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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 2: punctuation marks; order of operations; evaluating algebraic expressions; identifying factors, coefficients, and exponents; solving open sentences; and translating verbal mathematical relationships into algebraic expressions. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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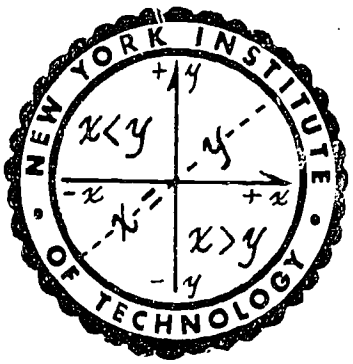

VOLUME

2

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

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P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME 2

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 2

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IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
1	1	$\frac{1}{1}$
1	2	$\frac{40}{1}$
1	3	$\frac{80}{1}$
1	4	$\frac{121}{2}$
1	5	$\frac{171}{2}$

VOLUME 2

This volume covers the following material as shown in this excerpt from the Syllabus:

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Punctuation marks Order of operations	1 - 8 1 - 9	1 - 6 1 - 5 (1 - 7)	2 - 7 4 - 10
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4	Solving open sentences	2 - 3	3 - 1 5 - 1	5 - 2
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D

READING ASSIGNMENT

VOLUME 2

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	19	24
2	35	37
3	40	41
4	44	46
5	49	54

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem.

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE.

These, too, should be entered in your NOTEBOOK.

E

HOMEWORK ASSIGNMENT

VOLUME NO. 2

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	21	1 - 15	02110 - 02112
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9	46	1 - 15	02410
10	47	1 - 10	02410
11	48	21 - 30 , 41 - 45	02420 , 02430
12	50	1 - 14	02510 , 02512
13	53	4 - 14	02520
14	54	1 - 25	02520
15	55	9 - 15	02520

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is not correct you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is correct then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

VOLUME 2 SEGMENT 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 6	(Segment Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 2	(Volume Number)
	66	and	68	0 1	(Segment Number)

Introductory Note

Suppose that a number of small gifts were divided among a few boys by placing each boy's gifts in a pile on the table, each pile belonging to a different boy. If one of the boys desired to protect his share or to keep it separate from the other shares, he might cup his hands and encircle the pile of gifts belonging to him. What is the shape of his two cupped hands? It is a parenthesis ( ) . Similarly, in mathematics parentheses are symbols of inclusion. It is a symbol used to show that the entire expression within the symbol is the name of one number.

In this segment you will learn about the various symbols of inclusion used in mathematics and how to work with them.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Apply the proper principle and select the letter which has the correct answer to the following question: Which is the largest number?

I. $(2 + 3) \times 4$

II. $3 \times (2 + 4)$

III. $4 \times (2 \times 3)$

(A) I

(B) II

(C) III

(D) They are all the same.

$\frac{2}{1}$

Sorry, but your choice is incorrect. When working with fractions, the line between the numerator and denominator is not only a division sign; but it is also a symbol of inclusion. You must first simplify the numerator and write it as a single number and then simplify the denominator and write it as a single number. Thus for example,

$$\frac{8\,(\,3\,+\,2\,+\,1\,)}{7\,(\,11\,-\,8\,)} = \frac{8\,(\,6\,)}{7\,(\,3\,)} = \frac{48}{21} = \frac{16}{7}$$

Please return to page $\frac{22}{2}$ and try this question again.

$\frac{2}{2}$

What procedure do we follow in simplifying an expression that has more than one symbol of inclusion? We first simplify the numbers that are in the innermost symbol. Thus, in this question you first write

$$(\,4\,+\,2\,) = 6$$

and then simplify the expression in the brackets.

Please continue.

Please return to page $\frac{20}{1}$ and try this question again.

Sorry, your choice is incorrect. The expression

$$4 + (2 + 3)$$

is the same as $4 + 5 = 9$

When simplifying expressions with parentheses, you first combine the numbers within the parentheses, and then perform the indicated operations. In this case, 4 is added to 5 , not multiplied.

Please return to page 17/2 and try this question again.

A number written immediately before a parenthesis means that this number multiplies the expression inside the parenthesis.

Thus, for example;

$$7 (3 + 6)$$

means 7×9

Please return to page 27/2 and try this question again.

$\frac{4}{1}$

I made the correct choice.

In the absence of symbols of inclusion multiplication and division are performed in order from left to right first.

Thus, the expression

$$18 \div 9 + 3 \times 4$$

is the same as $(18 \div 9) + (3 \times 4)$

which becomes: $2 + 12 = 14$

Please go on to question 12 below.

$\frac{4}{2}$

Question 12

Apply the proper principle and simplify the expression,

$$3[6 + 48 \div 6 - 4 \div 2]$$

Select the letter which labels the correct statement.

(A) $3\frac{1}{2}$

(B) $40\frac{1}{2}$

(C) 36

(D) None of these.

$\frac{5}{1}$

You must have made a careless error. Examine the given numerator,

$$49 - 5 (3 + 2)$$

Notice that the numbers inside the parenthesis are connected with a plus sign.

Thus, we have $49 - 5 (5) = 49 - 25$

Please continue.

Return to page $\frac{10}{2}$ and try this question again.

$\frac{5}{2}$

Sorry, but your choice is incorrect.

Please check your arithmatic. Decide which operations should be done before the others.

Return to page $\frac{24}{1}$ and try this question again.

$\frac{6}{1}$

We don't agree.

The choices represent different numbers. Please note that

$$(2 + 3) \times 4 = 5 \times 4$$

and

$$3 \times (2 + 4) = 3 \times 6$$

Please return to page $\frac{1}{2}$ and try this question again.

$\frac{6}{2}$

When we have more than one symbol of inclusion, we first simplify the innermost symbol.

In this question, we have to simplify

$$\left(\frac{72 - 18}{9}\right)$$

Write this fraction as a single number and continue.

Please return to page $\frac{13}{2}$ and try this question again.

If you examine the given expression, you will notice that it has three different symbols of inclusion. The innermost symbols are parentheses. We must therefore, simplify the numbers within these symbols. Thus, we begin as follows:

$$\{13 - [35 - (3 + 8 + 17)] - 19\} =$$

$$\{13 - [35 - 28] - 19\}$$

Please continue.

Return to page 28/2 and try this question again.

When the number of different symbols of inclusion increases, greater care must be taken to avoid making mistakes. You should proceed one step at a time, simplifying the innermost symbols first. We will do a question of equal difficulty to illustrate the procedure that you should follow.

Simplify the expression,

$$\{12 + [21 - 13 + 4(14 - 8)]\} - 20$$

We first simplify the innermost symbols of inclusion, and re-write the expression with this symbol removed:

$$\{12 + [21 - 13 + 4 \times 6]\} - 20$$

Now, we simplify the expression in the brackets, and again re-write the expression with this symbol removed:

$$\{12 + 21 - 13 + 24\} - 20$$

Finally, we remove the braces by performing the indicated operations within this symbol.

We get $44 - 20 = 24$

Return to page 38/2 and try this question again.

$\frac{8}{1}$

Yes, but your answer is not in lowest terms.

The fraction you chose can be reduced. Please do so.

Return to page $\frac{10}{2}$ and try this question again.

$\frac{8}{2}$

The expression

$$7 (5 + 1)$$

means

$$7 \times (5 + 1) = 7 \times 6 = 42$$

Please keep in mind that a number immediately preceeding a parenthesis means that this number multiplies the expression within the parenthesis.

