DOCUMENT RESUME

ED 081 642 SE 016 795

AUTHOR	Strachan, Florence
TITLE	Algebra 1r, Mathematics (Experimental): 5215.13.
INSTITUTION	Dade County Public Schools, Miami, Fla.
PUB DATE	71
NOTE	22p.; An Authorized Course of Instruction for the Quinmester Program
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	*Algebra; Behavioral Objectives; *Curriculum; Instruction; Mathematics Education; *Objectives; *Secondary School Mathematics; *Teaching Guides; Tests
IDENTIFIERS	*Quinmester Program

ABSTRACT

This third of six guidebooks on minimum course content for first-year algebra includes work with laws of exponents; multiplication, division, and factoring of polynomials; and fundamental operations with rational algebraic expressions. Course goals are stated, performance objectives listed, a course outline provided, testbook references specified which are keyed to the course outline, and teaching strategies suggested. Pretest and posttest items are included, plus a list of three references. For other booklets in this series, see ED 067 296, ED 067 283, ED 067 284, SE 016 504, and SE 016 505. (DT)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

ALGEBRA 1r
5215.13

Mathematics

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION • 1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

ALGEBRA 1r
5215.13

(Experimental)

Written by
Florence Strachan

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

DADE COUNTY SCHOOL BOARD

Mr. William Lehman, Chairman
Mr. G. Holmes Braddock, Vice-Chairman
Mrs. Ethel Beckham
Mrs. Crutcher Harrison
Mrs. Anna Brenner Meyers
Dr. Ben Sheppard
Mr. William H. Turner

Dr. E. L. Whigham, Superintendent of Schools
Dade County Public Schools
Miami, Florida 33132

Published by the Dade County School Board

Copies of this publication may be obtained through

Textbook Services
2210 S. W. Third Street
Miami, Florida 33135

PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

A continuation of work with polynomials and the development of ability to work with rational expressions. Includes work with negative exponents; multiplication, division, and factoring of polynomials; and fundamental operations with rational algebraic expressions.

Designed for the student who has mastered the skills and concepts of Algebra 1q.

TABLE OF CONTENTS

OVERALL GOALS

1. To develop further skills in working with polynomials.
2. To apply the skills of factoring in performing operations with rational expressions.
3. To apply the laws of exponents to integral exponents.
4. To use algebraic skills in solving verbal problems.

KEY TO REFERENCES
(* State Adopted)

* D - Dolciani, Mary; Wooten, William; Beckenbach, Edwin; Jurgensen, Ray; and Donnelly, Alfred. Modern School Mathematics, Algebra 1. New York: Houghton Mifflin, 1967.

N - Nichols, Eugene D. Modern Elementary Algebra. New York: Holt, Rinehart, and Winston, 1961.

* PL - Payne, Joseph; Zamboni, Floyd; and Lankford, Francis. Algebra One. New York: Harcourt, Brace, Jovanovich, 1969.

* PA - Pearson, Henen R. and Allen, Frank B.. Modern Algebra: A Logical Approach, Book One. Boston: Ginn and Co., 1964.

[1] - The number in the block preceeding an objective indicates the number of the state assessment standard to which the objective is related.

PERFORMANCE OBJECTIVES

I. Polynomials

The student will

[8] 1. Apply the laws of exponents in multiplying two or more monomials with integral exponents.

[8] 2. Apply the laws of exponents in raising to a power a monomial with integral exponents.

[8] 3. Apply the laws of exponents in simplifying a rational expression whose numerator and denominator are monomials with integral exponents.

[5] 4. Find the following products mentally:

a. the product of the sum and difference of two terms
b. the square of a binomial
c. the product of two binomials

[9] 5. Factor:

a. the difference of two squares
b. a perfect square trinomial
c. trinomials of the form $x^2 + bx + c$
d. trinomials of the form $ax^2 + bx + c$, $a \neq, o$
e. a given polynomial using one or more of the above methods

[5] 6. Find the quotient of a polynomial and a binomial.

