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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 7: products and quotients of powers, multiplying polynomials by monomials and by polynomials, and problems involving area. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 through SE 015 870.) (DT)

# PROGRAMMED MATH CONTINUUM

## level one

# ALGEBRA



VOLUME 7



NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

ED 075208

# PROGRAMMED MATH CONTINUUM

LEVEL ONE

# ALGEBRA

VOLUME 7

New York Institute of Technology

Old Westbury - New York

## PREFACE

A

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 7

# TABLE OF CONTENTS



## IN THE STUDY GUIDE:

# VOLUME 7

This volume covers the following material as shown in this excerpt from the Syllabus:

| SEGMENT | DESCRIPTION | REFERENCE BOOK SECTION DOLCIANI | DRESSLER | DODES |
|---|---|---|---|---|
| 1 | Product of powers | 6-3 | 7-3<br>7-4 | 4-2 |
| | Power of product | 6-4 | 7-4<br>11-3 | 4-2 |
| 2 | Multiplying Polynomial by monomial | 6-5 | 8-3<br>8-4 | 8-4 |
| 3 | Multiplying polynomial by polynomial | 6-6 | 8-5 | 8-4 |
| 4 | Problems about area | 6-7 | 10-11 | 11-4 |
| 5 | Quotient of powers | 6-9 | 7-5<br>7-6 | 4-3 |

D

READING ASSIGNMENT

VOLUME 7

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

| SEGMENT | FROM PAGE | TO PAGE |
|---|---|---|
| 1 | 203 | 206 |
| 2 | 206 | 208 |
| 3 | 209 | 210 |
| 4 | 211 | 215 |
| 5 | 215 | 221 |

Modern Algebra Book I
Dolciani, Berman and
Freilich Houghton
Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer-
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

E

HOMEWORK ASSIGNMENT

VOLUME NO. 7

BOOK: 597 DRESSLER

| HOMEWORK QUESTION NO. | PAGE NO. | EXAMPLE NUMBER | MBO REFERENCE |
|---|---|---|---|
| 1 | 204 | 2 , 5 | 07110 |
| 2 | 204 | 9 , 13 | 07110 |
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| 7 | 207 (B) | 10 , 11 , 13 | 07210 |
| 8 | 208 | 1 , 3 , 5 | 07219 |
| 9 | 210 | 7 , 9 | 07310 |
| 10 | 210 | 15 , 17 , 21 | 07310 |
| 11 | 210 | 23 , 25 , 29 | 07310 |
| 12 | 210 | 33 , 39 | 07310 |
| 13 | 212 | 3 | 07420 |
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F

# GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

Volume 7 Segment 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

| | | | | |
|---|---|---|---|---|
| COLUMNS | 48 and 50 | 3 | 1 | (Sequence Number) |
| | 54 and 56 | 0 | 4 | (Type of Punch Card) |
| | 60 and 62 | 0 | 7 | (Volume Number) |
| | 66 and 68 | 0 | 1 | (Segment Number) |

Your READING ASSIGNMENT for this Segment is pg: 203 - 206
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

In the expression $5x^2$, recognize which symbol is the base, and which is the exponent; and select the correct statement.

(A) 5 is the base and 2 is the exponent.

(B) "x" is the base and 2 is the exponent.

(C) 2 is the base and 5 is the exponent.

(D) 5 is the exponent and x is the base.

---



We don't agree. The expression contains only one variable, "x". Surely, the expression contains different powers of "x", but as long as you see only one letter in the polynomial; the expression is a polynomial in only one variable.

Return to page 6 and try question 2 again.

$\frac{2}{1}$

We don't agree.

The rule for multiplying quantities which have the same base is to keep the base and <u>add</u> the exponents.

$$x^a \cdot x^b = x^{a+b}$$

If you examine your choice, you will notice that the exponents are multiplied.

Return to page $\frac{30}{1}$ and try question 4 again.

---

$\frac{2}{2}$

This choice is correct.

Now proceed to question 10 which follows:

<u>Question 10</u>

Apply the proper principle and select the correct answer to the question: ( - 4a ) 3 equals ?

(A) $12a^3$

(B) $-64a^3$

(C) $64a^3$

(D) $-12a^3$

Wrong answer.

When a power is raised to a power, the powers are multiplied. Thus for example, $( x^2 )^7 = x^{14}$ . Note that the 7 and 2 were multiplied. It would be a good idea for you to write this rule down in your notebook.

$$( x^a )^b = x^{ab}$$

Return to page $\frac{31}{1}$ and try question 7 again.

---



We don't agree. The expression you selected, namely $5a^2$ is a monomial. A monomial is an expression consisting of one term. It may help you to remember this if you think of the word monocle, an eyeglass for one eye.

Return to page $\frac{135}{2}$ and try question 1 again.

$\frac{4}{1}$

The exponent of "a" is 1 ; the exponent of the letter "b" is 2 an exponent which is wri n to its right and above. When there is no visible exponent, it means that the exponent is 1 .

Return to page $\frac{45}{1}$ and try question 2 again.

---

$\frac{4}{2}$

This choice is correct.

Now proceed to question 3 which follows:

Question 3

Recognize each of the following 4 expressions and select the letter which labels the true statement.

(I.) $a^2 - b^2$ . (II.) $a^2b$. (III.) $a^2 + ab + b^2$. (IV.) $\frac{x^2y^2}{70}$

(A) I and II are monomials

(B) I and IV are binomials

(C) II is a monomial and IV is a trinomial

(D) I is a binomial and III is a trinomial

When we multiply 2 quantities having numerical coefficients, these coefficients also have to be multiplied. Thus, for example, in multiplying $9x^2y$ and $4xy$, the 9 and the 4 must be multiplied, thus making the coefficient of the product equal to 36 .

Return to page $\frac{35}{2}$ and try question 5 again.

---



This choice is correct.

Now proceed to question 11 which follows:

Question 11

Apply the proper principle and indicate which letter labels the correct answer to question, $(-4b^3)^2$ equals?

(A) $8b^6$

(B) $16b^5$

(C) $16b^6$

(D) $-16b^6$

$\frac{6}{1}$

When a product of 2 quantities is raised to a power, each of the quantities is raised to that power. Thus, for example, $(kl)^3$ means that both and l are raised to the third power. $(rst)^4$ means that s, t are each raised to the fourth power.

Return to page $\frac{36}{1}$ and try question 8 again.

---

$\frac{6}{2}$

This choice is correct.

Now proceed to question 2 which follows:

<u>Question 2</u>

Recognize which of the following expressions is a p~lynomial in three variables, and select the letter which labels the correct statement.

(A) $x^3 - 5x^2 + 6x$

(B) $ab + bc + ca$

(C) $a^2 + b^2 + a^2b^2$

(D) $x^3y^3$

The rule for multiplying quantities having the same base is: maintain the base and add the exponents. Since $a^7 \cdot a^2$ really means

$$\underset{7}{a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a}\cdot\underset{2}{a\cdot a} = \underset{9}{a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a}$$

Thus, $a^7 \cdot a^2 = a^{7+2} = a^9$ . Note that the base "a" was maintained, and the exponents 7 and 2 were added. You, of course, realize that a letter written without an exponent means that the letter has the exponent 1 .

Return to page $\frac{37}{1}$ and try question 3 again.

---



The commutative property is a property of <u>order</u>. Thus, the commutative property of multiplication states that the <u>order</u> in which two factors are multiplied does not affect the product. We say $ab = ba$ for all "a" and "b".

Return to page $\frac{11}{2}$ and try question 4 again.

$\frac{8}{1}$

When multiplying quantities with the same base, the exponents are added. This rule applies when the exponents are letters as well as when they are numbers. Thus, for example,

$$y^{2k-3} \cdot y^{3k+7} = y^{2k-3+3k+7}$$

$$= y^{5k + 4}$$

Return to page $\frac{38}{1}$ and try question 6 again.

---

$\frac{?}{?}$

This choice is correct.

Now proceed to question 12 which follows:

Question 12

Apply the proper principle and indicate which letter labels the correct answer to the question, $(-2xy)^3$ equals ?

(A) $-8x^3y^3$

(B) $8x^3y^3$

(C) $-6x^3y^3$

(D) $6x^3y^3$

When a power is raised to a power, the exponents are multiplied. The fact that an exponent is a letter instead of a number, does not change this rule.

Return to page $\frac{17}{2}$ and try question 9 again.

---



Please write down the following into your notebook. The distributive property states: if an indicated sum is to be multiplied by a number, <u>each</u> addend must be multiplied by that number.

For example,

$$\boxed{a(x + y) = \underline{a}\,x + \underline{a}\,y\ .}$$

Return to page $\frac{22}{2}$ and try question 5 again.

$\frac{10}{1}$

This choice is correct.

Now proceed to question 13 which follows:

Question 13

Determine which of the following statements is correct by checking the appropriate letter.

(A) $5^3 + 5^3 = 5^6$

(B) $5^3 + 5^3 = 10^3$

(C) $5^3 + 5^3 = 10^6$

(D) none of the above statements is correct.

---

$\frac{10}{2}$

This choice is correct.

Now proceed to question 6 which follows:

Question 6

Apply the proper principle and indicate which letter labels the correct answer to the product: $-9(a^2 - 2ab)$.

(A) $-9a^2 - 18ab$

(B) $-9a^2 + 18ab$

(C) $-9a^2 - 7ab$

(D) $-9a^2 + 11b$

$\frac{11}{1}$

Good. You made the right choice. The expression $( a^2 - b^2 )$ contains 2 terms, and is therefore called a binomial. The expression $( a^2 + ab + b^2 )$ contains three terms and is therefore called a trinomial.

Please go on to question 4 below.

---

$\frac{11}{2}$

Question 4

Recognize the property which enables you to state that $5(x + y) = 5x + 5y$, and select the correct letter.

(A) the commutative property of multiplication

(B) the associative property of addition

(C) the whole is equal to the sum of its parts

(D) the distributive property

$\frac{12}{1}$

The base is the letter (or number) to which the exponent is written. It is the number which is multiplied. Thus, for example, in the expression, $3a^5$ , the base is the letter "a," and the exponent is 5. $a^5$ means $a \cdot a \cdot a \cdot a \cdot a$ ; $3a^5$ means $3\ a \cdot a \cdot a \cdot a \cdot a$. Similarly, in the expression $7^4$ , the base is the number 7 and the exponent is 4. $7^4$ means $7 \cdot 7 \cdot 7 \cdot 7$.

Return to page $\frac{1}{1}$ and try question 1 again.

---

$\frac{12}{2}$

In order for an expression to be a polynomial in 3 variables, it is necessary that exactly 3 different letters appear in the expression.

Return to page $\frac{6}{2}$ and try question 2 again.

When multiplying 2 quantities with the same base, you add the exponents. You were right in adding the exponents of a , but when it came to the exponents of b , you multiplied then. It is quite probable that you were not sure of the rule just stated above. Don't you think that it would be a good idea to write this rule down in your notebook ?

$$\boxed{x^a \cdot x^b = x^{a+b}}$$

Please return to page $\frac{30}{1}$ and try question 4 again.

---



When a product of 2 quantities is raised to a power, each of the quantities is raised to that power. Let us illustrate this with a few examples:

$$(-5r)^3 \text{ means } (-5)^3 \cdot r^3$$

$$(7s)^4 \text{ means } (7)^4 \cdot s^4$$

Return to page $\frac{2}{2}$ and try question 10 again.

$\frac{14}{1}$

When a power is raised to a power, the exponents are multiplied, and not added. You don't seem to be sure of this rule, since, in one part of the question you multiplied the exponents, and in the second part, you added them. Please write this rule down in your notebook.

$$\boxed{(x^a)^b = x^{ab}}$$

Return to page $\frac{31}{1}$ and try question 7 again.

---

$\frac{14}{2}$

A monomial is an expression of one term. The one term may consist of the product of many letter, thus, $5a^2b^3c^7$ is still a monomial, even though it is made up of the product of 4 quantities.

Return to page $\frac{135}{2}$ and try question 1 again.

We don't agree. The letter " y " has no visible exponent. When this is the case, it means that " y " has the exponent 1.

We do the same with coefficients. When we write $x + 2y$ , we really mean $1(x) + 2y$.

Return to page $\frac{45}{1}$ and try question 2 again.

---



Wrong answer.

A monomial is an expression consisting of only one term. Thus $5xy$, $-3x^2$ , $7/3$ , are each monomials. However, once a plus or minus sign exists between terms, the expression is no longer a monomial. Thus, for example, $x - y$ , and $5a + 3b - 9$ are not monomials.

Return to page $\frac{4}{2}$ and try question 3 again.

$\frac{16}{1}$

In this problem " a " is multiplied by $a^2$. You remember that a letter written without an exponent means that the letter has the exponent 1. Thus, when you see a product like $(xy)\ (x^2y^3)$, you should immediately think of $xy$ as $x^1y^1$.

Return to page $\frac{35}{2}$ and try question 5 again.

---

$\frac{16}{2}$

Wrong answer.

When a term like $5x^2y$ is enclosed in parentheses, and raised to a power, each member of the term must be raised to that power.

For example, $(5x^2y)^3$ means $5^3$, times $(x^2)^3$, times $y^3$.

Return to page $\frac{5}{2}$ and try question 11 again.

Good. You made the correct choice. When the product of 2 or more quantities is raised to a power, each of the quantities is raised to that power. Thus, $(xy)^4$ means that both $x$ and $y$ are raised to the $4^{th}$ power.

Hence, the correct answer is

$$x^4y^4$$

Proceed to question 9 below.

---



Question 9

Apply the proper principle and indicate which letter labels the correct statement.

(A) $(r^3)^{2a} = r^{6a}$

(B) $(r^3)^{2a} = r^{3+2a}$

(C) $(r^3)^{2a} = r^{5a}$

(D) $(r^3)^{2a} = r^{6+a}$

$\frac{18}{1}$

When a letter is written without an exponent, it means that the letter has the exponent 1. Thus, the expression $xy^5$ really means $x^1 y^5$. You are probably thinking to yourself, why don't they write the exponent 1, and avoid the problem of remembering that a letter written without an exponent means the exponent is 1 ? Can you think of a good reason ? I suppose that a good reason is common usage. For example, when you say <u>a</u> dollar, you mean 1 dollar. Similarly, when you say a letter " x ", you mean $1x^1$

Return to page $\frac{37}{1}$ and try question 3 again.

---

$\frac{18}{2}$

Incorrect.

The associative property is a property of <u>grouping</u>.

That is, the way in which factors are grouped does not affect the product. We say $a(bc) = (ab)c$ for all a, b, and c.

Return to page $\frac{11}{2}$ and try question 4 again.

That's an incorrect choice.

You were probably hasty in your choice of the answer.

You know that $(a + 1) + (a - 1)$ does not equal 0.

Just to make sure, return to page $\frac{38}{1}$ and try question 6 again.

---

$\frac{19}{2}$

Incorrect.

Each of the members of the term in the parenthesis is raised to the third power. When a negative number is raised to an odd power, the sign of the result is negative.

Return to [illegible] t[illegible] tion 12 again.

When multiplying quantities with the same base, the exponents are added. This rule applies when the exponents are letters as well as when they are numbers. The nice thing about this rule is that it has no exceptions

In other words, if you are asked to calculate:

$$x^a \cdot x^b$$

then the answer would be:

$$x^a \cdot x^b = x^{a+b}$$

Please return to page $\frac{38}{1}$ and try question 6 again.

---

)

We don't agree.

Is it possible that you forgot how to multiply quantities with the same base ? This illustration should refresh your memory.

$$x^3y \quad (3x + 4y) = 3x^4y + 4x^3y^2$$

Note that when we multiply quantities having the same base, the exponents are added. Also note that a letter written without an exponent, means that the letter has the exponent 1 .

Please return to page $\frac{22}{2}$ and try question 5 again.

orrect. When er is raised to a power, the exponents are ltiplied. Did . make a careless take by saying that the product of 3 and 2a is 5a ?

Return to page $\frac{17}{2}$ and try question 9 again.

---



You fell for an obvious trap. The rule for adding exponents applies only when we are <u>multiplying</u> quantities with the same base. In this question we have $5^3$ <u>plus</u> $5^3$ , and the above rule does not apply. You are probably asking yourself this question: if we have no rule, how do we get an answer? We just have to resort to actual computation. Note that:

$$5^3 + 5^3 = 125 + 125 = 250.$$

Re rn page $\frac{10}{1}$ an question 3 again

Very good. You made the correct choice.

By the distributive property we are able to say that

$$5\ (x + y) = 5x + 5y$$

Please go on to the next question below.

---

Question 5

Apply the proper principle and indicate the correct answer to the product, $3a(a^2 + 5)$.

(A) $3a^3 + 5$

(B) $3a^3 + 15a$

(C) $3a^2 + 15a$

(D) $3a^3 + 8a$

The small number to the right and above is the exponent. The letter which has the exponent to its right and above is the base. The number in front of a term, in this case, the number 5, is called the coefficient. Thus, for example, in the term

$$8\,a^{5},$$

we note that 8 is the coefficient

"a" is the base

5 is the exponent

Return to page 1/1 and try question 1 again.

---



It is possible that you are confusing the power to which a variable is raised with the number of variables in an expression. Let us try to clear this up for you. A variable is a letter used to represent an unspecified element of a set of numbers. To determine the number of variables in an expression, we count the number of different letters that appear in the expression under consideration.

Please return to page 6/2 and try question 2 again.

2-

Wrong answer.

When multiplying 2 quantities having the same base, the exponents are added, not multiplied.

Thus for example, $x^3 \cdot x^4 = x^{3+4}$. Since you will have to use this rule very often in your future work in algebra, it would be a good idea for you to write it into your notebook.

$$\boxed{x^a \cdot x^b = x^{a+b}}$$

Please return to page $\frac{30}{1}$ and try question 4 again.

---

2-

Incorrect.

In the expression $(-4a)^3$, the number $-4$ is also raised to the third power. Furthermore, the product of 2 negative quantities is a positive quantity, and the product of a positive and negative quantity is negative. I wonder whether you can state a rule for the sign of a number raised to an odd power. Of course you can. "A negative number raised to an odd power has a negative sign."

Please return to page $\frac{2}{2}$ and try question 10 again.

Wrong answer.

The rule is:

When a power is raised to a power, the exponents are multiplied.

You should know that

$(x^2)^5$ is a short way of writing $x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2$

which equals $x^{10}$.

Do you see the reasoning behind the rule now?

Please return to page $\frac{31}{1}$ and try question 7 again.

---



A monomial is an expression of one term. The expression $-3a^3b$ consists of the product of $-3$, $a^3$, and b. A continued product of any number of quantities is still a single term. In order for an expression not to be a monomial, it must consist of a number of terms separated by plus or minus signs. Thus, for example, $3x - 5y$ is not a monomial.

Please return to page $\frac{135}{2}$ and try question 1 again.

$\frac{26}{1}$

The exponent applies only to the letter next to which it is written. Thus, only the letter "b" is raised to the second power. Since there is no visible exponent next to the letter "a", it means that "a" is raised to the first power.

Please return to page $\frac{45}{1}$ and try question 2 again.

---

$\frac{26}{2}$

Half right and half wrong. It is true that $(a^2 - b^2)$ is a binomial, but is $\frac{x^2y^2}{10}$ a binomial? Examine the two expressions and notice in what way they are different.

Please return to page $\frac{4}{2}$ and try question 3 again.

Wrong answer. You are multiplying exponents when you should be adding them. Remember the rule: When two quantities with the same base are multiplied, the exponents are added.

$$\boxed{x^a \cdot x^b = x^{a+b}}$$

Please return to page $\frac{35}{2}$ and try question 5 again.

---



This answer choice is wrong. When raising a power, the exponents are multiplied.

Please return to page $\frac{5}{2}$ and try this question again.

$\frac{28}{1}$

There is a difference between $(xy)^4$ and $xy^4$. The expression with the parenthesis means that both " x " and " y " are raised to the $4^{th}$ power. The second expression without the parenthesis means that only " y " is raised to the $4^{th}$ power. You see, the exponent only applies to the letter next to which it is written.

Please return to page $\frac{36}{1}$ and try question 8 again.

---

$\frac{28}{2}$

The mistake you made has to do with the rules for multiplying signed numbers. We are sure that you remember that the product of two negative quantities is a positive quantity. See if you can spot your mistake.

Please return to page $\frac{10}{2}$ and try question 6 again.

The rule for multiplying two quantities having the same base is to add the exponents. Your mistake was that you multiplied them in the first part of the question. Since you did choose the right answer to the second part of the question, the error may be due to an oversight on your part. Please be more careful.

Please return to page $\frac{37}{1}$ and try question 3 again.

---



We don't agree.

It would be helpful for you to study the various algebraic properties, such as, the commutative, associative and distributive properties. You should put these facts into your notebook.

Please return to page $\frac{11}{2}$ and try question 4 again.

$\frac{30}{1}$

This choice is correct.

Now proceed to question 4 which follows.

Question 4

Apply the proper principle to the product $(a^2b^3)\ (a^5b^4)$ and indicate the correct answer.

(A) $a^7b^7$

(B) $a^{10}b^{12}$

(C) $a^{10}b^7$

(D) $a^7b^{12}$

---

$\frac{30}{2}$

Each member of the term $(-2xy)$ is raised to the third power. You did not raise $-2$ to the third power. You know that a number raised to the third power means the continued product of this number 3 times.

Please return to page $\frac{8}{2}$ and try question 12 again.

This choice is correct.

Now proceed to question 7 which follows:

Question 7

Apply the proper principle and indicate the correct simplification of both expressions

I. $(a^3)^5$

II. $(x^2)^7$

| | | | |
|---|---|---|---|
| (A) I. $a^8$ | (B) I. $a^8$ | (C) I. $a^{15}$ | (D) I. $a^{15}$ |
| II. $x^9$ | II. $x^{14}$ | II. $x^{14}$ | II. $x^9$ |

---



The distributive law states that every term in the parenthesis should be multiplied by 3a. We are sure that you don't want to be a law breaker.

Return to page $\frac{22}{2}$ and try question 5 again.

$\frac{32}{1}$

When a power is raised to a power, the exponents are multiplied. We feel sure that you know this rule, and you didn't mean to choose $6 + a$ for the exponent.

Please return to page $\frac{17}{2}$ and examine the choices carefully, and select another letter.

---

$\frac{32}{2}$

This is not correct.

Simple computation would have shown you that $5^3 + 5^3 = 250$.
while $10^3 = 1000$.

Return to page $\frac{10}{1}$ and try question 13 again.

Sorry, that is not correct. Suppose that instead of using letters "y" and "m" to represent years and months, we used numbers. Here is the same problem restated with numbers.

One boy is 3 years and 5 months old. His brother is twice that age. How old is his brother?

Clearly, in doubling the boy's age, you must double both the years and months. Thus, the brother's age is

2 (3 years + 5 months) = 6 years and 10 months.

Please return to page $\frac{56}{1}$ and try question 7 again.

---



You overlooked the fact that a negative quantity multiplied by a positive quantity is negative. We are sure that you remember the rules for multiplying signed numbers. In any event, it would be a good idea to check whether you have these rules written down in your notebook. If they are not written down in your notebook, please do so.

Please return to page $\frac{64}{1}$ and try question 8 again.

$\frac{34}{1}$

This is a wrong answer.

The number 5 is a coefficient. The exponent is the number which is written smaller, to the right and above the base $x$. The 5 is not affected by the exponent; $5x^2$ means $5(x^2)$.

Return to page $\frac{1}{1}$ and try question 1 again.

---

$\frac{34}{2}$

This choice is correct.

Now proceed to question 10 which follows:

Question 10

Apply the proper principle and indicate which of the statements is the correct answer to the product, $5p^2r(2p - 3pr^2 + 7pr)$, by selecting the right letter.

