DOCUMENT RESUME

ED 075 202 24 SE 015 856

TITLE	Programmed Math Continuum, Level One, Algebra, Volume 1.
INSTITUTION	New York Inst. of Tech., Old Westbury.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-8-0157
PUB DATE	[73]
CONTRACT	OEC-0-8-080157-3691(010)
NOTE	195p.
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	*Algebra; *Computer Assisted Instruction; Curriculum; Individualized Instruction; *Instruction; Instructional Materials; Mathematics Education; Programed Instruction; Programed Materials; *Programed Texts; *Secondary School Mathematics

ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. Volume 1 includes general instructions for working with this system, and then covers the following topics in algebra: number line, comparing numbers, sets and set membership, and subsets. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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VOLUME

1

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

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P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME I

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 1

TABLE OF CONTENTS

COVER	PAGE
PREFACE	A
TABLE OF CONTENTS	B
SYLLABUS	C
READING ASSIGNMENT	D
HOMEWORK ASSIGNMENT	E
GENERAL INSTRUCTIONS	F

IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
Introduction Material		1 - 17
1	2	18/1
1	3	68/1
1	4	110/1
1	5	149/1

VOLUME 1

This volume covers the following material

as shown in this excerpt from the Syllabus.

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION DOLCIANI	DRESSLER	DODES
1	Introduction: general instructions			
2	Representing numbers on a line	1-1 (1-6)	1-3 (1-1)(1-2)	3-2 4-4 4-6
3	Comparing numbers, = or	1-2 1-3	1-4	2-5 3-1
4	Membership in a set Kinds of sets	1-4 1-5 1-6	2-1 2-2 2-6 3-2 5-10	1-2 (1-3)
5	Subsets	1-7	2-3	1-2

READING ASSIGNMENT

VOLUME 1

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1		
2	1 (16)	5 (17)
3	5	10
4	10	17
5	18	19

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

HOMEWORK ASSIGNMENT

VOLUME NO. 1

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	3 - 4	1 , 2 , 3 , 4	01215
2	4	7 , 8 , 9 , 10	01215
3	17	1 , 2 , 3 , 4	01230
4	17	5 , 6 , 7 , 8	01230
5	17	9 , 10 , 11 , 12	01230
6	6	2 , 3 , 4 , 6	01310
7	7	11 , 12 , 13 , 15	01331
8	9	1 , 2 , 6 , 7	01310 , 01320
9	9	12 , 14 , 18 , 21	01310 , 01320
10	10	25 , 26 , 36 , 38	01330
11	12	27 , 28 , 29 , 30	01410
12	14	1 , 2 , 3 , 4	01421 , 01430
13	15	6 , 7 , 8 , 9	01421 , 01430
14	15	11 , 12	01421 , 01430
15	15	14 , 15	01421 , 01430
16	19	1 , 2 , 3 , 4	01512
17	19	5 , 6 , 7 , 8	01510
18	19	9 , 10	01520
19	19	11 , 12	01520
20	19	13 , 14	01520

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

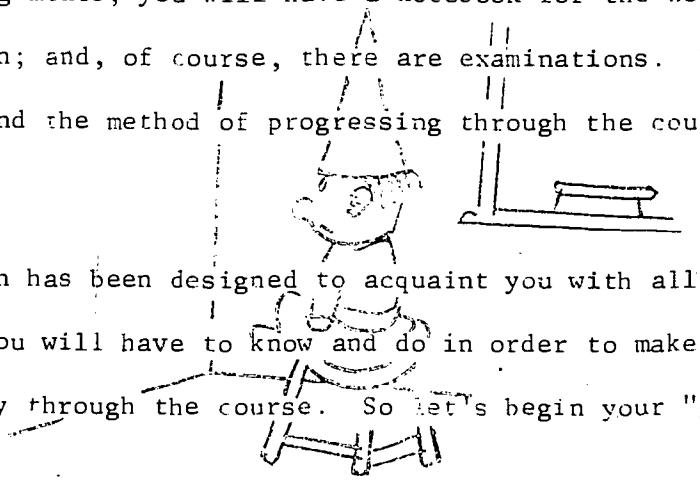
If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

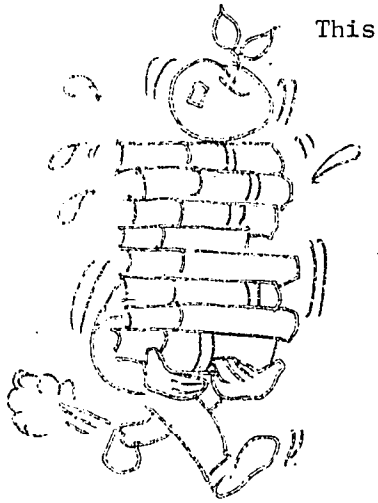
1. This is a course in ELEMENTARY ALGEBRA which is not too different from other subjects you have studied. There is a textbook; there are homework assignments; you will have a notebook for the notes you must take down; and, of course, there are examinations. However, you will find the method of progressing through the course quite different.



This first section has been designed to acquaint you with all of the things that you will have to know and do in order to make your way satisfactorily through the course. So let's begin your "guided tour":

1.1 The work of the entire course is divided into 18 parts, called VOLUMES (VOL). Each Volume consists of 5 parts, called SEGMENTS (SEG). Every VOL. deals with material which, under normal circumstances, would be covered in a two week period. However, if you are permitted to proceed at your own pace or on your own time schedule, you could probably finish a Volume in much less time.

This book, which you are now reading, is called a STUDY GUIDE, (SG). This is VOL. 1 of the SG; this portion of VOL. 1 is called SEG 1.



In addition to the SG, you will need:

(1) The basic text: Modern Algebra Book 1
By Dolciani, Berman and Freilich
Houghton Mifflin, 1965

(2) The remedial text: Comprehensive Ninth Year Mathematics
by Dressler
Amsco, 1966

(3) A Notebook: ...Where you will record specific items for further study.

And something new:

(4) Punch cards-

(5) Program control- You'll learn about these in a few minutes.

ABOUT THE PUNCH CARD:

Your progress through this course will be followed very carefully by the computer. It will record, for example, which questions you found easy and which caused you some trouble. In fact, it will keep track of all of the students in many ways and will analyze their various performances in such a way that your teacher will be able to know immediately just where extra attention should be directed to help all the students do as well as they can.

The way the computer finds out about you is through the PUNCH CARD. Those holes all mean something.

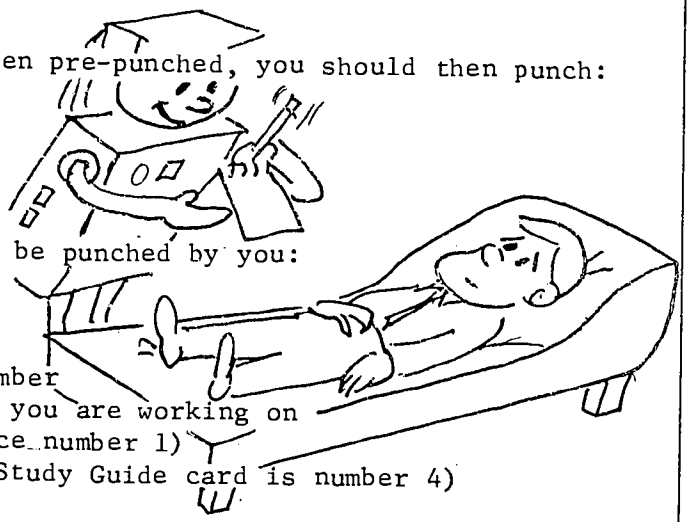
When the punch card is placed in the PROGRAM CONTROL, with the proper ANSWER MATRIX, you will see that as you punch a hole in response to a question in the SG, a light will go on in the PROGRAM CONTROL, the page to which you should turn will then be illuminated. So you see, the PUNCH CARD is very important.

HOW TO PREPARE THE PUNCH CARD:

(1) Take your PUNCH CARD and print: your LAST NAME, and FIRST NAME above the word "date."

If the card has not already been pre-punched, you should then punch:

(2) Your Course student number
(3) This Course number
(4) Your Student I.D. number



The following information must be punched by you:

(5) Month number
(6) Day number
(7) Last two digits of the Year Number
(8) The Sequence number of the SEG you are working on (VOL. 1, SEG. 1 has Sequence number 1)
(9) The type of Input card. (The Study Guide card is number 4)
(10) The Volume Number.
(11) The Segment Number.

NOTE: All of this information must be entered; if not, the computer will reject the card.

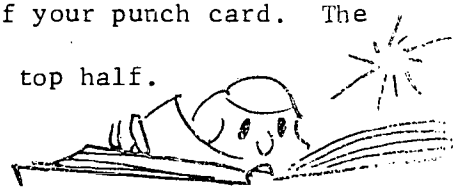
Punch the necessary information, being S L O W and C A R E F U L so as to punch only those holes which you intend to punch. Remember, you cannot make corrections, once you have punched the wrong hole. Check to see that the program control device shows that it has the answer matrix for volume 1, segment 1.

1.3 You are now ready to see how the study guide will work. Read question 1, and decide on an answer. Then examine the four choices offered.

Question 1: What is the product of 6 and 2 ?
(A) 8 (B) 12 (C) 4 (D) 3

Let us suppose that you believe the correct answer is 8, that is choice A. You would use the stylus to punch the hole on line 1 at A. Please do this now. As you see, a number appears on the program control device, the number $\frac{7}{2}$. This means turn, in your study guide, to page 7, bottom half. Do this now.

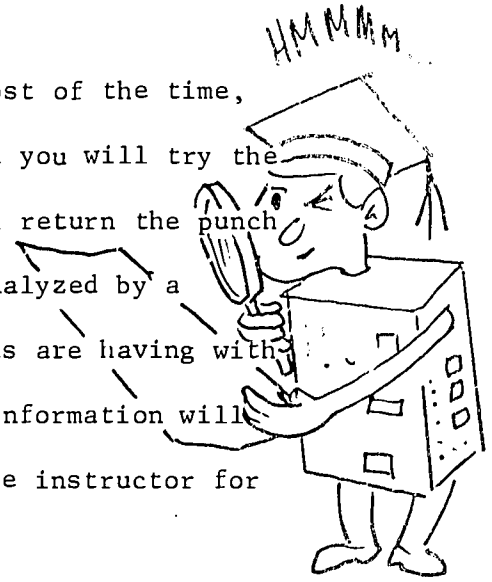
1.4 You see how the program control has directed you to return and try again, as well as telling you why your choice was not correct. Of course, looking back at question 1, you know that choice B is the correct answer. Punch B on line 1 of your punch card. The program control shows $\frac{9}{1}$. Turn to page 9, top half.



1.5 The procedure of this study guide is called programmed instruction. The purpose of the study guide is what its name indicates; it is a guide to your study. This is not a test in disguise. As you answer questions in the study guide, you should go back to your textbook, notebook, or material in the study guide itself, if you need to look up something which you are not sure of. The effect is that you will go over all the material of the course, in easy stages, with special emphasis on the more important points. The important points which the course contains are called M.B.O.'s, and each question you meet is designed to give you practice in dealing with one of these points.

1.5 Many abbreviations are used I'm sure you are curious about "M.B.O."

The letters "MBO" , stand for "Measurable Behavioral Objectives". They are the ideas, definitions and skills that are to be taught, hence they are called "objectives". Since they are stated in a way that questions can be asked about them, they are called "measurable". Because you have to demonstrate that you understand these, and can use them they are called "behavioral".



You will get the right answer on the first trial most of the time, but if you make a mistake you will find out why and you will try the problem again. At the end of the segment, you will return the punch card to your instructor. The cards will then be analyzed by a computer to find out the difficulties which students are having with the material of the course. The analysis of this information will indicate what remedial work should be planned by the instructor for the benefit of the students.

1.6 In general, at the beginning of a segment, you are directed to read a specific portion of Dolciani. In reading this, you MUST work out, on paper, any illustrative problems which the text contains, so that you are sure you can do the problem being demonstrated. Remember that the author may not have written every step as you would, so work on your own paper and try to follow the work shown in the book. In addition to the reading assignment in the text, there may be some notes in the study guide itself. You should enter in your notebook those items which seem to be of special importance, or those items which you may be directed to write down.

(continued on page 5)

Then, before you proceed with the segment, the instructor will insert the ANSWER MATRIX for your particular segment into the program control device. You will complete the necessary information items on a PUNCH CARD, insert it into the program control device, and then punch the necessary information into the card, using your stylus.

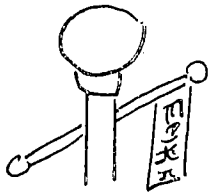
Now you proceed to the questions. You must take the questions in order. You'll have no choice, since the pages have been scrambled. That is, after question 1, you do not go ahead to the next page in order, but to whatever page the program control directs you. Each question will offer four choices, of which one, and only one, is correct. The three other choices, called distractors, are answers which you might well arrive at if your understanding of the principle were not completely correct. For every choice which you punch on your card, the program control directs you to a specific half of a particular page. Here you find a statement of why you were wrong (if you were), or a statement that you were right with instructions to proceed. This half-page is called a REM, (this is short for REMEDY) and the program control will direct you to a particular REM for any answer you choose for a question in the study guide. It should be of some help to you.

1.7 After this page, you must follow the instructions given to you by the program control. You must not go page by page, since the material is not in order.

Please remember, DO ... GUESS:

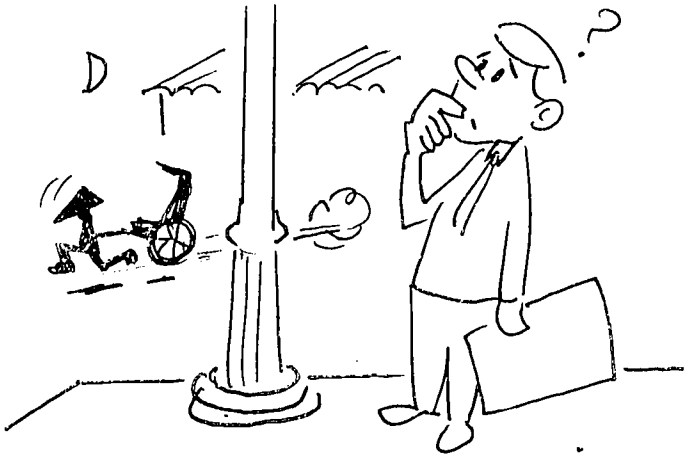
This guide is intended to help you to learn and practice correctly.

You should go back and reread pages 1 through 5 as often as necessary. When you have considered the question, punch your answer on the punch card and the program control will tell you where to turn next.



1.8 IF YOU " LOST YOUR PLACE "

If you lose your place for any reason, insert your stylus in the last hole that you had punched on your card. This will not affect any record that's being kept. The program control will put you back on the track.



Now turn to page $\frac{7}{1}$

Question 2: What type of instructional device is this book?

(A) M.B.O. (B) punch card

(C) answer matrix (D) study guide

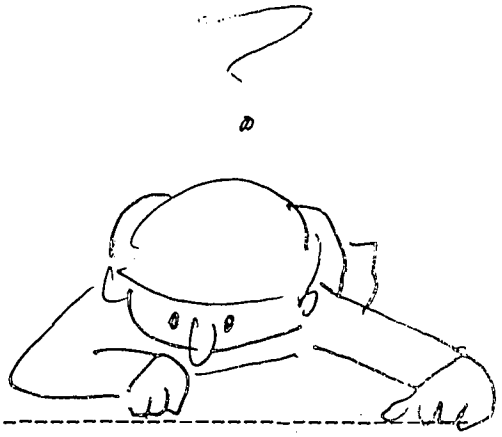
When you have decided on your answer, punch the letter of your choice on line 2 of the punch card, and follow the directions of the program control.

You have found the sum, but you were asked for the product, not the sum. The product of two numbers is the result obtained by multiplication.

Return to page 3, section 1.4

8
1

If you will refer to page 1, section 1.1, you will find that the study guide contains 18 volumes. You are correct.



Now proceed to question 4 which follows.

8
2

Question 4:

How many segments in each volume contain?

(A) 5

(B) 10

(C) 18

(D) 90

This choice is correct.

Proceed to page 3, section 1.5

The REM is the half page to which the program control refers you. What you are reading at this moment is a REM. In the study guide there are from 2 to 4 REMS for each question, and each punch card handles many questions. Therefore, this choice is not correct.

Return to page 12/2 and try question 5 again.

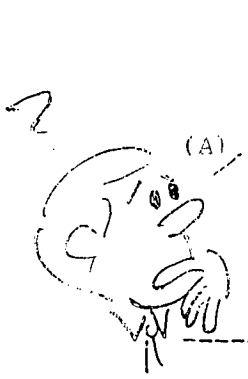
10
1

This choice is correct.

Now proceed to question 3 which follows:

Question 3

How many volumes is the study guide composed of?



(A) 5 (B) 10 (C) 18 (D) 90

10
2

The number of segments in a volume is discussed on page 1. If you reread that page, section 1.1, you will find that the correct number of segments is not the one you have chosen.

Return to page 8/2 and try question 4 again.

I

The punch card is the rectangular card in which you have punched choice B on line 2. Since it is not the book you are now reading, this choice is incorrect.

Return to page 7/1 and try question 2 again.

Since the course contains 18 volumes, and each volume contains 5 segments, you should be able to find out how many segments the entire course contains. Since the guide tells you that you will return the punch card to the instructor at the end of each segment, and that you will insert a new card into the program control device for each segment, it follows that you will need a card for each segment. Therefore, this choice is not correct.

Return to page 15/2 and try question 6 again.

12
1

According to section 1.1 on page 1 of the study guide, each volume contains 5 segments. Therefore, this choice is correct.

Proceed to question 5 which follows.

12
2

Question 5:

You will need a new punch card for each _____.

Which word should be inserted in the blank?

(A) segment

(B) question

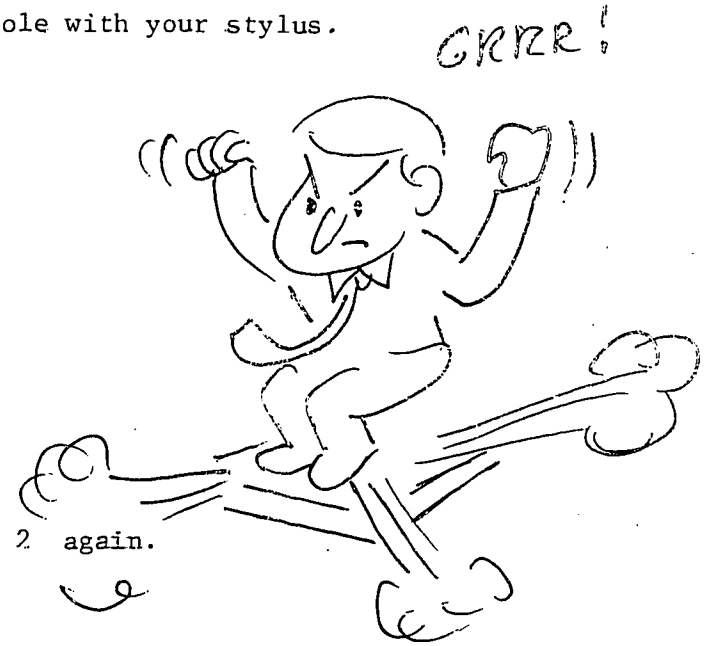
(C) REM

(D) M.B.O.

The number of volumes in the course is discussed on page 1 of the study guide. the study guide covers the entire course. Therefore, this choice is not correct.

Return to page 10/1 and try question 3 again.

The answer matrix is actually a code to the position of the REM's in the study guide, since the pages have been scrambled. The answer matrix is placed in the program control device, and the only way you can get to read any part of it, is to punch a hole with your stylus.



This choice is not correct.

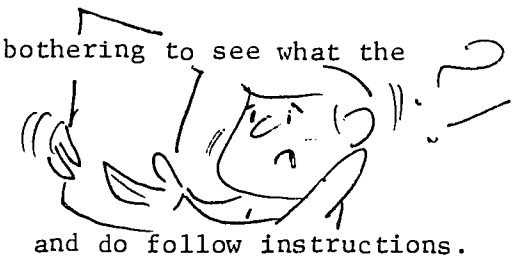
Return to page 7/1 and try question 2 again.

I

14
1

You have done some correct arithmetic, but I'm afraid you forgot the meaning of the word "product." The product of two numbers is the result of multiplying them.

Return to page 3, section 1.4, and please follow the study guide. You went ahead and chose an answer without bothering to see what the guide had to say.



Remember, return to section 1.4, page 3, and do follow instructions.

14
2

An M.B.O. is one of the important points which you will have learned as a result of studying this course. The meaning of "M.B.O." was discussed on page 4. It is true that the questions you meet are related to M.B.O.'s, but you do not use a punch card for them.

