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ABSTRACT

This study identified and defined 30 structural variables related to problem solving. Four dependent variables (only one, percent correct, is analyzed in this report) were derived from data retained by the computer from 172 arithmetic word problems presented to and solved by junior college students on IBM 2741 computer terminals. The structural and dependent variables were combined into a stepwise regression analysis in order to predict problem solving difficulty. The order of importance of the 30 structural variables was determined, and the goodness of fit of the model was analyzed by using F and Chi-square tests. Results showed that the variable "memory 2" (recall) was highly significant for predicting the difficulty of word problems and accounted for 47 percent of the variance. (DT)

# USING STRUCTURAL VARIABLES TO PREDICT WORD - PROBLEM SOLVING DIFFICULTY FOR JUNIOR COLLEGE ARITHMETIC STUDENTS



## - THE CAPS PROGRAM AT THE COAST COMMUNITY COLLEGE DISTRICT -

BY

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# PREFACE

This paper represents a larger study being conducted by the author at Golden West College, Huntington Beach, California. Since several parts here were used from almost directly that study, the present paper is rather sizable. The reader who wishes to obtain the essence of the paper is invited to skip to Chapter III, page 29. Furthermore, Appendices 2 and 3 are related to the study in that they describe some of the facilities at the Coast Community College District that motivated the study. The reader interested in CAI, and CAI related research will find the materials in these two Appendices quite interesting. Presently, there are more than 600 CAI units of instruction available to students at Golden West College and Orange Coast College, in the Coast District. Inquiries should be directed to the author, at Golden West College, or to Dr. Bernard Luskin, Office of Educational Development, Coast Community College District, Costa Mesa, California.

## CHAPTER I

## INTRODUCTION

Problem solving has fascinated research workers for a long time. Volumes have been written on the subject. Formal theories and rules of thumb have been put forth. Yet, our understanding of the process is still not very clear. And, in an increasingly complex society such as ours, the understanding of the problem solving process is of tantamount importance to the education of the young. The present study reflects an effort to contribute to such understanding.

This study was concerned with one facet of problem solving in mathematics education: word-problem solving difficulty. Past research in this area was divided into three broad categories based on the type of predictor variables used: (a) student variables, (b) presentation variables, and (c) structural variables. This study was of the third type.

In structural variables studies we assume the existence of factors in the structure of arithmetic word problems that contribute to the difficulty level of the problems. This leads to the identification, classification, and definition of a set of factors hereby called structural variables:

$$X = \{ x_1, x_2, x_3, \cdots, x_K \}$$

where each $x_i$ represents a particular structural characteristic of

a verbal arithmetic problem. Given n problems, each problem has a set of structural variables, thus we may rewrite the set X as:

$$X_i = \{x_{i1}, x_{i2}, \cdots, x_{in}\} \quad i = 1, 2, \ldots, m$$

Examples of structural variables are the number of words in the problem, the noun to verb ratio, and the number of different operations required to achieve a solution.

Once the set X has been identified and defined, each element $x_j$ of X, is paired with a number $a_j$, an element of the set $A = \{a_0, a_1, \cdots, a_n\}$ in such a way that $a_j$ reflects the relative importance of $x_j$ in predicting problem solving difficulty. Thus, if we hold all other variables constant, a unit increase in $x_j$ will cause a relative increase of $a_j$ units in problem difficulty. The set A can be derived by any number of statistical regression techniques which fall under the category of regression. In this study, the set A is derived by linear regression--in particular, stepwise linear regression.

Suppes, Hyman, and Jerman (1966) first introduced linear regression as a model for predicting difficulty in arithmetic problems. Subsequently, Suppes, Loftus, and Jerman (1969), Loftus (1969), and Jerman (1971) extended this model to arithmetic <u>word</u> problems. The present study followed this path closely, adding some innovations. A major extention planned in a more comprehensive study, of which this paper is only a part, is the application of canonical correlation and discriminant analysis to the response data.

## STATEMENT OF THE PROBLEM

This study identified and defined 30 structural variables. Four dependent variables (only one will be analyzed here) were derived from

data trapped by the computer from 172 arithmetic word problems presented to and solved by junior college students on IBM 2741 computer terminals. The structural and dependent variables were combined into a stepwise regression analysis in order to predict problem solving difficulty.

The objective of this study was to:

a. determine a set of weighs to be assigned to set X, and to determine the order of importance of the elements in X;

b. analyze the goodness of the fit of the model by using F and Chi-squared tests.

## SIGNIFICANCE OF STUDY

In California, as well as in many other states, the junior college typifies the ideal in American education--free education for all. The result of this precept of the junior college, known as the open door policy is a widely diversified student population. The most outstanding diversity is the academic background of the student population.

The mathematics curriculum at the junior college reflects this diversity by offering courses which range from sixth-grade arithmetic to fourth semester calculus; from mathematics for prospective elementary school teachers to probability and statistics; and from "tech math" to computer programming and elementary numerical analysis.

These students, indeed, have individual differences. Besides the "usual" individual differences they share with their university peers

(such as I.Q., attitude, and socio-economic background) junior college students differ in past and present academic performance. Whereas one student does B work in second semester calculus, another struggles through grade-school arithmetic--having difficulty with problems such as: "27 is what percent of 30?"

This study dealt with the latter type of students; for of all the subpopulations, as divided by the junior college mathematics curriculum, those enrolled in the arithmetic course are perhaps the most pedagogically curious. This set of students is separated from the rest by a score of four of less on the mathematics part of the SCAT*. Such a score implies that after twelve years of formal schooling these students cannot perform at a sixth-grade level of mathematics, or, at the very best, seventh-grade level mathematics.

In the Coast Community College District, where this study was conducted, and at other junior colleges, a minimum proficiency in arithmetic (SCAT score 4 at Coast) is required for the attainment of the Associate of Arts Degree. Thus, these students are forced through yet another try at arithmetic--an experience which is usually relished neither by them, nor by even the most concerned and conscientious of mathematics faculties. This problem, well known to secondary school and junior college mathematics educators, has recently come under scrutiny of the Mathematical Association of America, an organization of professional mathemticians heretofore in-

* School and College Achievement Test. The norms used for arriving at the cut-off score of four are local. Junior college students at Orange Coast College, Costa Mesa, California, were used to establish these norms. For national norms a score of four is equivalent to the 23 percentile. SCAT scores range from 0 to 10.

terested solely in the college and college preparatory mathematics curriculum. In a recent publication of the MAA's Committee of the Undergraduate Program in Mathematics (CUPM)* a panel of mathematicians described the problem as follows:

> Many students in basic mathematics courses have seen this subject matter in elementary and high school without apparent success in learning it there. It is often the case that a second exposure to essentially the same material similarly organized, is no more successful even though an attempt is sometimes made to present the subject in a more "modern" manner.
>
> There are certainly many complex reasons for this state of affairs. Some of these may be psychological and sociological and may require the work of learning theorists and others trained in the social sciences in order to lessen their influence... Nonetheless, we believe that it deserves very serious consideration by the mathematical community and hope that many different kinds of institutions will find our suggestions, wholly or in part, of good use when dealing with the type of students described.

The committee's reference to a "modern" manner is obviously directed to the "new math" movement, which has not been as successful with low achievers in mathematics as it has with the more mathematically talented students. However, the committee's jab at novelty of presentation goes beyond the modern math movement. For instance, much effort has recently been placed in multimedia approaches to learning. Schools are presenting segments and whole courses through such media as video-tapes, audio-tapes, sound-on-slides, computer-assisted instruction, and combinations of these. The guiding principle behind this effort is the individualization of instruction--a striving toward that seemingly magical, Socratic ratio of one--to-one: one student to one teacher. Though it is logical for educators to use the fruits of technology to try to improve learning, it is not logical

* Boas, Ralph (Ed.), "A Course in Basic Mathematics for Colleges," CUPM, 1971.

that they expect learning will indeed be improved. Unfortunately, educators do not, or do not want to recognize this simple fact; topics students find difficult in the classroom environment are the same ones they find difficult in the environment of other media (Fey, 1969).

The apparent failure of these innovative methods to improve instruction significantly cannot be blamed on any single characteristic. But one fault in the system presently is that the curriculum has not been properly adapted to the media. If an innovator uses the standard textbook curriculum in the new, innovative media, he cannot really expect improvement of instruction.

Groen and Atkinson (1966) called for such adaptations by offering, among others, a Dynamic Programming approach that may be used to predict the optimum time for the presentation of a new topic during a CAI segment of instruction. Suppes, Loftus and Jerman (1969), by virtue of conducting problem solving experiments in a CAI mode, also called for the adaptation of curricula to media. The fact that such approaches may tend to be readily labelled as esoteric by "practical" educators does not deny the fact that their use is sorely needed.

Three factors contribute to the significance of the present investigation. First, a pedagogical problem exists in the junior college mathematics program. Second, the writer has found that for the students involved, problem solving in the form of word problems is a difficult and frustrating topic. No effort known to this writer has been made to study problem-solving difficulty at the junior college level using the structural variables approach. Third, greater emphasis is being placed today on computer-assisted instruction (CAI) as exemplified by the Coast Community College District where this study will be conducted.

LIMITATIONS OF THE STUDY

This study was conducted totally in the computer-assisted instruction (CAI) mode. Subjects were presented the problems for solution via IBM 2741 computer terminals. Responses, as well as computations, were made only through the terminals, thus, subjects' computational skills were controlled heavily. By the same token, errors unique to CAI were introduced. The most common errors were typographical, and even though they usually did not affect the correctness of the solution they did increase student's latency of response.

The subjects included the entire (and only) arithmetic class conducted during the Fall semester of 1971 at Golden West College, Huntington Beach, California. Students enrolled in this class scored lower than four on the SCAT test and were required to take the class in order to achieve a minimum proficiency level in arithmetic--compulsory for the Associate of Arts degree.

This was a singular group, and randomization of subjects within this group would, of course, have made for a "tighter" experiment. Unfortunately, the small number of students starting the course (62) did not allow for this procedure. However, randomization of subjects was not as crucial to this study as it might be for other experiments.

The word problems in this study were limited to a level of sophistication not greater than ninth grade arithmetic The crucial limitations lie here, with the problem set; its make-up, selection, and presentation. Ideally, the experimenter should be able to present problems to subjects in such a way that variables increase in a uniform manner. This suggests that variables need to be analyzed individually. For example, to

test the effects of the variable which measures the verb to noun ratio the experimenter ought to have a set of problems available that steadily increases the ratio along a continuum while holding the other structural variables constant--an impossible task since the variables are not independent. An increase in the verb-noun ratio forces an increase in the number of words in the problem.

In view of this dilemma, and to avoid dealing with an enormously large set of problems, the 172 problems were randomized for presentation to subjects. In constructing the problems, the writer tried to allow the variables to take on diversified values in their domain of definition.

## CHAPTER II

## REVIEW OF THE LITERATURE

The set X of variables used in the present study reflects the results of these studies by including either the same variables thereof proved to be important or modified versions and combination of those variables. In this chapter the names of these variables will be capitalized and underlined.

Steffe (1967) explored the effect of using existential quantifiers at the beginning of problems, and the use of common versus different names for the elements of sets under consideration for problem solving presented to ninety first graders. He found only the latter variable statistically significant in affecting the correctness of response. For example, a word problem which requires arithmetic operations to be performed with feet and inches is considerably more difficult to solve than one which involves only inches or only feet. The implication here is that a <u>CONVERSION</u> variable should be included in the set X.

A superficial, yet logical variable to consider is the length (number of words) of a verbal problem. One can assume that longer problems in arithmetic are more difficult than short ones. Kilpatrick (1960) verified this assumption and found that the length of a problem was more significant in predicting difficulty in problem solving than vocabulary or length of sentences. Further evidence of the importance of this variable is given in

language studies by Braun-Lamesh (1962), Ervin (1964), and Miller and Ervin (1963). A natural extension of this variable as used in this study is LENGTH as measured by the number of letters in a word problem. Future studies might also include the number of syllables as an indication of length. To use all three, or even two of these variables in the same study would be redundant since their correlation is very high.

Burns and Yonally (1964) have shown that, for fifth graders, the sequence, or ORDER in which the data appears in a verbal problem is important. Problems where the data appeared in the problem in the same order as they were needed to achieve the correct solution were found to be significantly easier to solve than problems where the data did not appear in the needed sequence. Rosenthal and Resnick (1971) concurred with and strengthened the above results by showing that when data appears in the reverse order as needed for solution, the problem achieves maximum difficulty.

The VERBAL CLUE variable, and clue variables in general have had strong support from structural variables researchers. Clues in mathematical word problems seem to be a favorite topic for research. These studies offer diversified results, however, as a result of various types of clues being employed in the presentation of word problems for solution.

Wright (1968) found strong support for verbal clues in arithmetic word problems for 382 fifth-grade children. Both correct answers and correct process in the pursuit of an answer were heavily affected by the presence or absence of verbal clues. Early (1967), as well, found similar results with 296 sixth-grade pupils. The subjects, when considered as a whole, performed better in selecting correct answers and correct procedures in

solving the word problems that contained verbal clues; but in some subgroups, e.g. high performers in ability tests and those of average computational ability, word clues did not improve performance. Early found, however, that the most significant differences occurred for children of lower ability.

In related research, Lee (1971) showed that "visually presented cues are effective for the acquisition of a rule rather than for transfer." Visual clues combined with a weight clue allowed fourth-graders to acquire the concept of the coefficient rule (a F = S) of a linear function significantly faster. However, clues did not help on the learning of the total linear function (aF + C = S).

Sherrill (1970), also on visual clues, found that the "effect of presenting an accurate pictorial representation as an aid in a testing situation was so strong that the low grade average group...scored higher than any grade average group..."

These convincing results are not without blemish. For verbal clues especially, the fly in the ointment is controversy. What constitutes a verbal clue? Arriving at a sound definition of a verbal clue is difficult, since there are many exceptions to most definitions. One way around the problem is to list the set of terms which are considered verbal clues. Jerman (1971), and Suppes, Loftus, and Jerman (1969), did precisely this and the practice has been adopted in this study as well.

Pursuing the verbal clue variable further leads to another variable. One soon discovers that words used as clues, and even those commonly regarded as strong clues, e.g. the words "sum" and "total" can be misleading at times.

Consider the problem statement: "The sum of George's grades is 326. What grade must he receive on his next test to bring his total to 400?" Here the words "sum" and "total" imply addition yet the problem is solved by subtraction.

Suydam and Weaver (1970) warned researcher of the possibility of "grossly misleading" clues. A structural variable that will take up this slack is introduced and called the DISTRACTOR variable.

The difficulty of mathematics word problems is as much a function of linguistic complexity as it is of any single other variable. By its very nature, a word problem is a problem in communication. Students who solve a problem unsuccessfully often do not know what is wanted. Upon seeing the correct solution, typically these students will remark: "Is that what he wanted?"

Has the problem writer failed to communicate, or has the problem solver failed to receive properly? The answer to this question must certainly be complex--yet it seems logical to assume that a failure in communication is most likely to occur if a problem is linguistically complex rather that simple.

Research seems to bear this out. Linville (1969) found that scores of 408 fourth graders on verbal arithmetic problems were significantly higher on test items with easy vocabulary and/ or easy syntax. He also found that high intelligence and high reading achievers scored better that low intelligence and low reading achievers. This suggests that the breakdown in communication lies on the side of the problem solver; i.e. successful problem solvers seem to cut through the "noise" and get to the problem situation more easily than poor problem solvers. Assuming, of course, that if the

subjects knows what is wanted he can get the correct answer.

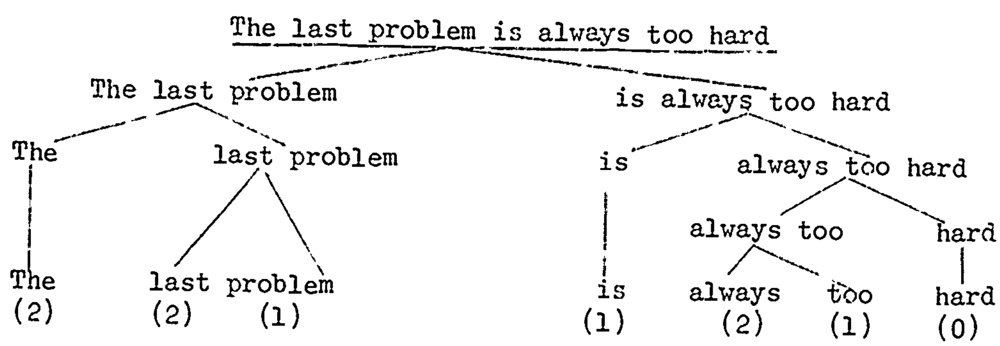
Further evidence on the importance of reading ability to problem solving in mathematics was offered by Cottrell (1967). He found arithmetic achievement and reading achievement to have a correlation coefficient of .860. Harvin and Gilchrist (1970), as well, found positive relationships between problem solving in arithmetic and reading. They concluded, rather interestingly, that arithmetic teachers should teach certain reading skills as well as arithmetic skills.

From the structural variables point of view the researcher would like to know what it is about language that makes students stumble when reading a word problem.

The analysis of language and its complexity is the endeavor of the fields of Linguistics and Psycholinguistics. These prolific researchers have already produced meaningful results.

From these researchers the DEPTH variable is drawn. Yngve (1960) proposed a model for sentence analysis which breaks a sentence down into its constituent parts by using a binary rewrite rule. A sentence is thus diagrammed as a binary "tree". The starting node represents the entire sentence while ending nodes represent each word in the sentence.

For example, the sentence "The last problem is always too hard" can be broken down as follows:



The numbers beneath each word are called "Yngve numbers" and indicate the depth of imbeddedness of the word in that sentence. This depth is measured by the number of nodes (sentence constituents) the reader must keep in his short-term memory when he is considering that word. In the example, for the word "last", the reader must use his short-term memory to recall that "last" preceded (and pertained to) the noun "problem" and the verb phrase "is always too hard," thus "last" has a depth of two. Conversely, upon focusing his attention on the word "last", the reader (or listener) has come to expect (because of our language habits) an noun and a predicate to follow. Martin and Roberts (1966), have shown using Yngve numbers, that any variations from these unwritten constraints "significantly affects sentence retention in the free learning situation."

As a rule of thumb, the Yngve number of a word in a sentence can be derived easily by counting the number of left branches in the binary tree beginning with the uppermost node and ending at the word.

Yngve applied his model to algebraic sentences quite successfully. This success was due to the consistency of mathematical language. When he extended it to the English language, however, the model turned out to be less than perfect--because of the well known inconsistency of the English language.

With this limitation noted, researcher use the Yngve "metric" for measuring the linguistic complexity of the sentences of a word problem as did Loftus (1969), who pioneered the use of Yngve numbers in problem solving research. Some measure of sentence complexity is bettei than none. To shore up the Yngve model with some statistical sandbags the experimenter and

Miss Alice Anderton, a graduate student in Linguistics at UCLA independently assigned Yngve numbers to the word problems used in the study. The correlation coefficient between the two vectors was .76.

In language studies there is an abundance of interesting experimental results as yet untapped by structural variables studies researchers. Consider, for example, measuring the "richness" of a sentence (Bruno, 1972) by using the ratio of the number of adjectives to the number of nouns. By reversing this variable, we get the NOUNS TO ADJECTIVES ratio used in this study. Inherently, a sentence with a high nouns to adjectives ratio is rather barren of description, and, therefore, harder to understand. No studies are offered to support or dispute this conjecture because apparently none have been conducted in mathematics problem solving. This type of variable should not only be used, but extended. Bruno (1972), in analyzing the poetic style of German poetry used other linguistic structural variables as well as the adjective to noun ratio. Some of these have been adopted in this study. Among these are the adverb to noun ratio, NOUN TO ADVERB; the number of verbs, VERBS; and the number of adverbs, ADVERB.

The role MEMORY plays in problem solving and learning in general has been explored rather thoroughly. Not all results have shown significant correlation between memory and problem solving. However, there seems to be some relationship between the two--especially between short-term memory (STM) and problem solving (Sieber, Kameya, and Paulson, 1969; Whimbey, 1968, 1970; Weir, 1965). Interviews with experienced mathematics teachers have substantiated this relationship. These teachers agreed that students able to recall a formula by memory generally have an advantage, during problem solving

and class discussion, over the ones who cannot recall the formula.

Returning now to the memory variable, one can include memory as a variable in set X being careful to define it in a structural variable sense not as a subject-oriented variable.

Several structural variables studies which analyzed problem solving difficulty for problems presented and solved in computer-assisted instruction mode (CAI) were conducted at Stanford University by Professor Patrick Suppes and associates.

In Suppes, Hyman, and Jerman (1966), and Suppes, Jerman and Brian (1968), the authors developed and appl ed the theory of multiple linear regression to the problem of predicting problem-solving difficulty and latency of response. Several analyses were conducted on data trapped by the computer on CAI drill-and-practice exercises presented to 270 subjects composed of third, fourth, fifth, and sixth graders. The data analyzed were from problems in addition, subtraction, and multiplication.

The structural variables used in the analysis of addition problems were MAGSUM (the magnitude of the sum), MAGSMALL (the magnitude of the smallest addend), and NSTEPS, itself the sum of the three sub-variables: TRANSFORMATIONS, OPERATIONS, and MEMORY. A detailed explanation of the relationships between these variables and their definitions is not in place here. Enough be said that a specific algorithm, well explained in the study, was used to obtain the values of the subvariables, and thus, the value of NSTEPS. The analysis of subtraction problems involved three variables: MAGDIF (magnitude of the difference), MAGSUM (magnitude of the subtrahend) and NSTEPS (same as in the addition problems).

In both analyses NSTEPS was by far the most significant variable for predicting percent of errors and success latency. The ranges of $R^2$ for the various populations were .16 to .74 for the proportion of errors and .19 to .74 for success latency. Thus in many cases the model, using only three variables, explained problem solving difficuly for addition and subtraction problems quite well.

In two analyses on multiplication problems the variables used were LARGER (magnitude of the larger factor), SMALLER (magnitude of the smaller factor), and three zero-one variables reflecting the mutually exclusive forms: a x b = ___, a x ___ = c, or ___ x b = c of the multiplication problem (these 0, 1 variables were used only in the first multiplication analysis). The most important variables were found to be, respectively: the zero-one variable reflecting the form a x b = ___, and SMALLER. Consequently, multiplication problems of the type a x b = ___ are more difficult (for fourth graders at least) than the other two forms; and, given two problems of this form, the one with the larger smaller factor will tend to have a longer response time. For example, 3 x 7 = ___, according to Suppes, is more difficult than 2 x 8 = ___.

In reaching for a finer grained analysis, the experimenters broke up the NSTEPS variable into its three component sub-variables: TRANSFORMATIONS, OPERATIONS, and MEMORY, and applied the regression equation to 80 fourth-grade addition problems. The order of importance of these variables was MEMORY, TRANSFORMATIONS, and OPERATIONS. Further refinement brought the MEMORY variable together with $O_1$ (the number of addition operations), and $O_2$ (the number of subtraction operations) in the analysis of 19 more problems. Results

showed the MEMORY variable again was more important than the other variables. These results support the earlier remarks on the MEMORY variable (page ) and have led the researcher to adopt this variable in this study.

These pioneering studies by Professor Suppes and his associates have had a doctrinally, far-reaching effect on subsequent structural variables investigations. In addition to the theory, structural variables studies concerned with the analysis of mathematics word problems have adopted most of the variables. The implications for the present investigation was to adopt a modified form of the original variables. These are: STEPS, MEMORY, NUMBER TYPE, and OPERATIONS.

In presenting the model under discussion at the Conference on Needed Research in Mathematics Education, held on the campus of the University of Georgia, Suppes (1967), attracted comments from three leading researchers: Ralph T. Heimer, Jack E. Forbes, and Joseph M. Scandura. The interested reader should consult the report of the conference in order to place the linear regression model as applied to structural variables studies in its proper perspective in the field of research in mathematics education.

The first structural variables study to analyze data accumulated from mathematics word problems presented and solved in computer-assisted instruction mode was conducted as Stanford University (Suppes, Loftus, and Jerman, 1969).

Twenty-seven above average fifth graders solved 68 word problems of sixth grade difficulty on ten commercially available teletype machines connected by private telephone wires to a PDP-1 computer at the Institute for Mathematical Studies in the Social Sciences.

The intent of the experiment was to extend the previously defined, and herein discussed, linear regression model (Suppes, 1966), to the realm of word problems in an effort to single out factors that contribute to the difficulty of such problems.

