

$\frac{88}{1}$

What number did you find? If you multiply your number by itself, do you get the number you started with as a result?

That is what this choice says, and it is not correct.

Please return to page  $\frac{97}{2}$  and try question 17 again.

---

$\frac{88}{2}$

When two numbers are added, the result is larger than either one.

Therefore,  $\frac{1}{2} + \frac{1}{2}$  cannot be smaller than  $\frac{1}{2}$ .

and this choice is not correct.

Please return to page  $\frac{99}{2}$  and try question 20 again.

$$4 + \frac{4}{3} = \frac{12}{3} + \frac{4}{3}$$

since it is necessary to get a common denominator when adding fractions.

When multiplying fractions, the procedure is different:

$$\frac{4}{1} \times \frac{4}{3} = \frac{4 \times 4}{1 \times 3}$$

$$= \frac{16}{3}$$

Then the results are equal, therefore, this choice is not correct.

Please return to page  $\frac{83}{2}$  and try question 13 again.

---

Since the number

$$9 + 3 \text{ equals } 12$$

and

$$12 \text{ divided by } 2 \text{ equals } 6$$

which is not larger than

6

This choice is not correct.

Please return to page  $\frac{107}{2}$  and try question 16 again.

$$\frac{90}{1}$$

Since this choice offers the same two numbers divided in different orders, the results are not equal.

Therefore, since we were asked for the statement which is not correct, this choice is the proper choice.

Please proceed to question 7 below.

---

$$\frac{90}{2}$$

Question 7

Consider the following numbers:

$$P \text{ is } 8 + 2$$

$$Q \text{ is } 5 \times 2$$

$$R \text{ is } 2 \div 2$$

Perform the calculations to find which statement is true.

- (A) All three are equal
- (B) Only P and Q are equal
- (C) Only Q and R are equal
- (D) Only P and R are equal

Since

5 divided by 3 is 1

which is smaller than 2 .

This choice is correct.

Proceed to question 9 below.

---

Question 9

Which statement do you recognize as correct?

(A)  $\frac{5}{13} > \frac{5}{12}$

(B)  $\frac{5}{13} = \frac{5}{12}$

(C)  $\frac{5}{13} < \frac{5}{12}$

(D)  $\frac{5}{13} = \frac{4}{12}$

$\frac{92}{1}$

Since one of the other choices is correct, this is not.

Return to page  $\frac{98}{2}$  and try question 13 again.

---

$\frac{92}{2}$

In order to subtract fractions, it is necessary to find a common denominator. In this case the common denominator would be 6 .

Showing the work in horizontal form, which we usually use in algebra;

we have:

$$\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

Since this value is not smaller than  $\frac{1}{2}$ , this choice is not correct.

Please return to page  $\frac{99}{2}$  and try question 20 again.

There are only two numbers with this special property:

$$1 \times 1 = 1$$

and

$$0 \times 0 = 0$$

No other number has this property.

Therefore, this choice is correct,

since

0 is less than 10 .

Please proceed to question 18 below.

---

Question 18

Apply your knowledge to find which statement is correct.

(A)  $\frac{6 + 4}{2} = 3 + 4$

(B)  $\frac{6 + 4}{2} = 3 + 2$

(C)  $\frac{6 + 4}{2} = 6 + 2$

(D) None of these.

$$\frac{94}{1}$$

In adding fractions, it is necessary to use a common denominator. In this case, the common denominator would be 77, and there is a good bit of arithmetic in completing the addition. However, it isn't necessary to do the addition to find the answer to this question. The fraction  $\frac{22}{7}$  is larger than 3, and adding a quantity to it gives a still larger result. Then it is not possible for the total to be equal to 2.

Therefore, this choice is not correct. Incidentally, if you are interested, the total of the two fractions is

$$\frac{291}{77}$$

or

$$3\frac{60}{77}$$

Please return to page  $\frac{83}{2}$  and try question 12 again.

---

$$\frac{94}{2}$$

One of the other choices is correct, therefore, this is not correct.

Please return to page  $\frac{100}{2}$  and try question 15 again.

A simple rule which you should know is that if two fractions have the same numerator, the one with the larger denominator is the smaller fraction.

Therefore,  $\frac{5}{13}$  is smaller than  $\frac{5}{12}$  and this choice is correct.

Please proceed to question 10 below.

---

Question 10

The number

$$S = \frac{1}{2} + \frac{1}{3}$$

and the number

$$T = \frac{1}{2} \times \frac{1}{3}$$

Perform the calculations to find the correct choice.

(A)  $T = S$

(B)  $T > S$

(C)  $S > T$

(D) None of these.

$\frac{96}{1}$

Since

$$6.2 \times 70 = 434$$

and

$$350 + 100 - 16 = 434$$

this choice does not make the inequality true.

Therefore, this choice is not correct.

Please return to page  $\frac{104}{2}$  and try question 14 again.

---

$\frac{96}{2}$

Finding the value of the numerator first, we get zero.

Then zero divided by 3 is 0, which is not larger than 3.

Therefore, this choice is not correct.

Please return to page  $\frac{102}{2}$  and try question 19 again.

In order for a division to give a result larger than 6, after dividing by 2; the number had to be larger than 12. But the numerator could not be larger than 12 unless the question mark is replaced by a number larger than 9. Since no such number was offered, this choice is the only correct one.

Please proceed to question 17 below.

---

Question 17

You know that

$$1 \times 1 = 1$$

Therefore, this is a number which when multiplied by itself gives the same number. Which of the following statements do you recognize is true?

- (A) There is no other number like this.
- (B) There are many other numbers like this.
- (C) There is only one other number like this and it is larger than 10.
- (D) There is only one other number like this and it is less than 10.

$\frac{98}{1}$

If you are sharp, you can save arithmetic on this one.

You know that 4 quarters equal one dollar. Then

$$25 \times 4 = 100$$

Since 25.47 is larger than 25, when you multiply by 4 the result is larger than 100.

Therefore, this choice is correct, and it is not even necessary to perform the complete multiplication.

Please proceed to question 13 below.

---

$\frac{98}{2}$

Question 13

Which numeral do you recognize may replace the question mark so as to make the following statement true?

$$\frac{3 + 2}{2} = ? + 1$$

- (A) 1.5
- (B) 2.5
- (C) 3
- (D) None of these.

Finding the value in the numerator first, we get 11 .

Now

11 divided by 3 is larger than 1

therefore, this choice is correct. If you foolishly decide to divide before doing the subtraction, you must make sure to divide each number by the 3 . You would still find that this choice is correct, but subtracting first is a safer procedure.

Please proceed to question 20 below.

---

Question 20

Apply your knowledge to find the statement in which the question mark can correctly be replaced by the sign  $<$  .

(A)  $\frac{1}{2} + \frac{1}{2} ? \frac{1}{2}$

(B)  $\frac{3}{5} \times \frac{4}{3} ? 1$

(C)  $\frac{5}{6} - \frac{1}{3} ? \frac{1}{2}$

(D)  $1.6 \times .8 ? 1$

$$\frac{100}{1}$$

Since

$$6.2 \times 70 = 434$$

and

$$350 + 70 = 420$$

this choice makes the inequality true.

Therefore, this choice is correct.

Please proceed to question 15 below.

