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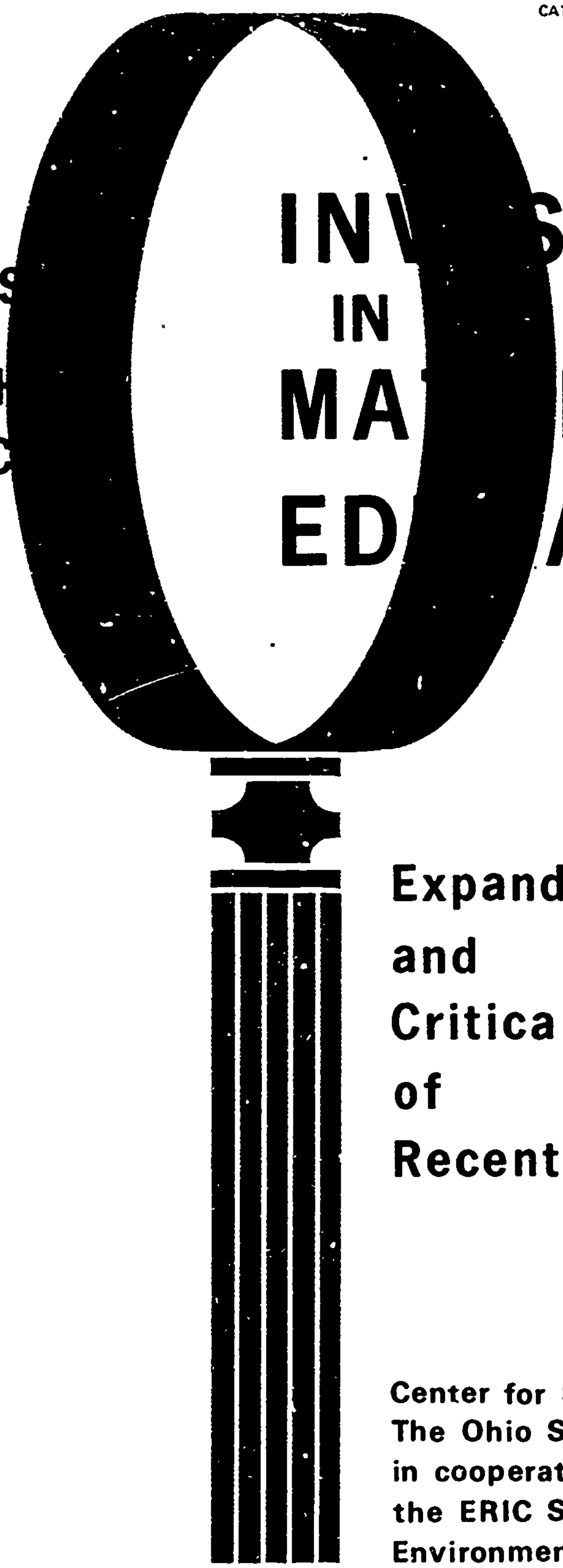
ABSTRACT

Expanded abstracts and critical analyses are given for each of 13 research articles. Six articles are concerned with evaluating methods of instruction, four deal with evaluation of student achievement and attitudes, one is concerned with test construction, and three articles investigate patterns of learning and learning hierarchies. (DT)

Vol. 6, No. 1, Winter, 1973



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INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts and Critical Analyses of Recent Research

Center for Science and Mathematics Education
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Winter, 1973

EJ 038 322 340 SE 503 384
DOES A TERMINAL MATHEMATICS COURSE CONTRIBUTE TO CHANGES IN ATTITUDES TOWARD MATHEMATICS? Aichele, Douglas B., Journal for Research in Mathematics Education, v2 n3, pp197-205, May 71
Descriptors--*College Mathematics, *Mathematics Instruction, *Mathematics Curriculum, *Student Attitudes, Attitudes, Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Phillip S. Jones, The University of Michigan

1. Purpose

To develop and apply to a particular course a test of changes in student's attitudes accompanying the taking of that course.

2. Rationale

Nonmathematics, nonscience majors in the College of Education and the College of Arts and Sciences of the University of Missouri-Columbia are required to take either College Algebra or Basic Concepts of Modern Mathematics. A major goal of the latter course is the promotion of positive attitudes toward mathematics. This article reports the process of developing and applying a test designed to measure the achievement of this goal. This study related both to previous attempts to measure attitudes toward mathematics such as those by L.R. Aiken and R.M. Dreger, NLSMA, and the International Study of Achievement in Mathematics, and to the design of some types of general or liberal education, collegiate mathematics courses. A survey of the literature of attitude testing and a report of a similar application is to be found in the 1970 unpublished University of Michigan doctoral dissertation of

David R. Duncan, The Effects of Instruction in Selected Mathematical Topics on Attitudes toward Mathematics of College General Mathematics Students.

3. Research Design and Procedure

Twenty-seven statements were derived from those in the International Study of Achievement in Mathematics. They were grouped to represent four categories of attitudes: I. Views concerning the learning of mathematics, II. Views concerning mathematics as a process, III. Views concerning the place of mathematics in society, IV. Views concerning school and learning generally.

Each of these statements was assigned a scale value on the basis of paired choices within the groups made by 117 students in the College Algebra course, and determined using a process derived from Thurstone's law of comparative judgments. The reliability of the scales was tested by administering the scale twice to a group of students not currently taking mathematics.

Each of the 65 students in the Basic Concepts course was assigned two scores derived from administrations of the scale at the first and last meetings of the class. The scores were the medians of the scale values of the items with which the students agreed.

4. Findings

There were no significant differences between the initial and terminal attitudes of the students when the replies were grouped into categories. There were significant changes in the replies to four specific items. Three of these represented deterioration in attitudes. The fourth was represented by the statement, "Anyone can learn mathematics." This improvement was from 27.7% to 43.1% favoring this view.

5. Interpretations

If the Basic Concepts course is to improve attitudes and opinions about the learning and nature of mathematics, it must be changed in some or all of its content, methodology, or activities.

Abstractor's Notes

It is particularly important today that attempts to test afferent goals of mathematical instruction be continued and improved and that attention be given to developing tests for higher order cognitive outcomes. Aichele's procedures for test construction seem to be thoughtfully planned and carefully executed. The lack of data on age, sex, ability, and previous training of his population probably does not affect the validity of his test for his immediate purpose but makes his scale less valuable as a contribution to a growing body of materials and knowledge for general use. His results are not inconsistent with those of Duncan and others who find attitudes toward school normally decline over a school year.

It is interesting to note the resurgence of interest in general education collegiate mathematics courses, but here, again, the lack of data about the particular course and its methodology restricts the utility and interest of this work.

Phillip S. Jones
The University of Michigan

EJ 042 088 110 SP 501 068
THE EFFICACY OF BEGINNING THE STUDY OF ALGEBRA AT DIFFERENT TIMES IN THE EIGHTH GRADE--A COMPARATIVE STUDY. Beal, Barry B., Colorado Journal of Educational Research, v10 n4, pp27-30, Sum 71
Descriptors--*Algebra, *Secondary School Mathematics, *Course Organization, Grade 8, Time Factors (Learning), Curriculum Design

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jeremy Kilpatrick, Teachers College, Columbia University

1. Purpose

To "determine the comparative efficacy" of beginning the study of algebra in the first versus the second semester of the eighth grade.

2. Rationale

Beginning the study of algebra in the first semester of the eighth grade frees time at the end of the high school program for additional pre-college mathematics. Postponing the beginning one semester gives pupils additional time "to mature and to develop a stronger arithmetic foundation." Selected eighth graders in the Denver Public Schools begin their study of algebra in the second semester. Some mathematics teachers suggested that it might be desirable to begin the study of algebra a semester earlier, and the Research Department of the Denver Public Schools was asked to undertake a comparative study.

3. Research Design and Procedure

The subjects were pupils of above-average intellectual ability (IQ at least 115) and above-average mathematical ability (on standardized tests and in former teachers' judgment) who were in the accelerated mathematics track at a six-year junior-senior high school in a high socio-economic area of Denver. Group E consisted of pupils who began algebra as eighth graders in the first semester of the 1967-68 school year; the next year they took a first course in geometry. Group A consisted of pupils who studied arithmetic during the first semester of the eighth grade in 1967-68 and who then began algebra in the second semester. Group G consisted of pupils a grade ahead who had begun algebra in the second semester of eighth grade, had taken three semesters of algebra, and took the first course in geometry as tenth graders at the same time (1968-69) the pupils in Group E were taking it as ninth graders. No information is given on how the groups were formed, how many classes or teachers there were, or how many subjects were in each group initially.

