$\frac{88}{1}$

Very good. You made the right choice. Multiplication and division can often be simplified by factoring. Thus, in this question, we write each of the numbers as a product of primes,

$$\frac{54 \times 98}{42 \times 63} = \frac{2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 2}{7 \cdot 3 \cdot 2 \cdot 7 \cdot 3 \cdot 3}.$$ Cancelling the like factors in the numerator and denominator, we are left with 2 in the numerator.

$\frac{88}{2}$

You have now completed this segment.

Hand in the PUNCH CARD.

Make sure that you have the following items in your notebook.

(1) Definition of the expression, factoring over a given set.

(2) Definition of the word prime number.

(3) Definition of the words common monomial factor, greatest common monomial factor.

You should now do question 9 - 12 of the homework assignment.

You showed some ingenuity in this choice.

$$y^2 - y + yz - z \text{ can be arranged to}$$

$$y^2 + yz - y - z$$

Now the first two terms $y^2 + yz$

can be factored into $y(y + z)$

and the second two terms $-y \quad -z$

can be factored into $-1(y + z)$

together they equal $y(y + z) - 1(y + z)$

But this expression can be factored further.

Return to page $\frac{86}{2}$ and try this question again.

Suppose we make up a little formula:

a number = (its square root) (its square root)

With this formula you can easily check whether you have made the correct choice of a square root. To illustrate, let us check whether

$.08ab^2$ is the square root of $.04a^2b^4$

$.04a^2b^4$ should equal $(.08ab^2)(.08ab^2)$

Since

$(.08)(.08) = .0064$ we know that we have made a bad choice.

Please return to page $\frac{109}{2}$ and choose another letter.

$\frac{90}{1}$

Try multiplying 11 x 11 over again. We have obtained a different answer than the one you did.

Please return to page $\frac{83}{1}$ and choose another letter.

$\frac{90}{2}$

You can check whether you have factored correctly by multiplying the two factors. Their product should equal the original expression. The question that you should answer is:

Does $(a - 2c)(a - 2c) = a^2 - 4c^2$?

Please return to page $\frac{93}{2}$ and try this question again.

The choice you made is not wrong, but adding or subtracting zero from a number is really equivalent to just writing the numbers as they were given in the question. There is a better choice and we are sure that you can find it.

Return to page $\frac{85}{2}$ and choose another letter.

To check whether you have factored correctly, you should multiply the two factors.

Thus, if your choice is correct,

$(y - \frac{1}{8})(y + \frac{1}{8})$ should equal $y^2 - \frac{1}{16}$, but it doesn't.

Please return to page $\frac{106}{2}$ and try this question again.

$\frac{92}{1}$

Very good. You made the correct choice. The product of

$(x + 3)(x - 3)$ can be obtained by

direct multiplication; thus,

$$\begin{array}{rrr} x & + & 3 \\ x & - & 3 \\ \hline x^2 & + & 3x \\ & - & 3x - 9 \\ \hline x^2 & - & 9 \end{array}$$

However, if we observe the two binomials whose product we see, we note that they are of the <u>form</u>

$(a + b)(a - b)$. These are called "conjugates".

The product of such binomials is the square of the first term minus the square of the second term.

This formula should be memorized.

$$\boxed{(a + b)(a - b) = a^2 - b^2}$$

Please go on to question 5 below.

$\frac{92}{2}$

<u>Question 5</u>

Apply the proper principle to find the product

$(b - \frac{1}{2})(b + \frac{1}{2})$ and select the letter which

labels the correct answer.

(A) $b - \frac{1}{4}$

(B) $b^2 - b + \frac{1}{4}$

(C) $b^2 - \frac{1}{4}$

(D) None of these.

Very good. You made the correct choice.

Since: $(.9x)(.9x) = .81x^2$,

therefore, $.9x$ is the square root of $.81x^2$.

Since: $(.2ab^2)(.2ab^2) = .04a^2b^4$,

therefore, $.2ab^2$ is the square root of $.04a^2b^4$.

Please go on to question 9 below.

Question 9

Apply the proper principle and write

$a^2 - 4c^2$ in factored form.

Select the letter which labels the correct answer.

