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ABSTRACT

A generalized framework for developing analytical and conceptual relationships involving the flow of information has been suggested. This paper provides further refinement, rigor, and extension for some of the earlier relationships suggested. In particular, a measure of the amount of information is defined as the difference of the value of the decision state of the decision-maker after and before receipt of the message. This measure is universally applicable for all information that is concerned with the effectiveness of the message upon the recipient. It is accordingly called "pragmatic information." The definition is a direct consequence of the interdependence between information and decision-making and of the definition that "information is data of value in decision-making." In order to evaluate this measure of information, it is convenient to use a "generalized information system model" which has previously been proposed and which has virtually universal applicability. Each data set or document has some average amount of information content for a decision-maker of any given "effectiveness." The relationship of this average amount of information as a function of the decision-maker effectiveness is suggested as an important functional relationship that exists for every document. It is called an "information profile" of that document or data set. A typical information profile is suggested. (Author/NH)

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# A GENERALIZED CONCEPTUAL DEVELOPMENT FOR THE ANALYSIS AND FLOW OF INFORMATION

by

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# COMPUTER & INFORMATION SCIENCE RESEARCH CENTER

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A generalized framework for developing analytical and conceptual relationships involving the flow of information has previously been suggested. This paper provides further refinement, rigor, and extension for some of the earlier relationships suggested. In particular, a measure of the amount of information is defined as the difference of the value of the decision state of the decision-maker after and before receipt of the message. This measure is universally applicable for all information that is concerned with the effectiveness of the message upon the recipient. It is accordingly called *pragmatic information*. The definition is a direct consequence of the interdependence between information and decision-making and of the definition that *information is data of value in decision-making*. In order to evaluate this measure of information, it is convenient to use a *generalized information system model* which has previously been proposed and which has virtually universal applicability.

The use of this model permits the evaluation of the measure of information in terms of the reduction of uncertainty to a decision maker. Six different types of uncertainty are identified. Specifically, a type of uncertainty which is generally overlooked in the decision-making literature is found to be important. This is called *executional uncertainty*. It is pointed out that the information science aspects of decision theory must cover comprehensively those decision-makers who not only are expert but those decision-makers who may be mediocre or even rather poor. Although any decision rule may be utilized in terms of the framework outlined, a suggested rule which is convenient and reasonable is proposed for evaluating the decision state of a decision-maker at any point in time. The measure of information suggested is situation, time, and decision-maker dependent. The framework and relationships developed are an important step toward the development of a true theory of information science.

It is further suggested that each data set or document might have some average (over time) amount of information content for a decision-maker of any given "effectiveness". The relationship of this average amount of information as a function of the decision-maker effectiveness is suggested as an important functional relationship that exists for every document. It is called an *information profile* of that document or data set. A typical information profile is suggested.

## PREFACE

This report is the result of research performed by M. C. Yovits and B. J. Whittemore. It was supported in part by Grant Number GN 534.1 from the Office of Science Information Service, National Science Foundation to the Computer and Information Science Research Center, The Ohio State University. B. J. Whittemore is currently at the Institute for Defense Analyses, Arlington, Virginia. All correspondence concerning this paper should be sent to Dr. M. C. Yovits, Department of Computer and Information Science, The Ohio State University, Columbus, Ohio 43210.

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INTRODUCTION

In several previous papers, much of which has been summarized in Reference [1], one of the authors discussed in detail some of the properties which should exist if information science is ever to become a "true science" similar to physics or chemistry. It was pointed out, among other requirements, that a number of analytical expressions and concepts which have wide applicability should exist. These concepts can then be used to generate principles which describe and analyze the flow of information. The principles can also suggest the limitations of the relationships developed. In Reference [1] a general framework has been suggested which permits the development of these concepts and expressions. This framework has been called a *generalized information system*.

It was suggested, that by use of this generalized model, it would be possible to define information quantitatively and in a rigorous, measurable and a repeatable way. It was suggested that information and other important quantities necessary for the development of a basic theory of an information "science" could be defined largely in terms of physical and measurable observable quantities. Suggestions were given regarding ways this generalized model would be used for developing the necessary fundamental relationships and limitations which govern the flow of information. Additional suggestions were made for the development of approaches leading toward a "true" information science.

Since the publication of Reference [1], much additional research has been performed which has permitted the extension of some of these basic ideas and the development of a number of analytical relationships leading toward an extended rigorous theoretical treatment. This particular paper then carries through many of the suggestions that were made in Reference [1] and provides considerable refinement for some of the definitions and recommended procedures that were suggested in this earlier paper. Also provided are some suggested directions for future research leading toward practical objectives which permit defining system and document parameters as well as criteria that would assist in the development of useful information systems.

## BACKGROUND AND OBJECTIVES

The word "information" is one of the most overused words in the English language, it has a large variety of meanings and is used in a large number of different contexts. Information is frequently, if not generally, considered to be almost synonymous with "knowledge". It is by and large in this context that information scientists are concerned with information. It is also essentially the public understanding of the meaning of information as well, as in, "May I have some 'information' please?".

Of course, information is frequently used rather specifically in the sense that Shannon and Weaver [2] have established in their treatment of "information theory" more accurately called "communication theory". In this sense the context of the message is of no significance and the theory is concerned with the probability of the receipt of any particular message for various conditions of the transmission system. While this may indeed be of interest in information science, it is certainly not the major nor even a large part of information science. Such a treatment does not consider the really important areas of concern, almost all of which are involved with the context, meaning, and effectiveness of the message.

On the one hand, the Shannon and Weaver approach is generally agreed to be too restrictive to be of wide interest with regard to the formulation of an information science. On the other hand, the treatment of information to be synonymous with knowledge, while this would be almost the broadest view that could be taken, appears to the authors to be far too broad to lead to principles and relationships that are meaningful and useful.