II. Rational Algebraic Expressions

The student will

1. Evaluate a rational expression for a given value of the variable.

2. Determine the restrictions on the variable for a given rational expression.

3. Reduce a rational expression to lowest terms.

4. Determine whether or not two rational expressions are equivalent.

5. Multiply rational expressions.

6. Divide rational expressions.

7. Find the LCM of two or more polynomials.

8. Add and subtract rational expressions with like and unlike denominators.

9. Solve open sentences involving rational algebraic expressions.

RELATED OBJECTIVES	COURSE OUTLINE
I.	I. Laws of Exponents
1-3	A. Product of powers B. Quotients of powers C. Powers of powers
	II. Polynomials
4	A. Products of binomials
	1. $(ax + b)(cx + d)$ 2. $(ax \pm b)^2$ 3. $(a + b)(ax - b)$ 4. Other products
5	B. Factoring
	1. Binomials
	a. Monomial factors b. Difference of two squares
	2. Perfect square trinomial 3. Other trinomials
6	C. Quotients of polynomials
	1. Long division 2. Simplifying by factoring
II.	III. Rational Algebraic Expressions
1-4	A. Rational Expressions
	1. Definition of rational expressions 2. Evaluating rational expressions 3. Determining the restrictions on variables 4. Reducing to lowest terms 5. Equivalent rational expressions
5-8	B. Operations with Rational Expressions
	1. Multiplication and division 2. Addition and subtraction
9	C. Solving open sentences containing rational algebraic expressions

TEXTBOOK REFERENCES

Course Outline	D	N	PL	PA
I.	267-270 310-313 324-326	312-324	257-270	340-350
II.	272-292	351-361	313-339	366-368
III.	328-338 350-353	152-161	Entire Chapter 9 386-409	397-406
	120-130	211-251	130-141	251-255

SUGGESTED STRATEGIES
(Keyed to Course Outline)

I.

1. Point out the difference between the expressions $-x^2$ and $(-x)^2$. Show that $-x^2 \neq x^2$, unless $x = 0$.

2. Work with powers that have numerals as bases: For example, $2^4 \cdot 2^5$. Students tend to multiply the bases together, $2^4 \cdot 2^5 = 4^9$. Show the correct multiplication by returning to the definition $x^m \cdot x^n = x^{m+n}$ and giving x the value of 2 and m an n the values 4 and 5. ($2^4 \cdot 2^5 = 2^9$.)

3. Provide students with many drills on the operations with powers.

II.

1. Demonstrate the following methods of multiplying polynomials:

   A. Foil Method (first, outer, inner and last terms)
   B. Distributive property
   C. Vertical multiplication

2. Use colored pen or chalk to indicate how the middle term is computed when showing horizontal multiplication of binomials.

3. Show that the pattern for common monomial factoring is based on a direct application of the distributive law. Example, $5n + 10 = 5(n + 2)$.

4. Have the student recognize a binomial such as $25y^2 - 9$ in terms of $(5y)^2 - (3)^2$ so that the idea of the difference of two squares will be more meaningful.

5. Compare the division algorithm for long division of numbers to one that's algebraic. This should facilitate the understanding of long long division of polynomials: For example:

$275 \div 25 \Rightarrow$

$$\begin{array}{r|l} & 10 + 1 \\ \hline 20+5 & 200 + 70 + 5 \\ & \underline{200 + 50} \\ & \quad 20 + 5 \\ & \underline{20 + 5} \end{array}$$

$2x^2 + 3x - 20 \div x + 4 \Rightarrow$

$$\begin{array}{r|l} & 2x - 5 \\ \hline x+4 & 2x^2 + 3x - 20 \\ & \underline{2x^2 + 8x} \\ & \quad -5x - 20 \\ & \underline{-5x - 20} \end{array}$$

III.

1. Point out that in defining a rational expression, variables in the denominator cannot be replaced by numbers that make the denominator equal to zero.

2. Reduction of rational expressions should be emphasized as the numerator and denominator being divided by the same non-zero polynomial such that an equal expression is obtained in lower terms.

3. Provide many exercises on the concept of reduction of rational expressions, pointing out that rational expressions are in simplest form only when both polynomials of the rational expression have no common factors other than 1 or -1.

4. Use colored chalk or pen to distinguish common errors made in simplifying algebraic expressions, in terms of students crossing out like terms without factoring. Re-emphasize Equivalent Expressions!

5. Provide the student with many exercises on finding the LCD before the addition and subtraction of rational expressions are introduced.

6. Students should be urged to read and re-read problems carefully and try to organize a detailed plan for solving the problem.

7. Have students formulate the plan by using variable(s), and writing a mathematical model for such (equation).

8. Finally, a check is necessary in the original problem to make sure all the conditions are satisfied and that the answer is reasonable.

