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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 11: multiplying and dividing fractions involving factoring, combining fractions with equal and with unequal denominators, mixed expressions, complex fractions, and solving open sentences having fraction coefficients. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

VOLUME 11

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

P R O G R A M M E D   M A T H   C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME 11

New York Institute of Technology

Old Westbury - New York

PREFACE

A

This volume is one of a set of 18
that form a complete course
in
ALGEBRA - LEVEL ONE

The volume has been structured
in a multiple choice question-answer format,
with the pagination scrambled
and
is to be used in conjunction with
a program control console
utilizing
punch card input.

It is one exhibit in the demonstration of a model
developed under the direction of
the U.S. Department of Health Education and Welfare
Project 8-0157

at the

New York Institute of Technology
Westbury, New York

VOLUME 11

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IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
1	1	$\frac{1}{1}$
1	2	$\frac{46}{1}$
1	3	$\frac{87}{1}$
1	4	$\frac{134}{1}$
1	5	$\frac{168}{1}$

C

VOLUME 11

This volume covers the following material

as shown in this excerpt from the Syllabus.

SEGMENT	DESCRIPTION	(CORE) DOLCIANI	(REMEDIAL) DRESSLER	(ENRICHMENT) DODES	OTHER TEXT
1	Multiplication and division of fractions involving factoring	8-7	12-4	4-10, 8-6 8-9	
2	Combination of fractions with equal denominators	8-8	12-5	4-11	
3	Combination of fractions with unequal denominators	8-9	12-6	4-11 8-6, 8-9	
4	Investigation of mixed expressions	8-10	12-7	4-6, 4-12	
	Investigation of complex fractions	8-11*			
5	Solution of open sentences with fraction coefficients	8-12	13-1 13-2 13-3	5-3	

* Optional topic for enrichment

D

READING ASSIGNMENT

VOLUME 11

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	296	
2	297	298
3	299	300
4	304	
5	306	

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

E

HOMEWORK ASSIGNMENT

Volume No. 11

Book: 597 Dolciani

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	297	3, 4	11114
2	297	5, 7	11115
3	297	11, 12, 14	11116
4	297	15, 19, 20	11116
5	299	5, 6, 8	11213
6	299	9, 15	11215
7	299	12, 13	11215
8	299	18, 19, 20	11226
9	301	3, 9, 10, 11	11323
10	301	13, 14, 20	11324
11	301	22, 23	11325
12	302	25, 26, 30	11327
13	303	2, 6, 10	11415
14	305	2, 3, 6	11434
15	305	11, 12, 15	11435
16	305	17, 18	11436
17	307	2, 5, 6	11515
18	307	15, 16, 20	11515
19	307	23, 24	11515
20	307	25, 26	11515

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is not correct you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is correct then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

VOLUME 11 SEGMENT 1 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0	6	(Sequence Number)
	54	and	56	0	4	(Type of Punch Card)
	60	and	62	1	1	(Volume Number)
	66	and	68	0	1	(Segment Number)

Your READING ASSIGNMENT for this SEGMENT is Page 296 in Dolciani.

In the previous volume, you learned the methods for multiplying fractions and for dividing fractions as separate operations. In this Segment, you will investigate the procedures for handling problems containing both operations. This will also include the applications of the rules for multiplying and dividing exponential forms. Actually, this Segment, as is the case in most parts of Algebra, draws heavily on the previous skills and rules you have learned while putting them together in one compound problem.

You will now be asked a series of questions to draw your attention to some more important points.

Question 1

Perform the necessary calculation to find the resultant value of

$$\frac{3}{8} \div \frac{5}{4} \cdot \frac{2}{9}$$

expressed as a fraction. Select the letter next to the correct choice.

(A) $\frac{5}{48}$

(B) $\frac{27}{20}$

(C) $\frac{1}{15}$

(D) None of these.

$\frac{2}{1}$

We don't agree!

Note that

$$\frac{x^4}{x^4} = 1$$

and not

$$x$$

A number divided by itself is equal to 1 .

(except zero)

Please return to page $\frac{24}{2}$ and try this question again.

$\frac{2}{2}$

You made an error in factoring. Please note that the expression

$$(x - 2)^2$$

means

$$(x - 2)(x - 2)$$

and that

$$x^2 - 4$$

is the difference of two squares.

Recall that

$$a^2 - b^2 = (a - b)(a + b)$$

Please return to page $\frac{28}{2}$ and try this question again.

$\frac{3}{1}$

We don't agree.

In dividing quantities having the same base, we subtract the exponents. This rule does not apply to coefficients unless they are in exponential form.

$$\frac{3}{72} \text{ does not equal } \frac{1}{69}$$

The numbers should be factored:

$$\frac{3}{72} = \frac{3 \cdot 1}{3 \cdot 24} = \frac{1}{24}$$

Please return to page $\frac{26}{1}$ and try question 5 again.

$\frac{3}{2}$

Your answer is not in lowest terms. Take another look at the choice you made and see if you can reduce it further.

Please return to page $\frac{31}{2}$ and try question 11 again.

$\frac{4}{1}$

Remember [illegible] the operation of division [illegible] that of multiplication, only the number immediately following the division sign is replaced by its reciprocal.

Please return to page $\frac{20}{2}$ and try question 2 again.

$\frac{4}{2}$

You forgot the main point of this segment.

Division by

$$\frac{y + 2}{5}$$

is the same as multiplication by

$$\frac{5}{y + 2}$$

Please return to page $\frac{11}{2}$ and try question 8 again.

$\frac{5}{1}$

To write the reciprocal of a monomial, we invert the entire monomial; not just the coefficient.

The reciprocal of

$$7x \quad \text{is} \quad \frac{1}{7x}$$

Thus, we have in this problem

$$\frac{x^3}{y^2} \cdot \frac{1}{7x} \cdot \frac{21}{xy}$$

Please return to page $\frac{21}{2}$ and try question 4 again.

$\frac{5}{2}$

We don't agree. You multiplied by the reciprocal of

$$\frac{a + b}{a - b}$$

Keep in mind that we change only the term immediately following the division sign to its reciprocal.

Please return to page $\frac{22}{2}$ and try question 1 again.

$\frac{6}{1}$

You made the wrong choice.

The fractio mediately following the division sign should be replaced by its ipr

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{6}{2}$

When we write this question as a continued product of three terms, we get

$$18pq \cdot \frac{1}{p^2q^2} \cdot \frac{p^3q^3}{3} = \frac{18p^4q^4}{3p^2q^2}$$

Continue from here and apply the rule for the quotient of powers.

Please return to page $\frac{17}{2}$ and try question 7 again.

$\frac{7}{1}$

What is the reciprocal of

$$\frac{6x}{y} \quad ?$$

Recall that the reciprocal of a number is the quotient of 1 divided by that number. For example;

the reciprocal of $\frac{5a}{8b}$ is $\frac{8b}{5a}$

The entire fraction is turned upside down, not just the coefficients.

Please return to page $\frac{24}{2}$ and try this question again.

$\frac{7}{2}$

You did not reduce to lowest terms.

Examine the choice you made and see if you can reduce it further.

Please return to page $\frac{28}{2}$ and try this question again.

$\frac{8}{1}$

It seems that you made a careless mistake.

Don't try to do this entire problem mentally.

Write down the problem and do one step at a time.

Please return to page $\frac{26}{1}$ and try this question again.

$\frac{8}{2}$

We don't agree. The expression

$$(m - n)^2$$

is not the same as $2(m - n)$

The expression: $(m - n)^2$

means $(m - n)(m - n)$

While the expression: $2(m - n)$

means the sum of $(m - n)$ and $(m - n)$

Please return to page $\frac{31}{2}$ and try this question again.

You forgot to multiply by the reciprocal of $\frac{5}{9}$.

Note that

$$\frac{5}{7} \cdot \frac{35}{45} \div \frac{5}{9}$$

is the same as

$$\frac{5}{7} \cdot \frac{35}{45} \cdot \frac{9}{5}$$

Please return to page $\frac{20}{2}$ and try this question again.

The expression

$$y^2 - 4$$

is the difference of two squares. Recall that

$$a^2 - b^2 = (a - b)(a + b)$$

Please return to page $\frac{11}{2}$ and try this question again.

$\frac{10}{1}$

You made an error in using the rule for the quotient of powers. Recall that

$$\frac{x^a}{x^b} = x^{a - b}$$

Please return to page $\frac{21}{2}$ and try this question again.

$\frac{10}{2}$

You made an error in factoring. Note that

$$a^2 - b^2 = (a - b)(a + b)$$

and that $$a^2 + 2ab + b^2$$

is a perfect square trinomial.

Please return to page $\frac{22}{2}$ and try this question again.

Only the fraction $\frac{5}{4}$ should be replaced by its reciprocal.

You should not have replaced $\frac{2}{9}$ by its reciprocal.

For example:

$$\frac{a}{b} \div \frac{c}{d} \cdot \frac{e}{f} = \frac{a}{b} \cdot \frac{d}{c} \cdot \frac{e}{f}$$

Please return to page $\frac{1}{2}$ and try this question again.

Question 8

Apply the proper principle and perform the indicated operations:

$$\frac{y^2 - 4}{2y^2} \cdot \frac{y}{y - 2} \div \frac{y + 2}{5}$$

Select the letter which labels the correct simplification.

(A) $\frac{(y + 2)^2}{10y}$ (B) $\frac{5(y - 2)}{2y(y + 2)}$

(C) $-\frac{10y}{y^2 - 4}$ (D) $\frac{5}{2y}$

$\frac{12}{1}$

You made an error in applying the rule for the quotient of powers. Recall that

$$\frac{x^a}{x^b} = x^{a - b}$$

you realize, of course, that

$$\frac{x^4}{4} \neq x$$

The 4's can't be divided since one of them is an exponent.

Please return to page $\frac{24}{2}$ and try question 3 again.

$\frac{12}{2}$

Your answer is not in lowest terms.

Examine the numerator and the denominator of the choice you made.

Each one can be factored.

Please return to page $\frac{38}{2}$ and try question 13 again.

You must check this mistake.

$$K \div 9abc \neq K \cdot \frac{abc}{9}$$

You seem to understand that division is replaced by multiplication by the reciprocal of the divisor. But that means the reciprocal of the <u>entire</u> divisor,

$$K \div 9abc = K \cdot \frac{1}{9abc}$$

Please return to page $\frac{26}{1}$ and try this problem again.

It appears that you made the mistake of considering

$$m^2 - n^2$$

the same as $(m - n)^2$

They are different!

One way of seeing this is to evaluate each expression with the same numerical values and to compare the results. Use the values $m = 5$ and $n = 2$ in the expressions below:

$$m^2 - n^2 \stackrel{?}{=} (m - 2)^2$$

$$5^2 - 2^2 \stackrel{?}{=} (5 - 2)^2$$

$$25 - 4 \stackrel{?}{=} (3)^2$$

$$21 \neq 9$$

Please return to page $\frac{31}{2}$ and rectify your mistake.

$\frac{14}{1}$

Recall that a number divided by itself is equal to 1 and not to zero. That is:

$$\frac{a}{a} \neq 0 \text{ rather } \frac{a}{a} = 1$$

Please return to page $\frac{20}{2}$ and try this question again.

$\frac{14}{2}$

We don't agree! One of the letters does have the correct answer next to it. Try factoring each expression where possible. Place similar factors one under the other.

Please return to page $\frac{28}{2}$ and try this question again.

Remember. The division sign affects only the fraction immediately following it.

e.g.

$$\frac{a}{b} \div \frac{c}{d} \cdot \frac{e}{f} = \frac{a}{b} \cdot \frac{d}{c} \cdot \frac{e}{f}$$

If both fractions were to be affected by the division sign, then a set of parentheses would have to be used.

e.g.

$$\frac{a}{b} \div \left(\frac{c}{d} \cdot \frac{e}{f}\right) = \frac{a}{b} \cdot \frac{d}{c} \cdot \frac{f}{e}$$

Please return to page $\frac{21}{2}$ and try this question again.

You must have made a careless error. Please go over your work; first make sure that you have copied the question correctly on your paper.

Please return to page $\frac{25}{2}$ and try this question again.

$\frac{16}{1}$

You must have made an unusual mistake.
One of the other letters does have the correct answer next to it.
Please reconsider.

Please return to page $\frac{1}{2}$ and try this question again.

$\frac{16}{2}$

You reduced individual terms in the numerator with individual terms in the denominator. In reducing a fraction to lowest terms, we divide both the numerator and denominator by <u>like factors</u>.

Consider the following illustration:

$$\frac{x^2 - 4}{4x} \cdot \frac{1}{x - 2} \div \frac{x + 2}{8}$$

This combined operation can be written as a product as follows:

$$\frac{x^2 - 4}{4x} \cdot \frac{1}{x - 2} \cdot \frac{8}{x + 2}$$

We now factor the numerator of the first fraction, and we get:

$$\frac{\overset{1}{\cancel{(x - 2)}}\,\overset{1}{\cancel{(x + 2)}}}{\underset{1}{\cancel{4}x}} \cdot \frac{1}{\underset{1}{\cancel{(x - 2)}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{(x + 2)}}}$$

To simplify, we divide the numerator and the denominator by the like factors that appear in both numerator and denominator. Thus, we divide by,

$$(x - 2) \quad (x + 2)$$

The answer in simplest form is $\frac{2}{x}$. Now use this technique on the given problem.

Please return to page $\frac{11}{2}$ and try this question again.

Good. Let's review one typical procedure in problems of this type.

$$\frac{6x^2y^3}{z} \div 9x^2y^4 \cdot \frac{3z^4}{x^2} = \frac{6x^2y^3}{z} \cdot \frac{1}{9x^2y^4} \cdot \frac{3z^4}{x^2}$$

$$= \frac{18}{9} \cdot \frac{x^2}{x^4} \cdot \frac{y^3}{y^4} \cdot \frac{z^4}{z}$$

$$= \frac{2}{1} \cdot \frac{1}{x^2} \cdot \frac{1}{y} \cdot \frac{z^3}{1}$$

$$= \frac{2z^3}{x^2y}$$

NOTE: The problem is expressed as a series of fractions having the same letters; these are reduced separately and then multiplied.

Now proceed to problem 7 below.

Question 7

Apply the proper principle and perform the indicated operations,

$$18pq \div p^2q^2 \cdot \frac{p^3q^3}{3}$$

Select the letter which labels the correct simplification.

(A) $6p^6q^6$

(C) $6p^2q^2$

(B) $6pq$

(D) $18p^2q^2$

$\frac{18}{1}$

Sorry!

You made a wrong choice.

The expression

$$\frac{a + b}{a - b}$$

does not equal (- 1) . Why not? Suppose that we

let $a = 4$

and $b = 1$

then

$$\frac{a + b}{a - b} = \frac{5}{3}$$

That should convince you that cancelling individual terms in the numerator with individual terms in the denominator is not permissable.

Please return to page $\frac{22}{2}$ and try question 10 again.

$\frac{18}{2}$

Sorry! You made the wrong choice. In order to obtain the answer that you chose, [illegible] lied by the reciprocal of

$$\frac{3yz - 9}{yz}$$

This is not the term immediately following the division sign.

Please return to page $\frac{38}{2}$ and try question 13 again.

What are the factors of

$$(x^2 - 2x - 3)\ ?$$

Before going ahead, make sure that you have factored the above trinomial correctly.

You can check your choice by multiplying the factors. See if their product equals the trinomial.

Please return to page $\frac{25}{2}$ and try this question again.

This question involves the factoring of six expressions.

It is easy to make a mistake in a question requiring all this work.

You must exercise great care in choosing the correct factors of each of the trinomials. You should check these factors by multiplying them to see whether their product equals the given trinomial.

Please return to page $\frac{40}{2}$ and try this question again.

s the correct answer.

damental rule is:

$$\frac{a}{b} \div \frac{c}{d} \cdot \frac{e}{f} = \frac{a}{b} \cdot \frac{d}{c} \cdot \frac{e}{f}$$

d rule that applies is:

if $\frac{a}{b} = \frac{pq}{pr}$

then $\frac{a}{b} = \frac{q}{r}$

ctor p is common to both the numerator and denominator, and it has the

of

$$\frac{p}{p} = 1$$

go on to question 2 below.

Question 2

Perform the necessary calculation to find the resultant value of

$$\frac{5}{7} \cdot \frac{35}{45} \div \frac{5}{9}$$

when it is reduced to a simple fraction. Select the letter which labels the correct answer.

(A) 1

(B) $\frac{81}{49}$

(C) $\frac{25}{81}$

(D) 0

Very good! You ... the correct ... .

We have

$$\frac{x^4}{4} \cdot \frac{}{x} \qquad \frac{6x}{}$$

We change the operation of division ... multiplication by multiplying by the reciprocal of the number immediately following the division sign. This number is called the divisor. Thus, we have

$$\frac{x^4}{4} \cdot \frac{3}{x^3} \cdot \frac{y}{6x} = \frac{3x^4y}{24x^4}$$

The result can be simplified further by dividing the numerator and the denominator by

$3x^4$. We get $\frac{y}{8}$ as the correct answer.

Please go on to question 4 below.

$\frac{21}{2}$

Question 4

Apply the proper principle and perform the indicated operations,

$$\frac{x^3}{y^2} \div 7x \cdot \frac{21}{xy}$$

Select the letter ...ich labels t... correct simplification.

(A) $\frac{3x}{y^3}$

(B) $\frac{3x^3}{y^3}$

(C) $\frac{3x^2}{y^3}$

(D) $\frac{x^3}{147y}$

XI

Very good! You made the correct choice. We first rewrite the combined operation of multiplication and division into an operation involving multiplication only.

Thus, we get:

$$\frac{(x-2)^2}{x+2} \cdot \frac{1}{x^2 - } \cdot (2x + 4) \qquad [\ \text{Factoring}$$

we get

$$\frac{(x-2)(x-2)}{(x-2)} \cdot \frac{1}{(x-2)(x+2)} \cdot 2(x+2)$$

Now, a careful technique is the following:

Write each factor of the numerator down first.

$$(x - 2) \cdot (x - 2) \cdot (2) \cdot (x + 2)$$

Then rearrange the factors of the denominator so that factors of the denominator are placed under like factors of the numerator.

$$\frac{(x-2)}{(x-2)} \quad (x-2) \cdot (2) \cdot \frac{(x-2)}{(x-2)} \cdot \frac{1}{(x+2)}$$

All fractions in the form

$$\frac{a}{a} = 1$$

leaving

$$\frac{2\ (x-2)}{(x+2)}$$

Now go on to question 10 below.

$\frac{2}{2}$

Question 10

Apply the proper principle and perform the indicated operations,

$$\frac{a^2 - b^2}{a^2} \cdot \frac{a+b}{a-b} \div (a^2 + 2ab + b^2)$$

Select the letter which labels the correct answer.

(A) $\frac{a-b}{a^2(a + b)}$

(B) $\frac{a+b}{a^2(a-b)}$

(C) $-\frac{a-b}{a^2(a+b)}$

(D) $\frac{1}{a^2}$

This answer is correct.

Please go on to question 6 [illegible] follows.

Question 6

Apply the proper principle and pe[illegible] the indicated operations,

$$\frac{6x^2y^3}{z} \div 9x^2y^4 \cdot \frac{3z^{[illegible]}}{[illegible]}$$

Select the letter which labels [illegible]rect simplification.

(A) $\frac{9z^3}{x^2y}$

(B) $2x^2yz^3$

(C) $\frac{2z^3}{x^2y}$

(D) None of these.

We don't agree.

One of the other letters identifies the correct answer.

Please reconsider your choice.

Please return to page $\frac{38}{2}$ and try this question again.

$\frac{24}{1}$

This is the correct answer.

To accomplish the same end as factoring each of the members of the numerator and denominator and placing them in the same position as

$$\frac{a}{a} = 1$$

it is sometimes more convenient to divide both the numerator and denominator by the same value.

For example:

Basic Method:

$$\frac{35}{45} = \frac{7 \cdot 5}{3 \cdot 3 \cdot 5}$$

$$= \frac{7}{9} \cdot \frac{\cancel{5}^{\,1}}{\cancel{5}_{\,1}}$$

$$= \frac{7}{9}$$

Alternate Method:

$$\frac{35}{45}$$ Divide numerator and denominator by the factor common to both. (5)

$$\frac{\cancel{35}}{\cancel{45}}$$ Write the result as a fraction.

$$\frac{7}{9}$$

Please go on to question 3 below.

$\frac{24}{2}$

Question 3

Apply the proper principle and perform the indicated operations,

$$\frac{x^4}{4} \cdot \frac{3}{x^3} \div \frac{6x}{[illegible]}$$

Select the letter which labels the correct simplification.

(A) $\frac{xy}{8}$

(B) $\frac{y}{8}$

(C) $\frac{x^2}{8y}$

(D) $\frac{y}{6x}$

This is the correct answer.

Let's review the method.

$$\frac{(m-n)^2}{mn} \cdot \frac{m+n}{m-n} \div \frac{m^2-n^2}{m^2}$$

Factor and change to multiplication by inverting the divisor.

$$\frac{\overset{1}{\cancel{(m-n)}}(m-n)}{\underset{1}{mn}} \cdot \frac{\overset{1}{\cancel{(m+n)}}}{\underset{1}{\cancel{(m-n)}}} \cdot \frac{\overset{1}{\cancel{m}}\, m}{\underset{1}{\cancel{(m+n)}}\underset{1}{\cancel{(m-n)}}}$$

Follow the principle that

$$\frac{a}{a} = 1$$

and reduce like factors.

The correct answer is $\frac{m}{n}$.

Please go on to question 12 below.

Question 12

Apply the proper principle and perform the indicated operations.

$$\frac{x}{x+1} \div \frac{2x^2}{2x+6} \cdot \frac{x^2-2x-3}{x^2-9}$$

Select the letter which labels the correct answer.

(A) $\frac{1}{x}$ (B) $\frac{2}{x^2}$

(C) $\frac{(x-1)(x+3)}{x(x+1)}$ (D) $\frac{2x+6}{x(x+3)}$

$\frac{26}{1}$

This is the correct answer.

Please go on to the next question which follows.

Question 3

Apply the proper principle and perform the indicated operations,

$$\frac{a^2 b^2}{c^2} \cdot \frac{3c^5}{8b^3} \div 9abc$$

Select the letter which labels the correct simplification.

(A) $\frac{ac^2}{69b^2}$ (C) $\frac{a^2 b^2 c^2}{24}$

(B) $\frac{ac^2}{24b^2}$ (D) $\frac{a^3 c^4}{24}$

$\frac{26}{2}$

Sorry, You did not make the correct choice.

Did you check the factors of each of the expressions?

It is important that you multiply the factors of each of the trinomials, especially, to see whether their product is equal to the trinomial.

That's where your mistake probably is.

Return to page $\frac{45}{2}$ and try this question again.

Very good. You made the correct choice. We first change the operation of division to that of multiplication. Thus, we have

$$\frac{x^2 + 7x + 10}{x^2 - 5x - 6} \cdot \frac{x^2 + x - 6}{x^2 - 25} \cdot \frac{x^2 - 3x - 10}{x^2 + 5x + 6}$$

We now factor each numerator and each denominator as follows:

$$\begin{aligned}
x^2 + 7x + 10 &= (x + 5)(x + 2)\\
x^2 - 5x - 6 &= (x - 6)(x + 1)\\
x^2 + x - 6 &= (x + 3)(x - 2)\\
x^2 - 25 &= (x - 5)(x + 5)\\
x^2 - 3x - 10 &= (x - 5)(x + 2)\\
x^2 + 5x + 6 &= (x + 3)(x + 2)
\end{aligned}$$

Note that the factoring was done in a separate place, making room for checking these factors before using them in the multiplication. We now have:

$$\frac{(x + 5)(x + 2)}{(x - 6)(x + 1)} \cdot \frac{(x + 3)(x - 2)}{(x - 5)(x + 5)} \cdot \frac{(x - 5)(x + 2)}{(x + 3)(x + 2)}$$

Dividing numerator and denominator by the like factors,

$$(x + 5),\ (x + 2),\ (x + 3),\ (x - 5)$$

we get:

$$\frac{(x + 2)(x - 2)}{(x - 1)(x - 6)} = \frac{x^2 - 4}{(x + 1)(x - 6)}$$

Please go on to question 16 below.