Given these considerations, it was desired, as with any theoretical development, to develop relationships which have as wide a generality as possible, although with perhaps somewhat less than general applicability.

In our formulation we adopt the definition that *information is data of value in decision-making*. While this may somewhat delimit the total range of interest in an intellectual sense, it *does* have virtually universal applicability with regard to any potential applications for information. The authors also feel that any more general definition is not really amenable to the quantification and conceptualization necessary to establish meaningful relationships. An implication of this definition then is that information is used *only* for decision-making and that the decision maker has *only* the resource of information available to him. Thus, information and decision-making, which might be defined to be purposeful activity or intelligent behavior, are very closely bound together, if not totally inextricable.

In Reference [2], Weaver refers to the three levels of communication problems:

Level 1. The technical problem.
(How accurately can the symbols of communication be transmitted?)

Level 2. The semantic problem.
(How precisely do the transmitted symbols convey the desired meaning?)

Level 3. The effectiveness (or behavioral) problem.
(How effectively does the received meaning affect conduct in the desired way?)

These three levels of communications research are perhaps most clearly and most simploy described by the questions: (1) what is the message? (2) what does the message mean? (3) what are the effects of the message on the recipient?

Problems at the first level are relatively well in hand. As we have indicated the significance of Shannon's work [2] on communication theory or information theory is, of course, widely recognized. The treatment of problems at the second level has been somewhat less complete than the treatment of those at the first level; nevertheless, significant and quite widely accepted work of a fundamental nature (e.g., the semantic information theory of Carnap and Bar-Hillel [3] and more recently the work of others such as Winograd [4]) does exist. It is at the third level that significant research of a fundamental nature is in short supply. No comprehensive and widely accepted theory of information at the third level has yet been published despite its clear significance. In the terminology of the linguist, this third level may be called the problem of *pragmatic* information.

The research which is described here and in Reference [1] may be considered to have as its major objective the development of a pragmatic (or effectiveness) information theory, which permits the analysis and quantification of information. It is suggested that the work reported here is a significant step toward the development of such a theory. Specifically this paper develops a framework which permits such analysis and quantification.

THE GENERALIZED MODEL AND THE FUNDAMENTAL HYPOTHESES

The generalized information systems model which has been referred to was described in some detail in Reference [1]. It is very briefly summarized here. There are *three basic hypotheses* on which the model is based. The *first hypothesis* is the close interrelationship between information and decision-making, giving rise to the definition of information stated above. The *second hypothesis* is related to the ability to measure or quantify information or even to know of its existence. In the physical world, the only way in which a real parameter, say a mass or a force, can be measured, is by its interaction with the environment. If this interaction does not exist, then it is impossible, almost by definition, to measure a quantity or even to establish its existence. One ought to be able to make the same statement about information. It is necessary that the information produce physical interaction with the real world. In any real situation it must give rise to an observable effect. The *third hypothesis* which the model is to satisfy is the necessity that the decision-maker must have some way of comparing predicted observables with the actual observables that result from any decision for which he is responsible. As is described below, the decision-maker generates a course of action which he believes will have certain observable effects. It is only after comparing the actual observables with those he has predicted, that he has some way of verifying the validity of his course of action. In other words, the third hypothesis dictates that the generalized model must have feedback between the resulting observables and the decision-maker. The resulting model is depicted schematically in Figure 1. It was originally suggested by Yovits and Ernst [5].

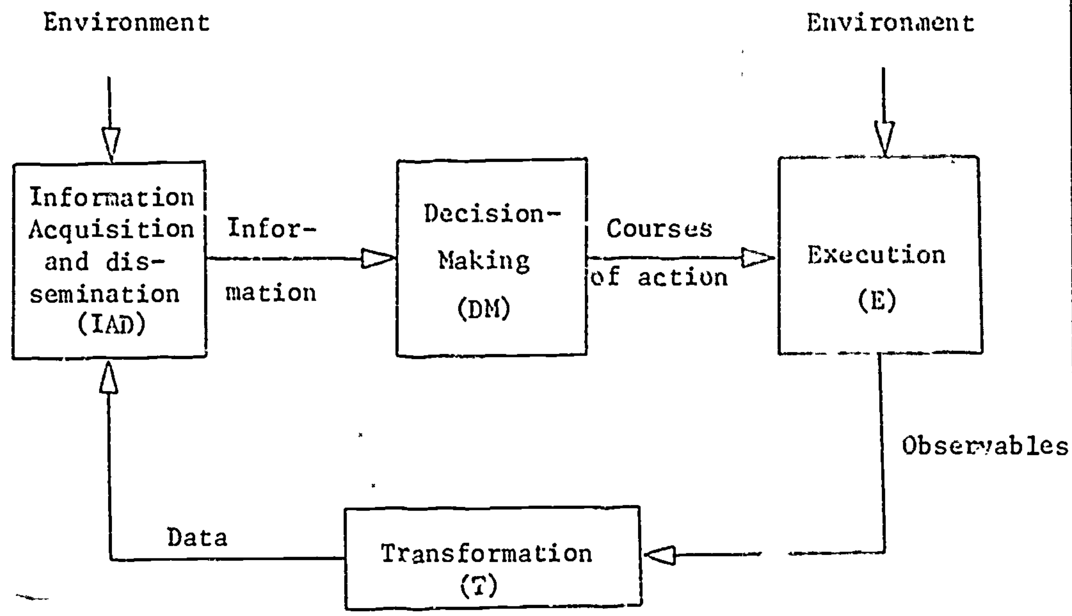


Figure 1 The generalized information system model

The generalized information system consists of four essential functional components. These include the *Information Acquisition and Dissemination* function, the *Decision-Making* function, the *Execution* function, and the *Transformation* function. Virtually all situations involving the flow of information in a decision system can be described by this model. These situations include the use of information by the research scientist or the development engineer, management of a large corporation, command and control of a military engagement, or such relatively straightforward and simple activities as the switching on or off of a thermostat-furnace system.