9. The teacher's working out most problems with the students makes for a better attitude toward verbal problems.

SAMPLE PRETEST ITEMS

I. Perform the indicated operations. Express all answers in simplest form.

1. $x^3 \cdot x^2 =$

2. $(x^2)^3 =$

3. $(x^{-2}) =$

4. $x^3 \div x^4 =$

5. $x^0 =$

6. $(3z - b)(3a + b) =$

7. $(3a - b)^2 =$

8. $(3a - b)(4a + 2b) =$

9. $(3x - 2)(x^2 - 2x + 3) =$

10. $(x^2 - 2x + 1) \div (x + 1) =$

11. $\frac{2}{3}x \cdot \frac{4}{9}x^2 =$

12. $\frac{x - 2}{x^2 - 4} =$

13. $\frac{3}{(2x - 4)} + \frac{2}{(x - 2)} =$

14. $\frac{x}{x^2} - \frac{1}{x} =$

15. $\frac{3}{4}x = -8,\ x =$

16. $(m^2 n^3)^5 =$

Factor:

17. $x^2 - y^2$

18. $3x^4 - 2x^3 - 5x^2$

19. $x^2 - 12x + 35$

20. $16x^2 + 8xy - y^2$

II. Solve the following verbal problems.

1. Mary is three times as old as her brother John. Four years from now Mary will be twice as old as John. How old is each of them now?

2. Four added to five times a number gives a result of 24. Find the number.

KEY TO PRETEST

I.

1. x^5
2. x^6
3. $\frac{1}{x^2}$
4. $\frac{1}{x}$
5. 1
6. $9a^2 - b^2$
7. $9a^2 - 6ab + b^2$
8. $12a^2 + 2ab - 2b^2$
9. $3x^3 - 8x^2 + 13x - 6$
10. $x - 3 + \frac{4}{x+1}$
11. $\frac{8}{27} x^3$
12. $\frac{1}{x+2}$
13. $\frac{7}{2(x-2)}$
14. 0
15. $-\frac{32}{3}$
16. $m^{10}n^{15}$
17. $(x+y)(x-y)$
18. $x^2(3x-5)(x+1)$
19. $(x-7)(x-5)$
20. $(4x+y)^2$

II.

1. Mary 12; John 4
2. 4

SAMPLE POSTTEST ITEMS
(Keyed to Objectives)

I. 1. Simplify each expression.

a. $x^2 . x^3 . x$

b. $-x^{-2} . x^{14}$

c. $x^3(-3x)$

d. $(-y^4)(-y^3)(y^{-7})$

e. $\left(\frac{1}{2}x^3\right)\left(\frac{2}{3}x^2\right)$

2. Simplify each expression.

a. $x^0 . (x^2)^2$

b. $(x^{-4})^3$

c. $(2a^2b)^2$

d. $\frac{1}{4}(m^2n^{-3})^3$

e. $(x^2y^{-3})^{-4}$

3. Divide

a. $2^3 \div 2$

b. $(x^3y^2) \div (xy)$

c. $y^{10} \div y^{-4}$

d. $-4^9 \div 4^9$

e. $17x^6 \div 34x$

4. Multiply.

a. $(5a - 2b)(5a + 2b)$

b. $(2x - 4)^2$

c. $(3n+4)(2n-5)$

d. $(a+4d)^2$

e. $(7+2a)(7-2a)$

5. Factor each polynomial completely.

   a. $25x^2-121$

   b. $a^2+14a+49$

   c. $12a^2-3y^2$

   d. $6-11a-10a^2$

   e. $12x^3+10x^2y-8xy^2$

6. Simplify each expression.

   a. $(x^2+4x+4)\div(x+2)$

   b. $(a^2-9)\div(a+3)$

   c. $(2x^2-x+2)\div(x-2)$

   d. $(x^3+4x^2-25x+12)\div(x-3)$

II. 1. Evaluate each expression for the values given.

   a. $\frac{a^2-b}{a+b}$ $a = 2, b = 1$

   b. $\frac{2x^2-xy}{y^2+4}$ $x = -2, y = 2$

   c. $\frac{3b-3a}{4a^3}$ $a = -1, b = \frac{1}{2}$

2. Indicate the excluded values of the variable for each expression.

   a. $\frac{4}{x+2}$

   b. $\frac{x-1}{x^2-4}$

   c. $\frac{x^2-5}{x^2+x-6}$

3. Write each expression in simplest form.

   a. $\frac{x^2-16}{3x+12}$ b. $\frac{y^2+y-12}{12-4y}$ c. $\frac{2a^2-8b^2}{4a^2-2ab-12b^2}$