Question 16

Apply the proper principle and perform the indicated operations,

$$\frac{x^2 - 4x + 4}{x^2 - 7x + 12} \cdot \frac{x^2 - 6x + 9}{x^2 - 4} \div \frac{x^2 - 5x - 6}{x^2 - 2x - 8}$$

Select the letter which labels the correct simplification.

(A) $\frac{(x - 2)(x - 3)}{(x - 6)(x + 1)}$

(B) $\frac{(x - 3)(x - 4)}{(x - 2)(x - 6)}$

(C) $\frac{(x - 6)}{(x - 4)}$

(D) None of these.

$\frac{28}{1}$

This is the correct answer.

Let's review our procedure.

$$\frac{y^2 - 4}{2y^2} \cdot \frac{y}{y - 2} \div \frac{y + 2}{5} \quad \text{becomes}$$

$$\frac{(y + 2)(y - 2)}{2 \cdot y \cdot y} \cdot \frac{y}{(y - 2)} \cdot \frac{5}{(y + 2)}$$

Now we can rearrange the factors by using the COMMUTATIVE property of multiplication in an effort to get expressions in the form $\frac{a}{a}$ which, of course, equals 1 .

$$\frac{(y + 2)}{(y + 2)} \cdot \frac{(y - 2)}{(y - 2)} \cdot \frac{y}{y} \cdot \frac{5}{2y} \quad \text{This then becomes}$$

$$\frac{5}{2y}$$

Please go on to question 9 below.

$\frac{28}{2}$

Question 9

Apply the proper principle and perform the indicated operations,

$$\frac{(x - 2)^2}{x + 2} \div (x^2 - 4) \quad (2x + 4)$$

Select the letter which labels the correct product.

(A) 2

(B) $\frac{2 (x - 2)}{x + 2}$

(C) $\frac{(x - 2)(2x + 4)}{(x + 2)^2}$

(D) None of these

Very good. You made the correct choice.

The question can be written as a product of terms, thus,

$$\frac{y^3}{3y} \cdot \frac{z}{y^2z^2 - 9} \cdot \frac{3yz - 9}{yz} \qquad \text{[Factor}$$

$$\frac{y^3}{3y} \cdot \frac{z}{(yz - 3)(yz + 3)} \cdot \frac{3(yz - 3)}{yz}$$

Dividing by the equal factors: (3), (y^2), (z), and $(yz - 3)$ in both the numerator and denominator, we get,

$$\frac{y}{yz + 3}$$

Please go on to question 14 below.

Question 14

Apply the proper principle and perform the indicated operations,

$$\frac{a^2 + 10a + 25}{a^2 + 10a} \cdot \frac{10a}{a^2 + 15a + 50} \div \frac{a + 5}{a + 10}$$

Choose the letter next to the correct simplification.

(A) $\frac{1}{a + 1}$ (C) $\frac{10(a + 5)}{10}$

(B) $\frac{10}{a + 10}$ (D) $\frac{10(a + 5)}{(a + 10)(a + 10)}$

$\frac{30}{1}$

You overlooked something. The numerator of the second fraction; namely,

$$x^2 + 5xy$$

can be written as $x(x + 5y)$

Check the factors of each of the expressions, especially the factors of

$$x^2 + 2xy - 8y^2$$ and the factors of

$$x^2 - xy - 2y^2$$

In answering a question that involves many operations, it is extremely important that you arrange your work neatly and proceed carefully.

Please return to page $\frac{39}{2}$ and try this question again.

$\frac{30}{2}$

Sorry, but you did not make the right choice.

Note that some of the expressions are written in descending order of the variable while some are written in ascending order of the variable.

You recall that

$(1 - 4x^2)$ is the same as

$-(4x^2 - 1)$ and

$(2x - x^2)$ is the same as

$-(x^2 - 2x)$

With this change, all the expressions are now in descending order of the variable x .

Please continue from here.

Please return to page $\frac{35}{2}$ and try this question again.

This is the correct answer.

Let's review the procedure:

$$\frac{a^2 - b^2}{a^2} \cdot \frac{a + b}{a - b} \div (a^2 + 2ab + b^2) = \frac{(a + b)(a - b)}{a^2} \cdot \frac{a + b}{a - b} \cdot \frac{1}{(a + b)(a + b)}$$

Note: We factor and employ the reciprocal simultaneously.

$$= \frac{\overset{1}{\cancel{(a + b)}}\overset{1}{\cancel{(a - b)}}}{a^2} \cdot \frac{\overset{1}{\cancel{(a + b)}}}{\underset{1}{\cancel{(a - b)}}} \cdot \frac{1}{\underset{1}{\cancel{(a + b)}}\underset{1}{\cancel{(a + b)}}}$$

Important: Instead of re-writing to put fractions in the form

$$\frac{a}{a}$$

we can reduce the similar factors.

$$= \frac{1}{a^2}$$

Please go on to question 11 below.

Question 11

Apply the proper principle and perform the indicated operations.

$$\frac{(m - n)^2}{mn} \cdot \frac{m + n}{m - n} \div \frac{m^2 - n^2}{m^2}$$

Select the letter which labels the correct reduction.

(A) $\frac{m^2}{mn}$

(C) $\frac{2m}{n}$

(B) $\frac{m}{n}$

(D) $\frac{m}{n} \cdot \frac{(m + n)}{(m - n)}$

$\frac{32}{1}$

This question requires the factoring of six expressions. Tabulate the factors of each of the expressions in a separate place on your paper. Check each of the factors by multiplying them. See if their product equals the polynomial that you have factored. When you have made sure that you have factored correctly, proceed to perform the indicated operations.

Please return to page $\frac{27}{2}$ and try this question again.

$\frac{32}{2}$

Sorry! We do not agree.

Suppose we try a similar problem together, and as you follow all the steps, find out where you made your error.

Find the value of

$$\frac{4a^2b^3}{c} \div 24a^2b^4 \cdot \frac{3c^4}{a^2} =$$

Change multiplication and division to only multiplication.

$$\frac{4a^2b^3}{c} \cdot \frac{1}{24a^2b^4} \cdot \frac{3c^4}{a^2} =$$

$$\frac{12a^2b^3c^4}{24a^4b^4c} =$$

From left to right,

$$\frac{12}{24} = \frac{1}{2},$$

$$\frac{a^2}{a^4} = \frac{1}{a^2},$$

$$\frac{b^3}{b^4} = \frac{1}{b}$$

$$\frac{c^4}{c} = c^3$$

$$\frac{c^3}{2a^2b}$$

Please return to page $\frac{23}{1}$ and try this question again.

Your answer is not in lowest terms. The numerator of the expression that you chose can be factored. You have made another error as well. Copy the problem carefully and factor one expression at a time.

Please return to page $\frac{25}{2}$ and try this question again.

$\frac{33}{2}$

We don't agree.

In changing the operation of division to that of multiplication, we multiply by the reciprocal of the fraction immediately following the division sign. In this question, the fraction immediately following the division sign because of the parentheses is the product of

$$\frac{3c^2}{3a + 2b} \quad \text{and}$$

$$\frac{2c - 3b}{c + b}$$

Therefore, you should first find this product, then multiply

$$\frac{3c}{3a + 2b}$$

by the reciprocal of this product.

Please return to page $\frac{43}{2}$ and try this question again.

$\frac{34}{1}$

Note that the expression

$$a^2 + 10a + 25$$

is a square trinomial; and is, therefore, the product of two equal factors. Furthermore, the factors of

$$a^2 + 15a + 50$$

are $(a + 10)$ and $(a + 5)$

Factor each expression carefully. Do one step at a time.

Please return to page $\frac{29}{2}$ and try this question again.

$\frac{34}{2}$

The fractions $\frac{1}{5}$ and $\frac{2}{10}$ are equivalent; that is, they have the same value. But they are not "like" fractions.

For example: $\frac{a}{b}$ and $\frac{c}{b}$

are "like" fractions since they have the same denominator.

Please return to page $\frac{46}{2}$ and reconsider the question.

Very good! You made the correct choice.

We first list each numerator and denominator in factored form. You can understand why you had to learn how to factor before tackling this topic. Thus,

$$4x^2 - 4x + 1 = (2x - 1)(2x - 1)$$
$$3x^2 + 4x - 4 = (3x - 2)(x + 2)$$
$$x^2 - 4 = (x - 2)(x + 2)$$
$$3x^2 + 13x - 10 = (3x - 2)(x + 5)$$
$$2x^2 - 5x + 2 = (2x - 1)(x - 2)$$
$$9x^2 - 12x + 4 = (3x - 2)(3x - 2)$$

Changing the combined operations of multiplication and division to multiplication only, we now have,

$$\frac{(2x - 1)\ (2x - 1)}{(3x - 2)\ (x + 2)}\ \frac{(x - 2)\ (x + 2)}{(3x - 2)\ (x + 5)}\ \frac{(3x - 2)\ (3x - 2)}{(2x - 1)\ (x - 2)} = \frac{2x - 1}{x + 5}$$

Some students avoid the confusion caused by crossing out the common factors by putting a check over the pairs.

Please go on to question 19 below.

$\frac{35}{1}$

<u>Question 19</u>

Apply the proper principle and perform the indicated operations,

$$\frac{1 - 4x^2}{2x^2 - 7x + 3} \cdot \frac{2x - x^2}{6x^2 + 13x + 5} \div \frac{2x^2 - 4x}{3x^2 - 4x - 15}$$

Select the letter which labels the correct simplified result.

(A) $\frac{2 - x}{3x + 5}$

(B) -1

(C) $\frac{1 - 2x}{2 - x}$

(D) None of these.

56

The rule for adding fractions with like denominators is:

Add the numerators for the new numerator and maintain the same denominator.

Thus, for example,

$$\frac{2}{7} + \frac{3}{7} =$$

$$\frac{2 + 3}{7} =$$

$$\frac{5}{7}$$

Please return to page $\frac{51}{2}$ and try this question again.

Yes, but your answer is not in lowest terms. It is important that you examine your answer to see whether it can be further simplified.

If $\quad \frac{p}{q} = \frac{ab}{cb} \qquad$ [Reduce $\frac{b}{b} = 1$

Then $\quad \frac{p}{q} = \frac{a}{c}$

Please return to page $\frac{65}{2}$ and try this question again.

Yes, but your answer is not in lowest terms.

You can remove a common monomial factor from the numerator of your choice.

For example:

$$\frac{9x}{3} = \qquad [\text{Factor}$$

$$\frac{3 \cdot 3x}{3} = \qquad [\text{Reduce } \frac{a}{a} = 1$$

$$3x$$

Please return to page $\frac{57}{1}$ and try this question again.

We do not agree.

This question requires the combined operation of adding and subtracting fractions with like denominators. The rule for combining such fractions is to add the numerators of those fractions that are connected by a plus sign and to subtract the numerators of the fractions that follow a minus sign. As with the operation of addition only, the same denominator is maintained in the answer.

Please return to page $\frac{67}{1}$ and try this question again.

$\frac{38}{1}$

This is the correct answer

First copy the problem: $\frac{x}{x + 1} \div \frac{2x^2}{2x + 6} \cdot \frac{[illegible]}{[illegible]} \quad \underline{-\ 3}$

Then factor and convert to multiplication: $\frac{x}{x + 1} \cdot \frac{2 (x -}{2x^2} \cdot \frac{(x - 3)(x + 1)}{(x - 3)(x + 3)}$

If "crossing out" is confusing, then re-arrange the factors of the denominator to match those of the numerator:

$$\frac{x}{x} \cdot \frac{2}{2} \cdot \frac{(x + 3)}{(x + 3)} \cdot \frac{(x - 3)}{(x - 3)} \cdot \frac{(x + 1)}{(x + 1)} \cdot \frac{1}{x}$$

The "unmatched" forms the answer, $\frac{1}{x}$

Please go on to question 13 below.

$\frac{38}{2}$

Question 13

Apply the proper principle and perform the indicated operations,

$$\frac{y^3}{3y} \div \frac{y^2z^2 - 9}{z} \cdot \frac{3yz - 9}{yz}$$

Select the letter which labels the correct answer.

(A) $\frac{y (3yz - 9)}{z (y^2z^2 - 9)}$

(B) $\frac{y^3 (yz + 3)}{9}$

(C) $\frac{y}{yz + 3}$

(D) None of these.

Very good. You have made the correct choice.

Let us first list the factors of all the numerators and denominators:

$$x^2 - 4x + 4 = (x - 2)(x - 2)$$
$$x^2 - 7x + 12 = (x - 4)(x - 3)$$
$$x^2 - 6x + 9 = (x - 3)(x - 3)$$
$$x^2 - 4 = (x - 2)(x + 2)$$
$$x^2 - 5x - 6 = (x - 6)(x + 1)$$
$$x^2 - 2x - 8 = (x - 4)(x + 2)$$

We now have:

$$\frac{(x - 2)(x - 2)}{(x - 4)(x - 3)} \cdot \frac{(x - 3)(x - 3)}{(x - 2)(x + 2)} \cdot \frac{(x - 4)(x + 2)}{(x - 6)(x + 1)}$$

Dividing by the like factors, $(x - 2)$, $(x - 3)$, $(x - 4)$, and $(x + 2)$:

The correct answer is $\frac{(x - 2)(x - 3)}{(x - 6)(x + 1)}$

Please go on to question 17 below.

$\frac{39}{2}$

Question 17

Apply the proper principle and perform the indicated operations.

$$\frac{x^2 - xy - 20y^2}{x^2 + 2xy - 8y^2} \cdot \frac{x^2 + 5xy}{x^2 - 25y^2} \div \frac{x^2 - xy - 2y^2}{x + y}$$

Select the letter which labels the correct simplification.

(A) $\frac{1}{(x - 2y)^2}$

(B) $\frac{x}{(x - 2y)^2}$

(C) $\frac{x}{x^2 - 4y^2}$

(D) $\frac{x - 5y}{x - 2y}$

$\frac{40}{1}$

Very good. You have chosen the correct answer.

This question can be done in the following manner:

$$\frac{a^2 - 10a + 25}{a^2 + 10a} \cdot \frac{10a}{a^2 + 15a + 50} \div \frac{a + 5}{a + 10}$$

Change from $(\div)$ to multiplication by using the reciprocal of the expression immediately following the division symbol, and factor all terms to prime factors.

$$\frac{(a + 5)(a + 5)}{a(a + 10)} \cdot \frac{10a}{(a + 5)(a + 10)} \cdot \frac{(a + 10)}{(a + 5)}$$

Divide both the numerators and denominators of the expression by their common factor. The result you got was

$$\frac{10}{(a + 10)}$$

Now proceed to question 15 which follows below.

....

$\frac{40}{2}$

Question 15

Apply the proper principle and perform the indicated operations,

$$\frac{x^2 + 7x + 10}{x^2 - 5x - 6} \cdot \frac{x^2 + x - 6}{x^2 - 25} \div \frac{x^2 + 5x + 6}{x^2 - 3x - 10}$$

Select the letter which labels the correct simplification.

(A) $\frac{(x - 2)(x + 2)}{(x + 5)(x - 1)}$ (B) $\frac{x + 2}{x + 3}$

(C) $\frac{x^2 - 4}{(x + 1)(x - 6)}$ (D) $\frac{(x + 5)(x + 2)}{(x + 1)(x - 6)}$

41/1

Your answer is partially correct. You ignored another possible choice. "Like" fractions have the same denominator.

Please return to page 46/2 and reconsider the question.

41/2

No, we do not add denominators in finding the sum of two fractions with like denominators. Re-read the text assignment for this segment and study the rule for adding such fractions.

Please return to page 57/1 and try this question again.

$\frac{42}{1}$

When adding algebraic fractions with like denominators, we do <u>not</u> add the denominators. Think of it this way:

The denominator is the name of the fraction. For example:

$$\frac{2}{7} \qquad + \qquad \frac{3}{7}$$

can be thought of as

2 sevenths + 3 sevenths

You are adding quantities called "sevenths" and you get 5 "sevenths."

We add <u>only</u> the numerators of fractions with like denominators.

Please return to page $\frac{68}{1}$ and try this question again,

---

$\frac{42}{2}$

We do not agree.

Before you make a choice, it is a good policy to examine the problem and your work for possible errors.

Please check the sum of the numerators

$$(2b + 3) + (5 - b)$$

Please return to page $\frac{54}{2}$ and try this question again.

Very good. You made the correct choice.

Since two of the expressions are not written in descending order of the variable, x, we first change them so that they are in descending order. Thus, we write,

$$1 - 4x^2 = -(4x^2 - 1) \quad \text{and}$$

$$2x - x^2 = -(x^2 - 2x)$$

Factoring each expressions, we get,

$$\frac{-(2x-1)(2x+1)}{(2x-1)(x-3)} \cdot \frac{-x(x-2)}{(3x+5)(2x+1)} \cdot \frac{(3x+5)(x-3)}{2x(x-2)} = \frac{1}{2}$$

It is always a good idea to check our answer. Let us, therefore, do so. If the original expression reduces to $\frac{1}{2}$ then it should be so for any value of x except those values that make a denominator factor equal zero. Let us try $x = 1$. Replacing x by 1 in the original question we get:

$$\frac{1-4}{2-7+3} \cdot \frac{2-1}{6+13+5} \cdot \frac{2-4}{3-4-15} = \frac{-3}{-2} \cdot \frac{1}{24} \cdot \frac{-16}{-2} =$$

$$\frac{48}{96} = \frac{1}{2}$$

Please go on to question 20 below.

$\frac{43}{2}$

Question 20

Apply the proper principle and perform the indicated operations,

$$\frac{3c}{3a+2b} \div \left[\frac{3c^2}{3a+2b} \cdot \frac{2c-3b}{c+b}\right]$$

Select the letter which labels the correct answer.

(A) $\frac{2c-3b}{c(b+c)}$ (B) $\frac{c(b+c)}{2c-3b}$

(C) $\frac{b+c}{c(2c-3b)}$ (D) None of these.

$\frac{44}{1}$

Very good. You made the correct choice.

The rule for adding fractions with like denominators is to add the numerators and maintain the same denominator.

The proto-type for this is $\frac{a}{d} + \frac{b}{d} + \frac{c}{d} = \frac{a + b + c}{d}$

$$\frac{2}{11} + \frac{3}{11} + \frac{5}{11} = \frac{2 + 3 + 5}{11} = \frac{10}{11}$$

Please go on to question 3 below.

$\frac{44}{2}$

<u>Question 3</u>

Apply the proper principle and express the sum of the fractions in lowest terms.

$$\frac{6}{5x} + \frac{1}{5x} + \frac{8}{5x}$$

Select the letter which labels the correct statement.

(A) $\frac{3}{x}$

(B) $\frac{15}{15x^3}$

(C) $\frac{15}{15x}$

(D) $\frac{1}{x}$

Very good. You made the correct choice.

Let us first write each of the expressions in factored form. We have:

$$x^2 - xy - 20y^2 = (x - 5y)(x + 4y)$$
$$x^2 + 2xy - 8y^2 = (x + 4y)(x - 2y)$$
$$x^2 + 5xy = x(x + 5y)$$
$$x^2 - 25y^2 = (x - 5y)(x + 5y)$$
$$x^2 - xy - 2y^2 = (x - 2y)(x + y)$$

The question can now be written as:

$$\frac{(x - 5y)(x + 4y)}{(x + 4y)(x - 2y)} \cdot \frac{x(x + 5y)}{(x - 5y)(x + 5y)} \cdot \frac{(x + y)}{(x - 2y)(x + y)}$$

If the order of the denominator factor is changed, then it will be easy to see the factors that are eliminated.

$$= \frac{(x - 5y)}{(x - 5y)} \cdot \frac{(x + 4y)}{(x + 4y)} \cdot \frac{x}{1} \cdot \frac{(x + 5y)}{(x + 5y)} \cdot \frac{(x + y)}{(x + y)} \cdot \frac{1}{(x - 2y)} \cdot \frac{1}{(x - 2y)}$$

$$= \frac{x}{(x - 2y)^2}$$

Please go on to question 18 below.

Question 18

Apply the proper principle and perform the indicated operations,

$$\frac{4x^2 - 4x + 1}{3x^2 + 4x - 4} \cdot \frac{x^2 - 4}{3x^2 + 13x - 10} \div \frac{2x^2 - 5x + 2}{9x^2 - 12x + 4}$$

Select the letter which labels the correct answer.

(A) $\frac{x + 2}{3x - 2}$ (B) $\frac{3x - 2}{x + 5}$

(C) $\frac{2x - 1}{x + 5}$ (D) $\frac{2x - 1}{3x - 2}$

$\frac{46}{1}$

VOLUME 11 SEGMENT 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 7	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	1 1	(Volume Number)
	66	and	68	0 2	(Segment Number)

Algebra is a tool-subject as it is used in all areas of science. Frequently formulas are encountered that contain fractions. Fractions (elements of the set of rational numbers) can be added, subtracted, multiplied, and divided. By the CLOSURE PROPERTY fractions can be converted into one fraction by various combinations of the four operations. In this segment we will investigate the addition and subtraction of fractions.

In this segment, you will learn how to add and subtract algebraic fractions that have the same denominator. Many formulas in geometry and in physics are expressed as the sum or difference of algebraic fractions. Many problems in algebra require the addition and subtraction of fractions for their solution. But the major reason is to be able to simplify algebraic expressions by making several fractions into one fraction. Fractions, as numerals, can be added, subtracted, multiplied, or divided.

Your READING ASSIGNMENT for this SEGMENT is page 297 - 298 . You will now be asked a series of questions to draw your attention to the more important points.

$\frac{46}{2}$

Recognize the set that contains " like " fractions and select the letter next to the correct choice.

$$P = \left\{\frac{1}{5}, \frac{2}{10}\right\}$$

$$Q = \left\{\frac{1}{5}, \frac{2}{5}\right\}$$

$$R = \left\{\frac{1}{5}, \frac{1}{5}\right\}$$

(A) Only P (C) Both Q and R

(B) Only Q (D) P , Q , and R

Sorry, but you made an incorrect choice.

Notice that there is a plus sign between the fractions and not a multiplication sign.

Please return to page $\frac{68}{1}$ and try this question again.

$\frac{47}{2}$

Sorry, but one of the other letters does have the correct answer next to it.

Consider a similar question:

Find the sum of the fractions,

$$\frac{3m - 2n}{3m + n} + \frac{6m + 5n}{3m + n}$$

The sum of the numerators is:

$$9m + 3n = 3(3m + n)$$

The sum of the fractions can be written as:

$$\frac{3(3m + n)}{(3m + n)}$$

Dividing numerator and denominator by the identical factor,

$$(3m + n)$$

we get 3 as our answer.

Please return to page $\frac{65}{2}$ and try this question again

$\frac{48}{1}$

What is the <u>sum</u> of $3x$ and $3y$?

Remember, unlike monomials can't be added, but the binomial formed by the combination can be factored.

Please return to page $\frac{57}{1}$ and try this question again.

$\frac{48}{2}$

Do not be hasty in choosing an answer until you have checked whether this answer is in lowest terms.

Note that the denominator is the difference of two squares and can be factored as the product of two binomials.

Please return to page $\frac{54}{2}$ and try this question again.

Congratulations!

You did a pretty difficult problem correctly.

Suppose we go over a method you could have used to do this question.

We first find the product of the two fractions in the parenthesis. Thus, we have:

$$\frac{3c^2}{3a + 2b} \cdot \frac{2c - 3b}{c + b} = \frac{3c^2 \ c - 3b)}{(3a + 2b)(c + b)}$$

This can't be reduced. Now, we take the reciprocal of this product and multiply it by the first fraction of this question. We have,

$$\frac{3c}{3a + 2b} \cdot \frac{(3a + 2b)\ (c + b)}{3c^2(2c - 3b)} = \frac{b + c}{c(2c - 3b)}$$

You have now finished this Segment. Hand in your PUNCH CARD. You should have entered in your NOTEBOOK the following definitions.

