DOCUMENT RESUME

ED 075 211	24	SE 015 865

TITLE	Programmed Math Continuum, Level One, Algebra, Volume 10.
INSTITUTION	New York Inst. of Tech., Old Westbury.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-8-0157
PUB DATE	[73]
CONTRACT	OEC-0-8-080157-3691(010)
NOTE	208p.
EDRS PRICE	MF-$0.65 HC-$9.87
DESCRIPTORS	*Algebra; *Computer Assisted Instruction; Curriculum; Individualized Instruction; *Instruction; Instructional Materials; Mathematics Education; Programed Instruction; Programed Materials; *Programed Texts; *Secondary School Mathematics

ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 10: solving equations having factors whose product is zero; solving polynomial equations by factoring; use of factoring in problem solving; and reducing, multiplying, and dividing algebraic fractions. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 through SE 015 870.) (DT)

PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

VOLUME

10

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

PROGRAMMED MATH CONTINUUM

LEVEL ONE

ALGEBRA

VOLUME 10

New York Institute of Technology

Old Westbury - New York

A

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUMF 10

TABLE OF CONTENTS

VOLUME 10

This volume covers the following material as shown in this excerpt from the Syllabus:

		REFERENCE BOOK SECTION		
SEGMENT	DESCRIPTION	DOLCIANI	DRESSLER	DODES
1	Solution of equations having factors whose product is zero	7-12	19-2	8-1
2	Solution of polynomial equations by factoring	7-13	19-1 19-2	8-7,8-9
3	Use of factoring in problem solving	7-14	19-11	8-7,8-9
4	Investigation of algebraic fractions Reduction of fractions	8-1 8-2	12-1 12-2	4-6 4-7,8-6
5	Multiplication of fractions Division of fractions	8-5 8-6	12-3 12-4	4-8 4-9

D

READING ASSIGNMENT

VOLUME 10

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	263	264
2	264	266
3	267	270
4	281	284
5	292	295

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE.

These, too, should be entered in your NOTEBOOK.

HEW
PMC

E

HOMEWORK ASSIGNMENT

VOLUME NO. 10

BOOK: 597 DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	264	1 , 3 , 4	10110
2	264	7 , 9 , 11	10110
3	264	13 , 15 , 16	10110
4	264	18 , 19 , 20	10110
5	266	4 , 7 , 9 , 11	10210
6	266	15 , 19 , 20 , 22	10210
7	266	25 , 27 , 30	10210
8	266	31 , 32 , 38	10210
9	269	1 , 2 , 7	10310
10	269	6 , 8 , 10	10310
11	270	16 , 20	10310
12	270	21 , 22	10310
13	282	15 , 17 , 22 , 25 , 28	10410
14	285	4 , 5 , 7 , 14 , 17	10420
15	285	28 , 30 , 31	10420
16	285	32 , 33 , 40 , 43	10410 - 10420
17	294	13 , 16 , 23 , 25 , 31	10510
18	294	33 , 35 , 40 , 43	10510
19	295	8 , 10 , 14	10520
20	296	17 , 22 , 27 , 29	10520

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is not correct you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is correct then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

VOLUME 10 SEGMENT 1 BEGINS HERE:

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0	1	(Sequence Number)
	54	and	56	0	4	(Type of Punch Card)
	60	and	62	1	0	(Volume Number)
	66	and	68	0	1	(Segment Number)

SUPPLEMENTARY NOTE:

Is it possible to know something about a closed box without opening it and looking in? You think not? We don't agree. Suppose that you were shown a tightly closed box, one inch deep, and you were told that it contained steel balls. Without opening the box, you immediately know that none of the balls is more than one inch in diameter. You have actually discovered something about the size of the balls without looking into the box. This situation is not unique. Consider this algebraic example.

If a and b are any two (real) numbers and you are told that

$$a^2 + b^2 = 0$$

then without any calculation, you immediately know that

$$a = 0 \text{ and } b = 0$$

Why? The square of a real number is never negative and the sum of two non-negative numbers cannot equal zero, unless each number equals zero.

In this segment, we will discuss two numbers whose product ab is zero. We will discover what can be said about a and b if this is the case. We will find a most useful application of the little equation,

$$ab = 0$$

Your READING ASSIGNMENT for this Segment is pages 263-264
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Recognize which of the following products equals zero. Select the letter next to the correct answer.

(A) $(-1)\ (+1)$

(B) $(\ 1)\ (-\frac{1}{1})$

(C) $(\frac{1}{2})\ (-\frac{1}{2})$

(D) $5\ (0)$

X

$\frac{2}{1}$

When we have an equation consisting of the product of two factors equal to zero, we equate each factor to zero and solve the resulting equations.

i. e.

if $ab = 0$ we set

$a = 0$ and

$b = 0$

Here a and b represent factors that contain the variable. We could not do this if either a or b were numerical constants. If you follow this procedure carefully, you will find that this choice is not correct.

Please return to page $\frac{19}{1}$ and try question 3 again.

$\frac{2}{2}$

You can use a little help.

In solving the equation

$$5y + 1 = 0$$

we must first subtract 1 from both sides of the equation in order to get 5y by itself. Thus,

$$5y = -1$$

and y will be negative.

In the equation $2y - 6 = 0$

we first add 6 to both sides and then divide by the <u>coefficient of y</u>.

Please continue from here. Choose the set that contains <u>both</u> values of y that solve the equation.

Please return to page $\frac{33}{1}$ and try question 8 again.

$\frac{3}{1}$

Very good, you made the correct choice. The solution follows this pattern:

$$(x + 2)(x - 7) = 0$$

$x + 2 = 0 \propto\!\!< -2$	$x - 7 = 0 \propto\!\!< +7$
$x = -2$	$x = 7$

Please proceed to question 6 below.

$\frac{3}{2}$

Question 6

Apply the proper principle and find the solution set of

$$z\,(3z + 1) = 0$$

(A) $\left\{0,\ \frac{1}{3}\right\}$

(B) $\left\{1,\ ^{-}\frac{1}{3}\right\}$

(C) $\left\{0,\ ^{-}\frac{1}{3}\right\}$

(D) $\left\{0,\ -3\right\}$

X

4/1

Sorry, but this i[illegible] answer. Suppose you were right, and x was equal to zero. Using this value of x in the given equation, we get:

$$3\,(0 - 1) = 0$$

$$3\,(\ \ - 1) = 0$$

$$-3 = 0$$

This is obviously not true.

Please return to page 21/2 and try question 2 again.

4/2

We don't agree.

In order to find the value of x which satisfies the given equation, we have to solve the equation

$$\frac{2}{x} - \frac{4}{3} = 0$$

Start by adding $\frac{4}{3}$ to both sides of the equation, and continue on your own from there.

Please return to page 17/2 and try question 10 again.

X

Sorry, this value is not correct. Since the product of 225 and $(18 - \frac{3}{t})$ equals zero, it is necessary that

$$18 - \frac{3}{t} = 0$$

If you replace t by the value you have chosen, you will find that this does not check.

Please return to page $\frac{10}{2}$ and try question 9 again.

Sorry, this answer is not correct.

Let us review the procedure for solving a linear equation. Consider the equation

$$13x + 9 = 0$$

Our objective is to get the term containing the variable by itself.

Then we get:

$$\begin{aligned} 13x + 9 &= 0 && \text{Subtract 9 from both sides} \\ 13x &= -9 && \text{Divide both sides by 13} \\ x &= -\frac{9}{13} \end{aligned}$$

Study this procedure and apply it to the question. Let each variable factor equal zero.

Please return to page $\frac{37}{2}$ and try question 12 again.

X

$\frac{6}{1}$

You made a hasty choice. You saw $+1$ and -1 and decided that their product is zero. The sum of $+1$ and -1 is zero, but their product is

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{6}{2}$

It is true that if we equate each factor to zero, we get the same solution in each case. However, in writing the elements of a set, we do not repeat the same element.

Please return to page $\frac{23}{1}$ and try question 7 again.

Very good. You made the correct choice. To obtain the values of x that satisfy the given equation, we use the following principle:

> If the product of two algebraic quantities is equal to zero, then each of the quantities may be zero. Thus,
>
> if $(x - 3)(x - 4)$ equals zero, either
>
> $x - 3 = 0$ or
>
> $x - 4 = 0$
>
> or both of them are zero

It is necessary for us to consider the possibility that both are zero, so that we have:

$$(x - 3)(x - 4) = 0$$

$x - 3 = 0$	$x - 4 = 0$
$x = 3$	$x = 4$

Thus, we have two solutions for the equation. Since 4 is one of the solutions, this choice is certainly correct.

Please proceed to question 4 below.

Question 4

Apply the proper principle and select the letter which gives the general solution to the equation,

$$(a - b)(a + b) = 0$$

(A) $a = 0$

(B) $b = 0$

(C) both a and b equal zero

(D) $a = b$, or $a = -b$

$\frac{8}{1}$

Almost right.

Please reconsider the following:

$$\text{if} \quad 3z + 1 = 0$$

what does $3z$ equal? What does z equal?

Please return to page $\frac{3}{2}$ and try question 6 again.

[illegible]

A long time may have elapsed since you had to work with decimals. This is probably the reason for your difficulty with this question. How do we deal with decimals in an equation? We have to solve the linear equations,

$$7x - .3 = 0 \quad \text{and} \quad 2x + .9 = 0$$

Consider the first equation. If we multiply both sides of this equation by 10 we get the equivalent equation

$$70x - 3 = 0$$

Now you have an equation without decimals.

Please continue from here.

Please return to page $\frac{20}{2}$ and try question 13 again.

X

We don't agree. You should keep the following thought in mind:

> If the product of two quantities is equal to zero, at least one of the quantities must equal zero.

We have here the product of 3 and $(x - 1)$. Since the number 3 cannot equal anything but 3 the only other possibility is that

$$x - 1 = 0$$

Please continue from here on your own.

Please return to page $\frac{21}{2}$ and try question 2 again.

No, the answer to this question is contained in one of the sets offered. You must solve the problem systematically. You are given that

$$10\left(\frac{2}{x} - \frac{4}{3}\right) = 0$$

That is of the form

$$ab = 0$$

however, you realize that although either

$$a = 0 \quad \text{or}$$

$$b = 0$$

in this case since $a = 10$ the other factor

$\left(\frac{2}{x} - \frac{4}{3}\right)$ must equal zero.

Solve this equation according to the principles that you have learned. You will find your answer in one of the sets.

Please return to page $\frac{17}{2}$ and do this problem again.

$\frac{10}{1}$

This is the correct answer.

Your work should be arranged in this fashion.

$$(5y + 1)(2y - 6) = 0$$

$5y + 1$	$= 0$	$\propto - 1$	$2y - 6$	$= 0$	$\propto +6$
$5y$	$= -1$	$\propto \div 5$	$2y$	$= 6$	$\propto \div 2$
y	$= -\frac{1}{5}$		y	$= 3$	

Please proceed to question 9 which follows.

$\frac{10}{2}$

Question 9

Apply the proper principle and find the value of t that satisfies the equation,

$$225\left(18 - \frac{3}{t}\right) = 0$$

Select the letter which labels the correct answer.

(A) $t \in \{0, 3, 6, 9, 12, 15\}$

(B) $t \in \{18, 36, 54, 72, \frac{225}{3}\}$

(C) $t \in \{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{18}\}$

(D) $t \in \{\frac{225}{18}, -\frac{1}{3}, -\frac{1}{5}, -\frac{1}{18}, -\frac{1}{225}$

The quotient of a positive number and a negative number is negative.

Thus,

$$\frac{1}{-1} = -1$$

and the product of 1 and -1 does not equal zero.

Please return to page $\frac{1}{2}$ and try question 1 again.

We don't agree. Applying the principle:

$$\text{if } ab = 0$$
$$\text{then } a = 0$$
$$\text{and } b = 0$$

we have to solve the linear equations:

$$x + 2 = 0 \text{ and } x - 7 = 0$$

It seems necessary at this point to refresh your memory on how to solve linear equations. We will, therefore, try a similar problem. Find the solution of

$$y + 7 = 10$$

Since we wish to find the value of y, it is necessary that the letter y should appear by itself on one side of the equation. We, therefore, subtract 7 from both sides. Thus:

$$y + 7 = 10 \quad \text{Subtract 7 from both sides}$$
$$y = 3$$

Surely, this example served to refresh your memory.

Please return to page $\frac{26}{2}$ and try question 5 again.

X

$\frac{12}{1}$

We don't agree.

Please reconsider the solution of the linear equation

$$2x + 5 = 0$$

You should have little trouble finding your mistake.

Please return to page $\frac{22}{2}$ and try question 11 again.

$\frac{12}{2}$

A solution set must contain all of the possible values. Since there were three factors, each different, it is expected that there would be three members in the solution set.

Please return to page $\frac{24}{2}$ and reconsider the problem.

X

How did you get 1 as a solution for this equation? When you decide on an answer, it is always a good idea to check it.

If $z = 1$ is a solution of

$$z\,(3z + 1) = 0$$

then replacing z by 1 in the equation, should result in a true statement. Let us try it:

$$z\,(3z + 1) = 0$$
$$1\,(3 + 1) = 0$$
$$1\,(4) = 0$$
$$4 \neq 0$$

Do you see why $z = 1$ does not satisfy the equation?

Please return to page $\frac{3}{2}$ and try question 6 again.

Decimals can be troublesome, but there is a way of eliminating them when solving an equation. An equation can be likened to a balance scale. You can maintain the balance by multiplying both sides of the scale by the same quantity. Thus, in the equation

$$.01x + 1.5 = 0$$

we get an equivalent equation by multiplying both sides by 100 . This new equation will not have decimals.

Please return to page $\frac{40}{2}$ and try question 17 again.

$\frac{14}{1}$

This is impossible. The number 3 cannot equal any other number. Certainly 3 doesn't equal zero.

Please return to page $\frac{21}{2}$ and try question 2 again.

$\frac{14}{2}$

It appears that you used the correct procedure. Then where did you make your mistake? Was it in handling signs or in arithmetic?

Be very careful to consider the operation you are performing at each step, and apply the proper rules.

Please return to page $\frac{29}{2}$ and try question 14 again.

If the product of an arithmetic number and an algebraic expression is equal to zero, then only the algebraic expression is equal to zero. For example,

$$\text{if} \quad 250\,(5y - 13) = 0$$
$$\text{then only} \quad 5y - 13 = 0$$

Please return to page $\frac{23}{1}$ and try question 7 again.

You decided that it can't be solved, but it <u>can</u> be done. Did you notice that the factor $(x - 1)$ appears in the first and in the second product? Since this is the case, the given equation can be written in the form:

$$(x - 1)(x + 3) + (x - 1)(2x + 5) = 0$$
$$(x - 1)[(x + 3) + (2x + 5)] = 0$$

Please continue from here and convince yourself that this equation can be solved.

Please return to page $\frac{35}{1}$ and try question 16 again.

X

$\frac{16}{1}$

We don't agree. When we multiply a positive number by a negative number, the product is a negative number. Furthermore, the product of two fractions is obtained by multiplying both numerators and multiplying both denominators. For example:

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7}$$
$$= \frac{10}{21}$$

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{16}{2}$

You need a clearer understanding of the principle that we are using. We say that if the product of two expressions is equal to zero, each of the expressions <u>may</u> equal zero. It does not matter how involved each expression is; the principle is still the same. For example, if

$$(x^2 - y^2 - 5x)(x + 5y - 7) = 0$$

then $x^2 - y^2 - 5x = 0$

and $x + 5y - 7 = 0$

Please return to page $\frac{7}{2}$ and try question 4 again.

Very good, you have made the correct choice. We can disregard the number 225 , since 225 $(18 - \frac{3}{t})$ = 0 implies that

$$18 - \frac{3}{t} = 0 \quad \propto + \frac{3}{t} \quad \text{Adding } \frac{3}{t} \text{ to both sides}$$

$$18 = \frac{3}{t} \quad \propto \cdot t \quad \text{Multiplying both sides by } t$$

$$18t = 3 \quad \propto \div 18 \quad \text{Dividing both sides by } 18$$

$$t = \frac{1}{6}$$

Please proceed to question 10 below.

Question 10

Apply the proper principle and find the value of x that satisfies the equation

$$10\left(\frac{2}{x} - \frac{4}{3}\right) = 0$$

Select the letter which labels the correct answer.

(A) $x \in \{0, 1, 2, 3, 4, 5\}$

(B) $x \in \left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}\right\}$

(C) $x \in \left\{\frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\right\}$

(D) None of these.

X

$\frac{18}{1}$

We don't agree. You seem to have some difficulty in solving the equation

$$3z + 1 = 0$$

To help you, we will solve a similar equation. Solve for y :

$$\begin{array}{lll} 5y + 7 = 22 & -7 & \text{(Subtract 7 from both sides)} \\ 5y = 15 & \div 5 & \text{(Divide both sides by 5)} \\ y = 3 & & \end{array}$$

Please return to page $\frac{3}{2}$ and try question 6 again.

$\frac{18}{2}$

Your solution set contains three members, which is to be expected since each of the factors were different. But why did you introduce the term x ? It has no definition in regards to the problem. The solution set must be derived from the problem and either be in terms of the other constants given, or i [illegible]erical form where that is possible.

Please return to page $\frac{24}{2}$ and reconsider the problem.

X

This is the correct answer.

Please proceed to question 3 which follows.

Question 3

Apply the proper principle and select the letter which correctly completes the statement:

IF

$$(x - 3)(x - 4) = 0$$

then the value of x that satisifies the equation is:

(A) $x = -3$

(B) $x = 4$

(C) $x = -4$

(D) $x = 0$

The presence of letters instead of numbers does not effect the procedure. Consider, for example, the equation

$$3mx + 8n = 0$$

Let us solve this equation for x . Our objective is to get the term containing x by itself. Thus:

$3mx + 8n = 0$ Subtract 8n from both sides

$3mx = -8n$ Divide by 3m on both sides

$x = \frac{-8n}{3m}$

If you apply this procedure to the problem, you should get the correct solution.

Please return to page $\frac{25}{2}$ and try question 18 again.

V... you made the correct choice. We have:

$y + 2)(5y - 3) = 0$	Since 5 does not equal z...
$y + 2 = 0 \propto\!\!< -2$	$5y - 3 = 0 \propto\!\!< +3$
$7y = -2 \propto\!\!< \div 7$	$5y = 3 \propto\!\!< \div 5$
$y = -\frac{2}{7}$	$y = \frac{3}{5}$

Please proceed to question 13 below.

13

App... the proper principle and find the solution set of

$$.9\,(7x - .3)\,(2x + .9) = 0$$

Select the letter which labels the correct statement.

(A) $\left\{\frac{3}{70},\ \frac{9}{10}\right\}$

(B) $\left\{\frac{3}{7},\ -\frac{9}{20}\right\}$

(C) $\left\{\frac{3}{70},\ -\frac{9}{20}\right\}$

(D) $\left\{\frac{3}{35},\ -\frac{9}{10}\right\}$

Very good. Y ade the corre ice.

The important point to notice is t the product of two numbe s is zero, if and only if at least one of the n rs is zero.

i. e.

$$ab = 0$$

if $a = 0$

or if $b = 0$

and if both a and $b = 0$

Please go on to question 2 below.

Question 2

Apply the proper principle and select the statement which correctly completes the statement:

If $3(x - 1) = 0$, then

(A) $x = 0$

(B) $x =$

(D) $3 = 0$

X

22

Since

$$10 \left(\frac{2}{x} - \frac{4}{3} \right) = 0$$

and 10 is not equal to zero, it follows that:

$$\frac{2}{x} - \frac{4}{3} = 0 \qquad \propto + \frac{4}{3}$$

$$\frac{2}{x} = \frac{4}{3} \qquad \propto \cdot 3x$$

$$6 = 4x \qquad \propto \div 4$$

$$\frac{3}{2} = x$$

Therefore, this choice is correct.

Please proceed to question 11 below.

22

Question 11

Apply the proper principle and find the solution set of

6 ([illegible] + 3) (3x + 5) = 0

Select the letter which labels the correct statement.

(A) { [illegible] , $-\frac{3}{5}$ }

(B) { [illegible] , 10 }

(C) { [illegible] , − [illegible] }

(D) None of these.

This is the correct answer.

Please go on to question 7 which f[illegible]s.

<u>Question 7</u>

Apply the proper principle and find [illegible] solution set of

$$4 (x - 2) (\quad - 2) = 0$$

Select the letter which labels [illegible] ect statement.

(A) [illegible]

(B) [illegible]

(C) [illegible]

(D) [illegible]

Well, you are partly correct.

When $x = 1$ the expression does become

$0 + 0 = 0$ But when

$x = -$ the expression becomes

$0 + (-4)(-1) \neq 0$

Therefore, this choice is incorrect.

You have learned how to factor expressions by removing a common factor.

Now, in this equation $(x - 1)$ is a common factor.

Apply your knowledge of factoring. Convert the problem to the form

$$ab = 0$$

and then attack it in the usual way.

Please return to page $\frac{35}{1}$ and try question 16 again.