4. State whether the pairs of expressions are equivalent or not.

   a. $\frac{5b^2c}{2b}$, $\frac{10b^2c^2}{c}$

   b. $\frac{2a^2}{3a}$, $\frac{4a^2}{6a}$

   c. $\frac{x+2}{x}$, $\frac{y+2}{y}$

5. Multiply. Express the answer in simplest form.

   a. $\frac{8a^4}{b^3} \cdot \frac{b^2}{14a^2}$

   b. $\frac{4x}{x^2-4} \cdot \frac{6x+12}{18x^2}$

   c. $\frac{x^2-y^2}{x^2-4y^2} \cdot \frac{4x-8y}{3x^2+3xy-6y^2}$

6. Divide. Express the answer in simplest form.

   a. $\frac{2xy^2}{3b^2} \div \frac{8x^3y}{9b^3}$

   b. $\frac{2x-2}{x^2-9} \div \frac{4x-4}{x+3}$

   c. $\frac{x^3-5x^2-6x}{3x^2+9x+6} \div \frac{2x^2-14x+12}{x^2+x-2}$

7. Find the LCM of each pair of polynomials.

   a. $(x+1)$, (x^2-1)

   b. $3ab^2$, $2a^2$

   c. (x^2-2x+1), $(4x^2-4)$

8. Perform the indicated operations.

   a. $\frac{5a-7}{2a} - \frac{4a-1}{2a}$

   b. $5 + \frac{2x}{x-1}$

c. $\frac{3}{y-3} + \frac{2y}{y^2-9}$

9. Find the solution set of each equation.

a. $\frac{x}{3} + 2 = 5$

b. $\frac{5}{y} - \frac{19}{2y} = 3$

c. $\frac{x+1}{x-1} = \frac{x-4}{x+2}$

KEY TO POSTTEST

I.

1. a. x^6
   b. $-x^{12}$
   c. $-3x^4$
   d. 1
   e. $\frac{1}{3}x^5$

2. a. x^4
   b. x^{-12}
   c. $4a^4b^2$
   d. $\frac{1}{4}m^6n^{-9}$
   e. $x^{-8}y^{12}$

3. a. 4
   b. x^2y
   c. y^{14}
   d. -1
   3. $\frac{x^5}{2}$

4. a. $25a^2-4b^2$
   b. $4x^2-16x+16$
   c. $6n^2-7n-20$
   d. $a^2+8ad+16d^2$
   e. $49-4a^2$

5. a. $(5x+11)(5x-11)$
   b. $(a+7)^2$
   c. $3(2a+y)(2a-y)$
   d. $(2-5a)(3+2a)$
   e. $2x(2x-y)(3x+4y)$

6. a. $x+2$
   b. $a-3$
   c. $2x+3+\frac{8}{x-2}$
   d. x^2+7x-4

II. 1. a. 1
   b. $\frac{3}{2}$
   c. $\frac{-9}{8}$

2. a. $x \neq -2$
   b. $x \neq \pm 2$
   c. $x \neq 2, -3$

3. a. $\frac{x-4}{3}$
   b. $\frac{-(y+4)}{4}$
   c. $\frac{a+2b}{2a+3}$

4. a. Not equivalent

   b. Equivalent

   c. Not equivalent

5. a. $\frac{4a^2}{7b}$

   b. $\frac{4}{3x(x-2)}$

   c. $\frac{4(x+y)}{3(x+2y)^2}$

6. a. $\frac{3by}{4x^2}$

   b. $\frac{1}{2(x-3)}$

   c. $\frac{x}{6}$

7. a. (x^2-1)

   b. $6a^2b^2$

   c. $4(x+1)(x-1)^2$

8. a. $\frac{a-6}{2a}$

   b. $\frac{7x-5}{x-1}$

   c. $\frac{5y+9}{y^2-9}$

9. a. $\{9\}$

   b. $\left\{\frac{-3}{2}\right\}$

   c. $\left\{\frac{1}{4}\right\}$

BIBLIOGRAPHY

Banks, J. Houston; Sobel, Max A.; and Walsh, William C. Algebra: Its Elements and Structure, Book 1. New York: McGraw Hill Book Company, 1971.

Denholm, Richard A.; Dolciani, Mary P.; and Cunningham, George E.. Elementary Algebra, Part 1. Boston: Houghton Mifflin Company, 1970.

Denholm, Richard A. and Dolciani, Mary P.. Elementary Algebra, Part 2. Boston: Houghton Mifflin Company, 1971.