(A) $(a - 4c) (a + 4c)$

(B) $(a - 2c) (a + 2c)$

(C) $(a - 2c) (a - 2c)$

(D) $(a^2 - 2c) (1 + 2c)$

$\frac{94}{1}$

Very good. You made the correct choice. The product of two such conjugate binomials can be written down at sight. These binomials are of the form

$$(a + b) (a - b)$$

Their product is $a^2 - b^2$, or the square of the first term minus the square of the second term.

Thus, we have

$$(xy - 1) (xy + 1) = (xy)^2 - 1^2$$

$$= x^2y^2 - 1$$

Please go on to question 7 below.

$\frac{94}{2}$

Question 7

Perform the necessary calculation to find the square roots of the numbers

$$0.01 \quad , \frac{1}{16}$$

Select the letter which has the correct set of square roots next to it.

(A) $.1 \quad , \frac{1}{8}$

(B) $.0001 \quad , \frac{1}{4}$

(C) $.1 \quad , \frac{1}{4}$

(D) $.005 \quad , \frac{1}{32}$

This choice is correct.

Now proceed to question 2 which follows:

Question 2

Perform the necessary operation to find the squares of the two monomials,

$$8r^2t \quad \text{and} \quad -3uv^2$$

Select the letter next to the correct answers.

(A) $64r^4t^2$
$9u^2v^4$

(B) $16r^4t^2$
$-6u^2v^2$

(C) $64r^4t^2$
$-9u^2v^4$

(D) $64r^4t$
$9uv^4$

A good question to ask yourself is:

how much is $\frac{1}{2}$ times $\frac{1}{2}$.

The answer is not 1 , but $\frac{1}{4}$. Remember that $\frac{1}{2}$ of a number is not the same as the square root of that number.

Please return to page $\frac{110}{1}$ and try this question again.

$\frac{96}{1}$

You are confusing the square root of a number with $\frac{1}{2}$ of a number. They are not the same. You are looking for a number, which multiplied by itself will equal $\frac{1}{16}$. It may help you find such a number if you keep in mind that the square root of a proper fraction is the square root of the numerator divided by the square root of the denominator.

Return to page $\frac{106}{2}$ and try this question again.

$\frac{96}{2}$

If you read the question, you will see that the choice you made does not meet the requirements. Each of the numbers must be written as a binomial. The binomials must consist of the same numbers; the only difference being that one binomial has a plus sign, while the other has a minus sign. Here is an illustration of the type of binomials that we are looking for:

$$(77 - 5) (77 + 5).$$

Please return to page $\frac{85}{2}$ and try this question again.

You did not make the right choice.

What is the square root of $4c^2$?

Do you see your mistake?

Return to page $\frac{93}{2}$ and try this question again.

When we multiply two decimal numbers each less than one, the product is a smaller number than the numbers we started with.

Thus, .2 times .2 = .04.

Clearly, .04 is smaller than .2. Thus, in order to obtain the number .09 as a result of multiplying two equal numbers, the equal numbers must be larger than .09.

Please return to page $\frac{107}{1}$ and try this question again.

$\frac{98}{1}$

Go over your work and pay close attention to signs. We are sure that you will find your error. However, with binomials, one of which is the sum of two quantitites, the other is the difference of the same two quantities; (that is, when they are conjugate binomials) multiplication is not necessary. The product can be written at sight.

Please go over your reading assignment and study the rule for finding the product of such binomials.

Return to page $\frac{92}{2}$ and try this question again.

$\frac{98}{2}$

Say this to yourself: if my answer is correct, then

(.0001) x (.0001) should equal .01

But, .0001 x .0001 = .00000001

Therefore, I am wrong.

Please return to page $\frac{94}{2}$ and try this question again.

We do not agree. We would like to refresh your memory on a few matters.

(1) The square of a quantity means the quantity multiplied by itself. Thus, the square of $-3axy^2$ is

$$(-3axy^2)(-3axy^2) = 9a^2x^2y^4$$

(2) The square of a number is always a positive quantity.

(3) When two quantities having the same base are multiplied, the exponents are added.

Please return to page $\frac{95}{2}$ and try this question again.

Please check the choice you made by multiplying the two factors.

Does $(1 - x)(1 + 9x)$ equal $(1 - 9x^2)$?

Return to page $\frac{110}{1}$ and choose another letter.

$\frac{100}{1}$

We do not agree [illegible] choices does represent the correct answer to this ques[illegible]

Please return to page $\frac{99}{2}$ and choose another letter.