X

$\frac{24}{1}$

This is the correct answer. Your solution should look like this:

$(2y + 3)$	$(y - 9)$	$(3y - 1) = 0$	Since any factor may be zero
$2y + 3 = 0$	$y - 9 = 0$	$3y - 1 = 0$	Add or subtract on both sides
$2y = -3$	$y = 9$	$3y = 1$	Divide on both sides
$y = -\frac{3}{2}$	$y = 9$	$y = \frac{1}{3}$	

Since the original equation had three factors we should expect to find three solutions.

Please proceed to question 15 below.

$\frac{24}{2}$

Question 15

Apply the proper principle and find the solution set with reference to the variable x of

$$(x + a)(x - a)(x + b) = 0$$

Select the letter which labels the correct statement.

(A) $x = \{a, -b\}$

(B) $x = \{a, -a, -b\}$

(C) $x = \{a, b, c\}$

(D) $x = \{a, b, -b\}$

X

Very good. You made the correct choice.

We have to solve the two linear equations:

$$(.3x - 1.2)(.01x + 1.5) = 0$$

$.3x - 1.2 = 0$ ⋉ ·10	$.01x + 1.5 = 0$ ⋉ ·100
$3x - 12 = 0$ ⋉ + 12	$1x + 150 = 0$ ⋉ −150
$3x = 12$	$x = -150$
$x = 4$	

Please proceed to question 18 below.

Question 18

Apply the proper principle and find the solution set for the variable in the equation:

$$(3x + a)(bx - c) = 0$$

Select the letter which labels the correct statement.

(A) $\{-\frac{a}{3}, \ \ \}$

(B) $\{-\frac{a}{3}, \ \ \}$

(C) $\{-3a, \ \ \}$

(D) $\{-\frac{3}{a}, \frac{b}{c}\}$

X

26
1

That's true. In the equation

$$(a - b)\ (a - b) = 0$$

we have a variation of the basic principle.

When two non-numerical factors are multiplied and their product equals zero, we let each of the factors equal zero, for this would meet the conditions. We then solve the two newly formed equations to determine which value of the variable would indeed make the entire factor equal zero. Therefore, we can proceed in this fashion.

Given $(a - b)\ (a + b) = 0$

$(a - b) = 0 \quad +b$	$(a + b) = 0 \quad -b$
$a = b$	$a = -b$

This can be interpreted to mean that in either case, one of the two factors would become zero and regardless of what the other factor turned out to be, the product of zero and the other factor would be zero.

Now proceed to question 5 below.

26
2

Question 5

Apply the proper principle and find the solution set of

$$(x + 2)\ (x - 7) = 0$$

(A) $\{2, 7\}$

(B) $\{-2, 7\}$

(C) $\{2, -7\}$

(D) $\{-2, -7\}$

X

It is necessary to repeat the principle used in solving equations consisting of the product of several quantities equal to zero. Here it is:

If the product of several quantities is equal to zero, each of the quantities may equal zero. We have, in the given equation, the product of

$$6 \quad , \quad 2x + 3 \quad \text{and} \quad 3x + 5$$

all equal to zero.

Now the number 6 cannot equal zero, therefore, the only possibilities are:

$$2x + 3 = 0 \quad \text{and} \quad 3x + 5 = 0$$

If you continue from here, you will find that this choice is not correct.

Please return to page 22/2 and try question 11 again.

You are only partially correct. x does equal 1 ; this can be easily verified by substitution. Did you arrive at this answer that way or did you make the mistake that

if $a + b = 0$

you thought that $a = 0$

$b = 0$?

That is not the same as the basic principle that we've been plugging away at throughout this segment. The basic principle is

if a <u>times</u> $b = 0$ then we let $a = 0$ and $b = 0$

In order to use the correct principle, you must convert the form of the given problem into the product of two factors. This can be accomplished by "factoring."

Notice that

$$(x - 1)$$

is a common factor. Remove that factor, combine the like terms, and you will find that you will have the proper form.

Please return to page 31/2 and tackle this problem again.

X

$\frac{28}{1}$

The two factors are the same. Equating each of the factors to zero cannot result in different answers. Please reconsider your choice.

Please return to page $\frac{23}{1}$ and try question 7 again.

$\frac{28}{2}$

You were fooled with this choice. It looks like your thinking is a bit confused. Yes, -3 does make the product of the first two factors equal to zero and $\frac{-5}{2}$ does make the second pair of factors equal to zero. However, you forgot a very important point. The <u>one</u> value must be substituted throughout the problem at one time. You can't substitute one value for x in one part of the problem and a different value for x in another part of the problem at the same time and expect any proper results.

Please return to page $\frac{73}{2}$ and solve this problem according to proper procedures.

There is a simple procedure for handling the decimals.

$.9\ (7x - .3)\ (2x + .9) = 0$		Since .9 does not equal 0
$7x - .3 = 0$	$2x + .9 = 0$	(Multiply both sides by 10)
$70x - 3 = 0$	$20x + 9 = 0$	(Add or subtract on both sides)
$70x = 3$	$20x = -9$	(Divide on both sides)
$x = \frac{3}{70}$	$x = -\frac{9}{20}$	

This this choice is correct.

Please proceed to question 14 below.

Question 14

Apply the proper principle and find the solution set of

$$(2y + 3)\ (y - 9)\ (3y - 1) = 0$$

Select the letter which labels the correct answer.

(A) $\left\{\frac{3}{2},\ -9,\ \frac{1}{3}\right\}$

(B) $\left\{\frac{2}{3},\ -9,\ -\frac{1}{3}\right\}$

(C) $\left\{-\frac{3}{2},\ 9,\ \frac{1}{3}\right\}$

(D) $\left\{9,\ -\frac{1}{3}\right\}$

X

$\frac{30}{1}$

No, your choice is not correct.

Although we don't have specific values given for a and b in the original problem, each member of the set that you chose must have the property of making at least one of the factors equal to zero no matter what the other factors become.

In that way, the entire product will equal zero which is the condition stated in the original problem.

Please return to page $\frac{24}{2}$ and try this problem again.

$\frac{30}{2}$

We don't agree. In order to find the solution set of the given equation, it is necessary to change the sum of the four terms into the form of a product of factors. In a previous segment, you learned how to factor expressions by grouping. This is what is needed here. Remember the technique?

Notice: $y^3 + y^2 = y^2 (y + 1)$

and $-y - 1 = -1 (y + 1)$

therefore $y^3 + y^2 - y - 1 = y^2 (y + 1) - 1 (y + 1)$

Please return to page $\frac{41}{2}$ and continue the solution.

Very good. You made the correct choice.

You were given

$$(3x + a)\ (bx - c) = 0$$

Therefore, we have to solve the two linear equations:

$3x + a = 0$ and	$bx - c = 0$
For the first equation	For the second equation
$3x + a = 0 \quad \propto\!< -a$	$bx - c = 0 \quad \propto\!< +c$
$3x = -a \quad \propto\!< \div 3$	$bx = c \quad \propto\!< \div b$
$x = -\frac{a}{3}$	$x = \frac{c}{b}$

Please proceed to question 19 below.

Question 19

Relate to the proper principle and find the solution set of the equation,

$$(x - 1)^2 + 3(x - 1) = 0$$

Select the letter which labels the correct statement.

(A) $x = 1$

(B) $\left\{1, \frac{1}{3}\right\}$

(C) $\left\{1, -2\right\}$

(D) None of these.

X

$\frac{32}{1}$

You made a hasty choice.

Reconsider the solution of the equation

$$3x + 5 = 0$$

Please return to page $\frac{22}{2}$ and try question 11 again.

$\frac{32}{2}$

The standard form of a polynomial is obtained by removing all parentheses, combining like terms and arranging the terms in descending powers of the variable. A polynomial equation in standard form is such an expression equal to zero. For example,

$$3x^3 - 7x^2 + 5x + 8 = 0$$

is a polynomial equation in standard form.

Please return to page $\frac{43}{1}$ and try question 1 again.

X

This is the correct answer.

Please go on to question 8 which follows.

Question 8

Apply the proper principle and find the solution set of

$$(5y + 1)(2y - 6) = 0$$

Select the letter which labels the correct statement.

(A) $y \in \{1, 2, 3, 4, 5, 6\}$

(B) $y \in \{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\}$

(C) $y \in \{1, \frac{-1}{2}, 3, \frac{-1}{3}, 5, \frac{-1}{5}\}$

(D) $y \in \{-1, \frac{1}{2}, -3, \frac{1}{3}, -5, \frac{1}{5}\}$

We don't agree.

Let us clarify the meaning of the word linear.

An equation is said to be linear in a particular variable if the highest power of that variable is 1 . For example,

$$5x - 18 = 0$$

is a linear equation in the variable x . Consider another example:

Is the equation

$$(x - 2)(x + 5) - x^2 = 0$$

linear?

Before deciding on an answer, it is a good idea to write the equation in standard form. Thus,

$$(x - 2)(x + 5) - x^2 = 0 \quad [\text{ D}$$

$$x^2 - 3x - 10 - x^2 = 0 \quad [\text{ C}^\wedge\text{A}$$

$$3x - 10 = 0$$

The equation is, therefore, linear.

Please return to page $\frac{58}{2}$ and try question 3 again.

X

$\frac{34}{1}$

Did you write the equation in standard form?

That should have been your first step. The equation

$$6z^2 = -z$$

when changed to standard form is

$$6z^2 + z = 0$$

Notice the middle term corresponding to bx in the general form

$$ax^2 + bx + c = 0$$

is missing. Can you write the left side of this equation as the product of two factors?

Please return to page $\frac{45}{2}$ and try question 6 again.

$\frac{34}{2}$

Since this equation is already in standard form, you should proceed by factoring and setting each factor equal to zero. If you work carefully, you should be able to correct your mistake.

The general form of the factoring type is

$$a^2 - b^2 = (a + b)(a - b)$$

Please return to page $\frac{52}{2}$ and try question 11 again.

This is the correct answer.

Please proceed to question 16 which follows.

Question 16

Relate to the proper principle and find the solution set of

$$(x - 1)(x + 3) + (x - 1)(2x + 5) = 0$$

Select the letter which labels the correct statement.

(A) Since it is not of the form $ab = 0$ it can't be solved.

(B) $\{1, -3\}$

(C) $\{-3, -\frac{5}{2}\}$

(D) $\{1, -\frac{8}{3}\}$

We don't agree.

Before we can say that a polynomial equation is in standard form, all like terms must be combined.

Please return to page $\frac{48}{2}$ and try question 2 again.

30/1

Sorry. One of the choices is correct.

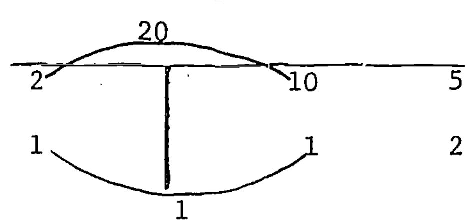
Concentrate your attention on the fact that (x - 1) is a common factor of both terms on the left side of the given equation.

Please return to page 31/2 and try question 19 again.

[illegible]

We don't agree. Try multiplying the two binomials and you will find that the sum of the inner and outer products does not equal the middle terms coefficient of the middle term in the equation.

Remember this form?



The sum of the inner and outer products is 21 , and it should be 9 .

Please return to page 50/2 and reconsider this problem.

This is the correct answer. The solution of the equation should look like this:

$6(2x + 3)(3x + 5) = 0$	Each factor, except 6, may be 0
$2x + 3 = 0 \quad \propto - 3$	$3x + 5 = 0 \quad \propto -5$
$2x = -3 \quad \propto \div 2$	$3x = -5 \quad \propto \div 3$
$x = -\frac{3}{2}$	$x = -\frac{5}{3}$

Since none of the other choices offered these two values, this choice is correct.

Please proceed to question 12 below.

Question 12

Apply the proper principle and find the solution set of

$$5\,(7y + 2)\,(5y - 3) = 0$$

(A) $y \in \left\{ \frac{-7}{2}, \frac{-5}{2}, \frac{+7}{2}, \frac{+5}{2}, \frac{+5}{7}, \frac{-5}{7} \right\}$

(B) $y \in \left\{ \frac{-2}{5}, \frac{+2}{5}, \frac{-2}{7}, \frac{+2}{7}, \frac{-3}{5}, \frac{+3}{5} \right\}$

(C) $y \in \left\{ \frac{+1}{2}, \frac{-1}{2}, \frac{+1}{3}, \frac{-1}{3}, \frac{+1}{5}, \frac{-1}{5} \right\}$

(D) $y \in \left\{ \frac{+2}{3}, \frac{-2}{3}, \frac{+3}{7}, \frac{-3}{7}, \frac{+5}{3}, \frac{-5}{3} \right\}$

X

Congratulations. You made the correct choice.

If we examine the given expression, we note that the first two terms have the common factor y^2, while the last two terms have the common factor -1. Thus we have:

$y^3 + y^2 - y - 1 = 0$	Factor in groups
$y^2(y + 1) - 1(y + 1) = 0$	Remove common factor
$(y^2 - 1)(y + 1) = 0$	Factor $y^2 - 1$
$(y + 1)(y - 1)(y + 1) = 0$	Set each factor equal to zero

$y + 1 = 0$	$y - 1 = 0$	$y + 1 = 0$	Solve each equation
$y = -1$	$y = 1$	$y = -1$	

Although there were three factors, there are only two distinct values for y. Then the solution set is $1, -1$

You have now finished this Segment. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following rule:

If the product of quantities is zero, then each quantity may equal zero.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT: 1 - 4

X

An equation is called quadratic if the highest power of the variable is 2 . Consider, for example,

$$(x + 2)(x + 3) = 8$$

In standard form, the equation becomes

$$x^2 + 5x - 2 = 0$$

This equation is quadratic since the highest power of x is 2 . What is the highest power in the equation you chose as quadratic?

Please return to page $\frac{58}{2}$ and try question 3 again.

We disagree.

Before writing a polynomial equation as the product of two binomials equal to zero, it is necessary to put the given equation into standard form. The equation in standard form would be

$$10y^2 - 11y + 3 = 0$$

The equation you have chosen is not equivalent to this.

You can easily check your answer by multiplying the two binomials and verifying that their product is the same as the transformed equation.

Please return to page $\frac{60}{2}$ and try question 5 again.

X

$\frac{40}{1}$

Good, this was a difficult one. Let's review the procedure.

The important feature of this problem is the factoring of the left side of the equation. We have a common factor, giving us:

$$(x - 1)(x + 3) + (x - 1)(2x + 5) = 0 \qquad [\ \text{factor}$$

$$(x - 1)\left[(x + 3) + (2x + 5)\right] = 0 \qquad [\ C^{\wedge}A$$

$$(x - 1)(3x + 8) = 0$$

$x - 1 = 0 \quad \propto\!\!< -1$	$3x + 8 = 0 \quad \propto\!\!< -8$
$x = 1$	$3x = -8 \quad \propto\!\!< \div 3$
	$x = -\frac{8}{3}$

Then this choice is correct.

Please proceed to question 17 below.

$\frac{40}{2}$

Question 17

Apply the proper principle and find the solution set of

$$(.3x - 1.2)(.01x + 1.5) = 0$$

Select the letter which labels the correct statement.

(A) $x \in \{.2,\ .4,\ .12,\ .15\}$

(B) $x \in \{2,\ 4,\ 120,\ 150\}$

(C) $x \in \{15,\ 20,\ 30,\ 40\}$

(D) $x \in \{40,\ 80,\ 120,\ 150\}$

Very good. You made the correct choice.

The important thing to notice is that the expression can be factored. We remove the common factor $(x - 1)$, remembering that $(x - 1)^2$ equals

$$(x - 1)(x - 1)$$

Then we have:

$$(x - 1)^2 + 3(x - 1) = 0 \qquad \text{[Factor}$$

$$(x - 1)[(x - 1) + 3] = 0 \qquad \text{[Combine terms}$$

$$(x - 1)(x + 2) = 0 \qquad \text{[Set each factor equal to 0}$$

$x - 1 = 0$	$x + 2 = 0$
$x = 1$	$x = -2$

Please proceed to question 20 below.

Question 20

Relate to the proper principle and find the solution set of the equation

$$y^3 + y^2 - y - 1 = 0$$

Select the letter which labels the correct statement.

(A) $\{1\}$

(B) $\{1, -1\}$

(C) $\{-1\}$

(D) Cannot be solved.

VOLUME 10 SEGMENT 2 BEGINS HERE:

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 2	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	1 0	(Volume Number)
	66	and	68	0 2	(Segment Number)

SUPPLEMENTARY NOTE:

In your study of algebra, you have been told many times that there is only one right answer, but many wrong answers to a problem. Now, we are going to modify this statement. We are going to say that it is possible to have two right answers (or even more). How is this possible? Consider the problem of finding two consecutive integers whose product is 12. Clearly, 3 and 4 are two such integers since 3 and 4 are consecutive, and 3 times 4 is 12. However, -3 and -4 are also two consecutive integers since -3 is one larger than -4. Furthermore $(-3)\ (-4) = 12$.

In this segment, you will be solving quadratic equations. You will find that such equations have two solutions, both of which are correct. The illustration with consecutive integers should convince you that two solutions to a problem is not a surprising event.

Your Reading Assignment for this Segment is pages 264 - 266. You will now be asked a series of questions to draw your attention to the more important points.

Please turn to page 43
1 .

Question 1

Recognize which one of the following equations is written in standard form. Select the letter next to the correct answer.

(A) $x^2 = 20x - 5$

(B) $x^2 - 20x = -5$

(C) $x^2 - 20x + 5 = 0$

(D) $20x - x^2 = 5$

We disagree.

Try multiplying the two binomials and you will find that the sum of the inner and outer products does not equal the middle term's coefficient.

Remember this form?

2	10	-5	5
1	1	-2	2

$-5 + -4 = -9$

The sum of the inner and outer products is negative 9 while it should be positive 9 .

Please return to page $\frac{50}{2}$ and reconsider the problem.

x

$\frac{44}{1}$

An equation is called cubic if the highest power of the variable is 3 . For example,

$$y^3 - 5y^2 - 10 = 0$$

is a cubic equation.

Why is this choice incorrect?

Please return to page $\frac{58}{2}$ and try question 3 again.

$\frac{44}{2}$

Surely, the first thing that you did was to put the equation into standard form. What did you get? You should have gotten

$$4x^2 + 8x - 140 = 0$$

This equation can be simplified. Each term can be divided by 4 . After doing this, you will have a simpler equation to work with, and you may avoid the mistake in arithmetic which seems to have occured. Besides, you should check each answer in the original equation before you make a decision about your results.

Please return to page $\frac{66}{2}$ and try question 8 again.

This is the correct answer.

Let us review the procedure.

$$10y^2 = 11y - 3 \qquad \propto - 11y$$

$$10y^2 - 11y = -3 \qquad \propto + 3$$

$$10y^2 - 11y + 3 = 0$$

$$(5y - 3)(2y - 1) = 0$$

(outer terms: -5; inner terms: -6)

to find the proper combination:

5	10	3	-3
2	1	1	-1

$-6 + -5 = -11$

verification:

$$(5y - 3)(2y - 1) \quad [\ D$$

$$5y(2y - 1) - 3(2y - 1) \quad [\ D$$

$$10y^2 - 5y - 6y + 3 \quad [\ A$$

$$10y^2 - 11y + 3$$

Please go on to question 6 which follows.

Question 6

Apply the proper principle and write the polynomial equation

$$6z^2 = -z$$

as the product of two factors equal to zero. Select the letter which labels the correct statement.

(A) $z(6z - 1) = 0$

(B) $z(6z + 1) = 0$

(C) $z(z - 6) = 0$

(D) $z(z + 6) = 0$

46/1

The given equation in standard form is

$$z^2 - 9z = 0$$

What are the factors of the expression on the left side of the equal sign? Remember, each of these factors should be set equal to zero. Both members of the solution set must appear as a subset in the set you choose.

Please return to page 59/2 and try question 10 again.

46/2

We don't agree. Before doing anything else, you should put the equation in standard form. Then we should have:

$$\begin{aligned} (x - 8)^2 &= 100 && \text{[Square the binomial } (x - 8) \\ x^2 - 16x + 64 &= 100 && \text{[Subtract 100 from both sides} \\ x^2 - 16x - 36 &= 0 && \text{Now you should continue} \end{aligned}$$

Remember, one value that checks does not necessarily fulfill the general requirements for the solution set of a quadratic. This particular type has two values in it's solution set.

Please return to page 77/2 and try question 14 again.

We don't agree.

First you should put the given equation into standard form.

Thus

$$x^2 - 3x = 18$$

should be written as

$$x^2 - 3x - 18 = 0$$

The next step is to express the left side of the equation as the product of two linear factors. Each of these factors is then set equal to zero, and the resulting equations are solved for x .

Please return to page $\frac{51}{2}$ and try question 7 again.

We disagree.

Since 2 is a root of the equation if we substitute 2 for x , the equation will check. You should now be able to continue and find the correct value of k .

Please return to page $\frac{62}{2}$ and try question 12 again.

X

$\frac{48}{1}$

Very good. You made the correct choice.

In the equation,

$$x^2 - 20x + 5 = 0$$

the variable x is written in descending order, and the expression is equated to zero.

The general or standard form can be expressed as

$$ax^2 + bx + c = 0$$

Please proceed to question 10 below.

$\frac{48}{2}$

Question 10

Recognize which one of the following polynomial equations is written in standard form. Select the letter which labels the correct answer.