(A) $10p^2r - 15p^2r^2 + 35p^2r$

(B) $7p^3r + 2p^3r^3 + 12p^3r^2$

(C) $10p^3r - 15p^3r^3 + 35p^3r^2$

(D) $10p^3r - 15p^3r^3 + 12p^3r^2$

Very good. You made the correct choice. By the associative law of multiplication, $(a^2b^3)\ (a^5b^4)$ can be written as $(a^2a^5)\ (b^3b^4)$. Now in each parenthesis, you have the product of two quantities with the same base. The rule for multiplying quantities with the same base is to maintain the base and add the exponents,

$$\boxed{x^a \cdot x^b = x^{a+b}}$$

Thus, the correct answer is $a^7b^7$.

Please go on to question 5 below.

---



Question 5:

Apply the proper principle and indicate which of the following is the correct answer to the product $(4a^2b^4)\ \ (7ab^3)$.

(A) $11a^3b^7$

(B) $28a^3b^7$

(C) $28a^2b^7$

(D) $28a^2b^{12}$

$\frac{35}{1}$

This choice is correct.

Now proceed to question 8 which follows:

Question 8

Apply the proper principle and indicate which of the following is a correct statement.

(A) $(xy)^4 = x^4y$

(B) $(xy)^4 = x^4y^4$

(C) $(xy)^4 = xy^4$

(D) $(xy)^4 = x^5y^5$

---

$\frac{35}{2}$

We don't agree. When the product of 2 or more quantities is raised to a power, each of the quantities is raised to that power. You should therefore raise -4 to the third power.

Return to page $\frac{2}{2}$ and try question 10 again.

This choice is correct.

Now proceed to question 3 which follows:

Question 3

Apply the proper principle and choose the letter where both statements are correct.

(A) $x^3 \cdot x^4 = x^{12}$
$k \cdot k^3 = k^3$

(B) $x^3 \cdot x^4 = x^7$
$k \cdot k^3 = k^3$

(C) $x^3 \cdot x^4 = x^{12}$
$k \cdot k^3 = k^4$

(D) $x^3 \cdot x^4 = x^7$
$k \cdot k^3 = k^4$

---

$\frac{37}{2}$

You were right in saying that $a^2b$ is a monomial,

but what about $\frac{x^2y^2}{70}$ ?

It is an expression consisting of the product of quantities with no plus or minus signs.

Return to page $\frac{4}{2}$ and try question 3 again.

$\frac{38}{1}$

This choice is correct.

Now proceed to question 6 which follows:

<u>Question 6</u>

Apply the proper principle and indicate the letter [illegible] gives the two correct statements.

I. $x^{n} \cdot x^{n+3} = x^{n^2 + 3n}$ II. $x^{n} \cdot x^{n+3} = x^{2n+3}$

III. $y^{a+1} \cdot y^{a-1} = y^{2a}$ IV. $y^{a+1} \cdot y^{a-1} = y^{0}$

(A) I and IV

(B) II and IV

(C) II and III

(D) I and III

---

$\frac{38}{2}$

When a quantity is squared, it means that the quantity is multiplied by itself. You therefore have the product of 2 negative quantities, that is, $(-4b^3)\ (-4b^3)$. It is always nice to have a rule. Suppose that a negative quantity is raised to an even power, what will the sign of the result be? You know the answer. Let us say it together. "When a quantity is raised to an <u>even</u> power, the sign of the result is positive."

Please return to page $\frac{5}{2}$ and try question 11 again.

The expression $(xy)^4$ is really a short way of writing (xy) (xy) (xy) (xy). Note that in each of the parentheses, "x" and "y" are raised to the first power. Now, you do remember the rule we learned for multiplying quantities with the same base? Prove it by returning to page $\frac{36}{1}$ and choosing the correct letter.

---



Wrong, wrong answer.

The reason that we wrote the word "wrong" twice is that you made 2 mistakes. In the first place, when 9 is multiplied by 2, the product is not 7. In the second place, when a negative quantity is multiplied by a negative quantity, the product is not a negative quantity. Please be more careful.

Return to page $\frac{10}{1}$ and try question 6 again.

$\frac{40}{1}$

You did all the difficult work correctly, but you made a mistake in signs. A negative quantity multiplied by a negative quantity is a positive quantity. Thus, $(-.3x)(-30xy^2)$ should have a plus sign in front of it.

Return to page $\frac{68}{1}$ and try question 9 again.

---

$\frac{40}{2}$

Descending powers of a variable means that the powers to which the variable is raised decreases with each succeeding term. Thus, for example, the expression $y^{13} + 7y^5 - 24y$ is written in descending powers of the variable $y$.

Return to page $\frac{76}{2}$ and try question 2 again.

This choice is correct.

Now proceed to question 11 which follows:

Question 11

Apply the proper principle to perform the indicated operation and select the correct answer:

$$3xy(x + y) - 5x^2(y - x)$$

(A) $-2x^2y - 5x^3 + 3x^2y$

(B) $3xy - 5x^2$

(C) $3x^2y + 3xy^2 - 5x^2y + 5x^3$

(D) $3xy^2 + 5x^3 - 2x^2y$

---



Each of the members of the term in the parenthesis is raised to the third power. Note that $(-2)^3$ means $(-2)(-2)(-2)$. Remember that the product of an odd number of negative quantities is a negative quantity.

Return to page $\frac{8}{2}$ and try question 12 again.

$\frac{42}{2}$

Very good. You made the correct choice. If a manufacturer produces $(x + 10)$ dresses and makes a profit of \$5 on each dress, then the total profit he makes on the dresses is the product of $5(x + 10) = 5x + 50$. Similarly, if he produces $(2x - 3)$ sweaters and makes a profit of \$2 on each sweater, the total profit he makes on the sweaters is $2(2x - 3) = 4x - 6$. Finally, the total profit from both the dresses and sweaters is $5x + 50 + 4x - 6 = 9x + 44$.

Please go on to question 14 below.

---

$\frac{42}{2}$

Question 14

You have a multiplying machine that the most it can handle is the product of two 4 digit numbers. Propose a procedure you could use to find the product of 17328 and 3415 on your machine. Which of the following is the procedure you adopted ?

(A) I gave up; it can't be done on my machine.

(B) The product can be written as 3415(8664 + 8664) and my machine can do this.

(C) I made the machine multiply 3415 by 10,000, then multiply 3415 by 7328, and added the two results.

(D) I multiplied the numbers without using the machine; it was much easier that way.

We don't agree.

The distance that an object travels equals the product of the rate and the time it travels. Thus, for example, if a car travels 70 miles an hour for 2 hours, the distance it will cover is

$$(70)\ (2) = 140 \text{ miles.}$$

Each expression for the rate must therefore be multiplied by the time in order to obtain the total distance.

Return to page $\frac{47}{2}$ and try question 12 again.

---



We don't agree. Treat this question as a product of two binomials.

Use the Distributive Law: $(a+b)\ (c+d) = a(c+d) \ +b(c+d)$

and combine the answers.

Return to page $\frac{49}{1}$ and try question 3 again.

$\frac{44}{1}$

Well done. You made the correct choice. Let us work together and examine each of the other choices.

$5^3 + 5^3$ does not equal $5^6$ because the rule for adding exponents applies only when we have the product of quantities with the same base.

$5^3 + 5^3$ does not equal $10^3$ because when you add two like quantities, you get twice that quantity; thus, $5^3 + 5^3 = 2(5^3) = 250$.

$5^3 + 5^3$ cannot equal $10^6$ since two mistakes are made; namely, exponents were added in a problem of addition, and the base was doubled.

Hence, your choice was the right one.

---

Notebook Instructions

$\frac{44}{2}$

You have now finished this Segment. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following definitions and formulas:

$\frac{\text{exponent}}{\text{base}} = \text{power}$ $\qquad x^a x^b = x^{a+b}$

$(-x)^{\text{odd}} = -$ $\qquad (x^a)^b = x^{ab}$

$(-x)^{\text{even}} = +$ $\qquad (x^a y^b)^c = x^{ac} y^{bc}$

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT:

questions 1 - 5

Volume 7, Segment 2 begins on page $\frac{135}{2}$

This choice is correct.

Now proceed to question 2 which follows:

Question 2

In the two expressions $ab^2$ and $x^3y$, recognize and select the correct statement.

(A) Both "a" and "b" are raised to the $2^{nd}$ power.

(B) "a" is raised to the first power and "y" is raised to the third power.

(C) "a" is raised to the first power and "y" is raised to the first power.

(D) "a" is raised to the $2^{nd}$ power and "y" is raised to the first power.

---



We don't agree.

By the distributive law,

$-9(a^2-2ab)$ is the same as $-9(a^2)$ plus $-9(-2ab)$.

Return to page $\frac{10}{2}$ and try question 6 again.

Your choice is the boy's age, and not his brother's age.

Return to page $\frac{56}{1}$ and try question 7 again.

---

$\frac{46}{2}$

You made a careless error.

You know that $-\frac{1}{4}$ times 4 is not -16.

Please be more careful.

Return to page $\frac{64}{1}$ and try question 8 again.

exponent means that the letter has the exponent 1. Thus, when you multiplied $5p^2r$ and $2p$, you should have said to yourself, I have to multiply $5p^2r^1$ and $2p^1$.

Return to page $\frac{34}{2}$ and try question 10 again.

---

$\frac{47}{2}$

This choice is correct.

Now proceed to question 12 which follows:

<u>Question 12</u>

Determine the polynomial which expresses the total distance traveled: A man travels for 2 hours at $(45 + x)$ miles per hour and 3 hours at $(100 + x)$ miles per hour.

(A) $145 + 2x$

(B) $390 + 5x$

(C) $290 + 5x$

(D) $145 + 5x$

something that can't be done?

Return to page $\frac{42}{2}$ and try question 14 again.

---

$\frac{48}{2}$

We don't agree. A polynomial written in ascending powers of a variable means that each succeeding term has a higher power of the variable. Furthermore, a number without a letter next to it is of lower power than a number with a letter. Thus, for example the number 3 is of lower order than 3y.

Return to page $\frac{60}{2}$ and try question 1 again.

Now proceed to question 3 which follows:

Question 3

Perform the indicated operation and choose the letter that labels the correct answer to the product,

$$\left( 5 + \frac{1}{4} \right) \cdot \left( 2 + \frac{1}{2} \right) =$$

(A) $10 \frac{1}{8}$

(B) $13 \frac{1}{8}$

(C) $10 \frac{1}{2}$

(D) $12 \frac{1}{2}$

---



We do not agree. The product of two binomials can be separated into two products of a monomial and a binomial. Each term of the first monomial multiplies each term in the second binomial.

For example,

$$(2m + 3n)\ (4m + 5n) = 2m\ (4m + 5n) + 3n\ (4m + 5n)$$

Return to page $\frac{70}{2}$ and try question 4 again.

Now proceed to question 13 which follows:

Question 13

Determine the polynomial which expresses the total profit made: a manufacturer makes \$5 on a dress and \$2 on a sweater. He sells $(x + 10)$ dresses and $(2x - 3)$ sweaters.

| | | | |
|---|---|---|---|
| (A) | $9x + 44$ | (B) | $12x - 5$ |
| (C) | $9x + 56$ | (D) | $3x + 7$ |

---

50
2

This choice is correct.

Now proceed to question 5 which follows:

Question 5

Apply the proper principle and select the correct answer to the problem

multiply:

$$\begin{array}{r} 3y + 5 \\ \underline{2y + 4} \end{array}$$

| | | | |
|---|---|---|---|
| (A) | $6y^2 + 20$ | (B) | $6y^2 + 10y + 9$ |
| (C) | $6y^2 + 22y + 20$ | (D) | $6y^2 + 22y + 9$ |

incorrect. In multiplying numbers with the same base, the exponents are added. Furthermore, when a letter is written without a visible exponent, it means that the exponent is 1.

Return to page $\frac{68}{1}$ and try question 9 again.

---

$\frac{51}{2}$

We don't agree. An arithmetic number such as 7, 15 is of lower order than a term containing a letter. Thus, for example, 15 is of lower order than 15a. You will learn later on that a number, say, 15 is equivalent to that number times a variable raised to the zero power. In other words, 15 and $15a^0$ are equivalent. That is why a number is said to have a lower power than a letter with any positive exponent.

Return to page $\frac{76}{2}$ and try question 2 again.

We don't agree. The boy's age is (y + m). Now, when you double this expression to obtain the age of the brother, you should double both the number of years and the number of months.

Return to page $\frac{56}{1}$ and try this question again.

---

$\frac{52}{2}$

You did not complete the multiplication. You forgot to multiply the second term of the first binomial with the second term of the second binomial.

Return to page $\frac{70}{2}$ and try question 4 again.

Too bad. You were almost right. We are not going to tell you where you made your mistake. We are just going to mention the kind of mistake you made. A negative multiplied by another negative is a positive quantity. We think that you can find your mistake without help.

Return to page $\frac{41}{1}$ and try question 11 again.

---



Notice that there is a plus sign between the terms and not a multiplication sign. We have no rule for adding powers of quantities with the same base. The rule we have applies only to products. A little arithmetic may help you arrive at the correct answer to this question. Don't give up.

Return to page $\frac{10}{1}$ and try question 6 again.

We don't agree. I believe that you did not read the question carefully You interchanged the profit made on each dress with profit made on a sweater.

Return to page $\frac{50}{1}$ and try question 13 again.

---

$\frac{54}{2}$

Ascending means going up. When we say ascending order, we mean that the powers of the variable in question, get larger. Thus, in the expression $x^5 - x^7 + x^{11}$, the powers of "x" are getting larger. This expression is therefore written in ascending powers of the variable "x".

Return to page $\frac{60}{2}$ and try question 1 again.

an answer. You made a mistake in arithmetic which we are sure is not the result of not knowing the answer, [illegible] result of being too hasty. Even though a question seems easy, it is a good idea to do the work in writing. You can always come back and check a written job, but you can't go back and check something that you did mentally.

Return to page $\frac{4[illegible]}{2}$, follow our advice, and try this question again.

---

$\frac{55}{2}$

We don't agree. If you replace the letter "y" by the number 2, the first binomial will equal 11. Similarly, the second binomial will equal 15. Surely you can go on from here without any more help.

Return to page $\frac{77}{2}$ and try question 6 again.

Now proceed to question 7 which follows:

Question 7

Determine the correct answer to the following: one boy is "y" years and "m" months old. His brother is twice as old. How old is his brother?

(A) 2ym

(B) y+m

(C) 2y + 2m

(D) 2y + m

---

$\frac{56}{2}$

The quantity in the first parenthesis can be expressed as (a+b).

The quantity in the second parenthesis can be expressed as (c+d).

All that you have to do is find the product of the above 2 binomials.

i.e. $(a+b)\ (c+d) = a(c+d) + b(c+d)$

- - - and combine the answers.

Return to page $\frac{49}{1}$ and try question 3 again.

We don't agree. You probably made a hasty choice. You know that

$-\frac{1}{4}$ times 8 is not the same as $-\frac{1}{2}$ times 8.

Return to page $\frac{64}{1}$ and try question 8 again.

---



We don't agree. Remember the meaning of an exponent?

| | | | | |
|---|---|---|---|---|
| if | $x = 2$ | | and if | $y = 3$ |
| then | $x^2 = 2^2$ | | then | $y^3 = 3^3$ |
| | $= 4$ | | | $= 27$ |

Please do your calculations over again.

Return to page $\frac{71}{2}$ and try question 7 again.

$(5p^2r)$ $(2p)$, the 5 and 2 are multiplied. One thing that we can say in your favor is that you were consistent; you made the same mistake with every one of the three terms of the question.
Take a good look at what you did wrong.

Please return to page $\frac{34}{2}$ and try this question again.

---

$\frac{58}{2}$

You forgot something. By the distributive law,

$$(2x + 3)(5x + 1) = 2x(5x + 1) + 3(5x + 1).$$

Perform the indicated operation and see if you can't find your mistake.

Now, return to page $\frac{61}{2}$ and try question 8 again.

You are asking the machine to do something that it cannot do. The number 10,000 is a 5 digit number and the machine cannot handle it. Give this question a little more thought.

Please return to page $\frac{42}{2}$ and try question 14 again.

---

$\frac{59}{2}$

We don't agree. When 2 signed numbers are added, the sum has the sign of the number with the greater absolute value. Examine the middle term of your choice and see if you can find your mistake.

Please return to page $\frac{66}{2}$ and try question 9 again.

$\frac{60}{1}$

Congratulations. You made the correct choice. The machine cannot multiply 17328 by 3415, since the first of these numbers has 5 digits. But, 17328 = 8664 + 8664, and all the machine has to do is to multiply 8664 by 3415 twice and add the two results. You have now completed this segment. Hand in your punch card. Before going ahead to the next segment, follow the notebook instructions; then do problem 6-8 of your homework assignment.

Notebook Instructions: 07-2

After the completion of this segment, you should have the following items entered into your notebook:

1. Definition of the words monomial, binomial, trinomial, and polynomial.
2. A description of the algebraic properties: commutative, associative, and distributive properties.

---

$\frac{60}{2}$

Volume 7 Segment 3 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

| | | | | | | |
|---|---|---|---|---|---|---|
| COLUMNS | 48 | and | 50 | 3 | 3 | (Sequence Number) |
| | 54 | and | 56 | 0 | 4 | (Type of Punch Card) |
| | 60 | and | 62 | 0 | 7 | (Volume Number) |
| | 66 | and | 68 | 0 | 3 | (Segment Number) |

Your READING ASSIGNMENT for this Segment is pg: 209 - 210
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Recognize which of the following polynomials are written in ascending powers of the variable and select the correct letter.

(A) $x^2 - 5x^3 + 25$ (C) $p^2 - p^3 + 50p$

(B) $x^4 - 100x + 75$ (D) $5r - 3r^2 - r^4$

We don't agree. In multiplying 2 binomials, each term of the first binomial is multiplied by each term of the second binomial, and the results are combined. You only multiplied 3y by 2y and 5 by 4. Now do you see what you not do?

Please return to page $\frac{50}{2}$ and try question 5 again.

---



This choice is correct.

Now proceed to question 8 which follows:

Question 8

Apply the proper principle and indicate the letter which labels the correct answer to the product, $(2x + 3)(5x + 1)$

(A) $10x^2 + 15x + 3$

(B) $10x + 17x + 3$

(C) $10x^2 + 17x + 3$

(D) $10x^2 + 17x + 4$

You seem to have a little trouble with the multiplication of decimals. Let me try to help you with these illustrations:

$$.15 \quad x \quad 3 \quad = \quad .45$$

$$1.5 \quad x \quad 3 \quad = \quad 4.5$$

$$15 \quad x \quad .3 \quad = \quad .045$$

Please return to page $\frac{68}{1}$ and try question 9 again.

---



Wrong answer. To write a polynomial in descending powers of a variable means that each succeeding term must have the variable in question raised to a lower power than the preceeding term.

Please return to page $\frac{76}{2}$ and try question 2 again.

You were hasty. Let's try this question together. We both know that the total profit on a sale is the product of the number of items sold and the profit on a single item. Thus, in this case, the profit on the dresses is the product of $(x + 10)$ and \$5. Similarly, the profit on the sweaters is the product of $(2x - 3)$ and \$2. You should be able to continue from here.

Please return to page $\frac{50}{1}$ and try question 13 again.

---



You overlooked something. Remember that the product of 2 binomials is obtained by multiplying each term of the first binomial by each of the terms of the second binomial. After doing this, like terms are combined. The mistake you made is in your choice of the second term. See if you can find it.

Please return to page $\frac{70}{2}$ and try question 4 again.

This choice is correct.

Now proceed to question 8 which follows:

Question 8

Apply the proper principle and indicate the letter which labels the correct answer to the product, $-\frac{1}{4}(8ab + 4a^2)$

(A) $-2ab - a^2$

(B) $-2ab + a^2$

(C) $-2ab - 16a^2$

(D) $-4ab - a^2$

---



As an example, suppose we multiply these 2 binomials together:

$$\begin{array}{lrrrr} & 5x & -3 & & \\ & \underline{4x} & \underline{+\ 2} & & \\ \text{line 1} & 20x^2 & -12x & & \\ \text{line 2} & & \underline{+10x} & \underline{-6} & \\ \text{line 3} & 20x^2 & -2x & -6 & \end{array}$$

Observe that line 1 was obtained by multiplying $(5x - 3)$ by $4x$. Line 2 was obtained by multiplying $(5x - 3)$ by 2. Notice that like terms were written in the same column. Finally, line 3 was obtained by adding line 1 and line 2.

Please return to page $\frac{66}{2}$ and try question 9 again.

Did you just take a guess? You did not work out the problem. Let's try it together:

Applying the distributive law to the first product, we get,

$$3xy(x + y) = 3x^2y + 3xy^2$$

Applying the distributive law to the second product, we get,

$$-5x^2 (y - x) = -5xy^2y + 5x^3$$

Continue from here, and see if you can't find the correct answer.

Please return to page $\frac{41}{1}$ and try question 11 again.

---



Ascending powers of a variable refers to the exponent of the variable and not to the size of the coefficient. Thus, for example, the term $x^2$ is of higher order than the term $875x$

Please return to page $\frac{60}{2}$ and try question 1 again.

We agree with you that it is easier to multiply the two numbers as is and not try to figure out how the machine could do it. However, if you had to do a few hundred multiplications, then it would certainly be to your advantage to know how the machine could do it. Moreover, the machine doesn't make human errors.

Give this question the thought it deserves and return to page 42/2 and try question 14 again.

---



This choice is correct.

Now proceed to question 9 which follows:

Question 9

Apply the proper principle and choose the letter which labels the correct answer to the product $(7a - 3)\ (5a + 2)$

(A) $35a^2 - a - 6$

(B) $35a^2 + a - 6$

(C) $35a^2 - a - 1$

(D) $35a^2 - a + 6$

That answer choice is incorrect.

The total distance traveled is given by the following expression.

$$2(45 \div x) + 3(100 + x)$$

Please continue and see if you can find the correct letter.

Please return to page $\frac{47}{2}$ and try question 12 again.

---

$\frac{67}{2}$

By the distributive law, each of the numbers of the second binomial should be multiplied by each of the numbers of the first binomial. Thus, you have to evaluate

$$5(2) + 5(\tfrac{1}{2}) + \tfrac{1}{4}(2) + \tfrac{1}{4}(\tfrac{1}{2}) =$$

Please complete the computation.

Please return to page $\frac{49}{1}$ and try question 3 again.

This choice is correct.

Now proceed to question 9 which follows:

Question 9

Apply the proper principle and indicate the letter which labels the correct answer to the question, $-.3x(5x^2y - 30xy^2)$ equals,

(A) $-1.5x^3y - 9x^2y^2$

(B) $-1.5x^3y + 9x^2y^2$

(C) $-1.5x^2y + 9x^2y^2$

(D) $-.15x^3y + .9x^2y^2$

---



In order to have obtained the answer you selected, you must have added the two binomials instead of multiplying them, as you were asked to do.

Please return to page $\frac{71}{1}$ and try question 7 again.

Sorry, we don't agree. You did well until you came to the last term. This last term is not $12\ p^3r^2$. Can you see your mistake?

Please return to page $\frac{34}{2}$ and try question 10 again.

---



We don't agree. Remember that when you multiply two quantities with the same base, the exponents are added.

Thus, for example:

$$5b^2 \cdot 3b = 15b^{2+1} = 15b^3$$

Please return to page $\frac{61}{2}$ and try question 8 again.

$\frac{70}{1}$

This choice is correct. Here is how you could have done this problem:

$$\left(5 + \frac{1}{4}\right)\left(2 + \frac{1}{2}\right) = 5(2) + 5\left(\frac{1}{2}\right) + \frac{1}{4}(2) + \frac{1}{4}\left(\frac{1}{2}\right)$$

$$= 10 + \frac{5}{2} + \frac{1}{2} + \frac{1}{8}$$

$$= 13\frac{1}{8}$$

Please proceed to question 4 below.

---

$\frac{70}{2}$

Question 4

Apply the proper principle and choose the letter next to the correct answer which is the product of $(4x + 5)(3x + 2)$:

(A) $4x(3x + 2) + 5(3x + 2)$

(B) $(4x)(3x) + (5)(2)$

(C) $12x^2 + 23x$

(D) $12x^2 + 15x + 10$

Very good: You made the correct choice.