$\frac{100}{2}$

You should think of the binomial

$$(2x - 3y) \text{ as a single term.}$$

Let us go a step further and denote this binomial by a single letter; thus, let

$$u = 2x - 3y$$

Then, we have to factor $u^2 - 16z^2$.

This is the difference of two squares.

Please continue from here.

Return to page $\frac{111}{2}$ and try this question again.

You misunderstood.

Twice the sum is not the sam twice the product.

Please return to page $\frac{118}{1}$ and choose another letter.

We not agree.

How do we factor an expression consisting of the difference of two squares? Let us do a problem.

Factor $9x^2 - 4y^2$

What is the square root of $9x^2$? Answer, $3x$.

What is the square root of $4y^2$? Answer, $2y$.

Now, one of the factors is the sum of these square roots, the other is the difference of these square roots;

thus, $(3x + 2y)(3x - 2y)$, is the answer.

Please return to page $\frac{93}{2}$ and try this question again.

$\frac{102}{1}$

We do not agree.

You should remember that the two binomial factors of the difference of two [illegible] e different signs.

Please return to page $\frac{106}{2}$ and try this question again.

$\frac{102}{2}$

We do not agree.

One of the choices does have the correct answer. Examine all the choices again.

Return to page $\frac{110}{1}$ and try this question again.

One of the choices does represent the correct answer. Examine each of the choices carefully.

Return to page $\frac{85}{2}$ and choose another letter.

The square root of a number is not th same as one-half that number.

You need the answer to this question:

what number multiplied by itself equals $\frac{1}{16}$?

Please return to page $\frac{94}{2}$ and try this question again.

104
1

Very good. You made the correct choice.

The square root of $.09r^2$ is $.3r$, and the square root of s^2 is s.

Hence, the factors are

$$(.3r + s) \text{ and } (.3r - s).$$

Please go on to question 13 below.

104
2

Question 13

Apply the proper principle to write

$$x^{2a} - y^{2a} \text{ in factored form.}$$

Select the letter which labels the correct statement.

(A) This is not the difference of two squares and cannot be factored.

(B) $(x^2 + y^a) (x^a - y^a)$

(C) $(x^a - y^a)^2$

(D) $x^a y^a (x^a - y^a)$

This choice is correct.

Now proceed to question 6 which follows:

Question 6

Apply the proper principle to find the product

$(xy - 1)(xy + 1)$ and select the letter next to the correct answer.

(A) $x^2 y^2$

(B) $x^2 y^2 - xy - 1$

(C) $x^2 y^2 - 2xy - 1$

(D) $x^2 y^2 - 1$

To find twice the product of two monomials, you first multiply them together and then double the result.

For example: to find twice the product of 3a and 4b. We first find the product

$(3a)(4b) = 12ab$, then we double the result

$2(12ab) = 24ab$.

Please return to page $\frac{125}{1}$ and try this question again.

$\frac{106}{1}$

A binomial in the form $a^2 - b^2$ can be factored as

$$(a + b)(a - b).$$

Therefore, $a^2 - 4c^2 = (a + 2c)(a - 2c)$

or $(a - 2c)(a + 2c)$

this can be verified by multiplying the two binomials.

$$\begin{array}{rrrr} a & - & 2c & \\ \underline{a} & \underline{+} & \underline{2c} & \\ a^2 & - & 2ac & \\ & + & 2ac & - \; 4c^2 \\ \hline a^2 & & & - \; 4c^2 \end{array}$$

Please go on to question 10 below.

$\frac{106}{2}$

Question 10

Apply the proper principle to write $y^2 - \frac{1}{16}$ in factored form and select the letter which labels the correct answer.

Write the fractions in the form $\frac{1}{8}$.

(A) $(y - \frac{1}{8})(y + \frac{1}{8})$

(B) $(y - \frac{1}{32})(y + \frac{1}{32})$

(C) $(y - \frac{1}{4})(y + \frac{1}{4})$

(D) $(y - \frac{1}{4})(y - \frac{1}{4})$

This choice is correct.

Now proceed to question 12 which follows:

Question 12

Apply the proper principle to write

$.09r^2 - s^2$ in factored form and select the letter which labels the correct answer.

(A) $(.0003r + s)(.0003r - s)$

(B) $(.003r + s)(.003r - s)$

(C) $(.03r + s)(.03r - s)$

(D) $(.3r + s)(.3r - s)$

Twice the product of two quantities means that the product of these quantities is to be doubled.