(A) $x^2 - (5x + 8x) - 12 = 0$

(B) $3x^2 - 7x - 13 = 0$

(C) $5y^2 - 3y^2 + 8y = 0$

(D) $4x^2 + 3x - 2 = 1$

X

Did you find the numbers rather large?

We should proceed as follows:

$4x^2 + 8x = 140$	[Subtract 140 from both sides
$4x^2 + 8x - 140 = 0$	[Divide both sides by 4
$x^2 + 2x - 35 = 0$	[Factor the left side (always reduce the numbers if possible).
$(x + 7)(x - 5) = 0$	[Set each factor equal to zero
$x + 7 = 0 \quad x - 5 = 0$	[Solve the equations
$x = -7 \quad x = 5$	

Check each answer in the original equation.

$4(-7)^2 + 8(-7) \stackrel{?}{=} 140$	$4(5)^2 + 8(5) \stackrel{?}{=} 140$
$4(49) - 56 \stackrel{?}{=} 140$	$4(25) + 40 \stackrel{?}{=} 140$
$196 - 56 \stackrel{?}{=} 140$	$100 + 40 \stackrel{?}{=} 140$
$140 = 140$	$140 = 140$

Then this choice is correct.

Please proceed to question 9 below.

Question 9

Apply the proper principle and find the set that contains the solution of the equation as a sub set.

$$x^2 = 9 = 6x$$

Select the letter which labels the correct answer.

(A) $\{1, 3, 5, 7, 9\}$

(B) $\{-1, -3, -5, -7, -9\}$

(C) $\left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}\right\}$

(D) $\{2, 4, 6, 8, 10\}$

50
1

This is the correct answer.

This chart summarizes the situation:

Type Equation:	Standard Form
linear	$ax + b = 0$
quadratic	$ax^2 + bx + c = 0$
cubic	$ax^3 + bx^2 + cx + d = 0$

50
2

Question 4

Apply the proper principle and write the polynomial equation,

$$2x^2 + 9x + 10 = 0$$

in factored form. Select the letter which labels the correct statement.

(A) $(2x + 5)(x + 2) = 0$

(B) $(x + 10)(2x + 1) = 0$

(C) $(2x - 5)(x - 2) = 0$

(D) $(2x + 10)(x + 1) = 0$

X

Very good. You made the correct choice.

We first write the given equation in standard form.

Thus, we have:

$$\begin{array}{rcll} 6z^2 & = & -z & \propto + z \\ 6z^2 + z & = & 0 & [\ D \\ z(6z + 1) & = & 0 & \end{array}$$

if $\quad ab = 0$ then $a = 0 \quad b = 0$

$$\begin{array}{r|ll} z = 0 & 6z + 1 = 0 & \propto - 1 \\ z = 0 & 6z = -1 & \propto \div 6 \\ z = 0 & z = -\frac{1}{6} & \end{array}$$

Please proceed to question 7 below.

$\frac{51}{2}$

Question 7

Apply the proper principle and find the solution set of the equation,

$$x^2 - 3x = 18$$

Select the letter which labels the correct statement.

(A) $\{3 , 18\}$

(B) $\{-6 , 3\}$

(C) $\{6 , -3\}$

(D) $\{-6 , -3\}$

X

52/1

We should proceed in this manner:

$9z = z^2$	[Subtract $9z$ from both sides
$0 = z^2 - 9z$	[Apply the symmetric property
$x^2 - 9z = 0$	[Factor the left side
$z(z - 9) = 0$	[Set each factor equal to zero
	[Solve the equations

$z = 0$	$z - 9 = 0$
$z = 0$	$z = 9$

Check both values in the original

$9(0) \stackrel{?}{=} (0)^2$	$9(9) \stackrel{?}{=} (9)^2$
$0 = 0$	$81 = 81$

Then this choice is correct.

Please proceed to question 11 below.

52/2

Question 11

Apply the proper principle and find the solution set of the equation,

$$t^2 - 16 = 0$$

Choose the set below that contains the solution set as a subset.

(A) $\{0, 4, 8, 12, 16\}$

(B) $\{0, -4, -8, -12, -16\}$

(C) $\{+8, -8, +12, -12, +16, -16\}$

(D) $\{0, +2, -2, +4, -4, +14\}$

X

Please reconsider. One of the necessary conditions for an equation to be in standard form is to have all the terms on one side of the equation with the other side equal to zero.

Please return to page $\frac{48}{2}$ and try question 2 again.

What is the first thing that we must do? The equation has to be put into standard form. If you recall the definition of the standard form of a polynomial equation, one of the needed steps is the removal of parentheses. We must, therefore, square the binomial $(x + 7)$.

Remember:

$$(x + a)^2 \neq x^2 + a^2$$

rather

$$(x + a)^2 = x^2 + 2ax + a^2$$

Please continue from here.

Please return to page $\frac{67}{2}$ and try question 13 again.

X

$\frac{54}{1}$

We don't agree.

If you put the equation into standard form, factor, and set each factor equal to zero, you get two values that are not in this solution set.

Please return to page $\frac{49}{2}$ and try question 9 again.

$\frac{54}{2}$

We don't agree.

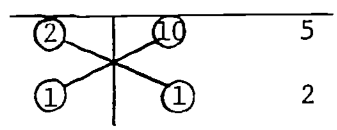
Although we can't factor as we have been doing, the value of k can be determined. You might consider the meaning of the phrase "root of the equation." That should suggest a method of procedure.

Please return to page $\frac{62}{2}$ and try question 12 again.

We don't agree.

Try multiplying the two binomials and you w[illegible] find that the sum of the inner and outer products [illegible] equal [illegible] coefficient of the middle term in the given equation.

Remember this form?



The sum of the [illegible] + [illegible] = 12

in[illegible] er pro[illegible] 12 and [illegible] 9 .

Please return to page $\frac{50}{2}$ and try question 4 again.

We disagree.

What were the factors you obtained for the trinomial

$$9x^2 - 4x - 1$$

Try multiplying them; you will find that there is an error.

Please return to page $\frac{73}{2}$ and try question 16 again.

X

$\frac{56}{1}$

When you obtain a solution set for a given equation, you should check each member of the set to see whether it satisfies the original equation. To illustrate: you selected a solution set which has the number 3 as one of its elements. Hence, 3 should satisfy the equation

$$x^2 - 3x = 18$$

that is: $(3)^2 - 3(3)$ should equal 18

Does it?

Please return to page $\frac{[illegible]}{2}$ and try question 7 again.

$\frac{56}{2}$

No, we don't agree.

Each of the equations must be transformed to the standard form first.

After factoring, the solution set of each can be compared.

You are looking for a value that appears in each of two of the solution sets.

Please return to page $\frac{65}{2}$ and tackle the problem.

Very good. You made the correct choice. The best procedure to follow is:

$(x - 8)^2 = 100$	[Square the binomial $(x - 8)$
$x^2 - 16x + 64 = 100$	[Subtract 100 from both sides
$x^2 - 16x - 36 = 0$	[Factor the left side
$(x - 18)(x + 2) = 0$	[Set each factor equal to zero.

$x - 18 = 0$	$x + 2 = 0$	Solve the equations
$x = 18$	$x = -2$	

Check:

$(18 - 8)^2 \stackrel{?}{=} 100$	$(-2 - 8)^2 \stackrel{?}{=} 100$
$10^2 \stackrel{?}{=} 100$	$(-10)^2 \stackrel{?}{=} 100$
$100 = 100$	$100 = 100$

Please proceed to question 15 below.

Question 15

Apply the proper principle and find the equation of lowest degree whose solution set is -1, -2. Select the letter which labels the correct answer.

(A) $x^2 + 3x = -2$

(B) $x^2 - 3x + 2 = 0$

(C) $2x^2 - x - 1 = 0$

(D) $x = -2 - x$

X

$\frac{58}{1}$

Very good. You made the correct choice.

The equation

$$3x^2 - 7x - 13 = 0$$

is in standard form.

Note the following: The expression contains no parentheses; there are no like terms to be combined, and the variable is in descending order. Furthermore, the right side of the equality contains a zero.

Please proceed to question 3 below.

$\frac{58}{1}$

<u>Question 3</u>

Recognize which of the polynomial equations is linear, which is quadratic, and which is cubic. Select the letter which labels the correct answer.

I. $3x + 5 = 2x - 1$ II. $x^3 = 300x^2 - 6$ III. $20x = 7 - x^2$

(A) I. is quadratic, II. is cubic, III. is linear

(B) I. is linear, II. is quadratic, III. is cubic

(C) I. is cubic, II. is quadratic, III. is linear

(D) I. is linear, II. is cubic, III. is quadratic

X

Very good. You made the correct choice.

We first put the given equation into standard form.

Thu. , we have:

$$x^2 + 9 = 6x \qquad \leftarrow -6x$$

$$x^2 - 6x + 9 = 0 \qquad [\ D' \text{ (SYMMETRIC DISTRIBUTIVE PROPERTY)}$$

$$(x - 3)(x - 3) = 0 \qquad \text{if } ab = 0 \text{ then } a = 0 \quad b = 0$$

$x - 3 = 0$	$x - 3 = 0$
$x = 3$	$x = 3$

Since we do not repeat like elements when listing the members of a set, the solution set is 3 . This set is a subset of the one you chose.

Please proceed to question 10 below.

Question 10

Apply the proper principle and find the set that contains the solution set of the equation as a subset

$$9z - z^2$$

Select the letter which labels the correct statement.

(A) $\{0, 1, 2, 3, 4, 5\}$

(B) $\{-1, -2, -3, -4, -5\}$

(C) $\{0, 3, 6, 9, 12\}$

(D) $\{0, -6, -7, -8, -9\}$

X

60
1

This is the correct answer.

If we arrange the factors of the coefficient of the first term and the factors of the constant in this form

2	10	-10	+5	-5
1	1	- 1	+2	-2
5	+	4	=	9

we can locate the proper combination of factors.

This can then be verified by multiplying the binomials we form.

+4

$$(2x + 5) (x + 2)$$

+5

We will find the product equals

$$2x^2 + 9x + 10$$

Please go on to question 5 which follows.

60
2

<u>Question 5</u>

Apply the proper principle and write the polynomial equation,

$$10y^2 = 11y - 3$$

as the product of two binomials equal to zero. Select the letter which labels the correct answer.

(A) $(10y - 3)(y - 1) = 0$

(B) $(5y - 3)(2y - 1) = 0$

(C) $(5y - 1)(2y - 3) = 0$

(D) $(5y + 3)(2y - 1) = 0$

You should transpose the form of the equation to the standard form, factor, and then equate each factor to zero. Then, after solving the two equations, check both answers.

Please check -6 as a solution of the given equation. You will find that this number does not satisfy the equation.

Please return to page $\frac{51}{2}$ and try question 7 again.

No.

We don't agree, although this set does contain one value that satisfies the equation. However, a quadratic equation has two values in its solution set and generally different values.

Did you square the binomial $(x + 7)$?

Let us do so now. We have:

$$(x + 7)^2 = x^2 + 14x + 49$$

and the given equation can now be written as

$$x^2 + 14x + 49 = 25$$

Please continue.

Please return to page $\frac{67}{2}$ and try question 13 again.

X

$\frac{62}{1}$

Since the equation is already in standard form, we begin by factoring.

Then we have:

$t^2 - 16 = 0$		Factor the left side $(a^2 - b^2) = (a+b)(a-b)$
$(t + 4)(t - 4) = 0$		Set each factor equal to zero
$t + 4 = 0$	$t - 4 = 0$	Solve the equations
$t = -4$	$t = 4$	

Check:

$(-4)^2 - 16 \stackrel{?}{=} 0$	$(+4)^2 - 16 \stackrel{?}{=} 0$
$16 - 16 \stackrel{?}{=} 0$	$16 - 16 \stackrel{?}{=} 0$
$0 = 0$	$0 = 0$

Then this choice is correct.

Please proceed to question 12 below.

$\frac{62}{2}$

Question 12

Apply the proper principle and find the value of k, if 2 is a root of the equation

$$x^2 - 6x + k = 0$$

Select the letter which labels the correct answer.

(A) $k = 4$

(B) $k = 6$

(C) $k = 8$

(D) Cannot be determined.

X

We disagree.

If you solve this equation, you will not get this solution set. You might also try checking one of the roots; you will discover that it does not check.

Please return to page $\frac{57}{2}$ and try question 15 again.

We don't agree.

Suppose that a 6-inch ruler was broken into two pieces at the 2-inch mark. How long will the larger piece be? 4 inches is correct. How did you get this answer?

You subtracted 2 inches from 6 inches and got 4 inches. Now, suppose that the ruler was x inches long and 2 inches are broken off. How would you represent the remaining piece?

The answer is

$$(x - 2) \text{ inches.}$$

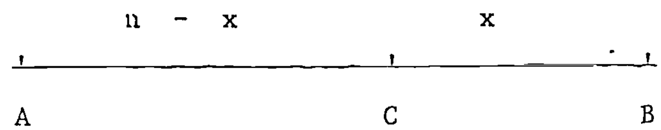
Please return to page $\frac{75}{1}$ and try question 1 again.

64/1

Sorry, you made an incorrect choice.

Let's consider a problem with a similar relationship.

Suppose that we have a rope AB which is n inches long.



Now, we cut off a piece at some point C so that CB is x inches long. How much rope do we have left? Clearly, we started with n inches, cut off x inches, so we must have (n - x) inches left.

In other words, the remaining piece is always the whole piece minus the first piece. Apply this conclusion to the question you are now doing.

Please return to page 93/2 and try question 3 again.

64/2

We have to find two numbers whose sum is 17 , and whose product is 60 . If we represent one of the numbers by x , how should we represent the other number?

Please return to page 81/2 and try question 5 again.

Very good. You made the correct choice.

Up to this point we can solve a quadratic equation only if the expression that is equal to zero can be factored. We know that there exist trinomials that cannot be factored. Such trinomials are called prime. This is exactly the situation that we have in this question. The trinomial

$$9x^2 - 4x - 1$$

is prime. Hence, with our present knowledge, we cannot solve this quadratic equation.

Now proceed to question 17 below.

Question 17

Apply the proper principle and choose the two equations that have one of the two roots the same.

$$\begin{aligned} \text{I.}\quad & 2x^2 + 24 = 14x \\ \text{II.}\quad & x^2 - 15 = 2x \\ \text{III.}\quad & 18 - 9x = -x^2 \\ \text{IV.}\quad & 2x^2 + 12x + 16 = 0 \end{aligned}$$

(A) I and II

(C) I and III

(B) I and IV

(D) II and IV

X

$\frac{66}{1}$

Very good. This choice is correct.

In solving this equation, we first put it in standard form, then factor, and next set each factor equal to zero. The solution should look like this:

$$x^2 - 3x = 18 \qquad \propto\!< -18$$

$$x^2 - 3x - 18 = 0 \qquad [\,D' \quad \text{symmetric form}$$

$$(x - 6)(x - 3) = 0 \qquad \text{if } ab = 0 \text{ then } a = 0 \ \ b = 0$$

$x - 6 = 0 \propto\!< +6$	$x + 3 = 0 \quad \propto\!< -3$
$x = 6$	$x = -3$

Then each value should be checked in the <u>original</u> equation.

$(6)^2 - 3(6) \stackrel{?}{=} 18$	$(-3)^2 - 3(-3) \stackrel{?}{=} 18$
$36 - 18 \stackrel{?}{=} 18$	$9 + 9 \stackrel{?}{=} 18$
$18 = 18 \checkmark$	$18 = 18 \checkmark$

Please proceed to question 8 below.

$\frac{66}{2}$

<u>Question 8</u>

Apply the proper principle and find the solution set of the equation,

$$4x^2 + 8x = 140$$

Select the letter which labels the correct statement.

(A) $\{-2, \ 35\}$

(B) $\{4, \ 5\}$

(C) $\{-7, \ 5\}$

(D) $\{7, \ -5\}$

X

$\frac{67}{1}$

Very good. This choice is correct.

Since 2 is a root of the equation, we can substitute 2 for x and the equation will check. Thus we have:

$$
\begin{aligned}
x^2 - 6x + k &= 0 && \text{[Substitution} \\
(2)^2 - 6(2) + k &= 0 && \\
4 - 12 + k &= 0 && \text{[Collect} \\
-8 + k &= 0 && +\,8 \\
k &= 8 &&
\end{aligned}
$$

Please proceed to question 13 below.

$\frac{67}{2}$

<u>Question 13</u>

Apply the proper principle and find the solution set of the equation,

$$(x + 7)^2 = 25$$

Select the letter which labels the set containing the solution as a subset

(A) $\{-7, -5, 3, 9\}$

(B) $\{-1, -2, -3, -9\}$

(C) $\{-2, -4, -10, -12\}$

(D) None of these.

X

68/1

This choice is not correct.

If two values are included in the solution set, it implies that the equation is at least the second degree. It could also mean that one of the roots repeated could raise the equation to a higher degree.

In this case you were asked to find the equation of the lowest degree. Therefore, you are looking for a quadratic equation.

Please return to page 57/2 and try question 15 again.

68/2

Please keep the following in mind:

If two numbers have a given <u>sum</u>, and one of these numbers is denoted by some letter, the second number is always the <u>sum</u> minus that letter.

Remember, we subtract from the sum.

Please return to page 81/2 and try question 5 again.

X

Very good. You made the correct choice.

When a quantity is divided into two parts, and one part is denoted by some letter, the other part is the original quantity minus that letter. Thus, in this question, 90° is divided into two parts. One part is m the other part is, therefore, the quantity 90° minus the part m .

The correct answer is

$$(90 - m)^\circ$$

Please proceed to question 2 below.

Question 2

Recognize and select the expression which correctly completes the statement:

If the perimeter of a rectangle is 54 feet, and its width is denoted by x , then the length is denoted by

(A) (54 - 2x) feet

(B) (27 - x) feet

(C) (27 + x) feet

(D) (2x - 54) feet

X

Very good. You really had to do a lot of work to solve this one.

Equation I:

$$2x^2 + 24 = 14x \qquad \propto\!<\ -14x$$
$$2x^2 - 14x + 24 = 0 \qquad \propto\!<\ \div 2$$
$$x^2 - 7x + 12 = 0 \qquad [D'$$
$$(x - 4)(x - 3) = 0$$

$x - 4 = 0 \quad \propto\!<\ + 4$	$x - 3 = 0 \quad \propto\!<\ + 3$
$x = 4$	$x = 3$

Equation II:

$$x^2 - 15 = 2x \qquad \propto\!<\ - 2x$$
$$x^2 - 2x - 15 = 0 \qquad [\ D'$$
$$(x - 5)(x + 3) = 0$$

$x - 5 = 0 \quad \propto\!<\ + 5$	$x + 3 = 0 \quad \propto\!<\ - 3$
$x = 5$	$x = -3$

Equation III:

$$18 - 9x = -x^2 \qquad \propto\!<\ + x^2$$
$$x^2 - 9x + 18 = 0 \qquad [\ D'$$
$$(x - 6)(x - 3) = 0$$

$x - 6 = 0 \quad \propto\!<\ + 6$	$x - 3 = 0 \quad \propto\!<\ + 3$
$x = 6$	$x = 3$

Equation IV:

$$2x^2 + 12x + 16 = 0 \qquad \propto\!<\ \div 2$$
$$x^2 + 6x + 8 = 0 \qquad [\ D'$$

The factors are: $(x + 4)$ and $(x + 2)$

$x + 4 = 0 \quad \propto\!<\ - 4$	$x + 2 = 0 \quad \propto\!<\ -2$
$x = -4$	$x = -2$

Please go on to page 71/1.

NOTE: D' means the symmetric form of the DISTRIBUTIVE PROPERTY which is "factoring."

Thus, we can see that Equations I and III have $x = 3$ for one root. No other equations have a common root.

You have now finished this Segment. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following items:

1. The definition of standard form of a polynomial.
2. The procedure used in solving a quadratic equation by factoring.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT: 5 - 8

We don't agree. Recall the following definition:

The reciprocal of a number is the quotient of 1 divided by that number.

This is a special case of the more general rule:

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

If a is an integer, then b is 1,

and the reciprocal of a is $\frac{1}{a}$.

Please return to page $\frac{181}{2}$ and try question 11 again.

Sorry, but one of the other letters does have the correct answer next to it. Expand the binomial, transform the equation to standard form, and then factor.

Please return to page $\frac{67}{2}$ and try question 13 again.

We don't agree.

Suppose that you had $2,500.00 to invest, and you invested $1,000.00 at 5% , and the rest at 6% . How much money would you invest at 6% ? How did you get $1,500.00 ? You subtracted $1,000.00 from the total of $2,500.00. Apply the same procedure and replace the amount $1,000.00 by the letter p .

Please return to page $\frac{76}{2}$ and try question 4 again.

Very good. You made the correct choice. In solving this equation, we proceed as follows:

$$
\begin{array}{rcll}
x^2 + 3x & = & -2 & +2 \\
x^2 + 3x + 2 & = & 0 & [D \\
(x + 1)(x + 2) & = & 0 &
\end{array}
$$

$$
\begin{array}{lll|lll}
x + 1 & = 0 & -1 & x + 2 & = 0 & -2 \\
x & = -1 & & x & = -2 &
\end{array}
$$

Check:

$$
\begin{array}{rcl|rcl}
(-1)^2 + 3(-1) & \stackrel{?}{=} & 2 & (-2)^2 + 3(-2) & \stackrel{?}{=} & -2 \\
1 - 3 & \stackrel{?}{=} & 2 & 4 - 6 & \stackrel{?}{=} & -2 \\
-2 & = & -2 \checkmark & -2 & = & -2 \checkmark
\end{array}
$$

Please proceed to question 16 below.