When $y = 2$,

the binomial $3y + 5 = 6 + 5$

$= 11$

Similarly, when $y = 2$,

the second binomial $7y + 1 = 14 + 1$

$= 15$

We therefore have the product of 11 and 15 , and the result is 165

Please proceed to question 7 below.

---



Question 7

Perform the necessary computations to find the value of the product:

$$( 3x^2 - 4 ) ( y^3 + 2 )$$

when

$$x = 2, \text{ and } y = 3$$

Choose the letter which labels the correct answer:

(A) 22

(B) 37

(C) 58

(D) 232

$\frac{72}{7}$

You chose an answer that has two mistakes in it. In the first place, you did not include the product of 4 and 3y. In the second place, in another part of the question, you added when you should have multiplied. Examine your work, and try to locate the errors that we pointed out

Please return to page $\frac{50}{2}$ and try question 5 again.

---

$\frac{72}{2}$

You made the correct choice.

Now proceed to question 11 below:

<u>Question 11</u>

Apply the proper principle and select the letter next to the correct answer to the product: $(y^2 + 8)\ (y^3 - 5)$

(A) $y^6 + 8y^3 - 5y^2 - 40$

(B) $y^5 + 8y^3 - 5y^2 - 40$

(C) $y^5 + 3y^2 - 40$

(D) $y^5 - 40$

Not quite right. Your mistake is in the last term of your choice. You added when you should have multiplied. Go over your work and see if you can find the mistake you made.

Please return to page $\frac{61}{2}$ and try question 8 again.

---



You made an error. We would like you to find the error without our help. Go back to your work, make sure that each term of the binomial (2p + 1) is multiplied by the trinomial. Also, see to it that like terms are written in the same column.

Return to page $\frac{79}{1}$ and try question 12 again.

$\frac{74}{1}$

If you sold 7 footballs and made a profit of \$2 on each ball, what would your total profit be? "That's easy," you say. "The answer is \$14."

Question 13 is just as easy. All you have to do is find the product of the number of items and the profit per item to get the total profit.

Please return to page $\frac{50}{1}$ and try question 13 again.

---

$\frac{74}{2}$

We don't agree. When we have to find the product of three binomials, the procedure is to multiply two binomials first. Thus, in this question, we first multiply $(x + y)(x - y)$. Then we multiply the third binomial by the product we obtained by the multiplication of the first two.

Return to page $\frac{136}{2}$ and try question 14 again.

It is a wrong answer. Let us refresh your memory regarding the multiplication of signed numbers:

| |
|---|
| When two quantities with unlike signs are multiplied together, the sign of the product is minus. |

Please take a look to see whether you have this rule written in your notebook. If not, write it down so that you will remember it.

Return to page $\frac{66}{2}$ and try question 9 again.

---



It is a wrong answer. When we say that a statement is true for all values of the letters, we mean that the two expressions are exactly alike. We call such expressions, identities.

Now, $(x + y)(x + y)$ is not identical to $x^2 + y^2$. Convince yourself by letting $x = 2$, $y = 3$

You can further convince yourself by actually multiplying $(x + y)(x + y)$.

Return to page $\frac{87}{2}$ and try question 15 again.

$\frac{6}{}$

There is nothing wrong with your work except that you did not finish the job. A good craftsman does not leave a job unfinished. To be more specific, like terms should be combined.

Take a look at the choice you made and notice the like terms that were not combined.

Return to page $\frac{41}{1}$ and try question 1 again

---

$\frac{6}{2}$

This choice is correct.

Now proceed to question 2 which follows:

Question 2

Recognize which of the following polynomials are written in descending powers of the variable "a", and select the letter of the correct answer choice:

(A) $a^3b - 5a^2b^2 + 10ab^3$

(B) $b^2a + 5a^2b + ?$

(C) $5 - ab$

(D) $a^3 + 8 + a^4b$

Very good. You made the right choice. The multiplication of the two binomials is performed as follows;

either:

$$
\begin{array}{rrrrr}
 & 3y & + & 5 & \\
 & \underline{2y} & \underline{+} & \underline{4} & \\
 & 6y^2 & + & 10y & \\
 & & + & 12y & + \; 20 \\
\hline
 & 6y^2 & + & 22y & + \; 20
\end{array}
$$

or: $(3y + 5)(2y + 4) = 3y(2y + 4) + 5(2y + 4)$

$$= 6y^2 + 12y + 10y + 20$$

$$= 6y^2 + 22y + 20$$

Please proceed to Question 6 below.

---

Question 6

Perform the necessary computation to find the value of the product

$$(3y + 5)(7y + 1), \text{ when } y = 2$$

The value you found was:

(A) 26

(B) 165

(C) 151

(D) 115

$\frac{78}{1}$

| |
|---|
| The product of two negative numbers is a positive number. |

Examine the final term of your choice and see if you can find your mistake.

Please return to page $\frac{86}{2}$ and try Question 10 again.

---

$\frac{78}{2}$

Sorry, your choice is not the right one. You should understand that when two expressions are equivalent, the coefficients of like powers of x, must be the same. Thus, when we multiply $(x - a)(x - b)$ we get $x^2 - ax - bx + ab = x^2 - (a + b)x + ab$

Now, ask yourself, what should the value of $(+a\ +b)$ be? What should ab equal? This is a pretty difficult problem, you will enjoy getting it right.

Please return to page $\frac{82}{2}$ and try Question 16 again.

Your choice is correct.

Now proceed to Question 12 which follows.

Question 12

Apply the proper principle and indicate by choosing the appropriate letter, the correct statement.

$$(2p + 1)(p^2 - 3p - 1) \text{ equals,}$$

(A) $2p^3 - 5p^2 - 5p - 1$

(B) $2p^3 - p^2 - p$

(C) $2p^3 - 6p - 1$

(D) $2p^3 + 7p^2 + 6p - 1$

---



If $x = 2$,

then $x^2 = 4$

and $3x^2 = 12$

We are sure that you made the choice in haste. Please be more careful.

Please return to page $\frac{71}{2}$ and try Question 7 again.

$\frac{80}{1}$

We don't agree.

> When two quantities with the same base are multiplied, the exponents are added.

We are sure that you recall that a letter written without an exponent means that the letter has the exponent 1.

For example: $x$ has the same meaning as $x^1$

Please return to page $\frac{86}{2}$ and try Question 10 again.

---

$\frac{80}{2}$

Before finding the area of a rectangle, it is necessary to change the measurements to the same unit. Thus, the width of 4 inches can be changed to $\frac{1}{3}$ of a foot. Then we can multiply the length and width to find the area. The area is then expressed in square feet.

Please return to page $\frac{89}{2}$ and try Question 1 again.

We don't agree. In a problem of this type, it is very important that you arrange your work neatly. Like terms should be written in the same column and combined. Your addition of signed numbers must be done with great care. Go over your work again.

Return to page $\frac{85}{2}$ and try Question 13 again.

---



This choice is correct.

Now proceed to Question 4 which follows.

Question 4

Apply the proper principle to express as a polynomial, the area of the rectangle whose length is $(5k - 3)$ inches, and whose width is $(3w + 1)$ inches. Choose the correct letter.

(A) $(15kw - 3)$ sq. in.

(B) $(15kw - 9w + 5k - 3)$ sq. in.

(C) $(15kw - 9w + 5k + 3)$ sq. in.

(D) $(15kw - 3w + 5k - 3)$ sq. in.

82
1

Very good. The expression $(x - y)(x - y)$ is identical to the expression $x^2 - y^2$, since

$$
\begin{array}{rrrr}
 & x & - \; y & \\
 & x & + \; y & \\
\hline
 & x^2 & - \; xy & \\
 & & + \; xy & - \; y^2 \\
\hline
 & x^2 & & - \; y^2
\end{array}
$$

Please proceed to question 16 below.

---

82
2

Question 16

Propose a procedure which enables you to find the numerical values of "a" and "b", if it is given that $(x - a)(x - b)$ is equivalent to the trinomial $x^2 - 8x + 15$

Choose the correct letter:

(A) $a = 4, b = 2$

(B) $a = 3, b = 5$

(C) $a = 8, b = 7$

(D) none of these

Wrong answer. You forgot the old rule.

If two quantities with the same base are multiplied, the exponents are added.

Thus, for example,

$$c^3 - c^2 = c^5 \text{ and } \underline{\text{not}} \; c^6$$

Please return to page $\frac{72}{2}$ and try question 11 again.

---

$\frac{83}{2}$

The area of a rectangle whose length and width are expressed in the same unit, is equal to the product of these two dimensions. We would suggest that you change the mixed numbers to improper fractions.

For example:

$$5\frac{2}{3} = 5 + \frac{2}{3}$$

$$= \frac{15}{3} + \frac{2}{3}$$

$$= \frac{17}{3}$$

Please return to page $\frac{91}{2}$ and try question 2 again.

$\frac{84}{1}$

Almost right. Your mistake is in the last term. We are not going to say more, because we have confidence in your ability to find your mistake

Please return to page $\frac{50}{2}$ and try question 5 again.

---

$\frac{84}{2}$

Let us ask you this question: If the product of two numbers is 18 and one of the n  rs is 6, what is the other number? That's easy, you say, the answer is 3. How did you get this answer? Clearly you said,

$$\frac{18}{6} = 3$$

In other words, if the product of two numbers is given, and one of these numbers is known, the other number is found by dividing the product by the known number. Now, the area of a rectangle is also the product of two numbers. We are sure that you can continue from here without our help,

Please return to page $\frac{103}{1}$ and try question 5 again.

This choice is correct since,

$$(2p + 1)(p^2 - 3p - 1) = 2p(p^2 - 3p - 1) + 1(p^2 - 3p - 1)$$
$$= 2p^3 - 6p^2 - 2p + p^2 - 3p - 1$$
$$= 2p^3 - 5p^2 - 5p - 1$$

Or using the other form:

$$\begin{array}{rrrr} & p^2 & - 3p & - 1 \\ & \underline{2p} & \underline{+ 1} & \\ 2p^3 & - 6p^2 & - 2p & \\ \underline{\phantom{2p^3}} & \underline{+ p^2} & \underline{- 3p} & \underline{- 1} \\ 2p^3 & - 5p^2 & - 5p & - 1 \end{array}$$

Now proceed to question 13 below.

---



Question 13

Apply the proper principle and choose the letter which labels the correct answer to the product:

$(3x^2 - x + 2)(x^2 + 2x + 1)$

(A) $3x^4 + 8x^2 + 3x + 2$

(B) $3x^4 + 7x^3 - 7x^2 + 3x + 2$

(C) $3x^4 + 5x^3 + 3x^2 + 3x + 2$

(D) $3x^4 + 5x^3 + 3x^3 + 5x + 2$

$\frac{86}{1}$

Very good.

You made the correct choice.

Here is how we do this problem: either,

$$\begin{array}{rrrr}
 & 7a & - & 3 \\
 & \underline{5a} & \underline{+} & \underline{2} \\
 & 35a^2 & - & 15a \\
 & & + & 14a - 6 \\
\hline
 & 35a^2 & - & a - 6
\end{array}$$

or,

$$\begin{aligned}
(7a - 3)(5a + 2) &= 7a(5a + 2) - 3(5a + 2) \\
&= 35a^2 + 14a - 15a - 6 \\
&= 35a^2 - a - 6
\end{aligned}$$

Please proceed to question 10 below.

---

$\frac{86}{2}$

Question 10

Apply the proper principle and choose the letter which labels the correct answer to: $( x - 3 ) ( 2x - 1 ) = ?$

(A) $2x^2 - 7x - 3$

(B) $2x^2 - 7x + 3$

(C) $2x - 7x + 3$

(D) $2x^2 - 6x + 3$

Very good. You made the correct choice. In multiplying three binomials, we first multiply any two. Thus, to calculate $(x + y)(x - y)(x + 1)$ we proceed as follows:

$$\begin{array}{rrr} x & + y & \\ \underline{x} & \underline{- y} & \\ x^2 & + xy & \\ & \underline{- xy} & \underline{- y^2} \\ x^2 & & - y^2 \end{array}$$

Then we find the product of the above result and the third binomial.

$$\begin{array}{llll} x^2 & - y^2 & & \\ \underline{x} & \underline{+ 1} & & \\ x^3 & - xy^2 & & \\ & & \underline{+ x^2} & \underline{- y^2} \\ x^3 & - xy^2 & + x^2 & - y^2 \end{array}$$

Please go on to the next question below.

---



Question 15

Determine which of the following statements is true for all values of the letters, and select the correct letter.

(A) $(x + y)(x + y) = x^2 + y^2$

(B) $(x - y)(x + y) = x^2 - y^2$

(C) $(x - y)(x - y) = x^2 - y^2$

(D) $(a + b)(c + d) = ac + bd$

Congratulations: You did it. When two expressions are equivalent, it implies that the coefficients of like powers of the variable are the same

The product of $(x - a)(x - b)$ is $x^2 - (a + b)x + ab$.

Thus, $a + b = 8$ and

$ab = 15$

We are therefore looking for two numbers "a" and "b" whose sum is 8 and whose product is 15. Clearly, the numbers $a = 5$, $b = 3$ fill the bill.

You have now completed this segment. Make sure that the following items are in your notebook:

1. Description of a polynomial written in ascending order of a variable.
2. Description of a polynomial written in descending order of a variable.
3. The rule for multiplying two binomials by applying the distributive property.
4. An example worked out illustrating the vertical method of multiplying two polynomials.

Hand in your punch card.

You should now do Question 9-12 of the homework assignment.

Volume 7 Segment 4 begins on page $\frac{89}{1}$

Volume 7 Segment 4 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

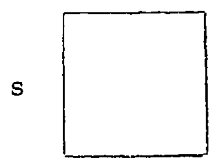
| | | | | | | |
|---|---|---|---|---|---|---|
| COLUMNS | 48 | and | 50 | 3 | 4 | (Sequence Number) |
| | 54 | and | 56 | 0 | 4 | (Type of Punch Card) |
| | 60 | and | 62 | 0 | 7 | (Volume Number) |
| | 66 | and | 68 | 0 | 4 | (Segment Number) |

Your READING ASSIGNMENT for this Segment is pg: 211-215

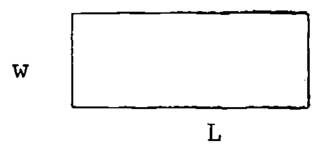
In your reading you have studied the application of the multiplication of polynomials to certain geometric figures and to other verbal problems.

Before starting to do the questions of this segment, you should review the following area formulas:

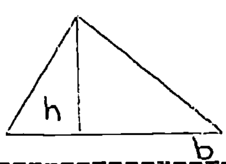
1. Area of a square: $A = s^2$



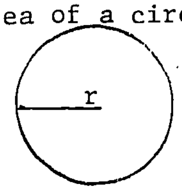

2. Area of a rectangle: $A = LW$




3. Area of a triangle: $A = \frac{1}{2} hb$




4. Area of a circle: $A = r^2$

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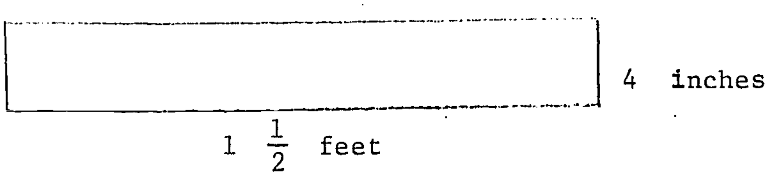
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Recognize the correct statement by choosing the appropriate letter.

The area of the rectangle drawn below is:




(A) 7 square feet

(B) 6 square inches

(C) 6 square feet

(D) $5\frac{1}{2}$ square feet

$\frac{90}{1}$

Try asking yourself the following questions:

What is the area of the outside (large) rectangle?

What is the area of the inside (small) rectangle?

What combination of the above two rectangles is the shaded part?

Return to page 115 and try this question again.

---

$\frac{90}{2}$

We don't agree.

The area of the race track consists of the area of a rectangle plus the area of two semi-circles. You can see from the figure that the rectangle measures 100 x 14, and the radius of each of the semi-circles is 7. Review your calculations.

Return to page $\frac{108}{1}$ and try question 10 again.

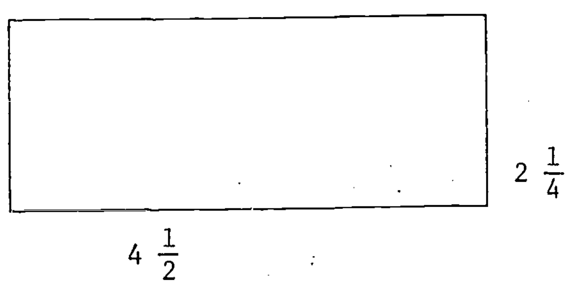
You overlooked something. When 2 binomials are multiplied, each term of the first binomial is multiplied by each term of the second binomial. You left out one of these multiplications. Go over your work and you will find the omission you made.

Return to page $\frac{86}{2}$ and try question 10 again.

---

$\frac{91}{2}$

Perform the necessary computation to find the area of the rectangle drawn below and select the letter that labels the correct answer.




(A) $10\frac{1}{8}$ square feet

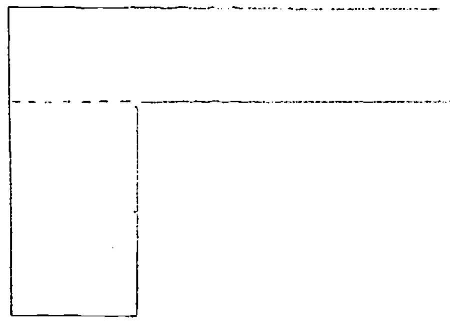
(B) $8\frac{3}{4}$ square feet

(C) $8\frac{1}{8}$ square feet

(D) 7 square feet

$\frac{92}{1}$

The area of the bracket can be divided into the area of two rectangles by drawing the dotted line shown in the figure. Examine the figure, and see if you can come up with the right answer.



Return to page $\frac{100}{2}$ and try question 6 again.

---

$\frac{92}{2}$

We do not agree.

The area of the rectangle is the product of the two binomials

( 5k - 3 ) and ( 3w + 1 ).

| |
|---|
| In multiplying the two binomials:<br>each term of the first binomial is multiplied<br>by each of the second binomial. |

As you perform each of these operations, you should be careful of the signs.

Please return to page $\frac{81}{2}$ and try question 4 again.

When we multiply the binomials ( x - y ) ( x - y ) , there are three terms in the answer. For example, suppose we multiply the following similar binomials,

$$\begin{array}{r} p - q \\ \underline{p - q} \\ p^2 - pq \qquad \\ \underline{\quad - pq + q^2} \\ p^2 - 2pq + q^2 \end{array}$$

NOTE: there are <u>three</u> terms in the answer:

Return to page $\frac{87}{2}$ and try question 15 again.

---



If you examine this figure, you will see that it consists of a rectangle and a triangle. The area of this taper pin is therefore the sum of the areas of the rectangle and the triangle. We are sure that you remember that

| |
|---|
| the area of a rectangle is the product of its length and width |

and that

| |
|---|
| the area of a triangle is $\frac{1}{2}$ the product the product of its base and altitude. |

Return to page $\frac{101}{2}$ and try question 8 again.

$\frac{94}{1}$

By the distributive law, the product of two binomials is made up of the product of each term of the first binomial by each term of the second binomial. Thus, the product of two binomials is an expression of four terms. Only in case there are like terms that can be combined, can the product of two binomials have fewer than four terms. In this question, there are no like terms; the answer must therefore contain four terms.

Return to page $\frac{72}{2}$ and try question 11 again.

---

$\frac{94}{2}$

If you examine the open box, you will observe that there are five rectangles whose areas must be found. The open box is composed of a front and back that measures 10 x 2. There are two sides that measure 2 x 8 and there is the bottom of the box.

Return to page $\frac{103}{2}$ and try question 11 again.

$\frac{95}{1}$

Very good. Since both the length and width are expressed in feet, the area of the rectangle is obtained by multiplying these 2 dimensions. Thus,

$$4\frac{1}{2} \times 2\frac{1}{4} = \frac{9}{2} \times \frac{9}{4}$$

$$= \frac{81}{8}$$

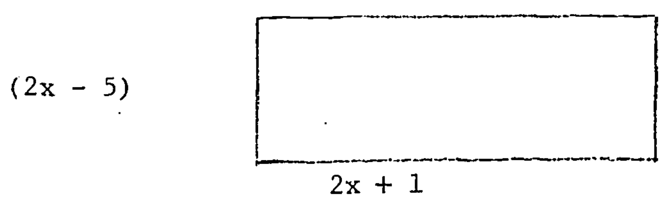
$$= 10\frac{1}{8}$$

Please go on to question 3 below.

---

$\frac{95}{2}$

Apply the proper principle and express the area of the rectangle drawn here, as a polynomial.




(A) $4x^2 - 5$

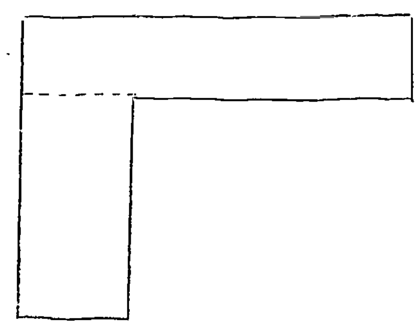
(B) $4x - 4$

(C) $4x^2 - 8x - 5$

(D) $4x^2 - 8x + 5$

$\frac{96}{1}$

We are just going to draw the figure and include a dotted line to help you see that the area of the bracket consists of the sum of the areas of two rectangles. We believe that you can do the rest yourself.



Return to page $\frac{100}{2}$ and try question 6 again.

---

$\frac{96}{2}$

Close! - but do you measure a side in <u>square</u> inches ?

Return to page $\frac{103}{1}$ and make another choice.

$\frac{97}{1}$

You of course know that the area of a circle is $r^2$, where r is the radius of the circle and equals $\frac{22}{7}$. Now ask yourself the following:

1. What is the area of the larger circle?
2. What is the area of the smaller circle?
3. What combination of the larger and smaller circle equals the area of the ring?

Return to page $\frac{105}{1}$ and try question 5 again.

---

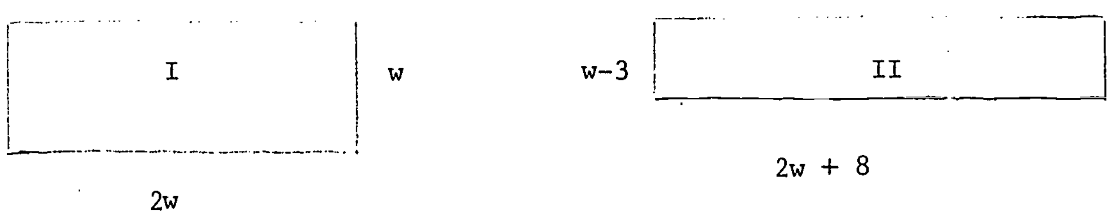
$\frac{97}{2}$

This choice is correct.

Now proceed to question 14 which follows:

<u>Question 14</u>

Apply the proper principle to find the dimensions of the first rectangle, and select the letter which labels the correct answer.




Given: the 2 rectangles are equal in area

(A) length: 5, width: $\frac{5}{2}$

(B) length: 24, width: 12

(C) length: 32, width: 9

(D) length: $\frac{5}{2}$, width: 5

$\frac{98}{1}$

We don't agree.

The area of a rectangle is the product of its length and width. The product of $(2x + 1)$ and $(2x - 5)$ is not $(4x^2 - 5)$. By the distributive law, each term of the first binomial must be multiplied by each term of the second binomial.

Please return to page $\frac{95}{2}$ and try question 3 again.

---

$\frac{98}{2}$

We don't agree.

A problem must be read carefully. We would suggest that you read a problem at least 3 times to make sure that you have not overlooked any of the facts involved. You have overlooked the fact that the second rectangle is 200 square feet greater in area than the first rectangle.

Return to page $\frac{112}{2}$ and try question 15 again.

We don't agree.

The left side of the equation is the product of two binomials, $(x + 5)$ and $(x + 3)$. This product equals $x^2 + 8x + 15$. The right side of the equation equals $x^2 + 7x + 20$. Note that the $x^2$ terms can be subtracted from both sides, thus leaving a linear equation in $x$. A linear equation in two variables is one in which the highest exponent of the variables is one.

