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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 3: solving problems with open sentences; axioms of equality; closure properties; commutative, associative, and distributive properties; and addition-subtraction and division-multiplication properties of equality. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

VOLUME 3

NEW YORK INSTITUTE OF TECHNOLOGY

OLD WESTBURY, NEW YORK

PROGRAMMED MATH CONTINUUM

LEVEL ONE

ALGEBRA

VOLUME 3

New York Institute of Technology

Old Westbury - New York

A

PREFACE

This volume is one of a set of 18
that form a complete course
in
ALGEBRA - LEVEL ONE

The volume has been structured
in a multiple choice question-answer format,
with the pagination scrambled
and
is to be used in conjunction with
a program control console
utilizing
punch card input.

It is one exhibit in the demonstration of a model
developed under the direction of
the U.S. Department of Health Education and Welfare
Project 8-0157

at the

New York Institute of Technology
Westbury, New York

VOLUME 3

TABLE OF CONTENTS

C

VOLUME 3

This volume covers the following material as shown in this excerpt from the Syllabus:

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION DOLCIANI	DRESSLER	DODES
1	Solving problems with open sentences	2-6	3-4	5-5
2	Axioms of equality	3-1	4-1	2-5
	Closure properties	3-2	4-2	2-8
3	Commut., Assoc. Prop.	3-3	4-3 4-4	4-13
	Distributive property	3-4	4-5 , 4-6 4-7	4-13
4	Add.-Subtr. properties of equality	3-5	5-4	4- 5
5	Division, multi., prop. of equality	3-6	5-6	4- 5

READING ASSIGNMENT

VOLUME 3

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	56	60
2	69	73
3	73	79
4	80	83
5	83	85

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer-
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

E

HOMEWORK ASSIGNMENT

VOLUME NO. 3

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	58	3 - 14	03111
2	59	4 - 12 , 19 - 21	03114
3	72	1 - 14	03220
4	73	1 - 12	03220
5	70	1 - 8	03411
6	74 - 75	9 - 26	03310
7	79	15 - 24 , 35 - 44	03330
8	82	1 - 10	03420
9	82	11 - 20	03430
10	83	1 - 10	03420
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12	101	24 - 31	03410
13	84	1 - 10	03521
14	84	11 - 18	03522
15	85	37 - 40	03512
16	101 - 102	32 - 36	03510

VOLUME 3 SEGMENT 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	1	1	(Sequence Number)
	54	and	56	0	4	(Type of Punch Card)
	60	and	62	0	3	(Volume Number)
	66	and	68	0	1	(Segment Number)

Your READING ASSIGNMENT for the segment is pg: 56 - 57 .

SUPPLEMENTARY NOTES:

It is extremely important for you to follow the procedures outlined in the text. Even though you may find that you can solve many of the problems without following the four steps, you must realize that you are being asked to learn a technique which will be <u>necessary</u> in handling more difficult problems. A little patience and a serious effort in this segment will make the rest of this course meaningful and useful to you.

Now you will be asked a series of questions to draw your attention to the more important points.

<u>Question 1</u>

If John has 5 cents more than Alfred, which do you recognize as the <u>best</u> meaning for variable x ?

(A) x is Alfred's money

(B) x is the amount of money Alfred has

(C) x is the number of cents Alfred has

(D) x is the number of coins Alfred has

$\frac{2}{1}$

The correct translation of

" m is "

into algebra is

" m = . . ."

Since this choice states

" m less 3 is equal to 5p "

which implies that m is larger that 5p .

It contradicts the given conditon that m was less than 5p .

Please return to page $\frac{24}{2}$ and try question 3 again.

$\frac{2}{2}$

This equation is a tranlation of the sentence,

" 5 less than double n is <u>n</u> greater than s ".

The underlined letter was missing from the original statement of the problem, therefore, this choice is not correct even though it is very close.

Please return to page $\frac{27}{2}$ and try question 6 again.

This choice can be read as

" w increased by 3 is 7 <u>which</u> is greater than n ".

The one underlined word was missing from the original problem, and that makes this choice incorrect. As a matter of fact, this choice is not <u>one</u> open sentence; but a combination of open sentences.

Return to page $\frac{20}{2}$ and try question 5 again.

None of the values included in your answer are wrong; but consider this:

If n were equal to 20 ,
then n - 10 would be 10
which is smaller than 28 .

Therefore, there is no need for n to be smaller than 18 . You have introduced an unnecessary restriction.

This choice is not correct since you omitted some of the possible answers.

Please return to page $\frac{30}{2}$ and try question 9 again.

$\frac{4}{1}$

If you reread the problem you will discover that Peter's age is described in terms of <u>Bob's age three years ago</u>. This quantity represents the independent variable. Therefore, it is <u>not</u> best to use y to represent Bob's age today. There is no question that it could be used in this manner, but that was not the question.

Return to page $\frac{19}{2}$ and try question 2 again.

$\frac{4}{2}$

The quantity on the left side of the open sentence is a correct representation of the first sentence in the problem. The quantity on the right is a correct representation of the first part of the second sentence. However, which of these two is the larger?

This choice is not correct.

Return to page $\frac{22}{2}$ and try question 7 again.

would you represent the number of boys?

Then doesn't this choice say that the number of boys is 25 ?

That is not the statement of the problem; you have omitted something.

Return to page $\frac{29}{2}$ and try question 4 again.

If x were equal to 1 , there would be no meaning to

$2x - 6$ since we cannot subtract

6 from 2 and arrive at positive numbers. Since 1 is a member of the set offered in this choice, this choice is not correct. Although there are other members of the set you chose that will produce a positive number when substituted, we cannot include them in the solution set. The solution set must be restricted to only possible answers.

Return to page $\frac{33}{2}$ and try question 11 again.

$\frac{6}{1}$

What exactly does this choice mean?

Remember, it is necessary for a letter to represent a number. But a number of what?

You can find a better choice.

Return to page $\frac{1}{2}$ and try question 1 again.

$\frac{6}{2}$

Since $6x$ means 6 times the value of x, and the smallest value for x in the replacement set is 10, it follows that the value of $6x$ alone would be larger than 42 for any of the numbers in this set.

Substitute each value of the set for x in the given equation, before you make your decision.

Therefore, this choice is not correct.

Return to page $\frac{17}{2}$ and try question 8 again.

This choice is a correct statement; it just happens not to be the correct answer to the question.

This statement says,

" w increased by 3 is greater than n ".

There is not any mention of the number 7 which appeared in the original problem.

Please return to page $\frac{20}{2}$ and try question 5 again.

One of the members of the set offered in this choice is the number 3 . If the value of x is 3 , then the width of the rectangle is

$$3 - 4$$

which has no meaning.

Since a member of the set gives us a meaningless expression, this choice is not correct.

Please return to page $\frac{21}{2}$ and try question 12 again.

$\frac{8}{1}$

You must include all the possible answers when you make an inequality statement.

In order to have a result smaller than 28 after subtracting 10 , it is necessary to start with a number less than 38 .

Then this choice is correct.

Please roceed to question 10 below.

$\frac{8}{2}$

Question 10

Apply your knowledge to find the solution of the open sentence

$$23 - n < n$$

(A) $n < 12$

(B) $n < 13$

(C) $n > 10$

(D) None of these.

You have mixed up something. Why should the total be added to the number of girls?

In any case, this equation is very strange. It says that the result of adding 5 to a number is the same as the result of adding 25 !

Please return to page $\frac{29}{2}$ and try question 4 again.

The expression

$$2(n - 5)$$

is a translation of

" double 5 less than n ".

But the statement of the problem had the words in a different order, therefore, this choice is not correct.

Please return to page $\frac{27}{2}$ and try question 6 again.

$\frac{10}{1}$

This is certainly a correct description, but it is not specific enough.

Does this mean the number of yards of money?

You must be specific in naming the type of quantity.

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{10}{2}$

This choice says that some quantity is greater than the <u>original number</u>. But the problem said that one quantity which came from the number is greater than a different quantity which also was obtained from the number. Therefore, this choice is not correct.

It is a good practice in complicated problems like this one to list a set of definitions of what each quantity is represented by.

Please return to page $\frac{22}{2}$ and try question 7 again.

Your choice is almost correct. In fact, the error was in your interpretation of English, not in the algebraic representation of interpretation.

There are two possible contradictory interpretations of a statement such as

" m is 3 less than 5 times p "

(1) m is (3 less than 5) (pause) times p

which you chose. It really means 2 times p

(2) m is 3 less than (pause) (5 times p)

which is correct.

Now here's the key. In the absence of emphasis by voice pacing or punctuation such as commas or parentheses, the multiplication of 5 and p takes precedence over the subtraction and is, therefore, to be done first.

Please return to page $\frac{24}{2}$ and reconsider the question.

$\frac{12}{1}$

You seem to have confused

" increased by 3 "

with

" 3 times "

3w

means

3 times w

Therefore, this choice is not correct.

Please return to page $\frac{20}{2}$ and try question 5 again.

$\frac{12}{2}$.

If you were to evaluate the expression

$$6x + x$$

for the largest value of x offered here, you would get

$$6(5.5) + 5.5 =$$

$$33.0 + 5.5 =$$

$$38.5$$

But this is too small to equal 42 . Therefore, 5.5 is not the solution. Now you can see that a smaller value for x will make the value of

$$6x + x$$

even smaller than 38.5 .

Therefore, no number in this set is the solution, and this choice is not correct.

Please return to page $\frac{17}{2}$ and try question 8 again.

Reading this problem we are led from Peter's age to Bob's age three years ago. It is, therefore, not best to use y to represent Peter's age if y is to represent the independent variable. Of course, if could be done, but the question asked for the best meaning for y as well as the identification of the independent variable.

Please return to page $\frac{19}{2}$ and try question 2 again.

The choice $n < 12$

says that any value less than 12 is a solution. Well, suppose we try a few as a check.

Now if $n = 11.6$

then $23 - n =$

11.4

Now 11.4 is smaller than the value we chose. So that is correct choice. However, suppose

$n = 10$

then $23 - n =$

13

But 13 is not smaller than the value (of 10) which we just chose for n .

So one possible value of our choice is correct and one is wrong. The solution must be some statement such, that any value which fits it, will also fit the original open sentence. This choice is not correct.

Please return to page $\frac{8}{2}$ and try question 10 again.

$\frac{14}{1}$

In order to get the number of children in the class, you ought to add the numbers of boys and girls. What did you do?

This choice is not correct.

Return to page $\frac{29}{2}$ and try question 4 again.

$\frac{14}{2}$

When you make a statement in algebra, you should check it by looking at it from a common sense point of view, asking yourself " what does this really mean "?

If n were smaller than 2.8 , what would be the meaning of n - 10 ?

Remember that n - 10 tells you to subtract 10 from the value of n , and this would only have meaning if n is at least 10 .

This choice is not correct.

Return to page $\frac{30}{2}$ and try question 9 again.

It is correct for x to represent a number, but why the number of coins? Is it the number of coins or the value of the money we are concerned with in this problem? In what units should we express the value of the money?

Return to page $\frac{1}{2}$ and try question 1 again.

This choice is a translation of the statement,

" Double n is 5 less than s ".

Since the problem didn't call for 5 less than s , this choice is not correct.

Return to page $\frac{27}{2}$ and try question 6 again.

$\frac{16}{1}$

If x were equal to 2 then

$$2x - 6$$

would equal

$$4 - 6$$

and we cannot subtract 6 from 4 and get a positive number. Since 2 is a member of the set offered in this choice, this choice is not correct.

Although there are other members of the set which will produce possible answers we cannot include in the solution set members that won't produce the desired answer.

Please return to page $\frac{33}{2}$ and try question 11 again.

$\frac{16}{2}$

If x is an integer, then

$$x - 4$$

is also an integer.

But there is no reason why the length of a rectangle has to be an integer; or why its width should be an integer.

Then this choice is not correct.

Please return to page $\frac{21}{2}$ and try question 12 again.

The quantity on the left side of this open sentence is a translation of

" double the result of increasing a number by 3 ".

The quantity on the right side is a translation of

" the result of increasing double a number by 3 ".

Since the one on the left is the larger, this statement agrees with the original problem and, therefore, this choice is correct.

Proceed to question 8 below.

Question 8

Apply your knowledge to find the set which contains the solution of the equation

$$6x + x = 42$$

(A) $\{10, 14, 18\}$

(B) $\{6, 10, 14\}$

(C) $\{3.5, 4.5, 5.5\}$

(D) $\emptyset$

$\frac{18}{1}$

When we have the phrase " less than " , we have to be very alert when we translate. The elements can not be kept in the same order in algebra as in English.

3 less than 5 times p

is not

$3 - 5p$

Instead, we must reverse the order to

5 times p diminished by 3

or

$5p - 3$

which is correct.

Please return to page $\frac{24}{2}$ and reconsider question 3 again.

$\frac{18}{2}$

The choice

$n < 13$

says that <u>any</u> value of n less than 13 is a solution to the open sentence.

However, if $n = 11$

we find that $23 - n = 12$

which is not less than the value which we just used for n . Of course, you should be able to find a value of n less than 13 for which the statement does check.

Since it does not check for <u>every</u> value of n less than 13 this choice is not correct.

Please return to page $\frac{8}{2}$ and try question 10 again.

Since it is necessary for x to represent a number, it is only necessary to decide what units would be most reasonable.

The only money units mentioned in the problem are <u>cents</u>, and therefore, this choice is correct.

Proceed to question 2 below.

<u>Question 2</u>

Peter's present age is twice the result of adding 9 to Bob's age three years ago. Which do you recognize as the <u>best</u> meaning for y if it is to represent the independent variable.

(A) y represents Bob's age 3 years ago

(B) y represents Bob's age today

(C) represents Peter's age today

(D) y represents Peter's age three years ago.

$\frac{20}{1}$

If there are $\underline{g}$ girls in the class, then the number of boys should be represented as

$$g + 5$$

since we were told that there are 5 more boys. Then we must add the number of boys and girls to get the total of 25 .

This gives us the equation of this choice, which is, therefore, correct.

Please proceed to question 5 below.

$\frac{20}{2}$

<u>Question 5</u>

If w increased by 3 is 7 greater than n , choose the open sentence which states this fact.

(A) $w + 3 = 7 > n$

(B) $w + 3 > n$

(C) $3w + 7 > n$

(D) $w + 3 = n + 7$

For any value of x larger than 3 the value of $2x$ is larger than 6 and then the value of

$$2x - 6$$

is larger than 0 .

Since positive numbers are greater than zero, this choice is correct.

Please proceed to question 12 below.

Question 12

If the width of a rectangle is

$$x - 4$$

where x represents the number of centimeters in the length, choose the number of centimeters in the length, choose the domain of x .

(A) $\{\text{all integers}\}$

(B) $\{\text{all integers} > 4\}$

(C) $\{\text{all numbers}\}$

(D) $\{\text{all numbers} > 4\}$

$\frac{22}{1}$

The translation of

" 5 less than double n "

is

$2n - 5$

The translation of " is greater than s "

is merely $> s$

Then this choice is correct.

You should note that the translation of

" is p greater than s "

is

$s + p$

The slight difference between this wording and the wording of the problem itself makes the difference between the equal sign and the inequality sign.

Please proceed to question 7 below.

$\frac{22}{2}$

Question 7

A number was doubled and the result increased by 3 . If the number had been increased by 3 and the result doubled, we would have obtained a larger number than before. Apply your knowledge to find the open sentence which states this relationship.

(A) $2(n + 3) > 2n + 3$

(B) $2n + 3 > 2(n + 3)$

(C) $2(n + 3) > n$

(D) $2n + 3 > n$

III

This choice says that there is no solution to the equation. Is that what you meant to say? It is not the correct choice.

If you substitute each of the values (or make a judgment concerning the probable size of the solution), you will find that there is one value that solves the equation.

Please return to page $\frac{17}{2}$ and try question 8 again.

$\frac{23}{2}$

The choice

$$n > 10$$

says that any value of n greater than 10 is a solution of the open sentence.

However, if

$$n = 11$$

we find that

$$23 - n = 12$$

which is not less than the value which we just used for n.

Of course, you should be able to find a value of n greater than 10 for which the statement does check, but since it does not check for every value of n greater than 10, this choice is not correct.

Please return to page [illegible] and try question 10 again.

The problem deals with Peter's age and Bob's age. In reading the problem you are led from Peter's age to Bob's age three years ago. This then would be the independent variable. It is therefore, best to use your independent variable to represent the underlined quantity.

Then ... choice is correct.

Proceed to question 3 below.

Question 3

If m is 3 less than 5 times p, choose the equation which states this fact.

(A) $m = 5p - 3$

(B) $m - 3 = 5p$

(C) $m = (5 - 3) \times p$

(D) $m = 3 - 5p$

Since one of the other choices is correct, this is not.

There must be at least one number from which you could subtract ten and get an answer less than 28, wouldn't you say?

Please return to page $\frac{30}{2}$ and try question 9 again.

You might like to try this approach.

We are looking for values of n which make the left-hand side smaller than the right-hand side.

Let us try the largest number offered to us since if the largest number works, the others will also.

Using 13 for n we have:

$$2 \cdot 13 + 5 \cdot 13 + 10$$

which equals

$$26 + 65 + 10 =$$

$$101$$

This certainly not smaller than 26. Since one value does not check, this is not the solution set.

Then, this choice is not correct.

Please return to page $\frac{31}{2}$ and try question 15 again.

III

$\frac{16}{2}$

Substituting 2 for x in the inequality, we get

$$2 + 5 \cdot 2 =$$
$$2 + 10 =$$
$$12$$

which is certainly less than [illegible] .

However, this is not the only member of the domain for which the inequality checks.

This choice is, therefore, incorrect.

Please return to page $\frac{43}{2}$ and try question 14 again.

$\frac{26}{2}$

If 10 is increased by half of itself, which is 5 , the result is 15 . Then if we used a number smaller than 10 , we would get a result smaller than 15 ; it certainly couldn't be 18 .

Then this choice is not correct.

Please return to page $\frac{49}{2}$ and try question 16 again.

The translation of

" w increased by 3 " is $w + 3$.

The phrase

" is 7 greater than n " translates as $= n + 7$.

Then the correct equation is

$$w + 3 = n + 7 ,$$

and this choice is correct.

You should realize that the sign $>$ means only " is greater than " , while $= \ldots - r$ means

" is r greater than " .

Proceed to question 6 below.

Question 6

If 5 less than double n is greater than s , choose the open sentence which states this fact.

(A) $2n - 5 = s + n$ (C) $2(n - 5) > s$

(B) $2n - 5 > s$ (D) $2n > s - 5$

$\frac{28}{1}$

If x is larger than 4 then

$$x - 4$$

is larger than zero.

Since both the length and the width of a rectangle must be larger than zero, this result is reasonable. This choice permits both the length and the width to be either whole numbers or fractions, which is correct.

Therefore, this choice is correct.

Please proceed to question 13 below.

$\frac{28}{2}$

Question 13

If a haberdasher has

$$3s + 4$$

shirts left in stock after selling some of the five dozen he started with, apply your knowledge to find the domain of s .

(A) $\{$ all positive integers $\}$

(B) $\{$ all numbers $\}$

(C) $\{$ all positive integers $< 19\}$

(D) $\{$ all numbers $< 19\}$

The sentence

" m is 3 less then 5 times p "

must be punctuated as

m is 3 less than (pause) 5 times p

m = (5p) − 3

The translation of

" is three less than " into algebra is

" = . . . − 3 "

5 times p is translated as 5p .

This choice is correct.