$\frac{73}{2}$

Question 16

Apply the proper principle and find the solution set of the equation,

$$9x^2 - 4x - 1 = 0$$

Select the letter that labels the correct answer.

(A) $\left\{\frac{1}{3}, -\frac{1}{3}\right\}$

(B) $\left\{-1, \frac{1}{9}\right\}$

(C) $\left\{\frac{2}{3}, -\frac{1}{3}\right\}$

(D) This equation cannot be solved by factoring.

X

Volume 10 Segment 3 begins here.

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0	3	(Sequence Number)
	54	and	56	0	4	(Type of Punch Card)
	60	and	62	1	0	(Volume Number)
	66	and	68	0	3	(Segment Number)

SUPPLEMENTARY NOTE:

Suppose that somebody asked you to guess the following: I have something in my hand. It is round, made of metal, and it has a hole in the middle. What is it? Now, what objects meet the requirements of the question? The object could be a car wheel, a ring, or a washer. All these objects are round, made of metal, and have a hole in the middle. Which one of these objects would you reject? The wheel, of course. Why? Simply because a wheel is too large to be concealed in a closed hand.

We have already learned that there are usually two solutions to a quadratic equation. Solving problems that lead to a quadratic equation sometimes results in two different solutions, one of which must be rejected for the same reason that we rejected the wheel. The nature of the problem will indicate which solution is impractical, unreal, and, therefore, not acceptable.

In this segment, you will solve problems where you will have to use common sense in rejecting the impractical and unreal solutions to such problems.

Your READING ASSIGNMENT for this Segment is pages 267 - 270 .
You will now be asked a series of questions to draw your attention to the more important points.

Please proceed to page 75 .

Question 1

Recognize and select the expression which correctly completes the statement:

If a right angle is divided into two parts and one part is denoted by $m°$, the other part is denoted by

(A) $90m°$

(B) $m - 90°$

(C) $90 - m°$

(D) $90 + m°$

We don't agree.

Let us start a similar problem together. The sum of the ages of John and Mary is 18 years and the product of their ages is 65 years. How old is each one?

Let x = John' age.

How shall we denote Mary's age? Mary's age is the sum of both their ages minus John's age. That is, Mary's age is

$$(18 - x)$$

Now, we can use the fact that the product of their ages is 65 to write an equation.

Please return to page $\frac{87}{2}$ and try question 6 again.

X

Very good. You made the correct choice.

The sum of the ages of father and son is 59 years. The son, Jack, is y years old. Hence, the father's age is the total of both ages minus the son's age.

The answer is, therefore,

$$(59 - y)$$

Please proceed to question 4 below.

Question 4

Recognize and select the expression which correctly completes the statement:

A man invests \$2,500.00 , part at 5% and the rest at 6% .

If the amount invested at 5% is denoted by p , then the amount invested at 6% is denoted by ,

(A) $2500 - p$

(B) $2500 - 6p$

(C) $p - 2500$

(D) $(\frac{5}{6})p$

Very good. You made the correct choice.

The work should appear this way:

$(x + 7)^2 = 25$	[Square the binomial (x + 7)
$x^2 + 14x + 49 = 25$	[Subtract 25 from both sides
$x^2 + 14x + 24 = 0$	[Factor the left side
$(x + 12)(x + 2) = 0$	[Set each factor equal to zero
$x + 12 = 0$ | $x + 2 = 0$	[Solve the equations
$x = -12$ | $x = -2$	

Check:

$(-12 + 7)^2 \stackrel{?}{=} 25$	$(-2 + 7)^2 \stackrel{?}{=} 25$
$(-5)^2 \stackrel{?}{=} 25$	$(5)^2 \stackrel{?}{=} 25$
$25 = 25$	$25 = 25$

Please proceed to question 14 below.

Question 14

Apply the proper principle and find the solution set of the equation,

$$(x - 8)^2 = 100$$

Select the letter which labels the set that contains the solution set as a subset

(A) $\{2, 4, 6, 8, 10\}$

(B) $\{-2, -4, -8, -10, +18\}$

(C) $\{-6, 12, -14, +18, -18\}$

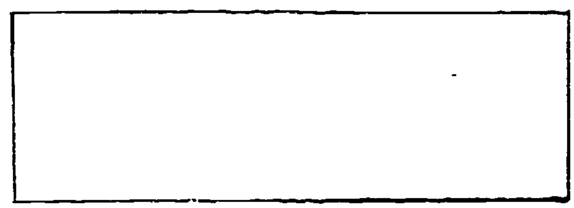
(D) $\{-2, -12, 14, 16, -18\}$

X

$\frac{78}{1}$

Did you make a picture of the rectangle?

It always helps to [illegible] with a sketch. Let us make one now.



The perimeter of a rectangle is the sum of all four sides.

If the perimeter is 54 feet, how much is one length plus one width? Clearly, the answer is 27 feet; that is, one half the perimeter. Please continue.

Please return to page $\frac{69}{2}$ and try question 2 again.

$\frac{78}{2}$

We don't agree.

If the perimeter of the rectangle is 44 inches, then the sum of one length and one width is 22 inches.

If we let

x equal the length of the rectangle

then its width is denoted by

$$(22 - x)$$

Please continue.

Please return to page $\frac{32}{2}$ and try question 7 again.

X

We don't agree.

We will help you by doing a similar problem together.

The sum of two numbers is 9 and their product is 20 .

What are the numbers?

Let x be one of the numbers.

The second number is the sum of both numbers minus the first number.

Hence, the second number is

$$(9 - x)$$

The equation that will solve this problem is

$$x(9 - x) = 20$$

Please return to page $\frac{81}{2}$ and try question 5 again.

If the larger integer were 5 , the smaller one would be 4 . The sum of the squares of these two integers must equal 61 .

Does

$$5^2 + 4^2 = 61 \ ?$$

Please return to page $\frac{105}{2}$ and try question 9 again.

X

$\frac{80}{1}$

If your choice is correct, it will meet the conditions of the problem. The perimeter is given as 26 inches, therefore, the length plus the width is 13 inches, and

$$7 + 6 = 13$$

So far, your answer checks. However, the area of the rectangle is 36 square inches, and

$$7 \times 6 \text{ does not equal } 36$$

Please return to page $\frac{99}{2}$ and try question 8 again.

$\frac{80}{2}$

If Bill's present age is 7 then his father's present age is 49 . Now, in 9 years, Bill will be 16 and his father will be 58 . But, 58 does not equal 3 times 16 . Hence, your choice does not meet the conditions of the problem.

Please return to page $\frac{91}{2}$ and try question 11 again.

Very good. You made the correct choice.

The total amount invested is \$2,500.00. If part of this money is invested at 5%, and this part is denoted by p , the remaining part is the total, \$2,500.00, minus this part. Hence, the answer is

$$\$2,500.00 - p$$

Please proceed to question 5 which follows below.

Question 5

Apply the proper principle and select the equation that can be used to solve the following problem. The sum of two numbers is 17 , and their product is 60 . Find the numbers.

(A) $17x = 60$

(B) $x(17 - x) = 60$

(C) $x(x - 17) = 60$

(D) $18x = 60$

X

$\frac{82}{1}$

Very good. You made the correct choice.

If we denote the age of the son by x , then the father's age is the sum of both ages minus the age of the son. Thus, the father's age is

$$(31 - x)$$

The product of their ages is

$$x(31 - x) = 168$$

Please proceed to question 7 below.

$\frac{82}{2}$

Question 7

Apply the proper principle and select the equation that can be used to solve the following problem:

The perimeter of a rectangle is 44 inches, and its area is 117 square inches. Find its dimensions.

(A) $x(44 - x) = 117$

(B) $x(117 - x) = 44$

(C) $x(22 - x) = 117$

(D) $2x + 2(22 - x) = 117$

X

We don't agree.

One length plus one width of the rectangle is equal to half the perimeter. It is, therefore, not possible for the length alone to be

$$(27 + x)$$

feet long.

Please return to page $\frac{69}{2}$ and try question 2 again.

At the beginning of this segment, we discussed the rejection of certain solutions to a problem. Although this is one of the two roots of the equation, common sense tells us that a triangle cannot have a side of negative length.

Please return to page $\frac{104}{2}$ and try question 12 again.

X

$\frac{84}{1}$

We don't agree.

I: THE VARIABLES:

If we

let x denote the larger number,

then $11 - x$ denotes the smaller number.

II: THE RELATIONSHIP:

Now, we are told that the square of the larger number x,
minus the smaller number $(11 - x)$ is equal to 45.

Please write the equation and continue from here.

Please return to page $\frac{97}{2}$ and try question 10 again.

$\frac{84}{2}$

We don't agree.

If your choice were correct, either the product of 11 and 12 would have to equal 462 or the product of 12 and 13 would have to equal 462. But they don't, do they?

Please return to page $\frac{107}{2}$ and try question 15 again.

X

We don't agree.

If we

let x = the length,

then $13 - x$ will be the width.

The area of the rectangle is the product of this length and width.

Please continue.

Please return to page $\frac{99}{2}$ and try question 8 again.

We don't agree.

What will be the ages of father and son in 9 years from now?

Clearly, each one will be 9 years older. Thus, the father will be

$$y^2 + 9$$

and the son will be $y + 9$ years old.

Read the last sentence of the problem again. It is this sentence that enables you to write the equation needed to solve this problem.

Please return to page $\frac{91}{2}$ and try question 11 again.

X

$\frac{86}{1}$

We don't agree.

Two integers are called consecutive when the larger one is one more than the smaller one. Such integers can be represented by x and $x + 1$. Furthermore, by the sum of their squares, we mean the square of the first integer plus the square of the second integer.

Please return to page $\frac{105}{2}$ and try question 9 again.

$\frac{86}{2}$

Suppose that you never had algebra, and someone asked you this question. How would you go about finding an answer. You would start with a triangle and discover that it has no diagonals. Then you would draw a four-sided figure and discover that it has two diagonals. Continuing in this manner, you would have little difficulty in discovering that a five-sided polygon has five diagonals, and a six-sided polygon has nine diagonals. Let us write these numbers down:

0 , 2 , 5 , 9 , ______. Do you see a pattern?

Yes. A 7 , 8 , 9 sided figure will have 15 , 21 , 27 diagonals, respectively. Now that you know the answer to this question, solve it algebraically and see if it checks.

Please return to page $\frac{110}{2}$ and try question 13 again.

X

Very good. You made the correct choice.

We are given the fact that the sum of the numbers is 17 .

If we represent one number by x , the other number is represented by

$$(17 - x)$$

Using the fact that the product of the numbers is 60 , we obtain the equation,

$$x(17 - x) = 60$$

Please proceed to question 6 below.

Question 6

Apply the proper principle and select the equation that can be used to solve the following problem. The sum of the ages of a father and son is 31 years, and the product of their ages is 168 years. Find their ages.

(A) $x(31 - x) = 168$

(B) $x(x - 31) = 168$

(C) $x(31 + x) = 168$

(D) None of these.

X

$\frac{88}{1}$

If the sum of two quantities is some number N , and one of these quantities is equal to x , then the second quantity is equal to

$$(N - x)$$

Please return to page $\frac{69}{2}$ and try question 2 again.

$\frac{88}{2}$

The area of a triangle is equal to $\frac{1}{2}$ the product of its base and height. Thus, if we denote the height by x the base should be denoted by

$$x + 7$$

One-half the product of these two quantities is equal to 72 . Set up the equation and solve it according to the procedures you have learned. Your answer is not correct.

Please return to page $\frac{104}{2}$ and try question 12 again.

X

It seems that you are confusing the meaning of the words "perimeter" and "area" of a rectangle. The perimeter of a rectangle is the sum of the measures of two lengths and two widths. The area of a rectangle is the product of the measures of a length and a width. Thus, in the rectangle drawn below, the perimeter is

$$7 + 7 + 4 + 4 = 22$$

and the area is $7 \times 4 = 28$

Please return to page $\frac{82}{2}$ and try question 7 again.

You should always check your answer with the conditions of the problem. Suppose that the sum of the numbers was less than 40 say, 39 . Then, the numbers would be 19 and 20 . The product of 19 and 20 is 380 and not 462 .

Please return to page $\frac{107}{2}$ and try question 15 again.

X

$\frac{90}{1}$

Sorry, your choice is incorrect.
Let us do a similar problem together to help you find out where you went wrong. The length plus the width of a certain rectangle is 8 inches, and its area is 15 square inches. What are the dimensions of this rectangle?

$$\begin{aligned} \text{Let} \qquad x &= \text{the length of the rectangle, and} \\ 8 - x &= \text{the width, and} \\ x(8 - x) &= \text{The area of the rectangle.} \end{aligned}$$

We are told that the area of this rectangle is 15 square inches. Hence, we have the equation

$$\begin{array}{ll} x(8 - x) = 15 & [\ \text{D} \\ 8x - x^2 = 15 & \propto\!\!<\ -15 \\ 8x - x^2 - 15 = 0 & [\ \text{C} \\ -x^2 + 8x - 15 = 0 & \propto\!\!<\ \cdot(-1) \\ x^2 - 8x + 15 = 0 & [\ \text{Factor} \\ (x - 5)(x - 3) = 0 & \end{array}$$

$$\begin{array}{l|l} x - 5 = 0 & x - 3 = 0 \\ x = 5 & x = 3 \end{array}$$

Since the length is usually considered to be the larger dimension, the length $x = 5$, and the width $8 - x = 3$.

Please return to page $\frac{99}{2}$ and try question 8 again.

$\frac{90}{2}$

We don't agree!
Surely, you had no trouble in writing the equation necessary to solve this problem. The equation is

$$\frac{n}{2}(n + 1) = 703 \quad \propto\!\!<\cdot 2$$

$$n(n + 1) = 1406 \quad \text{or}$$

$$n^2 + n - 1406 = 0$$

The difficulty is finding factors of a number as large as 1406. Here is a suggestion how to find such factors. We first look for <u>two equal</u> numbers whose product is approximately 1406. Now $40 \times 40 = 1600$ is a little too big. We try $35 \times 35 = 1225$ which is a little too small. However, we have located the <u>neighborhood</u> of the factors that we are seeking. They are somewhere between 35 and 40 . We have further clue. Since the coefficient of n is 1 , we know that the required factors differ by 1 . Can you find two numbers that are somewhere between 35 and 40 that differ by 1 have a product equal to 1406.

Please return to page $\frac{96}{2}$ and try question 14 again.

x

Very good. You made the correct choice.

I: THE VARIABLES:

We let

$$x = \text{the larger number},$$

and since the sum of both numbers is 11, the smaller number is denoted by $(11 - x)$

II. THE RELATIONSHIP:

The second sentence of the problem gives us the equation needed to solve this problem. The equation is

$$x^2 - (1 - x) = 45$$

Removing parentheses, we get

$$\begin{aligned} x^2 - 11 + x &= 45 && \propto -45 \\ x^2 - 11 + x - 45 &= 0 && \text{[Correct} \\ x^2 + x - 56 &= 0 && \text{[Factor} \\ (x + 8)(x - 7) &= 0 \end{aligned}$$

Therefore, $x = -8$

and $x = 7$

III: THE CHECK:

We reject -8 since we are dealing with positive numbers. The larger number is 7 and the smaller is 4. We now check these results to see whether they satisfy the condition of the problem. We see that the square of the larger number $7^2 = 49$ minus the smaller number 4, equals 45.

Please proceed to question 11 below.

Question 11

Apply the proper principle and select the letter which labels the correct statement. Bill is now y years old, while his father is y^2 years old. In nine years from now, the father's age will be three times Bill's age at that time.

(A) Bill's present age is 7 years.

(B) The sum of the father's and son's present ages is 45 years.

(C) Bill's father is 30 years older than Bill.

(D) None of these.

X

$\frac{92}{1}$

If x denotes the first of two consecutive integers, the second is denoted by

$$(x + 1)$$

Since it was given that the sum of the squares = 61

the equation that will solve this problem is.

$$x^2 + (x + 1)2 = 61$$

Please return to page $\frac{105}{2}$ and try question 9 again.

$\frac{92}{2}$

We don't agree.

If we denote the son's age by x , the father's age is x^2 .

In four years, the son will be

$$(x + 4)$$

years old, and the father will be

$$(x^2 + 4)$$

years old. Please read the problem again, and try to write an equation.

Please return to page $\frac{100}{2}$ and try question 16 again.

Very good. Your choice is correct.

Since the perimeter is 54 feet, one-half the perimeter is 27 feet.

But the length plus the width of the rectangle equals one-half the perimeter.

Therefore, the length is 27 minus the width.

Please proceed to question 3 below.

Question 3

Recognize and select the expression which correctly completes the statement:

If the sum of the ages of Jack and his father is 59 years, and Jack's age is represented by y years, then the father's age is represented by,

(A) $\frac{59}{y}$ years

(B) y - 59 years

(C) 59y years

(D) 59 - y years

X

$\frac{94}{1}$

You started out well, but you seem to have become confused. Are you using the letter x to represent the length, and $22 - x$ to represent the width of the rectangle? That is correct.

But, what rule did you use to get the area of the rectangle?

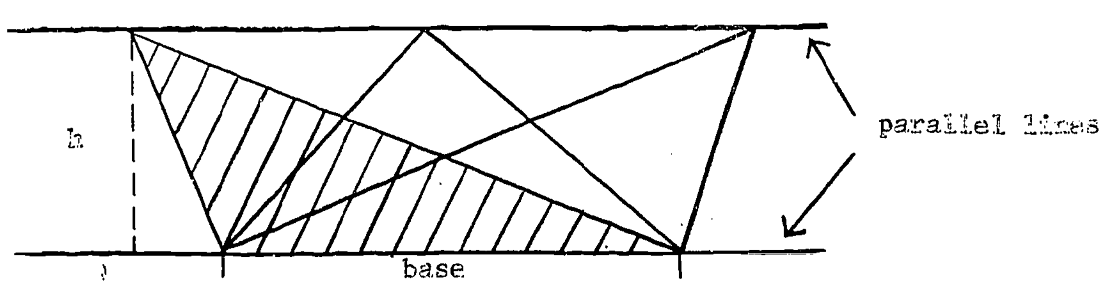
This choice is not correct.

Please return to page $\frac{82}{2}$ and try question 7 again.

$\frac{94}{2}$

No, the triangle need not be isosceles. You took a guess.

Actually, a triangle having a certain base, height, and area can have many shapes.



Please return to page $\frac{104}{2}$ and try question 12 again.

Did you check your solution to the quadratic equation?

If the smaller number is 23 the larger must be 24 .

However, the product of 23 and 24 does not equal 462.

Please return to page $\frac{107}{2}$ and try question 15 again.

You could have discovered for yourself that you did not make the correct choice. If there are 30 coins in all in the piggy bank, and 20 of these are quarters, then 10 coins must be nickels. Now,

20 quarters = 20 (.25) = \$5.00

10 nickels = 10 (0.5) = 0.50

However, the square of .50 does not equal \$5.00.

Please return to page $\frac{117}{2}$ and try question 17 again.

X

$\frac{96}{1}$

Very good! You made the correct choice. We have the equation,

$$\frac{n}{2}(n - 3) = 27$$ Multiplying both sides of the equation by 2,

we get $n(n - 3) = 54$. Clearing parentheses, we have

$$n^2 - 3n = 54, \text{ or } n^2 - 3n - 54 = 0 .$$

Factoring the trinomial, we obtain, $(n - 9)(n + 6) = 0$. Whence, $n = 9$, $n = -6$. Rejecting the negative answer, since it does not make sense to talk about a negative number of diagonals, we have $n = 9$ as the answer.

Please proceed to question 14 below.

$\frac{96}{2}$

Question 14

Apply the proper principle and select the letter which labels the correct statement. The sum of the first n consecutive integers, 1 , 2 , 3 ....,n is given by the formula, $\frac{1}{2}n(n + 1)$. How many of these consecutive integers must be added to obtain a sum of 703 ?

(A) 40 integers

(B) More than 38 , but less than 40 .

(C) 73 integers

(D) More than 35 , but less than 38 .

X

Very good You made the correct choice. Two consecutive integers can be represented by x and x + 1 .

The sum of the squares of these integers is 61 . We, therefore, have the equation,

$$x^2 + (x+1)^2 = 61 \qquad \text{[Square the binomial}$$

$$x^2 + x^2 + 2x + 1 = 61 \qquad \text{[Collect}$$

$$2x^2 + 2x + 1 = 61 \qquad \propto -61$$

$$2x^2 + 2x - 60 = 0 \qquad \propto \div 2$$

$$x^2 + x - 30 = 0 \qquad \text{[Factor}$$

$$(x+6)(x-5) = 0$$

therefore, $x = -6$

and $x = 5$.

Since the problem states that the integers must be positive, we reject the negative answer. Thus, $x = 5$ and $x + 1 = 6$. Checking, we find that

$$5^2 + 6^2 = 25 + 36$$
$$= 61$$

Please proceed to question 10 below.