Please go on from here.

Return to page 117/1 and try question 12 again.

---



The problem states that the length is four <u>times</u> the width. Do you know what you did? You said the length is four <u>more</u> than its width. That is not the same. If I have $x$ dollars and you have four times as much, you have $4x$ dollars. On the other hand, if I have $x$ dollars and you have 4 dollars more, you will have $(x + 4)$ dollars.

Return to page 105/2 and try question 16 again.

$\frac{100}{1}$

Incorrect. The iplying 2 binomials is not to multiply the first 2 ast 2 terms. Since this is a common mistake made by many students, let's check by examining a simple problem in arithmetic. Suppose you had to multiply 8 by 6 or using a binomial, for each of the numbers, such as, (5 + 3) x (4 + 2). According to your method, you would say that the answer is 5 x 4 + 3 x 2 which equals 20 + 6 or 26. But, the first binomial equals 8, and the second equals 6. We thus have the product of 8 and 6, which is 48 and not 26.

Return to page $\frac{72}{2}$ and try question 11 again.

---

$\frac{100}{2}$

This choice is correct.

Now proceed to question 6 which follows:

<u>Question 6</u>

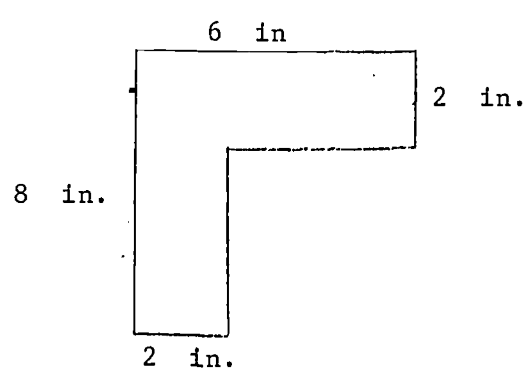
Perform the necessary computation to find the area of the figure drawn here, and choose the correct statement.

(A) 24 sq. in.

(B) 28 sq. in.

(C) 16 sq. in.

(D) 36 sq. in.



By the distributive law for multiplication of two binomials, each term of the first binomial is multiplied by each term of the second binomial. In your choice, there are no like terms to combine. Hence, the product should contain four terms.

Return to page 87/2 and try question 15 again.

---



This choice is correct.

Now proceed to question 8 which follows:

<u>Question 8</u>

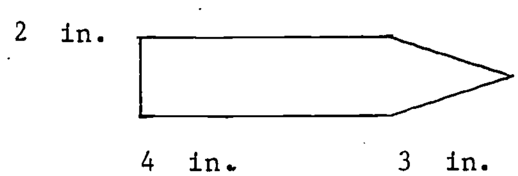
Perform the necessary computations to find the area of the taper pin drawn here, and select the letter which labels the correct answer.

(A) 11 sq. in.

(B) 14 sq. in.

(C) 10 sq. in.

(D) 9 sq. in.



$\frac{102}{1}$

The area of a rectangle is equal to the product of its length and its width. Do you know what you did? You took the sum instead of the product. In other words, you confused perimeter with area.

Return to page $\frac{95}{2}$ and try this question again.

---

$\frac{102}{2}$

Division by a number is the operation of multiplying by the reciprocal of that number. For example,

$$\frac{3}{7} \div \frac{5}{14} \text{ is the same as}$$

$$\frac{3}{7} \times \frac{14}{5}$$

It is important to note that the fraction <u>after</u> the division sign was changed to its reciprocal, while the first fraction remained unchanged.

Please return to page $\frac{130}{2}$ and try question 3 again.

This choice is correct.

Now proceed to question 5 which follows:

<u>Question 5</u>

Appl· · proper principle to e: ress the length of a rectangle whose area is $(x^2 + 16)$ sq. in. and its width is 4 in.

Choose the correct letter.

(A) $4(x^2 + 16)$ inches

(B) $(x + 4)$ inches

(C) $\frac{1}{4}(x^2 + 16)$ inches

(D) $\frac{1}{4}(x^2 + 16)$ Sq. inches

---



This choice is correct.

Now proceed to question 11 which follows:

<u>Question 11</u>

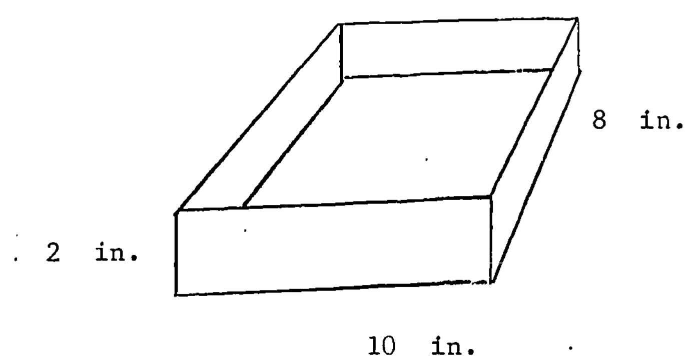
Perform the necessary computation to find the total area of the sides and bottom of the open box drawn here, and select the letter which indicates the correct answer.

(A) 56 sq. in.

(B) 72 sq. in.

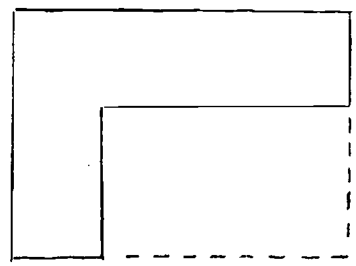
(C) 152 sq. in.

(D) 160 sq. in.



$\frac{104}{1}$

Examine the figure drawn below. You will see that the area of the bracket is the difference of the area of two rectangles.



Return to page $\frac{100}{2}$ and try question 6 again.

---

$\frac{104}{2}$

The correctness of the result of a division problem can be checked by multiplication.

e.g. $\frac{10}{2} = 5$ is true because $2 \times 5 = 10$

Now, $\frac{0}{2} = 0$ because $2 \times 0 = 0$

Therefore, zero can be divided by a number.

Your choice was wrong.

Return to page $\frac{122}{1}$ and try question 2 again.

This choice is correct.

Now proceed to question 9 which follows:

Question 9

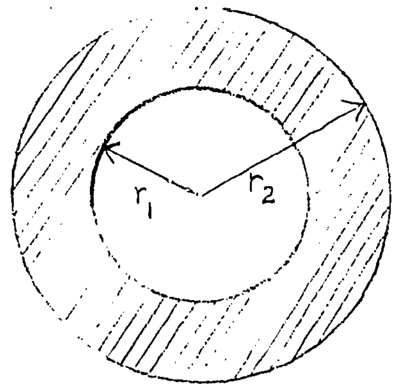
Perform the necessary computation to find the area of the ring that is shaded in the figure, and choose the correct letter.

(A) $115 \frac{1}{2}$ sq. in.

(B) $24 \frac{1}{2}$ sq. in.

(C) 154 sq. in.

(D) $63 \frac{9}{14}$ sq. in.




$r_1 = 3 \frac{1}{2}$ "

$r_2 = 7$"

---



This choice is correct.

Now proceed to question 16 which follows:

Question 16

Apply the proper principle to find the dimensions of the rectangle:

A rectangular field is four times as long as it is wide. If it were 5 feet shorter and 2 feet wider, its area would be increased by 20 square feet. Select the correct letter.

(A) length: 40 ft. width: 10 ft.

(B) length: $10\frac{2}{3}$ ft. width: $6\frac{2}{3}$ ft.

(C) length: 10 ft. width: 40 ft.

(D) length: $4\frac{1}{9}$ ft. width: $4\frac{4}{9}$ ft.

$\frac{106}{1}$

Very good. You made the correct choice. The equation we have to solve is, $(x + 5)(x + 3) = x(x + 7) + 20$

Multiplying the two binomials on the left side of the equation we get:

$8x + 15$

| | |
|---|---|
| $x^2 + 8x + 15 = x(x + 7) + 20$ | Use the Distributive Law |
| $x^2 + 8x + 15 = x^2 + 7x + 20$ | Subtract $x^2$ from both sides |
| $8x + 15 = + 7x + 20$ | Subtract 7x from both sides |
| $x + 15 = 20$ | Subtract 15 from both sides |
| $x = 5$ | |

Note: $x^2 + 7x + 15$ could have been subtracted all at once as an alternate method.

Please go to question 13 below.

---

$\frac{106}{2}$

Question 13

Choose the equation which describes the following verbal statement: The length of a rectangle is 5 less than twice its width, and its area is 96 square units. (Note, w = width)

(A) $w(2w - 5) = 96$

(B) $w(5 - 2w) = 96$

(C) $2w(w - 5) = 96$

(D) $3w - 5 = 96$

Volume 7 Segment 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked punch out the following:

| | | | | | |
|---|---|---|---|---|---|
| COLUMNS | 48 | and | 50 | 3 5 | (Sequence Number) |
| | 54 | and | 56 | 0 4 | (Type of Punch Card) |
| | 60 | and | 62 | 0 7 | (Volume Number) |
| | 66 | and | 68 | 0 5 | (Segment Number) |

Your READING ASSIGNMENT for this Segment is pg: 215 - 221

In your reading you considered division of monomials and the properties of reciprocals. The following questions will call your attention to the principle points.

Question 1

Recognize which of the following pairs of numbers are reciprocals of each other, and select the correct letter.

(A) 2,1 (C) $\frac{3}{4}, \frac{4}{3}$

(B) 1,-1 (D) $\frac{1}{2}, \frac{1}{4}$

---



It is a shame to be marked wrong when you really know how to do the problem. Look at your answer again and examine what you called length and what you called width.

Return to page $\frac{105}{2}$ and try question 16 again.

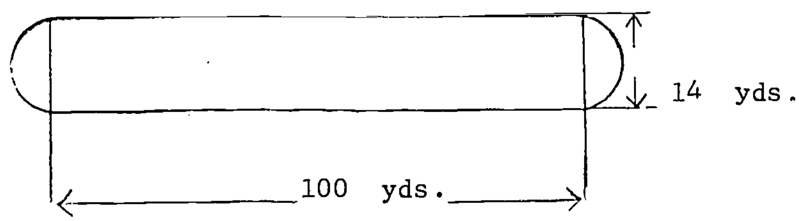
$\frac{108}{1}$

This choice is correct.

Now proceed to question 10 which follows:

Question 10

Perform the necessary computation to find the area of the space inside the race track with semicircular ends drawn here. Select the letter that indicates the correct answer.




(A) 1400 sq. yds.

(B) 1554 sq. yds.

(C) 1477 sq. yds.

(D) 1708 sq. yds.

---

$\frac{108}{2}$

Incorrect. You divided the exponents, when you should have subtracted them. It will be helpful to you to keep in mind the meaning of an exponent. Thus, for example:

$$\frac{2^5}{2^2} \text{ means } \frac{2.2.2.2.2}{2.2} = 2.2.2 = 2^3 = 8$$

or in other form:

$$\frac{32}{4} = 8$$

Please return to page $\frac{120}{1}$ and try question 5 again.

We don't agree.

Since the two rectangles are equal in area, we have to equate the expressions for the areas of these rectangles. In other words, we have to solve the equation,

$$(2w)(w) = (2w + 8)(w - 3).$$

Return to page $\frac{97}{2}$ and try question 14 again.

---



You overlooked something. Here is a good way to proceed. When you have the quotient of two monomials, for example,

$$\frac{16a^5b^3}{8a^2b}$$

you can think of the above as 3 separate questions. Thus, you have,

$$\frac{16}{8},\ \frac{a^5}{a^2},\ \frac{b^3}{b}$$

You can now work your way from left to right. The first part of the question is merely the quotient of two numbers; the other two parts of the question, involve the quotient of powers. You, of course, know the rule for dividing monomial exponential expressions having the same base - retain the base and subtract the exponent of the denominator from the exponent of the numerator.

i.e.: $x^a \div x^b = x^{a-b}$

Please return to page $\frac{124}{2}$ and try question 7 again.

$\frac{110}{1}$

Wrong answer. When we say that the length is 5 less than twice the width we mean to say that the length equals twice the width, but 5 less than that.

Return to page $\frac{106}{2}$ and try question 13 again.

---

$\frac{110}{2}$

Every number except zero has a reciprocal. The quotient of 1 by a number is the reciprocal of that number. For example,

the reciprocal of 2 is $\frac{1}{2}$,

the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

You should also note that the reciprocal of a number has the same sign as the number.

Return to page $\frac{107}{1}$ and try question 1 again.

This choice is correct.

Now proceed to question 4 which follows:

Question 4

Apply the proper principle to choose the letter which labels the correct statement.

(A) $\frac{x^8}{x^2} = x^4$ (B) $\frac{x^8}{x^2} = 6x$

(C) $\frac{x^8}{x^2} = x^6$ (D) $\frac{x^8}{x^2} = 4$

---



Wrong answer. Let us go back to the meaning of exponents.

$$\boxed{\begin{aligned} r^2 &= r.r \\ r^6 &= r.r.r.r.r.r \end{aligned}}$$

Hence, $\frac{r^2}{r^6} = \frac{r.r}{r.r.r.r.r.r} = ?$

since $\frac{r}{r} = 1$

The two r's in the numerator can be cancelled with two r's in in the denominator, thus leaving four r's in the denominator.

Please continue from here.

Return to page $\frac{131}{1}$ and try question 6 again.

$\frac{112}{1}$

Very good. You made the correct choice. The two rectangles are equal in area. Therefore, we have the equation,

| | |
|---|---|
| $(2w + 8)\ (w - 3) = (2w)\ (w)$ | Using the Distributive Law |
| $2w^2 + 2w - 24 = 2w^2$ | subtracting $w^2$ from both sides of the equation |
| $2w - 24 = 0$ | adding 24 to both sides |
| $2w = 24$ | dividing by 2 |
| $w = 12$ | |

---

$\frac{112}{2}$

Question 15

Apply the proper principle to find the dimensions of the rectangle:

A rectangle is 30 feet longer than it is wide. If its length were increased by 50 feet and its width were diminished by 8 feet, its area would be increased by 200 square feet. Select the correct letter.

(A) length: 80 ft. width: 30 ft.

(B) length: $51\frac{1}{3}$ ft. width: $21\frac{1}{3}$ ft.

(C) length: 50 ft. width: 20 ft.

(D) length: $45\frac{5}{21}$ ft. width: $15\frac{5}{21}$ ft.

We suggest that you draw 2 rectangles with the given dimensions as a guide to help you solve the problem.

You were a little careless. You made a mistake in the last term of your choice. Remember the rule for multiplying signed numbers, and see if you can find your mistake.

Return to page $\frac{95}{2}$ and try question 3 again.

---



It is important that you read problem until you are completely familiar with all the facts. We are sure that you can do this problem. We call your attention to the part of the problem which states, " if it were 5 feet shorter and 2 feet wider ". This means if the length was 5 feet less, and the width 2 feet more.

Return to page $\frac{105}{2}$ and try question 16 again.

$\frac{114}{1}$

You lost the base. The rule for finding the quotient of two powers with the same base is to <u>keep</u> the base and subtract the exponents.

$$\boxed{\frac{x^a}{x^b} = x^{a-b}}$$

Thus, for example:

$$\frac{2^5}{2^2} = 2^{5-2} = 2^3$$

Notice that the base 2 was retained in every step.

Please return to page $\frac{120}{1}$ and try question 5 again.

---

$\frac{114}{2}$

You forgot an old rule. The numerator is negative, while the denominator is positive. The quotient of two quantities whose signs are <u>unlike</u> is a negative number.

Return to pag[illegible] $\frac{8}{2}$ [illegible]d try question 8 again.

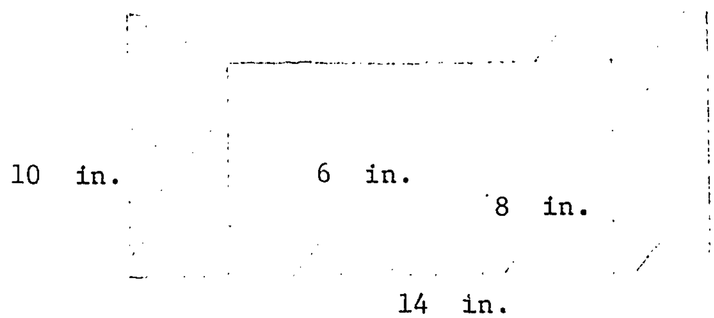
115/1

This choice is correct.

Now proceed to question 7 which follows:

<u>Question 7</u>

Perform the necessary computation to find the area of the shaded part of the frame drawn here, and select the letter which labels the correct answer.




(A) 72 sq. in.

(B) 80 sq. in.

(C) 84 sq. in.

(D) 92 sq. in.

---

115/2

The product of zero and any number is equal to zero. Multiplying a number by zero is therefore a defined operation. On the other hand, dividing a number by zero is not a permissable operation.

Return to page 122/1 and try question 2 again.

$\frac{116}{1}$

Wh- the quotient of 2 powers, both of which have the same base, and ont in the numerator is larger than the exponent in the deno ne rule is to keep the base and subtract the exponents:

$$\boxed{\frac{x^a}{x^b} = x^{a-b}}$$

Sin e sounds like a lot of words for a simple rule, let us illus with an example.

$$\frac{a^7}{a^3} = \frac{a.a.a.a.a.a.a}{a.a.a} \quad a.a.a.a = a^4$$

Or, ...

$$\frac{a^7}{a^3} = a^{7-3} = a^4$$

Re ge $\frac{111}{1}$ and try this question again.

---

$\frac{116}{2}$

This e is correct.

Now proceed to question 9 which follows:

Question 9

Apply the proper principle and select the letter which labels the correct answer to the quotient:

$$\frac{-9p^2q^3}{-36p^4q^5} =$$

$\frac{1}{4p^2q^2}$ (C) $-\frac{1}{27p^2q^2}$

(B) $\frac{1^2q^2}{4}$ (D) $\frac{1}{4p^2q^2}$

This choice is correct.

Now proceed to question 12 which follows:

Question 12

Apply the proper principle to solve following equation for x :

$$(x + 5)(x + 3) = x(x + 7) + 20$$

and select the letter indicating the correct answer.

(A) $x = -\frac{5}{7}$

(B) $x = -5$

(C) $x = 5$

(D) $x = \frac{1}{3}$

---



You will find it helpful to proceed from left to right and ask yourself the following questions:

What does $\frac{-6}{3}$ equal ?

What does $\frac{x^n}{x^2}$ equal ?

What does $\frac{y^m}{y^3}$ equal ?

Please return to page ... and ... question 10 again.

Ve  good. You made the corr  t choice. Here is how th  problem is done.

We  ave $\frac{20x^3y^2}{4xy}$

$$\frac{20}{4} = 5, \quad \frac{x^3}{x} = x^2 \quad \text{and} \quad \frac{y^2}{y} = y$$

, the answer is $5$

P  se go on to q  stion 8 below.

---

## Question 8

Apply the proper principle and select the letter which labels the correct answer to the quotient,

$$\frac{-7a^3b}{21a^2b^5}$$

(A) $\frac{-a}{3b^4}$

(B) $\frac{a}{3b^4}$

(C) $\frac{-a}{14b^4}$

(D) $\frac{-a^{\frac{3}{2}}b^{\frac{1}{5}}}{3}$

This answer choice is correct. Let's run through one method of solving the problem. A diagram is very helpful.


Area = L x W

$= 4x \cdot x$

$= 4x^2$

Area = L W

$= (4x - 5)(x + 2)$

$= 4x^2 + 3x - 10$

The area of rectangle II is 20 sq. ft. more than the area of rectangle I.

Therefore,

| | | |
|---|---|---|
| $4x^2 + 3x - 10 = 4x^2 + 20$ | to check your work |
| $3x - 10 = 20$ | substitute $x = 10$ |
| $3x = 30$ | in the original problem |
| $x = 10$ | |

---



You have now finished with Segment Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following definitions and formulas:

1. Definitions of words area, perimeter, circumference.
2. Formulas for the area of a square, rectangle and triangle.
3. Formulas for the area of a circle and semi-circle.
4. Formulas for the perimeter of a square and rectangle.
5. Formula for the circumference of a circle.
6. A verbal problem dealing with areas of rectangles worked out in detail.

You should now do questions 13-16 of the homework assignment.

Volume 7, Segment 5 begins on page $\frac{107}{1}$

$\frac{120}{1}$

This choice is correct.

Now proceed to question 5 which follows.

Question 5

Apply the power principle and choose the correct statement.

(A) $\frac{3^8}{3^2} = 3^4$

(B) $\frac{3^8}{3^2} = 3^6$

(C) $\frac{3^8}{3^2} = 6$

(D) $\frac{3^8}{3^2} = 1^6$

---

$\frac{120}{2}$

When you have a big job to do, it is usually a good idea to break it down to a number of smaller jobs. Thus, this question can be broken down to 3 smaller questions. Let us show you what we mean. You can think of

$$\frac{-7a^3b}{21a^2b^5} \quad \text{as} \quad \frac{-7}{21} \quad \frac{a^3}{a^2} \quad \frac{b}{b^5}.$$

Now, work your way from left to right, and find the answer to each of the three parts of the question, in turn.

Return to page $\frac{118}{2}$ and try question 4 again.

You misread t[illegible] [illegible]structions. A verbal problem is like a recipe for [illegible]aking a cake. [illegible]ou must follow th[illegible] instructions if you want to have a g[illegible]od cake.

The problem s[illegible] [illegible] that if the [illegible]th is $\underline{w}$, the length is twice that mu[illegible], but less [illegible] that is. [illegible] $-$ $\underline{3}$.

Try this problem again.

Return to page $\frac{1\,6}{}$ and try question 13 again.

---



The rule for [illegible]inding the quotient of powers can only be applied when the bases [illegible]. Notice that in the numerator, the base is 2, while in the [illegible]nominator, the base is 8. You should therefore try to change the denominator to a power of 2. Can 8 be changed to a power of 2? Think [illegible] for awhile. We are sure that you will get the idea.

Return to page $\frac{133}{3}$ and try ques[illegible] 11 again.

$\frac{122}{1}$

This choice is correct.

Now proceed to question 2 which follows:

<u>Question 2</u>

Recognize which of the following expressions represent an operation that is <u>not</u> defined. Choose the letter which labels the correct answer.

A) $\frac{0}{5}$

B) $5 \times 0$

C) $0 \times 0$

D) $\frac{5}{0}$

---

$\frac{122}{2}$

In this choice you made, you cancelled exponents. This is not one of the rules of the game. Let us help you get started.

$$6 = 2 \times 3$$

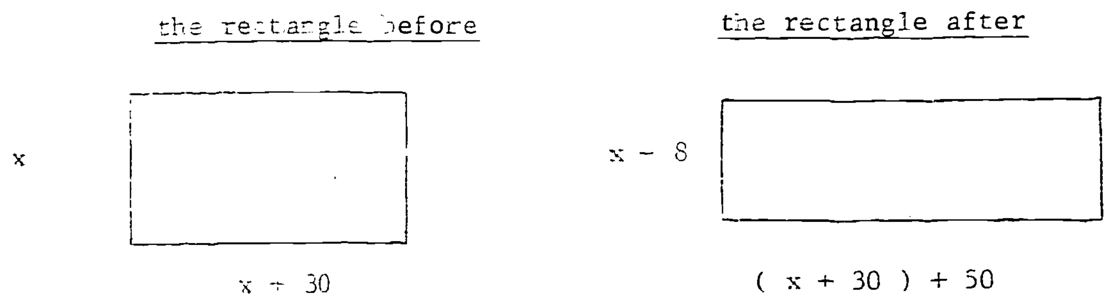
$$6^3 = (2 \times 3)^3$$

$$6^3 = 2^3 \times 3^3$$

Please continue from here.

Return to page $\frac{129}{2}$ and try this question again.

You have heard the expression: " a picture is worth a thousand words. " This expression applied here, means that a diagram can be of great help in solving a problem.




Now, we are told that the second rectangle is 200 square feet larger in area than the first. The area of the larger rectangle is:

$$(x + 80)(x - 8).$$

The area of the smaller rectangle is:

$$x(x + 30).$$

Please continue from here. Return to page $\frac{112}{2}$ and choose another letter.