---

$$\frac{100}{2}$$

Question 15

Perform the calculation to find which number may replace the question mark to make the inequality true:

$$5 \times ? + 2 < 4$$

(A) 1

(B)  $\frac{1}{2}$

(C)  $\frac{1}{5}$

(D) None of these.

If you examine each element of this set, you will discover that every number except one possesses a certain property. Can you find this property?

Please return to page  $\frac{110}{2}$  and try this question again.

---

What is the meaning of the three dots?

As you continue your study of mathematics, you will be impressed with the fact that mathematicians always look to express themselves more briefly. This is done by using symbols.

Thus, to enumerate the set of all natural numbers between 1 and 100 inclusive, we write

$$1, 2, 3, \dots, 100$$

The three dots are a symbol which indicates all the numbers up to 100 .

Please return to page  $\frac{127}{2}$  and try this question again.

$$\frac{102}{1}$$

The proper procedure is to find the value of the numerator first, then divide. If you do this, you will find that the division gives the result 5, and the addition on the right side gives result five.

Therefore, this choice is correct. Some students decide to divide before performing the addition. While foolish, it can be done if you are careful. You would have to divide each number in the numerator by the 2, not just one of them. Adding first avoids any complication.

Please proceed to question 19 below.

---

$$\frac{102}{2}$$

Question 19

Apply your knowledge to find in which statement the question mark can correctly be replaced by the sign  $>$ .

(A)  $\frac{15 - 4}{3} ? 1$

(B)  $\frac{6 + 4}{6} ? 5$

(C)  $\frac{2 \times 5 \times 0}{3} ? 3$

(D)  $\frac{1}{2} \div 2 ? 2$

We do not agree.

Four of the five elements of this set have something in common. One element describes something entirely different.

Please return to page  $\frac{116}{2}$  and try this question again.

---

The set of whole numbers between 100 and 115 inclusive, consists of these two numbers and all the counting numbers between 100 and 115.

Ask yourself the following question:

Is  $\frac{225}{2}$  a counting number?

Please return to page  $\frac{112}{2}$  and try this question again.

$\frac{104}{1}$

Since

$$\frac{3 + 2}{2} = \frac{5}{2} = 2.5$$

and

$$1.5 + 1 = 2.5$$

it follows that this choice is correct.

Please proceed to question 14 below.

---

$\frac{104}{2}$

Question 14

$$6.2 \times 70 = 350 + ?$$

Which numeral do you recognize may replace the question mark to make the inequality true.

(A) 84

(B) 70

(C) 12 x 7

(D) 100 - 10

In multiplying decimals, the rule is to multiply the numbers ignoring decimals, and then add the number of decimal places in each of the numbers you were given in order to get the number of decimal places in the result.

Multiplying

16 by 8 we get 128

Since each number had one decimal place, the result has

1 + 1

or

2 decimal places

Therefore, the result is 1.28, which is not smaller than 1.

Then this choice is not correct.

Please return to page  $\frac{99}{2}$  and try question 20 again.

Did you enumerate the elements of the set of natural numbers that are less than 25? If you didn't, we will do so now.

They are:

$A = \{1, 2, 3, \dots, 24\}$

Now, select the members of this set A that can be divided by 3 without a remainder.

Please return to page  $\frac{129}{1}$  and try this question again.

106  
1

When we say that a set is well-defined, we mean that we can determine all of its elements. In order to do this, there should be no ambiguity about what element is or is not a member of the set. Now, the expression "large number" is subject to opinion. You may consider one million as a large number, but the Treasury Department may not. Hence, the set of large numbers is not well-defined.

Please return to page 122 and try this question again.  
2

---

106  
2

We don't agree.

A set which contains a large number of elements is not an infinite set. As long as the number of elements of a set can be specified the set is not infinite.

Please return to page 131 and try question 8 again.  
2

$$5 \times \frac{1}{5} = 1$$

and

$$1 + 2 = 3$$

which is less than

4

Therefore, this choice is correct.

Please proceed to question 16 below.

---

Question 16

Perform the calculation to find which number may replace the question mark to make the inequality true:

$$\frac{? + 3}{2} > 6$$

(A) 7

(C) 9

(B) 8

(D) None of these.

$$\frac{108}{1}$$

The rule for dividing fractions is

" Invert the divisor and multiply ".

Then  $\frac{1}{2}$  divided by 2 equals  $\frac{1}{2}$  multiplied by  $\frac{1}{2}$ , and this has the value

$$\frac{1}{2} \times \frac{1}{2}$$

which is

$$\frac{1}{4}$$

It is not larger than 2 .

Therefore, this choice is not correct.

Please return to page  $\frac{102}{2}$  and try question 19 again.

---

$$\frac{108}{2}$$

We do not agree.

A finite set is a set that contains a specific number of elements. The specified number may be very large as, for example, the set of all counting numbers from 1 to 1 billion. However, as long as the number of elements can be specified that set is finite.

Now can you specify the number of elements in set P ?

Please return to page  $\frac{115}{2}$  and try question 7 again.

In multiplying two fractions, we should reduce when possible before doing the actual multiplication. Then the work would look this way:

$$\frac{1}{5} \times \frac{4}{1} = \frac{4}{5}$$

Since  $\frac{4}{5}$  is less than 1, this choice is correct.

You have now finished this Segment. Hand in the Punch Card.

You should have entered in your notebook the following:

When a calculation is performed there is always a single result if there is any. In subtraction, there is no result if the number being subtracted is larger than the number from which we are subtracting. In division, the divisor may not be zero.

You should now be able to complete the homework assignment

Number 1, examples 6-10

Volume 1 Segment 4 begins on page 110.

I

Volume 1 Segment 4 begins here.

Obtain a punch card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	<u>0</u>	<u>4</u>	(Sequence Number)
	54	and	56	<u>0</u>	<u>4</u>	(Type of Punch Card)
	60	and	62	<u>0</u>	<u>1</u>	(Volume Number)
	66	and	68	<u>0</u>	<u>4</u>	(Segment Number)

Your reading assignment for this segment is pg: 10 - 14

You have used the word Set on many occasions without being aware of the fact that this word has great significance in practically all branches of mathematics. You may have made the following statements:

I lost my pencil set.

I have a good set of teeth.

My mother has a beautiful set of dishes.

and so on. In this segment you will learn the meaning and use of the word set in mathematics.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Apply the proper principle and select the element that does not belong to the set A :

$\{2, 4, 6, 7, 8\}$

where A is the set of even ~~numbers~~  $< 10$

(A) 8

(B) 7

(C) 4

(D) 2

Sorry, but we do not agree. The elements of set S show no discernable pattern. Since this set cannot be described, we cannot be sure what elements belong or do not belong to this set.

Please return to page  $\frac{122}{2}$  and try this question again.

---

In listing the elements of a set, we do not repeat the like elements. Thus, for example, if we were listing the set of flowers in a vase which had three roses, five violets, and two lilies; we would only list rose, violet, lily.

Please return to page  $\frac{134}{1}$  and try this question again.

I

112  
1

Very good. You made the correct choice. The three dots after the number 8 mean that we continue to count all the even numbers until we reach 22 . In other words, the first few numbers give us the clue to the pattern; and the last number tells us how far to continue.