Question 10

Apply the proper principle and select the letter which labels the correct statement. The sum of two positive integers is 11 . The square of the larger number minus the smaller number is 45 .

(A) The product of the two numbers is less than 25 .

(B) The difference between the two numbers is 3 .

(C) The larger number is 8 .

(D) The smaller number is 5 .

X

$\frac{98}{1}$

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Construct a chart to organize your thinking.

	age now	age in 9 years
Bill		
Father		

Please return to page $\frac{91}{2}$ and try question 11 again.

$\frac{98}{2}$

If the smaller number is 8 then the larger one is 9 .

If we add 79 to the square of 9 we get

$$79 + 81 = 160$$

But, 160 does not equal to twice the square of 8 .

Hence, your choice does not check.

Please return to page $\frac{123}{2}$ and try question 18 again.

Very good. You made the correct choice.

If the perimeter of the rectangle is 44 inches, then the sum of one length and one width is 22 inches. If we

Let x = the length of the rectangle

then $22 - x$ = the width.

The area of a rectangle is the product of its length and width; that is,

$$x(22 - x)$$

We are told that this area is 117 square inches. Hence, we have the equation,

$$x(22 - x) = 117$$

Please proceed to the next question below.

Question 8

Apply the proper principle and select the letter which labels the correct statement. The perimeter of a certain rectangle is 26 inches, and its area is 36 inches. The dimensions of this rectangle are:

(A) length is 7 inches, width is 6 inches.

(B) The longer side is less than 8 inches.

(C) The shorter side is greater than 5 inches.

(D) The longer side exceeds twice the shorter side by 1 inch.

X

100
1

Very good. You r choice.

I: THE

We proceed as follows:

Let x = the first number,

$x + 1$ = the successor

II: THE RELATIONSHIP:

We are told that the product of these two numbers is 462 .

Hence, we have the equation,

$$
\begin{aligned}
x(x + 1) &= 462 \qquad [D \\
x^2 + x &= 462 \qquad \propto- 462 \\
x^2 + x - 462 &= 0 \\
(x - 21)(x + 22) &= 0 \\
x = 21 \qquad x &= -22
\end{aligned}
$$

Since the problem calls for positive integers only, we reject -22 , and the numbers are 21 and 22 .

III: THE CHECK:

$21 \times 22 = 462$

Please proceed to question 16 below.

100
2

Question 16

Apply the proper principle and select the letter which labels the correct statement.

Mr. Jones' present age is equal to the square of his son's age now. In four years, Mr. Jones will be four times as old as his son will be then.

(A) Their present ages are 30 and 6 .

(B) The sum of the present ages of father and son is 40 years.

(C) The difference of the present ages of father and son is 30 years.

(D) The father is now 7 times as old as his son is now.

X

You did the difficult part of the problem correctly, but you forgot the formula for the area of a triangle. I right triangle, the formula can be reformed from the basic formula

$$\text{area of triangle} = \frac{1}{2}\text{ base} \times \text{height}$$

to

$$\text{area of right triangle} = \frac{1}{2}\text{ leg}_1 \times \text{leg}_2$$

Return to page $\frac{109}{2}$ and try question 20 again.

Any number divided by itself is 1 . Then this fraction is defined.

Return to page $\frac{124}{2}$ and try question 1 again.

X

$\frac{102}{1}$

If you took the trouble to check your answer, you would have found out for yourself that you made an incorrect choice.

Please return to page $\frac{117}{2}$ and try question 17 again.

$\frac{102}{2}$

We don't agree. When we write

$$5 \div 3$$

we mean that the number 5 is divided by the number 3 . As a fraction, this is written as

$$\frac{5}{3}$$

Similarly,

$$(3x + y) \div 7z$$

is the fraction

$$\frac{3x + y}{7z}$$

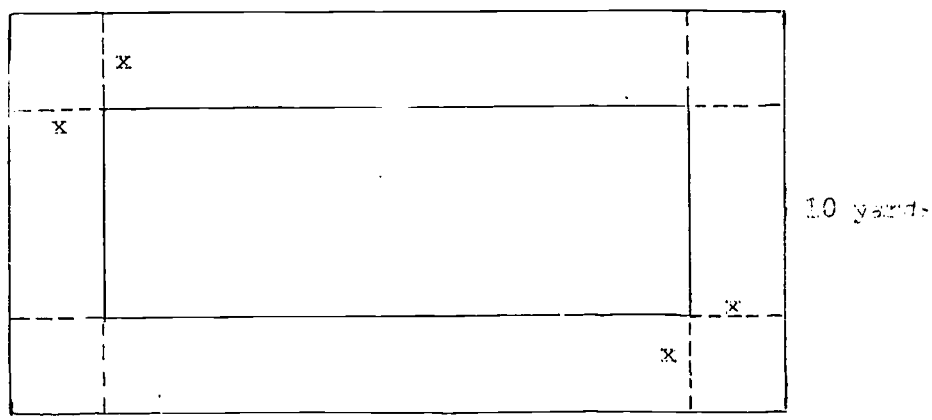
Notice that the expression before the division sign is the numerator of the fraction.

Please return to page $\frac{125}{2}$ and try question 4 again.

X

In a problem of this type, it is very helpful to make a sketch.

Examine the sketch drawn below.



Note that if we let

$$x = \text{the width of the walk,}$$

the dimensions of the inside rectangle are

$$(10 - 2x) \quad \text{and} \quad (20 - 2x)$$

Can you continue from here?

Please return to page $\frac{118}{2}$ and try question 19 again.

We don't agree. If you substitute the number 2 for a , the numerator will equal zero, but the denominator will not equal zero.

Please return to page $\frac{119}{2}$ and try question 2 again.

X

Very good. You made the correct choice.

I: THE VARIABLES:

We are told that the letter y represents Bill's age now, and y^2 represents Bill's father's age now. In 9 years, their ages will be $y + 9$ and $y^2 + 9$ respectively.

	Now	In 9 years
Bill	y	$y + 9$
Father	y^2	$y^2 + 9$

II: THE RELATIONSHIP:

Furthermore, in 9 years, the father will be three times as old as his son. Hence, we have the equation,

$$y^2 + 9 = 3(y + 9) \qquad [D$$

$$y^2 + 9 = 3y + 27 \qquad \propto - 3y - 27$$

$$y^2 + 9 - 3y - 27 = 0 \qquad [\text{ Collect}$$

$$y^2 - 3y - 18 = 0 \qquad [\text{ Factor}$$

$$(y - 6)(y + 3) = 0 \text{ and } y = 6 \quad y = -3$$

Rejecting the negative answer, we get $y = 6$ and

$$y^2 = 36$$

III: CHECK:

In 9 years Bill will be 15 years old and Bill's father will be 45 years old, and

$$45 = 3 \times 15$$

	Now	In 9 years
Bill	6	15
Father	36	45

Please proceed to question 12 below.

Question 12

Apply the proper principle and select the letter which labels the correct statement. The base of a triangle exceeds its height by 7 inches. The area of the triangle is 72 square inches.

(A) The base is -6 inches.

(B) The base plus the height is 27 inches.

(C) The height is less than 10 inches.

(D) The triangle is isosceles.

X

Very good. You made the correct choice.

We start as follows:

I: THE VARIABLES:

Let x = the length of the rectangle.

Then since the perimeter = 26 , the semi-perimeter = 13

Therefore, $13 - x$ = the width.

II: THE RELATIONSHIP:

Now, the area of a rectangle is the product of its length and width. Hence, we have the equation,

$$x(13 - x) = 36 \qquad [\ D$$

$$13x - x^2 = 36 \qquad \alpha\ -\ 36$$

$$13x - x^2 - 36 = 0 \qquad [\ C$$

$$-x^2 + 13x - 36 = 0 \qquad \alpha \cdot (-1)$$

$$x^2 - 13x + 36 = 0$$

We now factor the trinomial on the left side of the equality obtaining,

$$(x - 4)(x - 9) = 0$$

$x - 4 = 0$	$x - 9 = 0$
$x = 4$	$x = 9$

We usually consider the length to be the larger dimension.

Therefore, $x = 9$ is the length and $13 - x = 4$ is the width.

Please proceed to question 9 below.

Question 9

Apply the proper principle and select the letter which labels the correct statement. The sum of the squares of two consecutive positive integers is 61 .

(A) The larger integer is 5 .

(B) The product of the integers is 35 .

(C) The sum of the two integers is greater than 10 .

(D) The sum of the two integers is less than 10 .

X

$\frac{106}{1}$

After reading the problem carefully, you should begin by representing the unknowns. Thus, you should

let x = the son's present age,

x^2 = the father's pre[illegible]ge.

Now, the problem deals with the ages of father and son in four years. Therefore, you should represent algebraically their ages in four years from now. After you have done this, read the problem once more; this time your objective is to write an equation. Now that we have outlined what you should do, please do it.

Please return to page $\frac{100}{2}$ and try question 16 again.

$\frac{106}{2}$

We don't agree. If we denote the first number by x the successor is denoted by

$$(x + 1)$$

The square of the successor is

$$(x + 1)^2$$

Twice the square of the first number is $2x^2$. Can you write an equation?

Please return to page $\frac{123}{2}$ and try question 18 again.

Congratulations! You made the correct choice.

We write the equation,

$\frac{n}{2}(n + 1) = 703$ Multiplying each term by 2 we get,

$n(n + 1) = 1406$ Clearing parentheses and writing the equation in standard form we have,

$n^2 + n - 1406 = 0$ Factoring, we obtain,

$(n + 38)(n - 37) = 0$ Whence,

$n = -38 \quad n = 37$

Rejecting the negative answer, we have

$$n = 37$$

Please proceed to question 15 below.

Question 15

Apply the proper principle and select the letter which labels the correct statement. A positive number and its successor are multiplied and the product is found to be 462 .

(A) The sum of the numbers is 12 .

(B) The sum of the numbers is less than 40 .

(C) The sum of the numbers is greater than 40 .

(D) The smaller of the two numbers is 23 .

X

$\frac{108}{1}$

Sorry! You made the wrong choice.

You should have started this problem as follows:

Let

$$x = \text{the number of nickels in the piggy bank.}$$

$$30 - x = \text{the number of quarters in the piggy bank.}$$

Now, every nickel is equal to 5¢ . Hence, the value in pennies of x nickels is $5x$. Similarly, every quarter is equal to 25 pennies, and the value of

$$(30 - x)$$

quarters is

$$25 (30 - x)$$

Can you now write an equation?

Return to page $\frac{117}{2}$ and try question 17 again.

$\frac{108}{2}$

The expression

$$\frac{x - 0}{x}$$

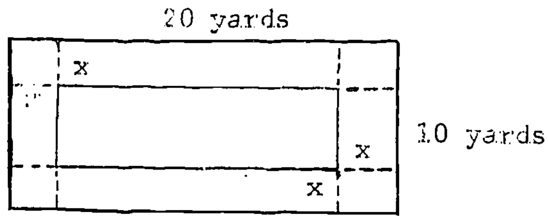
is the same as $\frac{x}{x}$

This is a number divided by itself, which is equal to 1 .

Then this choice is not correct.

Return to page $\frac{124}{2}$ and try question 1 again.

Very good. You made the correct choice. It's always a good idea to make a sketch.



I: THE VARIABLES:

If we let

x = the width of the walk, the dimensions of the inside rectangle are $(20 - 2x)$ and $(10 - 2x)$.

II: THE RELATIONSHIP:

The area of the inside rectangle is given as 96 square yards. We, therefore, can write the equation,

$(20 - 2x)(10 - 2x)$	$= 96$	[D
$200 - 60x + 4x^2$	$= 96$	$-$ 96 C
$4x^2 - 60x + 104$	$= 0$	$\div$ 4
$x^2 - 15x + 26$	$= 0$	[Factor
$(x - 13)(x - 2)$	$= 0$	and $x = 13$, $x = 2$

We have two positive answers. However, common sense tells us that we have to reject 13, since it is not possible to have a walk wider than the width of the entire plot.

III: CHECK: The dimensions of the inside rectangle are

$10 - 4 = 6$, and $20 - 4 = 16$, and $6 \times 16 = 96$

Please proceed to question 20 below.

109
2

Question 20

Apply the proper principle and select the letter which labels the correct statement.

The sum of two legs of a right triangle is 23". The sum of the squares of these legs is 289". Find the area of this right triangle.

(A) The area is 120 square inches. (C) The area is 51 square inches.

(B) The area is 60 square inches. (D) None of these.

X

110
1

Very good. You made the correct choice.

If we let x = the height of the triangle, then $x + 7$ = its base. Now, the area of a triangle is equal to one-half the product of its base and height, or better still, the product of the base and height of a triangle equals twice its area. Hence, we have the equation,

$$x(x + 7) = 2(72) \quad [\ D$$

$$x^2 + 7x = 144 \quad - 144$$

$$x^2 + 7x - 144 = 0 \quad [\ \text{Factor}$$

$$(x + 16)(x - 9) = 0$$

$$x = -16, \ x = 9 .$$

Rejecting the negative answer, we have the height, $x = 9$ and the base $x + 7 = 16$.

Check:

$$\text{The area} = \frac{1}{2} (\text{base}) (\text{height})$$

$$= \frac{1}{2} (16) \quad (9)$$

$$= 8 \quad (9)$$

$$= 72$$

Please proceed to question 13 below.

110
2

Question 13

Apply the proper principle and select the letter which labels the correct statement. The number of diagonals that can be drawn in a polygon of n sides is given by the formula, $\frac{n}{2}(n - 3)$. A certain polygon has 27 diagonals; how many sides does it have?

(A) 13 sides

(B) 10 sides

(C) more than 10 sides, but less than 13

(D) None of these

X

In order for the father to be seven times as old as his son, and at the same time be equal in age to the square of his son's age, the father would have to be 49 years old and his son would have to be 7 years old.

Please check these answers.

Please return to page $\frac{100}{2}$ and try question 16 again.

The important point to notice is not the value of the letter, but the value of the denominator. A fraction is not defined when its denominator is zero.

This choice is not correct.

Please return to page $\frac{119}{2}$ and try question 2 again.

X

$\frac{112}{1}$

Sorry. One of the letters does have the correct answer next to it. Please reconsider your choice.

Set up the values of the variables and substitute them in the second part of the problem to create the equation.

Return to page $\frac{123}{2}$ and try question 18 again.

$\frac{112}{2}$

This choice indicates a misconception about the meaning of the division sign.

When we write

$$27 \overline{\smash{)}514}$$

we mean that 514 is divided by 27 . Thus the following expressions are equivalent:

$$27 \overline{\smash{)}514} \quad , \quad 514 \div 27 \quad \text{and} \quad \frac{514}{27}$$

Return to page $\frac{125}{2}$ and try question 4 again.

The expression

$$\frac{x - x}{x}$$

is the same as

$$\frac{0}{x}$$

Zero divided by any non-zero number is equal to zero. Therefore, this choice is not correct.

Please return to page $\frac{124}{2}$ and try question 1 again.

We don't agree. Notice that the numerator and the denominator are exactly alike. We, therefore, have a number divided by itself.

Please return to page $\frac{122}{2}$ and try question 6 again.

X

$\frac{114}{1}$

We don't agree. The only time that a fraction is not defined is when the denominator is equal to zero.

Please return to page $\frac{131}{2}$ and try question 3 again.

$\frac{114}{2}$

We don't agree. Both the numerator and denominator can be divided by the same value.

Remember:

$$\frac{abc}{db} = \frac{a}{d} \times \frac{b}{b} \times \frac{c}{1}$$

Please return to page $\frac{136}{1}$ and try question 8 again.

X

You must have made an error in factoring the quadratic equation you obtained for the solution of this problem. You should have checked your results.

Please go over your work.

Return to page $\frac{109}{2}$ and try question 20 again.

You did not read the question very carefully.

You were asked to find what you divided by, not what the result of the division was.

Return to page $\frac{149}{2}$ and try question 10 again.

X

... late the value of a fraction when the numerical value of the ... given, we replace the letter by this numerical value and ... e indicated operations. Since you have obtained an incorrect ... have made some error in your calculation.

... find it?

Pl... eturn to page $\frac{138}{1}$ and try question 5 again.

146

... ut you did not make the correct choice. Think of the numerator ... product of

$$15 \quad , \quad a^3 \quad , \quad \text{and} \quad b^2$$

Think of the denominator as the product of

$$3 \quad , \quad a \quad , \quad b^2 \quad , \quad \text{and} \quad b^2$$

Thus we have:

$$\frac{(15)(a^3)(b^2)}{(3)(a)(b^2)(b^2)} = \frac{15}{3} \quad \frac{a^3}{a} \quad \frac{b^2}{b^2} \quad \frac{1}{b^2}$$

Now work from left to right, dividing as you go.

Please return to page $\frac{141}{2}$ and try question 7 again.

117
1

Very good. You made the correct ch[illegible] We proceed as follows:

	Present age	In – years
son	x	+ 4
father	x^2	+ 4

We are told that in four years the father will be four times his son's age at that time. Hence, we have the equation,

$$x^2 + 4 = 4\ (x + 4) \qquad [\ D$$

$$x^2 + 4 = 4x + 16 \qquad [\ -4x \quad -16$$

$$x^2 + 4 - 4x - 16 = 0\ , \qquad [\ C$$

$$x^2 - 4x - 12 = 0\ , \qquad [\ \text{Factor}$$

$$(x - 6)\ (x + 2) = 0\ , \text{ and } x = 6\ , \ x = -2\ .$$

Rejecting the negative age, we have $x = 6$, $x^2 = 36$, as the ages of son and father respectively.

Check: In four years, the son will be 10 years old and the father will be 40 years old, and 40 = 4 (10).

Please proceed to question 17 below.

117
2

Question 17

Apply the proper principle and select the letter which labels the correct statement. A boy has quarters and nickels in his piggy bank, 30 coins in all. The number of the value of the quarters is equal to the square of the number of the value of the nickels.

(A) The boy has twenty quarters in the bank,

(B) The boy has two nickels in the bank.

(C) [illegible] r times as many quarters as nickels.

(D) The number of quarters is greater than [illegible] but less than [illegible] .

Ve good You made the correct ch... Let us do th's problem together.

I ...E ...RIABLES:

Let $=$ the first number,

$+ 1 =$ the successor ... , then

$+ 1)^2 =$ the square of the successor of x .

II. ... ...LATIONSHIP:

... twice ... e square of the first number. We, therefore, obtain the ... on,

... the square of the first number $=$ 79 more than the square of the ... sor .

$\ldots = (x + 1)^2 - 79$ [Squa... the binomial

$\ldots = x^2 + 2x + 1 + 7\ldots$ $-x^2 \quad -2x - 80$

$- 2x - 80 = 0$ [Fact...r

$(x - \ldots)(x + 8) = 0$ and $x = 1\ldots$, $x = -8$.

... required integers are 1... and 11 .

III. CHECK:

$$2\,(10)^2 \stackrel{?}{=} 79 + (1\ldots)^2$$

$$2\,(100) \stackrel{?}{=} 79 + 121$$

$$200 = 200$$

Proceed to question 19 below.

$\frac{118}{2}$

Question 19

Apply the proper principle and select the letter which labels the correct statement. Within a garden 10 yards wide and 20 yards long, we wish to pave a walk of uniform ... th so as to leave an area of 96 square yards for flowe... . How wide should the walk be?

(A) The width ... the walk should be 3 yards.

(B) The width ... the walk should be 13 yards.

(C) The width ... the walk should be 4 ... ards.

(D) The width ... the walk should be 2 yards.

Very good. You made the correct choice.

The expression,

$$\frac{x}{x - x} = \frac{x}{0}$$

Division by zero is not a defined operation.

Please proceed to question 2 which follows.

Question 2

Perform the necessary calculation to find the value of the letter for which the fraction,

$$\frac{5a - 10}{a + 2}$$

is not defined. Select the letter next to the correct answer.

(A) $a = 2$

(B) $a = -2$

(C) $a = 0$

(D) None of these.

$\frac{120}{1}$

We don't agree. One of the letters does have the correct answer next to it. Please reconsider your choice.

(1) Set up the variables carefully.

(2) Substitute the variables in the equation, " the sum of the squares of these legs is 289 . "

And then,

(3) Substitute the values for the legs in the formula for the area of a right triangle.

Return to page $\frac{109}{2}$ and try question 20 again.

$\frac{120}{2}$

You are very close to being correct.

Can you find the oversight which you made?

Return to page $\frac{131}{2}$ and try question 3 again.

Examine the numerator of the fraction that you chose. Can it be factored? After removing the common monomial factor in the numerator of this fraction, you will observe that the numerator and denominator have a common factor.

$$\frac{ab + ac}{b + c} = \frac{a(b + c)}{b + c} = a \cdot \frac{(b + c)}{(b + c)}$$

Please return to page $\frac{136}{1}$ and try question 8 again.

We don't agree.

The procedure is to factor both numerator and denominator, then the same factor, if there is one, can be eliminated from both by division.

The denominator of the given fraction does not have x as a common factor.

Please return to page $\frac{149}{2}$ and try question 10 again.

$\frac{122}{1}$

Very good ... made the correct choice.