The variables used were of two types: "zero-one" variables and "continuous" variables. The first set contained the variables: SEQUENTIAL (the problem could or could not be solved by exactly the same steps and operations as the problem immediately preceding it), VERBAL CLUE (the problem did or did not contain all the verbal clues corresponding to the required operations), and CONVERSION (the solution did or did not call for the conversion of units). The second set contained the variables OPERATIONS (the minimum number of different operations* required to achieve the correct solution), STEPS (the minimum number of steps required to achieve the correct solution), and LENGTH (the number of words in a problem).

The multiple linear regression equation obtained was:

$$Z_i = -7.36 + .87X_{i1} + .18X_{i2} + .02X_{i3} + 2.13X_{i4} + .26X_{i5} + 1.42X_{i6}$$

* The value of this variable can be misleading. Consider the problem: "To have passed his history class, Ralph needed at least 300 points. By how many points did he miss passing if his scores were 78, 56, 44, and 89?" By definition, the value of the OPERATIONS variable is 1 (300 - 78 - 56 - 44 - 89). However, the more "usual" solution requires two operations: 300 - (78 + 56 + 44 + 89). Suppes comments on this point but does not state which value he would use: 1 or 2 (see page for the resolution of this dilemma in this study).

with three significant variables: SEQUENTIAL = $x_4$, CONVERSION = $x_6$, and OPERATIONS = $x_1$, in their respective order of importance. The variables STEPS = $x_2$, LENGTH = $x_3$, and VERBAL CLUE = $x_5$ did not contribute significantly to $R^2$. The model explained about 45% of the variance (R = .67) and had a Chi-squared value of 555.76, indicating a rather poor fit.

The contribution of this study was not its ability to explain word problem difficulty, for both $R^2$ and Chi-squared were by far not acceptable. Rather, this was the first step toward an organized branch of research in mathematics word-problem solving. As shown later, both $R^2$ and Chi-squared will improve in succeeding studies. However, an unsatisfactory amount of variance will still be left unexplained. Perhaps, as the experimenters themselves admit "...nothing short of a full syntactic and semantic analysis will suffice to predict all the details that must be accounted for in the behavior of students."(page 14).

Loftus (1970) tried to capture at least one facet of the syntactic structure of the sentences in word problems--the surface structure*--measured by the DEPTH variable (see page ). This measure, though admittedly crude was a step in the right direction. For it is intuitively clear that the linguistic structure of a sentence is important to its understanding; as Professor Suppes conjectured, linguistic analysis in mathematics education research is essential for the advancement of the science on the word problem solving front.

* A sentence may also be measured by a "deep" structure analysis. In this approach the sentence is subjected to defined linguistic transformations that will reduce it to a standard format. A measure of the number of tranformations becomes the index of the depth of the sentence.

Besides the DEPTH variable, Loftus used the ORDER variable (see page ). These two, in conjunction with the six variables used in the Suppes et al., brought the total number to eight: $x_1$ = OPERATIONS; $x_2$ = STEPS; $x_3$ = LENGTH; $x_4$ = DEPTH; $x_5$ = SEQUENTIAL; $x_6$ = VERBAL CLUE; $x_7$ = ORDER; and $x_8$ = CONVERSION.

Sixteen sixth-grade students from two "depressed" area schools completed the study (no indication is given on how many began). The average sixth-grade I.Q. was 93 in one school and 99 in the other. The students solved 100 word problems of appropriate sixth-grade difficulty after a practice period of four weeks working with computer teletypes (students at one school took longer). The regression equation was:

$$z_i = -3.24 + .48x_{i1} + .04x_{i2} + .02x_{i3} + .88x_{i4} + .61x_{i5} + .20x_{i6} + .13x_{i7} + .49x_{i8}$$

with $R^2$ = .70 and Chi-squared = 206.74. Thus, though the amount of variance accounted for was respectable, the fit of the model was rather poor. Table 2.1 compares the results of this study with those of Suppes et al.

Note that the robustness of the variables OPERATIONS and SEQUENTIAL--especially in light of the fact that the subjects in the former study were bright and the ones in the latter were average-- was quite good. The emergence of robust variables is clearly of tantamount importance if practitioners of education are to be helped by this

TABLE 2.1

Comparison of the importance* of variables in the studies on word-problem solving difficulty by Suppes, Loftus, and Jerman (1969), and Loftus (1970).

| Suppes, and others (1969) VARIABLES | ORDER OF IMPORTANCE | Loftus (1970) VARIABLES |
|---|---|---|
| SEQUENTIAL | 1 | SEQUENTIAL |
| CONVERSION | 2 | OPERATIONS |
| OPERATIONS | 3 | DEPTH |
| VERBAL CLUE | 4 | LENGTH |
| STEPS | 5 | CONVERSION |
| LENTGH | 6 | VERBAL CLUE |
| | 7 | ORDER |
| | 8 | STEPS |

* In the 1969 study importance was seemingly taken to correspond to the the magnitude of the regression coefficients. In the 1970, the size of the partial correlation coefficient was used as an index of importance.

type of research. This is not to say that applicability should dictate the path of research.

The reader is cautioned not to allow the apparent robustness to lead him to false conclusions. These are the results of just two studies; the first did not make clear how the importance of the variables was defined, and the second, assuming the partial correlation coefficient is a valid measure of importance, was limited to only 16 subjects.

Further evidence of the robustness of the variable OPERATIONS, and the appearance of new variables high on the ladder of importance was found in a follow-up study by Jerman (1971). Jerman re-analyzed the data in the Suppes, Loftus, and Jerman (1969) study with a larger set of predictor variables (21). The following new variables were added (underlined variables have been adopted in the present study):

<u>OPERATIONS 2</u>: The sum of the number of different operations, and
4 if one of the operations is division,
2 if one of the operations is multiplication,
1 if one of the operations is addition.

<u>ORDER 2</u>: The sum of $S_1$ and one point for each verbal clue necessary to establish a new order.

<u>FORMULA</u>: Has value 1 if knowledge of a formula is required, zero otherwise.

AVERAGE: Has value 1 if the word average appears in the problem statement; zero otherwise.

ADDITION: Has value 1 if the problem requires addition, zero otherwise.

SUBTRACTION: Similar to addition.

MULTIPLICATION: Similar to addition.

DIVISION: Similar to addition.

$S_1$: Measures the number of displacements of order of operations in successive problems.

$S_2$: Measures the number of displacements between order of operations required and that given in the problem statement itself.

RECALL: The sum of:
(a) formulas needed,
(b) steps in each formula,
(c) conversions to be used,
(d) facts recalled and used from previous problems.

After some adroit, but artificial manipulation of variables and data, Jerman arrived at the results summrized in Table 2.2. We find, there, more support for the variable OPERATIONS, and new support for LENGTH, DIVISION, $S_2$, and CONVERSION.

A second task pursued by Jerman et al., was to compare the importance of structural variables for the problems presented in CAI versus paper-and-pencil mode. Even though this study does not deal with the problem in depth*, the results are very interesting. This analysis is displayed in Table 2.3 . There is a definite trend of importance for the variables that reflect computational skill, in the paper-and-pencil mode of solution. However, the variable LENGTH is also high for both modes. In a subsequent, unpublished paper, which Professor Jerman was kind enough to send to the author, a different criterion of importance was used to rank the variables (amount of variance contributed

* Jerman candidly warns readers that the comparisons may not be valid. Among some questions one may raise are the proper use of the SEQUENTIAL variable. Also, three problems with large Chi-squared values were removed from the problem set; and, the problem set of the paper-and-pencil group was not the same as that of the CAI group, and for that matter, neither were the subjects the same in both groups.

TABLE 2.2

Importance of two sets of structural variables using stepwise regression analysis on 65 arithmetic problems ( Jerman, 1971 )

| RESULTS WITH 16 VARIABLES | ORDER OF APPEARANCE | RESULTS WITH 21 VARIABLES |
|---|---|---|
| OPERATIONS | 1 | OPERATIONS |
| VERBAL CLUE | 2 | CONVERSIONS |
| DIVISION | 3 | LENGTH |
| LENGTH | 4 | ORDER 2 |
| FORMULA | 5 | DIVISION |
| $S_1$ | 6 | $S_2$ |
| CONVERSION | 7 | ORDER |
| $S_2$ | 8 | MEMORY |
| Other variables | 9 | DISTRACTORS |
| Other variables | 10 | Other variables |

TABLE 2.3

Comparison of the importance of structural variables in predicting word-problem solving difficulty for problems solved in CAI versus paper-and-pencil mode using 65 CAI problems, 30 paper-and-pencil problems, and 19 variables

| CAI MODE | ORDER OF IMPORTANCE | PAPER-AND PENCIL MODE |
|---|---|---|
| OPERATIONS 2 | 1 | LENTGH |
| LENTGH | 2 | NOMC2* |
| ORDER 2 | 3 | QUOTIENT* |
| RECALL | 4 | DISTRACTOR* |
| $S_2$ | 5 | COLC2* |
| .56 | $R^2$ | .87 |

* These are computational variables introduced especially because these subjects had to perform their own computations. Briefly, NOMC2 measures regroupings in multiplications, QUOTIENT measures the number of digits in division, COLC2 measures the number of columns and regroupings in addition and subtraction problems.

to the total variance). For this criterion of importance, the CAI variables were ranked, respectively: OPERATIONS 2, CONVERSIONS, LENGTH, ORDER 2, and DIVISION. Whereby the paper-and-pencil variables were ranked, respectively: MULTIPLICATION (NOMC2), DIVISION (QUOTIENT), LENGTH, and DISTRACTORS. These new results strenghten the above conclusions regarding the computational variables.

The large amount of variance accounted for (.87) in the Jerman study for paper-and pencil mode must be looked at in the light of the ratio of variables to problems. Statisticians warn that if this ratio is allowed to approach one, then the dimension of the rank space approaches the number of variables, and the regression approaches a unique solution with $R^2 = 1$. Indeed, one may solve a set of simultaneous equations in this case, where the sum of squres is zero. The ratio for the paper-and-pencil mode was 1.6, and that of the CAI mode was 3.42. This, more than the effect of the variables may be the reason for the high $R^2$ value in the paper-and-pencil mode. Along these same lines, the study by Loftus(1970), with a ratio of 12.5 is least open to this type of criticism--including the present study, which has a ratio of 5.7.

This chapter closes with a look to future research on structural variables studies. More of these studies will be emanating from Professor Suppes at Stanford University, and Professor Jerman, now at the Pennsylvania State University. Suppes is presently running a more comprehensive study, while Jerman has revised his 1971 study in an unpublished report by Jerman and Rees. Ongoing studies at Penn State include: " Linguistic Variables in Verbal Arithmetic Problems," by Ed Beardslee; "Arithmetic, Linguistic, and Algebraic Structural Variables That Contribute to Problem Solving Difficulty of Word Problems in Algebra," by Blair Cook; "Predicting

the Relative Difficulty of Problem-Solving Exercises in Arithmetic," by Max Jerman; and "Structural Variables in Problem Solving Exercises Solved by Prospective Elementary School Teachers," by Max Jerman and Sanford Miram.

All these studies, as well as the ones reviewed in this paper, approach problem solving difficulty by examining the structure of the word problem. All basically assume that structural variables have the ability to predict difficulty and latency of response. There is no reason to doubt, at this time, taht this assumption is incorrect; for amidst the variety of variables and approaches there has been a visible thread of consistency. Much work needs to be done, however, to wean this young branch of research into maturity.

CHAPTER III

## DEFINITION OF VARIABLES AND THEORETICAL CONSTRUCTION

This section is used to define the structural variables and the dependent variables and develop the notation needed for applying three statistical models to the analysis of the data collected. We begin with the dependent variables.

Part of the APL/360 software package written exclusively for this study is described in Appendix2. The functions in this software package were designed to present the problems to each subject and trap and save response data. The following dependent variables were chosen from the response data as being representative of problem solving difficulty:

$p_i$ = PERCENT CORRECT: (the only one used in this paper), measures the percent of correct responses for problem i.

$l_i$ = LATENCY: measures the average number of seconds used by the experimental group to arrive at an answer for problem i.

$t_i$ = SUCCESS LATENCY: measures the average number of seconds used by that subset of the experimental group that arrived at the correct answer for problem i.

$s_i$ = STEPS: measures the average number of steps used by the experimental group to arrive at a solution for problem i.

Consideration is now given to the task of defining the structural variables -- the independent variables of this study. Thirty variables were selected on the basis of proved importance in previous studies and

conjectured importance based on surveys with junior college mathematics students and teachers. Before proceeding, however, it should be mentioned that the variables SUPERFLUOUS DATA and MISSING DATA were not discussed in the review of the literature. But they are included as variables in this study. The author apologizes for this oversight in the present paper, and assures the reader that in the dissertation in progress, of which this paper is only a part, these variables will be given their proper due. One can intuitively see, however, that these variables, especially MISSING DATA, play an important part in the problem solving process. Other variables not specifically mentioned in the review of literature chapter are derivatives of reviewd variables.

The set X of structural variables contained the following elements:

$X_1$ = <u>OPERATIONS</u>: The minimum number of different operations required to achieve the correct solution, plus:

1 if at least one addition is required,

2 if at least one subtraction is required,

3 if at least one multiplication is required, and

4 if at least one division is required.

Range of $X_1$ = $(2, 3, 4, \cdots, 14)$

$X_2$ = <u>DIVISION</u>: The minimum number of times division needs to be used in order to achieve the correct solution.

Range of $X_2$ = $(0, 1, 2, 3, \cdots, n)$

$X_3$ = HIERARCHY: has value 16,8,4,or 2, respectively as the first operation required to achieve the correct solution is division, multiplication, subtraction, or addition.

Range of $x_3$ = (2,4,8,16)

$X_4$ = STEPS: the minimum number of steps (or binary operations) required to achieve the correct solution.

Range of $x_4$ = ( 1, 2, 3, ... , n)

$X_5$ = LENGTH: the total number of letters and digits in the problem.

Range of $x_5$ = (1, 2, 3, ... , n)

$X_6$ = DEPTH 1: the sum of the Yngve numbers of all words in the problem.

Range of $x_6$ = (1, 2, 3, ... , n)

$X_7$ = DEPTH 2: the value of $x_5$ divided by the total number of words in the problem.

Range of $x_7$ = ( rational numbers between 0 and 4)

$X_8$ = VERBS: the number of verbs in the problem.

Range of $x_8$ = (1, 2, 3, ... , n)

$X_9$ = ADJECTIVES: the number of adjectives in the problem.

Range of $x_9$ = (0, 1, 2, 3, ... , n)

$X_{10}$ = NOUNS: The number of nouns in the problem.

Range of $X_{10}$= (1,2,3, ..., n)

$X_{11}$ = ADVERBS: The number of adverbs and adverbial clauses in the problem statement.

Range of $X_{11}$ = (0,1,2, ..., n)

$X_{12}$ = PRONOUNS: The number of pronouns in the problem statement.

Range of $X_{12}$ = (0,1,2, ..., n)

$X_{13}$ = NOUN TO VERB RATIO: Measures "inverse assertions" by the ratio $X_{10}/X_{8}$ = number of nouns/number of verbs.

Range of $X_{13}$ = (positive rational numbers less than 5)

$X_{14}$ = NOUN TO ADJECTIVE RATIO: Measures the inverse of the "richness" of the problem statement by the ratio $X_{10}/X_{9}$ = number of nouns/number of adjectives.

Range of $X_{14}$ = (positive rational numbers less than 10)

$X_{15}$ = PRONOUN TO NOUN RATIO: Measures the amount of "indirectness" in the problem statement by the ratio $X_{12}/X_{10}$ = number of pronouns/number of nouns.

Range of $X_{15}$ = (positive rational numbers less than 10)

$X_{16}$ = VERB TO ADVERB RATIO: Measures the inverse of "modification" in the problem statement by the ratio $X_8/X_{11}$ = number of verbs/number of adverbs.

Range of $X_{15}$ = (positive rational numbers less than 10)

$X_{17}$ = SEQUENTIAL: Has value one if the problem does not follow a problem that is mathematically equivalent to it, i.e., the problem cannot be solved in exactly the same manner as the preceding problem. The value of $X_{17}$ is zero otherwise.

Range of $X_{17}$ = (0,1)

$X_{18}$ = ORDER 1: Has value zero if the numbers in the statement of the problem appear in exactly the same order as they are needed for solving the problem, i.e., if they appear in the order customarily prescribed by $X_3$; one otherwise.

Range of $X_{18}$ = (0,1)

$X_{19}$ = ORDER 2: The minimum number of permutations required to change the sequence of the numbers in the statement of the problem to the sequence customarily required by $X_3$. A number missing from the statement of the problem is inserted into the solution sequence increasing $X_{19}$ by one.

Range of $X_{19} = (0,1,2, \ldots, n)$

$X_{20}$ = VERBAL CLUE 1: Has value zero if the problem has at least one word that is customarily associated with one of the four basic arithmetic operations -- and that operation is required in order to solve the problem: otherwise, the value of $X_{20}$ is one. Since this variable is questionable well defined, we include in Appendix , the words considered verbal clues in this study.

Range of $X_{20} = (0,1)$

$X_{21}$ = VERBAL CLUE 2: Measures the number of verbal clues missing from the problem statement.

Range of $X_{21} = (0,1,2, \ldots, n)$

$X_{22}$ = DISTRACTOR: Measures the number of words in the problem -- usually considered verbal clues -- that are misleading clues. Thus, if the word "sum" appears in

the problem statement and it does not refer to an addition operation then $X_{22}$ is increased by one.

Range of $X_{22} = (0,1,2, \ldots, n)$

$X_{23}$ = <u>SUPERFLUOUS DATA</u>: Has value one or zero, respectively as the problem statement does or does not contain data irrelevant to the solution of the problem.

Range of $X_{23} = (0,1)$

$X_{24}$ = <u>MISSING DATA</u>: Has value one if the problem solver himself needs to enter data necessary to the solution of the problem.

Range of $X_{24} = (0,1)$

$X_{25}$ = <u>REUSED DATA</u>: Has value one if any numbers in the problem statement must be used more than once in arriving at the solution; zero otherwise.

Range of $X_{25} = (0,1)$

$X_{26}$ = <u>NUMBER TYPE 1</u>: Has value one if more than 25% of the numbers involved in the solution of the problem (including the answer) are less than one or greater than 10,000. $X_{26}$ is zero otherwise.

Range of $X_{26} = (0,1)$

$X_{27}$ = NUMBER TYPE 2: Has value one if the correct answer is not a whole number or a decimal exact to the hundreth position (i.e., if the answer is in an "unusual" form such as $68.25\bar{8}$). $X_{27}$ is zero otherwise.

Range of $X_{27}$ = (0,1)

$X_{28}$ = CONVERSION: Has value one if conversion of units from one denomination to another is required to achieve the solution; if a percent has to be "changed" to a decimal (or vice versa). $X_{28}$ is zero otherwise.

Range of $X_{28}$ = (0,1)

$X_{29}$ = MEMORY 1: is the sum of:

(a) $X_1$, and

(b) the number of numerals in the problem and

(c) the number of formulas needed

(d) the number of conversions needed, and

(i) 1 if the conversions involve squared units

(ii) 2 if the conversions involve cubic units.

Range of $X_{29}$ = (1,2,3, ..., n)

$X_{30}$ = MEMORY 2: Is the sum of the number of

(a) formulas needed, and

(b) steps in each formula, and

(c) conversions.

Range of $X_{30} = (0,1,2, \ldots, n)$

The thirty variables just defined make up a rather formidable set -- certainly too large to be of practical value. Ideally, five or less variables should be used, and optimally, two or three.

One can readily see, however, that problem solving will not be pinned down to only a handful of variables. The imposing amount of research on problem solving has spawned many seemingly different variables that contribute to problem solving difficulty. Whether all these variables are indeed different or can be reorganized into a smaller subset is yet to be shown. The writer doubts that even this smaller subset can be less than five in cardinality.

Yet, proliferation of variables is not a disturbing problem in structural variables studies using the regression model. One can easily "skim off" a workable number of variables from the regression equation. For this, one does need some criterion of "importance". The articles by Darlington (1968), and Linn and Merts (1969) should provide excellent guidelines. This study will show that one can choose the first three variables entering the regression equation and come away with more than 60 percent of the variance explained.

The notation needed for the statistical models used in this study will now be developed. The primary model is Linear

Regression. Discriminant Analysis and Canonical Correlation comprise the secondary models.

Let $x_{ij}$ be the value of the $j^{th}$ structural variable for the $i^{th}$ problem; $x_{ij}$ is an element of the set X of Chapter 2:

$$X = \{x1, x2, \ldots, xk\},$$

where each element now takes on a second subscript, i, for i = 1,2, ..., n, and n is the total number of problems. We rewrite X as:

$$X_i = \{X_{i1}, X_{i2}, \ldots, X_{ik}\}, \quad i=i,2, \ldots, n.$$

The set A of Chapter 2:

$$A = \{a_1, a_2, \ldots, a_k\}$$

remaines unchanged. An element $a_j$ of set A is a statistical parameter derived from linear regression analysis and represents the weight given to variable $x_j$.

The stepwise linear regression model (BMDO2R) will be used to derive the set A for the set X of structural variables, and a constant $a_o$, such that a weighed linear combination $\hat{p}_i$, defined by the equation

$$\hat{p}_i = \sum_j a_j x_{ij} + a_o, \quad i = 1,2, \ldots, n$$

will predict the dependent variable $p_i$.

The reader should note that even though the observed data will always yield a $p_i$ of value greater than or equal to zero but less than or equal to one, the predicted value $\hat{p}_i$ need not be restricted in such a manner as Figure 31 shows in the case of one independent variable.

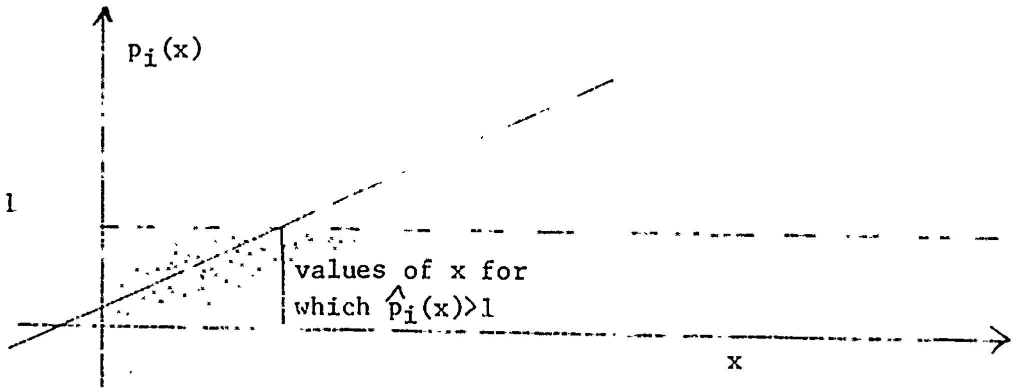



FIGURE 3.1

To avoid this statistical quirk a logarithmic transformation is used. This will allow the predicted values, $\hat{p}_i$, to stay within the range of values of a probability function, $0 \leq \hat{p}_i \leq 1$. The transformation* suggested by Suppes, Jerman and Brian (1968) is used:

$$Z_i = \log \frac{1-pi}{pi}, \quad 0 < pi < 1$$

The graphical effect of this transformation on the pi's can be seen in Figure 2

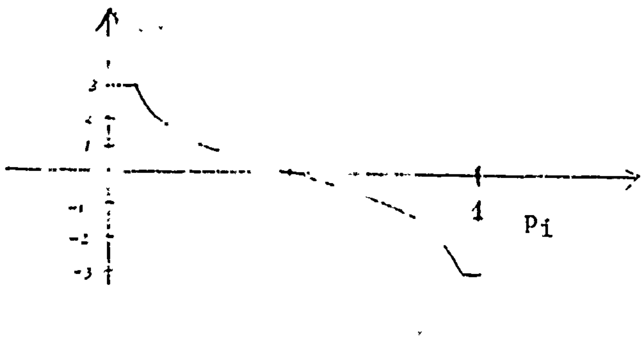



FIGURE 2

---

*Other transformations may be used. For example, $Z_i = \text{Cot}\ \pi\, pi$ will achieve similar results.

For cases where pi = 0 or pi = 1 ($Z_i = \pm\infty$) we adulterate the transformation by using pi = .001 and pi = .999 respectively. Also, $Z_i = 3$ for $0 \leq pi \leq .001$, and $Z_i = -3$ for $1 \geq pi \geq .999$.