This choice is not correct.

Return to page 12/2 and try question 5 again.

On page 3, in section 1.5, you are told that you will return your punch card to the instructor at the end of the segment. On page 4, you are instructed to insert a punch card into the program control device before starting a segment. Therefore, you need a punch card for every individual segment.

This choice is correct.

Now proceed to question 6 which follows.

Question 6

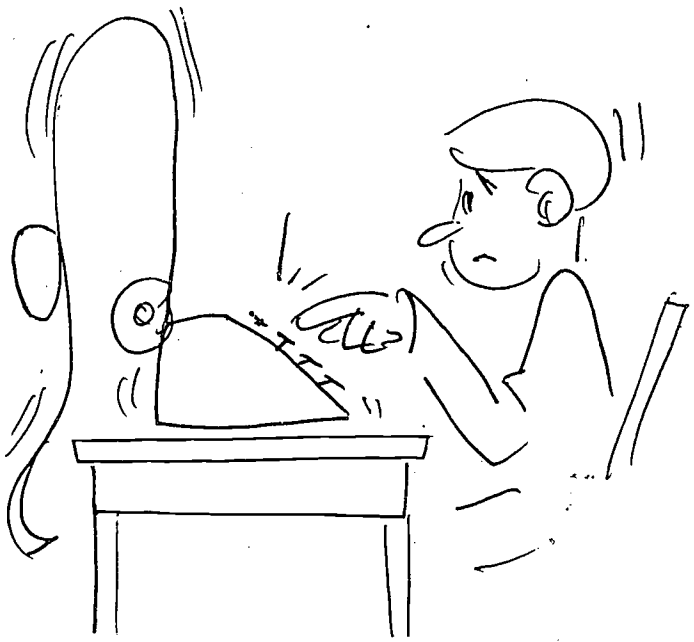
If we assume that no punch cards will be damaged, how many will a student require for the entire course?

(A) 5

(B) 10

(C) 18

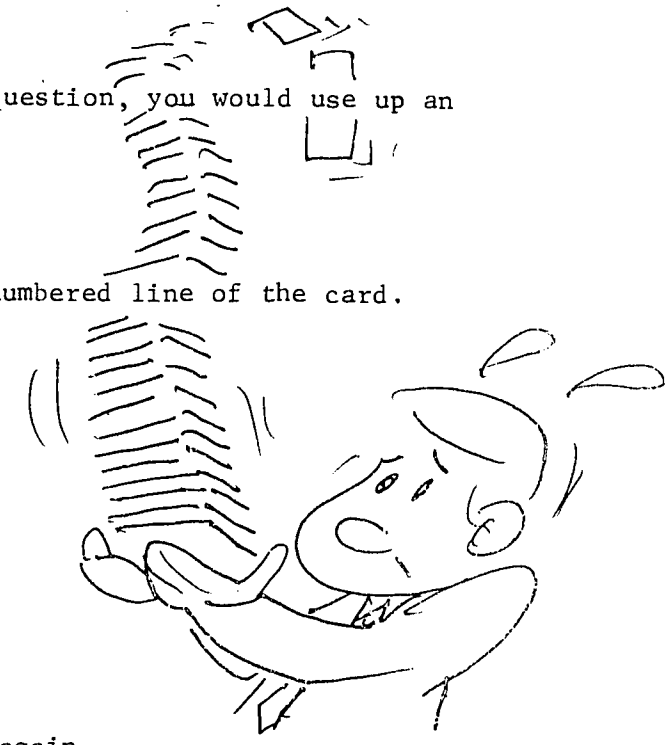
(D) 90



16
1

If you needed a new punch card for each question, you would use up an extrememly large number of cards.

Each question is answered on the proper numbered line of the card.



Return to page 12/2 and try question 5 again.

16
2

An M.B.O. is one of the important points which you will have learned as a result of studying this course. It is not the name of the book.

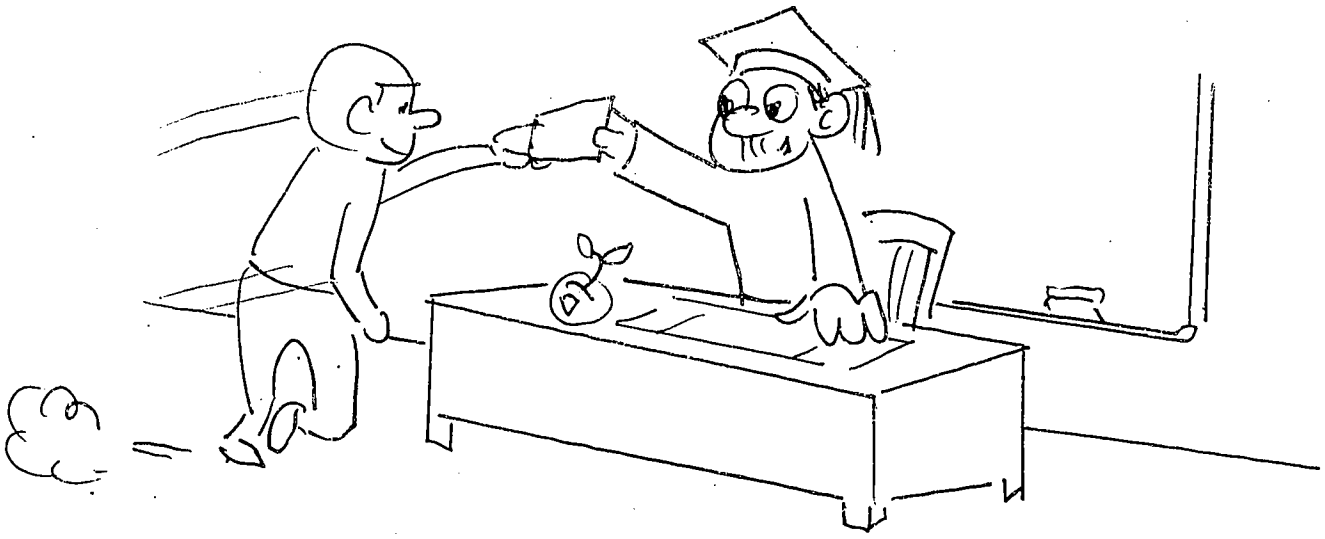
Return to page 7/1 and try question 2 again.

Since each segment calls for a new punch card, and there are 5 segments to a volume, a student needs 5 cards for each volume. There are 18 volumes, hence he needs 90 cards.

Therefore, this choice is correct.

You have now completed this segment. Before going on to Segment 2, be sure you have understood exactly what you are supposed to do. If you have punched more than one hole on a line of the punch card, except for line 1 where you should have punched 2, you have made an error. That is perfectly all right, provided that you learn from it. If you punched any hole without meaning to, you will have to practice being more careful.

When you are ready, ask your instructor for the answer matrix for Segment 2 and for a fresh punch card, and go ahead.



18
1

Volume 1 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48 and	50	0 2	(Sequence Number)
	54 and	56	0 4	(Type of Punch Card)
	60 and	62	0 1	(Volume Number)
	66 and	68	0 2	(Segment Number)

PRELIMINARY NOTES

Some of the streets of a certain city are numbered consecutively from 1 to 10. Let's assume that you happen to live on street number 3, while a friend of yours lives on street number 7. If you walk from your house to your friend's house, you will pass streets number 4, 5, and 6. Now each of these streets is a place. We say, in mathematics, that with every number we can associate a point (place) on a number line. In this segment, this simple, but very important idea, will be developed and studied in more detail.

Your READING ASSIGNMENT for this Segment is pp. 1 - 5.

You will now be asked a series of questions to draw your attention to the more important points.

18
2

Recognize which of the following numbers are natural numbers.

$$0 \quad , \quad 3 \quad , \quad \frac{1}{8} \quad , \quad 1\frac{1}{8} \quad , \quad \frac{8}{2}$$

Choose the letter which has only natural numbers, selected from the above set, written next to it.

(A) 0 , 3

(B) $1\frac{1}{3}$, 3 , $\frac{8}{2}$

(C) 3 , $\frac{8}{2}$

(D) 0 , 3 , $\frac{8}{2}$

Very good.

Whole numbers are the natural numbers with one addition. Zero is not a natural number, but it is a whole number. The number

$$\frac{2}{1} = 2, \text{ and } \frac{3}{1} = 3$$

Thus, any natural number written with a denominator of 1 is a whole number.

Finally, $\frac{1.1}{1.1} = 1$ since any number, other than zero, divided by itself is equal to one.

Please go on to question 4, below.

Question 4

Apply the proper principle and determine which of the following statements are true or false. Select the letter which labels the correct combination of truth values for your answer.

I. Every natural number can be written as a fraction.

II. There is a largest natural number.

III. Every fraction names a whole number.

(A) I is true
II is true
III

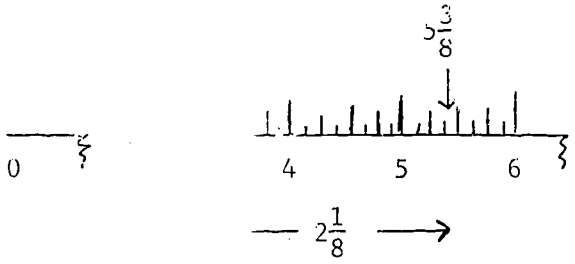
(C) I is true
II is false
III is false

(B) I is false
II is false
III is true

(D) I is true
II is false
III is true

20
1

our answer is



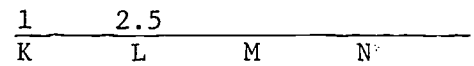
Please go on to question 8 below.

20
2

Question 8

Apply the principles you have learned and select the letter which correctly answers this question:

Points K , L , M , N are equidistant on the number line drawn below: what is the coordinate of N ?



(A) 5.0

(B) 10.0

(C) 7.5

(D) 5.5

21
1

Sorry, your choice is not correct.

Counting numbers and natural numbers are the same

Zero is not a counting number.

Please turn to page 33 and try question 2 again.
2

21
2

Did you mark of " tenths " on your number line?

Sorry, your answer is incorrect.

Please turn back to page 35 and try question 2 again.
2

$\frac{22}{1}$

If QR is [illegible] long PQ , we must first find the length of PQ. By subtracting P' coordinate from that of Q , we find PQ to be 4.75 units long. QR is twice that, or

2 (4.75) or 9.50 units long.

Thus, the coordinate of R is found by adding:

	9.25	(the coordinate of Q)
+	9.50	(the distance from Q to R)
	18.75	the coordinate of point R.

Your answer is correct.

Please go on to question 10 , below

$\frac{22}{2}$

Question 10

Apply the proper principle and select the letter which correctly answers this question.

What is the coordinate of point X on the number line drawn below if XY is $\frac{1}{3}$ as long as XY

X		Y	Z
		10.2	[illegible].9

(A) [illegible].1

(B) [illegible].1

(C) [illegible].3

(D) [illegible]

The natural numbers are 1 , 2 , 3 . . . . and so on.

Zero is not a natural number.

Please return to page 18/2 and try question 1 again.

We don't agree.

Is there a largest natural number? Suppose you write down the largest natural number you can:

We can always make it larger by adding 1 to it.

Thus, there is no largest natural number.

Please return to page 19/2 and try question 4 again.

$\frac{24}{1}$

Numbers such as

$\frac{1}{2}$ and $\frac{1}{3}$ are called fractions.

Fractions are not natural numbers.

Please return to page $\frac{36}{2}$ and try question 3 again.

$\frac{24}{2}$

Did you forget the meaning of " between " ?

Between means not including the end points.

For example,

T , U , V , and W are between S and X or in other words, there are four letters between S and X in the alphabet.

Return to page $\frac{38}{2}$ and try question 11 again.

$\frac{25}{1}$

Did you draw a number line to help you with this problem?

Find point $\frac{1}{2}$, now count 4 units to the left, and then another half unit to the left.

Please return to page $\frac{31}{2}$ and try question 5 again.

$\frac{25}{2}$

Draw a number line and mark off segments of $\frac{1}{4}$ units. Number all the integral values, then find

$2\frac{1}{2}$ and $8\frac{1}{2}$ on that line.

What are the natural numbers between them?

Please return to page $\frac{43}{-}$ and try question 12 again.

$\frac{26}{1}$

The number $1\frac{2}{3}$ is a mixed number, not a counting number. Mixed numbers as well as zero and decimal numbers are not considered " natural " numbers or counting numbers.

Please return to page $\frac{33}{2}$ and try question 2 again.

$\frac{26}{2}$

It's very easy to be careless!

Re-read the question. Does it say that R is double ? Or, does it rather say that QR is twice as long as PQ ?

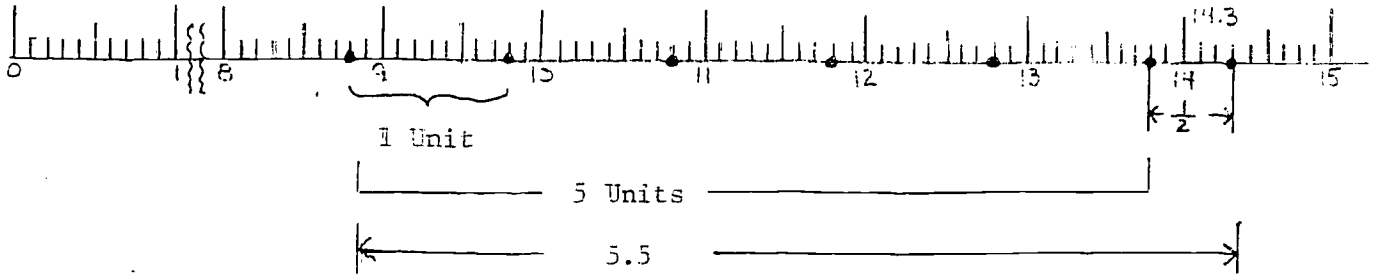
How would you determine how long PQ is?

Please return to page $\frac{40}{2}$ and try question 9 again.

After drawing the number line, we can mark off divisions of " tenths ".

Now we can begin at 8.8 and count 5.5 places.

Your answer is correct.



Please go on to question 7 below.

Question 7

Apply the proper principle and select the letter which correctly answers the question:

Two points $3\frac{1}{4}$ and $5\frac{3}{8}$ are chosen on a one-foot ruler; how far apart are these points?

(A) $2\frac{1}{8}$

(B) $2\frac{1}{4}$

(C) $1\frac{1}{8}$

(D) $12\frac{5}{8}$

$\frac{28}{1}$

We don't agree.

The number $1\frac{1}{8}$ is called a mixed number. It consists of a natural number and a fraction.

Please return to page $\frac{18}{2}$ and try question 1 again.

$\frac{28}{2}$

Sorry, we don't agree.

Can you represent the fraction one-third as a whole number?

Please consider your choice.

Return to page $\frac{19}{2}$ and try question 4 again.

$\frac{3}{1}$ is considered a whole number since it is written with a denominator of 1 .

Any number divided by one equates itself. However, if the numerator is one such as the number $\frac{1}{2}$, it is a fraction; not a natural number.

Please return to page $\frac{36}{2}$ and try question 3 again.

It's easy to be careless.

How large is the interval between points K and L ?

Please return to page $\frac{20}{2}$ and try question 8 again.

30
1

Good! Your answer is correct.

When we are asked for numbers " between " two other numbers, the end-points are not considered.

Please go on to question 13 below.

30
2

Question 13

Apply the proper principle and find how many natural numbers there are between .7 and 8.7 . Select the letter which labels the correct statement.

(A) 8

(B) 7

(C) 6

(D) 9

Yc . correctly omitted 0 and $1\frac{2}{3}$ which are not counting numbers. However, you included decimal numbers, which also are not " natural " or counting numbers.

Please return to page $\frac{33}{2}$ and try this question again.

Your answer is correct.

Please go on to question 5 which follows:

Question 5

Draw a segment of the number line for reference and label it appropriately. Apply the proper principle and find the coordinate of the point that is $4\frac{1}{2}$ units to the left of point 7 .

Select the letter which labels the correct statement.

(A) $3\frac{1}{2}$ (C) $1\frac{1}{2}$

(B) $2\frac{1}{2}$ (D) 3

I

$\frac{32}{1}$

What interval did you use on your number line?

Did you make each point $\frac{1}{8}$ of a unit away from the previous one?

$3\frac{1}{4}$ then, can be considered $3\frac{2}{8}$

Please return to page $\frac{27}{2}$ and try question 7 again.

$\frac{32}{2}$

It's very easy to be careless!

Please re-read the directions in question 10.

How long is segment YZ? What relationship does this have in our problem? If you find YZ, then find the length of XY. Then can you see how to find the coordinate of X?

Please return to page $\frac{22}{2}$ and try question 10 again.

This is correct.

Although $\frac{8}{2}$ looks like a fraction, it actually is a numeral for 4 and is, therefore, a natural number. The natural numbers are the " counting numbers ".

Please go on to question 2 below.

Question 2

Recognize which of the following numbers are counting numbers.

$$0 \ , \ 1 \ , \ 1.1 \ , \ 1.2 \ , \ 2 \ , \ 1\frac{2}{3}$$

Select the letter which has only counting numbers selected from the above set written next to it.

(A) 1 , 1.1 , 1.2 , 2

(B) 0 , 1 , 2

(C) 1 , $1\frac{2}{3}$, 2

(D) None of these other choices are correct.

$\frac{34}{1}$

We disagree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page $\frac{36}{2}$ and try question 3 again.

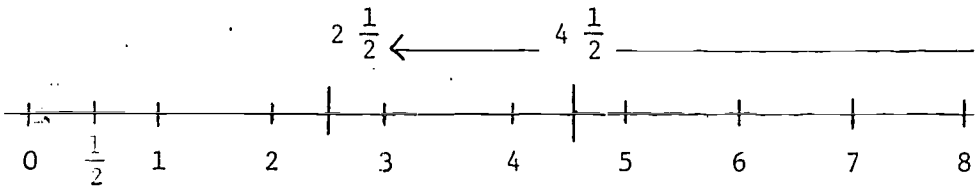
$\frac{34}{2}$

Please check your arithmetic!

Segment PQ's length can be determined by subtracting the coordinate of P from that of Q.

Please return to page $\frac{40}{9}$ and try question 9 again.

After drawing a number line, we can locate the point, 7 , and then count four units to the left, arriving at 3 . Now go one-half a unit farther to the left, $2\frac{1}{2}$.



Your answer is correct.

Please go on to question 6 below.

Question 6

Draw a segment of the number line and label it appropriately. Apply the proper principle and find the coordinate of a point that is 5.5 units to the right of point 8.8 . Select the letter which labels the correct statement.

(A) 15.3

(B) 14.3

(C) 3.3

(D) 13.3

$\frac{36}{1}$

Zero, the mixed number, $1\frac{2}{3}$, and the decimals, 1.1 and 1, 2 are not natural numbers.

Since each of the other answer choices contained at least one of these along with the counting numbers, your answer choice is correct.

Please go on to question 3 below.

$\frac{36}{2}$

Question 3

Recognize which of the following numbers are whole numbers.

$$0 \ , \ \frac{2}{1} \ , \ \frac{1}{2} \ , \ \frac{3}{1} \ , \ \frac{1}{3} \ , \ \frac{1.1}{1.1}$$

Select the letter which has only whole numbers selected from the above set, written next to it.

(A) $0 \ , \ \frac{2}{1} \ , \ \frac{3}{1} \ , \ \frac{1.1}{1.1}$

(B) $\frac{2}{1} \ , \ \frac{1}{2} \ , \ \frac{3}{1} \ , \ \frac{1}{3}$

(C) $0 \ , \ \frac{1}{2} \ , \ \frac{3}{1}$

(D) None of these other choices are correct.

Sorry, we don't agree.

Did you count <u>from</u> the point $3\frac{1}{4}$ to the [illegible] ?

It seems that you counted $3\frac{1}{4}$ <u>beyond</u> $5\frac{5}{8}$.

Please return to page $\frac{27}{2}$ and try question 7 again.

It's very easy to be careless.

The fraction $\frac{22}{7}$ does not symbolize the interval between 7 and 22 .

What does a fraction sygnify?

Please return to page $\frac{58}{2}$ and try question 15 again.

Segment YZ is 2.7 units in length. Segment XY is three times that; so, XY is 8.1 units in length. Since X is to the left of Y on the number line, it's coordinate must be less. Therefore, we must subtract the length of XY from the coordinate of Y to find the coordinate of X.

Since Y is 10.2 when we subtract the length of XY, the coordinate of X is 2.1 .

$$\begin{array}{r} 10.2 \\ -\ 8.1 \\ \hline 2.1 \end{array}$$

Your answer is correct.

Please go on to question 11 below.

Question 11

Apply the proper principle and find how many natural numbers there are between 10 and 15 . Select the letter which labels the correct answer.