Thus, this set consists of

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, and 11 is the correct answer.

Please go on to question 5 below.

---

112  
2

Question 5

Apply the proper principle and state whether

I.  $\frac{220}{2}$

II.  $\frac{225}{2}$

are members of the set of whole numbers between 100 and 115 indecisive. Select the letter which labels the correct statement.

- (A) I and II are members of this set.
- (B) I and II are not members of this set.
- (C) I is a member and II is not.
- (D) I is not a member and II is a member.

Very good. You made the correct choice. The set of all natural numbers that are divisible by 5 is the set

$$\{5, 10, 15, 20, \dots\}$$

Since there is no limit to the number of elements of this set, it is an infinite set. Similarly, the set of all prime numbers is

$$\{2, 3, 5, 7, 11, 13, \dots\}$$

It is the set of all whole numbers that have no exact divisors except 1 and the number itself. Euclid, a mathematician of ancient Greece, was the first man to prove that there is an infinite number of primes. Hence, this is an infinite set.

Please go on to question 9 below.

---

Question 9

Apply the proper principle and select the letter next to which all the statements about the following sets are correct.

$$P = \{5, 10, 15, 20, \dots, 550\}$$

$$Q = \{2, 4, 8, \dots, 128\}$$

$$R = \{3, 7, 11, 15, \dots, 31\}$$

- (A)  $100 \in P$ ,  $32 \notin Q$ ,  $21 \in R$ .  
(B)  $55 \in P$ ,  $32 \in Q$ ,  $24 \notin R$ .  
(C)  $100 \notin P$ ,  $64 \in Q$ ,  $23 \in R$ .  
(D)  $150 \in P$ ,  $24 \in Q$ ,  $27 \in R$ .

114  
1

The set  $\{0\}$  contains one element. This element is a member of the set of whole numbers. As long as a set contains one or more elements, it is not the null set.

Please return to page 125  
2 and try this question again.

---

114  
2

What do we mean when we say that two sets are equivalent? Consider, for example, the set of seats in your math class, and the set of students that attend this class. If every student in the class has a seat and all are present and there are no vacant seats left; then the set of seats and the set of students are equivalent sets.

Two sets are called equivalent if for every element of one set, we can match an element of the second set.

Please return to page 139  
2 and try this question again.

Very good. You made the correct choice. Let us list all the elements of this set. They are,

$$A = \{1, 2, 3, \dots, 24\}$$

The first element that is divisible by 3 is the element 3. The next one is 6. Yes, every time we add 3 to an element divisible by 3, we get another element that is divisible by 3.

Thus,  $B = \{3, 6, 9, 12, 15, 18, 21, \text{ and } 24\}$

The elements of this set are divisible by 3. Hence, 8 is the correct answer.

Please go on to question 7 below.

---

### Question 7

Apply the proper principle and select the finite set (or sets) from the following:

$P = \{\text{The set of all mixed numbers between } 1 \text{ and } 2\}$

$Q = \{1, 2, 3, 4, \dots, 10000\}$

$R = \{\text{The set of all whole numbers greater than } 1 \text{ million.}\}$

Choose the letter which labels the correct statement.

- (A) Set p is the only finite set.
- (B) Set P and Q are finite.
- (C) Set Q is the only finite set.
- (D) All the sets are finite.

116  
1

Very good. You made the correct choice. Every element of this set except the element 7, is a natural number divisible by 2. That is, dividing each number (except 7) by 2 results in a natural number plus a zero remainder.

Please go on to question 2 below.

---

116  
2

Question 2

Apply the proper principle of description of the set and select the one element that apparently does not belong to the set.

{French, German, English, London, Italian}

- (A) German
- (B) Italian
- (C) London
- (D) English

I

$\frac{117}{1}$

We do not agree. Consider the number . It is a natural number?  
Does it lie between 100 and 115? As soon as you find the answers  
to these questions, you will be able to select the correct letter.

Please return to page  $\frac{112}{2}$  and try this question again.

---

$\frac{117}{2}$

How many natural numbers less than 10 are divisible by 2? Did  
you write down all the elements of this set? Please do so.

Return to page  $\frac{114}{2}$  and try this question again.

I

118  
1

What is your definition of tall? Some people may consider 6 feet as being tall, while others may feel that one should be 6 feet and 3 inches in order to be considered tall. Since the word tall is not well-defined, the set  $R$  is not a well-defined set.

Please return to page 122 and try this question again.  
2

---

118  
2

The symbol  $\in$  means "belongs" or "is a member of."

Thus, we can say  $3 \in S$ , where

$$S = \{1, 2, 3, \dots\}$$

This same symbol with a slash through it,  $\notin$ , means does not belong or it is not a member of. Thus,  $6 \notin S$ . Now that you know the meaning of these symbols, you should be able to answer this question.

Please return to page 113 and try this question again.  
2

We do not agree. To describe the elements of a set accurately, is to specify a characteristic that they have in common. You should look for a characteristic that distinguishes these months from all other months of the year.

Please return to page 136  
2 and try this question again.

---

We don't agree. Keep this in mind.

A finite set cannot be equivalent to an infinite set.

One of the sets M, N, P is infinite.

Please return to page 133  
2 and try this question again.

20

Sorry, but your choice is not correct. A finite set is a set that contains a specified number of elements. Consider the set of mixed numbers.

$$(1\frac{1}{2}), (1\frac{1}{3}), (1\frac{1}{4}), (1\frac{1}{5}), \dots$$

Each one of these mixed numbers is greater than 1 and less than 2. How many of these can you write?

Yes, you can go on and on. Hence, P is not a finite set.

Please return to page  $\frac{115}{2}$  and try question 7 again.

---

120  
2

The set of primes consists of all whole numbers that are divisible only by 1 and by themselves. Since 2 is divisible by 1 and by 2, it is a prime number. Thus, the set Z is not the null set.

Please return to page  $\frac{125}{2}$  and try this question again.

I

121  
1

weight is 2000 pounds. Have you ever seen a dog that weighs  
2000 pounds?

Please reconsider your choice.

Please return to page 134 and try this question again.

---

121  
2

We disagree. Find the number of elements in each set.

If two of them have the same number of elements, then they ~~are~~  
equivalent.

Please return to page 139 and try question 14 again.

Very good. You made the correct choice. The elements French, German, English, and Italian can be thought of as languages. We can, for example, consider these elements as forming the set of languages taught in a high school. On the other hand, London is a city and, therefore, does not belong to this category.

Please go on to question 3 below.

---

Question 3

Apply the proper principle and select from the following the set which is well-defined:

P = {The set of large numbers.}

Q = {The set of presidents of the United States of America in office before 1968.}

R = {The set of tall students in your class.}

S = {7, 1, 16, 13}

(A)

(B)

(C)

(D)

$$\frac{123}{1}$$

In order for  $\frac{225}{2}$  to be a member of a set that consists of natural numbers, it must be a natural number; a counting number. Is it? On the other hand, when you reduce  $\frac{220}{2}$  don't you get a natural number?

Please return to page  $\frac{112}{2}$  and try this question again.

---

$$\frac{123}{1}$$

Think of the coins as being different. Let one of the coins be a penny and the other a nickel. Keep in mind that each coin can fall in two different ways.

