

6. Next Door Neighbors

Prepare small cards (about 4" square) and number from 1 to 100. Select enough consecutive cards so each child may have one. Teacher calls out a number, for example, "56". The child with this card comes to the front of the room and asks, Who are my next-door neighbors?". The neighbors, "55" and "57" stand beside "56". The game may be played without cards by having a child write a numeral on the chalkboard and ask other children to write the neighbors.

7. Jack-o! Lanterns

Spread out fingers on one hand and touch them as this rhyme is recited in unison:

Five little Jack-o' Lanterns sitting on a gate.
The first one said, "My it's getting late."
The second one said, "There are ghosts in the air."
The third one said, "Where, oh, where?"
The fourth one said, "We'd better run."
The fifth one said, "Oh, it's just Halloween fun."
"Whoooo," went the wind, and (puff) out went the light,
And away they all ran on Halloween night.

8. Beehives

Make one fist:

Here is a beehive
But where are the bees?
Hidden away where nobody sees.
Soon they come creeping
Out of the hive,
One, two, three, four, five. (Open fingers)

Make two fists:

Here are two beehives
But where are the bees?
Hidden away where nobody sees.
Here they come out again
One, two, three, four, five,
Six, seven, eight, nine, ten. (Open fingers)

9. Indians

Ten boys in a row, squat down and jump up one by one to dramatize the rhyme.

One little, two little, three little Indians;
Four little, five little, six little Indians;
Seven little, eight little, nine little Indians;
Ten little Indian boys.

The word "boy" may be changed to "girl" to give the girls a turn at the rhyme.

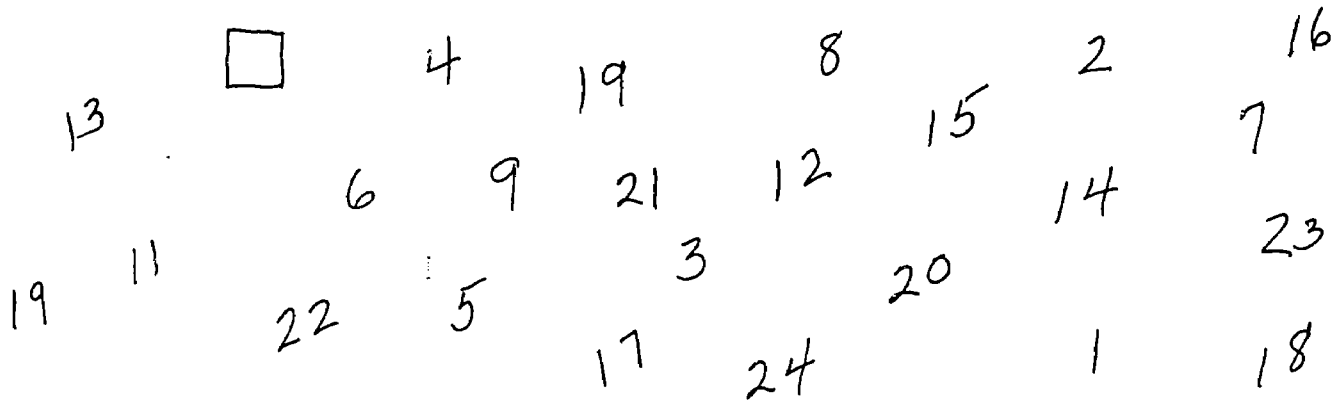
10. Ball Bounce

Child bounces and catches the ball on each count. Older children may keep ball bouncing continuously and clasp hands where there are no counting words.

One, two, buckle my shoe;
Three, four, shut the door;
Five, six, pick up sticks;
Seven, eight, lay them straight;
Nine, ten, a big fat hen.

11. Missing Number

A ditto sheet is prepared for each child. As the child counts the numbers to himself, he draws a ring around them. When he finds the missing number he writes it in the square or box. Extra Number may be played the same way.




12. Follow Directions

Call five children to the front of the room. Have the children form a line as if to walk across the front of the room. Listen carefully and see if you can follow my directions.