Return to page $\frac{24}{1}$ and try this question again.

We do not agree. How do we simplify this expression? In answer to this question, let us do a similar question together. Simplify the expression,

$$\frac{9\ (\ 7\ -\ 1\ )}{4\ (\ 6\ +\ 3\ )}$$

We start with the numerator. Combining the numbers in the parenthesis, we have

$$9\ (\ 6\ )\ =\ 54$$

Similarly, the denominator becomes

$$4\ (\ 9\ )\ =\ 36$$

Thus, we obtain

$$\frac{54}{36}$$

This fraction can be reduced to $\frac{3}{2}$ by dividing numerator and denominator by 18 .

Please return to page $\frac{29}{1}$ and try this question again.

Sorry, but your choice is not correct. Let us do a similar question together. Follow the procedure carefully, and you will discover your error. Simplify the expression,

$$\left\{\left(\frac{28\ -\ 12}{4}\right)\ \div\ 2\right\}\ +\ 8$$

We first simplify the expression within the innermost symbol, and this is:

$$\left(\frac{28\ -\ 12}{4}\right)\ =\ \frac{16}{4}\ =\ 4$$

We now have,

$$\left\{4\ \div\ 2\right\}\ +\ 8$$

Simplifying the numbers within the braces, we get

$$2\ +\ 8\ =\ 10$$

Please return to page $\frac{13}{2}$ and try this question again.

Very good. You made the correct choice. We have to simplify the expression,

$$\frac{2\ (\ 7\ +\ 5\ -\ 3\ )}{3\ (\ 9\ -\ 4\ )}$$

First, we simplify the numerator by combining all the numbers inside the parenthesis. We get,

$$2\ (\ 9\ )\ =\ 18$$

Then, we simplify the denominator by first writing all the numbers in the parenthesis as one number. We get,

$$3\ (\ 5\ )\ =\ 15$$

Hence, the fraction becomes

$$\frac{18}{15}$$

Finally, we divide the numerator and denominator of this fraction by 3 and obtain $\frac{6}{5}$ as the answer.

Please go on to question 5 below.

Question 5

Apply the proper principle and simplify the expression,

$$\frac{49\ -\ 5\ (\ 3\ +\ 2\ )}{7\ +\ 5}$$

Select the letter which labels the correct simplification.

(A) $\frac{19}{12}$

(B) $\frac{24}{12}$

(C) 2

(D) $2\ \frac{20}{12}$

$\frac{11}{1}$

You did not complete the question.

After simplifying the expression in the brackets, you should multiply this expression by $\frac{1}{2}$.

Please return to page $\frac{27}{2}$ and try this question again.

$\frac{11}{2}$

We do not agree.

The expression,

$$3\ [\ 6\ +\ 48\ \div\ 6\ -\ 4\ \div\ 2\ ]$$

is the same as

$$3\ [\ 6\ +\ (\ 48\ \div\ 6\ )\ -\ (\ 4\ \div\ 2\ )\ ]$$

Please continue.

Please return to page $\frac{4}{2}$ and try this question again.

$\frac{12}{1}$

Sorry, but your choice is not correct.

The expression

$$(2 + 3) \times 4$$

means

$$5 \times 4 = 20$$

One of the choices is a number greater than 20.

Please return to page $\frac{1}{2}$ and try this question again.

$\frac{12}{2}$

Have you considered the order of operations?

Please note that the expression

$$14 \times 2 - 2 \div 2$$

means

$$(14 \times 2) - (2 \div 2)$$

Please return to page $\frac{18}{1}$ and try this question again.

Very good. You made the correct choice. The other choices are wrong.

In simplifying an expression that has more than one symbol of inclusion, we first simplify the innermost symbol.

Thus, we have

$$[\ 30 + (\ 4 + 2\ )\ ] \div 12$$

The innermost symbol is the parenthesis. Hence, we write

$$(\ 4 + 2\ ) = 6$$

We now have

$$[\ 30 + 6\ ] \div 12 =$$

$$36 \div 12 = 3$$

Please go on to question 7 below.

Question 7

Apply the proper principle and simplify the expression,

$$\left\{\left(\frac{72 - 18}{9}\right) \div 3\right\} + 3$$

Select the letter which labels the correct statement.

(A) 9

(B) 5

(C) 6

(D) None of these.

14/1

When a number is written immediately before a parenthesis, that number multiplies whatever is inside that parenthesis.

For example,

$$2 (3 + 4 + 1)$$

means

$$2 \text{ times } (3 + 4 + 1)$$

We must perform the operations within the parentheses first;

that is,

$$(3 + 4 + 1) = 8$$

Therefore, the problem becomes

$$2 \times 8$$

Please return to page 17/2 and try this question again.

14/2

Sorry, but your choice is not correct.

The expression,

$$18 \div 9 + 3 \times 4$$

is the same as

$$(18 \div 9) + (3 \times 4)$$

Since the multiplication and divisions are done before the additions and subtractions.

Please re-read the top of page 23 of your text assignment.

Please return to page 26/2 and try this question again.

You must have made a mistake in your arithmetic.

Did you observe that this question has three different symbols of inclusion? Naming these symbols in order, that is; the innermost symbols first, we have parentheses, brackets, and braces.

In order to avoid errors, it is important to proceed carefully, one step at a time. We begin by removing the innermost symbol first.

Thus,

$$\{13 - [35 - (3 + 8 + 17)] - 19\} =$$
$$\{13 - [35 - (28)] - 19\} =$$
$$\{13 - [35 - 28] - 19\}$$

Now, proceed by simplifying the expression in the brackets.

Please continue.

Please return to page 28/2 and try this question again.

Did you find this question difficult? It is. That's all the more reason why you will enjoy getting it right.

If we examine the given expression, we note that there are four different symbols of inclusion. Which is the innermost symbol?

Yes, the bar over

$$3 + 2$$

You should, therefore, begin by simplifying this expression.

We can write,

$$2\{3 + 4[5 + 6(5 + 4 \times 5)]\} =$$
$$2\{3 + 4[5 + 6(25)]\}$$

Please continue from here.

Please return to page 35/2 and try this question again.

16/1

Examine the numerator of the given fraction. Note that the two numbers in the parentheses must be added first. That answer is multiplied by 5 .

Then this result is subtracted from 49 . The 5 cannot be subtracted from 49 before it multipleis the parentheses.

Please return to page 10/2 and try this question again.

16/2

We don't agree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Return to page 27/2 and try this question again.

Very good. You made the correct choice; we have,

$$(2 + 3) \times 4 =$$

$$5 \times 4 = 20$$

$$3 \times (2 + 4) =$$

$$3 \times 6 = 18$$

$$4 \times (2 \times 3) =$$

$$4 \times 6 = 24$$

Please go on to question 2 below.

Question 2

Apply the proper principle and select the letter which labels the correct statement.

(A) $4 + (2 + 3) = (4)(5)$

(B) $3(2 + 4) = (3)(6)$

(C) $4(2 \times 3) = 20$

(D) $5 + 3(4 + 2) = 8(6)$

18
1

This is the correct answer.

Please go on to question 10 below.

Question 10

Apply the proper principle and simplify the expression:

$$14 \times 2 - 2 \div 2$$

Select the letter which labels the correct statement.

(A) 0 (C) 27

(B) 13 (D) None of these.

18
2

Sorry, but you made an incorrect choice.

Recall that a number immediately before a symbol of inclusion means that this number multiplies the expression within this symbol.