Each of the four functions is seen to collect, store, operate, and disseminate. In any realizable and operational system, all the indicated functions must be present and they must be considered together in order to understand information flow or to establish principles, relationships, or guidelines for information flow.

This model is as general as it is possible to be and still satisfy the basic hypotheses that are suggested. Furthermore, this model can be used for all decision-making processes. The decision-making may be rational or irrational. The model may give rise to normative or descriptive procedures for decision-making and it encompasses all of the known variations that may be of significance with regard to decision-making in general.

Note that the decision-maker uses information in order to generate various observables. The decision-maker may be said to *transduce* information to observables. In order to do this, the decision-maker must have available some predictive model of the situation with which he is concerned. It may be a very poorly structured or poorly understood model, or it may be a very elaborate or detailed analytical model. Nevertheless the decision-maker must have some model (understanding) of the situation with which he is concerned. By observing the actual effects of his decisions, and comparing these with those he has predicted, the decision-maker is able to develop a judgement about the validity and effectiveness of his own predictive model. This is the only way in which a decision-maker can find out about the real situation with which he is concerned.

In general then, the generalized information system model and the associated concepts serve as a jumping-off point for the research discussed in this paper. What does it mean to say that information is data of value in decision-making? What value does a DM derive from information? Information resolves or reduces *uncertainty*. Uncertainty is the critical link between information and decision-making. To effect a meaningful analysis of pragmatic information, one must look in detail at that which makes decision-making such a challenging and oftentimes agonizing activity: *uncertainty*.

## INFORMATION, DECISION-MAKING, UNCERTAINTY

A very important part of this study involved a careful examination of existing models in the decision theoretic literature,[1] in general, and the extent to which these models provide an adequate representation of uncertainty, in particular. This investigation revealed that existing decision models do not provide a comprehensive representation of the uncertainty that exists in decision-making. Specifically, the group to which typical decision models are addressed consists of decision-makers who are assumed to have reached a rather advanced state of knowledge about the decision situation in question.

For example, the DM is frequently assumed to be able to enumerate exhaustive lists of the viable courses of action, the possible decision outcomes, and the relevant states of nature. Furthermore, the DM is assumed to possess a clear understanding of his goals as well as the relationship between the various decision outcomes and goal attainment. Finally, and perhaps most importantly, the DM is assumed to have a perfect understanding of the Execution component depicted in the generalized information system

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[1]Decision-making is a process; a decision is an event. As a process, decision-making includes all the intellectual activity that precedes and eventually culminates in the decision itself. The very essence of a decision situation is the requirement *to make a single choice* from among a set of at least two possible *alternatives* or *courses of action*. The entity (either man or machine) required to make this choice is the decision-maker (DM). The view of decision-making suggested here is a general one. As such, it includes, at least in principle, such choice behavior situations as response-selection and attitude-formation. In a general way we may equate decision-making to problem-solving.

model in Figure 1. That is, in typical decision models the joint occurrence of a state of nature (a description or representation of those environmental conditions which will prevail when the decision in question has its effect) and a course of action constitutes a decision outcome; presumably, there is no other uncertainty present besides this *environmental uncertainty*. In actuality, the joint occurrence of a state of nature and a course of action may not uniquely determine the decision outcome. Even if the state of nature is known with certainty, there may well be uncertainty associated with the implementation of a selected course of action. Various alternatives may not be equally efficient for the attainment of a given outcome. Hence, there is a need to consider in a formal wa[illegible] ..ncertainty associated with the execution of course of action (i.e., *executional uncertainty*).

Why are these shortcomings in existing decision models important here? Precisely because a formal and comprehensive analysis of the role of information in decision-making can be derived only from a decision model that provides a formal and comprehensive representation of uncertainty in decision-making.

In other words, the decision theoretic literature which exists in voluminous form treats the situation where the decision-maker already has a high degree of expertise. On the other hand, the information science aspects of decision theory must cover comprehensively those decision-makers who not only are expert but those decision-makers who may be mediocre or even rather poor. A large part of the development of information systems is clearly aimed at the non-expert or even the novice decision-maker. It is, therefore, most important in developing a formal role for information science that all levels of effectiveness of decision-makers be considered.

GENERALIZED DECISION MAKING

Consider now a decision-maker (DM) faced with a novel decision situation in which he is required to make a sequence of related decisions over a period of time. The DM's overall knowledge of the situation is assumed to be minimal or even nonexistent; the problem setting is initially thus ill-structured in that the situation is one for which the DM has no exact precedent..

Decision-making consists of formulating some sort of model of the decision situation and then making the "best" possible choice on the basis of that model. The DM faced with a new decision situation then must utilize any existing information to formulate a model fo that situation. When such immediate information is exhausted (or when a choice is required even if the immediate information is not exhausted), the DM must begin to learn by experience the relationships that exist between the decision elements in question.

In formulating a model of any system of interrelated components, two fundamental types of uncertainty must be considered: *structural uncertainty* and *relational uncertainty*. Of the two, structural uncertainty is more fundamental in that its consideration must precede that of relational uncertainty (although only to a limited extent). A DM must first attempt to identify the basic structure of the system: the type and number of components involved. Given some representation of this basic structure (albeit incomplete or even totally incorrect), the model-builder can then attempt to assess the nature of the relationships between these basic components.

Although some consideration of structural uncertainty clearly must come first (the model-builder must have an idea of exactly what it is he is trying to relate when he addresses relational uncertainty), it would most certainly be a mistake to imply that consideration of structural uncertainty ends where that of relational uncertainty begins. Subsequent experimentation with the model may well reveal that the understanding of both these fundamental considerations is seriously incomplete.

