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ABSTRACT

Since the development of causal path analysis by Wright, both the standardized and unstandardized procedures in path coefficient estimates have been advocated in the related literature, but the superiority of one technique over the other has not been demonstrated empirically or mathematically. In this study, the sampling properties of both coefficient estimates in a chain prediction model are compared under known conditions by Monte Carlo methods at systematically selected parameter points which include sample size and Var(E-sub-one, E-sub-two). The implications, interpretation, and problems of both methods are discussed with reference to the evaluation of learning hierarchies. (Author)

An Empirical Comparison of Two Path Coefficient Estimates

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AN EMPIRICAL COMPARISON OF TWO PATH COEFFICIENT ESTIMATES

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A. The Background

Since the development of path analysis by Wright (1918), the technique of path analysis has gradually become popular in the medical, biological, social and political sciences.

In the related literature, two types of path coefficient estimates have been advocated. Tukey (1954), Turner and Stevens (1959) have favored the use of unstandardized path coefficients. Wright (1960) restated some of the unique advantages of unstandardized procedure, but in general he favored the standardized form partly because of greater convenience in analysis. Applications of both types of approaches to path analysis have been reported.

Although two methods of calculating path estimates are available, the superiority of one method over the other has neither been demonstrated mathematically nor empirically. Zellner (1971) reported a Monte Carlo study of the sampling properties of the two-stage least square beta weight in a two-equation simultaneous equation model, but the only parameter he used was sample size and his empirical estimates were based on information from only 50 replications. Extending Zellner's work, the overall purpose of this study was to compare empirically under known conditions the sampling properties of both the unstandardized (i.e. derived from the beta weights) and the standardized (i.e. derived from the correlation coefficients) path coefficient estimates at selected parameter points.

B. The Problem

Turner and Stevens (1959) identified three basic models of open path networks, namely:

(1) the simple multiple prediction model,

(2) the simultaneous prediction model, and

(3) the chain prediction model.

An example of the first model is shown in Figure 1.

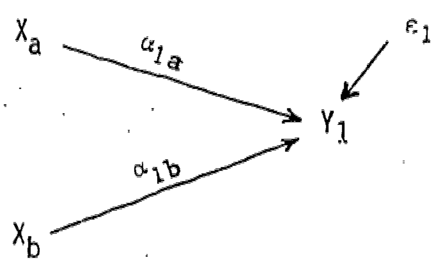

Figure 1

In this and the following path diagrams, the X's are the observed error free exogenous (independent) variables, and the Y's are the observed endogenous (dependent) random variables. The arrows indicate the direction of influence. In interpreting the path diagram, the variable at the head of one or more arrows is regarded as being a function of just those variables at the tails of these same arrows. The notation

'α' indicates that the path coefficient is derived from the regression or partial regression coefficients. When the variables are standardized or when the path coefficient is derived from the correlation or partial correlation coefficients, the notation for the path coefficient in this paper is 'π'.

The model equation in Figure 1 is:

(1) $$Y_1 = \alpha_1 + \alpha_{1a} X_a + \alpha_{1b} X_b + \varepsilon_1$$

This is the familiar multiple linear prediction model with two independent variables and the path coefficients are the ordinary partial regression coefficients. In symbols

(2) $$\hat{\alpha}_1 = b_1$$
$$\hat{\alpha}_{1a} = b_{1a.b}$$
$$\hat{\alpha}_{1b} = b_{1b.a}$$

This model is straightforward since the distribution of $\underline{b}$ (in matrix notation) is

(3) $$\underline{b} \overset{d}{=} N \; \{ \underline{B} \, , \, \sigma^2 \, (\underline{X}'\underline{X})^{-1} \}$$

where σ^2 is the variance of ε_1.

An example of the second path model, the simultaneous prediction model, is shown in Figure 2.

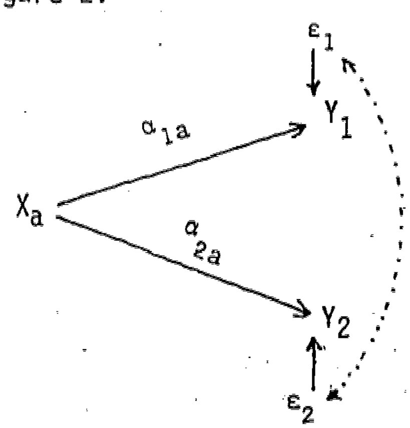

Figure 2

correlated errors

The model equations in this case are

$$(4) \qquad Y_1 = \alpha_1 + \alpha_{1a} X_a + \varepsilon_1$$

$$Y_2 = \alpha_2 + \alpha_{2a} X_a + \varepsilon_2$$

This model is also straightforward, since the distribution of $\underline{b}$ is

$$(5) \qquad \underline{b} \overset{d}{=} N \{ \underline{B} , \underline{\Sigma} (\underline{X}' \underline{X})^{-1} \}$$

where $\underline{\Sigma}$ in this case is a two-by-two error variance-covariance matrix. In the path technique as described by Turner and Stevens (1959, pp. 243 - 244), the errors of the two dependent variables (i.e. ε_1 and ε_2) were assumed to be uncorrelated.

The third model, i.e. the chain prediction model is more unusual. The path diagram of the simplest of this type is illustrated in Figure 3.

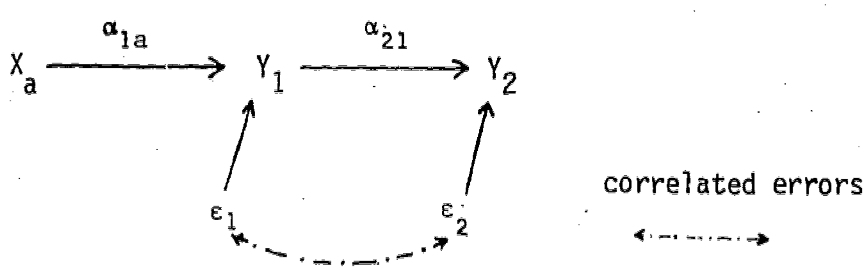

Figure 3

The model equations for this example are

$$(6) \qquad Y_1 = \alpha_1 + \alpha_{1a} X_a + \varepsilon_1$$

$$Y_2 = \alpha_2 + \alpha_{21} (Y_1 - \varepsilon_1) + \varepsilon_2$$

and the reduced regression equations are then found:

$$(7) \qquad Y_1 = \alpha_1 + \alpha_{1a} X_a + \varepsilon_1$$

$$Y_2 = (\alpha_2 + \alpha_1 \alpha_{21}) + (\alpha_{1a} \alpha_{21}) X_a + \varepsilon_2$$

The solution of the above path estimates can be approached by non-linear least square techniques, but due to the computations involved, linear least square technique is generally used. In terms of ordinary regression coefficients, the path estimates can be found as:

$$(8) \qquad \hat{\alpha}_{1a} = b_{1a}$$

$$\hat{\alpha}_{21} = \frac{b_{2a}}{b_{1a}}$$

where $\hat{\alpha}_{1a}$ is distributed as

$$(9) \qquad N \left\{ B_{1a} \; , \; \frac{\sigma^2_{\varepsilon 1}}{SS_X} \right\}$$

but no comfortable distribution theory is available for $\hat{\alpha}_{21}$ since it is a ratio of two correlated random variables. Should both $\hat{\alpha}_{1a}$, $\hat{\alpha}_{21}$ be expressed in the standardized form, i.e.

$$(10) \qquad \hat{\pi}_{1a} = r_{1a}$$

and

$$\hat{\pi}_{21} = \frac{r_{2a}}{r_{1a}}$$

then neither $\hat{\pi}_{1}$ nor $\hat{\pi}_{21}$ would have a comfortable distribution theory. It is partly because of this reason that Tukey (1954), and Turner and Stevens (1959) advocated the used of the unstandardized path coefficient estimates. Our conjectures were that both $\hat{\alpha}_{21}$ and $\hat{\pi}_{21}$ would have standard errors bigger than that of $\hat{\alpha}_{1a}$ and $\hat{\pi}_{1a}$ respectively and also that there might be some other undesirable sampling properties. Thus the specific objective of this study was to to investigate, by the Monte Carlo method, some of the empirical sampling behaviors of $\hat{\alpha}_{21}$ and $\hat{\pi}_{21}$ under selected parameter points, including sample size and differences in the error variance-covariance matrix of the dependent variables.