The regression analysis is then performed on $Z_i$:

$$\hat{Z}_i = \sum_j a_j x_{ij} + a_o$$

## CHAPTER IV

### DESIGN

The objective in the introductory chapter is here formalized:

OBJECTIVE: Use stepwise linear regression (BMD02R) and the logarithmic transformation $Zi = \log \frac{1 - pi}{pi}$ to obtain a set of weights, $A = \{a_0, a_1, \ldots, a_{30}\}$ that will yield a regression equation:

$$\hat{Z}i = \sum_{j=1}^{30} a_j x_{ij} + a_0 , \quad 1 \leq i \leq 172$$

Identify:

a. the order of importance for the elements of set X, and thus obtain the ordered set $X^1 = (X_1^1 > X_2^1 > \ldots > X_{30}^1)$.

b. the F-value which measures the goodness of the fit of the model.

SUBJECTS: Sixty-five students enrolled during the Fall semester of 1971 in the arithmetic course known as Math 005 at Golden West College, Huntington Beach, California, comprised the initial set of subjects. Because of normal academic attrition, 44 students

completed the course. And, since the study coincided with the course, 44 students completed the study. No response data of withdrawn students was used in the study.

The choice of experiment location was predicated primarily by the requirement of finding a junior college in Southern California with computer facilities capable of handling the study. Golden West College met this requirement.

Students take Math 005 at Golden West College because they wich to attain the Associate of Arts degree, but have scored less than four on the mathematics part of the SCAT.* A grade of C or better in the course rectifies this low score and fulfills the imposed minimum proficiency requirement in mathematics set by the college.

Generalizations based on the results of this study should be made keeping in mind that the subjects in this study were, insofar as mathematics was concerned, slow learners. Lest one be tempted to label all these students as slow learners in general, the experimenter can attest from experience that some excelled in other fields.

Golden West College draws its students primarily from the cities of Huntington Beach, Fountain Valley, Westminster, Garden Grove, and Seal Beach, California. The combined population of these cities is approximately 400,000. All cities are in the county of Orange, California, and comprise part of

---

*See the footnote on page of Chapter I .

what is facetiously called the "bedroom community" of Los Angeles. The socio-economic background ranges from lower to upper middle class.

HARDWARE. The problem in this study were presented to, and solved by students via IBM 2741 computer terminals located in the Multi-Media Center at Golden West College, Huntington Beach, California. Thirty-four terminals were available to students daily from 8 a.m. to 11 p.m. The terminals were hard-wired to an IBM 360/50 computer, located ten miles away at Orange Coast College, Costa Mesa, California. The system operated in OS mode, had 20 disk paks, three of which were used exclusively for CAI.

SOFTWARE. APL/360 was the language used to code the programs and analyze the data. It is a new, interactive language which is highly mathematically oriented, yet readily adaptable to computer-assisted instruction programs -- a spin-off not anticipated by the creator of the language, Dr. Kenneth Iverson (1967), presently with IBM. The functions were coded by Mr. John R. Clark, CAI consultant to the Coast Community College District under the direction of the experimenter. Some of these programs are listed in Appendix 2.

The presentation format used by this software package imitates closely that of the Stanford studies cited previously. But, the packages were coded totally independently. Imitation was by necessity rather than choice, since comparative investigations were planned as part of the present study.

Some changes in the software have been initiated by the experimenter, such as the addition of hints, in an effort to con-

duct more detailed, future studies in this area.

PROCEDURE. During the first meeting of the semester, students were oriented to the course by the regular instructor. He explained that part of the course requirement was the completion of twenty word problems per week to be presented and solved via computer (time factors were adjusted in order to make this unusual requirement fair to the students).

The experimenter introduced the entire group to general procedures and a video tape was used for illustration. In subsequent meetings the class was grouped, and each group was given a live demonstration on a computer terminal at the Multi-Media Center. Each student was assigned an individual computer number (with lockword) and instructed to use only that number for the entire semester. Each student chose a time slot of one hour per week convenient to him. This guaranteed the student a computer terminal for that hour.

The logic of carrying out the computer problem solving was itself taught through a pre-planned computer program as well as through demonstration and question and answer periods. Students used the first two months of the semester -- and, a total of 120 problems (not used in the analysis) to practice solving word problems on the computer in the context intended by the experiment.

The following examples, illustrate the sequence of steps one particular student used to solve a problem.

The student's responses are always preceded by the symbol "□", and have been underlined to facilitate reading.

1

*BILL HILTON'S CLASS PROGRAM AT CYPRESS COLLEGE LAST SEMESTER CONSISTED OF BIOLOGICAL SCIENCE, 3 UNITS; CALCULUS 2, 4 UNITS; ENGLISH 1B, 3 UNITS; PHYSICAL ED. 1 UNIT, AND U.S. HISTORY, 3 UNITS. HOW MANY UNITS IS HE TAKING THIS SEMESTER IF HE IS TAKING 3 MORE THAN LAST SEMESTER?*

```
                A← 3
                B← 4
                C← 1
⎕ A+B+A+C+A
                D← 14
⎕ D+3
                E← 17
⎕ E*
FIRST RATE,  YOU GOT THAT ONE
```

2

*AN AUTOMOBILE SALESMAN WILL ALLOW MR. ANDERSON A TRADE IN ALLOWANCE OF $1865 TOWARD THE PURCHASE OF A NEW GTO PRICED AT $3749. WHAT IS THE DIFFERENCE MR. ANDERSON HAS TO PAY FOR THE NEW CAR?*

```
                A← 1865
                B← 3749
⎕ B-A
                C← 1884
⎕ C*
VERY GOOD ,  YOU GOT THAT ONE
```

3

*TWO FLOCKS OF SHEEP WERE COUNTED. THE FIRST CONTAINED 40 SHEEP. THE SECOND HAD 10 MORE THAN THE FIRST. WHAT WAS THE TOTAL NUMBER OF SHEEP?*

```
                A← 40
                B← 10
⎕ A+B
                C← 50
⎕ C
                D← 50
⎕ D*
NEGATIVE,  YOU MISSED THIS ONE.
```

* The symbol ← is read "... is assigned the value ..."

The chronology of these problems is as follows: the computer typed out everything up to the first quad, including the numerals which appear under the problem. The computer then paused, giving control back to the student and was ready to execute any "legal" command given it by the student. For instance, in the first problem, the student commanded the computer to perform the addition A+B+A+C+A. The computer responded with D ←14. The remaining steps are self explanatory.

Note that the computer does not respond to an answer, right or wrong, unless the student commands it to recognize an answer by the preassigned method of typing a single letter followed by the asteriks (*). Thus, a student has the perogative to take as many steps as he wishes, as much time as he wishes.

<u>The Problems</u>

The 172 arithmetic word problems used in this study were selected as follows: 100 were identical to the problems used in the Loftus (1970) study. Minor changes in four problems had to be made. Since percent problems prove to be exceptionally difficult for Math 005 students, 30 problems on percent, similar to problems these students are required to solve in the course, were constructed and included in the study. The remaining 42 problems were of a general nature, covering concepts a person proficient in arithmetic should know, yet, these problems were still of the Math 005 caliber. Among the last 42 were 20 problems derived from problems suggested in the CUPM (1971) publication pertaining to this type of course.

## CHAPTER V

## RESULTS

This chapter presents the results of the experiment. To facilitate reading, the objective is repeated here, and then followed by the pertinent results. Before proceeding, the definition of variable importance, mentioned in Chapter IV, must be formalized:

> Definition of importance: Let $x_j$ and $x_k$ be two variables in the regression equation.
>
> $$y = \sum_i a_i x_i + a_0.$$
>
> Also, let $R_j^2$ and $R_u^2$ be the amount of variance accounted for by the respective regression equations:
>
> $$y = \sum_{\substack{i \\ i \neq j}} b_i x_i + b_0, \text{ and}$$
>
> $$y = \sum_{\substack{i \\ i \neq u}} c_i x_i + c_0.$$
>
> Then $x_j$ will be considered more important than $x_u$, written $x_j \succ x_u$, if $R_j^2 < R_u^2$.

This shall be the primary criterion of the importance for the variables present in the linear regression equations in this study. However, to test the "robustness" of the variables, other criteria of importance will be used in the sequel.

<u>Objective</u>. Use stepwise regression and the logarithmic transformation $z_i = \log (1 - p_i)/p_i$ to obtain a set of weights, $A = \{a_0, a_1, \ldots, a_{30}\}$ that will yield a linear regression equation:

$$\hat{z}_i = \sum_i a_i x_i + a_0, \quad 1 \leq i \leq 172$$

Identify:

a. the order of importance of the elements of set X using the definition stated above; thus, obtain the ordered set $X' = \{x_1' \succ x_2' \succ \cdots \succ x_{30}'\}$.

b. the significance of the model by the F-value in the analysis of variance for the regression; and the Chi-square value which measures the goodness of the fit.

The characteristics of the regression model are presented in Tables 1, 2, 3, and 4, as well as Figure 1. The coefficients of the regression equation: the elements of set A, are listed in the first column of Table 1. Also in Table 1 are the standard errors, computed t-values, and the amount of variance lost by the removal of each of the variables in turn from the model. Six variables had significant t-values, but the most impressive was that of the MEMORY 2 variable. This variable also had the largest drop in the R-squared when it was removed from the model. These two criteria have obvious theoretical ties, however, and it is not surprising that the variable scores highly in both cases. Table 2 shows the order of appearance of the variables into the regression, the multiple R, R-squared, and the increase in R-squared at each step. The variables MEMORY 2 and ORDER 1 alone account for over 50% of the variance in problem solving difficulty. It will be seen, however, that the first ten variables listed in Table 2 are not in the same order as given by the definition of importance adhered to in this study. The reason for this discrepancy is that the order of entry into the regression equation does not necessarily reflect the true contribution a variable makes to the total variance explained. Nevertheless, with a relatively small set of variables, 70% of the variance has been accounted for in problem solving difficulty for the junior college group sampled. Table 3 displays the analysis of variance for the regression.

TABLE 1

REGRESSION COEFFICIENTS, STANDARD ERRORS, COMPUTED T-VALUES, AND R-SQUARED DROP CAUSED BY THE REMOVAL OF EACH VARIABLE IN TURN FOR OBJECTIVE 1 (172 PROBLEMS, AND 30 VARIABLES.)

| VARIABLE | REGRESSION COEFFICIENT | STANDARD ERROR | COMPUTED T-VALUE | RSQ DROP |
|---|---|---|---|---|
| 0 CONSTANT | -2.6253 | | | |
| 1 OPERATIONS | 0.0354 | 0.0331 | 1.0711 | 0.0021 |
| 2 DIVISION | 0.1110 | 0.1076 | 1.0317 | 0.0020 |
| 3 HIERARCHY | -0.0026 | 0.0115 | -0.2226 | 0.0001 |
| 4 STEPS | -0.0132 | 0.0659 | -0.2010 | 0.0001 |
| 5 LENGTH | 0.0040 | 0.0022 | 1.8235 * | 0.0061 |
| 6 DEPTH 1 | -0.0060 | 0.0023 | -2.6147 ** | 0.0124 |
| 7 DEPTH 2 | 0.1044 | 0.1118 | 0.9334 | 0.0016 |
| 8 VERBS | 0.0410 | 0.0357 | 1.1486 | 0.0024 |
| 9 ADJECTIVES | 0.0054 | 0.0241 | 0.2239 | 0.0001 |
| 10 NOUNS | -0.0616 | 0.0227 | -2.7096 ** | 0.0133 |
| 11 ADVERBS | 0.1073 | 0.0616 | 1.7427 * | 0.0055 |
| 12 PRONOUNS | -0.0206 | 0.0548 | -0.3760 | 0.0003 |
| 13 NOUNS:VERBS | 0.0812 | 0.0585 | 1.3882 | 0.0035 |
| 14 NOUNS:ADJECTIVES | 0.0390 | 0.0327 | 1.1932 | 0.0026 |
| 15 PRONOUNS:NOUNS | 0.7056 | 0.6798 | 1.0380 | 0.0020 |
| 16 VERBS:ADVERBS | 0.0361 | 0.0294 | 1.2275 | 0.0027 |
| 17 SEQUENTIAL | 0.0344 | 0.0945 | 0.3637 | 0.0003 |
| 18 ORDER 1 | 0.3396 | 0.1194 | 2.8428 * | 0.0147 |
| 19 ORDER 2 | 0.0415 | 0.0696 | 0.5962 | 0.0007 |
| 20 VERBAL CLUE 1 | 0.1156 | 0.1257 | 0.9195 | 0.0016 |
| 21 VERBAL CLUE 2 | 0.0336 | 0.0540 | 0.6214 | 0.0007 |
| 22 DISTRACTORS | 0.0721 | 0.0711 | 1.0149 | 0.0019 |
| 23 SUPERFLUOUS DATA | 0.1591 | 0.1302 | 1.2218 | 0.0027 |
| 24 MISSING DATA | 0.1945 | 0.1727 | 1.1263 | 0.0023 |
| 25 REUSED DATA | 0.0296 | 0.1483 | 0.1996 | 0.0001 |
| 26 NUMBER TYPE 1 | 0.1180 | 0.1183 | 0.9975 | 0.0018 |
| 27 NUMBER TYPE 2 | 0.2200 | 0.1406 | 1.5654 | 0.0045 |
| 28 CONVERSION | -0.1784 | 0.1684 | -1.0590 | 0.0021 |
| 29 MEMORY 1 | 0.0741 | 0.0537 | 1.3806 | 0.0035 |
| 30 MEMORY 2 | 0.2876 | 0.0425 | 6.7595 *** | 0.0829 |

* $P<.1$    ** $P<.01$    *** $P<.001$

TABLE 2

SEQUENTIAL APPEARANCE OF THE VARIABLES INTO THE REGRESSION EQUATION FOR OBJECTIVE 1. ALSO SHOWN ARE THE MULTIPLE R AND R-SQUARED, THE INCREASE IN R-SQUARED (172 PROBLEMS, AND 30 VARIABLES)

| VARIABLE | MULTIPLE R | R SQUARED | INCREASE IN RSQ |
|---|---|---|---|
| 30 MEMORY 2 | 0.6898 | 0.4758 | 0.4758 |
| 18 ORDER 1 | 0.7559 | 0.5714 | 0.0956 |
| 21 VERBAL CLUE 2 | 0.7898 | 0.6238 | 0.0524 |
| 1 OPERATIONS | 0.8023 | 0.6437 | 0.0199 |
| 15 PRONOUNS:NOUNS | 0.8116 | 0.6587 | 0.0150 |
| 22 DISTRACTORS | 0.8177 | 0.6686 | 0.0099 |
| 11 ADVERBS | 0.8226 | 0.6767 | 0.0080 |
| 6 DEPTH 1 | 0.8281 | 0.6857 | 0.0091 |
| 16 VERBS:ADVERBS | 0.8351 | 0.6974 | 0.0116 |
| 23 SUPERFLUOUS DATA | 0.8397 | 0.7051 | 0.0077 |
| 27 NUMBER TYPE 2 | 0.8441 | 0.7125 | 0.0074 |
| 14 NOUNS:ADJECTIVES | 0.8456 | 0.7150 | 0.0025 |
| 2 DIVISION | 0.8468 | 0.7171 | 0.0020 |
| 26 NUMBER TYPE 1 | 0.8476 | 0.7184 | 0.0014 |
| 5 LENGTH | 0.8485 | 0.7200 | 0.0015 |
| 10 NOUNS | 0.8541 | 0.7295 | 0.0095 |
| 13 NOUNS:VERBS | 0.8554 | 0.7317 | 0.0022 |
| 29 MEMORY 1 | 0.8564 | 0.7334 | 0.0017 |
| 8 VERBS | 0.8576 | 0.7355 | 0.0021 |
| 24 MISSING DATA | 0.8588 | 0.7375 | 0.0021 |
| 28 CONVERSION | 0.8599 | 0.7394 | 0.0019 |
| 20 VERBAL CLUE 1 | 0.8609 | 0.7411 | 0.0017 |
| 7 DEPTH 2 | 0.8617 | 0.7425 | 0.0014 |
| 19 ORDER 2 | 0.8622 | 0.7434 | 0.0009 |
| 12 PRONOUNS | 0.8623 | 0.7436 | 0.0002 |
| 17 SEQUENTIAL | 0.8625 | 0.7439 | 0.0003 |
| 25 REUSED DATA | 0.8626 | 0.7441 | 0.0002 |
| 3 HIERARCHY | 0.8626 | 0.7442 | 0.0001 |
| 9 ADJECTIVES | 0.8626 | 0.7443 | 0.0001 |
| 4 STEPS | 0.8627 | 0.7443 | 0.0000 |

The F-value of 13.673 was significant at the .0001 level (see part b of Objective 1).

TABLE 3

ANALYSIS OF VARIANCE FOR THE REGRESSION ANALYSIS FOR OBJECTIVE 1

| SOURCE | D.F. | SUM OF SQUARES | MEAN SQUARES | F-RATIO |
|---|---|---|---|---|
| Regression | 30 | 114.605 | 3.820 | 13.673* |
| Residual | 141 | 39.396 | 0.279 | |
| Total | 171 | 154.001 | | |

*P<.001

The goodness of the fit of predicted versus observed $p_i$'s was calculated by means of the Chi-squared formula:

$$\chi^2 = \sum (f_i - p_iN)/(1 - p_i)p_iN,$$

where $f_i$ = observed frequency of correct responses, $p_i$ = the predicted percent of correct responses, and N = the number of students. The value of Chi-square was 1382.7, indicating a rather poor fit. However, a closer analysis showed that six problems had exceptionally high Chi-square values.

Figure 1 shows the graph of the predicted versus observed percent of correct responses ($\hat{p}_i$ and $p_i$). The graph was ranked according to the observed difficulty, thus the easiest observed problem is closest to the origin, while the most difficult problem is at the farthest point on the horizontal axis. Perusal of the graphs in Figure 1 will show that the model gives a much better account of the easier problems than the hard ones. Perhaps by choosing a more heterogeneous set of problems this difficulty can be ironed out. Nevertheless, the trends are the same.

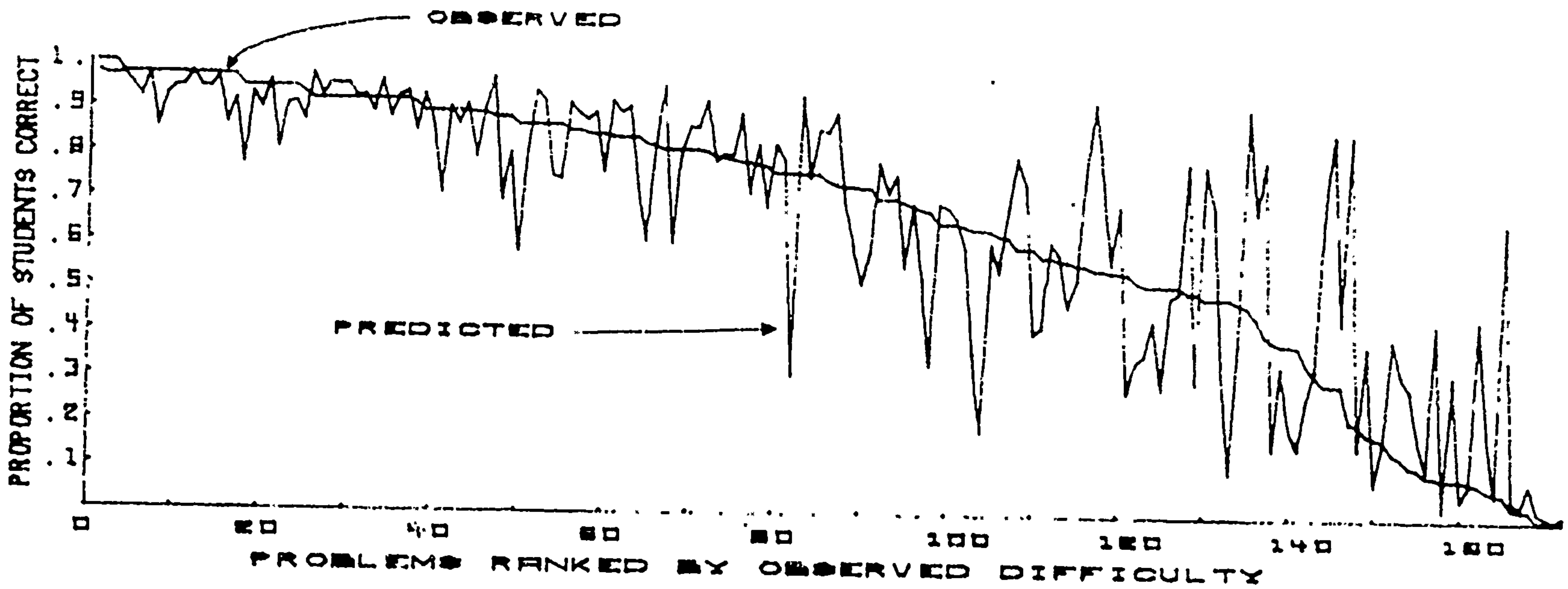



Fig. 1 Predicted versus observed $p_i$ for 172 problems and 30 structural variables of the objective in this study (44 junior college students)

Table 4 lists the variables according to four criteria of importance: the first, by the definition used in this study; the second, by order of introduction into the regression equation; the third, by size of the decoded beta weights; and the fourth, by the size of the partial correlation coefficient. Although in this study, the importance of a variable was defined according to the first criterion, that is, by the amount of R-squared drop caused by the removal of that variable from the regression model, Table 4 offers a measure of "robustness" of the variables, within the confines of this study. Note that the variable MEMORY 2, consistently appears first on all orderings of importance. The significance of this fact is underscored by the strong showing this variable made in Tables 1 and 2. The ordered set $X'$ (see part a of Objective 1), is shown in column one of Table 4. The ordered set $X'$ can be achieved by reading the column from top to bottom.

TABLE 4

ORDER OF IMPORTANCE OF THE 30 STRUCTURAL VARIABLES AS MEASURED BY FOUR CRITERIA OF IMPORTANCE. THE FIRST OF THESE IS THE CRITERION OF IMPORTANCE AS MEASURED BY THE DEFINITION, THE SECOND IS THE ORDER OF APPEARANCE OF THE VARIABLES INTO THE REGRESSION EQUATION, THE THIRD, BY THE DECODED BETA WEIGHTS, AND THE FOURTH BY THE SIZE OF THE PARTIAL CORRELATION COEFFICIENT (VARIABLES HAVE BEEN ABBREVIATED)

| DEFINITION OF OF IMPORTANCE | ORDER OF APPEARANCE | DECODED BETA WEIGHTS | PARTIAL CORR. COEFFICIENT |
|---|---|---|---|
| MEMORY 2 | MEMORY 2 | MEMORY 2 | MEMORY 2 |
| ORDER 1 | ORDER 1 | NOUNS | ORDER 1 |
| NOUNS | VERBAL CL2 | LENGTH | VERBAL CL2 |
| DEPTH 1 | OPERATIONS | DEPTH 1 | OPERATIONS |
| LENGTH | PRON:NOUN | ORDER 1 | PRON:NOUN |
| ADVERBS | DISTRACTOR | MEMORY 1 | VERB:ADVRB |
| NUM'R TYP2 | ADVERBS | ADVERBS | NOUNS |
| NOUN:VERB | DEPTH 1 | VERBS | DISTRACTOR |
| MEMORY 1 | VERB:ADVRB | NOUN:VERB | DEPTH 1 |
| SUP'S DATA | SUP'S DATA | NOUN:ADJTS | NUM'R TYP2 |
| VERB:ADVRB | NUM'R TYP2 | OPERATIONS | SUP'S DATA |
| NOUN:ADJTS | NOUN:ADJTS | PRON:NOUN | ADVERBS |
| VERBS | DIVISION | VERB:ADVRB | NOUN:ADJTS |
| MIS'G DATA | NUM'R TYP1 | NUM'R TYP2 | NOUN:VERB |
| OPERATIONS | LENGTH | MIS'G DATA | VERBS |
| CONVERSION | NOUNS | CONVERSION | MIS'G DATA |
| PRON:NOUN | NOUN:VERB | SUP'S DATA | DIVISION |
| DIVISION | MEMORY 1 | DIVISION | CONVERSION |
| DISTRACTOR | VERBS | NUM'R TYP1 | VERBAL CL1 |
| NUM'R TYP1 | MIS'G DATA | DISTRACTOR | MEMORY 1 |
| DEPTH 2 | CONVERSION | VERBAL CL1 | DEPTH 2 |
| VERBAL CL1 | VERBAL CL1 | ORDER 2 | LENGTH |
| VERBAL CL2 | DEPTH 2 | DEPTH 2 | NUM'R TYP1 |
| ORDER 2 | ORDER 2 | VERBAL CL2 | ORDER 2 |
| PRONOUNS | PRONOUNS | PRONOUNS | PRONOUNS |
| SEQUENT'L | SEQUENT'L | STEPS | SEQUENT'L |
| HIERARCHY | REUSE DATA | ADJECTIVES | STEPS |
| STEPS | HIERARCHY | SEQUENT'L | REUSE DATA |
| ADJECTIVES | ADJECTIVES | REUSE DATA | HIERARCHY |
| REUSE DATA | STEPS | HIERARCHY | ADJECTIVES |

## CHAPTER VI

## DISCUSSION AND SUGGESTIONS FOR FURTHER RESEARCH

The results of this study clearly indicate that the variable MEMORY 2 is highly significant for predicting the difficulty of word problems. By itself, the variable accounted for 47% of the variance. Its simple correlation with the dependent variable was a healthy .702. Furthermore, the variable was consistently first on all criteria of importance. The definition of MEMORY 2 is composed of basically two subvariables: (a) the number of formulas to be recalled in pursuing the correct solution, plus the number of steps in each formula, and (b) the number of conversions required to achieve the correct solution. For purposes of discussion, both parts can be combined in one word: recall.