A decision model must consist of a representation of certain *decision elements* including a set of courses of action, a set of possible decision outcomes, a decision goal or set of goals, a function which relates decision outcomes to goal attainment, and a set of states of nature.

Returning now to the general problem setting, one might imagine that the worst possible (i.e., the most ill-structured) decision situation is one in which the DM can view the situation only in terms of alternatives. He has no previous knowledge or related decision-making experience from which abstractions about any of the other decision elements can be derived. Since the essence of a decision situation is the requirement of a choice from among multiple alternatives, this is indeed some sort of lower bound for the conceptualization of a decision situation.

Such an alternatives-only conceptualization is not sufficiently rich to serve as a basis for intelligent and adaptive decision-making behavior. The reason is clear. Although a representation of structural uncertainty is present, it is somewhat degenerate in the sense that only one component of the situation has been identified. In this state no representation of relational uncertainty is possible. Without at least some representation of the way courses of action are related to decision outcomes, the DM has no vehicle for learning from the results of his decision.

A somewhat higher degree of conceptualization is achieved by the DM who views a complex decision situation in terms of courses of action and decision outcomes; relationally, he may be uncertain as how to represent the transformation of courses of action into observable decision outcomes. Yet the DM is faced with the task of making some sort of prediction about what outcomes will result from the selection of various courses of action. If he selects a particular course of action, he is uncertain about what outcome will occur when that action is executed. Hence, the DM is confronted with *executional uncertainty:* uncertainty associated with execution of various alternatives.

Actually making a decision requires the application of some sort of decision rule, however intuitive it might be. Such a rule can result only after the DM has considered the relative desirability of various decision outcomes. Although such consideration may be a very formal and explicit process in many cases, many decisions (perhaps most) are made by applying a decision rule that is the result of only a casual and intuitive assessment of the relative desirability of these outcomes.

Since decision-making is explicitly goal oriented, further refinement of the DM's conceptualization of a decision situation must include a more formal consideration of decision goals. A structural-relational dichotomy applies to *goal uncertainty* also. The DM may have only a vague, imprecise notion of the goals to which he aspires; that is, he may be uncertain as to the structural composition of his goal or goals. Also, even if the basic goal structure is relatively clear, the DM may be uncertain about the relationship between the various decision outcomes and goal attainment; that is, he may be uncertain as to how to assign values (commensurate with progress toward goal attainment) to the various outcomes. A natural byproduct of the DM's reduction of structural and relational uncertainty about his goals is his ability to formulate an explicit decision rule which will, consistent with his present understanding of the situation, guarantee the selection of that course of action which maximizes goal attainment.

According to Archer [6], a state of nature is a description of those environmental conditions within which the activities set in motion by the decision will operate. In discussing the decision elements alluded to previously, Ying [7] indicates that the definition and classification of the states of nature are to a large extent vague and arbitrary. The states can be so general that the outcome associated with a given act-state pair will not be unique. (This is indeed the perspective favored to here; this non-uniqueness associated with the outcome of a given act-state pair is due precisely to the existence of executional uncertainty.) In general, the DM may be uncertain about how the overall structure of the set of states; i.e., he may experience some difficulty in defining the nature and number of relevant states. Also, given some structural representation of the states, the DM may be uncertain as to the nature of the relationship between the set of states and the other decision elements. Hence, uncertainty about the states of nature (or *environmental uncertainty*) also has structural and relational considerations.

In general then, the view suggested here is that uncertainty in decision-making can be categorized in two possible ways. Uncertainty may be structural or relational in nature; at the same time, it may be executional, environmental, or goal associated. The classification of uncertainty that emerges is depicted in Figure 2.

UNCERTAINTY ABOUT:

| UNCERTAINTY ABOUT: | Execution of courses of action | Goals | Environment |
|---|---|---|---|
| Structural components of the decision model | structural-executional uncertainty | structural-goal uncertainty | structural-environmental uncertainty |
| Relationships among the structural components | relational-executional uncertainty | relational-goal uncertainty | relational-environmental uncertainty |

Figure 2 A classification of uncertainty in decision-making

In developing a model of a complex decision situation, a DM must recognize and cope with all six types of uncertainty. As a DM addresses the uncertainty that falls in one of the six categories, he is dealing with uncertainty about a particular part of his decision model. In general, however, as he addresses the six categories of uncertainty collectively, he is actually dealing with a higher order of uncertainty: uncertainty concerning his conceptualization of the decision situation.

The conceptual model suggested here then is a general one: a structure consisting of courses of action, decision outcomes, values of decision outcomes, and states of nature as well as the associated sources of uncertainty. The conceptual model is depicted schematically in Figure 3.