C. Methodology

Before the collection of empirical data, some of the theoretical values had first to be solved. In the following notations, ξ is an error free variable, α and π are the population parameters. According to the model (see Figure 3),

(11)
$$\mu_{Xa} = \mu_{\xi}$$
$$\mu_{Y1} = \alpha_1 + \alpha_{1a}\,\mu_{Xa}$$
$$\mu_{Y2} = \alpha_2 + \alpha_{21}\,\mu_{Y1}$$

By specifying the parameters of σ^2_X , α_1, α_{1a}, α_2, α_{21}, $\sigma^2_{\varepsilon_1}$, $\sigma^2_{\varepsilon_2}$ and $\sigma(\varepsilon_1\varepsilon_2)$, the theoretical var-covariances of X_a, Y_1 and Y_2 can be derived to be

(12)
$$\begin{array}{c|lll|}
X_a & \sigma^2_{Xa} & & \\
Y_1 & \alpha_{1a}\sigma^2_{Xa} & \alpha^2_{1a}\sigma^2_{Xa} + \sigma^2_{\varepsilon 1} & \\
Y_2 & \alpha_{21}\alpha_{1a}\sigma^2_{Xa} & \alpha_{21}\alpha^2_{1a}\sigma^2_{Xa} + \sigma(\varepsilon_1\varepsilon_2) & \alpha^2_{21}\,\alpha^2_{1a}\,\sigma^2_{Xa} + \sigma^2_{\varepsilon 2}
\end{array}$$

Based on this covariance matrix, the population standardized path coefficients of π_{1a} and π_{21} (in its proper context of α_{1a} and α_{21}) can be found as

(13)
$$\pi_{1a} = \rho_{1a}$$
$$\pi_{21} = \frac{\rho_{2a}}{\rho_{1a}}$$

The procedures of compiling the data in the Appendix are briefly summarized in the following:*

(1) The vector of independent variable (X) with specified μ and σ was generated using Marsaglia's technique as described by Jansson (1966, pp. 182 - 185). It has been found that this technique is reasonably fast and accurate. The pseudo-random numbers of this subroutine have been tested for normality and randomness. The two vectors of correlated error terms with specified μ's, σ's, and ρ were generated according to the multivariate law as described in Jansson (1966, pp. 186 - 187). This method is general and can be extended to any number of dimensions.

(2) The vectors of dependent variables (Y_1, Y_2) were computed as

$$
\begin{aligned}
\underline{Y}_1 &= \alpha_1 + \alpha_{1a}\,\underline{X} + \underline{\varepsilon}_1 \\
\underline{Y}_2 &= \alpha_2 + \alpha_{21}\,\underline{Y}_1 + \underline{\varepsilon}_2
\end{aligned}
\tag{14}
$$

where ε_1 and ε_2 are the vectors of error terms with correlation of ρ.

(3) The sample variance-covariance matrix from the generated vectors of X, Y1 and Y2 was then computed and all the path coefficent estimates of Figure 3 (i.e. $\hat{\alpha}_{1a}$, $\hat{\alpha}_{21}$, $\hat{\pi}_{1a}$, $\hat{\pi}_{21}$) calculated. In the expressions

$$\hat{\alpha}_{21} = \frac{b_{2a}}{b_{1a}} \tag{15}$$

and

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* Programmed in Fortran IV by Peter T.K. Tam

$$\hat{\pi}_{21} = \frac{r_{2a}}{r_{1a}}$$

it can be seen that as b_{1a} and r_{1a} approaches zero, the values of $\hat{\alpha}_{21}$ and $\hat{\pi}_{21}$ can approach infinity. To avoid such unusual possibilities, a sample of X, Y1 and Y2 was accepted only if its computed $| b_{1a} |$ was greater than BC and the corresponding $| r_{1a} |$ greater than CC where

$$\text{(16)} \qquad BC = (0.05) \left(\sigma^2_{\varepsilon 1} / SSx \right)^{\frac{1}{2}}$$

and

$$CC = (BC) \left(\sigma^2_X / \sigma^2_Y \right)^{\frac{1}{2}}$$

An example of the b_{1a}'s and r_{1a}'s that were accepted and rejected was shown in Table 5 in the Appendix.

(4). Steps (1) to (3) were replicated 200 times for each of the 15 different cycles of parameter values (see Tables 1 - 4 in Appendix) within each of the four different levels of sample sizes.

The main computer program and its subroutines have been carefully checked before its execution on the CDC 6500 at the Florida State University Computer Center. The CPU time is approximately 100 minutes and the CM is 50 K.

D. Results

As mentioned previously, the reason for using the Monte Carlo approach is that no existing comfortable distribution theory is available for $\hat{\alpha}_{21}$ and $\hat{\pi}_{21}$. It seems that our conjectures were confirmed. All the summary data were tabulated in Tables 1 - 4 in the Appendix. Some of the interesting findings were graphically illustrated in Figures 4 - 11 and discussed in the following:

(1) Figures 4 and 5 indicated some of the effects of sample size and error variance on the standard error of the path estimates. These graphs depicted the information of cycles 2, 6, and 12 of the Appendix Tables. In Figure 4, the broken line indicated the amount of standard error of $\hat{\alpha}_{21}$ across the four levels of sample sizes. The empirical and theoretical standard error of $\hat{\alpha}_{1a}$ are represented by the continuous and the red line respectively. It can be seen that the empirical and theoretical values are almost identical. Figure 4 is divided in three parts, namely A, B, and C, each of which has error variance (ε_1) of 50, 400, and 900 respectively. As error variance (ε_1) increased, the standard error of the estimates increased. But within each level of error variance, the standard error decreased with the increase of sample size. However, irrespective of the sample size and values of ε_1, the standard error of $\hat{\alpha}_{21}$ is larger than that of $\hat{\alpha}_{1a}$. This fact generally holds under other parameter points as indicated in the Tables in the Appendix.

(2) As illustrated in Figure 5, the same conclusions of above are generally applicable to that of $\hat{\pi}_{21}$. In terms of standard error, it's indecisive of the relative superiority of $\hat{\alpha}_{21}$ and $\hat{\pi}_{21}$. But both have empirical standard errors larger than that of $\hat{\alpha}_{1a}$ and $\hat{\pi}_{1a}$. It seems that except when the sample size is big or the error variance small , the standard error of $\hat{\alpha}_{21}$ or $\hat{\pi}_{21}$ may be too big to be tolerable.

(3) Figures 6 and 7 illustrated the effects of sample size and ε_1 on the empirical skewness of the path estimates. However, such effects, if there is any, are not obvious in this study. Figure 6 compared the skewness of $\hat{\alpha}_{21}$ with that of $\hat{\alpha}_{1a}$, and in this respect, two points might

be fairly obvious. First, the skewness of $\hat{\alpha}_{21}$ was in general larger than that of $\hat{\alpha}_{1a}$. Second, the skewness of $\hat{\alpha}_{21}$ was in general large and positive, although in some other cycles as indicated in the Tables in the Appendix, the skewness of $\hat{\alpha}_{21}$ was large and negative. As illustrated in Figure 7, the same conclusions were true with respect to the skewness of $\hat{\pi}_{21}$. Again the superiority of $\hat{\alpha}_{21}$ or $\hat{\pi}_{21}$ was indecisive but neither one of them was desirable in terms of the skewness of the estimates.

(4) Figure 8 and 9 compared $\hat{\alpha}_{21}$ and $\hat{\pi}_{21}$ in terms of the empirical bias of the estimates. The amount of bias in general decreased when the sample size was large or when ε_1 was small or both. Again the superiority of either $\hat{\alpha}_{21}$ or $\hat{\pi}_{21}$ was not obvious but both were inferior when compared with that of $\hat{\alpha}_{1a}$ and $\hat{\pi}_{1a}$ respectively.

(5) One of the assumptions made by Turner and Stevens (1959, p.241) in the solution of the path estimates was that the covariance among the error terms (ε_i) was zero. Although in our experiment we had introduced various levels of error covariance (i.e. $\sigma(\varepsilon_1\varepsilon_2)$), results of this present study indicated that, under the parameters we investigated, there were no definitive trends between error covariances and the sampling properties of $\hat{\alpha}_{21}$ and $\hat{\pi}_{21}$. This fact was illustrated in Figures 10 - 12.

In conclusion to the above, it seems that the results of this study do support our conjecture that both $\hat{\alpha}_{21}$ and $\hat{\pi}_{21}$ in the chain prediction model (see Figure 3) have indeed some unusual sampling properties. Greater care in the interpretation of the coefficients in path analysis should be exercised when the chain prediction model is involved. This point is particularly significant since most of the reported path networks in the related literature do have this chain prediction component.

Figure 4. Effects of sample size on the standard error of the unstandardized path estimates.

