DOCUMENT RESUME

ED 069 673 TM 002 124

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TITLE Linear and Curvilinear Models for Aptitude-Treatment Interactions.
PUB DATE 2 Sep 72
NOTE 9p.; Paper presented at American Psychological Association Annual Meeting (80th, Honclulu, Hawaii, Sept. 2, 1972).

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS *Aptitude Tests; Correlation; Factor Analysis; Interaction; *Interaction Process Analysis; *Mathematical Models; *Multiple Regression Analysis; Predictive Ability (Testing); Predictor Variables; *Research Methodology; Speeches; Statistical Analysis; Technical Reports; Test Interpretation

ABSTRACT

Statistical procedures are presented for determining ordinal and disordinal aptitude-treatment interactions with linear and curvilinear data. The paper presents a method for testing the homogeneity of group regressions for a single aptitude and provides models for expanding this test to linear and curvilinear regression planes. Procedures are presented for constructing appropriate tests of significance and for isolating the specific source of interaction in complex aptitude-treatment interactions for which there are multiple aptitudes. (Author)

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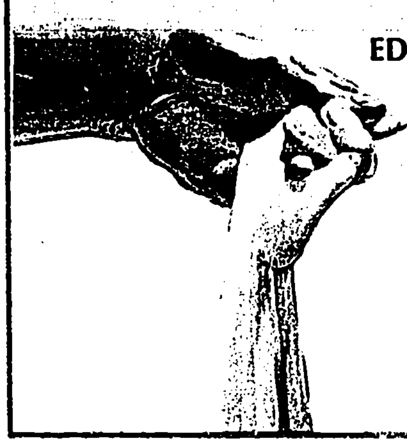

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# LINEAR AND CURVILINEAR MODELS FOR APTITUDE-TREATMENT INTERACTIONS

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Paper presented to the American Psychological Association at their 80th Annual Meeting, Honolulu, Hawaii, September 2, 1972.

## Abstract

### Linear and Curvilinear Models for Aptitude-Treatment Interactions

Statistical procedures are presented for determining ordinal and disordinal aptitude-treatment interactions with linear and curvilinear data. The paper presents a method for testing the homogeneity of group regressions for a single aptitude and provides models for expanding this test to linear and curvilinear regression planes. Procedures are presented for constructing appropriate tests of significance and for isolating the specific source of interaction in complex aptitude-treatment interactions for which there are multiple aptitudes.

LINEAR AND CURVILINEAR MODELS FOR

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Walker and Lev (1953) and Edwards (1968) illustrate a method for testing the homogeneity of group regressions for the case in which there is one linear predictor. Studies that have investigated aptitude-treatment interactions (see Cronbach and Snow, 1969) have adopted the homogeneity of regressions test as standard methodology for determining ordinal and disordinal interactions. The statistical model for this test, however, is inappropriate for the case in which there are two or more aptitude variables and the case in which aptitudes are suspected or known to be curvilinear. The potential usefulness of aptitude-treatment analyses for curvilinear data is suggested by the small number of significant findings from linear analyses reported by Bracht (1970) and the fact that linear analyses for even slightly curvilinear data yield conservative and inappropriate tests of significance. Practical applications of curvilinear aptitude-treatment analyses have been described by Borich (1971) and rationale for the use of more sensitive and appropriate methodologies has been suggested by Bracht. The purpose of this paper is to suggest additional statistical methodology by which the homogeneity of group regressions can be tested when two or more predictors are present and when the regression of the predictor on the criterion is suspected or known to be curvilinear. A brief review of the homogeneity of regression lines test is used to illustrate the general model. After which, the model is extended to test for homogeneity of regressions when multiple covariates and curvilinearity are present.[1]

*Homogeneity of group regressions, linear single-aptitude model.* To test the hypothesis that the regressions for two groups are parallel (i.e., the slopes are equal) the standard linear prediction model is constructed:

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{3i} + b_4X_{4i} + e_i;$$
$$i = 1\ldots,n$$

where $Y_i$ is the criterion; $a$ is the intercept; $b_1$ is the slope for the first group membership vector, $X_1$ (scored 0 if $S$, $i$, is in Group 1, scored 1 if not); $b_2$ the slope of a second group membership vector, $X_2$; $b_3$ the slope of the product of $X_1$ and the aptitude vector, $X_3$; and $b_4$ the slope of the product of $X_2$ and the aptitude vector.