Thus,

$$0 (8 - 2)$$

means

$$0 \times (8 - 2) =$$

$$0 \times 6$$

Now, you know that zero times any number is equal to zero.

Please continue.

Please return to page 32/1 and try this question again.

Very good. You made the correct choice.

We have to simplify the expression:

$$62 - 2\left\{39 - [\,4\,(\,7 - 6\,) + 4\,(\,1 + 6\,)\,] + 9\right\}$$

We first simplify the expression within the innermost symbols of inclusion. We get:

$$62 - 2\left\{39 - [\,(\,4 \times 1\,) + (\,4 \times 7\,)\,] + 9\right\} =$$

$$62 - 2\left\{39 - [\,4 + 28\,] + 9\right\}$$

Next we simplify the expression within the brackets.

We get:

$$62 - 2\left\{39 - [\,32\,] + 9\right\} =$$

$$62 - 2\left\{39 - 32 + 9\right\} =$$

$$62 - 2\left\{16\right\} =$$

$$62 - 32 =$$

$$30$$

Since this is not one of the numbers given, " none of these " is correct.

Please go on to question 16 below.

Question 16

Apply the proper principle and simplify the expression,

$$\left\{21 + 3\,[\,9\,(\,6 - 4\,) - 3\,]\right\} - 26\,(\,8 - 6\,)$$

t t[illegible]er which labels the correct statement.

(A) 10

(B) 40

(C) 14

(D) None of these.

$\frac{20}{1}$

This is the correct answer.

Please go on to the next question.

Question 6

Apply the proper principle and simplify the expression

$$[30 - (4 + 2)] \div 12$$

Select the letter which labels the correct result.

(A) $30 - \frac{1}{2}$

(B) 15

(C) 3

(D) None of these.

$\frac{20}{2}$

We do not agree.

Go over your work and check your order of operations.

Please return to page $\frac{13}{2}$ and try this question again.

We first combine the numbers in the parenthesis. Thus,

$$3 \times (2 + 4) =$$

$$3 \times 6$$

This is not the large number.

Please return to page 1/2 and try this question again.

We do not agree.

The bar over 3×14

is a symbol of inclusion, just like the parenthesis.

This expression should, therefore, be simplified first.

Thus,

$$[\overline{3 \times 14} - 8 (9 - 6)] =$$

$$[42 - 8 (3)] =$$

$$[42 - 24]$$

Please continue.

Please return to page 24/1 and try this question again.

Very good. You made the correct choice. The expression

$$\frac{6(3 + 5)}{2(12 - 3)} = \frac{6(8)}{2(9)} = \frac{48}{18} = \frac{8}{3}$$

The last fraction was obtained by dividing the numerator and denominator of the previous fraction by 6 .

Please go on to question 4 below.

Question 4

Apply the proper principle and simplify the expression,

$$\frac{3(7 + 5 - 3)}{3(9 - 4)}$$

Select the letter which labels the correct simplification

(A) $\frac{2}{3}$ (C) $\frac{30}{39}$

(B) $\frac{6}{5}$ (D) $\frac{16}{23}$

We do not agree.

One of the letters d[illegible] the correct answer next to it

Please reconsider your [illegible]

Please return to page 18/2 and try this question again.

23/2

Sorry, but your choice [illegible] incorrect.

Did you begin by first simplifying the innermost symbols of inclusion?

You should have started as follows:

$$\{21 \div 3\ [9(6 - 4) - 3]\} - 26(8 - 6) =$$

$$\{21 \div 3\ [9 \times 2 - 3]\} - 26 \times 2 =$$

$$\{21 \div 3\ [18 - 3]\} - 52$$

Please continue.

Please return to page 19/2 and try this question again.

24/1

This is the correct answer.

Please go on to the next question below.

Question 3

Apply the proper principle and simplify the expression.

$$7(5 + \quad - \sqrt{3 \times 14} - 8(9 - \quad)]$$

Select the letter which is the correct statement.

(A)

(B) 21

(C) 63

(D) 63

24/2

If a variable has a replacement set which contains only a single quantity, it is called a constant. Did you have any information about the replacement set? Since you did not, you do not know if it is a constant.

Therefore, this choice is not correct.

Please return to page 42/2 and try question 1 again.

do not agree. The expression

$$5 + 3(4 + 2)$$

means

$$5 + 3(6) =$$

$$5 + 18 =$$

$$23$$

The procedure we use is as follows:

We combine the numbers within the parenthesis; thus,

$$4 - 2 = 6$$

Then we multiply this number by the number before the paremthesis, getting

$$3(6) = 18$$

The 5 is not added to the 3 before multiplying. Finally, we obtain

$$5 + 18 = 23$$

Please return to page 7/2 and try this question again.

No. One of the other answers is correct.

Please re-check your work and make the correction.

Please return to page 35/2 and re-do the question.

Very good. You made the correct choice.

Mathematicians have agreed on the following rule:

If symbols of inclusion are omitted, multiplication and division are performed in order from left to right. Thus,

$$14 \times 2 - 2 \div 2$$

is equivalent to

$$(14 \times 2) - (2 \div 2) =$$

$$28 - 1 = 27$$

Note that a number divided by itself is equal to 1 .

Please go on to question 11 below.

Question 11

Apply the proper principle and simplify the expression,

$$18 \div 9 + 3 \times 4$$

Select the letter which labels the correct statement.

(A) 14

(B) $\frac{6}{7}$

(C) 20

(D) None of these.

Very good. You made the correct choice.

In simplifying an expression which has more than one symbol of inclusion, we first simplify the innermost symbol.

Thus, we have:

$$\left\{ \left(\frac{72 - 18}{9} \right) \div 3 \right\} + 3$$

$$\{ (\quad) \div 3 \} + 3$$

$$\{ \quad 2 \quad \} + 3$$

$$5$$

Simplifying $\left(\frac{72 - 18}{9} \right)$

we get $\frac{54}{9} = 6$

Please go on to question 8 below.

Apply the proper principle and simplify the expression

$$\frac{1}{2} \quad [5 (3 + 5) + 2 (7 - 4)]$$

Select the letter which labels the correct statement.

(A) 23 (C) 46

(E $\frac{19}{2}$ None of these.

28
1

Very good. You made the correct choice. The expression,

$$3\ [6\ +\ 48\ \div\ 6\ -\ 4\ \div\ 2\ ]$$

is the same as

$$3\ [6\ +\ (48\ \div\ 6)\ -\ (4\ \div\ 2)]$$

Simplifying the innermost symbols of inclusion first, we get

$$3\ [6\ +\ 8\ -\ 2]$$

$$3\ [\ 12\ \ ]$$

$$36$$

Please go on to question 13 below.

28
2

Question 13

Apply the proper principle and simplify the expression,

$$\{13\ -\ [35\ -\ (3\ +\ 8\ +\ 17)]\ +\ 19\}$$

Select the letter which labels the correct statement.

(A) 22

(B) 25

(C) 26

(D) None of these.

This is the correct answer.

Please go on to the next question.

Question 3

Apply the proper principle and simplify the expression

$$\frac{6(3 + 5)}{2(12 - 3)}$$

Select the letter which labels the correct simplification.

(A) $\frac{8}{3}$ (C) $\frac{4}{9}$

(B) $\frac{23}{21}$ (D) None of these other choices.

Sorry, but we do not agree.

Keep in mind the three numbers that came up on the dice and follow the instructions.

The first number doubled is 4 . Adding 5 , we get 9 .