Please proceed to question 4 below.

Question 4

The number of boys in a class is 5 more than the number of girls. If g represents the number of girls, choose the open sentence which expresses the fact that there are 25 children in the class.

(A) g + 5 = 25

(B) g + (g + 5) = 25

(C) g + 5 = g + 25

(D) g (g + 5) = 25

30
1

Since $6x$ means 6 times x, it follows that the value of $6x$ is 36 when x is 6.

Then	$6x$	+	x
would be	36	+	6
which is	42		

Therefore, this choice is correct.

Notice that you were to find the set which contained the solution. It didn't mean that every member of the set would be a solution.

Please proceed to question 9 below.

30
2

Question 9

Apply your knowledge to find the solution of the open sentence

$$n - 10 < 28$$

(A) $n < 38$

(B) $n < 18$

(C) $n < 2.8$

(D) None of these

Remember, the solution set must include all of the possible values and none of the impossible.

$$\text{Since} \quad x \div 5x =$$

$$6x$$

$$\text{we have} \quad 6x < 20$$

This inequality checks for

$$x = 0$$

and also for

$$x = 2$$

It does not check for

$$x = 4$$

or

$$x = 6$$

Therefore, this choice is correct.

Please proceed to question 15 below.

$\frac{31}{2}$

Question 15

If the domain is $\{\text{odd integers between } 3 \text{ and } 15\}$

apply your knowledge to find the solution set of the open sentence

$$2n + 5n + 10 < 26$$

(A) $\{5, 7, 9, 11, 13\}$

(B) $\emptyset$

(C) $\{5\}$

(D) $\{3, 5, 7, 9, 11, 13, 15\}$

$\frac{32}{1}$

Careful reading is a must in algebra.

A correct translation for this choice is,

" If 3 times a number is increased by 3 times a number, the result is 19 ".

But the original problem did not state that, therefore, this choice is not correct.

How do you represent

" 3 more than a number " ?

Please return to page $\frac{51}{2}$ and try question 18 again.

$\frac{32}{2}$

You have correctly translated

" two less than five times the number of kitchen workers "

as 5n - 2

In expressing the fact that the number of campers was

four times the number of counselors

you have made an error. Why did you use

4 times 5n

and subtract 2 ?

Did you make a mistake in the way you used the parentheses?

This choice is not correct.

Please return to page $\frac{55}{2}$ and try question 20 again.

$\frac{33}{1}$

Since none of the other choices is correct, this one is.

You should note that if n were to be

$$\frac{11.5}{23 - n}$$

would equal n .

Since we wanted the subtraction to give a smaller result, we needed a larger value for n . That is, the solution is all values of n greater than 11.5 .

Since all the other choices permitted some values of n less than

$$11.5$$

they were not correct.

There was no choice calling for values of n greater than

$$11.5$$

that is why we are forced to make this choice.

Please proceed to question 11 below.

$\frac{33}{2}$

Question 11

A positive number is represented as

$$2x - 6$$

Apply your knowledge to find the domain of x .

(A) {positive integers}

(B) {positive numbers}

(C) {numbers greater than 1}

(D) {numbers greater than 3}

$\frac{34}{1}$

The cost of 3 bookcovers at 48 cents each is \$1.44 . Since there were two more notebooks than bookcovers, we would have 5 notebooks. The cost of each notebook was 4 times the cost of each bookcover; Then each notebook cost \$1.92 . The total cost of 5 notebooks at \$1.92 each is \$9.60 . Without doing any more arithmetic, we can see that the total for all these purchases is more than \$2.76 . Therefore, this choice is not correct.

Return to page $\frac{42}{2}$ and try question 17 again.

$\frac{34}{2}$

Since he began with 5 dozen shirts, and he has sold some, he now has less than 60 shirts. The number 20 is a member of the set offered in this choice.

But if s is 20 , then

$$3s + 4 =$$

$$3 \cdot 20 + 4 \text{ which is } 64 .$$

This is impossible; he started with only 60 .

Then this choice is not correct.

Return to page $\frac{28}{2}$ and try question 13 again.

If a number is smaller than 12 , then half of it is smaller than 6 . This would give us a total smaller than 18 for the number increased by half of itself.

Then this choice is not correct since we wanted a total of 18 .

Please return to page $\frac{49}{2}$ and try question 16 again.

The problem reads,

" Twice a number increased by 5 less than half the number . . . "

This choice can be translated as,

" Twice a number increased by 5 and the result decreased by half the number . . . "

Please note that the underlined phrase must be kept together as one phrase to get meaning from the sentence.

Since there is an error in the translation, this choice is not correct.

Please return to page $\frac{46}{2}$ and try question 19 again.

$\frac{36}{1}$

It is true that

$$9 = 6 + 3$$

but this statement merely shows an equivalence: the possibility of replacing

$$9 \text{ by } 6 + 3$$

This illustrates another fundamental principle, the SUBSTITUTION PRINCIPLE.

You were asked to choose an example of the REFLEXIVE PRINCIPLE (which merely states that a thing is equal to itself.)

Return to page 53 and reconsider the question.

$\frac{36}{2}$

Of course it is true that

$$6 + 9$$

is the same as

$$15$$

and if

$$6 + 9 = x$$

then 15 will equal x

but this is an example of the SUBSTITUTION PRINCIPLE.

Check the definition of the SYMMETRIC PROPERTY in your text before you

Return to page $\frac{68}{2}$ and reconsider the problem.

Remember the solution set must contain all of the possible values and none of the impossible.

Although the set you chose does contain a correct value, substituting 4 for x in the inequality, we get

$$4 + 5 \cdot 4 =$$
$$4 + 20 =$$
$$24$$

which is not less than 20 .

Then this choice is not correct.

Please return to page $\frac{43}{2}$ and try question 14 again.

Sometimes it helps to translate your algebra back to English.

A correct translation for this choice is

" 3 more than a number is 19 more than 3 times the number ".

But that is not what the problem stated.

Therefore, this choice is not correct.

Return to page $\frac{51}{2}$ and try question 18 again.

III

$\frac{38}{1}$

Do you understand that you must substitute the chosen value in the original statement to test its correctness?

Combining the two terms

$$2n + 5n$$

gives us $7n$

Then if we substitute 5 for n we get

$$7 \cdot 5 + 10$$

which equals $35 + 10$

or 45

This is certainly not smaller than 26 .

Therefore, this choice is not correct.

Return to page $\frac{31}{2}$ and try question 15 again.

$\frac{38}{2}$

The cost of five book covers at 12 cents each is 60 cents. Since there were two more notebooks than book covers, we would have seven notebooks. But the cost of each notebook was 4 times the cost of one book cover; then each notebook cost 48 cents. The total cost of seven notebooks at 48 cents each is \$3.36 .

Without doing any more arithmetic, we can see that the total for all these purchases is more than \$2.76 .

Therefore, this choice is not correct.

Return to page $\frac{42}{2}$ and try question 17 again.

A number less than 19 could be a fraction such as $12\frac{1}{2}$.

But then

$3s - 4$

would also be a fraction and since this is a number of shirts; that is impossible.

Therefore, this choice is not correct.

Please return to page $\frac{28}{2}$ and try question 13 again.

If a number is larger than 12, half of it is larger than 6. Then the number increased by half of itself would be larger than 18. However, we wanted the total to be 18.

Therefore, this choice is not correct.

Return to page $\frac{49}{2}$ and try question 16 again.

III

$\frac{40}{1}$

This open sentence shows very good translation from English into algebra. However, there is one item you overlooked. The problem mentioned the number 36 , which does not appear in this choice.

Did u overlook it in reading the problem?

Since it has been omitted, this choice is not correct.

Return to page $\frac{46}{2}$ and try question 19 again.

$\frac{40}{2}$

Of course it is true that

$$9 = 3 \times 3$$

This statement shows that 9 is equivalent to

$$3 \times 3$$

and that it can be replaced by

$$3 \times 3$$

This is an illustration of a different fundamental principle, the SUBSTITUTION PRINCIPLE. You were asked to choose an example of the REFLEXIVE PRINCIPLE which merely states that a thing is equal to itself.

Please return to page $\frac{53}{2}$ and reconsider question 1

Sometimes it helps to translate your algebra back into English.

A correct translation for this choice is

" 3 more than 3 times a number is 19 ".

But that is not what the problem stated.

Therefore, this choice is not correct.

Return to page $\frac{51}{2}$ and try question 18 again.

Careful reading is necessary.

The problem stated that the number of campers was 4 times the number of counselors, but you have represented the number of campers as 4n , which would be 4 times the number of persons employed in the kitchen.

Therefore, this choice is not correct.

Please return to page $\frac{55}{2}$ and try question 20 again.

42
1

If a number is larger than 10 , then half of it is larger than 5 . This would give us a total larger than 15 for the number increased by half of itself. Since we wanted a total of 18 , this is reasonable. Of course, until we have found the number, we can't be sure that there is such a number. If you try the next larger even number, you will find the correct one.

Since the solution is an even number which is larger than 10 , this choice is correct.

Please proceed to question 17 below.

42
2

Question 17

Jay bought some notebooks and book covers at a total cost of $2.76 . Each notebook cost 4 times as much as a book cover, and he bought two more notebooks than book covers. Apply your knowledge to check the following choices in order to find the number of book covers he bought and the price he paid for each book cover.

(A) Three book covers at 12 cents each

(B) Three book covers at 48 cents each

(C) Five book covers at 12 cents each

(D) Five book covers at 48 cents each

According to this choice, the largest value s could have is 18 .

Then	$3s + 4$
would be	$3 \cdot 18 + 4$
or	$54 + 4$
which equals	58

It is possible that he has 58 shirts left out of the 60 . Therefore, this choice is correct. Of course, we must recognize that " s " must be an integer so that

$$3s + 4$$

will be an integer since this represents a number of shirts.

Please proceed to question 14 below.

Question 14

For the open sentence

$$x + 5x < 20$$

where the domain is $\{0, 2, 4, 6\}$

apply your knowledge to find the solution set.

(A) $\{0, 2\}$

(B) $\{2\}$

(C) $\{0, 4\}$

(D) $\{2, 4, 6\}$

$\frac{44}{1}$

Combining the two terms

$$2n + 5n$$

gives us $7n$

Now we are looking for the solution set of an inequality where the left side is to be smaller than 26 . Let us try the smallest element of the domain 3 as the value for n .

$$\begin{aligned} \text{We have} \quad & 7 \cdot 3 + 10 \\ \text{which equals} \quad & 21 + 10 \\ \text{or} \quad & 31 \end{aligned}$$

It is certainly not smaller than 26 . If we used a larger value for n , the result would be even larger than 31 .

Therefore, this choice is not correct.

How did you arrive at this choice? Did you select it because it is the domain itself? Remember, each element must satisfy the conditions. Did you check any of them?

Please return to page $\frac{31}{2}$ and try question 15 again.

$\frac{44}{2}$

Sorry, but we do not agree. The relation

" is larger than " ,

is not reflexive. It is not possible for a quantity to be larger than itself; that is, $a \not> a$

The REFLEXIVE PROPERTY relates a quantity to itself.

Please return to page $\frac{58}{1}$ and try this question agian.

Yes

$$6 + 9$$

has the same value as

$$9 + 6$$

they both equal 15 , but this is an example of the COMMUTATIVE PRINCIPLE:

$$a + b = b + a$$

However, the SYMMETRIC PROPERTY is somewhat different.

Check the explanation in your text before you return to page $\frac{68}{2}$ to try the question again.

Although $17 + 9$ does equal 26

and $39 \cdot 13$ also equals 26

your choice is equivalent to the following:

$$\begin{array}{lccc} \text{if} & a & = & c \\ \text{and} & b & = & c \\ \hline \text{then} & c & = & c \end{array}$$

This is not the TRANSITIVE PROPERTY, which is as follows:

$$\begin{array}{lccc} \text{if} & a & = & b \\ \text{and} & b & = & c \\ \hline \text{then} & a & = & c \end{array}$$

Please return to page $\frac{64}{2}$ and reconsider the question.

III

$\frac{46}{1}$

Since 3 more than a number is represented as

$$n + 3$$

and 3 times a number is represented by

$$3n$$

the expression $n + 3 + 3n$

represents 3 more than a number increased by 3 times the number, but that is exactly what the problem called for.

Therefore, this choice is correct.

Please proceed to question 19 below.

$\frac{46}{2}$ Question 19

Twice a number increased by 5 less than one-half the number is 36 greater than the original number.

Apply your knowledge to find the open sentence which fits the problem.

(A) $2n + 5 - \frac{1}{2}n = 36$

(B) $2n + (\frac{1}{2}n - 5) > n$

(C) $2n + (\frac{1}{2}n - 5) = n + 36$

(D) $2n + \frac{1}{2}(n - 5) = n + 36$

The cost of five book covers at 48 cents each is \$2.40 . Since each notebook cost 4 times a bookcover, the price of a notebook would be \$1.92 . Then with only one notebook and the five book covers we would have spent more than the \$2.76 stated in the problem.

Therefore, this choice is not correct.

Please return to page $\frac{42}{2}$ and try question 17 again.

You have done a fine job of representing the number of counselors as

$$5n - 2$$

In attempting to represent the number of campers, however, you ran into difficulty. You have represented the number of c--pers as 2 less than 4 times 5 times the number of persons employed in the kitchen.

Of course, that was not what you intended. What must you do to correct this error?

That's right; read very carefully!

Please return to page $\frac{55}{2}$ and try question 20 again.

III

$\frac{48}{1}$

It is true that your choice is a true statement, for 9 does equal

$$\frac{23 - 5}{2}$$

However, this is not an illustration of the REFLEXIVE PRINCIPLE. That principle merely states that a thing is equal to itself (not some other equivalent form - - which would be the application of the SUBSTITUTION PRINCIPLE.)

Please return to page $\frac{53}{2}$ and reconsider the choices offered for question 1

$\frac{48}{2}$

Your choice doesn't pass the test for the SYMMETRIC PROPERTY. The test is to determine whether the left and right sides of the relation symbol can be interchanged resulting in a true statement without altering the relation symbol.

For example: if a (R) b is true and
b (R) a is true.
then the relation is symmetric.

Let's take a practical example: Suppose (R) stood for " parallel " then if it were true that for two lines a and b

a / / b

it would also be true that

b / / a

Therefore, the relation / / is said to have the SYMMETRIC PROPERTY.

Please return to page $\frac{59}{2}$ and reconsider the question.

There is some tricky reasoning in this explanation.

Combining the two terms	2n	+	5n
gives us	7n		
Then we have	7n	+	10
is smaller than	26		
which means that	7n		
is smaller than	16		

But of all the choices offered for the domain of n the smallest element was 3 and $7 \cdot 3$ is not smaller than 16. Then there is no element of the various domains which satisfies the open sentence; that is, its solution set is the empty set.

Therefore, this choice is correct.

Please proceed to question 16 below.

Question 16

A number increased by half of itself is 18. Apply your knowledge to find which set contains the number, n.

(A) $\{\text{even numbers} < 10\}$

(B) $\{\text{odd numbers} < 12\}$

(C) $\{\text{even numbers} > 10\}$

(D) $\{\text{odd numbers} > 12\}$

$\frac{50}{1}$

You represented the number of counselors correctly as

$$5n - 2$$

In representing the number of campers as

$$4(5n - 2)$$

you did an excellent job of using parentheses. Since the total was 68 ; and you expressed that in your equation.

This choice is correct.

You have now finished Segment <u>1</u> .

Hand in the Punch Card.

You should enter in your Notebook the following:

<u>Standard Translations</u>:

is greater than p	$> p$
is less than p	$< p$
is m greater than p	$= p + m$
is m less than p	$= p - m$
w less than p	$p - w$
w more than p	$p + w$
is, are, was, were, etc.	$=$

You should now be able to complete Homework Assignment 3 , problems 1-14

If he bought three book covers at 12 cents each, the total cost would be 36 cents. From the information in the problem, the number of notebooks is two more than the number of book covers; then there were five notebooks.

Since each notebook cost 4 times as much as a book cover, the price of a notebook would be 48 cents. Then five notebooks at 48 cents each cost a total of \$2.40 .

Adding the two purchases together, we get \$2.76 which is the figure given in the problem.

Therefore, this choice is correct.

Please proceed to question 18 below.

Question 18

If 3 more than a number is increased by 3 times the number, the result is 19 . Apply your knowledge to find the open sentence which fits the problem.

(A) $3n + 3n = 19$

(C) $n + 3 = 3n + 19$

(B) $n + 3 + 3n = 19$

(D) $3n + 3 = 19$

$\frac{52}{1}$

Remember, we warned you about "less than" a bit earlier. The key phrase in this problem is,

" 5 less than one-half the number."

The expression

$\frac{1}{2}$ (n - 5) is a translation of

"one-half of 5 less than the number."

Since this is not the same as the original, this choice is not correct.

Please return to page $\frac{46}{2}$ and try question 19 again.

$\frac{52}{2}$

Since the REFLEXIVE PROPERTY relates a quantity to itself, the question that you should ask yourself is:

Can $a < a$ be true?

Please return to page $\frac{58}{1}$ and try this question again.

VOLUME 3 SEGMENT 2 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	1 2	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 3	(Volume Number)
	66	and	68	0 2	(Segment Number)

Your READING ASSIGNMENT for this Segment is page 69 - 71

SUPPLEMENTARY NOTES:

You have heard expressions such as: Mr. A is older than Mr. B; Jack is as tall as Bill; seven is greater than five and so on. Each of these expressions indicates a comparison between two things. We call this comparison "a RELATION". Now, there are many different types of relations, but the ones most frequently encountered in mathematics are the relations "is equal to", "is greater than", and "is less than".

Some relations have different properties than others. For example, if a and b are two numbers; then a is equal to b implies that b is equal to a . However, if a is larger than b does not imply that b is larger than a . Thus, the relation is equal to has different properties than the relation is larger than. In this segment you will learn the various properties of relations and their usefulness in mathematics.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Recognize which of the following choices demonstrates the REFLEXIVE PROPERTY OF EQUALITY.

(A) $9 = 6 + 3$

(B) $9 = 9$

(C) $9 = 3 \times 3$

(D) $9 = \frac{23 - 5}{2}$

$\frac{54}{1}$

This choice is an example of the ASSOCIATIVE LAW FOR ADDITION.

When given three or more quantities to add, two adjacent quantities can be grouped and added first before the remaining quantities are added.

However, you were directed to choose an example of the SYMMETRIC PROPERTY OF EQUALITY.

Please return to page $\frac{68}{2}$ and try the question again.

$\frac{54}{2}$

Sorry, but we do not agree.

For example, if Mr. X is the father of Y , and Y is the father of Z ; then Mr. X is the grandfather of Z .

Therefore, the relation "is the father of" is never a transitive relation.

Please return to page $\frac{69}{2}$ and try this question again.

The key phrase in this problem is

" 5 less than one-half the number. "

It is correct to translate it as

$$\frac{1}{2}n - 5$$

Then the statement,

" is 36 greater than the original number "

should be translated as

$$\ldots = n + 36$$

Thus, the open sentence which expresses the relationship is

$$2n + \frac{1}{2}n - 5 = n + 36$$

Therefore, this choice is correct.

Please proceed to question 20 below.

Question 20

At a certain camp there were n persons employed in the kitchen. The number of counselors was two less than 5 times the number of kitchen workers, while the number of campers was four times the number of counselors. If the total number of individuals in these three categories was 68 , apply your knowledge to select the equation which states the relationship in the problem.