(A) 4

(B) 5

(C) 6

(D) An infinite number.

Good! Your answer is correct.

The first natural number after .7 is 1. Before you reach 8.7 , you have counted through 1 , 2 , 3 , 4 , 5 , 6 , 7 , and 8. Thus, there are eight natural numbers between .7 and 8.7 .

Please go on to question 14 below.

Question 14

Apply the proper principle and find a number between 1 , 4 and 1.5 . Select the letter which labels the correct statement.

(A) 2.9

(B) 1.14

(C) 1.54

(D) 1.45

40
1

You can find the distance between K and L by subtracting their coordinates. Since each of the points are equidistant, the coordinate of N is

$$2.5 + 1.5 + 1.5$$

Coordinate of N is 5.5 .

Your answer is correct.

Please go on to question 9 below.

40
2

Question 9

Apply the proper principle and select the letter which correctly answers this question:

What is the coordinate of point R on the number line drawn below if QR is twice as long as PQ ?

(A) 27.75

(B) 13.75

(C) 18.75

(D) 18.50

The improper fraction $\frac{22}{7}$ can be expressed as a mixed number. A mixed number is made up of the natural number which is the largest possible quotient and a remainder. As a mixed number $\frac{22}{7}$ is expressed as $3\frac{1}{7}$

Please return to page $\frac{58}{2}$ and try question 15 again.

We're sorry, but we disagree.

$\frac{3}{4}$ is not between 1 and $\frac{3}{2}$. Did you draw a number line to help you?

Please return to page $\frac{52}{2}$ and try question 19 again.

Between $1\frac{1}{4}$ and 2 exclusive, there are two marks; one at $1\frac{1}{2}$ the other at $1\frac{3}{4}$. Between 2 and 3 inclusive, there are five marks. There are two more at $3\frac{1}{4}$ and $3\frac{1}{2}$. Thus, there are

$$2 + 5 + 2$$

or nine marks in all.

On looking at it in another way,

$$1\frac{1}{4} = \frac{5}{4}$$

and

$$3\frac{3}{4} = \frac{15}{4}$$

Between 5 fourths and 15 fourths there are 9 fourths.

Your answer is correct.

Please go on to question 17 below.

Question 17

Apply the proper principle and find how many whole numbers there are between $\frac{72}{6}$ and $\frac{6}{2}$ inclusive, that are also divisible by 3.

Select the letter which labels the correct answer.

(A) 2

(B) 3

(C) 4

(D) None of these.

Right! There are four natural numbers between 10 and 15 .

The idea of "between" means that the end points are not to be part of the segment.

Thus, we only consider the numbers,

11, 12, 13, and 14

Please go on to question 12 below.

Question 12

Apply the proper principle and find how many natural numbers there are between

$2\frac{3}{4}$ and $8\frac{1}{2}$.

Select the letter which labels the correct answer.

(A) 8

(B) 7

(C) 6

(D) 5

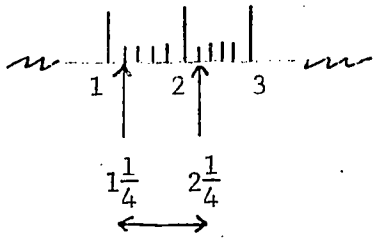
$\frac{44}{1}$

It's very easy to be careless. You were not asked for the sum of the coordinates, you were asked to find a number between 1.4 and 1.5 .

Please return to page $\frac{39}{2}$ and try question 14 again.

$\frac{44}{2}$

Sorry, we disagree. Did you sketch a number line to help you with this problem?



Now we're assuming that you can find the midpoint.

Please return to page $\frac{56}{2}$ and try question 18 again.

It seems as though $\frac{1}{5}$ would be the midpoint between $\frac{1}{4}$ and $\frac{1}{6}$ because 5 is the midpoint between 4 and 6.

However, it is not true. The point midway between two points could be considered as the "average" of the two points.

Please return to page 62/2 and try question 20 again.

This is only one of three choices which you were offered which have the value 14.

Therefore, this choice is not correct.

Return to page 69/1 and try question 1 again.

I

46
1

It's very easy to be careless! The information that QR is twice as long as PQ does NOT mean add the coefficients of P and Q to find R .

Please return to page 40 2 and try question 9 again.

46
2

Did you draw a number line to help you with this question? Find the point corresponding to .7 ; find the spot that corresponds to 8.7 and then count the number of natural numbers that fall between your two points.

Please return to page 30 2 and try question 13 again.

I

Since the fraction

$\frac{21}{7}$ equals 3 ,

and

$\frac{28}{7}$ equals 4 ,

then the fraction

$\frac{22}{7}$

is between the natural numbers

3 and 4 .

Your answer is correct.

Please go on to question 16 below.

Question 16

Apply the proper principle and select the letter which correctly answers the following question:

A 6-inch ruler has a mark for every one-quarter of an inch. How many of these marks are there between $1\frac{1}{4}$ and $3\frac{3}{4}$?

(A) 12

(B) 11

(C) 10

(D) 9

48
1

It's easy to be careless. Did you draw a number line to assist you? Divide your number line into fourths.

Please return to page 52/2 and try question 19 again.

48
2

What is the value of 4.5 + 3 ? And what is the value of 3 + 4.5 ? If you still get his choice, you must have made a mistake in addition since this choice is not correct.

Please return to page 60/2 and try question 2 again.

I

Good guess! If the question had asked "Find how many numbers there are between 10 and 15 " you would have been correct. You overlooked the adjective, "Natural" in the description. The natural numbers between 10 and 15 can be counted and thus are not Infinite.

Please return to page 38/2 and try question 11 again.

Let us ask you a similar question.

Is $1.14 between $1.40 and $1.50 ?

Please return to page 39/2 and try question 14 again.

I

$\frac{50}{1}$

Sorry, we don't agree. The number two can be expressed as $\frac{14}{7}$ and the number 3 as $\frac{21}{7}$. The number $\frac{22}{7}$ is not between those two.

Please return to page $\frac{58}{2}$ and try question 15 again.

$\frac{50}{2}$

Sorry, we don't agree. Did you forget the meaning of "inclusive"? Suppose you correct the given fraction to whole numbers and then list the numbers between them on the number line. The re-read the rest of the question before you make your answer choice.

Please return to page $\frac{42}{2}$ and try question 17 again.

I

If you are asked to tally (count) a number of items, a convenient way of doing it is to put down a mark for each item.
To make it easier to total, it is customary to place every fifth mark across the other four. This choice then offers the number five plus five plus four, which is certainly 14. Then this choice is not correct because its value is the same as two other choices.

Please return to page 69/1 and try question 1 again.

You are probably used to doing subtractions with one number written under the other. If you write it this way:

$$\begin{array}{r} 5.2 \\ \underline{-1.7} \end{array}$$

Do you now see your error? This choice is not correct.

Please return to page 65/2 and try question 3 again.

I

$\frac{52}{1}$

A synonym for "midway" is halfway. For example, midway between one and two is the point $1\frac{1}{2}$. Now another common method used to find the midpoint, or average of two numbers is to add them and divide by two. The sum of $1\frac{1}{4}$ and $2\frac{1}{2}$ is $3\frac{2}{4}$ or $3\frac{1}{2}$. One-half of $3\frac{1}{2}$ can be found by converting the mixed number to an improper fraction.

$3\frac{1}{2}$ equals $\frac{7}{2}$.

$\frac{1}{2}$ of $\frac{7}{2}$ is $\frac{7}{4}$.

This can be reformed as a mixed number $1\frac{3}{4}$

Your answer is correct.

Please go on to question 19 below.

$\frac{52}{2}$

Question 19

Apply the proper principle and find the coordinate of the point that lies midway between 1 and $\frac{3}{2}$. Select the letter which labels the correct statement.

(A) $\frac{3}{4}$

(B) $\frac{6}{5}$

(C) $\frac{4}{2}$

(D) $\frac{5}{4}$

Let us ask you a similar question.

Is $1.54 between $1.40 and $1.50 ?

Please return to page 39/2 and try question 14 again.

Sorry, we disagree!

Make a sketch of such a ruler to help you.

Return to page 47/2 and try question 16 again.

I

54
1

It's easy to be careless. To find an average of two points or the midpoint, you take <u>half</u> the sum of their coordinates.

Please return to page 62/2 and try question 20 again.

54
2

Since the sum of any two numbers is a number, both of the additions are possible.

Therefore, this choice is not correct.

Return to page 60/2 and try question 2 again.

I

The proper way to check a division example is to multiply the quotient by the divisor to see if you get the dividend. Using this fact, does

$$? = 0 \times 7 \ ?$$

It does not. Therefore, this choice is not correct.

Please return to page 80/2 and try question 5 again.

What did you get for the value of R ? If you had trouble dividing 2 by .2 , you should remember that it is necessary to move the decimal point in the divisor to the right of the number. Since that moves it one place to the right, you must then move the decimal point in the dividend one place to the right as well. Since the dividend was 2 .

What did you get when you moved the decimal point?

$$\text{e.g.} \quad \frac{2}{.2} = \frac{2.0}{.2} = \frac{20}{2}$$

Please return to page 90/2 and try question 7 again.

$\frac{56}{1}$

Congratulations! Your answer shows that you recognized that the word "inclusive" means that the end points of a segment are included in the interval. Let us review this problem.

$\frac{72}{6}$ is the same as 12

$\frac{6}{2}$ is the same as 3

The whole numbers between 12 and 3 inclusive are namely,

12, 11, 10, 9, 8, 7, 6, 5, 4, 3 .

Of these 12, 9, 6, and 3 are divisible by 3 .

Please continue with question 18 below.

$\frac{56}{2}$

Question 18

Apply the proper principle and find the coordinate of the point midway between $1\frac{1}{4}$ and $2\frac{1}{4}$.

Select the letter which labels the correct number.

(A) $1\frac{2}{4}$ (C) $1\frac{1}{3}$

(B) $1\frac{3}{4}$ (D) 1.2

Sorry, we don't agree. When two fractions are added, we do NOT merely combine their denominators.

Please return to page 62/2 and select another answer for question 20 .

Since this choice offers two different subtractions, and one of them is not possible to do, this choice is not correct.

Return to page 71/2 and try question 4 again.

I

$\frac{58}{1}$

Your answer is correct. Congratulations!

You can always find a number between two other numbers by finding their average, that is $\frac{1}{2}$ of their sum.

Please go on to question 15 below.

$\frac{58}{1}$

Question 15

Apply the proper principle and find between what two successive natural numbers $\frac{22}{7}$ lies. Select the letter which labels the correct answer.

(A) 7 and 22

(B) 3 and 4

(C) 3 and $3\frac{1}{7}$

(D) 2 and 3

I

Your answer choice indicated that $\frac{4}{2}$ is midway between 1 and $\frac{3}{2}$, did you check your statement? Did you draw a number line to help you?

Please return to page $\frac{52}{2}$ and try question 19 again.

Writing the example in the way you are used to seeing it:

$$\begin{array}{r} 1.7 \\ -5.2 \end{array}$$

You should recognize that the larger number is being taken away from the smaller, which is impossible for you to do (at this time).

Therefore, this choice is not correct.

Return to page $\frac{65}{2}$ and try question 3 again.

I

60
1

In Roman numerals this number has the value 10 (for X) plus 5 (for V) plus 1 (for I). Its value is, therefore, 16, which is not the same as the value of the other choices.

Therefore, this choice is correct.

Proceed to question 2 below.

60
2

Question 2

If P is 4.5 + 3 , and Q is 3 + 4.5 , which statement do you recognize is correct?

(A) P is larger than Q

(B) P is smaller than Q

(C) P has the same value as Q

(D) these additions are meaningless

This choice says that 2 divided by 4 equals 2 . The best way to check a division is by multiplying quotient by divisor to see if you get the dividend. But 2 does not equal 4 times 2 . Therefore, this choice is not correct.

Return to page 80/2 and try question 5 again.

Did you calculate the value of each of the letters? You have a mistake in arithmetic somewhere, since this choice is not correct.

Return to page 90/2 and try question 7 again.

$\frac{62}{1}$

Congratulations! Your answer is correct. If you divide your number line into fourths, 1 is the fourth mark from zero,

$\frac{3}{2}$ is the sixth mark from zero.

The point midway between these two is the fifth mark which corresponds to $\frac{5}{4}$.

Please go on to question 20 below.

$\frac{62}{2}$

Question 20

Apply the proper principle and find the coordinate of the point that lies midway between $\frac{1}{4}$ and $\frac{1}{6}$

(A) $\frac{1}{5}$

(B) $1\frac{5}{12}$

(C) $\frac{1}{10}$

(D) $\frac{5}{24}$

I

Since this choice offers two different divisions, where the order of the numbers is changed, the results are not equal. In general, the division of any two numbers is possible, but the result depends on the order of the numbers. In any event, this choice is incorrect.

Return to page 71/2 and try question 4 again.

Since the sum of two numbers does not depend on the order of addition, this statement is correct. But you were asked for the one which is not correct.

Therefore, this is not the correct choice.

Return to page 85/2 and try question 6 again.

$\frac{64}{1}$

Since 5 divided by 3 is $1\frac{2}{3}$ which is not larger than 2, this choice is not correct.

Return to page $\frac{73}{2}$ and try question 8 again.

$\frac{64}{2}$

In adding two fractions, the result is larger than either one. However, in multiplying two fractions (proper fractions), the reverse is true, since part of a quantity is smaller than the original. Then it is impossible for S and T to have the same value, and this choice is not correct.

Return to page $\frac{95}{2}$ and try question 10 again.

Since the value of P is 7.5 , and the value of Q is 7.5, this is correct.

Proceed to question 3 below.

Question 3

Which choice do you recognize is correct?

(A) $5.2 - 1.7 = 4.5$

(B) $1.7 - 5.2 = 4.5$

(C) $1.7 - 5.2 = 3.5$

(D) $5.2 - 1.7 = 3.5$

I

66
1

A simple rule which you should know is that if two fractions have the same numerator, the one with the larger denominator is the smaller fraction.

Therefore, this choice is not correct.

Return to page 91/2 and try question 9 again.

66
2

As pointed out in the Preliminary Notes to this segment, you may not divide by zero. Therefore, 5 divided by zero is not a number, and it cannot be compared in size to a number. Then this choice is not correct.

Return to page 77/2 and try question 11 again.

The midpoint between $\frac{1}{4}$ and $\frac{1}{6}$ can be determined by averaging the coordinates $\frac{1}{4}$ and $\frac{1}{6}$

The sum of $\frac{1}{4}$ and $\frac{1}{6}$ is $\frac{5}{12}$

Next, we take one-half of this, thus, the coordinates of the midpoint are

$$\frac{5}{24}$$

Your answer is correct.

You have now finished this SEGMENT. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following:

1. counting numbers are also called natural or whole numbers.
2. " between " means that the end points of the segment are not included.
3. " inclusive " means the end points are also part of the segment.
4. including means that only one end point is included.

You should now be able to complete the following from your homework:

pg. 3 - 4 questions 1 - 10

pg. 17 questions 1 - 12

Volume 1 Segment 3 begins here:

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following.

COLUMNS	48	and	50	0 3	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 1	(Volume Number)
	66	and	68	0 3	(Segment Number)

Preliminary Notes:

The four fundamental operations of arithmetic (addition, subtraction, multiplication and division) are familiar to you. However, there are certain facts that we ought to review, even though you may feel that they are obvious.

1. When any two numbers are added, the result is a number, and there is only one result.
2. When any two numbers are multiplied, the result is a number, and there is only one result.
3. When one number is subtracted from another and there is a result, there is only one result.
4. When one number is divided by another and there is a result, there is only one result.

You understand, of course, that a number may be large or small, integral (whole number) or fractional, or even zero.

Now the first two statements guarantee that addition of two numbers or multiplication of two numbers is always possible and give a single result. Statement number 3 suggests that subtraction is not always possible, and this is true.

continued

The number being subtracted must not be larger than the number we are subtracting it from. May the two be equal ? Of course, since a number subtracted from itself gives zero.

Division also does not guarantee a result. When may a division be impossible ? The answer is that any division is possible, except ... ..., but we'll get to that. You know that

$$6 \div 2 = 3, \text{ and}$$

$$2 \div 6 = \frac{1}{3} .$$

The division of one number by another, in that order, gives a single result. In a different order, it would be a different divison. Now for the exception: division by zero is impossible! Why is that ?

Let's look at $6 \div 3$.

We say the result is 2 because

$$6 = 3 \times 2 .$$

Then, if

$$6 \div 0 = (\text{a number})$$

It must follow that

$$6 = 0 \times (\text{a number})$$

But zero times any number equals zero. Therefore, YOU MAY NOT DIVIDE BY ZERO.

Now turn to your reading assignment for this segment: pages 5 - 8.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Which choice do you recognize is a numerical expression for a number different from the others ?

A. 14
B. XVI
C. fourteen
D. ~~IIII IIII IIII~~

$\frac{70}{1}$

Since the product of two numbers does not depend on the order of multiplication, this statement is correct; but you were asked for the one which is not correct. Therefore, this is not the correct choice.

Please return to page $\frac{85}{2}$ and try question 6 again.

$\frac{70}{2}$

In adding two fractions, the result is larger than either one. However, in multiplying two proper fractions, the reverse is true, since part of a quantity is smaller than the original.

Then: S is larger than $\frac{1}{2}$,,

while

T is smaller than $\frac{1}{2}$.

Therefore, this choice is not correct.

Please return to page $\frac{95}{2}$ and try question 10 again.

Writing the example in the more usual form, we have:

$$\begin{array}{r} 5.2 \\ \underline{-1.7} \end{array}$$

Performing the subtraction and being careful with our arithmetic, we get the result you chose.

Therefore, this choice is correct.

Please proceed to question 4 which follows.

71
1

Question 4

Which choice do you recognize is correct?

(A) $4 - 3 = 3 - 4$

(B) $4 \div 3 = 3 \div 4$

(C) $4 \times 3 = 3 \times 4$

(D) None of these.

I

72
1

If you add these numbers keeping the decimal points in line, the result is

1.2

which is certainly not smaller than

1

Therefore, this choice is not correct.

Please return to page 83/2 and try question 12 again.

72
2

This choice says that the value on the right side is

3.5

Is that the value you got for the left side? The left side said that you were to divide the total of

3 + 2 by 2

If you insist on dividing before you add, you must divide <u>each</u> number by

2

This choice is not correct.

Please return to page 98/2 and try question 13 again.

The value of

$P = 8 + 2$

is 10

The value of $Q = 5 \times 2$

is 10

The value of $R = 2 \div .2$

can be calculated as follows:

$.2 \overline{)2}$

$2. \overline{)20.}$ { shifting the decimal point one place to the right.

Then the value of R is also 10.

This choice is correct.

Please proceed to question 8 below.

Question 8

Which one of the statements do you recognize as correct ?

(A) $5 \div 3 > 2$

(B) $5 \div 3 < 2$

(C) $5 - 3 < 2$

(D) $5 \times 3 < 2$

74
1

This choice says that 0

divided by 7

equals 7

The best way to check a division is by multiplying quotient by divisor to see if you get the dividend. But

0

does not equal 7 x 7

Therefore, this choice is not correct.

Please return to page 80/2 and try question 5 again.

74
2

Since 6.2 x 70 = 434

and 350 + 84 = 434

this choice does not make the inequality true. Therefore, this choice is not correct.

Please return to page 104/2 and try question 14 again.

75/1

Since the same number, 5 , is divided by two different numbers; the results must have different values. Therefore, this choice is not correct.

Please return to page 91/2 and try question 9 again.

75/2

Since the number 7 + 3

equals 10

and 10

divided by 2

equals 5

which is not larger than 6

this choice is not correct.

Please return to page 107/2 and try question 16 again.

I

76
1

Since one of the other choices is correct, this is not.

Please return to page 71 and try question 4 again.
2

76
2

Since one of the other choices is correct, this is not.

Please return to page 85 and try question 6 again.
2

I

In adding two fractions, the result is larger than either one. However, in multiplying two proper fractions, the reverse is true since part of a quantity is smaller than the original.

Then

S is larger than $\frac{1}{2}$,

while

T is smaller than $\frac{1}{2}$.

Therefore, this choice is correct.

Please proceed to question 11 below.

Question 11

Which choice do you recognize as correct ?

(A) $5 \div 0 > 5$

(B) $5 \div 0 = 5$

(C) $5 \div 0 < 5$

(D) None of these.

78/1

Since $5 - 3$

is not smaller than 2

this choice is not correct.

Please return to page 73/2 and try question 8 again.

78/2

Since $5 \times 1 = 5$

and $5 + 2 = 7$

which is not smaller than 4

this choice is not correct.

Please return to page 100/2 and try question 15 again.