Multiplying this result by 5 , we get 45 . Now, we add the number on the second die, and obtain 49 . Multiplying this number by 10 , we have 490.

Finally, we add the number on the third die and end up with 495 .

Now simplify the expressions next to the letters and find out which one equals 495 .

Please return to page 30/2 and try this question again.

30
1

Very good. You made the correct choice. We have a simplify the expression

$$2\left\{3 + 4\ [5 + 6(5 + 4 \times \overline{3 + 2})]\right\}$$

We proceed by first simplifying the expression within the innermost symbol of inclusion; that is,

We get:
$$2\left\{3 + 4\ [5 + 6(5 + 4 \times \overline{5})]\right\} =$$
$$2\left\{3 + 4\ [5 + 6(5 + 20)\ ]\right\}$$

Now, we simplify the expression within the parenthesis. We obtain,

$$2\left\{3 + 4\ [5 + 6 \times 25]\right\} =$$
$$2\left\{3 + 4\ [5 + 150]\right\}$$

Next, we simplify the expression within the brackets. Thus, we get

$$2\left\{3 + 4 \times 155\right\} =$$
$$2\left\{3 + 620\right\} =$$
$$2\left\{623\right\} =$$
$$1246$$

Please go on to question 18 below.

30
2

Question 18

Relate to the proper principle and select the letter which correctly describes the following instructions: You have tossed three dice and the numbers 2, 4, and 5 came up. Take the number of the first die, double it and add 5 to the result; multiply the number you now have 5, add the number on the second die, and multiply the number you now have by 10.

Finally, add to this the number on the third die.

(A) $10\ [2(2 + 5) + 4] + 5$

(B) $10\ [4(5 \times 2) + 4] + 5$

(C) $10\ [[5(2 \times 2 + 5) + 4] + 5$

(D) None of these.

A variable represents any of the elements of a certain set.

While the set may have many elements, it also may not.

It might have only one element; therefore, this choice is not correct.

Please return to page 42/2 and try question 1 again.

An expression is called an open expression if it contains a variable, not a constant. Since this expression contains a variable, it <u>is</u> an open expression.

Therefore, this choice is not correct.

Please return to page 49/2 and try question 4 again.

$\frac{32}{1}$

This is the correct answer.

Please go on to question 14 which follows.

Question 14

Apply the proper principle and simplify the expression,

$$\left\{21 + [\,12 - 0(8 - 2)\,]\right\}$$

Select the letter which labels the correct statement.

(A) 33

(B) 27

(C) 35

(D) None of these.

$\frac{32}{2}$

There are seven days in the week; that is true, but that is not what the question asked.

Please return to page $\frac{54}{2}$ and try question 6 again.

Since the length of the ladder represented by c is always the same, c is a constant.

Therefore, this choice is not correct.

Please return to page 51
2 and try question 2 again.

This is a very nice value, but it is not the value you were asked to find.

Did you fail to see the decimal point?

Please return to page 46
2 and try question 8 again.

34/1

This is a fine value, but is it the only one? How many values did the replacement set for w contain?

You cannot decide to ignore some values and use others.

Please return to page 68/2 and try question 7 again.

34/2

In order to perform this subtraction it is necessary to write the number 6 as 6.00.

You would do best to put 4.73 under the 6.00 with the decimal points in line. If you did that, you probably would not make the mistakes in borrowing which you have made.

This choice is not correct.

Please return to page 69/1 and try question 9 again.

Very good. You made the correct choice. We have to simplify the expression

$$\{21 + 3[9(6 - 4) - 3]\} - 26(8 - 6)$$

We begin by simplifying the expression within the innermost symbol of inclusion. We get

$$\{21 + 3[18 - 3]\} - 26 \times 2$$

We continue by simplifying the expression in the brackets. Thus,

$$\{21 + 3 \times 15\} - 52 =$$
$$\{21 + 45\} - 52 =$$
$$66 - 52 =$$
$$14$$

Please go on to question 17 which follows.

Question 17

Apply the proper principle and simplify the expression,

$$2\{3 + 4[5 + 6(5 + 4 \times \overline{3 + 2})]\}$$

Select the letter which labels the correct statement.

(A) 766

(B) 1246

(C) 716

(D) None of these.

Congratulations!

You did a difficult problem correctly.

We follow the given instructions. The first number doubled is

$$2 \times 2$$

Add 5 and multiplying the result by 5 , yields

$$5(\overline{2 \times 2} + 5)$$

Adding to this, the number on the second die and multiplying the result by 10 yields

$$10\ [5(\overline{2 \times 2} + 5) + 4]$$

Finally, we add the number on the third die and obtain

$$10\ [5(\overline{2 \times 2} + 5) + 5] + 5$$

If we simplify this expression, we get 495 . It is interesting to note that

$$\begin{array}{r} 495 \\ -250 \\ \hline 245 \end{array}$$

That is, if we subtract 250 from the result obtained by following the above instructions; the numbers reading from left to right are the numbers of the dice.

Incidentally, this is the basis for a good trick if you ask a friend to roll a die three times and to follow those instructions and tell you the answer. You can tell him what numbers he rolled by merely subtracting 250 from his answer. The digits will be the numbers in the same order.

You have now finished this segment. Hand in your Punch Card.

You should have entered in your notebook, the following definitions and formulas:

1. Symbols of inclusion are used in mathematics to avoid ambiguity.

2. A number before a symbol of inclusion means that this number multiplies the numbers within the symbol.

3. In simplifying an expression containing more than one symbol of inclusion, we first simplify the innermost symbol.

4. In the absence of symbols, multiplications and divisions are performed from left to right.

You should now be able to complete the following problems from your homework assignment:

Problems 1, 2, 3, and 4 .

38
1

Very good. You made the correct choice. We have to simplify the expression

$$\{21 + [12 - 0(8 -$$

Examining the innermost symbols of inclusion, we note that

$$- 0(8 - 2) =$$

$$0$$

since zero times any number is equal to zero.

Thus we have:

$$\{21 + [12 - 0\} =$$

$$\{21 + 12 \} =$$

$$33$$

Please go on to question 15 below.

38
2

Question 15

Apply the proper principle and simplify the expression,

$$62 - 2\{39 - [4(7 - 6) + 4(1 + 6)] + 9\}$$

Select the letter which labels the correct statement.

(A) 18

(B) 55

(C) 46

(D) None of these.

Since this letter represents the elements of a set which contains only one element; it is a constant. Of course, you could call it a variable with only one value; but that would not be what the question asked for. You were asked for the one choice which is a variable.

There is a better choice.

Please return to page 62/2 and try question 3 again.

This set is actually a subset of the domain, but we were told that the domain is the set of even integers less than 10.
Therefore, we need the set which contains all such integers.

Please return to page 44/2 and try question 5 again.

VOLUME 2 SEGMENT 2 begins here:

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS				
48	and	50	0 7	(Sequence Number)
54	and	56	0 4	(Type of Punch Card)
60	and	62	0 2	(Volume Number)
66	and	68	0 2	(Segment Number)

Your reading assignment for this Segment is page 35-37 .

Supplementary Notes:

The new methods of indicating multiplication are very important. In the previous segment where you learned to use parenthesis, the point was made that

$3(2 + 4)$ means

multiply 3 by the sum of 2 and 4 . It is not only in this case that the X sign for multiplication is not used. In algebra, we very rarely use the X sign. An obvious reason for this is that we often use the letter x as a variable, and the difference between x and X is difficult to distinguish.