Please return to page  $\frac{128}{2}$  and try this question again.

I

124  
1

We do not agree. The set  $P$  is composed of all natural numbers between 5 and 550 inclusive, that are multiples of 5.

Is 100 a multiple of 5?

Yes, since  $100 = 20 \times 5$

Please continue.

Please return to page 113  
2 and try question 9 again.

---

124  
2

It is true that the number of books in the main library is quite large, and so is the number of people living in the United States. However, unless these numbers are exactly alike, the sets are not equivalent.

Please return to page 133  
2 and try this question again.

I

Very good. You made the correct choice. The set of distinct letters needed to spell Mississippi is

$$\{m, i, s, p\}$$

Hence, this set has 4 elements.

The set of natural numbers less than 19 that are divisible by 2 is

$$\{2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

Hence, this set has 9 elements.

The set of dogs that weigh 1 ton = 2000 pounds has no elements.

The word "republican" consists of 10 letters with no letter repeated.

Hence, this set has the greatest number of elements. By the way, can

you find another 10-letter word with all of its letters different?

It is not easy. Another such word is phlegmatic.

Please go on to question 11 below.

---

### Question 11

Apply the proper principle and select the null set (or sets) from the following sets:

$$X = \{0\}$$

$$Y = \{\text{The set of natural numbers less than 1}\}$$

$$Z = \{\text{The set of primes divisible by 2}\}$$

Choose the letter which labels the correct statement.

- (A) X and Y
- (B) X and Z
- (C) Y
- (D) Y and Z

$\frac{126}{1}$

How many numbers are there greater than 1 million? No matter how large a number you write, it is always possible to write a larger number simply by adding one to it.

Please return to page  $\frac{115}{2}$  and try question 7 again.

---

$\frac{126}{2}$

The set of all stars in the sky is a very large set. However, if all the stars were counted, we would have a specified number. Thus, this set is finite. On the other hand, the set of all even numbers is infinite.

Please return to page  $\frac{142}{2}$  and try this question again.

Since the conditions for being included or excluded from the set are quite specific, this set is said to be "well-defined."

Therefore, this is the correct answer.

Please go on to question 4 below.

---

Question 4

Apply the proper principle and find the number of elements of the set

$$T = \{2, 4, 6, 8, \dots, 22\}$$

Select the letter which labels the correct statement.

- (A) 22
- (B) 11
- (C) 7
- (D) None of these.

I

Very good. You made the correct choice. These 4 months are different from the other 8 months of a year in the fact that they are the only ones having 30 days.

Please go to question 13 below.

---

Question 13

Apply the proper principle and select the letter which lists all the elements of the set described by the rule:

The set of all possible outcomes of tossing two coins

- (A) {Heads, Tails}
- (B) {2 heads, 2 tails}
- (C) {2 heads, 2 tails, 1 tail and 1 head}
- (D) {2 heads, 1 head and 1 tail, 1 tail and 1 head, 2 tails}

This is the correct answer.

Please go on to question 6 which follows.

Question 6

Apply the proper principle and find the number of elements in the set of natural numbers that are less than 25 and are divisible by 3. Select the letter which labels the correct statement.

- (A) 8
- (B) 7
- (C) 12
- (D) 10

Sorry, but your choice is not correct. Examine the set  $R$ . How is this set generated? Can you see a pattern? Compare each number with the one following it.

Please return to page  $\frac{113}{2}$  and try this question again.

I

$\frac{130}{1}$

We do not agree. The number 2 is a prime.

Please return to page  $\frac{125}{2}$  and try this question again.

---

$\frac{130}{2}$

We do not agree. Please fill in all the members of this set until you reach 233 . Check your addition.

Please return to page  $\frac{144}{2}$  and try this question again.

Very good. You made the correct choice. Of the three given sets, Q is the only finite set. A finite set has a specified number of elements. The set Q consists of all the natural numbers between 1 and 10000 inclusive. This is a specified number.

Please go on to question 8 below.

---

Question 8

Apply the proper principle and select the infinite sets from the following:

$$\begin{aligned} X &= \{ \text{The set of all natural numbers divisible by 5} \} \\ Y &= \{ \text{The set of all whole numbers less than 1,000,000} \} \\ Z &= \{ \text{The set of all prime numbers} \} \end{aligned}$$

Choose the letter which labels the correct statement.

- (A) Y is an infinite set.
- (B) X and Y are infinite sets.
- (C) Y and Z are infinite sets.
- (D) X and Z are infinite sets.

Very good. You made the correct choice. The sets X and Z are infinite sets. When are two infinite sets equivalent? This is a very interesting question.

Infinite sets are said to be equivalent if the members of the two sets can be put in a 1- to -1 correspondence; that is, if we can match each element of one set with an element of the second set and vice versa.

Thus,

$$\begin{array}{r} X = \{1, 2, 3, 4, \dots\} \\ \updownarrow \updownarrow \updownarrow \updownarrow \\ Z = \{2, 4, 6, 8, \dots\} \end{array}$$

Since we can always match an even number with a natural number, the sets are equivalent. The unusual feature is that we can say that there are as many even numbers as there are natural numbers.

Please go on to question 17 below.

---

Question 17

Apply the proper principle and find the cardinal number of the set that consists of the number of ways we can obtain a sum of 7 with one toss of two dice. Select the letter which labels the correct statement.

- (A) 3
- (B) 4
- (C) 5
- (D) 6

Very good. You made the correct choice. Two sets are called equivalent if we can match every element of 1 set with an element of the second set. When dealing with finite sets, it is easy to determine whether they are equivalent. All we have to do is to count the members of each set. If they have the same number, they are equivalent.

Please go on to question 15 below.

---

Question 15

Apply the proper principle and select the sets that are equivalent from the following sets:

- M = {The set of all natural numbers divisible by 3 .}
- P = {The set of all people living in the United States}
- N = {The set of all books in the main library.}

Select the letter which labels the correct statement.

- (A) All three sets are equivalent.
- (B) N and P are equivalent
- (C) None of the sets are equivalent.
- (D) M and P are equivalent.

I

This is the correct answer.

Please go on to the next question.

Question 10

Apply the proper principle and select the set that has the greatest number of elements.

$S = \{ \text{The set of letters that spell Mississippi} \}$

$T = \{ \text{The set of natural numbers less than 100 and divisible by 2} \}$

$U = \{ \text{The set of dogs that weigh 1 ton} \}$

$V = \{ \text{The set of letters that spell Republican} \}$

Choose the letter that labels the correct statement.

(A) S

(B) T

(C) U

(D) V

Your answer is incorrect. Have you forgotten the meaning of "between"?

Please return to page 149 and try question 1 again.

$\frac{135}{1}$

Sorry, we don't agree. Did you try listing every combination of two  
... ?

Please return to page  $\frac{151}{2}$  and try question 3 again.

---

$\frac{135}{2}$

Sorry, we disagree. In mathematics if one example can be found as  
an "exception to a proposed rule" then the rule is out. The number  
4 is an even integer but it is not divisible by 30.

Please return to page  $\frac{161}{2}$  and try question 5 again.

I

136  
1

Very good. You made the correct choice.

The set of natural numbers start with 1. Thus, there are no natural numbers less than 1.

Therefore, this set is empty. A set with no member is called the null set.