Groups E and A took the Lankton First Year Algebra Test at the end of their first semester of algebra. Groups E and G took the Howell Geometry Test at the end of their year of geometry. Group E took the Contemporary Mathematics Test: Algebra (CMT:A) at the end of eighth grade; Group G took the CMT:A at the beginning of tenth grade. School records were apparently used to obtain IQ scores (test or tests unspecified).

Data were analyzed for 29 Group E and 50 Group A pupils after one semester of algebra, and for 27 Group E and 32 Group G pupils after a year of geometry.

A two-way analysis of covariance (IQ as covariate; sex and treatment as factors) was used to compare the performance of Groups E and A on the Lankton test. A second two-way analysis of covariance (IQ and CMT:A as covariates; sex and treatment as factors) was used to compare the performance of Groups E and G on the Howell test. The .05 level of significance was used.

4. Findings

(a) The difference between adjusted means on the Lankton test was not significant (34.7 and 34.3 for Groups E and A, respectively, before adjustment; 33.9 and 34.7 after adjustment).

(b) The difference between adjusted means on the Howell test was not significant (33.0 and 37.1 for Groups E and G, respectively, before adjustment; 34.1 and 36.3 after adjustment).

(Also, no sex differences or interactions were significant.)

5. Interpretations

"Due to certain limitations of the study, sample selection and sample size, generalization must be made with care. However, there is some evidence to indicate that a similar population of pupils would not be limited in their studies of algebra and geometry by starting algebra in the first semester of eighth grade.

"Indeed, it may even be advantageous for these pupils to do so if one considers the additional semester of mathematics study in the junior year that this course of action would provide."

Abstractor's Notes

The sample and design were clearly inadequate for the issues posed. Subjects were not assigned randomly to treatments; no account was taken of the potential dependence among responses to the treatments by subjects who were in the same classroom. The investigator appears to be unaware of the various assumptions underlying analysis of covariance.

The classification variable of sex was included "to provide for a more efficient test," but its use remains a mystery in the absence of hypotheses or questions concerning sex differences or sex-treatment interactions.

The logic of using the CMT:A as a covariate is faulty. Even if the analysis of covariance were justified, the CMT:A would not be an appropriate covariate since it was administered during the treatment. Group G had one more semester of algebra than Group E; using the CMT:A as a covariate has the potential for removing any advantage to the learning of geometry that this extra semester might have conferred.

The investigator makes no reference to previous research. Had he looked at the research literature on the effects of acceleration, he might have skirted some of the pitfalls to be found in comparative studies of this kind. It's unfortunate that, at the very least, he didn't pay a call on the nearby Laboratory of Educational Research at the University of Colorado before undertaking this two-year study.

Jeremy Kilpatrick
Teachers College
Columbia University

ED 049 056 SE 011 040
STRATEGIES IN LEARNING MATHEMATICAL STRUCTURES, A BRIEF REPORT OF THE STUDY. Branca, Nicholas A., Stanford University, California. Pub Date Apr 71, Note--12p. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics (49th, April 14-17, 1971, Anaheim, California), EDRS Price MF-$0.65 HC Not Available from EDRS

Descriptors--*Algebra, Females, Groups, *Learning Characteristics, *Learning Processes, *Learning Theories, Mathematical Experience, *Mathematics, Student Characteristics

(Editor's Note: An addition report of this research appears in the Journal for Research in Mathematics Education, v3 n3 (May 72) under the title "The Consistency of Strategies in the Learning of Mathematical Structures.")

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Gerald R. Rising, State University of New York at Buffalo

1. Purpose

The study was undertaken to test findings of Dienes and Jeeves concerning game analysis "strategies and to determine whether subjects are consistent across structures and embodiments in the strategies they use and the evaluations they give."

2. Rationale

This is essentially a replication study to confirm or deny findings of Dienes and Jeeves as reported in Thinking in Structures (London: Hutchinson Educational Press, 1965).

3. Research Design and Procedure

From the author's description: "Each of the one hundred adolescent girls was given three experimental tasks in game-playing situations. The tasks were presented in three interviews with approximately two weeks between them. In each task the subject manipulated an apparatus that embodied a mathematical structure. The goal was to learn the rules of the game so as to make correct predictions about the outcome of each move. The interviewer kept track of the subject's moves in learning the game, the predictions she made, and the evaluations she gave of how the game worked.

"The first task, a Color game, had been used by Dienes and Jeeves and was based on the Klein group. The second task, a Map Game, had a network structure. The third task, a Light Game, also embodied the Klein group.

"For the two group structure tasks, Dienes and Jeeves's scheme was used for classifying subjects' evaluation--their views of how the game worked. Evaluations fell into three major categories, Operator, Pattern, or Memory, depending on whether subjects saw their moves as operators, looked for patterns in the plays, or merely attempted to memorize the plays. Evaluations of the network structure game were categorized as either Individual Roads or Detour Routes depending whether the subject focused on parts or the network or considered it as a whole.

"The sequence of a subject's moves on each task was taken as a measure of the strategy she was using, and strategy scores were calculated from this sequence. Dienes and Jeeves's system was used for the two group structure tasks. A strategy scoring system was devised for the network structure task."

4. Findings

"Distributions and cross-tabulations of the evaluations and the strategy scores confirmed Dienes and Jeeves's findings that the distribution of evaluations is ordered in decreasing frequency of occurrence as Pattern, Memory, Operator, and in decreasing efficiency, as measured by the mean length of play, as Operator, Pattern, Memory. Other findings by Dienes and Jeeves, such as zero correlation between measures of performance and intelligence and the existence of a relationship between evaluations and strategies as measured by their scoring system, were not supported. Consistency across tasks was most pronounced for measures of success and failure. Evaluations showed some consistency across tasks, especially across the two group structure tasks. Strategies tended not to be consistent across tasks, (possibly) owing to inadequacies in the strategy scoring system."

5. Interpretations

None.

Abstractor's Notes

Replication studies, given the tendency toward "overclaim" and "undercontrol" of today's scientists, are very much in order. This one appears carefully designed and carried out although as always an abstract, by its very nature, leaves details open to question. For example, while the data provided supports the first sentence of 4 above. the data for other findings is sketchy or absent. Internal evidence suggests that the author has this supportive data.

I am not at all put off by the author's lack of interpretation or extension. He has provided valuable evidence in support of Dienes' and Jeeves' work. It is to these other productive experimenters and theory builders that this task is left.

Gerald R. Rising
State University of New York at Buffalo

EJ 046 714 520 SE 504 303
COMPARISON OF TEACHER-WRITTEN AND EMPIRICALLY DERIVED DISTRACTORS TO MULTIPLE-CHOICE TEST QUESTIONS. Coppedge, Floyd L.; Hanna, Gerald S., Journal for Research in Mathematics Education, v2 n4, pp299-303, Nov 71
Descriptors--*Geometry, *Mathematics Education, *Multiple Choice Tests, *Test Construction, Objective Tests, Research, Teacher Developed Materials, Testing

Expanded Abstract and Analysis Prepared Especially for I.M.E. by L.D. Nelson, University of Alberta

1. Purpose

To determine the ability of experienced teachers and student teachers to select the most discriminating distractors for multiple-choice items in evaluating certain aspects of geometry.

2. Rationale

The authors point out the dilemma of teachers who prefer to use completion-type items in constructing tests but who often use a multiple-choice format instead because of ease and reliability of scoring. One way that teachers can attempt to improve the quality of any particular multiple-choice items is to administer the item to pupils first in completion form and then select from among the actual errors those distractors which have the most discriminating power. Teachers often try to do this but instead of choosing for distractors those erroneous responses that are the best discriminators they choose the responses which occur most frequently. The authors imply that the most frequent erroneous response is not necessarily the most discriminating.

3. Research Design and Procedure

To check the ability of experienced and student teachers to construct the most discriminating distractors for multiple-choice test items, a 33-item geometry test in completion format was first administered to 357 students.

An item analysis (or more correctly a response analysis) was then done in which the responses of the top 27% of the students were compared with the responses of the bottom 27%. There were among these, of course, some correct answers and some incorrect answers. In any case the percent of each group that gave each response to each item was calculated. A discrimination index for each response was obtained by subtracting the percent of the low group giving the response from the percent of the high group giving that response. This index would normally be positive for correct responses but negative for incorrect responses. The best distractors among the incorrect responses would presumably be those with the highest negative values. Sometime after this analysis was done 13 experienced secondary mathematics teachers and 18 university seniors (student teachers) were supplied with the 33-item test and the responses of the 357 geometry students. They were not supplied the item analysis data, however.