(A) $n + (5n - 2) + (4)5n - 2 = 68$

(B) $n + (5n - 2) + 4(5n - 2) = 68$

(C) $n + (5n - 2) + 4n = 68$

(D) $n + (5n - 2) + 4 \cdot 5n - 2 = 68$

$\frac{56}{1}$

It is true that

if	$9 + 5 = 14$	(by subtracting 5 f...
and	$\underline{\quad 5 = 5}$	both sides of the equation)
then	$9 = 14 - 5$	

But this is an illustration of the SUBTRACTION AXIOM.

The TRANSITIVE PROPERTY is symbolized as follows:

$$\begin{array}{ll} \text{if} & a = b \\ \text{and} & \underline{b = c} \\ \text{then} & a = c \end{array}$$

Please return to page $\frac{64}{2}$ and reconsider the question.

$\frac{56}{2}$

In order for a set to be closed under an operation, each combination of the elements of the set must result in a number that is itself a member of the same set.

Now when two odd numbers are subtracted, the result is an even number.

Your set doesn't contain any even numbers, just odd.

Try all of the combinations in the set you choose when you try this problem again before you make a commitment.

Please return to page $\frac{70}{2}$ and reconsider the question.

You said that

$$\begin{array}{lrcl} \text{if} & x & = & y + 2 \\ \text{and} & w & = & y + 2 \\ \hline \text{then} & y + 2 & = & y + 2 \end{array}$$

Of course, by the REFLEXIVE PROPERTY, any quantity equals itself; but this does not illustrate the TRANSITIVE PROPERTY.

Please return to page $\frac{78}{2}$ and try the question again.

When the elements of a set are listed individually, it does not mean that the same element can't be used more than once. Suppose, for example, you were asked to compose words that were spelled with T and O . Of course, you would choose TO ; but couldn't you write TOO , or TOOT , or OTTO ?

In this choice you were given the set $\{0, 1\}$. Now keeping in mind what we just said, couldn't you choose

$$1 + 1 \quad ?$$

Since the answer 2 is not an element of the given set, the set is not closed under addition.

Now return to page $\frac{76}{2}$ and try this question again.

$\frac{58}{1}$

Your answer is correct. The REFLEXIVE PRINCIPLE states that a quantity is equal to itself.

Please go on to question 2 which follows.

Question 2

Apply your knowledge and select the letter next to the relation that is REFLEXIVE.

(A) is larger than

(B) is smaller than

(C) is unequal to

(D) is equal to

$\frac{58}{2}$

Remember, one exception makes the statement false.

What is the product of 5 and 7 ?

Is 35 an element of set S ?

Please return to page $\frac{80}{2}$ and try question 9 again.

The SYMMETRIC PROPERTY merely states that the quantities on the two sides of the equal sign can change sides.

That is, if $a = b$

then $b = a$

This choice was the actual proto-type of the property.

Please proceed to the next question below.

<u>Question 4</u>

Apply your knowledge and select the letter next to the relation that is symmetric.

(A) Is greater than

(B) Is less than

(C) Is equal in area to

(D) Is a factor of

$\frac{60}{1}$

It is true that

$$\begin{array}{llr}
\text{if} & 87 - 29 = 71 - 13 & \\
\text{and} & \underline{\quad\quad 29 = \quad\quad 29} & \text{(by adding 29 to both sides of the equality)} \\
\text{then} & 87 \quad\quad = 71 + 16 &
\end{array}$$

But this is an illustration of the ADDITION AXIOM.

The TRANSITIVE PROPERTY is symbolized as follows:

$$\begin{array}{ll}
\text{if} & a = b \\
\text{and} & \underline{b = c} \\
\text{then} & a = c
\end{array}$$

Please return to page $\frac{64}{2}$ and try question 5 again.

$\frac{60}{2}$

You saw the pattern for the TRANSITIVE PROPERTY, but you overlooked one small detail. The middle term must be exactly the same.

You have no way of knowing whether

$$x + a$$

is the same as $\quad x + 2$

It might be, but then again, it might not. When this situation exists, we have to consider $\quad x + a =$

$$x + 2$$

as not being true.

Please return to page $\frac{78}{2}$ and reconsider question 7.

It is time consuming to be thorough.

In problems of this sort, it is not enough to try a few cases in order to see if they are correct. Every possible case must be considered. If only one exception is found, then it throws out the statement. Did you try

$$5 - 3 \text{ ?}$$

Since 2 is not an element of the set, we cannot say that it is closed under subtraction.

Please return to page $\frac{70}{2}$ and try the question again.

The set that you chose $\{0, 1, 2, 3\}$ was to be tested for closure under addition. In situations like this, it is wise to check all the possibilities systematically.

$0 + 0 = 0$	$1 + 1 = 2$	
$0 + 1 = 1$	$1 + 2 = 3$	
$0 + 2 = 2$	$1 + 3 = \underline{4}$!	which is <u>not</u> an
$0 + 3 = 3$		element of the set.

Of course with a little thinking, you could try one certain combination and find an exception immediately.

($3 + 3 = 6$, right ?)

Please return to page $\frac{76}{2}$ and try question 10 again.

$\frac{62}{1}$

Relations such as "likes" or "admires" are not necessarily transitive. Consider, for example, the following situation:

> If you like your teacher, and your teacher likes the dean, then it does not automatically follow that you like the dean.

Therefore, this is not always a TRANSITIVE RELATIONSHIP.

Please return to page $\frac{69}{2}$ and try this question again.

$\frac{62}{2}$

Sorry, we disagree.

It is true that when a larger number of the set is divided by a smaller, the result is a member of the set, but a smaller could be divided by a larger.

For example, what is the result of dividing 4 by 8 ? Is it an element of the set?

Each of the elements of the set are integers. Will the answers to each division operation be an integer? The second group mentioned will produce fractions.

Return to page $\frac{72}{2}$ and try question 11 again.

The REFLEXIVE PROPERTY refers a quantity to itself.

Since it is not possible for a quantity to be unequal to itself, the relation " is unequal to " cannot be REFLEXIVE.

Please return to page $\frac{58}{1}$ and try this question again.

Remember, only one exception makes a mathematical statement false. Let's find one.

What is the product of 6 and 5 ?

Is 30 an element of the set of natural numbers between 1 and 7 ?

Please return to page $\frac{80}{2}$ and try question 9 again.

III

$\frac{64}{1}$

Very good. You made the correct choice.

The relation "is equal in area to" is symmetric. Suppose, for example, that we were dealing with two rectangles. If the first rectangle is equal to the second, then the second rectangle would be equal to the first. This would then meet the requirement in order for the relation to be called SYMMETRIC.

If a (R) b is true

and b (R) a is also true where the relation (R)

has not been changed, then the relation (R) is said to have the SYMMETRIC PROPERTY.

Please proceed to question 5 below.

$\frac{64}{2}$

Question 5

Recognize which of the following choices demonstrates the TRANSITIVE PROPERTY OF EQUALITY.

(A) If 3 + 7 = 10, and 10 = 8 + 2, then 3 + 7 = 8 + 2

(B) If 17 + 9 = 39 - 13, then 26 = 26

(C) If 9 + 5 = 14, then 9 = 14 - 5

(D) If 87 - 29 = 71 - 13, then 87 = 31 + 16

Here is a case where you can say something that is true and still be wrong.

If $x = y - 3$

then we know that $y - 3 = x$

by the SYMMETRIC PROPERTY of equality.

In the case you considered,

if $x = y - 3$

and $\underline{y - 3 = a}$

the conclusion should be

$$x = a$$

if we are to apply the TRANSITIVE PROPERTY.

Please return to page $\frac{78}{2}$ and try question 7 again.

It looks like you forgot that an element of a set can be repeated in the operation. For example:

$$2 \times 2 = 4$$

and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Neither $\frac{1}{4}$ nor 4 are members of the original set and, therefore, the set is not closed under multiplication.

Please return to page $\frac{81}{2}$ and try question 12 again.

Sorry, we disagree.

There are many exceptions. Consider 3 and 2 .

The result of dividing 3 by 2 is not an integer, and is not an element of the set:

The order could be reversed

$$2 \div 3$$

and it still is an exception.

Please return to page $\frac{76}{2}$ and try question 10 again.

Sorry, we disagree. You analyzed the first set correctly, but you missed something in the second set.

When 2 and 6 are added, the sum is 8 . Is 8 an element of the set shown in II ?

Please return to page $\frac{86}{2}$ and try question 13 again.

This set is composed of the first 5 odd numbers.

When two odd numbers are added, the result is an even number. No even number is an element of the set. A set is closed under an operation if the result of performing that operation on every pair of elements is also an element of that set.

Please return to page $\frac{72}{2}$ and try question 11 again.

It's easy to be careless. Did you forget that "open" means that the result of performing the operation is NOT an element of the set? For example, the set

$$\{3, 5, 7\}$$

is open under <u>addition</u> because

$$3 + 5 \text{ is } 8$$

and 8 does not belong to the set.

Please return to page $\frac{82}{2}$ and try question 19 again.

$\frac{68}{1}$

Very good. You made the correct choice. The relation "is equal to" is reflexive.

The REFLEXIVE PROPERTY states that

$$a = a$$

or in other words, a quantity equals itself.

Please go on to question 3 below.

$\frac{68}{2}$

Question 3

Recognize which of the following choices demonstrates the SYMMETRIC PROPERTY OF EQUALITY.

(A) If $6 + 9 = x$, then $15 = x$

(B) $6 + 9 = 9 + 6$

(C) $(A + B) + C = A + (B + C)$

(D) If $a = b$, then $b = a$

Very good. Your choice parallels the form that illustrates the TRANSITIVE PROPERTY OF EQUALITY.

if	$3 + 7 = 10$	if	$a = b$
and	$\underline{\quad 10 = 8 + 2}$	and	$\underline{b = c}$
then	$3 + 7 = 8 + 2$	then	$a = c$

Now proceed to question 6 below.

Question 6

The logical relations are not restricted to algebra; they apply to other types of relationships, too.

Apply your knowledge and select the letter next to the relation that is transitive.

(A) Is the father of

(B) Likes

(C) Is included in

(D) Is a friend of

III

$\frac{70}{1}$

Your choice is correct.

In words we can say that the TRANSITIVE PROPERTY states:

"if the first quantity equals a second, and the second quantity equals a third; then the first quantity equals the third."

Please proceed to question 8 below.

$\frac{70}{2}$

Question 8

Apply your knowledge of the closure property of sets to the sets below and select the true statement.

(A) $S = \{1, 3, 5, 7, 9\}$ is closed under subtraction of the form $a - b$ where $a > b$.

(B) $T = \{1, 3, 4, 5, 6\}$ is closed under subtraction of the form $a - b$ where $a > b$.

(C) $U = \{1, 2, 3, 5, 6\}$ is closed under subtraction of the form $a - b$ where $a > b$.

(D) $V = \{2, 4, 6, 8, 10\}$ is closed under subtraction of the form $a - b$ where $a > b$.

We can test your choice by not taking two small even numbers that fit; like 2 times 6 equals 12 , but by testing two large numbers near the extreme end of the set; like 98 and 96 .

What is the product of 98 and 96 ?

Since 9408 is not an even number less than 100, we have found an exception.

Please return to page $\frac{80}{2}$ and try question 9 again.

You struck out on this one. Let us review the important points about this lesson.

In order for a set to be closed under some operation, every possible pair of elements (including an element used twice) must be tested to see whether the answer resulting from that operation is itself a member of the set. If it is, then the set is closed.

If there is an exception, even just one, then the set is said to be open.

Please return to page $\frac{86}{2}$ and reconsider this entire problem.

III

$\frac{72}{1}$

Congratulations! Your answer is correct.

The possible products of 0 and 1 are

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$

Since these answers are elements of the set, the set is closed under multiplication.

Please go on to question 11 below.

$\frac{72}{2}$

Question 11

Apply your knowledge of the closure properties of sets to the sets below and select the true statement.

(A) The set $\{1, 2, 4, 8, 16\}$ is closed under division.

(B) The set $\{1, 3, 5, 7, 9\}$ is closed under addition.

(C) The set $\{0, 1, 2, 3, \ldots\}$ is closed under addition.

(D) The set $\{1, 2, 4, 8, 16\}$ is closed under multiplication.

When the set contains the three dots . . .

It doesn't mean that the set is " open " under an operation.

It means the set has no end, but the succeeding elements are known to follow a certain pattern or rule.

For example, the multiples of 2 could be written as

2 , 4 , 6 , 8 , 10 , 12 , 14 (going on forever)

or as a set:

$$\{2 , 4 , 6 , 8 , . . .\}$$

Now with this in mind, reconsider problem 14 on page $\frac{125}{2}$

Did we confuse you? The question asked for the set that was OPEN under the operation of subtraction.

A set is open under the operation of subtraction if the result of subtracting two elements is NOT an element of the set. Remember a smaller number is to be subtracted from a larger. Here the same element can't be repeated.

Please return to page $\frac{84}{2}$ and try question 16 again.

$\frac{74}{1}$

If Arnold is a friend of Barbara, and Barbara is a friend of Charlie; it doesn't necessarily mean that Arnold is a friend of Charlie. They may not even know each other.

Therefore, "is a friend of" is not a true TRANSITIVE relation.

Please return to page $\frac{69}{2}$ and try this question again.

$\frac{74}{2}$

You are half right. The first set is not closed under multiplication; that's correct, but the second set is not closed under division either. For one thing,

$$\frac{1}{2} \div 2 =$$

$$\frac{1}{2} \times \frac{1}{2} =$$

$$\frac{1}{4}$$

Since $\frac{1}{4}$ is not an element of the set, the set is not closed.

Please return to page $\frac{81}{2}$ and choose the correct answer.

In mathematics there is a very important principle that you must keep in mind. If a statement is to be called true, then there can be no exception; not even <u>one</u> .

In the set you chose, there are many answers to the subtraction of the elements that are themselves members of the set. But in order for the set to be closed under subtraction, there can be no exception.

Did you try

6 - 2 ?

Please return to page 70/2 and try question 8 again.

Did we confuse you? The question asked for the set that was OPEN under the operation of addition.

A set is OPEN under addition if the sum of two elements is NOT a member of the set.

In the unlimited set you chose, the addition of any two elements will be one of the other elements of that set.

Please return to page 79/2 and try question 15 again.

III

$\frac{76}{1}$

If you can establish a rule for determining the results, you will be spared endless trials.

The product of two odd numbers is itself an odd number. Thus, every product of two odd numbers greater than three will also be an odd number greater than three. Your answer is correct.

Please go on to question 10 below.

$\frac{76}{2}$

Question 10

Apply your knowledge of the closure properties of sets to the sets below and select the true statement.

(A) The set $\{0, 1\}$ is closed under multiplication.

(B) The set $\{0, 1\}$ is closed under addition.

(C) The set $\{0, 1, 2, 3\}$ is closed under addition.

(D) The set $\{1, 2, 3, 4\}$ is closed under division.

Although each succeeding element is found by multiplying the previous element by 2 , if you test the multiplication of any two of the larger elements you will agree that the product is far beyond the limits of the given set.

Please return to page $\frac{72}{2}$ and try question 11 again.

You analyzed the second set correctly, but you missed something about the first set.

When any two even numbers are multiplied, their product is an even number. It is interesting to note that it is not necessary for all even numbers to be the product of other even numbers; just that the product of two even numbers is an even number and, therefore, the product is a number of the set. Thus, set I is closed under multiplication.

Please return to page $\frac{86}{2}$ and try question 13 again.

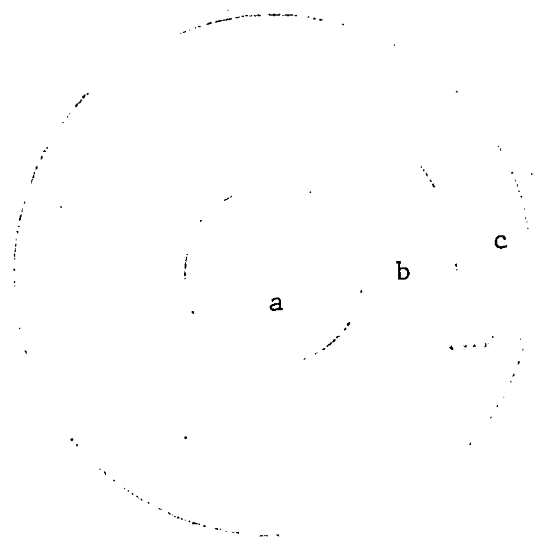
$\frac{78}{1}$

Very good. You made the correct choice. The relation

" is included in "

is always transitive. Consider the figure below:

If a is included in b , then the circle a must be inside the circle b . Furthermore, if b is included in c ; then circle b must be inside circle c . Thus, circle a is, of necessity, inside circle c .



Please go on to question 7 below.

$\frac{78}{2}$

Question 7

Apply the principle of equality and choose which form is an example of the TRANSITIVE PROPERTY.

(A)

$x = y + 2$

$\underline{w = y + 2}$

$y + 2 = y + 2$

(B) if $x = (a + b)$

and $\underline{(a + b) = y}$

then $x = y$

(C) if $x = y + 2$

and $\underline{y + a = z}$

then $x = z$

(D) if $x = y - 3$

and $\underline{y - 3 = a}$

then $y - 3 = x$

Your answer is correct in the set

$$\{0, 1, 2\}$$

When 2 is multiplied by itself, the result is 4, and 4 does not belong to the set

$$\{0, 1, 2\}$$

Please go on to question 15 below.

Question 15

Apply your knowledge of sets that are " open " or " closed " with respect to an operation and select the set below that is " open " under the operation of addition.

(A) $\{2, 4, 6, 8\}$

(B) $\{2, 4, 6, 8, \ldots\}$

(C) $\{1, 2, 3, 4, \ldots\}$

(D) $\{3, 6, 9, 12, \ldots\}$

$\frac{80}{1}$

Congratulations! Before you can state that a set [illegible]sed under an operation, you must either try every combination possible, or arrive at some observation that relates to all of the possible combinations.

In this case, since the set was a small one composed of even numbers; you can verify that when two different even numbers are subtracted the result is an even number smaller than the larger number.

Since this set contains all of the even numbers from 10 on down, every possible answer to the subtractions is contained [illegible] the set and is, therefore, truly closed.

Please proceed to question 9 below.

$\frac{80}{2}$

Question 9

Apply your knowledge of the closure property of sets to the sets below and select the true statement.

(A) The set $\{1, 3, 5, 7, 9\}$ is closed under multiplication.

(B) The set $\{$odd numbers greater than $3\}$ is closed under multiplicatio[illegible]

(C) The set of natural numbers between 1 and 7 is closed under multiplication.

(D) The set of even numbers less than 100 is closed under multiplication.

The three dots ... in the set indicate that the sequence of numbers continues according to the same rule of formation. This set represents all of the positive integers.

Congratulations! Your answer is correct. The result of adding two integers is another integer.

Please go on to question 12 below.

Question 12

Apply your knowledge of the closure property of sets to the statements below and select the letter with the proper truth value.

I. the set $\left\{\frac{1}{2}, 1, 2\right\}$ is closed under multiplication.

II. the set $\left\{\frac{1}{2}, 1, 2\right\}$ is closed under division.

(A) I. True
II. True

(B) I. True
II. False

(C) I. False
II. False

(D) I. False
II. True

$\frac{82}{1}$

When two even numbers are multiplied, their product is another even number. Thus, the set described in I is closed under multiplication. When the two elements 4 and 6 are added, their sum is 10. 10 is not an element of the set described in II, and thus; II is not closed under addition.