For example, find twice the product of

$$-4xy^2 \text{ and } 6x^2$$

The product is $(-4xy^2)(6x^2) = -24x^3 y^2$. Notice the signs

The product doubled is $2 (-24x^3 y^2) = -48x^3 y^2$.

Let us try another example.

Find twice the product of

$$(-3p^2 q) \text{ and } (-6pq^2).$$

The product is $(-3p^2 q)(-6pq^2) = 18p^3 a^3$. Notice the signs.

The product doubled is $2 (18p^3 q^3) = 36p^3 q^3$.

Return to page $\frac{112}{2}$ and choose another letter.

108
1

This choice is correct.

Now proceed to question 4 which follows:

Question 4

Apply the proper principle to find the product

$(x + 3)(x - 3)$ and select the letter which labels the correct answer.

(A) $x^2 - 9$

(B) $x^2 + 6x - 9$

(C) $x^2 - 6x - 9$

(D) x^2

108
2

You seem to have mixed up different methods.

$(2x - 3y)^2$ is not the same as $(2x)^2 - (3y)^2$

Why don't you let $(2x - 3y) = a$ and $(16z) = b$.

Then the problem would become $a^2 - b^2$, which you know how to factor.

Please return to page 111/2 and try this question again.

Do not let the fact that the exponents are letters upset you.

You still would like to know the following:

Is x^{2a} the square of a quantity?

Is y^{2a} the square of a quantity?

If so, we have the difference of two squares, and it can be factored.

Please return to page 104/2 and try this question again.

This choice is correct.

Now proceed to question 8 which follows:

Question 8

Perform the necessary computation to find the square roots of each of the monomials:

$$.81x^2 \quad , \quad .04a^2 b^4$$

Select the letter which labels the correct answer.

(A) $.9x$, $.08ab^2$

(B) $.09x$, $.02ab^2$

(C) $.9x$, $.2a^2 b^2$

(D) $.9x$, $.2ab^2$

$\frac{110}{1}$

This choice is correct.

Now proceed to <u>question 11</u> which follows:

<u>Question 11</u>

Apply the proper principle to write $1 - 9x^2$ in factored form and select the letter which labels the correct answer.

(A) $(1 + 3x)(1 - 3x)$

(B) $(\frac{1}{2} - 3x)(\frac{1}{2} + 3x)$

(C) $(1 - x)(1 + 9x)$

(D) None of these.

$\frac{110}{2}$

The square of a binomial is a trinomial. It is very important for you to remember this statement.

Keep in mind that there are three terms in the expansion of $(x + y)^2$.

$(a + b)^2 \neq a^2 + b^2$ check by letting: $a = 10$

$b = 2$

$(10 + 2)^2 \neq 100 + 4$

Please return to page $\frac{114}{1}$ and try this question again.

Very good. The fact that we have literal exponents does not change the rule.

We note that $x^{2a} = (x^a)^2$, $y^{2a} = (y^a)^2$.

Thus, we have to factor the difference of two squares.

The factors are,

$$(x^a + y^a) \text{ and } (x^a - y^a).$$

Please go on to question 14 below.

Question 14

Determine the proper principle and express

$(2x - 3y)^2 - 16z^2$ in factored form.

Choose the letter next to the correct answer.

(A) $(2x - 3y - 4z)^2$

(B) $(2x - 3y)(2x + 3y) - 16z^2$

(C) $(2x - 3y + 4z)(2x - 3y - 4z)$

(D) $4z(2x + 3y) - 4z(2x - 3y)$

$\frac{112}{1}$

We do not agree.

We are looking for <u>twice</u> the product, not merely the product.

Return to page $\frac{118}{1}$ and choose another letter.

$\frac{112}{2}$

This choice is correct.

Now proceed to <u>question 3</u> which follows:

<u>Question 3</u>

Perform the necessary calculation to find twice the product of

$$-3ab^2 \quad \text{and} \quad 6a.$$

Select the letter which labels the correct statement.

(A) $-9a^2b^2$

(B) $-18a^2b^2$

(C) $18a^2b^2$

(D) $-36a^2b^2$

Although $(x^a)^2$ equals x^{2a}

It is not true that

$$(x^a - y^a)^2 = x^{2a} - y^{2a}$$

The square of a binomial is a trinomial.

$$(a - b)^2 = a^2 - 2ab + b^2$$

Please return to page 104/2 and try this question again.