From this information they were each asked to change the completion form to multiple-choice form and to supply what they considered to be the three best distractors for each item.

The results of the item analysis for a few selected items (7 out of 33) are given in a table. There is a separate section in the table for the analysis of the students' responses, another for those of the experienced teachers, and a third for those of the student teachers.

4. Findings

In the student section of the table there is a column which shows the percent of students passing each of the 7 items, the next gives the discrimination index of the correct response and the last gives the total of the discrimination index of the best three distractors.

For both the experienced teachers and student teachers there is one column which shows the percent choosing the best distractor. Another column gives the total of the discrimination index of the three distractors they chose. Whether the authors selected items which showed as great a variety as possible or not, the tabulated information indicates there was a great deal of variation in the efficiency with which experienced and student teachers chose distractors. On some items there was little difference between the collective judgment of the teachers and the actual results of the response analysis. On others they chose responses which discriminated in the wrong way. The authors state that both groups of teachers found it difficult to separate the popular.distractors from best discriminating factors.

They also give further information that does not appear in the table. They state that there is a "loss in discriminatory potential of completion format student errors" of 72% for experienced teachers and 68% for student teachers. They also report that each group of teachers selected the single best distractor for items only about once out of four.

The experienced teachers are reported to have selected the best distractor slightly less accurately than did the student teachers. This finding was attributed to either 1. chance factors, 2. the student teachers' greater conscientiousness in research activities, 3. the student teachers' greater identification with the mental processes of examinees, 4. the student teachers' ability to distinguish between most popular distractors and the most discriminating ones.

5. Interpretations

The authors recommend that if one defines the best multiple-choice item distractors as those which discriminate best in completion test format one might follow this procedure:

1. Administer items in completion format
2. Analyze the items
3. Select the most discriminating errors as distractors for multiple-choice items

They point out the need, however, of determining the relationship between completion items and parallel multiple-choice items.

Finally they suggest that even though teachers might not follow the recommended procedure all the time in constructing multiple-choice items, it might be done from time to time. The advantages would be that teachers might learn to spot items with very poor distractors, they might increase skill in anticipating student errors, and they might improve their insight into the mental processes of high and low ability students alike.

Abstractor's Notes

This is a neat little study which points out the possibility of selecting multiple-choice distractors on the basis of discriminatory power rather than frequency. The underlying assumption in the whole study is that the procedure outlined would produce scores on multiple-choice tests that would probably correlate higher than usual with parallel tests in completion form. It is a pity that the investigators did not use the information they already had to go on and test this assumption. One still has the uneasy feeling that the most discriminating errors which show up in completion format may not discriminate in the same way if they are used as distractors in multiple-choice format.

This is a short report (only 4 pages in the journal) and probably was designed that way. However, more specific information about what data were provided to the teachers would have made the report easier to read. One wonders, too, why the complete analysis is shown for only 7. of the 33 items. An example or two of a completion test item, the kinds of response students made to them, and the kind of distractors chosen by teachers (were they the most frequent errors?) in making multiple-choice items would have improved the report immeasurably.

L.D. Nelson
University of Alberta

EJ 046 537 450 SE 504 301
LEARNING HIERARCHIES--NUMERICAL CONSIDERATIONS. Eisenberg, Theodore A.; Walbesser, Henry H., Journal for Research in Mathematics Education, v2 n4, pp244-256, Nov 71

Descriptors--*Behavioral Objectives, *Mathematics Education, *Research Methodology, *Sequential Learning, *Statistics, Curriculum, Evaluation Techniques, Learning Processes

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Richard J. Shumway, The Ohio State University

1. Purpose

To discuss numerical considerations dealing with the validation and comparison of behaviorally stated learning hierarchies.

2. Rationale

The construction of learning hierarchies has been proposed as a viable technique for learning research and curriculum development. Statistical procedures for the validation and comparison of such learning hierarchies for research are needed.

3. Research Design and Procedure

For a given hierarchy, let

$f(1,1)$ = the frequency of all completed tasks and subtasks prefaced by Ss success on all subordinate tasks,*

$f(1,0)$ = the frequency of all completed tasks and subtasks prefaced by Ss failure on at least one subordinate task,

*assumes at least one subordinate task

$f(0,1)$ = the frequency of all failed tasks prefaced by Ss success on all subordinate tasks,*

$f(0,0)$ = the frequency of all failed tasks prefaced by Ss failure on at least one subordinate task.

*assumes at least one subordinate task

Five ratios for learning hierarchies were proposed.

1. Consistency ratio

$$T \Rightarrow \forall Sb \qquad \frac{f(1,1)}{f(1,1) + f(1,0)}$$

2. Adequacy ratio

$$\forall Sb \overset{\text{(Instr.)}}{\Rightarrow} T \qquad \frac{f(1,1)}{f(1,1) + f(0,1)}$$

3. Inverse consistency ratio

$$\sim T \Rightarrow \sim\forall Sb \qquad \frac{f(0,0)}{f(0,0) + f(0,1)}$$

4. Inverse adequacy ratio

$$\sim\forall Sb \overset{\text{(Instr.)}}{\Rightarrow} \sim T \qquad \frac{f(0,0)}{f(0,0) + f(1,0)}$$

5. Completeness ratio

$$\frac{f(1,1)}{f(1,1) + f(0,0)}$$

In order to order hierarchies by "magnitude" an ordered triple,

(N, cc, ℓ),

was used, where,

N = the number of tasks with subordinate tasks,

cc = average number of subordinate tasks per task,

ℓ = number of subordinate levels.

A learning hierarchy associated with the triple (N_1, cc_1, ℓ_1) is of "higher magnitude" than a learning hierarchy associated with (N_2, cc_2, ℓ_2) if and only if two of the following are strict inequalities:

$$N_1 \geq N_2 \text{ , } cc_1 \geq cc_2 \text{ , or } \ell_1 \geq \ell_2 \text{ .}$$

4. Findings

None reported.

5. Interpretations

Ratios 1-5 can enable researchers to analytically determine whether or not a particular learning sequence achieves its objectives.

Given two equally valid hierarchies, the one of lesser magnitude is the more desirable.

The numerical ratios and magnitude vectors proposed will help researchers answer questions related to the effects for hierarchical learning sequences.

Abstractor's Notes

1. It is certainly appropriate to develop some machinery for objectively studying learning hierarchies.
2. What relationships exist among ratios 1-5? Some logical statements appear to be contrapositives of others. Author says no. What about linear dependence of measures?

3. What limitations can be given for the numerical considerations proposed? Is the magnitude ordering transitive?
4. What research should be conducted to study the value and properties of the proposed ratios and triples?

Richard Shumway
The Ohio State University

EJ 055 233 450 SE 505 358
DEFINITIONS AND EXAMPLES AS FEEDBACK IN A CAI STIMULUS-CENTERED MATHEMATICS PROGRAM. Keats, John B.; Hansen, Duncan N., Journal for Research in Mathematics Education, v3 n2, pp113-122, Mar 72
Descriptors--*Algebra, *Computer Assisted Instruction, *Instruction, *Learning, *Secondary School Mathematics, Definitions, Feedback, Numbers

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Max Jerman, The Pennsylvania State University

1. Purpose

The purpose of the study was to compare the efficacy of a numerical-example type of correctional feedback in a proof-constructing task, with a verbal-definition type of correctional feedback in a computer-assisted instructional (CAI) program.

2. Rationale

Correctional feedback following an error in a learning task is believed to facilitate acquisition to a greater degree than is providing reinforcement for a correct response. However, the precise form and content of the correctional feedback, following an error, that will optimize learning has not been established. Typically classroom teachers are observed to give as corrective feedback a restatement of the problem giving analogous examples or to state the solution rule in verbal form or simply to give the correct answer.

The present study was related to previous studies which were concerned with such things as inductive vs. deductive learning, discovery vs. expository learning, and rule vs. example learning in that although the use of verbal rules and examples was the subject of the study, "the rules and examples were used solely in CAI feedback as an aid in justifying the steps in a mathematical proof and not as a technique for problem solving."

The study also sought to investigate "the feasibility of structuring the program in much the same way that the classroom unit is developed."