Please go on to question 12 below.

---

136  
2

Question 12

Apply the proper principle and select the letter which most accurately describes the following set designated by the roster method:

$$T = \{ \text{September} , \text{April} , \text{June} , \text{November} \}$$

- (A) Four months of the year
- (B) The months that have 30 days
- (C) The months that have the most legal holidays
- (D) None of these

Sorry, your answer is not correct.

The NULL SET, or the EMPTY SET, as it is sometimes called, is a subset of every set except itself.

Please return to page  $\frac{158}{2}$  and try question 2 again.

---

It's easy to be careless.

Have you forgotten that every set is a subset of itself?

Please return to page  $\frac{166}{2}$  and try question 6 again.

I

138  
1

The set of natural numbers that are divisible by 5 are an infinite set. On the other hand, the population of the United States is a specified number and is not infinite.

Remember, a finite set cannot be equivalent to an infinite set.

Please return to page 133 and try this question again.  
2

---

138  
2

By the cardinal number of a set, we mean the number of elements that it has. Hence, the cardinal number of this set is the number of ways of obtaining a 7 with a single throw of 2 dice.

There are more than the number of ways you chose.

Please return to page 132 and try this question again.  
2

Very good. You made the correct choice.

Let us suppose that one coin is made of silver and one is made of copper.

Now, each coin has two faces and can therefore come up in 2 different ways. Thus, we can tabulate the possibilities as follows:

<u>Silver</u>	<u>Copper</u>
head	head
tail	tail
head	tail
tail	head

Please go on to question 14 below.

---

Question 14

Apply the proper principle and select the two sets that are equivalent from the following sets:

$$\begin{aligned} S &= \left\{ \text{The set of all natural numbers greater than } 7.5 \text{ and less than } 19.3 \right\} \\ T &= \left\{ 1, 4, 9, 16, 25, \dots, 100 \right\} \\ U &= \left\{ \text{The set of the months of a year.} \right\} \end{aligned}$$

Choose the letter which labels the correct statement.

- (A) S and T                      (C) T and U
- (B) S and U                      (D) None of these.

$\frac{140}{1}$

Congratulations!

There are 10 subsets of set R which contain two elements each.

These subsets are:

$$\begin{array}{cccc} \{5, 6\} & \{6, 7\} & \{7, 8\} & \{8, 9\} \\ \{5, 7\} & \{6, 8\} & \{7, 9\} & \\ \{5, 8\} & \{6, 9\} & & \\ \{5, 9\} & & & \end{array}$$

Please continue below with question 4.

-----

$\frac{140}{2}$

Question 4

S is the set  $\{1, 3, 4, 8\}$

Perform the calculation to find the total number of subsets of S .

(A) 16

(B) 15

(C) 11

(D) 7

$\frac{141}{1}$

Have you forgotten?

Every set is a subset of itself.

Please return to page  $\frac{158}{2}$  and try question 2 again.

---

$\frac{141}{2}$

Have you forgotten the meaning of "proper subset"?

Please go back and look it up.

Return to page  $\frac{155}{2}$  and try question 9 again.

142  
1

This is the correct answer.

Please go on to question 16 which follows:

Question 16

Apply the proper principle and choose the sets that are equivalent from the following sets:

$$X = \{\text{The set of all counting numbers.}\}$$

$$Y = \{\text{The set of all the stars in the sky.}\}$$

$$Z = \{\text{The set of all even numbers.}\}$$

Select the letter which labels the correct statement.

(A) X and Z

(C) X and Y

(B) Y and Z

(D) None of these.

---

142  
2

We don't agree on how many days there are in July.

Please return to page 177 and try question 11 again.  
2

$\frac{143}{1}$

We disagree.

All the whole numbers between 3 and 9 include a number which is not an element of set P.

Please return to page  $\frac{149}{1}$  and try question 1 again.

---

$\frac{143}{2}$

Sorry, we disagree.

In mathematics, if only "one exception to a proposed rule" can be found, then the rule is disproved.

The number 10 is a multiple of 5, but 10 is not divisible by 30.

Please return to page  $\frac{161}{2}$  and try question 5 again.



$\frac{145}{1}$

It's easy to be careless.

Have you forgotten that the null, or empty set is a subset of every set?

Please return to page  $\frac{166}{2}$  and try question 6 again.

---

$\frac{145}{2}$

Have you forgotten the meaning of the term "improper subset"?

Please go back and look it up.

Return to page  $\frac{163}{2}$  and try question 8 again.

146  
1

It's easy to be careless.

Take another look at set  $N$ . Is zero an element of set  $N$  ?

Please return to page 150 and try question 7 again.  
1

---

146  
2

Sorry, we disagree.

The odd multiples of 3 are such numbers as

3 , 9 , 15 , 21 . . . .

These numbers are not integral multiples of 6 , and thus cannot be elements of set  $W$ .

Please return to page 170 and try question 10 again.  
2

We disagree.

The odd integers between 2 and 10 are the integers,

3 , 5 , 7 , and 9 .

Every set is a subset of itself.

Please return to page  $\frac{158}{2}$  and try question 2 again.

Did you forget the empty set?

The empty set is a subset of all sets except itself.

Did you forget the set S itself?

Each set is a subset of itself except the empty set.

Please return to page  $\frac{140}{2}$  and try question 4 again.

Very good You made the correct choice.

Did you fill in the members of the set until you reached 233? Let us do so together:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233

Hence, 13 is the cardinal number of this set. It is interesting to note that the set of numbers formed by the above rule is quite famous. The elements of this set are called Fibonacci Numbers. Those numbers have many interesting properties and in your future study of mathematics, you are going to hear more about these numbers.

You have now finished this segment. Hand in your punch card.

You should have entered in your notebook the following definitions and formulas:

- (1) Definition of a set.
- (2) Definition of a finite and infinite set.
- (3) Definition of a null set.
- (4) Definition of the cardinal number of a set.
- (5) Definition of equivalent sets.
- (6) Definition of Roster Method and rule for describing a set.

You should now be able to complete the following problems from your homework assignment: Problems 13, 14, 15, and 16.

Volume 1 Segment 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	<u>0</u>	<u>5</u>	(Sequence Number)
	54	and	56	<u>0</u>	<u>4</u>	(Type of Punch Card)
	60	and	62	<u>0</u>	<u>1</u>	(Volume Number)
	66	and	68	<u>0</u>	<u>5</u>	(Segment Number)

Your READING ASSIGNMENT for this segment is pg: 18  
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Which of the choices do you recognize as a subset of the set

$$P = \{4, 6, 7, 8\} ?$$

- (A)  $\{\text{all whole numbers between } 4 \text{ and } 8\}$
- (B)  $\{\text{all whole numbers between } 3 \text{ and } 9\}$
- (C)  $\{2, 4, 6, 8\}$
- (D)  $\{4, 6, 7, 8\}$

150  
1

Your answer is correct.

Please go on to question 7 which follows:

Question 7

Which choice do you recognize as a subset of

$$N = \{1, 2, 4\} ?$$

(A)  $\{0, 1\}$

(B)  $\{0, 1\}$ .

(C)  $\{0, 1, 2, 4\}$

(D) None of these.

---

150  
2

It's very easy to be careless.