There are two ways we use to indicate multiplications (as your text points out.) The first, and the least used method is the dot. Thus, $5 \cdot 2$ means 5 times 2 and $5a$ means 5 times a . This method might lead to confusion between $5 \cdot 2$ and the decimal 5. 2 . If you check carefully, you will see a difference in the height of the dot from the line of print in which it appears.

Please continue on page 41

By far the more important method of showing multiplication of two quantities is to write the quantities next to each other with NO SIGN BETWEEN THEM. Thus 5a or xy indicates a multiplication. But you will notice that two numbers cannot be written in such a way since our number system would cause us to read them as a single number. That is, 1928 does not mean 19 times 28 . We add one detail to avoid this difficulty; we use parenthesis. Thus, (19)(28) could not be a single number. However, since there is no sign between the quantities, this indicates multiplication. Actually, we could do with only one set of parentheses; the same meaning would exist if we wrote 19(28) or (19)28 . Notice at the bottom of page 36 in your text, we proceed from $\frac{1}{2}$ r to $\frac{1}{2}$ (2) , inserting parentheses when we need them to give meaning to an expression.

You will often be called upon to evaluate an algebraic expression. Remember that in order to do this, you must be given a replacement set (domain) for the variable. Frequently the domain will contain only one element, but this is not always the case.

Did you learn the definition of the word term , as listed in the text on page 37 ? Don't look back just yet. A useful statement concerning terms is:

Any two terms are separated either by a plus sign or a minus sign. Therefore, if an algebraic expression contains no addition or subtraction to separate it into parts it is a single term.

Now you should re-read the book's definition.

Please continue on page 42.

42
1

You see that only multiplication (product) or division (quotient) can be involved within a term. Then how can

$$\frac{y - x}{5}$$

be a term as stated on line 6 of page 37 ?

The answer is that the fraction line acts like parentheses so that we have

(y - x) divided by 5

which is one term.

In the same way

$$3(x - y)$$

is one term.

Would you believe that this is one term?

$$\frac{5xy - 7ab + 3w}{2m + p + q}$$

Since this is just one huge division, it is a single term.

Now that you have learned about variables, replacement set, domain, constant, algebraic expression, and so on; you will be asked a series of questions to draw your attention to the more important points.

42
2

Question 1

According to your reading, which statement do you recognize is true?

(A) a variable is a constant

(B) a constant is a variable

(C) a variable has many values

(D) a constant has many values

Since the top of the ladder will slide down the wall, the height, which is represented by b , is not a constant.

Therefore, this choice is not correct.

Please return to page 51/2 and try question 2 again.

This choice is correct.

Congratulations on handling the decimals correctly.

Please proceed to question 10 which follows.

Question 10

Perform the calculation to evaluate the open expression

$$\frac{x}{y}$$

when x and y have the values 15 and 5 respectively.

(A) 1 and 1

(B) $\frac{1}{3}$

(C) 3

(D) 3 and $\frac{1}{3}$

44
1

If the value of an expression can be found merely by performing indicated calculations, it is not an open expression. Since this is merely an addition of two numbers, it is not open.

Then this choice is correct.

Please proceed to question 5 below.

44
2

Question 5

If the domain of n is $\{\text{even integers less than } 10\}$

which set do not recognize as the domain?

(A) $\{2, 4, 6, 8\}$

(B) $\{2, 4, 6\}$

(C) $\{2, 4\}$

(D) $\{6\}$

You seem to have missed something which was explained in the text and again in the notes at the beginning of this segment. This choice is not correct. Reread this material.

Please return to page 66/2 and try question 11 again.

The plus sign separates this expression into two parts, each being a term. Therefore, this does not contain only one term.

Please return to page 77/2 and try question 13 again.

Since the domain of w contained two values, the expression

$$2w + 3$$

will also have two values. You have evaluated the expression correctly, and this is the correct choice.

Please proceed to question 8 below.

Question 8

Perform the calculation to evaluate the open expression

$$.3 + x$$

when x has the value 5 .

(A) 8

(B) 5.3

(C) 1.5

(D) .35

Since a constant is defined as a variable which has only a single value, this choice is not correct.

Please return to page 42/2 and try question 1 again.

You weren't thinking when you chose this answer.

What operation is indicated when we write 9w ?

Now you should get the correct answer.

Please return to page 61/2 and try question 12 again.

48
1

There are two days in the usual weekend, but that is not what the question asked. Did you remember that domain and replacement set have the same meaning?

Please return to page 54/2 and try question 6 again.

48
2

Have you understood the question correctly? The problem asked you to do one calculation; how do you get two answers?

This choice is not correct.

Please return to page 43/2 and try question 10 again.

Since R has a replacement set which contains more than one element, R is a variable and not a constant.

Therefore, this choice is correct.

Please proceed to question 4 below.

49
1

Question 4

Which do you recognize is NOT an open expression?

(A) $3x$

(B) $2.4n + 9$

(C) $5 + 17.1$

(D) $2p + 2q$

50/1

As it stands, the expression has signs which separate it into parts. Each part is called a term .

Therefore, this choice is not correct.

Please return to page 64/2 and try question 14 again.

50/2

The value of the fraction

$$\frac{h - g}{3}$$

must be calculated properly. It calls for a divison by 3 , but the value of the numerator must be divided by the 3 .
Did you do this? Every division calculation has a result. What was the result of your division? You probably made a mistake there; this choice is not correct.

Return to page 65/2 and try question 16 again.

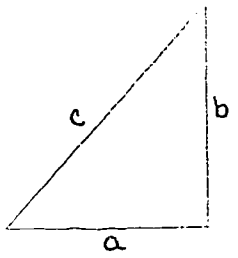
Since a constant is defined as a variable which has only a single value, it is correct to say that a constant is a variable.

Therefore, this choice is correct.

Please proceed to question 2 below.

Question 2

In the diagram c represents the length of a ladder leaning against a wall. The top of the ladder is a distance b above the ground, while the foot (bottom) of the ladder is a distance a from the wall. If the bottom of the ladder is pulled outward from the wall, the top of the ladder will slide down the wall.



Which statement do you recognize is correct?

(A) a , b , and c , are variable: none is constant

(B) a and c are variable; b is constant

(C) b and c are variable; a is constant

(D) a and b are variable; c is constant

52
1

In order to find the value of the expression, it is necessary to replace the letter p by its value wherever it appears. Since there are three terms, you must do this for each term, and then add the results. There are three separate parts to the problem, but you appear to have done only a portion of each part. You should notice that the value of p must be the same wherever it appears.

Please return to page 73/2 and try question 17 again.

52
2

It is correct to find the value of each separate quantity in parentheses before doing anything else. However, you appear to have made a mistake, perhaps in not being able to understand what you wrote.

This choice is not correct.

Please return to page 75/2 and try question 19 again.

Your arithmetic is very good, but you did the wrong calculation.

What operation is involved in

$$.3 + x \quad ?$$

Please return to page 46/2 and try question 8 again.

The minus sign separates this expression into two parts, each being a term.

Therefore, this choice is not correct.

Please return to page 77/2 and try question 13 again.

54
1

The domain is the set of even integers less than 10 .

Then this choice, which lists the roster of that set, is correct.

Please proceed to question 6 below.

54
2

Question 6

If the domain of x is $\{$ weekdays $\}$, choose the number of elements in the replacement set for x .

(A) 7

(B) 5

(C) 2

(D) impossible to tell

Since the bottom of the ladder is being pulled outward from the wall, the distance from the wall; which is represented by a is not a constant.