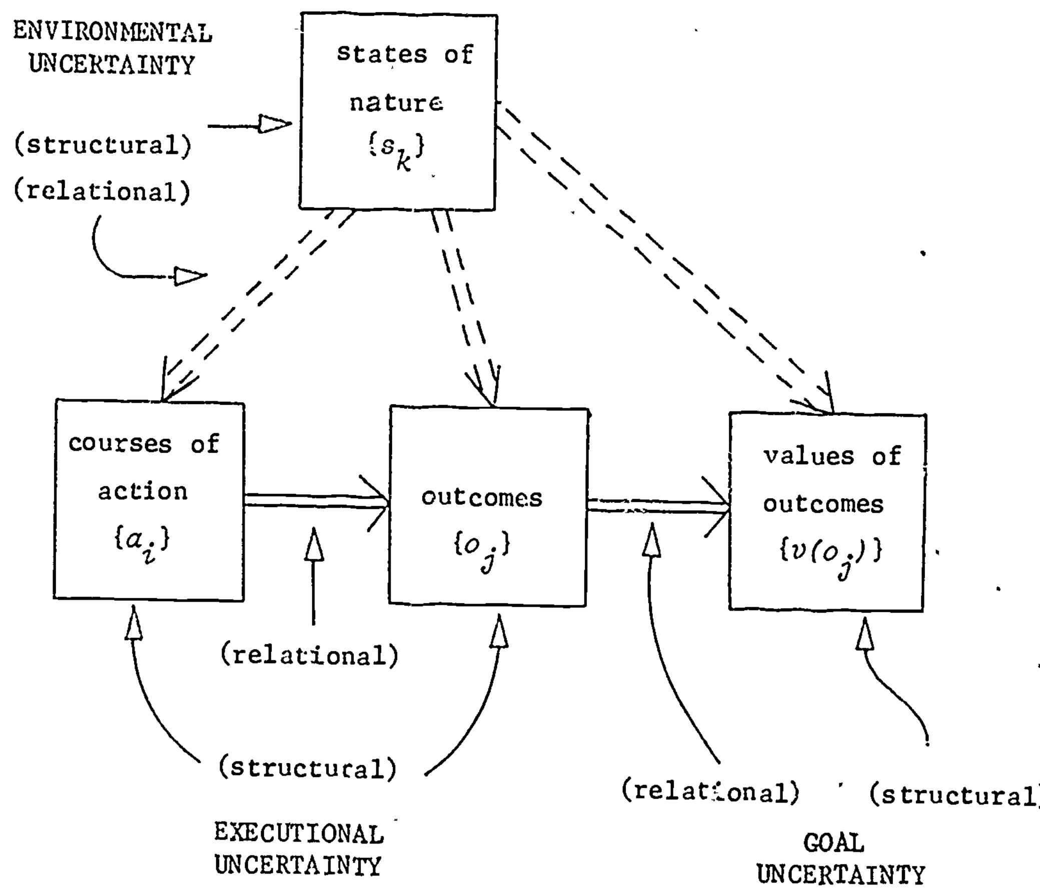


Figure 3 The conceptual decision model

A decision-maker is required to make a sequence of related choices from among a discrete set of alternatives $A = \{a_1, a_2, \ldots, a_m\}$. Since the DM may be uncertain about the nature and number of elements in this set, $m$ is not, in the long run, a fixed and/or known constant. The execution of a particular course of action results in the occurrence of one of a set of possible outcomes $O = \{o_1, o_2, \ldots o_n\}$. To allow for unexpected outcomes, $n$ too must to interpreted as variable over time. The DM may be uncertain as to the relationship between a particular course of action $a_i$ and an outcome

$o_j$. The likelihood that the execution of course of action $a_i$ will result in outcome $o_j$ can be denoted by the subjective probabilistic estimate $w_{ij}$.

After a DM has determined his overall goal structure, he must then assign numbers (in terms of value units) to the various decision outcomes that reflect the relative value of these outcomes with respect to goal attainment. These relative values can be denoted by $\{v(o_j)\}$.

The set of relevant states of nature can be denoted by $S = \{s_1, s_2, \ldots, s_r\}$. As with $m$ and $n$, $r$ must also be interpreted as a number whose value may vary according to the DM's current understanding of the decision situation. The probabilities of occurrence for the various states of nature can be denoted by the subjective estimates $P(s_1), P(s_2), \ldots, P(s_r)$.

The general interpretation of $m$, $n$, and $r$ as variables suggests obvious problems with respect to the probabilities $w_{ij}$ and $P(s_k)$. Recall, however, that these values depend of the DM's *current* understanding of the situation. In general, this understanding changes over time; nevertheless, at any one point in time at which the DM is required to use his decision model to make a decision, $m$, $n$, and $r$ assume whatever values reflect his current understanding of the situation. Hence, at any particular point in time,

$$\sum_{j=1}^{n} w_{ij} = 1 \text{ for } i = 1, 2, \ldots, m \text{ and } \sum_{k=1}^{r} P(s_k) = 1.$$

The decision elements $A$, $O$, $\{w_{ij}\}$, and $\{v(o_j)\}$ are dependent upon the state of the external environment. Courses of action which seem quite reasonable under one set of conditions may be totally nonviable under other circumstances; similarly, a decision outcome which is very possible in one state may be quite impossible in another state. Also it is clear that (a) the probability with which a particular course of action results in a particular outcome and (b) the value of a particular decision outcome are both dependent upon the state of nature. These dependencies can be indicated as follows: the sets $A$ and $O$ can be defined so as to reflect the DM's current understanding of all possible courses of action and all possible decision outcomes respectively (i.e., for all states of nature); also, a set of $\{w_{ij}^k\}$ and $\{v_k(o_j)\}$ can be assumed to exist for each state of nature. For state of nature $s_k$, the suggested mathematical representation is depicted in Figure 4.

| | | | | | | |
|---|---|---|---|---|---|---|
| Relative Values | $v_1^k$ | $v_2^k$ | ... | $v_j^k$ | ... | $v_n^k$ |
| Outcomes | $o_1$ | $o_2$ | ... | $o_j$ | ... | $o_n$ |
| Courses of Action | | | | | | |
| $a_1$ | $w_{11}^k$ | $w_{12}^k$ | | $w_{1j}^k$ | | $w_{1n}^k$ |
| $a_2$ | $w_{21}^k$ | $w_{22}^k$ | | $w_{2j}^k$ | | $w_{2n}^k$ |
| ⋮ | | | | | | |
| $a_i$ | $w_{i1}^k$ | $w_{i2}^k$ | | $w_{ij}^k$ | | $w_{in}^k$ |
| ⋮ | | | | | | |
| $a_m$ | $w_{m1}^k$ | $w_{m2}^k$ | | $w_{mj}^k$ | | $w_{mn}^k$ |