3. Research Design and Procedure

The subjects were 45 ninth-grade student volunteers, 18 boys and 27 girls ranging in age from 14 to 15 years, selected from two Florida junior high schools. The students had just finished a year-long course in Algebra I in which the field properties for the real numbers had been introduced, but they had not been given formal instruction in complex numbers prior to the study. The SRA Primary Mental Abilities Test - Grades 9-12 was administered to assess the "entry behavior" of each student since "group differences in reasoning ability could account for group differences in performance of the CAI task."

The students were given a 24-item list of mathematical rules with abbreviations (e.g. CA:commutativity with respect to addition in R) and a brief review of the rules by the author of the study. The students were told that they would use the abbreviations of these rules in responding to the questions in their CAI programs. Those students who were unable to complete the program in one sitting returned to complete the treatment within two to five days following their first session.

The program itself consisted of 11 mathematical proofs which required a total of 64 steps. The number of steps per proof ranged from three to nine. The proofs established that the set of complex numbers was a mathematical field under the operations of multiplication and addition. Complex numbers were defined as ordered pairs of real numbers.

The 45 students were randomly assigned to one of three treatment groups. Group I, the verbal-definition group, received a nine-minute introductory session on formal definitions of a complex number, equality of complex numbers, and addition and multiplication of complex numbers. Each definition was illustrated by a single example. A brief review of the field properties of the real numbers was also included in the introduction, which made no provision for student response. Immediately following the introductory material the 11 proofs were presented consecutively. No review or preliminary information was given between proofs. In this treatment an incorrect response was followed by a few short sentences describing the rule which should have been applied (e.g. "No. Numbers are added two at a time. This rule tells us that it makes no difference which two are added first. Try again").

Group II, the numerical-example group, was given the same introduction and set of proofs except that the feedback following an incorrect response was in the form of a numerical example of the rule which was to be applied in that step of the proof, (e.g. "No. Why does (6,8) + (3,4) = (9,12)? Try again").

Group III, the detailed program with combination feedback, received a lengthy introduction to complex numbers in which students were asked to respond to specific questions in the introduction 29 times. Feedback was given in each case. Both formal definitions of complex numbers and equality of complex numbers were given and illustrated by short numerical exercises. A detailed review of the field properties of real numbers was given. In all, the introductory program included four exercises in addition and 11 exercises in multiplication.

The feedback used in the treatment for this group consisted of 36 statements of formal definitions and 28 numerical examples. In each case the feedback itself was identical to that received by one of the other groups for the same step. The form of feedback for a particular step varied between verbal definition and numerical example "according to the intuitions of an experienced mathematics instructor. . ."

4. Findings

A repeated measures analysis of variance using errors per step per student as the dependent variable, was performed. A significant group-by-proof interaction was found between Group I and Group III. With Group III removed, Groups I and II differed significantly with significant interaction, using the same statistic as before. A sign test for the total number of successes on the first trial of each step indicated a significant difference in favor of Group I over Group II ($p<.001$). Group I correctly identified 32 out of the last 38 steps in the treatment.

The average time spent on the program by each of Groups I, II, and III respectively was as follows: 3.04, 3.13, and 4.12 hr. No significant differences on mean posttest scores were found for the three groups. Significant negative correlations ($p<.01$) were found between total errors on the programmed unit and the posttest scores for both Group III (-.655) and for all groups combined (-.429).

5. Interpretations

Scores on the posttest were quite low for all groups (overall mean 11.3, 20 possible). The high error rate may have been due in part to the fact that some of the items were repeated from the CAI unit out of context. It is possible that one reason for the low reliability ($r=.38$) of the posttest was due to the fact that there were only 20 items on the test, 10 true-false and 10 multiple choice. "With nearly identical scores for all these groups on an unreliable instrument, it is difficult to draw any conclusions about treatment effects with respect to acquisition and retention of knowledge." However, using the information derived from error analysis, it appears that, with respect to CAI presentations, "providing correctional feedback in the form of verbal definition is of more benefit to the learner than using a numerical example."

The performance by Group III was below expectation. "The longer and more detailed treatment of mathematical proofs within a CAI framework may be unwarranted."

Group II exhibited very poor performance on two proofs (6 and 9) which required the use of multiplication concepts. Proof 6 was the first proof which involved multiplication concepts. The first five proofs did not. Proof 9 was the first proof involving steps where both sides of the equation were operated upon. "It is suspected that the numerical examples were not very effective in illustrating operations performed to both sides of an equation."

Abstractor's Notes

The author's statement "With nearly identical scores for all three groups on an unreliable instrument, it is difficult to draw any conclusions about treatment effects with respect to acquisition and retention of knowledge" should be taken seriously. The study was of very short duration, 3-4 hours.

Some questions that come to mind are the following.

1. Did the treatments themselves permit the testing of the hypotheses of the study? The feedback for Group I focused the student's attention on a step in a proof by telling the student what was being done on the step and asking him to give the rule. The treatment given Group II used a simple numerical example and expected the student

to transfer the information back to the step in the proof. Perhaps if the numerical example had been a direct numerical substitution rather than a simpler case, some transfer would have occurred.

2. What was the effect on achievement or error rate of a student's not being able to complete the study in one sitting? We are not told the number of students who were unable to complete the experiment in one sitting.
3. What attempt was made to control for prior learning?
4. Would a different mix of reinforcement feedback for Group III have altered the performance of that group?
5. Was it feasible to structure the program in the same way as a classroom unit is developed?

Max Jerman
The Pennsylvania State University

ED 037 345 88 SE 008 012
UTILIZING AN ELECTRONIC CALCULATOR TO FACILITATE INSTRUCTION IN MATHEMATICS IN THE 11TH AND 12TH GRADES, FINAL REPORT. Keough, John J.; Burke, Gerard W., Suffolk County Regional Center, Patchogue, N.Y. Spons Agency--Office of Education (DHEW), Washington, D.C. Bureau of Elementary and Secondary Education. Pub Date Jul 69, Note--60p, EDRS Price MF-$0.65 HC-$3.29
Descriptors--Curriculum Development, *Individualized Instruction, *Instruction, *Mathematics, Research, *Secondary School Mathematics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by T.E. Kieren, University of Alberta

1. Purpose

The purpose of this study was threefold.

- Will the use of calculators improve instruction in eleventh and twelfth grade mathematics?
- Can adequate software be developed for its use?
- Are there peripheral applications in chemistry, physics, etc.?

2. Rationale

The rationale for this study was a practical one. It is thought that computational aids, in this case the Wang Laboratory Calculator, can improve instruction in mathematics. This improvement should arise through more thorough understanding of concepts, through individualization and through increased motivation. This study sought, for these school districts, to evaluate these claims for this particular piece of hardware.

3. Research Design and Procedure

The study used a quasi-experimental design. Six regular and accelerated grade 11 and 12 mathematics classes with 161 students formed the experimental group. A cohort group of seven classes in a similar school with 146 students was labeled as control students.

The experimental students would, if the topic lent itself, work in pairs with the teacher-prepared materials in a calculator laboratory. These exercises were similar to those used in regular classroom work. The intent was usually to introduce or consolidate work on a particular topic. Generally the teacher would give a brief introduction and then the students would work on the problems using the calculators with teacher assistance. In addition to class time students could use the calculators during free periods and after school. This procedure was carried out over the course of the 1968-69 school year, and included considerable teacher in-service training and meetings.

The formal two year study, the first year of which is discussed in this report, attempted to answer the questions posed above through three stated hypotheses:

a. The use of the calculator as an instructional tool will result in significantly greater achievement, by the experimental group, on standardized tests in eleventh and twelfth grade mathematics.

b. The use of the machine by the experimental group will have a positive effect on student motivation.

c. Students in the experimental group will study a greater number of topics in the required and optional syllabus for mathematics of the eleventh and twelfth grades.

To test these hypotheses pre and post treatment data were collected on the mathematics section of the Sequential Tests of Educational Progress, Form 2A. In addition the Strong Vocational Interest Blank was used in evaluating student motivation (the post testing in this case was to be done at the end of the second year of the program). As a source of information on the two samples, scores on 1968 Regents Examinations and Otis I.Q. scores were gathered. Using t-tests. no mean differences were uncovered between experimental and control groups on these two sets of tests. However, the data from the Otis I.Q. substantiated the fact that the students were generally of above average ability.