Thus, statement I is true, and II is false. Your answer is correct.

Please go on to question 14 below.

$\frac{82}{2}$

Question 14

Apply your knowledge of sets and the concept of closure to select the set below that is "open" under the operation of multiplication.

(A) $\{0, 1\}$

(B) $\{0, 1, 2\}$

(C) $\{0, 1, 2, \ldots\}$

(D) $\{2, 4, 6, \ldots\}$

If you were directed to substitute

$$\begin{array}{rcl} & a & \text{for } x, \\ & -b & \text{for } y, \\ \text{and} & 3 & \text{for } w \end{array}$$

then your substitution should have had only a's , b's , 3's , and other numbers in it.

Please return to page $\frac{89}{2}$ and try the problem again.

This choice suggests that it is correct to change the order of the numbers. The COMMUTATIVE PROPERTY states that this can be done in addition and multiplication; does it apply here?

This choice is not correct.

Please return to page $\frac{90}{2}$ and try question 1 again.

$\frac{84}{1}$

Congratulations!

A set is open under an operation if the result of performing an operation on two elements of the set is NOT an element of the set. Thus, since the sum of 4 and 6 is 10, and 10 is not an element of the set, the set

$$\{2, 4, 6, 8\}$$

is open under addition.

Proceed to question 16 below.

$\frac{84}{2}$

Question 16

Apply your knowledge of sets and the concept of closure to select the set below that is " open " under the operation of subtraction of the form

$$a - b \quad \text{where} \quad a > b$$

(A) $\{0, 1, 2, 3, \ldots\}$

(B) $\{0, 2, 4, 6, \ldots\}$

(C) $\{1, 3, 5, 7, \ldots\}$

(D) $\{2, 4, 6 \}$

85
1

The COMMUTATIVE PROPERTY FOR ADDITION states that the sum of two numbers does not change if you change the order of addition.

Or

$$\boxed{a + b = b - a}$$

Then this choice is not the <u>incorrect</u> one you were to locate.

Please return to page 96/1 and try question 2 again.

85
2

The ASSOCIATIVE PROPERTY states that you may group the numbers in any way in an addition example.

$$\boxed{\begin{array}{l} a + b + c = \\ (a + b) + c = \\ a + (b + c) \end{array}}$$

If you regroup the quantities properly, you will see that this choice is not correct.

Please return to page 110/2 and try question 4 again.

$\frac{86}{1}$

The product of $\frac{1}{2}$ and $\frac{1}{2}$ is $\frac{1}{4}$.

Since $\frac{1}{4}$ is not an element of set I , set I is not closed under multiplication.

When $\frac{1}{2}$ is divided by 2

the quotient is $\frac{1}{4}$

Since $\frac{1}{4}$ is not an element of set II , set II is not closed under division. Thus the statements that I and II are closed are false.

Your answer is correct.

Please go on to question 13 below.

$\frac{86}{2}$

Question 13

Apply your knowledge of the closure property of sets to the statements below and select the letter with the proper truth value.

I. the set $\{0, 2, 4, 6, \ldots\}$ is closed under multiplication.

II. the set $\{0, 2, 4, 6\}$ is closed under addition.

(A) I. True
II. True

(B) I. False
II. True

(C) I. True
II. False

(D) I. False
II. False

You chose

$$3a + 3 - 5b =$$
$$3 - 3b$$

but didn't you lose a pair of parentheses in the process? Wasn't there a negative sign with b ?

Please return to page $\frac{89}{2}$ and try the problem again.

Since the ASSOCIATIVE PROPERTY does not hold for the process of subtraction, it is necessary to perform the calculations inside the parentheses first. Thus, we get for x

$$(5 - 4) - 1 =$$
$$1 - 1 = 0$$

Apply this principle to the other statements and you will see why this choice is not correct.

Please return to page $\frac{109}{2}$ and try question 6 again.

$\frac{88}{1}$

It is certainly true that

$$1 \cdot 7 = 7 \cdot 1$$

However, this is not the only value that x might have which would make the statement true. Therefore, this choice is not correct.

Please return to page $\frac{105}{2}$ and try question 3 again.

$\frac{88}{2}$

The given problem

$$16 \div 4 \div 2$$

cannot be rearranged by taking the numbers in any order. Two rules are broken at once:

Division is not COMMUTATIVE; and

Division is not ASSOCIATIVE.

Without even referring to these rules, you will see that a scrambled order of operations will produce different answers for the same problem. Why don't you try this in this problem and see for yourself?

Please return to page $\frac{102}{2}$ and reconsider the problem.

Very good.

The result of subtracting two odd numbers is always an even number. Thus, the set of odd numbers (even though unlimited) is <u>open</u> under subtraction.

Please go on to question 17 below.

<u>Question 17</u>

If you are told to apply the substitution principle and to

<u>let</u> $x = a$

$y = -b$

$w = 3$

in the following expression:

$$3 (x + 3) - 5y = w - 3y$$

Choose the correct result.

(A) $w (a + w) - 5 (-b) = 3 - w (-b)$

(B) $3 (a + 3) - 5 (-b) = 3 - 3 (-b)$

(C) $3a + 3 - 5b = 3 - 3b$

(D) $3 (a + 3) - 5 - b = 3 - 3 - b$

III

Volume 3 Segment 3 begins here:

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	1 3	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 3	(Volume Number)
	66	and	68	0 3	(Segment Number)

Your reading assignment for this Segment - page 73 - 77 .

SUPPLEMENTARY NOTES:

There are three properties of arithmetic whose names you must learn in addition to learning the properties themselves.

(1) For addition and for Multiplication:

COMMUTATIVE PROPERTY: the order of the numbers may be changed.

(2) For Addition and Multiplication:

ASSOCIATIVE PROPERTY: the groups of numbers may be changed.

Before proceeding to the third property, let us consider this:

The basic operations of arithmetic fit into two different levels--addition and subtraction, on one level, are the simplest: multiplication and division, on a higher level, are more complex operations.

Now let us examine the next property.

(3) For a combination of two operations on different levels:

DISTRIBUTIVE PROPERTY: multiplication (or division) is distributive over addition (or subtraction)

Please turn to page 91 .
1

It is simple to remember the distributive law by learning that you should "distribute the multiplier."

That is, in the problem

$$a(x + y)$$

we distribute or use the multiplier <u>a</u> with each term inside the parentheses, getting

$$ax + ay$$

The statement of these properties and the illustrations you have found in your text should be listed in your notebook.

You will now be asked a series of questions to draw your attention to the more important points.

$\frac{91}{2}$

<u>Question 1</u>

Which of the statements do you recognize is always true?

(A) $a - 5 = 5 - a$

(B) $a + 2 = 2 + a$

(C) $a \div 5 = 5 \div a$

(D) $8 \div 2 = 2 \div 8$

$\frac{92}{1}$

The COMMUTATIVE PROPERTY FOR MULTIPLICATION states that the product of two numbers does not change if you change the order of multiplication.

or

$$\boxed{a \cdot b = b \cdot a}$$

Then this choice is not the <u>incorrect</u> one you were to locate.

Return to page $\frac{96}{2}$ and try question 2 again.

$\frac{92}{2}$

In calculating the value of z, we perform the operations as we meet them, getting

$$z = 5 - 4 - 1 = 1 - 1 = 0$$

For the other two expressions, we recognize that the ASSOCIATIVE PROPERTY does not hold in subtraction, and we perform first the operations inside the parentheses.

When we do this, we find that this choice is not correct.

Return to page $\frac{109}{2}$ and try qyestion 6 again.

When you are to substitute a negative value, such as

" -b " for y

you must be careful that you haven't accidentally changed the negative sign into a subtraction sign. They do look alike; but they have different meanings.

In this problem; for example, in the part of the expression where you had to substitute -b for y in the expression

$$-3y$$

you wrote $-3 - b$

(which means that -3 and -b are to be added algebraically, or that you are to subtract b from -3)

But doesn't -3y really mean that -3 is to multiply y or the value for y ? To avoid this mistake a pair of parentheses are used in this manner.

$$-3y \text{ becomes } -3(-b)$$

Please return to page $\frac{89}{2}$ and reconsider problem 17.

The COMMUTATIVE PROPERTY FOR MULTIPLICATION states that the product of two numbers does not depend on the order of the numbers. Since there is no change in the order of multiplication in this example, this choice is not correct.

Please return to page $\frac{107}{2}$ and try question 8 again.

$\frac{94}{1}$

This statement is certainly true for any whole value of x, but isn't it true for fractions?

For example:

$$\frac{1}{2} \cdot 7 = 7 \cdot \frac{1}{2}$$

Since you limited the possible answers unnecessarily, your choice is not correct.

Please return to page $\frac{105}{2}$ and try question 3 again.

$\frac{94}{2}$

Since the first operation involved changing the order of two numbers, it cannot involve the ASSOCIATIVE PROPERTY. The ASSOCIATIVE PROPERTY involves grouping of numbers in a calculation.

Then this choice is not correct.

Please return to page $\frac{115}{2}$ and try question 9 again.

The ASSOCIATIVE law does <u>not</u> apply to division; that is,

$$(a \div b) \div c \quad \text{does NOT equal} \quad a \div (b \div c)$$

A good reason is that the same problem would have different answers; for example,

$$\underbrace{16 \div 4}_{} \div 2$$
$$4 \div 2$$
$$2$$

$$16 \div \underbrace{4 \div 2}_{}$$
$$16 \div 2$$
$$8$$

In a problem such as

$$a \div b \div c$$

the proper procedure is to perform the operations one at a time from left to right.

Please return to page $\frac{102}{2}$ and reconsider the problem.

You had to apply the DISTRIBUTIVE PROPERTY

$$\boxed{a(b + c) = ab + ac}$$

where $a = p$

and $b = 2$

Since p^2 means $p \cdot p$ rather than $p \cdot 2$, this choice is not correct.

Please return to page $\frac{108}{2}$ and try question 10 again.

$\frac{96}{1}$

The COMMUTATIVE PROPERTY FOR ADDITION states that the order of the two numbers in an addition may be changed without changing the result. Or

$$\boxed{a + b = b + a}$$

Therefore, this choice is correct.

Please proceed to question 2 below.

$\frac{96}{2}$

Question 2

Which of the statements do you recognize is NEVER true?

(A) $5 + p = p + 5$

(B) $5 \cdot p = p \cdot 5$

(C) $6 - 2 = 2 - 6$

(D) $6 \div 2 = 2 + 1$

Very good. You realize that when substituting given values, you must retain the parentheses in the original expression and use others where a misunderstanding in operations might result.

You have now finished Segment 2 . Hand in the PUNCH CARD. You should have entered in your NOTEBOOK the following definitions and formulas.

Properties of Equality.

REFLEXIVE		x = x
SYMMETRIC	if	a = b
	then	b = a
TRANSITIVE	if	a = b
	and	b = c
	then	a = c

Open set under an operation.

Closed set under an operation.

You should be able to complete the HOMEWORK ASSIGNMENT: through problem 4 .

$\frac{98}{1}$

Well it is true that

$$x = 0$$

then $\quad x \cdot 7 = 7 \cdot x$

would become $\quad 0 \cdot 7 = 7 \cdot 0$

which is a true statement.

However, can't you think of other possible values for x ?

Please return to page $\frac{105}{2}$ and try question 3 again.

$\frac{98}{2}$

The ASSOCIATIVE PROPERTY does not apply in subtraction. Hence, in calculating x and y, it is necessary to perform first those operations inside the parentheses. Since there are no parentheses in the expression for z, we perform the operations in the order in which we meet them. If you do a careful job of calculation, you will find that they are not all equal values, and that this choice is not correct.

Please return to page $\frac{109}{2}$ and try question 6 again.

This question deals with the application of the ASSOCIATIVE PROPERTY of addition.

$$\boxed{a + b + c = (a + b) + c = a + (b + c)}$$

Using it, we get:

$P = x + (7 + 2)$

or $P = x + 9$

Also $Q = x + 9$

And $R = x + (7 + 2) =$

$x + 9$

Then P, Q, and R are all equal quantities.

Please proceed to question 5 below.

Question 5

Perform the calculations and select the correct statement:

$P = x \cdot (2 \cdot p) \quad Q = (x \cdot 2) \cdot p \quad R = x \cdot (p^2)$

(A) only P and Q are equal

(B) only P and R are equal

(C) only Q and R are equal

(D) all three are equal

$\frac{100}{1}$

Since the result of division on the left side is 3 , and the result of the addition on the right side is also 3 , this choice is not the incorrect one you were to locate.

Return to page $\frac{96}{2}$ and try question 2 again.

$\frac{100}{2}$

The ASSOCIATIVE PROPERTY refers to grouping of the quantities.

Since there is no grouping involved in this problem, this choice is not correct.

Return to page $\frac{107}{2}$ and try question 8 again.

RIBUTIVE PROPERTY involves two operations of different levels

.n as multiplication and additio.

For example:

$$\boxed{a(b + c) = ab + ac}$$

Since this calculation did not have more than a single type of operation, it cannot involve the DISTRIBUTIVE PROPERTY.

Then this choice is not correct.

Please return to page $\frac{115}{2}$ and try question 9 again.

Since this problem involves both multiplication and addition, it is necessary to use the DISTRIBUTIVE PROPERTY. That is,

$$\boxed{a(b + c) = ab + ac}$$

Note:

$a(b + c)$ does not equal abc

This choice is not correct.

Please return to page 108 and try question 10 again.

$\frac{102}{1}$

Since the ASSOCIATIVE PROPERTY does not hold for subtraction, it is nece... ary to perform first that calculation inside the parentheses. Thus,

$$x = (5 - 4) - 1$$

$$1 - 1 = 0$$

Also

$$y = 5 - (4 - 1) =$$

$$5 - 3 = 2$$

Since the expression for z contains no parentheses, we must perform the calculations as we meet them, getting

$$z = 5 - 4 - 1 =$$

$$1 - 1 = 0$$

It, therefore, follows that y is the largest value; and this choice is correct.

Please proceed to question 7 below.

$\frac{102}{2}$

Question 7

Apply the proper principle and choose which operation in the given problem should be performed first.

$$16 \div 4 \div 2$$

(A) $4 \div 2$

(B) $16 \div 4$

(C) $16 \div 2$

(D) It makes no difference.

Since this problem involves both subtraction and division, the DISTRIBUTIVE PROPERTY should be applied. However, you did not distribute the divisor.

$$\frac{a - b}{c}$$

can be considered as

$$\frac{1}{c}(a - b)$$

which equals

$$\frac{1}{c}a - \frac{1}{c}b$$

Please return to page $\frac{119}{2}$ and try question 11 again.

According to this problem we are looking for a number which when multiplied by any quantity, does not change that quantity. We call this the identity element for multiplication.

However, zero is not that identity element.

What does $0 \cdot x$ equal? You have mixed two ideas together; this choice is not correct.

Please return to page $\frac{129}{2}$ and try question 13 again.

$\frac{104}{1}$

Since

$$p^2 = p \cdot p$$

it is not always equal to

$$2 \cdot p$$

The only exceptions are:

when $p = 0$, or $p = 2$

because $0^2 = 2 \cdot 0$

$2^2 = 2 \cdot 2$

Then this choice is not correct.

Please return to page $\frac{99}{2}$ and try question 5 again.

$\frac{104}{2}$

The DISTRIBUTIVE PROPERTY involves two different levels of operations in one problem. Since this problem has only one operation, multiplication; this choice is not correct. The form must be

$$a\ (b \pm c)$$

to apply the DISTRIBUTIVE PROPERTY.

Please return to page $\frac{124}{2}$ and try question 12 again.

Since the COMMUTATIVE PROPERTY does not apply to subtraction, the statement is not true.

That is

$$a - b \neq b - a$$

Then this choice is the correct choice.

Proceed to question 3 below.

Question 3

If $x \cdot 7 = 7 \cdot x$, which do you recognize as the true statement?

(A) x must be 1

(B) x must be an integer

(C) x must be any number

(D) x must be zero

$\frac{106}{1}$

It is true that

$$1 + x =$$

$$x + 1$$

but how can you be sure that 1 is the value of m ?

Couldn't it have another value?

This choice is not correct.

Return to page $\frac{125}{2}$ and try question 14 again.

$\frac{106}{2}$

Applying the DISTRIBUTIVE PROPERTY,

$$p(x + m) =$$

$$px + pm$$

Then, if

$$m = 1$$

we have

$$px + p$$

which does not equal px .

Then this choice is not correct.

Return to page $\frac{130}{2}$ and try question 15 again.

The ASSOCIATIVE PROPERTY does not apply to division. Therefore, it is necessary to perform the calculations as written from left to right. That is,

$$16 \div 4 \div 2$$

becomes

$$(16 \div 4) \div 2$$

$$4 \div 2$$

$$2$$

Then this choice is correct.

Please proceed to question 8 below.

Question 8

Choose the property illustrated in the statement:

$$p + 5x = 5x + p$$

(A) COMMUTATIVE PROPERTY for multiplication

(B) COMMUTATIVE PROPERTY for addition

(C) ASSOCIATIVE PROPERTY for multiplication

(D) ASSOCIATIVE PROPERTY for addition

$\frac{108}{1}$

Since the first step in the calculation is changing the order of two numbers in multiplication; e. g.,

$$\boxed{a \cdot b = b \cdot a}$$

This illustrates the COMMUTATIVE PROPERTY FOR MULTIPLICATION. The second step grouped the numbers differently; e. g.,

$$\boxed{(a \cdot b)c = a(b \cdot c)}$$

And, therefore, involves the ASSOCIATIVE PROPERTY FOR MULTIPLICATION.

Then this choice is correct.

Please proceed to question 10 below.

$\frac{108}{2}$

<u>Question 10</u>

Perform the calculation and select the result:

$$p(2 + w)$$

(A) $p^2 + pw$

(B) $2p + wp$

(C) $2wp$

(D) $p2w$

In this question, we apply the ASSOCIATIVE PROPERTY and then the COMMUTATIVE PROPERTY. Thus, we have:

$$P = x \cdot (2 \cdot p) = (x \cdot 2) \cdot p = (2 \cdot x) \cdot p = 2xp$$

$$Q = (x \cdot 2) \cdot p = (2 \cdot x) \cdot p = 2xp$$

However, $R = x \cdot (p^2) = x \cdot p \cdot p = xp^2$

Then R is not the same value as the other two quantities, and this choice is correct.

Please proceed to question 6 below.

Question 6

Perform the calculations and select the largest value:

$x = (5 - 4) - 1 \qquad y = 5 - (4 - 1) \qquad z = 5 - 4 - 1$

(A) x

(B) y

(C) z

(D) all three are equal

$\frac{110}{1}$

The COMMUTATIVE PROPERTY states that the product of any two numbers is the same regardless of the order of multiplication.

that is

$$\boxed{a \cdot b = b \cdot a}$$

Therefore, in this example,

a is replaced by x

and b is replaced by 7

and the rule becomes

$x \cdot 7 = 7 \cdot x$

Therefore, this choice is correct.

Please proceed to question 4 below.

$\frac{110}{2}$

Question 4

If

$P = (x + 7) + 2$ and $Q = x + (7 + 2)$ and $R = x + 7 + 2$

choose the correct statement.

(A) P is the largest quantity

(B) P and Q are not equal

(C) P and Q are equal, but R has a different value

(D) P , Q , and R are equivalent

Since this problem involves both multiplication and addition, it is necessary to use the DISTRIBUTIVE PROPERTY.