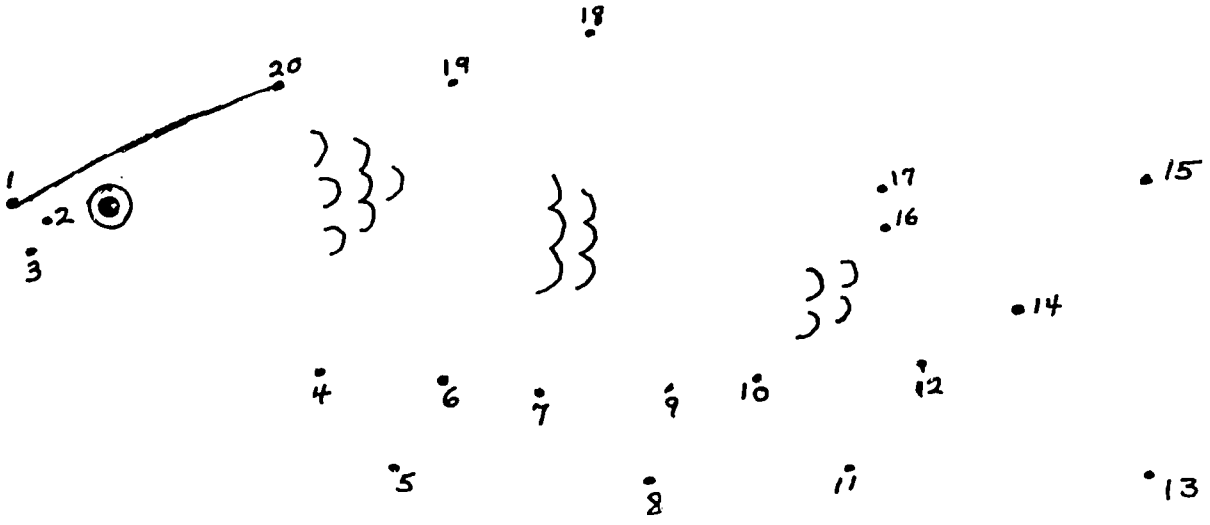
Will the second person clap his hands?
Will the fifth person turn around in place?
Will the first person jump up and down?
Will the fourth person touch his toes?
Will the third person wave his hands?

Call five different children to the front of the room. Give a new set of directions.

Find and sit on the eighth chair in the row.
Find and open the 7th book on the shelf.
Find and point to the third frog in the picture.
Find the fourth turtle.
Find the 6th volume in the set of encyclopedias.
Which child is fifth in line? Turn line, who is fifth now?

13. Dot Picture

Draw from dot to dot following the numbers in order.



## VOCABULARY

### Comparison Words

#### Size:

narrow, wide, widest, width, big, bigger, biggest, small, smaller, smallest, long, longer, longest, length, short, shorter, shortest, large, larger, largest, tiny, giant, little, greater, least, difference, height, high, higher, highest, tall, taller, inch, foot, yard

#### Distance:

far, farther, near, nearer, nearest, miles

#### Quantity:

none, not any, few, some, several, more, less, much, dozen, number, each, any, all, once, twice, three times, most, fewer, fewest, empty set, cupful, part, many, whole, half, half-full, empty, as many as, single, pair, least, half-dozen, dozen, some, same, halves, basketful

#### Weight:

light, lighter, lightest, heavy, heavier, heaviest, ounce, pound, ton

### Order or position in space:

in, inside, into, out, outside, on, off, beside, forward, backward, after, before, around, between, right, left, up, down, center, below, above, over, under, upon, low, lower, high, higher, highest, first, second, third, fourth, fifth, sixth, seventh, eighth, ninth, tenth, last, middle, behind, in front of, end, beginning, next

Shape or form:

circle, circular, square, rectangle, rectangular, round, triangle, triangular, point, side, straight, star, ring

Time:

today, yesterday, tommorrow, now, winter, autumn, fall, soon, old, older, oldest, young, younger, youngest, age, day, week, month, year, season, hour, minute, second, noon, midnight, spring, summer, early, late, then

Others:

slow, slower, slowest, soft, softer, softest, shallow, deep, one-to-one, hard, hardest, temperature, below zero, above zero, freezing, hot, cool, cold, chilly, warm

## EVALUATION

Evaluation of any program is judging its effectiveness. It is objective measurement in the sense that it seeks to determine how many of the initial objectives have been achieved. It is also non-objective in that many side effects have been experienced.

Questions come to mind. Were the essential goals satisfied? Did the details lead away from the main concepts? Was the program interesting? challenging? a "leading-on" or a "sudden-stop" action? Was the attitude such that the children were eager to tackle new areas? Did each concept tie into the next smoothly? Did a great many individuals need extra help? Did the activites provide enough practice to firmly fix the ideas, or was it necessary to do much reteaching? Which of the offered suggestions did not work?

What are the pitfalls?

What changes are necessary for improving the old and for creating new ideas for teaching ORDER IN NUMBER?

## ASTERISKED RESOURCE LIST

| | |
|---|---|
| RAILBOARD BOARD | Michigan Products, Inc.<br>1200 Keystone Avenue<br>Lansing, Michigan 48909<br>Telephone: 517 - 393-0440 |
| OAKTAG or MANILLA PAPER | Michigan Products, Inc.<br>1200 Keystone Avenue<br>Lansing, Michigan 48909<br>Telephone: 517 - 393-0440 |
| CONSTRUCTION PAPER | Michigan Products, Inc.<br>1200 Keystone Avenue<br>Lansing, Michigan 48909<br>Telephone: 517 - 393-0440 |
| WOODEN BEADS | Ideal School Supply Co.<br>Oak Lawn, Illinois 60453 |
| WOODEN BLOCKS | Playskool Educational Materials #702<br>Milton Bradley Company<br>Springfield, Mass. 01101 |
| GRADUATED SHAPES<br>FIT A SQUARE<br>FIT A CIRCLE | Michigan Products Company<br>1200 Keystone Avenue<br>Lansing, Michigan 48909<br>Telephone: 5 17 - 393-0440 |
| ADDING MACHINE TAPE | Michigan Products Company<br>1200 Keystone Avenue<br>Lansing, Michigan 48909<br>Telephone: 517 - 393-0440 |
| CONTACT PLASTIC | Michigan Products Company<br>1200 Keystone Avenue<br>Lansing, Michigan 48909<br>Telephone: 517- 393-0440 |
| LAMINATING FILM | D.M.I. Industries, Inc.<br>4305 Delemere Blvd.<br>Royal Oak, Michigan 48073 |