Part of the rule for squaring a binomial states " twice the product ". To make sure that your middle term is right, multiply the two terms of the binomial and then double this product.

Please return to page 134/1 and try this question again.

$\frac{114}{1}$

This choice is correct.

Now proceed to question 4 which follows:

Question 4

Apply the proper principle to find $(p + q)^2$ and select the letter which labels the correct statement.

(A) $p^2 + q^2$

(B) $2pq$

(C) $p^2 + 2pq + q^2$

(D) $p^2 q^2$

$\frac{114}{2}$

What is the square of 3r? Please note that the first term of the binomial has the letter " r " raised to the second power. Squaring this term will therefore result in increasing the power of " r ".

Please return to page $\frac{127}{2}$ and try this question again.

Your answer is not wrong since you did factor the first term. However, you failed to realize that

$$(2x - 3y)^2 - 16z^2$$ is actually a case

of " the difference of two squares ", and it can be factored the same way that $a^2 - b^2$ can be factored.

But here, of course, " a " stands for

$$(2x - 3y).$$

With this hint you should be able to continue.

Please return to page $\frac{111}{2}$ and try this question again.

We do not agree.

As you are finding the square of a binomial, you should be saying the following to yourself:

The square of the first term, plus twice the product of the first and second terms, plus the square of the second term. Thus, in expanding

$$(3rs + t)^2$$ for example we notice that the square

of the first term is

$$(3rs)^2 = 9r^2 s^2 .$$

Twice the product of the first and second terms is

$$2(3rs)(t) = 6rst.$$

The square of the second term is

$$(t)^2 = t^2 .$$

Please return to page $\frac{135}{2}$ and try this question again.

le for finding the square of a binomial is to square the first

d twice the product of the first and second terms; and add th

f the second term.

our choice carefully. We are sure that you will discover the

at you made.

return to page $\frac{126}{2}$ and try this question again.

---

er that part of the rule states, " twice the product " of the two

of the binomial. What is twice the product of (5x) and (-3y

Please reconsider your choice.

to page $\frac{123}{2}$ and choose another letter.

Check your answer by multiplication.

Note:

$x^a y^a \quad) = x^{2a} y^a$ this should convince you that you made a poor choice.

Please return to page $\frac{104}{2}$ and try this question again.

We do not agree.

Please remember that the square of a binomial is a trinomial.

You have a term missing.

Please return to page $\frac{134}{1}$ and try this question again.

1

Volume 8 Segment 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	5[illegible]	4	0	(Sequence Number)
	54	and	5[illegible]	0	4	(Type of Punch Card)
	60	and	6[illegible]	0	8	(Volume Number)
	66	and	6[illegible]	0	5	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 248 - [illegible]9
You will now be asked a series of questions to draw your attention to the most important points.

Question 1

Perform the necessary calculation to find twice the product of 5 and 7.

Select the letter which labels the correct answer.

(A) 24 (C) 4

(B) 35 (D) 70

1[illegible]
2

The square of the sum of two terms is equal to the square of the first term, plus twice the product of the first and the second, plus the square of the second term.

For example, $(x + 3)^2$ equals;

x^2, the square of the first term,

$+ 6x$, twice the product of the first and second terms,

$+ 9$, the square of the last term.

This follows the rule:

$$a + b^2 = a^2 + 2ab + b^2$$

Please return to page 1 and choose another letter.

Excellent. You answered a pretty difficult question correctly.

If we consider the binomial (2x - [illegible]) as a single term, we have to factor the difference of two squares.

Now, the square root of

(2x - [illegible] is 2x - 3y;

and the square root of

16[illegible] [illegible] 4z.

Hence, the factors are:

2x - [illegible] + 4z

and 2x - [illegible] - 4z.

You have now completed this segment. Make sure that you have the following items in your notebook:

(1) The product of the sum and difference of two numbers is the square of the first number minus the square of the second number. Thus,

$$(a + b) (a - b) = a^2 - b^2 .$$

(2) The difference of the squares of two numbers can be factored as illustrated by the following example:

$$a^2 - b^2 = (a + b) (a - b).$$

You should now do questions 13 - 16 of the homework assignment.

