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ABSTRACT

This guidebook on minimum content for a study of probability and its applications was designed for the student who has mastered second-year algebra. Covered in this booklet are sample spaces; tree diagrams, permutations, combinations, probability concepts and applications; and game theory. The course goals and performance objectives are specified, and a course outline along with suggested sources and strategies are provided. Sample pretest and posttests and an annotated list of 14 references are included. (DT)

ED 079131

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

PROBABILITY

5293.38

MATHEMATICS

SE 016 508

DIVISION OF INSTRUCTION•1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

PROBABILITY

5293.38

(EXPERIMENTAL)

Written by

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for the

DIVISION OF INSTRUCTION
Dade County Public Schools
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PREFACE

The following course of study has been designed to set a <u>minimum standard</u> for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a <u>minimum</u>, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

A study of probability and its applications, including the application to Game Theory and Voting Power.

Designed for the student who has interest in higher mathematics and has mastered the skills and concepts of Algebra 2a, b, c, and d.

TABLE OF CONTENTS

<ins>GOALS</ins>

I. The student will develop an understanding of the background concepts for finite probability and insights into the formal definitions that follow.

II. The student will develop an understanding of the concepts of relative frequency and probability as related to various types of sample spaces. He will analyze these concepts by solving problems dealing with them.

III. The student will apply the concept of probability to problems in game theory, voting power, Markov chains, and/or other topics commensurate with his background.

PERFORMANCE OBJECTIVES

GOAL I.

The student will:

1. Intuitively solve simple problems involving vocabulary including: probability, mathematical model, insurance, experiment, events, frequency, and relative frequency.
2. Define a finite sample space, a sample point, and an event.
3. Establish a model of the sample space for a given experiment and use the model to estimate the probability of events intuitively.
4. Describe the concept of successive choices by tree diagrams and describe counting arrangements using factorial notation.
5. Define Cartesian product and permutation and compute numbers of choices using the resulting formulas.
6. Identify and find the power set, and the number of elements in the power set of a given finite set.
7. Define, synthesize the formula for, and compute the combinations on any finite set.
8. Prove various theorems related to permutations and combinations.
9. State the binomial theorem and apply it to various problems of binomial expansion and approximation of powers.

GOAL II.

The student will:

1. Formally define, identify, and find: elementary events, mutually

exclusive events, complementary events, and the probability of each.

2. Define and identify mathematical certainty and mathematical impossibility.
3. Find the probability of event A on the condition that event B has occurred.
4. Find the probability of the union or intersection of events.
5. Define and identify independent events and develop the formula for the probability of independent events.
6. Define and identify partition sets; develop and prove Bayes' theorem and apply the formula to various problems.
7. Compute the probability of an event A occurring exactly 'x' times during 'n' trials, given the probability of A.

GOAL III.

The following objectives have been written primarily for use with the text Probability and Statistics, by Willoughby, but should not be interpreted as limitations on the goal. Other applications of probability may be used for this section.

The student will:

1. Define two player zero sum games, strategy, best strategy, payoff, and the value of the game.
2. Given a simple matrix or tabular description of a two player zero sum game, find the best strategy for each player, the value of the game, and all equilibrium pairs.

3. Define and find mathematical expectations, and define and identify a fair game.
4. Determine the voting power of each member, for a given voting situation.

SEQUENCE, SOURCES, AND STRATEGIES

The following material is keyed to the Willoughby text with Glicksman as the main resource. None of the other (state adopted) texts was found to contain all the material necessary to sustain a 9-week quinmaster. A key to the text code will be found in the bibliography on pages 19 and 20.

SEQUENCE	SOURCES AND STRATEGIES
I. Introduction A. Two Definitions of Probability B. Applications 1. Insurance 2. Miscellaneous Applications C. Sample Spaces - Mathematical Models	W - pp. 1-17, G - pp. 262-268, N - pp. 99-100, P - pp. 566-568. I - Covers insurance. Supplement assignments with experiments such as tossing tacks, coins or dice, or drawing cards to illustrate rélative frequency and experimental probability.
II. **Counting, Permutations, Combinations, and the Binomial theorem.** A. Successive Choices and Tree Diagrams B. The Fundamental Principle of Counting and Permutations C. Subsets D. Combinations E. The Binomial Theorem	W - pp. 19-41, G - pp. 241-261, D - pp. 599-616, H - pp. 481-499, P - pp. 554-565, S - pp. 783-838. G - very thorough. W - Permutations with elements not all distinct is not mentioned. S - A very thorough introduction to permutations and combinations and development of the formulas.