I

You may not have been able to think of it, but there is more than just the number 1 which has this property.

Will this hint help? A number fits this property is a very familiar one, but one which you often do not think of as a number.

Please return to page 97/2 and try question 17 again.

The proper procedure is to find the value of the numerator first, and then divide. If you do this, you will find that the fraction on the left does not have the same value as the sum on the right. Therefore, this choice is not correct.

Did you decide to divide one of the numbers by 2 before adding? That is not correct. If you insist on dividing before performing the addition, you must divide <u>each</u> number by the 2 .

Please return to page 93/2 and try question 18 again.

I

80
1

Since the result in a multiplication does not depend on the order of the numbers, this choice is correct.

Please proceed to question 5 below.

80
2

Question 5

Which choice do you recognize as correct ?

(A) $7 \div 7 = 0$

(B) $2 \div 4 = 2$

(C) $0 \div 7 = 7$

(D) $2 \div 4 = .5$

I

Since 5×3

is not smaller than 2

this choice is not correct.

Please return to page 73/2 and try question 8 again.

Since $6.2 \times 70 = 434$

and $350 + 12 \times 7 = 350 + 84 = 434$

this choice does not make the inequality true.

Therefore, this choice is not correct.

Please return to page 104/2 and try question 14 again.

$\frac{82}{1}$

If one is subtracted from both the numerator and the denominator of a fraction, the resulting fraction does not have the same value as the original fraction.

e.g. $\frac{9}{3}$ (which equals 3)

would become $\frac{9 - 1}{3 - 1} = \frac{8}{2}$ (which equals 4)

Please return to page $\frac{91}{2}$ and try question 9 again.

$\frac{82}{2}$

Since the number 8 + 3 equals 11

and 11 divided by 2

is $5\frac{1}{2}$

which is not larger than 6

this choice is not correct.

Please return to page $\frac{107}{2}$ and try question 16 again.

I

Since none of the other choices is correct, this choice is correct.

Notice that 5 divided by 0 is not a number, and it cannot be compared to size to any number.

Please proceed to question 12 below.

Question 12

Perform the calculation to find which choice is correct.

(A) $.4 + .8 < 1$

(B) $25.47 \times 4 > 100$

(C) $4 + \frac{4}{3} \neq 4 \times \frac{4}{3}$

(D) $\frac{22}{7} \div \frac{7}{11} = 2$

84
1

Have you found some numbers which have the property mentioned? When you multiply your number by itself, you are supposed to get the same number again. This choice says that you found many numbers like this. Since that is impossible, you have misunderstood the problem in some way.

This choice is not correct.

Please return to page 97/2 and try question 17 again.

84
2

If you find the value in the numerator first, you get 10 .

But 10 divided by 6 is less than 2 ;

it is certainly not larger that 5 .

Therefore, this choice is not correct. Note that it is important to add before doing the division.

Please return to page 102/2 and try question 19 again.

This choice says that

2 divided by 4 equals .5

The best way to check a division is by multiplying quotient by divisor to see if you get the dividend.

Does 2 equal 4 times .5 ? Yes, it does.

Therefore, this choice is correct.

Please proceed to question 6 below.

Question 6

Which statement do you recognize is NOT correct?

(A) $8.3 \div 2 = 2 \div 8.3$

(B) $8.3 \times 2 = 2 \times 8.3$

(C) $8.3 + 2 = 2 + 8.3$

(D) None of these.

$\frac{86}{1}$

Since one of the other choices is correct, this is not.

Please return to page $\frac{95}{2}$ and try question 10 again.

$\frac{86}{2}$

$$5 \times \frac{1}{2} = 2.5$$

and

$$2.5 + 2 = 4.5$$

which is not smaller than 4 .

Therefore, this choice is not correct.

Please return to page $\frac{100}{2}$ and try question 15 again.

This choice says that the value on the right side is 4 . Is that the value you got for the left side? The left side said that you were to divide the total of

$$3 + 2 \text{ by } 2$$

If you insist on dividing before you add, you must divide each number by 2 .

This choice is not correct.

Please return to page 98/2 and try question 13 again.

Since one of the other choices is correct, this is not.

Please return to page 93/2 and try question 18 again.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE

11320 ...recognize and apply the principle: to combine fractions with unlike denominators, first change the fractions to equivalent fractions with like denominators.

11321 ...calculate the least common multiple (LCM) of 2 binomials.
e.g. the LCM of $(a + b)$ and $(a^2 - b^2)$ is $a^2 - b^2$

11322 ...combine fractions with different monomial denominators consisting of multiples of one letter, e.g.

$$\frac{b}{2a} + \frac{c}{3a} - \frac{d}{4a} = \frac{6b + 4c - 3d}{12a}$$

11323 ...combine fractions with different monomial denominators consisting of more than one letter, e.g.

$$\frac{3x}{2ab} + \frac{5y}{3a^2b} - \frac{7z}{6ab^2} = \frac{9abx + 10by - 7az}{6a^2b^2}$$

11324 ...combine fractions with different monomial denominators and binomials in the numerators, e.g.

$$\frac{x + 4}{3a} - \frac{2 - x}{2b} = \frac{2b(x + 4) - 3a(2 - x)}{6ab}$$

11324 ...combine fractions with different binomials as denominators, e.g.

$$\frac{3a}{2a + 6} - \frac{a - 1}{a + 3} = \frac{3a - 2(a - 1)}{2(a + 3)}$$

11326 ...combine fractions containing a denominator which is the difference of 2 squares, e.g.

$$\frac{1}{a^2 - b^2} + \frac{3}{a + b} = \frac{1 + 3(a - b)}{(a + b)(a - b)}$$

11327 ...combine fractions with a trinomial in the denominator, e.g.

$$\frac{6x - 13}{x^2 - 5x + 6} - \frac{5}{x - 3} = \frac{6x - 13 - 5(x - 2)}{(x - 3)(x - 2)}$$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
11410	...combine mixed expressions.
11411	...recognize that a mixed number can be written as the sum of an integer and a fraction, e.g. $2\frac{1}{3} = 2 + \frac{1}{3}$
11412	...change a mixed number to an improper fraction, e.g. $4\frac{2}{5} = \frac{22}{5}$
11413	...express a mixed number as a fraction, e.g. $a + \frac{2}{a} = \frac{a^2 + 2}{a}$
11414	...express a fraction as a mixed expression, e.g. $\frac{3x - 2}{x} = 3 - \frac{2}{x}$
11415	...express a mixed expression as a single fraction, when the denominator is a binomial, e.g. $x - \frac{2x}{x - 2} = \frac{x^2 - 4x}{x - 2}$
11416	...multiply 2 mixed expressions, e.g. $\left[x - \frac{1}{x}\right]\left[1 + \frac{1}{x + 1}\right] = \frac{x^2 - 1}{x} \cdot \frac{x + 2}{x + 1} = \frac{(x - 1)(x + 2)}{x}$
11420	...divide 2 mixed expressions, e.g. $\left[2 + \frac{4}{x - 2}\right] \div \left[\frac{x}{x - 2} + 1\right] = \frac{2x}{x - 2} \cdot \frac{x - 2}{2x - 2} = \frac{x}{x - 1}$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE

11430 ...simplify complex fractions.

11431 ...recognize the meaning of a complex fraction, e.g.

$$\frac{\frac{4}{2}}{3}, \quad \frac{\frac{1}{a}}{a}, \quad \frac{1 - \frac{1}{a}}{a + 1}$$

are called complex fractions.

11432 ...simplify a complex fraction consisting of the quotient of a fraction and a number, e.g.

$$\frac{\frac{2}{3}}{4} = \frac{2}{12} = \frac{1}{6}$$

11433 ...simplify the complex fraction consisting of the quotient of a number and a fraction, e.g.

$$\frac{4}{\frac{5}{6}} = 4 \cdot \frac{6}{5} = \frac{24}{5}$$

11434 ...simplify the quotient of 2 fractions, e.g.

$$\frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$$

11435 ...simplify a fraction where the numerator is a mixed expression, e.g.

$$\frac{1 + \frac{a}{b}}{c} = \frac{1}{c} + \frac{a}{bc}$$

11436 ...simplify a fraction where both numerator and denominator is a mixed expression, e.g.

$$\frac{1 - \frac{1}{1 - x}}{1 - \frac{x}{x - 1}} = \frac{1 - x - 1}{1 - x} \div \frac{1 - x + x}{1 - x}$$

$$= -x$$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
11510	...solve open sentences with fractional coefficients.
11511	recognize and apply the principle: two open sentences with the same solution set are called equivalent equations.
11512	recognize and apply the principle: an open sentence can be changed to an equivalent open sentence by multiplying both sides by the same quantity. Thus, $\frac{x}{2} + \frac{x}{3} = 5$ (multiplied by 6) becomes: $3x + 2x = 30$ and the two are equivalent
11513	find the lowest common denominator of an open sentence with fractional coefficients. e.g the L.C.D of $\frac{x}{2} + \frac{x}{3} = 5$ is 6
11514	change an open sentence with fractional coefficients to an equivalent sentence with integral coefficients.
11515	solve the linear equation resulting from changing an equation with fractional coefficients to one with integral coefficients.
11516	check the solution set obtained in solving the open sentence that contained fractional coefficients.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
12110	...solve verbal investment problems
12111	recognize and apply the principle: simple interest (I) on an amount of money (P), invested at a certain annual rate (R) for one year, is the product of that amount and the rate. i. e. $I = P \cdot R$
12112	recognize and apply the principle: simple interest on an amount of money invested at a certain annual rate for (t) years is given by the formula: $I = P \cdot R \cdot t$
12113	recognize and apply the principle: at the end of one year, an investment of (P) dollars at (R) annual rate of interest will result in the investor having $(P + PR)$ dollars.
12114	express the percent interest rate as decimal.
12115	recognize and apply the principle: if the sum (N) of the 2 quantities is given, and one of the quantities is denoted by (x), the other quantity is $(N - x)$
12116	recognize and apply the principle: the income from 2 investments is the sum of the incomes from each investment.
12117	solve the equations that result from analysis of interest problems having decimal coefficients.
12118	check the results obtained with the original problem.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
12210	...solve verbal percent mixture problems
12211	recognize and apply the principle: the addition of pure water represents the addition of a substance with no percent concentration of the other ingredient. e.g. in adding water to alcohol, water has 0% alcohol.
12212	recognize and apply the principle: the addition of a pure substance to a mixture is the addition of 100% concentration of that substance.
12213	recognize and apply the principle: the addition of an amount of a substance to a mixture increases the total mixture by that amount e.g. if 1 gallon of water is added to a mixture of x gallons, the total mixture now contains (1 + x) gallons.
12214	recognize and apply the principle: if an amount of water has been evaporated, the new mixture is reduced by that amount. e.g. if 20 ounces of water is evaporated from a solution of x ounces, the new mixture contains (x - 20) ounces.
12215	calculate the amount of pure ingredient in a mixture of a certain percent pure of that ingredient.
12216	recognize and apply the principle: the percent pure of a mixture is found by dividing the amount of total pure by the amount of total mixture and multiplying by 100 .
12217	check the results obtained with the original problem.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE

12310 ...solve fractional equations using the following sequence of operations:

12311 recognize and apply the principle: a fractional equation is an equation with variables in the denominators,

e.g. $\frac{3}{2x} + \frac{1}{x} = \frac{5}{9}$

12312 find the lowest common multiple of the denominators of the equation (LCD).

12313 change the fractional equation to an equation with integral coefficients by multiplying all terms by the lowest common denominator of the equation.

12314 recognize and apply the principle: multiplying a fractional equation by a variable expression which can represent zero, may produce extraneous roots.

12315 recognize and apply the principle: the solutions obtained must be checked by substituting their values in the original equation.

12316 ...solve fractional equations with binomial expressions in their denominator,

e.g.

$$4x - \frac{x}{3x - 1} = 3$$

12317 ...solve fractional equations with trinomial expressions in the denominator,

e.g. $\frac{1}{2y + 3} + \frac{1}{2y + 1} = \frac{2}{4y^2 + 8y + 3}$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
12410	...solve work problems
12411	recognize and apply the principle: the rate of working means the amount of work done in some specified unit of time. e.g. if a man can do a certain job in (a) days, in <u>one</u> day, he does $\frac{1}{a}$ of the job.
12412	recognize and apply the principle: the amount of work done is the product of the time spent working and the rate of doing work. e.g. if a man does $\frac{1}{a}$ of the job in one day he does $\frac{x}{a}$ of the job in x days.
12413	recognize and apply the principle: if several persons are working together on a job, the amount of work done in one day is the sum of the individual amounts. e.g. $\frac{1}{a} + \frac{1}{b} +$ ---- is the total amount of work done in one day.
12414	recognize and apply if several persons are working together on a job, the amount of work done in (x) days is the sum of (x) times their daily amounts. e.g. $\frac{x}{a} + \frac{x}{b} +$ --- = amount done in x days.
12415	recognize and apply the total job can be represented by 1 .
12416	write the equation indicating the amount of work performed in x days by each operator or operation to complete the job. e.g. $\frac{x}{a} + \frac{x}{b} +$ --- = 1
12417	solve the resulting equation having fractional coefficients.
12418	check the results obtained with the original problem.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE

12510 ...solve motion problems involving the use of fractional equations.

12511 recognize and apply the principle: distance equals the product of rate and time.
i.e. $d = r \cdot t$

12512 recognize and apply the principle: time equals the quotient of distance and rate.
i.e. $t = \frac{d}{r}$

12513 recognize and apply the principle: rate equals the quotient of distance and time.
i.e. $r = \frac{d}{t}$

12514 calculate the combined rate of an object going in the same direction of a wind or a current.
e.g. if the simple rate is "r", and rate of current is "c" then rate with current is (r + c)

12515 calculate the combined rate of an object going against a wind or current.
e.g. rate against current is (r - c)

12516 tabulate the given data of a motion problem in a chart form,

	R	x	T	=	d
with the wind					
against the wind					

12517 write a fractional equation involving the relations given between time or distance.

12518 solve the resulting fractional equation.

12519 check the results obtained with the original problem.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
13110	...write an equation in two variables to express the relationships in a verbal problem.
13111	...choose letters to represent the two variables.
13112	...translate the verbal statement of a relationship into an algebraic equation, by using the chosen variables and equivalents for standard words such as "equal to" increased by, is, twice, etc.
13120	...find the ordered pairs of numbers which are solution sets of a given equation.
13121	...recognize and apply the principle: the numbers of an ordered pair represent the x value and the y value in that order. e.g $(5, -2)$ means $x = 5$, $y = -2$
13122	...transform a given equation of the form $ax + by = c$ into one of the form $y = mx + k$
13123	...find the value of y which corresponds to each value of x in its replacement set.
13124	...express the solution set of an equation as a roster of ordered pairs.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
13210	...locate the point in a plane with reference to a set of Cartesian coordinate axes which corresponds to an ordered pair of numbers.
13211	...recognize and use the words axis, Cartesian coordinate axis, origin, horizontal, vertical, plot, abscissa and ordinate, in the creation and interpretation of graphs of points in a plane.
13212	...recognize the difference of position of points in a Cartesian plane naming abscissas of zero, positive or negative values.
13213	...recognize the differences of position of points in a Cartesian plane having ordinates of zero, positive or negative values.
13220	...find the ordered pair of numbers corresponding to a given point in a Cartesian plane.
13221	...recognize that the abscissa of a point means the distance, parallel to the axis, to the right or left from the Y axis to the point.
13222	...recognize that the ordinate of a point means the distance parallel to the Y axis, up or down from the X axis to the point.
13223	...recognize that a point which is on the Y axis has an abscissa of zero.
13224	...recognize that a point which is on the X axis has an ordinate of zero.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
13230	...draw the graph on a Cartesian plane of a given linear equation in two variables, x and y.
13231	...construct a table of values composed of the coordinates of at least 3 points whose coordinates satisfy the equation.
13232	...plot the location of the points from the table of values on graph paper containing Cartesian coordinates and connect them with a straight line.
13233	...draw the graph of an equation of the form $x = c$, by drawing a line through any 3 points each having an abscissa of c .
13234	...draw the graph of an equation of the form $y = c$, by drawing a line through any 3 points each having an ordinate of c .
13235	...draw the graph of an equation of the form $ax + by = c$, by locating 3 points whose coordinates fit the equation, and by connecting them with a straight line.

OPTIONAL

MBO LIST

Upon completion of this segment the student should be able to:

MBO CODE

13310 ...find the slope of a given line.

13311 ...define the value of the slope of a line as $\frac{\triangle y}{\triangle x}$

13312 ...find the value of delta y , $(\triangle y)$ from two points, (x_1, y_1) and (x_2, y_2) by subtracting the y values: $y_2 - y_1$

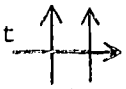
13313 ...find the value of delta x , $(\triangle x)$ from two points, (x_1, y_1) and (x_2, y_2) by subtracting the x values: $x_2 - x_1$

13314 recognize from the appearance of the line when the slope is positive:

positive:

negative:

zero

does not exist 

13320 ...determine whether or not 3 points A, B, C are on a straight line.

13321 ...calculate the slopes of AB and BC.

13322 ...recognize that equal slopes of two lines through one point indicate that the lines and the points on the lines are collinear.

13323 ...recognize that unequal slopes of two lines through one point indicate that the lines and the points on the lines are not collinear.

13324 ...recognize that, if neither segment (AB or BC) through the same point, B, has a slope, the points A , B , and C , are collinear.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
13330	...use slope to determine the value of a variable.
13331	...find a second point on a line, given one point and the slope: e.g. $m = 3$, (2,1), (a,4), find a.
13332	...find the value of a variable given the slope: e.g. $m = 2$, (a, 1), (2a, 5), find a.
13340	...graph a line by the slope-intercept method, given its equation in the form $ax + by = c$, $b \neq 0$.
13341	...transform the equation from $ax + by = c$ to $y = mx + b$.
13342	...recognize the value of the slope, $m = \frac{\Delta y}{\Delta x}$ m from the form $y = mx + b$.
13343	...recognize the value of the y intercept, b, from the form $y = mx + b$.
13344	...locate the y intercept, (b), on the graph.
13345	...use the slope $m = \frac{\Delta y}{\Delta x}$, to locate a second point, $(\Delta x , b + \Delta y)$
13346	...draw the line through the y intercept and the 2nd point found by using the slope.

MBO LIST

Upon completion of this segment, the student should be able to:

13410 ...graph a line given its equation in the form $ax + by = c$.

13411 ...graph a line when $b = 0$, given the form $ax + by = c$

13412 ...graph a line when $a = 0$, given the form $ax + by = c$

13413 ...graph a line of the form $ax + by = c$ when $a \neq 0$, $b \neq 0$.

13420 ...graph a linear inequality in two variables.

13421 ...recognize and apply the principle that $y \quad mx + b$ indicates all points above the line $y = mx + b$.

13422 ...recognize and apply the principle: $y \quad mx + b$ indicates all points below the line $y = mx + b$.

13423 ...recognize and apply the principle: $x \quad \ldots$ means all points to the right of the line.

13424 ...recognize and apply the principle: $x \quad \ldots$ means all points to the left of the line.