$\frac{120}{1}$

You found twice the difference.

In order to find twice the product, you first [illegible] the product and then double this product.

Please return to page $\frac{118}{1}$ and choose another letter.

$\frac{120}{2}$

The square of a binomial is <u>not</u> equal to the square of the first term plus the square of the second term. In other words.

$$(2a + 3b)^b \text{ does } \underline{\text{not}} \text{ equal } (2a)^2 + (3b)^2 .$$

It is possible that you are saying to yourself, " why not "? Let us show why not. Suppose that we have $(3 + 7)^2$. Had we added the sum of the squares of the terms, we would have

$$3^2 - 7^2 = 9 + 49 = 58$$

But,

$$3 - 7 = 10 \quad \text{and} \quad 10^2 = 100.$$

Certainly, 100 does not equal 58.

Please return to page $\frac{126}{2}$ and try this question again.

Did you guess?

If you had read the assignment, you would have known that the square of a binomial is a expression of three terms.

This rule implies: $(a + b)^2 = a^2 + 2ab + b^2$

Return to page 111 and try this question again.

You did not make the right choice.

What is the square of 3k ?

Remember that the square of a number is the number multiplied by itself and not the number multiplied by 2.

Please return to page 134 and choose another letter.

$\frac{122}{1}$

What is [illegible] the product of $3x^2$ and [illegible] ?

Please re-examine the middle term of your choice.

Return to page $\frac{127}{2}$ and choose another letter.

$\frac{122}{2}$

We do not agree.

We have no quarrel with the first and last terms of your answer, but the middle term is not correct. The middle term is found by taking twice the product of the two terms of the binomial.

What does

$$2\,(\,a\,)\,(\,-\frac{b}{2}\,)$$ equal to?

Do you see your mistake?

Please return to page $\frac{136}{2}$ and try this question again.

Although one method of squaring a binomial is to employ the rule which you should memorize:

$$\boxed{(a + b)^2 = a^2 + 2ab + b^2}$$

Some students find it easier to say:

" the square of the first term

plus two times the product of the first and last term

plus the square of the last term ".

Your answer is correct.

Please go on to question 9 below.

Question 9

Apply the proper principle to expand $(5x - 3y)^2$

Select the letter next to the correct answer.

(A) $25x^2 - 15xy + 9y^2$

(B) $15x^2 - 30xy + 6y^2$

(C) $25x^2 - 30xy + 9y^2$

(D) $25x^2 + 9y^2$

$\frac{124}{1}$

We do not agree. You forgot an old rule.

When <u>a power</u> is raised to a power, the exponents are multiplied.

i.e. $\boxed{(x^a)^b = x^{ab}}$

For example, $(3x^2 y)^2 = 3^2 (x^2)^2 (y^1)^2 = 9x^4 y^2$.

Please return to page $\frac{133}{1}$ and try this question again.

$\frac{124}{2}$

Let us give you a small hint.

Suppose you re-write

$$(2x - 3y + 1)^2 \quad \text{as}$$

$$[(2x - 3y) + 1]^2$$

In other words, think of $(2x - 3y)$ as a single term and apply the rule for squaring a binomial.

Please return to page $\frac{132}{2}$ and try this question again.

This choice is correct.

Now proceed to question 2 which follows.

Question 2

Perform the necessary calculation to find twice the product of $3m$ and $2n$. Select the letter which is next to the correct answer.

(A) $6\ mn$

(B) $12\ mn$

(C) $24\ mn$

(D) $6\ mn + 4\ n$

This choice is correct.

Now proceed to question 12 which follows.

Question 12

Apply the proper principle to expand $(x^a + y^b)^2$

Select the letter which labels the correct statement.

(A) $x^{a^2} + 2x^a y^a + y^{a^2}$

(B) $x^{2a} + y^{2b}$

(C) $x^{2a} + 2x^a y^b + y^{2b}$

(D) $x^{a^2} + y^{a^2}$

Very good. You made the correct choice.

$$(p + q)^2 \text{ is the same as } (a + b)^2$$

i.e. $$(p + q)^2 = p^2 + 2pq + q^2 .$$

This rule should be memorized.

Please go on to question 5 below.

Question 5

Apply the proper principle to expand $(3x + 1)^2$. Select the letter next to the correct answer.