You have evidently confused the terms "proper" and "improper" subsets.

Please return to page 155 and try question 9 again.  
2

Congratulations! Your answer is correct.

Six is a one-digit multiple of three, and is therefore an element of the set described by choice B. Six is not an element of set Q. Thus, the set described in B cannot be a subset of Q.

Please go on to question 3 below.

---

Question 3

R is the set

$$\{5, 6, 7, 8, 9\}$$

Perform the calculation to find the number of subsets of R which contain 2 elements each.

- (A) 10
- (B) 8
- (C) 4
- (D) 2

152  
1

In the Northern Hemisphere, summer arrives in late June and leaves in late September. In the Southern Hemisphere, summer arrives in late December, and leaves in late March.

Do you still think that November is a summer month?

Please return to page 177 and try question 11 again.  
2

---

152  
2

It's easy to be careless.

Please re-read choice A again.

Does 1<sup>st</sup> year algebra contain all the other subjects given in high school?

Please return to page 180 and try question 15 again.  
1

You recognized that the null set is a subset of every set. However, your choice

B ,  $\{ \emptyset \quad 1 \}$  shows the null set as an

element of set N.

The null set is not an element of set N.

Please return to page  $\frac{150}{1}$  and try question 7 again.

---

Sorry, we disagree.

Some of the spectators in the PDQ seats might be the parents of the football players. These parents are not students at the college.

Please return to page  $\frac{179}{2}$  and try question 13 again.

$\frac{154}{1}$

It's very easy to be careless.

Is the element 2 in set P ?

Please return to page  $\frac{149}{1}$  and try question 1 again.

---

$\frac{154}{2}$

We don't agree.

Try to list sets following a systematic pattern.

Please return to page  $\frac{140}{2}$  and try question 4 again.

One of the subsets that may be formed from any set is that set itself, we call it an "improper subset". Each set can have one, and only one, improper subset.

Your answer is correct.

Please go on to question 9 below.

---

Question 9

Perform the calculation to find the number of proper subsets of

$$V = \{a, e, i, o, u\}$$

(A) 0

(B) 1

(C) 32

(D) 31

156  
1

Sorry, we disagree.

In mathematics, if only "one exception to a proposed rule" then the rule is out.

The number 40 is a multiple of 20, but 40 is not divisible by 30.

Please return to page 161 and try question 5 again.  
2

---

156  
2

We fooled you this time.

The empty set contains no elements, but it is a subset of every set except itself.

The symbol for the empty set is  $\emptyset$ . The set  $\{\emptyset\}$  is the set containing the empty set.

Please return to page 166 and try question 6 again.  
2

Sorry, we disagree.

The odd multiples of 2 include such numbers as

$$2 \times 5$$

$$2 \times 7$$

$$2 \times 11$$

$$2 \times 13$$

These numbers:

10 , 14 , 22 , and 26

are not multiples of

6

and are not members of set W

Please return to page  $\frac{170}{2}$  and try question 10 again.

---

Sorry, we disagree.

Some holidays fall on Saturdays, others on Sundays.

Please return to page  $\frac{183}{2}$  and try question 14 again.

158  
1

Congratulations! Your answer is correct.

Every set is a subset of itself.

Please go on to question 2 below.

---

158  
2

Question 2

Which choice do you recognize is NOT a subset of set

$$Q = \{3, 5, 7, 9\} \quad ?$$

- (A)  $\emptyset$
- (B)  $\{\text{all multiples of } 3 \text{ which contain one digit}\}$
- (C)  $\{3, 5, 7, 9\}$
- (D)  $\{\text{all odd integers between } 2 \text{ and } 10\}$

$\frac{159}{1}$

Did you misread the question?

You were asked which set was set Z a subset of.

Z is not a subset of the empty set.

Please return to page  $\frac{172}{1}$  and try question 12 again.

---

$\frac{159}{2}$

Your choice was not correct.

The product of two odd numbers is an odd number not an even number.

Please return to page  $\frac{168}{2}$  and try question 16 again.

$\frac{160}{1}$

Haven't you forgotten something?

You found the number of total subsets possible. This includes improper as well as proper subsets.

What did the question ask you to find?

Please return to page  $\frac{155}{2}$  and try question 9 again.

---

$\frac{160}{2}$

Sorry, we disagree.

The set of two digit numbers contains many elements that are not the sum of two one-digit numbers. The largest sum of two one-digit numbers is

$$9 + 9 = 18$$

There are 81 other two digit numbers which don't fit the description.

Please return to page  $\frac{181}{2}$  and try question 17 again.

Congratulations! Your answer is the correct one.

To find the total number of subsets of a set you may wish to follow this procedure: Let  $n$  represent the number of elements in a set.

If  $n = 1$  that is, for set  $T = \{x\}$

the possible subsets are two:  $A = \{x\}$  and  $B = \emptyset$

If  $n = 2$  that is, for a set  $V = \{x, y\}$

the possible subsets are four:  $A = \{x\}$

$B = \{y\}$

$C = \{x, y\}$  and

$D = \emptyset$

If  $n = 3$  that is, for a set  $W = \{x, y, z\}$

the possible subsets are eight:  $A = \{x\}$   $E = \{x, z\}$

$B = \{y\}$   $F = \{y, z\}$

$C = \{z\}$   $G = \{x, y, z\}$  and

$D = \{x, y\}$   $H = \emptyset$

There is a definite pattern here,

<u>number of elements</u>	<u>use 2 in multiplication</u>	<u>number of subsets</u>
1	1 (2)	= 2
2	2 (2)(2)	= 4
3	3 (2)(2)(2)	= 8
4	4 (2)(2)(2)(2)	= 16
.....	.....	.....
$n$	$n \quad 2^n$	= $2^n$

Please go on to question 5 below.

Question 5

T is the set

$\{ \text{all the integers divisible by } 30 \}$

Which of the following do you recognize is a subset of T?

(A)  $\{ \text{even integers} \}$  (C)  $\{ \text{multiples of } 20 \}$

(B)  $\{ \text{multiples of five} \}$  (D)  $\{ \text{multiples of } 60 \}$

162  
1

Sorry, we disagree.

One of the previous three choices is the correct one.

Please return to page 177 and try question 11 again.  
2

---

162  
2

Sorry, we disagree.

The S is not a subset of the set containing the empty set.

Please return to page 180 and try question 15 again.  
1

Your answer is correct.

None of the choices, A , B , or C is a subset of set N.

Choices A and C , have zero as elements of the subsets, yet zero is not an element of N.

Choice B has the empty set as an element, and the empty set is not an element of N.

Congratulations!

Now please go on to question 8 below.

---

Question 8

Perform the calculation to find the number of improper subsets of

$$P = \{2.4, 2.5, 2.7, 2.8\}$$

(A) 0

(B) 1

(C) 10

(D) 16

$\frac{164}{1}$

When we say divisible by 2, we mean that a number may be divided by two without leaving a remainder.

The number 1 does not fit that description.

Please return to page  $\frac{172}{1}$  and try question 12 again.

---

$\frac{164}{2}$

Sorry, we disagree.

The sum of two odd numbers is an even number.

Please return to page  $\frac{168}{2}$  and try question 16 again.

Whose band played at half-time?