The findings in this study, then, indicate that for the Junior College students sampled, an arithmetic word problem which requires some recall of previously learned facts is more difficult to solve than one that does not. Of course, there is the question: "If the needed recall facts were made available to students, say, via a hint, could students then carry the problem to solution?" This is a meaningful research question. To give this question deserving attention in future research, the driving functions in the software package used for this study have already been

"patched" to present hints and trap student response data pre and post hints. No comments will be made here regarding results since the data have not yet been organized into proper form.

It is indeed encouraging and rewarding to discover that a memory-related variable is so potent. Suppes (1966, page 248) also found that his memory variable was quite important in predicting the difficulty of fourth-grade addition problems. He goes on to state (p. 254):

> From a psychological standpoint, the most suggestive single finding is probably the importance of the process variable NSTEPS, or of its component variables, particularly memory, in all the relevant analyses. . . If. . . the dominant variables had turned out to be magnitude variables, then a less significant first step would have been taken, because anyone would immediately ask what characteristics of the processing done internally by the students made these magnitude variables so significant. In postulating process variables and being able to establish their direct importance, we have already been able to move part this first step.

As stated at the beginning of this paper, the results under discussion are but one-sixth of a larger study, representing the author's doctoral dissertation at UCLA. Space does not permit a full discussion of all results in that dissertation, but it is most appropriate at this point to allow for a passing comment. Regression analyses were also run on subsets of the 172 problems (toward more heterogenous problem sets); on average latency of response, and average success latency for all 172 problems. Appendix I displays the analyses of the data unaccompanied by discussion.

Perusal of these data will show that the variable MEMORY 2 is also of premiere importance!

Professor Suppes has eloquently pointed out the psychological importance of this variable. But there is another important point to be made. The variable MEMORY 2 is well defined! This fact makes MEMORY 2 a agmatic variable and ought to facilitate its implementation into the world of educational practice.

This success is somewhat dulled, however, by the variable's multiplicity of definition. Is the recall of formulas what makes it a strong variable? Or is it the need to recall appropriate conversion units? A finer grained analysis was not performed in this study to help answer this question. Neither can an answer be derived from the present results since "pure" conversion and formula variables were present in the set X of structural variables (CONVERSION, and MEMORY 1). Hopefully, future research will answer this question.

The variable ORDER 1 showed a respectable amount of robustness. It was second on three of the four criteria of importance, including the stated definition of importance used in this study. The variable's showing should not be disappointing even to those who abide by the beta weight as an index of importance. Here, ORDER 1 was fifth (see Table 1, Chapter V). The simple correlation of ORDER 1 with the dependent variable, the percent of correct responses, was .478.

The implication of the ORDER 1 variable to problem solving difficulty is that students did not "internalize" problems.

Item analysis relating to this variable showed that often students knew a problem should be solved by division. But when the correct answer was to be achieved by commanding the computer to perform $A \div B$, very often they asked the machine to perform $B \div A$, and promptly lablelled the latter as the correct answer. Again there appears a dichotomous query: "was this carelessness, or a more deep pedagogical reason?"

In any event, note this is another well defined variable and thus easily implementable into educational practice. This, together with the fact that the set {MEMORY 2, ORDER 1} alone accounts for 57% of the variance, makes for a significant contribution to predicting problem solving difficulty for Junior College arithmetic students.

It is time to turn to the unpleasant task of discussing suppresor variables. The next two variables, NOUNS and DEPTH 1 set the stage. Both have positive simple correlations with the dependent variable (.301 and .486, respectively); both are, by definition and other criteria, important; yet both have significantly high negative regression coefficients--and, therefore, negative beta weights.

It is tempting to offer the explanation: more nouns and deeper sentences reduce the difficulty of arithmetic word problems. But this would be a spurious interpretation in view of the positive, simple correlations. It is more sensible to say that because of the intercorrelatedness of the independent variables, the regression procedure, in trying to minimize the sum of squares, achieves this goal by assigning negative coefficients to these two variables.

This conclusion is further supported by glancing at the simple correlations between variables. The correlation between NOUNS and LENGTH was .961! Furthermore, NOUNS was entered into the regression equation immediately after LENGTH.

DEPTH 1 was correlated with VERBAL CLUE 2 (.576), MEMORY 2 (.561), and NOUNS (.731). All these variables entered the regression before DEPTH 1.

Before we lower the life boats, it should be noted that the ship may be listing, but still floating. The goal in this study was to predict problem solving difficulty. This can be accomplished even with the appearance of negative regression coefficients. If the concern was with the more delicate question of cause-effect, then the appearance of negative weights would be more damaging.

The variable LENGTH, though not as robust as the other four, was high on the list of predictive worth. Its simple correlation with the dependent variable was .382. This correlation is too small to venture a cause-effect relationship. But with the other variables present, it can be said that the length of a problem does affect its difficulty.

Recall that for the students involved in this study, mathematics has been a difficult subject. These are people who after twelve years of formal schooling are not able to perform at a level of eight, or at most ninth grade arithmetic. Word problems in particular, by record and the student's own admission, are particularly difficult. In view of this, it's not hard to understand why a lengthy problem, with unordered data, and requiring recall of previously learned facts, has a tendency to--if we may use the current vernacular--pshyc the students out.

Again, note that the length variable is the most well defined of the entire set X, of structural variables, and the easiest to measure. Such an asset should prove valuable in its implementation into practice.

The first five variables of the ordered set X' have been discussed. Their robustness and highly acceptable definitions were encouraging in the effort to predict problem solving difficulty for Junior College arithmetic students. It would not be meaningful to belabor the discussion to the remaining 25 variables, for their intercorrelatedness would make the interpretation rather artificial. The interested reader is referred to Table 4, Chapter V, the other tabulated data, and Appandix 1. These are presented clearly enough for anyone to reach his own conclusions. In the author's opinion, the accomplishment of the study rests with pinning down the set {MEMORY 2, ORDER 1} whose two elements are structural variables in predicting problem solving difficulty of CAI-presented word problems for Junior College arithmetic students.

Before concluding this section, it will be informative to summarize in tabular form these results along with those in previous structural variables studies. The latter have been already analyzed in the review of the literature chapter, thus there is no need for further comment here. Table 5 presents this summary. Let the reader beware that the table is highly artificial. Place, time, subjects, variables, and criteria of importance differ in each study.

TABLE 5

S[illegible]ry [illegible]e [illegible]ing di[illegible]ent [illegible]dic[illegible] structural variables using the regression model suggested by Suppes, Hyman, and Jerman (1966)

| STUDY | Suppes, Hyman and Jerman (1966) | Suppes, Loftus and Jerman (1969) | Loftus (1970) | Jerman (1971) | Jerman (1971) | Segalla (1973) |
|---|---|---|---|---|---|---|
| TYPE | + − × Arithmetic Facts (C.A.I.) | Arithmetic word Problems (CAI) | Arithmetic word Problems (CAI) | Arithmetic word Problems (CAI) | Arithmetic word problems (paper & Pencil) | Arithmetic word Problems (CAI) |
| # of Variables / # of Numbers | 3 / 19-95 | 6 / 68 | 8 / 100 | 22 / 65 | 19 / 30 | 30 / 172 |
| SUBJECTS | 270 above average 3rd, 4th, 5th, and 6th graders | 27 bright fifth graders | 16 below average sixth graders | Same as Suppes, Loftus, and Jerman (1969) | 20 and 161 average fifth graders | 44 low mathematics ability Junior College Students |
| TOTAL $R^2$ | Variable; minimum = .16 maximum = .74 | .45 | .70 | .73 | .91 | .74 |
| Criterion of Importance | Size of regression coefficient | Size of regression coefficient | Size of the partical correlation coefficient | Order of entrance into the regression equation | Contribution to total varieance | Size of $R^2$ drop caused by removing the variable |
| Significant Variables | Steps (transformation, operation, and memory) | SEQUENTIAL CONVERSION OPERATIONS | SEQUENTIAL OPERATIONS DEPTH LENGTH CONVERSION | OPERATIONAL CONVERSIONS LENGTH ORDER 2 DIVISION | MULTIPLICATION DIVISION LENGTH DISTRACTOR | MEMORY 2 ORDER 1 NOUNS DEPTH 1 LENGTH |

Thus comparisons should be made...carefully, if at all. The inability to make valid comparisons from data like in Table 5 brings to attention a sad state of affairs in this young branch of educational research. A certain amount of standardization is sorely needed. Criteria of importance, for example, ought to be the same or equivalent for all structural variables studies. A first step toward this goal was taken in the present study by listing variables according to four criteria of importance. A pool of varibles and problems ought to be made available to researchers interested in this field. For CAI oriented studies, similar software ought to be used. Toward this end, the APL software used in this study is available to anyone who wishes it. Appendix 2 presents only some of the functions, for illustrative purposes.

Furthermore, research studies are needed to identify, define, and categorize structural variables. From these studies, of factor analytic nature, more orthogonal variables can be discovered. Also, more measures of dependent variables are needed; for problem solving difficulty is not only reflected by percent of correct responses. Suppes, Hyman, and Jerman (1966), for example, analyzed average success latency. In the present study, of which this paper is only a part, average total latency and average success latency were used as dependent variables. In this same study, percent of correct responses together with average total latency were used in a canonical correlation analysis. In the author's opinion, canonical analysis offers real possibilities for these types of studies.

Perhaps the vehicle to be used in achieving normalization of process is a structural variables journal--aware though the author is of the present proliferation of journals.

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APPENDIX 1

Tables and graphs of other regression analyses related to the results in this paper. These are offered without further comments, save that three regression runs are represented.

TABLE 9

REGRESSION COEFFICIENTS, STANDARD ERRORS, COMPUTED T-VALUES, AND R-SQUARED DROP CAUSED BY THE REMOVAL OF EACH VARIABLE IN TURN FOR OBJECTIVE 3 ( 72 PROBLEMS, AND 30 VARIABLES.)

| VARIABLE | REGRESSION COEFFICIENT | STANDARD ERROR | COMPUTED T-VALUE | RSQ DROP |
|---|---|---|---|---|
| 0 CONSTANT | ¯2.48986 | | | |
| 1 OPERATIONS | -0.0268 | 0.0667 | -0.4011 | 0.0010 |
| 2 DIVISION | -0.0302 | 0.1936 | -0.1559 | 0.0002 |
| 3 HIERARCHY | 0.0053 | 0.0228 | 0.2318 | 0.0004 |
| 4 STEPS | 0.0177 | 0.1001 | 0.1764 | 0.0002 |
| 5 LENGTH | 0.0080 | 0.0040 | 1.9851 | 0.0172 |
| 6 DEPTH 1 | -0.0095 | 0.0070 | -1.3592 | 0.0095 |
| 7 DEPTH 2 | 0.1467 | 0.2509 | 0.5848 | 0.0018 |
| 8 VERBS | 0.0825 | 0.0590 | 1.3973 | 0.0104 |
| 9 ADJECTIVES | 0.0000 | 0.0000 | 1.0000 | 0.0000 |
| 10 NOUNS | -0.0949 | 0.0359 | -2.6402 | 0.0369 |
| 11 ADVERBS | 0.0294 | 0.1211 | 0.2424 | 0.0004 |
| 12 PRONOUNS | -0.0674 | 0.0828 | -0.8135 | 0.0035 |
| 13 NOUNS:VERBS | 0.1263 | 0.1306 | 0.9674 | 0.0050 |
| 14 NOUNS:ADJECTIVES | 0.0293 | 0.0324 | 0.9025 | 0.0022 |
| 15 PRONOUNS:NOUNS | 2.1116 | 1.3149 | 1.6058 | 0.0036 |
| 16 VERBS:ADVERBS | 0.0230 | 0.0520 | 0.4416 | 0.0011 |
| 17 SEQUENTIAL | 0.0278 | 0.2199 | 0.1265 | 0.0001 |
| 18 ORDER 1 | 0.0378 | 0.2459 | 0.1535 | 0.0002 |
| 19 ORDER 2 | -0.0153 | 0.1230 | -0.1247 | 0.0001 |
| 20 VERBAL CLUE 1 | 0.2190 | 0.2623 | 0.8351 | 0.0038 |
| 21 VERBAL CLUE 2 | 0.0632 | 0.1397 | 0.4523 | 0.0010 |
| 22 DISTRACTORS | 0.1426 | 0.1298 | 1.0986 | 0.0063 |
| 23 SUPERFLUOUS DATA | 0.0204 | 0.1944 | 0.1049 | 0.0001 |
| 24 MISSING DATA | 0.4886 | 0.3264 | 1.4968 | 0.0110 |
| 25 REUSED DATA | 0.1679 | 0.2620 | 0.6410 | 0.0023 |
| 26 NUMBER TYPE 1 | 0.1186 | 0.2110 | 0.5621 | 0.0017 |
| 27 NUMBER TYPE 2 | 0.3488 | 0.1886 | 1.8489 | 0.0177 |
| 28 CONVERSION | -0.3409 | 0.2712 | -1.2569 | 0.0084 |
| 29 MEMORY 1 | 0.1316 | 0.1003 | 1.3123 | 0.0090 |
| 30 MEMORY 2 | 0.2133 | 0.0721 | 2.9562 | 0.0462 |

TABLE 10

SEQUENTIAL APPEARANCE OF THE VARIABLES INTO THE REGRESSION EQUATION FOR OBJECTIVE 3. ALSO SHOWN ARE THE MULTIPLE R, R-SQUARED, AND THE INCREASE IN R-SQUARED (72 PROBLEMS, AND 30 VARIABLES)

| VARIABLE | MULTIPLE R | R SQUARED | INCREASE IN RSQ. |
|---|---|---|---|
| 30 MEMORY 2 | 0.6453 | 0.4164 | 0.4164 |
| 18 ORDER 1 | 0.7307 | 0.5339 | 0.1175 |
| 21 VERBAL CLUE 2 | 0.7583 | 0.5750 | 0.0411 |
| 15 PRONOUN:NOUN | 0.7755 | 0.6014 | 0.0264 |
| 12 PRONOUNS | 0.7898 | 0.6238 | 0.0224 |
| 22 DISTRACTOR | 0.7997 | 0.6395 | 0.0157 |
| 27 NUMBER TYPE 2 | 0.8082 | 0.6532 | 0.0137 |
| 13 NOUN:VERB | 0.8193 | 0.6713 | 0.0181 |
| 14 NOUN:ADJECTIVE | 0.8252 | 0.6810 | 0.0097 |
| 24 MISSING DATA | 0.8280 | 0.6856 | 0.0046 |
| 28 CONVERSION | 0.8329 | 0.6937 | 0.0081 |
| 11 ADVERBS | 0.8361 | 0.6991 | 0.0053 |
| 6 DEPTH 1 | 0.8403 | 0.7061 | 0.0070 |
| 5 LENGTH | 0.8521 | 0.7261 | 0.0200 |
| 10 NOUNS | 0.8642 | 0.7468 | 0.0208 |
| 8 VERBS | 0.8680 | 0.7534 | 0.0066 |
| 29 MEMORY 1 | 0.8741 | 0.7641 | 0.0106 |
| 20 VERBAL CLUE 1 | 0.8765 | 0.7683 | 0.0042 |
| 25 REUSED DATA | 0.8785 | 0.7718 | 0.0035 |
| 26 NUMBER TYPE 1 | 0.8793 | 0.7732 | 0.0014 |
| 7 DEPTH 2 | 0.8803 | 0.7749 | 0.0018 |
| 1 OPERATIONS | 0.8810 | 0.7762 | 0.0012 |
| 16 VERB:ADJECTIVE | 0.8817 | 0.7774 | 0.0012 |
| 3 HIERARCHY | 0.8818 | 0.7776 | 0.0002 |
| 2 DIVISION | 0.8819 | 0.7777 | 0.0002 |
| 17 SEQUENTIAL | 0.8819 | 0.7777 | 0.0002 |
| 4 STEPS | 0.8820 | 0.7779 | 0.0002 |
| 19 ORDER 2 | 0.8820 | 0.7779 | 0.0001 |
| 23 SUPERFLUOUS DATA | 0.8820 | 0.7779 | 0.0001 |
| 11 ADJECTIVES | 0.8820 | 0.7780 | 0.0000 |

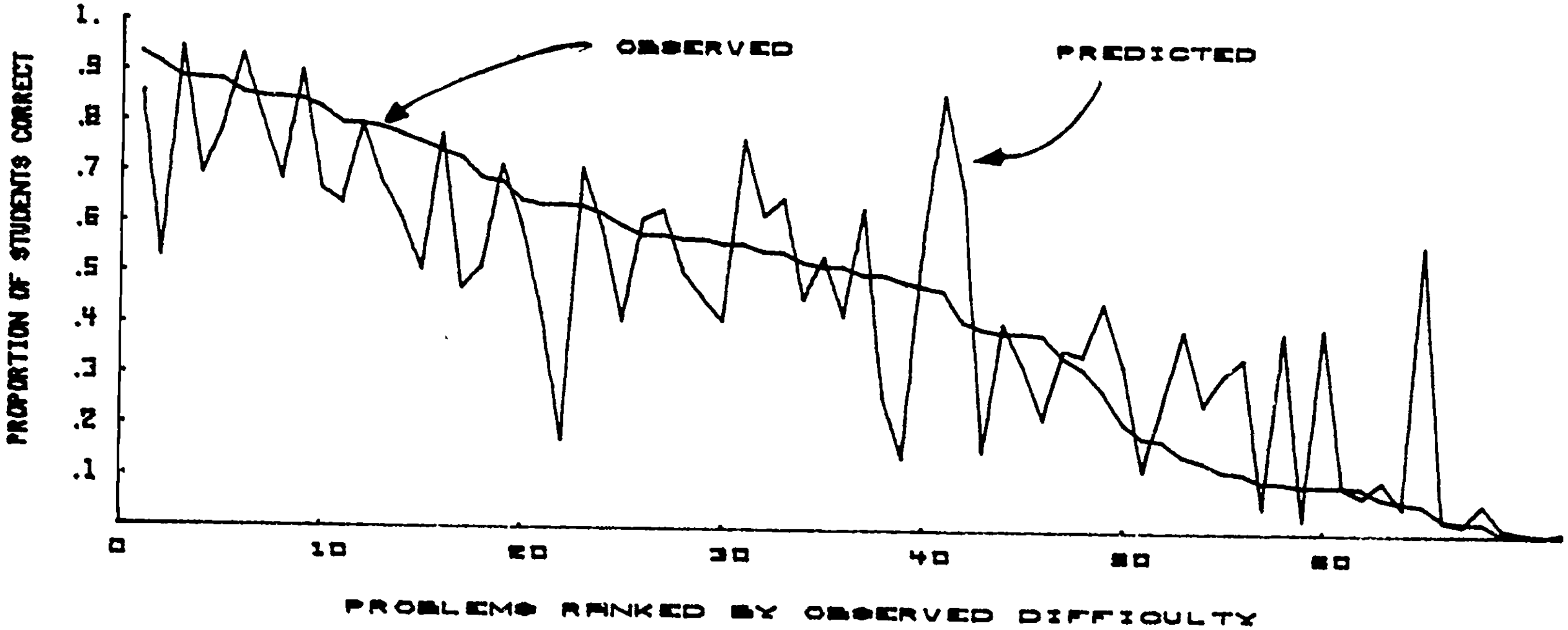

OBSERVED
PREDICTED
PROPORTION OF STUDENTS CORRECT
1.
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
10
20
30
40
50
60
PROBLEMS RANKED BY OBSERVED DIFFICULTY

TABLE 12

ORDER OF IMPORTANCE OF THE 30 STRUCTURAL VARIABLES AS MEASURED BY FOUR CRITERIA OF IMPORTANCE. THE FIRST OF THESE IS THE CRITERION OF IMPORTANCE AS MEASURED BY THE DEFINITION, THE SECOND IS THE ORDER OF APPEARANCE OF THE VARIABLES INTO THE REGRESSION EQUATION, THE THIRD, BY THE DECODED BETA WEIGHTS, AND THE FOURTH BY THE SIZE OF THE PARTIAL CORRELATION COEFFICIENT (VARIABLES HAVE BEEN ABBREVIATED)

| DEFINITION OF OF IMPORTANCE | ORDER OF APPEARANCE | DECODED BETA WEIGHTS | PARTIAL CORR. COEFFICIENT |
|---|---|---|---|
| MEMORY 2 | MEMORY 2 | NOUNS | MEMORY 2 |
| NOUNS | ORDER 1 | LENGTH | ORDER 1 |
| NUM'R TYP2 | VERBAL CL2 | DEPTH 1 | VERBAL CL2 |
| LENGTH | PRON:NOUN | VERBS | OPERATIONS |
| MIS'G DATA | PRONOUNS | MEMORY 2 | PRON:NOUN |
| VERBS | DISTRACTOR | MEMORY 1 | VERB:ADVRB |
| DEPTH 1 | NUM'R TYP2 | MIS'G DATA | NOUNS |
| MEMORY 1 | NOUN:VERB | PRON:NOUN | DISTRACTOR |
| CONVERSION | PRON:ADJTS | NUM'R TYP2 | DEPTH 1 |
| DISTRACTOR | MIS'G DATA | CONVERSION | NUM'R TYP2 |
| NOUN:VERB | CONVERSION | NOUN:VERB | SUP'S DATA |
| VERBAL CL1 | ADVERBS | PRONOUNS | ADVERBS |
| PRON:NOUN | DEPTH 1 | DISTRACTOR | PRON:ADJTS |
| PRONOUNS | LENGTH | PRON:ADJTS | NOUN:VERB |
| REUSE DATA | NOUNS | VERBAL CL1 | VERBS |
| PRON:ADJTS | VERBS | REUSE DATA | MIS'G DATA |
| DEPTH 2 | MEMORY 1 | VERBAL CL2 | DIVISION |
| NUM'R TYP1 | VERBAL CL1 | OPERATIONS | CONVERSION |
| VERB:ADVRB | REUSE DATA | DEPTH 2 | VERBAL CL1 |
| OPERATIONS | NUM'R TYP1 | NUM'R TYP1 | MEMORY 1 |
| VERBAL CL2 | DEPTH 2 | VERB:ADVRB | DEPTH 2 |
| HIERARCHY | OPERATIONS | ADVERBS | LENGTH |
| ADVERBS | VERB:ADVRB | STEPS | NUM'R TYP1 |
| DIVISION | HIERARCHY | HIERARCHY | ORDER 2 |
| STEPS | DIVISION | ORDER 2 | PRONOUNS |
| ORDER 1 | SEQUENT'L | DIVISION | SEQUENT'L |
| SEQUENT'L | STEPS | ORDER 1 | STEPS |
| ORDER 2 | ORDER 2 | SEQUENT'L | REUSE DATA |
| SUP'S DATA | SUP'S DATA | SUP'S DATA | HIERARCHY |
| ADJECTIVES | ADJECTIVES | ADJECTIVES | ADJECTIVES |