| | |
|---|---|
| MILK BOTTLE TOPS | D. M. I. Industries, Inc.<br>4305 Delemere Blvd.<br>Royal Oak, Michigan 48073 |
| MAGNETIC STRIPS | Instructo Corporation<br>Paoli, Penna. 19301 |

## SUGGESTED READINGS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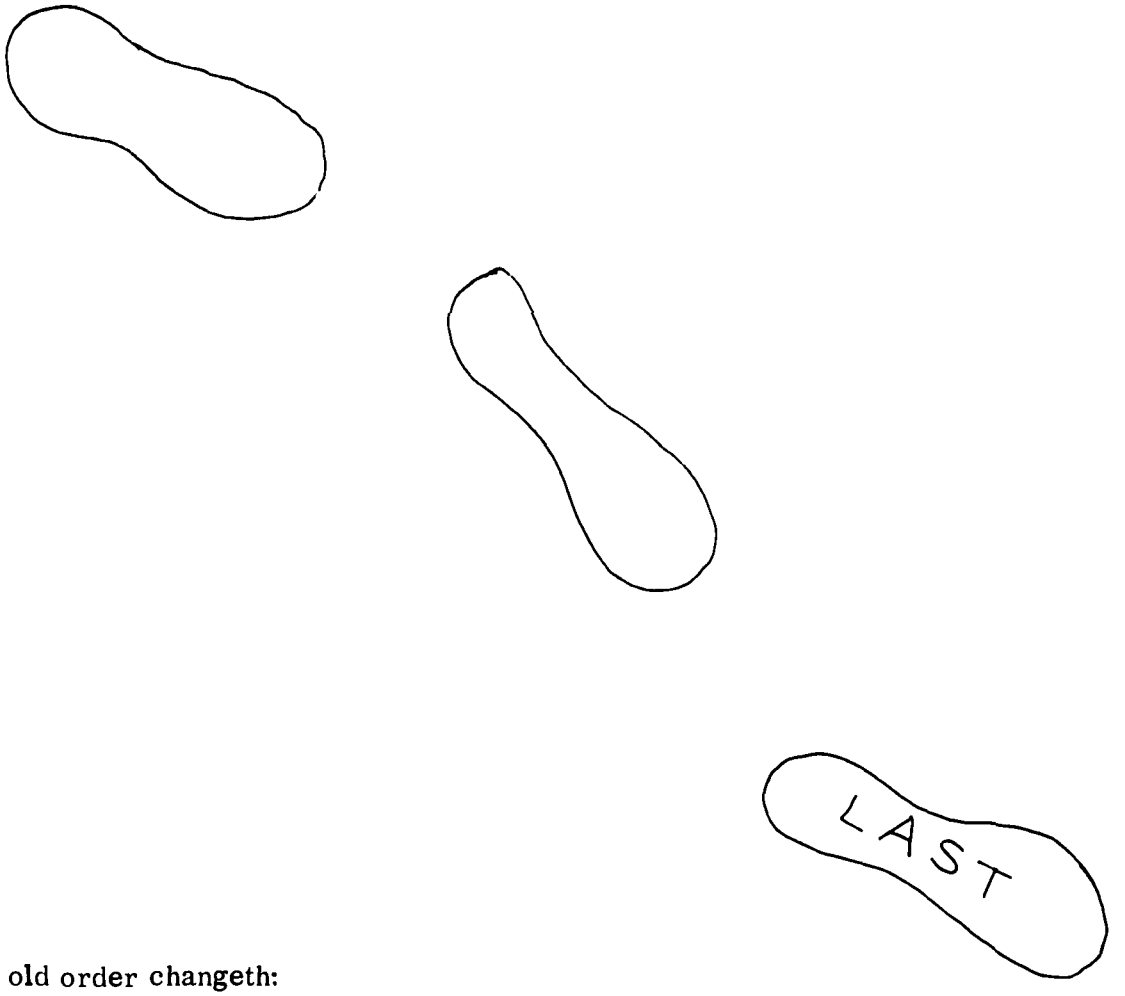


The old order changeth:

Our Moon Men made the journey of thousands of miles before they took that famous first step.

"One small step for man

One giant step for mankind."

Neil Armstrong

The following were members of the Guidelines Committee for Quality Mathematics Teaching during the development of this monograph:

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| Thersea Denman | Detroit |
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Your comments and criticisms of this monograph as well as suggestions for other monographs (or manuscripts for them) can be sent to

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