---



We all get carried away sometimes. You were so busy subtracting exponents, that you also subtracted the numerical coefficients.

The quotient $\frac{9}{36}$ is not affected by the rule for the quotient of powers. This fraction still equals $\frac{1}{4}$.

Please return to page $\frac{116}{2}$ and try question 9 again.

12-

The general rule for finding the quotient of exponential terms with the same base is to retain the base and subtract the exponents. However, this rule may lead us to a situation where we will end up with a negative exponent. This will occur when the exponent in the numerator is smaller than the one in the denominator as in $\frac{x^3}{x^5}$,

where the answer would be $x^{3-5} = x^{-2}$ Now, to avoid negative exponents we revert to the basic meaning or

$$\frac{x^3}{x^5} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^2}$$

Therefore, please write this rule in your notebook:

$$\boxed{\frac{x^a}{x^b} = \frac{1}{x^{b-a}} \quad \text{when} \quad a < b.}$$

Proceed to the next question below.

---

$\frac{124}{2}$

Question 7

Apply the proper principle to find the correct answer to the question:

What is the value of the quotient $\frac{20x^3y^2}{4xy} = ?$

(A) $5x^3y^2$

(B) $16x^2y$

(C) $5x^2y$

(D) $5x^4y^3$

You cancelled the base 3. It is wrong to do that. The question is: When is it alright to cancel and when is it wrong to cancel? When the numerator of a fraction is written as a product of factors, and the denominator is also written as a product of factors, and there are factors in the numerator that are the same as the ones in the denominator, then these like factors may be cancelled.

e.g.
$$\frac{a \cdot b \cdot c}{a \cdot d \cdot e} = \frac{a}{a} \cdot \frac{b\ c}{d \cdot e}$$
$$= 1 \cdot \frac{b \cdot c}{d \cdot e}$$
$$= \frac{bc}{de}$$

Please return to page $\frac{120}{1}$ and try question 5 again.

---



You are not playing the game according to the rules when you <u>divide</u> the bases. Your first job is to change the denominator so that it is written as a power of 2. Notice that $8 = 2^3$. By substitution $8^4$ is therefore $(2^3)^4$. Your second job is to apply the rule for finding the quotient of powers. You are asked to continue from here by yourself.

Return to page $\frac{133}{2}$ and try question 11 again.

Any number multiplied by zero is equal to zero and that includes zero multiplied by zero. When it comes to division, there is an exception. Division by zero is not a defined operation. It would be a good idea for you to write this down in your notebook.

$$\boxed{\frac{0}{0} \text{ is not defined}}$$

Please return to page $\frac{122}{1}$ and try this question again.

---

$\frac{126}{2}$

We must first examine the expression to see if we have like bases in the numerator and denominator. Clearly, we have the quotient of powers of 10. However, we also have the quotient of a power of 4 and a power of 2. It is therefore necessary to change the power of 4 to a power of 2.

Note that $4 = 2^2$. Please continue from here.

Return to page $\frac{128}{2}$ and try question 13 again.

You overlooked something. What is the sign of the quotient of two negative quantities? Let us both review quotients of signed numbers. When both the numerator and the denominator have the same signs, the quotient is positive.

i.e.

$$\frac{+a}{+b} = +\frac{a}{b}$$

$$\frac{-a}{-b} = +\frac{a}{b}$$

When the numerator and the denominator have different signs, the quotient is negative.

i.e.

$$\frac{+a}{-b} = -\frac{a}{b}$$

$$\frac{-a}{+b} = -\frac{a}{b}$$

Please return to page $\frac{116}{2}$ and try question 9 again.

---



It is alright to subtract exponents, but not until the bases are the same. In the numerator, the base is 6 , while in the denominator, the base is 2. Think of 6 as 2 x 3 and see if this gives you an idea how to continue.

Return to page $\frac{129}{2}$ and try question 12 again.

$\frac{128}{1}$

Very good. You made the right choice.

Here is how this question can be answered.

We have $\frac{6^3}{2^3}$ Now, $6 = 2 \times 3$

$$6^3 = (2 \times 3)^3$$

$$6^3 = 2^3 \times 3^3$$

Hence, the question can be rewritten as,

$\frac{2^3 \cdot 3^3}{2^3}$ We can now cancel the like factors $2^3$

and the answer is $3^3$

Check: $\frac{6^3}{2^3} = \frac{216}{8} = 27$

Please go on to question 13 which follows.

---

$\frac{128}{1}$

Question 13

Use the principle of the quotient of powers to determine the value of the expression

$$\frac{10^4 \times 4^3}{10^3 \times 2^5}$$

and select the letter which labels the correct answer.

(A) 200

(B) 5

(C) 20

(D) 12

This choice is correct.

Now proceed to question 10 which follows:

Question 10

Apply the proper principle and choose the letter which labels the correct answer to the question $-\frac{6x^n y^m}{3x^2 y^3}$ equals ?

(A) $-\quad 2x^{\frac{n}{2}} y^{\frac{m}{3}}$

(B) $-\ 2x^{n-2}y^{-3}$

(C) $-\ 9x^{n-2}y^{m-3}$

(D) $2x^{n-2}y^{m-3}$

---



This choice is correct.

Now proceed to question 12 which follows:

Question 12

Apply the proper principle and select the letter which labels the correct statement.

(A) $\frac{6^3}{2^3} = 3^3$

(B) $\frac{6^3}{2^3} = 3$

(C) $\frac{6^3}{2^3} = 3^0$

(D) $\frac{6^3}{2^3} = 4$

$\frac{130}{1}$

This answer choice is correct.

The correctness of the result of a division problem can be checked by multiplication.

e.g. $\frac{12}{3} = 4$ is correct because $3 \times 4 = 12$ now if

$\frac{12}{0} = x$ (some number), that would mean that $0 \cdot x = 12$,

but 0 times any number is zero! There is no number possible that is the result of dividing by zero. Therefore, division by zero is not possible.

Proceed to problem 3 below.

---

$\frac{130}{2}$

Question 3

Perform the indicated operation and select the correct answer:

$$\frac{3}{5} \div \frac{1}{10} =$$

(A) $\frac{3}{50}$

(B) 6

(C) $\frac{50}{3}$

(D) $\frac{1}{6}$

This choice is correct.

Now proceed to question 6 which follows:

Question 6

Apply the proper principle and choose the correct answer to the question:

what does $\frac{r^2}{r^6}$ equal ?

(A) $\frac{1}{r^3}$

(B) $\frac{1}{r^4}$

(C) $r.\frac{1}{3}$

(D) $\frac{1}{3}$

---



Wrong answer.

When you have the quotient of powers, the rule is to retain the base and subtract the exponents. If you examine your choice, you will notice that you divided the exponents. Please study this rule in your text book and write it down in your notebook.

Return to page $\frac{118}{2}$ and try question 8 again.

This answer choice is not correct.

The area of a rectangle is the <u>product</u> of its length and width. You used the <u>sum</u> of the length and width.

Please return to page $\frac{106}{2}$ and try question 13 again.

---

Allow us to refresh your memory concerning the rule for finding the quotient of powers when the denominator has the larger power.

Consider the quotient

$$\frac{b^3}{b^7} = \frac{1}{b^{7-3}} = \frac{1}{b^4}$$

Notice that the numerator is 1 , and the denominator has the letter "b" raised to the larger power minus the smaller, or

$$\boxed{\frac{x^a}{x^b} = \frac{1}{x^{b-a}}}$$

when $a < b$.

Return to page $\frac{116}{2}$ and try question 9 again.

You made the correct choice.

Here is how this question can be done.

We have $\frac{-6x^n y^m}{3x^2 y^3}$

Now, $\frac{-6}{3} = 2$

$$\frac{x^n}{x^2} = x^{n-2}$$

$$\frac{y^m}{y^3} = y^{m-3}$$

Putting all these together, we get

$$-2x^{n-2}y^{m-3}$$

Please go on to question 11 which follows.

---



Question 11

Apply the proper principle and choose the letter which labels the correct statement.

(A) $\frac{2^{16}}{8^4} = 2^{12}$

(B) $\frac{2^{16}}{8^4} = \frac{1}{4^{12}}$

(C) $\frac{2^{16}}{8^4} = \frac{1^4}{4}$

(D) $\frac{2^{16}}{8^4} = 2^4$

$\frac{134}{1}$

We do not agree. Examining the choice you made, you did get the right equation, but you made a mistake in multiplying the binomials

$$(x + 80)\ (x - 8)$$

Go over your work and see if you can find the mist...

Return to page $\frac{12}{2}$ and try question 15 again.

---

$\frac{134}{2}$

In order to apply the rule for the quotient of powers, the bases in the numerator and denominator must be the same. In this question, this is not the case. Your job is to see whether you can do something to the numerator to make it resemble the denominator. Suppose you think of 6 as $2 \times 3$, then $6^3$ can be expressed as powers of 2 and 3. Think about what was just said; it should give you an idea how to go ahead.

Return to page $\frac{129}{2}$ and try question 12 again.

We don't agree.

The rule for finding the quotient of powers which have the same base is to retain the base and subtract the exponents. Please examine the following two illustrations:

$$\frac{a^{11}}{a^{7}} = a^{11-7} = a^{4}$$

$$\frac{x^{5}}{x^{8}} = \frac{1}{x^{8-5}} = \frac{1}{x^{3}}$$

Before applying the rule illustrated above, make sure that the bases are the same.

Return to page $\frac{133}{2}$ and try question 11 again.

---



Volume 7 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

| | | | | | | |
|---|---|---|---|---|---|---|
| COLUMNS | 48 | and | 50 | 3 | 2 | (Sequence Number) |
| | 54 | and | 56 | 0 | 4 | (Type of Punch Card) |
| | 60 | and | 62 | 0 | 4 | (Volume Number) |
| | 66 | and | 68 | 0 | 2 | (Segment) |

Your READING ASSIGNMENT for this Segment is pg: 206 - 209

In your reading you have followed a discussion about monomials, binomials and other polynomials and have worked out problems involving multiplications of these polynomials. You will now be asked several questions designed to draw your attention to the more important points.

Question 1

Recognize which of the following expressions is not a monomial, and select the correct letter.

(A) $5a^2$ (B) $\frac{1}{2}xy^2$

(C) $-.3a^3b$ (D) $x^3 - 4x + 50$

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We can use the multiplication algorithm to multiply

$(x^2 + 2x + 1)$ by $(3x^2 - x + 2)$ as follows:

$$
\begin{array}{rrrrr}
x^2 & + 2x & + 1 & & \\
3x^2 & - x & + 2 & & \\
\hline
3x^4 + 6x^3 & + 3x^2 & & & \\
- x^3 & - 2x^2 & - x & & \\
 & + 2x^2 & + 4x & + 2 & \\
\hline
3x^4 + 5x^3 & + 3x^2 & + 3x & + 2 &
\end{array}
$$

Therefore, your answer choice was correct.

Now proceed to question 14 below.

---

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2

Question 14

Apply the proper principle to choose the letter which labels the correct answer to the product of the three binomials:

$(x + y)(x - y)(x + 1) =$

(A) $x^3 - y^2$

(B) $x^3 - xy^2 + x^2 - y^2$

(C) $x^3 - xy^2 - y^2$

(D) $2x^3 + x^2 - y^2$

Congratulations. You made the correct choice. We can break the problem up into the product of 2 quotients, namely,

$$\frac{10^4}{10^3} \quad \text{and} \quad \frac{4^3}{2^5}$$

Consider the first fraction:

$$\frac{10^4}{10^3} = 10^{4-3}$$

$$= 10^1$$

$$= 10$$

now consider the second fraction:

$$\frac{4^3}{2^5} = \frac{(2^2)^3}{2^5} \quad (\text{since } 4 = 2^2)$$

$$= \frac{2^6}{2^5} \quad (\text{since } (2^2)^3 = 2^6$$

$$= 2^{6-5}$$

$$= 2^1$$

$$= 2$$

The product of the two quotients is therefore, $10 \times 2 = 20$

We are sure that you appreciate the fact that a knowledge of exponents can help simplify multiplication and division of numbers.

You have now completed this segment. Hand in your PUNCH CARD. Make sure that the following information is in your notebook.

1. Definition of the reciprocal of a number.
2. Meaning of division.
3. The rule for finding the quotient of powers when the numerator has the larger power.
4. The rule for finding the quotient of power, when the denominator has the larger power.

You should now do questions 17-20 of the homework assignment.

Be sure to complete the entire assignment questions 1-20, in preparation for the test on volume 7.

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 7

Attach to
Back Cover

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected. Listed below are misprints that will affect the mathematics of the problems or the progress through the book. Make a careful correction of each misprint as follows:

| PAGE | MISPRINT | CORRECTION | CHECK WHEN CORRECTION MADE |
| --- | --- | --- | --- |
| $\frac{53}{2}$ | Return to page $\frac{10}{1}$ and try question 6 again | Return to page $\frac{10}{2}$ and try question 13 again. | |
| $\frac{97}{1}$ | Return to page $\frac{105}{1}$ and try question 5 again. | Return to page $\frac{105}{1}$ and try question 9 again. | |
| E | Homework assignment Dressler | Dolciani | |
| $\frac{39}{2}$ | $\frac{10}{1}$ | $\frac{10}{2}$ | |

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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 8: dividing a polynomial by a monomial and by a polynomial, factoring, identifying common factors, multiplying sum and difference of two numbers, factoring differences of two squares, and squaring a binomial. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

# PROGRAMMED MATH CONTINUUM

## *level one*

# ALGEBRA



VOLUME

8

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

# P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

# A L G E B R A

VOLUME 8

New York Institute of Technology

Old Westbury - New York

A

# PREFACE

This volume is one of a set of 18
that form a complete course
in
ALGEBRA - LEVEL ONE

The volume has been structured
in a multiple choice question-answer format,
with the pagination scrambled
and
is to be used in conjunction with
a program control console
utilizing
punch card input.

It is one exhibit in the demonstration of a model
developed under the direction of
the U.S. Department of Health Education and Welfare
Project 8-0157

at the

New York Institute of Technology
Westbury, New York

VOLUME 8

TABLE OF CONTENTS



IN THE STUDY GUIDE:

C

VOLUME 8

This volume covers the following material as shown in this excerpt from the Syllabus:

| SEGMENT | DESCRIPTION | REFERENCE BOOK SECTION DOLCIANI | DRESSLER | DODES |
|---|---|---|---|---|
| 1 | Dividing polynomial by monomial | 6 - 11 | 8 - 6 | 8 - 5 |
| 2 | Dividing polynomial by polynomial | 6 - 12 | 8 - 7 | 8 - 5 |
| 3 | Factoring in algebra | 7 - 1 | 11- 1 | |
| | Indentifying common factors | 7 - 2 | 11- 2 | 8 - 6 |
| 4 | Multiplying sum and difference of two numbers | 7 - 3 | 11- 4 | 8 - 4 |
| | Factoring difference of two squares | 7 - 4 | 11- 5 | 8 - 6 |
| 5 | Squaring a binomial | 7 - 5 | 11- 5 | 8 - 4 |

D

READING ASSIGNMENT

VOLUME 8

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

| SEGMENT | FROM PAGE | TO PAGE |
|---|---|---|
| 1 | 219 | 221 |
| 2 | 221 | 223 |
| 3 | 237 | 244 |
| 4 | 245 | 248 |
| 5 | 248 | 251 |

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghtom Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co. 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

E

HOMEWORK ASSIGNMENT

VOLUME NO. 8

BOOK: DOLCIANI

| HOMEWORK QUESTION NO. | PAGE NO. | EXAMPLE NUMBER | MBO REFERENCE |
|---|---|---|---|
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| 2 | 220 | 8 , 11 , 13 | 08110 |
| 3 | 220 | 15 , 18 , 19 , 20 | 08110 |
| 4 | 221 | 25 , 29 , 30 , 31 | 08110 |
| 5 | 223 | 2 , 4 | 08210 |
| 6 | 223 | 5 , 9 | 08210 |
| 7 | 223 | 17 , 19 , 21 | 08210 |
| 8 | 223 | 23 , 30 , 32 | 08210 |
| 9 | 239 | 15 , 17 , 18 , 20 | 08310 |
| 10 | 240 | 1 , 2 , 4 , 6 , 8 | 08310 |
| 11 | 242/2 | 3 , 11 , 18 , 21 | 08320 |
| 12 | 243 | 23 , 25 , 30 | 08320 |
| 13 | 246/1 | 16 , 19 , 20 , 23 | 08410 |
| 14 | 246/2 | 4 , 9 , 17 , 19 | 08410 |
| 15 | 247/2 | 2 , 5 , 9 | 08420 |
| 16 | 247/2 | 15 , 17 , 20 | 08420 |
| 17 | 249 | 3 , 4 , 5 , 7 | 08510 |
| 18 | 249 | 9 , 10 , 12 , 14 , 16 | 08510 |
| 19 | 250 | 17 , 20 , 22 , 24 | 08510 |
| 20 | 250 | 25 , 26 , 27 , 28 | 08510 |

F

# GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

Volume 8 Segment 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

| | | | | |
|---|---|---|---|---|
| COLUMNS | 48 and 50 | 3 | 6 | (Sequence Number) |
| | 54 and 56 | 0 | 4 | (Type of Punch Card) |
| | 60 and 62 | 0 | 8 | (Volume Number) |
| | 66 and 68 | 0 | 1 | (Segment Number) |

Your READING ASSIGNMENT for this Segment is pg: 219 - 221
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Recognize which one of the following expressions is the same as

$$\frac{42 + 9}{3}$$

and choose the letter next to the correct answer.

(A) $39 + 6$ (C) $13 + 3$

(B) $\frac{42}{3} + \frac{9}{3}$ (D) None of these.

---



You made a wrong choice. You did not divide each of the terms in the numerator by the denominator. Consider, for example, the following quotient

$$\frac{39p - q}{q}$$

By the distributive property, $39p$ should be divided by $q$,

and $-q$ should also be divided by $q$.

Please return to page $\frac{27}{1}$ and try this question again.

$\frac{2}{1}$

Very good. You made the right choice. Here is how this question can be answered:

$$\frac{15x \quad + \quad 25y}{5}$$

is the same as,

$$\left(\frac{1}{5}\right)\left(15x \quad + \quad 25y\right)$$

this equals,

$$\left(\frac{1}{5}\right)\left(15x\right) \quad + \left(\frac{1}{5}\right)\left(25y\right)$$ by the distributive property.

which equals:

$$3x \quad + \quad 5y$$

Please proceed to question 5 below.

---

$\frac{2}{2}$

Question 5

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{12x^3 \; + \; 6x}{3x} \quad \text{equals:}$$

(A) $4x^3 + 2x$

(B) $9x^2 + 3$

(C) $6x^2$

(D) $4x^2 + 2$

Very good. You made the correct choice. Here is how the quotient

$$\frac{8m - n}{n}$$

can be simplified: We re-write the quotient as $\left(\frac{1}{n}\right)(8m - n)$. Then applying the distributive property, we get

$$\frac{8m}{n} - \frac{n}{n}$$

Now, $\frac{8m}{n}$ cannot be simplified any further, since the numerator and the denominator consist of different letters. The second term is a number divided by itself, with a negative sign in front of it. Therefore, it equals $-1$. Hence, the correct answer is

$$\frac{8m}{n} - 1$$

Please proceed to question 7 below.

---

$\frac{3}{2}$

<u>Question 7</u>

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{8a^3 - 4a}{2a}$$

equals:

(A) $4a^2 - 2$

(B) $6a^2 - 2$

(C) $4a^2 - 2a$

(D) $2a^2$

$\frac{4}{1}$

To simplify a ession of the type:

$$\frac{75 - 45 + 15}{5}$$

instead of combining the terms of the numerator and then dividing we can use the distributive property. The above expression can be re-written as:

$$\left(\frac{1}{5}\right)(75 - 45 + 15) = \left(\frac{1}{5}\right)(75) - \left(\frac{1}{5}\right)(45) + \left(\frac{1}{5}\right)(15)$$

$$= 15 - 9 + 3$$

$$+ 9$$

It is important to observe the signs of the terms in the numerator.

Return to page $\frac{23}{1}$ and try this question again.

---

$\frac{4}{2}$

We don't agree. Let us try to make this business of dividing a polynomial by a number a little more real to you. Suppose you were told that a certain child was 4 years, 8 months, 2 weeks and 6 days old, and tha a second child was half the age of the first child. How could you find the age of the second child? To obtain the age of the second child, you would divide the number of years by 2, the number of months by 2, the number of weeks by 2 and the number of days by 2. Well, you have actually followed the rule for dividing a polynomial by a number. The age of the first child is a polynomial; namely;

$4y + 8m + 2w + 6d$, where the letters stand for years, months, weeks, and days.

Similarly, in question 8, you should divide each term of the numerator by 3.

Return to page $\frac{17}{2}$ and try this question again.

$\frac{5}{1}$

You subtracted the numerical coefficients when you should have divided them. Let us try to make this point clear to you.

Consider for example: $\frac{27y^3}{3y}$

This quotient can be written as $\frac{27}{3} \cdot \frac{y^3}{y}$

Now, the numerical coefficient simply is 27 divided by 3, and that equals 9. The expression $\frac{y^3}{y}$ is the quotient of 2 powers with the same base. Only to this quotient do we apply the rule for the quotient of powers.

Please return to page $\frac{2}{2}$ and choose another letter.

---

$\frac{5}{2}$

You did not make the right choice. The mistake you made is that you subtracted the numerical coefficients. The question can be broken up into the quotient of 3 monomials, thus;

$$\frac{24b^4 - 12b^3 + 15b^2}{3b^2} = \frac{24b^4}{3b^2} - \frac{12b^3}{3b^2} + \frac{15b^2}{3b^2}$$

The numerical coefficients of each of these terms, namely;

$$\frac{24}{3} - \frac{12}{3}, \text{ and } \frac{15}{3}$$

represent division of numbers.

Please return to page $\frac{11}{2}$ and choose another letter.

$\frac{6}{1}$

You should have remembered an old rule which states that only like terms can be combined. The terms $8a^3$ and $-4a$ are not like terms. The quotien[illegible] a binomial and a monomial can be broken up into 2 quotients of [illegible]omi[illegible] Thus,

$$\frac{8a^3 - 4a}{2a} = \frac{8a^3}{2a} - \frac{4a}{2a}$$

Now, each of these quotients can be simplified. Please refer to your notebook and study the rules for simplifying the quotient of 2 monomials

Return to page $\frac{3}{2}$ and try this question again.

---

$\frac{6}{2}$

Almost right, but in mathematics, almost right means wrong. We are going to let you find your own mistake without our help. All that we will ask you is to recall the rules for the quotient of signed numbers.

Please return to page $\frac{8}{1}$ and try this question again.

This choice is correct.

Now proceed to question 4 which follows:

ion 4

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{15x + 25y}{5} \text{ equals:}$$

(A) $3x + 5y$

(B) $10x + 20y$

(C) $5x + 3y$

(D) $3x - 20y$

---

$\frac{7}{2}$

You made a wrong choice. A number divided by itself is not equal to 0.

$$\frac{a}{a} = 1 \quad \text{since} \quad a = 1\,a$$

Thus, a number divided by itself equals 1.

Please return to page $\frac{18}{2}$ and try this question again.

$\frac{8}{1}$

This choice is correct.

Now proceed to question 10 which follows:

Question 10

Apply the proper principle to simplify the quotient.

$$\frac{12x^3y^3 - 6x^2y^2 + 18xy}{-6xy}$$

and select the letter which labels the correct answer.

(A) $-2x^2y^2 - xy - 3$

(B) $6x^2y^2 - xy - 3$

(C) $-2x^3y^3 + x^2y^2 - 3$

(D) $-2x^2y^2 + xy - 3$

---

$\frac{8}{2}$

Suppose we first simplify this quotient:

$$\frac{8x^3 - 4x^2 - 2x}{2x} = \frac{8x^3}{2x} - \frac{4x^2}{2x} - \frac{2x}{2x}$$

$$= 4x^2 - 2x - 1$$

Now, we can substitute 2 for x and evaluate the expression.