TABLE 13

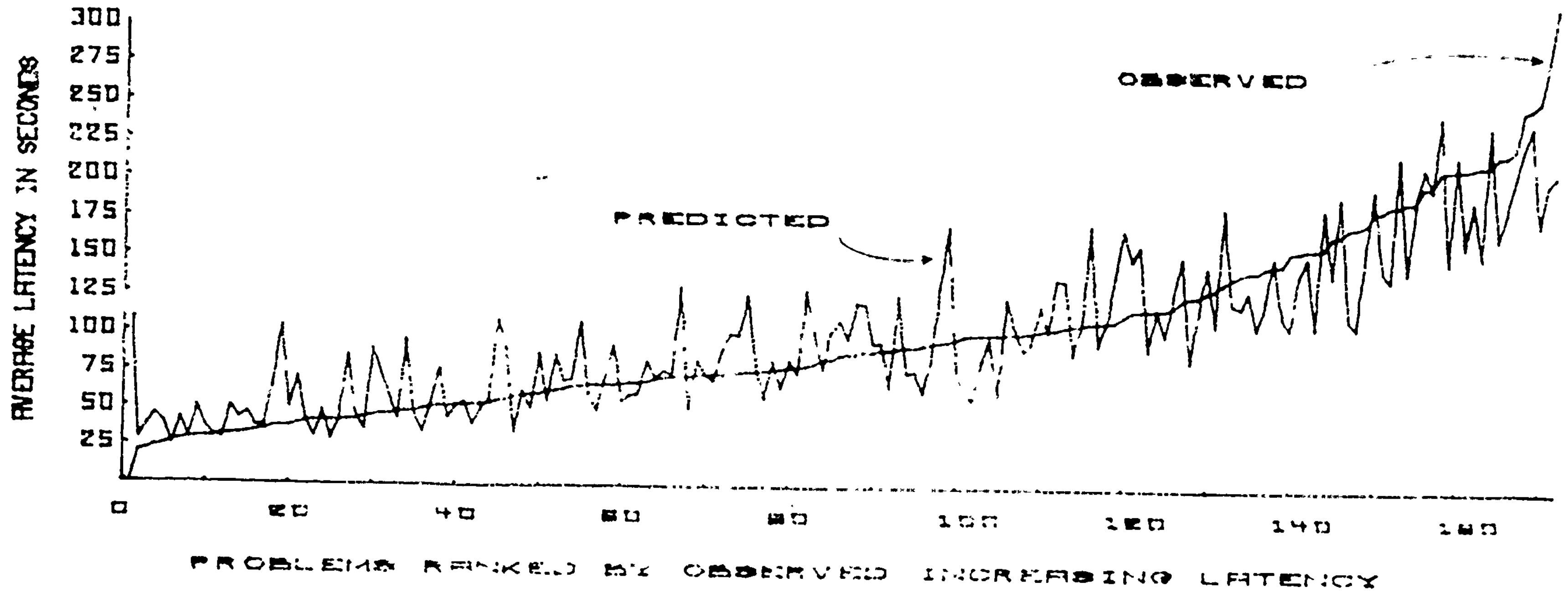
REGRESSION COEFFICIENTS, STANDARD ERRORS, COMPUTED T-VALUES, AND R-SQUARED DROP CAUSED BY THE REMOVAL OF EACH VARIABLE IN TURN FOR OBJECTIVE 4 ( 172 PROBLEMS, AND 30 VARIABLE)

| VARIABLE | REGRESSION COEFFICIENT | STANDARD ERROR | COMPUTED T-VALUE | RSQ DROP |
|---|---|---|---|---|
| 0 CONSTANT | -1.81545 | | | |
| 1 OPERATIONS | 1.3915 | 2.0236 | 0.6876 | 0.0008 |
| 2 DIVISION | 11.2315 | 6.5496 | 1.7148 | 0.0050 |
| 3 HIERARCHY | 0.2804 | 0.7035 | 0.3985 | 0.0002 |
| 4 STEPS | -2.4272 | 4.0291 | -0.6024 | 0.0006 |
| 5 LENGTH | -0.1236 | 0.1349 | -0.9160 | 0.0014 |
| 6 DEPTH 1 | 0.1255 | 0.1400 | 0.8967 | 0.0003 |
| 7 DEPTH 2 | -1.9487 | 6.8071 | -0.2863 | 0.0001 |
| 8 VERBS | 1.2650 | 2.1774 | 0.5810 | 0.0005 |
| 9 ADJECTIVES | -0.5891 | 1.4608 | -0.4033 | 0.0002 |
| 10 NOUNS | 2.0563 | 1.3631 | 1.5085 | 0.0036 |
| 11 ADVERBS | 6.2436 | 3.7602 | 1.6604 | 0.0047 |
| 12 PRONOUNS | -2.8404 | 1.7534 | -1.6200 | 0.0010 |
| 13 NOUNS:VERBS | 1.7998 | 3.5783 | 0.5030 | 0.0004 |
| 14 NOUNS:ADJECTIVES | -2.8388 | 2.0026 | -1.4176 | 0.0034 |
| 15 PRONOUNS:NOUNS | 0.0000 | 0.0000 | 1.0000 | 0.0000 |
| 16 VERBS:ADVERBS | -0.4220 | 1.7973 | -0.2348 | 0.0000 |
| 17 SEQUENTIAL | -8.1001 | 5.7798 | -1.4014 | 0.0033 |
| 18 ORDER 1 | 11.5196 | 7.3092 | 1.5760 | 0.0042 |
| 19 ORDER 2 | -2.7088 | 4.2556 | -0.6365 | 0.0006 |
| 20 VERBAL CLUE 1 | 17.5574 | 7.6031 | 2.3092 | 0.0090 |
| 21 VERBAL CLUE 2 | -0.9429 | 3.2865 | -0.2869 | 0.0001 |
| 22 DISTRACTORS | 10.8660 | 4.3330 | 2.5077 | 0.0106 |
| 23 SUPERFLUOUS DATA | 7.4960 | 7.8964 | 0.9493 | 0.0014 |
| 24 MISSING DATA | 5.1643 | 10.5370 | 0.4901 | 0.0003 |
| 25 REUSED DATA | 24.1036 | 9.0717 | 2.6570 | 0.0120 |
| 26 NUMBER TYPE 1 | -6.4511 | 7.2375 | -0.8913 | 0.0013 |
| 27 NUMBER TYPE 2 | 17.2618 | 8.5853 | 2.0106 | 0.0069 |
| 28 CONVERSION | -1.5282 | 10.2591 | -0.1490 | 0.0000 |
| 29 MEMORY 1 | 9.0147 | 3.2847 | 2.7444 | 0.0128 |
| 30 MEMORY 2 | 6.8064 | 2.5960 | 2.6219 | 0.0117 |

TABLE 14

SEQUENTIAL APPEARANCE OF THE VARIABLES INTO THE REGRESSION EQUATION FOR OBJECTIVE 4. ALSO SHOWN ARE THE MULTIPLE R, R-SQUARED, AND THE INCREASE IN R-SQUARED (172 PROBLEMS, AND 30 VARIABLES)

| VARIABLE | MULTIPLE R | R SQUARED | INCREASE IN RSQ |
|---|---|---|---|
| 29 MEMORY 1 | 0.6853 | 0.4696 | 0.4696 |
| 30 MEMORY 2 | 0.7570 | 0.5730 | 0.1034 |
| 18 ORDER 1 | 0.7875 | 0.6202 | 0.0471 |
| 2 DIVISION | 0.8077 | 0.6524 | 0.0322 |
| 11 ADVERBS | 0.8248 | 0.6803 | 0.0279 |
| 25 REUSED DATA | 0.8349 | 0.6971 | 0.0168 |
| 22 DISTRACTORS | 0.8438 | 0.7120 | 0.0149 |
| 10 NOUNS | 0.8489 | 0.7206 | 0.0086 |
| 27 NUMBER TYPE 2 | 0.8527 | 0.7271 | 0.0065 |
| 23 SUPERFLUOUS DATA | 0.8558 | 0.7324 | 0.0053 |
| 12 PRONOUNS | 0.8584 | 0.7369 | 0.0045 |
| 20 VERBAL CLUE 1 | 0.8614 | 0.7420 | 0.0052 |
| 17 SEQUENTIAL | 0.8634 | 0.7455 | 0.0034 |
| 14 NOUNS:ADJECTIVES | 0.8647 | 0.7477 | 0.0022 |
| 5 LENGTH | 0.8659 | 0.7498 | 0.0021 |
| 6 DEPTH 1 | 0.8668 | 0.7513 | 0.0016 |
| 4 STEPS | 0.8676 | 0.7527 | 0.0014 |
| 1 OPERATIONS | 0.8682 | 0.7538 | 0.0010 |
| 26 NUMBER TYPE 1 | 0.8689 | 0.7550 | 0.0012 |
| 19 ORDER 2 | 0.8692 | 0.7555 | 0.0005 |
| 24 MISSING DATA | 0.8693 | 0.7557 | 0.0002 |
| 3 HIERARCHY | 0.8694 | 0.7559 | 0.0002 |
| 9 ADJECTIVES | 0.8696 | 0.7562 | 0.0003 |
| 7 DEPTH 2 | 0.8696 | 0.75[illegible] | 0.0001 |
| 8 VERBS | 0.8697 | 0.7564 | 0.0002 |
| 13 NOUNS:VERBS | 0.8699 | 0.7567 | 0.0003 |
| 21 VERBAL CLUE 2 | 0.8700 | 0.7569 | 0.0002 |
| 16 VERBS:ADJECTIVES | 0.8700 | 0.7569 | 0.0001 |
| 28 CONVERSION | 0.8700 | 0.7569 | 0.0000 |
| 15 PRONOUNS:NOUNS | 0.8700 | 0.7569 | 0.0000 |



AVERAGE LATENCY IN SECONDS
300
275
250
225
200
175
150
125
100
75
50
25
0
20
40
60
80
100
120
140
160
PROBLEMS RANKED BY OBSERVED INCREASING LATENCY
OBSERVED
PREDICTED

TABLE 16

ORDER OF IMPORTANCE OF THE 30 STRUCTURAL VARIABLES AS MEASURED BY FOUR CRITERIA OF IMPORTANCE. THE FIRST OF THESE IS THE CRITERION OF IMPORTANCE AS MEASURED BY THE DEFINITION, THE SECOND IS THE ORDER OF APPEARANCE OF THE VARIABLES INTO THE REGRESSION EQUATION, THE THIRD, BY THE DECODED BETA WEIGHTS, AND THE FOURTH BY THE SIZE OF THE PARTIAL CORRELATION COEFFICIENT (VARIABLES HAVE BEEN ABBREVIATED)

| DEFINITION OF OF IMPORTANCE | ORDER OF APPEARANCE | DECODED BETA WEIGHTS | PARTIAL CORR. COEFFICIENT |
|---|---|---|---|
| MEMORY 1 | MEMORY 1 | REUSE DATA | MEMORY 1 |
| REUSE DATA | MEMORY 2 | NUM'R TYP2 | MEMORY 2 |
| MEMORY 2 | ORDER 1 | VERBAL CL1 | ORDER 1 |
| DISTRACTOR | DIVISION | ORDER 1 | DIVISION |
| VERBAL CL1 | ADVERBS | DIVISION | ADVERBS |
| NUM'R TYP2 | REUSE DATA | SUP'S DATA | REUSE DATA |
| DIVISION | DISTRACTOR | MIS'G DATA | DISTRACTOR |
| ADVERBS | NOUNS | DISTRACTOR | NOUNS |
| ORDER 1 | NUM'R TYP2 | SEQUENT'L | NUM'R TYP2 |
| NOUNS | SUP'S DATA | NUM'R TYP1 | VERBAL CL1 |
| PRON:ADJTS | PRONOUNS | MEMORY 1 | SUP'S DATA |
| SEQUENT'L | VERBAL CL1 | ADVERBS | PRONOUNS |
| LENGTH | SEQUENT'L | MEMORY 2 | SEQUENT'L |
| SUP'S DATA | PRON:ADJTS | CONVERSION | PRON:ADJTS |
| NUM'R TYP1 | LENGTH | DEPTH 2 | LENGTH |
| PRONOUNS | DEPTH 1 | ORDER 2 | DEPTH 1 |
| OPERATIONS | STEPS | STEPS | STEPS |
| STEPS | OPERATIONS | NOUN:VERB | OPERATIONS |
| ORDER 2 | NUM'R TYP1 | PRON:ADJTS | NUM'R TYP1 |
| VERBS | ORDER 2 | PRONOUNS | ORDER 2 |
| NOUN:VERB | MIS'G DATA | VERBAL CL2 | NOUN:VERB |
| DEPTH 1 | HIERARCHY | OPERATIONS | MIS'G DATA |
| MIS'G DATA | ADJECTIVES | NOUNS | ADJECTIVES |
| HIERARCHY | DEPTH 2 | VERBS | HIERARCHY |
| ADJECTIVES | VERBS | ADJECTIVES | VERBAL CL2 |
| DEPTH 2 | NOUN:VERB | VERB:ADVRB | DEPTH 2 |
| VERBAL CL2 | VERBAL CL2 | HIERARCHY | VERBS |
| PRON:NOUN | VERB:ADVRB | DEPTH 1 | VERB:ADVRB |
| VERB:ADVRB | CONVERSION | LENGTH | CONVERSION |
| CONVERSION | PRON:NOUN | PRON:NOUN | PRON:NOUN |

TABLE 17

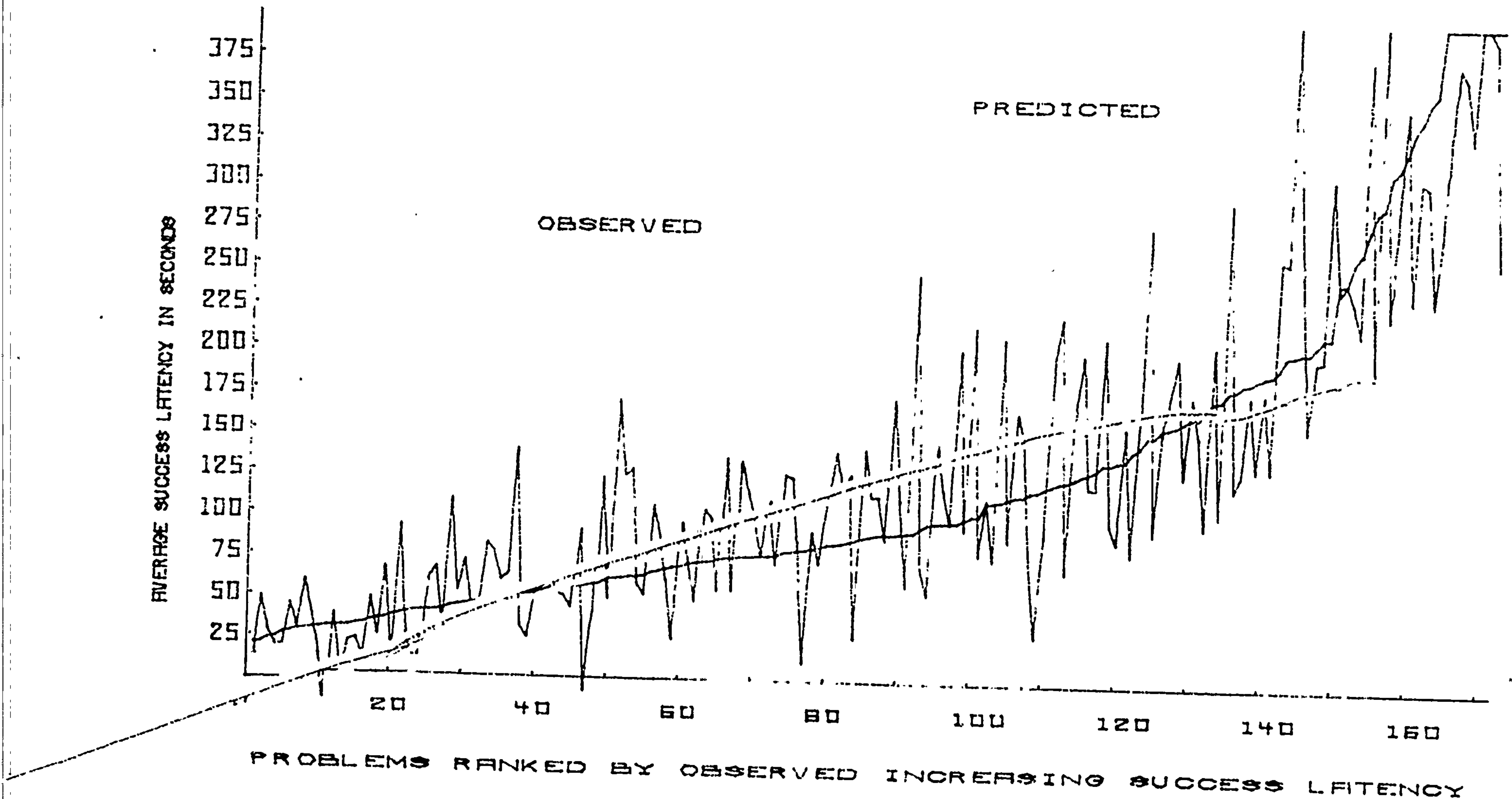
REGRESSION COEFFICIENTS, STANDARD ERRORS, COMPUTED T-VALUES, AND R-SQUARED DROP CAUSED BY THE REMOVAL OF EACH VARIABLE IN TURN FOR OBJECTIVE 5 ( 172 PROBLEMS, AND 30 VARIABLE)

| VARIABLE | REGRESSION COEFFICIENT | STANDARD ERROR | COMPUTED T-VALUE | RSQ DROP |
|---|---|---|---|---|
| 0 CONSTANT | -7.87572 | | | |
| 1 OPERATIONS | -4.2436 | 4.3395 | -0.9779 | 0.0018 |
| 2 DIVISION | 29.4222 | 14.4636 | 2.0342 | 0.0080 |
| 3 HIERARCHY | -0.7129 | 1.5316 | -0.4655 | 0.0004 |
| 4 STEPS | 1.7242 | 8.5338 | 0.2020 | 0.0001 |
| 5 LENGTH | 0.0000 | 0.0000 | 1.0000 | 0.0000 |
| 6 DEPTH 1 | -0.1096 | 0.2949 | -0.3717 | 0.0003 |
| 7 DEPTH 2 | -15.4948 | 15.0388 | -1.0303 | 0.0021 |
| 8 VERBS | 9.1322 | 4.6566 | 1.9611 | 0.0070 |
| 9 ADJECTIVES | 0.0000 | 0.0000 | 1.0000 | 0.0000 |
| 10 NOUNS | -4.5618 | 2.3356 | -1.9532 | 0.0046 |
| 11 ADVERBS | 15.5051 | 7.9277 | 1.9558 | 0.0068 |
| 12 PRONOUNS | 8.5458 | 7.3464 | 1.1633 | 0.0027 |
| 13 NOUNS:VERBS | 10.9615 | 7.6996 | 1.4237 | 0.0037 |
| 14 NOUNS:ADJECTIVES | -7.2293 | 3.0029 | -2.4075 | 0.0050 |
| 15 PRONOUNS:NOUNS | -127.5630 | 90.6954 | -1.4065 | 0.0039 |
| 16 VERBS:ADVERBS | -0.7102 | 3.9182 | -0.1812 | 0.0001 |
| 17 SEQUENTIAL | -9.6372 | 12.5878 | -0.7656 | 0.0011 |
| 18 ORDER 1 | 26.4054 | 16.0919 | 1.6409 | 0.0053 |
| 19 ORDER 2 | 1.2532 | 9.2653 | 0.1353 | 0.7199 |
| 20 VERBAL CLUE 1 | 29.9320 | 16.9363 | 1.7673 | 0.0061 |
| 21 VERBAL CLUE 2 | 4.6386 | 7.2309 | 0.6415 | 0.0008 |
| 22 DISTRACTORS | 8.9944 | 9.4294 | 0.9539 | 0.0017 |
| 23 SUPERFLUOUS DATA | -8.7771 | 17.3760 | -0.5051 | 0.0005 |
| 24 MISSING DATA | 28.3436 | 23.1524 | 1.2242 | 0.0029 |
| 25 REUSED DATA | 31.9389 | 19.8707 | 1.6073 | 0.0050 |
| 26 NUMBER TYPE 1 | -11.7147 | 15.5997 | -0.7510 | 0.0010 |
| 27 NUMBER TYPE 2 | 62.8473 | 18.9060 | 3.3242 | 0.0215 |
| 28 CONVERSION | 15.5708 | 22.6403 | 0.6877 | 0.0010 |
| 29 MEMORY 1 | 12.5838 | 7.1886 | 1.7505 | 0.0059 |
| 30 MEMORY 2 | 28.6438 | 5.7153 | 5.0117 | 0.0489 |

TABLE 18

SEQUENTIAL APPEARANCE OF THE VARIABLES INTO THE REGRESSION EQUATION FOR OBJECTIVE 5. ALSO SHOWN ARE THE MULTIPLE R, R-SQUARED, AND THE INCREASE IN R-SQUARED (172 PROBLEMS, AND 30 VARIABLES)

| VARIABLE | MULTIPLE R | R SQUARED | INCREASE IN RSQ |
|---|---|---|---|
| 30 MEMORY 2 | 0.6832 | 0.4668 | 0.4667 |
| 1 OPERATIONS | 0.7336 | 0.5382 | 0.0714 |
| 27 NUMBER TYPE 2 | 0.7644 | 0.5843 | 0.0461 |
| 11 ADVERBS | 0.7902 | 0.6244 | 0.0401 |
| 21 VERBAL CLUE 2 | 0.8067 | 0.6508 | 0.0263 |
| 18 ORDER 1 | 0.8164 | 0.6665 | 0.0157 |
| 24 MISSING DATA | 0.8226 | 0.6767 | 0.0102 |
| 14 NOUNS:ADJECTIVES | 0.8263 | 0.6828 | 0.0061 |
| 2 DIVISION | 0.8295 | 0.6881 | 0.0053 |
| 29 MEMORY 1 | 0.8334 | 0.6946 | 0.0065 |
| 20 VERBAL CLUE 1 | 0.8354 | 0.6979 | 0.0033 |
| 7 DEPTH 2 | 0.8372 | 0.7009 | 0.0030 |
| 22 DISTRACTORS | 0.8381 | 0.7024 | 0.0015 |
| 25 REUSED DATA | 0.8392 | 0.7043 | 0.0018 |
| 10 NOUNS | 0.8401 | 0.7058 | 0.0015 |
| 8 VERBS | 0.8414 | 0.7080 | 0.0022 |
| 13 NOUNS:VERBS | 0.8433 | 0.7112 | 0.0032 |
| 17 SEQUENTIAL | 0.8444 | 0.7130 | 0.0019 |
| 26 NUMBER TYPE 1 | 0.8451 | 0.7142 | 0.0012 |
| 15 PRONOUNS:NOUNS | 0.8457 | 0.7152 | 0.0010 |
| 12 PRONOUNS | 0.8468 | 0.7171 | 0.0019 |
| 28 CONVERSION | 0.8474 | 0.7181 | 0.0010 |
| 23 SUPERFLUOUS DATA | 0.8479 | 0.7189 | 0.0008 |
| 3 HIERARCHY | 0.8482 | 0.7194 | 0.0005 |
| 6 DEPTH 1 | 0.8483 | 0.7196 | 0.0002 |
| 4 STEPS | 0.8484 | 0.7198 | 0.0002 |
| 16 VERBS:ADJECTIVES | 0.8484 | 0.7198 | 0.0001 |
| 28 ORDER 2 | 0.8485 | 0.7200 | 0.0001 |
| 5 LENGTH | 0.8485 | 0.7200 | 0.0000 |
| 9 ADJECTIVES | 0.8485 | 0.7200 | 0.0000 |



AVERAGE SUCCESS LATENCY IN SECONDS
375
350
325
300
275
250
225
200
175
150
125
100
75
50
25
OBSERVED
PREDICTED
20
40
60
80
100
120
140
160
PROBLEMS RANKED BY OBSERVED INCREASING SUCCESS LATENCY