Replacing the letter y by 3 in the given fraction, we have:

$$\frac{9y - 18}{y^2 + 9y + 14} = \frac{9(3) - 18}{(3)^2 + 9(3) + 14}$$

$$= \frac{27 - 18}{9 + 27 + 14}$$

$$= \frac{9}{50}$$

Please proceed to question 6 below.

$\frac{122}{2}$

Question 6

Apply the proper principle and find the value of the fraction

$$\frac{3x + 1}{3x + 1}$$

when x = 0 . Select the letter which labels the correct value.

(A) 0

(B) 1

(C) 2

(D) Cannot be determined.

Very good. You made the correct choice. Here is how this problem could be solved.

I: THE VARIABLES:

Let x = the number of nickels, then

$30 - x$ = the number of quarters,

$5x$ = the value expressed in pennies of the nickels,

$25\ (30 - x)$ = the value expressed in pennies of the quarters.

II: THE RELATIONSHIP:

The problem states that the square of the value of the nickels is equal to the value of the quarters. Writing the above algebraically, we get the equation,

$$(5x)^2 = 25\ (30 - x) \qquad [\ \text{Distribution}$$
$$25x^2 = 750 - 25x \qquad \propto +25x - 750$$
$$25x^2 + 25x - 750 = 0 \qquad \propto \div 25$$
$$x^2 + x - 30 = 0 \qquad [\ \text{Factor}$$
$$(x + 6)\ (x - 5) = 0\ ,\ \text{and } x = -6\ ,\ x = 5.$$

Since it does not make sense to talk about a negative number of nickels in a piggy bank, we reject the negative answer. Thus, we have 5 nickels and 25 quarters.

III: CHECK:

The value of 25 quarters is \$6.25 , and the value of 5 nickels is 25¢; and $(25)^2 = 625$.

Proceed to question 18 below.

Question 18

Apply the proper principle and select the letter which labels the correct statement. Adding 79 to the square of the successor of a positive integer, yields a result that is twice the square of the integer.

(A) The smaller integer is 8 .

(B) The sum of the 2 integers is 21 .

(C) The product of the 2 integers is 80 .

(D) None of these.

X

VOLUME 10 SEGMENT 4 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0	4	(Sequence Number)
	54	and	56	0	4	(Type of Punch card)
	60	and	62	1	0	(Volume Number)
	66	and	68	0	4	(Segment Number)

In the previous segment you discovered that although you can start with a correct equation and solve it according to the algebraic laws, it was necessary to reject some answers due to the conditions of the original verbal problem. Usually this meant that a negative answer in a problem involving the dimensions of a geometric figure would have to be rejected.

In the first part of this segment you will investigate another restriction that occurs in algebra, the "impossible fraction." Later on you will learn how to reduce a fraction to lowest terms and this technique of simplification will make your work easier.

Your READING ASSIGNMENT for this segment is pages 281 - 284 . You will now be asked a series of questions to draw your attention to the more important points.

124/1

Question 1

If x is any number other than zero, recognize which of the following fractions is not defined. Select the letter which labels the correct answer.

(A) $\frac{x}{x}$ (B) $\frac{x - 0}{x}$ (C) $\frac{x - x}{x}$ (D) $\frac{x}{x - x}$

Very good. You made the correct choice.

To determine the values of the variable for which the fraction is not defined, we set the denominator equal to zero. Thus, we have:

$$9 - x^2 = 0 \quad [\text{ Factor}$$
$$\underline{(3 + x)(3 - x) = 0}$$

$3 + x = 0$	$3 - x = 0$
$x = -3$	$3 = x$

Please proceed to question 4 below.

Question 4

Apply the proper principle and write the expressions,

I. $(x^3 + 4) \div 3x$

II. $x - 3\overline{\smash{)}\, x^2 - 5x + 10}$

as algebraic fractions. Select the letter which labels the correct result.

(A) I. $\dfrac{3x}{x^3 + 4}$ II. $\dfrac{x^2 - 5x + 10}{x - 3}$

(B) I. $\dfrac{x^3 + 4}{3x}$ II. $\dfrac{x - 3}{x^2 - 5x + 10}$

(C) I. $\dfrac{3x}{x^3 + 4}$ II. $\dfrac{x^2 - 5x + 10}{x - 3}$

(D) I. $\dfrac{x^3 + 4}{3x}$ II. $\dfrac{x^2 - 5x + 10}{x - 3}$

$\frac{126}{1}$

Sorry. One of the letters does have a value of the letter a for which the fraction is not defined.

Return to page $\frac{119}{2}$ and try question 2 again.

$\frac{126}{2}$

You have forgotten the rule for the quotient of powers.

When dividing powers with the same base, we keep the base and subtract the exponents.

$$\boxed{\frac{x^m}{x^n} = x^{m-n}}$$

Return to page $\frac{141}{2}$ and try question 7 again.

Did you divide the individual terms in the numerator by the individual terms in the denominator? This is wrong. If the numerator and denominator are each written as a product of factors, then we can divide by like factors to reduce the fraction. This is a very important point and an illustration is in order. Consider the fraction:

$$\frac{6x^2 - 6}{6x^2 + 12x + 6}$$

As the fraction stands, we cannot cancel any term in the numerator with any term in the denominator even though the temptation to do so exists. However, we can factor and then reduce as follows.

$$\frac{\cancel{6}\,\cancel{(x+1)}\,(x-1)}{\cancel{6}\,\cancel{(x+1)}\,(x+1)} = \frac{x-1}{x+1}$$

Return to page $\frac{122}{2}$ and try question 6 again.

You have found a common factor. But it is not the <u>greatest</u> common factor. What have you left out?

Please return to page $\frac{135}{2}$ and try question 11 again.

X

Very good. You made the correct choice. Let us go over the solution of this problem together.

I: THE VARIABLES:

Let x = the length of one leg of the right triangle, then
$23 - x$ = the length of the other leg.

II: THE RELATIONSHIP:

We are told that the sum of the squares of the legs is equal to 289 .
We, therefore, have the equation,

$$\begin{array}{lcll}
x^2 + (23 - x)^2 & = & 289 & \text{[Square} \\
x^2 + 529 - 46x + x^2 & = & 289 & \propto\!< - 289 \\
2x^2 + 529 - 46x - 289 & = & 0 & \text{[Combine} \\
2x^2 - 46x + 240 & = & 0 & \propto\!< \div 2 \\
x^2 - 23x + 120 & = & 0 & \text{[Factor} \\
(x - 15)(x - 8) & = & 0 & \text{and} \\
x = 15, \text{ or } x = 8 & & &
\end{array}$$

Hence, the legs of the right triangle are equal to 8 and 15 .
The area of a right triangle is equal to one-half the product of its legs.
Hence, the area is 60 .

You have now finished this Segment. Hand in your PUNCH CARD. You should have entered in your NOTEBOOK the following definitions and formulas:

1. If the sum of two numbers is N , and one of the numbers is denoted by x , the other number is denoted by $N - x$.
2. A verbal problem that leads to a quadratic equation will, in general, have two solutions, one of which may have to be rejected. The nature of the problem will indicate which solution should be rejected.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT:

Problems 9 , 10 , 11 , and 12 .

This is the correct answer. There are no common factors in the numerator and denominator.

Please proceed to question 9 which follows.

Question 9

Choose the process which can be used to transform the fraction

$$\frac{6x^2 - 12x}{x^2 - 4}$$

into an equivalent fraction.

(A) Subtract the x^2 in both numerator and denominator

(B) Divide the numerator by $6x$

(C) Multiply numerator and denominator by 5

(D) None of these

We don't agree. The prime factors of 35 are 7 and 5 . What are the factors of 63 ? The fraction that you chose can be simplified.

Please return to page $\frac{146}{2}$ and try question 12 again.

X

$\frac{130}{1}$

You did reduce the fraction but not far enough.

Since both numerator and denominator of this choice can be divided by the same quantity, it is not in lowest terms.

That is if $\frac{p}{q} = \frac{xy}{zy}$

$$= \frac{x}{z} \cdot \frac{y}{y}$$

$$= \frac{x}{z} \cdot 1$$

$$= \frac{x}{z}$$

Please return to page $\frac{141}{2}$ and try question 7 again.

$\frac{130}{2}$

Did you cancel individual terms in the numerator with individual terms in the denominator? That is wrong. It is necessary to factor the numerator and the denominator first, and then to divide like factors.

Please return to page $\frac{156}{1}$ and try question 13 again.

X

Since this value makes the denominator zero, this choice is correct. Remember, a fraction is not defined if its denominator has the value zero.

Please proceed to question 3 below.

Question 3

Perform the necessary calculation to find the value or values of the variable for which the fraction

$$\frac{4 - x}{9 - x^2}$$

is not defined. Select the letter which labels the correct statement.

(A) $x = 4$

(B) $x = +3$

(C) $x = 3, x = -3$

(D) $x = 0, x = 9$

X

Even though you do not know the value of x, it is possible to determine the value of the fraction.

Please return to page $\frac{122}{2}$ and try question 6 again.

You factored the numerator, but you failed to factor the denominator. The procedure used to simplify a fraction is to factor both the numerator and the denominator and then divide by any factors common to both.

Please return to page $\frac{148}{2}$ and try question 14 again.

X

$\frac{133}{1}$

This choice is incorrect.

The denominator of a fraction is the dividend. Thus, for example,

$$(y^2 - 4) \div (y - 2)$$

is the same as

$$\frac{y^2 - 4}{y - 2}$$

Furthermore, $z - 3\overline{)z^2 - 9}$

is the same as $\frac{z^2 - 9}{z - 3}$

Please return to page $\frac{125}{2}$ and try question 4 again.

$\frac{133}{2}$

Sorry. You made an incorrect choice.

Both the numerator and the denominator of this fraction are trinomials which can be factored. It is possible that you experienced some trouble in finding the factors. Please refer to your NOTEBOOK on the topic of factoring trinomials.

Pay close attention to the signs in the binomial factors.

Please return to page $\frac{158}{2}$ and try question 17 again.

X

$\frac{134}{1}$

Sorry, this choice is not correct. In order to transform a fraction only multiplication or division can be used.

Adding the same value to the numerator and denominator or subtracting changes the value of the fraction.

$$\frac{a}{b} \neq \frac{a \div k}{b + k}$$

If it were true that $\frac{a}{b} = \frac{a + k}{b - k}$

then,
$$a\,(b + k) = b\,(a + k)$$
$$ab + ak = ba + bk \quad \propto - ab$$
$$ak = bk \quad \propto \div k$$
$$a = b$$

This is, it is true only when $a = b$.

Please return to page $\frac{129}{1}$ and try question 9 again.

$\frac{134}{2}$

Your answer is not in lowest terms. If you examine the numerator, you will see that it is the difference of two squares. Factor this numerator and you will observe that further simplification is possible.

Please return to page $\frac{154}{2}$ and try question 15 again.

X

Very good. This choice is correct.

The procedure is to factor first, and then divide out any factors common to both numerator and denominator. Then we have:

$$\frac{x^2 - 5x}{x^2 - 25} = \frac{x(x - 5)}{(x - 5)(x + 5)}$$

Since the factor common to numerator and denominator is $(x - 5)$ this is the quantity we use as a divisor.

Please proceed to question 11 below.

Question 11

Apply the proper principle and find the greatest factor common to both numerator and denominator of the fraction:

$$\frac{6x^2 - 6x - 36}{2x^2 + 8x + 8}$$

(A) 2

(B) $x + 2$

(C) $2(x + 2)$

(D) $2x^2 + 2x + 4$

X

$\frac{136}{1}$

This is the correct answer.

Please go on to question 8 which follows.

Question 8

Apply the proper principle and select the letter next to which the fraction is in lowest terms.

(A) $\frac{3xy}{2x}$

(B) $\frac{21ab^2}{11c}$

(C) $\frac{5s^2t}{15t^2}$

(D) $\frac{7x + 7y}{x + y}$

$\frac{136}{2}$

We don't agree. You seem to be quite mixed up.

There should be a division in your work. Did you divide 18n by 9n? You should not have done that, but if you did, you would not get 2n as a result.

Please return to page $\frac{156}{1}$ and try question 13 again.

X

You have forgotten the rule for dividing powers. Recall that

$$\frac{x^7}{x^3} = x^{7-3} = x^4$$

Furthermore, keep in mind that a letter written without any exponent has the exponent 1 .

Please return to page $\frac{146}{2}$ and try question 12 again.

What are the factors of

$$n^2 + 5n - 6$$

One of the factors is <u>not</u> $(n - 6)$.

This choice is not correct.

Please return to page $\frac{147}{2}$ and try question 18 again.

X

$\frac{138}{1}$

This is the correct answer. Please go on to the next question, which follows.

Question 5

Perform the necessary calculation to find the value of the fraction

$$\frac{9y - 18}{y^2 + 9y + 14} \quad \text{when } y = 3$$

Select the letter which labels the correct answer.

(A) $\frac{9}{50}$

(B) $\frac{9}{47}$

(C) $\frac{9}{44}$

(D) $\frac{9}{68}$

$\frac{138}{2}$

Remember that

$$a^2 - b^2$$

is the difference of two squares.

You should know the factors for the difference of two squares.

Please return to page $\frac{148}{2}$ and try question 14 again.

X

In order to transform a fraction by division, it is necessary to divide both numerator and denominator by the same quantity.

That is,

$$\frac{p}{q} = \frac{p}{q} \times 1$$

$$= \frac{p}{q} \times \frac{\frac{1}{k}}{\frac{1}{k}} = \frac{\frac{p}{k}}{\frac{q}{k}}$$

Return to page $\frac{129}{1}$ and try question 9 again.

The numerator is a trinomial square. The factors of this trinomial are

$$(x - 2)(x - 2)$$

The denominator of the fraction is the difference of two squares and its factors are

$$(2 - x)(2 + x)$$

It would be nice if $(x - 2)$

were a factor of the denominator instead of

$$(2 - x)$$

How can we accomplish this?

Return to page $\frac{159}{2}$ and try question 19 again.

X

$\frac{140}{1}$

How did you arrive at this result?

Did you forget to factor the entire numerator and the entire denominator?

Please return to page $\frac{135}{2}$ and try question 11 again.

$\frac{140}{2}$

Sorry. You made the wrong choice.

Let us do a similar problem. Reduce

$$\frac{y^2 - 36}{3y + 18}$$

to lowest terms. The numerator of the fraction is the difference of two squares, and can be factored. The denominator has the common monomial factor, 3 , which can be factored out. Then we have:

$$\frac{y^2 - 36}{3y + 18} = \frac{\overset{1}{\cancel{(y + 6)}}(y - 6)}{3\underset{1}{\cancel{(y + 6)}}}$$

$$= \frac{y - 6}{3}$$

Please return to page $\frac{154}{2}$ and try question 15 again.

X

Since $x > 0$ the numerator and denominatror have the same value and they are positive. But any number divided by itself equals 1 .

Therefore, this choice is correct.

Please proceed to question 7 below.

Question 7

Apply the proper principle and write the fraction

$$\frac{15a^3b^2}{3ab^4}$$

in lowest terms. Select the letter which labels the correct answer.

(A) $\frac{5a^3b^2}{ab^4}$

(B) $\frac{5a^2b}{b^3}$

(C) $5a^2b^{\frac{1}{2}}$

(D) $\frac{5a^2}{b^2}$

$\frac{142}{1}$

One of the letters does have the correct answer next to it. Start by factoring the numerator and the denominator.

Remember,

$$\frac{ab}{ac} = \frac{b}{c}$$

Return to page $\frac{156}{1}$ and try question 13 again.

$\frac{142}{2}$

We don't agree. Please check the factors of each of the trinomials by multiplying them to see whether their product equals the trinomial.

Return to page $\frac{147}{2}$ and try question 18 again.

We don't agree.

One of the letters does have the correct answer next to it.

Return to page $\frac{148}{2}$ and try question 14 again.

Your answer is not in lowest terms.

You " cancelled " the 5's but it is possible to factor the denominator and find other factors which are common to both the numerator and denominator. As long as the numerator and the denominator can both be divided by the same number, your answer can be reduced.

Return to page $\frac{160}{2}$ and try question 1 again.

X

$\frac{144}{1}$

Perhaps you were too hasty. One of the other choices is correct.

Please return to page $\frac{129}{1}$ and try question 9 again.

$\frac{144}{2}$

If you examine the terms in the numerator, you will notice that it is in descending powers of y. The denominator is in ascending powers of y. How can we change the denominator so that it starts with plus y^2 and is in descending powers of the letter? This should suggest the proper procedure.

Please return to page $\frac{153}{2}$ and try question 20 again.

X

Very good. This choice is correct.

The work should look this way:

$$\frac{s^2 - 16}{3s + 12} = \frac{\overset{1}{\cancel{(s+4)}}(s-4)}{3\underset{1}{\cancel{(s+4)}}} \quad \left[\text{Divide by } (s+4)\right.$$

$$= \frac{s-4}{3}$$

Please proceed to question 16 below.

Question 16

Apply the proper principle and reduce the fraction,

$$\frac{z^3 - z}{z^2 - 2z - 3}$$

to lowest terms. Select the letter which labels the correct result.

(A) $\frac{z(z-1)}{z-3}$

(B) $\frac{z-1}{z+2}$

(C) $\frac{z+1}{z+3}$

(D) $\frac{z(z+1)}{z+3}$

$\frac{146}{1}$

We should proceed as follows:

$$\frac{6x^2 - 6x - 36}{2x^2 + 8x + 8} = \frac{6(x^2 - x - 6)}{2(x^2 + 4x + 4)}$$

$$= \frac{6(x - 3)(x + 2)}{2(x + 2)(x + 2)}$$

Since the 6 in the numerator is 2 times 3, the greatest common factor is

$$2(x + 2)$$

and this choice is correct.

Please proceed to question 12 below.

$\frac{146}{2}$

<u>Question 12</u>

Apply the proper principle and reduce the fraction

$$\frac{-35x^2y^3}{63xy^3}$$

to lowest terms. Select the letter which labels the correct result.

(A) $\frac{-35x}{63}$

(B) $\frac{-5x^2}{9}$

(C) $\frac{-5x}{9}$

(D) $\frac{7x}{9}$

Very good. You made the correct choice.

The procedure is to factor both the numerator and denominator, as follows:

$$\frac{p^2 - 2p - 8}{p^2 - p - 6} = \frac{(p - 4)\overset{1}{\cancel{(p + 2)}}}{(p - 3)\underset{1}{\cancel{(p + 2)}}}$$

$$= \frac{p - 4}{p - 3}$$

Please proceed to question 18 below.

$\frac{147}{2}$

Question 18

Apply the proper principle and reduce the fraction,

$$\frac{n^2 + 5n - 6}{n^2 - 2n + 1}$$

to its lowest terms. Select the letter which labels the correct result.

(A) $\frac{n + 6}{n - 1}$

(B) $\frac{n - 6}{n + 1}$

(C) $\frac{n - 6}{n - 1}$

(D) $\frac{n + 6}{n + 1}$

X

$\frac{148}{1}$

Very good. You made the correct choice.

The procedure is to factor the numerator and the denominator.

The numerator has the common monomial factor 9, and the denominator has the common monomial factor 9 . Then we have:

$$\frac{9m - 18n}{18m + 9n} = \frac{9(m - 2n)}{9(2m - n)}$$

Divide by the common factor 9

$$= \frac{m - 2n}{2m - n}$$

Please proceed to question 14 below.

$\frac{148}{2}$

Question 14

Apply the proper principle and reduce the fraction,

$$\frac{6a + 6b}{a^2 - b^2}$$

to lowest terms. Select the letter which labels the correct result.

(A) $\frac{6(a + b)}{a^2 - b^2}$

(B) $\frac{6}{a - b}$

(C) $\frac{6}{a + b}$

(D) None of these.

X

If you multiply both numerator and denominator of a fraction by the quantity 5 , the result will be an equivalent fraction. True, there doesn't seem to be any reason why we would want to multiply by 5 , but it would be correct to do so. This choice is correct.

This follows the rule:

$$\frac{a}{b} = \frac{a}{b} \times 1$$

$$= \frac{a}{b} \times \frac{c}{c}$$

$$= \frac{ac}{bc}$$

Please proceed to question 10 below.

Question 10

Apply the proper principle and select the letter which labels the correct statement. In order to reduce the fraction,

$$\frac{x^2 - 5x}{x^2 - 25}$$

to lowest terms, the numerator and denominator must be divided by:

(A) $x - 5$

(B) $\frac{x}{x + 5}$

(C) $x (x - 5)$

(D) $x (x + 5)$

X

$\frac{150}{1}$

In order to simplify this fraction, we first write the denominator in descending powers of y using the commutative property and then we factor out -1 .

$$\frac{y^2 - 3y - 10}{15 + 2y - y^2} = \frac{y^2 - 3y - 10}{-y^2 + 2y + 15}$$

$$= \frac{y^2 - 3y - 10}{-1(y^2 - 2y - 15)} \qquad \text{[Factor}$$

$$= \frac{\overset{1}{\cancel{(y-5)}}(y+2)}{-1\underset{1}{\cancel{(y-5)}}(y+3)} \qquad \text{[Divide by (y-5)}$$

$$= \frac{y + 2}{-1(y + 3)}$$

$$= -\frac{y + 2}{y + 3}$$

Then this choice is correct.

$\frac{150}{2}$

You have now finished this Segment. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following definitions:

(1) Division by zero is not a defined operation.