Figure 4 The decision matrix for the $k$th state of nature

The mathematical decision model suggested here consists of this decision matrix together with an appropriate decision rule. A decision rule is a rule that, when applied to the decision matrix, results in the selection of one course of action. The exact nature of this decision rule is dependent entirely upon the decision-maker's personal attitude toward uncertainty. For example, if the DM is conservative, he might choose to select that course of action which maximizes his minimum possible gain. For illustrative purposes, assume that the DM selects an alternative on the basis of maximum expected (relative) value. That is, he selects $a^*$ such that

$$EV(a^*) = \max_i \{EV(a_i)\}, \qquad (1)$$

where $EV$ is defined by

$$EV(a_i) = \sum_{k=1}^{r} P(s_k) \sum_{j=1}^{n} w_{ij}^{k} \, v_k(o_j). \qquad (2)$$

It is important to note that nothing yet indicated is really dependent upon the type of decision rule which the decision-maker may use. This decision rule may be totally a descriptive one or it may be a normative rule of any type, or it may be a combination of the two. The above equations are representative only and suggest a specific decision rule that one might apply. It is, however, a very reasonable rule and one that it is convenient to consider in somewhat more detail later[2].

[2] In the interest of conserving space, the mathematical details which are summarized here are given in more detail in other references. These have not yet been completely established.

ANALYSIS OF INFORMATION

The generalized decision model discussed provides a framework for a formal and comprehensive representation of uncertainty in decision-making. As such, it also then provides a suitable framework for examining the role of information in decision-making in a way that is also formal and comprehensive. A plausible approach to analyzing information is to look separately at its impact on the various types of uncertainty.

The information contained in a set of data (either unexpected new data from the external environment or feedback data from past decisions) has either structural value or relational value or most likely both. If the data indicate to the DM that his understanding of the structural components of the decision situation is incomplete and/or incorrect, then these data contain *structural information*. For example, the occurrence of a previously unknown decision outcome or the discovery of a new, viable course of action are informative in that they enhance the DM's understanding of the structural components of the situation. Similarly, if the data cause the DM to reassess his overall goal structure (e.g., perhaps he is being too conservative) or to recognize another relevant state of nature, then the data are *structurally* informative.

On the other hand, data that help the DM refine his model of the relationships that exist between known structural components contain *relational information*. For example, a refining of the DM's understanding of the states of nature probabilities or the probabilities that the execution of a course of action will result in the various outcomes occurs as a result of using relational information. Similarly, data that allow the DM to assess more accurately the relative values of outcomes according to a given goal structure are *relationally* informative.

In general then, the effect of information is to change the DM's representation of the various types of uncertainty; his decision model at time $t+1$ will be a revised version of his model at time $t$. Structural information either changes the overall goal structure or the nature and number of components in the sets $A$, $O$, or $S$. Given that the DM has resolved a certain amount of structural uncertainty, the effects of relational information are to change the probabilities associated with the execution of a course of action

$$[w_{ij}^{k}]_{t+1} = [w_{ij}^{k}]_{t} + \Delta w_{ij}^{k} ; \tag{3}$$

the probabilities associated with the state of nature

$$[P(s_k)]_{t+1} = [P(s_k)]_{t} + \Delta P(s_k) ; \tag{4}$$

and/or the relative values of outcomes

$$[v_k(o_j)]_{t+1} = [v_k(o_j)]_{t} + \Delta v_k(o_j) . \tag{5}$$

The way in which a particular DM actually utilizes information to revise his representation of the various types of uncertainty is highly individualistic. The generalized decision model is amenable to the application of a number of possible *learning rules*. Whether a DM will actually use any of these formal learning rules is somewhat doubtful. Hence, explicit enumeration of possible learning rules and a detailed discussion of their application would not add anything to the discussion at this point. It is important to note, however, that the generalized decision model provides a framework in which the DM can apply whatever learning rules he desires.

Although the separate effects of the various types of information are clearly important, they are really only of significance in their combined effect on the DM's understanding of the situation. The amount or value of the information contained in a set of data cannot be meaningfully expressed except in terms of how the data effects the DM's model of the decision situation. Specific measures of the amount of each type of information do not have great significance if they are not directly related to an overall measure of information value. Accordingly we suggest a measure of pragmatic information that subsumes any possible measure of amount of information in that it is, in fact, a measure of the *value* of an *amount* of information.

Regardless of what decision rule a DM is utilizing, it is possible to obtain a distribution that reflects his overall inclination toward the various courses of action. For example, suppose the DM is maximizing expected relative value. He will then select $a^*$ as indicated by Eqs. (1) and (2). If $P(a_i)$ is defined by

$$P(a_i) = \frac{EV(a_i)}{\sum_{h=1}^{m} EV(a_h)}, \tag{6}$$

then selecting $a^*$ such that

$$P(a^*) = \max_i \{P(a_i)\} \tag{7}$$

is an identical decision rule.