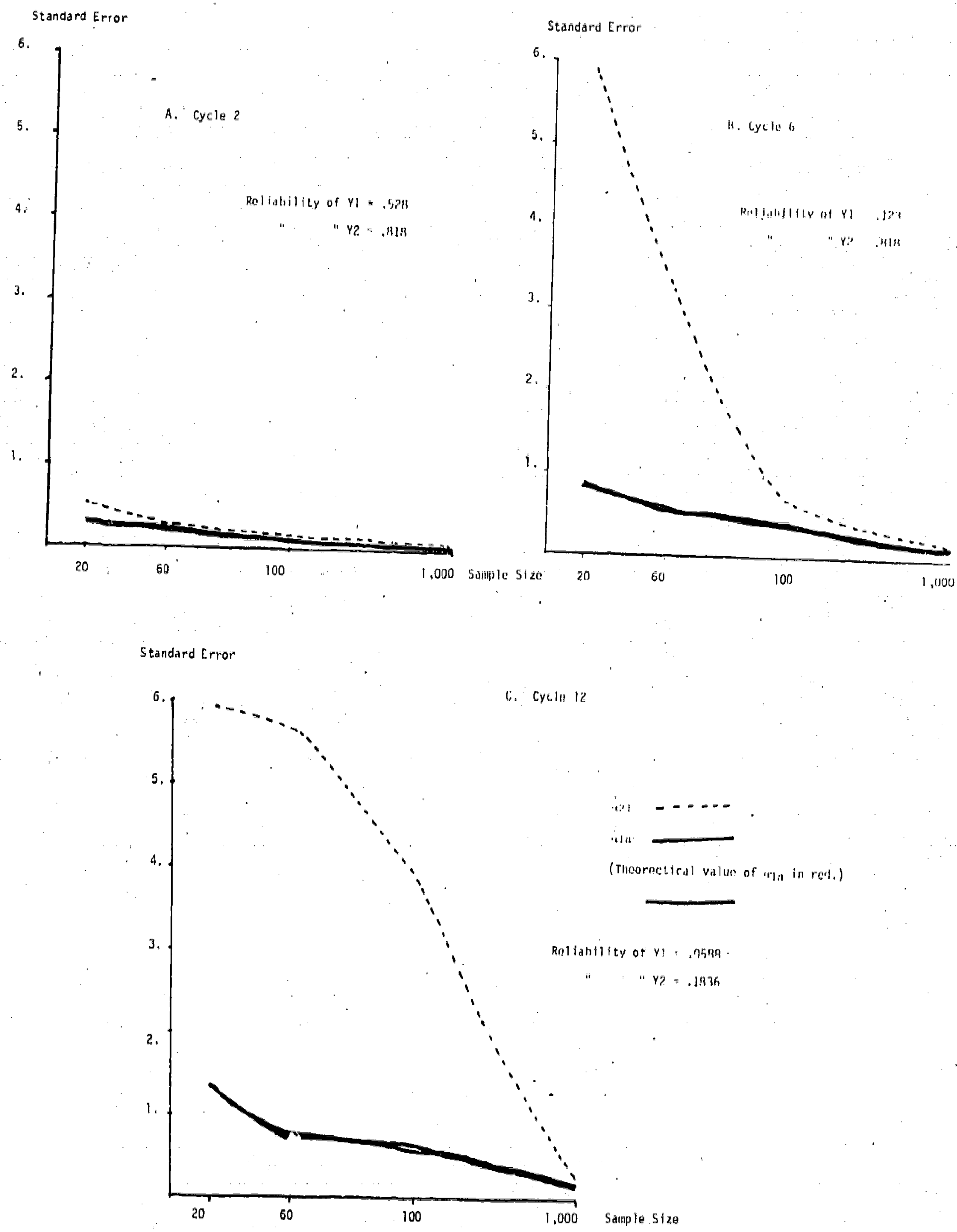

Figure 5. Effects of sample size on the standard error of the standardized path estimates.

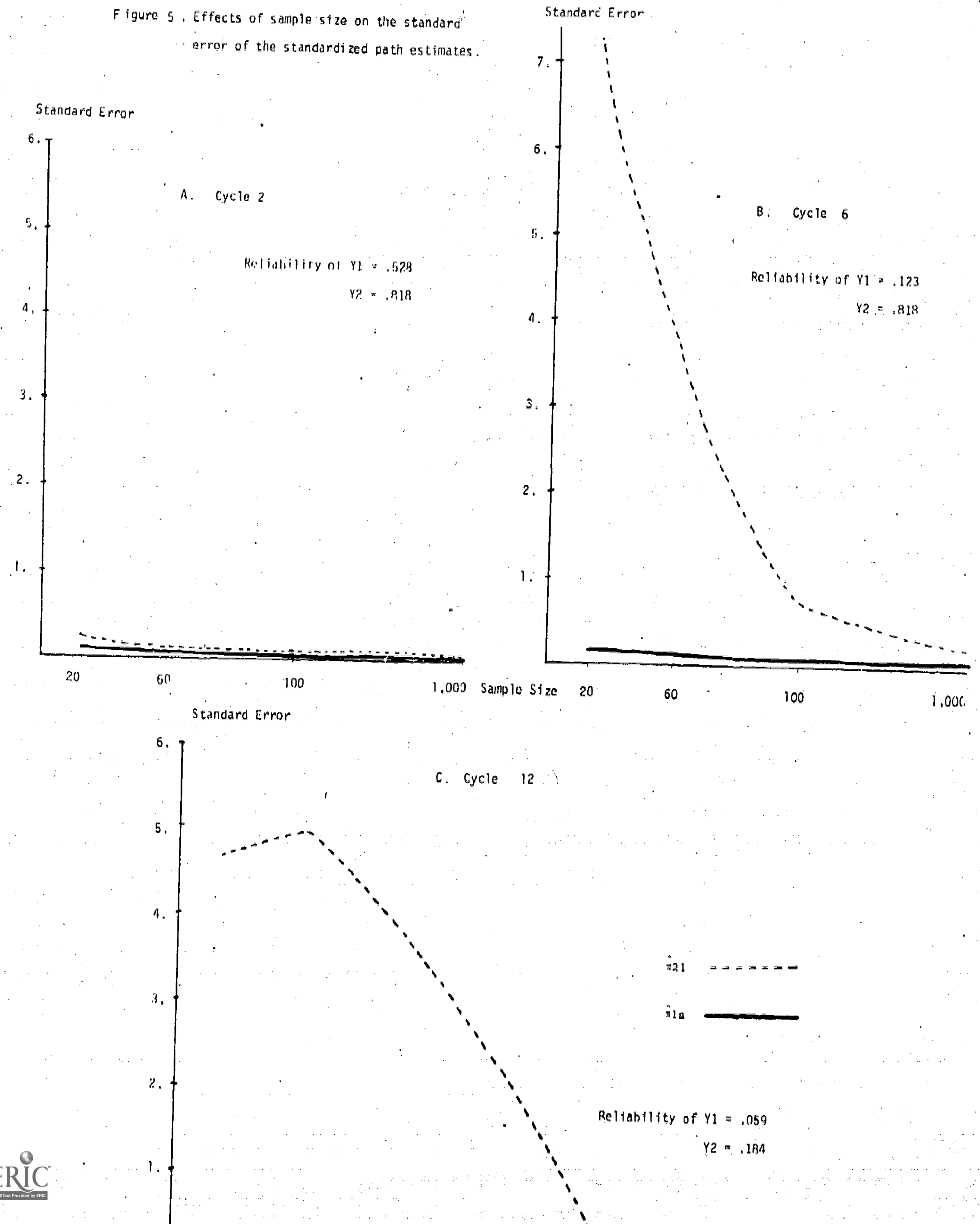

Figure 6. Effects of sample size on the skewness of the unstandardized path estimates.