SEQUENCE	SOURCES AND STRATEGIES
III. Probability A. Basic Definitions 1. Elementary Event (singleton) 2. Mutually Exclusive Events 3. Probability of an Event B. Complementary Events C. Probability of the Union of Events D. Conditional Probability E. Independent Events F. Bayes' Theorem (A - posteriori probability) G. Random Selections	W - pp. 43-67, G - pp. 262-296 (A-F), D - pp. 617-628 (A-E), H - pp. 503-509 (A,C,E, Brief), N - pp. 100-119 (A-E), P - pp. 566-588 (A-E). If there is time, a review of, or introduction to, mathematical induction prior to this unit would give the students a method of proof for many of the theorems. N - A good discussion of independent trials (pp. 119-124) which is lacking in W. (See also: G - pp. 296-303) Try using a tree diagram to illustrate Bayes' Theorem. The text diagrams are very complex.
IV. Game Theory A. Introduction B. Equilibrium Pairs and Pure Strategies. C. Mathematical Expectation D. Mixed Strategies E. Shapley-Shubik Power Index	W - pp. 163-184, G - pp. 303-308 (C only), N - pp. 214-234. N - Includes a discussion of Markov Chains but approaches the topics from the viewpoint of matrices. The arguments are simple enough for the student to follow without matrices if the teacher details matrix multiplication as an algorithm.

SAMPLE PRE-TEST

A. Evaluate each:

1. $5!$ 2. $\frac{7!}{4!\ 3!}$ 3. $\frac{3!}{0!\ 3!}$

4. $(3 - .2)^3$ 5. $\frac{1}{21} + \frac{2}{21} - \frac{4}{7}$ 6. $\frac{3}{4} + \frac{7}{9}$

7. $\frac{4}{5} \div \frac{8}{7}$ 8. $\frac{1\frac{3}{4}}{\frac{4}{5}} + \frac{\frac{3}{11}}{\frac{7}{11}}$

B. Simplify each:

9. $(n - 1) \cdot n \cdot \left[(n - 2)!\right]$ 10. $\frac{m!}{(m - 3)!}$

11. $(3x + y)^4$ 12. $\frac{a}{b} + \frac{a + 1}{b + 1}$

13. $\frac{\frac{x + 3}{2x + 2}}{\frac{x^3 + 27}{x + 1}}$ 14. $\left(\frac{m^2 - n^2}{3m}\right)\left(\frac{4n}{m + n}\right)$

C. Answers each of the following questions:

15. 42% of what number is 25? 16. What is 42% of 25?

17. Three percent of all the students at Ringaling High School enroll in a course in acient Mayan culture. Four and two-thirds per cent of all the students take high-speed sleight-of-hand. How many students take high-speed sleight-of-hand if 63 students take ancient Mayan culture?

Sample Pre-Test Continued:

18. An article which normally sells for twenty dollars is marked up 20 per cent and then reduced 20 per cent. Find the new sale price.

D. Given the following sets: U (the universal set) = $\{-1, 0, 1, 2, 3\}$, $A = \{-1, 0, 1, 3\}$, $B = \{0\}$, $C = \{-1, 1, 2\}$, $D = \{2, 3\}$.

Write a roster for each of the following:

19. $A \cup B$	20. $A \cap C$	21. $D \cup \emptyset$	22. $\overline{A}$
23. $\overline{A \cup C}$	24. $\overline{A} \cup \overline{C}$	25. $B \times D$	26. $D \times D$

ANSWERS TO THE PRE-TEST:

A.

1. 120	2. 35	3. 1	4. 21.952
5. $-\frac{3}{7}$	6. $\frac{55}{36}$	7. $\frac{7}{10}$	8. $\frac{293}{112}$

B.