1. In questions involving the combined operation of multiplication and division of fractions, we replace only the fraction immediately following the division sign by its reciprocal.
2. Fractions can be reduced only when, after factoring, the same factor appears in the numerator and denominator;

   if $\frac{N}{M} = \frac{ab}{cb}$

   then $\frac{N}{M} = \frac{a}{c}$ since $\frac{b}{b} = 1$

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT:

Problems 1 , 2 , 3 , and 4 .

Your answer is not in lowest terms.

An answer that is not reduced to lowest terms is like a job left unfinished.

Please finish the job.

Please return to page $\frac{60}{2}$ and try this question again.

$\frac{50}{2}$

You made an error.

Please pay close attention to what follows, and you will avoid making this same mistake again.

Consider a similar question: Express in lowest terms the difference,

$$\frac{4a}{3b} - \frac{3 - 8a}{3b}$$

The minus sign between the two fractions means that we have to subtract the entire numerator, $(3 - 8a)$ from $4a$. Thus, we have:

$$\frac{4a - (3 - 8a)}{3b} \qquad \text{[Distribution}$$

$$= \frac{4a - 3 + 8a}{3b} \qquad \text{[Combine like terms}$$

$$= \frac{12a - 3}{3b} \qquad \text{[Factor}$$

$$= \frac{3(4a - 1)}{3b} \qquad \text{[Reduce } \frac{3}{3} = 1$$

$$= \frac{4a - 1}{b}$$

Please return to page $\frac{79}{1}$ and try this method on the question again.

You are correct. Like fractions are defined as fractions having the same denominator. There is no restriction on the numerator.

Please go on to question 2 below.

Question 2

Apply the proper principle and find the sum of the fractions,

$$\frac{2}{11} + \frac{3}{11} + \frac{5}{11}$$

Select the letter which labels the correct statement.

(A) $\frac{10}{33}$

(B) $\frac{10}{11}$

(C) $\frac{30}{11}$

(D) None of these.

$\frac{52}{1}$

We do not agree.

One of the letters does have the correct answer next to it.

Apply the basic rule which can be expressed as:

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

Please return to page $\frac{68}{1}$ and try this question again.

$\frac{52}{2}$

We do not agree. Instead of pointing out the error you made, we will work out a similar problem and have you discover what you did wrong.

Express the difference of the fractions,

$$\frac{a^2}{a - b} - \frac{b^2}{a - b}$$

in lowest terms.

We subtract the numerators and keep the same denominator. Thus, we get,

$$\frac{a^2 - b^2}{a - b}$$

Now, we note that the numerator is the difference of two squares. Hence, we can write the above as:

$$\frac{(a - b)(a + b)}{(a - b)}$$

Both the numerator and the denominator can be divided by the identical factor

$$(a - b)$$

The answer is, therefore,

$$(a + b)$$

Please return to page $\frac{74}{1}$ and try this question again.

You made an error in trying to combine the denominators.

Check the basic rule.

Note: the denominators are not altered. $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$

Return to page $\frac{44}{2}$ and try this question again.

We do not agree.

Note that $\frac{a}{b} - \frac{c}{d}$

is the same as $\frac{a}{b} + \frac{-c}{d}$

Thus, the subtraction of two fractions can be treated as the addition of these fractions following the minus sign is multiplied by -1.

In this question,

$$\frac{7}{5mn} - \frac{1 - 3m}{5mn}$$

is the same as

$$\frac{7}{5mn} + \frac{-(1 - 3m)}{5mn}$$

Please continue.

Return to page $\frac{62}{2}$ and try this question again.

$\frac{54}{1}$

Good. Let's review the method that is generally used in problems of this type.

$$= \frac{3p - 5q}{2p + q} + \frac{p + 7q}{2q + q} \qquad \left[\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \right.$$

$$= \frac{3p - 5p + p + 7q}{2p + q} \qquad [\text{ C and A}$$

$$= \frac{4p + 2q}{2p + q}. \qquad [\text{ Factor}$$

$$= \frac{2(2p + q)}{(2p + q)} \qquad \left[\frac{ab}{b} = a \right.$$

$$= 2$$

Now proceed to question 7 below.

$\frac{54}{2}$

<u>Question 7</u>

Apply the proper principle and express the sum of the fractions in lowest terms.

$$\frac{2b + 3}{b^2 - 64} + \frac{5 - b}{b^2 - 64}$$

Select the letter which labels the correct statement.

(A) $\frac{7b + 2}{b^2 - 64}$ (C) $\frac{b + 8}{b^2 - 64}$

(B) $\frac{b + 8}{b^2 - 64}$ (D) $\frac{1}{b - 8}$

Sorry, but you made an incorrect choice.

Note that the numerator of the difference of the two fractions is a square trinomial.

Return to page $\frac{71}{2}$ and try this question again.

You did not combine the numerators correctly.

Note that

$$5 - a - (a - 3) =$$

$$5 - a - a + 3 =$$

$$8 - 2a$$

Please continue.

Return to page $\frac{83}{2}$ and try this question again.

Please check the addition and subtraction of the numerators of the given fractions. The result is not 36 .

Please return to page $\frac{60}{2}$ and try this question again.

The difference between the numerators of the given fractions is

$$(2 - x) - (5 - 2x)$$

Note that the minus sign between fractions effects each term of the numerator of the fraction following the minus sign.

Please return to page $\frac{81}{1}$ and try this question again.

This is the correct answer. Please go on to question 5 which follows.

Question 5

Apply the proper principle and express the sum of the fractions in lowest terms:

$$\frac{3x}{x + y} + \frac{3y}{x + y}$$

Select the letter which labels the correct statement.

(A) $\frac{3x + 3y}{x + y}$

(B) $\frac{3x + 3y}{2(x + y)}$

(C) $\frac{3xy}{x + y}$

(D) 3

We do not agree.

If you examine the denominators of the two fractions, you will notice that they are not alike. It is, however, possible to make them alike by multiplying one of them by a certain number.

Can you discover that number?

Return to page $\frac{90}{2}$ and try this question again.

You did not add the fractions correctly.

The rule for adding fractions with like denominators is to add the numerators for the new numerator and write this numerator over the same denominator.

Thus, for example,

$$\frac{9}{5y} + \frac{7}{5y} + \frac{3}{5y} = \frac{9 + 7 + 3}{5y} = \frac{19}{5y}$$

Return to page $\frac{44}{2}$ and try this question again.

Yes, but your answer is not in lowest terms.

Examine the numerator of your choice and see if you can factor it.

Return to page $\frac{74}{1}$ and try this question again.

Sorry, but you did not make the correct choice.

When two fractions with like denominators are added, the numerators are added for a new numerator and the denominator of the sum is the same as the denominators of each of the fractions.

After this is done, the resulting fraction is examined to see whether it can be simplified. This requires factoring and dividing both numerator and denominator by identical factors.

Please return to page $\frac{54}{2}$ and try this question again.

Please note that when you combine all the numerators of the given fractions, you get

$$\begin{aligned} 3 + 1 - (10 - 2x) &= 4 - 10 + 2x \\ &= 2x - 6 \\ &= 2(x - 3) \end{aligned}$$

Please return to page $\frac{73}{2}$ and try this question again.

$\frac{60}{1}$

Very good. You made the correct choice.

In combining fractions connected by plus and minus signs, we perform the indicated operation to the numerators and keep the same denominator.

Thus,

$$\frac{17}{13} + \frac{3}{13} - \frac{9}{13} = \frac{17 + 3 - 9}{13}$$

$$= \frac{11}{13}$$

Please go on to question 9 below.

$\frac{60}{2}$

Question 9

Apply the proper principle and perform the indicated operations.

$$\frac{10}{7x} + \frac{22}{7x} - \frac{4}{7x}$$

Select the letter which expresses the result in lowest terms.

(A) $\frac{28}{7x}$ (C) $4x$

(B) $\frac{36}{7x}$ (D) $\frac{4}{x}$

$\frac{60}{1}$

Very good. You made the correct choice.

In combining fractions connected by plus and minus signs, we perform the indicated operation to the numerators and keep the same denominator.

Thus,

$$\frac{17}{13} + \frac{3}{13} - \frac{9}{13} = \frac{17 + 3 - 9}{13}$$

$$= \frac{11}{13}$$

Please go on to question 9 below.

$\frac{60}{2}$

Question 9

Apply the proper principle and perform the indicated operations.

$$\frac{10}{7x} + \frac{22}{7x} - \frac{4}{7x}$$

Select the letter which expresses the result in lowest terms.

(A) $\frac{28}{7x}$ (C) $4x$

(B) $\frac{36}{7x}$ (D) $\frac{4}{x}$

$\frac{62}{1}$

Very good. You made the correct choice.

It is important to note that the minus sign before a fraction is equivalent to multiplying the numerator of the fraction that follows by (-1) .

That is,

$$\frac{2x}{9y} - \frac{3 - 7x}{9y}$$

is the same as $\frac{2x}{9y} + \frac{-(3 - 7x)}{9y}$ [D

$= \frac{2x - 3 + 7x}{9y}$ [Combine

$= \frac{9x - 3}{9y}$ [Factor the common monomial

$= \frac{3(3x - 1)}{3 \cdot 3y}$ [Divide numerator and denominator by 3 .

$= \frac{3x - 1}{3y}$

Please go on to question 12 below.

$\frac{62}{2}$

Question 12

Apply the proper principle and perform the indicated operations.

$$\frac{7}{5mn} - \frac{1 - 3m}{5mn} - \frac{2m - 6}{5mn}$$

Select the letter which expresses the result in lowest terms.

(A) $\frac{1}{5n}$

(B) $\frac{12}{5n}$

(C) $\frac{m + 12}{5mn}$

(D) None of these.

$\frac{63}{1}$

We do not agree. It appears that you added the denominators and then reduced the sum.

Please refer to your text and re-read the rule for adding fractions with like denominators.

Please return to page $\frac{44}{2}$ and try this question again.

$\frac{63}{2}$

Good, but you did not finish the job.

Your answer is not in lowest terms.

Examine the numerator of your choice and see if you can simplify the result.

Please return to page $\frac{73}{2}$ and try this question again.

$\frac{64}{1}$

The numerator of the difference of two fractions is the first numerator minus the second. Thus, the numerator of the difference of these fractions is

$$p^2 - q^2$$

Please return to page $\frac{74}{1}$ and try this question again.

$\frac{64}{2}$

Recall that

$$a^2 - 2ab + b^2 = (a - b)(a - b)$$

Please continue.

Please return to page $\frac{71}{2}$ and try this question again.

Very good. You made the correct choice.

In finding the sum of

$$\frac{3x}{x + y} + \frac{3y}{x + y}$$

We first note that the denominators are the same. Therefore, we can add the numerators and write this sum over the same denominator.

Thus, we get

$$\frac{3x + 3y}{x + y}$$

The numerator of this fraction can be factored by removing the common monomial factor 3 . We get

$$\frac{3(x + y)}{(x + y)}$$

Dividing by the identical factor,

$$(x + y)$$

we obtain 3 as our answer.

Please go on to question 6 below.

Question 6

Apply the proper principle and express the sum of the fractions in lowest terms.

$$\frac{3p - 5q}{2p + q} + \frac{p + 7q}{2p + q}$$

Select the letter which labels the correct statement.

(A) $\frac{4p + 2q}{2p + q}$

(C) 2

(B) 1

(D) None of these.

$\frac{66}{1}$

Your answer is not in lowest terms. The numerator (8 - 2a) can be written as

$$-(2a - 8) = -2(a - 4)$$

Also note that the denominator is the difference of two squares and should be factored.

Please return to page $\frac{83}{2}$ and try this question again.

$\frac{66}{2}$

No, you made an incorrect choice.

We do not multiply denominators when we add fractions.
Please re-read the text assignment for this segment and learn the rule for combining fractions with like denominators.

Please return to page $\frac{90}{2}$ and try this question again.

This is the correct answer.

$$\frac{2b + 3}{b^2 - 64} + \frac{5 - b}{b^2 - 64}$$

[$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$

$$\frac{2b + 3 + 5 - b}{b^2 - 64}$$

[C^ A

$$\frac{b + 8}{b^2 - 64}$$

[Factor: Difference of two squares.

$$\frac{b + 8}{(b + 8)(b - 8)}$$

[$\frac{a}{ab} = \frac{1}{b}$

$$\frac{1}{b - 8}$$

Please go on to the next question.

Question 8

Apply the proper principle and perform the indicated operations,

$$\frac{17}{13} + \frac{3}{13} - \frac{9}{13}$$

Select the letter which labels the correct statement.

(A) $\frac{29}{13}$

(B) $\frac{11}{39}$

(C) $\frac{11}{26}$

(D) None of these.

$\frac{68}{1}$

This is the correct answer.

The first sum is $\frac{15}{5x}$ which becomes $\frac{3}{x}$ when it is reduced.

Please go on to question 4 which follows.

Question 4

Apply the proper principle and find the sum of the fractions,

$$\frac{2x + 1}{5} + \frac{7x - 5}{5}$$

Select the letter which labels the correct statement.

(A) $\frac{9x - 4}{10}$ (C) $\frac{14x^2 - 3x - 5}{25}$

(B) $\frac{9x - 4}{5}$ (D) None of these.

$\frac{68}{2}$

We do not agree.

The numerator of the difference of the two given fractions is:

$$(2 - x) - (5 - 2x) = 2 - x - 5 + 2x$$
$$= x - 3$$

Keep in mind that the minus sign before a fraction means that every term in the numerator following the minus sign is multiplied by -1.

Thus, the difference of the two fractions can be written as

$$\frac{2 - x}{x^2 - 2x - 3} - \frac{5 - 2x}{x^2 - 2x - 3} = \frac{2 - x}{x^2 - 2x - 3} + \frac{-(5 - 2x)}{x^2 - 2x - 3}$$

This procedure may help you to avoid making the mistake that you made.

Please return to page $\frac{81}{1}$ and try this question again.

Sorry, but you made an incorrect choice.

Let us do a similar question together to help you discover where you made the error. Combine the fractions,

$$\frac{19}{6y} + \frac{11}{6y} - \frac{12}{6y}$$

and write the answer in lowest terms.

Since the fractions have the same denominator, we can proceed directly. Combining the numerators, we get

$$19 + 11 - 12 = 18$$

Thus, the numerator of our answer is 18 . The denominator remains the same. Therefore, we have

$$\frac{18}{6y}$$

Now,

$$18 = 6 \times 3$$

Thus, we have

$$\frac{6(3)}{6y}$$

Dividing both numerator and denominator by 6 , we get

$$\frac{3}{y}$$

as our answer.

Please return to page $\frac{60}{2}$ and try this question again.

When the numerators are combined, we get

$$1 + m - (3m - m^2 + 4)$$
$$= 1 + m - 3m + m^2 - 4$$
$$= m^2 - 2m - 3$$

Remember, a fraction cannot be reduced unless the same factor exists in both the numerator and the denominator.

Please return to page $\frac{75}{2}$ and try question 17 again.

$\frac{70}{1}$

Yes, but you did not try to factor the denominator

$$6x^2 + 5x - 14$$

It is possible that one of the factors of this trinomial is

$$(6x - 7)\ ?$$

Please return to page $\frac{77}{2}$ and try this question again.

$\frac{70}{2}$

If we are given any fraction, say $\frac{2}{3}$, an equivalent fraction can be obtained by multiplying both numerator and denominator of this fraction by the same number. Thus,

$$\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

is an equivalent fraction since we multiplied numerator and denominator by 3 . Similarly, if we divide the numerator and denominator of a fraction by the same number, the new fraction will be equivalent to the original fraction.

For example

$$\frac{36}{45} = \frac{4}{5}$$

since we divide the numerator and denominator by the same number 9 .

Please return to page $\frac{87}{2}$ and try question 1 again.

Very good. You made the correct choice.

We have to combine the fractions,

$$\frac{7}{5mn} - \frac{1 - 3m}{5mn} - \frac{2m - 6}{5mn}$$

It is important to note that the minus sign between two fractions effects each term of the numerator of the fraction following the minus sign. Thus, combining the numerators, we have

$$\frac{7 - (1 - 3m) - (2m - 6)}{5mn} = \frac{7 - 1 + 3m - 2m + 6}{5mn}$$

$$= \frac{12 + m}{5mn}$$

Please go on to question 13 below.

Question 13

Apply the proper principle and express the difference

$$\frac{a^2 + b^2}{a - b} - \frac{2ab}{a - b}$$

Select the letter which labels the correct statement.

(A) $\frac{1}{a - b}$

(B) $\frac{a + b}{a - b}$

(C) $a - b$

(D) 1

$\frac{72}{1}$

True; the denominators are not alike. But, they can be made alike.

Ask yourself this question:

How are the denominators $(3x - 2y)$ and $(2y - 3x)$ related?

Return to page $\frac{90}{2}$ and try this question again.

$\frac{72}{2}$

After combining the numerators of the given fractions, we get,

$$\frac{ab - a + b - 1}{(a + 1)^2}$$

The question arises as to whether this answer is in simplest form. It is not. The numerator can be factored. Recall a method called " factoring by grouping. "

For example:

$$rs - r + s - 1 =$$

$$r(s - 1) + (s - 1) =$$

$$(r + 1)(s - 1) =$$

Now see if you can write this numerator as the product of two linear factors.

Return to page $\frac{92}{2}$ and try this question again.

This is the correct answer.

Let's review the procedure:

$$\frac{2 - x}{x^2 - 2x - 3} - \frac{5 - 2x}{x^2 - 2x - 3} = \frac{2 - x - (5 - 2x)}{x^3 - 2x - 3} \qquad \text{[D}$$

$$= \frac{2 - x - 5 + 2x}{x^2 - 2x - 3} \qquad \text{[C A}$$

$$= \frac{x - 3}{x^2 - 2x - 3} \qquad \text{[Factor}$$

$$= \frac{(x - 3)}{(x - 3)(x + 1)} \qquad \text{[Reduce}$$

$$= \frac{1}{x + 1}$$

Please go on to question 15 below.

<u>Question 15</u>

Apply the proper principle and combine the fractions,

$$\frac{3}{x - 3} + \frac{1}{x - 3} - \frac{10 - 2x}{x - 3}$$

Select the letter which expresses the result in lowest terms.

(A) 2

(B) 6

(C) $\frac{2x - 6}{x - 3}$

(D) -2

$\frac{74}{1}$

This is the correct answer.

Please go on to the next question.

Question 10

Apply the proper principle and express the difference in lowest terms

$$\frac{p^2}{p + q} - \frac{q^2}{p + q}$$

Select the letter which labels the correct statement.

(A) 0

(B) $\frac{p^2 - q^2}{p + q}$

(C) $\frac{p - q}{p + q}$

(D) $p - q$

....

$\frac{74}{2}$

The question that you should ask yourself is: By what number must $(3a)$ be multiplied in order to obtain $(9a^2)$?

Once you determine that number, the numerator must then be multiplied by the same number.

Please return to page $\frac{94}{2}$ and try question 2 again.

This is the correct answer.

Follow this technique:

$$\frac{5 - a}{a^2 - 16} - \frac{a - 3}{a^2 - 16} = \frac{5 - a - (a - 3)}{a^2 - 16} \qquad [\ D$$

$$= \frac{5 - a - a + 3}{a^2 - 16} \qquad [\ C \text{ and } A$$

$$= \frac{8 - 2a}{a^2 - 16} \qquad [\ \text{Factor}$$

$$= \frac{2(4 - a)}{(a + 4)(a - 4)} \qquad [\ 4 - a = -1(a - 4)$$

$$= \frac{2(-1)(a - 4)}{(a + 4)(a - 4)} \qquad \left[\ \frac{b}{b} = 1\right.$$

$$= \frac{-2}{a + 4}$$

or $$= -\frac{2}{a + 4}$$

Please go on to question 17 below.

Question 17

Apply the proper principle and combine the fractions,

$$\frac{1 + m}{m^2 - 3m - 4} - \frac{3m - m^2 + 4}{(m - 4)(m + 1)}$$

Select the letter which expresses the result in lowest terms.

(A) $\frac{m - 3}{m - 4}$ (B) $\frac{m + 4}{m + 1}$

(C) $\frac{m + 1}{m - 4}$ (D) $\frac{m - 1}{m + 1}$

$\frac{76}{1}$

We do not agree. The numerator of the difference of the two given fractions is a square trinomial. The factors of this trinomial are

$$(a - b) \quad \text{and} \quad (a - b)$$

Please reconsider your choice.

Please return to page $\frac{71}{2}$ and try this question again.

$\frac{76}{2}$

The lowest common multiple of two or more numbers is the smallest number that is exactly divisible by each of the given numbers.
For example, given the numbers 2, 3, 4, the smallest number that can be divided by 2, 3, and 4 without a remainder is 12 . Thus, 12 is the least common multiple of these numbers.

Please return to page $\frac{107}{2}$ and try question 4 again.

Very good. You made the correct choice.

Upon first observation, the denominators of the two fractions are different, and you are tempted to say that these fractions cannot be combined. However, upon closer examination, we note that the second denominator,

$$(2y - 3x) \text{ equals } -(3x - 2y)$$

That is, the second denominator is equal to the first multiplied by (-1). The question can, therefore, be written as

$$\frac{13x - 3y}{3x - 2y} + \frac{4x + 3y}{-(3x - 2y)}$$

$$= \frac{13x - 3y}{3x - 2y} + \frac{-(4x + 3y)}{3x - 2y} \qquad \left[\; + \frac{a}{-b} = + \frac{-a}{b}\right.$$

$$= \frac{13x - 3y - (4x + 3y)}{3x - 2y} \qquad [\; D$$

$$= \frac{13x - 3y - 4x - 3y}{3x - 2y} \qquad [\; C \wedge A$$

$$= \frac{9x - 6y}{3x - 2y} \qquad [\; \text{Factor}$$

$$= \frac{3(3x - 2y)}{(3x - 2y)} \qquad \left[\; 3 \cdot \frac{a}{a} = 3\right.$$

$$= 3$$

Please go on to question 19 below.

Question 19

Apply the proper principle and combine the fractions,

$$\frac{2x - x^2 - 2}{6x^2 + 5x - 14} - \frac{5 - x^2 - 4x}{6x^2 + 5x - 14}$$

Select the letter which expresses the result in lowest terms.

(A) $\frac{6x - 7}{6x^2 + 5x - 14}$ (B) $\frac{1}{x + 2}$

(C) $\frac{6x - 7}{3x + 7}$ (D) $\frac{1}{6x - 7}$

$\frac{78}{1}$

You must have made a careless error. Combining numerators of the given fraction, we get

$$3 + 1 - (10 - 2x) = 4 - 10 + 2x$$

It is always advisable to write the positive term first. Thus, instead of writing $-6 + 2x$ write $2x - 6$.

Please return to page $\frac{73}{2}$ and try question 15 again.

$\frac{78}{2}$

You made an incorrect choice. Note that the required denominator

$$(x^2 - y^2)$$

is the difference of two squares and is equal to

$$(x - y)(x + y)$$

Please return to page $\frac{99}{2}$ and try question 3 again.

This is the correct answer.

Please go on to the next question.

Question 11

Apply the proper principle and express the difference in lowest terms

$$\frac{2x}{9y} - \frac{3 - 7x}{9y}$$

Select the letter which labels the correct statement.

(A) $\frac{-5x - 3}{9y}$ (B) $\frac{5x - 3}{9y}$

(C) $\frac{x - 3}{y}$ (D) $\frac{3x - 1}{3y}$

....

The rule for finding the lowest common multiple of two or more expressions is not altered if the expressions are binomials or trinomials. Thus, the binomial $(x + 3)$ is prime; that is,

$$x + 3 = (x + 3)\cdot 1$$

The binomial $x^2 - 9 = (x + 3)(x - 3)$

Please continue.

Please return to page $\frac{98}{2}$ and try question 7 again.

$\frac{80}{1}$

It is nice to see that you observed that the denominators of the fractions are alike. Did you factor the first denominator before you decided?

However, you did not factor the numerator correctly

Please check these factors by multiplying them.

Please return to page $\frac{75}{2}$ and try this question again.