The residual sum of squares ($\Sigma\hat{e}_i^2$) has degrees of freedom given by the number $S$s minus the number of linearly independent parameters (Bottenberg and Ward, 1963). Therefore, we have $N - 4$ *df* or for additional group vectors, $N - 2k$ *df*, where $k$ equals the number of treatment groups.

To test that $b_3 = b_4$, the data are fitted to a second more restrictive model which represents observations within each treatment group about the regression lines with a common slope given by:

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + (b_3X_{3i} + b_4X_{4i}) + f_i.$$

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[1]Computer programs for the statistical procedures and representations reported in this paper are available from the author.

For the residual sum of squares ($\Sigma\hat{f}_i^2$) we have $N - k + 1$ or $N - 3$ *df*.

Since the restricted model combines product vectors, $\Sigma\hat{f}_i^2$ is expected to be greater than $\Sigma\hat{e}_i^2$. These can be equal if the hypothesis is true, but $\Sigma\hat{f}_i^2$ cannot be less than $\Sigma\hat{e}_i^2$.

To test for parallel slopes, an hypothesis sum of squares is formed, given by $SS_{\text{hyp}} = \Sigma\hat{f}_i^2 - \Sigma\hat{e}_i^2$ with $(N - 4) - (N - 3) = 1$ *df* or for models with greater than two treatment groups, $k - 1$ *df*. An $F$ test is constructed with the observations within each group about the regression for the group given by:

$$F(k - 1, N - 2k) = \frac{SS_{\text{hyp}}/(k - 1)}{\Sigma\hat{e}_i^2/(N - 2k)}.$$

*Homogeneity of group regressions, curvilinear single-aptitude model.* To test the hypothesis that the curvilinear regressions for two groups are parallel, the curvilinear model is used, given by:

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{3i} + b_4X_{4i} + b_5X_{3i}^2 + b_6X_{4i}^2 + e_i$$

with $N - 3k$ or $N - 6$ *df*, where $X_3$ is the product of $X_1$ and the aptitude vector and similarly, $X_4$ is the product of $X_2$ and the aptitude vector.

To test that $b_5 = b_6$, i.e., the curvilinear slopes for the treatment groups are parallel, the restricted model is fitted:

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{3i} + b_4X_{4i} + (b_5X_{3i}^2 + b_6X_{4i}^2) + f_i$$

with $N - 2k + 1$ or $N - 5$ *df*.

The $F$ test for the homogeneity of curvilinear group regressions reflects the addition of the curvilinear interaction vectors to the full and restricted model. To test for parallel slopes, the hypothesis sum of squares is formed, given by $SS_{\text{hyp}} = \Sigma\hat{f}_i^2 - \Sigma\hat{e}_i^2$ with $(N - 3k)$ -

$(N - 2k + 1)$ or $(N - 6) - (N - 5) = 1$ *df*. The *F* ratio is constructed in the usual manner.

*Homogeneity of group regressions, linear multiple-aptitude model.* By extending the linear single-aptitude model, the investigator can test combinations of aptitudes and isolate specific aptitudes which account for nonparallel slopes. For greater than one aptitude, regression planes and hyperplanes (three or more aptitudes) are analogous to regression lines. For two groups each with two aptitude variables, the linear model may be extended to fit the hypothesis represented in Figure 1 given by the model:

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{3i} + b_4X_{4i} + b_5Z_{1i} + b_6Z_{2i} + e_i$$

with $N - 3k$ or $N - 6$ *df*, where $X_{1,4}$ is defined as above, $Z_1$ is the product of $X_1$ and the second aptitude vector and similarly, $Z_2$ is the product of $X_2$ and the second aptitude vector.