(A) $9x^2 + 3x + 1$

(B) $9x^2 + 6x + 1$

(C) $9x^2 + 1$

(D) $6x + 2$

You should memorize the rule.

$$\boxed{(a - b)^2 = a^2 - 2ab + b^2}$$

Then $(3k - 2)^2$ would have $a - 3k$

$b = 2$

and substituting in the rule

$$(3k - 2)^2 = (3k)^2 - 2(3k)(2) + (2)^2$$
$$= 9k^2 - + 4$$

Your answer choice was correct.

Please go on to question 7 below.

Question 7

Apply the proper principle to expand $(3r^2 - 7)^2$. Select the letter which labels the correct statement.

(A) $9r^2 - 42r + 49$

(B) $6r^4 - 42r + 49$

(C) $9r^4 - 49$

(D) $9r^4 - 42r^2 + 49$

$\frac{128}{1}$

The square of a number is the number multiplied by itself. Thus, the square of $5x$ is $(5x)(5x)$, and the square of $3y$ is $(3y)(3y)$. Look over the choice you made. We are sure that you will discover your error.

Return to page $\frac{123}{2}$ and choose another letter.

$\frac{128}{2}$

By just looking at your choice, you know that it is not correct.

All that you did was to square each term.

You must apply the rule

$$(x - y)^2 = x^2 - 2xy + y^2 .$$

Please return to page $\frac{136}{1}$ and try another letter.

The square of a binomial is a trinomial. Since your choice contains only two terms, please reconsider.

Return to page $\frac{133}{1}$ and try this question again.

Allow us to refresh your memory concerning the rule for raising a power to a power. When a power is raised to a power, the exponents are multiplied.

For example: $(x^3)^2 = x^6$. Similarly,

$$(r^a)^2 = r^{2a}, \text{ and not } r^{a^2}.$$

Please return to page $\frac{125}{2}$ and try this question again.

$\frac{130}{1}$

The square of a binomial does <u>not</u> equal the square of the first term plus the square of the second term. Since the square of a binomial is a trinomial, you have a term missing. Please find it.

Return to page $\frac{127}{2}$ and try this question again.

$\frac{130}{2}$

What is the square of 12? That's easy you say. The square of 12 is 144. Now, $12 = 10 + 2$, so that $(10 + 2)^2$ should also equal 144. If we take the sum of the squares of the two terms of the above binomial we get, $10^2 + 2^2 = 100 + 4 = 104$. This answer is 40 less than the correct answer. How do we get this number 40 ? We must add twice the product of the two terms of the binomial; thus, $2(10)(2) = 40$. What is the moral of the above illustration? Don't forget the middles term when you square a binomial.

Please return to page $\frac{123}{2}$ and try this question again.

The square of a number is not the same as twice that number. The expression

$$(3x + 1)^2$$

is not the same as

$$2(3x + 1).$$

Return to page $\frac{126}{2}$ and try this question again.

The rule for squaring a binomial says in part, "twice the product". We are sure that you made your choice hastily.

Please return to page $\frac{133}{1}$ and choose another letter.

13.
1

Very good. You made the correct choice. We have to expand

$$(x^a + y^b)^2 .$$

The square of x^a is $(x^a)^2 = x^{2a}$.

Twice the product of x^a and y^b is $2x^a y^b$.

The square of y^b is $(y^b) = y^{2b}$.

Please go on to question 13 below.

13.
2

Question 13

Relate to the proper principle to simplify the expression,

$$(2x - 3y + 1)^2 .$$

Select the letter next to the correct statement.

(A) $4x^2 + 9y^2 + 1$

(B) $4x^2 - 12xy + 9y^2 + 1$

(C) $4x^2 - 12xy + 9y^2 + 4x - 6y + 1$

(D) None of these.

This choice is correct.

Now proceed to question 10 which follows.

Question 10

Apply the proper principle to expand $(x^2 - 3y^2)^2$.

Select the letter which labels the correct statement.

(A) $x^2 - 6x^2y^2 + 9y^2$

(B) $x^4 - 6x^2y^2 + 9y^4$

(C) $x^4 - 9y^4$

(D) $x^4 - 3x^2y^2 + 9y^4$

We do not agree. One of the letters has the correct answer to this question. Do you remember the rule for squaring a binomial? Square the first term; add twice the product of the first and second terms; add the square of the second term.