Sometimes a local high school band is invited to play during half-time.

The members of the high school band would not be students at the college.

Return to page  $\frac{179}{2}$  and try question 13 again.

---

Sorry, we don't agree.

What is the square of 6 ?

Please return to page  $\frac{173}{2}$  and try question 18 again.

Congratulations! Your answer is correct.

Perhaps you would like to see one way in which this problem can be handled.

The set  $T$  can be written in roster form as follows:

$$T = 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, \\ 330, 360, 390, 420, 450, \dots$$

The set of multiples of 60 can be written:

$$X = 60, 120, 180, 240, 300, 360, 420, \dots$$

Beginning with the second element of  $T$ , every other element of  $T$  belongs to  $X$ .

Thus, the set of multiples of 60 is a subset of  $T$ .

Please go on to question 6 below.

---

Question 6

Which choice do you recognize as containing all the subsets of

$$M = \{3, 5\} \quad ?$$

- (A)  $\{3\}$  ,  $\{5\}$
- (B)  $\{3\}$  ,  $\{5\}$  ,  $\{3, 5\}$
- (C)  $\{3\}$  ,  $\{5\}$  ,  $\{3, 5\}$  ,  $\emptyset$
- (D)  $\{3\}$  ,  $\{5\}$  ,  $\{3, 5\}$  ,  $\{\emptyset\}$

$\frac{167}{1}$

Sorry, we don't agree.

One of the choices is a subset of W.

Please return to page  $\frac{170}{2}$  and try question 10 again.

---

$\frac{167}{2}$

We disagree here.

Some weekdays are not school days. Election Day is a weekday yet there is no school on Election Day.

Please return to page  $\frac{183}{2}$  and try question 14 again.

Congratulations!

First year algebra is one of the subjects given in high school; thus, the set containing first year algebra is a subset of the set of all subjects given in high school.

Please go on to question 16 below.

---

Question 16

Which of the following statements do you choose as correct?

- (A)  $\{5, 11, 17\}$  is a subset of  $\{2, 5, 8, 11, \dots\}$
- (B)  $\{\text{product of two odd numbers}\}$  is a subset of  $\{\text{even numbers}\}$
- (C)  $\{\text{sum of two odd numbers}\}$  is a subset of  $\{\text{odd numbers}\}$
- (D)  $\{0\}$  is a subset of  $\{\text{whole numbers between 5 and 6}\}$

It's easy to be careless.

How many digits are there in the number 10 ?

Please return to page  $\frac{172}{1}$  and try question 12 again.

---

We disagree.

If a number is an element of a subset of a set, then it must be an element of the larger set, too.

The number 1 will always divide another number. Thus, 1 will be an element of S and thus of T.

Please return to page  $\frac{178}{2}$  and try question 19 again.

170  
1

The number of proper subsets is one less than the total number of subsets because the set itself is counted as a subset, but it is an improper subset. When a set has five elements, the number of total subsets is found by multiplying five 2's:

$$2 \times 2 \times 2 \times 2 \times 2 = 32$$

Now, deduct 1 from that figure because one of those subsets will not be a proper subset.

Thus,  
 $32 - 1 = 31$  represents the number of proper subsets of set V.

Your answer was correct. Congratulations!

Now please continue with question 10 below.

---

170  
2

Question 10

If  $W = \{\text{multiples of } 6\}$

apply your knowledge to find which choice is a subset of W.

(A)  $\{\text{even multiples of } 3\}$

(B)  $\{\text{odd multiples of } 3\}$

(C)  $\{\text{odd multiples of } 2\}$

(D) none of these

$\frac{171}{1}$

Sorry, but we disagree. There are many two digit numbers which are not the sum of two one-digit numbers.

Please return to page  $\frac{181}{2}$  and try question 17 again.

---

$\frac{171}{2}$

I'm sorry, we disagree. What integer squared, gives 3 as a result ?

Please return to page  $\frac{173}{2}$  and try question 18 again.

I

172  
1

Your answer is correct.

Please go on to question 12 which follows.

Question 12

$$\text{Set } Z = \{1, 2, 10\}$$

is a subset of one of the following sets.

Which do you recognize as correct?

- (A)  $\emptyset$
  - (B)  $\{\text{one digit numbers}\}$
  - (C)  $\{\text{numbers divisible by } 2\}$
  - (D)  $\{\text{numbers with 3- letter English names.}\}$
- 

172  
2

Sorry, we disagree. The empty set is a subset of every set. However, the set containing the empty set is NOT a subset of every set.

To designate the empty set, we use the symbol,  $\emptyset$ .

To designate the set containing the empty set, we use the notation  $\{\emptyset\}$ .

Please return to page 180  
1 and try question 15 again.

Congratulations! The elements of set  $T$  contain the numbers between 0 and 18, inclusive. Certainly, the numbers between 2 and 18 are in this interval. You realize that the subset doesn't have to include all of the possible numbers.

Please go on to question 18 below.

---

Question 18

The set

$$N = \{\text{squares of integers}\}$$

is a subset of  $R$ . Apply your knowledge to find which statement is correct.

- (A)  $R = \{\text{numbers whose last digit is } 0, 1, 4, \text{ or } 9\}$
- (B)  $R = \{\text{numbers whose last digit is } 0, 1, 4, 5, 6, 9\}$
- (C)  $R = \{\text{numbers whose last digit is } 0, 1, 2, 3, 5, 6, 9\}$
- (D)  $R = \{\text{numbers with any digit as last digit}\}$

174  
1

Sorry, we disagree. You are confusing the set containing the element 0 with the empty set,  $\emptyset$ .

Please return to page 168 and try question 16 again.  
2

---

174  
2

It's easy to be careless.

The square root of

25 is 5 ,

this cannot be an element of the set described in B. Thus, the square root of 25 would not be an element of N, yet this contradicts the statement of the problem.

Please return to page 185 and try question 20 again.  
2

I

$\frac{175}{1}$

It's very easy to be careless. You found the total number of possible subsets, not the number of improper ones.

Please return to page  $\frac{163}{2}$  and try question 8 again.

---

$\frac{175}{2}$

It's easy to be careless. Are Memorial Day and New Year's Day the only holidays in the year ?

Please return to page  $\frac{183}{2}$  and try question 14 again.

I

$\frac{176}{1}$

Sorry, we disagree. One of the previous choices is the correct one.

Please return to page  $\frac{153}{2}$  and try question 13 again.

---

$\frac{176}{2}$

Sorry, we disagree. The number 19 is an element of the set described in C, yet, 19 does not belong to set T.

Please return to page  $\frac{181}{2}$  and try question 17 again.

I

Congratulations! Your choice is correct. One of the ways you might have determined this would be to realize that the set of even multiples of 3, is formed by listing the elements:

$$\begin{array}{ll} 3 \times 2 & (3 \times 2) \\ 3 \times 2 \times 2 & (3 \times 2) \times 2 \\ 3 \times 2 \times 2 \times 2 & (3 \times 2) \times 2 \times 2 \\ 3 \times 2 \times 2 \times 2 \times 2 & (3 \times 2) \times 2 \times 2 \times 2 \end{array}$$

etc.

These elements are

$$6, 12, 18, 24, \dots$$

or

$$6, 6 \times 2, 6 \times 3, 6 \times 4, \dots$$

which are the elements of set  $W$ .