TABLE 20

ORDER OF IMPORTANCE OF THE 30 STRUCTURAL VARIABLES AS MEASURED BY FOUR CRITERIA OF IMPORTANCE. THE FIRST OF THESE IS THE CRITERION OF IMPORTANCE AS MEASURED BY THE DEFINITION, THE SECOND IS THE ORDER OF APPEARANCE OF THE VARIABLES INTO THE REGRESSION EQUATION, THE THIRD, BY THE DECODED BETA WEIGHTS, AND THE FOURTH BY THE SIZE OF THE PARTIAL CORRELATION COEFFICIENT (VARIABLES HAVE BEEN ABBREVIATED)

| DEFINITION OF OF IMPORTANCE | ORDER OF APPEARANCE | DECODED BETA WEIGHTS | PARTIAL CORR. COEFFICIENT |
|---|---|---|---|
| ORDER 2 | MEMORY 2 | MEMORY 2 | MEMORY 2 |
| MEMORY 2 | OPERATIONS | VERBS | OPERATIONS |
| NUM'R TYP2 | NUM'R TYP2 | NOUNS | NUM'R TYP2 |
| DIVISION | ADVERBS | MEMORY 1 | ADVERBS |
| VERBS | VERBAL CL2 | NUM'R TYP2 | VERBAL CL2 |
| ADVERBS | ORDER 1 | ADVERBS | ORDER 1 |
| VERBAL CL1 | MIS'G DATA | PRON:ADJTS | MIS'G DATA |
| MEMORY 1 | PRON:ADJTS | DIVISION | MEMORY 1 |
| ORDER 1 | DIVISION | PRON:NOUN | PRON:ADJTS |
| PRON:ADJTS | MEMORY 1 | NOUN:VERB | DIVISION |
| REUSE DATA | VERBAL CL1 | PRONOUNS | VERBAL CL1 |
| NOUNS | DEPTH 2 | ORDER 1 | NOUN:VERB |
| PRON:NOUN | DISTRACTOR | REUSE DATA | DEPTH 2 |
| NOUN:VERB | REUSE DATA | VERBAL CL1 | VERBS |
| MIS'G DATA | NOUNS | MIS'G DATA | SEQUENT'L |
| PRONOUNS | VERBS | OPERATIONS | REUSE DATA |
| DEPTH 2 | NOUN:VERB | DEPTH 2 | PRONOUNS |
| OPERATIONS | SEQUENT'L | DISTRACTOR | DISTRACTOR |
| DISTRACTOR | NUM'R TYP1 | CONVERSION | NOUNS |
| SEQUENT'L | PRON:NOUN | VERBAL CL2 | NUM'R TYP1 |
| NUM'R TYP1 | PRONOUNS | NUM'R TYP1 | PRON:NOUN |
| CONVERSION | CONVERSION | DEPTH 1 | CONVERSION |
| VERBAL CL2 | SUP'S DATA | SEQUENT'L | SUP'S DATA |
| SUP'S DATA | HIERARCHY | SUP'S DATA | HIERARCHY |
| HIERARCHY | DEPTH 1 | HIERARCHY | DEPTH 1 |
| DEPTH 1 | STEPS | STEPS | STEPS |
| STEPS | VERB:ADVRB | VERB:ADVRB | VERB:ADVRB |
| VERB:ADVRB | ORDER 2 | ORDER 2 | ORDER 2 |
| LENGTH | LENGTH | LENGTH | LENGTH |
| ADJECTIVES | ADJECTIVES | ADJECTIVES | ADJECTIVES |

APPENDIX 2

# COMPUTER ASSISTED PROBLEM SOLVING ( C A P S ) AT GOLDEN WEST COLLEGE

ANGELO SEGALLA

The CAPS program was designed to augment the mathematics curriculum at Golden West College. Its existence is a result of a mutual interest on the parts of the college administration and the mathematics staff to foster innovation in the learning process. CAPS was supported by four (4) general objectives:

1. To appropriate the computer in the curriculum as an innovative tool,

2. To provide instructors with a complete chronicle of each student's path through the program reflecting his thought patterns on each word problem;

3. To provide instructors with an item-analysis option of each word problem in the program; and

4. To identify and define structural variables for each problem in an effort to predict the difficulty level of the problem (difficulty level derived from regression analysis).

The CAPS program is in many ways similar to the pioneering CAI work conducted by Professor Suppes at Stanford University (Suppes, Jerman and Bryan, 1968), but it does offer several important differences, reflecting the fact that it was coded independently of the Stanford materials. The most obvious

difference is the language: CAPS was coded in APL, the Stanford program is in BASIC. Further, the analysis functions, the heart of the CAPS program, seem to be a great deal more sophisticated in their ability to trap even the most minute bits of response data, such as the number of seconds it takes student I to accomplish step J in problem K, and the exact steps he used in achieving his answer. The purpose here is not to compare the two programs, especially in view of the fact that our knowledge of the Stanford materials is limited to the information published by Professor Suppes and his associates, and these publications usually do not refer to the internal workings of the programs.

HARDWARE

The problems in CAPS are presented to and solved by students via IBM 2741 terminals located in the Multi-Media Center and the Math-Science building at Golden West College. Thirty-four (34) terminals are available to students daily from eight a.m. to ten p.m. The terminals are hardwired to an IBM 370/155 computer located ten miles away at Orange Coast College, Costa Mesa, California. Orange Coast College is the "sister" college to Golden West, and both comprise the Coast Community College District.

SOFTWARE

The CAPS program is coded in APL, a new, interactive language, highly mathematically oriented, yet readily adaptable to computer assisted instruction; a spinoff not anticipated by the creator of the language, Dr. Kenneth Iverson, presently with IBM. Mr. John R. Clark, CAI consultant to the Coast Community College District, coded the functions with direction from the author. Figure 1 displays 9 of the thirty-six (36) functions used in the CAPS program.

FIG. 1

```
      ∇ALGCHECK[⎕]∇
    ∇ Z←ALGCHECK ST;MV;Q;OPCODE;SW
[1]   SW←1+[illegible](ρST←ST,' ')ρ[illegible]←0×ρOPCODE←'+-×÷'
[2]   MV[1]←~ST[1]∊'(',ALP,'0123456789.'
[3]   MV[Q]←MV[Q]∨~ST[1+Q←(ST∊OPCODE)/[illegible]ιρST]∊'(',[illegible]ALP,'0123456789.'
[4]   MV[Q]←MV[Q]∨~ST[1+Q←¯1↓(ST∊[illegible]ALP)/[illegible]]∊' )',OPCODE
[5]   MV[Q]←MV[Q]∨~ST[[illegible](ST∊'0123456789.')/[illegible]]∊' )',OPCODE,'0123456789./'
[6]   MV[Q]←MV[Q]∨~ST[1+Q←(ST='(')/[illegible]]∊'(',ALP,'0123456789.'
[7]   →11×ι~Z←∨/MV←MV PARMATCH ST
[8]   SW/'THIS IS INCORRECT PLEASE REENTER'
[9]   ' ↓'[1+SW\¯1↓MV]
[10]  →0,ρ⎕←SW\¯1↓ST
[11]  MV←(ρINS←INS,' ')ρ0
[12]  MV[Q]←MV[Q]∨~INS[Q← ¯1 1 ∘.+(INS='/')/ιρINS]∊'0123456789'
[13]  →0×ι~Z←∨/MV
[14]  'YOU ARE NOT ALLOWED SPACES AROUND THE ''/'' WHEN REPRESENTING FRACTIONS.
      '
[15]  ' ↓'[1+MV]
[16]  INS
[17]  ⍝THIS FUNCTION CHECKS STUDENT INPUT FOR ALGEBRAIC CORRECTNESS
[18]  ⍝OPERATORS BETWEEN OPERANDS
[19]  ⍝MATCHED PARENTHESIS  ETC
    ∇

      ∇APOLISH[⎕]∇
    ∇ W←APOLISH I;L
[1]   W←((~I∊'()')/I)[⍋1,,⍉(2,ρL)ρL,(~W∊L)/W←1+ι2×ρL←(¯1↓L+ρL)-⌊2⍟¯1+2⊥⍉W∧L∘.<L
      ←Φ⍋⍋Φ-2⊥⍉(L∘.≥L)∧W←W∘.<W←ιρL←((I∊OPS)/(RANK,0)[OPSιI]+(W∘.≥W←ιρI)+.×
      99 ¯99 +.×'()'∘.=I),0]
[2]   ⍝THIS PROGRAM CONVERTS THE ALGEBRAIC STRING TO POLISH NOTATION
[3]   ⍝THIS PROGRAM DEVELOPED BY WILEY GRINER
    ∇

      ∇CAPS[⎕]∇
    ∇ CAPS XZ;K;LCH;Q;H1;H2;H3
[1]   →10×ι¯23=W,0ρQ←ι0
[2]   →11×ιQ[1]≠' '
[3]   INITIAL 1
[4]   →6×ιW=DQ 0
[5]   COMANDρQ←Q,TV[1+2↑ 60 60 60 60 ⊤I20]
[6]   SVMSG
[7]   ''
[8]   W←¯23
[9]   SAVEMESSAGE
[10]  →0,ρ⎕←'ALL THE PROBLEMS IN THIS SET HAVE BEEN ANSWERED.  COME BACK
      NEXT WEEK WHEN THERE IS A NEW SET OF CAPS .'
[11]  →4×ι(I29)=10⊥¯1+'0123456789'ιQ[2 3 4 5 6 7]
[12]  'WARNING:   YOU ARE NOT SIGNED ON TO YOUR PROPER NUMBER'
[13]  '
      PLEASE CALL THE LAB ASSISTANT IMMEDIATELY!'
[14]  →
    ∇
```

Fig 1 (Continued)

```
     ∇APOLISH[⎕]∇
     ∇ W←APOLISH I;L
[1]    W←((~I∊'()')/I)[⍋1,,⍉(2,ρL)ρL,(~W∊L)/W←1+⍳2×ρL←(¯1↓L+∇ρL)-⌊2⍟
       ¯1↓2⊥⍉W∧L∘.<L←⌽⍋⍋⌽-2⊥⍉(L∘.≥L)∧W←W∘.<W←⍳ρL←((I∊OPS)/(RANK,0)[OPS⍳I]+(
       W∘.≥W←⍳ρI)+.× 99 ¯99 +.×'()'∘.=I),0]
     ∇

     ∇ANALYSIS[⎕]∇
     ∇ ANALYSIS;INS;T;PS;T;ST;P
[1]    →(2 3 15)[1+VALCK INS←(' '≠INS)/INS←(1↑('⍞'=1↑INS)/2)↓INS,0ρT←(⌶
       20)-T,0ρINS←⍞,0ρT←(⌶20),0ρ⍞←'[] ']
[2]    →1,ρQ←Q,(CVTM T),'⍞',INS
[3]    →ANS×⍳(2=ρINS)∧('*'=¯1↑INS)∧(1↑INS)∊27↑ALP
[4]    →2×⍳ALCHECK INS
[5]    Q←Q,(CVTM T),INS←INS,(0=ρINS)/'0'
[6]    SETNUM
[7]    ST←(P←32)ρ0
[8]    GOEVAL PS←APOLISH INS
[9]    →1
[10] ANS:→14×⍳VAL[ALP⍳1↑INS]≠VAL[1]
[11]   CHAT[?8;];',  YOU GOT THAT ONE'
[12]   '

       '
[13]   →0,ρQ←Q,(CVTM T),INS,'⌽'
[14]   WHAT[?6;];',  YOU MISSED THIS ONE.'
[15]   →23×⍳1=HP←HP+1
[16]   Q←Q,(CVTM T),'?',INS
[17]   →(0 18 20 22)[HP]
[18]   'LETS REREAD THE PROBLEM.'
[19]   →1,ρ⍞←H1
[20]   'LOOK'
[21]   →1,ρ⍞←H2
[22]   H3
[23]   Q←Q,(CVTM T),INS,'Δ'
[24]   '

       '
     ∇

     ∇NEWLIT[⎕]∇
     ∇ Z←NEWLIT A;B
[1]    Z←(¯1*1ρA∊'-¯')×(10*1+B-(⌽A)⍳'.')×10⊥¯1+'0123456789'⍳(A∊'0123456789'
       )/A←A,(~'.'∊A←(¯1+A⍳'E')ρA)ρ'.',0ρB←(¯1*1ρB∊'¯-')×10⊥¯1+'0123456789'
       ⍳(1ρB∊'¯-')↓B←(A⍳'E')↓A,(2×~'E'∊A←(A∊'0123456789.¯-E')/A)ρ'00'
     ∇
```

Fig 1 (continued)

```
      ∇PRESENT[⎕]∇
    ∇ PRESENT X;I;L;J;V
[1]   K←(V←¯1↓2⌈((DQ X)+ι-/DQ(X+1),X])ιS,0ρLCH←H1←H2←H3←,' '
[2]   V←K↓V,0ρ⎕←K↑V
[3]   VAL[1]←NEWLIT ¯1↓(K←VιS)↑V,0ρVAL←32ρ0×I←2
[4]   I←2
[5]   →((0=ρV),(HP←'Δ'=1↑V←K↓V))/ 8
[6]   '                    ';(AL⌈[I]);'← ';VAL[I]←NEWLIT ¯1↓(K←VιS)↑V
[7]   →5,I←I+ρρLCH←LCH,ALP[I]
[8]   H1←1↓(K←(1↓V)ι'Δ')↑V
[9]   H2←1↓(K←(1↓V)ι'Δ')↑V←K↓V
[10]  H3←1↓K↓V
    ∇


      ∇PARMATCH[⎕]∇
    ∇ Z←PARMATCH ST;K;L
[1]   →0×ι∧/0=Z←(ST∘.='()')+.× 1 ¯1
[2]   →5×ι(K>ρZ)∨K<L←(+\(K←Zι¯1)↑Z)ι1
[3]   →2×ι~∧/0=Z,0ρZ[(1+K-L),K]←0
[4]   →0
[5]   SW←∨/MV
[6]   'PARENTHESIS MISSMATCH ',SW/'AND'
[7]   Z[]←0
[8]   →(2÷I26)×ιK<ρZ
[9]   →0,Z[1+K-L]←1
[10]  Z[K]←1
    ∇


      ∇VALCK[⎕]∇
    ∇ Z←VALCK INS
[1]   →6×ι~∧/('STOP'=4↑INS)∨('→'=1↑INS)∨HR←('?'=1↑INS)∨∧/'HINT'=4↑INS
[2]   →14
[3]   Q[I+ιρQ]←Q←Q,(CVTN T),TV[1+2↑ 60 60 60 60 TI20]
[4]   I←I+ρQ
[5]   SAVEMESSAGE
[6]   →11×ι(26=ρLCH)∨(100≤ρQ)∨32≤+/INS∊ALP
[7]   →0×ιZ←∧/V←INS∊LCH,'+-×÷()0123456789.*'
[8]   'THIS IS INCORRECT PLEASE REENTER'
[9]   (~V)\'↓'
[10]  →0,ρ⎕←INS
[11]  'YOU ARE PLAYING WITH MY BUTTONS AND NOT WORKING THE PROBLEM'
[12]  'I AM GOING TO SIGN YOU OFF AND WE CAN RESTART AGAIN NEXT TIME.'
[13]  →3,ρQ[1]←'⍞'
[14]  →16×ιHP
[15]  →3,ρQ[1]←'S'
[16]  →(17 18)[1+0≠HP]
[17]  →Z←0×ρ⎕←'THERE ARE NO HINTS FOR THIS PROBLEM'
[18]  Z←2
    ∇
```

PRESENTATION

The CAPS program has now been operational for three years, accummulating a total of 6000 on line student-hours. The program is used almost exclusively by students enrolled in Math 005, the remedial mathematics course at both Golden West and Orange Coast Colleges.

There are eight units in the CAPS program corresponding to the eight chapters in the Math 005 syllabus. Each unit consists of twenty (20) word problems and deals with the subject matter presented in class and on the audio-tutorial and video tapes. Thus all media are synchronized to present the same concepts over the duration of two weeks.

A student who is ready to do the CAPS program for any one unit reports to the Multi-Media Center or the computer room in the Math-Science Building, signs on to the computer and calls for the program appropriate to that unit. The interaction for any given problem is illustrated in Figure 2. Student responses are always preceded by the symbol "□", called "quad." Everything else is computer generated. In the first problem of Figure 2, the computer typed everything up to the first quad. The student then typed B - C and returned the carriage, giving control back to the computer. The latter in turn responded with D← 40. The student, in turn, typed C ÷ B. This time the computer came back with E←.2222345686. The student typed E x 100; the computer responded with F← 22.22345686. Note that the computer does not commit itself to recognize the answer; the student must command it to recognize the answer by typing a single letter followed by an asterisk (F*).

The forte of the CAPS program is its ability to trap student responses and

FIG 2

```
15
A CALCOLO 400 DESK CALCULATOR HAS BEEN REDUCED FROM 179.99
TO 139.99 DOLLARS. WHAT IS THE PERCENT OF DISCOUNT?

                A← 400
                B← 179.99
                C← 139.99
⎕ B-C
                D← 40
⎕ D÷B
                E← 0.2222345686
⎕ E×100
                F← 22.22345686
⎕ F*
CORRECT
16
 FIND 47 PERCENT OF 34.

                A← 0.47
                B← 34
⎕ A×B
                C← 15.98
⎕ C*
FIRST RATE,  YOU GOT THAT ONE



17

WHAT IS .25 PERCENT OF .015?

                A← 0.0025
                B← 0.015
⎕ A×B
                C← 3.75E¯5
⎕ C*
VERY GOOD ,  YOU GOT THAT ONE



18

37 PERCENT OF WHAT NUMBER IS 61?

                A← 0.37
                B← 61
                C← 0
⎕ A×B
                D← 22.57
⎕ D*
NEGATIVE,  YOU MISSED THIS ONE.
```

save them for analysis by still other functions in the software package. For example, if the instructor wishes to see how student I performed on problem J, the dyadic function "BREAKDOWN" gives him this information, as illustrated in Figure 3, where I=1 and J=15. The computer immediately types the information upon receiving the command " 1 BREAKDOWN 15."

Fig. 3

```
          1 BREAKDOWN 15
STEP 1      88 SEC     B÷C
STEP 2     118 SEC     B-C
STEP 3      31 SEC     E÷B
STEP 4      13 SEC     F
TOTAL TIME  250 SEC              CORRECT
```

The information in Figure 3 can be read as follows: "On problem #15, student #1 took 88 seconds to decide on his first step toward his solution; that step was B ÷ C. (The computer assigned this result to the letter D, in other words, D is the result of B ÷ C.) After 118 seconds, the student typed B − C (seemingly he changed his strategy), and the computer assigned this number to E. It took 31 seconds for the student to command the computer to calculate E ÷ B, which the computer called F. Finally, it took 13 seconds for the student to decide F was the correct answer: He typed F*. Total time: 250 seconds, and the answer was correct."

This is but one of the varied analysis functions performed by the CAPS program. As an example, consider the display in Figure 4. By typing "ANALYSIS 23" the instructor is able to item analyze problem #23 in quite a lot of detail. This function first types the number and responses of students who answered question #23 incorrectly. Then it types the entire groups' responses. The time vectors (these are two digit symbols in BASE 60) are not translated by this function.

FIG. 4

coded time vectors

⍙ means student 5004 responded incorrectly

þ means student 5000 responded correctly.

```
UESTION 23
 002
5004     ⊃⊃B+C⊂_A-D⊂,E⍙
 017     ⊂/A-B+C⊂⊤D⍙
 018     ⊂∘C⍙
5051     ⊂○B⍙
 066     ⊂:C+F⊂⌊I-1⊂⍵J⍙
  5000    2    2   12   35   1   2   1   ⊂,B+C⊂KD-A⊂,Eþ
  5001    2    2   16   28   1   2   1   ⊂LB+C⊂\D-A⊂∪Eþ
  5002    1    0    0    0   0   0   0
  5004    2    2   61   95   0   2   1   ⊃⊃B+C⊂_A-D⊂,E⍙S⊂L/⍲⍲/⍲01K⊖
  5011    1    2   23   43   1   2   1   ⊂⌊(B+C)-A⊂ΔDþ
  5012    2    2   13   28   1   2   1   ⊂]B+C⊂KD-A⊂⊥Eþ
  5017    1    2   10   15   0   2   2   ⊂/A-B+C⊂⊤D⍙
  5018    0    0   19   19   0   0   5   ⊂∘C⍙
  5026    2    1   33   56   1   1   2   ⊂⍳40⊂]D-A⊂/Eþ
  5029    2    2   27   49   1   2   1   ⊂⍵B+C⊂LL-A⊂|Eþ
  5030    2    2   29   46   1   2   1   ⊂⍴B+C⊂KD-A⊂|Eþ
  5031    2    2   12   32   1   2   1   ⊂,B+C⊂KD-A⊂\Eþ
  5037    2    2   12   21   1   2   1   ⊂,B+C⊂⊤D-A⊂⊥Eþ
  5042    3    3   23   38   1   2   1   ⊂⌊B+C⊂|A-D⊂|D-A⊂∪Fþ
  5044    3    2   29   78   1   2   1   ⊂⍴B+C⊂,D-A⊂:0⊂~Eþ
  5050    1    0   24   32   1   0   5   ⊂⌈15⊂:Dþ
  5051    0    0   34   34   0   0   5   ⊂○B⍙
  5052    1    1   16   20   1   1   2   ⊂L40-A⊂⊥Dþ
  5053    3    3   52   83   1   2   1   ⊂<B+C⊂LA-D⊂KD-A⊂⊥Fþ
  5054    2    2   20   36   1   2   1   ⊂ΔB+C⊂,D-A⊂⊥Eþ
  5057    3    3   12   30   1   3   3   ⊂,⌊B×⊂|B+C⊂|D-A⊂|Eþ
  5060    1    2   13   18   1   2   1   ⊂]B+C-A⊂⊤Dþ
  5126    1    1   35   38   1   1   2   ⊂*40-25⊂∪Dþ
  5065    2    2   42   57   1   2   1   ⊂QB+C⊂KD-A⊂⊥Eþ
  5066    2    2    7   24   0   2   1   ⊂:C+F⊂⌊I-1⊂⍵J⍙
  5067    2    2    7   21   1   2   1   ⊂:C+B⊂:D-A⊂:Eþ
  5070    1    0   43   49   1   0   5   ⊂Q15⊂|Dþ
  5071    2    2   64   79   1   2   1   ⊃⊥B+C⊂KD-A⊂⊥Eþ
  5074    1    0   50   54   1   0   5   ⊂¨15⊂⊥Dþ
  5081    2    2   37   53   1   2   1   ⊂←B+C⊂,D-A⊂⊥Eþ
  5100    2    2   15   35   1   2   1   ⊂NB+C⊂KD-A⊂\Eþ
  5128    1    0   18   22   1   0   5   ⊂'15⊂⊥Dþ
  5137    2    2   12   30   1   2   1   ⊂,B+C⊂⌊D-A⊂⊥Eþ
  5140    6    6   48  264   1   2   2   ⊂IB-A⊂,A-C∩⊛D+E⊂QA+B⊂:B+C⊂KH-/
  5142    2    2    8   20   1   2   1   ⊂:B+C⊂:D-A⊂⊥Eþ
OK?  YES
```

| STEPS | OPERATIONS | COMP TIME | TOTAL TIME | R-W | NUMBER OF DIFFERENT OPERATIONS | FIRST OP + - × ÷ 1 2 3 4 |
|---|---|---|---|---|---|---|

```
QUESTION  23

6   60   876   1582   29   54   65
1.852941176   1.764705882   25.76470588   46.52941176   0.8529411765   1.588235294   1.9
  )
```

TOTALS.

AVERAGES

INTERRUPTED REMAINING STATISTICS

Thus, the steps for student 5004 were: B + C; A - D; and E: He did not answer this question correctly. Student 5000, on the other hand, answered correctly and typed: B + C; D - A; and E. This function further gives the totals, averages, and other descriptive statistics not shown in Figure 4. If the instructor wishes to find out how any one student has performed, he may type " FIND J ", (See Figure 5.) This analysis "cross-sections" the problems.

The CAPS program contains other functions designed for even closer analysis, but time and space do not allow a full discussion here.