$\frac{80}{2}$

Yes, but $54a^3b^2$ is not the <u>lowest</u> common multiple of $6ab$ and $9a^2b$.
Can you find a smaller common multiple?

Please return to page $\frac{91}{2}$ and try question 5 again.

This is the correct answer.

Please go on to the next question.

Question 14

Apply the proper principle and express the difference

$$\frac{2 - x}{x^2 - 2x - 3} - \frac{5 - 2x}{x^2 - 2x - 3}$$

in lowest terms. Select the letter which labels the correct statement.

(A) $\frac{1}{x + 1}$

(B) $\frac{3x - 3}{(x - 3)(x + 1)}$

(C) $\frac{x - 3}{x^2 - 2x - 3}$

(D) $\frac{3}{x + 3}$

Sorry, but you made the wrong choice.

The least common denominator of several fractions is the lowest common multiple of the denominators of these fractions. As before, each denominator should be written as a product of its prime factors and the rule for finding the lowest common multiple should be applied.

Please return to page $\frac{88}{2}$ and try question 8 again.

$\frac{82}{1}$

What are the factors of the trinomial

$$6x^2 + 5x - 14 \quad ?$$

After you decide upon the factors, please check them by multiplication.

Their product must equal the trinomial.

Please return to page $\frac{77}{2}$ and try this question again.

$\frac{82}{2}$

We do not agree.

Listen carefully and you will avoid making this mistake again.

We have two fractions; one has $(x - y)$ as its denominator, and one has $(x^2 - y^2)$ as its denominator. Now,

$$x^2 - y^2 = (x - y)(x + y)$$

Thus, the second denominator is $(x + y)$ times the first.

What must we, therefore, do to the first fraction to obtain an equivalent fraction having $x^2 - y^2$ for a denominator?

Please return to page $\frac{99}{2}$ and try question 3 again.

This is the correct answer.

Compare your work with this.

$$\frac{3}{x-3} + \frac{1}{x-3} - \frac{10-2x}{x-3}$$

$$= \frac{3+1-(10-2x)}{x-3} \qquad [\ \text{D}$$

$$= \frac{4-10+2x}{x-3} \qquad [\ \text{C}$$

$$= \frac{2x-6}{x-3} \qquad [\ \text{Factor}$$

$$= \frac{2(x-3)}{(x-3)} \qquad [\ \frac{2a}{a} = 2$$

$$= 2$$

Please go on to question 16 below.

Question 16

Apply the proper principle and combine the fractions,

$$\frac{5-a}{a^2-16} - \frac{a-3}{a^2-16}$$

Select the letter which expresses the result in lowest terms.

(A) $\frac{8}{a^2-16}$

(B) $\frac{2}{a^2-16}$

(C) $-\frac{2}{a+4}$

(D) $\frac{8-2a}{a^2-16}$

Congratulations. You did a fairly difficult problem correctly.

Let's review this:

$$\frac{ab}{(a + 1)^2} - \frac{a - b + 1}{(a + 1)^2}$$

$$= \frac{ab - (a - b + 1)}{(a + 1)^2}$$

After combining the numerators of the the two given fractions, we get:

$$\frac{ab - a + b - 1}{(a + 1)^2}$$ [Factor "a" out of the first two terms.

$$\frac{a(b - 1) + (b - 1)}{(a + 1)^2}$$ [Factor the (b - 1)

$$\frac{(b - 1)(a + 1)}{(a + 1)^2}$$ [Divide by (a + 1)

Thus we have: $\frac{b - 1}{a + 1}$

You have now finished this Segment. Hand in your PUNCH CARD. You should have entered in your NOTEBOOK the following definitions and formulas:

(1) The sum of fractions with equal denominators is a fraction whose numerator is the sum of the numerators and whose denominator is the denominator common to the given fractions.

(2) When two fractions are connected by a minus sign, the numerator of the fraction following the minus sign is multiplied by (- 1) .

(3) After several fractions have been combined, the resulting fraction should be reduced if possible.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT:

Problems 5 , 6 , 7 , and 8 .

When the numerators are combined, we get

$$1 + m - (3m - m^2 + 4) = 1 + m - 3m + m^2 - 4$$
$$= m^2 - 2m - 3$$

The factors of this trinomial are

$$(m - 3)(m + 1)$$

Please continue.

Please return to page $\frac{75}{2}$ and try question 17 again.

The lowest common multiple should not be confused with the lowest common divisor. Let us explain with a similar example:

The smallest number that divides into both 10 and 15 is 5 ; this is called the lowest (or least) common divisor of 10 and 15 .

The multiples of 10 are: 10, 20, 30, 40, 50, 60, etc.

The multiples of 15 are: 15, 30, 45, 60, 75, 90, etc.

The common multiples of 10 and 15 are: 30, 60, etc.

The lowest common multiple of 10 and 15 is: 30 .

Now return to page $\frac{107}{2}$ and reconsider this problem.

$\frac{86}{1}$

Did you combine the numerators of the given fractions correctly?
We have

$$2x - x^2 - 2 - (5 - x^2 - 4x)$$

Please continue.

Please return to page $\frac{77}{2}$ and try this question again.

$\frac{86}{2}$

We do not agree. In the first place, before combining fractions each of the fractions should be written with the same denominator. In the second place, fractions are not combined by adding numerators and adding denominators.

Your choice clearly shows that you have forgotten the method for adding fractions. Review the material in your text.

Please return to page $\frac{106}{2}$ and try question 9 again.

VOLUME 11 SEGMENT 3 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 8	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	1 1	(Volume Number)
	66	and	68	0 3	(Segment Number)

Your READING ASSIGNMENT for this SEGMENT is page 299 - 300. You will be asked a series of questions to draw your attention to the more important points.

INTRODUCTORY NOTE

In the previous segment, you learned how to add and subtract algebraic fractions having like denominators. The rule was simple: combine the numerators for a new numerator and place it above the same denominator. Now, in this coming segment, you will be asked to combine fractions with unlike denominators. You know that any fraction can be changed to an equivalent fraction with a different denominator. Can you anticipate how fractions with unlike denominators will be combined? Proceed to the first question, and you will soon find out whether you anticipated correctly.

Question 1

Apply the proper principle and change the fraction $\frac{5}{7}$ to an equivalent fraction having a denominator of 28. Select the letter which labels the correct answer.

(A) $\frac{26}{28}$ (C) $\frac{24}{28}$

(B) $\frac{25}{28}$ (D) $\frac{20}{28}$

$\frac{88}{1}$

Very good. You made the correct choice.

In finding the lowest common multiple of two or more binomial expressions, we proceed as before.

First, we write each expression as a product of prime factors.

Thus,

$$x + 3 = (x + 3)$$
$$x^2 - 9 = (x + 3)(x - 3)$$

Since each of the binomials, $(x + 3)$ and $(x - 3)$ appear but once, the lowest common multiple is

$$(x + 3)(x - 3) = x^2 - 9$$

Please proceed to question 8 below.

....

$\frac{88}{2}$

<u>Question 8</u>

Apply the proper principle and find the lowest common denominator of the fractions $\frac{1}{(x + 5)^2}$ and $\frac{1}{(x^2 - 25)}$

Select the letter which labels the correct answer.

(A) $(x + 5)(x - 5)$
(B) $(x + 5)^2$
(C) $(x + 5)(x - 5)^2$
(D) $(x + 5)^2(x - 5)$

$\frac{89}{1}$

We do not agree. One of the letters does have the correct answer next to it.

Apply the basic rule:

$$\frac{a}{b} = \frac{a}{b} \cdot 1 = \frac{a}{b} \cdot \frac{c}{c} = \frac{ac}{bc} \qquad \left[\frac{c}{c} = 1 \right.$$

Please return to page $\frac{99}{2}$ and try question 3 again.

$\frac{89}{2}$

We do not agree. What is the least common denominator of

$3x$, 3 , and $2x^2$? It is not $3x^2$, Do you see why?

Please return to page $\frac{113}{2}$ and try question 10 again.

$\frac{90}{1}$

Let's review the problem:

$$\frac{1 + m}{m^2 - 3m - 4} - \frac{3m - m^2 + 4}{(m - 4)(m + 1)}$$ [C, Arrange in descending order

$$\frac{m + 1}{m^2 - 3m - 4} - \frac{-m^2 + 3m + 4}{(m - 4)(m + 1)}$$ [Factor denominator

$$\frac{m + 1}{(m - 4)(m + 1)} - \frac{(-m^2 + 3m + 4)}{(m - 4)(m + 1)}$$ [Since fractions are "like", combine.

$$\frac{m + 1 - (-m^2 + 3m + 4}{(m - 4)(m + 1)}$$ [D

$$\frac{m + 1 + m^2 - 3m - 4}{(m - 4)(m + 1)}$$ [C^A

$$\frac{m^2 - 2m - 3}{(m - 4)(m + 1)}$$ [Factor

$$\frac{(m - 3)(m + 1)}{(m - 4)(m + 1)}$$ [$\frac{ab}{cb} = \frac{a}{c}$

$$\frac{m - 3}{m - 4}$$

Please go on to question 18 below.

$\frac{90}{1}$

Question 18

Apply the proper principle and combine the fractions,

$$\frac{13x - 3y}{3x - 2y} + \frac{4x + 3y}{2y - 3x}$$

Select the letter which expresses the result in lowest terms.

(A) $\frac{17x}{3x - 2y}$

(B) $\frac{17x}{(3x - 2y)(2y - 3x)}$

(C) 3

(D) Cannot be combined because the denominators are not alike.

Very good. You made the correct choice.

In order to obtain the lowest common multiple of two or more numbers, we express each number as a product of primes.

Thus,

$$8 = 2 \cdot 2 \cdot 2$$

$$12 = 2 \cdot 2 \cdot 3$$

Now, we take each factor the greatest number of times and find their product. The factor 2 appears three times while the factor 3 appears only once. Hence, the lowest common multiple is

$$2 \cdot 2 \cdot 2 \cdot 3 = 24$$

Please proceed to question 5 below.

Question 5

Apply the proper principle and find the least common multiple of $6ab$ and $9a^2b$. Select the letter which labels the correct answer.

(A) $54a^3b^2$

(B) $18a^2b$

(C) $15a^2b$

(D) $18a^2b^2$

92
2

Very good. You made the correct choice.

We first check to see if the fractions are "like" fractions and then combine the numerators.

Thus, we have

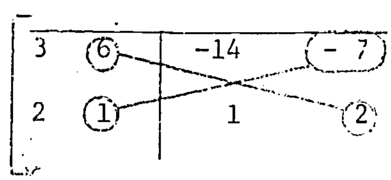
$$\frac{2x - x^2 - 2 - (5 - x^2 - 4x)}{6x^2 + 5x - 14} = \frac{2x - x^2 - 2 - 5 + x^2 + 4x}{6x^2 + 5x - 14} \quad [CA$$

$$= \frac{6x - 7}{6x^2 + 5x - 14}$$

Next, we examine the denominator to see if it can be factored.

The factors of $6x^2 + 5x - 14$ are: $(6x - 7)(x + 2)$

We can now write: $\dfrac{(6x - 7)}{(6x - 7)(x + 2)}$



Dividing numerator and denominator by the identical factor, we get: $\dfrac{1}{x + 2}$

Please go on to question 20 below.

92
2

Question 20

Apply the proper principle and combine the fractions,

$$\frac{ab}{(a + 1)^2} - \frac{a - b + 1}{(a + 1)^2}$$

Select the letter which expresses the result in lowest terms.

(A) $\dfrac{ab - a + b - 1}{(a + 1)^2}$ (B) $\dfrac{a(b - 1)}{a + 1}$

(C) $\dfrac{b - 1}{a + 1}$ (D) None of these.

We do not agree.

It is not enough to find an expression into which each of the given terms

$$3a \quad , \quad 8ab \quad , \quad 12a^2b^2$$

will divide exactly. We are looking for the smallest number that will do this job. Please write each of the numbers as a product of primes and follow the rule for finding the lowest common multiple.

Return to page $\frac{112}{2}$ and try question 6 again.

Not bad for a first attempt. However, your answer is not in lowest terms. You can start by removing a common momomial factor from the numerator of your choice and see if the same factor appears in the denominator. If it does, then the familiar rule

$$\frac{ab}{ac} = \frac{b}{c} \quad \text{applies.}$$

Return to page $\frac{106}{2}$ and try question 9 . again.

$\frac{94}{1}$

Very good. You made the correct choice.

We have to change $\frac{5}{7}$ to an equivalent fraction with a denominator of 28 .

Now,

$$28 = 7 \times 4$$

Hence, the numerator must also be multiplied by 4 . We, therefore, have

$$\frac{5 \times 4}{7 \times 4} = \frac{20}{28}$$

Please proceed to question 2 below.

$\frac{94}{2}$

Question 2

Apply the proper principle and change the fraction $\frac{5}{3a}$ to an equivalent fraction with a denominator $9a^2$.

Select the letter which labels the correct answer.

(A) $\frac{15}{9a^2}$

(C) $\frac{15a}{9a^2}$

(B) $\frac{8a}{9a^2}$

(D) None of these.

Please note that

$$x^2 - 25 = (x - 5)(x + 5)$$

while

$$(x + 5)^2 = (x + 5)(x + 5)$$

Your choice does not take into account the factor $(x - 5)$

Return to page $\frac{88}{2}$ and try question 8 again.

We do not agree.

Please observe that there is a minus sign between the fractions.

Thus, we have

$$\frac{3m}{2(m + 3)} - \frac{(m - 1)}{(m + 3)} \cdot \left(\frac{2}{2}\right) = \frac{3m - 2(m - 1)}{2(m + 3)}$$

Please continue.

Return to page $\frac{120}{2}$ and try question 12 again.

$\frac{96}{1}$

We do not agree.

If we divide 15 by 6 we get a remainder.

Similarly,

if we divide 15 by 9 we get a remainder.

Please return to page $\frac{91}{2}$ and try question 5 again.

$\frac{96}{2}$

We do not agree.

The least common denominator of

$$x^2y \quad \text{and} \quad xy^2$$

is not x^3y^3

Note that $x^2y = x \cdot x \cdot y$

and $xy^2 = x \cdot y \cdot y$

However, it is possible to combine fractions by using a common multiple of the denominator that is not the <u>least</u> common multiple.

If this is done, then you can be sure that the fraction answer can be reduced.

This is the case here.

Please return to page $\frac{108}{2}$ and try question 11 again.

Yes, but your answer is not in lowest terms. The least common denominator of $3x$, 3, and $2x^2$ is obtained by writing each of these terms as a product of primes; thus,

$$3x = 3 \cdot x$$

$$3 = 3$$

$$2x^2 = 2 \cdot x \cdot x$$

Hence, the least common denominator is

$$2 \cdot 3 \cdot x \cdot x = 6x^2$$

Please return to page $\frac{113}{2}$ and try question 10 again.

You did choose the correct least common denominator, but you made an error in multiplying the two binomials. Please check the multiplication of

$$(x - 2) \quad \text{and} \quad (x - 3)$$

Please return to page $\frac{116}{2}$ and try question 14 again.

$\frac{98}{1}$

Very good. You made the correct choice.

We write each of the given terms in prime factor form:

$$3a = 3a = 3a$$

$$8ab = 2 \cdot 2 \cdot 2 \cdot a \cdot b = 2^3ab$$

$$12a^2b^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b = 2^2 3a^2b^2$$

In order to find an expression into which each of the given terms can be divided, we merely have to multiply all three terms. However, if we wish to find the smallest expression into which each of the terms can be divided, we must construct an expression which is the product of each of the different factors with their respective exponents equal to the highest value that appears in any of the factorizations..

Now the different factors are: 2 , 3 , a, and b .

The highest exponent of 2 is 3 ; the highest exponent of 3 is 1 ; the highest exponent of a is 2 and the highest exponent of b is also 2 .

The product of each of these different factors with their respective highest exponents is the required " least common multiple. " Thus, we get

$$2^3 \cdot 3 \cdot a^2 \cdot b^2$$

which becomes $24a^2b^2$

$\frac{98}{2}$

Question 7

Apply the proper principle and find the lowest common multiple of

$$(x + 3) \text{ and } (x^2 - 9)$$

Select the letter which labels the correct answer.

(A) $x - 3$ (C) $x^2 - 27$

(B) $x^2 - 9$ (D) $x^3 - 27$

Very good. You made the correct choice.

The given fraction has a denominator of (3a) .

The new fraction has a denominator of ($9a^2$) .

Now,

$$(3a)(3a) = 9a^2$$

Since the denominator is multiplied by 3a , the numerator must also be multiplied by this number in order for the fraction to remain the same.

Thus,

$$\frac{5}{3a} \cdot \frac{3a}{3a} = \frac{15a}{9a^2}$$

Please proceed to question 3 below.

Question 3

Apply the proper principle and change the fraction

$$\frac{8}{x - y}$$

to an equivalent fraction with a denominator ($x^2 - y^2$) .

Select the letter which labels the correct answer.

(A) $\frac{8(x - y)}{x^2 - y^2}$

(C) $\frac{8xy}{x^2 - y^2}$

(B) $\frac{8(x + y)}{x^2 - y^2}$

(D) None of these.

$\frac{100}{1}$

What is the least com of the given fractions?

The denominators are

$$3 \quad , \quad 6a \quad , \quad 2a$$

Yes, 6a is the least common denominator. Examine the first fraction

$$\frac{2a}{3}$$

and ask yourself the following question: By what number must 3 be multiplied to get 6a ? The numerator 2a must then be multiplied by this same number.

i.e.

$$\frac{2a}{3} \cdot \left(\frac{?}{?}\right) = \frac{\quad}{6a}$$

Return to page $\frac{106}{2}$ and try question 9 again.

$\frac{100}{2}$

You used the correct lowest common denominator, but you made an error in combining the numerators.

Return to page $\frac{120}{2}$ and try question 12 again.

Did you write each of the denominators as a product of prime factors?

You did.

Did you note that the factor (x - 5) appears only once?

Please return to page $\frac{88}{2}$ and try question 8 again.

When is the difference between two numbers equal to zero?

When two numbers are equal, their difference is zero. Examine the two given fractions; are they identical?

No! Your choice is, therefore, incorrect.

Common sense cannot always tell us when we have the right answer, but it surely can help us to know when we have the wrong answer.

Please return to page $\frac{131}{2}$ and try question 15 again.

$\frac{102}{1}$

Almost right.

Did you write each of the terms as a product of its prime factors?

ıat

$$6ab = 2 \cdot 3 \cdot a \cdot b$$

and

$$9a^2b = 3 \cdot 3 \cdot a \cdot a \cdot b$$

Return to page $\frac{91}{2}$ and try question 5 again.

$\frac{102}{2}$

You overlooked something.

Notice that there is a minus sign between the two given fractions.

Do you recall that this minus sign effects each member of the numerator of the fraction following the minus sign. In other words, the two fractions can be written as

$$\frac{4}{x^2y} + \frac{-(2 - x)}{xy^2}$$

which equals

$$\frac{4}{x^2y} + \frac{-2 + x}{xy^2}$$

Return to page $\frac{108}{2}$ and try question 11 again.

You haven't been paying attention.

We do not combine fractions by adding or subtracting numerators and denominators. Why not? Suppose we follow your method and add the frac

$$\frac{5}{2} + \frac{3}{5} + \frac{9}{10}$$

Adding numerators and denominators as you did, we get

$$\frac{5 + 3 + 9}{2 + 5 + 10} = \frac{17}{17} = 1$$

But, the first fractions, $\frac{5}{2}$ is $2\frac{1}{2}$ and is much larger than 1 .

Please re-read the text assignment.

Please return to page $\frac{113}{2}$ and try question again.

You have the correct denominator, but we do not agree with your numerator. When the fractions are changed to equivalent fractions with denominators equal to

$$x^2 - 4$$

the numerators become $2(x - 2)$, $3(x + 2)$,

and $-5x$

Please go over your work and find the mistake you made in combining these numerators.

Please return to page $\frac{109}{2}$ and try question 16 again.

$\frac{104}{1}$

What are the factors of

$$p^2 - q^2 \quad ?$$

You know that they are

$$(p - q)$$

and $(p + q)$

Hence, the nume ator and denominator of the first fraction must be multiplied by $(p - q)$

in order for both fractions to have identical denominators.

We, therefore, have:

$$\frac{p}{p + q} \cdot \frac{(p - q)}{(p - q)} + \frac{pq}{(p + q)(p - q)} \qquad \frac{p - q}{p - q} = 1$$

Please continue.

Return to page $\frac{110}{2}$ and try question 13 again.

$\frac{104}{2}$

We do n agree.

Note tha ch of the denominators of the given fractions can be factored.

Thus,

$$6x + 6 = 6(x - 1)$$

$$2x - 2 = 2(x - 1)$$

$$3 - 3x^2 = 3(1 - x^2) = -3(x^2 - 1)$$

$$= -3(x + 1)(x - 1)$$

The questi can now be re-written as follows:

$$\frac{1}{6(x - 1)} - \frac{1}{2(x - 1)} - \frac{4}{3(x + 1)(x - 1)}$$

Please continue.

Return to page $\frac{122}{2}$ and try question 19 again.

Sorry, but your choice is not correct.

Suppose that we do a similar problem together to help you find your error.

Combine the fractions

$$\frac{5x}{3x + 9} - \frac{2x - 1}{x + 3}$$

The denominator of the first fraction can be written as $3(x + 3)$.

Hence, the least common denominator is $3(x + 3)$. Thus, we have:

$$\frac{5x}{3(x + 3)} - \frac{(2x - 1)}{(x + 3)} \cdot \left(\frac{3}{3}\right)$$

$$= \frac{5x - 3(2x - 1)}{3(x + 3)}$$

$$= \frac{5x - 6x + 3}{3(x + 3)}$$

$$= \frac{3 - x}{3(x + 3)}$$

NOTE: the order of the terms in the numerator is changed to have the minus sign between the terms.

Please return to page $\frac{120}{2}$ and try question 12 again.

$\frac{105}{2}$

You did choose the correct denominator, but your numerator is wrong.

You, of course, realized that

$$x^2 - 5x + 6 = (x - 3)(x - 2)$$

Hence, we must multiply the numerator and denominator of the first fraction by

$$(x - 2)$$

in order that both fractions should have the same denominator.

Please continue and discover your error.

Please return to page $\frac{118}{2}$ and try question 17 again.

$\frac{106}{1}$

Very good. You made the correct choice.

We proceed as follows:

$$(x + 5)^2 = (x + 5)(x +$$

$$(x^2 - 25) = (x + 5)(x - 5)$$

The factor $(x + 5)$ appears twice in the first identity, and the factor $(x - 5)$ appears once in the second identity. Hence, the lowest common denominator of the given fractions is

$$(x + 5)(x + 5)(x - 5) = (x + 5)^2(x - 5)$$

Please proceed to question 9 below.

$\frac{106}{2}$

Question 9

Apply the proper principle and combine the fractions,

$$\frac{2a}{3} + \frac{5}{6a} - \frac{3}{2a}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{2a - 2}{8a + 3}$ (C) $\frac{a - 2}{3a}$

(B) $\frac{4a^2 - 4}{6a}$ (D) $\frac{2(a^2 - 1)}{3a}$

Very good. You made the correct choice.

In order to change the fraction

$$\frac{8}{x - y}$$

to a fraction with a denominator

$$x^2 - y^2$$

we must multiply the numerator and denominator by

$$(x + y)$$

Thus,

$$\frac{8}{x - y} = \frac{8(x + y)}{(x - y)(x + y)} = \frac{8(x + y)}{x^2 - y^2}$$

Please proceed to question 4 below.

Question 4

Apply the proper principle and find the lowest common multiple of

8 and 12

Select the letter which labels the correct answer.

(A) 96 (C) 24

(B) 20 (D) 2

$\frac{108}{1}$

Very good. You made the correct choice.