Therefore, this choice is not correct.

Please return to page 51/2 and try question 2 again.

You have decided correctly that this is a multiplication problem. However, you've mixed up something.

Did you use a rule involving inverting some fraction? There is no such procedure in multiplication; that belongs to a division problem.

Please return to page 66/2 and try question 11 again.

$\frac{56}{1}$

You have become much too generous.

Since the replacement set for w contained the numbers 3 and 12, they should be used for w in the evaluation.

But are they values of the expression

$$2w + 3 \quad ?$$

This choice is not correct.

Please return to page $\frac{68}{2}$ and try question 7 again.

$\frac{56}{2}$

The value of the fraction

$$\frac{h - g}{3}$$

must be calculated properly.

It calls for a division by 3, but the value of the numerator must be divided by the 3. Did you do this? Or did you find a faster way which did not involve getting the value of the numerator first?

You have done something wrong since this choice is not correct.

Please return to page $\frac{65}{2}$ and try question 16 again.

You are quite right in recognizing that there is a calculation which you are to perform, but what operation did you choose? What operation is intended when 9w is given?

This choice is not correct.

Please return to page 61/2 and try question 12 again.

The numerator and the denominator contain the same type of expressions. In each case, it is necessary to find the value of the part in the parentheses before doing anything with the number on the outside.

You appear to have forgotten that and have, therefore, chosen an incorrect answer.

Please return to page 60/2 and try question 18 again.

$\frac{58}{1}$

Since the replacement set has the same meaning as the domain, you should find this question easy to answer.

You do know the meaning of weekday don't you? It does not mean " days of the week ".

This choice is not correct.

Please return to page $\frac{54}{2}$ and try question 6 again.

$\frac{58}{2}$

Division is an operation which must be done in the order indicated.

Remember that

$$\frac{28}{7} = 4$$

while

$$\frac{7}{28} = \frac{1}{4}$$

This choice is not correct.

Return to page $\frac{43}{2}$ and try question 10 again.

You didn't look very hard, did you?

One of the other choices is correct.

Please return to page 62/2 and try question 3 again.

What does the word respectively mean? It cannot be ignored.

The meaning is: in that order.

Therefore, the value of m is 6 , and the value of h is 5 . You didn't use those values, did you?

In any case, this choice is not correct.

Please return to page 70/2 and try question 15 again.

$\frac{74}{1}$

A prime factor is a factor that is divisible only by itself and 1 .

Thus, for example, 7 and 3 are prime factors of 21 since

$$7 \times 3 = 21$$

and both 7 and 3 are numbers which are exactly divisible only by themselves and 1 .

Please return to page $\frac{111}{2}$ and try this question again.

$\frac{74}{2}$

Sorry, but your choice is not correct.

To obtain a number when its factors are given, we multiply the given factors. Their product is the required number. You should also keep in mind that

$$1 \times 1 = 1 \quad \text{and}$$

$$1 \times 1 \times 1 = 1$$

That is, no matter how many times 1 is multiplied by itself, the product is 1 .

Please return to page $\frac{100}{2}$ and try this question again.

In the numerator and in the denominator you meet the same type of expression. The part in parentheses must be calculated first, then you can proceed with the rest.

The calculation looks like this:

$$\frac{t\,(m - n)}{5\,(m + n)} = \frac{3\,(9 - 5)}{5\,(9 + 5)}$$

$$= \frac{3\,(4)}{5\,(14)}$$

$$= \frac{12}{70}$$

$$= \frac{6}{35}$$

Therefore, this choice is correct.

Please proceed to question 19 below.

$\frac{75}{1}$

Question 19

If the area of a ring is

$$\pi\,(R + r)(R - r)$$

apply your knowledge to find the area when

$$\pi = \frac{22}{7}$$

$$R = 10$$

$$r = 4$$

(A) 264

(B) 572

(C) $\frac{22}{7}\,(84)$

(D) none of these

$\frac{76}{1}$

15m does mean that 15 multiplies m .

However, 15 can be written as $\frac{15}{1}$. When it multiplies a fraction the 15 remains in the numerator and multiplies the numerator of the other fraction.

Please return to page $\frac{66}{2}$ and try question 11 again.

$\frac{76}{2}$

What is the meaning of 2m ? It does not mean the value of 2 followed by the value of m . There is a specific calculation you are to do. If necessary, reread the text and your notes; this choice is not correct.

Please return to page $\frac{70}{2}$ and try question 15 again.

This choice is correct.

You have decided correctly that this problem calls for multiplication since there is no sign between the 9 and the w . Your arithmetic is good; the 5 does not divide into 63 evenly.

The answer is left in improper fraction form.

Please proceed to question 13 below.

Question 13

Which choice do you recognize contains exactly one term?

(A) f + m

(B) 1 + m

(C) 17xy

(D) 2x - y

78
1

Since p is an element of a set containing three elements, it is necessary to use each value for p . If you start with the first value for p and use it to get the value of each term and then add the three terms together, you will have the first value for the expression. Then it will be necessary to do the same thing over using the second value for p , and once more using the third value.

If you do this, you should find that one of the other choices is correct.

Please return to page 73/2 and try question 17 again.

78
2

If two or more numbers (or expressions) are multiplied, each one is a factor of the product. To obtain the number when its factors are given, we multiply the given factors. A number is equal to the product of its factors.

Please return to page 104/1 and try this question again.

Substituting the values given (be sure to note the difference between R and r), we get:

$$(R + r)(R - r) = \frac{22}{7}(10 + 4)(10 - 4)$$

$$= \frac{22}{\cancel{7}_{1}}\left(\cancel{14}^{2}\right)(6)$$

$$= 22(12)$$

$$= 264$$

Therefore, this choice is correct.

Please proceed to question 20 below.

Question 20

If the open expression

$$5x - 2$$

has the values which are elements of

$$\{8, 13, 18\}$$

apply your knowledge to find the domain of x .

(A) $\{38, 63, 88\}$

(B) $\left\{\frac{6}{5}, \frac{11}{5}, \frac{16}{5}\right\}$

(C) $\{3, 4, 5\}$

(D) $\{2, 3, 4\}$

VOLUME 2 SEGMENT 3 begins here:

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 8	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 2	(Volume Number)
	66	and	68	0 3	(Segment Number)

Your reading assignment for this segment is page 40 .

Introductory Note

Every human activity such as business, science or sports, has a language of its own with certain words having special meanings. Similarly, mathematics has a vocabulary of its own where certain words have a special meaning. Just as you cannot be a good football player without knowing the vocabulary of this sport, you cannot be a good student of algebra without a knowledge of its vocabulary.

In this segment, you will learn the meaning of such words as factor, coefficient, exponent and base. Your mathematical vocabulary will be enriched to help you become a better student of algebra.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1 is on page 81
1

Question 1

Apply the proper principle and select the letter which correctly completes the statement:

A pair of factors of 30 is,

(A) 3 and 27

(B) 15 and 15

(C) 6 and 5

(D) 3 , 5 , and 2

We do not agree. Set S is defined by the roster method.

Thus, the number whose factors are

4 , 5 , 5 , 7 is

4 x 5 x 5 x 7

The other two sets are defined by a rule. Please list the elements of each of these two sets and then multiply them.

Please return to page 86/2 and try this question again.

82
1

Did you examine every member of the set you chose? If you do, you will notice that 3 is not a factor of 28 . The number 3 does not divide 28 exactly.