The numbers $P(a_1)$, $P(a_2)$, ..., $P(a_m)$, can be interpreted as the DM's probabilities of choice regarding the $m$ courses of action. Notice that these probabilities of choice are dependent upon *all* the components of the generalized decision model; specifically, the effects of any of the six types of uncertainty are manifested in this one distribution. Consequently, the extent to which information reduces any of these six types of uncertainty will be reflected in this distribution. Hence, at any time $t$, the probabilities of choice $\{P_t(a_i)\}$ defines the *decision state* of the DM. A function of the impact of information on this distribution can serve as a pragmatic information measure that is both a measure of information *amount* and information *value* (since it measures the effect of information on a distribution which, by definition, depends directly on the $v_k(o_j)$ elements). *Such a measure thus adds operational significance to the conceptual notion that information effects a change in the state of the DM.*

Assuming the DM has resolved sufficient structural uncertainty to enumerate an exhaustive list of courses of action (by inserting a "catch-all alternative", if necessary), then the expression (originally suggested by Ackoff [8])

$$\sum_{i=1}^{m} \left| P(a_i) - \frac{1}{m} \right|$$

may provide a measure of the "distance" a DM is from a state of indeterminism. (This expression too, is only illustrative. It is a convenient and a reasonable measure to use, but other measures might also be used.) Specifically, if each $P(a_i) = \frac{1}{m}$, then the DM has no basis for choice and the value of the expression is zero. At the other extreme, if one of the probabilities of choice is unity and all others are zero, then the situation is completely determined and the value of the expression is maximal:

$$\sum_{i=1}^{m} \left|P(a_i) - \frac{1}{m}\right| = (1-\frac{1}{m}) + (m-1)\ \left|0-\frac{1}{m}\right| = 2 - \frac{2}{m}. \qquad (8)$$

The value $V$ of the decision state $DS$ at time $t$ is a function of the probabilities $\{P_t(a_i)\}$ and can be expressed by

$$V(DS_t) = \frac{\sum_{i=1}^{m}\left|P_t(a_i) - \frac{1}{m}\right|}{2 - \frac{2}{m}}. \qquad (9)$$

If the decision state is completely indetermined (i.e., the DM has no information on which to base a choice), then $V(DS_t)=0$; if the state is completely determined (i.e., the DM has no uncertainty whatsoever about which alternative to select), then $V(DS_t)=1$. Thus $V(DS_t)$ will go from a minimum of $0$ to a maximum of $1$. The greater the amount of information available the closer will the decision state be to unity.

The probabilities of choice $\{P_t(a_i)\}$ which define the decision state at time $t$ are functions of the various components of the decision model. Specifically from Eqs. (1), (2) and (6), at time $t$ we have

$$P_t(a_i) = \frac{EV(a_i)}{\sum_{h=1}^{m} EV(a_h)} = \frac{\sum_{k=1}^{r} P(s_k) \sum_{j=1}^{n} w_{ij}^{k}\ v_k(o_j)}{\sum_{h=1}^{m}\sum_{k=1}^{r} P(s_k) \sum_{j=1}^{n} w_{hj}^{k}\ v_k(o_j)} \qquad (10)$$

(For the sake of clarity, the $t$ subscripts on the $P(s_k)$, $w_{ij}^k$, and $v_k(o_j)$ are omitted. They are understood to be evaluated at time, $t$ .) Substitution in Eq. (9) then provides a measure for the value of the decision state at any time.

It is seen that the treatment of all the types of uncertainty depicted in Figure 2 is reflected in the probabilities of choice $\{P_t(a_i)\}$. Information has its impact on the various components of the decision model. Consequently, the pragmatic information $I$ contained in a set of data $D$ can be defined by the impact of this data on the value of the DM's decision state:

$$I(D) = V(DS_{t+1}) - V(DS_t) \quad . \tag{11}$$

Stated in words, Eq. (11) says that the measure of pragmatic information in a set of data or a message is equal to the difference of the value of the decision state of the decision-maker after and before receipt of the message, where the value of the decision state is a function of the determinism of the decision maker. Many different approaches may be used to determine the decision state, either descriptively or analytically. However in our suggested formulation this is obtained directly from Eqs. (9) and (10).

This measure is suggested as one of major importance. At the heart of the theories that already exist for the treatment of communications problems at the first and second levels are *measures* of information. If a comprehensive theory of pragmatic information (at the third level) is to be developed, the need for an analogous measure of pragmatic information is clear.

## DISCUSSION AND SUMMARY

We have now suggested a formal measure for the amount of information which exists in a set of data or in a message or in a document. It quantifies information in terms of its effect on the state of the decision-maker, where a decision state is defined so that it represents a complete description of the decision-maker's overall level of understanding about a particular decision situation at a particular point in time. This measure, it is claimed, is universally applicable for *pragmatic information* where pragmatic information is equivalent to Weaver's level three which is concerned with the effects of the message upon the recipient.

In order to evaluate this measure of information, it is convenient to use a *generalized information systems model* which it is claimed has virtually universal applicability. The use of this model then permits the evaluation of the measure of information in terms of the reduction of uncertainty. Six different types of uncertainty are identified. This evaluation can be made in terms of any kind of a decision rule. We have suggested in this paper a reasonable decision rule that can be used for illustrative purposes. It is to be noted that the probabilities of choice which define the decision state at a particular time are complicated functions of the various types of uncertainty involved. They are not a simple linear combination of the various types of uncertainty.

Virtually any other decision rule can be used for evaluating the effects of the various uncertainties referred to. On the other hand, it is also possible to evaluate the decision state of the decision-maker in a purely descriptive sense, if desired.

Several properties of the proposed information measure merit consideration and discussion. The measure is a function of the effect that a set of data has on a DM's decision state. This decision state is defined in such a way that it reflects the DM's understanding of a particular decision situation at a particular point in time. Hence, *I(D)* is definitely a situation dependent and time dependent measure. Clearly it must be time and situation dependent. The same data will have different significance to different decision-makers at any time or to the same decision maker at different times.

Note that function value of *I* may be either positive or negative. In general, positive information sharpens or refines the DM's understanding of the situation in that it either reduces the number of structural components in the model (e.g., the expulsion from the model of a nonviable alternative or an impossible decision outcome) or reduces the dispersion in one or more of the various probability distributions in the model. On the other hand, negative information either increases the number of structural components (e.g., the addition to the model of a previously unknown alternative or outcome) or increases the dispersion in the various distributions.