*FIND 1*

```
|555000.~
⊃∊B:C⊃∨B-C⊂↑÷B⊂]Fϕ
⊂>A×B⊃>CΔ
⊂∘B÷A∩ΔA×B⊂↑Dϕ
⊂(B×A⊂[Cϕ
⊃⊃A×B⊂CCϕ
⊂[B×A⊂;Cϕ
⊂≥A÷B⊂≥B×A⊂|DΔ
⊃A.005⊂Δ+B⊂]E÷C⊂∊[]STOP⊂DFΔS⊃[.∨⍟.∨01.ω
⊂→C÷A⊂|Dϕ
∪←B×A∩⋆A:B⊂:Dϕ
⊂/B×A⊂..CΔ.G
⊂ωA×A⊂|B×B⊂;D+E⊂[C×F⊂⊤GΔ
⊃E.35⊃⋆[]]A×B⊂∧A×B
```

*FIND 2*

```
|555001,:
⊂EA×B⊂⌊Cϕ
⊂⍳A÷B⊂:CΔ
⊂GB-⊂↑D÷B⊂:Eϕ
⊂ρA×B⊂]B+C⊂|DΔ
⊂⎕A×B⊂A+C⊂⊥DΔ
⊃\A×C⊂|DΔ
⊂'C÷A⊂⊥Dϕ
⊂A0.10⊂←A×C⊂:A+D⊂⊥EΔ
⊃⊂A×B⊂]Cϕ
⊂FA×B⊂/A+C⊂∇D-A⊂Eϕ
⊂)A×B⊂⊤CΔ
⊂⍳A+B⊂⍟×F⊂[D+E⊂⊤FΔ
⊃⎕A×C⊂~A×D⊂¨C×F⊂,F-G⊂⊛HΔ
⊂\A×B⊃∘A-D⊂B30⊂[E÷F⊂|GO-
⊂CD÷C⊂⊥EΔ
⊂≤A-B⊂(B-A⊃:A÷D⊂:EΔ
⊂_×B⊂\Co
```

*FIND 3*

```
|55002]∘
⊂↓×B⊂/A+C⊂;DΔ
∩⊃C×A⊂↓C-⊂HE/.
⊂\B×A⊂⊛Cϕ
⊂⌊A×B⊂⊤Cϕ
⊃⊃B÷⊂⍳B-C⊂>B÷E⊂]FΔ
⊂BB×A⊂[B+C⊂;DΔ
∪⍳A÷C∪,A-C⊂AC÷E⊃;A÷E⊂'GΔ
∪↓A×.10⊂ωA+C⊂¯DΔ
⊂[B×A⊂[Cϕ
⊃:A×B⊂]Cϕ
∩[A×4⊂(C
```

APPENDIX 3

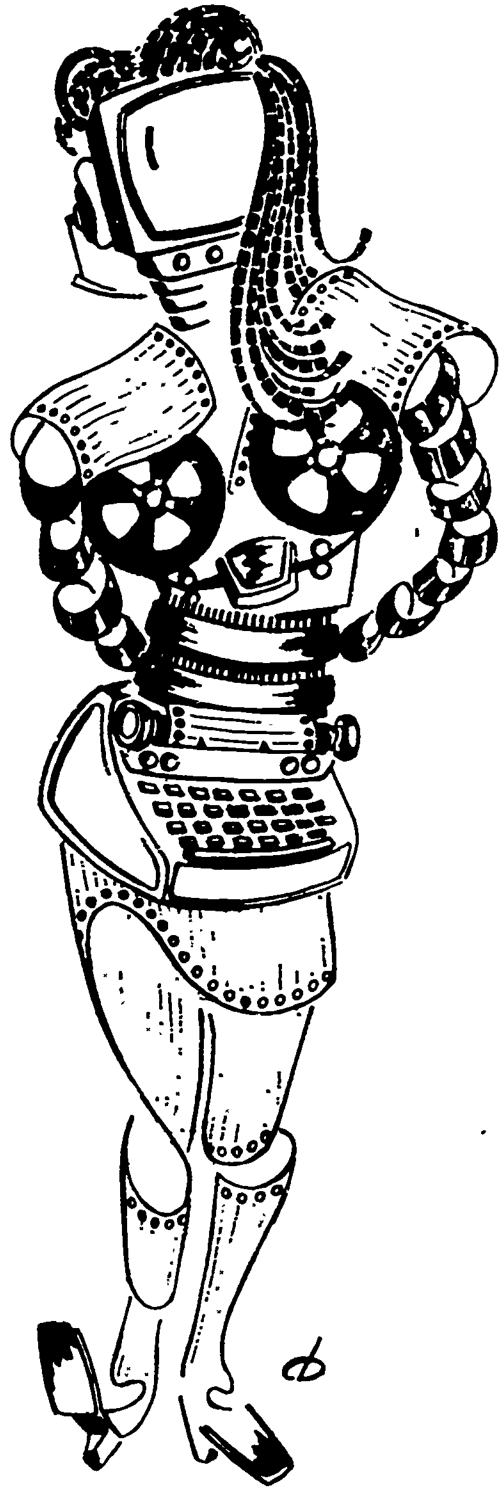
Examples of Computer Assisted Instruction Programs At

The Coast Community College District

*Coast Community College district*

1370 ADAMS AVENUE • COSTA MESA • CALIFORNIA 92626

NORMAN E. WATSON - CHANCELLOR



# COMPUTERS IN INSTRUCTION AT THE COAST COMMUNITY COLLEGES

BY

JOHN CLARK DICK MERCER

PREPARED FOR THE

ASSOCIATION FOR THE DEVELOPMENT OF INSTRUCTIONAL SYSTEMS

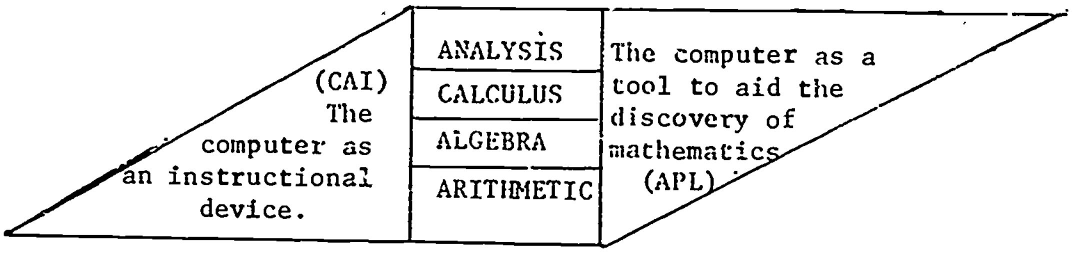
JAN 30, 1973

ORANGE COAST COLLEGE
COSTA MESA

GOLDEN WEST COLLEGE
HUNTINGTON BEACH

THE SEGALLA MODEL FOR RELATING
THE COMPUTER TO MATHEMATICS INSTRUCTION

Both Orange Coast College and Golden West College have integrated APL into the mathematics curriculum following a model developed by Professor Angelo Segalla.




As a student advances through the mathematics curriculum, the use of CAI segments diminishes, but student use of the computer as a tool, increases. Special classes have been added to the curriculum which parallel the classes in Algebra and Calculus. These classes introduce the student to the CAI segments available for the subject. And, also to techniques of programing in APL.

Using this model, it is possible for the instructors to concentrate their CAI developments at the lower levels of mathematics where most of the students are; and, where the best chance of cost effectiveness is. Still, they are able to provide enrichment opportunities for more advanced students.

It is hoped that with instruction in APL programing, the student will be able to "discover" mathematics. The discovery method, while theoretically sound, has in general failed, due to the computational work involved. The student who has mastered APL will find that calculations are no longer a hinderance and should, in a symbiotic relationship with the computer, actually be able to do some real discovery oriented learning.

The attached terminal printouts represent simulated Student Terminal Sessions. They provide examples of the variety of CAI segments used in the Coast Community College District. Each Segment presupposes some advance preparation on the part of the student. Most of the printouts do not include the initial interaction in which the student may obtain specific instruction on the control of the segment. In each segment the student input is underlined.

MEDTERMS
A drill-and-practice segment dealing with medical terminology and the meanings of commonly used medical prefixes. Questions and choices may be presented randomly. A retort is given for each student choice. A series of 20 similar segments have been developed which use a computer-driven Microfiche Projector.

SPANISH
A comprehensive series of instructional segments covering the whole of Spanish grammar, including the formation of all verb tenses; gender, plural and uses of nouns, pronouns, adjectives and articles; uses of adverbs, prepositions, and verbs; language syntax and other items related to Spanish sentence strategies. The series includes a 3,000 word vocabulary and approximately 200 idioms. The organizational format follows that of "Foundation Course in Spanish" by Turk and Espinosa. There are 193 segments providing 100 hours of instruction. Eleven hundred (1,100) student hours resulted from 1,200 student contacts during the Fall semester.

MODIFIERS
A tutorial program in English which requires the student to find and reposition a misplaced modifier in a sentence. There are 30 segments providing 21 hours of English instruction in such areas as spelling, paragraph coherence, sentence fragments, modifiers, transitions and often confused words. Eleven hundred (1,100) student hours resulted from 1,300 student contacts during the Fall semester.

PROBLEM
One of a series of segments which present word problems, and allows the student several options in its solution. The student may supply the solution, request more data, or enter an analysis mode in which he is asked to perform in a manner similar to what he would in an office session with an instructor. For this example the instructions, a problem and a series of analysis steps are shown. The software for this segment is suitable for use with Physics, Chemistry, or other subjects using word problems.

LIMIT
A segment designed to help a Calculus student with delta-epsilon limit proofs for polynomial functions. This is the first of a series of segments devoted to limits and limit proofs. The student has the option in this segment of requesting the proof and an explanation of any step in the proof. The student inputs the coefficients of the polynomial of his choice and the point where the limit is to be evaluated.

SUPERGRAPH

One of a series of plotting routines designed to aid the student in the discovery of mathematical relationships. The student may choose the function to be plotted and the domain of evaluation. Included in this workspace are routines to provide polar and parametric plots as well as routines to locate the real zeros of the function.

ZPLOT

A segment which visually displays complex roots and multiple roots. The student may use standard textbook notation for defining the function. By observing the function as the junction of various regions, the zeros may be obtained. In this example, a multiple root occurs at 4, real roots occur at -3.1 and -0.8, and complex roots of ($1 \pm 1.5i$) and ($4.7 \pm 3.5i$). Other segments in this workspace will locate the zeros to a higher degree of accuracy.

LEASTFIT

A physics workspace used for analysis of experimental data. In this example the student receives both descriptive statistics and a histogram comparing his data with a theoretical Gaussian Distribution having the same mean and standard deviation. The * printed in this Histogram represents points through which the normal curve would pass. Included in this workspace are other curve-fitting and analysis routines.

ELECTRONICS

A simulation segment which provides the student an opportunity to trouble-shoot a randomly generated failure in a given circuit. The student is given the symptoms of the failure and has the opportunity to test and/or replace various components at his descresion. The trouble persists until the failing component is replaced.

4. WHICH PREFIX MEANS 'HEART'?

CEPHAL-
CONTRA-
CARDIA-
CHOL-

M: CARDIA

CORRECT-THAT WAS CLOSE! AS USED IN CARDIOGRAM WHICH IS A TRACING OF HEART RHYTHM AND PATTERN. PROCEED TO NEXT ITEM.

5. WHICH OF THE FOLLOWING PREFIXES DENOTES A 'BLOOD VESSEL'?

CEPHAL-
CHOLE-
BRADY-
ANA-
ANGIO-

M: ANA

NO, NO! ANA- MEANS TO 'RENEW OR UNITE' AS IN ANASTOMOSIS WHICH MEANS A JOINING OF TWO VESSELS OR NERVES. TRY AGAIN.

M: BRADY

INCORRECT! BRADY- MEANS 'SLOW' AS IN BRADYCARDIA WHICH MEANS SLOW HEART. TRY AGAIN.

M: ANA
YOU HAVE MADE THIS CHOICE BEFORE
M: ANGIO

CORRECT! AS IN ANGIOSPASM, WHICH MEANS A SPASMODIC CONTRACTION OF BLOOD VESSELS. PLEASE CONTINUE.

6. WHICH PREFIX MEANS 'AGAINST'?

BRADY-
CON-
BI-
ANTI-
CLEIDO-

M: CONTRA
YOU MUST CHOOSE ONE OF THE ABOVE CHOICES
M: ANTI

HOW NICE! AS IN ANTIBIOTIC, WHICH IS A SUBSTANCE AGAINST VACTERIAL LIFE. PLEASE CONTINUE.

7. WHICH PREFIX MEANS 'CARTILAGE'?

CEPHAL-
OSTEO-
CARDIA-
CHOL-
CHONDRA-

PLEASE NOTE :

°........THE SMALL CIRCLE IS USED TO INDICATE INCORRECT SPELLING.
EX.: TRIAL MISPELLED = TRI°L.

↓........THE ARROW POINTS TO INCORRECT PUNCTUATION.
EX.: ME LLAMO PEPE↓ (NO PERIOD).

_........UNDERLINE LETTERS THAT NEED AN ACCENT OR TILDE.
EX.: LAPIZ MANANA

WHICH SECTION DO YOU WANT?... 5

GIVE IN SPANISH. DON'T USE THE SUBJECT PRONOUNS.

WE SPEND A SHORT TIME HERE EVERY DAY.
?... GASTAMOS UN ????? AQUI TODOS LOS DIAS.
°AS°°°° UN °°°° AQU° TODOS LOS D°AS.

WE SPEND A SHORT TIME HERE EVERY DAY.
?... PASAMOS UN RATO AQUI TODOS LOS DIAS.
!ESO ES!

WHAT TIME IS IT?
?... QUE HORA ES?
↓QUE HORA ES?

WHAT TIME IS IT?
?... ?QUE HORA ES?
THAT'S CORRECT!

I HAVE A LOT OF TIME TO STUDY.
?... HELP

DICK, HOW ABOUT:

TENGO MUCHO TIEMPO PARA ESTUDIAR.

I'M NOT GOING WITH MY MAMA THIS TIME.
?... NO VOY CON MI MAMA ESTA VEZ.
!BRAVISIMO!

I GO TWO TIMES EVERY WEEK.
?... VOY DOS TIEMPOS CADA SEMANA.
VOY DOS °°°°° CADA SEMANA.

I GO TWO TIMES EVERY WEEK.
?... VOY DOS VECES CADA SEMANA.
!ESO ES!

DO YOU ALWAYS ARRIVE ON TIME?
?... STOP

OK DICK. WE'LL STOP HERE FOR NOW.

DO YOU WANT TO SEE THE SCORE BOARD?...SI

| | |
|---|---|
| LESSON | 5 |
| TIME | 4 |
| PROBLEMS TRIED | 5 |
| ANSWERED CORRECTLY | 4 |

TOMORROW MANANA

MANY MUY
MANY MUCHO
**ANSWER: MANY-MUCHOS

NORTH AMERICAN AMERICANO
NORTH AMERICAN NORTEAMERICANO

IN THE AFTERNOON (HOUR GIVEN)
DE LA TARDE

MANY MUCHOS

TIME LA VEZ
TIME LA HORA

CHARLES CARLO
CHARLES CARLO
**ANSWER: CHARLES-CARLOS

URUGUAY EL URUGUAY

THERE IS HAY

CHARLES CARLOS

SON STOP

BEER SHOULD NOT BE SOLD TO STUDENTS CONTAINING MORE THAN 3.2 PERCENT ALCOHOL.

TYPE THE FIRST AND LAST WORDS OF THE MISPLACED MODIFIER.

SOLD STUDENTS

NO, DICK, YOU HAVEN'T GOT THE RIGHT MODIFIER *****
TRY AGAIN.

CONTAINING ALCOHOL

GOOD, YOU HAVE IDENTIFIED THE MODIFIER.

WHERE SHOULD 'CONTAINING MORE THAN 3.2 PERCENT ALCOHOL'
BE POSITIONED IN THE SENTENCE?

BEER^

BEER CONTAINING MORE THAN 3.2 PERCENT ALCOHOL SHOULD NOT BE SOLD TO STUDENTS.
VERY GOOD, DICK, ON TO THE NEXT QUESTION.

SHE WORE A RIBBON IN HER HAIR WHICH WAS A LIGHT PINK.

TYPE THE FIRST AND LAST WORDS OF THE MISPLACED MODIFIER.

WHICH PINK

GOOD, YOU HAVE IDENTIFIED THE MODIFIER.

WHERE SHOULD 'WHICH WAS A LIGHT PINK'
BE POSITIONED IN THE SENTENCE?

SHE^

SHE WHICH WAS A LIGHT PINK WORE A RIBBON IN HER HAIR.

NO, DICK, THE PROBLEM WITH THE SENTENCE IS THAT HER HAIR APPEARS TO BE 'LIGHT PINK' WHEN THIS SHOULD BE A DESCRIPTION OF WHAT? TRY AGAIN.

WHERE SHOULD 'WHICH WAS A LIGHT PINK'
BE POSITIONED IN THE SENTENCE?

RIBBON^

SHE WORE A RIBBON WHICH WAS A LIGHT PINK IN HER HAIR

```
    )91051
051) 21.41.49 10/20/72 P510CC

   A P L \ 3 7 0

    )LOAD 1234 PROBLEM
SAVED  21.35.51 10/20/72

      GO
```

IN THIS WORKSPACE YOU WILL BE PRESENTED A PROBLEM. YOU MAY CHOOSE TO ANSWER THE PROBLEM DIRECTLY, OR BE ASKED KEY QUESTIONS ABOUT IT.

THERE ARE FOUR DIFFERENT MODES BY WHICH YOU MAY USE THE TERMINAL IN SOLVING THE PROBLEM.

SOLUTION MODE — TYPE THE LETTER S FOLLOWED BY THE NUMBER THAT YOU BELIEVE TO BE THE SOLUTION OF THE PROBLEM.

HINT MODE — TYPE THE LETTER H AND I WILL PROVIDE YOU WITH A HINT ABOUT THE SOLUTION OF THE PROBLEM. IF YOU TYPE HS I WILL PROVIDE YOU WITH THE SOLUTION, BUT THIS WILL COST YOU 100 POINTS.

INQUIRY MODE — TYPE THE LETTER I FOLLOWED BY ONE OF THE LETTERS BELOW TO OBTAIN THE FOLLOWING INFORMATION:

```
    D   --  DEFINITION OF TERMS USED IN THE PROBLEM
    DD  --  DIRECT DATA INVOLVED IN THE PROBLEM
    ID  --  INDIRECT DATA INVOLVED IN THE PROBLEM
    E   --  THE EQUATION FOR THE SOLUTION
    C   --  THE CONCEPT INVOLVED IN THE PROBLEM
    U   --  THE UNKNOWN INVOLVED IN THE PROBLEM
    F   --  ANY SPECIAL FORMULAE NEEDED FOR THE SOLUTION
```

ANALYSIS MODE — TYPE THE LETTER A AND I WILL ASK YOU THE QUESTIONS YOU SHOULD ASK YOURSELF AS YOU ATTEMPT TO SOLVE THE PROBLEM

YOU MAY REQUEST THE CALCULATOR MODE BY TYPING THE LETTER C
THE CALCULATOR MODE ASSISTS YOU IN YOUR CALCULATIONS. TO EXIT THIS MODE YOU SHOULD TYPE THE WORD END

YOU START EACH PROBLEM WITH 100 POINTS. WHEN YOU ASK FOR ANY MODE OTHER THAN THE SOLUTION OR CALCULATOR MODE, I MAY SUBTRACT POINTS FOR THE HELP I GIVE YOU. IN THE ANALYSIS MODE I WILL NOT SUBTRACT POINTS IF YOU ANSWER MY QUESTIONS CORRECTLY.

YOUR SOLUTION MUST BE CORRECT IN THE SOLUTION MODE BEFORE YOU ARE ALLOWED TO ADVANCE TO THE NEXT PROBLEM.

TO STOP THIS PROGRAM, TYPE THE WORD STOP

GOOD LUCK ON YOUR PROBLEMS!

HOW MANY GALLONS OF BRANDY CONTAINING 40 PERCENT ALCOHOL
(80 PROOF) SHOULD BE ADDED TO 100 GALLONS OF WINE CONTAINING
12 PERCENT ALCHOL TO PRODUCE AN INTOXICATING BEVERAGE
WHICH HAS 20 PERCENT ALCOHOL (40 PROOF)?

WHEN THE SYMBOL @ APPEARS YOU MAY ENTER YOUR MODE CHOICE. I WILL
ALWAYS RETURN TO THE MODE CHOICE STATUS EACH TIME.

@

A
IF X IS THE NUMBER OF GALLONS OF BRANDY, WRITE AN EXPRESSION INVOLVING
X WHICH IS EQUIVALENT TO THE NUMBER OF GALLONS OF ALCOHOL IN THE BRANDY.

.4X
FULLY GOOD
@

A
HOW MANY GALLONS OF ALCOHOL ARE IN THE WINE?

12
YOU ARE CORRECT ON THIS PORTION OF THE ANALYSIS.
@

A
REMEMBERING THAT X IS THE NUMBER OF GALLONS OF BRANDY, WRITE AN EXPRESSION
INVOLVING X WHICH EXPRESSED THE NUMBER OF GALLONS OF THE RESULTING
BEVERAGE.

X+100
VERY GOOD
@

A
WRITE AN EXPRESSION , INVOLVING X, WHICH EXPRESSED THE AMOUNT OF
ALCOHOL IN THE RESULTING VEVERAGE.

.2×(100 + X)
VERY GOOD
@

A
WRITE AN EXPRESSION, INVOLVING X, WHICH IS EQUIVALENT TO THE
NUMBER OF GALLONS OF ALCOHOL IN THE BRANDY PLUS THE NUMBER OF
GALLONS OF ALCOHOL IN THE WINE.

8+4+2×.2X
EXCELLENT
@

A
WE KNOW THE TOTAL NUMBER OF GALLONS OF ALCOHOL IN THE BRANDY
AND IN THE WINE ADDED TOGETHER IS EQUAL TO THE TOTAL NUMBER OF
GALLONS OF ALCOHOL IN THE RESULTING BEVERAGE. WRITE AN EQUATION,
INVOLVING X, WHICH EXPRESSED THIS FACT.

12 + .4X = .2×(X+100)
YOUR EQUATION HAS THE SAME SOLUTION SET AS MINE. TAKE A LOOK AT MINE

LIMIT

THIS FUNCTION IS DESIGNED TO ASSIST YOU IN WRITING 'Δ ε' PROOFS FOR POLYNOMIALS. WHEN 'F(X)' APPEARS ENTER THE COEFFICIENTS OF THE POLYNOMIAL. WHEN 'X → ' APPEARS ENTER THE POINT THE DOMAIN WHERE THE LIMIT OF THE FUNCTION IS TO BE EVALUATED.

```
F(X)=
⎕:
      1 0 ¯2 4 2
X →
⎕:
      2


 LIM  X*4 - 2X*2 + 4X + 2 = 18
X→2

 CHOOSE Δ < ε / 59  WHEN 0 < ε < 59  DO YOU WANT PROOF?    YES

WHEN: ε>0; 0<Δ<1; F(X)=X*4 - 2X*2 + 4X + 2

A=2; L=18 HERE IS HOW TO FIND A Δ AS A FUNCTION OF ε  Δ<φ(ε)

                                SUCH THAT

            IF 0<|X - A|<Δ    THEN  |F(X) - L|<ε

1)0 < |X - 2| < Δ ; |X*4 - 2X*2 + 4X + 2 - 18| < ε

2)1 < X < 3         ; |X*4 - 2X*2 + 4X - 16| < ε

3)                    |X*3 + 2X*2 + 2X + 8|×|X - 2| < ε

4)|X - 2|   <   ε / |X*3 + 2X*2 + 2X + 8|

 CHOOSE Δ < ε / 59  WHEN 0 < ε < 59
ENTER 0  OR THE LINE NUMBER YOU WISH CLARIFIED
⎕:
      3
IN STEP 3  WE ARE EXPRESSING

|F(X)-L| OR |X*4 - 2X*2 + 4X - 16|, AS THE PRODUCT OF

|X-A| OR |X - 2| WHICH MUST FACTOR EVENLY INTO |F(X)-L|, AND ITS

COFACTOR: |(F(X)-L)/(X-A)| OR |X*3 + 2X*2 + 2X + 8|

ENTER 0  OR THE LINE NUMBER YOU WISH CLARIFIED
⎕:
      2
IN STEP 2  WE ARE DETERMINING

THE DOMAIN OF X  ;  THE CONSTANT IN F(X)-L=2-18=¯16

ENTER 0  OR THE LINE NUMBER YOU WISH CLARIFIED
⎕:
      0
```

SUPERGRAPH

INSTRUCTIONS? YES

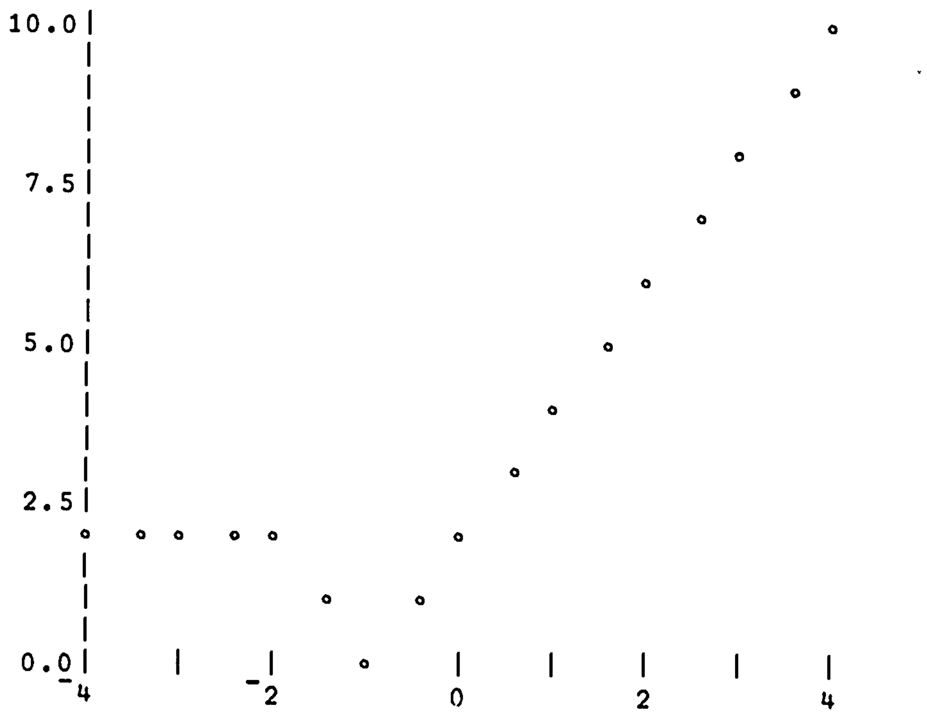
YOU MAY ENTER ANY EXPRESSION USING + - × ÷ ⋆ | SIN COS TAN AND I WILL PRODUCE A GRAPH OF THE EXPRESSION. STANDARD RULES OF ALGEBRA ARE USED TO DETERMINE THE ORDER OF EVALUATION.