We have to combine the fractions,

$$\frac{2}{3x} + \frac{x}{3} - \frac{1}{2x^2}$$

The lowest common denominator of these fractions is $6x^2$. We now replace each of the fractions by equivalent fractions with $6x^2$ as denominator. Thus, we have:

$$\frac{2}{3x} \cdot \frac{2x}{2x} + \frac{x}{3} \cdot \frac{2x^2}{2x^2} - \frac{1}{2x^2} \cdot \frac{3}{3}$$

$$= \frac{4x}{6x^2} + \frac{2x^3}{6x^2} - \frac{3}{6x^2}$$

$$= \frac{2x^3 + 4x - 3}{6x^2}$$ [COMMUTATIVE PROPERTY

Please proceed to question 11 below.

$\frac{108}{2}$

Question 11

Apply the proper principle and combine the fractions,

$$\frac{4}{x^2y} - \frac{2 - x}{xy^2}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{4xy^2 - 2x^2y + x^3y}{x^3y^3}$

(B) $\frac{4y - 2x - x^2}{x^2y^2}$

(C) $\frac{4y - 2x + x^2}{x^2y^2}$

(D) None of these.

Very good. You made the correct choice. We have to combine the fractions,

$$\frac{x - y}{x + y} - \frac{x + y}{x - y}$$

Since the denominators of these fractions are relatively prime, the least common denominator is their product,

$$(x + y)(x - y) = x^2 - y^2$$

To change the first fraction to an equivalent fraction with this denominator, we multiply numerator and denominator by $(x - y)$. To change the second fraction to an equivalent fraction with the required denominator, we multiply numerator and denominator by $(x + y)$. Thus, we have:

$$\frac{x - y}{x + y} \cdot \frac{x - y}{x - y} - \frac{x + y}{x - y} \cdot \frac{x + y}{x + y}$$

$$= \frac{(x - y)^2}{x^2 - y^2} - \frac{(x + y)^2}{x^2 - y^2}$$

$$= \frac{x^2 - 2xy + y^2 - (x^2 + 2xy + y^2)}{x^2 - y^2}$$

$$= \frac{x^2 - 2xy + y^2 - x^2 - 2xy - y^2}{x^2 - y^2}$$

$$= \frac{4xy}{x^2 - y^2}$$

Please proceed to question 11 below.

Question 11

Apply the proper principle and combine the fraction,

$$\frac{2}{x + 2} + \frac{3}{x - 2} - \frac{5x}{x^2 - 4}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{5 - 5x}{x^2 - 4}$ (C) $\frac{4}{x^2 - 4}$

(B) $\frac{-10}{x^2 - 4}$ (D) $\frac{2}{x^2 - 4}$

110
1
Very good. You made the correct choice.

We have to combine the fractions

$$\frac{3m}{2m + 6} - \frac{m - 1}{m + 3}$$

First, we observe that

$$2m + 6 = 2(m + 3)$$

Hence, the least common denominator of the given fractions is,

$$2(m + 3)$$

To change the second fraction to an equivalent fraction with this denominator, we multiply the numerator and denominator of this fraction by 2 which is the same as multiplying the fraction by $\frac{2}{2}$ or 1 which does not change its value.

$$\frac{3m}{2(m + 3)} - \frac{2}{2} \cdot \frac{(m - 1)}{(m + 3)} = \frac{3m - 2m + 2}{2(m + 3)}$$

$$= \frac{m + 2}{2(m + 3)}$$

It is important to note that the minus sign before the second fraction changes the sign of each term of the numerator of the fraction that follows to its opposite.

Please proceed to question 13 below.

110
2
Question 13

Apply the proper principle and combine the fractions,

$$\frac{p}{p + q} + \frac{pq}{p^2 + q^2}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{p^2 + 2pq}{p^2 - q^2}$ (C) $\frac{p + q}{p^2 - q^2}$

(B) $\frac{p^2}{p^2 - q^2}$ (D) $\frac{1}{q^2}$

Before deciding upon the least common denominator of these fractions, we must first write the trinomial

$$x^2 - 5x + 6$$

in factored form. Thus,

$$x^2 - 5x + 6$$

$$= (x - 3)(x - 2)$$

The first fraction has one of these factors for its denominator. Hence, the least common denominator is

$$(x - 2)(x - 3)$$

Please return to page $\frac{118}{2}$ and try question 17 again.

You expressed $9\frac{3}{8}$ as an improper fraction.

That is not what the question called for.

Please return to page $\frac{134}{2}$ and try question 1 again.

$\frac{112}{1}$

Very good. You made the correct choice.

As we did with arithmetic numbers, we write each of the terms as a product of prime factors.

Thus, we have

$$6ab = 2 \cdot 3 \cdot a \cdot b = 2^1 \cdot 3^1 \cdot a^1 \cdot b^1$$

$$9a^2b = 3 \cdot 3 \cdot a \cdot a \cdot b = 3^2 \cdot a^2 \cdot b^1$$

We pick each different factor that appears and include the highest exponent of that factor that appears.

We, therefore, have

$$2^1 \cdot 3^2 \cdot a^2 \cdot b^1$$

$$= 18a^2b$$

The product of these factors is the " least common multiple " of the original terms.

Please proceed to question 6 below.

$\frac{112}{2}$

Question 6

Apply the proper principle and find the lowest common multiple of

$$3a \quad , \quad 8ab \quad , \quad \text{and} \quad 12a^2b$$

Select the letter which labels the correct answer.

(A) $24a^2b^2$ (C) $36a^2b^2$

(B) $144a^4b^3$ (D) $24a^3b^3$

Very good. You made the correct choice.

We have to combine the fractions,

$$\frac{2a}{3} + \frac{5}{6a} - \frac{3}{2a}$$

We first find the least common denominator of these fractions.

We have:

$$3 = 3$$

$$6a = 2 \cdot 3 \cdot a$$

$$2a = 2 \cdot a$$

So that $(2 \cdot 3 \cdot a)$

is the least common denominator.

Next, each of the given fractions must be written with $6a$ as their denominator.

Thus,

$$\frac{2a}{3} = \frac{2a}{3} \cdot \frac{2a}{2a} = \frac{4a^2}{6a}$$

$$\frac{5}{6a} = \frac{5}{6a} \cdot \frac{1}{1} = \frac{5}{6a}$$

$$\frac{3}{2a} = \frac{3}{2a} \cdot \frac{3}{3} = \frac{9}{6a}$$

Please proceed to question 10 below.

Question 10

Apply the proper principle and combine the fractions

$$\frac{2}{3x} + \frac{x}{3} - \frac{1}{2x^2}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{x^3 + 2x^2 - 3}{3x^2}$

(B) $\frac{12x^2 + 6x^3 - 9x}{18x^3}$

(C) $\frac{2x^3 + 4x - 3}{6x^2}$

(D) $\frac{1 + x}{3x + 3 - 2x^2}$

$\frac{114}{1}$

We do not agree.

Since both denominators are prime, the least common denominator is the product of the two denominators,

$$(x + y)(x - y) = (x^2 - y^2)$$

Changing both fractions to equivalent fractions with this denominator, we get

$$\frac{x - y}{x + y} \cdot \frac{x - y}{x - y} \quad - \quad \frac{x + y}{x - y} \cdot \frac{x + y}{x + y}$$

$$\frac{(x - y)(x - y)}{x^2 - y^2} \quad - \quad \frac{(x + y)(x + y)}{x^2 - y^2}$$

Please continue.

Please return to page $\frac{131}{2}$ and try question 15 again.

$\frac{114}{2}$

How do we express a mixed number as an improper fraction? One basic method is to convert the whole number into a fraction having the same denominator as the fraction part of the mixed number, and then add the fractions.

e. g. $$3\frac{5}{8} = 3 + \frac{5}{8} = \frac{24}{8} + \frac{5}{8}$$

$$= \frac{29}{8}$$

This can be accomplished by a short algorithm. We multiply the denominator of the fraction by the whole number and add the numerator. Thus, for example,

$$3\frac{5}{8} = \frac{8 \cdot 3 + 5}{8} = \frac{29}{8}$$

Please return to page $\frac{129}{2}$ and try question 2 again.

Sorry, but one of the letters does have the correct answer next to it.

Please reconsider your choice.

Return to page $\frac{108}{2}$ and try question 11 again.

Although at first glance it appears that all three denominators are relatively prime, a closer look shows that the $(c^2 - 1)$ is the difference of two squares. When this is factored it is noticed that

$$(c + 1)(c - 1)$$

are in the reverse order compared to the first two denominators. This can be adjusted by multiplying the fraction by $\frac{-1}{-1}$.

Thus,

$$\frac{c^2}{c^2 - 1} = \frac{c^2}{c^2 - 1} \cdot \frac{-1}{-1}$$

$$= \frac{-c^2}{1 - c^2}$$

Please continue.

Return to page $\frac{128}{2}$ and tackle problem 18 again.

116
1

Very gc . . . ade the correct choice.

Since

$$^2 - q^2 = (p + q)(p - q)$$

the least ... enominator is

$$- q^2$$

In order ... first fraction to have this denominator, we must multiply numerator ... ominator by $(p - q)$.

Thus, we ha

$$\frac{p}{(p + q)} \cdot \frac{(p - q)}{(p - q)} + \frac{pq}{p^2 - q^2}$$

$$= \frac{p^2 - pq + pq}{p^2 - q^2}$$

$$= \frac{p^2}{p^2 - q^2}$$

Note: the D... BUTIVE PROPERTY ... in the part p (...)

Please ... question 14 below.

116
2

Question 14

Apply the proper principle and combine the fractions,

$$\frac{x - 2}{2x - 1} + \frac{x}{x - 3} - \frac{1}{2x - 1}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{3x^2 - 3x + 3}{(\ldots - 1)(x - 3)}$

(B) $\frac{\ldots + 1)}{2x - 1}$

(C) $\frac{3x^2 - 7x + 9}{(2x - 1)(x - 3)}$

(D) None of these.

Ve y good. You made the correct choice 1

art by factoring each of the denom tors. We have,

$$6x - 6 = 6(x + 1$$

$$2x - 2 = 2(x - 1$$

$$3 - 3x^2 = 3(1 - x = -3(x^2 - 1)$$

$$= -3(x + 1)(x - 1)$$

can now re-write the fractions as follows:

$$\frac{1}{6(x+1)} - \frac{1}{2(x-1)} - \frac{4}{3(x+1)(x-1)}$$

e lowest common denominator of these fractions is

$$6(x + 1)(x - 1)$$

nging all the fractions to equivalent fractions with this denominator, get,

$$\frac{1}{6(x+1)} \cdot \frac{x-1}{x-1} - \frac{1}{2(x-1)} \cdot \frac{3(x+1)}{3(x+1)} - \frac{4}{3(x+1)(x-1)} \cdot \frac{2}{2} =$$

$$\frac{x - 1 - 3x - 3 - 8}{6(x^2 - 1)} = \frac{-2x - 12}{6(x^2 - 1)} = \frac{-2(x + 6)}{6(x^2 - 1)} = \frac{-(x + 6)}{3(x^2 - 1)}$$

This could also be written as

$$\frac{x - 6}{3(1 - x^2)}$$

Thus applying the minus to the denominator and having it remove the

$$x^2 - 1 \quad \text{to} \quad -(1 - x^2)$$

You have now finished this Segment. Hand in your PUNCH CARD. $\frac{117}{2}$
You should have entered in your NOTEBOOK the following definitions and formulas:

1 To add or subtract fractions with unlike denominators, we first find the lowest common denominator of the fractions. Then, we write each fraction as an equivalent fraction with this denominator. Finally, we apply the same rule used in combining fractions with like denominators.

2 The denominator of every fraction should be factored if possible.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT: Problems 9 , 10 , 11 , and 12 .

1

Very [illegible] made the correct choice.

The le[illegible]st co[illegible]mon denominator of the gi[illegible] fractions is

$$x^2 - 4 = (x - 2)(x + 2)$$

In order [illegible] change each of the fractions to this denominator, we must multiply [illegible] numerators and denominators as follows:

$$\frac{2}{x + 2}\cdot\left[\frac{x - 2}{x - 2}\right] + \frac{3}{x - 2}\cdot\left[\frac{x + 2}{x + 2}\right] - \frac{5x}{x^2 - 4}$$

$$= \frac{2(x - 2) + 3(x + 2) - 5x}{x^2 - 4}$$

$$= \frac{2x - 4 + 3x + 6 - 5x}{x^2 - 4}$$

$$= \frac{2}{x^2 - 4}$$

Please proceed to question 17 below.

118
2

Question 17

Apply the proper principle and combine the fractions

$$\frac{1}{x - 3} - \frac{1}{x^2 - 5x + 6}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{x^2 - 6x + 9}{x^2 - 5x + 6}$ (B) $\frac{1}{x - 2}$

(C) $\frac{1}{x - 3}$ (D) $\frac{x - 2}{x - 3}$

Please check the multiplication of the bin . . s .

$$(x - y)(x -$$

and $$(x + y)(x + y$$

Please return to page $\frac{131}{2}$ and try question 5 again.

Yes, but $\frac{11}{8}$ is not a proper fraction since the numerator is larger than the denominator.

Please return to page $\frac{134}{2}$ and try question 1 again.

$\frac{120}{1}$

Very good. You made the correct choice.

The least common denominator of the given fractions is x^2y^2 since

$$x^2y = x \cdot x \cdot y \quad \text{and} \quad xy^2 = x \cdot y \cdot y$$

To change the first fraction to an equivalent fraction with x^2y^2 as denominator, we multiply numerator and denominator by y. To change the second fraction to an equivalent fraction with x^2y^2 as a denominator, we multiply numerator and denominator by x. Thus, we have

$$\frac{4}{x^2y} \cdot \frac{y}{y} - \frac{(2-x)}{xy^2} \cdot \frac{x}{x} = \frac{4y}{x^2y^2} - \frac{(2-x)}{x^2y^2}$$

$$= \frac{4y - 2x + x^2}{x^2y^2}$$

Note that the minus sign before the second fraction changes each term in the numerator to its opposite.

Please proceed to question 12 below.

$\frac{120}{2}$

Question 12

Apply the proper principle and combine the fractions

$$\frac{3m}{2m+6} - \frac{m-1}{m-3}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{m+3}{2(m-3)}$ (B) $\frac{2m+1}{2m+6}$

(C) $\frac{m-2}{2(m-3)}$ (D) $\frac{m-2}{m-3}$

You took a gue

This choice is far from the correct answer, that selecting it indicates guess work on your part.

Please work out the problem and do not go on the assumption that the simplest looking answer is the right answer.

Please return to page $\frac{116}{2}$ and try question 14 again.

This question is the same as one in addition of two fractions.

We can write

$$x + \frac{6}{x}$$

as

$$\frac{x}{1} + \frac{6}{x}$$

Now, find the least common denominator of the two fractions and add the fractions.

Please return to page $\frac{136}{2}$ and try question 3 again.

$\frac{122}{1}$

Very good. [illegible] correct choice.

The product [illegible] denominators of the first two fractions is

$$(1 - c^2)$$

The denominator of the third fraction is

$$c^2 - 1 = -(1 - c^2)$$

Hence, we can write

$$\frac{c}{1 + c} \cdot \frac{1 - c}{1 - c} + \frac{2c}{1 - c} \cdot \frac{1 + c}{1 + c} - \frac{c^2}{(1 - c)(1 + c)}$$

$$\frac{c - c^2 - 2c + 2c^2 - c^2}{1 - c^2} = \frac{3c}{1 - c^2}$$

Please proceed to question 19 below.

$\frac{122}{2}$

Question 19

Apply the proper principle and combine the fractions,

$$\frac{1}{6x + [illegible]} - \frac{1}{2x - 2} + \frac{[illegible]}{3 - 3x^2}$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{4}{3(+ x}$ [illegible]

(B) $\frac{[illegible]}{3}$

(C) $\frac{-(x + 4)}{3(x^2 - 1)}$

(D) $\frac{-(x + 6)}{3(x^2 - 1)}$

Sorry, but you made the wrong choice. Please note that

$$x^2 - 5x + 6 = (x - 3)(x - 2)$$

When the two fractions are written with this denominator, we have:

$$\frac{1}{x - 3} \cdot \frac{x - 2}{x - 2} = \frac{1}{(x - 3)(x - 2)}$$

Please continue.

Please return to page $\frac{118}{2}$ and try question 17 again.

$\frac{123}{2}$

Yes, but you did not answer the question.

You expressed the given product as a mixed expression and not as a single fraction.

Finish the job by combining the terms of your choice.

Please return to page $\frac{132}{2}$ and try question 0 again.

$\frac{124}{1}$

We do not agree.

One of the letters does have the correct answer next to it.

Read the number $9\frac{3}{8}$ to yourself.

You are saying nine and three eighths.

Now write that in numerals.

Please return to page $\frac{134}{2}$ and try question 1 again.

$\frac{124}{2}$

A complex fraction is a fraction whose numerator or denominator (or both) contains one or more fractions.

Please return to page $\frac{141}{1}$ and try question 8 again.

We do not agree. Let us do a similar question together.

Write as a single fraction,

$$a - \frac{4a}{2a - 3}$$

We can write the above as

$$\frac{a}{1} - \frac{4a}{2a - 3}$$

This now becomes an addition of fractions problem.

$$= \frac{a}{1} \cdot \frac{2a - 3}{2a - 3} - \frac{4a}{2a - 3}$$

Changing the first fraction to an equivalent fraction with this denominator, we get

$$= \frac{2a^2 - 3a}{2a - 3} - \frac{4a}{2a - s}$$

$$= \frac{2a^2 - 3a - 4a}{2a - 3}$$

$$= \frac{2a^2 - 7a}{2a - 3}$$

Please return to page $\frac{155}{2}$ and try question 5 again.

You seem to be creating your own rules. We have to simplify the complex fraction

$$\frac{\frac{5}{9}}{\frac{20}{3}}$$

Note that both the numerator and the denominator of this fraction are fractions. The line separating the numerator and denominator of a fraction means division. Thus, we have $\frac{5}{9}$ divided by $\frac{20}{3}$. This is the same as the product of $\frac{5}{9}$ and the reciprocal of $\frac{20}{3}$, or

$$\frac{5}{9} \cdot \frac{3}{20}$$

Please continue.

Please return to page $\frac{138}{2}$ and try question 11 again.

$\frac{126}{1}$

We do not agree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page $\frac{116}{2}$ and try question 14 again.

$\frac{126}{2}$

Before you can perform the operation of division, the expressions in the parentheses should be written as a single fraction. Thus,

$$2 + \frac{4}{y - 1} = \frac{2}{1} + \frac{4}{y - 1}$$

$$= \frac{2}{1} \cdot \frac{(y - 1)}{(y - 1)} + \frac{4}{y - 1}$$

$$= \frac{2y - 2 + 4}{y - 1}$$

$$= \frac{2y + 2}{y - 1} \qquad [\text{ Factor}$$

$$= \frac{2(y + 1)}{y - 1}$$

Do the same to the expression in the second pair of parentheses, and then perform the operation of division of two fractions.

Please return to page $\frac{147}{2}$ and try question 7 again.

Very good. You made the correct choice.

The sum of x and $\frac{6}{x}$ can be written as

$$\frac{x}{1} + \frac{6}{x} \quad \text{The least common denominator (L.C.D) is } x.$$

$$= \frac{x}{1} \cdot \left(\frac{x}{x}\right) + \frac{6}{x}$$

$$= \frac{x^2}{x} + \frac{6}{x}$$

$$= \frac{x^2 + 6}{x}$$

Please proceed to question 4 which follows.

Question 4

Apply the proper principle and express the fraction

$$\frac{3x^2 - 5x + 2}{x}$$

as a mixed expression. Select the letter which labels the correct answer.

(A) $3x - 5 + \frac{2}{x}$

(B) $\frac{3x^2 - 5x}{x} + \frac{2}{x}$

(C) $3x - \frac{5x + 2}{x}$

(D) $(3x - 2) \cdot \frac{x - 1}{x}$

$\frac{128}{1}$

Very good. You made the correct choice.
We have to combine the fractions

$$\frac{1}{x - 3} - \frac{1}{x^2 - 5x + 6}$$

The denominator of the second fraction is a trinomial, and the first job is to see whether it can be factored. Now, $x^2 - 5x + 6 = (x - 3)(x - 2)$. Hence, the least common denominator of the given fractions is $(x - 3)(x - 2)$. We write:

$$\frac{1}{x - 3} \cdot \frac{x - 2}{x - 2} - \frac{1}{(x - 3)(x - 2)}$$

Combining the fractions, we get:

$$\frac{x - 2 - 1}{(x - 3)(x - 2)}$$

$$= \frac{(x - 3)}{(x - 3)(x - 2)}$$

$$= \frac{1}{x - 2}$$

It is always a good idea to check the answer. Let us do so by letting $x = 4$. Substituting this value of x in the given fractions, we get

$$\frac{1}{(4) - 3} - \frac{1}{(16) - (20) + 6} = \frac{1}{1} - \frac{1}{2} = \frac{1}{2}$$

Substituting $x = 4$ in our answer, we get

$$\frac{1}{4 - 2} = \frac{1}{2}$$

Please proceed to question 18 below.

$\frac{128}{2}$

Question 18

Apply the proper principle and combine the fractions,

$$\frac{c}{1 + c} + \frac{2c}{1 - c} + \frac{c^2}{c^2 - 1}$$

Select the letter which labels the correct result.

(A) $\frac{3c}{1 - c^2}$ (B) $\frac{2c^2 + 3c}{c^2 - 1}$

(C) $\frac{3c - 4c^2}{1 - c^2}$ (D) $\frac{c^2 + 3c}{c^2 - 1}$

Yes, your choice is correct.

The mixed number $9\frac{3}{8}$ is read 9 and $\frac{3}{8}$.

The word and means plus. Hence, the correct answer is

$$9 + \frac{3}{8}$$

Please proceed to question 2 below.

Question 2

Recognize and express the mixed number $7\frac{8}{9}$ as an improper fraction.

Select the letter which labels the correct answer.

(A) $\frac{80}{9}$

(B) $\frac{64}{9}$

(C) $\frac{81}{9}$

(D) $\frac{71}{9}$

$\frac{130}{1}$

After changing each expression in parentheses to a single fraction, you forgot another rule.

You did not convert the division by a fraction to multiplication by its reciprocal.

$$\frac{a}{b} \div \frac{c}{d} =$$

$$\frac{a}{b} \times \frac{d}{c}$$

Please return to page $\frac{147}{2}$ and try question 7 again.

$\frac{130}{2}$

We do not agree.

A complicated looking fraction is not necessarily a complex fraction.

In order for a fraction to be called complex, either the numerator or the denominator or both contains one or more fractions. Thus, for example,

$$\frac{a + \frac{1}{b}}{c}$$

is a complex fraction because the numerator is a mixed expression, and contains one fraction.

Please return to page $\frac{141}{1}$ and try question 8 again.

$\frac{131}{1}$

Very good. You made the correct choice. The lowest common denominator of the fraction is $(2x - 1)(x - 3)$.
Thus, we have:

$$\frac{x-2}{2x-1}\cdot\frac{x-3}{x-3} + \frac{x}{x-3}\cdot\frac{(2x-1)}{(2x-1)} - \frac{1}{2x-1}\cdot\frac{(x-3)}{(x-3)} =$$

$$\frac{x^2-5x+6}{(2x-1)(x-3)} + \frac{2x^2-x}{(2x-1)(x-3)} - \frac{(x-3)}{(2x-1)(x-3)} =$$

$$\frac{3x^2-7x+9}{(2x-1)(x-3)}$$

When an answer to a question turns out to be a fairly complicated expression like the above, it is only natural to suspect that we may have made an error. How can we check our result? Since we claim that

$$\frac{x-2}{2x-1} + \frac{x}{x-3} - \frac{1}{2x-1} \text{ is the same as } \frac{3x^2-7x+9}{(2x-1)(x-3)}$$

the two expressions should yield the same numerical result for a given numerical value of x. Let us take $x = 4$. Substituting 4 for x in <u>each</u> of the expressions, we get

$$\frac{29}{7}$$

<u>Hence, this answer checks.</u>

$$\frac{(4)-2}{2(4)-1} + \frac{(4)}{(4)-3} - \frac{1}{2(4)-1} \overset{?}{=} \frac{3(4)^2-7(4)+9}{[2(4)-1][(4)-3]}$$

$$\frac{2}{7} + \frac{4}{1} - \frac{1}{7} \overset{?}{=} \frac{48-28+9}{7\cdot 1}$$

$$4\frac{1}{7} = \frac{29}{7}$$

Please proceed to question 15 below.