*Negative information*, despite a possible connotation of the term, does represent information that is of major significance to the DM. For example, a DM's initial model of the situation may have been too simplistic in that he failed to include some viable course of action. Or perhaps he mistakenly assumed that the execution of a given course of action always resulted in the same outcome. Information that caused the DM to change his model in order to rectify either of these mistaken beliefs would show up negatively in *I(D)*; yet such information actually contributes to a more accurate model of the situation and is, therefore, clearly significant.

We have defined the amount of information in terms of the change of the decision state of a decision-maker. There is clearly a minimum amount of data which will be required in order to change the state of a decision-maker. This is a quantum of information and it is suggested that this has a particularly significant role to play in the analysis of information. We may call this a unit of information (which as we have already indicated, must be relative to any particular situation and to any particular decision-maker at a particular time). For want of a better term, we may call this elementary unit an *informon.*

To most people the fact that a measure of information is completely relative may be somewhat unsettling. It is, of course, quite different from the situation in the physical universe where a unit is absolute and once defined essentially remains unchanged forever. That is, a mass of one gram is always a mass of one gram. One second is always one second. On the other hand, the amount of information in a message is completely situation, time, and decision-maker dependent. However, as we have pointed out, this is the way it must be and once it is accepted that information is a relative quantity, the relationships that follow are quite reasonable and intuitively satisfying.

Because of the strong interdependence between information and decision-making we have found it necessary in order to evaluate a measure for pragmatic information to consider a generalized formulation of decision-making processes. In this consideration a type of uncertainty of great importance to the information science formulation but generally overlooked in the decision theory literature has been identified. This has been called executional uncertainty. This executional uncertainty has great implications in information science development as well as in regard to understanding decision-making in general. The identification of this new type of uncertainty is felt in itself to be one of the more important results of this work described here.

## CURRENT RESEARCH

One of the purposes of any research project is to suggest possible directions for future research. This section briefly discusses one important area in which considerable research effort is needed. This is an area toward which efforts here at Ohio State are currently directed.

The information measure suggested in the preceding section is a measure of the pragmatic information content of a set of data for a particular DM at a particular point in time. The data acquired, processed, stored, and disseminated by an information system will be used, however, as a resource by *various* decision-makers at *various* points in time. Hence, in the design and development of information systems, there exists a problem whose level of complexity is conceptually an order of magnitude above that of the primary problems addressed in this study: the problem of quantifying the information contained in a set of data in terms of its overall usefulness for a *range of decision-makers over a period of time.*

Little [9] recently discussed the concept of a decision calculus as a model-based set of procedures for processing "data and judgments" to assist a DM in decision-making. Even more than that, such a decision calculus might include a set of functional representations and, ultimately, analytical expressions that apply to information and the processing of that information for *any* decision-making situation. *If such a decision calculus were formulated and verified for a reasonable range of diverse decision situations, it could serve as a set of fundamental guidelines for the design and development of any information system.*

Unfortunately, no such decision calculus is suggested in this paper. Nevertheless, the study discussed in this paper has fostered several conceptual notions which may well be of some significance in this regard.

One approach to the problem of assigning a number to a set of data that indicates the composite value or composite information content of this data would be to start by determining the relationship between the "effectiveness" of a DM and the pragmatic information content of the data for this DM. In fact, since what is really desired is some indication of the value of this data set (or "data source" or document or message) for this DM over a period of time, one might attempt to determ: e some index $\overline{I(D)}$ of the average (over time) information contained in data set $D$ (or derived from data source $D$). Then, if it were possible to assess the effectiveness of each of the decision-makers for whom this data set serves as a resource, it would be possible, by determining $\overline{I(D)}$ for each DM, to formulate an *information profile* for data set $D$.

Although a discussion of the details are postponed until a later publication, preliminary investigations have suggested that a relationship similar to that depicted in Figure 5 is likely to hold.
Specifically, for a typical data set $D$ the DM who is not very effective cannot appreciate the significance of $D$; hence, the overall information value of $D$ for him will be low. This decision-maker is not sufficiently effective to develop any reasonable predictive model of the situation. Similarly, the value of $D$ will be low for the very effective DM who already knows most or all of the information contained in $D$. Between these extremes, $D$ will be of more value for those who are capable of understanding the information in $D$ and who do not already know this information.

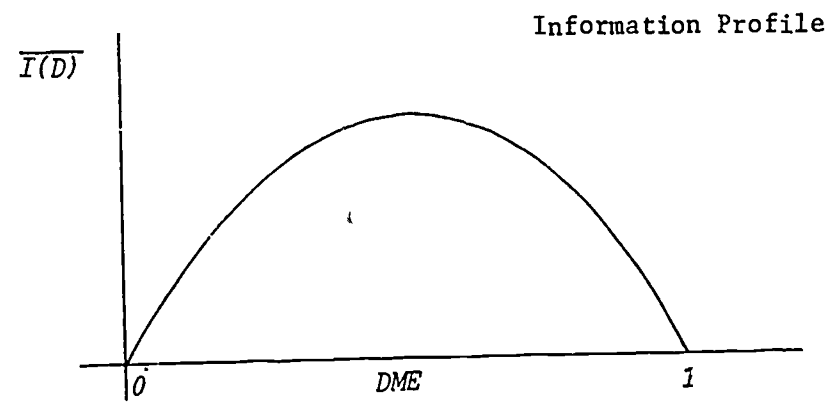


Figure 5  A possible relationship between composite information value $\overline{I(D)}$ and decision-maker effectiveness *DME* for a typical set of data *D*

If such a profile can be determined for every major data set or document in the information system, then this profile can serve as an index of the composite value of this data set or document. Each data set would have an information profile attached to it which would then help to establish the criteria for developing the information system to be used.

Further work along these lines in both a theoretical and experimental setting is currently underway at Ohio State. Application of this formalism to several practical examples are also being studied.

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