(2) A fraction can be simplified by factoring its numerator and denominator and dividing both by their greatest common factor.

(3) A fraction is in lowest terms when the numerator and denominator have no common factors.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT: 13 - 16 .

The number 35 can be written as 7 times 5 , while 63 can be written as 7 times 9 . Furthermore, a negative quantity divided by a positive quantity is negative.

The basic proto-type is

$$\frac{p}{q} = \frac{ab}{ac} = \frac{a}{a} \cdot \frac{b}{c}$$

$$= 1 \cdot \frac{b}{c}$$

Return to page $\frac{146}{2}$ and try question 12 again.

We don't agree. Did you find (n + 1) to be one of the factors of

$$n^2 - 2n + 1 \quad ?$$

That is not correct.

Return to page $\frac{147}{2}$ and try question 18 again.

X

$\frac{152}{1}$

We don't agree. In factoring the numerator, we first remove the common monomial factor z .

$$\text{Thus,} \qquad z^3 - z = z(z^2 - 1)$$

The expression in the parentheses is the difference of two squares and can be factored. Similarly, the denominator can be written as the product of two linear factors.

Please continue.

Return to page $\frac{145}{2}$ and try question 16 again.

$\frac{152}{2}$

We don't agree. The number 7 is the same as $\frac{7}{1}$.

Thus, the problem is equivalent to

$$\left(\frac{7}{1}\right)\left(\frac{5}{14}\right)\left(\frac{14}{25}\right)$$

Return to page $\frac{170}{1}$ and try question 2 again.

Very good. You made the correct choice. Since the numerator of this fraction is in descending powers, while the denominator is in ascending powers, one of them should be changed. Let us write the numerator in ascending powers.

$$x^2 - 4x + 4 = 4 - 4x + x^2$$

he Commutative Property.

en we have:

$$\frac{x^2 - 4x + 4}{4 - x^2} = \frac{4 - 4x + x^2}{4 - x^2} \quad \text{[Factoring}$$

$$= \frac{(2 - x)\cancel{(2 - x)}^{1}}{(2 + x)\cancel{(2 - x)}_{1}}$$

$$= \frac{2 - x}{2 + x}$$

Please proceed to question 20 below.

<u>Question 20</u>

Apply the proper principle and reduce the fraction

$$\frac{y^2 - 3y - 10}{15 + 2y - y^2}$$

to lowest terms. Select the letter which labels the correct result.

(A) $\frac{y + 5}{y + 3}$ (C) $-\frac{y + 2}{y + 3}$

(B) $\frac{y + 2}{y + 3}$ (D) $-\frac{y + 2}{y + 5}$

X

$\frac{154}{1}$

The proper procedure is to factor and then reduce, as follows:

$$\frac{6a + 6b}{a^2 - b^2} = \frac{6\,\overset{1}{\cancel{(a + b)}}}{\underset{1}{\cancel{(a + b)}}(a - b)}$$

$$= \frac{6}{(a - b)}$$

Then this choice is correct.

Please proceed to question 15 below.

$\frac{154}{2}$

Question 15

Apply the proper principle and reduce the fraction,

$$\frac{s^2 - 16}{3s + 12}$$

to lowest terms. Select the letter which labels the correct result.

(A) $\frac{s^2 - 16}{3(s + 4)}$

(B) $\frac{s - 4}{3(s + 4)}$

(C) $\frac{s - 4}{3}$

(D) $\frac{s}{3} - 4$

X

155
1

You overlooked a rule, namely the one dealing with the multiplication of signed numbers.

When multiplying factors of like signs, the product will be positive (+) ; factors of unlike signs, the product will be negative (−) .

Please return to page $\frac{160}{2}$ and try question 1 again.

155
2

Sorry. Your answer is not in lowest terms. It is important that you read the question carefully and answer exactly what is asked.

Please return to page $\frac{185}{2}$ and try question 4 again.

X

This is the correct answer.

Please go to question 13 which follows:

Question 13

Apply the proper principle and reduce the fraction

$$\frac{9m - 18n}{18m + 9n}$$

to lowest terms. Select the letter which labels the correct answer.

(A) $\frac{m - 2n}{2m + n}$

(B) $-1\frac{1}{2}$

(C) $\frac{m}{2} - 2n$

(D) None of these.

How do we factor the difference of two squares?

Recall that

$$a^2 - b^2 = (a - b)(a + b)$$

Please return to page $\frac{183}{2}$ and try question 6 again.

Can you find a number which divides both 35 and 25 without leaving a remainder? There is such a number. Therefore, your answer is not in lowest terms.

Return to page 170/2 and try question 2 again.

What are the factors of

$$x^2 - 25 \quad ?$$

Recall that the factors of

$$a^2 - b^2$$

are

$$(a + b)(a - b)$$

Note that the signs between the terms of the binomial factors are not alike.

Return to page 186/2 and try question 8 again.

X

$\frac{158}{1}$

Very good. You made the correct choice. We should proceed as follows:

$$\frac{z^3 - z}{z^2 + 2z - 3} = \frac{z(z^2 - 1)}{(z + 3)(z - 1)} \quad \text{[Factor}$$

$$= \frac{z(z + 1)\cancel{(z - 1)}^{1}}{(z + 3)\cancel{(z - 1)}_{1}} \quad \text{[Divide by } (z - 1)$$

$$= \frac{z(z + 1)}{z + 3}$$

Please proceed to question 17 below.

$\frac{158}{2}$

Question 17

Apply the proper principle and reduce the fraction,

$$\frac{p^2 - 2p - 8}{p^2 - p - 6}$$

to lowest terms. Select the letter which labels the correct result.

(A) $\frac{p + 4}{p + 2}$

(B) $\frac{p - 4}{p - 3}$

(C) $\frac{p + 4}{p - 3}$

(D) None of these.

Very good. This choice is correct. We proceed as follows:

$$\frac{n^2 + 5n - 6}{n^2 - 2n + 1} = \frac{(n + 6)\overset{1}{\cancel{(n - 1)}}}{(n - 1)\underset{1}{\cancel{(n - 1)}}}$$

$$= \frac{n + 6}{n - 1}$$

Please proceed to question 19 below.

Question 19

Apply the proper principle and reduce the fraction,

$$\frac{x^2 - 4x + 4}{4 - x^2}$$

to lowest terms. Select the letter which labels the correct result.

(A) -1

(B) $\frac{2 - x}{2 + x}$

(C) 1

(D) $\frac{2 + x}{2 - x}$

X

$\frac{160}{1}$

VOLUME 10 SEGMENT 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 5	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	1 0	(Volume Number)
	66	and	68	0 5	(Segment Number)

INTRODUCTORY NOTE

In this segment, you will learn how to multiply and divide algebraic fractions. Are you a bit surprised? When you studied arithmetic, you learned the four basic operations: addition, subtraction, multiplication, and division, in the order just mentioned. Why then are we going to learn how to multiply and divide algebraic fractions before learning how to add and subtract them? We will leave this question unanswered for the moment. After you have studied all the operations with algebraic fractions, you will know the answer.

Your READING ASSIGNMENT for this segment is pages 292 - 295 . You will now be asked a series of questions to draw your attention to the more important points.

Please go on to question 1 below.

$\frac{160}{2}$

Question 1

Calculate the product of the fractions,

$$\left(\frac{3}{5}\right)\left(\frac{5}{8}\right)\left(-\frac{2}{9}\right)$$

Select the letter which labels the correct answer written in lowest terms.

(A) $-\frac{6}{72}$ (C) $-\frac{1}{12}$

(B) $\frac{1}{12}$ (D) $-\frac{6}{63}$

X

It is true that in finding the quotient of powers, we keep the base and subtract the exponents. But you applied this rule to the division of arithmetic numbers. You know that $\frac{7}{28}$ does not equal $\frac{1}{21}$ but rather, it equals $\frac{1}{4}$.

Please return to page $\frac{185}{2}$ and try question 4 again.

What are the factors of

$$x^2 + 4x + 3 \quad ?$$

Remember that the factors of a trinomial with only plus signs between its terms consists of binomial factors with only plus signs between their terms.

Please return to page $\frac{186}{2}$ and try question 8 again.

X

$\frac{162}{1}$

Sorry. One of the letters does have the correct answer next to it. Please reconsider your choice.

Carefully factor each composite number and remove identical factor pairs from the numerator and denominator.

Remember,

$$\frac{a}{a} = 1 \qquad [\ a \neq 0$$

Please return to page $\frac{170}{2}$ and try question 2 again.

$\frac{162}{2}$

Your answer is not in lowest terms. Examine the numerator of your choice and find how it can be factored.

Please return to page $\frac{183}{2}$ and try question 6 again.

We don't agree. Each of the expressions,

$$x^2 - 49 \quad \text{and} \quad y^2 - 25$$

is the difference of two squares. Recall that

$$a^2 - b^2 = (a - b)(a + b)$$

Please return to page $\frac{172}{2}$ and try question 5 again.

Did you make an error in factoring?

A fraction can be simplified only by dividing both numerator and denominator by the same <u>factor</u>. Remember you may not cross out <u>terms</u>.

For example,

$$\frac{x + 7}{x + 2} \text{ does not equal } \frac{7}{2}.$$

"x" is not a <u>factor</u> common to both the numerator and the denominator and so it can not be crossed out.

Please return to page $\frac{175}{2}$ and try question 9 again.

X

$\frac{164}{1}$

The product of

$$\frac{6m}{11m} \cdot \frac{77n}{15m}$$

can be written as $\dfrac{(6)\ (77)\ (m)\ (n)}{(11)\ (15)\ (m)\ (n)}$ [associative law of multiplication

Instead of multiplying the numbers, we first examine the numerator and the denominator to see whether they can both be divided by the same number. Thus, for example, we can divide the numerator and the denominator by 11. Can you find other numbers that will divide both numerator and denominator?

Please return to page $\frac{176}{2}$ and try question 3 again.

$\frac{164}{2}$

You made a serious mistake.

When a fraction multiplication problem is of the form,

$$\frac{a}{bc} \cdot \frac{b}{ad}$$

the answer is $\frac{1}{cd}$

not cd

$$\left[\begin{aligned} \frac{a}{bc} \cdot \frac{b}{ad} &= \frac{\cancel{a}}{\cancel{a}} \cdot \frac{\cancel{b}}{\cancel{b}} \cdot \frac{1}{cd} \\ &= \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{cd} \\ &= \frac{1}{cd} \end{aligned}\right.$$

The c and d remain in the denominator.

Please return to page $\frac{173}{2}$ and reconsider the problem.

X

You made a mistake in multiplication. You should study the multiplication of

$$8 \times 7, \quad 8 \times 9, \quad \text{and} \quad 7 \times 9$$

Please return to page $\frac{160}{2}$ and try question 1 again.

You made a mistake in applying the rule for the quotient of powers. To refresh your memory, note that

$$\frac{x^a}{x^b} = x^{a-b}$$

Please return to page $\frac{193}{2}$ and try question 12 again.

X

$\frac{166}{1}$

It is necessary to remind you that a variable written without a visible exponent means that the variable has the exponent 1 .

Thus, for example,

$$\frac{a^2b^2}{ab} = a^{2-1}\ b^{2-1} = ab$$

Please return to page $\frac{185}{2}$ and try question 5 again.

$\frac{166}{2}$

Check the factors of

$$p^2 - 4p - 5$$

Please note that the linear factors of this trinomial must have different signs between their terms, since -5 was obtained by multiplying unlike signed numbers.

Please return to page $\frac{187}{2}$ and try question 15 again.

You made a careless error. Of course, you realize that you must factor each expression before you proceed to find common factors. By what did you divide both numerator and denominator?

Please return to page $\frac{183}{2}$ and try question 6 again.

Write this definition in your NOTEBOOK:

> The quotient of 1 divided by a number is the reciprocal of that number.

Zero is the only quantity which has no reciprocal. That is, the reciprocal of a is $\frac{1}{a}$. The reverse is true, the reciprocal of $\frac{1}{a}$ is a. Zero is the only quantity which has no reciprocal.

Please return to page $\frac{188}{2}$ and try question 10 again.

$\frac{168}{1}$

One of the other letters does have the correct answer next to it.

Please reconsider your choice.

Factor carefully.

Please return to page $\frac{186}{2}$ and try question 8 again.

$\frac{168}{2}$

When we divide one fraction by a second fraction, we multiply the first fraction by the reciprocal of the second fraction. Note carefully,

$$a \div b \quad \text{means} \quad a \cdot \frac{1}{b}$$

What is the reciprocal of

$$(x + 3)$$

Of course, it is $\frac{1}{(x + 3)}$.

Please return to page $\frac{189}{2}$ and try question 13 again.

X

As long as there is a number that can divide both numerator and denominator of a fraction evenly, that fraction is not in lowest terms. In the choice you made, we can for example, divide both the numerator and denominator by m .

Please return to page $\frac{176}{2}$ and try question 3 again.

What are the factors of

$$y^2 - 5y - 6 \quad ?$$

After you have decided on the factors of the trinomial, check your result by multiplying the factors to see if their product equals the original trinomial.

Please return to page $\frac{175}{2}$ and try question 9 again.

X

$\frac{170}{1}$

This is the correct answer.

A careful analysis of this problem indicates the underlying method to be used in all problems of multiplication of fractions.

$\frac{3}{5} \cdot \frac{5}{8} \cdot \frac{-2}{9}$	[	Factor
$\frac{3}{5} \cdot \frac{5}{2 \cdot 2 \cdot 2} \cdot \frac{-1 \cdot 2}{3 \cdot 3}$	[	Commutative Property
$(\frac{3}{3}) \cdot (\frac{5}{5}) \cdot (\frac{2}{2}) \cdot \frac{-1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3}$	[	Associative Property
$1 \cdot 1 \cdot 1 \cdot \frac{-1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3}$	[	Identity; multiply
$\frac{-1}{12} = -\frac{1}{12}$		

Of course, this method can be shortened by "dividing" both numerator and denominator by the same number that appears as a common factor of both numerator and denomination.

$$\overset{1}{\cancel{3}} / \underset{1}{\cancel{5}} \quad \overset{1}{\cancel{5}} / \underset{4}{\cancel{8}} \quad \overset{-1}{\cancel{-2}} / \underset{3}{\cancel{9}} = -\frac{1}{12}$$

Please go on to question 2 which follows.

$\frac{170}{2}$

Question 2

Calculate the product of the fractions,

$$(7) \cdot (\frac{5}{14}) \cdot (\frac{14}{25})$$

Select the letter which labels the correct answer written in lowest terms.

(A) $\frac{7}{5}$ (B) $\frac{1}{35}$

(B) $\frac{35}{25}$ (D) None of these.

X

Very good. This is the correct answer. The correct work follows:

$$\frac{x^2 - 3x - 10}{(x - 2)^2} \div \frac{x - 5}{x - 2} = \frac{x^2 - 3x - 10}{(x - 2)^2} \cdot \frac{x - 2}{x - 5}$$

$$= \frac{\overset{1}{\cancel{(x - 5)}}\,(x + 2)\,\overset{1}{\cancel{(x - 2)}}}{\underset{1}{\cancel{(x - 2)}}\,(x - 2)\,\underset{1}{\cancel{(x - 5)}}}$$

$$= \frac{x + 2}{x - 2}$$

Since neither $(x + 2)$ nor $(x - 2)$ can be factored, this fraction cannot be reduced any further.

Please proceed to question 18 below.

Question 18

Apply the proper principle to express the quotient

$$\frac{6w + 6}{8w} \div (5w^2 + 3w - 2)$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{4w(5w - 2)}{3}$

(B) $\frac{3}{4w(5w + 2)}$

(C) $\frac{3}{4(5w - 2)(w + 1)}$

(D) $\frac{3}{4w(5w - 2)}$

X

172
1

This is the correct answer. The work might be done as follows:

$$\frac{7xy}{5ab}\cdot\frac{15a^2b^2}{28x^2y^2} = \frac{\overset{1}{\cancel{(7)}}\,\overset{1}{\cancel{(x)}}\,\overset{1}{\cancel{(y)}}\,(3)\,\overset{1}{\cancel{(5)}}\,\overset{1}{\cancel{(a)}}\,(a)\,\overset{1}{\cancel{(b)}}\,(b)}{\underset{1}{\cancel{(5)}}\,\underset{1}{\cancel{(a)}}\,\underset{1}{\cancel{(b)}}\,(2)\,(2)\,\underset{1}{\cancel{(7)}}\,\underset{1}{\cancel{(x)}}\,(x)\,\underset{1}{\cancel{(y)}}\,(y)}$$

$$= \frac{3ab}{4xy}$$

If you can do a bit more work mentally, it would be better to do the problem this way:

$$\frac{\overset{1\cdot 1\cdot 1}{\cancel{7}\cdot\cancel{x}\cdot\cancel{y}}}{\underset{1\cdot 1\cdot 1}{\cancel{5}\cdot\cancel{a}\cdot\cancel{b}}}\cdot\frac{\overset{3\cdot a\cdot b}{\cancel{15}\cancel{a^2}\cancel{b^2}}}{\underset{4xy}{\cancel{28}\cancel{x^2}\cancel{y^2}}} = \frac{3ab}{4xy}$$

Please proceed to question 5 below.

172
2

Question 5

Apply the proper principle to express the product of

$$\frac{x^2 - 49}{y^2 - 25}\cdot\frac{y - 5}{x + 7}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{x + 7}{y - 5}$

(B) $\frac{x - 7}{y + 5}$

(C) $\frac{y - 7}{y - 5}$

(D) None of these

Very good. You made the correct choice. After factoring the numerator and the denominator of the first fraction, we get:

$$\frac{\overset{1}{\cancel{3}}\ \overset{1}{\cancel{(2p - q)}}}{\underset{1}{\cancel{(2p + q)}}\ \underset{1}{\cancel{(2p - q)}}} \cdot \frac{\overset{1}{\cancel{(2p + q)}}}{\underset{1}{\cancel{3}}} \qquad \left[\frac{abc}{cba} = 1\right.$$

We now note that the numerator and the denominator are equal. Hence, the quotient is 1 .

Please proceed to question 7 below.

Question 7

Apply the proper principle to express the product of

$$\frac{a - b}{15} \cdot \frac{5}{a^2 - 2a\text{[illegible]} \cdot b^2}$$

in lowest terms.

Select the letter which labels the correct result.

(A) $3 (a - b)$

(B) $\frac{1}{3 (a - b)}$

(C) $\frac{a - b}{3 (a + b)}$

(D) $\frac{1}{3 (a + b)}$

$\frac{174}{1}$

We should proceed in this manner:

$$\frac{x^2 + 6x + 9}{x} \div (x + 3) = \frac{x^2 + 6x + 9}{x} \cdot \frac{1}{x + 3} \qquad \left[\frac{a}{b} \div c = \frac{a}{b} \cdot \frac{1}{c}\right.$$

$$= \frac{\overset{1}{\cancel{(x + 3)}}(x + 3)}{x} \cdot \frac{1}{\underset{1}{\cancel{(x + 3)}}} \qquad \left[\frac{c}{c} = 1\right.$$

$$= \frac{x + 3}{x}$$

Since $(x + 3)$ cannot be factored, this is in simplest form.

Therefore, this choice is correct.

Please proceed to question 14 below.

$\frac{174}{2}$

Question 14

Apply the proper principle to express the quotient

$$\frac{1}{y^2 - 16} \div \frac{y - 4}{y + 4}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{1}{y^2 - 16}$

(B) $\frac{1}{(y - 4)^2}$

(C) $\frac{1}{y^2 - 16}$

(D) None of these.

Factoring first, we get:

$$\frac{x^2 + 4x + 3}{x^2 - 25} \cdot \frac{x - 5}{x + 3} = \frac{(x + 3)(x + 1)}{(x + 5)(x - 5)} \cdot \frac{(x - 5)}{(x + 3)}$$

$$= \frac{\overset{1}{\cancel{(x + 3)}}(x + 1)}{(x + 5)\underset{1}{\cancel{(x - 5)}}} \cdot \frac{\overset{1}{\cancel{(x - 5)}}}{\underset{1}{\cancel{(x + 3)}}}$$

$$= \frac{x + 1}{x + 5}$$

Since neither $(x + 1)$ nor $(x + 5)$ can be factored, this answer cannot be simplified; and this choice is correct.

Please proceed to question 9 below.

Question 9

Apply the proper principle to express the product of

$$\frac{y^2 - 5y - 6}{y^2 + 3y} \cdot \frac{y + 3}{y - 6}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{1}{y}$

(B) $\frac{y - 1}{y}$

(C) $\frac{(y + 3)(y + 1)}{y^2 + 3y}$

(D) $\frac{y + 1}{y}$

X

$\frac{176}{1}$

This is the correct answer.