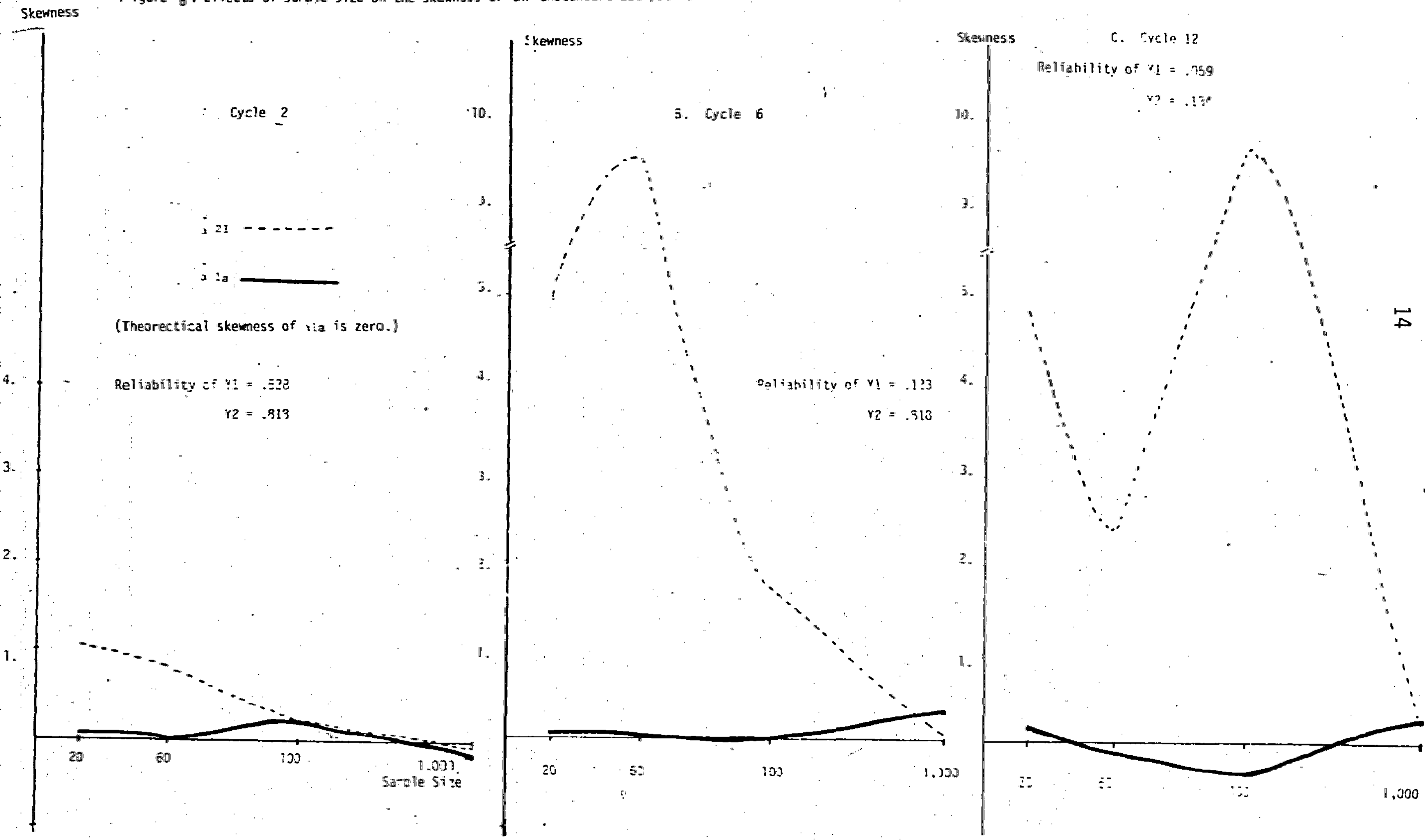

Figure 7. Effects of sample size on the skewness of the standardized path estimates.