$\frac{131}{2}$

<u>Question 15</u>

Apply the proper principle and combine the fractions,

$$\frac{x-y}{x+y} - \frac{x+y}{x-y}$$

Select the letter which labels the correct answer written in lowest terms.

(A) 0 (B) 1

(C) $\frac{-4xy}{x^2-y^2}$ (D) $\frac{2(x^2+y^2)}{x^2-y^2}$

XI

$\frac{132}{1}$

Very good.

You made the correct choice. We have to combine.

The least common denominator of these two fractions is $y - 2$.

$$y - \frac{2y}{y - 2}$$

$$= \frac{y}{1} - \frac{2y}{y - 2}$$

We have,

$$= \frac{y}{1} \cdot \frac{y - 2}{y - 2} - \frac{2y}{y - 2}$$

$$= \frac{y^2 - 2y}{y - 2} - \frac{2y}{y - 2}$$

$$= \frac{y^2 - 4y}{y - 2} = \frac{y(y - 4)}{(y - 2)}$$

Please [illegible]ed to question 6 which follows.

$\frac{132}{2}$

Question [illegible]

Apply the proper principle and find the product

$$(1 - \frac{1}{x}) \quad (1 + \frac{1}{x})$$

Select the letter which expresses this product as a single fraction.

(A) $1 - \frac{1}{x^2}$ (B) $x^2 - 1$

(C) $\frac{x^2 - 1}{x^2}$ (D) $- \frac{1}{x^2}$

We do not agree.

Start by changing $2\frac{2}{9}$ to an improper fraction; thus,

$$2\frac{2}{9} = \frac{2}{1} + \frac{2}{9}$$

$$= \frac{2}{1} \cdot \frac{9}{9} + \frac{2}{9}$$

$$= \frac{18 + 2}{9}$$

$$= \frac{20}{9}$$

This fraction is the numerator. The denominator is 3 . Thus, you have to divide $\frac{20}{9}$ by 3 . Please continue.

Please return to page $\frac{150}{2}$ and try question 9 again.

$\frac{133}{2}$

Yes, but your answer is not in lowest terms.

Please return to page $\frac{138}{2}$ and try question 11 again.

$\frac{134}{1}$

VOLUME 11 SEGMENT 4 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS:	48	and	50	0	9	(Sequence Number)
	54	and	56	0	4	(Type of Punch Card)
	60	and	62	1	1	(Volume Number)
	66	and	68	0	4	(Segment Number)

Your READING ASSIGNMENT for this segment is page 304. You will now be asked a series of questions to draw your attention to the more important points.

INTRODUCTORY NOTE

In this segment we will learn how to simplify complex fractions. Does the word complex mean the same as the word complicated? Not necessarily. When something is complex, it is composed of many parts; when something is complicated, it is difficult to understand. Simplifying a complex fraction consists of simplifying an ordinary fraction, but repeating the process a few times. You will actually be doing what you did before, but more often. Do not let the word complex scare you. Proceed with this segment with confidence and optimism.

$\frac{134}{2}$

Question 1

Recognize and express the mixed number $9\frac{3}{8}$ as the sum of a natural number and a proper fraction. Select the letter next to the correct answer.

(A) $\frac{75}{8}$

(B) $9 + \frac{3}{8}$

(C) $8 + \frac{11}{8}$

(D) None of these.

We do not agree. One of the letters does have the correct answer next to it.

There are two parts to this problem:

(1) Convert each expression in parentheses into a single fraction.

(2) Convert division by a fraction to multiplication by the reciprocal of that fraction.

Please return to page $\frac{147}{2}$ and try question 7 again.

We do not agree.

The numerator of the given complex fraction is a mixed expression.

How can we write this expression as a single fraction?

Note that $2 + \frac{6}{x}$

is the same as $\frac{2}{1} + \frac{6}{x}$

and the least common denominator of these fractions is x.

Please return to page $\frac{151}{2}$ and try question 12 again.

$\frac{136}{1}$

Yes, your choice is correct.

One basic method is to convert the whole number into a fraction having the same denominator as the fractional part of the mixed number:

$$7 + \frac{8}{9} = \frac{63}{9} + \frac{8}{9}$$

$$= \frac{71}{9}$$

A shorter algorithm accomplishes the same result, as follows:

To express a mixed number as an improper fraction, we multiply the denominator of the fraction by the whole number and add the numerator. Thus,

$$7\ \frac{8}{9} = \frac{9 \cdot 7 + 8}{9}$$

$$= \frac{63 + 8}{9}$$

$$= \frac{71}{9}$$

Please proceed to question 3 which follows.

$\frac{136}{2}$

Question 3

Apply the proper principle and write the mixed number

$$x + \frac{6}{x}$$

as a single fraction. Select the letter which labels the correct answer.

(A) $\frac{x + 6}{x}$

(B) $\frac{7}{x}$

(C) $\frac{x^2 + 6}{x}$

(D) None of these.

You forgot something. If we write the expression

$$1 - \frac{1}{x}$$

as a single fraction, we get

$$\frac{x}{x} - \frac{1}{x} = \frac{x - 1}{x}$$

Similarly, if we combine

$$1 + \frac{1}{x}$$

we get

$$\frac{x + 1}{x}$$

Now, do you see what you forgot?

Please return to page $\frac{132}{2}$ and try question 6 again.

$\frac{137}{2}$

We do not agree. The expression that you chose is called a mixed expression.

$$a + \frac{b}{c}$$

is a mixed expression.

$$\frac{a + \frac{b}{c}}{d}$$

is a complex fraction.

Please return to page $\frac{141}{1}$ and try question 8 again.

$\frac{138}{1}$

Very good. You made the correct choice.

We simplify the fraction,

$$\frac{14}{\frac{28}{5}}$$

How do we express this question in words? The number 14 is divided by $\frac{28}{5}$. But, this is equivalent to the number 14 multiplied by $\frac{5}{28}$. Hence, we have to find the product,

$$14 \cdot \frac{5}{28} = \frac{14 \cdot 5}{28}$$

$$= \frac{5}{2}$$

Please proceed to question 11 which follows.

$\frac{138}{2}$

Question 11

Apply your knowledge and simplify the fraction,

$$\frac{\frac{5}{9}}{\frac{20}{3}}$$

Select the letter next to the correct answer.

(A) $\frac{100}{27}$　　(B) $\frac{15}{180}$

(C) $\frac{1}{12}$　　(D) $\frac{3}{4}$

A mixed expression means a number and a fraction. Thus, in arithmetic,

$$3 + \frac{2}{7}$$

is a mixed expression. Similarly, in algebra,

$$a - \frac{2}{b}$$

is an example of a mixed expression.

Please return to page $\frac{127}{2}$ and try question 4 again.

We do not agree. Try multiplying both the numerator

$$(x + 3)$$

by x and the denominator

$$(1 + \frac{3}{x})$$

by x .

Please return to page $\frac{157}{2}$ and try question 13 again.

Very good. You made the correct choice.

The first job is to change each mixed expression to a single fraction. Thus,

$$\left[2 + \frac{4}{y-1}\right] \div \left[\frac{1}{y+2} + y\right]$$

$$\left[\frac{2}{1} + \frac{4}{y-1}\right] \div \left[\frac{1}{y+2} + \frac{y}{1}\right]$$ [Convert to like fractions

$$\left[\frac{2}{1}\cdot\frac{y-1}{y-1} + \frac{4}{y-1}\right] \div \left[\frac{1}{y+2} + \frac{y}{1}\cdot\frac{y+2}{y+2}\right]$$ [D

$$\left[\frac{2y-2+4}{y-1}\right] \div \left[\frac{1+y^2+2y}{y+2}\right]$$ [Combine

$$\left[\frac{2y+2}{y-1}\right] \div \left[\frac{y^2+2y+1}{y+2}\right]$$ [Factor and convert to multiplication.

$$\left[\frac{2(y+1)}{y-1}\right] \cdot \left[\frac{y+2}{(y+1)(y+1)}\right]$$ [Reduce

$$\frac{2(y+2)}{y^2-1}$$

Please proceed to question 8 on page $\frac{141}{1}$.

Question 8

Recognize and select the letter next to the expression which is called a complex fraction.

(A) $\dfrac{x^5 - 1}{x^7 - 1}$

(B) $\dfrac{1 - \frac{1}{x}}{x}$

(C) $\dfrac{y}{y^3 - y^2 + y + 1}$

(D) $1 + \dfrac{abc}{xyz}$

We do not agree. Let us do a similar question together. Simplify

$$\frac{2 - \frac{1}{p}}{4 - \frac{1}{p^2}}$$

Examine the numerator and the denominator of this complex fraction. By what number can we multiply both numerator and denominator to no longer have the quotient of two mixed expressions? The answer is $\frac{p^2}{1}$

Thus,

$$\frac{\left[2 - \frac{1}{p}\right]}{\left[4 - \frac{1}{p^2}\right]} \cdot \frac{\frac{p^2}{1}}{\frac{p^2}{1}} = \frac{2p^2 - p}{4p^2 - 1} \qquad [\ \text{Factor}$$

$$= \frac{p(2p - 1)}{(2p + 1)(2p - 1)} \qquad [\ \text{Reduce}$$

$$= \frac{p}{2p + 1}$$

Please return to page $\frac{162}{2}$ and try question 14 again.

$\frac{142}{1}$

Are you still cancelling individual terms in the numerator with individual terms in the denominator? Keep in mind: a fraction can be reduced by dividing the numerator and denominator by identical factors and in no other way.

Please return to page $\frac{132}{2}$ and try question 6 again.

$\frac{142}{2}$

Let us first consider the numerator of this complex fraction. How can we write it as a single fraction? The least common denominator of

$$\frac{1}{a} + \frac{1}{b}$$

is ab . Hence, we have

$$\frac{1}{a} \cdot \left(\frac{b}{b}\right) + \frac{1}{b} \cdot \left(\frac{a}{a}\right)$$

$$= \frac{b}{ab} + \frac{a}{ab}$$

$$= \frac{b + a}{ab}$$

Use the same method to combine the denominator of the given complex fraction and proceed by dividing the numerator by the denominator.

Please return to page $\frac{164}{2}$ and try question 15 again.

Very good. You made the correct choice.

To simplify a complex fraction, we first change the numerator of this fraction to a single fraction. Thus,

$$2\,\frac{2}{9} = \frac{20}{9}$$

We have to find the quotient,

$$\frac{\frac{20}{9}}{3}$$

Now, dividing by 3 is the same as multiplying by $\frac{1}{3}$.

Hence, we get

$$\frac{20}{9} \cdot \frac{1}{3} = \frac{20}{27}$$

Please proceed to question 10 which follows.

Question 10

Apply your knowledge and write the complex fraction

$$\frac{14}{\frac{28}{5}}$$

in simplest form. Select the letter next to the correct answer.

(A) $\frac{70}{28}$ (B) $\frac{5}{14}$

(C) $\frac{14}{5}$ (D) $\frac{5}{2}$

$\frac{144}{1}$

Your choice is incorrect.

The minus sign before a fraction changes every term in the numerator to its opposite. Thus,

$$-\frac{5x + 2}{x} = \frac{-5x - 2}{x}$$

The Distributive Law can apply here, too.

$$-\frac{5x + 2}{x} = \frac{-1(5x \div 2)}{x}$$

This part of your answer does not agree with the given problem.

Please return to page $\frac{127}{2}$ and try question 4 again.

$\frac{144}{2}$

The quotient of two quantities is equal to 1 if, and only if, the numerator is equal to the denominator, except if the number is zero. Since this is not the case here, your choice is incorrect.

Please return to page $\frac{157}{2}$ and try question 13 again.

We do not agree. One way of doing this problem is to multiply the numerator and denominator of the given fraction by 9 .

$$\frac{\frac{5}{9}}{\frac{20}{3}} \cdot \frac{\frac{9}{1}}{\frac{9}{1}}$$

What do you get after multiplying $\frac{5}{9}$ by 9 ?

What do you get after multiplying $\frac{20}{3}$ by 9 ?

Please continue.

Please return to page $\frac{138}{2}$ and try question 11 again.

We do not agree. Start by combining the numerator of the given fraction as follows:

$$\frac{p^2 + q^2}{2pq} + 1 = \frac{p^2 + q^2}{2pq} + \frac{2pq}{2pq} \circ (1)$$

$$= \frac{p^2 + q^2 + 2pq}{2pq} \quad [\ \text{COMMUTATIVE}$$

$$= \frac{p^2 + 2pq + q^2}{2pq}$$

You now multiply this last fraction by the reciprocal of

$$\frac{p^2 - q^2}{2pq}$$

Please continue.

Please return to page $\frac{152}{2}$ and try question 17 again.

$\frac{146}{1}$

We do not agree. Start by combining the numerator

$$\frac{x^2}{y^2} - 1$$

as a single fraction. Thus,

$$\frac{x^2}{y^2} - 1 = \frac{x^2 - y^2}{y^2}$$

Do the same to the denominator of the given fraction and continue.

Please return to page $\frac{154}{2}$ and try question 15 again.

$\frac{146}{2}$

We do not agree. Start by combining the denominator,

$$2 + \frac{3}{4 + d}$$

into a single fraction as follows:

$$= 2 \cdot \frac{(4 + d)}{(4 + d)} + \frac{3}{4 + d}$$

$$= \frac{8 + 2d}{4 + d} + \frac{3}{4 + d}$$

$$= \frac{11 + 2d}{4 + d}$$

Please continue.

Please return to page $\frac{156}{2}$ and try question 19 again.

Very good. You made the correct choice. We note that the product

$$\left(1 - \frac{1}{x}\right)\left(1 + \frac{1}{x}\right)$$

is of the form $(a - b)(a + b)$

Hence, the product is equal to the square of 1 minus the square of $\frac{1}{x}$;

that is, $\left(1 - \frac{1}{x^2}\right)$

Now, $= \quad 1 - \frac{1}{x^2} = \frac{x^2}{x^2} - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$

An alternate method is the conversion of the binomials to single fractions:

$$\left(1 - \frac{1}{x}\right)\left(1 + \frac{1}{x}\right) = \left(\frac{x - 1}{x}\right)\left(\frac{x + 1}{x}\right)$$

$$= \frac{x^2 - 1}{x^2}$$

Please proceed to question 7 that follows.

Question 7

Apply your knowledge and find the quotient of

$$\left[2 + \frac{4}{y - 1}\right] \div \left[\frac{1}{y + 2} + y\right]$$

Select the letter next to the correct answer.

(A) $\frac{2(y + 2)}{y^2 - 1}$

(B) $\frac{2(y - 1)}{y + 2}$

(C) $\frac{(y + 1)^2}{2(y + 2)}$

(D) None of these.

$\frac{148}{1}$

Yes, but your answer is not in simplest form. Write the numerator and the denominator as a product of primes and divide numerator and denominator by identical factors.

Please return to page $\frac{143}{2}$ and try question 10 again.

$\frac{148}{2}$

Start by multiplying numerator and denominator by y^2.

This will eliminate denominators. Thus,

$$\frac{y^2\left[\frac{x^2}{y^2} - 1\right]}{y^2\left[\frac{x^2}{y^2} - \frac{2x}{y} + 1\right]} = \frac{x^2 - y^2}{x^2 - 2xy + y^2}$$

The next step is to factor this numerator and denominator.

Please return to page $\frac{160}{2}$ and try question 18 again.

You changed the given expression into a product and incorrectly at that. Please note that a mixed expression consists of a whole number and a fraction.

Please return to page $\frac{127}{2}$ and try question 4 again.

Did you check your answer?

If the given complex fraction

$$\frac{x + 3}{1 + \frac{3}{x}}$$

equals $(x + 3)^2$ as you claim, then we should get the same numerical answer if we let $x = 3$ (for example) in the given fraction and in your choice. We have

$$\frac{3 + 3}{1 + \frac{3}{3}} = \frac{6}{2} = 3$$

and

$$(3 + 3)^2 = 6^2 = 36$$

Do you see now why it is important to check your answer?

Please return to page $\frac{157}{2}$ and try question 13 again.

$\frac{150}{1}$

Very good. You made the correct choice. A complex fraction is a fraction whose numerator or denominator or both contain one or more fractions. In the given expression, the numerator

$$1 - \frac{1}{x}$$

contains the fraction $\frac{1}{x}$

Hence,

$$\frac{1 - \frac{1}{x}}{x}$$

is a complex fraction.

Please proceed to question 9 which follows.

$\frac{150}{2}$

Question 9

Apply your knowledge and write the complex fraction,

$$\frac{2\frac{2}{9}}{3}$$

as a single common fraction in lowest terms. Select the letter next to the correct answer.

(A) $\frac{20}{27}$ (B) $\frac{20}{3}$

(C) $\frac{59}{27}$ (D) None of these.

Very good. You made the correct choice. We have to simplify the fraction,

$$\frac{\frac{5}{9}}{\frac{20}{3}}$$

The line between the two fractions means division; that is, we have the fraction $\frac{5}{9}$ divided by the fraction $\frac{20}{3}$. This is the same as the product,

$$\frac{5}{9} \cdot \frac{3}{20} = \frac{5 \cdot 3}{\text{[illegible]}}$$

$$= \text{[illegible]}$$

Dividing numerator and denominator by 15 we get $\frac{1}{12}$.

Please proceed to question 12 which follows.

Question 12

Apply your knowledge and simplify the fraction

$$\frac{2 + \frac{6}{x}}{2y}$$

Select the letter next to the correct answer.

(A) $\frac{x + 3}{xy}$ (B) $\frac{xy + 3}{y}$

(C) $\frac{8}{2xy}$ (D) $\frac{y + 3}{xy}$

Good. Which technique did you use?

(1) Combine fractions separately first and then simplify.

$$\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a^2}-\frac{1}{b^2}} = \frac{\frac{b+a}{ab}}{\frac{b^2-a^2}{a^2b^2}}$$ [Convert to a multiplication problem

$$= \frac{b+a}{ab} \cdot \frac{a^2b^2}{b^2-a^2}$$ [Factor

$$= \frac{(b+a)}{ab} \cdot \frac{ab \cdot ab}{(b+a)(b-a)}$$ [Reduce

$$= \frac{ab}{b-a}$$

(2) Or multiply by a^2b^2

$$\frac{\left[\frac{1}{a}+\frac{1}{b}\right]\cdot\frac{a^2b^2}{1}}{\left[\frac{1}{a^2}-\frac{1}{b^2}\right]\frac{a^2b^2}{1}} = \frac{ab^2+a^2b}{b^2-a^2}$$ [Factor

$$= \frac{ab(b+a)}{(b-a)(b+a)}$$ [Reduce

$$= \frac{ab}{(b-a)}$$

Both methods are proper.

Now proceed to question 17 below.

Question 17

Apply your knowledge and simplify the fraction,

$$\frac{\frac{p^2+q^2}{2pq}}{\frac{p^2-q^2}{2pq}}$$

Select the letter next to the correct statement.

(A) $\frac{p^2-q^2}{p^2-q^2}$ (B) 2

(C) $\frac{p+q}{p-q}$ (D) $\frac{2(p+q)}{p^2+q^2}$

Your choice is not correct. Let us do a similar question together.

Simplify the complex fraction

$$\frac{18}{\frac{12}{7}}$$

Now, the value of the fraction remains unaltered if we multiply both numerator and denominator by the same number.

If we multiply

$$\frac{18}{\frac{12}{7}} \cdot \frac{7}{7} = \frac{18(7)}{12} \qquad [\ \text{Factor}$$

$$= \frac{6 \cdot 3 \cdot 7}{6 \cdot 2} \qquad \div \frac{6}{6}$$

$$= \frac{21}{2}$$

Please return to page $\frac{143}{2}$ and try question 10 again.

Equations are said to be equivalent if they have the same solution set.

To find the solution set of the equation,

$$3x - 5 = 7 \qquad + 5$$

we add 5 to both members and obtain

$$3x = 12 \qquad \div 3$$

Next, we divide both members of the equation by 3 and get

$$x = 4$$

Now, is 4 the solution set of the second given equation,

$$6x - 10 = 17 \ ?$$

Please return to page $\frac{168}{2}$ and try question 1 again.

$\frac{154}{1}$

Very good. You made the correct choice.

We have to simplify the complex fraction,

$$\frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}} =$$

To do this, multiply the numerator and denominator by x^2.

We choose a value that will absorb the separate denominators.

$$= \frac{(1 - \frac{1}{x})}{(1 - \frac{1}{x^2})} \cdot \frac{x^2}{x^2}$$

$$= \frac{x^2 - x}{x^2 - 1}$$

$$= \frac{x(x - 1)}{(x - 1)(x + 1)}$$

$$\frac{x}{x + 1}$$

Please proceed to question 15 which follows.

$\frac{154}{2}$

Question 15

Apply your knowledge and simplify the fraction,

$$\frac{\frac{x^2}{y^2} - 1}{\frac{x}{y} - 1}$$

Select the letter next to the correct answer.

(A) $x - 1$ (B) $\frac{1}{x - 1}$

(C) $\frac{x + y}{y}$ (D) $\frac{y}{x - y}$

Very good. You made the correct choice.

A mixed expression consists of a number plus a fraction. For an algebraic expression to be called mixed, part of it is a fraction and part is not fractional. The given expression,

$$\frac{3x^2 - 5x + 2}{x} = \frac{3x^2}{x} - \frac{5x}{x} + \frac{2}{x}$$

$$= (3x - 5) + \frac{2}{x}$$

Please proceed to question 5 which follows.

Question 5

Apply the proper principle and write the expression

$$y - \frac{2y}{y - 2}$$

as a single fraction. Select the letter which labels the correct answer.

(A) $\frac{y^2}{y - 2}$

(B) $\frac{y^2 - 4y}{y - 2}$

(C) $y(y + 2)$

(D) None of these.

$\frac{156}{1}$

Very good. You made the correct choice.

This question can be done in more than one way. Let us work out two different ways. We can multiply the numerator and denominator of this fraction by y^2 to eliminate denominators.

Thus,

$$\frac{y^2\left[\frac{x^2}{y^2} - 1\right]}{y^2\left[\frac{x^2}{y^2} - \frac{2x}{y} + 1\right]} = \frac{x^2 - y^2}{x^2 - 2xy + y^2}$$

$$= \frac{(x - y)(x + y)}{(x - y)(x - y)}$$

Dividing the numerator and denominator by $(x - y)$, we obtain

$$\frac{x + y}{x - y}$$

Another way to do this question is to note that the numerator and denominator of the given complex fraction can be factored as follows:

$$\frac{(\frac{x}{y} - 1)(\frac{x}{y} + 1)}{(\frac{x}{y} - 1)(\frac{x}{y} - 1)} = \frac{\frac{x}{y} + 1}{\frac{x}{y} - 1}$$

Multiplying numerator and denominator of the last quotient by y, we get

$$\frac{x + y}{x - y}.$$

Please proceed to question 19 which follows.

$\frac{156}{2}$

Question 19

Relate to the proper principle and find the value of d in the equation below.

$$\frac{1}{2 + \frac{3}{4 + d}} = \frac{11}{25}$$

Select the letter next to the correct answer.

(A) $d = 5$ (B) $d = \frac{11}{5}$

(C) $d = 7$ (D) $d = \frac{7}{11}$

Very good. You made the correct choice.

There are two good ways of simplifying a complex fraction.

1st Method: Multiply the numerator and denominator by the least common denominator of all the fract . Thus, the least common denomi or of

$$\frac{2 + \frac{6}{x}}{\frac{2y}{1}}$$

is (x) multiplying the numerator and denominator by x , we get

$$\frac{\frac{x}{1}\left[2 + \frac{6}{x}\right]}{\frac{x}{1}\left[\frac{2y}{1}\right]} = \frac{2x + 6}{2xy}$$

$$= \frac{2(x + 3)}{2xy}$$

$$= \frac{(x + 3)}{xy}$$

2nd Methoa: Express the fraction as the numerator divided by the denominator using the $\div$ sign.

$$\frac{2 + \frac{6}{x}}{2y} = \frac{\frac{2x + 6}{x}}{2y}$$

$$= \frac{2(x + 3)}{x} \div 2y$$

$$= \frac{2(x + 3)}{x} \cdot \frac{1}{2y}$$

$$= \frac{2(x + 3)}{2xy}$$

$$= \frac{(x + 3)}{xy}$$

Please proceed to question 13 which follows.