That is,

$$\boxed{a(b + c) = ab + ac}$$

NOTE:

$$a(b + c) \neq abc$$

Moreover, it is not correct form in algebra to write p2w . When the product of two letters and one number is being written, the number is put first. Thus, it would be written

2pw or 2wp

However, this choice is not correct no matter how it is written.

Please return to page $\frac{108}{2}$ and try question 10 again.

This calculation involves two different operations: multiplication and division. Since they are not different levels of operation (see the Supplementary Notes), the DISTRIBUTIVE PROPERTY does not apply unless the form is equivalent to:

$$a(b \pm c)$$

Please return to page $\frac{124}{2}$ and try question 12 again.

$\frac{112}{1}$

How did you get a single term in the result of dividing two different terms by the same quantity? Since the terms could not be combined before doing the division, it is impossible to combine the results after division.

Therefore, this choice is not correct.

Return to page $\frac{119}{2}$ and try question 11 again.

$\frac{112}{2}$

Applying the DISTRIBUTIVE PROPERTY,

$$p(x + m) =$$
$$px + pm$$

Then we are looking for a value of m such that

$$px + pm =$$
$$px$$

How many different values did you try for m ?

This choice is not correct.

Return to page $\frac{130}{2}$ and try question 15 again.

We are looking for a number which when multiplied by any quantity, does not change that quantity. This is called the identity element for multiplication.

Since there is such a number this choice is not correct.

Return to page $\frac{129}{2}$ and try question 13 again.

$\frac{113}{2}$

You have done a fine job of combining terms and keeping separate terms which could not be combined, but you did slip up on one detail. Can you find it?

Return to page $\frac{123}{2}$ and try question 16 again.

$\frac{114}{1}$

It is true

$$0 + x = x + 0$$

but how can you be sure that 0 is the value of m ?

Couldn't it have another value?

This choice is not correct.

Please return to page $\frac{125}{2}$ and try question 14 again.

$\frac{114}{2}$

This problem involves a double division.

The longer fraction line tells us that the fraction

$$\frac{24y - 72}{12}$$

is to be divided by 2 . We apply the DISTRIBUTIVE PROPERTY to the fraction, and then apply it again to the final division by 2 .

However, we do not get this choice.

Please return to page $\frac{139}{2}$ and try question 18 again.

The COMMUTATIVE PROPERTY for addition states that the sum of two numbers does not depend on the order of the numbers. Since this problem concerns the addition of p and 5x with a change of order, this illustrates the COMMUTATIVE PROPERTY for addition.

$$\boxed{a + b = b + a}$$

NOTE: This property is not to be confused with the SYMMETRIC PROPERTY which states: (in one form)

if $a + b = c + d$

then $c + d = a + b$

Then this choice is correct.

Please proceed to question 9 below.

Question 9

The following calculation uses two properties of arithmetic operations:

$$(4 \cdot 378) \cdot 25 =$$
$$(378 \cdot 4) \cdot 25 =$$
$$378 \cdot (4 \cdot 25)$$

:ply your knowledge to select the properties in the order in which they were used.

(A) associative, commutative

(B) commutative, distributive

(C) associative, distributive

(D) commutative, associative

$\frac{116}{1}$

The first step in this problem is to apply the DISTRIBUTIVE PROPERTY

$$3(x + 2)$$

Did you make a mistake at this point?

You should have gotten

$$3x + 6$$

Then combine like terms. You have an error somewhere, since this choice is not correct.

Please return to page $\frac{134}{2}$ and try question 17 again.

$\frac{116}{2}$

Since the REFLEXIVE PROPERTY states that a quantity is equal to itself, and the expression in II is not exactly the same as the expression in I ; this choice is not correct.

The operation actually involved an application of the DISTRIBUTIVE PROPERTY.

Please return to page $\frac{121}{2}$ and try question 20 again.

This calculation involves two different operations: division and addition.

Since they are different levels of operation (see the Supplementary Notes) , the DISTRIBUTIVE PROPERTY should be applied. However, you do not appear to have distributed the divisor.

$$\frac{a + b}{c} = \frac{1}{c}(a + b)$$

$$= \left(\frac{1}{c}\right)a + \left(\frac{1}{c}\right)b$$

$$= \frac{a}{c} + \frac{b}{c}$$

This choice is not correct.

Please return to page $\frac{124}{2}$ and try question 12 again.

Applying the DISTRIBUTIVE PROPERTY,

$$p(x + m) =$$

$$px + pm$$

Then we are looking for a value of m such that

$$px + pm = px$$

What value would pm have if that were true?

There is a value possible; this choice is not correct.

Please return to page $\frac{130}{2}$ and try question 15 again.

$\frac{118}{1}$

You have reached a decision without much arithmetic.

Have you tried to find a value for x which would make the statement true?

Why even,

$$7 + x = x + 7$$

for example.

This choice is not correct.

Return to page $\frac{125}{2}$ and try question 14 again.

$\frac{118}{2}$

You have done an excellent job of combining like terms, but is it not possible to combine <u>unlike</u> terms?

This choice is not correct.

Return to page 1[illegible]3 / [illegible] and try question 16 again.

The DISTRIBUTIVE PROPERTY tells you to distribute the multiplier p.

This gives

$$p \cdot 2 + p \cdot w$$

Then, using the COMMUTATIVE PROPERTY FOR MULTIPLICATION, we get

$$2p + wp$$

and this choice is correct.

Please proceed to question 11 below.

Question 11

Perform the calculation and then select the correct result:

$$\frac{15x - 9}{3}$$

(A) $5x - 9$

(B) $15x - 3$

(C) $2x$

(D) $5x - 3$

$\frac{120}{1}$

We are looking for a nubmer which when multiplied by any quantity, does not change the quantity. This is called the identity element for multiplication. There is such a number, but only one.

Therefore, this choice is not correct.

Return to page $\frac{129}{2}$ and try question 13 again.

$\frac{120}{2}$

No, your choice is incorrect.

The statement:

if $5 + 2 = 7$ and $7 = 11 - 4$ then $5 + 2 = 11 - 4$

illustrates the TRANSITIVE PROPERTY OF EQUALITY. This is shown in symbolic form:

if	a	=	b
and	b	=	c
then	a	=	c

But you were asked to pick the example that illustrates the ADDITION PROPERTY OF EQUALITY.

Return to page $\frac{134}{1}$ and try this question again.

This problem involves double division. The longer fraction line tells us that the fraction

$$\frac{\frac{24y - 72}{12}}{2}$$

is to be divided by . Applying the DISTRIBUTIVE PROPERTY to the fraction, we get its value to be

$$2y - 6$$

Then dividing that and again applying the DISTRIBUTIVE PROPERTY, we get

$$y - 3$$

Therefore, this choice is correct. You must realize that the divisions cannot be performed in any order that you please the length of the fraction lines tells you the order. If the lengths had been reversed, it would have been a different problem.

Please proceed to question 19 below.

Question 19

In simplifying the expression

$$5 (x + 3) + x)$$

the successive steps are:

I. $5 (x + 3) - 3 x)$ IV. $5x + (3x + 15)$

II. $(5x + 15) - 3x$ V $(5x + 3x) + 15$

III. $5x + (15 + 3x)$ VI. $8x + 15$

Apply your knowledge to select the correct choice below.

(A) from step I to step II , the REFLEXIVE PROPERTY was applied.

(B) from step II to step III , the DISTRIBUTIVE PROPERTY was applied.

(C) from step II to step IV , the DISTRIBUTIVE PROPERTY was applied.

(D) from step IV to step V , the ASSOCIATIVE PROPERTY was applied.

$\frac{122}{2}$

Your choice is an example of the COMMUTATIVE PROPERTY OF ADDITION, which states that

$$x + y = y + x$$

Please return to page $\frac{142}{2}$ and try this question again.

$\frac{22}{2}$

We don't agree.

If you are given an equation of the form

$$x - a = b$$

in order to find x we must remove -a from that side of the equation. We must use the inverse operation and add a to both sides. But this is not the solution set; the final value of x is the answer.

Please return to apge $\frac{145}{2}$ and try this question again.

It is certainly true that

$$x + 0 = x$$

and thus it follows that

$$p(x + 0) = px$$

Then this choice is true.

You should see that the original problem involved the DISTRIBUTIVE PROPERTY.

Thus

$$p(x + m) = px + pm$$

Now, since we wanted the result to be px, we are looking for a value of m which will make

$$pm = 0$$

This indicates that m must equal zero, since px exists

$$p \neq 0$$

Of course, this gives us the same result, but this suggests a procedure which would lead to the answer, rather than trying to check the values offered to us.

Please proceed to question 16 below.

Question 16

Apply the DISTRIBUTIVE PROPERTY to combine similar terms in the expression:

$$5m + m + 7p + q - 2m$$

(A) $6m + 7p + q - 2m$

(B) $4m + 8pq$

(C) $4m + 7p + q$

(D) $5m^2 + 7p + q - 2m$

III

$\frac{12-}{1}$

Since this problem involves both subtraction and division, the DISTRIBUTIVE PROPERTY should be applied.

$$\frac{a-b}{c} = \frac{1}{c}(a-b) \qquad \frac{15x-9}{3} = \frac{1}{3}(15x-9)$$

$$= \frac{1}{c}a - \frac{1}{c}b \qquad = \frac{1}{3}(15x) - (\frac{1}{3})9$$

$$= \frac{a}{c} - \frac{b}{c} \qquad = 5x - 3$$

When you distribute the divisor, you do get the result

$$5x - 3$$

Then this choice is correct.

Please proceed to question 12 below.

$\frac{124}{2}$

Question 12

Apply your knowledge to select the correct application of the DISTRIBUTIVE PROPERTY.

(A) $5(3 \cdot 7) = 15 \cdot 35$

(B) $5(7 - 3) = 35 - 15$

(C) $\frac{8 \cdot 12}{4} = 2 \cdot 3$

(D) $\frac{8 + 12}{4} = 2 + 12$

According to this problem we are looking for a number which when multiplied by any quantity, does not change that quantity. We call this the identity element for multiplication, and its value is 1 .

That is:

$$\boxed{a \cdot 1 = a}$$

Please proceed to question 14 below.

Question 14

Apply your knowledge to determine the value of m

if $m + x = x + m$

(A) 1

(B) 0

(C) no value is possible

(D) any value is possible

You handled the DISTRIBUTIVE PROPERTY alright, but after combining 5x and 3x the parentheses can be dropped; and the remaining similar x term also combined.

This choice is not correct.

Please return to page $\frac{134}{2}$ and try question 17 again.

Your choice is an example of the SYMMETRIC PROPERTY OF EQUALITY which in symbolic form is expressed this way:

if $a = b$

then $b = a$

However, you were asked to choose an example of the ADDITION PROPERTY OF EQUALITY which states:

If the same number is added to equal numbers, the sums are equal.

Please return to page $\frac{136}{1}$ and try this question again.

From step II to step III , there has been a change only in the grouping of the quantities being added. The property used was the ASSOCIATIVE PROPERTY. The COMMUTATIVE PROPERTY refers to a change of order of the numbers.

Then this choice is not correct.

Return to page $\frac{121}{2}$ and try question 20 again.

Your choice is incorrect.

It appears that you performed a division. Let us do a similar question together. Find the solution set of

$$x - 13 = 18$$

In this equation, we know the value of

$$x - 13$$

How do we find the value of x ?

By the ADDITION PROPERTY OF EQUALITY, we obtain an equivalent equation by adding the same number to both sides of the given equality. We add the inverse of

$$-13$$

that is, we add $+13$

Thus, we get

$$x - 13 + 13 = 18 + 13$$

and

$$x = 18 + 13$$

$$x = 31$$

Return to page $\frac{145}{2}$ and try this question again.

$\frac{128}{1}$

Combining

$$5m + m$$

the DISTRIBUTIVE PROPERTY gives us

$$(5 + 1)m$$

which is

$$6m$$

It is extrememly important for you to remember that in addition, terms can be combined only when their letter parts are <u>identical</u>, and that the letter parts never change in addition.

This choice is not correct.

Return to page $\frac{123}{2}$ and try question 16 again.

$\frac{128}{2}$

Sorry, your choice is incorrect.

This is an example of a mistake in the application of the addition property.

$$3b + 3 \neq 6b$$

If

$$a = 3b$$

then

$$a + \underline{3} =$$

$$3b + \underline{3}$$

Please note that if we have an equality, then we maintain equality by adding the same number to the equal numbers.

Return to page $\frac{142}{2}$ and try this question again.

The DISTRIBUTIVE PROPERTY involves two different levels of operations in one problem. Since this problem contains both multiplication and subtraction, which are on different levels of operation (see Supplementary Notes) , the DISTRIBUTIVE PROPERTY should be applied.

$$\boxed{a(b + c) = ab + ac} \qquad \text{here:}\quad \begin{array}{rcl} 5 & = & a \\ 7 & = & b \\ -3 & = & c \end{array}$$

$$5(7 - 3) = 5 \cdot 7 + 5(-3)$$

$$= 35 - 15$$

Then this choice is correct.

Please proceed to question 13 below.

Question 13

Apply your knowledge to determine the value of m

if $m \cdot x = x$

(A) 1

(B) 0

(C) no value is possible

(D) any value is possible

$\frac{130}{1}$

This problem concerns an addition of two numbers where we want the same total regardless of the order of addition. But the COMMUTATIVE PROPERTY OF ADDITION says that this is so for any two numbers.

That is:

$$\boxed{a + b = b + a}$$

Therefore, this choice is correct.

Please proceed to question 15 below.

$\frac{130}{2}$

Question 15

Apply your knowledge to determine the value of m

if $p(x + m) = px$

(A) 0

(B) 1

(C) any value is possible

(D) no value is possible

Not so!

Each of the other three choices is an example of a fundamental principle.

One of the letters does have the correct answer next to it.

Return to page $\frac{136}{1}$ and try this question again.

There are subtraction signs in the statements you chose, but your choice is an example of the TRANSITIVE PROPERTY OF EQUALITY.

Please note that the subtraction property requires the subtraction of the same number from two equal numbers.

This can be expressed in symbolic form:

$$\text{if} \quad a = b$$

$$\text{then} \quad a - c = b - c$$

Return to page $\frac{149}{2}$ and try this question again.

$\frac{132}{1}$

From step III to step IV there has been a change in the order of addition of the quantities which is COMMUTATIVE PROPERTY.

The DISTRIBUTIVE PROPERTY involves a combination of operations, which were not part of step III .

This choice is not correct.

Please return to page $\frac{121}{2}$ and try question 20 again.

$\frac{132}{2}$

If

$$x - \frac{1}{2} = \frac{1}{4}$$

and we are to solve for x we must isolate x by eliminating the $-\frac{1}{2}$

We choose the additive inverse $+\frac{1}{2}$ and apply the ADDITION PROPERTY OF EQUALITY, we have

$$x - \frac{1}{2} + \frac{1}{2} =$$

$$\frac{1}{4} + \frac{1}{2}$$

Please continue.

Please return to page $\frac{138}{2}$ and try this question again.

Your choice is a true statement, but you actually used the substitution principle.

That is, if $p = 2q$

and $q = 5$

when we replace q by 5 and multiply, then

$$p = 10$$

However, this is not an example of the ADDITION PROPERTY OF EQUALITY, which you were asked to locate.

Return to page $\frac{142}{2}$ and try this question again.

We do not agree.

One of the letters does have the correct answer next to it. Please reconsider your choice.

Apply the addition property to eliminate the -7 .

Return to page $\frac{145}{2}$ and try this question again.

134
1

The DISTRIBUTIVE PROPERTY is symmetric and, therefore, can be written and used in reverse order. That is:

$$a(b + c) = ab + ac$$

can also be written as:

$$ab + ac = a(b + c)$$

or by using the COMMUTATIVE PROPERTY,

$$ba + ca = (b + c)a$$

Now this has an interesting application in the combining of like terms.

In the expression $3x + 2x$

We can let: $a = x$

$b = 3$

$c = 2$ in the above rule.

Therefore,

$$3x + 2x =$$
$$(3 + 2)x =$$
$$5x$$

In the choice you made on this problem,

$5m + m - 2m$ can be treated in this way.

$$(5 + 1 - 2)m =$$
$$4m$$

Since $7p$ and q don't have a common factor, the SYMMETRIC DISTRIBUTIVE property cannot be used and these terms cannot be combined. Your choice was, therefore, correct.

Please proceed to tne next question below.

134
2
Question 17

Apply your knowledge to express as simply as possible:

$$5x + 3(x + 2) - 2x$$

(A) $6x + 2$ (C) $(8x + 2) - 2x$

(B) $3(x + 2) + 3x$ (D) $6x + 6$

VOLUME 3 SEGMENT 4 begins here:

Obtain a PUNCH CARD from your instru ctor addition to the other identifying information that must be ... by you, you are asked to punch out the following:

COLUMNS	48	and	50	1 -	(Seque ce Nu ber)
	54	and	5-	0 4	(Type of ... Card)
	60	and	62	0 3	(Volume Numbe
	66	and	6-	0 4	(Segment Numb

Your READING ASSIGNMENT for this Segment is pgs: 8 - 82 in Dolciani.

SUPPLEMENTARY NOTES:

If you were ever in an old-fashioned grocery store, you would have noticed a balance scale consisting of two pans and some metal weights. These weights can be placed on either or both pans. Clearly, if the scale is in balance, the weights on each of the pans are equal. This situation resembles an equation. If you add a weight on one pan, an equal weight must be added to the other pan in order to maintian balance. Similarly, an equality can be maintained by adding the same amount to both sides of the equation.

In this segment, we will study the ADDITION and the SUBTRACTION PROPERTIES OF EQUALITY. We will learn how these properties can be used to find the solution set of certain equations, and how we can apply this knowledge to solve problems.

You will now be asked a series of questions to draw your attention to the more important points.

Turn the page for Question 1.

TTT

$\frac{136}{1}$

Question 1

Recognize the ADDITION PROPERTY OF EQUALITY and select the letter next to the statement which illustrates it.

Place each equation on a separate line; for example,

$$17 = (13 + 4)$$
$$17 + 5 = (13 + 4) + 5$$

(A) If $5 + 2 = 7$
and $7 = 11 - 4$
then $5 + 2 = 11 - 4$

(B) If $11 = 6 + 5$
then $6 + 5 = 11$

(C) If $17 = (13 - 4)$
then $17 + 5 = (13 + 4) + 5$

(D) None of these.

$\frac{136}{2}$

You are confusing the DISTRIBUTIVE PROPERTY OF MULTIPLICATION with respect to subtraction with the SUBTRACTION PROPERTY OF EQUALITY.

The SUBTRACTION PROPERTY OF EQUALITY can be expressed in symbolic form this way:

if $a = b$
then $a - c = b - c$

Please return to page $\frac{149}{2}$ and try this question again.

From step I to II , the DISTRIBUTIVE PROPERTY was applied.

From step I to III , the ASSOCIATIVE PROPERTY OF ADDITION was used.

From step III to IV , the problem illustrates the COMMUTATIVE PROPERTY OF ADDITION.

From IV to V , we use the ASSOCIATIVE PROPERTY OF ADDITION once more.

From V to VI , we have applied the substitution principle.

Therefore, this choice is correct.

You have now finished this Segment. Hand in the Punch Card.
Be sure that you have entered in your notebook the items listed in the Supplementary Notes at the beginning of this Segment.

You should now be able to complete Assignment 3 , problems 9 - 12 .

$\frac{138}{1}$

You made the correct choice.