Please continue from here.

Return to page $\frac{28}{2}$ and try this question again.

$\frac{9}{1}$

You did not make the correct choice. Let us help you get started.

$$\frac{x + y}{y} = 3$$

$$\frac{x}{y} + \frac{y}{y} = 3$$

Note that a number divided by itself equals 1.

We are sure that you can continue from here on your own.

Please return to page $\frac{19}{2}$ and consider question 13 again.

---

$\frac{9}{2}$

Remember this formula:

$$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = \text{QUOTIENT} + \frac{\text{REMAINDER}}{\text{DIVISOR}}$$

and it will help you decide which is the correct answer.

Each word used in mathematics (with the exception of some simple words) has a definite definition. It is dangerous to assume that you know what a word means unless you have already learned its official definition.

Please return to page $\frac{30}{2}$ and choose another letter.

$\frac{10}{1}$

It is true that the coefficients are in a decreasing order. However, you were asked to choose the polynomial whose variable was in a decreasing order.

Return to $\frac{25}{2}$ and try question 3 again.

---

$\frac{10}{2}$

You merely re-wrote the quotient in another way. As amatter of fact, you will come across three different ways of writing a problem in division; namely,

$$\frac{745}{15}, \quad 15\overline{)745}, \quad 745 \div 15$$

The above three expressions all indicate the same operation. We will give you a start to help you make the right choice. Think of the decimal representation of integers. Thus, for example:

$$864 = 8(100) + 6(10) + 4$$

Please continue from here.

Return to page $\frac{26}{2}$ and try this question again.

Very good. You made the right choice. Here is how this quotient can be simplified:

$$\frac{18x - 9y + 6z - 3w}{3} = \frac{18x}{3} - \frac{9y}{3} + \frac{6z}{3} - \frac{3w}{3} \quad \text{Now,}$$

$$\frac{18x}{3} = 6x$$

$$-\frac{9y}{3} = -3y$$

$$\frac{6z}{3} = 2z$$

$$-\frac{3w}{3} = -w$$

Thus, the correct answer is:

$$= 6x - 3y + 2z - w$$

Please proceed to question 9 below

---



Question 9

Apply the proper principle to simplify the quotient:

$$\frac{24b^4 - 12b^3 + 15b^2}{3b^2}$$

and select the letter which labels the correct answer.

(A) $21b^2 - 9b + 12$

(B) $8b - 4b^{\frac{3}{2}} + 5$

(C) $8b^2 - 4b + 5$

(D) $27b^2$

$\frac{12}{1}$

To simplify a numerical expression of the type:

$$\frac{36 + 18 + 6}{3}$$

instead of adding first and then dividing, as an alternate method, we can use the distributive property. The above expression can be re-written as

$$\frac{1}{3}(36 + 18 + 6) \text{ which equals}$$

$$\frac{1}{3}(36) + \frac{1}{3}(18) + \frac{1}{3}(6) =$$

$$12 + 6 + 2 = 20$$

Please return to page $\frac{1}{1}$ and try this question again.

---

$\frac{12}{2}$

We don't agree.

You made two mistakes. Let us try this question together.

$$\frac{12x^3y^3 - 6x^2y^2 + 18xy}{-6xy}$$

can be separated into 3 quotients:

$$\frac{12x^3y^3}{-6xy} \qquad \frac{-6x^2y^2}{-6xy} \quad \text{and} \quad \frac{18xy}{-6xy}$$

Now, ask yourself:

What is the value of $\frac{12}{-6}$ ?

What is the value of $\frac{-6}{-6}$ ?

Return to page $\frac{8}{1}$ and try this question again.

The expression $\frac{15x + 25y}{5}$ signifies that each term of the numerator is divided by the denominator. Thus, the expression can be re-written as

$$\frac{15x}{5} + \frac{25y}{5} .$$

We are sure that you can continue from here.

Return to page $\frac{7}{1}$ and choose another letter.

---



We do not agree. Let us separate this quotient into three quotients.

Thus: $$\frac{8a^3b}{4a^2b} - \frac{16a^2b}{4a^2b} + \frac{4a^2b}{4a^2b} .$$

Let us take the first quotient and work our way from left to right.

Thus, we have:

$$\frac{8}{4} \qquad \frac{a^3}{a^2} \qquad \frac{b}{b}$$

Now: $$\frac{8}{4} = 2 \qquad \frac{a^3}{a^2} = a \qquad \frac{b}{b} = 1 .$$

this becomes:

2a

Proceed in the same way with the other 2 quotients.

Return to page $\frac{18}{2}$ and try the question again.

$\frac{14}{1}$

Exponents are subtracted only when we have the quotient of the same letters raised to powers.

For example:

$$\frac{y^5}{y^2} = y^{5-2}$$

$$= y^3$$

However, a numerical quotient such as $\frac{18}{3}$ is merely the division of 18 by 3, and the answer is 6 and <u>not 15</u>. Now, examine your choice, and you will see that you subtracted the numerical coefficients when you should have divided them.

Return to page $\frac{3}{2}$ and try this question again.

---

$\frac{14}{2}$

You did not choose the right answer. It is always a good idea to check a problem in division by multiplying the quotient by the divisor and then adding the remainder. This should be equal to the dividend.

Look at this example.

$$\begin{array}{r l}
 & \underline{\quad 245} \\
25\ ) & 6138 \\
 & \underline{-50\quad} \\
 & 113 \\
 & \underline{-100} \\
 & 138 \\
 & \underline{-125} \\
 & 13 \text{ remainder}
\end{array}$$

Now, we multiply the quotient 245 by the divisor 25

$$\begin{array}{r}
245 \\
\underline{25} \\
1225 \\
\underline{490\;\,} \\
6125
\end{array}$$

Finally, we add the remainder to the above product; thus, 6125 + 13

This sum must equal the divident, 6138

Return to page $\frac{20}{2}$ and try this question again.

This choice is correct.

Now proceed to <u>question 3</u> which follows:

<u>Question 3</u>

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{ab^2 + ab + a}{a} \quad \text{equals ?}$$

(A) $b^2 + b$

(B) $ab^2 + ab + a$

(C) $b^2 + b + 1$

(D) $ab^2 + ab$

---



By missing terms we mean the missing <u>powers</u> of the variable, counting from the highest power down to the lowest power. For example, consider the polynomial,

$$x^7 - 3x^5 + x^2 - 3$$

Note that the highest power of $x$ is $7$. Therefore, we start from $7$ and count down to $0$. Thus, we get $7, 6, 5, 4, 3, 2, 1, 0$. Now, the polynomial does <u>not</u> have $x$ raised to the powers $6, 4, 3, 1$. Hence, the missing terms are $x^6, x^4, x^3, x$. Note, $3$ can be considered $3x^0$.

Please return to page $\frac{43}{2}$ and choose another letter.

$\frac{16}{1}$

You did not make the right choice. When you have to divide a polynomial by a monomial, it is a good idea to break up the expression into quotients of monomials. Let us illustrate this. Suppose that we have to simplify the quotient,

$$\frac{33a^4 + 22a^2}{11a}$$

The above can be written as the sum of the quotient of 2 monomials

$$\frac{33a^4}{11a} + \frac{22a^2}{11a}$$

Now, the numerical coefficients are divided, and the exponents of the letter are subtracted.

Return to page $\frac{2}{2}$ and try this question again.

---

$\frac{16}{2}$

We don't agree. The rule for finding the quotient of powers is to retain the base and <u>subtract</u> the exponents.

i.e. $\boxed{\frac{x^a}{x^b} = x^{a-b}}$ when $a > b$

Please return to page $\frac{11}{2}$ and try this question again.

Very good. Here is how this question can be answered:

$$\frac{8a^3 - 4a}{2a} = \frac{1}{2a}(8a^3 - 4a) \qquad \text{(By the Distributive Law)}$$

$$= \frac{8a^3}{2a} - \frac{4a}{2a} \qquad \text{Note:} \quad \frac{8}{2} = 4$$

$$= 4a^2 - 2 \qquad \frac{a^3}{a} = a^2$$

$$\frac{-4}{2} = -2$$

$$\frac{a}{a} = 1$$

Now, proceed to question 8 below.

---



Question 8

Apply the proper principle to simplify the quotient,

$$\frac{18x - 9y + 6z - 3w}{3}$$

and select the letter next to the correct answer.

(A) $6x - 3y + 2z - 3w$

(B) $6x - 3y + 2z - w$

(C) $12x - 3y + 2z - w$

(D) $15x - 6y + 3z - 6w$

$\frac{18}{1}$

The expression $\frac{ab^2 + ab + a}{a}$ can be re-written as:

$$\frac{1}{a}(ab^2 + ab + a) = \frac{ab^2}{a} + \frac{ab}{a} + \frac{a}{a}$$

Each of the three terms is a quotient of powers. If you refer to your notebook, you will find the rule for finding the quotient of powers.

Return to page $\frac{15}{1}$ and try this question again.

---

$\frac{18}{2}$

This choice is correct.

Now, proceed to question 11 which follows.

Question 11

Apply the proper principle to simplify the quotient,

$$\frac{8a^3 b - 16a^2 b^2 + 4a^2 b}{4a^2 b}$$

and select the letter which labels the correct answer.

(A) $2a - 4b + 1$

(B) $2a - 4b$

(C) $2ab - 4b^2 + b$

(D) $2a^{\frac{3}{2}} - 4ab^2 + ab$.

Very good. You made the right choice. One way to do this problem is to simplify the quotient first. Thus,

$$\frac{8x^3 - 4x^2 - 2x}{2x} = \frac{8x^3}{2x} - \frac{4x^2}{2x} - \frac{2x}{2x}$$

$$= 4x^2 - 2x - 1$$

Now, when $x = 2$, we have

$$= 4(2^2) - 2(2) - 1$$

$$= 16 - 4 - 1$$

$$= 11$$

It is interesting to note that the above procedure can be used to <u>check</u> your work. Let us explain. To check whether the quotient

$$\frac{8x^3 - 4x^2 - 2x}{2x} = 4x^2 - 2x - 1,$$ we evaluate the

quotient for any numerical value of "x", and we evaluate the answer for the same value of "x". These two answers should be the same. That is,

$$\frac{8(2^3) - 4(2^2) - 2(2)}{2(2)} =$$ should also equal 11.

If we complete the calculations, we find out that the above does equal 11. Please proceed to question 13 below.

---



<u>Question 13</u>

Apply the proper principle to find the ratio $x:y$, if:

$$\frac{x + y}{y} = 3$$

Se[illegible] next to the right answer.

(A) 1:2 (B) 2:1 (C) 1:3 (D) 3:1

made the correct choice. It will be helpful in remembering the...

if we write the formula,

$$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = \text{QUOTIENT} + \frac{\text{REMAINDER}}{\text{DIVISOR}}$$

in this question, we have:

$$\frac{645}{23} = 28 + \frac{1}{23}$$

...at 645 is the dividend, 23 is the divisor, 28 is the quotient,

1 is the remainder.

...se go on to question 2 below.

---

...ion 2:

...orm the indicated operation and select the letter which labels the

...rect answer, 3487 $\div$ 43

(A) $78\frac{3}{43}$

(B) 79

(C) 81

(D) $81\frac{4}{43}$

$\frac{21}{1}$

The variable is in an <u>ascending</u> order from left to right. True, it is in descending order the other way but the "left to right" order is the one that is always meant whether or not it is stated.

Return to page $\frac{25}{2}$ and try question 3 again.

---

$\frac{21}{2}$

We are dealing with integers that are represented in the decimal system. In this system, <u>position</u> is important. Consider the integer 372. This number is not the same as $3 + 7 \div 2$, because 372 means $300 + 70 + 2$. Let us try another number; say, 4378.

$$4378 = 4000 + 300 + 70 + 8$$

Please return to page $\frac{26}{2}$ and try this question again.

$\frac{\ldots}{1}$

Y did not divide correctl Maybe you should go back to the text assignment and study the division of a polynomial by a polynomial on page 222.

If $x + 5$ were the correct answer, then

$(x + 5)(x - 2)$ should equal

$x^2 - 7x + 10$ Check the multiplication.

You will see that you get

$x^2 + 3x - 10$

Return to page $\frac{37}{1}$ and try this question again.

---

$\frac{22}{\ldots}$

Question 7:

Apply the proper principle to find the quotient,

$$(3x^2 + 7x - 6) \div (3 + x)$$

and select the letter which labels the correct answer.

(A) $3x + 16$

(B) $3x + 16$ and remainder $42$

(C) $3x - 2$

(D) $3x - 2$ and remainder $\frac{-12}{x + 3}$

This choice is correct.

Now proceed to question 2 which follows.

Question 2

Recognize which one of the following expressions is the same as

$$\frac{4[illegible] - [illegible]}{16}$$

and choose the letter next to the correct answer.

(A) [illegible]

(B) 3[illegible]

(C) [illegible]

(D) None of these.

---



An old mathematics rule is true no matter when you learned it. We are referring to the rule for finding the quotient of powers. The rule is: to find the quotient of powers having the same base, we keep the base and subtract the exponent of the denominator from the exponent of the numerator. Thus,

$$\frac{b^7}{b^3} = b^{7-3} = b^4$$

$$\frac{c^5}{c} = c^{5-1} = c^4$$

Please return to page $\frac{8}{1}$ and try this question again.

$\frac{24}{1}$

The rule for finding the quotient of powers having the same base is to retain the base and subtract the exponents. For example.

$$\frac{21a^3}{3a} = 7a^2$$

$$\frac{18b}{3b} = 6$$

Note that a letter written without an exponent means that the letter has the exponent 1.

Please return to page $\frac{2}{2}$ and try this question again.

---

$\frac{24}{2}$

You divided the exponents; you should have subtracted them. Please keep in mind the rule for finding the quotient of powers:

$$\boxed{\frac{x^a}{x^b} = x^{a-b}} \quad \text{where } a > b$$

Return to page $\frac{18}{2}$ and try this question again.

If you re-write the quotient as the difference of the quotient of two monomials, you will get,

$$\frac{8a^3 - 4a}{2a} = \frac{8a^3}{2a} - \frac{4a}{2a}$$

Now, the second quotient is not equal to 2a. Give this question a little more thought.

Please return to page $\frac{3}{2}$ and choose another letter.

---



This choice is correct.

Now proceed to question 3 which follows.

Question 3

Recognize which polynomial is written in decreasing order of the variable x, and select the letter next to such a polynomial.

(A) $6x^4 - 5x^3 + 4x^5 - 3$

(B) $5x^2 - x + 10$

(C) $7x^2 \quad 5x^3 + 2x^4$

(D) $4 - 3x^2 + 2x$

$\frac{26}{1}$

We don't agree.

If you divided \$5 among 5 people; how much money would each person get? That's easy, you say. Each person will get \$1. If you divided 8 pieces of gum among 8 children; how many pieces would each child get? Again, the answer is 1. In other words, a number divided by itself is equal to 1.

Coming back to our question, we have

$$\frac{ab^2 + ab + a}{a} = \left(\frac{1}{a}\right)\left(ab^2 + ab + a\right) =$$

$$\frac{ab^2}{a} + \frac{ab}{a} + \frac{a}{a}$$

Observe that the last term is a number divided by itself.

Return to page $\frac{15}{1}$ and choose another letter.

---

$\frac{26}{2}$

This choice is correct.

Now proceed to question 5 which follows.

Question 5

Recognize which of the following represents the quotient $\frac{693x}{3}$ re-written as the quotient of a polynomial of three terms and a monomial and select the letter which labels the correct statement.

(A) $3\overline{)693}$

(B) $\frac{6(100)x + 9(10)x + 3x}{3}$

(C) $\frac{6x}{3} + \frac{9x}{3} + \frac{3x}{3}$

(D) $\frac{3(231x)}{3}$

This choice is correct.

Now proceed to <u>question 6</u> which follows:

<u>Question 6</u>

Apply the proper principle to simplify the quotient and select the letter which labels the correct answer.

$$\frac{8m - n}{n} \text{ equals ,}$$

(A) $\frac{8m}{n}$

(B) $8m - 1$

(C) $8m - n$

(D) $\frac{8m}{n} - 1$

---



You did not make the right choice. Let us do a part of this problem together:

$$\begin{array}{r|l} & 3x \\ \hline x + 3 & 3x^2 + 7x - 6 \\ & \underline{3x^2 + 9x} \end{array}$$

At this point, you should keep in mind that $(3x^2 + 9x)$ is subtracted from $(3x^2 + 7x)$. When we subtract two polynomials, we change the signs of the terms of the polynomial being subtracted, the subtrahend, and then apply the addition rules.

Please return to page $\frac{22}{2}$ and try this question again.

$\frac{28}{1}$

Each of the terms in the numerator are different since they have different powers of b. The first term has the letter b raised to the $4^{th}$ power; the second term has the letter b raised to the $3^{rd}$ power; and the last term has b raised to the $2^{nd}$ power. Therefore, these are unlike terms and cannot be combined. It would help you if you re-read the text assignment.

Return to page $\frac{11}{2}$ and try this question again.

---

$\frac{28}{2}$

This choice is correct.

Now proceed to question 12 which follows:

Question 12

Perform the indicated calculation to find the value of the quotient

$$\frac{8x^3 - 4x^2 - 2x}{2x},$$ when $x = 2$, and select the letter next to the correct statement.

(A) 20

(B) 12

(C) 22

(D) 11

Let us explain how an expression of this type can be simplified.

Consider the expression

$$\frac{x^3y^2 + x^2y + y}{y} .$$

This expression can be written as,

$$\left(\frac{1}{y}\right)\left(x^3y^2 + x^2y + y\right)$$

By the distributive property, the above equals,

$$\frac{x^3y^2}{y} + \frac{x^2y}{y} + \frac{y}{y}$$

Using the rule for finding the quotient of powers, we get

$$\frac{x^3y^2}{y} = x^3y , \quad \frac{x^2y}{y} = x^2 . \quad \frac{y}{y} = 1$$

Note that the last term is a number divided by itself and is, therefore, equal to 1.

Please return to page $\frac{15}{1}$ and try this question again.

---



In a Long division problem where the dividend consists of many terms many operations have to be performed. We subtract and multiply quite a few times. Hence, there are more chances for making mistakes. In a situation like this, it is very important to work carefully and arrange your work neatly. Furthermore, when you have completed such a problem it is important to check your answer. You should multiply the quotient by the divisor and see if the product equals the dividend.

Please return to page $\frac{31}{2}$ and try this question again.

$\frac{30}{1}$

Congratulations, you made the right choice. Here is how this question can be answered.

$$\frac{x + y}{y} = 3$$

$$\frac{x}{y} + \frac{y}{y} = 3 \quad \text{Now,} \quad \frac{y}{y} = 1 \quad \text{Hence, we have}$$

the equation

$$\frac{x}{y} + 1 = 3$$

$$\frac{x}{y} = 2 \text{ , or}$$

$$\frac{x}{y} = \frac{2}{1}$$

You have now completed this segment, hand in the PUNCH CARD. Make sure that you have written the following items in your notebook.

1. A problem worked out in detail involving the quotient of a polynomial and a monomial.

   For example: $\dfrac{8x^3y - 16xy^2 + 4x^2y}{4x^2y}$

You should now do questions 1-4 of the homework assignment.

---

$\frac{30}{2}$

Volume 8 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you you are asked to punch out the following:

| | | | | | | |
|---|---|---|---|---|---|---|
| COLUMNS | 48 | and | 50 | 3 | 7 | (Sequence Number) |
| | 54 | and | 56 | 0 | 4 | (Type of Punch Card) |
| | 60 | and | 62 | 0 | 8 | (Volume Number) |
| | 66 | and | 68 | 0 | 2 | (Segment Number) |

Your READING ASSIGNMENT for this Segment is pg: 221 - 223.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

In the following division, $645 \div 23 = 228\frac{1}{23}$, recognize which is the dividend, divisor, quotient, and remainder. Select the letter next to the correct statement.

| | dividend | divisor | quotient | remainder |
|---|---|---|---|---|
| (A) | 645 | 23 | 28 | $\frac{1}{23}$ |
| (B) | 23 | 645 | 28 | $\frac{1}{23}$ |
| (C) | 645 | 23 | 28 | 1 |
| (D) | 28 | 23 | 645 | 1 |

This choice is correct.

Now proceed to question 8 which follows:

Question 8

Apply the proper principle to find the quotient,

$$( 2x^3 - 3x^2 + 5x - 2 ) \div ( 2x - 1 ) \text{, and}$$

select the letter which labels the correct answer.

(A) $x^2 + 2$

(B) $x^2 - 2x + \frac{7}{2}$ remainder $-5\frac{1}{2}$

(C) $x^2 + x - 2$ and remainder $4$

(D) $x^2 - x + 2$

---



You did very well until the last step. Please note that in the final step of this problem, you have to subtract the 2 binomials,

$$\begin{array}{r} 15x + 8 \\ \underline{15x - 20} \end{array}$$

Examine your work and see if you can find the error you made.

Return to page $\frac{36}{2}$ and try this question again.

$\frac{32}{1}$

$a^2$ is followed by "a" , and that is descending order. However, 4 is of a lower order than "a" . It could be written $4a^0$ . At this point in the course, $a^0$ could be interpreted to mean " no a " ; therefore, the 4 is out of order.

Return to page $\frac{25}{2}$ and answer the question again.

---

$\frac{32}{2}$

It is true that $\frac{3(231x)}{3} = \frac{693x}{3}$ . However, you were asked to chose a <u>polynomial</u> of three terms divided by monomial. 3(231x) is a monomial.

Please return to page $\frac{26}{2}$ and answer the question again.

You made a mistake. It is always a good idea to check your answer. If your choice were correct, then $(x - 9)(x - 2)$ would be equal to the dividend, $x^2 - 7x + 10$. Multiply these two binomials and see that their product does not equal the dividend.

Return to page $\frac{37}{1}$ and try this question again.

---



We do not agree. Please note that the dividend has two missing terms. It should be re-written as

$x^3 + 0x^2 + 0x - 8$ since an $x^2$ term and an "x" term will appear in the division process.

Do this question over again; and when you have completed the division, check your answer by multiplying the quotient by the divisor. This product should equal the dividend.

Return to page $\frac{47}{2}$ and choose another letter.

$\frac{34}{1}$

This choice is correct.

Now proceed to <u>question 10</u> which follows:

<u>Question 10</u>

Apply the proper principle to find the quotient,

$$\frac{3x + 2x - 2}{2x - 1}$$

and select the letter which labels the correct answer.

(A) $\frac{3}{2} + x$ and remainder $x - \frac{1}{2}$

(B) $\frac{4x - 2}{-1}$

(C) $x + 2$

(D) $x + 1$ and remainder $-3$

---

$\frac{34}{2}$

You cancelled individual terms in the numerator with individual terms in the denominator. You can only cancel like <u>factors</u> in the numerator with like factors in the denominator. Thus, for example, in the quotient

$$\frac{x^4 + x^2 + 1}{x^2 + x + 1},$$

you cannot cancel the $x^2$ terms. Go back to the text assignment and study the procedure for dividing two polynomials.

Return to page $\frac{40}{2}$ and try this question again.

We do not agree. We are sure you noticed that there is a missing term in the dividend. Please re-write the dividend as

$$4x^2 + 0x - 1$$ since an $x$ term will appear in the division.

Also, make sure that in subtracting, you change the sign of the <u>lower</u> term and add.

Please return to page $\frac{53}{2}$ and try this question again.

---



We do not agree. It seems that the letter "a" is the cause of your difficulty. Well, it shouldn't be. Proceed as you would in any problem in division, as follows,

$$x - 1 \overline{\smash{)}\, x^2 + 2x - a}$$

Eventually, you will get a remainder expressed in terms of the letter "a". Since we are told that the remainder is zero, you can equate the expression for the remainder to zero. Solving the equation will lead us to the correct answer.

Please return to page $\frac{50}{2}$ and try this question again.