Question 13

Apply your knowledge and simplify the fraction,

$$\frac{x + 3}{1 + \frac{3}{x}}$$

Select the letter next to the correct answer.

(A) $\frac{1}{x}$

(B) x

(C) 1

(D) $(x + 3)^2$

$\frac{158}{1}$

We do not agree.

In words, the question states that 14 is divided by $\frac{28}{5}$.

This is equivalent to saying that 14 is multiplied by the reciprocal of

$\frac{28}{5}$.

Please continue.

Please return to page $\frac{143}{2}$ and try question 10 again.

$\frac{158}{2}$

We do not agree.

The solution set of the equation

$$7y - 3 = 4$$

is $\{1\}$

while the solution set of $2y = 1$

is $\left\{\frac{1}{2}\right\}$

Since the two equations have different solution sets, they are not equivalent.

Please return to page $\frac{168}{2}$ and try question 1 again.

Ask yourself the following question:

By what number should I multiply the numerator and denominator of the complex fraction so that denominators are cleared?

Yes, y^2 is that number.

Please return to page $\frac{154}{2}$ and try question 15 again.

We do not agree.

The least common denominator is the lowest common multiple (L C M) of the denominators and is divisible by each of the denominators evenly.

Note that $\frac{15}{2}$ is not an integer and $\frac{15}{6}$ is not an integer, yet denominators should divide into the (L C M) evenly; that is, with no remainder.

Please return to page $\frac{184}{2}$ and try question 3 again.

$\frac{160}{1}$

Very good. You made the correct choice.
We have to simplify the fraction,

$$\frac{\frac{p^2 + q^2}{2pq} + 1}{\frac{p^2 - q^2}{2pq}}$$

One method is to handle the numerator and denominator separately. Combining the numerator of this fraction, we get

$$\frac{p^2 + q^2}{2pq} + 1 = \frac{p^2 + q^2 + 2pq}{2pq} \quad [\quad C \quad \text{and factor}$$

$$= \frac{(p + q)(p + q)}{2pq}$$

The denominator of the given fraction can be written as

$$\frac{p^2 - q^2}{2pq} = \frac{(p - q)(p + q)}{2pq}$$

Multiplying the numerator by the reciprocal of the denominator, we obtain

$$\frac{(p + q)(p + q)}{2pq} \cdot \frac{2pq}{(p - q)(p + q)}$$

We now divide numerator and denominator by the identical factors 2pq and (p + q). We get

$$\frac{p + q}{p - q}$$

Please proceed to question 18 which follows.

$\frac{160}{2}$

Question 18

Apply your knowledge and simplify the fraction,

$$\frac{\frac{x^2}{y^2} - 1}{\frac{x^2}{y^2} - \frac{2x}{y} + 1}$$

Select the letter next to the correct answer.

(A) $\frac{x^2 - y^2}{(x - y)^2}$ (B) $\frac{x - y}{x + y}$

(C) $2x^4$ (D) $\frac{x + y}{x - y}$

How can we discover whether two equations are equivalent without actually finding the solution set of each equation?

One way is to observe that if each term of an equation is multiplied by the same number, an equivalent equation is obtained.

Thus, for example, the equation,

$$\frac{x}{2} + \frac{x}{3} = 5$$

is equivalent to the equation,

$$3x + 2x = 30$$

since the second equation was obtained from the first by multiplying each term by 6 .

Do you now see why your choice is incorrect?

Please return to page $\frac{173}{2}$ and try question 2 again.

We do not agree.

Let us do a similar question together. Solve the open sentence,

$$\frac{2}{5}z - \frac{1}{3}z = \frac{11}{15}$$

The lowest common denominator of the fractional coefficients

$$\frac{2}{5} , \frac{1}{3} , \frac{11}{15} \text{ is } 15$$

We, therefore, multiply each term of the open sentence by 15 . We have,

$$15 \left(\frac{2}{5}z - \frac{1}{3}z \right) = 15 \left(\frac{11}{15} \right)$$

Using the distributive property, we get

$$\begin{aligned} 15 \left(\frac{2}{5}z \right) - 15 \left(\frac{1}{3}z \right) &= 15 \left(\frac{11}{15} \right) \\ 6z - 5z &= 11 \\ z &= 11 \end{aligned}$$

You should now check this answer by substituting 11 in place of z in the given equation.

Please return to page $\frac{180}{2}$ and try question 5 again.

$\frac{162}{1}$

Very good. You made the correct choice.

We have to simplify the complex fraction,

$$\frac{x + 3}{1 + \frac{3}{x}}$$

The denominator of this fraction is more complicated than the numerator. We can simplify this denominator by multiplying it by x. Thus,

$$x(1 + \frac{3}{x}) = x + 3$$

In order not to change the value of the given fraction, we must also multiply the numerator by x. We get,

$$\frac{x(x + 3)}{x(1 + \frac{3}{x})} = \frac{x(x + 3)}{(x + 3)} = x$$

The final result was obtained by dividing numerator and denominator by the identical factor $(x + 3)$.

Please proceed to question 14 which follows.

$\frac{162}{2}$

Question 14

Apply your knowledge and simplify the fraction,

$$\frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}}$$

Select the letter next to the correct answer.

(A) $\frac{x}{x + 1}$ (B) $\frac{x - 1}{x}$

(C) $\frac{x - 1}{x + 1}$ (D) None of these.

What is the solution set of the first equation,

$$4r + 8 = r$$

What is the solution set of the second equation,

$$3r = 8$$

The above equations have different solution sets and are, therefore, not equivalent.

Please return to page $\frac{168}{2}$ and try question 1 again.

Examine the denominators of each of the terms in the open sentence. Ask yourself this question:

By what number must I multiply each of the terms in order to obtain an equivalent equation with integral coefficients?

That is another way of asking,

"what is the L C M of 5, 10, and 3 ?"

Please return to page $\frac{175}{2}$ and try question 4 again.

$\frac{164}{1}$

Very good. You made the correct choice.

We have to simplify the fraction,

$$\frac{\frac{x^2}{y^2} - 1}{\frac{x}{y} - 1}$$

Here is a different technique that can be used in this special case. Note that the numerator of this fraction is the difference of two squares,

$$\frac{(\frac{x}{y} - 1)(\frac{x}{y} + 1)}{(\frac{x}{y} - 1)}$$

Dividing numerator and denominator by the identical factor, we obtain:

$$\frac{x}{y} + 1$$

Combining this expression, we get:

$$\frac{x + y}{y}$$

The usual method will produce the same answer. Multiply numerator and denominator by y^2 (the L C M of the denominators).

$$\frac{\left[\frac{x^2}{y^2} - 1\right] \cdot \frac{y^2}{1}}{\left[\frac{x}{y} - 1\right] \cdot \frac{y^2}{1}} = \frac{x^2 - y^2}{xy - y^2} \quad \text{[Factor}$$

$$= \frac{(x + y)(x - y)}{y(x - y)} \quad \text{[Reduce}$$

$$= \frac{x + y}{y}$$

Please proceed to question 16 which follows.

$\frac{164}{2}$

Question 16

Apply your knowledge and simplify the fraction,

$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{b^2}}$$

Select the letter next to the correct answer.

(A) $\frac{1}{b - a}$ (B) $\frac{b + a}{b - a}$

(C) $ab(b + a)$ (D) $\frac{ab}{b - a}$

Your choice is incorrect.

The Least Common Denominator is the Lowest Common Multiple, (L C M), of the denominators and is divisible by each of the denominators. Note that $\frac{12}{5}$ is not an integer. Recall that the least common denominator is the smallest number that is divisible by each denominator in the set of denominators. That means that the denominator should divide into the L C M evenly; that is, with no remainder.

Please return to page $\frac{184}{2}$ and try question 3 again.

Did you check your answer? You forgot. Let us do so now.

We are given the open sentnece,

$$\frac{3p - 5}{2} - \frac{p}{3} = 8$$

Replacing p by your choice $\frac{2}{3}$ we get,

$$\frac{3\left(\frac{2}{3}\right) - 5}{2} - \frac{\frac{2}{3}}{3} \stackrel{?}{=} 8$$

Simplifying, we obtain,

$$\frac{2 - 5}{2} - \frac{2}{9} \stackrel{?}{=} 8$$

$$-\frac{3}{2} - \frac{2}{9} \stackrel{?}{=} 8$$

We need go no further. A negative quantity cannot equal 8 . Your choice is not correct.

Please return to page $\frac{171}{2}$ and try question 6 again.

Very good. You made the correct choice.

We start by combining the expression,

$$2 + \frac{3}{4 + d}$$

into a single fraction.

Now, $$2 = \frac{2(4 + d)}{(4 + d)}$$

and we have $$\frac{2(4 + d)}{(4 + d)} + \frac{3}{4 + d} = \frac{11 + 2d}{4 + d}$$

The reciprocal of the fraction is

$$\frac{4 + d}{11 + 2d}$$

We, therefore, have to solve the equation,

$$\frac{4 + d}{11 + 2d} = \frac{11}{25}$$

for the letter d.

Multiplying both members of the equation by $25(11 + 2d)$ we get

$$\begin{aligned} 25(4 + d) &= 11(11 + 2d) && [D \\ 100 + 25d &= 121 + 22d && - 22d \\ 100 + 3d &= 121 && - 100 \\ 3d &= 21 && \div 3 \\ d &= 7 \end{aligned}$$

You have now finished this Segment. Hand in your PUNCH CARD.

You should be able to complete the following problems from your HOMEWORK ASSIGNMENT:

Problems 13, 14, 15, and 16.

You have forgotten how to add fractions.
The sum of

$$\frac{r}{2} + \frac{5r}{7} \text{ is not } \frac{6r}{9}$$

To refresh your memory: when we add fractions with unlike denominators, we must first find the lowest common denominator of the fractions. Since 2 and 7 are relatively prime numbers, the lowest common denominator is

$$2 \times 7 = 14$$

Now, each fraction is written as an equivalent fraction with 14 as a denominator.
Thus,

$$\frac{r}{2} = \frac{7r}{14} \quad \text{and} \quad \frac{5r}{7} = \frac{10r}{14}$$

Hence,

$$\frac{r}{2} + \frac{5r}{7} = \frac{7r}{14} + \frac{10r}{14}$$

$$= \frac{17r}{14}$$

Please return to page $\frac{173}{2}$ and try question 2 again.

....

You multiplied the left member of the equation by 15 , but you did not multiply the right member by this number.
Please keep in mind that both members must be multiplied by the same number in order to obtain an equivalent equation, and maintain the equality of both members of the equation.

Please return to page $\frac{180}{2}$ and try question 5 again.

$\frac{168}{1}$

Volume 11 Segment 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch the following:

COLUMNS	48	and	50	1 0	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	1 1	(Volume Number)
	66	and	68	0 5	(Segment Number)

Your READING ASSIGNMENT for this Segment is page 306
You will now be asked a series of questions to draw your attention to the more important points.

Introductory Note

In previous segments we concentrated on the solution of equations in which the coefficients of the variables were integers. We will now proceed and consider methods of solving equations in which one or more of the terms includes a fraction. Why is it important for us to learn how to solve equations with fractional coefficients? The answer is simple: many problems which we will study lead to such equations. For example: How old is Mr. Jones if

$\frac{1}{3}$ of his age 6 years ago is equal to $\frac{1}{5}$ of his age 10 years hence.

If we let x = Mr. Jones' present age the equation that will solve this problem is,

$$\frac{1}{3}(x - 6) = \frac{1}{5}(x + 10) \qquad (I)$$

$$\frac{x}{3} - 2 = \frac{x}{5} + 2$$

Thus, in order to solve this problem it is necessary to know how to solve equations with fractional coefficients.

Please proceed to question 1 below.

$\frac{168}{2}$

Question 1

Recognize and select the letter next to which both equations are equivalent.

(A) $3x - 5 = 7$; $6x - 10 = 17$

(B) $2x + 3 = x + 8$; $3x - 10 = 5$

(C) $7y - 3 = 4$; $2y = 1$

(D) $4r + 8 = r$; $3r = 8$

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Multiply both sides of the equation by the (LCM) of the denominators and then solve the derived equivalent equation using the operations indicated.

Please return to page $\frac{175}{2}$ and try question 4 again.

....

We do not agree. Did you forget to multiply the right member of the equation by the same number that you multiplied the left member?

Please go over your work.

Please return to page $\frac{171}{2}$ and try question 6 again.

$\frac{170}{1}$

The Least Common Denominator is the Least Common Multiple (LCM) of the denominators and is divisible by each of the denominators. That means that each denominator should divide into the (LCM) evenly without any remainder.

The quotient $\frac{10}{6}$ is not an integer.

Your choice is, therefore, incorrect.

Please return to page $\frac{184}{2}$ and try question 3 again.

. . . .

$\frac{170}{2}$

Do you know what you forgot to do?

You forgot to multiply the right side of the open sentence by the same number that you multiplied the left side.

Please return to page $\frac{179}{2}$ and try question 9 again.

Very good. Your choice is correct.

If we examine the denominators 3 , 5 , 15 ; we note that the lowest common denominator is 15 . Multiplying each term of the open sentence by 15 will result in an equivalent open sentence with integral coefficients.

Thus,

$$15\left(\frac{2}{3}y - \frac{3}{5}y\right) = 15\left(\frac{7}{15}\right) \qquad [\ \text{D}$$

$$15\left(\frac{2}{3}y\right) - 15\left(\frac{3}{5}y\right) = 15\left(\frac{7}{15}\right)$$

$$10y - 9y = 7 \qquad [\ \text{Combine}$$

$$y = 7$$

CHECK:

$$\frac{2}{3}y - \frac{3}{5}y = \frac{7}{15}$$

$$\frac{2}{3}(7) - \frac{3}{5}(7) \stackrel{?}{=} \frac{7}{15}$$

$$\frac{14}{3} - \frac{21}{5} \stackrel{?}{=} \frac{7}{15}$$

$$\frac{70}{15} - \frac{63}{15} \stackrel{?}{=} \frac{7}{15}$$

$$\frac{7}{15} = \frac{7}{15}$$

Please proceed to question 6 which follows.

Question 6

Apply your knowledge to solve the open sentence,

$$\frac{3p - 5}{2} - \frac{p}{3} = 8$$

Select the letter next to the correct answer.

(A) $p = \frac{2}{3}$ (C) $p = \frac{23}{7}$

(B) $p = \frac{23}{16}$ (D) $p = 9$

$\frac{172}{1}$

Again you failed to check your answer.

If you took the trouble to do so, you would avoid the unpleasant feeling of being told that you are wrong.

Please return to page $\frac{188}{2}$ and try question 8 again.

....

$\frac{172}{2}$

Did you check your answer?

You didn't.

Everybody can make a mistake, but the student who checks his answer finds his mistake before his teacher finds it for him.

Please return to page $\frac{181}{2}$ and try question 11 again.

You made the correct choice.

The equations are said to be equivalent if they have the same solution set.

Now, we have

$$\begin{aligned} 2x + 3 &= x + 8 && \propto - x \\ x + 3 &= 8 && \propto - 3 \\ x &= 5 \end{aligned}$$

For the second equation, we have

$$\begin{aligned} 3x - 10 &= 5 && \propto + 10 \\ 3x &= 15 && \propto \div 3 \\ x &= 5 \end{aligned}$$

Both equations have the solution set $\{5\}$; and are, therefore, equivalent.

Please proceed to question 2 which follows.

$\frac{173}{2}$

Question 2

Recognize and select the letter next to which both equations are equivalent.

(A) $\frac{2y}{3} + \frac{y}{5} = 1$ and $10y + 3y = 5$

(B) $\frac{r}{2} + \frac{5r}{7} = 0$ and $\frac{6r}{9} = 10$

(C) $\frac{n}{3} + \frac{n}{4} = 2$ and $4n + 3n = 24$

(D) $t - \frac{t}{4} = 11$ and $3t = 11$

$\frac{174}{1}$

We do not agree.

The lowest common denominator of the fractional coefficients of the given open sentence is 6 . Start by multiplying each member by this number. Thus,

$$6\left(\frac{x}{3} + \frac{3x - 1}{6} \right) = 6(x - 5)$$

Please continue.

Please return to page $\frac{182}{2}$ and try question 7 again.

....

$\frac{174}{2}$

We don't agree.

To find the lowest common denominator, you have only to consider the fractions $\frac{3}{4}$ and $\frac{5}{8}$. Hence, to obtain integral coefficients, you should start by multiplying the given equation by 8 .

Please return to page $\frac{187}{2}$ and try question 10 again.

Very good. Your choice is correct.

The lowest common denominator is the smallest number that is divisible by each of the denominators. Thus, 30 is the LCD since

$$\frac{30}{2} = 15 \quad , \quad \frac{30}{5} = 6 \quad , \text{ and } \frac{30}{6} = 5$$

To find the lowest common denominator of two numbers, we multiply them and then divide by all the common factors they might have.

For example; in this case, we can take the first two denominators, 2 and 5 . Their product is 10 . Since there is no common factor of 2 and 5 , then 10 is also their lowest common multiple.

To find the LCM of three numbers, we find the LCM of the first two and then the LCM of that value and the third number. In this case, the LCM of 2 and 5 is 10 ; and the LCM of 10 and 6 is the product of 10 and 6 (which is 60) divided by the common factor, 2 , which is 30 . Thus, 30 is the lowest common denominator of 2, 5, and 6 .

Now proceed to question 4 which follows.

....

Question 4

Apply your knowledge to solve the open sentence,

$$\frac{2x}{5} - \frac{x}{10} = \frac{10}{3}$$

Select the letter next to the correct answer.

(A) $x = \frac{3}{10}$

(B) $x = \frac{100}{9}$

(C) $x = \frac{100}{3}$

(D) None of these.

$\frac{176}{1}$

When there is a minus sign between two fractions, every term in the numerator of the fraction following the minus sign is changed to its opposite.

Please be more careful.

Please return to page $\frac{179}{2}$ and try question 9 again.

....

$\frac{176}{2}$

We do not agree.

In order to change the given equation with decimal coefficients to integral coefficients, each term must be multiplied by 100 .

Please do not forget to multiply the right side of the equation by 100 .

Please return to page $\frac{190}{2}$ and try question 13 again.

You made a mistake. We will help you get started.

The lowest common denominator of the fractional coefficients is 6 .

We, therefore, multiply each term of the open sentence by this number.

Thus,

$$6\left(\frac{3p - 5}{2} \right) - 6\left(\frac{p}{3} \right) = 6(8)$$ Note: We must use the D property.

$$3(3p - 5) - 2(p) = 48$$

Please continue.

Please return to page $\frac{171}{2}$ and try question 6 again.

We do not agree.

To help you get started, examine the given open sentence,

$$x - \frac{5}{12} = \frac{1}{3}\left(x + \frac{3}{2}\right)$$

If we multiply both members by 12 , the L C M of all of the denominators, we will no longer have fractional coefficients. Thus,

$$12\left(x - \frac{5}{12} \right) = 12\left(\frac{1}{3} \right)\left(x + \frac{3}{2}\right) \qquad [\ D$$

Please return to page $\frac{181}{2}$ and try question 11 again.

$\frac{178}{1}$

We do not agree. The difference

$$t - \frac{t}{4}$$

does not equal $3t$

Remember t can be written as

$$\frac{t}{1}$$

Since the expression is now considered as a subtraction of fraction problems, it becomes

$$\frac{4t}{4} - \frac{t}{4}$$

which equals $\frac{3t}{4}$

Please return to page $\frac{173}{2}$ and try question 2 again.

. . . .

$\frac{178}{2}$

Not quite. You did multiply each term of the formula by 9 and you obtained

$$5(F - 32) = 9C$$

Please go over your work from this point on.

Please return to page $\frac{193}{2}$ and try question 16 again.

Very good. "None of these" is the correct choice.

Now, you could have done this problem in two ways.

Solution 1

$$\frac{5x}{6} - \frac{1}{2} = \frac{3}{2} - \frac{x}{6} \qquad + \frac{1}{2}$$

$$\frac{5x}{6} = \frac{3}{2} + \frac{1}{2} - \frac{x}{6} \qquad [\text{ Collect}$$

$$\frac{5x}{6} = 2 - \frac{x}{6} \qquad + \frac{x}{6}$$

$$\frac{6x}{6} = 2$$

$$x = 2$$

Solution 2 Multiply each term of the given open sentence by 6 . Thus ,

$$5x - 3 = 9 - x \qquad + 3$$

$$5x = 12 - x \qquad + x$$

$$6x = 12 \qquad \div 6$$

$$x = 2 .$$

Please proceed to question 9 which follows.

Question 9

Apply your knowledge to solve the open sentence,

$$\frac{y + 3}{8} - \frac{y - 2}{6} = 1$$

Select the letter next to the correct answer.

(A) y is greater than -10 but less than 0 .

(B) $y = 16$

(C) $y = -23$

(D) $y = -31$

$\frac{180}{1}$

Very good. Your choice is correct. We have the open sentence,

$$\frac{2x}{5} - \frac{x}{10} = \frac{10}{3}$$

The lowest common denominator of all the terms is 30 . To obtain an equivalent equation with integral coefficients, we multiply each term by 30 .

$$30\left(\frac{2x}{5} - \frac{x}{10}\right) = 30\left(\frac{10}{3}\right) \qquad [\ \mathrm{D}$$

$$30\left(\frac{2x}{5}\right) - 30\left(\frac{x}{10}\right) = 30\left(\frac{10}{3}\right)$$

$$12x - 3x = 100 \qquad [\ \text{Combine}$$

$$9x = 100 \qquad [\ \div\ 9$$

$$x = \frac{100}{9}$$

We now proceed to check this solution by replacing x by $\frac{100}{9}$ in the given equation. We get

$$\frac{2}{5}\left(\frac{100}{9}\right) - \frac{1}{10}\left(\frac{100}{9}\right) \stackrel{?}{=} \frac{10}{3}$$

$$\frac{40}{9} - \frac{10}{9} \stackrel{?}{=} \frac{10}{3}$$

$$\frac{30}{9} = \frac{10}{3}$$

Hence, the left member equals the right member.

Please proceed to question 5 which follows.

$\frac{180}{2}$

Question 5

Apply your knowledge to solve the open sentence,

$$\frac{2}{3}y - \frac{3}{5}y = \frac{7}{15}$$

Select the letter next to the correct answer.

(A) $y = 7$

(B) $y = -\frac{7}{4}$

(C) $y = \frac{7}{15}$

(D) $y = \frac{2}{3}$

Very good. You made the correct choice.

We have to solve the open sentence,

$$\frac{3}{4}(2x + 5) - \frac{5}{8}(3x - 1) = 1$$

The lowest common denominator of the fractional coefficients is 8 .
Multiplying both members of the equation by 8 we get,

$$
\begin{aligned}
6(2x + 5) - 5(3x - 1) &= 8 && [\ \text{D} \\
12x + 30 - 15x + 5 &= 8 && [\ \text{Collect} \\
-3x + 35 &= 8 && \measuredangle\ -35 \\
-3x &= -27 && \measuredangle\ \div\ -3 \\
x &= 9
\end{aligned}
$$

Check:

The left member of the open sentence will be equal to the right member if your solution, (x = 9) is correct. Substitute 9 in place of each x in the original equation.

$$
\begin{aligned}
\frac{3}{4}(2x + 5) - \frac{5}{8}(3x - 1) &= 1 \\
\frac{3}{4}(\ 23\ ) - \frac{5}{8}(\ 26\ ) &\overset{?}{=} 1 \\
\frac{69}{4} - \frac{65}{4} &\overset{?}{=} 1 \\
\frac{4}{4} &= 1
\end{aligned}
$$

Please proceed to question 11 which follows.