Your work can be arranged in either of two ways:

Factor Method:

$$7 \cdot \frac{5}{14} \cdot \frac{14}{25} =$$ [Factor

$$7 \cdot \frac{5}{7 \cdot 2} \cdot \frac{7 \cdot 2}{5 \cdot 5} =$$ [Rearrange factors to form identities

$$\frac{7}{1} \cdot \frac{7}{7} \cdot \frac{2}{2} \cdot \frac{5}{5} \cdot \frac{1}{5} =$$ [Reduce the form $\frac{a}{a} = 1$

$$\frac{7}{1} \cdot \frac{1}{5} =$$ [Multiply fractions

$$\frac{7}{5}$$

OR

$$7 \cdot \frac{5}{14} \cdot \frac{14}{25}$$

[Divide both numerator & denominator by the common factors 5 and 14 .

$$7 \cdot \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{14}}} \cdot \frac{\overset{1}{\cancel{14}}}{\underset{5}{\cancel{25}}}$$

$$\frac{7}{1} \cdot \frac{1}{5}$$

[Multiply the remaining fractions.

$$\frac{7}{5}$$

Please go on to question 3 which follows.

$\frac{176}{2}$

Question 3

Apply the proper principle and express the product of

$$\frac{6m}{11n} \cdot \frac{77n}{15m}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{462}{165}$

(B) $\frac{14mn}{3mn}$

(C) $\frac{14}{5}$

(D) $\frac{42}{15}$

X

$\frac{177}{1}$

Did you factor first, and then attempt to divide numerator and denominator by the same factor? If you do this correctly, you will find that one of the other choices is correct.

Please return to page $\frac{172}{2}$ and try question 5 again.

$\frac{177}{2}$

We disagree.

Remember that division by a quantity means multiplication by the reciprocal of <u>that</u> quantity.

$a \div b$ means $a \cdot \frac{1}{b}$ [the reciprocal of b is $\frac{1}{b}$

Please return to page $\frac{195}{2}$ and try question 19 again.

X

$\frac{178}{1}$

Sorry. You did not make the correct choice. The denominator of the second fraction is a trinomial square. Recall that

$$a^2 - 2ab + b^2 = (a - b)(a - b)$$

Please return to page $\frac{173}{2}$ and try question 7 again.

$\frac{178}{2}$

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page $\frac{174}{2}$ and try question 14 again.

X

We don't agree. This choice is not in lowest terms, since the denominator of the fraction that you chose can be factored. Remove the common monomial factor.

Please return to page $\frac{175}{2}$ and try question 9 again.

Your answer can be further simplified. For example, the numerator of your answer contains the letter y raised to the fifth power, while the denominator contains y raised to the third power.

Please continue.

Please return to page $\frac{193}{2}$ and try question 12 again.

X

$\frac{180}{1}$

If we were to reduce the fraction $\frac{42}{15}$ we would write the numbers 42 and 15 as the product of prime factors. We get,

$$\frac{42}{15} = \frac{7 \times 3 \times 2}{5 \times 3}$$

$$= \frac{7}{5} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}} \times \frac{2}{1}$$

$$= \frac{14}{5}$$

or in other words, we could have divided both 42 and 15 by 3 in this manner,

$$\frac{\overset{14}{\cancel{42}}}{\underset{5}{\cancel{15}}} = \frac{14}{5}$$

Can the numerator and denominator of your choice be divided by the same number?

Please return to page $\frac{176}{2}$ and try question 3 again.

$\frac{180}{2}$

This question requires the factoring of four expressions. Two of the expressions are the difference of two squares and two are trinomials that require factoring by trial. Surely, you know how to factor the difference of two squares. Hence, to get you started, note that the factors of

$$x^2 + 5xy + 6y^2$$

are $(x + 3y)(x + 2y)$

Please return to page $\frac{192}{2}$ and try question 20 again.

Very good. You made the correct choice.

The reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{1}{3}$ is $\frac{1}{\frac{1}{3}}$ which is equal to

$$1 \cdot \frac{3}{1} = 3$$

In proto-type form, we can state:

the reciprocal of a is $\frac{1}{a}$

the reciprocal of $\frac{1}{a}$ is a

Please proceed to question 11 below.

Question 11

Apply the proper principle to find the reciprocals of

I. $7xy$ II. $\frac{5b}{a^2}$

Select the letter which labels the correct result.

(A) I. $\frac{7}{xy}$ II. $\frac{5a^2}{b}$

(B) I. $\frac{1}{7xy}$ II. $\frac{a^2}{5b}$

(C) I. $\frac{xy}{7}$ II. $\frac{b}{5a^2}$

(D) None of these.

X

$\frac{182}{1}$

Very good. You made the correct choice. We note that;

$$\frac{x^2 - 2x - 3}{x^2 - 9} \div \frac{x^2 + 5x}{x^2 + 8x + 15} = \frac{x^2 - 2x - 3}{x^2 - 9} \cdot \frac{x^2 + 8x + 15}{x^2 + 5x}$$

$$= \frac{\overset{1}{\cancel{(x - 3)}}(x + 1)}{\underset{1}{\cancel{(x - 3)}}\underset{1}{\cancel{(x + 3)}}} \cdot \frac{\overset{1}{\cancel{(x + 5)}}\overset{1}{\cancel{(x + 3)}}}{x\underset{1}{\cancel{(x + 5)}}}$$

$$= \frac{x + 1}{x}$$

Since $\frac{x + 1}{x}$ cannot be reduced, this is our final answer.

Please proceed to question 17 below.

$\frac{182}{2}$

<u>Question 17</u>

Apply the proper principle to express the quotient:

$$\frac{x^2 - 3x - 10}{(x - 2)^2} \div \frac{x - 5}{x - 2}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{x - 2}{x + 2}$ (C) $\frac{x + 2}{x - 2}$

(B) $\frac{x + 5}{x - 5}$ (D) 1

It is necessary to factor first. Thus we have:

$$\frac{x^2 - 49}{y^2 - 25} \cdot \frac{y - 5}{x + 7} = \frac{(x - 7)(x + 7)}{(y - 5)(y + 5)} \cdot \frac{(y - 5)}{(x + 7)}$$

$$= \frac{(x - 7)\overset{1}{\cancel{(x + 7)}}}{\underset{1}{\cancel{(y - 5)}}(y + 5)} \cdot \frac{\overset{1}{\cancel{(y - 5)}}}{\underset{1}{\cancel{(x + 7)}}}$$

$$= \frac{x - 7}{y + 5}$$

In proto-type this would be:

$$\frac{a}{\underset{1}{\cancel{c}}} \cdot \frac{\overset{1}{\cancel{b}}}{d} \cdot \frac{\overset{1}{\cancel{c}}}{\underset{1}{\cancel{b}}} = \frac{a}{d}$$

Therefore, this choice is correct.

Please proceed to question 6 below.

Question 6

Apply the proper principle to express the product of

$$\frac{6p - 3q}{4p^2 - q^2} \cdot \frac{2p + q}{3}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{2p - q}{2p + q}$

(C) $\frac{2p + q}{2p - q}$

(B) $\frac{6p - 3q}{3(2p - q)}$

(D) 1

$\frac{184}{1}$

Dividing by the fraction

$$\frac{48y^3}{7z^4} \quad \text{means multiplying by}$$

the reciprocal of this quantity, namely $\frac{7z^4}{48y^3}$.

Did you perform this multiplication?

Please return to page $\frac{193}{2}$ and try question 12 again.

$\frac{184}{2}$

Before you decide on the factors of a given trinomial, check those factors by multiplying them. Their product should equal the original trinomial. You probably made your error in the factoring.

Please return to page $\frac{195}{2}$ and try question 19 again.

X

This choice is correct. The product of

$$\frac{6m}{11n} \cdot \frac{77n}{15m} \quad \text{can be written as}$$

$$\frac{(6)\ (77)\ (m)\ (n)}{(11)\ (15)\ (m)\ (n)} \qquad \text{[Associative law of multiplication}$$

Factoring into prime factors, we have:

$$\frac{6m}{11n} \cdot \frac{77n}{15m} = \frac{(2)\ (3)\ (7)\ (11)\ (m)\ (n)}{(11)\ (3)\ (5)\ (m)\ (n)} \qquad \text{[Divide by common factors}$$

$$= \frac{(2)\ \overset{1}{(\cancel{3})}\ (7)\ \overset{1}{(\cancel{11})}\ \overset{1}{(\cancel{m})}\ \overset{1}{(\cancel{n})}}{\underset{1}{(\cancel{11})}\ \underset{1}{(\cancel{3})}\ (5)\ \underset{1}{(\cancel{m})}\ \underset{1}{(\cancel{n})}} \qquad \text{[Multiply remaining factors}$$

$$= \frac{14}{5}$$

<u>Note:</u> it is not necessary to pair the same factors as division identities since we accomplish the same result by division.

Please proceed to question 4 below.

<u>Question 4</u>

Apply the proper principle and express the product of

$$\frac{7xy}{5ab} \cdot \frac{15a^2b^2}{28x^2y^2}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{21ab}{28xy}$

(B) $\frac{3ab}{4xy}$

(C) $\frac{5ab}{21xy}$

(D) $\frac{3a^2b^2}{4x^2y^2}$

$\frac{186}{1}$

Very good. This is the correct answer.

The work is as follows:

$$\frac{a - b}{15} \cdot \frac{5}{a^2 - 2ab + b^2} = \frac{a - b}{15} \cdot \frac{5}{(a - b)(a - b)}$$

$$= \frac{\overset{1}{\cancel{(a - b)}}}{\underset{3}{\cancel{15}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{(a - b)}}(a - b)}$$

$$= \frac{1}{3(a - b)}$$

Please proceed to question 8 below.

$\frac{186}{2}$

Apply the proper principle to express the product of

$$\frac{x^2 + 4x + 3}{x^2 - 25} \cdot \frac{x - 5}{x + 3}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{x + 1}{x + 5}$ (C) $\frac{x - 1}{x + 5}$

(B) $\frac{x + 1}{x - 5}$ (D) None of these.

This is the correct answer. Since the reciprocal of

$$\frac{y - 4}{y + 4} \text{ is } \frac{y + 4}{y - 4}$$

the correct work would look like this:

$$\frac{1}{y^2 - 16} \div \frac{y - 4}{y + 4} = \frac{1}{y^2 - 16} \cdot \frac{(y + 4)}{(y - 4)}$$

$$= \frac{1}{\underset{1}{\cancel{(y + 4)}}(y - 4)} \cdot \frac{\overset{1}{\cancel{(y + 4)}}}{(y - 4)}$$

$$= \frac{1}{(y - 4)(y - 4)}$$

$$= \frac{1}{(y - 4)^2}$$

Please proceed to question 15 below.

Question 15

Apply the proper principle to express the quotient

$$\frac{p^2 + 4p + 3}{p^2 - 4p - 5} \div \frac{p + 3}{p - 5}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{p + 1}{p - 1}$ (C) 1

(B) $\frac{p + 4}{p + 3}$ (D) $\frac{p - 1}{p + 1}$

$\frac{188}{1}$

This is the correct answer. Factoring, we have:

$$\frac{y^2 - 5y - 6}{y^2 + 3y} \cdot \frac{y + 3}{y - 6} = \frac{(y - 6)(y + 1)}{y(y + 3)} \cdot \frac{(y + 3)}{(y - 6)}$$

$$= \frac{\overset{1}{\cancel{(y - 6)}}(y + 1)}{y\underset{1}{\cancel{(y + 3)}}} \cdot \frac{\overset{1}{\cancel{(y + 3)}}}{\underset{1}{\cancel{(y - 6)}}}$$

$$= \frac{y + 1}{y}$$

Since $(y + 1)$ cannot be factored, this is in simplest form.

Please proceed to question 10 below.

$\frac{188}{2}$

Question 10

Apply the proper principle to find the reciprocals of

I. 5 II. $\frac{1}{3}$

(A) I. $\frac{5}{1}$ II. $\frac{3}{1}$

(B) I. $\frac{1}{5}$ II. $\frac{2}{3}$

(C) I. $\frac{1}{5}$ II. 3

(D) I. -4 II. $-\frac{2}{3}$

X

Very good. You made the correct choice.

The division rule states:

To divide by an expression, multiply by its reciprocal.

We have:

$$\frac{24y^5}{35z} \div \frac{48y^3}{7z^4} = \frac{24y^5}{35z} \cdot \frac{7z^4}{48y^3}$$

the reciprocal of $\frac{48y^3}{7z^4}$ is $\frac{7z^4}{48y^3}$

$$= \frac{24 \cdot 7y^3z \cdot y^2z^3}{24 \cdot 7y^3z \cdot 5 \cdot 2}$$

Regrouping to pair common factors.

$$= \frac{y^2z^3}{10}$$

Please proceed to question 13 below.

Question 13

Apply the proper principle to express the quotient

$$\frac{x^2 + 6x + 9}{x} \div (x + 3)$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{x}{x + 3}$

(B) $\frac{(x + 3)^3}{x}$

(C) $\frac{x + 6}{x}$

(D) $\frac{x + 3}{x}$

X

$\frac{190}{1}$

Sorry, you made the wrong choice. Note that the factors of

$$x^2 - 2x - 3 \text{ are } (x - 3)(x + 1)$$

and the factors of

$$x^2 - 9 \qquad \text{are } (x + 3)(x - 3)$$

Please continue.

Please return to page $\frac{197}{2}$ and try question 16 again.

$\frac{190}{2}$

You multiplied the two fractions instead of dividing the first fraction by the second.

Please read the question more carefully.

Please return to page $\frac{189}{2}$ and try question 13 again.

$\frac{191}{1}$

You must have factored

$$p^2 + 4p + 3$$

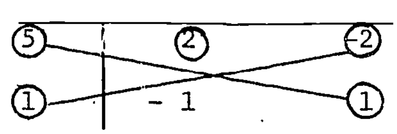
incorrectly. Make sure that you have the correct factors by multiplying them. Their product should equal the given trinomial.

Please return to page $\frac{187}{2}$ and try question 15 again.

$\frac{191}{2}$

We disagree. It is necessary to factor completely and then to divide both numerator and denominator by the same factor. You have made an error somewhere. To factor

$$5w^2 + 3w - 2 \text{ use the form:}$$



Please return to page $\frac{171}{2}$ and try question 18 again.

X

$\frac{192}{1}$

Very good. You made the correct choice.

We have:

$$\frac{2x^2 - 5x + 2}{12} \div \frac{x^2 - 4x + 4}{9} = \frac{2x^2 - 5x + 2}{12} \cdot \frac{9}{x^2 - 4x + 4}$$

$$= \frac{(2x - 1)\overset{1}{\cancel{(x - 2)}}}{\underset{4}{\cancel{12}}} \; \frac{\overset{3}{\cancel{9}}}{(x - 2)\underset{1}{\cancel{(x - 2)}}}$$

$$= \frac{3(2x - 1)}{4(x - 2)}$$

Please proceed to question 20 below.

$\frac{192}{2}$

Question 20

Apply the proper principle to express the quotient

$$\frac{x^2 + 5xy + 6y^2}{x^2 - y^2} \div \frac{x^2 - 9y^2}{x^2 - 2xy - 3y^2}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{x + 3y}{x - 3y}$ (C) $\frac{x + 6y}{x + 3y}$

(B) $\frac{x + 2y}{x - y}$ (D) $\frac{x + 6y}{x - 3y}$

Very good. You made the correct choice. The reciprocal of

$$7xy \text{ is } \frac{1}{7xy}$$

and the reciprocal of

$$\frac{5b}{a^2} \text{ is } \frac{a^2}{5b}$$

Please proceed to question 12 below.

Question 12

Apply the proper principle to express the quotient

$$\frac{24y^5}{35z} \div \frac{48y^3}{7z^4}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{y^2z^3}{10}$

(B) $\frac{y^2z^4}{10}$

(C) $\frac{y^5z^4}{10y^3z}$

(D) $\frac{y^8}{10z^5}$

X

$\frac{194}{1}$

We disagree. It appears that you made a mistake in factoring. Did you check by multiplying your factors to see if the result is the original expression? Remember that

$$(x - 2)^2$$

means that $(x - 2)$ is multiplied by $(x - 2)$

Please return to page $\frac{182}{2}$ and try question 17 again.

$\frac{194}{2}$

We don't agree.

The quotient

$$\frac{y - 4}{y + 4}$$

does not equal -1

You cannot cancel the y's . A fraction can be simplified only by dividing numerator and denominator by the same <u>factor</u> .

Please return to page $\frac{174}{2}$ and try question 14 again.

Since division by a quantity means multiplication by the reciprocal of that quantity, we have:

$$\frac{6w + 6}{8w} \div (5w^2 + 3w - 2) = \frac{6w + 6}{8w} \cdot \frac{1}{5w^2 + 3w - 2}$$

$$= \frac{\overset{3}{\cancel{6}}\overset{1}{\cancel{(w + 1)}}}{\underset{4}{\cancel{8}}w} \cdot \frac{1}{(5w - 2)\underset{1}{\cancel{(w + 1)}}}$$

$$= \frac{3}{4w(5w - 2)}$$

Therefore, this choice is correct.

Please proceed to question 19 below.

Question 19

Apply the proper principle to express the quotient

$$\frac{2x^2 - 5x + 2}{12} \div \frac{x^2 - 4x + 4}{9}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{4(x - 2)}{3(2x - 1)}$

(B) $\frac{3(2x - 1)}{4(x - 2)}$

(C) $\frac{3(2x + 1)}{4(x - 2)}$

(D) $\frac{3(2x^2 - 5x + 2)}{4(x - 2)^2}$

X

$\frac{196}{1}$

What were ɔr

$$x^2 + 6x + 9 \ ?$$

Didn't you notice that this is a trinomial square?

Apply the principle that the factors of

$$a^2 + 2ab + b^2$$

are $(a + b)(a + b)$

Please return to page $\frac{189}{2}$ and try question 13 again.

$\frac{196}{2}$

You can't drop your attention at any time.

You evidently thought that

$$x^2 + 5xy + 6y^2 = (x + 6y)(x - y)$$

When you check this by multiplying, you will get

$$x^2 + 5xy - 6y^2$$

Now be careful.

Please return to page $\frac{192}{2}$ and try this problem again.

X

The first step is to change to multiplication; then we factor, getting:

$$\frac{p^2 + 4p + 3}{p^2 - 4p - 5} \div \frac{p \quad 3}{\quad 5} = \frac{p^2 + 4p + 3}{p^2 - 4p - 5} \cdot \frac{p - 5}{p + 3}$$

$$= \frac{\overset{1}{\cancel{(p + 3)}}\,\overset{1}{\cancel{(p + 1)}}}{\underset{1}{\cancel{(p - 5)}}\,\underset{1}{\cancel{(p + 1)}}} \cdot \frac{\overset{1}{\cancel{(p - 5)}}}{\underset{1}{\cancel{(p + 3)}}}$$

$$= \frac{1}{1}$$

$$= 1$$

Therefore, this choice is correct.

Please proceed to question 16 below.

Question 16

Apply the proper principle to express the quotient

$$\frac{x^2 - 2x - 3}{x^2 - 9} \div \frac{x^2 + 5x}{x^2 + 8x + 15}$$

in lowest terms. Select the letter which labels the correct result.

(A) $\frac{x + 3}{x}$

(C) $\frac{x - 1}{x}$

(B) $\frac{(x + 3)(x + 1)}{x(x - 3)}$

(D) $\frac{x + 1}{x}$

X

$\frac{198}{1}$

Sorry. You made the wrong choice. The denominator of the first fraction is the difference of two squares. Recall that

$$a^2 - b^2 = (a + b)(a - b)$$

Please return to page $\frac{174}{2}$ and try question 14 again.

$\frac{198}{2}$

Division by

$$(5w^2 + 3w - 2)$$

means multiplying by the reciprocal of this expression; namely,

$$\frac{1}{5w^2 + 3w - 2}$$

Check the factors of this carefully.

Please return to page $\frac{171}{2}$ and try question 18 again.

Very good. You made the correct choice. Here is how this problem can be done:

$$\frac{x^2 + 5xy + 6y^2}{x^2 - y^2} \div \frac{x^2 \quad 9y^2}{x^2 \quad 2xy - 3y^2} = \frac{x^2 + 5xy + 6y^2}{x^2 - y^2} \cdot \frac{x^2 - 2xy - 3y^2}{x^2 - 9y^2}$$

$$= \frac{\overset{1}{\cancel{(x + 3y)}}(x + 2y)}{\underset{1}{\cancel{(x + y)}}(x - y)} \cdot \frac{\overset{1}{\cancel{(x - 3y)}}\overset{1}{\cancel{(x + y)}}}{\underset{1}{\cancel{(x + 3y)}}\underset{1}{\cancel{(x - 3y)}}}$$

$$= \frac{x + 2y}{x - y}$$

You have now finished this segment. Hand in your PUNCH CARD.

$\frac{199}{2}$

You should have entered in your NOTEBOOK the following definitions and rules:

(1) The product of two or more algebraic fractions is a fraction whose numerator is the product of the numerators, and whose denominator is the product of the denominators. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

(2) When algebraic fractions are multiplied, factoring is used to make it possible to divide the numerator and denominator by identical factors, thus reducing the final result. $\frac{ab}{ac} = \frac{b}{c}$

(3) The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

(4) Dividing by a number is equivalent to multiplying by the reciprocal of that number. $R - \frac{a}{b} = R \ \frac{b}{a}$

You should now complete ASSIGNMENT 10 , problems 16-20 in preparation for the test on Volume 10 . .

X

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 10

1/16/70
Attach to
Back Cover

To the users of this book

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected. Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE	MISPRINT	CORRECTION	CHECK WHEN CORRECTION MADE
48/2	.... question 10	....question 2	
186/2	Question number omitted	Question 8	
124/2	SPECIAL INSERT		