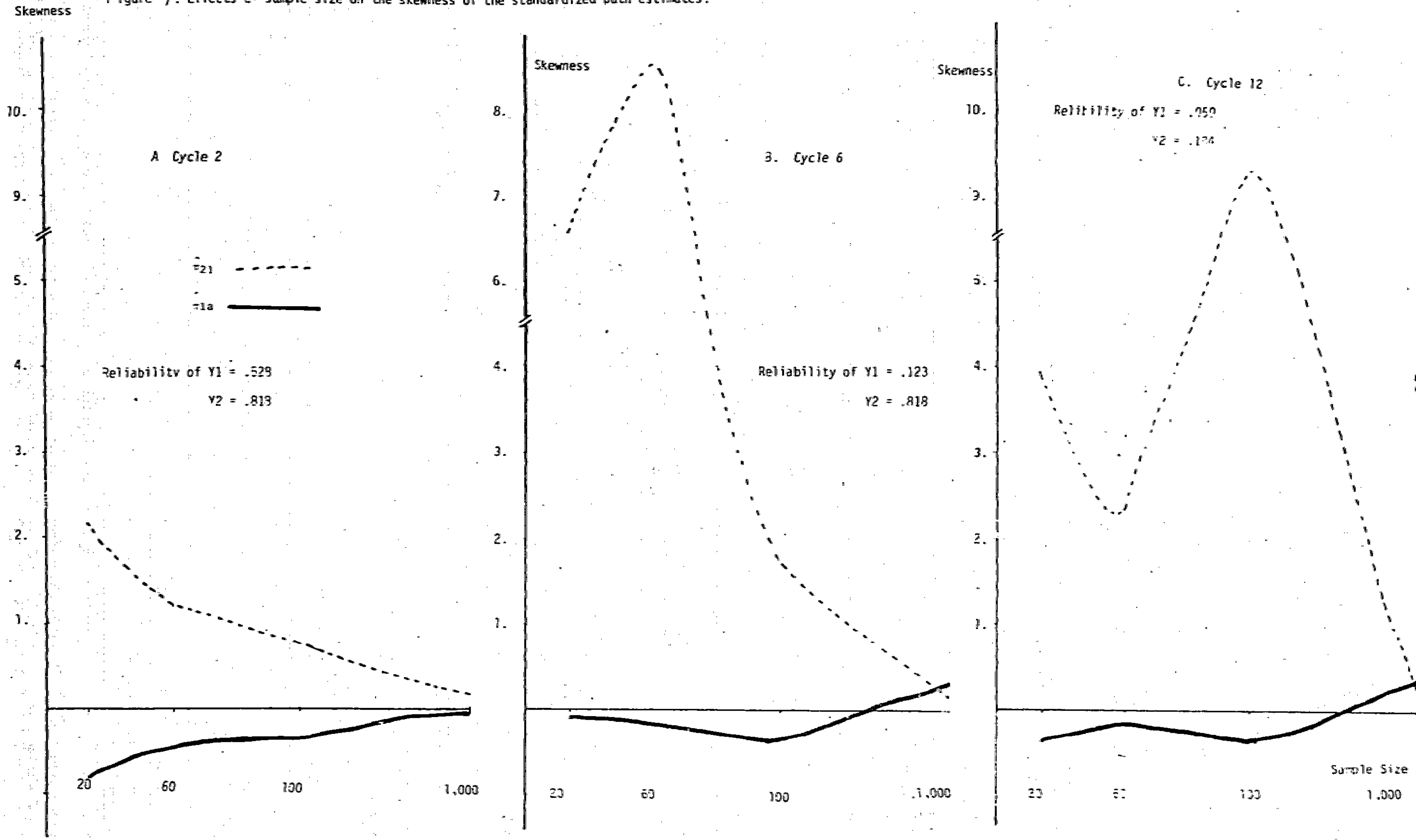

Figure 8. Effects of sample size on bias of unstandardized path estimates

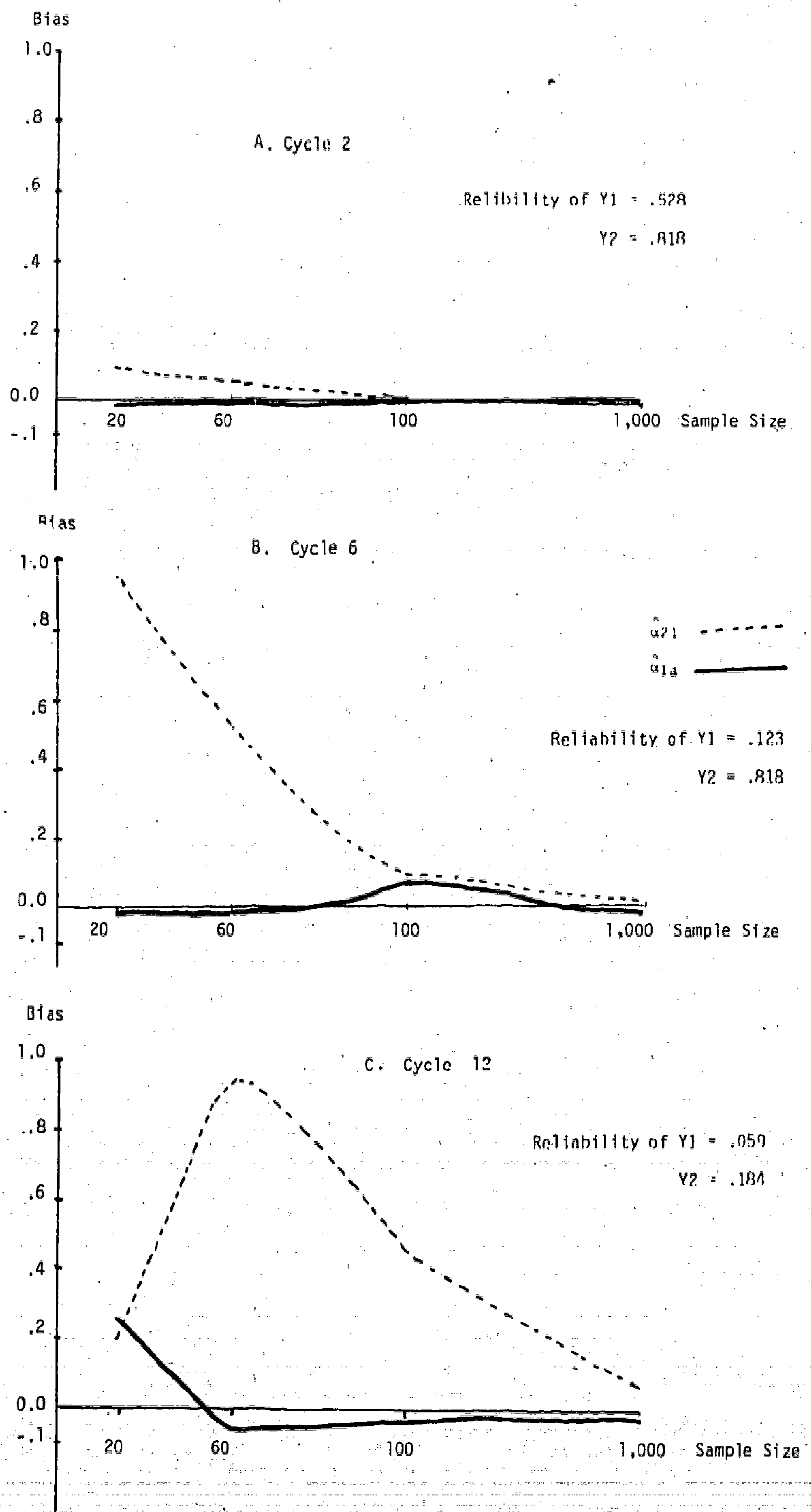

Figure 9. Effects of sample size on bias of standardized path estimates

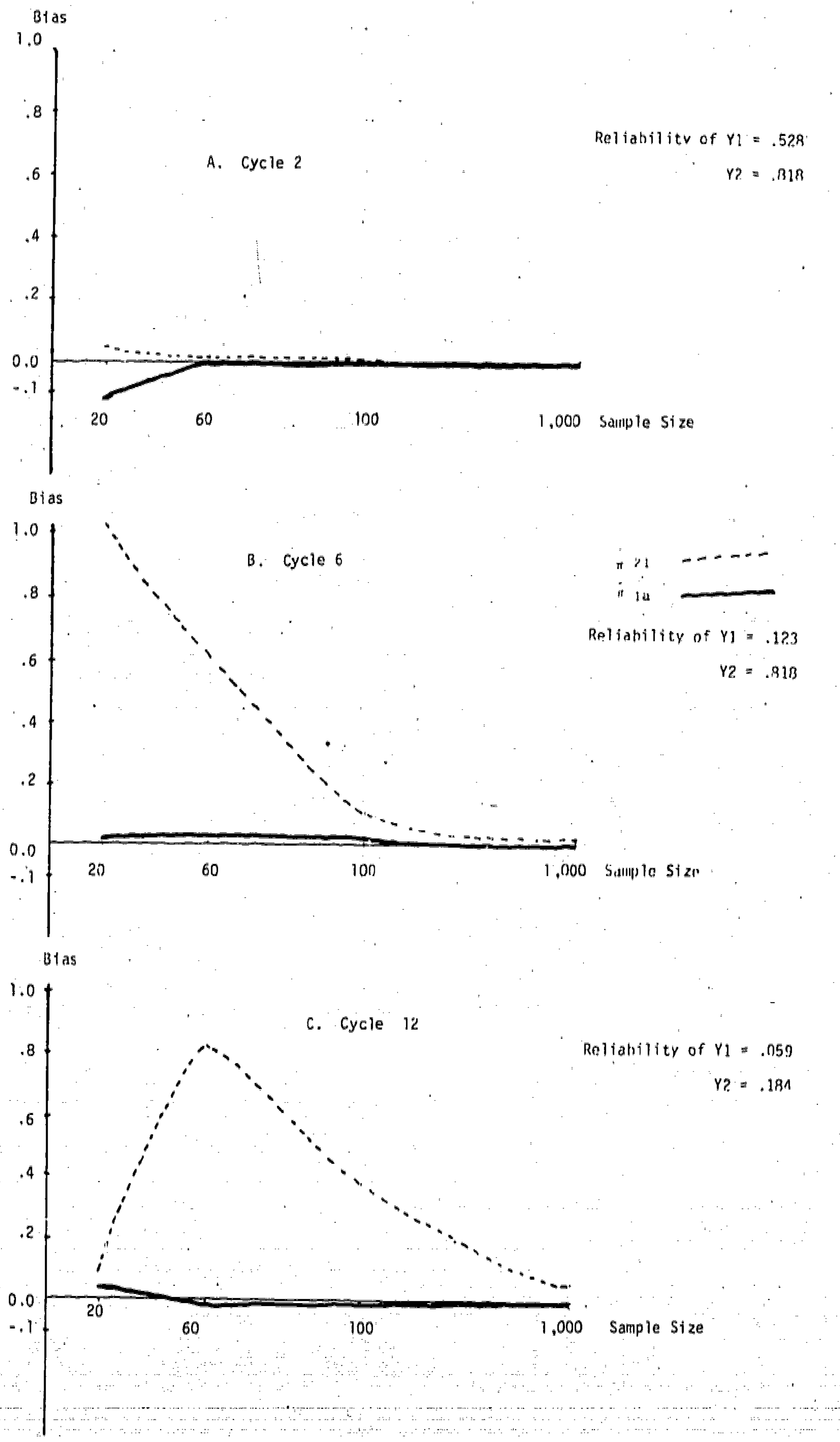

Figure 10. Effects of error covariance and sample size on the standard error and bias of $\hat{\alpha}_{21}$ (A,B) and $\hat{\pi}_{21}$ (C,D). Information based on cycles 1 - 4.