4. Findings

With respect to the achievement hypothesis, it was found that there was no significant difference in pre-treatment means (t-test) between the experimental and control groups (E = 290.77, C = 288.71), on STEP-2A. On the post test there was a significant difference at the .01 level (t-test) between the means of the experimental and control groups (E = 295.1, C = 291.0) on STEP-2A.

With respect to the motivation hypothesis, the data is still not available. However, a majority of the experimental groups expressed the opinion that the calculator work did not help improve their achievement on standardized tests. But 97 per cent of these students wished this experience continued.

With respect to numbers of topics covered, only the accelerated grade 12 students appeared to cover more topics than in previous years. In all other groups there were no differences in the number of topics covered using the calculator and the number covered in previous years.

5. Interpretations

The investigators felt that the use of calculators does significantly increase achievement on standardized tests. Although software for 23 topics was developed, there is a great need in the area. It is recommended that further study be done at lower grade levels in the school and with low achievers.

Abstractor's Notes

1. Were the achievement test results more a statistical artifact than an educational reality?
2. The abstractor would have liked to see detailed results of calculator contributions to achievement in particular topics. The instrument used tends to obscure this.
3. Was there a differential effect between average and high ability students with respect to the use of the calculators?

4. Were there particular calculator uses which proved effective? Ineffective? What were their characteristics?

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EJ 039 003 110 SE 503 280
ON SCRAMBLING INSTRUCTIONAL STIMULI. King, Bruce W., Journal for Research in Mathematics Education, v1 n4, pp233-240, Nov 70
Descriptors--*Curriculum, *Mathematics Education, *Programed Instruction, *Research Reviews (Publications), Learning

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert E. Pingry, The University of Illinois, Urbana-Champaign

1. Purpose

The author reviewed recent research (1960-1969) concerning sequencing of instructional stimuli so as to learn what had been found from the research, and to see if implications existed for classroom instruction.

2. Rationale

In an experiment in 1960 a student failed to read the instructions for use of a programed textbook and read down the page instead of from page to page. Despite the fact that he responded to the items in the order 1, 40, 79, 118, 157, 2, 41, and so on rather than 1, 2, 3, and so on, he still received a high score on the criterion test. This incident seemed to stimulate several studies on sequencing of items on programed instruction units. These studies continued throughout the sixties. The latest study reviewed was one in 1969. Over the years researchers gradually refined the questions asked, and improved the quality of the experiments.

3. Research Design and Procedure

The research studies reviewed experiments with several aspects of the sequencing of items. Some of the research questions arose out of an apparent conflict between Gagne's work showing that mastery of higher order tasks usually occurred only if subordinate tasks were mastered, and several studies showing that there were no significant differences in the effectiveness of scrambled and logically ordered sequence of items. Other studies arose out of questions related to the size of the unit. One study related to macro-order, where the program elements appear to have been discrete levels of a hierarchical structure, and to micro-order, where the program elements were frames leading to a particular task in a hierarchy.

4. Findings

Many of the studies reported no significant differences in the achievement resulting from various sequencing of items in a learning program. It appears that varying the micro-order of instructional stimuli does not make much difference in the effectiveness of instruction. However, disruption of macro-order sequencing did affect learning. The fact still remains that in the large, some things must be learned before other things, but in the small, there appears to be considerable latitude in the sequencing of learning stimuli without affecting learning.

The author of the review, after his study of the research reports, used considerable caution in making inferences from the research for classroom instruction. He was willing to conclude "that varying highly refined sequences of instructional stimuli does not make much difference in effectiveness of instruction, as long as concept order is preserved."

5. Interpretations

The author of the review pointed out that if it is true that sequencing makes little difference, then "one might continue organizing instruction along strictly logical lines, since it seems to do little harm, and is convenient for the instructor." In addition, the organizing of mathematical information into logical sequences is a valuable learning experience for its own sake.

The author pointed out that sequencing stimuli in an order that first considers intuitive and exploratory aspects of a topic before a formal logical order may be done without excessive concern for the order.

Abstractor's Notes

Educational research literature is filled with reports of experiments where the results are reported but "no significant difference was found." It is not uncommon for the naive reader to conclude "there is no difference." Mr. King, the author of the review, warned his readers of this fallacy when he stated, "Such results might be due to the fact that these studies had certain methodological flaws."

A researcher does not report that he has proved that there is no difference. He reports that he has not found a sample difference so large that he believes that there is a difference in the population. The reasons the researcher has not found a difference may be that none exists, but it also may be because his:

1. measuring instrument is too crude.
2. sample is too small related to the variance of his criterion measure.
3. control of the experimental conditions were not adequate.

It would have been helpful in a review of research studies on the scrambling of instructional stimuli for some evaluation to be made of the conclusion "no significant difference was found." Were these conclusions the result of sloppy experimentation or results from carefully controlled, adequately designed experiments? Some of this evaluation was done (p. 235), but more would have been helpful.

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EJ 044 687 310 AA 510 557
MATHEMATICS LEARNING AND PUPIL CHARACTERISTICS. Liedtke, Werner, Alberts Journal of Educational Research, v17 n3, pp143-153, Sep 71

Descriptors--*Learning Activities, *Mathematics, *Student Characteristics, *Teaching Techniques, *Concept Formation, Hypothesis Testing, Data Collection, [*Piaget (Jean)]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Kenneth J. Travers, University of Illinois, Urbana-Champaign

1. Purpose

The major purpose was to isolate some pupil characteristics which might facilitate learning of mathematics and to examine the role of these characteristics in a self-directed (SD), a partially teacher-directed (PTD) and a teacher-directed (TD) setting.

2. Rationale

The work of Piaget and his followers has suggested that the child learns from active interaction with his environment and from doing things that are interesting to him. When applied to elementary school mathematics, Piaget's theory suggests a learning atmosphere described by various authors as "narrow(ing) the gap between real life learning and traditional school learning" (page 143). Writings by Duckworth (1964), Harrison (1969) and Biggs and MacLean (1969) are in this vein. Merits of activity or discovery approaches to mathematics are reported by Worthen (1967) and Biggs (1968).

The work of these and other investigators has isolated many variables which relate to mathematics achievement in a variety of instructional settings.

3. Research Design and Procedure

Three experimental groups at the grade five level were established:

a. Self-Directed (SD) (N=51, 30 males, 21 females). These pupils worked in small groups. They solved problems and participated in activities without receiving formal instruction from the teachers. Answers were provided, but checking of work was voluntary.

b. Partially-Teacher-Directed (PTD) (N=53, 32 males, 21 females). Teachers were assigned more tasks here than for the SD group. Topics were formally introduced. Answers from previous day's work were read. Examples and assignments from new work were given. Pupils worked in small groups as in SD.

c. Teacher-Directed (TD) (N=37, 16 males, 21 females). A more formal setting. Teacher supervised checking of work and explained new topics to pupils.

Ss were selected for the SD treatment from a school which utilized an approach similar to that of the treatment. This was done in order to avoid spending a great portion of the treatment time on orientation and familiarization. Ss for the PTD treatment were selected from the same school as the SD group, but were from classes where past experience was textbook oriented. Ss for the TD group were from another school in the city. Their past experience was textbook oriented. The treatments lasted for four weeks.

4. Findings

No significant correlations were found between intelligence, socioeconomic status, personal adjustment, social adjustment, reading ability and the scores on the initial learning and retention test for the SD group. Significant correlations ($p<.01$) were found between intelligence, reading ability and the criterion tests for both the PTD and the TD groups. For the PTD group only, significant correlations ($p<.01$) were found between personal adjustment, social adjustment and the criterion measures. Within the SD and the TD treatment groups, no significant differences between means were found between the reflective and impulsive students (classified by sex) on the criterion measures. For the PTD subjects a significant t ($p<.01$) was obtained for mean differences between impulsive and reflective groups on the retention test (but not the initial learning test).

5. Interpretations

a. For the self-directed (SD) group, the absence of relationships between the selected variables and the learning outcomes suggests that changes in role or leadership style of the teacher result in different behavior reactions from individual pupils.

b. The partially teacher-directed (PTD) setting appears to benefit most (in terms of mathematics achievement) those students who make personal and social adjustments readily and have high reading ability.

Each student was provided with an activity booklet on the various mathematics topics. This booklet was used by the SD and PTD groups and modified slightly to conform to the format of the TD treatment. Teams of teachers were used with each treatment group. The teachers took turns in teaching in an attempt to minimize teacher effects. (The report does not make it clear whether the same team of teachers worked with each group.)