9. $n!$ 10. $m^3 - 3m^2 + 2m$ and $m \geq 3$ and $m \in$ {all integers}

11. $81x^4 + 36x^3y + 54x^2y^2 + 12xy^3 + y^4$

12. $\frac{a + 2ab + b}{b^2 + b}$ 13. $\frac{1}{2x^2 - 6x + 18}$ and $x \notin \{-1, -3\}$

14. $\frac{4mn - 4n}{3m}$ and $m + n \neq 0$

Answers to Sample Pre-Test Continued:

C.

15. $\frac{1250}{21} = 5\frac{20}{21}$

16. $\frac{21}{2} = 10\frac{1}{2}$

17. 98 students

18. \$19.20

D.

19. $A = \{-1, 0, 1, 3\}$

20. $\{-1, 1\}$

21. $\{D = 2, 3\}$

22. $\{2\}$

23. $\emptyset$

24. $\{0, 2, 3\}$

25. $\{(0, 2), (0, 3)\}$

26. $\{(2, 2), (2, 3), (3, 2), (3, 3)\}$

SAMPLE POST-TEST

GENERAL DIRECTIONS: There is exactly one best answer to each of the problems below except for the proof in problem 30.
Numbers in parenthesis preceeding the test item indicate the objective or objectives that are being tested by that question.

(I-3) 1. If a coin is tossed, the probability of it landing head up is most likely (greater than/equal to/less than) the probability of a thumbtack landing head up (point down).

(I-2, I-3) 2. The number of ways that a heart can be drawn from a bridge deck in a single draw is ___. The probability of drawing an ace of hearts in a single draw from a fair deck is ___.

(3-6) A fair coin has been tossed ten times with the following results: H, H, H, T, T, H, T, H, H, H

(I-3) 3. The probability of heads on the eleventh throw is (more than/equal to/less than) if it were on the first throw.

(I-1) 4. The frequency of heads in this experiment is ___.

(I-1) 5. The relative frequency of heads in this experiment is ___.

(I-1, I-3) 6. If the coin is tossed 100,000 times, you would expect the relative frequency of heads to be (more than/equal to/less than) it is in this experiment.

Sample Post-Test Continued:

(7-8) From a set of colored cards, we select one at random: a pink, P, or a blue, B, and turn it over revealing a number: 1, 2, or 3 printed on the reverse side. Five-hundred trials on this experiment produced the following results:

	1	2	3
P	75	145	50
B	125	65	40

(I-1, I-5) 7. Find the frequency of (a) pink cards printed 1; (b) blue cards.

(I-1) 8. Find the sum of the relative frequencies.

(9-11) If one die of a pair is normal, but the other has two faces printed three and none printed five;

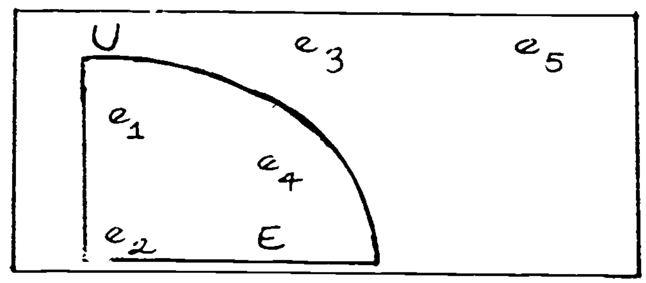
(I-3, I-4, I-5) 9. Draw a table to describe the sample space.

(I-3) 10. Find the probability of rolling a seven with the pair if each face of each die is equally likely.

(I-3) 11. Find the probability of rolling four (total) with these dice if each face of each die is equally likely.

Sample Post-Test Continued:

(12-15) If the 'e' in the diagram represents elements of sample space U, and E is an event in U as shown:



(II-1) 12. If the e_i are equally likely, find the probability of the complement of E, $P(\bar{E})$.

(II-1, II-2, II-4) 13. Why is it impossible for $P(e_1) = 0.1$, $P(e_2) = 0.2$, $P(e_3) = 0.3$, $P(e_4) = 0.4$, and $P(e_5) = 0.5$?