Please return to page 118 and try this question again.
1

82
2

Please write the following definition in your notebook:

A base is a number raised to a power, which is indicated by an exponent. The exponent indicates how many times the base is used as a factor.

In 8^3 , the base 8 is used three times as a factor.

Please return to page 113 and try this question again.
1

You understood the problem although it was set up in reverse order, but you have a mistake somewhere since this choice is not correct.

Please return to page 79/2 and try question 20 again.

If the factors of a number are listed as 1 , 2 , 5 , and 5 , all three of the factors must be multiplied; i.e.,

$$2 \times 5 \times 5 = 50$$

It is true that the divisors of 50 other than itself are only

1 , 2 , and 5

Please return to page 100/2 and try question 5 again.

$\frac{84}{1}$

The number 15 is not a prime factor because 15 can be divided exactly by 5 and by 3 .

A prime factor is a number divisible only by itself and by 1 .

Please return to page $\frac{111}{2}$ and try this question again.

$\frac{84}{2}$

ry, but your choice is not correct.

Each factor of a product is the coefficient of the product of the remaining factors. Think of the number

$$\frac{5}{6}xyz$$

as the product of

$$\frac{5}{6} \text{ , } x \text{ , } y \text{ , and } z$$

Please return to page $\frac{99}{2}$ and try this question again.

Almost right. In the term $39y$, 39 is the coefficient; and y is the base. But the exponent of y is not zero.

Ask yourself:

How many times is y used as a factor in the term $39y$?

Please return to page 97/2 and try this question again.

Sorry, but we do not agree.

The square of a number means that number multiplied by itself.

Thus, for example, the square of 8 means

$$8 \times 8$$

and the square of $3y$ means

$$(3y)(3y)$$

Please return to page 110/2 and try this question again.

Very good. You made the correct choice. It is necessary to have a clear idea of the meaning of the word coefficient. Each factor of a product is the coefficient. Each factor of a product is the coefficient of the product of the remaining factors.

Thus,

$27xy$ has the factors

27, x, and y

The coefficient of x is the product of the remaining factors

27 and

y

Please go on to question 7 which follows.

Question 7

Apply the proper principle and select the letter which correctly answers the following question:

The elements of the following sets are factors of a number. Which set represents the largest number?

S = {4, 5, 5, 7}

T = {The set of all natural numbers less than 7}

U = {The set of positive whole numbers divisible by 5 and less than 20.}

(A) Set S

(B) Set T

(C) Set U

(D) Sets S and U are equal.

It appears that you interpreted the problem correctly and found values that you felt could be substituted for x that would produce the given set of values.

But you evidently made a mistake in solving the equation. This could have been detected by substituting your answer in the original open expression.

Please return to page 79/2 and try question 20 again.

The expression "the cube of x" or "x cubed" means

$$x \cdot x \cdot x$$

Since the factor x is used three times, we write

$$x^3$$

Please return to page 115/2 and try this question again.

184
1

The statement you chose

$$x = y + y$$

can be interpreted as

" x is 7 more than y ."

That isn't what the problem stated.

Please return to page 196 (1) and try question 9 again.

184
2

Your answer is correct.

Please go on to question 19 which follows.

Question 19

The sum of the ages of John and his father is 5 smaller than twice his mother's age. If the ages of John, father, and mother are represented respectively by x , y , and z apply your knowledge to select the choice which states this fact.

(A) $x + y = 2z - 5$

(B) $x + y < 2z - 5$

(C) $x + y < 2z$

(D) $x + y = 5 - 2z$

The sentence,

q is 7 less than x .

can be written in different ways. It is preferred that you write it in the form

$$q = x - 7$$

However, the other forms, such as

$$q + 7 = x$$

and $$x - q = 7$$

are equivalent.

Please go on to question 13 below.

Question 13

Choose the equation which is a correct representation of the statement:

" y is x less than z ."

(A) $x = z + y$

(B) $y - x = z$

(C) $x = z - y$

(D) $y - z = x$

Congratulations!

Mary has an age.

She is greater than zero. Thus, John must be greater than 8 .

The set of ages greater than 8 is the set ,

$$\{9 , 10 , 11 , 12 , \ldots\}$$

The upper limit is not specified.

Please go on to question 3 below.

Question 3

Choose the domain below for the variable x in the following relationship:

"If x is my age, then

$$2x - 4$$

is my brother's age."

(A) $\{3 , 4 , 5 , \ldots\}$

(B) $\{2 , 3 , 4 , \ldots\}$

(C) $\{1 , 2 , 3 , \ldots\}$

(D) $\{0 , 1 , 2 , \ldots\}$

No, we disagree.

When we write the symbol for larger than; the arrow $>$, we must be sure that it points in the proper direction. The arrow should always point towards the smaller number.

For example,

$$5 > 4$$

or $$4 < 5$$

Please return to page 200/1 and try question 17 again.

Have you forgotten the meaning of the word "quotient" ? Perhaps you will recall it if we tell you that the quotient of

10 and 2 is 5 .

Please return to page 195/2 and try question 18 again.

188
1

There is a misunderstanding here. The phrase "is smaller than" can be interpreted in at least two ways.

Let's take for example:

" 5 is smaller than 8 "

It is written

$5 < 8$

but the phrase

" 5 is 3 smaller than 8 "

can be written as

$5 = 8 - 3$

Please return to page 184/2 and try question 19 again.

188
2

I'm sorry, we disagree.

Let's consider your answer.

$w = 5z + 2$

Since w represents the cost of one pencil, and z the cost of one pen; you have written:

The cost of a pencil is two more than five times the cost of one pen.

Is this the statement you were asked to translate?

Please return to page 199/2 and try question 20 again

This choice is correct.

Now proceed to question 7 which follows.

Question 7

If y represents the number of freshmen, and w represents a quantity twice as large; choose the correct statement.

(A) $w < y$

(B) $w > y$

(C) $2w = 2y$

(D) $w = 2y$

In verbal problems you must be careful to represent the meaning of each variable used and the units of each number used.

In this problem,

if w represents the cost of one pencil;

then 5w and not 5 + w represents the cost of 5 pencils.

Please return to page 199/2 and try question 20 again.

190
1

Congratulations!

The most direct translation of the equation,

$$x = 2y - 3$$

can be arrived at in the following manner,

the part " x = " becomes " x is "

the part " -3 " becomes " three less than "

the part " 2y " becomes " double the number y . "

Thus, $c = 2y - 3$

is best translated as,

" x is three less than double the number y . "

Please go on to question 15 below.

190
1

Question 15

Choose the equation which is a translation of the statement:

" x is the difference between 28 and z .

(A) $x = z - 28$

(B) $x = 28 - z$

(C) $z = x - 28$

(D) $z + 28 = x$

There are different ways of representing the same idea. In order to write that 11 is 7 more than 4 , we could write,

$$11 = 4 + 7$$

We could also write, $11 - 7 = 4$

And we could also write, $11 - 4 = 7$

Each of these is valid. To write the statement,

"y is 7 more than x"

we could write, $y = x + 7$

or we could write, $y - 7 = x$

or we could write, $y - x = 7$

Your choice is correct. Please go on to question 10 below.

Question 10

Choose the equation which is a correct representation of the statement:

" a is b more than y . "

(A) $a + y = b$

(B) $a + b = y$

(C) $a = b - y$

(D) $a - y = b$

192
1

Congratulations! Your answer is correct.

You realized that the word "represents" can be translated as "equals", and the phrase w represents a quantity twice as large as y can be translated as:

$$w = 2(y)$$

or simply $w = 2y$

Please go on to question 8 below.