F(X) = .5×|2X+|2X+4||

FOR WHAT VALUES OF X ARE WE TO EVALUATE THE FUNCTION

⎕:

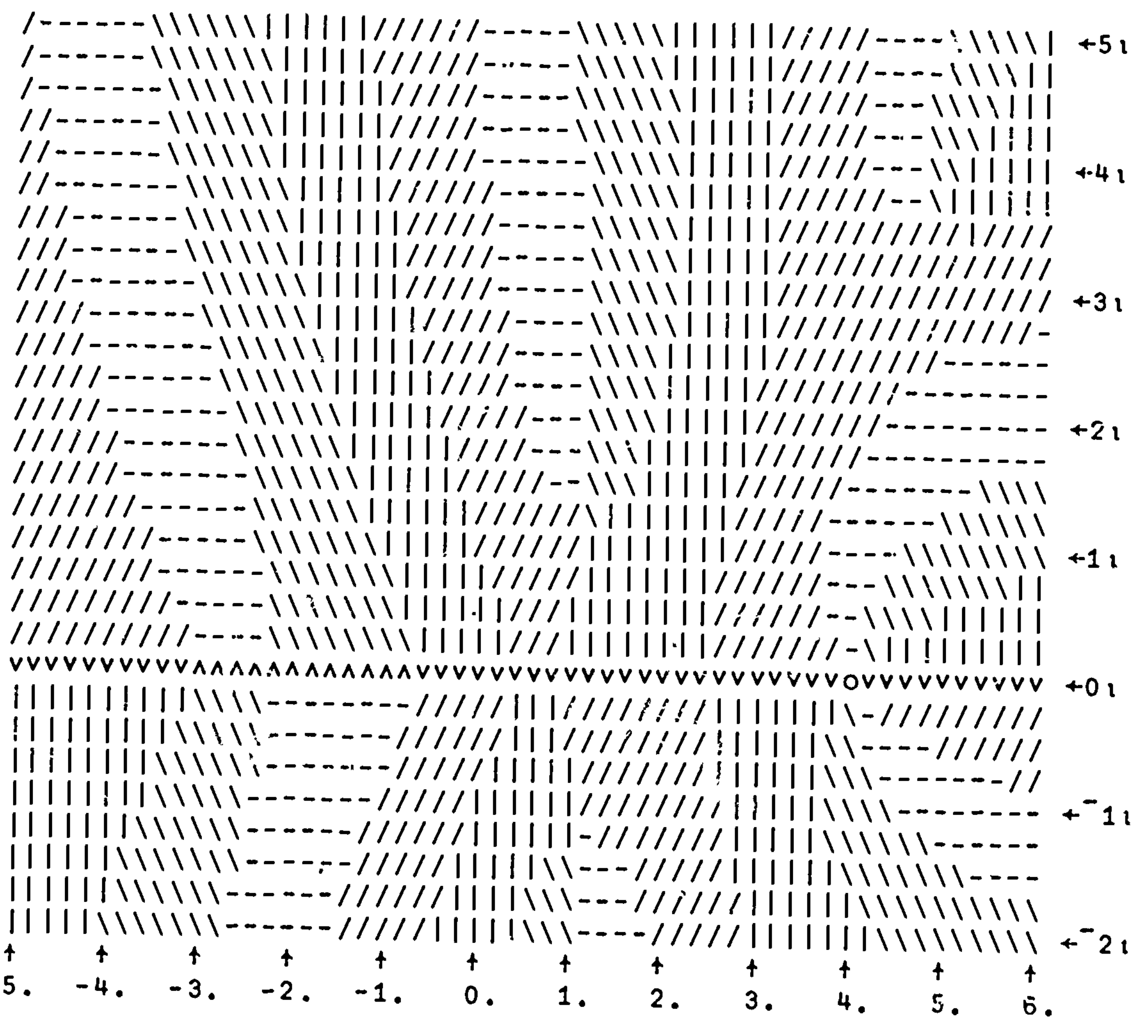
STEP ¯4 4 .5



DO YOU WANT A TABLE OF THE X AND F(X) VALUES? YES

| | |
|---|---|
| ¯4 | 2 |
| ¯3.5 | 2 |
| ¯3 | 2 |
| ¯2.5 | 2 |
| ¯2 | 2 |
| ¯1.5 | 1 |
| ¯1 | 0 |
| ¯0.5 | 1 |
| 0 | 2 |
| 0.5 | 3 |
| 1 | 4 |
| 1.5 | 5 |
| 2 | 6 |
| 2.5 | 7 |
| 3 | 8 |
| 3.5 | 9 |
| 4 | 10 |

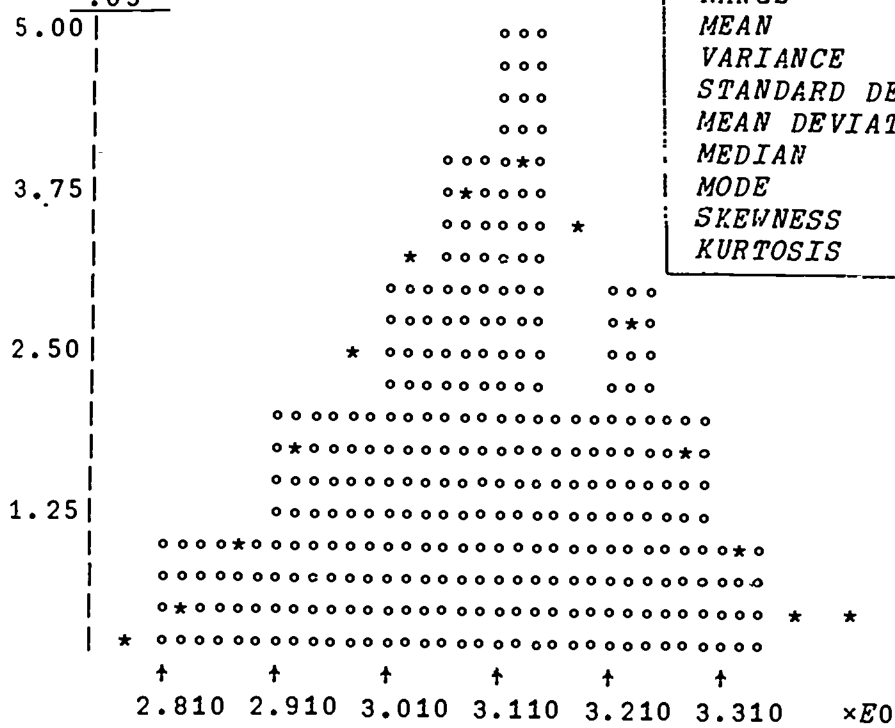
```
      )LOAD 1234 ZPLOT
SAVED  20.01.58 11/14/72

     ZPLOT

F(X) = (EXP X - X SIN X)(X*2 -8X + 16)
WHAT IS THE DOMAIN OF THE REAL VALUES?
⎕:
      STEP ¯5 6 .2
WHAT IS THE DOMAIN OF THE IMAGINARY VALUES?
⎕:
      STEP ¯2 5 .25
```




$$f(x) = (e^x - x \sin x)(x^2 - 8x + 16)$$

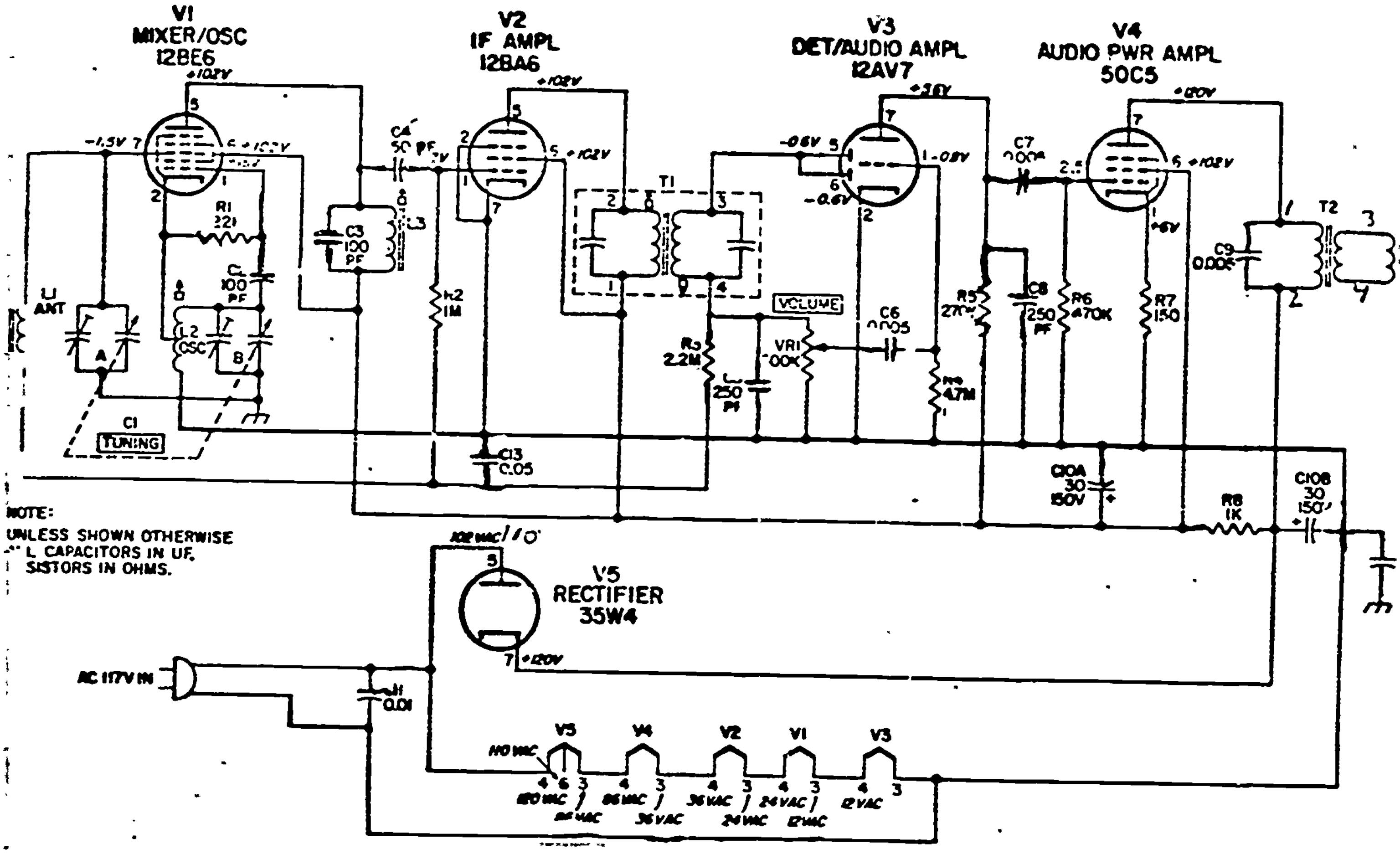
```
      )LOAD 1234 LEASTFIT
SAVED  11.55.38 12/12/72

      M1←2.81 2.86 3.31 3.26 3.22 2.91 2.98 3.19 3.17 3.16 3.00 3.01 3.24 3.01
      M2←2.93 3.11 3.12 2.96 3.39 3.20 3.10 3.06 3.09 3.07 3.08 3.07 3.12

      MASS←M1,M2

      GAUSSFIT
ENTER DATA VECTOR
⎕:
      MASS
INTERVAL WIDTH
⎕:
      .05
```

| DSTAT MASS | |
|---|---|
| SAMPLE SIZE | 27 |
| MAXIMUM | 3.39 |
| MINIMUM | 2.81 |
| RANGE | 0.58 |
| MEAN | 3.09 |
| VARIANCE | 0.01859230769 |
| STANDARD DEVIATION | 0.1363536127 |
| MEAN DEVIATION | 0.1051851852 |
| MEDIAN | 3.09 |
| MODE | 3.01 3.07 3.12 |
| SKEWNESS | ¯3.908272332E¯14 |
| KURTOSIS | 2.530063807 |




DO YOU WANT A TABLE OF VALUES? YES

| INTERVAL CENTER | FREQUENCY | EXPECTED FREQUENCY |
|---|---|---|
| 2.81 | 1 | 0.4831319234 |
| 2.86 | 1 | 0.9660180035 |
| 2.91 | 2 | 1.667516917 |
| 2.96 | 2 | 2.500424743 |
| 3.01 | 3 | 3.307342057 |
| 3.06 | 4 | 3.844173135 |
| 3.11 | 5 | 3.885170268 |
| 3.16 | 2 | 3.447080891 |
| 3.21 | 3 | 2.669859935 |
| 3.26 | 2 | 1.827862884 |
| 3.31 | 1 | 1.09026796 |
| 3.36 | 0 | 0.5613267593 |




)LOAD 1234 ELECTRONICS
SAVED 12.41.00 01/04/73

TROUBLE
DO YOU WANT INSTRUCTIONS? NO

NO AUDIO ON LOW BAND
NOISY AUDIO NEAR TOP OF HIGH BAND
SQUELLING ON HIGH BAND
⎕:
SIGNAL TUBE 3 PIN 5
NO SIGNAL
⎕:
VOLTAGE TUBE 3 PIN 5
-4.0 VOLTS
⎕:
REPLACE TUBE 3
***TROUBLE PERSISTS***
⎕:
RESISTANCE RES 4
4.7M OHMS
⎕:
REPLACE CAP 13
●●●TROUBLE GONE●●●
WANT ANOTHER PROBLEM? NO

# TOTALS

624 INSTRUCTIONAL SEGMENTS

IN

37 SUBJECT AREAS

308 HOURS OF INSTRUCTION

DEVELOPED BY

85 DIFFERENT FACULTY MEMBERS

# FALL 1972

2000 STUDENTS

ACCUMULATED

20000 HOURS

DURING

21000 TERMINAL SESSIONS

| SUBJECT AREA | SEGMENTS | TIME |
|---|---|---|
| ACCOUNTING | 8 | 3:35 |
| ALGEBRA | 37 | 20:10 |
| ANATOMY | 20 | 10:00 |
| ARITHMETIC | 44 | 19:30 |
| ART | 1 | :05 |
| BIOLOGY | 16 | 6:10 |
| BUSINESS LAW | 14 | 9:45 |
| BUSINESS | 1 | 1:00 |
| CAI | 3 | :40 |
| CALCULUS | 42 | 15:05 |
| CHEMISTRY | 31 | 11:25 |
| COMPUTERS | 13 | 7:45 |
| ECONOMICS | 7 | 8:05 |
| ELECTRONICS | 5 | 2:15 |
| ENGINEERING | 1 | :50 |
| ENGLISH | 38 | 20:30 |
| FINITE MATH | 9 | 3:20 |
| GEOGRAPHY | 2 | 1:00 |
| GEOLOGY | 1 | :30 |
| GEOMETRY | 7 | 2:00 |
| GERMAN | 3 | 1:30 |
| GRAPHIC ARTS | 3 | 1:30 |
| LIBRARY | 1 | :30 |
| LOGIC | 5 | 12:20 |
| MATH | 1 | :20 |
| MUSIC | 1 | :40 |
| PHOTOGRAPHY | 1 | :30 |
| PHYSICS | 28 | 11:25 |
| PLOTTING | 6 | 2:40 |
| POLICE SCI. | 31 | 9:20 |
| POLITIC SCI. | 17 | 9:00 |
| PSYCHOLOGY | 3 | 2:00 |
| SEC. SCI. | 8 | 3:50 |
| SLIDE RULE | 14 | 6:30 |
| SPANISH | 193 | 99:30 |
| STATISTICS | 8 | 2:15 |
| TRIGONOMETRY | 1 | :20 |

# WHY APL FOR CAI?

## I. APL IS A NOTATION NOT A COMPUTER LANGUAGE.

A. IT WAS DESIGNED TO CONVEY IDEAS IN THE CLASSROOM RATHER THAN DRIVE A COMPUTER THROUGH CALCULATIONS.

B. THE COMPUTER IMPLEMENTATION CLOSELY FOLLOWS THE NOTATION AND THEREFORE IS NOT LIMITED BY THE HARDWARE CONCEPTS OF FIRST AND SECOND GENERATION VON NEUMANN MACHINES.

## II. APL IS THE ONLY AVAILABLE TELEPROCESSING SYSTEM THAT WILL SUPPORT THE CAI NEEDS OF THE COMMUNITY COLLEGE.

A. DRILL & PRACTICE SEGMENTS.

B. PROBLEM SOLVING COMPUTATIONAL MODE.

C. TUTORIAL WITH MULTIPLE BRANCHING.

D. SIMULATION

## III. APL IS THE RICHEST LANGUAGE AVAILABLE.

A. THERE ARE OVER 70 PRIMITIVE FUNCTIONS AND 3 OPERATORS AS OPPOSED TO LESS THAN 1/2 DOZEN PRIMITIVES FOR BASIC, FORTRAN ETC.

B. EXTENSION OF DATA TYPES TO TENSORS (SCALARS, VECTORS, MATRICES, ARRAYS ETC.) ELIMINATES THE NEED FOR LOOPING IN MANY CASES AND REDUCES THE PROGRAMING TO SIMPLE DESCRIPTIONS OF THE PROBLEM.

## IV. APL SUPPORTS THE CREATION OF SPECIALIZED CAI LANGUAGES.

A. FUNCTION CALL STRUCTURE ALLOWS FOR EASY COMMUNICATION AND TRANSFER OF DATA BETWEEN FUNCTIONS.

B. FUNCTIONS WRITTEN FOR ONE C A I SEGMENT MAY BE EASILY COPIED AND USED FOR ANOTHER SEGMENT.

C. UNDER PROGRAM CONTROL, IT IS EASY TO TURN THE SYSTEM OVER TO THE CALCULATOR MODE IF REQUESTED BY THE STUDENT.

V. APL ALSO SUPPORTS AND MEETS STUDENTS NEEDS FOR A PROGRAMING LANGUAGE.

A. THE LANGUAGE IS EASY TO LEARN.

B. STUDENT MAY PRETEST EACH STEP OF PROGRAM BEFORE ENTERING IT.

C. STUDENT IS NOT BOGGED DOWN WITH FLOWCHARTING, DATA STRUCTURES, LOOPING TECHNIQUES, AND OTHER TRIVIA AND IS ALLOWED TO SOLVE PROBLEMS IN A NATURAL WAY.

D. INPUT/OUTPUT IS EASY TO HANDLE AND REQUIRES ONLY PRIMITIVE FUNCTIONS.

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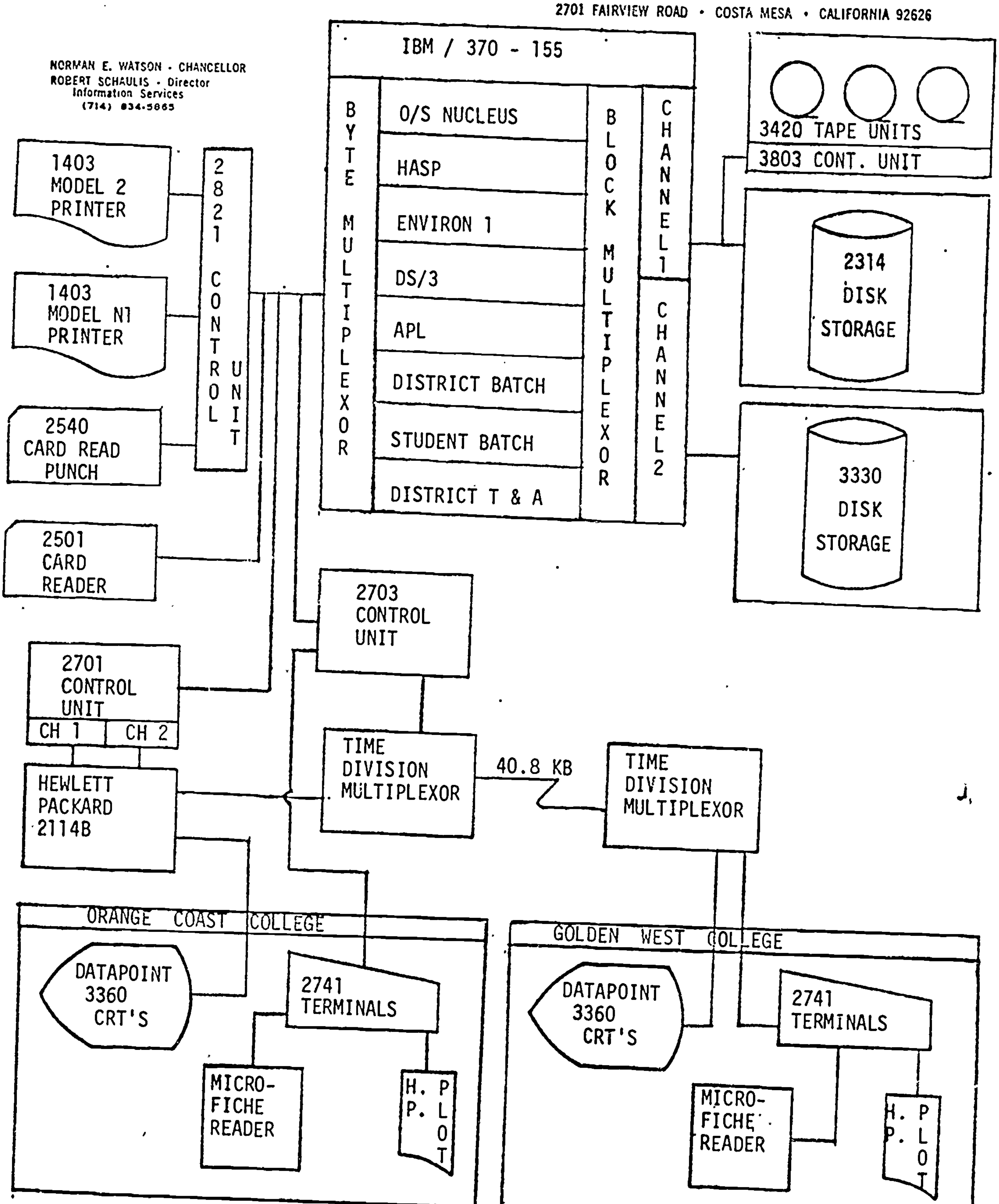

# R E F E R E N C E S

## CAI

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|---|---|
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