Question 11

Apply your knowledge to solve the open sentence,

$$x - \frac{5}{12} = \frac{1}{3}\left(x + \frac{3}{2}\right)$$

Select the letter next to the correct answer.

(A) $x \in \left\{\frac{5}{3}, \frac{7}{3}, \frac{11}{3}\right\}$ (C) $x \in \left\{\frac{11}{24}, \frac{13}{24}, \frac{17}{24}\right\}$

(B) $x \in \left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right\}$ (D) $x \in \left\{\frac{9}{8}, \frac{11}{8}, \frac{13}{8}\right\}$

$\frac{182}{1}$

Very good. You made the correct choice.

We are given the open sentence,

$$\frac{3p - 5}{2} - \frac{p}{3} = 8$$

The lowest common denominator of the fractional coefficients is 6 .

We, therefore, multiply each term of the open sentence by 6 . We get,

$$\begin{array}{rcll}
6\left(\frac{3p - 5}{2}\right) - 6\left(\frac{p}{3}\right) & = & 6(8) & \\
3(3p - 5) - 2p & = & 48 & [\ \text{D} \\
9p - 15 - 2p & = & 48 & [\ \text{Combine} \\
7p - 15 & = & 48 & +\ 15 \\
7p & = & 63 & \div\ 7 \\
p & = & 9 &
\end{array}$$

Check: We replace the letter p by 9 .

$$\begin{array}{rcl}
\frac{27 - 5}{2} - \frac{9}{3} & \stackrel{?}{=} & 8 \\
\frac{22}{2} - \frac{9}{3} & \stackrel{?}{=} & 8 \\
11 - 3 & \stackrel{?}{=} & 8 \\
8 & \stackrel{?}{=} & 8
\end{array}$$

Please proceed to question 7 which follows.

$\frac{182}{2}$

Question 7

Apply your knowledge to solve the open sentence,

$$\frac{x}{3} + \frac{3x - 1}{6} = x - 5$$

Select the letter next to the correct answer.

(A) $x = -1$

(B) $x = 1$

(C) $x = 29$

(D) None of these.

You surprised us by making the mistake you made.
Please note that

$$- 4(y - 2) = - 4y + 8$$

The number -4 in front of the parentheses multiplies each term within the parentheses.

Please return to page $\frac{179}{2}$ and try question 9 again.

....

In order to obtain an equivalent equation with integral coefficients, it is necessary to multiply each term by 100 .
Why do we have to multiply by 100 ?

Examine the term 0.25

Please return to page $\frac{196}{2}$ and try question 12 again.

$\frac{184}{1}$

Very good. You made the correct choice.

It is not always necessary to find the solution sets of two equations to determine whether they are equivalent or not. If each term of an equation is multiplied by the same number, we obtain an equivalent equation. Thus, if we multiply each term of the equation

$$\frac{n}{3} + \frac{n}{4} = 2$$

by 12 , we get, $12\left(\frac{n}{3}\right) + 12\left(\frac{n}{4}\right) = 12(2)$

$$4n + 3n = 24$$

It is important to keep in mind that the right member of the equation must also be multiplied by the same number as the left side.

Please proceed to question 3 which follows.

....

$\frac{184}{2}$

Question 3

Apply your knowledge to find the least common denominator of the fractional coefficients of the open sentence,

$$\frac{x}{2} - \frac{3x}{5} + \frac{5x}{6} = 4$$

Select the letter next to the correct answer.

(A) 15 (B) 12

(C) 10 (D) 30

Very good. You made the correct choice.

To solve the open sentence,

$$.04y + .06(2000 - y) = 96$$

we multiply each term by 100 . Thus, we get

$$4y + 6(2000 - y) = 9600 \quad [\text{ D}$$
$$4y + 12000 - 6y = 9600 \quad [\text{ Combine}$$
$$-2y + 12000 = 9600 \quad \text{α} - 12{,}000$$
$$-2y = -2400 \quad \text{α} \div -2$$
$$y = 1200$$

Check:

$$.04(1200) + .06(2000 - 1200)$$
$$\approx 48 + .06(800)$$
$$\approx 48 + 48$$
$$\approx 96$$

which is the same as the right member of the equation.

Please proceed to question 14 which follows.

Question 14

Apply your knowledge to solve the open sentence,

$$.08(4z + 5) - .03(2z - 3) = .36$$

Select the letter next to the correct answer.

(A) $z \in \left\{\frac{36}{5}, \frac{33}{5}, \frac{31}{5}\right\}$ (C) $z \in \left\{-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}\right\}$

(B) $z \in \left\{\frac{17}{13}, \frac{15}{13}, \frac{11}{13}\right\}$ (D) $z \in \left\{\frac{5}{26}, \frac{7}{26}, \frac{9}{26}\right\}$

$\frac{186}{1}$

You must have made a careless error.
A likely place is in the application of the DISTRIBUTIVE PROPERTY. Please go over your work and check your answer before making a choice.

Please return to page $\frac{181}{2}$ and try question 11 again.

$\frac{186}{2}$

It is very important that you learn how to solve this open sentence. In a future segment, you will be asked to do problems whose solutions will lead to this type of equation. Let us, therefore, do a similar question together.
Solve the open sentence for x :

$$.06x + .04(5000 - x) = 280$$

We first note that the decimal coefficients are hundredths. To change these coefficients to integers, we must, therefore, multiply by 100 .
We get,

$$\begin{aligned} 6x + 4(5000 - x) &= 28000 && [\ \text{D} \\ 6x + 20000 - 4x &= 28000 && [\ \text{Combine} \\ 2x + 20000 &= 28000 && -\ 20{,}000 \\ 2x &= 8000 && \div\ 2 \\ x &= 4000 \end{aligned}$$

Check:

$$\begin{aligned} & .06(4000) + .04(5000 - 4000) \\ = {} & 240 + .04(1000) \\ = {} & 240 + 40 \\ = {} & 280 \end{aligned}$$

This checks with the right side of the original equation.

Please return to page $\frac{190}{2}$ and try question 13 again.

Very good. You made the correct choice.

We have to solve the open sentence,

$$\frac{y + 3}{8} - \frac{y - 2}{6} = 1$$

The lowest common denominator is 24 . We, therefore, multiply both members of the equality by 24 . Thus ,

		Check:
$24\left[\frac{y+3}{8} - \frac{y-2}{6}\right] = 24\,(1)$	[D	$\frac{y+3}{8} - \frac{y-2}{6} = 1$
$3(y + 3) - 4(y - 2) = 24$	[D	$\frac{-7+3}{8} - \frac{-7-2}{6} \stackrel{?}{=} 1$
$3y + 9 - 4y + 8 = 24$	[Collect	$\frac{-4}{8} - \frac{-9}{6} \stackrel{?}{=} 1$
$-y + 17 = 24$	[- 17, (-1)	$\frac{-1}{2} + \frac{9}{6} \stackrel{?}{=} 1$
$y = -7$		$\frac{-1}{2} + 1\frac{1}{2} \stackrel{?}{=} 1$
		$1 = 1$

Please proceed to question 10 which follows.

Question 10

Apply your knowledge to solve the open sentence,

$$\frac{3}{4}(2x + 5) - \frac{5}{8}(3x - 1) = 1$$

Select the letter next to the correct answer.

(A) x is greater than 8 , but less than 12

(B) $x \in \{-9, -11, -13\}$

(C) $x \in \left\{\frac{17}{3}, \frac{19}{3}, \frac{23}{3}\right\}$

(D) None of these.

188
1

Very good. Your choice is correct.
We have to solve the open sentence,

$$\frac{x}{3} + \frac{3x - 1}{6} = x - 5$$

We multiply each term by the lowest common denominator of the fractional coefficient which is 6 . Thus,

$$\begin{array}{rcrcll}
6\left(\frac{x}{3}\right) + 6\left(\frac{3x-1}{6}\right) & = & 6(x - 5) & & [\ \mathrm{D} \\
2x + 3x - 1 & = & 6x - 30 & & [\ \mathrm{C} \\
5x - 1 & = & 6x - 30 & & - 6x \\
-x - 1 & = & - 30 & & + 1 \\
-x & = & - 29 & & (-1) \\
x & = & 29 & &
\end{array}$$

Check:

$$\frac{29}{3} + \frac{3(29) - 1}{6} = \frac{29}{3} + \frac{86}{6}$$

$$= \frac{58}{6} + \frac{86}{6}$$

$$= \frac{144}{6}$$

$$= 24$$

and the right member of the equation is

$$29 - 5 = 24$$

Please proceed to question 8 which follows.

188
2

Question 8

Apply your knowledge to solve the open sentence,

$$\frac{5x}{6} - \frac{1}{2} = \frac{3}{2} - \frac{x}{6}$$

Select the letter next to the correct answer.

(A) x is more than 3 , but less than 6. (C) x = 6

(B) x = 3 (D) None of these.

If instead of the letter a , we used some number, say, 2 , this question would have given you no trouble.

Don't let the appearance of an extra letter disturb you. The procedure for solving this open sentence is the same as before. What is the lowest common denominator of the fractional coefficients $\frac{a}{3}$ and $\frac{a}{5}$?

By what number must each term of the open sentence be multiplied to obtain integral coefficients? You know the answers to these questions.

Well then, go ahead.

Please return to page $\frac{198}{2}$ and try question 15 again.

You forgot to do something.
The lowest common denominators of the fractions on the left side of the equality is

$$(a + 1)(a - 1)$$

To obtain an equivalent equation with integral coefficients, you multiplied the left side of the equality by

$$(a + 1)(a - 1)$$

but you didn't multiply the right side of the equality by this quantity.

Please return to page $\frac{199}{2}$ and try question 17 again.

190
1

Very good. you made the correct choice. We have to find the solution set of the open sentence,

$$3x - 0.2 = 0.25 - 6x$$

To change the decimal coefficients to integral coefficients, each term must be multiplied by 100 . Thus,

$$\begin{array}{rcrcrcrl}
100(3x) & - & 100(0.2) & = & 100(0.25) & - & 100(6x) & \\
300x & - & 20 & = & 25 & - & 600x & + 600x \\
900x & - & 20 & = & 25 & & & + 20. \\
900x & & & = & 45 & & & \div 900 \\
x & & & = & \frac{45}{900} & & & [\ \text{Reduce} \\
x & & & = & \frac{1}{20} & \text{or} & 0.05 &
\end{array}$$

CHECK:

$$\begin{array}{rcrcrcr}
3x & - & 0.2 & = & 0.25 & - & 6x \\
3(0.05) & - & 0.2 & \stackrel{?}{=} & 0.25 & - & 6(0.05) \\
0.15 & - & 0.2 & \stackrel{?}{=} & 0.25 & - & .30 \\
 & - & .05 & \stackrel{?}{=} & - .05 & &
\end{array}$$

Please proceed to question 13 which follows.

190
2

Question 13

Apply your knowledge to solve the open sentence,

$$.04y + .06(2000 - y) = 96$$

Select the letter next to the correct answer.

(A) $y \in \{900, 100, 1200\}$ (C) $y \in \{-240, -260, -280\}$

(B) $y \in \{620, 640, 660\}$ (D) None of these.

Did you take a guess? If you did, it was a bad one.

Consider the formula

$$\frac{5}{9}(F - 32) = C$$

as you would an open sentence with fractional coefficients. By what number must each member be multiplied so as to obtain integral coefficients?
Yes, we must multiply by 9 . We get

$$9\left(\frac{5}{9}\right)(F - 32) = 9(C)$$

and

$$5(F - 32) = 9C$$

Please continue.

Please return to page $\frac{193}{2}$ and try question 16 again.

We do not agree. Let us refresh your memory on the rule for multiplying two binomials at sight. Recall that

$$(x - a)(x - b) = x^2 - ax - bx + ab \qquad [D$$

and

$$(x + a)(x + b) = x^2 + ax + bx + ab \qquad [D$$

Thus,

$$(x - \tfrac{1}{3})(x - \tfrac{1}{2}) = x^2 - (\tfrac{1}{3})x - (\tfrac{1}{2})x + \tfrac{1}{6} \qquad [D$$

Please continue.

Please return to page $\frac{195}{2}$ and try question 18 again.

By this time you should know the procedure to follow in solving an open sentence with decimal coefficients. Note that .08, .03, and .36 are each hundredths. You should, therefore, multiply each term by 100 to obtain an open sentence with integral coefficients.

Please be more careful when you remove parentheses and combine like terms.

Please return to page $\frac{185}{2}$ and try question 14 again.

We do not agree.

In a previous segment, you learned how to combine fractions with different binomial denominators. Is it possible that you no longer remember how to perform this operation? To combine the fractions,

$$\frac{x}{a+1} + \frac{x}{a-1}$$

we first find the lowest common denominator of the fractions. Since $(a + 1)$ and $(a - 1)$ are relatively prime, the lowest common denominator is the product $(a + 1)(a - 1)$. We change each fraction to an equivalent fraction with this denominator. Thus,

$$\frac{x}{a+1} \cdot \frac{a-1}{a-1} + \frac{x}{a-1} \cdot \frac{a+1}{a+1}$$

$$= \frac{x(a-1)}{(a+1)(a-1)} + \frac{x(a+1)}{(a+1)(a-1)}.$$

Please continue.

Please return to page $\frac{199}{2}$ and try question 17 again.

Very good. You made the correct choice. We have to solve the open sentence,

$$\frac{ax}{3} - \frac{ax}{5} = 4$$

To obtain integral coefficients, we multiply each term by 15 and get,

$$15\left(\frac{ax}{3}\right) - 15\left(\frac{ax}{5}\right) = 15(4)$$

$$5ax - 3ax = 60 \quad \text{[Combine}$$

$$2ax = 60 \quad [\div 2a$$

$$x = \frac{60}{2a} \quad \text{[Reduce}$$

$$= \frac{30}{a}$$

<u>Check:</u>

$$\frac{a\left(\frac{30}{a}\right)}{3} - \frac{a\left(\frac{30}{a}\right)}{5} \stackrel{?}{=} 4$$

$$\frac{30}{3} - \frac{30}{5} \stackrel{?}{=} 4$$

$$10 - 6 \stackrel{?}{=} 4$$

$$4 = 4$$

Please proceed to question 16 which follows.

$\frac{193}{2}$

<u>Question 16</u>

Apply your knowledge to solve the formula

$$\frac{5}{9}(F - 32) = C$$

for F Select the letter next to the correct answer.

(A) $F = 9C + 160$

(B) $F = \frac{9}{5}(C + 32)$

(C) $F = \frac{9}{5}C + 32$

(D) None of these.

$\frac{194}{1}$

You probably tried to change the decimal coefficients to fractions. It is not necessary to do so.

Please note that when we multiply a decimal number by 10 the decimal point moves one place to the right.

When we multiply a decimal number by 100 , the decimal point moves two places to the right.

Thus, for example,

$$100(0.25) = 25$$

Please return to page $\frac{196}{2}$ and try question 12 again.

....

$\frac{194}{2}$

We do not agree.

One of the letters does have the correct answer next to it.

Please return to page $\frac{190}{2}$ and try question 13 again.

Very good. You made the correct choice. The lowest common denominator of the given equation,

$$\frac{x}{a+1} + \frac{x}{a-1} = 2a$$

is $(a+1)(a-1)$

To obtain an equivalent equation with integral coefficients, we multiply each member of the equation by the lowest common denominator. Thus,

$$
\begin{aligned}
(a+1)(a-1)\left[\frac{x}{a+1} + \frac{x}{a-1}\right] &= 2a(a+1)(a-1) && \text{[D} \\
x(a-1) + x(a+1) &= 2a(a+1)(a-1) && \text{[D} \\
xa - x + xa + x &= 2a(a^2-1) && \text{[Collect} \\
2ax &= 2a(a^2-1) && \div\ 2a \\
x &= a^2 - 1
\end{aligned}
$$

Check:

$$
\begin{aligned}
\frac{a^2-1}{a+1} + \frac{a^2-1}{a-1} &\stackrel{?}{=} 2a && \text{[Factor and reduce} \\
(a-1) + (a+1) &\stackrel{?}{=} 2a && \text{[Combine} \\
2a &= 2a
\end{aligned}
$$

Please proceed to question 18 which follows.

$\frac{195}{2}$

Question 18

Apply your knowledge to solve the open sentence,

$$(x - \tfrac{1}{3})(x - \tfrac{1}{2}) = (x + \tfrac{1}{3})(x + \tfrac{1}{2}) - 20$$

Select the letter next to the correct answer.

(A) x is a value less than 10

(B) x is a value greater than 15

(C) x is an element of the set $\{6, 8, 10, 14\}$

(D) x is less than 15 but more than 10 .

$\frac{196}{1}$

Very good. ...rect choice. We have to solve the open sentence,

$$\frac{5}{12} = \frac{1}{3}(x + \frac{3}{2})$$

To obtain an equivalent open sentence with integral coefficients, we multiply both members by 12 .

$$\begin{array}{rcll}
12(x - \frac{5}{12}) & = & 12(\frac{1}{3})(x + \frac{3}{2}) & [\ \mathrm{D} \\
12x - 5 & = & 4(x + \frac{3}{2}) & [\ \mathrm{D} \\
12x - 5 & = & 4x + 6 & \sphericalangle 4x, + 5 \\
8x & = & 11 & \sphericalangle \div 8 \\
x & = & \frac{11}{8} &
\end{array}$$

Check:

$$\begin{array}{rcll}
x - \frac{5}{12} & = & \frac{1}{3}(x + \frac{3}{2}) & \\
\frac{11}{8} - \frac{5}{12} & \stackrel{?}{=} & \frac{1}{3}(\frac{11}{8} + \frac{3}{2}) & [\ \mathrm{D} \\
\frac{33}{24} - \frac{10}{24} & \stackrel{?}{=} & \frac{11}{24} + \frac{3}{6} & [\ \mathrm{C} \\
\frac{23}{24} & \stackrel{?}{=} & \frac{11}{24} + \frac{12}{24} & [\ \mathrm{C} \\
\frac{23}{24} & = & \frac{23}{24} &
\end{array}$$

Please proceed to question 12 which follows.

$\frac{196}{2}$

Question 12

Apply your knowledge to solve the open sentence,

$$3x - 0.2 = 0.25 - 6x$$

Select the letter next to the correct answer.

(A) $x \in \{\frac{1}{2}, \frac{1}{3}, \frac{1}{5}\}$

(B) $x \in \{\frac{1}{10}, \frac{1}{20}, \frac{1}{100}\}$

(C) $x \in \{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}\}$

(D) None of these.

"None of these" is not correct.

Use the same procedures as before to solve an equation with fractional coefficients.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page $\frac{193}{2}$ and try question 16 again.

We have a different answer than you have. One of us must be wrong. Lets see who.

We will check your answer by replacing x by $a(a - 1)$ in the given open sentence.

$$\frac{a(a-1)}{a+1} + \frac{a(a-1)}{(a-1)} = \frac{a(a-1)}{a+1} + a$$

$$= \frac{a(a-1) + a(a+1)}{a+1}$$

$$= \frac{a^2 - a + a^2 + a}{a+1}$$

$$= \frac{2a^2}{a+1}$$

But, this does not equal $2a$, therefore, your choice is incorrect.

Please return to page $\frac{199}{2}$ and try question 17 again.

$\frac{198}{1}$

Very good. You made the correct choice. To solve the open sentence,

$$.08(4z + 5) - .03(2z - 3) = .36$$

we multiply term by 100 . Thus, we get

		Check:
$8(4z + 5) - 3(2z - 3) = 36$	[D	$.08(-2 + 5) - .03(-1 - 3) =$
$32z + 40 - 6z + 9 = 36$	[Combine	$.08\ (3) - .03(-4) =$
$26z + 49 = 36$	-49	$.24 + .12 =$
$26z = -13$	$\div 26$	$.36$
$z = -\frac{1}{2}$		

Please proceed to question 15 which follows.

$\frac{198}{2}$

Question 15

Apply your knowledge and solve the open sentence,

$$\frac{ax}{3} - \frac{ax}{5} = 4$$

for x .

Select the letter next to the correct answer.

(A) $x = \frac{2}{a}$

(B) $x = \frac{30}{a}$

(C) $x = 30a$

(D) $x = \frac{15}{a}$

Very good. You made the correct choice.

The given formula is

$$\frac{5}{9}(F - 32) = C$$

[illegible] each member by 9 to obtain an equivalent formula with integral coefficients. Thus,

$$9\left(\frac{5}{9}\right)(F - 32) = 9(C)$$

$$5(F - 32) = 9C \qquad [\ D$$

$$5F - 160 = 9C \qquad \propto\ +\ 160$$

$$5F = 9C + 160 \qquad \propto\ \div\ 5$$

$$F = \frac{9}{5}C + \frac{160}{5}$$

$$F = \frac{9}{5}C + 32$$

Do you happen to know what this formula stands for? It is a most interesting and useful formula. F stands for temperature in the Fahrenheit scale, and C stands for the temperature in the [illegible]grade scale. In the formula of your answer, if you were given a temperature in the Centigrade scale, you would be able to find the equivalent temperature in the Fahrenheit scale. Thus, for example, if the temperature in Paris, France is 30° C, then

$$F = \left(\frac{9}{5}\right)(30) + 32$$

$$F = 54 + 32$$

$$F = 86° \qquad \text{or } 86° F$$

Please proceed to question 17 below.

Question 17

Apply your knowledge to solve the open sentence,

$$\frac{x}{a+1} + \frac{x}{a+1} = 2a$$

for x .

Select the letter next to the correct answer.

(A) $x = 1$ (B) $x = 2a^2$

(C) $x = a(a - 1)$ (D) $x = a^2 - 1$

$\frac{200}{1}$

We do not agree. One of the letters does have the correct answer next to it.

Please reconsider your choice.

Return to page $\frac{196}{2}$ and try question 12 again.

$\frac{200}{2}$

Very good. You made the correct choice. We have to find the solution set of the open sentence,

$$(x - \tfrac{1}{3})(x - \tfrac{1}{2}) = (x + \tfrac{1}{3})(x + \tfrac{1}{2}) - 20$$

Expanding the binomials, we obtain,

$$x^2 - (\tfrac{1}{3})x - (\tfrac{1}{2})x + \tfrac{1}{6} = x^2 + (\tfrac{1}{3})x + (\tfrac{1}{2})x + \tfrac{1}{6} - 20$$

Note that $x^2 + \frac{1}{6}$ appears on both sides of the equation. Subtracting this expression from both sides, we get

$$\begin{aligned}
-(\tfrac{1}{3})x - (\tfrac{1}{2})x &= (\tfrac{1}{3})x + (\tfrac{1}{2})x - 20 && (6)\\
-2x - 3x &= 2x + 3x - 120 && [C\\
-5x &= 5x - 120 && -5x\\
-10x &= -120 && -10\\
x &= 12
\end{aligned}$$

Check

$$\begin{aligned}
(12 - \tfrac{1}{3})(12 - \tfrac{1}{2}) &= (12 + \tfrac{1}{3})(12 + \tfrac{1}{2}) - 20\\
(\tfrac{35}{3})(\tfrac{23}{2}) &= (\tfrac{37}{3})(\tfrac{25}{2}) - \tfrac{120}{6}\\
(\tfrac{805}{6}) &= \tfrac{805}{6}
\end{aligned}$$

You have now completed this volume. Hand in your PUNCH CARD
You should have entered in your NOTEBOOK the following definition:

To solve an open sentence where the numerical coefficients are fractions, we find the lowest common denominator of the fractions and multiply each term of the open sentence by this number.

You should now complete the rest of your HOMEWORK ASSIGNMENT in preparation for a test on Volume 11 .

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 11

Attach to
Back Cover

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected. Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE	MISPRINT	CORRECTION	CHECK WHEN CORRECTION MADE
46/"2"	Ques. # omitted	Question 1	
5/2	...try question 1 again	... try question 10 again	
37/2	...return to page 67/1	...return to page 67/2	
98/1	"go on" omitted	.. go on to Ques 7 below.	
109/2	Question 11	Question 16	
109/1	...proceed to Ques.11...	...proceed to Ques. 16..	
142/2	...try Ques. 15 again	...try Ques. 16 again	