(II-1, II-4) 14. If $P(e_1) = 0.1$, $P(e_2) = 0.2$, $P(e_3) = 0.3$, and $P(e_4) = 0.4$;

(a) Find $P(e_5)$ (b) Find $P(E)$

(II-2, II-4) 15. Find $P(E \cap \bar{E})$

(16-23) A bag is chosen at random and a marble selected at random from the chosen bag. Bag M has 6 red, 2 white, and 2 blue marbles, while bag N has 2 red, 1 white, and 2 blue marbles.

(I-4) 16. Draw a tree diagram to describe the sample space.

Sample Post-Test Continued:

17. Find the probability that a red marble will be drawn, given that bag M has been selected. (P (R/M).

(II-4) 18. Find $P(R \cap M)$.

(II-1) 19. Find P(B).

(II-3) 20. Find P(W/M).

(II-5) 21. Events M and ____ are independent.

(II-1) 22. Events M and ____ are mutually exclusive.

(II-6) 23. If it is known that a blue marble was selected, what is the probability that it was drawn from bag M?

(24-25) The probability of Place-hitter Palmer getting a hit is 0.300. If his times at bat form independent trials, find:

(I-9, II-7) 24. The probability of his getting exactly three hits in four times at bat.

(I-9, II-7, II-4) 25. The probability that he will get at least three hits in four times at bat.

(26-27) In a zero sum two player game, each player has three strategies. Player A's strategies are given as A_1, A_2, and A_3 on the table below; thus, for the pair of strategies (A_1, B_3), B pays A two units.

(III-1) 26. Find A's best strategy.

(III-2) 27. List all equilibrium pairs, if there are any. If there are none, write "none".

Sample Post-Test Continued:

	B_1	B_2	B_3
A_1	0	1	2
A_2	-1	4	0
A_3	0	2	-2

(III-3) 28. On a one dollar bet in a game of roulette, a player wins one dollar, loses one dollar, or ties and "breaks even". The probability of winning is $\frac{18}{38}$, the probabilit of losing is $\frac{19}{38}$, and the probability of tying is $\frac{1}{38}$.

Find the player's mathematical expectation.

(II-4, II-5) 29. If A and B are events, write the formula for the probability of (a) either A or B, (b) both A and B.

(II-4, II-5) 30. Given:

$$P(A/B) = P(A).$$

Prove:

$$P(B/A) = P(B)$$

POST-TEST ANSWER KEY

1. greater than
2. 13, $\frac{1}{52}$
3. equal to
4. 7
5. $\frac{7}{10}$
6. less than
7. (a) 75, (b) 230
8. one
9.
10. $\frac{1}{6}$
11. $\frac{1}{9}$
12. $\frac{2}{5}$
13. $\sum_{i=1}^{5} P(e_i) > 1$
14. (a) 0, (b) 0.7
15. 0
16.
17. $\frac{3}{5}$
18. $\frac{3}{10}$
19. $\frac{3}{10}$
20. $\frac{1}{5}$
21. W
22. N
23. $\frac{1}{3}$
24. ${}_4C_3\ (.3)^3\ (.7) = .0756$
25. ${}_4C_4\ (.3)^4\ (.7)^0 + {}_4C_3\ (.3)^3\ (.7)^1 = .0837$
26. A_1
27. (A_1, B_1)
28. $\frac{-1}{38}$
29. (a) $P(A \cup B) = P(A) + P(B) - P(A/B)$
30. (b) $P(A \cap B) = P(A/B)\ P(B)$ or

    $P(B/A)\ P(A)$
31.
    1. $P(A/B) = P(A)$
    2. $P(A/B) = \frac{P(A \cap B)}{P(B)}$
    3. $\therefore\ P(A) = \frac{P(A \cap B)}{P(B)}$
    4. $P(A)\ P(B) = P(A \cap B)$
    5. But: $P(A \cap B) = P(B \cap A)$

Sample Post-Test Answer Key Continued:

6. $\therefore P(A)\,P(B) = P(B \cap A)$

7. $P(B) = \frac{P(B \cap A)}{P(A)}$

8. But: $P(B/A) = \frac{P(B \cap A)}{P(A)}$

9. $\therefore P(B/A) = P(B)$

REFERENCES

STATE ADOPTED TEXTS

Listed in Sources as:

D — Dolciana, Mary P.; Wooton, William; Beckenbach, Edwin F.; and Sharron, Sidney. Algebra 2 and Trigonometry. Modern Mathematics Series. Boston: Houghton Mifflin Co., 1968. pp. 599-629. Good illustrations, good source for problems.