How do we find the solution set of the equation

$$n - 7 = 14 \quad ?$$

We know that $n - 7$

is equal to 14

and we want to find the value of

$$n$$

By the ADDITION PROPERTY OF EQUALITY, we obtain an equivalent equation by adding 7 to both sides of the given equality. Thus, we have

$$n - 7 + 7 = 14 + 7$$

Now -7

and $+7$

are inverses and their sum is equal to zero.

Hence, we get $n = 14 + 7$

or $n = 21$

Please go on to question 5 below.

$\frac{138}{2}$

Question 5

Apply the principles of equality and find the solution set for x in the open sentence $x - \frac{1}{2} = \frac{1}{4}$

Select the letter next to the correct value for x .

(A) $\left\{ \frac{1}{2} \right\}$ (C) $\left\{ \frac{3}{4} \right\}$

(B) $\left\{ \frac{5}{4} \right\}$ (D) $\left\{ -\frac{1}{4} \right\}$

You are generally told by us to show all your work and not to skip steps because this is the surest way to avoid making mistakes. However, here we are going to surprise you. Let's examine the steps:

$5x + 3(x + 2) - 2x$	Distributive Law
$5x + 3x + 6 - 2x$	Commutative Law
$5x + 3x - 2x + 6$	Associative Law
$(5x + 3x - 2x) + 6$	Symmetric Distributive Law
$(5 + 3 - 2)x + 6$	Addition
$(8 - 2)x + 6$	Subtraction
$6x + 6$	

You should be able to justify every little step taken as above, but you are advised to actually do the entire set of steps mentally and reduce the written operation to just two steps:

$5x + 3(x + 2) - 2x$	Distributive Law
$5x + 3x + 2 - 2x$	" Combine Like Terms "
$6x + 6$	

Please proceed to question 18 below.

Question 18

Apply your knowledge to find the choice which has the same value as:

$$\frac{\frac{24y - 72}{12}}{2}$$

(A) $4y - 12$ (C) $y - 3$

(B) $2y - 6$ (D) None of these.

III

You will probably say that this is unfair. It is true that if

$$x^2 - 1 = 24$$

then

$$x^2 = 25$$

and

$$x = 5$$

Then if you apply the subtraction property and subtract 2 from each side of the equation, you will get

$$x - 2 = 3$$

We are asking for a simple one-step illustration of the subtraction property. This problem is not an example of only the subtraction property and is, therefore, not a good choice.

Please return to page $\frac{149}{2}$ and try this question again.

It is true that in step I the COMMUTATIVE PROPERTY was employed to change the order of the terms for convenience in grouping. It is also true that this was done on both sides of the equation. But these were separate operations, independent of each other. One operation of the COMMUTATIVE PROPERTY can be done on either side of the equation alone.

e.g.

$$a + b + c$$

can be altered to $a + c + b$

Please return to page $\frac{160}{2}$ and reconsider the problem.

You subtracted .7 from both sides of the given equality when you should have added this number. Please keep in mind that in finding the solution set of an equation, you need to replace the given equation by an equivalent equation where the left side has the variable only.

In equations of the form:

$$\begin{array}{lr|ccccccc|}
\hline
 & & x & - & a & = & k & & \\
\text{we add} & a & & & & & & & \\
\text{to both sides} & & & & a & = & a & & \\
\cline{3-9}
 & & x & & & = & k & + & a \\
\hline
\end{array}$$

NOTE: " a " and " -a " are additive inverses

Return to page $\frac{154}{2}$ and try this question again.

By this time you probably realize that the basic technique of solving equations of this type

$$x + a = k$$

is to either use the addition property employing the additive inverse of a or to use the subtraction property and subtract a from both sides. Therefore, your error must be in your handling of fractions. Shall we remind you to convert both fractions to equivalent fractions with the same denominator? In this case, they both become twelfths.

Return to page $\frac{157}{2}$ and do the problem carefully.

III

$\frac{142}{1}$

You made the correct choice.

The ADDITION PROPERTY OF EQUALITY states that if the same number is added to equal numbers, the sums are equal. In this case, we have

$$17 = (13 + 4) \quad \text{and}$$

the same number 5 is added.

Thus, $17 + 5 = (13 + 4) + 5$

In symbolic form, this can be stated:

if	a			=	b		
then	a	+	c	=	b	+	c

Please go on to question 2 below.

$\frac{142}{2}$

<u>Question 2</u>

Recognize the ADDITION PROPERTY OF EQUALITY and select the letter next to the statement which illustrates it.

(A) If $x + y = 8$, then $y + x = 8$

(B) If $a = 3b$, then $a + 3 = 6b$

(C) If $p = 2q$ and $q = 5$, then $p = 10$

(D) If $m = 3n$, then $m + 8 = 3n + 8$

It appears that you used the proper principle, but that you made an error in the arithmetic,

What is the sum of $\frac{1}{2}$ and $\frac{1}{4}$?

Please change $\frac{1}{2}$ into quarters and find the correct sum of these 2 fractions.

Please return to page $\frac{138}{2}$ and try this question again.

In this question, in order that the left side of the given equation should contain the letter z only, it is necessary to eliminate 82 from that side of the equation.

This is an example of the form where we employ the additive inverse

$$\begin{array}{rcl} x + a & = & k \\ \underline{\quad - a} & \underline{=} & \\ x & = & k - a \end{array}$$

Since subtracting a is equivalent to adding −a , we can use the subtraction property and subtract a from both sides of the equation. Check your arithmetic before you make a decision.

Please return to page $\frac{167}{2}$ and try this question again.

$\frac{144}{1}$

This operation is possible, but if 17 is subtracted from both sides of the equation

$$\begin{array}{rcr} 17 - x & = & 3 \\ \underline{-17} & \underline{=} & \underline{-17} \\ - x & = & -14 \end{array}$$

The result does not isolate x , but rather negative x . Consider which <u>one</u> operation will do the job.

Please return to page $\frac{165}{2}$ and try problem 7 again.

$\frac{144}{2}$

It appears that you think that you should add 8.01 and 7.6 . May we ask why? Since the equation was of the form

$$k = x + a$$

the method you should use is to isolate x by subtracting a from both sides. This is an example of the SUBTRACTION PROPERTY OF EQUALITY.

Please return to page $\frac{152}{2}$ and try the question again.

You made the correct choice.

If $18 = a$

then $18 - 5 =$

$a - 5$

The SUBTRACTION PROPERTY OF EQUALITY states:

If the same number is subtracted from equal numbers, the differences are equal.

Please go on to question 4 below.

Question 4

Apply the principles of equality and find the solution set of the equation

$$n - 7 = 14$$

Select the letter next to the correct answer.

(A) $\{7\}$ (C) $\{2\}$

(B) $\{21\}$ (D) None of these.

III

$\frac{146}{1}$

You were careless in the addition of two decimal numbers. When we add decimal numbers, the decimal points must be in the same column.

Thus, for example,

$$.7 \quad + \quad .05$$

should be written as

$$\begin{array}{r} .7 \\ \\ \underline{.05} \end{array}$$

The underlying principle is the rule for adding fractions. Only like fractions (those having the same denominator can be added) Here, the decimals are considered decimal fractions;

$$.7 \quad = \quad \frac{7}{10}$$

and

$$.05 \quad = \quad \frac{5}{100}$$

But by making .7 into .70 , it becomes

$$\frac{70}{100}$$

and can be added to

$$\frac{5}{100}$$

This can be accomplised by merely aligning the decimal points.

Please return to page $\frac{154}{2}$ and try this question again.

$\frac{146}{2}$

It is true that in step II the ASSOCIATIVE PROPERTY was used to group the like terms. By coincidence, it was also used on both sides of the equation, but this is not a requirement of the property which merely states

$$a \; + \; b \; + \; c$$

can be grouped as $(a + b) + c$

Since you were asked to identify the operation that must be performed on both sides of the equation, your choice is not right.

Please return to page $\frac{160}{2}$ and reconsider the equation.

Evidently you used the ADDITION PROPERTY OF EQUALITY, but to eliminate $\frac{1}{4}$ by this property, you should add the additive inverse $-\frac{1}{4}$. Of course, the same result could be achieved by subtracting $\frac{1}{4}$ itself from both sides of the equation.

Note: $\frac{1}{4}$ is smaller than $\frac{1}{3}$ and when the first is subtracted from the second the result is positive.

Return to page $\frac{157}{2}$ and try this question again.

$\frac{147}{2}$

Sorry, but we do not agree.

Part of your difficulty was changing the fraction $\frac{3}{8}$ to a decimal. To refresh your memory, we will change $\frac{5}{8}$ to a decimal. Note that $\frac{5}{8}$ is the same as

$$8\overline{)\;5}$$

We place a decimal point after the 5 and add a few zeros. Thus,

$$\begin{array}{r} .625 \\ 8\overline{)\,5.000} \\ \underline{4\;8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

Return to page $\frac{163}{1}$ and try this question again.

$\frac{148}{1}$

Sorry, but your choice is incorrect. To eliminate the $-\frac{1}{2}$ we must first determine its additive inverse. Then we use the ADDITION PROPERTY OF EQUALITY and add $\frac{1}{2}$ to both sides of the given equation.

Please return to page $\frac{138}{2}$ and try this question again.

$\frac{148}{2}$

It is always possible to subtract 3 from both sides of any equation. The point to consider is; will it help towards getting a solution. The second point is that if you do decide to subtract 3 , then do it properly!

The left side of the equation was

$$3x + 10$$

$$\text{Now} \quad (3x + 10) - 3 =$$

$$3x + 7$$

The 3 can't be subtracted from 3x ; they are unlike terms. The correct result of subtracting 3

$$\begin{array}{rrcl} \text{from:} & 3x + 10 & = & 17 \\ & 3 & = & 3 \\ \hline \text{is:} & 3x + 7 & = & 14 \end{array}$$

which shows that subtracting 3 didn't help much in finding the solution.

Now return to page $\frac{169}{2}$ and reconsider the problem.

You made the correct choice.

Equality is maintained by adding the same number 8 to each side.

Thus, we have the equality:

$$\begin{array}{rcl} m & = & 3m \\ \underline{\quad 8} & \underline{=} & \underline{\quad 8} \\ m + 8 & = & 3m + 8 \end{array}$$

equality is maintained by adding the same number 8 to each side.

Please go on to question 3 below.

Question 3

Recognize the SUBTRACTION PROPERTY OF EQUALITY and select the letter next to the statement which illustrates it.

(A) If $x - 4 = a$ and $a = y - 6$, then $x - 4 = y - 6$

(B) The expression $5(6 - 3)$ equals $5 \times 6 - 5 \times 3$

(C) If $18 = a$, then $18 - 5 = a - 5$

(D) If $x^2 - 1 = 24$, then $x - 2 = 3$

$\frac{150}{1}$

It is possible to subtract 3 from both sides of the equation

$$17 - x = 3$$

but the result will be

$$14 - x = 0$$

This did not do what you were supposed to do, isolate x .

Please return to page $\frac{165}{2}$ and try this question again.

$\frac{150}{2}$

No, we don't agree. One of the letters does have the correct answer next to it. Let's review the technique.

This equation has the form:

$$x + a = k$$

To solve for x we must isolate x on one side of the equation. The SUBTRACTION PROPERTY can be used to eliminate the value of a . Suppose you apply this principle to the problem presented to you.

Please return to page $\frac{167}{2}$ and try this question again.

In order to decide how to eliminate -6 from the left side of the equation, it is necessary to use the additive inverse of -6 ; which, of course, is +6 . But this does not require an equation to be determined. The additive inverse of a is -a .

Since you were asked to identify the operation that must be performed on both sides of an equation, your choice is not correct.

Please return to page 160/2 and reconsider the equation.

An equation can always be solved by substituting different values for the unknown; performing the arithmetic and finding the value that satifies. But this is a " hit-or-miss " system. It could be an almost endless procedure. Except in the simplest of equations, it is not an efficient system.

Please return to page 161/2 and reconsider the other choices.

$\frac{152}{1}$

You made the correct choice. Let's review the technique. We have to find the solution set of the equation

$$x + \frac{1}{4} = \frac{1}{3}$$

Since we want the left side of the equality to contain the letter x only, we subtract $\frac{1}{4}$ from both sides and obtain the equivalent equation

$$x + \frac{1}{4} - \frac{1}{4} =$$

$$\frac{1}{3} - \frac{1}{4}$$

or $$x = \frac{1}{3} - \frac{1}{4}$$

converting the fractions $$x = \frac{4}{12} - \frac{3}{12}$$

to like fractions $$x = \frac{1}{12}$$ this value is a member of the set you chose.

Please go on to question 11 below.

$\frac{152}{2}$

Question 11

Apply the principles of equality and choose the correct statement concerning the solution of this equation

$$8.01 = t + 7.6$$

Select the letter next to the correct answer.

(A) t = 15.61

(B) t = 0.41

(C) 7.6 should be added to each side.

(D) t should be subtracted from each side.

No, it is possible to solve an equation of the form

$$x + a = b$$

where one of the two letters a and b is a fraction; and the other is a decimal. But one must be changed to the other so that either both are fractions or both decimals.

Return to page $\frac{163}{1}$ and try to solve the equation.

In practical problems, you must be able to construct your own equation interpreting the data before you can apply the principle of equality to solve the equation. It is always a good idea to make a common-sense estimate of the correct solution to a problem. If you have to add money to the piggy bank in order to get \$5, then the bank had less than \$5 in it to begin with.

The answer must, therefore, be a sum of money less that \$5. Is this the case with the choice you made?

A second good practice is to examine the equations that you wrote and to translate it back into English as a check on its correctness.

Return to page $\frac{168}{2}$ and try this question again.

$\frac{154}{1}$

Very good. You made the correct choice. We have the equation

$$x - \frac{1}{2} = \frac{1}{4}$$

In order to find the solution set of this equation, we must isolate the x . Since $-\frac{1}{2}$ is on the same side of the equal sign, we must do something that will eliminate the $-\frac{1}{2}$.

What number do we add? We add $\frac{1}{2}$, since

$$-\frac{1}{2} + \frac{1}{2} = 0$$

$$\begin{array}{lcl}
x - \frac{1}{2} & = & \frac{1}{4} \\
+ \frac{1}{2} & = & + \frac{1}{2} \\
\hline
x - \frac{1}{2} + \frac{1}{2} & = & \frac{1}{4} + \frac{1}{2} \\
x & = & \frac{1}{4} + \frac{2}{4} \\
x & = & \frac{3}{4}
\end{array}$$

Please go on to question 6 below.

$\frac{154}{2}$

Question 6

Apply the principles of equality and find the solution set of the equation

$$y - .7 = 2.2$$

(A) 1.5 (C) 2.9

(B) 9.2 (D) None of these.

It is always possible to add 10 to both sides of an equation. But it should be done only as a step in isolating the variable we are solving for.

In the problem	3x	+	10	=	17
if you chose to add			10	=	10
10 should get	3x	+	20	=	27

As you can see, this was not much help. You can also see that you did not apply the rule correctly.

Please return to page $\frac{169}{2}$ and try this problem again.

Let us make a common-sense estimate of the answer to this question. If we had \$4.75 left after spending some money, we must have had more than \$4.75 to start with. The correct answer must, therefore, be an amount greater than \$4.75. However, if you solve the equation that you chose,

$$x = \$2.70$$

Please return to page $\frac{174}{2}$ and try this question again.

$\frac{156}{1}$

This choice, add x to both sides, is possible.

But the question is, did it do what it was intended to do?

Let's check and see.

$$17 - x = 3$$

if x is added $\quad 17 - x + x = 3 + x$

$$17 = 3 + x$$

Since this did not isolate x in one step, you will have to try this question again.

Please return to page $\frac{165}{2}$ and reconsider.

$\frac{156}{2}$

In solving an equation of the type

$$x + a = b$$

where a is a fraction and b is a decimal, it is possible to solve it by changing the decimal to a fraction and then to apply the subtraction property arriving at the solution

$$x = b - a$$

In the problem

$$s + \frac{3}{8} = .45$$

when these fractions are subtracted, the answer is not the same as the one you chose. It is generally easier to change the fraction $\frac{3}{8}$ to the decimal equivalent instead.

Please return to page $\frac{163}{1}$ and re-do the problem.

To find the solution set of the equation

$$z + 82 = 175$$

we use the SUBTRACTION PROPERTY OF EQUALITY. Thus, we obtain in symbolic form:

$$\begin{array}{lrcl} & x + a = k & & z + 82 = 175 \\ & \underline{\quad a = a \quad} & & \underline{\quad - 82 = -82 \quad} \\ \text{by subtraction} & x \quad = k - a & \text{and} & z = 93 \end{array}$$

This value was one of the elements of the set you chose.

Please go on to question 10 below.

<u>Question 10</u>

Apply the principles of equality and find the set that contains the value of x that is the solution of the equation

$$x + \frac{1}{4} = \frac{1}{3}$$

Select the letter next to the correct answer.

(A) $\left\{-\frac{1}{7}, -\frac{2}{7}, -\frac{3}{7}, -\frac{4}{7}\right\}$

(B) $\left\{\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}\right\}$

(C) $\left\{\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}\right\}$

(D) $\left\{-\frac{1}{12}, -\frac{5}{12}, -\frac{7}{12}, -\frac{11}{12}\right\}$

$\frac{158}{1}$

In an equation of the form

$$k = x + a$$

when the object is to solve for x the side of the equation that contains x is examined. Whatever other values are with x are to be eliminated in order to isolate it; therefore, in this form the value represented by a is to be subtracted from both sides. Subtracting the x will not lead to an immediate solution.

Please return to page $\frac{152}{2}$ and reconsider the question.

$\frac{158}{2}$

This method will solve the problem, but handling the x's separately and treating the numbers one at a time is a slow procedure. It is more efficient to combine the x's and to combine the numbers before they are operated on by the addition or subtraction properties. In general, the fewer the necessary steps, the more efficient is the procedure.

Please return to page $\frac{161}{2}$ and reconsider the other choices.

Sorry, but we don't agree. One of the letters does have the correct answer next to i . Please reconsider your choice.

Remember, equations of the form

$$x - a = k$$

can be solved for variable x by finding the additive inverse of $-a$ and adding it to both sides.

Return to page $\frac{154}{2}$ and try this question again.

We don't agree. All you have to do is to interpret the facts, one fact at a time. We wish to find out how much money you have in a piggy bank. We represent this amount by x . Now, we are told that \$1.15 is added to this amount. After this is done, we have

$$x + 1.15$$

in the bank. What else do we know?

Please continue.

Return to page $\frac{168}{2}$ and try this question again.

$\frac{160}{1}$

Good going! This question was a bit tricky. We had to apply the addition property to eliminate the negative x from the left side, and also use the subtraction principle to eliminate the +3 from the right side where the x would end up.

This dual operation can be combined into one,

"add $x - 3$ to both sides of the equation."

Please proceed to question 8 below.

$\frac{160}{2}$

Question 8

Determine which principles used in finding the solution of the equation below require a simultaneous operation on both sides of the equation,

$$3x + 2 - 2x - 8 = 10 - 2 + 11$$

(1)	$3x - 2x + 2 - 8$	$=$	$10 + 11 - 2$
(2)	$(3x - 2x) + (2 - 8)$	$=$	$(10 + 11) - 2$
(3)	$x - 6$	$=$	$21 - 2$
(4)	$x - 6$	$=$	19
(5)	$+ 6$	$=$	$+6$
(6)	x	$=$	25

(A) COMMUTATIVE PROPERTY

(B) ASSOCIATIVE PROPERTY

(C) ADDITIVE INVERSE

(D) ADDITION PROPERTY

Your answer is correct. You realize that the ADDITION and SUBTRACTION PROPERTIES OF EQUALITY apply to any equation and any value. However, the object of their application is to simplify the equation in an effort to isolate the variable.