13425 ...recognize and apply the principle: a dotted line is the graph of only the inequality; a solid line is the graph of an equation combined with an inequality,

i.e. $y \quad mx + b$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
13510	...recognize from the graph of two equations whether the equations are independent or inconsistent.
13511	...determine that the equations are independent by finding that the lines intersect.
13512	...determine that the equations are dependent by finding the two lines coincide.
13513	...determine that the equations are inconsistent by finding that the two lines are parallel
13520	...find the solution set of a pair of equations from the graph of the equations.
13521	...graph the two lines on the same set of cartesian coordinate axes
13522	...find the coordinates of the point of intersection and call them the solution set of the two equations.
13523	...check the solution set by substituting the values in each equation.
13524	...recognize that the solution set is the empty set if the lines are parallel.
13525	...recognize that the solution set is infinite if the two lines coincide.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
14110	...solve a pair of linear simultaneous equations in two unknowns by the addition method.
14111	...recognize the conditions under which adding the two equations together will eliminate one variable.
14112	...solve for one variable, after eliminating the other by addition.
14113	...solve for the second variable, by substituting the value already found for the first in one of the equations.
14114	...check the solution by substituting both values in both equations.
14120	...solve a pair of linear simultaneous equations in two unknowns by the subtraction method.
14121	...recognize the conditions under which subtracting one equation from the other will eliminate one variable.
14122	...solve for one variable, after eliminating the other by subtraction.
14123	...solve for the second variable, by substituting the value already found for the first in one of the equations.
14124	...check the solution by substituting both values in both equations.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
14130	...solve a pair of linear simultaneous equations in two unknowns by eliminating one variable by the addition or subtraction method after multiplying each equation by the appropriate number.
14131	...find the lowest common multiple of the x coefficients and of the y coefficients.
14132	...find what multiplier is needed in the first equation, and what multiplier is needed in the second, so that equal coefficients will be produced for one of the variables.
14133	...multiply all terms in each equation by the appropriate number
14134	...determine whether to add or subtract the resulting equations so as to eliminate one variable.
14135	...add or subtract the two equations so as to eliminate one variable.
14136	...solve for the second variable by substituting the value already found for the first in one of the equations.
14137	...check the solution by substituting both values in both equations.
14140	...transform each equation of the pair of simultaneous linear equations in two variables into the form $ax + by = c$.
14141	...multiply both members of the equation by the lowest common denominator to remove any fractions.
14142	...combine simultaneous and add or subtract appropriate quantities by using the addition and subtraction - properties of equality to give a result in the form $ax + by = c$.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
14210	...solve a pair of simultaneous linear equations in two unknowns by the substitution method.
14211	...determine in which equation of a pair of simultaneous linear equations in two unknowns, solution for one variable in terms of the other would be simpler.
14212	...solve one equation for one variable in terms of the other variable.
14213	...substitute the algebraic value obtained for one variable from one equation in the other equation, so as to produce a third equation in one variable.
14214	...solve the resulting equation in one variable for that variable.
14215	...substitute the solution for this variable in one of the original equations and solve for the remaining variable.
14216	...check by substituting the values found for each variable in both of the original equations.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
14310	..graph in the Cartesian plane an inequality in one or two variables.
14311	...transform an inequality to an equality by replacing the inequality sign with an equals sign.
14312	...recognize and apply the principle: the equation derived by transforming an inequality to an equation is called the equation of the reference line.
14313	...graph the reference line
14314	...recognize that the graph of an inequality in two variables is one of the two half-planes formed by the reference line.
14315	...recognize that the graph of an inequality which contains only the variable y is that half-plane either above or below the horizontal reference line.
14316	...recognize that the graph of an inequality which contains only the variable x is that half-plane either to the right or left of the vertical reference line.
14317	...recognize that the reference line is to be dotted unless the equal sign also appeared with the inequality.
14320	..solve graphically a set of linear inequalities in two unknowns.
14321	...represent the solution of each inequality by a different method of shading of the half-plane
14322	...recognize the region where the individual solutions overlap, as the solution of the set of the two given inequalities.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
14410	...solve verbal problems by using two variables and one of the three methods for solving simultaneous linear equations.
14411	...solve number relationship problems using two variables.
14412	...solve perimeter problems using two variables.
14413	...solve purchasing problems using two variables.
14414	...solve investment problems using two variables.
14415	...solve mixture problems using two variables.
14416	...solve coin problems using two variables.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
14510	.. determine the equation of a line whose graph in the Cartesian plane goes through two points which have the same abscissa.
14511	.. recognize that points with the same abscissa lie on a line parallel to the Y axis
14512	.. recognize that the equation of a line parallel to the Y axis is of the form x = k
14520	...determine the equation of a line whose graph in the Cartesian plane goes through two points which have the same ordinate.
14521	.. recognize that points with the same ordinate lie on a line parallel to the X axis.
14522	...recognize that the equation of a line parallel to the X axis is of the form y = k
14530	...determine the equation of a line whose graph in the Cartesian plane passes through two points which have different abscissas or different ordinates or both.
14531	...recognize and apply the principle: the equation of a line can be written in the form y = mx + b.
14532	..substitute the values of x and y, which are the coordinates of the two points on the graph of the line in the equation after it has been put in the form y = mx + b

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
14533	...solve the simultaneous equations to find the values of m and of b.
14534	...write the equation of the line.
14535	...check that the coordinates of the points fit the resulting equation.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
15110	...solve verbally posed digit problems concerning the relationships of the digits, the reverse number and the number itself.
15111	...express a two digit number in the form: $10t + u$.
15112	...recognize and apply the principle: the place value of the units digit is equal to its face value.
15113	...recognize and apply the principle: the place value of the tens digit is ten times its face value.
15114	...express the sum of the digits in the form: $t + u$
15115	...express the reversed number in the form: $10u + t$
15116	...recognize and apply the principle: changing the order of the digits changes the place value but not the face value of the digits.
15117	...express verbal statements of relations between the digits as equations.
15118	...translate into verbal form any equation concerning the digits, the number, or the reversed number.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
15210	...solve verbally posed constant motion problems involving compound rates along the same line of motion, e.g. "boat-in-stream" problems.
15211	...recognize and apply the principle: the relationship involved in constant linear motion problems is rate times time equals distance.
15212	...recognize and apply the principle: the effective rate downstream of a boat is the sum of two rates: the basic rate of the boat in stillwater plus the rate of the stream.
15213	...recognize and apply the principle: the effective rate upstream of a boat is the difference of two rates: the basic rate of the boat in still water minus the rate of the stream.
15214	...recognize and apply the principle: a "round trip" indicates that the distance upstream equals the distance downstream.
15215	...recognize and apply the principle: the units in a constant motion problem must be consistent. e.g. m miles per hour for h hours equals mh miles; a feet per second for b seconds equals ab feet; s miles per hour in still water + c miles per hour of the current equals s + c miles per hour downstream.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
15310	...solve verbally posed age problems involving two variables.
15311	...express the age of each person by a different variable.
15312	...recognize and apply the principle: at a time"a" years in the future, the age of each person will be "a" greater than at present.
15313	...recognize and apply the principle: at a time "b" years ago, the age of each person was "b" less than at present.
15314	...recognize and apply the principle: the difference between the ages of two people is an invariant at any time of their lives.
15315	...recognize and apply the principle: the ratio of the ages of two people varies with time.
15316	...express as an algebraic equation a given relationship between the ages of two persons.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
15320	...solve verbally posed problems about fractions.
15321	...represent numerator and denominator by different variables.
15322	...recognize that the value of a fraction determines only the ratio between the numerator and the denominator.
15323	...express the reciprocal of a given fraction as a function of the original numerator and denominator.
15324	...express as an algebraic equation a given relationship between the numerator and denominator of a fraction.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
15410	...recognize the characteristics of the set of natural numbers.
15411	...recognize that the set of natural numbers is equivalent to the set of positive integers.
15412	...recognize that the set of positive integers is closed under addition and multiplication.
15413	...recognize that the set of positive integers is not closed under subtraction or division.
15420	...recognize the characteristics of the set of rational numbers.
15421	...define a rational number as the quotient of two integers $\frac{a}{b}$ where the $b \neq 0$.
15422	...recognize that division by zero is not defined.
15423	...recognize that the set of rational numbers is closed under addition, subtraction, multiplication and division.
15430	...recognize that any two rational numbers are either equal, or can be put in order of size.
	..reduce a fraction to lowest terms.
[illegible]	...recognize whether or not two rational numbers are equal by reducing them to lowest terms.
15433	...recognize which of two unequal rational numbers is larger by using the principle: $\frac{a}{b} > \frac{b}{d}$ if and only if $ad > bc$.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE:

15434 ...recognize that there is an infinite set of rational numbers between two given numbers.

15435 ...find a rational number between two given unequal rational numbers.

15440 ...recognize that every rational number can be expressed as a decimal, and it will either be terminating or repeating but non-terminating.

15441 ...transform a rational number into decimal form by division.

15442 ...represent a repeating decimal by means of the bar notation,

e.g. $\frac{1}{12} = .08333... = .08\overline{3}$

15450 ...recognize that every finite decimal, or infinite repeating decimal is a rational number and can be written as a common fraction.

15451 ...transform a finite decimal to a common fraction by learning that the number of decimal places indicates the power of 10 in the denominator.

e.g. $.173 = \frac{173}{1000}$

15452 ...transform an infinite repeating decimal to a common fraction:

e.g. $\overline{.43} = \frac{43}{99}$

15453 ...round off an infinite decimal to a specified number of places.

$.\overline{6} = .6666... = .67$ to two decimal places

MBC LIST

Upon completion of this segment, the student should be able to:

MBC CODE	
15510	..recognize the meaning of a root of a number for an index of 2, 3 or 4.
15511	...find the positive and negative square root of a perfect square.
15512	..recognize and apply the principle: the radical alone indicates the positive value of the square root, e.g. $\sqrt{16} = +4$
15513	...calculate the square root of a product by taking the square root of each factor, e.g. $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$
15514	...calculate the square root of a quotient by taking the square root of a numerator and divisor, e.g. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
15520	...define an irrational number as a non-terminating, non-repeating decimal.
15521	...recognize that, if a number is not a perfect square, its square root is irrational.
15522	...calculate the value of an irrational square root to the nearest tenth.
15523	...find the square root of a number from a table.
15530	...define a real number as a number which is either rational or irrational.
15531	...recognize that all the numbers dealt with in this course are real numbers.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
16110	...calculate the length of the missing side of a right triangle given the length of two sides.
16111	...recognize and distinguish between the hypotenuse, which is the side opposite the right angle in a right triangle, and the legs, which are the sides that form the right angle, of a right triangle.
16112	...substitute the given values for the sides of a right triangle, in the Pythagorean theorem formula: $leg^2 + leg^2 = hyp^2$, and calculate the missing value.
16120	...construct a diagram representing the data as a geometric diagram involving a right triangle.
16121	...recognize that each given length is represented as a straight line segment.
16122	...recognize and apply the principle: in verbal problems involving the solution of right triangles, a wall, telephone pole, tree, and such, are assumed to form right angles with the ground, unless otherwise stated.
16123	...recognize that, in verbal problems, a ladder leaning against a wall is assumed to form the hypotenuse of a right triangle.
16124	...recognize that, in verbal problems, a shadow is assumed to be on the horizontal ground.
16125	...recognize the position of the "foot of the pole" or tree or buildings as the point of intersection of the line representing them with the line representing the ground.
16126	...recognize that, if a point is on the ground, its distance from the foot of the pole is the distance measured along the ground.

MBO LIST

Upon co[illegible]n of this segment, the stu[illegible]ould be able to:

MBO CODE

16210 ..recognize and apply the principle: the product of the indicated square roots of two positive numbers is the square root of the product of those numbers,
e.g. $\sqrt{a}\sqrt{b} = \sqrt{ab}$

16211 ...recognize and apply the principle: a radical multiplied by itself equals the radicand,
e.g. $(\sqrt{a})^2 = \sqrt{a}\sqrt{a} = a$

16212 ...recognize and apply the principle: the product of two radicals having coefficients is the product of the coefficients times the radical whose radicand is the product of the radicands,
e.g. $a\sqrt{b} \cdot c\sqrt{d} = ac\sqrt{bd}$

16213 ...recognize and apply the principle: the square root of the square of a positive number is the number itself,
e.g. $\sqrt{a^2} = a$

16214 ...recognize and apply the principle: a radical having a radicand with a perfect square as a factor can be transformed to the product of the square root of the perfect square and the radical whose radicand contains the remaining factors,
e.g. $\sqrt{a^2b} = a\sqrt{b}$

16215 ...recognize and apply the principle: the symbol $\sqrt{a}$ means the square root of "a," the symbol "$\sqrt{\ }$" is called a "radical sign" and "a" is called the "radicand." In this course "a" cannot be negative

16216 ...recognize and apply the principle: the indicated square root of a number will be referred to as a "radical" in this course,
e.g. $\sqrt{a}$ is a radical
and it can be referred to as "radical a ."

P. III

LIST

Upon completion of this segment, the student should be able to:

MI CODE

16221 ...recognize and apply the principle: the quotient of two square root radicals is the square root of their quotient

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad \text{where } b \neq 0$$

16222 ...recognize and apply the principle: a radical divided by itself = 1

$$\frac{\sqrt{a}}{\sqrt{a}} = 1 \quad \text{where } a \neq 0$$

16222 ...recognize and apply the principle: the quotient of two radicals having coefficients is the quotient of the coefficients times the radical whose radicand is the quotient of the radicands, e.g.

$$\frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c}\sqrt{\frac{b}{d}} \quad \text{where } d \neq 0$$

16223 ...recognize and apply the principle: a fraction containing a radical in the denominator can be rationalized by multiplying the numerator and denominator by a radical that makes the denominator a rational number

e.g. $\frac{a}{\sqrt{b}} = \frac{a\left(\sqrt{b}\right)}{\sqrt{b}\left(\sqrt{b}\right)} = \frac{a\sqrt{b}}{b}$ where $b > 0$

16224 ...find the smallest number which can be used as a multiplier in rationalizing the denominator of a fraction containing a radical

MBO LIST

...on completion of this segment, ...e student should be able to:

MBO CODE	
...0	...add and subtract like radicals.
...011	...recognize that two radicals are alike when the radicands are identical.
...012	...recognize and apply the principle: adding terms with radicals is analagous to adding terms with letters.
...013	...recognize that unlike radicals can be combined only if they can be simplified to produce like radicals.
...014	...recognize that unlike radicals cannot be combined if they cannot be transformed into like radicals.

MSG LIST

Upon completion of this segment, the student should be able to:

MSG CODE	
16-1	..multiply binomials containing radicals.
1641	...recognize that $a + \sqrt{b}$ or $\sqrt{a} + b$ are binomials.
1642	...recognize and form the conjugate of a binomial containing a radical. e.g. the conjugate of $a + \sqrt{b}$ is $a - \sqrt{b}$ and the pair are called conjugate binomials.
16-13	...recognize and apply the principle: the product of conjugate binomials is a rational quantity e.g. $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$
16-14	...recognize that squaring a binomial containing radicals of the form $a + \sqrt{b}$ where b is not a square produces an irrational result e.g. $(a + \sqrt{b})^2 = a^2 + b + 2a\sqrt{b}$
16415	...multiply a radical by a binomial containing radicals by using the distributive law. e.g. $\sqrt{a}(b + \sqrt{c}) = b\sqrt{a} + \sqrt{ac}$
16416	...multiply two non-conjugate binomials containing radicals. e.g. $(a + \sqrt{b})(c + d\sqrt{b}) = (ac + bd) + (ad + c)\sqrt{b}$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
16420	...rationalize the denominator of a fraction when that denominator is a binomial containing radicals. e.g. $\frac{a}{b + \sqrt{c}} = \frac{a(b - \sqrt{c})}{b^2 - c}$
16421	...find the conjugate of the denominator of a fraction that is a binomial containing radicals.
16422	...rationalize the denominator of a fraction when the numerator is rational.
16423	...rationalize the denominator of a fraction when the numerator is a monomial radical. e.g. $\frac{\sqrt{a}}{+ \sqrt{c}} = \frac{b\sqrt{a} - \sqrt{ac}}{b^2 - c}$
16424	...rationalize the denominator of a fraction when the numerator is a binomial radical. e.g. $\frac{\sqrt{a} + \sqrt{b}}{c - \sqrt{d}} = \frac{c\sqrt{a} + c\sqrt{b} + \sqrt{ad} + \sqrt{bd}}{c^2 - d}$
16430	...find the solution of a radical equation, where only a single term contains the radical. e.g. $a + \sqrt{x} = b$
16431	...transform an equation containing a radical so that the term containing the radical is alone on one side of the equation. e.g. $a + \sqrt{x} = b$ because $\sqrt{x} = b - a$
16432	...transform an equation containing a radical by squaring both sides of the equation. e.g. $\sqrt{x} = b$ because $x = b^2$
16433	...solve a non-radical equation.
16434	...check each solution obtained from an equation containing a radical and discard any value which does not check in the original equation.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
16510	...define a relation between two variables as a set of ordered pairs. e.g. $y = 2x + 1$ is the set $\{(0,1), (1,3), (2,5)\ldots\}$.
16511	...define the domain of the relation as the first element of the relation.
16512	...define the range of the relation as the second element of the relation.
16513	...recognize that a relation can often be expressed as an equation.
16514	...write an equation, given a roster of a simple relation, e.g. $\begin{array}{c\|cccc} x & 0 & 1 & 2 & 3 \\ \hline y & 1 & 3 & 5 & 7 \end{array}$
16520	...define a function as a special kind of relation, where each member of the domain is paired with only one member of the range.
16521	...find the domain, given a graph of the function.
16522	...find the range, given a graph of the function.
16523	...write the equation of a linear function which is described in words.
16524	...graph the function (linear) if it is expressed as an equation.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
16530	...solve verbal problems dealing with direct variation, by means of the equation $y = kx$.
16531	...recognize and apply the principle: $y = kx$ expresses direct variation.
16532	...recognize that the graph of direct variation is a straight line.
16533	...express as an equation a verbal statement involving direct variation.
16534	...recognize that the principle of direct variation is involved where one variable increases when the other increases, and where the first decreases when the other decreases.
16540	...solve verbal problems dealing with direct variation by means of a proportion.
16541	...recognize that the proportion $\frac{Y_1}{Y_2} = \frac{X_1}{X_2}$ indicates direct variation.
16542	...recognize that a proportion is an equation of two ratios.
16543	...define the term "means" as Y_2 and X_1 in the proportion $\frac{Y_1}{Y_2} = \frac{X_1}{X_2}$
16544	...define the term "extremes" as Y_1 and X_2 in the proportion $\frac{Y_1}{Y_2} = \frac{X_1}{X_2}$
16545	...recognize that in a proportion the product of the means equals the product of the extremes.
16546	...express as a proportion a verbal statement involving direct variation.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
17110	...solve verbal problems involving inverse variation between the two variables x and y by means of the equation $y = \frac{k}{x}$ where k is a constant.
17111	...express as an equation a verbal statement involving inverse variation.
17112	...recognize and apply the principle: $y = \frac{k}{x}$ expresses inverse variation.
17113	...recognize that the graph on Cartesian Coordinates, of inverse variation is a hyperbola. (not a straight line)
17114	...recognize and apply the principle: inverse variation is involved where one variable increases when the other decreases, while the product of the paired values remains constant.
17120	...solve verbal problems dealing with inverse variation by means of a proportion.
17121	...recognize and apply the principle: the proportion: $\frac{Y_1}{Y_2} = \frac{X_2}{X_1}$ indicates inverse variation involving the ordered pair, (X_1Y_1) and (X_2Y_2)
17122	...express as a proportion a verbal statement involving inverse variation.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
17130	...solve verbal problems involving the joint variation, of the variables x, y, and z, by means of the equation $z = kxy$ where k is a constant.
17131	...express as an equation a verbal statement involving the joint variation of three variables.
17132	...recognize that $z = kxy$ indicates joint variation, i.e. z varies directly as the product of x and y.
17140	...solve verbal problems involving the joint variation of three variables by means of a proportion.
17141	...express as a proportion a verbal statement involving joint variation.
17142	...recognize that joint variation of the two order triple $(X_1Y_1Z_1)$ and $(X_2Y_2Z_2)$, can be expressed as a proportion, e.g. $\frac{Z_1}{Z_2} = \frac{X_1 Y_1}{X_2 Y_2}$
17150	...solve verbal problems involving combined variation of three variables.
17151	...recognize and apply the principle: combined variation can be expressed by the equation: $ZY = KX$ where K is a constant.
17152	...recognize and apply the principle: combined variation of the two ordered triples, $(X_1Y_1Z_1)$ and $(X_2Y_2Z_2)$ can be expressed by the proportion: $\frac{Z_1}{Z_2} = \frac{X_1Y_2}{X_2Y_1}$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
17210	...solve a quadratic equation of the type $ax^2 - c = 0$ for the variable x where a and c are constant, and $a > 0$, $c > 0$
17211	...recognize and apply the principle: $x = +s$, $x = -s$ are solutions to the quadratic equation $x^2 = s^2$
17212	...transform an equation from the form $ax^2 - c = 0$ to the form $x^2 = +\frac{c}{a}$ where $c > 0$, $a > 0$
17213	...recognize that the solution of $x^2 = +\frac{c}{a}$ may be rational or irrational and in the form: $x = \pm\sqrt{\frac{c}{a}}$ where $c > 0$, $a > 0$
17214	...check the solutions by substitution in the original quadratic equation.
17215	...recognize that some quadratic equations of the form: $ax^2 - c = 0$ do not have solutions in the real number system. e.g. $x^2 = \frac{+c}{a}$ when: $c < 0$, $a > 0$
17220	...solve quadratic equations of the form $(ax + b)^2 = c$ for x where $a > 0$, $c > 0$, by taking the square root of both sides of the equations.
17221	...recognize that in the solving of equations of the form $(ax + b)^2 = c$, for x $a \neq 0$, $c > 0$, two linear equations result. i.e. $ax + b = +\sqrt{c}$ and $ax + b = -\sqrt{c}$
17222	...solve the derived linear equations, of the form $ax + b = \pm\sqrt{c}$ resulting from taking the square root of both sides of the quadratic equation.
17223	...check the solutions by substituion, in the original quadratic equation.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
17310	...transform a quadratic polynomial of the form $x^2 + bx$ into a perfect trinomial square by "completing the square".
17311	...recognize and apply the principle: the square of $\frac{1}{2}b$ must be added to $x^2 + bx$ to produce a perfect trinomial square.
17312	...express a perfect trinomial square as the square of a binomial e.g. $x^2 + bx + (\frac{1}{2}b)^2 = (x + \frac{1}{2}b)^2$
17320	...solve a quadratic equation of the form $x^2 + bx + c = 0$ by completing the square (real roots), using the following sequence of operations:
17321	transform the equation $x^2 + bx + c = 0$ into the form $x^2 + bx = -c$ by subtracting c from both sides.
17322	add $\frac{b^2}{4}$ to both sides of the equation, $x^2 + bx = -c$
17323	express the left side as the square of a binomial: $(x + \frac{b}{2})^2$
17324	express the right side as a single term: $\frac{b^2 - 4c}{4}$
17325	write the two linear equations resulting from taking the square roots of both sides.
17326	solve the linear equations.
17327	check the solutions by substitution.
17328	calculate the value of irrational roots correct to the nearest tenth.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
17410	...solve quadratic equations of the form $ax^2 + bx + c = 0$ by completing the square, using the following sequence of operations:
17411	transform the equation so that the coefficient of x^2 is 1 by dividing both sides of the equation by a.
17412	transform the equation by completing the square
17413	solve the resulting equation by taking the square roots of both sides.
17414	check the solutions by substitution.
17415	recognize that the process of completing the square permits the solution of any quadratic equation with real roots.
17420	...solve fractional equations which transform into quadratic equations, using the following sequence of operations:
17421	transform a fractional equation by multiplying both sides of the equation by the LCD of all the fractions.
17422	transform the equation into the form $ax^2 + bx = c$.
17423	solve the resulting equation by completing the square.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
17430	...solve verbal problems leading to quadratic equations, where the roots are irrational, in the following applications:
17431	solve area problems.
17432	solve problems involving right triangles.
17433	solve purchasing problems.
17434	solve motion problems.