To test that there are parallel slopes for both aptitudes, the restricted model is formed by setting $b_3$ equal to $b_4$ and $b_5$ equal to $b_6$ in the model:

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + (b_3X_{3i} + b_4X_{4i}) + (b_5Z_{1i} + b_6Z_{2i}) + f_i$$

with $N - k + 2$ or $N - 4$ *df*. Should the null hypothesis be rejected, three possibilities remain: slopes may be parallel for Aptitude A but not parallel for Aptitude B, i.e., $b_3 = b_4$ but $b_5 \neq b_6$; or the reverse, $b_3 \neq b_4$ but $b_5 = b_6$; or both Aptitudes A and B have nonparallel slopes. Restricted models which test for parallel slopes at Aptitudes A and B, respectively, are given by:

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + (b_3X_{3i} + b_4X_{4i}) + b_5Z_{1i} + b_6Z_{2i} + f_i$$

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{3i} + b_4X_{4i} + (b_5Z_{1i} + b_6Z_{2i}) + f_i$$

with $N - 2k + 1$ or $N - 5$ *df*.

*Homogeneity of group regressions, curvilinear multiple-aptitude model.* For curvilinear as well as linear data, simpler models can be extended to test combinations of variables and isolate specific sources of nonparallelism. For two groups each with two predictor variables a curvilinear model may be extended to fit the hypothesis represented in Figure 2 given by the model:

$$Y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{3i} + b_4X_{4i} + b_5Z_{1i} + b_6Z_{2i}$$
$$+ b_7X^2_{3i} + b_8X^2_{4i} + b_9Z^2_{1i} + b_{10}Z^2_{2i} + e_i$$

with $N - 5k$ or $N - 10$ *df*, where $X_{1,4}$ is defined as above, $Z_1$ is the product of $X_1$ and the second aptitude vector and similarly, $Z_2$ is the product of $X_2$ and the second aptitude vector.

Procedures are identical to the linear multiple-aptitude model for restricting the model over both aptitudes and isolating ordinal or disordinal interactions for any one aptitude. Hypothesis sum of squares and significance tests are computed in the usual manner.

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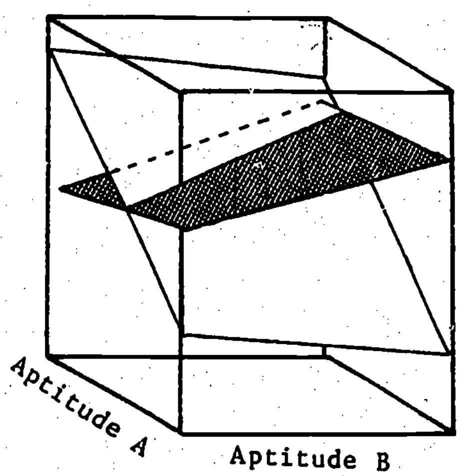


Figure 1. Regression planes in which the aptitudes are linearly related to the criterion.

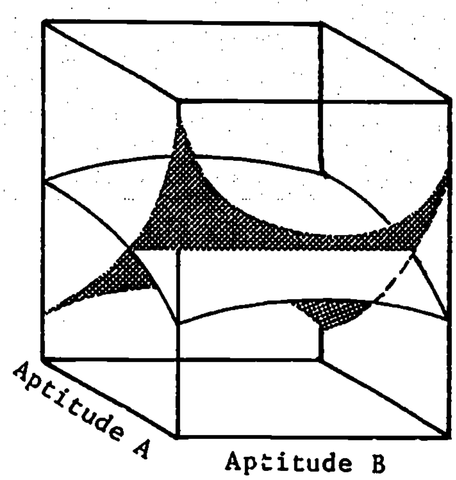


Figure 2. Regression planes in which the aptitudes are curvilinearly related to the criterion.