Please go on to question 14 below.

161
2

Question 14

Of the following legitimate sequence of operations, choose the one that is most efficient in solving the equation for x

solve for x:

$$3x - x - 7 + 3 = x$$

A. 1. substitute different values for x
   2. perform the arithmetic
   3. see which value satisfies the equation

B. 1. combine like terms.
   2. add 4 to both sides
   3. subtract x from both sides

C. 1. add x to both sides
   2. add 7 to both sides
   3. subtract 3 from both sides
   4. subtract 2x from both sides

D. 1. subtract x from both sides
   2. subtract 3 from both sides
   3. combine the x terms
   4. add 7 to both sides

$\frac{162}{1}$

Your choice is incorrect. If you interpret the equation you chose you will see that it states the original amount plus what you have left equals what you spent. That just doesn't make sense, does it?

If we let x equal the amount of money you started with, then $x - 2.05$ represents the money you have left after spending \$2.05. Please continue.

Return to page $\frac{174}{2}$ and try this question again.

$\frac{162}{2}$

It is true that if

$$8 = \frac{16}{2}$$

then

$$\frac{16}{2} = 8$$

However, this is an illustration of the SYMMETRIC PROPERTY of the equality relationship, not of the DIVISION PROPERTY OF EQUALITY.

Please return to page $\frac{172}{2}$ and try question 1 again.

Your answer is correct. Please go on to question 12.

Question 12

Apply the principles of equality and find the solution set of the equation,

$$s + \frac{3}{8} = .45$$

Select the letter next to the correct answer.

(A) $\{ .075 \}$

(B) $\{ .75 \}$

(C) $\left\{ \frac{18}{100} \right\}$

(D) An equation containing fractions and decimals can't be solved.

When you interpreted the matching by the father you didn't mean that the amount that the father gave multiplied the amount the boy earned did you? But that is what x^2 means in the choice you made.

Please return to page $\frac{179}{2}$ and reconsider the problem.

$\frac{164}{1}$

Another technique of testing an equations correctness, is to solve it and interpret the answer.

Did you find the solution set of the equation you chose? Let us do so now.

$$x - 3.85 = 1.15$$

Using the ADDITION PROPERTY OF EQUALITY, we get

$$x - 3.85 + 3.85 = 1.15 + 3.85,$$

and

$$x = 1.15 + 3.85 = 5.00.$$

Clearly, this cannot be correct, because this states that we had $5 in the piggy bank to begin with.

Please return to page $\frac{168}{2}$ and try this question again.

$\frac{164}{2}$

It looks like the

$$x + a \text{ and } b + c$$

being divided by y influenced you into thinking of the DISTR[illegible]VE LAW. This law was discussed several segments ago. It can be expressed symbolically like this,

"for all non-zero c,

$$\frac{a + b}{c} =$$

$$\frac{a}{c} + \frac{b}{c}."$$

Now the statement in question 2 does not illustrate the DISTRIBUTIVE LAW OF DIVISION with respect to addition.

It is a matter of timing. To arrive at the expression

$$\frac{x + a}{y} = \frac{b + c}{y}$$

when given

$$x + a = b + c,$$

we use another property.

However, to take that expression and simplify it we would use the distributive law and get

$$\frac{x}{y} + \frac{a}{y} =$$

$$\frac{b}{y} + \frac{c}{y}$$

Please return to page $\frac{183}{2}$ and try question 2 again.

Very good. You made the correct choice.

We have the equation $y - .7 = 2.2$

We obtain an equivalent equation by adding the inverse of

$$-.7$$

to both sides of the equality. The inverse of

$$-.7 \quad \text{is} \quad +.7$$

$$\text{since} \quad -.7 + .7 = 0$$

Thus, we get

$$y - .7 + .7 = 2.2 + .7 \quad \text{and}$$

$$y + 0 = 2.9$$

$$y = 2.9$$

Please go on to question 7 which follows.

Question 7

Apply the principles of equality and choose the correct operation that will isolate the variable in one step in the solution of the equation

$$17 - x = 3$$

Select the letter next to the correct answer.

(A) Subtract 17 from both sides of the equation

(B) Subtract 3 from both sides of the equation

(C) Add x to both sides of the equation

(D) Add $x - 3$ to both sides of the equation

$\frac{166}{1}$

The REFLEXIVE PROPERTY of equality states that

$$x = x$$

Choice B illustrates that principle, not the DIVISION PROPERTY OF EQUALITY.

Please return to page $\frac{172}{2}$ and try question 1 again.

$\frac{166}{1}$

The SYMMETRIC PROPERTY of the equality relationship states that if

$$a = b \text{ then } b = a$$

This is what has been illustrated in choice B ,

If $10 = (5)(2)$

then $(5)(2) = 10$

the question asked you to recognize with MULTIPLICATION PROPERTY OF EQUALITY.

Did you write an example of this property in your notebook?

The MULTIPLICATION PROPERTY states that:

"For every a , every b , and every c ,

if $a = b$ then $ac = bc$"

Please return to page $\frac{188}{2}$ and try question 3 again.

Good. You figured this problem out correctly. Each of the other properties can be employed on any collection of terms, whether or not they are in an equation. But the ADDITION PROPERTY OF EQUALITY states that if an quantity is added to one side of the equation then it must be added to the other side as well.

that is,

$$\boxed{\begin{array}{l} \text{if} \quad x = k \\ \text{then} \quad x + a = k + a \end{array}}$$

of course, this rule, combined with the additive inverse can be used to solve equations of the form

$$\boxed{\begin{array}{rcl} x - b & = & k \\ b & = & b \\ x & = & k + b \end{array}}$$

as well.

Now proceed to question 9 below.

Question 9

Apply the principles of equality and find the set that contains the value of z that is the solution of the equation,

$$z + 82 = 175$$

Select the letter next to the correct answer.

A. $\{247, 257, 267, 277\}$

B. $\{73, 83, 93, 103\}$

C. $\{173, 183, 193, 203\}$

D. None of these sets contains the correct value

III

168
1

We agree. This is the most efficient method in mathematics. When more than one method of solution is possible the shortest is usually considered the preferable one. The word "elegant" is reserved for a particularly concise proof.

Now proceed to question 15 below.

168
2

Question 15

Apply the principles of equation formation and select the letter next to the equation whose solution set is the answer to the following question:

How much money did you have in your piggy bank, if after adding, \$1.15, you have \$5.00?

(A) $x - 1.15 = 5.00$

(B) $x + 1.15 = 5.00$

(C) $x - 3.85 = 5.00$

(D) $x - 3.85 = 1.15$

Very good. You made the correct choice. It is nice to see that you remembered how to change the fraction $\frac{3}{8}$ to a decimal.

Since

$$\frac{3}{8} = .375$$

we can write,

$$s + .375 = .45$$

Now, we apply the subtraction property of equality and we get

$$s + .375 - .375 = -.375 + .45$$

and

$$s = .45 - .375$$

$$s = .075$$

Please go on to the next question, below.

Question 13

Apply the principles you have learned and select the operation that was properly applied in the first step in solving this equation for x:

$$3x + 10 = 17$$

(A) Subtract 3 from both sides $\quad x + 10 = 14$

(B) Add 10 to both sides $\quad 3x = 27$

(C) Subtract 10 from both sides $\quad 3x = 7$

(D) Subtract 3 from both sides $\quad x + 7 = 14$

$\frac{170}{1}$

There is nothing wrong with the thought behind your equation. But there are two mistakes in your algebraic representation of the ideas.

First, the dollar signs are not included in the equation. They just add confusion. The statement, "Let x equal the number of dollars" implies that all of the other numbers represent numbers of dollars too.

Secondly, the "x" stands for the the number of dollars whatever it turns out to be, decimals, fractions, or whole numbers. It is poor practice to write x.00 when "x" will do just as well.

Please return to page $\frac{174}{2}$ and try the question again.

$\frac{170}{2}$

Division is not an associative operation.

Consider the following

36 divided by 6 divided by 3

$$36 - 6 - 3 \neq 36 - (6 - 3)$$

$$6 - 3 \qquad 36 - 2$$

$$2 \qquad 18$$

To avoid this contradiction the rule for order of operation states that divisions are performed in order, from left to right. Since division is not an associative operation, the statement cannot be an illustration of the ASSOCIATIVE PROPERTY of division.

Please return to page $\frac{183}{2}$ and try question 2 again.

Oops, an oversight on your part.

After he spent five dollars, he had a dollar <u>more</u> than his original pay check.

Now you didn't handle that properly, did you?

Return to page $\frac{179}{2}$ and reconsider the question.

The DISTRIBUTIVE PROPERTY of multiplication deals not only with the operation of multiplication, but also with another operation that the multiplication is to be distributed over.

Algebraically, we represent the distributive law of multiplication over addition as

$$a(b + c) = ab + ac$$

and the distributive law of multiplication over subtraction as

$$a(b - c) = ab - ac$$

Neither of these is the example in the problem.

Please return to page $\frac{193}{2}$ and try question 4 again.

$\frac{172}{1}$

VOLUME 3 SEGMENT 8 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS:	48	and	50	1 5	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 3	(Volume Number)
	66	and	68	0 5	(Segment Number)

READING ASSIGNMENT: Pages 83 - 84

SUPPLEMENTARY NOTES:

You have just finished investigating the ADDITION PROPERTIES OF EQUALITY and how the additive inverse figures in the application of these.

We will now extend our discussion to include the operations of multiplication and division in the solution of equations. These two operations also have an inverse.

$\frac{172}{2}$

Question 1

Recognize which of the following illustrates the DIVISION PROPERTY OF EQUALITY:

(A) if $8 = \frac{16}{2}$ then $\frac{16}{2} = 8$

(B) $\frac{16}{2} = \frac{16}{2}$

(C) if $16 = y$ then $\frac{16}{3} = \frac{y}{3}$

(D) if $\frac{16}{2} = 8$ then $\frac{24}{3} = 8$

Although this choice contains a multiplication and is an equality, it does not illustrate the MULTIPLICATION PROPERTY OF EQUALITY. Rather

$$(8)\ (7) = (8)\ (7)$$

is an illustration of the REFLEXIVE PROPERTY of the equality relationship.

Please return to page $\frac{188}{2}$ and try question 3 again.

The statement

$$\frac{84}{\frac{12}{3}} = \frac{\frac{84}{12}}{3}$$

is NOT correct.

The expression on the left side of the equal sign is equal to 21 . It is evaluated by first dividing the 3 into 12 , then dividing that quotient into 84 . The expression on the right is equal to

$$\frac{7}{3}$$

The fact that the left and right sides are not equal shows that this could be an example of why division is NOT associative.

Please return to page $\frac{185}{2}$ and try question 5 again.

174
1

You made the correct choice.

If we let x equal the amount of money in the piggy bank, then

$$x + 1.15$$

is the amount after 1.15

has been added, but this amount is \$5 ; hence the equation is

$$x + 1.15 = 5.00$$

Please go on to question 16 below.

174
2

Question 16

Apply the principles of equation formation and select the letter next to the equation whose solution set is the answer to the following question:

> How much money did you start with, if after spending \$2.05 you have \$4.75 left?

(A) $x - 2.05 = 4.75$

(B) $x + 2.05 = 2.75$

(C) $x + 4.75 = 2.05$

(D) $\$x.00 - \$2.05 = \$4.75$

You have the correct idea, but you broke a rule or two.

. you can't introduce a letter into an equation unless it has been defined.

You didn't tell what "p" and "y" stood for.

Secondly, if an idea or variable is used in more than one place in an equation the same letter should be used to stand for it.

Return to page $\frac{179}{2}$ and try this question again.

Evidently you attempted to apply the principle of division of both sides of the equation by 2.

But when one side of the equation is a binomial you must employ the DISTRIBUTIVE PRINCIPLE, as well, that is,

$a + b$ when divided by 2

can be written as:	<u>cannot</u> be written as:
$(a + b) \div 2$	$a + b \div 2$
$\frac{a + b}{2}$	$a + \frac{b}{2}$
$\frac{1}{2}(a + b)$	$\frac{a}{2} + b$

Please return to page $\frac{189}{1}$ and reconsider the problem.

$\frac{176}{1}$

The operation of addition is commutative because

$$a + b = b + a$$

The operation of multiplication is commutative because

$$(a)(b) = (b)(a)$$

Let's consider division

$$\frac{a}{b} \stackrel{?}{=} \frac{b}{a}$$

Take a simple example:

Does $\frac{8}{4} \stackrel{?}{=} \frac{4}{8}$

This shows that the operation of division is not commutative. T[illegible] statement cannot be an illustration of the COMMUTATIVE PROPERTY OF DIVISION since there is no such principle.

Please return to page $\frac{183}{2}$ and try question 2 again.

$\frac{176}{2}$

The COMMUTATIVE PROPERTY OF MULTIPLICATION deals with the reversability of the factors in a multiplication example. If we say that

$$(a)(b) = x$$

then $(b)(a)$

will also equal x

This property is not the one represented in the example.

Please return to page $\frac{193}{2}$ and try question 4 again.

It is true that $\frac{16}{2} = 8$,

and it is also true that $\frac{24}{3} = 8$

but this does not represent the DIVISION PROPERTY OF EQUALITY.

The DIVISION PROPERTY OF EQUALITY states that if

$$a = b,$$

and c does not equal zero, then

$$\frac{a}{c} = \frac{b}{c} .$$

Please return to page $\frac{172}{2}$ and try question 1 again.

The MULTIPLICATIVE PROPERTY OF EQUALITY states that ac = bc.

$$\begin{array}{rrcl} \text{if} & a & = & b \\ & \underline{c} & \underline{=} & \underline{c} \\ & ac & = & bc \end{array}$$

thus,

$$\begin{array}{rcl} x & = & 4y \\ \underline{2} & \underline{=} & \underline{2} \\ 2x & = & 8y \end{array}$$

The statement is correct since it parallels the MULTIPLICATIVE PROPERTY OF EQUALITY; and thus, the correct truth value for statement I is TRUE.

Please return to page $\frac{185}{2}$ and try question 5 again.

$\frac{178}{1}$

Although this choice contains both a multiplication and an equals sign it is not an illustration of the MULTIPLICATION PROPERTY OF EQUALITY.

The MULTIPLICATION PROPERTY OF EQUALITY states that "for every a, every b, and every c,

if $a = b$ then $ac = bc$."

The statement in choice D, if

$$(6)\ (4) = 24,$$

then

$$(4)\ (6) = 24$$

is an illustration of the commutativity of multiplication.

Please return to page $\frac{188}{2}$ and try question 3 again.

$\frac{178}{2}$

Your choice is not correct.

The object is to solve for t.

Choose the proper inverse operation to eliminate the 4 and check your work carefully.

Return to page $\frac{194}{1}$ and try this question again.

You made the correct choice.

If we let x equal the amount of money you had to start with, then

$$x - 2.05$$

represents the amount of money left after spending

$2.05

Now, this amount is equal to

$4.75

Hence, the equation is:

$$x - 2.05 = 4.75$$

By using the ADDITION PROPERTY, adding $2.05 to both sides,

$$x = \$6.80$$

Please go on to question 17 below.

Question 17

A boy got a job as a messenger. His father said that he would match whatever he was paid. After payday the boy received his fathers gift and later spent 5 dollars. To his surprise he had a dollar more than his original paycheck left. What was his pay for being a messenger. Let x equal the number of dollars he received in his paycheck. Choose the equation that describes the situation.

(A) $x + x - 5 = x + 1$

(B) $x^2 - 5 = x + 1$

(C) $2x - 5 = 1$

(D) $x + y - 5 = p + 1$

180
1

Associativity is a property of an operation. The ASSOCIATIVE PROPERTY OF ADDITION states that

$$a + (b + c) =$$

$$(a + b) + c.$$

It can be expressed verbally as "the addition of three elements may be performed by grouping any two of the elements and adding the third to the sum of the grouping." The ASSOCIATIVE PROPERTY OF MULTIPLICATION states that

$$a\ (bc) = (ab)\ c.$$

There is no such thing as the multiplication property of associativity.

Your answer is incorrect.

Please refer to your notes and review all the different properties. Then, return to page $\frac{193}{2}$ and try question 4 again.

180
2

After you have applied what you think is the correct principle to solve the problem, you should check the reasonableness of your answer. Suppose that you ask yourself the following question:

If 12y equals $\frac{1}{3}$

will y equal more or less than $\frac{1}{3}$?

Clearly, the answer is less than $\frac{1}{3}$.

Thus, any choice greater than $\frac{1}{3}$ must be wrong.

Please return to page $\frac{186}{2}$ and try this question again.

You will be surprised at this, because you probably though that you had analyzed the problem correctly.

It's true that the DIVISION PRINCIPLE OF EQUALITY means that both sides of the equation can be divided by the same value. But it is part of the principle that zero cannot be used as a division.

Therefore, the answer should have read:

$$\frac{2x + 4}{a} = \frac{14}{a} \text{ if } a \neq 0$$

Return to page $\frac{189}{1}$ and reconsider the problem.

In solving an equation involving decimals and fractions, the procedure is to change all of the numbers to the same type, that is, all decimals or all fractions. Then the principles of equality can be used to isolate the variable.

In your choice:

$.5r = \frac{3}{4}$ we change $\frac{3}{4}$ to .75

$.5r = .75$

$r = \frac{.75}{.5}$ (divide both sides by .5)

$r = 1.5$

Since the choice you made is a proper method and you were asked to choose an incorrect statement you are not right.

Return to page $\frac{198}{2}$ and reconsider the problem.

III

$\frac{182}{1}$

You seem to be having trouble with decimals in di ions.

Let's review some of the principles involved.

A fraction can be multiplied by 1 or an expression equaling 1, without affecting its value.

i.e.

$$\frac{a}{b} = \frac{a}{b} \cdot (1)$$

now since $\frac{c}{c} = 1$, we can also write

$$\frac{a}{b} = \frac{a}{b} \cdot \frac{c}{c}$$

We can use this principle to remove the decimal from a denominator by choosing a power of ten for c equivalent to the decimal position.

e.g.

"tenths" $$\frac{3}{.2} = \frac{3}{.2} \times \left(\frac{10}{10}\right) = \frac{30}{2}$$

"hundredths" $$\frac{3}{.25} = \frac{3}{.25} \times \left(\frac{100}{100}\right) = \frac{300}{25} \qquad \text{etc.}$$

Please return to page $\frac{191}{2}$ and reconsider the question.

$\frac{182}{2}$

Even though we have rules for finding the solution set of an equation, it is important to make a common sense estimate of the answer to a question. We are told that when s is divided by 14 the result is 28.

Thus, the value of s must be much larger than 28. Keep this in mind.

Please return to page $\frac{203}{2}$ and try this question again.

The DIVISION PROPERTY OF EQUALITY states that if equal quantities are divided by equal, non-zero quantities, then their quotients are equal: that is,

if	$x = y$	$16 = y$
and	$a = a$	$3 = 3$
than	$\frac{x}{a} = \frac{y}{a}$	$\frac{16}{3} = \frac{y}{3}$

NOTE: $a \neq 0$

Your answer is correct.

Please go on to question 2 below.

Question 2

Choose the statement below that best describes the following:

if	$x + a = b + c$
and if	$y \neq 0$
then	$\frac{x + a}{y} = \frac{b + c}{y}$

(A) illustration of the DISTRIBUTIVE LAW OF DIVISION with respect to addition.

(B) illustration of the DIVISION PROPERTY OF EQUALITY.