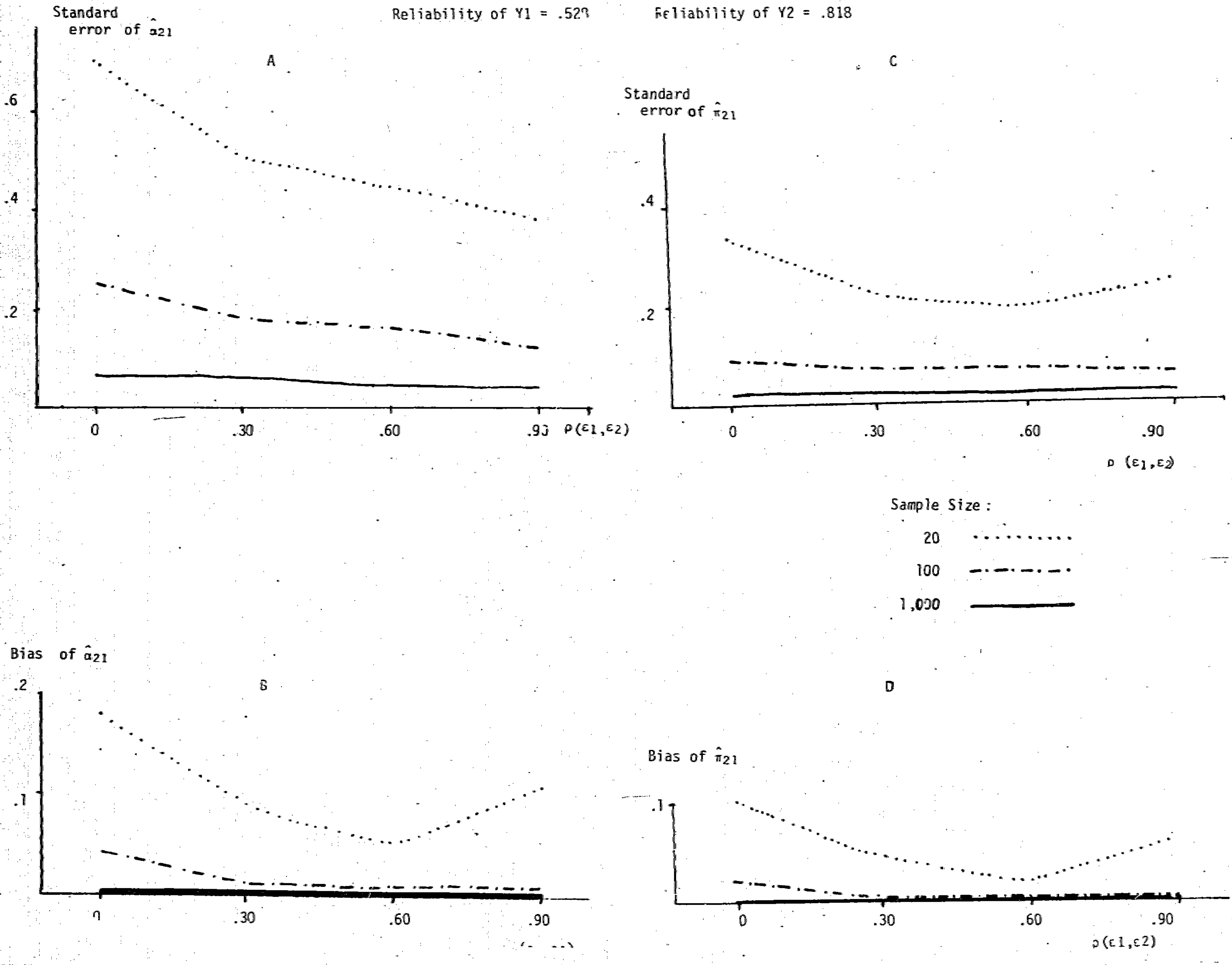

Figure 11. Effects of error covariance and sample sizes on the standard error of $\hat{\alpha}_{21}$ (A) and $\hat{\pi}_{21}$ (b).
Information based on cycles 5 - 8

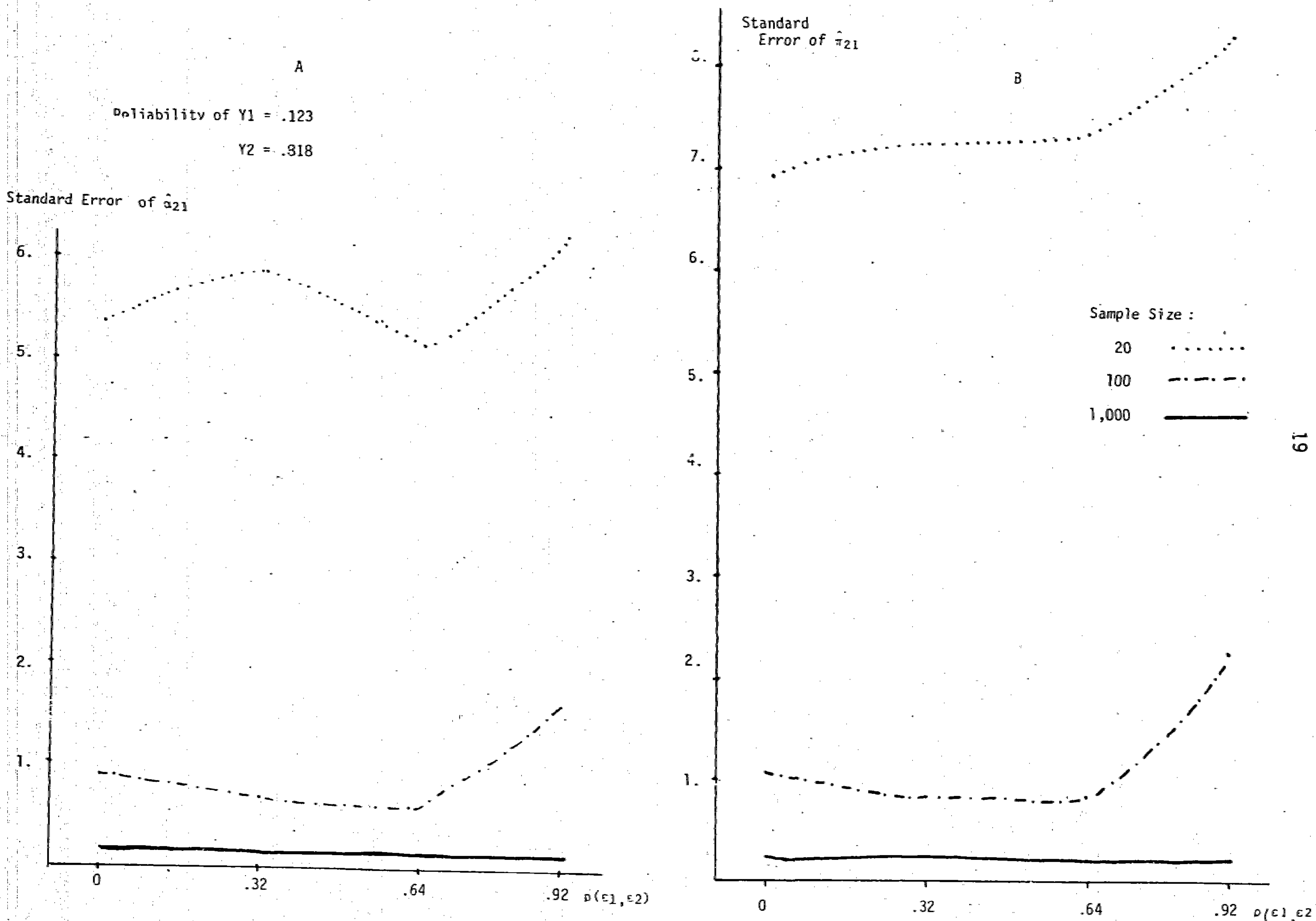

Figure 12. Effects of error covariance on the bias of $\hat{\alpha}_{21}$ (A) and $\hat{\pi}_{21}$ (B). Information based on cycles 5 - 8.

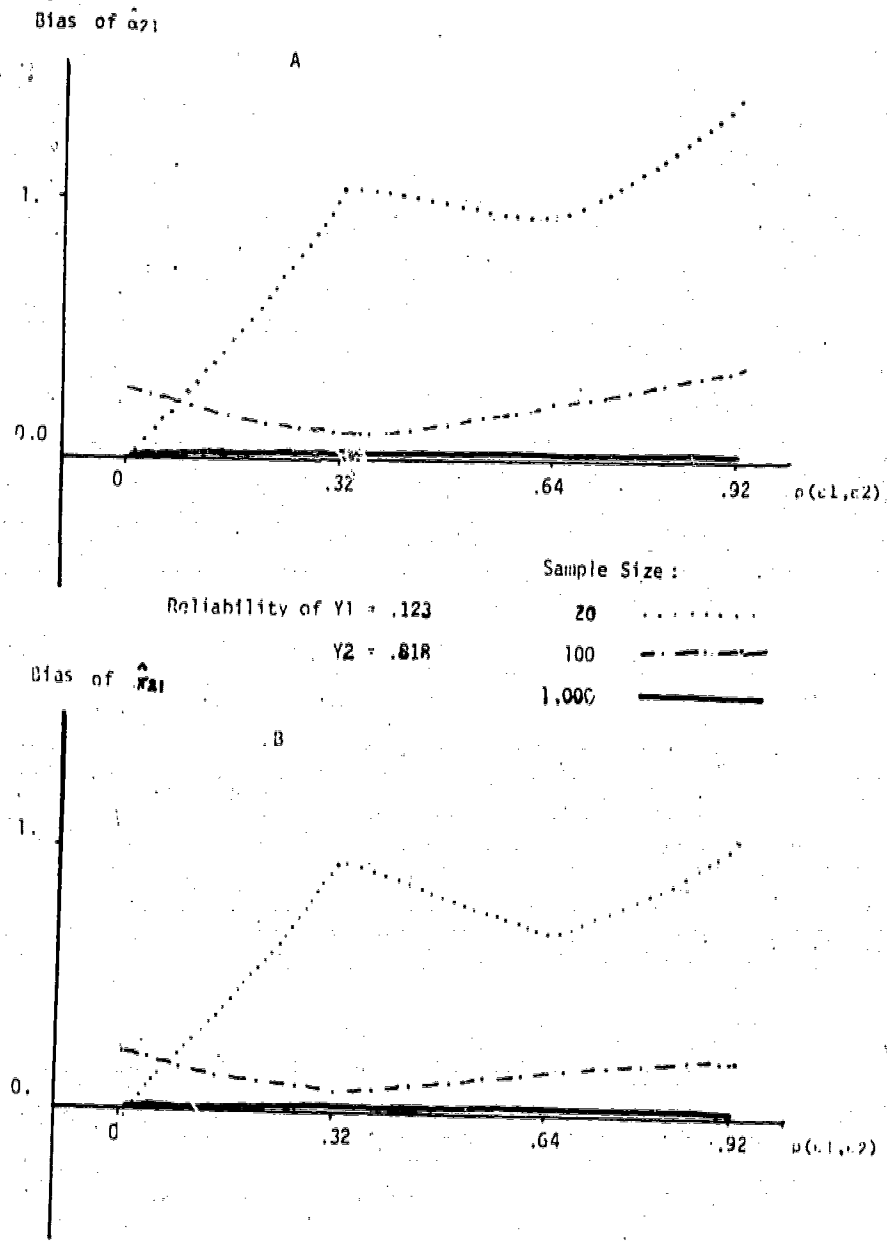

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Table 1. Summary of statistics for sample size of 20.