6

Very good. You made the correct choice. Here is one way the problem can be done.

$$
\begin{array}{r}
x^2 - x + 2 \\
2x - 1 \overline{\smash{)}\, 2x^3 - 3x^2 + 5x - 2} \\
\underline{2x^3 - x^2} \qquad\qquad \\
-2x^2 + 5x \qquad \\
\underline{-2x^2 + x} \qquad \\
4x - 2 \\
\underline{4x - 2}
\end{array}
$$

Please proceed to question 9 below.

---

Question 9

Apply the proper principle to perform the indicated operation:

$$( 6x^2 + 7x + 8 ) \div ( 3x - 4 ) \text{, and}$$

select the letter next to the correct answer.

(A) $2x + 5 + \frac{28}{3x}$ and remainder $\frac{112}{3x}$

(B) $2x + 5$ and remainder $28$

(C) $2x - 5$ and remainder $-12$

(D) $2x + 5$ and remainder $\frac{28}{3x-4}$

This choice is correct.

Now proceed to <u>question 6</u> which follows:

<u>Question 6</u>

Apply the proper principle to find the quotient,

$( x^2 - 7x + 11 ) \div ( x - 2 )$ and select the letter

which labels the correct answer.

(A) $x + 5$

(B) $x - 9$

(C) $x - 9$ and remainder 8

(D) $x - 5$

$\frac{38}{1}$

Before you start to divide two polynomials, you must first make sure that both the dividend and the divisor are written in descending order of the variable. In this question, the divisor should be written as ( ... 3 ). Furthermore, you made a mistake in subtraction of polynomials.

Let us illustrate with another example:

$$\begin{array}{r l} & \underline{\quad 5y \qquad\qquad\qquad} \\ y + 5 \,) & 5y^2 \;-\; 8y \;+\; 10 \\ & \underline{5y^2 \;+\; 25y} \end{array}$$

When you subtract $5y^2 + 25y$ from $5y^2 - 8y$, the difference is $-33y$

Please refresh your memory with regard to the rule for subtracting two polynomials.

| "Change the signs of the subtrahend and apply the rules for addition." |
|---|

Return to page $\frac{22}{2}$ and try this question again.

---

$\frac{38}{2}$

You did not make the right choice. If you examine the dividend, you will notice that two facts must be attended to.

First, the dividend is not written in descending order of the letter $x$.

Second, the dividend has missing terms. The $x^3$ and the $x$ terms are missing. Hence, before starting to divide, the dividend should be re-written as

$$x^4 + 0x^3 + x^2 + 0x + 1$$

Please continue from here.

Return to page $\frac{40}{2}$ and try this question again.

The choice you made indicates that you forgot to do something important. Before starting a problem involving the division of two polynomials, you should make sure that both the dividend and the divisor are written in descending order of the variable. Please examine the dividend and arrange it in descending order of " : " .

Return to page $\frac{34}{1}$ and try this question again.

---



You did not make the right choice. Suppose we do a similar problem:

$$(9y^2 - 1) \div (3y - 1)$$

First, we notice that there is a missing term in the dividend. There is no " y " term. Therefore, we write the dividend as follows:

$$9y^2 + 0y - 1 \qquad \text{We obtain}$$

$$\begin{array}{r l}
 & \underline{\;3y\qquad\qquad\qquad\;} \\
3y - 1 \,) & 9y^2 + 0y - 1 \\
 & \underline{9y^2 - 3y}
\end{array}$$

Now, you must be careful. In subtracting $-3y$ from $0y$ , the sign of the lower term changes, and we add $3y + 0y$.

We are sure that you can continue from here without our help.

Return to page $\frac{53}{2}$ and try this question again.

$\frac{40}{1}$

Very good. You made the correct choice. In this question, we first have to re-write the dividend to include the two missing terms. Thus, we write

$$x^3 - 8 = x^3 + 0x^2 + 0x - 8$$

$$
\begin{array}{rrrrrr}
 & x^2 & + 2x & + 4 & & \\
\cline{2-6}
x - 2 \,) & x^3 & + 0x^2 & + 0x & - 8 & \\
 & \underline{x^3} & \underline{- 2x^2} & & & \\
 & & 2x^2 & + 0x & - 8 & \\
 & & \underline{2x^2} & \underline{- 4x} & & \\
 & & & 4x & - 8 & \\
 & & & \underline{4x} & \underline{- 8} & \\
 & & & & 0 & \text{remainder}
\end{array}
$$

Please proceed to question 13 below.

---

$\frac{40}{2}$

Question 13

Apply the proper principle to find the quotient,

$( x^2 + x^4 + 1 ) - ( x^2 + x + 1 )$ , and select the letter which labels the correct statement.

(A) $x^3$

(B) $x^2 - x + 1$

(C) $x^2 + x - 1$ and remainder 1

(D) None of these.

Volume 8 Segment 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that may be furnished by you you are asked to punch out the following:

| | | | | | | |
|---|---|---|---|---|---|---|
| COLUMNS | 48 | and | 50 | 3 | 8 | (Sequence Number) |
| | 54 | and | 56 | 0 | 4 | (Type of Punch Card) |
| | 60 | and | 62 | 0 | 8 | (Volume Number) |
| | 66 | and | 68 | 0 | 1 | (Segment Number) |

SUPPLEMENTARY NOTES

What is factoring?

In the book, Alice in Wonderland, the Mad Hatter makes the point that it would be more fun to celebrate one's 364 unbirthdays rather than the single birthday. Don't you agree? In your study of mathematics, you have been asked to multiply and multiply. Now, for the first time, you will be asked to "unmultiply". Let us explain. If you are given the numbers, say, 3,5,7, and you are asked to find their product, you get $3 \times 5 \times 7 = 105$; this is multiplying. If on the other hand you are given the number 105, and you are asked to find the numbers 3,5,7 whose product will equal 105; this is unmultiplying. However, instead of the word unmultiplying, we use the word factoring. The numbers 3,5,7 are called the factors of 105.

Why do we factor?

The process of factoring comes under the heading of "tricks of the trade." Just as craftsmen, such as carpenters, electricians, or plumbers, have ways of doing things that make their job easier, so the mathematician uses factoring to solve certain problems with much less effort. Hence, if you want to become a good craftsman in algebra, you must learn how to factor.

Your READING ASSIGNMENT for this Segment is pg: 237 - 244
You will now be asked a series of questions to draw your attention to the more important points.

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Question 1

Perform the necessary operation to express each of the numbers 210, 252 as a product of numbers and select the letter which labels the correct statement.

(A) $210 = 7 \times 6 \times 2 \times 5$
$252 = 4 \times 7 \times 6 \times 2$

(B) $210 = 7 \times 5 \times 3 \times 2$
$252 = 21 \times 2 \times 3 \times 3$

(C) $210 = 7 \times 5 \times 3 \times 2$
$252 = 7 \times 3 \times 2 \times 3 \times 2$

(D) None of these.

$\frac{42}{1}$

You went too far. A good question to ask is: when is the operation of division completed? In other words, when do we stop dividing? The answer to this question is very important.

The operation of division is completed when the remainder is zero; or, when the remainder is of a degree less than that of the divisor.

Suppose we do a problem to illustrate the above statement.

The first remainder is $-4x + 3$. This remainder is of the same degree as the divisor. Hence, we have to continue. The second remainder is 2, which is of lower degree than the divisor. Therefore, we stop dividing.

$$\begin{array}{r|l} & 2x - 1 \\ \hline 4x - 1 & 8x^2 - 6x + 3 \\ & \underline{8x^2 - 2x} \\ & \quad - 4x + 3 \\ & \quad \underline{- 4x + 1} \\ & \qquad\quad 2 \quad \text{remainder} \end{array}$$

Please return to page $\frac{36}{2}$ and try this question again.

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$\frac{42}{2}$

You are not playing the game according to the rules. You are not allowed to cancel individual terms in the numerator of a fraction with individual terms in the denominator. Only equal factors can be cancelled. Thus, $\frac{2x^2yz}{2y}$ is a product of numbers divided by a product of numbers and the 2's and y's can be cancelled. However, in the quotient,

$$\frac{2x^2 + y - z}{2x^2 + y}$$, you <u>cannot</u> cancel any of the like terms in the numerator with the like terms in the denominator. Go back to your text assignment, and study the procedure for dividing two polynomials.

Return to page $\frac{34}{1}$ and try this question again.

You are correct.

A polynomial written in decreasing order of a variable means that the exponents of the variable get smaller as you go from left to right. All we are concerned with is the exponents, not the size of the coefficients. For example, the following polynomial is written in decreasing order of the variable r ;

$$3r^4 + r^3 - 50r^2 + 75r + 100$$

You should also note that a number without a letter is of lower order than a number with a letter. Thus, 100 is of lower order than 75r since 100 could be written as $100r^0$

At this point in the course, $r^0$ could be interpreted as "no r ".

Please proceed to question 4 below.

---



Question 4

Recognize the missing terms that must be inserted in the polynomial

$$5x^4 - x^2 + 1$$

to make it a complete polynomial of order 4 .

Select the letter which labels the correct answer.

(A) $x^3$ , $x$

(B) $x^5$ , $x^3$

(C) $x^3$

(D) $x$

$\frac{44}{1}$

It looks as if you forgot how to divide.

Perhaps you should review the textbook instruction on Page 2?2. In the long division method you have to subtract. Remember, when you subtract, you must change the signs of the subtrahend and then use the addition rules.

Return to page $\frac{37}{1}$ and try this question again.

---

$\frac{44}{2}$

You cancelled individual terms in the numerator with individual terms in the denominator. We cannot do this. You may ask, " why not? " Let us illustrate " why not " with an example.

Consider the quotient

$$\frac{4x^2 - 6}{2x - 3}$$

and let us find the value of this quotient when $x = 2$.

We get,

$$\frac{4(2^2) - 6}{2(2) - 3} = 10$$

Had we cancelled as you did, the quotient would have become $2x + 2$, which equals $\underline{6}$ when $x = 2$.

Please return to page $\frac{52}{2}$ and try this question again.

You probably made a mistake in your work, and your answer does not match any of the given choices. However, one of the given choices is a correct answer to this question. Let me help you get started. The dividend has two things wrong with it. It is not written in descending order of $x$, and there are missing terms. Re-write the dividend as

$$x^4 + 0x^3 + x^2 + 0x + 1$$

and do this problem over again.

Return to page $\frac{40}{2}$ and work out the problem again.

---



Although 48 does equal

$$4 \times 4 \times 3$$

these are not the prime factors of 48 ; the number 4 is not a prime number since it can be expressed as the product of two smaller numbers.

$$4 = 2 \times 2 .$$

4 is a composite number.

Return to page $\frac{78}{1}$ and try this question again.

$\frac{46}{1}$

Good! You did a strange problem correctly.

Check through this method:

$$
\begin{array}{r|l}
 & x + 3 \\
\cline{2-2}
x - 1 & x^2 + 2x - a \\
 & \underline{x^2 - x} \\
 & \qquad 3x - a \\
 & \qquad \underline{3x - 3} \\
 & \qquad\ 3 - a
\end{array}
$$

Now, we were told that the remainder is zero!

Therefore, $3 - a$ must equal zero.

That is, $3 - a = 0$

or, $3 = a$

Check this by multiplying $(x - 1)$ by $(x + 3)$

and see if the result is $x^2 + 2x - (3)$.

---

$\frac{46}{2}$

You have now completed this segment. Hand in your PUNCH CARD. Make sure that the following items are in your notebook.

(1) Definition of the words dividend, divisor, quotient, and remainder.

(2) The identity $\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$

(3) A question involving the quotient of two polynomials worked out in detail; for example,

$$(4x^2 + x - 5) \div (2x - 1)$$

You should now do question 5 - 8 of the homework assignment.

You added when you should have subtracted. Thus,

$$2x - 1 \overline{\big) \, 2x^2 + 3x - 2} \quad \text{quotient } x$$
$$\underline{2x^2 - x}$$

$(2x^2 - x)$ is subtracted from $2a^2 + 3x)$. The rule for subtracting 2 polynomials is to change the signs of the terms of the lower polynomial and add.

Return to page 34 and try this question again.

---



This choice is correct.

Now proceed to question 12 which follows.

Question 12

Apply the proper principle to find the quotient, $(x^3 - 8) - (x - 2)$, and select the letter which labels the correct answer.

(A) $x^2 - 4$

(B) $x^2 + 2x + 4$

(C) $x^2 + 4x + 4$

(D) $x^2$ and remainder $\frac{2x^2 - 8}{x - 2}$.

You chose the correct answer. The procedure for dividing two polynomials is carefully explained in the text assignment, Page 222. However, since the division of polynomials is a process consisting of many steps, let's go over this procedure very carefully, one step at a time and do this problem together.

$$( 2x^2 + 5x - 3 ) - ( x + 3 )$$

Steps

(1) We re-write the quotient as a problem in long division.

$$x + 3 \,\overline{)\, 2x^2 + 5x - 3}$$

(2) We find the quotient of the first term of the dividend and the first term of the divisor, $\frac{2x}{x} = 2x$. This becomes the first term of the quotient and is written over the division sign.

$$\begin{array}{r} 2x \phantom{+ 5x - 3} \\ x + 3 \,\overline{)\, 2x^2 + 5x - 3} \end{array}$$

(3) Next, we multiply $2x(x + 3) = 2x + 6x$, and write this under the first two terms of the dividend. Are you still with us?

$$\begin{array}{r} 2x \phantom{+ 5x - 3} \\ x + 3 \,\overline{)\, 2x^2 + 5x - 3} \\ \underline{2x^2 + 6x} \phantom{- 3} \end{array}$$

(4) We now subtract, getting $-x$ $-3$.

$$\begin{array}{r} 2x \phantom{+ 5x - 3} \\ x + 3 \,\overline{)\, 2x^2 + 5x - 3} \\ \underline{2x^2 + 6x} \phantom{- 3} \\ - x - 3 \end{array}$$

(5) As before, we consider the quotient $\frac{-x}{x}$ which equals $-1$ and write this number above the division sign.

$$\begin{array}{r} 2x - 1 \phantom{- 3} \\ x + 3 \,\overline{)\, 2x^2 + 5x - 3} \\ \underline{2x^2 + 6x} \phantom{- 3} \\ - x - 3 \end{array}$$

(6) We multiply $( -1 ) ( x + 3 )$ and subtract again. As you can see, there is no remainder; hence,

$2x - 1$ is the answer.

$$\begin{array}{r} 2x - 1 \phantom{- 3} \\ x + 3 \,\overline{)\, 2x^2 + 5x - 3} \\ \underline{2x^2 + 6x} \phantom{- 3} \\ - x - 3 \\ \underline{- x - 3} \\ 0 \text{ remainder} \end{array}$$

(7) Now, to make sure, check the division by multiplying $( 2x - 1 ) ( x + 3 )$. This product should equal the dividend $2x^2 + 5x - 3$.

$$\begin{array}{rrr} & x & + 3 \\ & \underline{2x} & \underline{- 1} \\ 2x^2 & + 6x & \\ & \underline{- x} & \underline{- 3} \\ 2x^2 & + 5x & - 3 \end{array}$$

Copy this page into your notebook.

Proceed to question 7 on page $\frac{22}{2}$

We do not agree. If we subtract (-2x - 6) from (-2x - 6), the difference is zero. The rule for subtracting two polynomials is to change the signs of the terms of the subtrahend and apply the rules for addition.

There is one other item that needs clearing up. You confused the remainder with the quotient of the remainder and the divisor. It will help you to remember this formula:

$$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = \text{QUOTIENT} = \frac{\text{REMAINDER}}{\text{DIVISOR}} .$$

Note that the word remainder appears only in the numerator.

Please return to page $\frac{22}{2}$ and try this question again.

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$\frac{49}{2}$

The quotient is right; the remainder is wrong. Recall the formula.

$$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = \text{QUOTIENT} = \frac{\text{REMAINDER}}{\text{DIVISOR}}$$

Note that the remainder is <u>not</u> a quotient.

Please return to page $\frac{36}{2}$ and try this question again.

$\frac{50}{1}$

Very good. Check this method against your work.

$$
\begin{array}{r}
x^2 - x + 1 \phantom{+ 0x + 1} \\
x^2 + x + 1 \overline{\smash{)}\, x^4 + 0x^3 + x^2 + 0x + 1} \\
\underline{x^4 + x^3 + x^2 \phantom{+ 0x + 1}} \\
- x^3 + 0x^2 + 0x + 1 \\
\underline{- x^3 - x^2 - x \phantom{+ 1}} \\
x^2 + x + 1 \\
\underline{x^2 + x + 1}
\end{array}
$$

Proceed to question 14 below.

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$\frac{50}{2}$

Question 14

You are told that when $x^2 + 2x - a$ is divided by $x - 1$, there is no remainder. Do the division and determine a value of the letter a. Select the correct answer.

(A) $a = 0$

(B) $a = 3$

(C) $a = 1$

(D) Cannot be determined.

We don't agree. Let me help you get started. First consider the number 238 and express it as the product of prime numbers. Do the same to the number 255. Examine the factors of each of these numbers and select the largest factor that they have in common.

Please return to page $\frac{59}{2}$ and try this question again.

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You made one error. If you examine the middle term of the given polynomial, you will notice that the letter "c" is raised to the first power. The greatest common factor cannot have the letter "c" raised to a larger power.

Please return to page $\frac{58}{2}$ and try this question again.

$\frac{52}{1}$

We do not agree. In order to find the factors of a number, say 252, we first observe that this number is even. Hence, 2 is a factor and $\frac{252}{2} = 126$. Now, 126 is still even, and we can divide by 2 again $\frac{126}{2} = 63$.

Clearly, 63 can be divided by 3 and the quotient is 21.

We are sure that you can continue from here. You should note that with each division the number gets smaller, and it is easier to discover factors of smaller numbers.

Please return to page $\frac{41}{2}$ and try this question again.

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$\frac{52}{2}$

We do not agree. If you raise 2x to the $3^{rd}$ power, both 2 and x are each raised to this power.

Thus, $(2x)^3 = 2^3 \cdot x^3 = 8x^3$.

Clearly, 8 does not divide 12 exactly.

Please return to page $\frac{61}{2}$ and try this question again.

Very good. You made the correct choice.

Let us do this problem together, step by step.

$$2x - 1 \overline{\smash{)}\,3x + 2x^2 - 2}$$

rearrange in descending order

$$2x - 1 \overline{\smash{)}\,2x^2 + 3x - 2}$$

$$\frac{2x^2}{2x} = x$$

$$\begin{array}{r}
x \phantom{+ 3x - 2} \\
2x - 1 \overline{\smash{)}\,2x^2 + 3x - 2}
\end{array}$$

multiply divisor $(2x - 1)$ by quotient $(x)$

$$\begin{array}{r}
x \phantom{+ 3x - 2} \\
2x - 1 \overline{\smash{)}\,2x^2 + 3x - 2} \\
\underline{2x^2 - x \phantom{- 2}}
\end{array}$$

subtract

$$\begin{array}{r}
x \phantom{+ 3x - 2} \\
2x - 1 \overline{\smash{)}\,2x^2 + 3x - 2} \\
\underline{2x^2 - x \phantom{- 2}} \\
4x - 2
\end{array}$$

$$\frac{4x}{2x} = 2$$

$$\begin{array}{r}
x + 2 \phantom{x - 2} \\
2x - 1 \overline{\smash{)}\,2x^2 + 3x - 2} \\
\underline{2x^2 - x \phantom{- 2}} \\
+ 4x - 2
\end{array}$$

multiply divisor $(2x - 1)$ by $2$

$$\begin{array}{r}
\underline{4x - 2} \\
0
\end{array}$$

subtract;
remainder $= 0$

Proof: $(2x - 1)(x + 2) = 2x^2 + 3x - 2$

Please go on to question 11 below.

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Question 11

Apply the proper principle to find the quotient,

$(4x^2 - 1) - (2x - 1)$, and select the letter

which labels the correct answer.

(A) $2x - 1$ and remainder $- 2$

(B) $2x - 1$

(C) $2x$

(D) $2x + 1$

$\frac{54}{1}$

You did not make the correct choice. In order to find the factors of a number, we start with dividing this number by the smallest divisor first. Thus, we first try dividing by 2:

$$\frac{246}{2} = 123.$$

Then we try dividing by 3 and so on.

We are sure that you can continue from here by yourself.

Return to page $\frac{66}{2}$ and try this question again.

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$\frac{54}{2}$

This choice is correct.

Now proceed to question 8 which follows.

Question 8

Apply the proper principle to find the greatest common monomial factor of the monomials: s: $7r^2 s^2 t^2$ and $-21r^3 st^2$ . Then select the letter which labels the correct answer.

(A) $-3rst$

(B) $7r^2st^2$

(C) $-7r^2st^2$

(D) $3r^2st^2$

You made the wrong choice.

You have to consider only the numbers 6[illegible] [illegible] 48 . Express each of these numbers as a produ[illegible] [illegible]s. [illegible] [illegible]ving done this, examine the factors to see what numbers they have in common.

Please return to page $\frac{62}{2}$ and try this question again.

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You did not make the right choice. In order for $c^3$ to be a factor each of the terms of the given polynomial must have the letter "c" raised to the third power or better.

Since this is not the case, please reconsider your choice.

Return to page $\frac{58}{2}$ and try this question again.

$\frac{56}{1}$

Although 84 does equal 21 x 2 x 2 , this is not the correct answer; the question called for prime factors.

The number 21 is not prime, since it can be written as a product of 2 other numbers.

Thus, $21 = 7 \times 3$ .

A prime number can be written only as a product of 1 and itself.

For example, $19 = 1(19)$.

Please return to page $\frac{8}{1}$ and try this question again.

---

$\frac{56}{2}$

If you raise 3x to the third power, both 3 and x are each raised to this power.

Thus, $(3x)^3 = 3^3 \cdot x^3 = 27x^3$ .

We are sure that you can see that the number 27 is too large to be a factor of 12 .

Please return to page $\frac{61}{2}$ and try this question again.

We do not agree.

It is true that 5 is a factor of 45 and 15 . However, in trying to find the greatest common monomial factor of two monomials, first determine the largest common factor of the numerical coefficients. Examine the numbers 45 and 15 more carefully and decide what their largest common factor is.

Please return to page $\frac{60}{2}$ and try the question again.

---



We do not agree. To check whether you factored correctly, the product of the two factors should equal the original polynomial. Thus,

$$7(2x - 3y) = 14x - 21y,$$

and this is not the original polynomial. You are missing the number 7 . Now, ask yourself, "what number times 7 equals 7 ." Certainly $7 \times 0$ does not equal 7 .

Return to page $\frac{69}{2}$ and try this question again.

$\frac{58}{1}$

Very good. You made the correct choice. Here is how this question can be answered. We first consider the numerical coefficients, and we find that 7 is the largest common factor of 7 and 21 . Then we see that $r^2$ is the largest power of $r$ that is common to both monomials. Similarly, the largest power of $s$ is $s$ , and the largest power of $t$ is $t^2$ . Hence the answer is $7r^2 st^2$ .

Please go on to question 9 below.

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$\frac{58}{2}$

<u>Question 9</u>

Apply the proper principle to find the greatest common monomial factor of the polynomial, $5c^3 d - 10cd^2 + 15c^2 d^2$ .

Then, select the letter which labels the correct answer.

(A) $5cd$

(B) $5c^2d$

(C) $5c^3d^2$

(D) None of these.

Very good. You made the correct choice. Using the algorithm for finding the prime factors of a number,

| | | |
|---|---|---|
| 2 | 246 | We first divide the given number by 2 $\frac{246}{2} = 123$ |
| 3 | 123 | Now we divide the quotient by 3 $\frac{123}{3} = 41$ |
| | 41 | |

Clearly, 41 is a prime number

Please go on to question 5 below.

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Question 5

Perform the necessary computation and select the letter which correctly completes the statement: The largest common factor of 238 and 255 is:

(A) 3

(B) 7

(C) greater than 13, but less than 24.

(D) greater than 24, but less than 51.