192
2

Question 8

Choose the equation which is a correct representation of the statement:

" x is 4 less than m ."

(A) $x = 4 - m$

(B) $x = m - 4$

(C) $x - 4 = m$

(D) $x - m = 4$

$12\frac{2}{3}$ means 12 plus $\frac{2}{3}$

or an addition problem.

$\frac{2}{3}$ of 12

implies a multiplication problem.

Please return to page 197/1 and try question 16 again.

I'm sorry, we disagree.

Your choice actually says that W is larger than the quotient of m and r, because when 1 is added to the quotient, the sum equals W.

This contradicts the problem.

Please return to page 195/2 and try question 18 again.

194
1

Your answer is correct.

The phrase "more than" indicates an addition operation. Just as the sentence,

	7	is	3	more than	4
can be written	7	=	4	+	3	
so the phrase	x	is	3	more than	y
can be written	x	=	y	+	3	

or in symbols

$$\boxed{x} = \boxed{\frac{3}{y}}$$

Please go on to question 12 below.

194
2

Question 12

Choose the equation which is a correct representation of the statement:

" q is 7 less than x ."

(A) $7 - q = x$

(B) $q + 7 = x$

(C) $q = 7 - x$

(D) $x = 7 - q$

Your answer is correct.

The term "product" indicates a multiplication operation. When we multiply algebraic terms, we may omit the symbol of operation; the multiplication sign.

To express the phrase "larger than" we may use the symbol as long as the symbol points towards the smaller value.

Please go on to question 18 below.

Question 18

Choose the statement which says that W is smaller than the quotient of m and r .

(A) $W < m + r$

(B) $W = \frac{m}{r} + 1$

(C) $W < \frac{r}{m}$

(D) $W < \frac{m}{r}$.

196
1

Your .

Please continue on to question 9 which follows:

Question 9

Choose the equation which is a correct representation of the statement:

" y is 7 more than x ."

(A) $y + 7 = x$

(B) $y = 7 (x)$

(C) $x = y + 7$

(D) $y - x = 7$

196
2

I'm sorry; we disagree.

If w represents the cost of one pencil, and z the cost of one pen, then your choice means.

"the cost of one pencil is 5 cents
plus the cost of one pen plus 2 cents."

This isn't the statement that you were given to interpret.

Please return to page 199 and try question 20 again.
2

Good. your answer is correct.

Please go on to question 16 which follows.

Question 16

Apply your knowledge to calculate two-thirds of 12 .

(A) $\frac{2}{3}(12)$

(B) 18

(C) $12\frac{2}{3}$

(D) 8

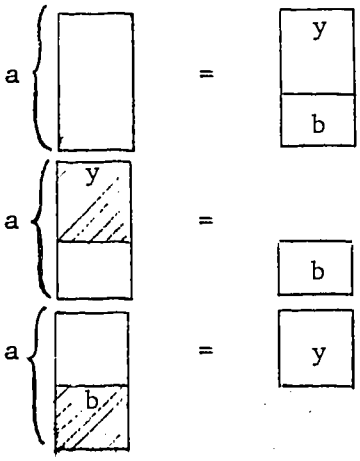
I'm sorry, we disagree. Merely saying that the sum of the ages of John and his father is less than twice his mother's age is not enough in this case. While true, it doesn't give a satisfactory picture of the problem, it ignores the fact that the difference between the two amounts is 5 .

Please return to page 184/2 and select the better answer to question 19.

198
1

Your answer is correct.

Y ght be interested in seeing that the relationship can be described in three ways:

$a = b + y$ 

$a - y = b$

$a - b = y$

Please go on to question 11 below.

198
2

Question 11

Choose the equation which is a correct representation of the statement

" x is 3 more than y ."

(A) $x + 3 = y$

(B) $x = 3\ (y)$

(C) $x = y + 3$

(D) $y = 3 + x$

Congratulations! Your answer is correct.

The sum of ages of John and his father is

$$x + y$$

five smaller than twice his mother's age is

$$2z - 5$$

and the correct relationship is

$$x + y = 2z - 5$$

Although the phrase "smaller than" is used, the inequality sign is not employed because the actual difference is shown. When the difference is applied to the larger amount, it makes both amounts equal.

Please go on to question 20 below.

Question 20

The cost of a pen exceeds the cost of 5 pencils by 2 cents. Apply your knowledge to find the equation which states this relationship if w represents the cost of one pencil in cents, and z represents the cost of one pen in cents.

(A) $w = 5z + 2$

(B) $w = 5 + z + 2$

(C) $z = 5w + 2$

(D) $z = 5 + w + 2$

200
1

Your answer is the correct one. When we say

"two - thirds of ..."

or any other fraction of a number, we are indicating that the fraction is to be multiplied by that number. Thus, two-thirds of 12 is evaluated by

$$\frac{2}{3}(12) = \frac{24}{3} = 8$$

Please go on to question 17 below.

200
2

Question 17

Choose the inequality which states that y is larger than the product of p and q.

(A) $y > pq$

(B) $y > p + q$

(C) $y < p + q$

(D) $y < pq$

I'm sorry; we disagree.

The quotient of m and r is not expressed as $\frac{r}{m}$.

Perhaps if you tried to express the fact that the quotient of 8 and 3 is $\frac{8}{3}$, you would be helped in the expression of the quotient of m and r.

Please return to page 195/2 and try question 18 again.

Sorry, we disagree.

You have chosen one translation of the statement,

" z plus 28 equals x "

which is another form of

" 28 is the difference between x and z "

Please return to page 190/2 and try question 15 again.

202
1

It's easy to be careless. How do we represent the phrase

"five smaller than twice his mother's age"?

Perhaps if you look at a similar situation you might get a hint.

How would you write

13 is smaller than two times 9 ?

If we begin by writing two times 9 : 2 (9)

then saying

" 5 smaller than 2 (9) " : 2 (9) - 5

then saying, 13 is, or 13 = 2 (9) - 5

Please return to page 184 (2) and try question 19 again.

202

You were asked to calculate two-thirds of 12 . Your answer represents three-halves of twelve.

Please return to page 197 (1) and try question 16 again.

Congratulations! Your answer is correct.

The problem can be translated in the following manner; (the cost of a pen) exceeds the cost of 5 pencils by 2 cents.

let z = cost of a pen

w = cost of a pencil

$5w$ = cost of 5 pencils

Thus, we have $z = 5w + 2$

You have now finished Segment 5 and the Volume. Hand in the PUNCH CARD. You should have entered in your NOTEBOOK the following definitions and formulas:

a is k more than b	$a = k + b$
	$a - b = k$
	$a - k = b$

a is k less than b	$a = b - k$
	$a + k = b$
	$k = b - a$

The difference between a and b is k

$a - b = k$

You should be able to complete all of the problems from your HOMEWORK ASSIGNMENT:

You are now advised to review all of your notes and to re-read the reading assignment in order to prepare yourself for the volume test.

II

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 2

Attach to
Back Cover

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected.

Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE:	MISPRINT:	CORRECTION:	CHECK WHEN CORRECTION MADE
$\frac{13}{1}$		Thus we have: "in a similar problem"	
$\frac{35}{2}$		$(5 \div 4 \times \overline{3 + 2})\big]\big\}$ Add Bracket	
$\frac{44}{2}$	which set do not recognize	which set do you recognize.	
$\frac{160}{2}$		$3(3x + 5) \leq 12$ Add Sign	