CYCLE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\sigma_{\varepsilon 1}$	50	50	50	50	400	400	400	400	50	50	900	900	900	900	900
$\sigma_{\varepsilon 2}$	50	50	50	50	50	50	50	50	400	400	1000	1000	1000	1000	1000
$\sigma_{\varepsilon 1 \varepsilon 2}$	0	15	30	45	0	45	90	130	45	90	0	250	500	650	850
$\rho_{\varepsilon 1 \varepsilon 2}$	.00	.30	.60	.90	.00	.32	.64	.92	.32	.64	.00	.26	.53	.69	.90
X, Y1, Y2 ρ	.73 .90 .66	.75	.83	.92	.35 .90 .32	.44	.57	.68	.73 .60 .61	.79	.24 .43 .10	.33	.57	.70	.89
Standard Error *															
$\hat{\alpha}_{1a}$	.36	.32	.33	.32	1.00	.81	1.08	1.00	.38	.35	1.45	1.36	1.57	1.46	1.43
$\hat{\alpha}_{21}$	.70	.51	.44	.38	5.38	5.88	5.14	6.22	.83	.55	5.76	5.90	4.91	5.94	2.61
$\hat{\pi}_{1a}$	.13	.10	.10	.12	.20	.18	.23	.20	.13	.12	.21	.20	.24	.22	.21
$\hat{\pi}_{21}$	.34	.22	.19	.24	6.92	7.28	7.36	8.33	.28	.18	4.97	4.65	4.26	4.34	2.40
Skewness *															
$\hat{\alpha}_{1a}$	-.06	.02	.08	-.20	.34	.01	.18	-.20	.15	-.27	.04	.16	-.47	-.13	-.34
$\hat{\alpha}_{21}$	1.72	1.08	2.68	3.15	-2.91	4.91	2.64	3.48	.43	.48	-4.13	4.88	-1.31	4.99	1.23
$\hat{\pi}_{1a}$	-.93	-.80	-.75	-.89	-.42	-.10	-.41	-.69	-1.32	-1.15	-.39	-.34	-.54	-.28	-.55
$\hat{\pi}_{21}$	2.66	2.13	2.72	4.10	-2.53	6.59	3.25	3.70	-.24	-.77	-3.07	3.95	-1.48	1.12	1.81
Bias from parameter *															
$\hat{\alpha}_{1a}$	-.04	-.01	.02	-.05	.03	.01	-.09	-.04	-.00	-.01	.20	.27	-.09	.08	.12
$\hat{\alpha}_{21}$	.18	.09	.05	.11	-.06	.95	.70	1.06	.01	.01	-.69	.20	-.41	-.26	-.06
$\hat{\pi}_{1a}$	-.03	-.12	[illegible]	-.02	-.01	.01	-.03	-.02	-.01	[illegible]	.02	.03	-.02	[illegible]	.01
$\hat{\pi}_{21}$	.10	.05	.02	.06	.00	1.02	.94	1.36	-.02	.02	-.69	.09	-.38	-.39	-.05

* Each cell entry in Tables 1 - 4 was based on 200 replications.

Table 2. Summary of statistics for sample size of 60.

LE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\sigma_{\varepsilon 1}$	50	50	50	50	400	400	400	400	50	50	900	900	900	900	900
$\sigma_{\varepsilon 2}$	50	50	50	50	50	50	50	50	400	400	1000	1000	1000	1000	1000
$\sigma_{\varepsilon 1, \varepsilon 2}$	0	15	30	45	0	45	90	130	45	90	0	250	500	650	850
$\rho_{\varepsilon 1, \varepsilon 2}$	.0	.30	.60	.90	.0	.32	.64	.92	.32	.64	.0	.26	.53	.69	.90
X Y_1 Y_2 ρ	.73 .90 .66	.75	.83	.92	.35 .90 .32	.44	.57	.68	.73 .60 .61	.79	.24 .43 .10	.33	.57	.70	.89
Standard Error															
$\hat{\alpha}_{1a}$	.18	.21	.19	.17	.55	.51	.50	.52	.18	.20	.73	.74	.78	.80	.74
$\hat{\alpha}_{21}$	.29	.27	.23	.15	3.11	3.24	1.91	3.02	.37	.28	3.05	5.63	3.72	3.76	6.21
$\hat{\pi}_{1a}$	.06	.06	.07	.06	.12	.11	.11	.12	.06	.07	.11	.12	.12	.12	.11
$\hat{\pi}_{21}$	.10	.10	.10	.09	3.94	3.71	2.62	4.01	.11	.09	2.85	4.97	3.15	3.51	5.61
Skewness															
$\hat{\alpha}_{1a}$	.06	-.00	-.05	-.07	.19	.00	-.17	.16	.15	.01	-.27	-.12	-.12	-.22	-.08
$\hat{\alpha}_{21}$	.73	.81	.87	1.02	-4.73	9.57	6.15	10.66	.18	.30	2.46	2.41	-5.78	5.19	1.62
$\hat{\pi}_{1a}$	-.22	-.48	-.41	-.39	-.23	-.17	-.33	.08	-.26	-.40	-.30	-.12	-.28	-.42	-.25
$\hat{\pi}_{21}$	.84	1.20	.86	.96	-3.03	8.64	4.71	10.38	-.09	-.23	1.83	2.36	-5.08	5.09	1.75
Bias from parameter															
$\hat{\alpha}_{1a}$	.02	.01	-.01	-.02	-.01	.03	.03	-.10	.01	.00	.05	-.07	-.07	.05	.01
$\hat{\alpha}_{21}$	.01	.04	.01	.03	.29	.51	.22	.66	-.02	.02	.37	.95	.06	.29	.29
$\hat{\pi}_{1a}$	-.00	.00	-.01	-.01	-.01	.01	.01	-.02	-.00	.00	.01	-.01	-.01	.01	-.00
$\hat{\pi}_{21}$	.00	.01	-.02	.02	.38	.60	.26	.90	-.02	[illegible]	.27	.81	.06	[illegible]	.26

Table 3. Summary of statistics for sample size of 100.

CYCLE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\sigma_{\varepsilon 1}$	50	50	50	50	400	400	400	400	50	50	900	900	900	900	900
$\sigma_{\varepsilon 2}$	50	50	50	50	50	50	50	50	400	400	1000	1000	1000	1000	1000
$\sigma_{\varepsilon 1 \varepsilon 2}$	0	15	30	45	0	45	90	130	45	90	0	250	500	650	850
$\rho_{\varepsilon 1 \varepsilon 2}$	.00	.30	.60	.90	.00	.32	.64	.92	.32	.64	.00	.26	.53	.69	.90
X Y1 ρ Y2	.73 .90 .66	.75	.83	.92	.35 .90 .32	.44	.57	.68	.73 .60 .61	.79	.24 .43 .10	.33	.57	.70	.89
$\hat{\alpha}_{1a}$	.16	.14	.14	.14	.41	.43	.37	.43	.14	.15	.61	.61	.64	.60	.59
$\hat{\alpha}_{21}$	.25	.18	.16	.12	.89	.69	.60	1.61	.27	.21	4.63	3.85	3.13	1.73	.78
$\hat{\pi}_{1a}$	.05	.04	.04	.05	.09	.09	.08	.09	.05	.05	.09	.10	.10	.09	.09
$\hat{\pi}_{21}$	.09	.07	.07	.06	1.06	.83	.83	2.27	.09	.07	4.10	3.34	2.86	1.45	.73
$\hat{\alpha}_{1a}$	.05	.24	-.08	-.31	.06	-.01	.05	-.35	.23	.21	.27	-.34	.11	.15	.33
$\hat{\alpha}_{21}$	.39	.23	.69	.78	3.97	1.71	1.59	9.42	-.14	.20	.26	9.70	6.21	3.10	3.08
$\hat{\pi}_{1a}$	-.27	-.36	-.33	-.64	-.11	-.35	-.08	-.58	-.32	-.52	.05	-.39	.04	.04	.24
$\hat{\pi}_{21}$	.71	.72	.61	1.20	4.15	1.79	1.86	9.74	-.12	-.41	.71	9.30	6.26	2.51	3.30
$\hat{\alpha}_{1a}$	-.01	.00	-.00	.00	-.01	.07	-.03	.03	.02	-.01	.02	-.03	.04	-.01	.03
$\hat{\alpha}_{21}$	.04	.01	.01	.01	.22	.08	.18	.24	-.03	.00	.37	.47	.44	.32	.23
$\hat{\pi}_{1a}$	.01	.00	[illegible]	-.00	-.01	.01	-.00	.00	.00	[illegible]	.00	[illegible]	.01	[illegible]	-.00
$\hat{\pi}_{21}$	.02	.00	.00	.00	.26	.10	.22	.35	-.00	[illegible]	.30	[illegible]	.38	[illegible]	.20

Table 4. Summary of statistics for sample size of 1,000.