Upon completion of this segment, the student should be able to:

MBO CODE

17510 ...derive the quadratic formula: if $ax^2 + bx + c = 0$, then $x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$ using the following sequence of operations:

17511 transform the equation $ax^2 + bx + c = 0$ by dividing both sides of the equation by a.

17512 transform the derived equation by subtracting $\frac{c}{a}$ from both sides of the equation.

17513 transform the resulting equation by adding $\frac{b^2}{4a^2}$ to both sides.

17514 express the resulting equation in the form: $\left[x + \frac{b}{2a}\right]^2 = \frac{b^2 - 4ac}{4a^2}$

17515 transform the resulting equation by taking the square root of both sides, getting two linear equations.

17516 transform the resulting equation by subtracting $\frac{b}{2a}$ from both sides.

17520 ...solve any quadratic equation with real roots by means of the quadratic formula, using the following sequence of operations:

17521 write the quadratic formula.

17522 transform a quadratic equation into the form $ax^2 + bx + c = 0$

17523 find the values of a, b, and c.

17524 substitute the values of a, b, and c into the formula.

17525 calculate the exact values of the solutions.

17526 calculate values to the nearest tenth, when the solutions are irrational.

PMC IX

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
17530	solve verbal problems leading to quadratic equations with irrational roots in the following applications:
17531	solve area problems.
17532	solve problems involving right triangles.
17533	solve purchasing problems.
17534	solve motion problems.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
18110	...recognize certain basic geometric concepts, as follows:
18111	recognize that a point has a position, but no size.
18112	recognize that a line has only length and that it extends indefinitely.
18113	recognize that a plane has only two dimensions and that it extends indefinitely.
18114	recognize that a line segment is a portion of a line with two end points.
18115	recognize that an angle is an amount of rotation between two intersecting lines.
18120	...recognize proper nomenclature for line segments and angles, as follows:
18121	recognize a point named by a capital letter placed near it.
18122	recognize a line segment named by the capital letters of its endpoints.
18123	recognize a line segment named by a small letter placed near the segment.
18124	recognize an angle named by the capital letter at the vertex.
18125	recognize an angle named by a small letter placed inside the angle near the vertex.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
18210	...solve problems involving similar triangles using the following sequence of operations:
18211	recognize that triangles are similar if two pairs of corresponding angles are equal.
18212	recognize that if two triangles are similar, their corresponding sides are in proportion.
18213	write a proportion involving the sides of two similar triangles.
18214	solve the proportion.
18220	...solve verbal problems involving similar triangles, using the following sequence of operations:
18221	recognize the position of the right angles in shadow problems.
18222	recognize that all right angles are equal.
18223	recognize the vertical angles in the diagram for finding the distance across a stream.
18224	recognize and apply the principle: vertical angles are equal.
18225	find the similar triangles in the diagram made from the verbal problem.
18226	write a proportion involving the sides of the similar triangles.
18227	solve the proportion.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
18310	...recognize and apply the principle: a ratio of two numbers is the quotient of one divided by the other.
18311	...recognize and apply the principle: a ratio of two denominate quantities expressed in the same units is the quotient of one divided by the other.
18312	...reduce a ratio to lowest terms.
18313	...recognize that the ratio between two numbers does not determine the value of either number.
18314	...recognize that the order of the numbers in a ratio is important.
18320	...solve verbal problems involving ratios.
18321	...represent two quantities in the ratio a:b as ax and bx .
18322	...represent two quantities where one is a given fraction of the other: e.g. if one is $\frac{2}{3}$ of the other number, represent them as $2x$ and $3x$.

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE

18410 ...define the tangent function of an acute angle in a right triangle as the ratio of the length of the opposite leg to the length of the adjacent leg.

18411 ...recognize that the tangent function refers to a particular angle in a right triangle.

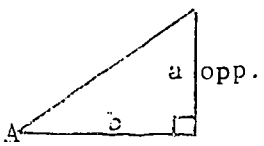
18412 ...recognize that the hypotenuse is the side opposite the right angle.

18413 ...recognize the "opposite leg" is the side opposite the particular acute angle.

18414 ...recognize that the "adjacent leg" is the leg adjacent to the particular acute angle.

18420 ...find a missing leg in a right triangle when given one acute angle and the other leg, using the following sequence of operations:

18421 locate and label the particular angle involved, the hypotenuse, the opposite leg, and the adjacent leg.



18422 write the equation of the form $\tan A = \frac{a}{b}$

18423 substitute values of the angle and the known leg in the equation.

18424 find the value of tan A in the table.

18425 substitute the value for tan A in the equation.

18426 solve the equation for the missing quantity and round off as directed.

18427 recognize that when the denominator involves the variable, a long division will result.

18428 solve an equation of the type:

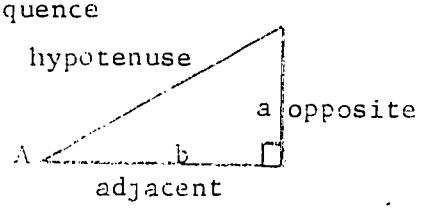
$$.4012 = \frac{24}{x}$$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE

18430 ..find an acute angle of a triangle given the two legs, using the following sequence of operations:



18431 locate and label the hypotenuse, opposite leg, and adjacent leg.

18432 write the equation in the form $\tan A = \frac{a}{b}$

18433 convert the fraction $\frac{a}{b}$ to a four place decimal.

18434 write the equation in the form tan A = decimal.

18435 locate the nearest entry in the tan column of the table.

18436 write the result in the form A = ... expressing the angle to the nearest degree.

18440 ...solve verbal problems involving an angle of elevation or an angle of depression in a properly labeled diagram using the following sequence of operations:

18441 find the horizontal line in the diagram, drawing it in, if necessary.

18442 recognize that the angle of elevation is at the bottom of the diagram rotating upward from the horizontal.

18443 recognize that the angle of depression is at the top of the diagram rotating downward from the horizontal.

18444 recognize that the angle of elevation and the angle of depression are equal since they form alternate interior angles.

18445 choose the right triangle in which either one leg and one acute angle are known, or two legs are know.

18446 write an equation using the tangent function.

18447 solve the equation, and round off properly.

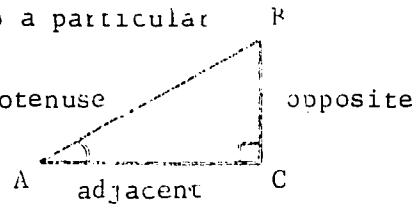
Upon completion of this segment, the student should be able to:

MBO CODE

18510 ...define the sine function of an acute angle in a right triangle as the ratio of the opposite leg to the hypotenuse. The cosine function of an acute angle in a right triangle is the ratio of the adjacent leg to the hypotenuse.

18511 ...recognize that each function refers to a particular acute angle in a right triangle.

18512 ...locate and label hypotenuse, opposite leg and adjacent leg.



18520 ...find a missing leg in a right triangle in which hypotenuse and one acute angle are given, using the following sequence of operations:

18521 locate and label the particular angle, opposite leg, hypotenuse, adjacent leg.

18522 choose the sine function if the opposite leg is to be found

18523 choose the cosine function if the adjacent leg is to be found.

18524 write the equation $\sin A = \frac{a}{c}$ or $\cos A = \frac{b}{c}$

18525 substitute the values for A and C.

18526 find the value of sin A or cos A in the table.

18527 substitute the value in the equation.

18528 solve the equation.

18530 ...find the hypotenuse in a right triangle in which one leg and one acute angle are given, using the following sequence of operations:

18531 locate and label the particular angle, hypotenuse, opposite leg and adjacent leg

18532 choose sine function if opposite leg is known.

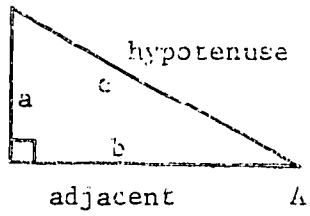
18533 choose cosine if adjacent leg is known.

18534 write the equation $\sin A = \frac{a}{c}$ or $\cos A = \frac{b}{c}$

MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
18535	substitute values for A and a or b in the equation.
18536	find the value of sin A or of cos A in the table.
18537	substitute the value in the equation.
18538	recognize that there will be a long division in the solution of the equation.
18539	solve the equation.
18540	...find an acute angle in a right triangle, given the hypotenuse and one leg, using the following sequence of operations:
18541	locate and label the particular angle, hypotenuse, opposite leg and adjacent leg.
18542	choose sine or cosine depending on whether the known leg is opposite leg or adjacent leg.
18543	write the equation $\sin A = \frac{a}{c}$ or $\cos A = \frac{b}{c}$.
18544	substitute values for a and c or b and c.
18545	divide to change the fraction to a 4 place decimal.
18546	write the equation in the form sin A = decimal or cos A = decimal.
18547	locate nearest entry in the proper column of the table.
18548	write the result, A = ... giving value to the nearest degree.



MBO LIST

Upon completion of this segment, the student should be able to:

MBO CODE	
18550	...solve verbal problems dealing with right triangles where one acute angle and two sides are involved, using the following sequence:
18551	draw diagrams and label with given information.
18552	choose the right triangle in which two quantities are known (not both angles.)
18553	locate and label the particular angle, hypotenuse, opposite leg and adjacent leg
18554	decide which two sides of the triangle are involved.
18555	choose the sine, cosine or tangent function depending on the pair of involved sides.
18556	write the equation using the proper function.
18557	solve the equation, rounding off properly.

DOCUMENT RESUME

ED 075 202 24 SE 015 856

TITLE Programmed Math Continuum, Level One, Algebra, Volume 1.
INSTITUTION New York Inst. of Tech., Old Westbury.
SPONS AGENCY Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO BR-8-0157
PUB DATE [73]
CONTRACT OEC-0-8-080157-3691(010)
NOTE 195p.

EDRS PRICE MF-$0.65 HC-$6.58
DESCRIPTORS *Algebra; *Computer Assisted Instruction; Curriculum; Individualized Instruction; *Instruction; Instructional Materials; Mathematics Education; Programed Instruction; Programed Materials; *Programed Texts; *Secondary School Mathematics

ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. Volume 1 includes general instructions for working with this system, and then covers the following topics in algebra: number line, comparing numbers, sets and set membership, and subsets. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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EDUCATION & WELFARE
OFFICE OF EDUCATION
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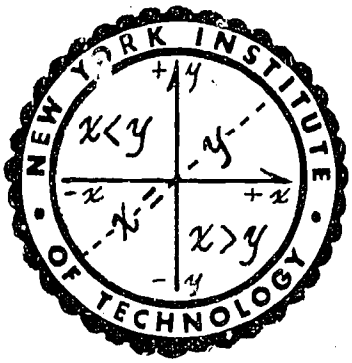


VOLUME

1

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

FILMED FROM BEST AVAILABLE COPY

ED 075202

P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME I

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 1

TABLE OF CONTENTS

COVER	PAGE
PREFACE	A
TABLE OF CONTENTS	B
SYLLABUS	C
READING ASSIGNMENT	D
HOMEWORK ASSIGNMENT	E
GENERAL INSTRUCTIONS	F

IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
Introduction Material		1 - 17
1	2	18/1
1	3	68/1
1	4	110/1
1	5	149/1

VOLUME 1

This volume covers the following material

as shown in this excerpt from the Syllabus.

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Introduction: general instructions			
2	Representing numbers on a line	1-1 (1-6)	1-3 (1-1) (1-2)	3-2 4-4 4-6
3	Comparing numbers, = or	1-2 1-3	1-4	2-5 3-1
4	Membership in a set Kinds of sets	1-4 1-5 1-6	2-1 2-2 2-6 3-2 5-10	1-2 (1-3)
5	Subsets	1-7	2-3	1-2

READING ASSIGNMENT

VOLUME 1

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1		
2	1 (16)	5 (17)
3	5	10
4	10	17
5	18	19

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

HOMEWORK ASSIGNMENT

VOLUME NO. 1

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	3 - 4	1 , 2 , 3 , 4	01215
2	4	7 , 8 , 9 , 10	01215
3	17	1 , 2 , 3 , 4	01230
4	17	5 , 6 , 7 , 8	01230
5	17	9 , 10 , 11 , 12	01230
6	6	2 , 3 , 4 , 6	01310
7	7	11 , 12 , 13 , 15	01331
8	9	1 , 2 , 6 , 7	01310 , 01320
9	9	12 , 14 , 18 , 21	01310 , 01320
10	10	25 , 26 , 36 , 38	01330
11	12	27 , 28 , 29 , 30	01410
12	14	1 , 2 , 3 , 4	01421 , 01430
13	15	6 , 7 , 8 , 9	01421 , 01430
14	15	11 , 12	01421 , 01430
15	15	14 , 15	01421 , 01430
16	19	1 , 2 , 3 , 4	01512
17	19	5 , 6 , 7 , 8	01510
18	19	9 , 10	01520
19	19	11 , 12	01520
20	19	13 , 14	01520

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

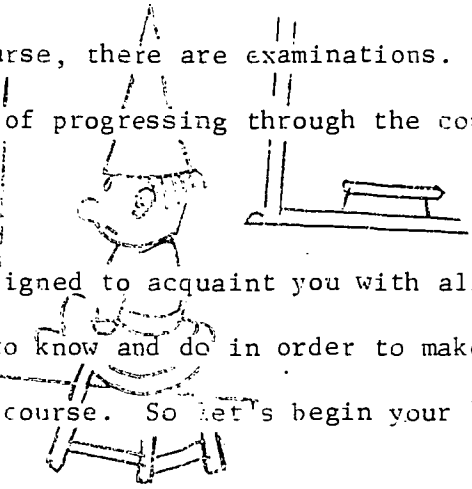
If your choice is not correct you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is correct then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

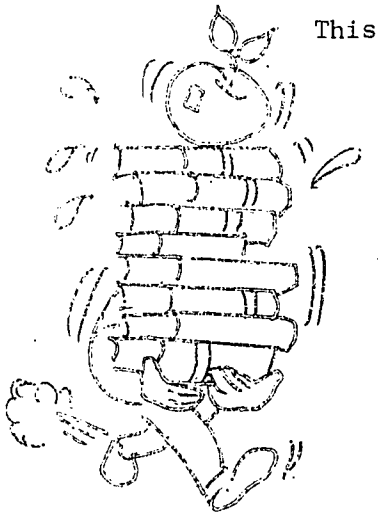
1. This is a course in ELEMENTARY ALGEBRA which is not too different from other subjects you have studied. There is a textbook; there are homework assignments; you will have a notebook for the notes you must take down; and, of course, there are examinations. However, you will find the method of progressing through the course quite different.



This first section has been designed to acquaint you with all of the things that you will have to know and do in order to make your way satisfactorily through the course. So let's begin your "guided tour":

1.1 The work of the entire course is divided into 18 parts, called VOLUMES (VOL). Each Volume consists of 5 parts, called SEGMENTS (SEG). Every VOL. deals with material which, under normal circumstances, would be covered in a two week period. However, if you are permitted to proceed at your own pace or on your own time schedule, you could probably finish a Volume in much less time.

This book, which you are now reading, is called a STUDY GUIDE, (SG). This is VOL. 1 of the SG; this portion of VOL. 1 is called SEG 1.



In addition to the SG, you will need:

(1) The basic text: Modern Algebra Book 1
By Dolciani, Berman and Freilich
Houghton Mifflin, 1965

(2) The remedial text: Comprehensive Ninth Year Mathematics
by Dressler
Amsco, 1966

(3) A Notebook: ...Where you will record specific items for further study.

And something new:

(4) Punch cards-

(5) Program control- You'll learn about these in a few minutes.

ABOUT THE PUNCH CARD:

Your progress through this course will be followed very carefully by the computer. It will record, for example, which questions you found easy and which caused you some trouble. In fact, it will keep track of all of the students in many ways and will analyze their various performances in such a way that your teacher will be able to know immediately just where extra attention should be directed to help all the students do as well as they can.

The way the computer finds out about you is through the PUNCH CARD. Those holes all mean something.

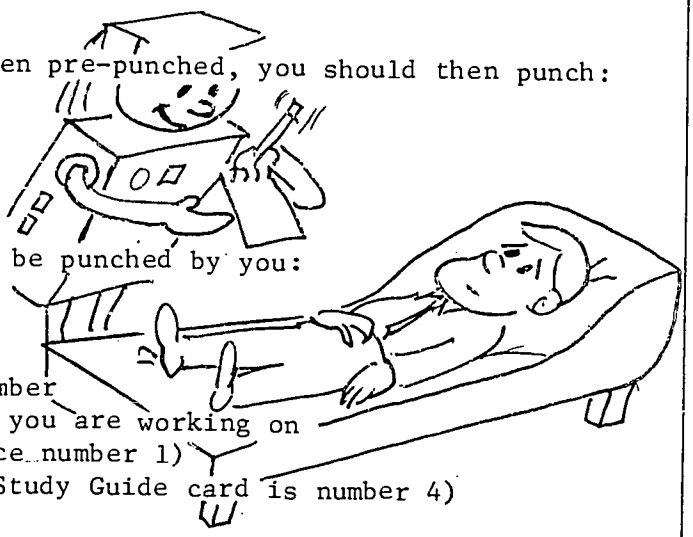
When the punch card is placed in the PROGRAM CONTROL, with the proper ANSWER MATRIX, you will see that as you punch a hole in response to a question in the SG, a light will go on in the PROGRAM CONTROL, the page to which you should turn will then be illuminated. So you see, the PUNCH CARD is very important.

HOW TO PREPARE THE PUNCH CARD:

(1) Take your PUNCH CARD and print: your LAST NAME, and FIRST NAME above the word "date."

If the card has not already been pre-punched, you should then punch:

(2) Your Course student number
(3) This Course number
(4) Your Student I.D. number



The following information must be punched by you:

(5) Month number
(6) Day number
(7) Last two digits of the Year Number
(8) The Sequence number of the SEG you are working on (VOL. 1, SEG. 1 has Sequence number 1)
(9) The type of Input card. (The Study Guide card is number 4)
(10) The Volume Number.
(11) The Segment Number.

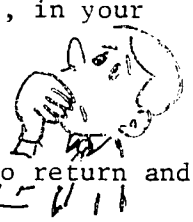
NOTE: All of this information must be entered; if not, the computer will reject the card.

Punch the necessary information, being S L O W and C A R E F U L so as to punch only those holes which you intend to punch. Remember, you cannot make corrections, once you have punched the wrong hole. Check to see that the program control device shows that it has the answer matrix for volume 1, segment 1.

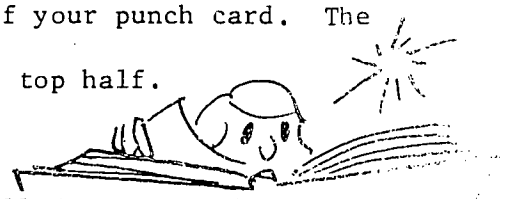
1.3 You are now ready to see how the study guide will work. Read question 1, and decide on an answer. Then examine the four choices offered.