Data were collected using the following instruments:

California Short-Form Test of Mental Maturity (Level 2)

California Test of Personality (Elementary--Form AA)

Paragraph Meaning Test, Stanford Achievement Test--Reading (Intermediate I)

Kagan's Matching Familiar Figures Test (Kagan, 1965). Six items from this test were administered individually to each S. This test was used to classify students as impulsive or reflective.

Pretest and Posttest on Mathematics. These tests were based on the content of the topics presented during the treatments.

Data are summarized in tables of descriptive statistics for the various measures, test results on the criterion measure (mathematics achievement), intercorrelations among variables and criteria for Ss in each treatment group, and in a narrative report of differences between impulsive and reflective Ss within each treatment group.

c. For the teacher-directed (TD) treatment, intelligence was the most important predictor of mathematics achievement.

d. No advantages to learning were apparent for either the impulsive or the reflective child in either the SD or the TD settings.

e. Achievement tests administered after short durations of self-directed activity are not good indications of the apparent productivity (easy adjustment, eager participation and work, etc.) which attends independent group work.

Abstractor's Notes

The topic of this research is extremely important in view of the widespread interest in Piaget, and the implications which his work appears to have for curriculum, instruction and learning.

The outcomes of this study are not surprising. In traditional, or even "partially traditional" settings, those students who do best by conventional standards (such as performance on achievement tests) are those who have done best all along by conventional standards (performance on tests of mental maturity, etc.). Of more interest is the finding that in nonconventional settings having high pedagogic merit (encouraging participation, discussion, activity) the conventional predictors of success are no longer useful. Bloom et al. (1971, p. 45) have noted that when the kind and quality of instruction and the amount of time available for learning are made appropriate to characteristics and needs of each student, mastery of the subject may be expected and correlation between aptitude and achievement (for normal distribution of aptitude) should approach zero.

The researcher is to be commended for the apparent thought and effort put into developing the criterion measure. The instrument is described as having three parts: (1) recall of material, (2) ability to generalize about material presented, and (3) problem solving requiring operations and solutions beyond those presented in the treatments. The important chapter by Wilson in Bloom et al. (1971) provides a valuable conceptual framework and a rich source of item types for others wishing to develop achievement tests which purport to measure a suitably comprehensive set of intended learning outcomes.

It is entirely likely, however, the conventional research procedures are not able to tease out the kinds of information pertinent to this sort of study: learning styles of individual students; effects of interpersonal relationships upon learning outcomes; conceptual development as it relates to the teaching setting; and so on. A more productive type of research may be the so-called "ethnographic approach" as exemplified by Smith and Geoffrey (1968). This technique, devised and used by anthropologists, includes the monitoring of events over long periods of time, the use of informant rather than respondent interviews and the determination of usages and meanings of words and behaviors by close observation of the contexts in which they are employed.

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EJ 036 629 270 AA 509 252
TYPE OF MATHEMATICS TEACHING, MATHEMATICAL ABILITY AND DIVERGENT THINKING IN JUNIOR SCHOOL CHILDREN. Richards, P.N.; Bolton, N., British Journal of Educational Psychology, v41 pt1. pp32-37, Feb 71

Descriptors--*Mathematics Instruction, *Divergent Thinking, *Test Results, *Tests. *Children

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lewis R. Aiken, Guilford College

1. Purpose

To assess the effects of activity and discovery methods of teaching mathematics on the divergent thinking abilities and cognitive performances of British junior school pupils.

2. Rationale

In both the United States (e.g., the Madison Project) and Britain (e.g., the Nuffield Project), discovery methods have been increasingly applied in recent years to the teaching of mathematics. Although these curricular projects are designed to encourage creativity and critical thinking, the findings of studies concerned with the effects of activity and discovery methods have not been conclusive.

3. Research Design and Procedure

The following tests of intelligence, divergent thinking, and mathematical ability were selected to measure a wide range of convergent and divergent thinking abilities in junior school samples:

Intelligence: Moray House Verbal Reasoning Tests 81 and 82 (1968 versions).

Divergent Thinking: Minnesota Tests of Creative Thinking--Circles Test; Guilford's Uses for Things Test, Torrance's Consequences Test; Wallach & Kogan's Pattern Meanings; Getzels and Jackson's Make-Up Problems Test.

Mathematical Ability: NFER Concept Test, Part A; NFER Arithmetic Progress Test C1; NFER Intermediate Mathematics Test 1; Series Completion; Gap-Filling Test; Easy Problems Test.

In addition, a simple scale was used to measure attitudes toward reading, mathematics, writing stories, and physical education.

Scores on the Moray tests were available at the beginning of the study. The remaining tests were administered during three sessions to 265 children (mean age of 11 years) in their final year at three junior schools in Northeastern England. The three groups did not differ significantly in average age, IQ, or social class. Mathematics was taught one hour a day to the 102 children in School A, the 71 children in School B, and the 92 children in School C. The schools differed, however, in their method of teaching mathematics. The mathematics teaching in School A represented an attempt to maintain a balance between traditional and discovery methods, whereas the teaching in School B was more traditional and oriented to the class as a whole. Finally, the Nuffield Project school, School C, was committed to a discovery approach.

4. Findings

The 24 scores obtained from the 12 tests, the six attitude scores, and the sex variable were factored analyzed (varimax solution). Six factors were extracted, the first two of which accounted for 32.7% and 14.8% of the variance. The intelligence and mathematical ability tests had high loadings on Factor I, labeled "general ability." Although several of the divergent thinking tests loaded moderately on the first factor, their loadings were higher on Factor II, labeled "divergent thinking." The other four factors are not easily interpreted, but the largest percentage of the remaining variance (10.4%) was accounted for by Factor V. All of the tests of divergent thinking ability had negative loadings on this factor.

The means of School C were lower than those of Schools A and B on 10 of the 12 mathematics tests, and significantly so on the NFER Arithmetic Progress C1 tests. On the other hand, the mean of School C was higher than those of the other two schools on the Circles Test from the Minnesota Tests of Creative Thinking. In addition, Schools A and C had higher means than School B on all of the divergent thinking tests, but the three schools did not differ appreciably on the Easy Problems Test. With regard to the scores of the three schools on the measure of attitudes toward mathematics, the school means were not significantly different.

5. Interpretations

The first conclusion drawn from the results of the factor analysis is that a general ability factor was largely responsible for performance on the tests of mathematical ability. In contrast, on only a few of the mathematics tests did divergent thinking ability make a significant contribution to performance, and in those cases it was relatively small. Consequently, it was concluded that teaching which emphasizes divergent thinking ability will not greatly affect achievement in mathematics. Furthermore, the results were interpreted as supporting a previous finding that activity methods result in poor computational and problem-solving skills, but as offering no support for the assertion that such methods lead to a higher level of creative thinking. It is suggested that further studies of open-ended instructional situations be conducted, and that it be determined which kinds of information are best learned by authority and which by creative teaching methods.

Abstractor's Notes

Obvious difficulties with this investigation concern the omission of information about characteristics of the three groups of children and how the samples were selected. Also, we are not told anything specific about the time intervals between administrations of the various tests, how and by whom they were administered, etc. Finally, differences among the means of the various tests were presumably analyzed by some means, but the procedures employed (t tests, F tests with multiple comparisons?) and the specific results are not clearly described in the paper.

These omissions, however, are of relatively minor importance. The major problem with this investigation, as with many non-experimental studies in education, is the attempt to draw causal conclusions from what are basically correlational data. The children were not randomly selected or randomly assigned to three groups. If the schools were randomly selected, then the unit of analysis is the school mean rather than the individual test scores. In any case, there is no way of determining what uncontrolled organizmic variables may have influenced the results. True, the three groups did not differ significantly in age, IQ, or social class, but other influential individual and group differences may have existed. A potentially more illuminating approach, although still correlational rather than experimental, would be to conduct a pretest-posttest study of comparative changes in factor structure and mean scores on the various tests over the school year. Such a study would, of course, entail a greater amount of testing and a more elaborate analysis of the data.

In spite of the shortcomings of this investigation, the fact that both the mathematics tests and the intelligence tests had high loadings on the same factor is consistent with other evidence of a substantial overlap between quantitative reasoning ability and general intelligence. As has been found in other instances, tests of divergent thinking tend to have rather small loadings on this general ability factor. Also, the results of this investigation reinforce the abstractor's preference for a balanced approach in teaching mathematics, an approach which attempts to foster a "critical, creative turn of mind" without sacrificing basic skills and concepts.