$\frac{60}{1}$

Very good. You made the correct choice. Here is how this question can be answered. We express each of the numbers 60 , 48 as a product of primes; thus,

$$60 = \overline{2 \times 2 \times 3} \times 5$$

$$48 = 2 \times 2 \times 2 \, \overline{\times 2 \times 3}$$

Now we match like factors of the first number with the second number as indicated by the lines over the numbers. We see that $2 \times 2 \times 3 = 12$ match. Hence, the largest numerical coefficient that is a factor of both binomials is 12 .

Please go on to question 7 below.

---

$\frac{60}{2}$

Question 7

Apply the proper principle to find the greatest common monomial factor of the monomials, $45x^3y^2$ and $15x^2y^5$ . Then, select the letter which labels the correct answer.

(A) $5x^2y^2$

(B) $9x^2y^2$

(C) $15x^3y^5$

(D) $15x^2y^2$

Although 150 does equal 25 x 3 x 2 , this is not the correct answer. The question called for prime factors.

The number 25 is not a prime number since it can be expressed as the product of two smal numbers. Thus, 25 = 5 x 5 . Any number that can be expressed as a product of other numbers is called composite. For example, 66 is a composite number, since

$$66 = 2 \times 3 \times 11$$

Please return to page $\frac{78}{1}$ and try this question again.

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This choice is correct.

Now proceed to question 10 which follows.

Question 10

Apply the proper principle to find the highest power of the first monomial that is a factor of the second monomial.

$$2x, \quad 12x^3y^2 ,$$

and select the letter which labels the correct answer.

(A) $( 2x )^3$

(B) $( 2x )^2$

(C) $((3x )^3$

(D) None of these.

$\frac{62}{1}$

Very good. You made the correct choice. Let us do this question together. First, we take 238 and write it as a product of primes: thus,

$$238 = 2 \times 7 \times 17$$

Now, we work with 255. We start by dividing this number by 3 . We find

$$255 = 3 \times 5 \times 17$$

and the largest common factor of two given numbers is, therefore, 17 . 17 is greater than 13 and less than 24 .

Please go on to question 6 below.

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$\frac{62}{2}$

Question 6

Perform the necessary computation and select the letter which correctly completes the statement: The largest number that is a factor of the two monomials

$$60y^3z \text{ and } 48y^2z^2 \text{ is:}$$

(A) 6

(B) 12

(C) 16

(D) 30

Very good. You made the right choice. Here is how we proceed to express a number as a product of other numbers.

We start with the smallest possible division of the number.

A convenient form ( or algorithm ) is illustrated here:

a) $\begin{array}{r|l} 2 & \underline{210} \\ & \ 105 \end{array}$ Thus, $\frac{210}{2} = 105$

b) $\begin{array}{r|l} 2 & \underline{210} \\ 5 & \underline{105} \\ & \ \ 21 \end{array}$ Now, a number ending in 5 is divisible by 5, $\frac{105}{5} = 21$

c) $\begin{array}{r|l} 2 & \underline{210} \\ 5 & \underline{105} \\ 3 & \ \ \underline{21} \\ & \ \ \ \ \underline{7} \end{array}$ and $\frac{21}{3} = 7$

d) Hence, $210 = 7 \times 5 \times 3 \times 2$

and the factors of 252 can be found a similar way.

$$\begin{array}{r|l} 2 & \underline{252} \\ 2 & \underline{126} \\ 3 & \ \ \underline{63} \\ 3 & \ \ \underline{21} \\ & \ \ \ \ 7 \end{array}$$

$252 = 7 \times 3 \times 3 \times 2 \times 2$

Please go on to question 2 below.

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Question 2

Recognize which of the following represent a factoring of the number 90 over the set of positive integers and select the letter which labels the correct choice.

(A) $9 \times 5 \times 4 \times \frac{1}{2}$

(B) $9 \times 5 \times 2$

(C) $( -1 ) ( -2 ) ( 5 ) ( 9 )$

(D) $( 18 ) ( 5 ) ( \frac{1}{9} ) ( 9 )$

$\frac{64}{1}$

We do not agree. One of the choices contains the correct answer.

Please return to page $\frac{58}{2}$ and try this question again.

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$\frac{64}{2}$

You did not make the correct choice. One of the letters has the correct answer next to it. It may be helpful if you recall the rule:

$$(ab)^n = a^n b^n .$$

For example,

$$(5y)^2 = 5^2 \cdot y^2 = 25y^2$$

Please return to page $\frac{61}{2}$ and try this question again.

A good way to proceed in answering this question is to start from the left and work your way to the right. First find the largest common factor of 7 and 21 . Then consider the letter "r" and find the largest power of this letter that is common to both monomials. Proceed and do the same for the letters "s" and "t" .

Please return to page $\frac{54}{2}$ and try this question again.

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If you multiply the two factors that you chose;

namely, 3ab and ab - 3b,

you will find that this product does not equal the original polynomial. Hence, you did not make the right choice.

The following illustration should be helpful.

Write $x^3 + x^2 + x$

in factored form.

Since x divides each of the above terms, x is a factor; we can write,

$x ( - - - ).$

To fill in the dashes you should ask yourself,

"what must multiply x to give me $x^3$ ?"

The answer is $x \; x^2 = x^3$ .

Next you ask yourself,

"what must multiply x to give me $x^2$ ?"

The answer is $x \; x = x^2$ .

Finally,

what must multiply x to give x ?

The answer is $x \; 1 = x$ .

Hence, the answer is

$$x ( x^2 + x + 1 )$$

Please return to page $\frac{79}{2}$ and try this question again.

$\frac{66}{1}$

Your choice,

$$231 = 3 \times 11 \times 7$$ is the correct answer;

the question specified prime factors.

A prime number is a number whose only whole number divisors are 1 and the number itself. There is one exception; the number 1 is not considered a prime.

For future reference, we will write the first 10 prime numbers,

2, 3, 5, 7, 11, 13, 17, 19, 23, 29 .

Note that each of these numbers has no exact divisors except 1 and the number itself. You should write these in your notebook.

Go on to question 4 below.

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$\frac{66}{2}$

Question 4

Perform the necessary computation and select the letter which completes the following statement correctly.

The largest prime factor of 246 is:

(A) 3

(B) 13

(C) greater than 13 but less than 32.

(D) greater than 32 but less than 42.

To check a factorization you just have to multiply the two factors and see whether their product is the original polynomial.

Thus $7(2x - 3y + 1) = 14x - 21y + 7$.

Therefore, you are correct.

Proceed to question 12 below.

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Question 12

Apply the proper principle and write in factored form,

$$5p^2q - 10pq^2.$$

Select the letter which labels the correct answer.

(A) $5pq(p - 2q)$

(B) $5pq(p - 2)$

(C) $5p(pq - 2q^2)$

(D) $5q(p^2 - 2pq)$

$\frac{68}{1}$

The number 9 is an exact divisor of 45, but it does not divide 15 exactly. You are looking for the largest number that will divide both 45 and 15 exactly.

Please return to page $\frac{60}{2}$ and try this question again.

---

$\frac{68}{2}$

You misunderstood. Writing an expression in factored form means writing it as a product of simpler expressions. You cleared parenthesis and combined like terms; that is not factoring. Examine the question to see if there is an expression that is common to both terms. This expression is one of the factors. Can you find the other?

Please return to page $\frac{81}{2}$ and try this question again.

Since

$$( 2x )^2 = 4x^2$$

we must first check to see if $4x^2$ is a factor of $12x^3 y^2$ ; that is, does $4x^2$ divide into $12x^3 y^2$ evenly.

i.e. $4x^2 \overline{) 12x^3 y^2}$ with quotient $3x\ y^2$ , so it is a factor.

Then we check to see if $2x$ is the highest power that divides $12x^3 y^2$ evenly. We try $( 2x )^3$ which equals $8x^3$ . But $8x^3$ doesn't divide into $12x^3 y^2$ .

Therefore, your choice, $( 2x )^2$ is correct.

Proceed to question 11 below.

---



Question 11

Apply the proper principle and write the polynomial

$$14x - 21y + 7$$

as the product of a numerical coefficient and a polynomial.

Select the letter which labels the correct statement.

(A) $7 ( 2x - 3y )$

(B) $7x ( 2 - 3y )$

(C) $7 ( 2x - 3y + 1 )$

(D) None of these.

$\frac{70}{1}$

The product $3ab\ (a - 5b) = 3a^2b - 15ab^2$ . This product should equal the original polynomial. Since this is not the case, please reconsider your choice.

Return to page $\frac{79}{2}$ and try this question again.

---

$\frac{79}{2}$

You must arrange your work in an organized fashion. Start with the situation: The shaded area is the area of the rectangle minus the area of the semi-circle. Now, here are some of the questions you must answer:

What is the formula for the area of a rectangle? $A = l \cdot w$

What is the formula for the area of a circle? $A = \pi r^2$

What is the formula for the area of a semi-circle?

Return to page $\frac{77}{2}$ and try this question again.

If you examine the first two terms of the question; namely,

$y^2 - y$ you will notice that they can

be written in factored form as

$$y(y - 1)$$

If you examine the remaining two terms,

$yz - z$ you will notice that they can

also be written in factored form as

$$z(y - 1)$$

Please continue from here.

Return to page $\frac{86}{2}$ and try this question again.

---



You did not answer the question. The slide rule is a very fine instrument, you were asked to simplify the computation by using factoring.

Please return to page $\frac{84}{2}$ and try this question again.

To remove the highest common monomial factor, it is necessary to examine the numerical coefficients first, and find the largest number which will divide both these coefficients exactly. This part of the question you did correctly. Then, the letters should be examined and the highest power of each letter that is common to all terms should be removed. For example, in the polynomial

$$x^3 y^5 + x^4 y^4 + x^2 y^7$$

the highest power of $x$ that can be removed is $x^2$, because that is the highest power of $x$ that is common to all the terms of the polynomial. Similarly, the highest power of $y$ that can be removed is $y^4$.

Please return to page $\frac{67}{2}$ and try this question again.

---



The square of a number means the number multiplied by itself. For example, the square of 6 is

$$6 \times 6 = 36$$

The square of 9 is

$$9 \times 9 = 81$$

Please do not confuse the square of a number with multiplying a number by 2.

Return to page $\frac{83}{1}$ and choose another letter.

If your choice were correct, then

$$7x\ (\ 2\ -\ 3y\ )$$ would equal the original polynomial.

Since this is not the case, please reconsider your choice.

Return to page $\frac{69}{2}$ and try this question again.

---



We do not agree. Your error may be due to careless reading of the question. Note that we have

$$x\ (\ x\ -\ 2\ )\ +\ 3\ (\ x\ -\ 2\ )$$

This is in the form

$$xa\ +\ 3a$$

which equals

$$a\ (\ x\ +\ 3\ )$$

Please return to page $\frac{81}{2}$ and try this question again.

$\frac{74}{1}$

You did not make the right choice. Factoring a number over the set of positive integers means that only positive integers can be used as factors. This eliminates factors that are fractions and negative numbers. However, the factors do not have to be prime numbers.

Please return to page $\frac{63}{2}$ and try this question again.

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$\frac{74}{2}$

You did not make the correct choice. You are looking for the highest powers of both $x$ and $y$, which are common to both the given monomial. Consider, for example, the two monomials, $a^5 b^3$ and $a^2 b^4$. The largest power of $a$ that is common to both monomials is $a^2$. A larger power of $a$, say $a^3$, is a factor of the first monomial; but not of the second. Similarly, the highest power of $b$ that is a factor of both monomials, is $b^3$.

Return to page $\frac{60}{2}$ and try this question again.

Very good. You made the right choice. Suppose we do this question together.

The area of the shaded part is obtained by subtracting the area of the semi-circle from the area of the rectangle.

Area of shaded part =

| | | |
|---|---|---|
| = Area of Rectangle | − Area of Semi-Circle | Situation |
| = length x width | $- \frac{1}{2} \cdot \pi \cdot (\text{radius})^2$ | Formula |
| $= 2r \cdot r$ | $- \frac{1}{2} \pi r^2$ | Substitution |
| $= 2r^2$ | $- \frac{1}{2} \pi r^2$ | Simplify |
| $= r^2 \left(2 - \frac{1}{2}\pi\right)$ | | Factor |

Please go on to question 17 below.

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Question 17

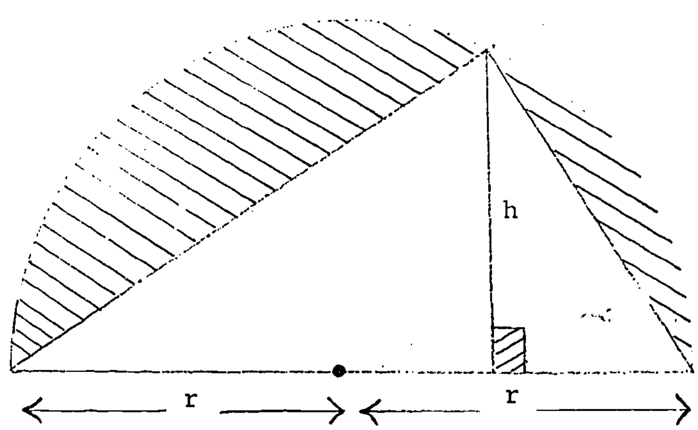
In the figure below, a triangle is inscribed in a semi-circle of radius r. Apply the proper principle to express the area of the shaded part in factored form. Select the letter which labels the correct answer.

(A) $r \left( \frac{\pi r}{2} - h \right)$

(B) $r \left( \pi r - h \right)$

(C) $r \left( \frac{\pi}{2} - 2h \right)$

(D) $r \left( \pi - h \right)$



$\frac{76}{1}$

Writing the polynomial

$$3a^2b - 15ab^2 + 3ab$$ in factored form means that

the largest common factor should be removed. Now, 3a does not fill this requirement. If you look at the trinomial of your answer, you will see that there is another letter that is common to all the three terms.

Please return to page $\frac{79}{2}$ and try this question again.

---

$\frac{76}{2}$

We do not agree.

Instead of multiplying large numbers like 98 , 54 , and then dividing, it is much easier to perform mathematical operations using smaller numbers. Furthermore, if you write the numerator and denominator as products of smaller numbers, many numbers may cancel, thus making your job much easier.

Please return to page $\frac{84}{2}$ and try this question again.

Very good. You made the right choice. This is a two-step problem. Here is how this question can be answered. We write the first two terms as

$$y ( y - 1 )$$

and the second two terms as

$$z ( y - 1 )$$

Therefore, $y^2 - y + yz - z = y ( y - 1 ) + z ( y - 1 )$.

Now,

$( y - 1 )$ is a common factor and can be removed. Hence, we get

$$( y - 1 ) ( y + z ).$$

Please go on to question 16 below.

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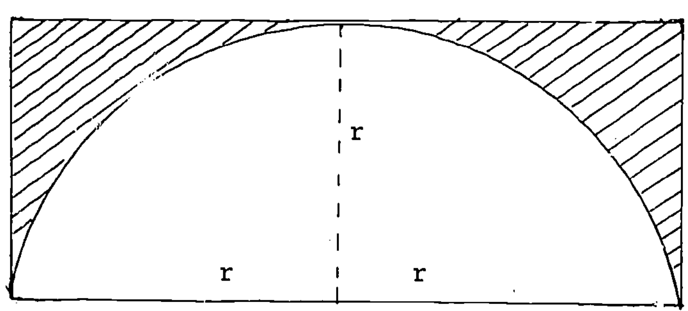
Question 16 77/2

In the figure below, a rectangle is circumscribed about a semi-circle of radius r. Apply the proper principle to express the area of the shaded part in factored form and select the letter next to the correct answer.




(A) $r^2 ( \frac{\pi}{2} - 2 )$

(B) $r^2 ( \pi - 2 )$

(C) $r^2 ( 2 - \frac{\pi}{2} )$

(D) $r^2 ( 3 - \pi )$

$\frac{78}{1}$

This choice is correct.

Now proceed to Question 3 which follows:

Question 3

Recognize which of the following represent the number on the left side expressed as a product of prime factors, and select the letter next to the correct answer.

| | | | |
|---|---|---|---|
| (A) | 48 | = | 4 x 4 x 3 |
| (B) | 84 | = | 21 x 2 x 2 |
| (C) | 150 | = | 25 x 3 x 2 |
| (D) | 231 | = | 3 x 11 x 7 |

---

$\frac{78}{2}$

The correct answer is written next to one of the letters. Examine each of the coefficients 14 , 21 , 7 and choose the largest number that divides each of these 3 numbers exactly.

Please continue from here.

Return to page $\frac{69}{2}$ and try this question again.

We can always check a factorization by multiplying the factors.

$$5pq\ (p - 2q) = 5p^2q - 10pq^2$$

It is always advisable to check the factor chosen to see whether any other common factor can be removed.

Your answer is correct.

Proceed to question 13 below.

---

$\frac{79}{2}$

Question 13

Apply the proper principle and write the following polynomial in factored form:

$$3a^2b - 15ab^2 + 3ab$$

Select the letter which labels the correct answer.

(A) $3ab\ (ab - 3b)$

(B) $3ab\ (a - 5b)$

(C) $3a\ (ab - 5b^2 + b)$

(D) $3ab\ (a - 5b + 1)$

The area of the two shaded segments can be found by subtracting the area of the triangle from the area of the semi-circle.

The formula for the area of a triangle is $\frac{1}{2}$ b h

The formula for the area of a circle is $\pi r^2$

Please return to page $\frac{75}{2}$ and try this question again.

---



Not bad, you have the right idea.

However, you did not go far enough. By far enough, we mean that you should express each number as a product of prime numbers.

Please return to page $\frac{84}{2}$ and try this question again.

This choice is correct.

Now proceed to questic 14 which follows:

Question 14

Apply the proper principle and write in this expression in factored form:

$$x(x - 2) + 3(x - 2)$$

Select the letter which labels the correct answer.

(A) $x^2 + x - 6$

(B) $(x + 3)(2x - 4)$

(C) $4x(x - 2)$

(D) $(x - 2)(x + 3)$

---



We do not agree. Notice that the two binomials whose product we seek are exactly alike with the exception that the first binomial has a plus sign between its terms, while the second binomial has a minus sign. The product of such binomials can be written down immediately. It is the square of the first term minus the square of the second term.

Let us illustrate:

$$(3a + b)(3a - b) = 9a^2 - b^2$$

Please return to page $\frac{108}{1}$ and try this question again.

$\frac{82}{1}$

You are right as far as you went.

$y^2 - y + yz - z$ does equal $y(y - 1) + z(y - 1)$

But this result can be factored further. Since the

$(y - 1)$ is common to both terms;

it, too, can be factored,

$$(y - 1)(y + z)$$

Return to page $\frac{86}{2}$ and try this question again.

---

$\frac{82}{2}$

We do not agree. First, let us make sure that we have the same definition of <u>the square root of a number</u>. If a number N is written as the product of two equal numbers, than each of the equal numbers is called the square root of the number N.

Thus; for example,

(1) if $81 = 9 \times 9$, then 9 is the square root of 81

(2) if $49a^2 = (7a)(7a)$, then $7a$ is the square root of $49a^2$

You can always check whether you have the correct square root by multiplying the square root by itself.

For example, if $\frac{1}{8}$ is the square root of $\frac{1}{16}$, then $\frac{1}{8}$ times $\frac{1}{8}$ should equal $\frac{1}{16}$ which is not true.

Please return to page $\frac{94}{2}$ and try this question again.

Volume 8 Segment 4 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

| | | | | | | |
|---|---|---|---|---|---|---|
| COLUMNS | 48 | and | 50 | 3 | 9 | (Sequence Number) |
| | 54 | and | 56 | 0 | 4 | (Type of Punch Card) |
| | 60 | and | 62 | 0 | 8 | (Volume Number) |
| | 66 | and | 68 | 0 | 4 | (Segment Number) |

Your READING ASSIGNMENT for this Segment is pg: 245 - 248
You will now be asked a series of questions to draw your attention to the more important points.

In your reading you learned about multiplying two binomials of the form

$( a + b )$ and $( a - b )$ , and the reverse operation; the factoring

of their product $a^2 - b^2$.

You will now be asked several questions to call your attention to the more important points.

Question 1
Perform the necessary operation to find the squares of the following numbers; 7 , 11 , 13. Select the letter indicating the correct set of squares.

(A) $\{14 , 22 , 26\}$ (C) $\{49 , 121 , 169\}$

(B) $\{49 , 121 , 143\}$ (D) $\{49 , 111 , 169\}$

---



Suppose we try a similar problem:

find the product of $( 2ab - 1 ) ( 2ab + 1 )$.

The only difference between the two binomials, is that one of them has a minus sign between its terms, while the other has a plus sign between its terms. The product of two such conjugate binomials is the square of the first term minus the square of the second term. Thus,

$$( 2ab )^2 - 1^2 = 4a^2 b^2 - 1$$

is the answer. This rule is a great time saver; make a note of it in your notebook. You may write

$$\boxed{( x + y ) ( x - y ) = x^2 - y^2}$$

Return to page $\frac{105}{1}$ and try this question again.

$\frac{84}{1}$

Very good. You made the right choice.

Let us do the problem together.

The area of the shaded part

$$= \text{the area of the semi-circle} - \text{the area of the triangle}$$

$$= \frac{1}{2}\pi \text{ times the radius squared} - \frac{1}{2} \text{ base times height}$$

$$= \frac{1}{2}\pi r^2 - \frac{1}{2} b\ h$$

$$= \frac{1}{2}\pi r^2 - \frac{1}{2}(2r)h$$

$$= r\left(\frac{\pi r}{2} - h\right)$$

Please go on to question 18 below.

---

$\frac{84}{2}$

Question 18

Propose how you could simplify the computation of

$\frac{54 \times 98}{42 \times 63}$ by using factoring and find the value of the

above expression without performing the actual calculation. Select the letter which best describes the procedure you used.

(A) The best procedure is to use a slide rule.

(B) The best procedure is to multiply and divide as indicated.

(C) $\frac{54 \times 98}{42 \times 63} = \frac{9 \times 6 \times 98}{7 \times 6 \times 9 \times 7} = \frac{98}{49} = 2$

(D) $\frac{54 \times 98}{42 \times 63} = \frac{2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 2}{7 \cdot 3 \cdot 2 \cdot 7 \cdot 3 \cdot 3} = 2$

Very good. You made the right choice. The square of the first monomial is:

$$(8r^2t)(8r^2t) = 8 \cdot 8\, r^2 \cdot r^2 \cdot t \cdot t = 64r^4t^2$$

Similarly, the square of the second monomial is:

$$(-3uv^2)(-3uv^2) = (-3)(-3)\, u \cdot u \cdot v^2 \cdot v^2 = 9u^2v^4$$

Note that the product of two negative quantities is a positive quantity.

Please go on to question 3 below.

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<u>Question 3</u>

Recognize that each of the numbers of the product (51)(49) can be written as binomials.

Select the letter which labels the correct answer that has the same two terms and differs only in the sign between the terms.

e.g. $(a + b)(a - b)$

(A) $(51 + 0)(49 - 0)$

(B) $(50 + 1)(50 - 1)$

(C) $(49 + 2)(50 - 1)$

(D) None of these.

$\frac{86}{1}$

A binomial can also be the common factor.

$x ( x - 2 ) + 3 ( x - 2 )$ has $( x - 2 )$ as the common factor.

Just as in

$$x^{a + 3a} = a ( x + 3 )$$ this common factor can be removed and placed as a multiplier of the remaining polynomial. Therefore, the correct answer is

$$( x - 2 ) ( x + 3 )$$

Please go on to question 15 below.

---

$\frac{86}{2}$

<u>Question 15</u>

Apply the proper principle and write the following expression in completely factored form.

$$y^2 - y + yz - z$$

Select the letter which labels the correct answer.

(A) $yz ( y - 1 )$

(B) $y ( y - 1 ) + z ( y - 1 )$

(C) $( y - 1 ) ( y + z )$

(D) $y ( y + z ) - ( y + z )$

Try multiplying 13 x 13 over again. We don't agree with the result you obtained.

Return to page $\frac{83}{1}$ and choose another letter.

---



You did not make the correct choice. You neglected an old rule.

> When multiplying two quantities having the same base, keep the base and add the exponents.

For example,

$$p \cdot p = p^{1+1} = p^2$$

Return to page $\frac{92}{2}$ and try the question again.