Question 1: What is the product of 6 and 2 ?
(A) 8 (B) 12 (C) 4 (D) 3

Let us suppose that you believe the correct answer is 8, that is choice A. You would use the stylus to punch the hole on line 1 at A. Please do this now. As you see, a number appears on the program control device, the number $\frac{7}{2}$. This means turn, in your study guide, to page 7, bottom half. Do this now.



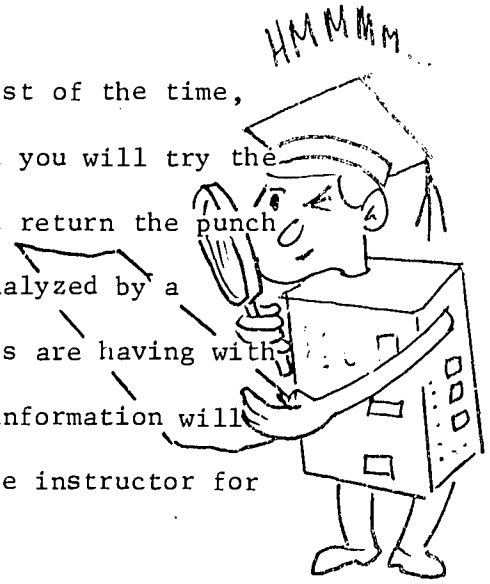
1.4 You see how the program control has directed you to return and try again, as well as telling you why your choice was not correct. Of course, looking back at question 1, you know that choice B is the correct answer. Punch B on line 1 of your punch card. The program control shows $\frac{9}{1}$. Turn to page 9, top half.



1.5 The procedure of this study guide is called programmed instruction. The purpose of the study guide is what its name indicates; it is a guide to your study. This is not a test in disguise. As you answer questions in the study guide, you should go back to your textbook, notebook, or material in the study guide itself, if you need to look up something which you are not sure of. The effect is that you will go over all the material of the course, in easy stages, with special emphasis on the more important points. The important points which the course contains are called M.B.O.'s, and each question you meet is designed to give you practice in dealing with one of these points.

viations are used in this course. I'm sure you are curious M.B.O."

The letters "MBO" , stand for "Measurable Behavioral Objectives". They are the ideas, definitions and skills that are to be taught, hence they are called "objectives". Since they are stated in a way that questions can be asked about them, they are called "measurable". Because you have to demonstrate that you understand these, and can use them they are called "behavioral".



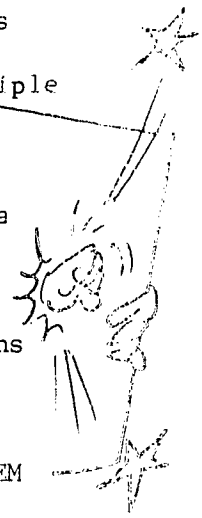
You will get the right answer on the first trial most of the time, but if you make a mistake you will find out why and you will try the problem again. At the end of the segment, you will return the punch card to your instructor. The cards will then be analyzed by a computer to find out the difficulties which students are having with the material of the course. The analysis of this information will indicate what remedial work should be planned by the instructor for the benefit of the students.

1.6 In general, at the beginning of a segment, you are directed to read a specific portion of Dolciani. In reading this, you MUST work out, on paper, any illustrative problems which the text contains, so that you are sure you can do the problem being demonstrated. Remember that the author may not have written every step as you would, so work on your own paper and try to follow the work shown in the book. In addition to the reading assignment in the text, there may be some notes in the study guide itself. You should enter in your notebook those items which seem to be of special importance, or those items which you may be directed to write down.

(continued on page 5)

Then, before you proceed with the segment, the instructor will insert the ANSWER MATRIX for your particular segment into the program control device. You will complete the necessary information items on the CARD, insert it into the program control device, and then punch the necessary information into the card, using your stylus.

Now you proceed to the questions. You must take the questions in order. You'll have no choice, since the pages have been scrambled. That is, after question 1, you do not go ahead to the next page in order, but to whatever page the program control directs you. Each question will offer four choices, of which one, and only one, is correct. The three other choices, called distractors, are answers which you might well arrive at if your understanding of the principle were not completely correct. For every choice which you punch on your card, the program control directs you to a specific half of a particular page. Here you find a statement of why you were wrong (if you were), or a statement that you were right with instructions to proceed. This half-page is called a REM, (this is short for REMEDY) and the program control will direct you to a particular REM for any answer you choose for a question in the study guide. It should be of some help to you.



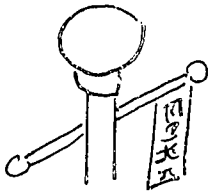
I

1.7 After this page, you must follow the instructions given to you by the program control. You must not go page by page, since the material is not in order.

Please remember, DO NOT GUESS:

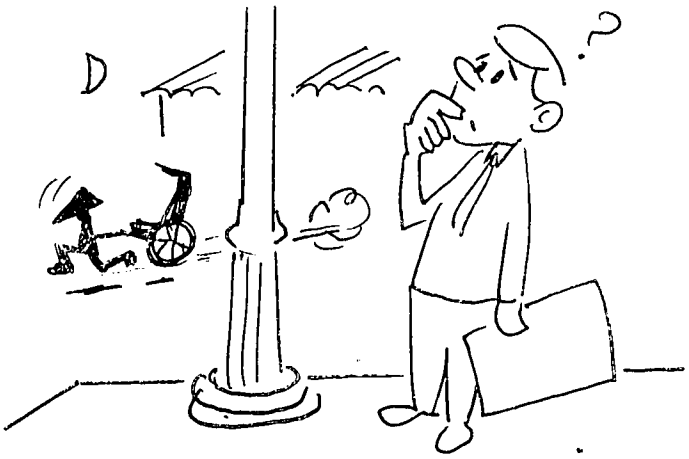
This guide is intended to help you to learn and practice correctly.

You should go back and reread pages 1 through 5 as often as necessary. When you have considered the question, punch your answer on the punch card and the program control will tell you where to turn next.



1.8 IF YOU " LOST YOUR PLACE "

If you lose your place for any reason, insert your stylus in the last hole that you had punched on your card. This will not affect any record that's being kept. The program control will put you back on the track.



Now turn to page $\frac{?}{1}$

Question 2: What type of instructional device is this book?

(A) M.B.O. (B) punch card

(C) answer matrix (D) study guide

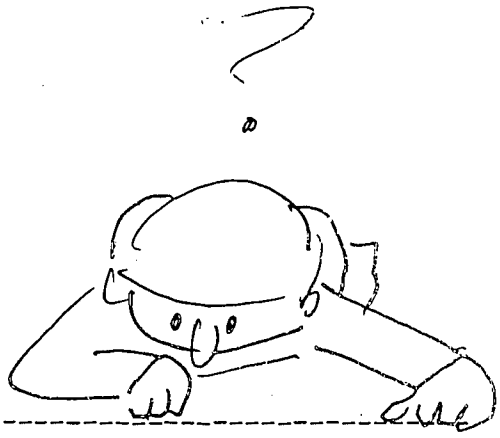
When you have decided on your answer, punch the letter of your choice on line 2 of the punch card, and follow the directions of the program control.

You have found the sum, but you were asked for the product, not the sum. The product of two numbers is the result obtained by multiplication.

Return to page 3, section 1.4

8
1

If you will refer to page 1, section 1.1, you will find that the study guide contains 18 volumes. You are correct.



Now proceed to question 4 which follows.

8
2

Question 4:

How many segments does each volume contain?

(A) 5

(B) 10

C) 18

D) 90

This choice is correct.

Proceed to page 3, section 1.5

The REM is the half page to which the program control refers you. What you are reading at this moment is a REM. In the study guide there are from 2 to 4 REMS for each question, and each punch card handles many questions. Therefore, this choice is not correct.

Return to page 12/2 and try question 5 again.

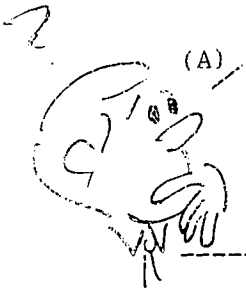
10
1

This choice is correct.

Now proceed to question 3 which follows:

Question 3

How many volumes is the study guide composed of?



(A) 5 (B) 10 (C) 18 (D) 90

10
2

The number of segments in a volume is discussed on page 1. If you reread that page, section 1.1, you will find that the correct number of segments is not the one you have chosen.

Return to page 8/2 and try question 4 again.

I

11/1

The punch card is the rectangular card in which you have punched choice B on line 2. Since it is not the book you are now [illegible] is incorrect.

Return to page 7/1 and try question 2 again.

11/2

Since the course contains 18 volumes, and each volume contains 5 segments, you should be able to find out how many segments the entire course contains. Since the guide tells you that you will return the punch card to the instructor at the end of each segment, and that you will insert a new card into the program control device for each segment, it follows that you will need a card for each segment. Therefore, this choice is not correct.

Return to page 15/2 and try question 6 again.

12
1

According to section 1.1 on page 1 of the study guide, each volume contains 5 segments. Therefore, this choice is correct.

Proceed to question 5 which follows.

12
2

Question 5:

You will need a new punch card for each _____.

Which word should be inserted in the blank?

(A) segment

(B) question

(C) REM

(D) M.B.O.

$\frac{13}{1}$

The number of volumes in the course is discussed on page 1 of the study guide. the study guide covers the entire course. Therefore, this choice is not correct.

Return to page $\frac{10}{1}$ and try question 3 again.

$\frac{13}{2}$

The answer matrix is actually a code to the position of the REM's in the study guide, since the pages have been scrambled. The answer matrix is placed in the program control device, and the only way you can get to read any part of it, is to punch a hole with your stylus.

This choice is not correct.

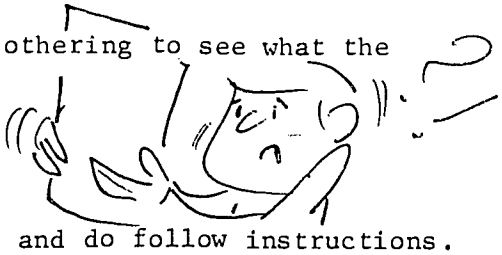
Return to page $\frac{7}{1}$ and try question 2 again.

I

14/1

You have done some correct arithmetic, but I'm afraid you forgot the meaning of the word "product." The product of two numbers is the result of multiplying them.

Return to page 3, section 1.4, and please follow the study guide. You went ahead and chose an answer without bothering to see what the guide had to say.



Remember, return to section 1.4, page 3, and do follow instructions.

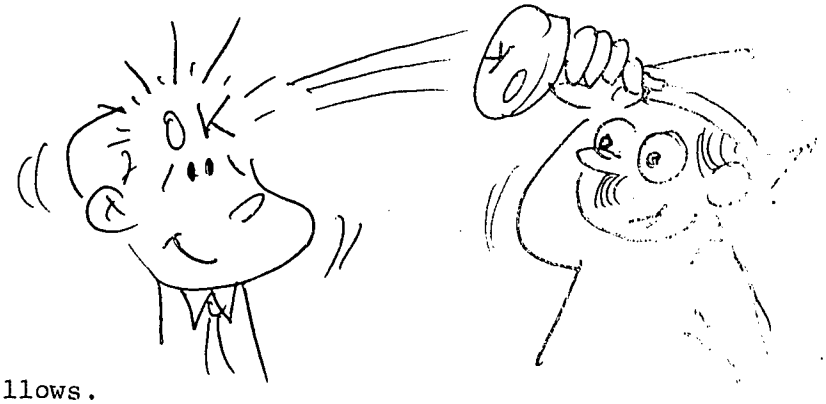
14/2

An M.B.O. is one of the important points which you will have learned as a result of studying this course. The meaning of "M.B.O." was discussed on page 4. It is true that the questions you meet are related to M.B.O.'s, but you do not use a punch card for them.

This choice is not correct.

Return to page 12/2 and try question 5 again.

On page 3, in section 1.5, you are told that you will return your punch card to the instructor at the end of the segment. On page 4, you are instructed to insert a punch card into the program control device before starting a segment. Therefore, you need a punch card for every individual segment.



This choice is correct.

Now proceed to question 6 which follows.

Question 6

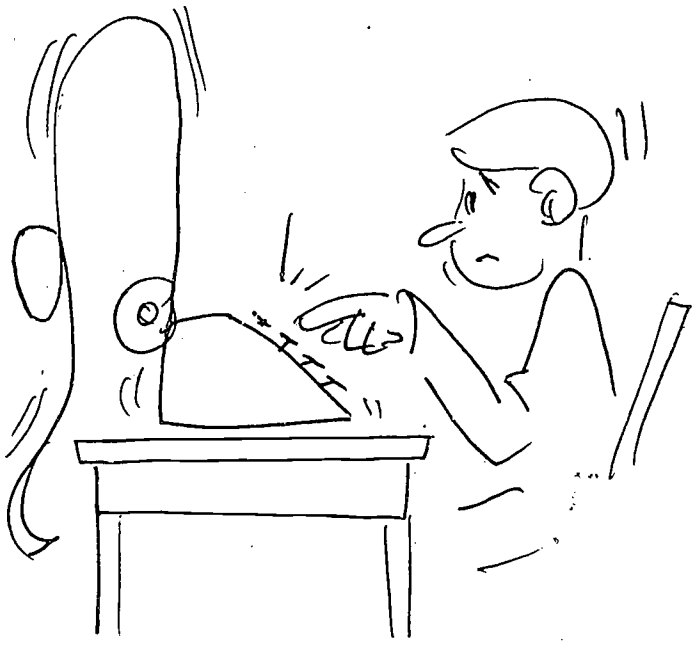
If we assume that no punch cards will be damaged, how many will a student require for the entire course?

(A) 5

(B) 10

(C) 18

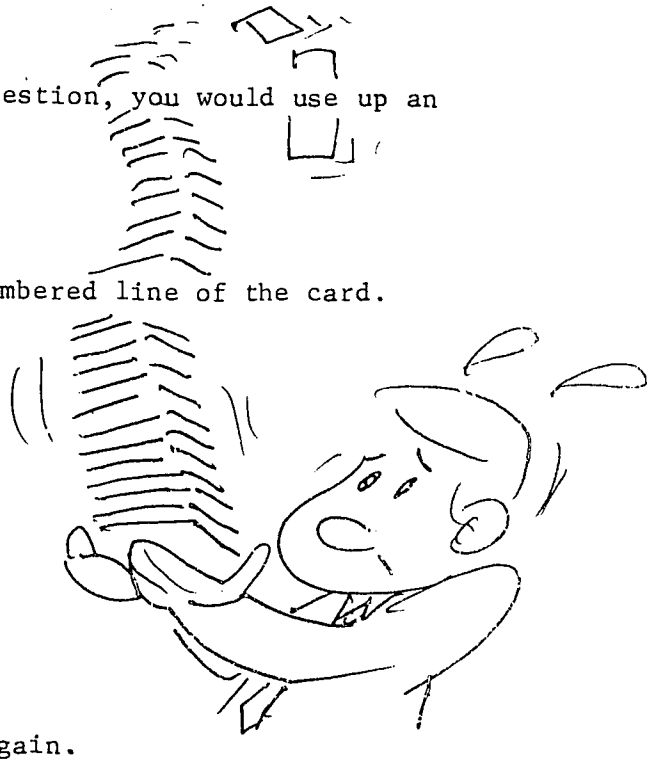
(D) 90



16
1

If you needed a new punch card for each question, you would use up an extrememly large number of cards.

Each question is answered on the proper numbered line of the card.



Return to page 12/2 and try question 5 again.

16
2

An M.B.O. is one of the important points which you will have learned as a result of studying this course. It is not the name of the book.

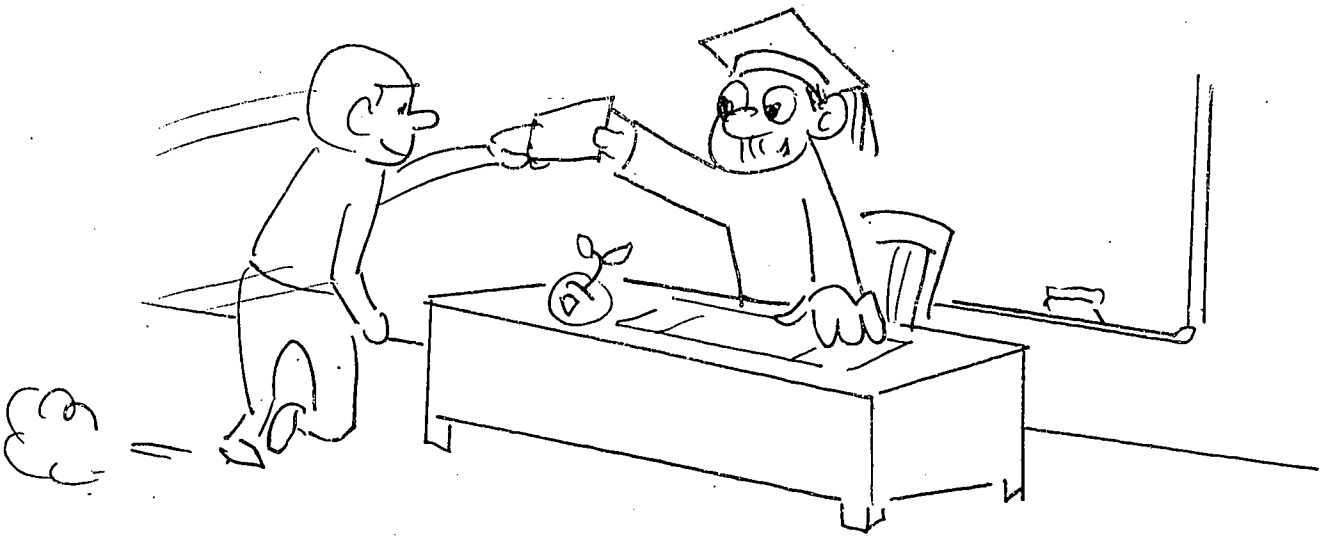
Return to page 7/1 and try question 2 again.

Since each segment calls for a new punch card, and there are 5 segments to a volume, a student needs 5 cards for each volume. There are 18 volumes, hence he needs 90 cards.

Therefore, this choice is correct.

You have now completed this segment. Before going on to Segment 2, be sure you have understood exactly what you are supposed to do. If you have punched more than one hole on a line of the punch card, except for line 1 where you should have punched 2, you have made an error. That is perfectly all right, provided that you learn from it. If you punched any hole without meaning to, you will have to practice being more careful.

When you are ready, ask your instructor for the answer matrix for Segment 2 and for a fresh punch card, and go ahead.



Volume 1 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 2	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 1	(Volume Number)
	66	and	68	0 2	(Segment Number)

PRELIMINARY NOTES

Some of the streets of a certain city are numbered consecutively from 1 to 10. Let's assume that you happen to live on street number 3, while a friend of yours lives on street number 7. If you walk from your house to your friend's house, you will pass streets number 4, 5, and 6. Now each of these streets is a place. We say, in mathematics, that with every number we can associate a point (place) on a number line. In this segment, this simple, but very important idea, will be developed and studied in more detail.

Your READING ASSIGNMENT for this Segment is pp. 1 - 5.

You will now be asked a series of questions to draw your attention to the more important points.

$\frac{18}{2}$

Recognize which of the following numbers are natural numbers.

$$0 \quad , \quad 3 \quad , \quad \frac{1}{8} \quad , \quad 1\frac{1}{8} \quad , \quad \frac{8}{2}$$

Choose the letter which has only natural numbers, selected from the above set, written next to it.

(A) 0 , 3

(B) $1\frac{1}{3}$, 3 , $\frac{8}{2}$

(C) 3 , $\frac{8}{2}$

(D) 0 , 3 , $\frac{8}{2}$

Whole numbers are the natural numbers with one addition. Zero is not a natural number, but it is a whole number. The number

$$\frac{2}{1} = 2 \text{ , and } \frac{3}{1} = 3$$

Thus, any natural number written with a denominator of 1 is a whole number.

Finally, $\frac{1.1}{1.1} = 1$ since any number, other than zero, divided by itself is equal to one.

Please go on to question 4 , below.

Question 4

Apply the proper principle and determine which of the following statements are true or false. Select the letter which labels the correct combination of truth values for your answer.