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EJ 055 231 450 SE 505 354
THE EFFECT OF TRAINING ON LENGTH ON THE PERFORMANCE OF KINDERGARTEN CHILDREN ON NONSTANDARD BUT RELATED TASKS. Romberg, Thomas A.; Gilbert, Lynn E., Journal for Research in Mathematics Education, v3 n2, pp69-75, Mar 72

Descriptors--*Conservation (Concept), *Geometric Concepts, *Instruction, *Mathematics Education, *Research, Achievement Tests, Kindergarten, Mathematical Concepts

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James K. Bidwell, Central Michigan University

1. Purpose

The purpose of the study was an examination of the effect on kindergarten children that the learning of the concept of length as an attribute of objects would have on the performance of related tasks involving length.

2. Rationale

Young children have perceptual misconceptions about the stability of the different attributes of an object. "The study was undertaken from a process-by-attribute instructional approach." The child focuses on a relevant attribute of objects and makes comparisons between objects (decides whether two objects are or are not the same with respect to the particular attribute).

3. Research Design and Procedure

The design used was a non-equivalent pretest-posttest control group design (Group I--treatment, Group II--control). A third group (III) was used to test instructional effects of the pretest. Three morning kindergarten classes were used as the three groups and were assumed to have no significant difference between them. The group sizes were 24, 23, 24 (42 boys and 29 girls) respectively.

Groups I and III were given a training program consisting of three 20-minute group lessons on successive mornings. Lesson one dealt with characteristics of length adopted from The Child's Conception of Geometry. Lesson two was concerned with comparing lengths of two objects. The third lesson compared lengths requiring an intermediate representation of one of the lengths.

Groups I and II were given a pretest. All three groups were given a posttest. The same 12-item individually administered test was used as the pretest and as the posttest. The items were problems that could be solved using the training procedures. Several of the items were based on questions described in The Child's Conception of Geometry. They involved comparing lengths, conservation of length, comparing via representation of length, and continuous and discontinuous objects.

Group means were found on the tests, a three-factor (training, sex, age) ANOVA was done on the posttest scores for Groups I and II, the same analysis was done for Groups I and III, reliability coefficients were found for all three groups on both tests, item difficulties were found, and separate ANOVA's were done for each item on the test.

4. Findings

Group I had a pretest mean of 3.38 and a posttest mean of 5.63, Group II had means of 3.00 and 3.20 respectively and Group III had a mean of 5.00 on the posttest. The effects due to training between Groups I and II were significant ($p<.004$). No significant difference was found between Groups I and III. indicating that the pretest had no instructional effect. There was, however, a mental set towards length, caused by the pretest.

Approximately 75% of the pretest mean was attributable to four test items. When these items were eliminated the test means became 0.84 and 2.37 for Group I and 0.73 and 0.96 for Group II, respectively. Group I children showed some understanding of the remaining eight items after training.

The separate ANOVA's showed significant or marginally significant gains on five of the twelve test items because of the training sessions. Only one of these five items was judged to have been affected by pre-treatment understanding.

5. Interpretations

"This instructional experiment reveals the difficulties of influencing children's misconceptions." The results attest to the phenomenon of perceptual misconception among children. A mean of 5.3 out of 12 after treatment does not indicate real comprehension of the concepts involved. There were, however, significant gains--even though the instruction was brief and directed to the whole group.

Abstractor's Notes

Although the task and instruction descriptions were very brief, it appears that the significant gains were made on items most closely related to the training program. The authors do not say how close in time the testing was to the treatment. Assuming the tests were administered on the days previous to and following the training program, it is clear that the instruction was insufficient. A more prolonged training period may have resulted in much higher scores approaching mastery level. This study is apparently an example of a well-known difficulty of teaching young children: lack of mastery due to insufficient exposure time and duration of training. It also indicates the weakness of group instruction and the (probable) lack of individual student involvement.

The child's perceptual "misconceptions," on the other hand, simply points out the tenacity of the mental schemas of even an immature mind and the need to find appropriate phenomena and activities to modify what we consider to be "inaccurate" schemas. We must be careful to remember that children do not have wrong concepts, only different or incomplete concepts from adults.

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EJ 050 619 270 SE 504 656

THE INFLUENCE OF TEACHER BEHAVIOR AND DYAD COMPATIBILITY ON CLINICAL GAINS IN ARITHMETIC TUTORING. Schultz, Edward W., Journal for Research in Mathematics Education, v3 n1, pp33-41, Jan 72

Descriptors--*Mathematics Education, *Research, *Student Teacher Relationships, *Teacher Attitudes, *Tutoring, Elementary School Students, Instruction, Remedial Arithmetic, Teacher Behavior, Teacher Characteristics

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas J. Cooney, The University of Georgia

1. Purpose

The collective hypotheses examined in this study were that neither a tutor's level of predisposition for offering facilitative conditions nor a tutor's degree of interpersonal compatibility between tutor and student, nor both, would be significantly related to student achievement in arithmetic, change in student self-concept of arithmetic ability, student judgment of the relationship provided by tutors, tutor judgment of the relationship provided students, or tutor-offered levels of facilitative conditions.

2. Rationale

Researchers have been concerned with the relationship between student learning and personal growth and the quality of the interpersonal relationship between teacher and student. It is felt that a teacher's ability to empathize and relate to students is a central ingredient in changing students' cognitive and affective behavior. However, the research has not been conclusive concerning the interpersonal relationship between students and teacher and student growth in arithmetic. In this study the effects of facilitative behavior and interpersonal compatibility between teacher and student on selected educational outcomes were investigated. Children diagnosed as having learning problems in relation to arithmetic concepts and principles related to numbers and non-negative rationals were used as subjects.

3. Research Design and Procedure

Tutors were selected from 30 University students enrolled in a course titled "Diagnosis and Remedial Teaching of Arithmetic." These students completed three instruments: Predisposition Index of Facilitative Conditions (PIFC), Fundamental Interpersonal Relations Orientation-Behavior (FIRO-B) and the mathematical knowledge section of the Callahan Test of Mathematical and Professional Knowledge. Apparently, the 10 students who ranked highest and the 10 students who ranked lowest on the PIFC instrument were selected as participants in the study. The difference of means of these two groups was significant for the PIFC instrument ($p < .01$) but not significant on the Callahan instrument ($p < .01$.)

Eighty-six 5th, 6th, and 7th grade school children were identified on the basis of tests indicating a 10 month discrepancy between observed (Stanford Diagnosis Arithmetic Test - SDAT) and expected grade level performance. From these 86 students, 40 were selected as subjects on the basis of those most and least compatible with the 20 tutors as determined by the FIRO instrument. Thus there were 20 most compatible pairs possible and 20 least compatible pairs possible of student-tutor teams. Nine one-hour weekly tutoring sessions were conducted from November through January.

Prior to the first tutoring session, each child was administered the Brookover Self-Concept of Academic Abilities-Arithmetic (BSCA-A) instrument. Following the tutorial sessions the BSCA-A and SDAT (equivalent form) were readministered. Tutors, upon completion of all tutorial sessions were given inventories to measure their relationship with students. Measures of facilitative conditions (Accurate Empathy, Interpersonal Respect) of the tutorial sessions were obtained by rating the tape recordings of selected tutorial sessions. Correlation coefficients between raters ranged from .87 to .97.

A 2x2 analysis of variance (fixed effects model) was used to test the main and interaction effects specified in the hypotheses (tested at .05 level).

4. Findings

a. The evidence did not indicate a significant relationship between achievement and tutor predisposition level of facilitative behavior and/or degree of interpersonal compatibility between tutor and student.

b. Although positive, the relationship between tutor level of facilitative behavior and student self-concept growth was not statistically significant.

c. Degree of interpersonal compatibility was found to be significantly related to student judgment of the teacher-student relationship.

d. Tutor perception of the existence of a helping relationship was not significantly related to teacher level of predisposition for offering facilitative behavior and/or degree of interpersonal compatibility between tutor and student.

e. Tutors did not differ significantly in the level of facilitative conditions they offered students (although tutors were selected to maximize differences).