Please return to page $\frac{136}{2}$ and choose another letter.

$\frac{134}{1}$

This choice is correct.

Now proceed to question 6 which follows.

Question 6

Apply the proper principle to expand $(3k - 2)^2$

Select the letter which labels the correct answer.

(A) $9k^2 - 12k + 4$

(B) $9k^2 - 6k + 4$

(C) $9k^2 - 4$

(D) $6k^2 - 12k + 4$

$\frac{134}{2}$

You handled the squaring of x^a and y^b properly, but you forgot the rule for squaring a binomial.

Please return to page $\frac{125}{2}$ and try this question again.

The expression $(3r^2 - 7)^2$ can be considered a form of $(a - b)^2$ which equals $a^2 - 2ab + b^2$ when expanded.

By careful substitution when $3r^2 = a$ and $b = 7$,

we get $(3r^2)^2 - 2 (3r^2) (7) - (7)^2$

or $9r^4 - 42r + 49$

which is our answer choice.

Please go on to question 8 below.

Question 8

Apply the proper principle to expand $(2ax + b)^2$.

Select the letter next to the correct answer.

(A) $4a^2x^2 + b^2$

(B) $4a^2x^2 + 4axb + b^2$

(C) $4a^2x^2 + 2axb + b^2$

(D) $4abx$

136
1

To expand $(x^2 - 3y^2)^2$, we refer to the basic formula

$$(a - b)^2 = a^2 - 2ab + b^2$$

Therefore, $(x^2 - 3y^2)^2 = (x^2)^2 - 2(x^2)(3y^2) + (3y^2)^2$

$$= x^4 - 6x^2y^2 + 9y^4$$

Your choice was correct.

Please go on to question 11 below.

136
2

Question 11

Apply the proper principle to expand $(a - \frac{b}{2})^2$

Select the letter which labels the correct statement.

(A) $a^2 - \frac{ab}{2} + \frac{b^2}{4}$

(B) $a^2 - \frac{b^2}{4}$

(C) $a^2 - ab + \frac{b^2}{4}$

(D) None of these.

Excellent. You did a pretty difficult problem correctly. Let us do this problem together. We have

$$(2x - 3y + 1)^2$$

We will consider $(2x - 3y)$ as a single term and rewrite the above as,

$$\left[(2x - 3y - 1)\right]^2$$

This can be considered as the square of a binomial; thus,

$$(2x - 3y)^2 + 2(2x - 3y) + 1$$

Expanding again, we get,

$$4x^2 - 12xy + 9y^2 + 4x - 6y + 1$$

You have now finished this segment.

PUNCH CARD: Before you hand in your punch card to the instructor, check to see that you have entered the required information; particularly your course student number, and this course number and the sequence number of the segment just completed which is 40.

NOTEBOOK: You should have entered in your notebook the following formulas and definitions:

(1) $(a + b)^2 = a^2 + 2ab + b^2$

(2) $(a - b)^2 = a^2 - 2ab + b^2$

(3) $(a + b + c)^2 = \left[(a + b) + (c)\right]^2$

$= (a + b)^2 + 2(a + b) (c) + c^2$

HOMEWORK ASSIGNMENT: You are now able to complete the homework assignment. Hand in your homework before you take the volume test. You are now advised to review all of your notes and to re-read the reading assignment in order to prepare yourself for the volume test.

1/16/70

Programmed Mathematics Continuum — ERRATA SHEET — Attach to back Cover

ALGEBRA - LEVEL ONE — VOLUME 8

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected. Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE		MISPRINT	CORRECTION	CHECK WHEN CORRECTION MADE
$\frac{44}{2}$		$\frac{52}{2}$	$\frac{53}{2}$	
$\frac{100}{1}$		$\frac{99}{2}$	$\frac{92}{2}$	
$\frac{48}{1}$	Line 6	___	$\frac{\cdot}{\cdot}$	
$\frac{22}{2}$	Line 2	___	$\frac{\cdot}{\cdot}$	
$\frac{36}{2}$	Line 2	___	$\frac{\cdot}{\cdot}$	
$\frac{53}{2}$	Line 2	___	$\frac{\cdot}{\cdot}$	
$\frac{40}{2}$	Line 2	___	$\frac{\cdot}{\cdot}$	
$\frac{86}{2}$	Line 4	x^{a+3a}	$xa + 3a$	