I. Every natural number can be written as a fraction.

II. There is a largest natural number.

III. Every fraction names a whole number.

(A) I is true
II is true
III is true

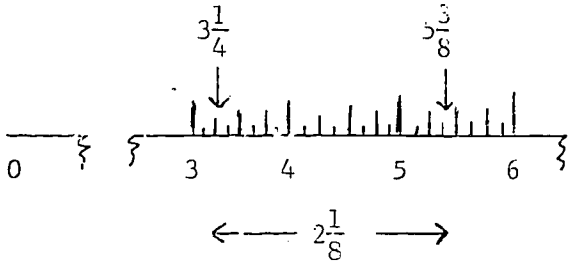
(B) I is false
II is false
III is true

(C) I is true
II is false
III is false

(D) I is true
II is false
III is true

I

Your answer is correct.



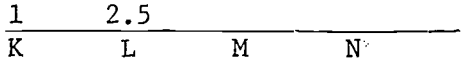
Please go on to question 8 below.

$\frac{20}{2}$

Question 8

Apply the principle and select the letter which correctly answers this question:

Points K , L , M , N are equidistant on the number line drawn below: what is the coordinate of N ?



(A) 5.0

(B) 10.0

(C) 7.5

(D) 5.5

Sorry, but your choice is not correct.

Counting numbers and natural numbers are the same.

Zero is not a counting number.

Please return to page $\frac{33}{2}$ and try question 2 again.

$\frac{21}{2}$

Did you mark off " tenths " on your number line?

Sorry, your answer is incorrect.

Please go back to page $\frac{35}{2}$ and try question 5 again.

I

If QR is twice as long PQ we must first find the length of PQ. By subtracting P's coordinate from that of Q , we find PQ to be 4.75 units long. Then QR is twice that, or

(4.75) or units long.

Thus, the coordinate of R is found by adding:

	9.25	(the coordinate of Q)
+	4.50	(the distance from Q to R)
	13.75	the coordinate of point R.

Your answer is correct.

Please go on to question 10 , below.

22/2

Question 10

Apply the proper principle and select the letter which correctly answers this question:

What is the coordinate of point X on the number line drawn below if YZ is $\frac{1}{3}$ as long as XY ?

X	Y	Z
	10.2	12.9

(A) 2.1

(B) 8.1

(C) -1.3

(D) 3.4

The natural numbers are 1 , 2 , 3 . . . . and so on.

Zero is not a natural number.

Please return to page 18/2 and try question 1 again.

23/2

We don't agree.

Is there a largest natural number? Suppose you write down the largest natural number you can:

We can always make it larger by adding 1 to it.

Thus, there is no largest natural number.

Please return to page 19/2 and try question 4 again.

I

Numbers such as

$\frac{1}{2}$ and $\frac{1}{3}$ are called fractions.

Fractions are not natural numbers.

Please return to page 36 and try question 3 again.

$\frac{24}{2}$

Did you forget the meaning of " between " ?

Between means not including the end points.

For example,

T , U , V and W are between S and X , or in other words, there are four letters between S and X in the alphabet.

Return to page $\frac{38}{2}$ and try question 11 again.

Did you draw a number line to help you with this problem?

Find point 7 , count 4 units to the left, and then another half unit to the l

lease return to page $\frac{31}{2}$ and try question 3 again

$\frac{25}{2}$

Draw a number line and mark off segments of $\frac{1}{4}$ units. Number all the integral values, then find

$2\frac{3}{4}$ and $8\frac{1}{2}$ on that line.

What are the natural numbers between them?

Please return to page $\frac{43}{2}$ and try question 12 again.

I

The number $1\frac{2}{3}$ is a mixed number, not a counting number. Mixed numbers as well as zero and decimal numbers are not considered " natural " numbers or counting numbers.

Please return to page $\frac{38}{2}$ and try question 2 again.

$\frac{26}{2}$

It's very easy to be careless!

Re-read the question. Does it say that R is double Q ? Or, does it rather say that QR is twice as long as PQ ?

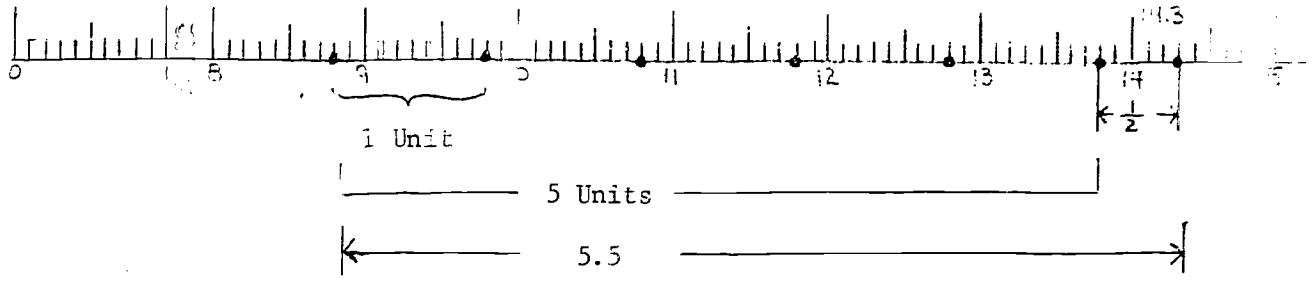
How would you determine how long PQ is?

Please return to page $\frac{40}{2}$ and try question 9 again.

After drawing the number line, we can mark off divisions of " tenths ".

Now we can begin at 8.8 and count 5.5 places.

Your answer is correct.



Please go on to question 7 below.

Question 7

Apply the proper principle and select the letter which correctly answers the question:

Two points $3\frac{1}{4}$ and $5\frac{3}{8}$ are chosen on a one-foot ruler; how far apart are these points?

(A) $2\frac{1}{8}$

(B) $2\frac{1}{4}$

(C) $1\frac{1}{8}$

(D) $12\frac{5}{8}$

We don't agree.

The number $1\frac{1}{8}$ is called a mixed number. It consists of a natural number and a fraction.

Please return to page $\frac{18}{2}$ and try question 1 again.

$\frac{28}{2}$

Sorry, we don't agree.

Can you represent the fraction one-third as a whole number?

Please consider your choice.

Return to page $\frac{19}{2}$ and try question 4 again.

$\frac{}{1}$ is considered a whole number since it is written with a denominator of 1 .

Any number divided by one equates itself. However, if the numerator is one such as the number $\frac{1}{2}$, it is a fraction; not a natural number.

Please return to page $\frac{36}{2}$ and try question 3 again.

$\frac{29}{2}$

It's easy to be careless.

How large is the interval between points K and L ?

Please return to page $\frac{20}{2}$ and try question 8 again.

I

Good! Your answer is correct.

When we are asked for numbers " between " two other numbers, the end-points are not considered.

Please go on to question 13 below.

Question 13

Apply the proper principle and find how many natural numbers there are between .7 and 8.7 . Select the letter which labels the correct statement.

(A) 8

(B) 7

(C) 6

(D) 9

You correctly omitted 0 and $1\frac{1}{3}$ which are not counting numbers. However, you included decimal numbers, which also are not " natural " or counting numbers.

Please return to page $\frac{33}{2}$ and try this question again.

$\frac{31}{2}$

Your answer is correct.

Please go on to question 5 which follows:

Question 5

Draw a segment of the number line for reference and label it appropriately. Apply the proper principle and find the coordinate of the point that is $4\frac{1}{2}$ units to the left of point 7 .

Select the letter which labels the correct statement.

(A) $3\frac{1}{2}$ (C) $1\frac{1}{2}$

(B) $2\frac{1}{2}$ (D) 3

I

What interval did you use on your number line?

Did you make each point $\frac{1}{8}$ of a unit away from the previous one?

$3\frac{1}{4}$ then, can be considered $3\frac{2}{8}$

Please return to page $\frac{27}{2}$ and try question 7 again.

$\frac{32}{2}$

It's very easy to be careless!

Please re-read the directions in question 10.

How long is segment YZ? What relationship does this have in our problem? If you find YZ, then find the length of XY. Then can you see how to find the coordinate of X?

Please return to page $\frac{22}{2}$ and try question 10 again.

This is correct.

Although $\frac{8}{2}$ looks like a fraction, it actually is a numeral for 4 and is, therefore, a natural number. The natural numbers are the " counting numbers ".

Please go on to question 2 below.

Question 2

Recognize which of the following numbers are counting numbers.

$$0 \ , \ 1 \ , \ 1.1 \ , \ 1.2 \ , \ 2 \ , \ 1\frac{2}{3}$$

Select the letter which has only counting numbers selected from the above set written next to it.

(A) 1 , 1.1 , 1.2 , 2

(B) 0 , 1 , 2

(C) 1 , $1\frac{2}{3}$, 2

(D) None of these other choices are correct.

We disagree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page $\frac{36}{2}$ and try question 3 again.

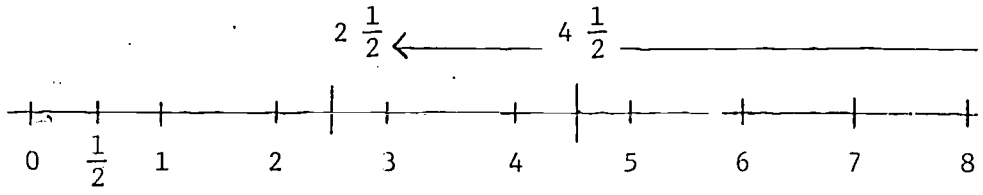
$\frac{34}{2}$

Please check your arithmetic!

Segment PQ's length can be determined by subtracting the coordinate of P from that of Q.

Please return to page $\frac{40}{9}$ and try question 9 again.

After drawing a number line, we can locate the point, 7 , and then count four units to the left, arriving at 3 . Now go one-half a unit farther to the left, $2\frac{1}{2}$.



Your answer is correct.

Please go on to question 6 below.

Question 6

Draw a segment of the number line and label it appropriately. Apply the proper principle and find the coordinate of a point that is 5.5 units to the right of point 8.8 . Select the letter which labels the correct statement.

(A) 15.3

(B) 14.3

(C) 3.3

(D) 13.3

I

Zero, the mixed number, $1\frac{2}{3}$, and the decimals, 1.1 and 1,2 are not natural numbers.

Since each of the other answer choices contained at least one of these along with the counting numbers, your answer choice is correct.

Please go on to question 3 below.

Question 3

Recognize which of the following numbers are whole numbers.

$$0 \ , \ \frac{2}{1} \ , \ \frac{1}{2} \ , \ \frac{3}{1} \ , \ \frac{1}{3} \ , \ \frac{1.1}{1.1}$$

Select the letter which has only whole numbers selected from the above set, written next to it.

(A) $0 \ , \ \frac{2}{1} \ , \ \frac{3}{1} \ , \ \frac{1.1}{1.1}$

(B) $\frac{2}{1} \ , \ \frac{1}{2} \ , \ \frac{3}{1} \ , \ \frac{1}{3}$

(C) $0 \ , \ \frac{1}{2} \ , \ \frac{3}{1}$

(D) None of these other choices are correct.

Sorry, we don't agree.

Did you count from the point $3\frac{1}{4}$ to the $5\frac{3}{8}$?

It seems that you counted $3\frac{1}{4}$ beyond $5\frac{5}{8}$.

Please return to page $\frac{27}{2}$ and try question 7 again.

$\frac{37}{2}$

It's very easy to be careless.

The fraction $\frac{22}{7}$ does not symbolize the interval between 7 and 22 .

What does a fraction sygnify?

Please return to page $\frac{58}{2}$ and try question 15 again.

I

segment YZ is 2.7 units in length. Segment XY is three times that; so, XY is 8.1 units in length. Since X is to the left of Y on the number line, it's coordinate must be less. Therefore, we must subtract the length of XY from the coordinate of Y to find the coordinate of X.

Since Y is 10.2 when we subtract the length of XY the coordinate of X is 2.1 .

$$\begin{array}{r} 10.2 \\ -\ 8.1 \\ \hline 2.1 \end{array}$$

Your answer is correct.

Please go on to question 11 below.

Question 11

Apply the proper principle and find how many natural numbers there are between 10 and 15 . Select the letter which labels the correct answer.

(A) 4

(B) 5

(C) 6

(D) An infinite number.

I

Good! Your answer is correct.

The first natural number after .7 is 1. Before you reach 8.7 , you have counted through 1 , 2 , 3 , 4 , 5 , 6 , 7 , and 8. Thus, there are eight natural numbers between .7 and 8.7 .

Please go on to question 14 below.

Question 14

Apply the proper principle and find a number between 1 , 4 and 1.5 . Select the letter which labels the correct statement.

(A) 2.9

(B) 1.14

(C) 1.54

(D) 1.45

I

You can find the distance between K and L by subtracting their coordinates Since each of the points are equidistant, the coordinate of N is

$$2.5 + 1.5 + 1.5$$

Coordinate of N is 5.5 .

Your answer is correct.

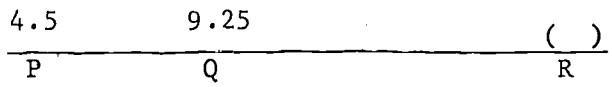
Please go on to question 9 below.

$\frac{40}{2}$

Question 9

Apply the proper principle and select the letter which correctly answers this question:

What is the coordinate of point R on the number line drawn below if QR is twice as long as PQ ?



(A) 27.75

(B) 13.75

(C) 18.75

(D) 18.50

I

number is made up of the natural number which is the largest possible quotient and a remainder. As a mixed number $\frac{22}{7}$ is expressed as $3\frac{1}{7}$.

Please return to page $\frac{58}{2}$ and try question 15 again.

$\frac{41}{2}$

We're sorry, but we disagree.

$\frac{3}{4}$ is not between 1 and $\frac{3}{2}$. Did you draw a number line to help you?

Please return to page $\frac{52}{2}$ and try question 19 again.

I

Between $1\frac{1}{4}$ and 2 exclusive, there are two marks; one at $1\frac{1}{2}$ the other at $1\frac{3}{4}$. Between 2 and 3 inclusive, there are five marks. There are two more at $3\frac{1}{4}$ and $3\frac{1}{2}$. Thus, there are

$$2 + 5 + 2$$

or nine marks in all.

On looking at it in another way,

$$1\frac{1}{4} = \frac{5}{4}$$

and

$$3\frac{3}{4} = \frac{15}{4}$$

Between 5 fourths and 15 fourths there are 9 fourths.

Your answer is correct.

Please go on to question 17 below.

Question 17

Apply the proper principle and find how many whole numbers there are between $\frac{72}{6}$ and $\frac{6}{2}$ inclusive, that are also divisible by 3.

Select the letter which labels the correct answer.

(A) 2

(B) 3

(C) 4

(D) None of these.

I

Right! There are four natural numbers between 10 and 15 .

The idea of "between" means that the end points are not to be part of the segment.

Thus, we only consider the numbers,

11, 12, 13, and 14

Please go on to question 12 below.

Question 12

Apply the proper principle and find how many natural numbers there are between

$2\frac{3}{4}$ and $8\frac{1}{2}$.

Select the letter which labels the correct answer.

(A) 8

(B) 7

(C) 6

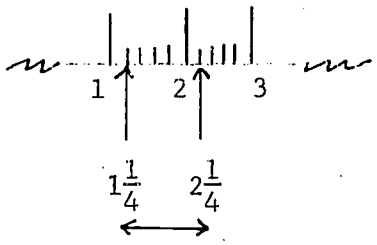
(D) 5

It's very easy to be careless. You were not asked for the sum of the coordinates, you were asked to find a number between 1.4 and 1.5 .

Please return to page $\frac{39}{2}$ and try question 14 again.

$\frac{44}{2}$

Sorry, we disagree. Did you sketch a number line to help you with this problem?



Now we're assuming that you can find the midpoint.

Please return to page $\frac{56}{2}$ and try question 18 again.

It seems as though $\frac{1}{5}$ would be the midpoint between $\frac{1}{4}$ and $\frac{1}{6}$ because 5 is the midpoint between 4 and 6.

However, it is not true. The point midway between two points could be considered as the "average" of the two points.

Please return to page $\frac{62}{2}$ and try question 20 again.

$\frac{45}{2}$

This is only one of three choices which you were offered which have the value 14.

Therefore, this choice is not correct.

Return to page $\frac{69}{1}$ and try question 1 again.

I

Please return to page $\frac{40}{2}$ and try question 9 again.

$\frac{46}{2}$

Did you draw a number line to help you with this question? Find the point corresponding to .7 ; find the spot that corresponds to 8.7 and then count the number of natural numbers that fall between your two points.

Please return to page $\frac{30}{2}$ and try question 13 again.

I

3 and · 4 .

Your answer is correct.

Please go on to question 16 below.

Question 16

Apply the proper principle and select the letter which correctly answers the following question:

A 6-inch ruler has a mark for every one-quarter of an inch. How many of these marks are there between $1\frac{1}{4}$ and $3\frac{3}{4}$?

(A) 12

(B) 11

(C) 10

(D) 9

I

$\frac{48}{2}$

What is the value of 4.5 + 3 ? And what is the value of 3 + 4.5 ? If you still get his choice, you must have made a mistake in addition since this choice is not correct.

Please return to page $\frac{60}{2}$ and try question 2 again.

I

Please return to page $\frac{38}{2}$ and try question 11 again.

Let us ask you a similar question.

Is $1.14 between $1.40 and $1.50 ?

Please return to page $\frac{39}{2}$ and try question 14 again.

I

Please return to page $\frac{58}{2}$ and try question 15 again.

$\frac{50}{2}$

Sorry, we don't agree. Did you forget the meaning of "inclusive"? Suppose you correct the given fraction to whole numbers and then list the numbers between them on the number line. The re-read the rest of the question before you make your answer choice.

Please return to page $\frac{42}{2}$ and try question 17 again.

I

Please return to page $\frac{69}{1}$ and try question 1 again.

You are probably used to doing subtractions with one number written under the other. If you write it this way:

$$\begin{array}{r} 5.2 \\ \underline{-1.7} \end{array}$$

Do you now see your error? This choice is not correct.

Please return to page $\frac{65}{2}$ and try question 3 again.

I

$\frac{1}{2}$ of $\frac{7}{2}$ is $\frac{7}{4}$.

This can be reformed as a mixed number $1\frac{3}{4}$

Your answer is correct.

Please go on to question 19 below.

Question 19

Apply the proper principle and find the coordinate of the point that lies midway between 1 and $\frac{3}{2}$. Select the letter which labels the correct statement.

(A) $\frac{3}{4}$

(B) $\frac{6}{5}$

(C) $\frac{4}{2}$

(D) $\frac{5}{4}$

Please return to page $\frac{39}{2}$ and try question 14 again.

$\frac{53}{2}$

Sorry, we disagree!

Make a sketch of such a ruler to help you.

Return to page $\frac{47}{2}$ and try question 16 again.

I

Please return to page $\frac{62}{2}$ and try question 20 again.

$\frac{54}{2}$

Since the sum of any two numbers is a number, both of the additions are possible.

Therefore, this choice is not correct.

Return to page $\frac{60}{2}$ and try question 2 again.

I

Please return to page [illegible] try question 5 again.

$\frac{55}{2}$

What did you get for the value of R ? If you had trouble dividing 2 by .2 , you should remember that it is necessary to move the decimal point in the divisor to the right of the number. Since that moves it one place to the right, you must then move the decimal point in the dividend one place to the right as well. Since the dividend was 2 .

What did you get when you moved the decimal point?

e.g. $\frac{2}{.2} = \frac{2.0}{.2} = \frac{20}{2}$

Please return to page $\frac{90}{2}$ and try question 7 again.

I

The whole numbers between 12 and 3 inclusive are namely,

12, 11, 10, 9, 8, 7, 6, 5, 4, 3 .

Of these 12, 9, 6, and 3 are divisible by 3 .

Please continue with question 18 below.

Question 18

Apply the proper principle and find the coordinate of the point midway between $1\frac{1}{4}$ and $2\frac{1}{4}$.

Select the letter which labels the correct number.

(A) $1\frac{2}{4}$ (C) $1\frac{1}{3}$

(B) $1\frac{3}{4}$ (D) 1.2

I

Please return to page $\frac{62}{2}$ and select another answer for question 20 .

$\frac{57}{2}$

Since this choice offers two different subtractions, and one of them is not possible to do, this choice is not correct.

Return to page $\frac{71}{2}$ and try question 4 again.

I

Please go on to question 15 below.

Question 15

Apply the proper principle and find between what two successive natural numbers $\frac{22}{7}$ lies. Select the letter which labels the correct answer.

(A) 7 and 22

(B) 3 and 4

(C) 3 and $3\frac{1}{7}$

(D) 2 and 3

I

Please return to page $\frac{52}{2}$ and try question 19 again.

Writing the example in the way you are used to seeing it:

$$\begin{array}{r} 1.7 \\ -5.2 \end{array}$$

You should recognize that the larger number is being taken away from the smaller, which is impossible for you to do (at this time).

Therefore, this choice is not correct.

Return to page $\frac{65}{2}$ and try question 3 again.

I