(C) illustration of the ASSOCIATIVE PROPERTY OF DIVISION.

(D) illustration of the COMMUTATIVE PROPERTY OF DIVISION.

184
1

You dug out the algebraic relationship quite well. Sometimes a picture of the problem helps clarify the relationships:

PAY	+	GIFT	−	SPENT	=	PAY	+	1
x	+	x	−	5	=	x	=	1

always list what the unknown stands for.

Let x = amount earned

x = father's gift (same amount as earned)

Let's solve this, then we can check the answer against the original problem.

$x + x - 5 = x + 1$	combine
$2x - 5 = x + 1$	subtract x
$x - 5 = 1$	add 5
$x = 6$	

CHECK:

$6 + $6 gift = $12
less spent $ 5
left $7 which is $1 more than pay.

184
2

You have now finished this Segment. Hand in your Punch Card.
You should have entered in your notebook the following definitions and formulas:

(1) The addition property of equality: If a, b, c are any numbers

and $a = b$

then $a + c =$
$b + c$

(2) The subtraction property of equality: If a, b, c are any numbers

and $a = b$

then $a - c =$
$b - c$

You should now be able to complete the following problems from your homework assignment: Problems 13, 14, 15, and 16

The statement

"For each a , each b , and each c , if

$$a = b$$

$$\text{then} \quad ac = bc"$$

is the symbolic representation of the MULTIPLICATION PROPERTY OF EQUALITY.

This is an important tool used in solving equations containing a fraction coefficient of the variable.

$$\text{e.g.} \quad \frac{x}{a} = b$$

then

$$a = a$$

$$a \quad \frac{x}{a} = a \quad b$$

$$x = ab$$

Please proceed to question 5 below.

Question 5

Apply your knowledge of the MULTIPLICATION AND DIVISION PROPERTIES OF EQUALITY to the statements below, then select the choice containing the correct truth value:

I The statement that if $x = (4)(y)$

then $2x = 8y$

is correct because of the MULTIPLICATIVE PROPERTY OF EQUALITY.

II The statement: $\dfrac{\frac{84}{12}}{3} = \dfrac{\frac{84}{12}}{3}$

is correct because of the DIVISION PROPERTY OF EQUALITY.

(A)	I.	false	(C)	I.	false
	II.	true		II.	false
(B)	I.	true	(D)	I.	true
	II.	true		II.	false

III

$\frac{186}{1}$

Since the object is to find the value of t, we must isolate the variable on one side of the equation. In this case this means that 4 must be eliminated.

Since 4t means t is multiplied by 4,

we must choose the inverse operation of multiplication, which is division.

Therefore, we apply the DIVISION PRINCIPLE OF EQUALITY to

$$4t = 32$$

and divide both sides by 4

$$\frac{4t}{4} = \frac{32}{4}$$

giving

$$t = 8$$

The set you chose included this value as a member.

Please proceed to question 8 below.

$\frac{186}{2}$

Question 8

Apply the proper principle and find the set that contains the solution of the equation,

$$12y = \frac{1}{3}$$

Select the letter next to the correct answer.

(A) $\{ 2, 4, 9, 36 \}$

(B) $\{ 11\frac{2}{3} , 12\frac{1}{3} \}$

(C) $\{ \frac{1}{3} , \frac{1}{4} \}$

(D) $\{ \frac{1}{12} , \frac{1}{15} , \frac{1}{36} \}$

Very good. You made the correct choice. We have to find the solution set of the equation

$$\frac{w}{3} = 4\frac{2}{3}$$

Now,

$$4\frac{2}{3} = \frac{14}{3}$$

Thus, the equation becomes,

$$\frac{w}{3} = \frac{14}{3}$$

Multiplying both sides of the equation by 3, we obtain the equivalent equation,

$$\frac{w}{3}(3) = \frac{14}{3}(3)$$

and

$$w = 14$$

It might be interesting to note that a different way of looking at the problem can arrive at the correct answer. If two fractions are equal, and they have the same denominator as

$$\frac{w}{3} \text{ and } \frac{14}{3}$$

have; then their numerators are equal. That is, if

$$\frac{a}{b} = \frac{c}{b}$$

then

$$a = c$$

Please go on to question 14 below.

Question 14

Apply the proper principle and find the set that contains the solution of the equation $8x - 7 = 41$

Select the letter next to the correct answer.

(A) $\{46, 48, 50\}$

(B) $\{4, 6, 8, 10\}$

(C) $\left\{\frac{34}{8}, \frac{41}{8}, \frac{48}{8}\right\}$

(D) $\{19, 29, 40\}$

III

$\frac{188}{1}$

"Equal quantities divided by equal quantities give equal quotients."

This is another way of expressing the DIVISION PROPERTY OF EQUALITY. Algebraically, the property can be expressed as

"if $x = b$

and if y does not equal zero,

then

$\frac{x}{y} = \frac{b}{y}$ "

You are correct then in saying that B is the best completion of the statement.

Please go on to question 3 below.

$\frac{188}{2}$

Question 3

Recognize which of the following illustrates the MULTIPLICATION PROPERTY OF EQUALITY:

(A) if $5 = (3 + 2)$ then $5(4) = (3 + 2)(4)$

(B) if $10 = (5)(2)$ then $(5)(2) = 10$

(C) $(8)(7) = (8)(7)$

(D) if $(6)(4) = 24$ then $(4)(6) = 24$

This choice is correct.

Please proceed to question 6 which follows.

Question 6

Apply the Division Principle of Equality and consider the following statements in relation to

$$2x + 4 = 14$$

Choose the correct derived equation.

(A) $2x + 4 \div 2 = 14 \div 2$

(B) $\frac{2x}{2} + 4 = \frac{14}{2}$

(C) $\frac{1}{2}(2x + 4) = \frac{14}{2}$

(D) $\frac{2x + 4}{a} = \frac{14}{a}$

You will find that the value for x is the solution set of the equation. It checks, but the other important consideration is whether the equation itself fits the problem.

Careful re-reading will show that your interpretation is wrong.

Please return to page $\frac{201}{2}$ and try this question again.

$\frac{190}{1}$

To eliminate the decimal in an equation, one method is to multiply both sides of the equation by 10 or 100 or some other power of 10 large enough to counteract the decimal's magnitude.

In your choice: $.5r = \frac{3}{4}$ NOTE: Since .5 is 5 tenths when it is multiplied by 10, it would become a whole number.

$$5r = \frac{30}{4} \quad \text{(multiply both sides by 10)}$$

$$r = \frac{6}{4} \quad \text{(divide both sides by 5)}$$

$$r = \frac{3}{2}$$

Since the choice you made is a proper method, and you were asked to choose an incorrect statement, you are not right.

Please return to page $\frac{198}{2}$ and try the question again.

$\frac{190}{2}$

Your idea of multiplying both sides of the equation by 10 indicates that you intended to get rid of the decimals first. Ordinarily, that is not a bad idea though it does show that you don't like to work with decimals.

But $\frac{n}{.4}$ when multiplied by 10

does not become $\frac{n}{4}$

Let's examine this more carefully:

$\frac{n}{.4}$ means $n \div \frac{4}{10}$

this equals $n \times \frac{10}{4}$

now, if we multiply this by 10 ,

we get $\frac{n \times 100}{4}$ or $25n$

Doesn't that surprise you?

Of course, on the other side of the equation, 3.6 when multiplied by 10 becomes 36 .

Please return to page $\frac{197}{2}$ and do this problem over again.

Very good. You made the correct choice. We have to find the solution set of the equation,

$$12y = \frac{1}{3}$$

In order to obtain the value of one y , we must eliminate the 12 . Therefore, since 12 multiplies y we use the inverse operation and employ the DIVISION PROPERTY OF EQUALITY dividing both sides of the equation by 12 .

Thus we have

$$\frac{12y}{12} = \frac{\frac{1}{3}}{12}$$

$$y = \frac{1}{3} \times \frac{1}{12}$$

$$y = \frac{1}{36}$$

Now, dividing by 12 is the same as multiplying by the reciprocal of 12 which is $\frac{1}{12}$

Please go on to question 9 below.

Question 9

Apply the proper principle and find the solution set of the equation

$$.4z = 4.8$$

Select the letter next to the correct answer.

(A) $\{ 12 \}$

(B) $\{ .12 \}$

(C) $\{ 1.2 \}$

(D) $\{ 4.4 \}$

$\frac{192}{1}$

The statement says

"none of these."

But there is a number offered, which when divided by 14 equals 28 .

Check your calculations.

Please return to page $\frac{203}{2}$ and choose the correct answer.

$\frac{192}{2}$

It is advisable that you change the mixed number

$$4\frac{2}{3}$$

to an improper fraction before you start to solve this equation.

You see,

$$4\frac{2}{3}$$

does not mean $4 \times \frac{2}{3}$

rather it means $4 + \frac{2}{3}$

which can be changed to $\frac{14}{3}$

Once this is done, you can use the MULTIPLICATION PRINCIPLE OF EQUALITY.

Please return to page $\frac{202}{2}$ and try this question again.

The MULTIPLICATION PROPERTY OF EQUALITY may be represented as

"for every a, every b, and every c, if

$$a = b$$

then $$ac = bc$$ "

Thus, if we think of the 5 as a, the $(3 + 2)$ as b, and the 4 as c; we can reason along the following lines:

if	$a = b$	Likewise	if	$5 = (3 + 2)$
then	$ac = bc$		then	$5(4) = (3 + 2)(4)$

Your answer is correct.

A is the illustration of the MULTIPLICATION PROPERTY OF EQUALITY.

Please go on to question 4 below.

Question 4

Choose the statement below that best completes the following:

"for each a, each b, and each c,

if $a = b$

then $ac = bc$ "

is the symbolic representation of the algebraic rule:

(A) the DISTRIBUTIVE PROPERTY OF MULTIPLICATION.

(B) the COMMUTATIVE PROPERTY OF MULTIPLICATION.

(C) the MULTIPLICATION PROPERTY OF EQUALITY.

(D) the MULTIPLICATION PROPERTY OF ASSOCIATION.

III

$\frac{194}{1}$

Your choice is correct.

Of course, division by 2 can be also written as multiplication by $\frac{1}{2}$

Now we proceed to question 7 which follows.

Question 7

Apply the proper principle of equality and choose the set that contains the value of t that satisfies the equation

$$4t = 32$$

Select the letter which has the correct statement.

(A) $\{32, 34, 36, 38\}$

(B) $\{24, 26, 28, 30\}$

(C) $\{4, 6, 8, 12\}$

(D) $\{120, 124, 128, 132\}$

$\frac{194}{2}$

In order to find the solution set of this equation, it is necessary to use two properties of equality.

First apply the addition property, to eliminate the 7, and then use the division property.

Please return to page $\frac{187}{2}$ and try this question again.

You were the victim of incomplete analysis.

Usually when a number is divided by another number the quotient is smaller than the number that you started with.

For example,

$$\frac{10}{2} = 5$$

But if the number is divided by a fraction or by a decimal, the quotient is larger.

Consider this

$$\frac{10}{1} = 10$$

Now if 1 goes into 10 , 10 times, would a fraction smaller than 1 go into 10 more than 10 times?

e.g. $$\frac{10}{\frac{1}{2}} = 20$$

Please return to page $\frac{197}{2}$ and reconsider this problem.

In solving an equation involving decimals and fractions, the procedure is to change all of the numbers to the same type; that is, all decimals or all fractions. Then the principles of equality can be used to isolate the variable.

In your choice

$$.5r = \frac{3}{4} \quad \text{we change .5 to } \frac{5}{10} \text{ or } \frac{1}{2}$$

$$\frac{1}{2}r = \frac{3}{4} \quad \text{(multiply both sides by 2)}$$

$$r = \frac{3}{2}$$

Since the choice you made is a proper method, and you were asked to choose an incorrect statement; you are not right.

Please return to page $\frac{198}{2}$ and try the question again.

$\frac{196}{1}$

Your choice is correct.

Let's review the procedure.

We let x equal the number in question

one-third of the number = $\frac{1}{3}x$

Translating:

one-third of a number is 15 less than 51

$$\frac{1}{3}x = 51 - 15$$

$$\frac{1}{3}x = 36$$

multiply by 3

$$x = 108$$

Please proceed to the next question below.

$\frac{196}{2}$

Question 16

Apply your knowledge, set up the equation and solve it.

Select the letter which correctly answers the following question:

In 8 years from now, Jack will be $\frac{1}{2}$ his father's present age.

If Jack's father is now 38 years old, how old is Jack now?

(A) Between 5 and 10.

(B) More than 15 but less than 18 .

(C) More than 10 but less than 15 years old.

(D) 15 years old.

You made ice.

We have to the solution set of the equation,

$$\frac{s}{14} = 28$$

The letter s whose value we are trying to determine is divided by a number.

When this is the case, we use the inverse operation of division and employ the multiplication property of equality. We multiply both sides of the equation by 14 and obtain the equivalent equation,

$$(14)\frac{s}{14} = 14\ (28)$$
$$s = 392$$

Please go on to question 12 below.

Question 12

Apply the proper principle and find the solution set of the equation,

$$\frac{n}{.4} = 3.6.$$

Select the letter next to the correct statement.

(A) The value of n is less than $1\frac{1}{2}$

(B) Divide both sides of the equation by .4

(C) Multiply both sides of the equation by 10 to eliminate the decimals first.

(D) The value of n must be greater than 3.6

$\frac{198}{1}$

Very good. You made the correct choice. We have to find the solution set of the equation

$$.4z = 4.8$$

In order to find the value of z when we have the value of .4z, it is necessary to eliminate the .4 which multiplies x by dividing both sides of the equation by .4

Thus,

$$.4z = 4.8$$

$$.4 = .4$$

$$z = \frac{4.8}{.4}$$

$$z = 12$$

Hence, z = 12 is the correct answer.

Please go on to question 10 below.

$\frac{198}{2}$

Question 10

Apply the proper principles that can be used to find the solution set of the equation

$$.5r = \frac{3}{4}$$

Select the letter next to the wrong statement.

(A) Change $\frac{3}{4}$ to .75 and divide both sides by .5

(B) Change .5 to $\frac{1}{2}$ and multiply both sides by 2

(C) Multiply both sides by 10 and then divide both sides by 5

(D) Decimals and fractions can't be in the same equation.

No matter how a problem has been solved you should check your answer against the original verbal problem as well as against the equation you structructed.

that Jack was 10 , in eight years he'd be 18 . Is that half of the fathers present age ? No, it isn't.

Return to page $\frac{196}{2}$ and reconsider the problem.

You didn't apply the principle carefully.

$$\frac{n}{.4} = 3.6 \quad \text{when divided by } .4$$

would become

$$\frac{n}{.4} \cdot \frac{1}{.4} = \frac{3.6}{.4}$$

Although this is a possible operation it is not one that will lead immediately to a solution.

Consider the proto-type

$$\frac{x}{a} = b \quad \text{(multiply by a)}$$

$$x = ab$$

Now return to page $\frac{197}{2}$ and reconsider the problem.

$\frac{200}{1}$

You are not using the correct property of equality necessary to find the solution set of the given equation. When the letter that we are solving for has a numerical coefficient, the equation is divided by this coefficient.

Please return to page $\frac{191}{2}$ and try this question again.

$\frac{200}{2}$

Evidently you misread the problem and think that in eight years Jack will be half of his father's age. No, Jack will be half of his father's present age at that time.

Return to page $\frac{196}{2}$ and reconsider the problem.

Very good. You made the correct choice. We have to find the solution set of the equation,

$$8x - 7 = 41$$

First, we use the ADDITION PROPERTY OF EQUALITY and add 7 to both sides of the equation. We get,

$$8x - 7 + 7 = 41 + 7$$

and $$8x = 48$$

Now we use the DIVISION PROPERTY, and divide both sides by 8

$$\frac{8x}{8} = \frac{48}{8}$$

$$x = 6$$

Please go on to the next question.

Question 15

Apply your knowledge and select the equation whose solution set is the correct answer to this question:

If one third of a certain number is 15 less than 51, what is this number?

(A) $\frac{x}{3} = 15 - 51$, $x = -108$

(B) $\frac{x}{3} = 51 - 15$, $x = +108$

(C) $\frac{1}{3}(x - 15) = 51$, $x = 168$

(D) $15 - 3x = 51$, $x = -12$

III

$\frac{202}{1}$

To solve an equation of the form

$$\frac{x}{a} = b$$

we multiply both sides by a

$$x = ab$$

If a happens to be a decimal, it doesn't change the procedure at all.

Hence, in

$$\frac{n}{.4} = 3.6 \quad \text{(multiply by } .4)$$

$$n = (3.6)(4)$$

$$n = 1.44$$

Since this value is less than $1\frac{1}{2}$ your answer is correct.

Proceed to the next question below.

$\frac{202}{2}$

Question 13

Apply the proper principle and find the solution set of the equation

$$\frac{w}{3} = 4\frac{2}{3}$$

Select the letter next to the correct answer.

(A) $\{ 8 \}$

(B) $\{ 12 \}$

(C) $\{ \frac{7}{3} \}$

(D) $\{ 14 \}$

Your choice is correct.

Although it is not the most common situation, it is perfectly alright to have decimals and fractions in the same equation. For that matter, decimals are a form of fractions.

The mixture just makes it more imperative to choose an efficient procedure before solving the equation.

Now proceed to the next question below.

Question 11

Apply the proper principle of equality and choose the set that contains the value of s that satisfies the equation

$$\frac{s}{14} = 28$$

Select the letter next to the correct answer

(A) $\{2\}$ $\{1,2,3,4\}$

(B) $\{14\}$ $\{12,14,16,18\}$

(C) $\{392\}$ $\{382,392,394\}$

(D) None of these

III

One of the best ways to solve a verbal problem is to set up a chart in which you locate the vari... quantities that relate to the problem. In this problem, for example, you would write:

	Age now	Age in eight years
Jack	x	x + 8
Father	38	46

These then are the facts needed to set up the equation.

In 8 years, Jack will be $\frac{1}{2}$ his father's present age.

$$x + 8 = \frac{1}{2} (38)$$

$$x + 8 = 19 \quad \text{subtract } 8$$

$$x = 11$$

Then this should be checked:
Jack will be 19, which is half of his father's present age.
You have now finished the volume. Complete the homework assignment.
Study your notes for the volume test.

Hand in your punch card. You should have entered in your notebook, the following definitions and formulas:

(1) The DIVISION PROPERTY OF EQUALITY: when equal numbers are divided by the same number, the quotients are equal. (you may not divide by zero) .

(2) The MULTIPLICATION PROPERTY OF EQUALITY: when 2 equal numbers are multiplied by the same number, the products are equal.

You should now complete the remaining problems of your homework assignment, in preparation for a test on Volume 3 .

PROGRAMMED MATHEMATICS CONTINUUM — ERRATA SHEET — Attach to Back Cover
ALGEBRA - LEVEL ONE — VOLUME 3

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected.

Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE:	MISPRINT:	CORRECTION:	CHECK WHEN CORRECTION MADE
$\frac{45}{2}$	39.13	39-13	
$\frac{170}{2}$	$36-6-3 \neq 36-(6-3)$	$36 \div 6 \div 3 \neq 36 \div (6 \div 3)$	
	$6-3 \neq 36-2$	$6 \div 3 \neq 36 \div 2$	
	$2 \neq 18$	$2 \neq 18$	
$\frac{139}{1}$	$\frac{\frac{24y - 72}{12}}{2}$	$\frac{\frac{24y - 72}{12}}{2}$	