CYCLE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\sigma_{\varepsilon 1}$	50	50	50	50	400	400	400	400	50	50	900	900	900	900	900
$\sigma_{\varepsilon 2}$	50	50	50	50	50	50	50	50	400	400	1000	1000	1000	1000	1000
$\sigma_{\varepsilon 1 \varepsilon 2}$	0	15	30	45	0	45	90	130	45	90	0	250	500	650	850
$\rho_{\varepsilon 1 \varepsilon 2}$	.00	.30	.60	.90	.00	.32	.64	.92	.32	.64	.00	.26	.53	.69	.90
X, Y_1, Y_2 (ρ)	.73 .90 .66	.75	.88	.92	.35 .90 .32	.44	.57	.68	.73 .60 .61	.79	.24 .43 .10	.33	.57	.70	.89
$\hat{\alpha}_{1a}$	.05	.04	.05	.05	.13	.13	.13	.13	.04	.05	.19	.19	.19	.19	.18
$\hat{\alpha}_{21}$	.07	.06	.05	.04	.17	.16	.15	.15	.09	.06	.32	.26	.25	.20	.16
$\hat{\pi}_{1a}$	.01	.01	.02	.02	.03	.03	.03	.03	.01	.01	.03	.03	.03	.03	.03
$\hat{\pi}_{21}$	.02	.02	.02	.02	.21	.21	.20	.21	.03	.02	.27	.23	.21	.18	.15
$\hat{\alpha}_{1a}$	-.02	-.11	.10	-.10	.10	.35	.22	.01	-.23	.31	-.18	.26	-.29	-.12	-.39
$\hat{\alpha}_{21}$	.15	-.02	-.08	.22	.22	.09	.20	.43	-.12	.15	1.09	.25	.90	.91	1.12
$\hat{\pi}_{1a}$	-.54	-.04	-.22	-.20	.03	.30	.12	-.03	-.32	-.17	-.28	.18	-.34	-.21	-.37
$\hat{\pi}_{21}$	.57	.14	.26	.34	.36	.17	.28	.50	-.16	.15	1.17	.37	.95	.87	1.26
$\hat{\alpha}_{1a}$	.00	-.01	-.00	.00	.01	-.01	-.01	-.01	-.00	-.00	-.01	-.02	-.01	-.02	-.02
$\hat{\alpha}_{21}$	-.00	.00	.00	-.00	.00	.02	.02	.02	.00	.00	.06	.06	.03	.04	.03
$\hat{\pi}_{1a}$	.00	.00	-.00	.00	.00	-.00	-.00	-.00	-.00	[illegible]	-.00	-.00	-.00	[illegible]	-.00
$\hat{\pi}_{21}$	-.00	-.00	.00	-.00	.01	.03	.03	.03	.00	.00	.0	.05	.03	[illegible]	.0

Table 5. Examples of the b_{1a} and r_{1a} accepted and rejected during the experiment in Cycle 5 for a sample of 20

AVERAGE SSX= 474.01

Theorectical SSX = 500
B_{1a} = 1.500
R_{1a} = .3511

$\varepsilon 1$ = 400
$\varepsilon 2$ = 50
$\rho_{\varepsilon_1 \varepsilon_2}$ = 0

σ^2_{Y1} ~~= 465.25~~ ~~X is error free~~
σ^2_{Y2} = 275.00

BY1X ACCEPTED

.3547	.6159	3.2475	2.2429	1.1153	.8039	2.8362	-.2698	1.8909	5.4131
1.7799	2.6348	2.1019	2.5556	1.6472	2.0039	2.5473	2.5418	.7284	1.0703
2.2225	-.6533	1.3576	2.6234	3.1159	1.3156	3.2317	2.9938	1.4938	.9244
.2476	-.1902	.2858	.5911	2.2416	1.0339	2.1497	.1892	2.9608	-.1920
1.7209	1.2756	2.3146	1.4151	2.2313	1.4433	1.1096	1.9436	1.1481	3.1364
1.1727	.6632	1.0535	1.5879	1.3517	1.5741	1.7237	2.0131	1.5392	-.8453
3.1268	.4670	2.3773	2.3833	-.2223	2.4352	1.7596	.9369	1.7096	1.5197
1.5138	1.0569	2.4055	1.5535	1.3422	.6696	.7367	2.6128	.1903	1.9840
.4397	2.6224	.9297	1.0775	1.5394	1.7477	1.9211	1.9725	.4599	2.3004
4.2455	1.3342	.3874	.5149	1.6511	1.2463	1.7466	1.2970	3.1164	2.0964
2.8272	2.1310	2.5057	3.2001	2.0437	1.0018	1.1733	2.4703	1.3054	.1465
2.4557	2.2674	3.7714	.4327	.3712	1.0327	1.5743	3.7340	.9540	1.4124
1.4451	-.6744	2.0756	.3475	1.2493	1.7986	1.3186	-.3958	2.0038	1.8557
1.7191	1.4583	1.9538	-.0972	2.9013	2.1752	-.7433	.6744	.8385	2.1332
2.7427	1.2511	1.1066	.3516	1.3429	1.2901	1.6440	1.7001	2.1097	-.7250
.2896	.4400	.8995	1.3019	2.7728	1.3622	1.7096	1.8333	2.3802	-.2568
.2119	.8639	1.3500	2.0253	1.4230	1.0099	.2094	1.5248	1.5655	.5269
1.8431	.9369	.5175	2.5971	2.6541	1.5516	.5362	2.6016	.3270	.9560
1.2056	2.0910	1.1692	1.7441	.6334	2.4455	2.3631	.7349	1.6148	2.6124
.7527	.6381	1.3356	2.1736	1.2936	[illegible].4914	1.6264	.1694	1.1972	.7170

BY1X REJECTED, CRITERION = .04588 IN ABS.VAL
.0005 .0109 .0133 -.0270

RY1X ACCEPTED

.0734	.1908	.6751	.5872	.2385	.2002	.6385	-.0659	.4492	.6241
.3787	.6831	.3727	.5157	.4960	.4352	.5415	.4349	.2240	.3456
.4165	-.0165	.3757	.3354	.6280	.2217	.4617	.6151	.3199	.1736
.0553	-.0439	.0305	.2258	.3564	.2573	.6042	.0418	.5077	-.0687
.4115	.4051	.5125	.2279	.5456	.2743	.2786	.4430	.3637	.6322
.2659	.1612	.4558	.5233	.3564	.5563	.3687	.4379	.4376	-.1955
.6974	.1798	.3116	.4422	-.0595	.3375	.3620	.3143	.4295	.3804
.4185	.2226	.5646	.3935	.3265	.1557	.1795	.4779	.0347	.4245
.1213	.4343	.2575	.2736	.4390	.4276	.4227	.5243	.1192	.4959
.5191	.3370	.0332	.1733	.5145	.2705	.4092	.2878	.5847	.3963
.6763	.4951	.5253	.5433	.3952	.2375	.2443	.6028	.3609	.0260
.4528	.6215	.5281	.1123	.1533	.3971	.5212	.5037	.2225	.3581
.2058	-.1402	.5579	.2835	.3483	.3213	.2856	-.0673	.4683	.3516
.3370	.3488	.4332	-.0276	.4433	.5368	-.2359	.3537	.2242	.5476
.4746	.2613	.2504	.1337	.2197	.2162	.3722	.4599	.5361	-.1675
.0811	.1176	.2615	.4176	.4432	.3529	.3344	.3722	.4634	-.0729
.0464	.2169	.3077	.5165	.3591	.2917	.0396	.4031	.4368	.1818
.4632	.2044	.1333	.5635	.4503	.2703	.1027	.5336	.0765	.2854
.4101	.3670	.3297	.4437	.1177	.4320	.6661	.2015	.2994	.6960
.1806	.1495	.2316	.3772	.3526	.6430	.4071	.1652	.2438	.1876

RY1X REJECTED, CRITERION= .01074 IN ABS. VAL
.0031 .0043 .0033 -.0053