Please go on to question 11 below.

---

Question 11

$$\text{Set } Y = \{\text{July, November}\}$$

is a proper subset of one of the following sets.

Which do you recognize is correct?

- (A) {30 day months}
- (B) {summer months}
- (C) {months containing more than 29 days}
- (D) none of these

178  
1

Congratulations! In the process of squaring any number, the first step will always be to get the product of the units digit.

This will involve one of the following products:

$$1 \times 1 = 1$$

$$6 \times 6 = 36$$

$$2 \times 2 = 4$$

$$7 \times 7 = 49$$

$$3 \times 3 = 9$$

$$8 \times 8 = 64$$

$$4 \times 4 = 16$$

$$9 \times 9 = 81$$

$$5 \times 5 = 25$$

$$10 \times 10 = 100$$

The units digits of these squares are:

1, 4, 5, 6, 9, 0

Please go on to question 19 below.

---

178  
2

Question 19

The last two digits of a number are 20 and its divisors are in set  $S$ . If  $S$  is a subset of  $T$ , apply your knowledge to find  $T$ .

(A)  $\{2, 5, 10, 20\}$

(B)  $\{1, 2, 5, 10, 20\}$

(C)  $\{1, 2, 4, 5, 10, 20\}$

(D) None of these.

Congratulations! Your answer is the correct one. In English the names of the numbers,

- 1 is one
- 2 is two
- 10 is ten

There are three letters in each name.

Please go on with question 13 below.

---

Question 13

Which choice do you recognize as a subset of

- { all the students at PDQ College } ?
- (A) { The spectators in the PDQ seats at the football game. }
- (B) { The players on the PDQ team at the football game. }
- (C) { The players in the band at half time at the game. }
- (D) None of these.

180  
1

Your answer is correct.

Please go on to question 15 which follows.

Question 15

Set  $S = \{\text{subjects given in high school.}\}$

Which choice do you recognize is a correct statement ?

- (A)  $\{S\}$  is a subset of  $\{\text{1st Year Algebra}\}$
  - (B)  $\{\text{1st Year Algebra}\}$  is a subset of  $\{S\}$
  - (C)  $\{S\}$  is a subset of  $\{\emptyset\}$
  - (D)  $\{\emptyset\}$  is a subset of  $\{S\}$
- 

180  
2

We disagree. The number 4, will always divide a number ending in 20 because,

- 20 divided by 4 is 5
- 120 divided by 4 is 30
- 220 divided by 4 is 55
- 320 divided by 4 is 80

and; if you are dividing by 4 they are the only possible combinations.

Please return to page 178  
2 and try question 19 again.

Congratulations! The set

$$\{2, 5, 8, 11, \dots\}$$

is an infinite set. We recognize a pattern, or rule for the formation of successive elements.

Each new element is three more than the previous. The next two elements in the set are 14 and then 17. Thus, the set

$$\{5, 11, 17\}$$

is a subset of the set

$$\{2, 5, 8, 11, \dots\}$$

Please go to question 17 below.

---

Question 17

The set

$$T = \{\text{sum of two one-digit numbers}\}$$

Apply your knowledge to find which choice lists the set which is a subset of T .

- (A)  $\{\text{two-digit numbers}\}$
- (B)  $\{\text{one or two-digit numbers}\}$
- (C)  $\{\text{numbers between 1 and 20}\}$
- (D)  $\{\text{numbers between 2 and 18}\}$

I

182  
1

I'm sorry but we disagree. What integer squared gives 2 as a result?

Please return to page 173  
2 and try question 18 again.

---

182  
2

I'm afraid we have confused you. The empty set (referred to as the null set, and noted as  $\emptyset$ ) is a subset of every other set, but it has no subsets of itself.

Please return to page 185  
2 and try question 20 again.

I

Your answer is correct. According to the rules of the game.

In order to play on a college football team, you must be a student at that college.

Please go on to question 14 below.

---

Question 14

Which choice do you recognize as correct ?

- (A)  $\{\text{holidays}\}$  is a subset of  $\{\text{weekdays}\}$
- (B)  $\{\text{weekdays}\}$  is a subset of  $\{\text{school days}\}$
- (C)  $\{\text{Memorial Day, New Year's Day}\}$  is a subset of  $\{\text{holidays}\}$
- (D)  $\{\text{holidays}\}$  is a subset of  $\{\text{Memorial Day, New Year's Day}\}$

$\frac{184}{1}$

Sorry, the correct answer is one of the other three choices.

Please return to page  $\frac{178}{2}$  and try question 19 again.

---

$\frac{184}{2}$

We disagree. One of the other choices is the correct one.

Please return to page  $\frac{185}{2}$  and try question 20 again.

I

Congratulations! Your answer is the correct one. If the divisors of the number are in set  $S$ , then set  $T$ , having set  $S$  as a subset, must contain all possible divisors of  $20$ , too.

Please go on to question 20 below.

---

Question 20

The square of an integer  $N$  ends in  $25$ . Then  $N$  is a subset of one of the following.

Apply your knowledge to find the correct choice.

- (A)  $\{ \text{numbers ending in } 5 \}$
- (B)  $\{ \text{numbers ending in } 10 \}$
- (C)  $\emptyset$
- (D) None of these.

Congratulations! Your answer is correct.

You have now finished Segment 5. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following definitions:

- (1) The definitions of a subset.
- (2) The definition of the empty set, or null set.
- (3) The definition of proper and improper subset.

You should be able to complete the following problems from your HOMEWORK ASSIGNMENT; page 19, questions 1 - 14

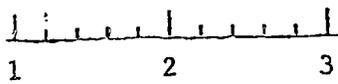
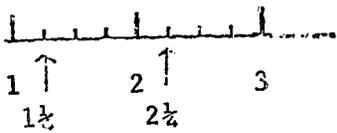
You are now advised to review all of your notes and to re-read the reading assignment in order to prepare yourself for the volume test.

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented.

Certain typographical errors already exist and will also be corrected.

Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

<u>PAGE</u>	<u>MISPRINT</u>	<u>CORRECTION</u>	<u>CHECK WHEN CORRECTION MADE</u>
$\frac{34}{2}$	page $\frac{40}{9}$	page $\frac{40}{2}$	
$\frac{35}{1}$	see diagram	The line $4\frac{1}{2}$ should run from 7 to $2\frac{1}{2}$	
$\frac{39}{2}$	1 , 4	1.4	
$\frac{44}{2}$			
$\frac{46}{1}$	coefficients	coordinates	
$\frac{90}{2}$	$2 \div 2$	$2 \div .2$	
$\frac{91}{1}$	1	$1\frac{2}{3}$	
$\frac{104}{2}$	=	≠	
$\frac{109}{1}$	$\frac{1}{5} \times \frac{4}{1}$	$\frac{1}{5} \times \frac{4}{\cancel{1}}$	
$\frac{112}{2}$	indecisive	inclusive.	
$\frac{161}{1}$	B $\emptyset$ D $\emptyset$	B = $\emptyset$ D = $\emptyset$	
$\frac{69}{1}$	Ques. I-D: <del>111</del> <del>111</del> <del>111</del>	<del>111</del> <del>111</del> 1111	