5. Interpretations

a. After nine tutoring sessions, only 15 of the original 40 students continued to meet screening criteria. Total group growth was 1.31 years. The advantages inherent in tutorial settings may have been a pervading influence in determining student growth.

b. The BSCA-A instrument is generally not used in research over a short period of time. It has been suggested that when this instrument is used on a short term basis, significant results are less likely to occur. Nevertheless, the observed trend in self-concept growth is encouraging.

c. From findings c and d it appears that students are more perceptive than tutors in analyzing the student-teacher relationship. Even though a positive relationship between teacher and student can be identified, such a relationship does not seem to be related to student growth.

d. The PIFC instrument may have assessed an "ideal" rather than "actual" level of facilitative conditions among the tutors. This is consistent with other findings which indicate that both "good" and "poor" teachers can describe the ideal student-teacher relationship, but only the "good" teachers could translate the ideal into classroom practice.

Abstractor's Notes

This is an excellent study dealing with student-tutor relationships and selected educational outcomes. Further studies involving a more extensive relationship between tutor (or teacher) and student might elucidate some of the questions raised in this study. Nine tutorial sessions one week apart may not be sufficient time to allow a tutor's facilitative behavior to manifest itself. One might conjecture that a tutor's primary concern, particularly in the initial stages of instruction, is with the content, i.e., diagnosing student's weaknesses in content, writing content objectives and evaluating the student's progress in arithmetic achievement. One might also conjecture that while the tutor may be subject matter oriented, the student may be more apt to focus on the interpersonal relationship between himself and his instructor. These phenomena may be inherent in tutors' comments such as, "He can't square a binomial," as opposed to students' comments "I like the tutorial arrangement because my tutor is interested in me." This writer has observed these and other similar statements in overseeing various tutorial situations. If valid, such conjectures may help explain findings c and d. It might also explain, at least in part, finding a which indicates that a student's arithmetic growth (see Interpretation a) is not seemingly related to a tutor's facilitative behavior.

It is astonishing that in nine tutorial sessions the average increase was 1.31 grade equivalents. Is such an increase retained? At what levels did the gains occur; that is, how much below grade level were the subjects operating?

The author points out that the BSCA-A instrument is more appropriate to research involving an academic year's work. In view of this, one has to question the usefulness of this instrument when applied to a situation involving only nine weekly tutorial sessions. There is also the question of the extent to which the instruments utilized in the study measure the nature of interpersonal relationships that are the crucial variables in the teaching-learning process.

Even though this study would probably be judged inconclusive, it is this writer's opinion that much can be learned about teaching and learning when investigations are housed in clinical settings.

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EFFECTS OF TRAINING ON THE CONCEPTS OF WATER LEVEL AND HORIZONTALITY IN THE CLASSROOM. Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning. Technical Report No. 128. Spons Agency--National Center for Educational Research and Development (DHEW/CE), Washington, D.C. Pub Date May 70, Note--92p, EDRS Price MF-$0.65 HC-$3.29

Descriptors--*Concept Formation, *Elementary School Mathematics, Instruction, *Learning, Perception, *Research, Space Orientation, *Visual Learning

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Marilyn N. Suydam, The Ohio State University

1. Purpose

To ascertain whether specific training on the water level concept and the concept of horizontality, with and without specific language labels, would significantly affect performance on a test of understanding of these concepts.

2. Rationale

The concept of horizontality is important as a developmental process necessary so that the child is able to coordinatize the plane; until this is done, Euclidean geometry would be unintelligible. Piagetian research on the development of the concept of space is cited; he states that spatial representations are built up slowly through the organization of actions performed on objects in space. Piaget "apparently believes that if something is given in the child's environment we can automatically assume that he is attending to it"; the researcher contends that we cannot make this assumption.

Piaget postulates that the stage that culminates with the discovery of the horizontal axis as part of the coordinate system does not originate until children are 7-8 years old and may continue through ages 9-12. If this were true, it would effectively delay the geometry program until grade 6 or beyond. However, further research indicates that (a) age levels for each stage of development might be somewhat lower and (b) achievement of the stages is governed more by other factors than by age.

3. Research Design and Procedure

Using a pretest-posttest nonequivalent control group design, the treatments were administered to 206 children in kindergarten and grade 2 in one school and in grade 1 in another school, with assignment of treatment by class within each grade. In treatment 1, the language was general; in treatment 2, "horizontal" and "vertical" were stressed; the control group took only the tests. The treatments consisted of three 20-minute sessions (plus two test days) over a 10-day period, with all grade levels receiving the same treatments, taught by one teacher. The first day was an introduction; instances of horizontality and verticality were found in the room and the relationship between horizontal and vertical lines was introduced. Water level was shown to be horizontal in all positions of a bottle. On the second day a carpenter's level was shown and explained, and the parallel drawn between it and the waterline in a very full bottle. Children used a "carpenter's level" to find surfaces which were horizontal. On the third day the original introduction was repeated using transparencies. The teacher illustrated the idea of horizontal using a non-gravitational vertical axis.

A 16-item written test was used pre- and post-treatment; it contained five items where children were to draw the waterline of a partially-obscured bottle, four items in which they were to select the correct illustration, three matching items, three items using the word "horizontal," and a general item with shapes.

A least squares estimate for class and grade effects on the pretest was done; F-tests then showed that the classes were substantially different both within and across grade levels. A permutation test indicated that the treatments were effective; multivariate analysis of variance on pre- and posttest scores was made "using student data as the basic unit affected by treatment," followed by an analysis of variance on posttest differences corrected for pretest differences. Analysis of variance on gain scores was also done in an attempt to eliminate the influence of the initial class effects and to establish the magnitude of the effectiveness of the treatments.

Appendices include the lesson plans and test items used.

4. Findings

The treatments were significantly more effective than no treatment, but there was little differentiation between the two treatments. There was a significant improvement in gain scores in treatment groups at all grade levels, with a definite grade effect. Kindergarteners did better than expected.

5. Interpretations

Children can be taught the horizontality concept, or at least to use techniques which enable them to use their abilities more effectively. Language labels might better be used after the child has learned the concept, rather than when he is forming it.

Abstractor's Notes

1. At points, the writing in this report is very direct; at other times, however, it is, to say the least, non-succinct. Use of semi-colons in place of commas would help the reader immeasurably.
2. Apparently no pilot testing of the instrument was done. It is noted that items 10-12 and 16 were much too easy. Analysis of tables of item data indicate that other items (e.g., item 1) also appear to have been answered correctly by most children on the pretest as well as on the posttest. Items 14-15 were "possibly too confounded by the logic necessary to grasp the concept to be adequate test items for children at this age level" and were, moreover, admittedly valid (with item 13) only for treatment 2. Nevertheless, data were analyzed for all items, as well as for Parts A (items 1-12, 16) and B (items 13-15) separately. It is noted that discussion of Part

B is "problematical given the invalidity of the items," but nevertheless continues, "There was a statistical grade effect 'on this part'." Over one-third of the test items were questionable: how much credence should be placed in results which are derived from it? How much can a multiplicity of statistical tests compensate for it?

3. Consider: "Each class received the treatments as a whole so that the scores on the posttest are not independent" (p. 21). And: "Pretest scores were independent except for a linear class effect nested within each grade" (p. 21). And: "Assumptions made of the independence of effects on each individual student in the analyses are clearly not true" (p. 42). <u>Yet</u>, the _n_ of pupils, not classes, was used in the data analysis, and this statement is made: "While the assumptions of independence were violated, these were the best procedures that could have been employed" (p. 43). Some discussion on these points (in context, as well as out of context) appears warranted: not all would agree with the conclusion so positively stated.

4. This paragraph also warrants discussion:

> The research could justifiedly be criticized if these differences [in pupil behaviors] were real. A more thorough project would have been to have had three grade levels in each of three separate schools. But the research was done in a "real" situation, in the same way any curriculum that would be developed would be taught and while not as professional for statistical purposes as it might have been, the experimenter felt the project justified in that it was developed in its appropriate setting. (p. 43)

What is the purpose of research, and particularly of research on a question such as that posed in this study? We do indeed need to conduct some research in real classrooms and to test research in real classrooms--but does that mean that all research, of whatever type, for whatever purpose, should be conducted in the real classroom where control of variables may be impossible? The amount that could be said in the discussion section which related back to the rationale for this study is perhaps the best answer to this question. No definite conclusions can be stated, except that the study might be replicated with a reliable test and better controls. Justifying research design errors on the grounds that we have to expect errors to appear in research in classroom settings is not warranted. Problems there are: being unable to assign pupils to treatment at random is one of these. But errors can be avoided, even in "real classroom" research: something more than a statement from a school that classes are equivalent needs to be secured; tests can be piloted. "Real" and "sloppy" are not equivalent.

6. A positive note: the treatments are carefully documented, so that a replication, with improvements, appears possible.

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