H — Herberg, Theodore; and Bristol, James D. Elementary Mathematical Analysis. 2d ed. Boston: D.C. Heath and Co., 1967. pp. 481-512. Very brief but some problems are useful.

N — Nahikian, Howard M. Topics in Modern Mathematics. London: Macmillan Co., 1966. pp. 99-124 and pp. 214-234. A good reference book for problems. Might work as a student text if supplemented for permutations and combinations. Game Theory uses matrices.

P — Payne, Joseph N.; Zamboni, Floyd F.; and Lankford, Francis G., Jr. Algebra Two with Trigonometry. Harbrace Mathematics Series. New York: Harcourt, Brace and World, Inc., 1969. pp. 553-558. Readable to students.

W — Willoughby, Stephen S. Probability and Statistics. Morristown, N. J.: Silver Burdett Co., 1968. pp. 1-67 and pp. 163-184. Most complete reference, suggested for use as the student text.

OTHER BOOKS

G — Glicksman, Abraham M.; and Ruderman, Harry D. Fundamentals for Advanced Mathematics. New York: Holt, Rinehart, and Winston, Inc., 1964. pp. 241-311. Excellent resource, suggested as a secondary text for students if available in quantity. Contains many advanced level problems.

References Continued:

Listed in
Sources as: Other Books Continued:

S — School Mathematics Study Group. Intermediate Mathematics Part II. Stanford, Calif.: A.C. Vroman, Inc., 367 Pasadena Avenue, Pasadena, Calif., 1960. pp. 783-941. Nothing on probability, but may serve as an introduction to permutations and combinations to supplement Nahikian.

PAMPHLETS

I — Sets, Probability, and Statistics - The Mathematics of Life Insurance. pp. 17-35. These booklets are available at no cost from Educational Division, Institute of Life Insurance, 277 Park Avenue, New York, N. Y. 10017. Free teacher's manual is sent with each class set.

OTHER SOURCES

- Earl, Boyd; Moore, J. William; and Smith, Wendell I. Introduction to Probability. New York: McGraw-Hill Book Co., Inc., 1963. Good for problems, tests, and make-up or remedial work for students. Programmed text.

- Meserve, Bruce E.; Pettofrezzo, Anthony J.; and Meserve, Dorothy T. Principles of Advanced Mathematics. Syracuse: The L. W. Singer Co., (a Division of Random House, Inc.) 1964. pp. 45-96. Very good on permutations with special attention to circular and ring permutations and arrangements with repetitious elements.

- Mosteller, Frederick; Rourke, Robert E. K.; and Thomas, George B. Probability with Statistical Applications. Reading, Mass. Addison-Wesley Publishing Co., Inc., 1961. pp. 1-275. Excellent for details of a rigorous approach. Many problems with statistical approach, material far beyond the intentions of this course. Binomial experiments and successive trials very good.

- Shanks, Merrill E.; Brumfiel, Charles F.; Fleenor, Charles R.; and Eicholz, Robert E. Pre-Calculus Mathematics. Reading, Mass. Addison, Wesley Publishing Co., 1965. Excellent for accellerated students; theoretical and indepth.

- Vannatta, Glen D.; Carnahan, Walter H.; and Fawcett, Harold P. Advanced High School Mathematics, Expanded Edition. Columbus, Ohio: Charles E. Merrill Books, Inc., 1956. Easy to read. Lots of good examples; good for clarification.

- Western, Donald W. and Haag, Vincent H. An Introducation to Mathematics. New York: Henry Holt and Co., Inc., 1959. pp. 214-245. A very modern approach. Treats probability as a function. Good for permutations, combinations, binomial experiments, and repeated trials.